Floating net reduction: calculate the capacitance matrix of the signal nets: C.

$$\begin{bmatrix} Q \\ 0 \end{bmatrix} = \begin{bmatrix} A & X \\ Y & Z \end{bmatrix} \begin{bmatrix} V \\ v \end{bmatrix}$$

$$Q = (A - XZ^{-1}Y)V = CV$$

C is a symmetric matrix, whose diagonal elements are positive while off diagonal elements are negative, and the sum of each row/column is 0.

Let T be the diag(1.0/diag(Z)), $K = I - ZT$

$$(I - K)^{-1} = T^{-1}Z^{-1}$$

Each element of K is smaller than 1.0. Using neumann series: $(I - K)^{-1} = I + K + K^2 + K^3 + \cdots$

$$Z^{-1} = T(I + K + K^2 + K^3 + \cdots)$$

$$C = A - XT(I + K + K^2 + K^3 + \cdots)Y$$

Using random walk to estimate:

$(-XT)_{in} = -X_{in}/Z_{nn}$, which are positive <1.0

$X = Y^T : (TY)^T = Y^T T = XT$

$$1 \le i,j \le N_{signal},$$

$$1 \le m,n \le N_{floating}$$

$$XZ^{-1}Y = (-XT)(I + K + K^2 + K^3 + \cdots)T^{-1}(-TY)$$

$$(XZ^{-1}Y)_{ij} = \sum_{k=1}^{N_{floating}} p_{ki}p_{kj}Z_{kk} + \sum_{k=1,l=1}^{N_{floating}} p_{ki}p_{kl}p_{lj}Z_{ll} + \sum_{k=1,l=1,u}^{N_{floating}} p_{ki}p_{kl}p_{lu}p_{uj}Z_{uu}$$

Where $p_{ki} = -\frac{X_{ik}}{Z_{kk}} = -\frac{X_{ik}}{A_{ii}}\frac{A_{ii}}{Z_{kk}} = \frac{A_{ii}}{Z_{kk}}p_{ik},\ p_{kj} = -\frac{X_{jk}}{Z_{kk}}, p_{kl} = -\frac{Z_{kl}}{Z_{kk}}, p_{uj} = -\frac{X_{ju}}{Z_{uu}}$

Random walk to estimate row i:
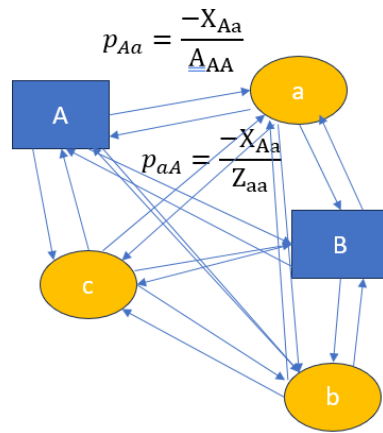
Calculate the transient probability matrix of
$\begin{bmatrix} A & X \\ Y & Z \end{bmatrix}$. $(Each\ column\ divided\ by\ the\ diagonal\ elements\ and\ set\ diagonal\ to\ 0)$

Start from signal net i, random walk until arrives a signal net j, get the accumulated value from i to j:
$E_{ij} += \frac{A_{ii}}{Z_{kk}}Z_{ww}$. Where k represents the first floating id and w represents the last floating id. $E_{ij} += 0$ if there is no floating id on the path.
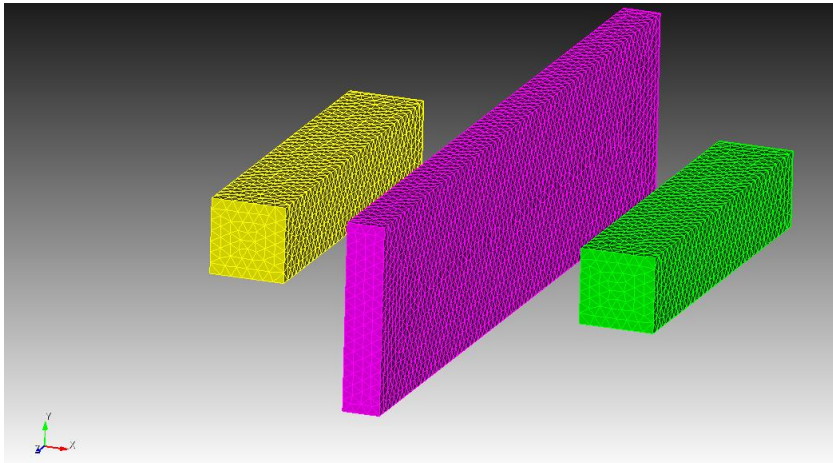
$(XZ^{-1}Y)_{ij} = E_{ij}/N_{walk}$

