

# Letters

## A New Reconstruction Method for a Source Above an Arbitrarily Shaped Ground Plane

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**Abstract**—Source reconstruction using equivalent dipole moments to replace the original noise source is widely used in radio frequency interference. The conventional reconstruction methods only work well in the cases of free space or with a large ground plane. In this article a new method is proposed for source reconstruction over an arbitrarily shaped ground plane. It uses the field below the ground plane to eliminate the arbitrary ground effect. Comparing to the method of solving surface integral equation, it is simple and does not need the ground geometric information. The new method is validated in the full-wave simulations.

**Index Terms**—Image theorem, radio frequency interference (RFI), source reconstruction, surface equivalence theorem.

### I. INTRODUCTION

SOURCE reconstruction is widely used in radio-frequency interference (RFI) to characterize the noise source. If the source is in free space, the equivalent source can be solved from Maxwell equations with fields on observation points. To improve accuracy, the least square method is often used. When the source is above an infinitely large ground plane, the image theorem can be used. An image of the unknown equivalent source is added to replace the ground [1].

However, the source reconstruction will become challenging if the ground plane has finite size. Besides the image theorem, the diffraction caused by the truncation also needs to be considered. A source reconstruction method based on Green's function and surface integral equations can model the finite size ground plane and represent it as fictitious surface currents [2]. This method requires generating meshes on the ground plane for solving

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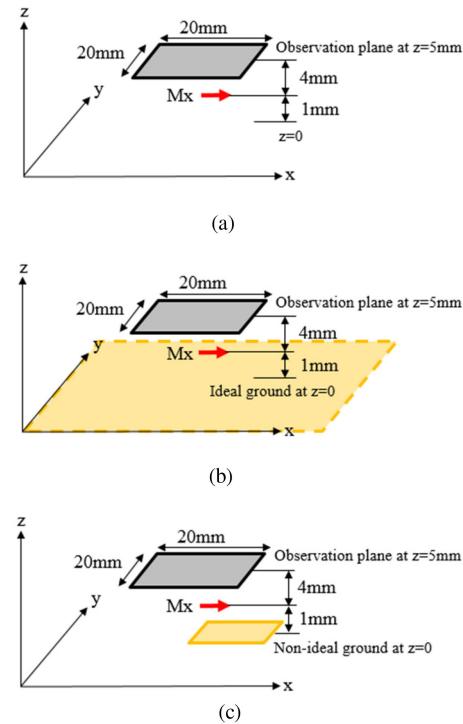


Fig. 1. Test cases for the conventional equivalent source reconstruction. (a) Free space case. (b) Infinitely large ground case. (c) Finite size ground case.

the integral equations. If the geometry of the ground plane is unknown or complicated for modeling, this method will be difficult to implement. Another method in near-to-far-field transformation uses the auxiliary dipoles at the edges of the ground plane to characterize the diffraction [3]. But those auxiliary dipoles are used to compensate the missing near field data on the unscanned area. They are not the equivalent source.

For the source above an arbitrarily shaped ground plane, and assuming the geometry is unknown, the conventional source reconstruction methods in [1] result in large error. Fig. 1 shows three cases for the source reconstruction: in free space, over ideal ground, and over nonideal ground, respectively. For comparing the source reconstruction accuracy, the noise source was selected as the unit horizontal magnetic dipole in the commercial full-wave simulation tool, Ansys HFSS. It was placed at the height of

