Floating net reduction: calculate the capacitance matrix of the signal nets: C.

$$\begin{bmatrix} Q \\ 0 \end{bmatrix} = \begin{bmatrix} A & X \\ Y & Z \end{bmatrix} \begin{bmatrix} V \\ v \end{bmatrix}$$

$$Q = (A - XZ^{-1}Y)V = CV$$

C is a symmetric matrix, whose diagonal elements are positive while off diagonal elements are negative, and the sum of each row/column is 0.

Let T be the diag(1.0/diag(Z)), $K = I - ZT$

$$(I - K)^{-1} = T^{-1}Z^{-1}$$

Each element of K is smaller than 1.0. Using neumann series: $(I - K)^{-1} = I + K + K^2 + K^3 + \cdots$

$$Z^{-1} = T(I + K + K^2 + K^3 + \cdots)$$

$$C = A - XT(I + K + K^2 + K^3 + \cdots)Y$$

Using random walk to estimate:

$(-XT)_{in} = -X_{in}/Z_{nn}$, which are positive <1.0

$X = Y^T : (TY)^T = Y^T T = XT$

$$1 \le i, j \le N_{signal},$$

$$1 \le m, n \le N_{floating}$$

$$XZ^{-1}Y = (-XT)(I + K + K^2 + K^3 + \cdots)T^{-1}(-TY)$$

$$(XZ^{-1}Y)_{ij} = \sum_{k=1}^{N_{floating}} p_{ki}p_{kj}Z_{kk} + \sum_{k=1,l=1}^{N_{floating}} p_{ki}p_{kl}p_{lj}Z_{ll} + \sum_{k=1,l=1,u}^{N_{floating}} p_{ki}p_{kl}p_{lu}p_{uj}Z_{uu}$$

Where $p_{ki} = -\frac{X_{ik}}{Z_{kk}} = -\frac{X_{ik}}{A_{ii}}\frac{A_{ii}}{Z_{kk}} = \frac{A_{ii}}{Z_{kk}}p_{ik},\ p_{kj} = -\frac{X_{jk}}{Z_{kk}}, p_{kl} = -\frac{Z_{kl}}{Z_{kk}}, p_{uj} = -\frac{X_{ju}}{Z_{uu}}$

Random walk to estimate row i:

Calculate the transient probability matrix of
$\begin{bmatrix} A & X \\ Y & Z \end{bmatrix}$. $(Each\ column\ divided\ by\ the\ diagonal\ elements\ and\ set\ diagonal\ to\ 0)$

Start from signal net i, random walk until arrives a signal net j, get the accumulated value from i to j: $E_{ij} \mathrel{+}= \frac{A_{ii}}{Z_{kk}}Z_{ww}$. Where k represents the first floating id and w represents the last floating id. $E_{ij} \mathrel{+}= 0$ if there is no floating id on the path.

$(XZ^{-1}Y)_{ij} = E_{ij}/N_{walk}$

Problem: slow and non-symmetric

```
yz@yz:~/Cap3D/toy_random_walk$ ./main
before floating net reduction:
[[44.63423 -8.81106 -0.00347 -0.00108 -0.00533]
 [-8.81106 18.20445 -0.00398 -0.00829 -0.97816]
 [-0.00347 -0.00398 37.99677 -9.22764 -1.39234]
 [-0.00108 -0.00829 -9.22764 45.53753 -5.26923]
 [-0.00533 -0.97816 -1.39234 -5.26923 64.72104]]
after floating net reduction:
[[ 32.14754 -10.36438  -4.07649  -5.22793 -12.47874]
 [-10.36438  17.13217  -1.09229  -1.26467  -4.41083]
 [ -4.07649  -1.09229  30.23036 -15.85921  -9.20237]
 [ -5.22793  -1.26467 -15.85921  36.44533 -14.09352]
 [-12.47874  -4.41083  -9.20237 -14.09352  40.18547]]
matrix inversion method: 0.0001716613769531 25 seconds
using random walk:
[[ 31.93609 -10.3894   -4.0306   -5.25567 -12.06584]
 [-10.38406  17.13539  -1.10022  -1.2779   -4.24824]
 [ -4.38622  -1.0925   30.18184 -15.94699  -8.83161]
 [ -5.51731  -1.28374 -15.94721  36.3517  -13.65442]
 [-14.2404   -4.72554 -10.2285  -15.301    39.08867]]
Random walk method: 13356.541240930557 seconds
```