Analysis of Volatility Modelling in VaR Estimation

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Outline

- Introduction
- Volatility Models
- Model Comparison
- Conclusion

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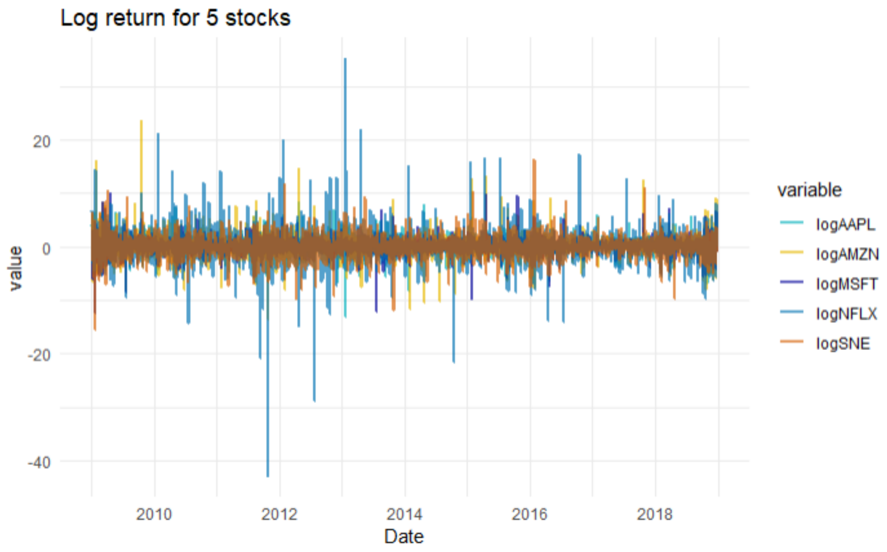
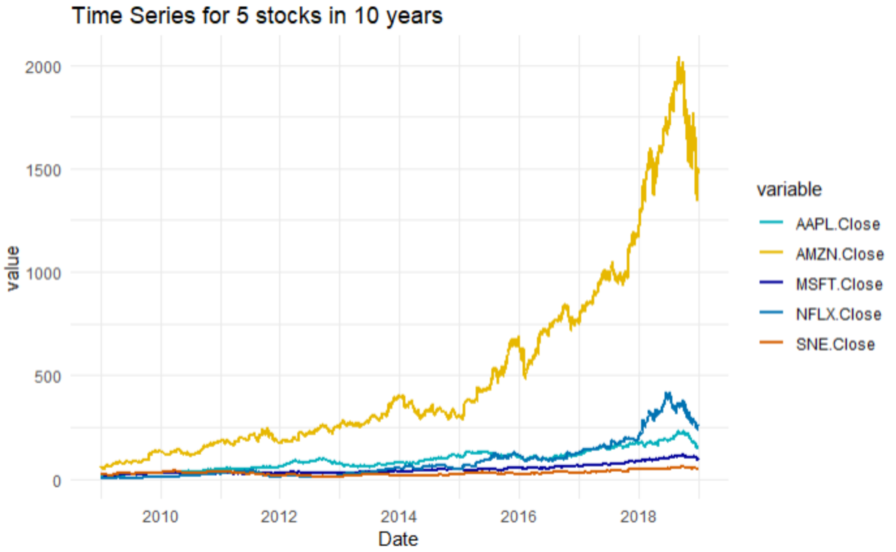
Exploratory Data Analysis

- Data: 3 kinds of return data, 10 years
  - 5 Stocks and 10 year Treasury bond return data:

Summary statistics	AAPL	AMZN	NFLX	MSFT	SNE	10yTsy
Mean	0.099	0.132	0.046	0.064	0.031	0.0106
Median	0.090	0.091	0.165	0.048	0.000	-0.1708
Maximum	8.502	23.740	35.223	10.001	16.291	207.94
Minimum	-13.188	-13.533	-42.918	-12.458	-15.541	-165.82
SD	1.676	2.138	3.315	1.562	2.163	15.78
Skewness	-0.142	0.843	-0.272	-0.172	0.146	0.554
Kurtosis	3.774	11.967	23.652	7.580	4.960	43.17

Exploratory Data Analysis





Exploratory Data Analysis

- Data Introduction: 3 kinds of return data, 10 years
  - 4 Exchange return data:

Summary statistics	USD/CAD	USD/CHF	USD/JPY	EUR/USD
Mean	0.004	-0.003	0.007	-0.008
Median	0.008	0.000	0.000	0.000
Maximum	2.853	9.239	3.464	3.733
Minimum	-2.188	-17.145	-2.772	-2.653
SD	0.560	0.725	0.620	0.598
Skewness	0.161	-4.230	-0.055	0.021
Kurtosis	1.679	129.207	3.610	1.833

Exploratory Data Analysis

- Data Introduction: 3 kinds of return data, 10 years
  - 5 Indices return data:

Summary statistics	CAC40	DAX30	HSI	N225	S&P500
Mean	0.065	0.176	0.137	-0.018	0.039
Median	0.123	-0.080	0.137	0.168	0.058
Maximum	179.176	783.400	273.942	778.447	6.837
Minimum	-98.225	-782.004	-269.634	-778.406	-6.896
SD	13.971	62.697	16.366	60.970	1.048
Skewness	0.868	-0.550	0.219	-0.132	-0.328
Kurtosis	26.964	75.726	114.095	69.920	5.003

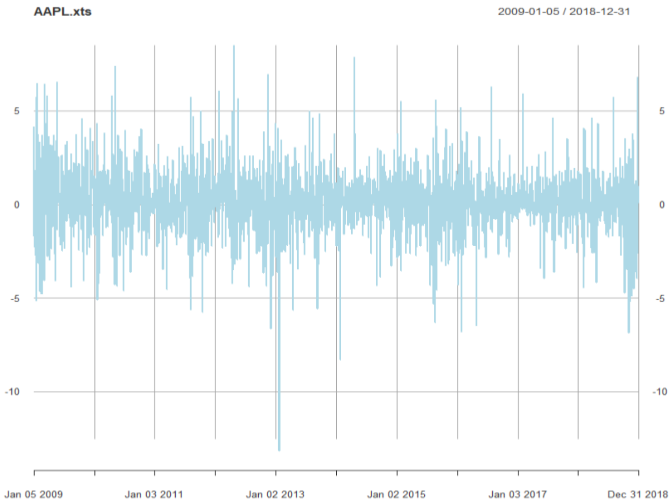
VaR (Value at Risk)