Random Walk: Signal → Floating → Signal

$$\begin{bmatrix} Q \\ 0 \end{bmatrix} = \begin{bmatrix} A & X \\ Y & Z \end{bmatrix} \begin{bmatrix} V \\ v \end{bmatrix}$$

$$p_{Aa} = \frac{-X_{Aa}}{A_{AA}}$$

$$p_{aA} = \frac{-X_{Aa}}{Z_{aa}}$$

Example paths:
A→a→c→b→c→B
A→c→A
A→B
…

```
[Mesh] constructor: Elem_num=11036, node_num=5524
Quadrature degree is 6
The area of metal objective 1: 0.879994m^2
The area of metal objective 2: 0.879994m^2
The area of metal objective 3: 2.49999m^2
Load the mesh takes 0.139477 seconds
Calculate the rhs vector takes 0.009728 seconds
Calculate the coefficient matrix takes 143.212 seconds
Solve the matrix equation takes 72.0022 seconds
The capacitance matrix:
3.88349e-11  -1.76782e-12  -2.18726e-11
-1.76782e-12  3.88349e-11  -2.18726e-11
-2.18726e-11  -2.18726e-11  7.7696e-11
Post processing takes 0.010208 seconds
Total time: 215.386 seconds
```

```
C^-1:
[[3.25147162e+10 7.88582434e+09 1.13733637e+10]
 [7.88582434e+09 3.25147162e+10 1.13733637e+10]
 [1.13733637e+10 1.13733637e+10 1.92742235e+10]]
Q:
[ 1.e-10 -1.e-10  0.e+00]
V:
[ 2.46288919e+00 -2.46288919e+00  2.22044605e-16]
before floating net reduction:
[[ 3.88349e-11 -1.76782e-12]
 [-1.76782e-12  3.88349e-11]]
after floating net reduction:
[[ 3.26774320e-11 -7.92528796e-12]
 [-7.92528796e-12  3.26774320e-11]]
```

```
the env CAP_MATRIX is unset: calculate the chargen distribution
==================start==================
[Config] constructor:
Homogeneous background permitivity : 8.854e-12F/m
The electrostatic potential of metal objective 1: 2.46289V
The electrostatic potential of metal objective 2: -2.46289V
The electrostatic potential of metal objective 3: 2.22e-16V
Thread numbers: 4
[Mesh] constructor: Elem_num=11036, node_num=5524
Quadrature degree is 6
The area of metal objective 1: 0.879994m^2
The area of metal objective 2: 0.879994m^2
The area of metal objective 3: 2.49999m^2
Load the mesh takes 0.116973 seconds
Calculate the rhs vector takes 0.002535 seconds
Calculate the coefficient matrix takes 146.45 seconds
Solve the matrix equation takes 71.9716 seconds
Total electric charge on the metal objective 1: 1e-10C
Total electric charge on the metal objective 2: -1e-10C
Total electric charge on the metal objective 3: -7.14066e-17C
Post processing takes 0.163949 seconds
Total time: 218.716 seconds
==================finish==================
```


Charge density
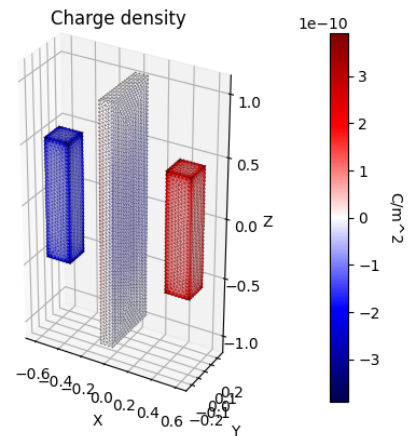
```
C:
[[ 3.883490e-11 -1.767820e-12 -2.187260e-11 -1.519448e-11]
 [-1.767820e-12  3.883490e-11 -2.187260e-11 -1.519448e-11]
 [-2.187260e-11 -2.187260e-11  7.769600e-11 -3.395080e-11]
 [-1.519448e-11 -1.519448e-11 -3.395080e-11  6.433976e-11]]
C^-1:
[[-1.54742505e+26 -1.54742505e+26 -1.54742505e+26 -1.54742505e+26]
 [-1.54742505e+26 -1.54742505e+26 -1.54742505e+26 -1.54742505e+26]
 [-1.54742505e+26 -1.54742505e+26 -1.54742505e+26 -1.54742505e+26]
 [-1.54742505e+26 -1.54742505e+26 -1.54742505e+26 -1.54742505e+26]]
Q:
[ 1.e-10 -1.e-10  0.e+00  0.e+00]
V:
[ 0. -4. -2. -2.]
before floating net reduction:
[[ 3.88349e-11 -1.76782e-12]
 [-1.76782e-12  3.88349e-11]]
after floating net reduction:
[[ 3.26774320e-11 -7.92528796e-12]
 [-7.92528796e-12  3.26774320e-11]]
```