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CS383-HW1

1.

(a).

$$\mu_1 = \frac{-2-5-3+0-8-2+1+5-1+6}{10} =-0.9$$
$$\mu_2 = \frac{1-4+1+3+11+5+0-1-3+1}{10} = 1.4$$

$$\sigma_1 = \sqrt{\frac{1}{10-1} \times \left|-2-\mu_1\right|^2 + \left|-5-\mu_1\right|^2 + ... + \left|6-\mu_1\right|^2} = \sqrt{\frac{1609}{90}} = 4.2282$$
$$\sigma_2 = \sqrt{\frac{1}{10-1} \times \left|1-\mu_2\right|^2 + \left|-4-\mu_2\right|^2 + ... + \left|1-\mu_2\right|^2} = \sqrt{\frac{274}{15}} = 4.2740$$

Standardized data is:

$X' \times X \div (N-1)$

$$\begin{bmatrix} -0.2602 & -0.9697 & -0.4967 & 0.2129 & -1.6792 & -0.2602 & 0.4494 & 1.3954 & -0.0237 & 1.6319 \\ -0.0936 & -1.2635 & -0.0936 & 0.3744 & 2.2462 & 0.8423 & -0.3276 & -0.5615 & -1.0295 & -0.0936 \end{bmatrix}$$

$\times$

$$\begin{bmatrix} -0.2602 & -0.0936 \\ -0.9697 & -1.2635 \\ -0.4967 & -0.0936 \\ 0.2129 & 0.3744 \\ -1.6792 & 2.2462 \\ -0.2602 & 0.8423 \\ 0.4494 & -0.3276 \\ 1.3954 & -0.5615 \\ -0.0237 & -1.0295 \\ 1.6319 & -0.0936 \end{bmatrix} \div 9 = \begin{bmatrix} 1 & -0.4083 \\ -0.4083 & 1 \end{bmatrix}$$

Eigenvalue :

$$\begin{bmatrix} 1 & -0.4083 \\ -0.4083 & 1 \end{bmatrix} - \lambda I = 0$$

$$(1-\lambda)^2 + (0.4083)^2 = 0$$

$$\lambda = \begin{bmatrix} 0.5917 \\ 1.4083 \end{bmatrix}$$

Eigenvector:

$\lambda = 0.5917$

$I_1 = \begin{bmatrix} -0.7071 \\ -0.7071 \end{bmatrix}$

$\lambda = 1.4083$

$I_2 = \begin{bmatrix} -0.7071 \\ 0.7071 \end{bmatrix}$

(b).

$$\begin{bmatrix} -0.2602 & -0.0936 \\ -0.9697 & -1.2635 \\ -0.4967 & -0.0936 \\ 0.2129 & 0.3744 \\ -1.6792 & 2.2462 \\ -0.2602 & 0.8423 \\ 0.4494 & -0.3276 \\ 1.3954 & -0.5615 \\ -0.0237 & -1.0295 \\ 1.6319 & -0.0936 \end{bmatrix} \times \begin{bmatrix} -0.7071 \\ 0.7071 \end{bmatrix} = \begin{bmatrix} 0.1178 \\ -0.2077 \\ 0.2850 \\ 0.1142 \\ 2.7756 \\ 0.7796 \\ -0.5494 \\ -1.3838 \\ -0.7112 \\ -1.2201 \end{bmatrix}$$

2. (a)

Remainder$_1$ = $\frac{2}{10}H(\frac{1}{2},\frac{1}{2}) + \frac{4}{10}H(\frac{1}{1},\frac{0}{1}) + \frac{4}{10}H(\frac{0}{1},\frac{1}{1}) = 0.2$

$H(\frac{1}{2},\frac{1}{2}) = 1$

$H(1,0) = 1log1 - 0 = 0$

$H(0,1) = 0 - 1log1 = 0$

IG(1) = 1 - 0.2 = 0.8

Remainder$_2$ = $\frac{3}{10}H(\frac{2}{3},\frac{1}{3}) + \frac{8}{10}H(\frac{1}{1},\frac{0}{1}) = 0.2755$

$H(\frac{2}{3},\frac{1}{3}) = \frac{2}{3}log\frac{2}{3} - \frac{1}{3}log\frac{1}{3} = 0.9183$

$H(1,0) = 1log1 - 0 = 0$

IG(2) = 1 - 2755 = 0.7245

(b)

IG(1)>IG(2), so class 1 is more discriminating.

(c)

$C_1 = \begin{bmatrix} -0.2602 & -0.0936 \\ -0.9697 & -1.2635 \\ -0.4967 & -0.0936 \\ 0.2129 & 0.3744 \\ -1.6792 & 2.2462 \end{bmatrix}$ $C2 = \begin{bmatrix} -0.2602 & 0.8423 \\ 0.4494 & -0.3276 \\ 1.3954 & -0.5615 \\ -0.0237 & -1.0295 \\ 1.6319 & -0.0936 \end{bmatrix}$

Mean for C_1 is [-0.6386 0.2340] Mean for C_2 is [0.6386 -0.2340]

$$\sigma_i^2 = (N-1)cov(C_i)$$

$$\sigma_1^2 = 4cov(C_1) = \begin{bmatrix} 2.0808 & -1.6490 \\ -1.6490 & 6.5255 \end{bmatrix}$$

$$\sigma_2^2 = 4cov(C_2) = \begin{bmatrix} 2.8415 & -0.5312 \\ -0.5312 & 1.9270 \end{bmatrix}$$

$$\sigma_1^2 + \sigma_2^2 = \begin{bmatrix} 4.9223 & -2.1803 \\ -2.1803 & 8.4526 \end{bmatrix}$$

$$(\sigma_1^2 + \sigma_2^2)^{-1} = \begin{bmatrix} 0.2294 & 0.0592 \\ 0.0592 & 0.1336 \end{bmatrix}$$

Eigenvalue of this product is $\begin{bmatrix} 0.3327 \\ 0 \end{bmatrix}$

Eigenvector of this product is $\begin{bmatrix} 0.9998 \\ 0.0493 \end{bmatrix}$

(d)

$$\begin{bmatrix} -0.2602 & -0.0936 \\ -0.9697 & -1.2635 \\ -0.4967 & -0.0936 \\ 0.2129 & 0.3744 \\ -1.6792 & 2.2462 \end{bmatrix} \times \begin{bmatrix} 0.9998 \\ 0.0493 \end{bmatrix} = \begin{bmatrix} -0.2645 \\ -1.0308 \\ -0.5007 \\ 0.2311 \\ -1.5664 \end{bmatrix}$$

$$\begin{bmatrix} -0.2602 & 0.8423 \\ 0.4494 & -0.3276 \\ 1.3954 & -0.5615 \\ -0.0237 & -1.0295 \\ 1.6319 & -0.0936 \end{bmatrix} \times \begin{bmatrix} 0.9998 \\ 0.0493 \end{bmatrix} = \begin{bmatrix} -0.2184 \\ 0.4327 \\ 1.3660 \\ -0.0744 \\ 1.6253 \end{bmatrix}$$

(e)

Yes. Because IG(previous) seems larger than IG(1), which was better than IG(2).