Consider a single asset or portfolio. Let R be its rate of return and $\mathrm{L} = -\mathrm{R}$ the loss. The VaR of level α or confidence $1-\alpha$ is denoted by that satisfies the following:

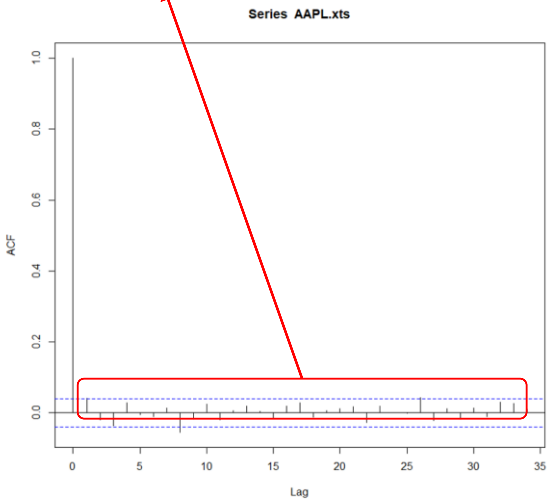
$$P(L > VaR(\alpha)) = \alpha$$

In the case of a portfolio of size S dollars, the VaR becomes $\mathrm{S} * \mathrm{VaR}(\alpha)$ dollars.

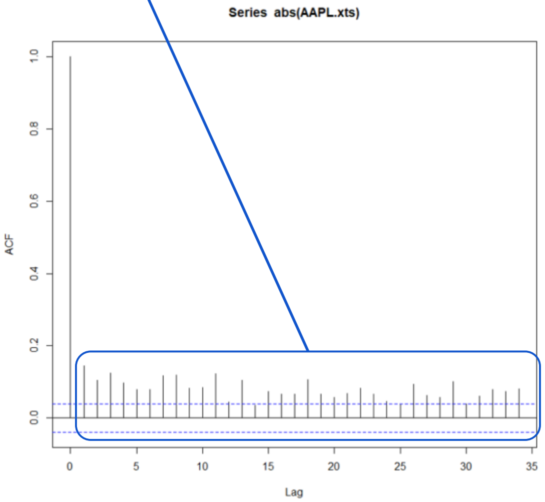
Return Data as Time Series



Uncorrelated



Not independent !



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Volatility Models

- ARCH(p) (AutoRegressive Conditional Heteroscedasticity):

$$Z_t = \sqrt{h_t} e_t \,, \qquad \{e_t\} \sim IID(0,1)$$
$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i Z_{t-i}^2 \,,$$

Where $\alpha_0 > 0$ and $\alpha_i \geq 0\,,\; i = 1, \cdots, p$

- GARCH(p, q) (Generalized ARCH):

$$Z_t = \sqrt{h_t} e_t, \qquad \{e_t\} \sim IID(0,1)$$
$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i Z_{t-i}^2 + \sum_{i=1}^{q} \beta_i h_{t-i} \,,$$

Where $\alpha_0 > 0$ and $\alpha_i \geq 0\,, \beta_i \geq 0\,,$ for each i.

Volatility Models

- ARMA(pM, qM) + GARCH(pV , qV) Model

ARMA(pM, qM) model specifies the conditional mean while GARCH(pV, qV) model specifies the conditional variance of the process :

$$X_t = \mu + \sum_{j=1}^{p} \phi_j X_{t-j} + \sum_{j=1}^{q} \theta_j Z_{t-j}$$

$$Z_t = \sqrt{h_t} e_t, \qquad \{e_t\} \sim IID(0,1)$$

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i Z_{t-i}^2 + \sum_{i=1}^{q} \beta_i h_{t-i}$$

The Innovations

The GARCH model

$$Z_t = \sqrt{h_t} e_t, \qquad \{e_t\} \sim IID(0,1)$$

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i Z_{t-i}^2 + \sum_{i=1}^{q} \beta_i h_{t-i}$$

is weakly stationary iff. $\sum \alpha_i + \sum \beta_i < 1$. When $\alpha + \beta = 1$, GARCH(1,1) becomes IGARCH(1,1).

It is usually assumed in addition that either

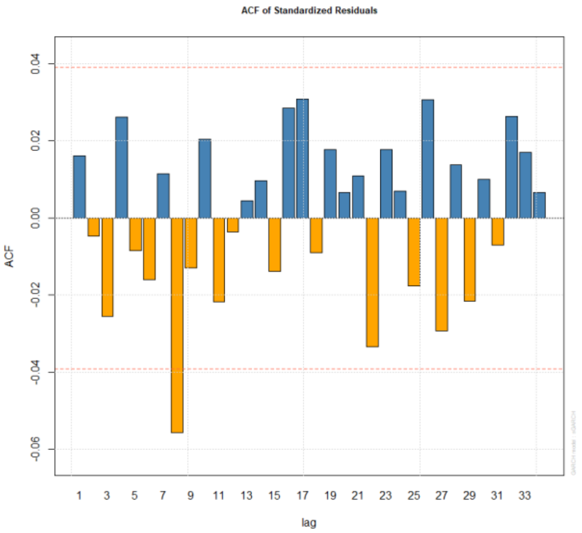
$$e_t \sim N(0,1)$$

or $\sqrt{\frac{v}{v-2}} e_t \sim t_v \, , \, v > 2$

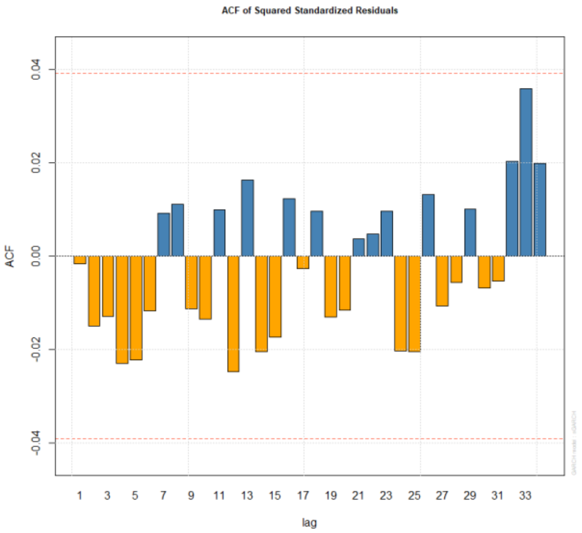
where t_v denotes scaled Student's t distribution with v degrees of freedom.