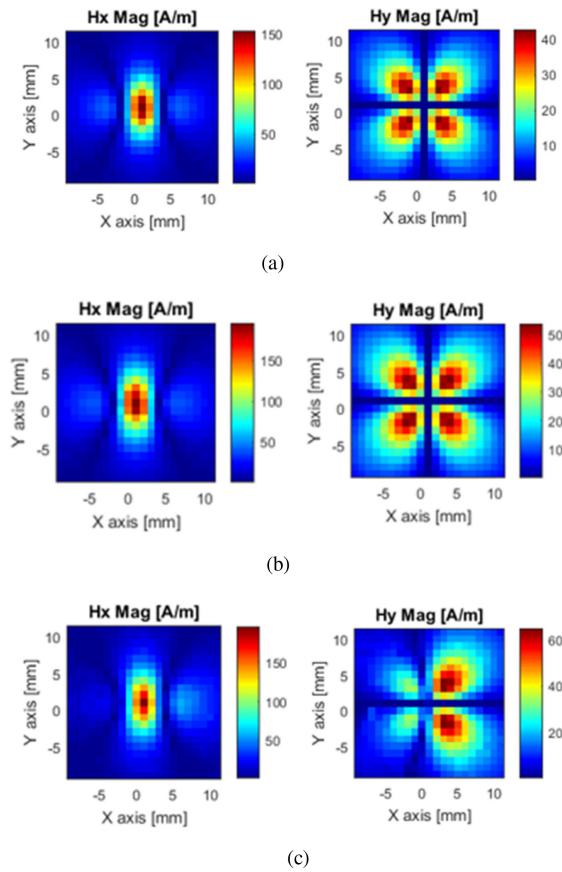


Fig. 2. Magnitudes of field components on the observation plane. (a) Free space case. (b) Infinitely large ground case. (c) Finite size ground case.

$z = 1$  mm. The observation plane consisted of  $21 \times 21$  points and its center was 4 mm above the source location. The field components  $H_x$  and  $H_y$  at 1 GHz were calculated by the finite element method with sufficient mesh density. In Fig. 1(b), the ground plane was infinitely large. In Fig. 1(c), the source was placed above an edge of the finite sized ground plane.

The magnitudes of field components are shown in Fig. 2. The patterns of free space case and ideal ground case are almost the same except the last one is stronger than the former one because of the additional reflection from the ground plane. For the finite-size ground case, Fig. 2(c) is no longer symmetrical to the y-axis because the scattering field in the right half region is stronger than that in the left half region.

Based on the known field components and transfer functions, the solutions of conventional source reconstruction methods were listed in Table I. The magnitude and phase of the six basic dipoles were calculated. Results in the first and second rows indicate that the conventional methods work well for the free space case and the infinite ground case. The magnitudes of reconstructed  $M_x$  dipole moments were 1.0 V·m with negligible errors. However, the conventional methods failed in the finite size ground case. In fact, one assumption, either free space or infinitely large ground plane, had to be made to characterize the finite size ground before using the conventional method. The corresponding reconstruction results are shown in the last two rows of Table I. If using the free space reconstruction

TABLE I  
SOURCE RECONSTRUCTION RESULTS

Magnitude $P:[A \cdot m]$ , $M:[V \cdot m]$	$P_x$	$P_y$	$P_z$	$M_x$	$M_y$	$M_z$
Free space case	$1.2 \times 10^{-5}$	$8.8 \times 10^{-5}$	$5.5 \times 10^{-5}$	1.0	$4.2 \times 10^{-4}$	$7.0 \times 10^{-4}$
Ideal ground case	$4.4 \times 10^{-5}$	$1.3 \times 10^{-5}$	$5.9 \times 10^{-5}$	1.0	$4.9 \times 10^{-4}$	$4.4 \times 10^{-4}$
Non-ideal ground case (assuming free space)	$2.0 \times 10^{-5}$	$1.8 \times 10^{-3}$	$6.6 \times 10^{-5}$	1.2	$1.2 \times 10^{-3}$	0.2
Non-ideal ground case (assuming ideal ground)	$1.3 \times 10^{-4}$	$2.3 \times 10^{-3}$	$4.3 \times 10^{-5}$	0.9	$6.2 \times 10^{-5}$	0.3

method, the magnitude of reconstructed  $M_x$  was 1.2 V·m with the relative error of 20%; the magnitude of reconstructed  $M_z$  was 0.2 V·m, which was supposed to be 0. If using the infinite ground reconstruction method, the magnitude of reconstructed  $M_x$  was 0.9 V·m with the relative error of 10%. Also, the magnitude of reconstructed  $M_z$  was 0.3 V·m.

From these examples, it can be observed that conventional source reconstruction methods are not suitable for the finite ground case. However, the source is often positioned above a such kind of nonideal ground plane in realistic problems. Although the method solving integral equations can reconstruct the source accurately if the geometric information of the ground plane is known, it may be too complicated to generate high quality meshes, select proper basis and test functions and solve singularity issue of integration. An efficient method for the source reconstruction over an arbitrary ground plane is needed.

