**Parameters:**

1. Qij – flow in link ij defined based on the branched network configuration, predetermined
2. Lij – length of link ij [km]
3. CPk – Cost of piping $/m for each pipe size k
4. C – Cost of pump, $
5. CMP – Operations and maintenance cost for pump station, $;
6. HLmax – upper bound of hydraulic loss [m].
7. Hout – head pressure at link ij [m] – pipes flowing out
8. Hin – head pressure at link ij [m] – pipes flowing in
9. ein – pipe elevation at link ij [m] – pipes flowing in
10. eout – pipe elevation at link ij [m] – pipes flowing out
11. Vmin = 0.6 [m/s] (equivalent to 2 ft/s);
12. Vmax = 3 [m/s] (equivalent to 10 ft/s);
13. hi – height of node i [m].

**Objectives:**

1. Pipe cost:
2. Pump cost:
3. Operation and maintenance costs:

**Decision variable:**

1. dijk -- binary variable {0,1} for each discrete size k [m], defines the pipe diameter for link ij [m]
2. HLij -- continuous variable representing the hydro loss in link ij [m]
3. pij – binary variable {0,1} indicating if a pump is needed in link ij
4. prij -- continuous variable representing the pressure injected by the pump at link ij [m]
5. ei – pipe elevations at node i [m]
6. Hi – continuous variable representing the head pressure at node i [m]

**Constraints:**

1. where and 10700 and 140 are defined by the Hazen Williams equation and units of analysis (See Mays textbook P472)
2. where the bounds are set first to be non-binding and can be changed later.
3. only select one pipe size k for each link
4. if pump is set in link ij, compute the power it produces. Otherwise, the pressure by pump is zero.
5. energy balance.
6. Velocity must be between 0.6 and 3 m/s. (Inclusion of minimum pipe sizes and minimum velocity constraints results in no feasible solutions; therefore, the minimum velocity constraint was removed.
7. upper bound is arbitrary maximum depth assuming 1 foot or 0.3048 meters of cover beneath the surface is needed for the pipes, and lower bound is minimum depth assuming 30m.
8. the head pressure at each node must be between 10 psi and 80 psi and convert them to meter.

\*\* the model report shows 7) is constraint causing infeasibility:

Optimization was stopped with status 3

IIS computed: 2 constraints, 0 bounds

IIS runtime: 0.00 seconds (0.00 work units)

Warning: variable name "hydro loss[0,1]" has a space

Warning: constraint name "Hydro loss" has a space

Warning: default variable names used to write mps file

Warning: default constraint names used to write mps file

Read MPS format model from file m.mps

Reading time = 0.00 seconds

pipe: 268 rows, 420 columns, 1236 nonzeros

PIPE ELV LB

PIPE ELV UB

My understanding is that: the gravity force, controlled by pipe elevation, is first used to satisfy energy requirement. But pipe elevation is constrained with lower and upper bound, and energy produced by gravity force is also constrained, so sometimes pump is needed when pipe elevation reaches the bounds.