Model Fitting

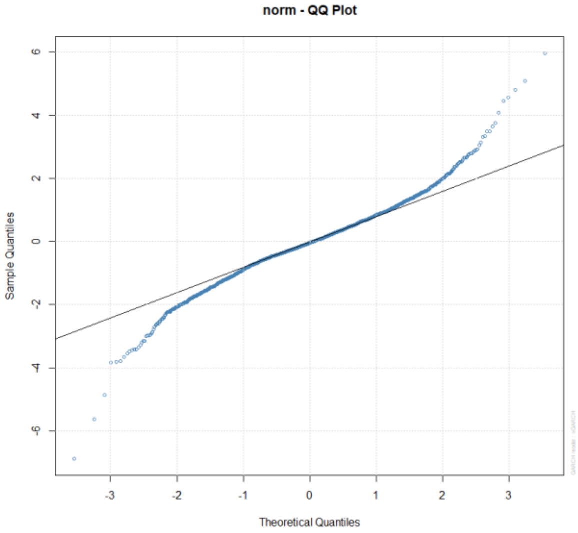
Residuals plot (Normal innovation)



ACF



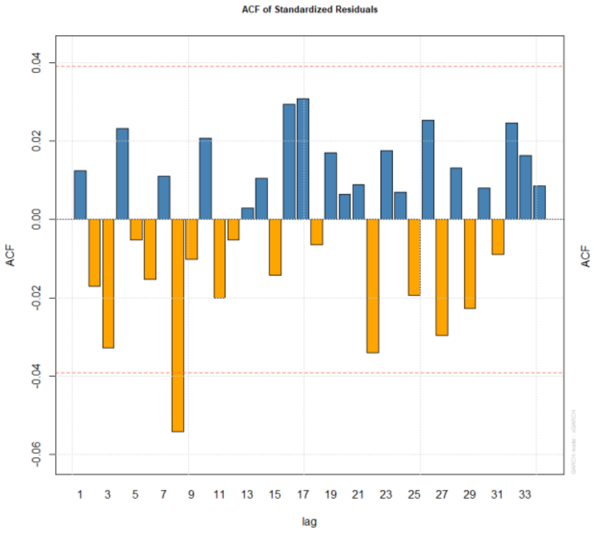
ACF of squared



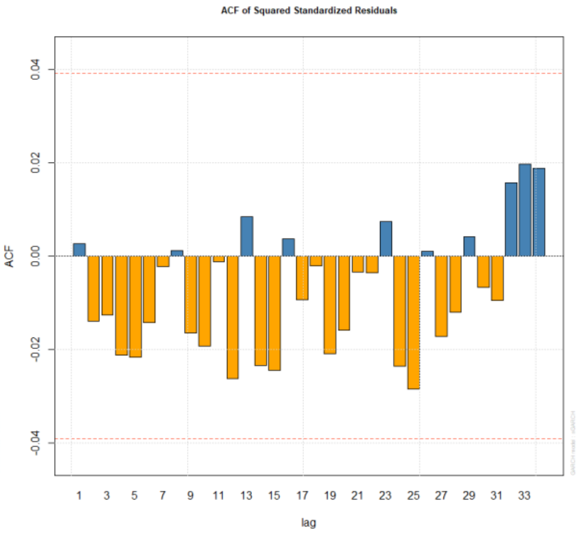
QQnorm

Model Fitting

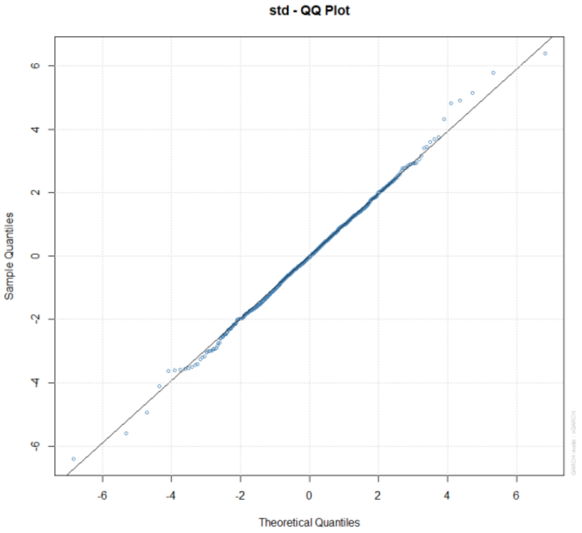
Residuals plot (t innovation)