## II. COMPENSATION METHOD

The failure of conventional source reconstruction methods was caused by the inaccurate estimation of the scattering field by the arbitrary ground plane, that is, either ignore it (assuming free space) or overestimate it (assuming infinitely large ground). The solution to this problem is straightforward: find a way to accurately determine and eliminate the effects of arbitrary ground plane before the source reconstruction.

Instead of exactly calculating the surface current densities on the ground plane, a compensation method is proposed to eliminate the arbitrary ground effects at the observation points by subtracting the field quantities at the compensation points. As shown in Fig. 3, the compensation point was symmetrical to the observation points about the ground plane. Their coordinates were represented by  $(x_c, y_c, z_c)$  and  $(x_o, y_o, z_o)$ , respectively. For convenience, set the ground plane at  $z = 0$ . This ground plane was formed by a finite size perfect electric conductor with apertures. There was a source at  $(x_s, y_s, z_s)$  above the ground plane. The total fields at observation point and compensation point were denoted as  $\vec{E}_o^{NGND}, \vec{H}_o^{NGND}$  and  $\vec{E}_c^{NGND}, \vec{H}_c^{NGND}$ . The superscript “NGND” indicates that they came from the finite ground plane model.

First, assuming there is another model similar as that in Fig. 3 except the ground plane becomes complete and infinitely

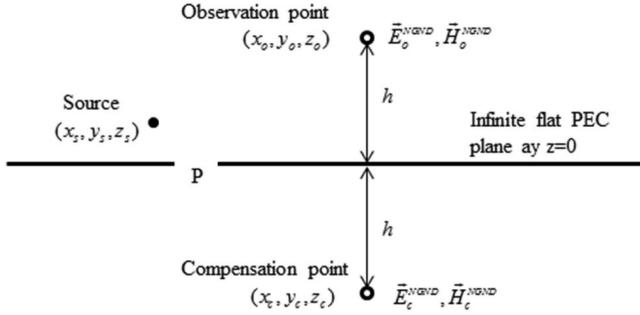


Fig. 3. Intentionally adding the compensation point below the ground plane.

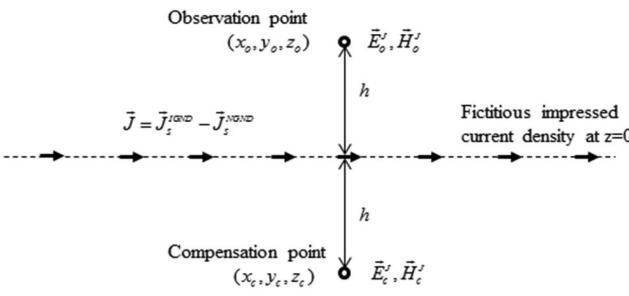


Fig. 4. Subtraction of the infinitely large ground plane model and the arbitrary ground equivalent model.

large. The infinite ground plane can be replaced by equivalent surface electric current density,  $\vec{J}_s^{IGND}$ . For this case, denote the total fields at observation point and compensation point as  $\vec{E}_o^{IGND}, \vec{H}_o^{IGND}$  and  $\vec{E}_c^{IGND}, \vec{H}_c^{IGND}$ . Similarly, the surface equivalence theorem can be applied in the finite ground model shown in Fig. 3. Denote the equivalent surface current density as  $\vec{J}_o^{NGND}$ . The total fields shown in Fig. 3 are the sum of the incident fields from the noise source and scattering fields generated by the equivalent surface current density.

Let us consider the effects of the finite ground plane. Both the infinite ground plane and the finite ground plane can be represented by the model consists of the original source and the equivalent surface current. Thus, the difference of these two models can be characterized by subtracting the finite ground model from the infinite ground model. The result is shown in Fig. 4. Since the original source at  $(x_s, y_s, z_s)$  exists in both ideal ground and nonideal ground cases, it disappears after the subtraction. The subtraction of surface current densities is represented by  $\vec{J}$  and it becomes the only source that generates fields  $\vec{E}_o^J, \vec{H}_o^J$  at the observation point and  $\vec{E}_c^J, \vec{H}_c^J$  at the compensation point. The subtraction also applies on the fields. The fields at observation point and compensation point satisfy the following equations:

$$\begin{cases} \vec{E}_o^{IGND} = \vec{E}_o^{NGND} + \vec{E}_o^J \\ \vec{H}_o^{IGND} = \vec{H}_o^{NGND} + \vec{H}_o^J \end{cases}, \quad (1)$$

and

$$\begin{cases} \vec{E}_c^{IGND} = \vec{E}_c^{NGND} + \vec{E}_c^J = 0 \\ \vec{H}_c^{IGND} = \vec{H}_c^{NGND} + \vec{H}_c^J = 0 \end{cases}. \quad (2)$$

