CS-521-900, Midterm

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1. **I)** $a = 4, b = 4, n^{\log_b a} = n, f(n) = 4n + 4, f(n) = O(n)$ Therefore $f(n) = n^{\log_b a}$, thus case 2. $T(n) = \theta(n^{\log_b a \log n)} = \theta(n \log n)$, which is $c.\theta(n \log n)$.

   II) Since $T(n/3) \geq T(n/4)$, Therefore we can rewrite $T(n)$ to $T(n) \leq 3T(n/3) + n$ In this case, $a = 3, b = 3, n^{\log_b a} = n, f(n) = n$, thus case 2. $T(n) = \theta(n^{\log_b a \log n)} = \theta(n \log n)$, which is $c.\theta(n \log n)$.

   III) $T(n) = T(n-2)+n^2+n = T(n-4)+n^2+n^2+n+n = T(n-6)+n^2+n^2+n^2+n+n+n = ... = T(\frac{n}{2}(n-2)+\frac{n}{2}n^2+\frac{n}{2}n) = \frac{n^3}{2}+\frac{n^2-2n}{2}+\frac{n^2}{2} = O(n^3)$Therefore, it is $e.\theta(n^3)$.

   III) $T(0) = 1, T(3) = 5, T(6) = 5^2, T(9) = 5^3, T(12) = 5^4 ... T(n) = 5^{n/3}$ Therefore, it is $g.\theta(5^{n/3})$.

2. **I)**

```
Select(A,p,q,i){
  Divide A to n/5 groups of size 5
  Find the median of each group of 5 by brute force
    and store them in a set A'of size n/5
   Use  Select(A',p,q,i) to find the median x of n/5 medians
   Partition the n elements around x. Let k=q-p+1(rank of x)
   if (i==k):
     return A[n/3] to A[2n/3]
   if(i>n/3):
     Select(A,n/3,q,(i-n/3))
   if(i<2n/3):
     Select(A,2n/3,q,i)
}
```

   Similar to the original *Select* Algorithm, since we only need n/3 of the elements between n/3 to 2n/3, so we don't have to sort the elements smaller then the pivot n/3 or the elements larger than pivot 2n/3. Therefore the complexity is $T(n) = T(n/5) + T(3n/4) + \theta(1/3n) = \theta(n)$

   II)

```
Select(A,p,q,i){
  Divide A to n/5 groups of size 5
  Find the median of each group of 5 by brute force
    and store them in a set A'of size n/5
   Use  Select(A',p,q,i) to find the median x of n/5 medians
   Partition the n elements around x. Let k=q-p+1(rank of x)
   if (i==k):
     return A[n/3] to A[2n/3]
   if(i>n/3):
     Select(A,n/3,q,(i-n/3))
   if(i<2n/3):
     Select(A,2n/3,q,i)
 }
 Quicksort(res,p,r)
```

Since the size of *res* is $n/3$, therefore the overall complexity is $\theta(n + n/3 \log n/3)$.

3.
```
Let y be the person number
Select(A,p,q,i,y){
  Divide A to n/5 groups of size 5
  Find the median of each group of 5 by brute force
    and store them in a set A'of size n/5
   Use  Select(A',p,q,i) to find the median x of n/5 medians
   Partition the n elements around x. Let k=q-p+1(rank of x)
   if (i==y):
     return x
   if(i>y):
      Select(A,p,k,i,y+1)
   if(i<y):
     Select(A,k.q.i-k,y-1)
 }
```

The overall complexity is $\theta(\log n)$.

4. **I)**

```
A = combine(A,B), n = 2m
Select(A,p,q,i){
  Divide A to n/5 groups of size 5
  Find the median of each group of 5 by brute force
    and store them in a set A'of size n/5
   Use  Select(A',p,q,i) to find the median x of n/5 medians
   Partition the n elements around x. Let k=q-p+1(rank of x)
   if (i==k):
     return x
   if(i<k):
     Select(A,p,k-1,i)
  else:
    Select(A,k,q,i-k)
}
```

Combine A and B to create the union, it takes constant time. Then use $Select()$ to find the median of the union. The complexity of the $Select()$ is $\theta(n)$. Therefore the overall complexity is $\theta(n)$.

II)

```
Make A a min heap
Make B a min heap
Let newRoot = -infinity
For each key level:
 newHeap.add(A.currentLevelElements)
 newHeap.add(B.currentLevelElements)
End
Heap-Extract-Max(newHeap)
Find the median x of middle key level by brute force
Return x
```

Since A and B are sorted array, it takes constant time to build new heap(simply copy paste). A and B has the same size, therefore the size of the new heap is $2n$. Find the median x in the middle level by brute force take constant time. The $Heap - Extract - Max(newHeap)$ function calls $Heapify()$ which takes $O(\log n)$ time, therefore the overall complexity is $\theta(\log n)$

5.

	A	B	O	o	Ω	ω	Θ
a.	$lg^k n$	n^ϵ	True	True	False	False	False
b.	n^k	C^n	True	True	False	False	False
c.	$\sqrt{n}$	$n^{sin(n)}$	False	False	False	False	False
d.	2^n	$2^{n/2}$	False	False	True	True	False
e.	n^{lgc}	c^{lgn}	True	False	True	False	True
f.	$lg(n!)$	$lg(n^n)$	True	False	True	False	True

6.