ACF

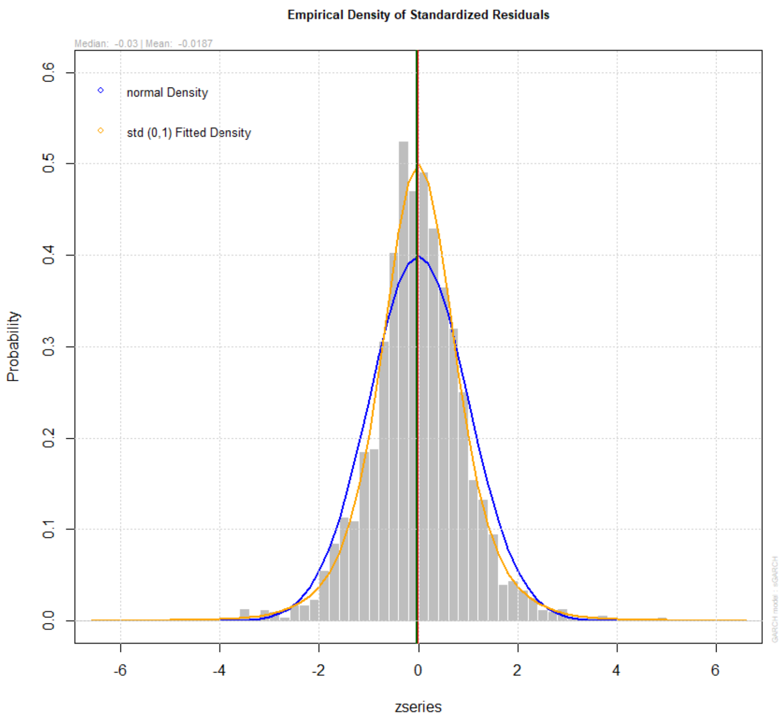


ACF of squared



QQ-t plot

Model Fitting



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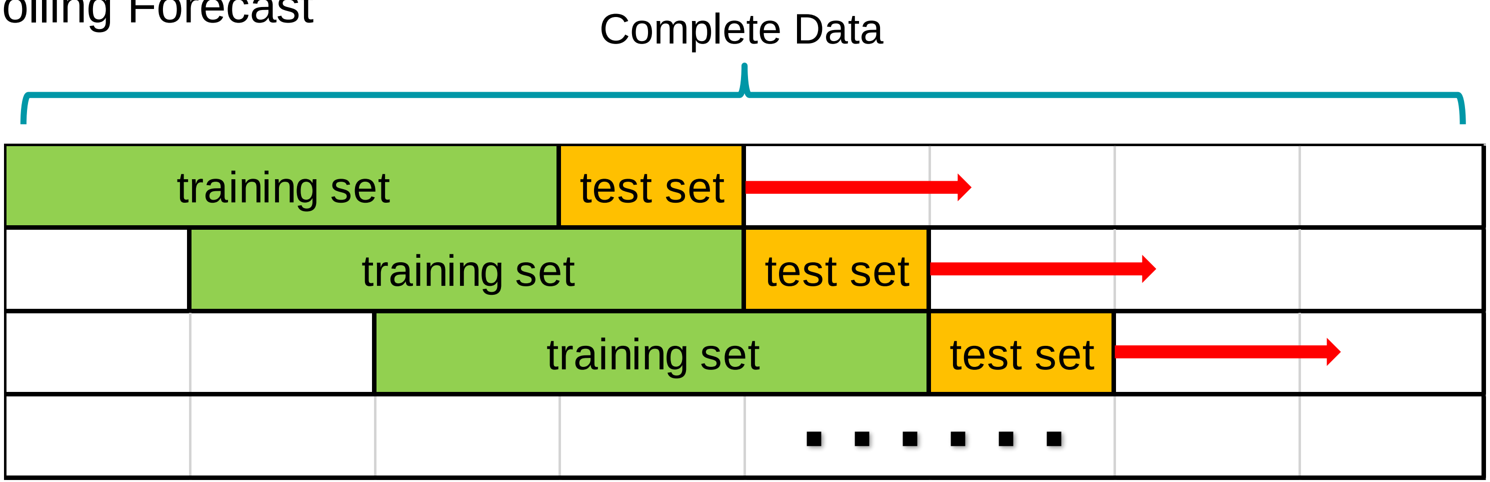
Model Assessment

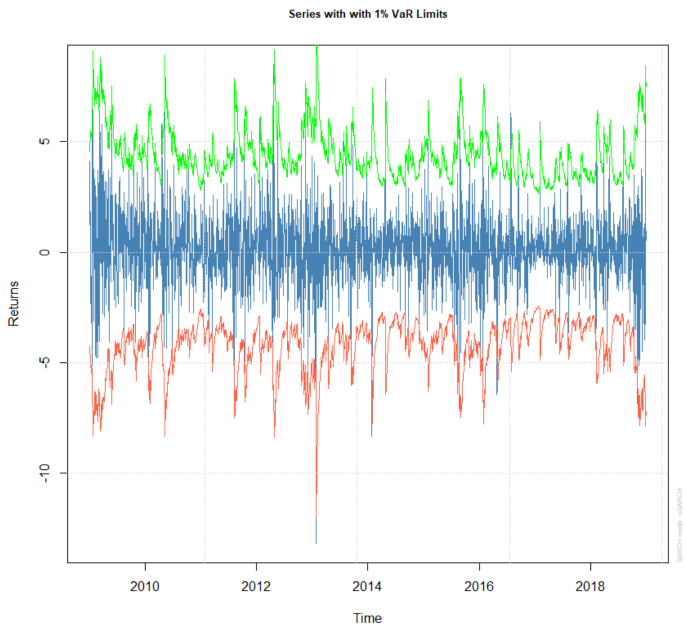
- Rolling Forecast

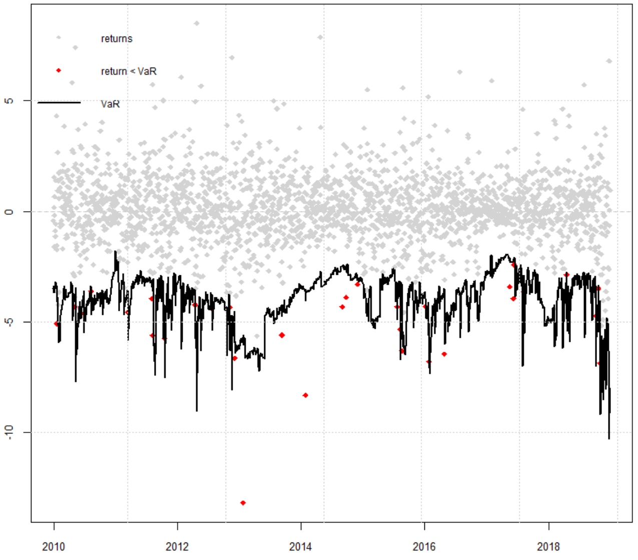


- Violation Test

$$E(\text{Loss} > \text{VaR})$$

Model Assessment

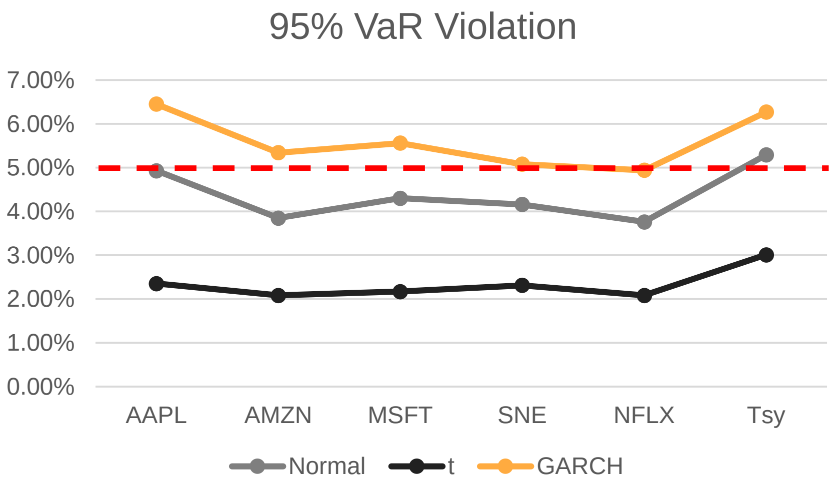


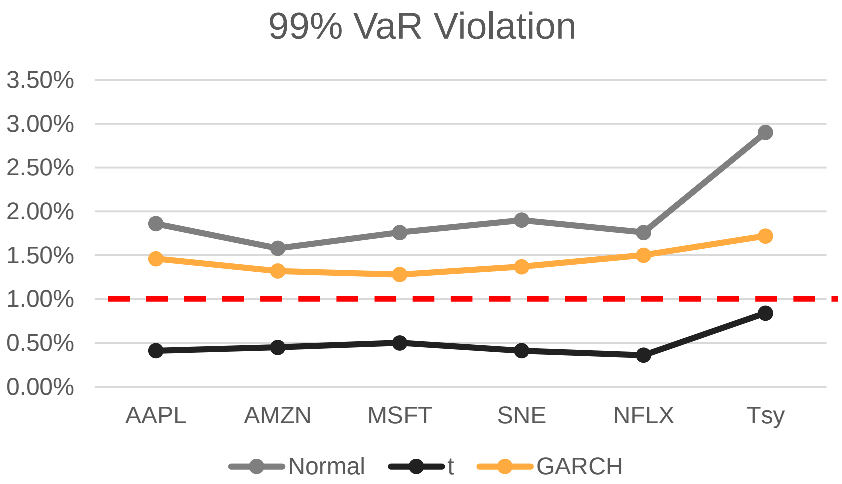


Model Assessment

- Fixed vs Conditional Volatility Models

Rolling sample size=250, Forecast size=10

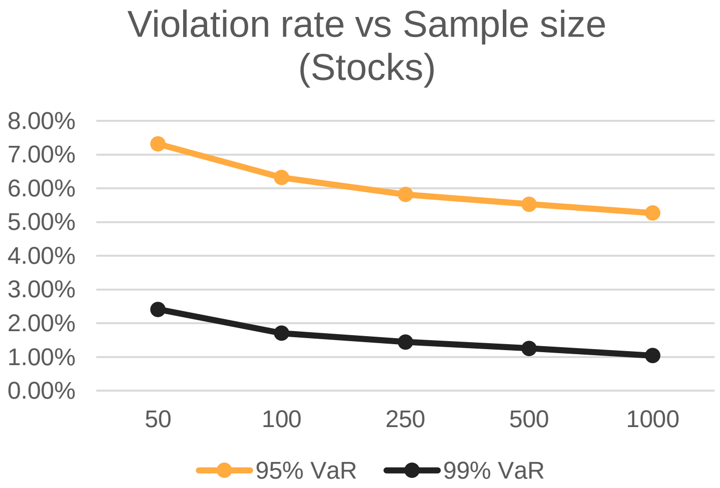


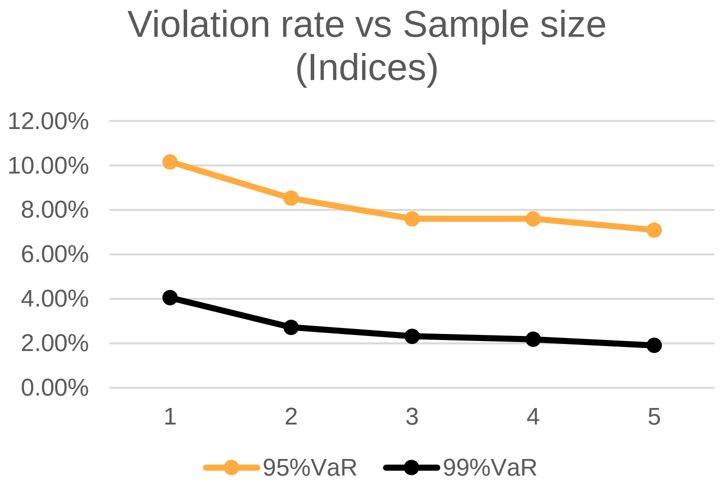


Model Assessment

❖ Choice of Sample Size

Rolling sample size: {50, 100, 250(1 year), 500, 1000}

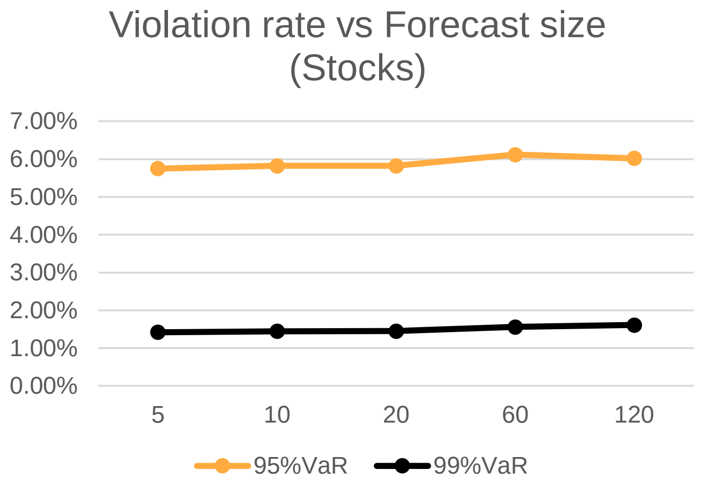


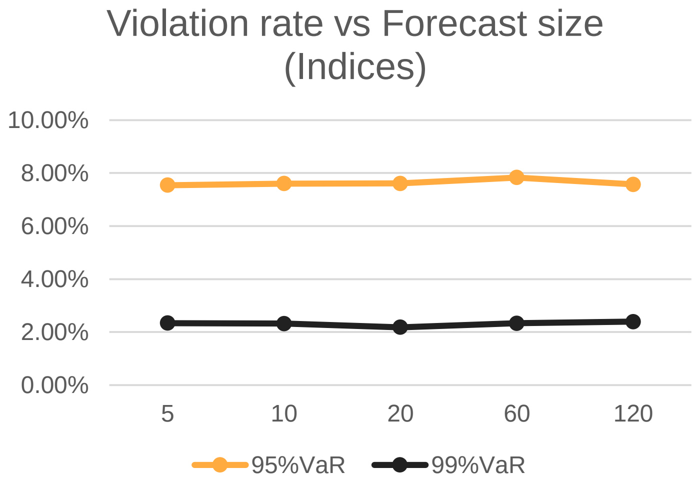


Model Assessment

❖ Choice of Forecast Length

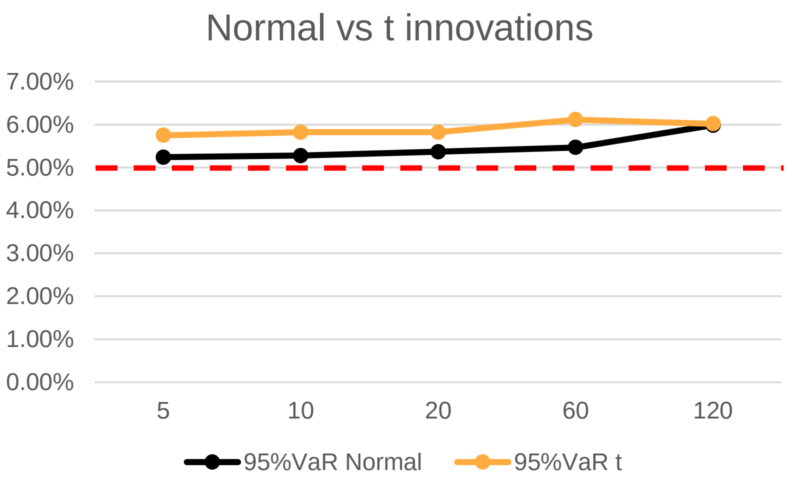
Forecast size: {5, 10(2 weeks), 20, 60 ,120}

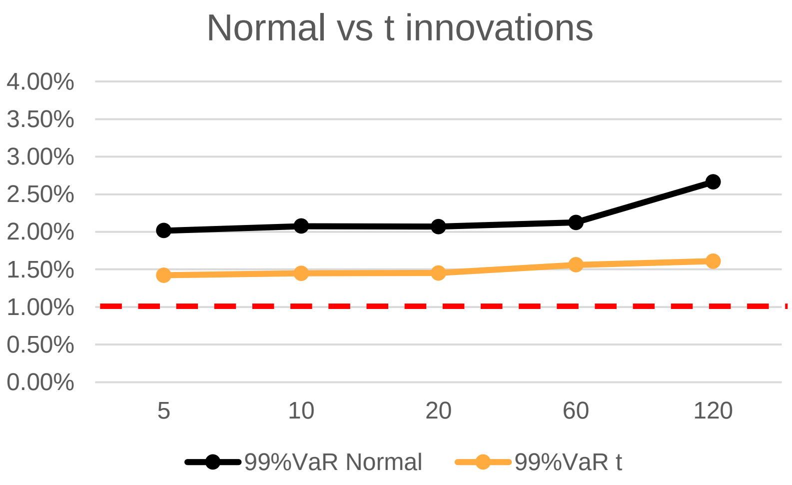




Model Comparison

❖ Assumptions on Innovations





Model Comparison

- Effect of Conditional Mean

ARMA(0,0)+GARCH(1,1)

ARMA(1,0)+GARCH(1,1)

ARMA(0,1)+GARCH(1,1)

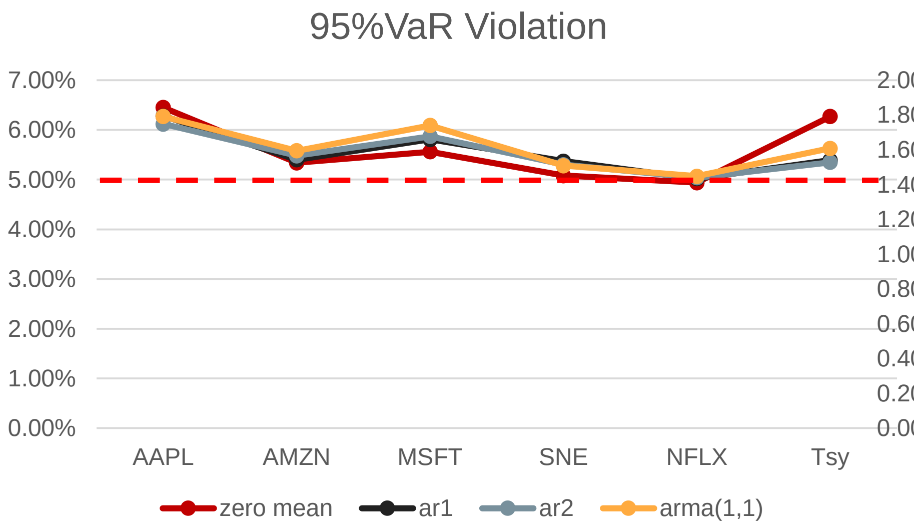
ARMA(1,1)+GARCH(1,1)

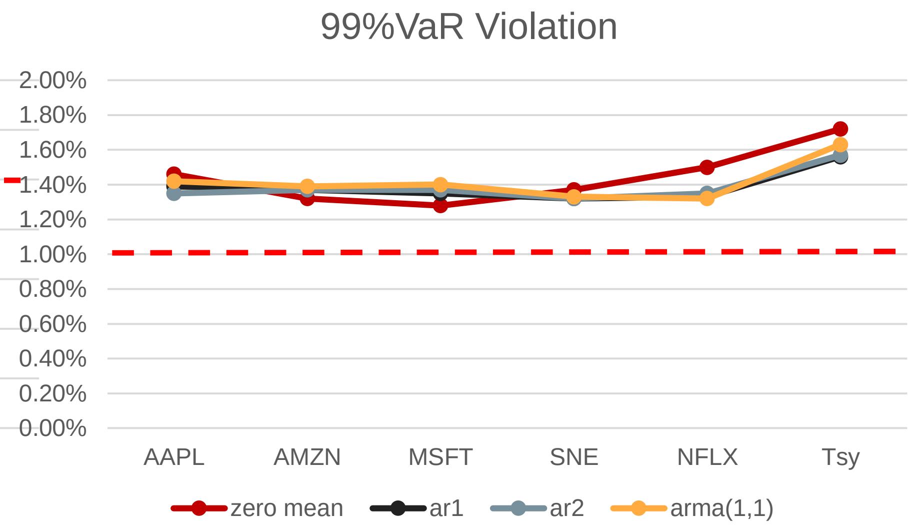
ARMA(2,0)+GARCH(1,1)

	AAPL		AMZN		MSFT		SNE		NFLX		10yTsy	
Model	95%VaR	99%VaR	95%VaR	99%VaR	95%VaR	99%VaR	95%VaR	99%VaR	95%VaR	99%VaR	95%VaR	99%VaR
arma(0,0)+garch(1,1)	6.45%	1.46%	5.34%	1.32%	5.56%	1.28%	5.08%	1.37%	4.94%	1.50%	6.27%	1.72%
arma(0,1)+garch(1,1)	6.31%	1.41%	5.43%	1.37%	5.74%	1.37%	5.43%	1.28%	4.86%	1.50%	6.53%	1.72%
arma(1,0)+garch(1,1)	6.31%	1.37%	5.47%	1.32%	5.70%	1.37%	5.47%	1.28%	4.86%	1.46%	6.53%	1.72%
arma(1,1)+garch(1,1)	6.31%	1.41%	5.65%	1.50%	6.05%	1.32%	5.39%	1.28%	4.90%	1.41%	6.62%	1.77%
arma(2,0)+garch(1,1)	6.09%	1.32%	5.56%	1.37%	5.78%	1.37%	5.34%	1.28%	4.86%	1.50%	6.58%	1.68%
arma(1,0)+garch(1,0)	6.05%	1.37%	5.39%	1.19%	5.74%	1.28%	5.25%	1.32%	5.03%	1.19%	5.13%	1.37%
arma(1,0)+garch(1,1)	6.31%	1.37%	5.47%	1.32%	5.70%	1.37%	5.47%	1.28%	4.86%	1.46%	5.40%	1.64%
arma(1,0)+garch(2,1)	6.31%	1.46%	5.34%	1.50%	5.87%	1.37%	5.43%	1.32%	5.03%	1.50%	5.40%	1.59%
arma(1,0)+garch(1,2)	6.40%	1.32%	5.39%	1.32%	5.74%	1.37%	5.39%	1.32%	5.08%	1.28%	5.40%	1.64%
arma(1,0)+garch(2,2)	6.31%	1.41%	5.43%	1.50%	6%	1.37%	5.30%	1.37%	5.12%	1.24%	5.62%	1.55%
average	6.28%	1.39%	5.40%	1.37%	5.81%	1.35%	5.37%	1.32%	5.02%	1.33%	5.39%	1.56%
arma(2,0)+garch(1,0)	5.87%	1.41%	5.56%	1.24%	5.83%	1.37%	5.08%	1.28%	4.99%	1.24%	5.09%	1.37%
arma(2,0)+garch(1,1)	6.09%	1.32%	5.56%	1.37%	5.78%	1.37%	5.34%	1.28%	4.86%	1.50%	5.44%	1.68%
arma(2,0)+garch(2,1)	6.23%	1.46%	5.39%	1.46%	5.92%	1.37%	5.30%	1.28%	5.08%	1.46%	5.40%	1.59%
arma(2,0)+garch(1,2)	6.23%	1.28%	5.56%	1.32%	5.83%	1.32%	5.43%	1.37%	5.08%	1.32%	5.40%	1.64%
arma(2,0)+garch(2,2)	6.18%	1.28%	5.34%	1.46%	6.00%	1.41%	5.34%	1.37%	5.21%	1.24%	5.44%	1.55%
average	6.12%	1.35%	5.48%	1.37%	5.87%	1.37%	5.30%	1.32%	5.04%	1.35%	5.35%	1.57%
arma(1,1)+garch(1,0)	6.14%	1.55%	5.52%	1.19%	6.00%	1.37%	4.94%	1.37%	5.25%	1.15%	5.17%	1.33%
arma(1,1)+garch(1,1)	6.31%	1.41%	5.65%	1.50%	6.05%	1.32%	5.39%	1.28%	4.90%	1.41%	6.62%	1.77%
arma(1,1)+garch(2,1)	6.31%	1.46%	5.61%	1.55%	6.23%	1.46%	5.34%	1.28%	5.03%	1.50%	5.53%	1.64%
arma(1,1)+garch(1,2)	6.36%	1.32%	5.65%	1.32%	6.09%	1.37%	5.43%	1.37%	5.08%	1.28%	5.35%	1.72%
arma(1,1)+garch(2,2)	6.23%	1.37%	5.47%	1.41%	6.09%	1.46%	5.30%	1.37%	5.08%	1.28%	5.48%	1.68%
average	6.27%	1.42%	5.58%	1.39%	6.09%	1.40%	5.28%	1.33%	5.07%	1.32%	5.63%	1.63%

Model Comparison

- Effect of Conditional Mean

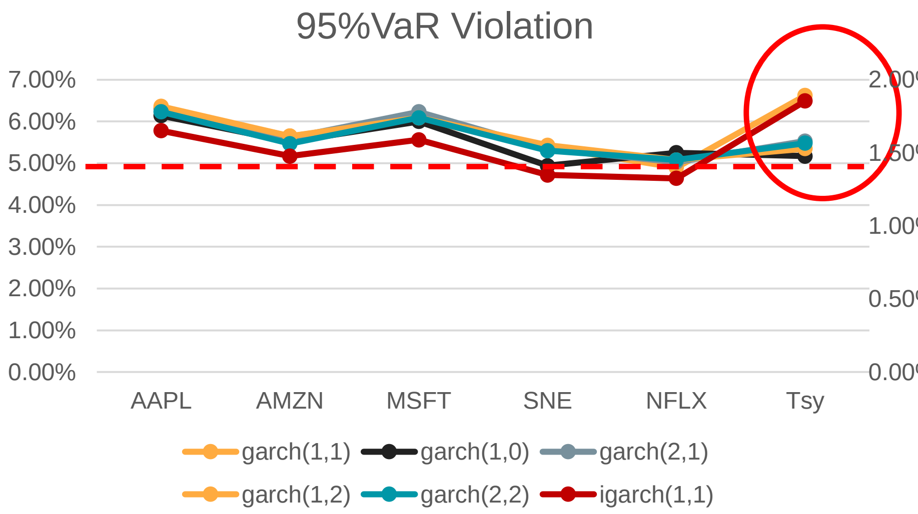


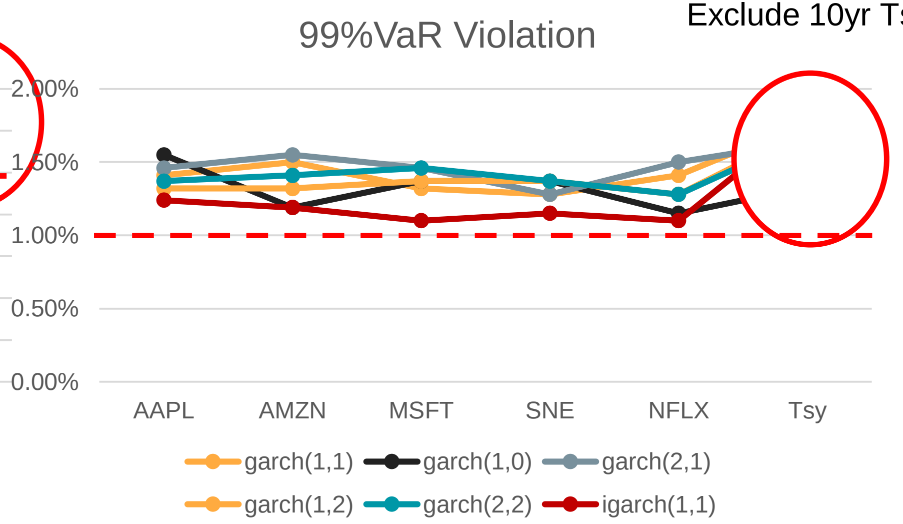


Model Comparison

- Choice of Volatility Model: Stocks

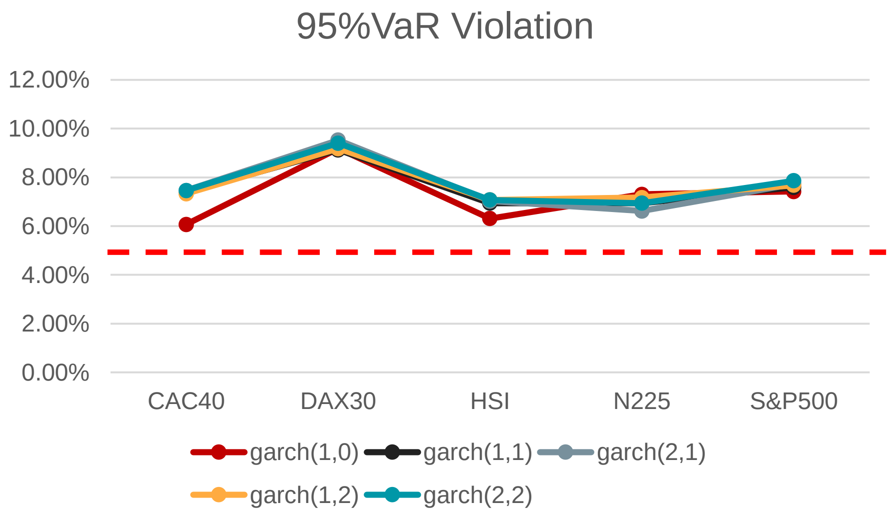
Exclude 10yr Tsy

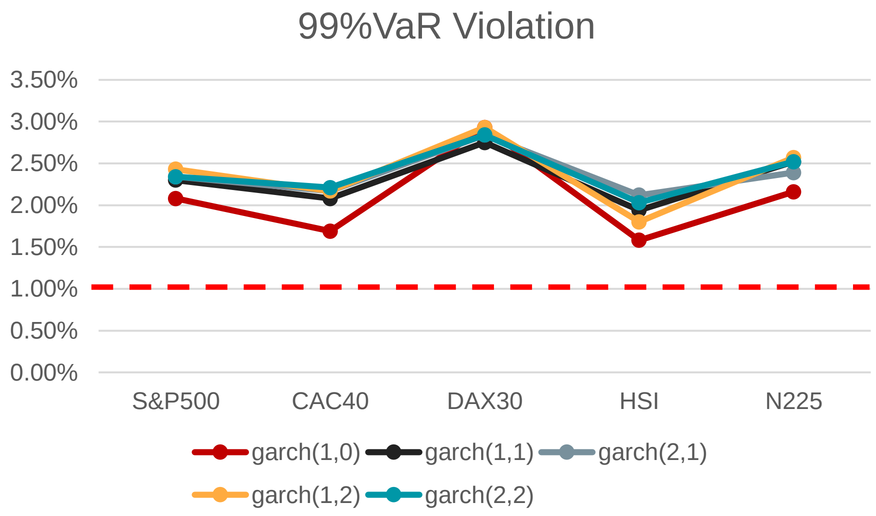




Model Comparison

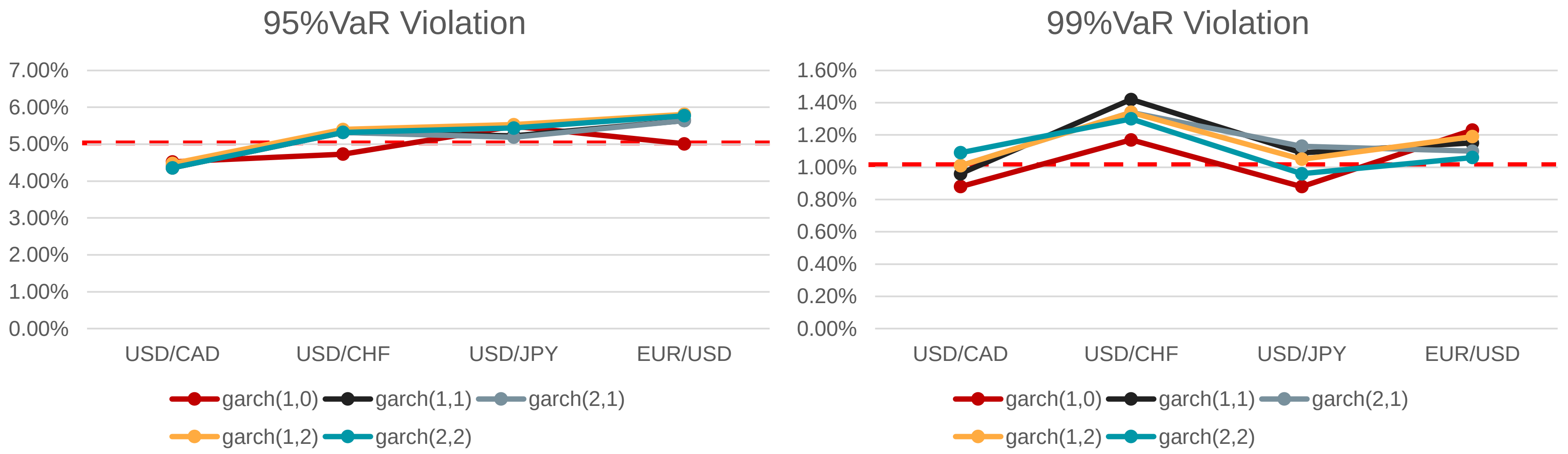
- Choice of Volatility Model: Indices





Model Comparison

- Choice of Volatility Model: FX rates



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Suggested Models for Our Data

- Stocks: IGARCH(1,1) (Integrated GARCH)
- 10yr Tsy Bonds: All models not working well, though ARCH(1,0) is slightly better; Need further study
- Indices: ARCH(1,0)
- FX rates: GARCH(1,1)

Conclusion

For our selected financial data:

- Conditional volatility models provide better forecasts
- Increasing Sample size improves forecast accuracy, while forecast size within a small range won't affect
- Innovation using t distribution outperforms normal on 99% VaR forecast
- Choice of conditional mean is almost indifferent
- Best volatility model depends on data

Thank You!

Q&A