In (2), for the infinite ground plane model, electromagnetic fields at the compensation point are 0. Therefore, the fields in the finite ground model can be represented as the fields generated by the fictitious surface current in Fig. 4

$$\begin{cases} \vec{E}_c^{NGND} = -\vec{E}_c^J = -\hat{x}E_{cx}^J - \hat{y}E_{cy}^J - \hat{z}E_{cz}^J \\ \vec{H}_c^{NGND} = -\vec{H}_c^J = -\hat{x}H_{cx}^J - \hat{y}H_{cy}^J - \hat{z}H_{cz}^J \end{cases}. \quad (3)$$

In (3), the fields are further expanded in terms of the field components in rectangular coordinates. Since the observation point and compensation point are symmetric about the fictitious current  $\vec{J}$ , the field components generated by the fictitious current satisfy specific relationships

$$\begin{cases} E_{ox}^J = E_{cx}^J, E_{oy}^J = E_{cy}^J, E_{oz}^J = -E_{cz}^J \\ H_{ox}^J = -H_{cx}^J, H_{oy}^J = -H_{cy}^J, H_{oz}^J = H_{cz}^J \end{cases}. \quad (4)$$

This is concluded from the theoretical equation that calculating fields in free space. Combining (3) and (4), the scattering fields at observation point can be expressed as the total fields at the compensation point, given by (5).

$$\begin{cases} \vec{E}_o^J = -\hat{x}E_{cx}^{NGND} - \hat{y}E_{cy}^{NGND} + \hat{z}E_{cz}^{NGND} \\ \vec{H}_o^J = \hat{x}H_{cx}^{NGND} + \hat{y}H_{cy}^{NGND} - \hat{z}H_{cz}^{NGND} \end{cases}. \quad (5)$$

The quantities of  $\vec{E}_o^J$  and  $\vec{H}_o^J$  represent the diffractions from the finite ground plane. The final step before source reconstruction is to eliminate such effects. Substituting (5) into (1), the observed total fields in the infinite ground model can be explicitly calculated from the total fields at the observation and compensation points in the finite ground model

$$\begin{cases} \vec{E}_o^{IGND} = F_e \left\{ \vec{E}_o^{NGND}, \vec{E}_c^{NGND} \right\} \\ = \hat{x}(E_{ox}^{NGND} - E_{cx}^{NGND}) + \hat{y}(E_{oy}^{NGND} - E_{cy}^{NGND}) \\ + \hat{z}(E_{oz}^{NGND} + E_{cz}^{NGND}) \\ \vec{H}_o^{IGND} = F_h \left\{ \vec{H}_o^{NGND}, \vec{H}_c^{NGND} \right\} \\ = \hat{x}(H_{ox}^{NGND} + H_{cx}^{NGND}) + \hat{y}(H_{oy}^{NGND} + H_{cy}^{NGND}) \\ + \hat{z}(H_{oz}^{NGND} - H_{cz}^{NGND}) \end{cases}. \quad (6)$$

Note that the original source in the infinite ground model is the same as that in the finite ground model. It means that the equivalent source reconstructed from the fields  $\vec{E}_o^{IGND}$  and  $\vec{H}_o^{IGND}$  calculated by (6) is the same as the source above the finite ground. Thus, equations in (6) provide an approach to convert the finite ground model into the infinite ground model. Thus, the source above an arbitrary ground plane can be accurately reconstructed.

Based on the compensation method, the source over an arbitrary ground plane can be reconstructed from the measured field components at observation points and compensation points. In Fig. 5(a), an unknown source is positioned at  $(x_s, y_s, z_s)$  and an arbitrary ground plane is located at  $z = 0$ . The observation plane at  $z = H$  composes of an observation point array. The compensation plane and observation plane are symmetric to the ground plane. The total fields at observation points and compensation points are represented by  $\vec{E}_o^{GND}, \vec{H}_o^{GND}$  and

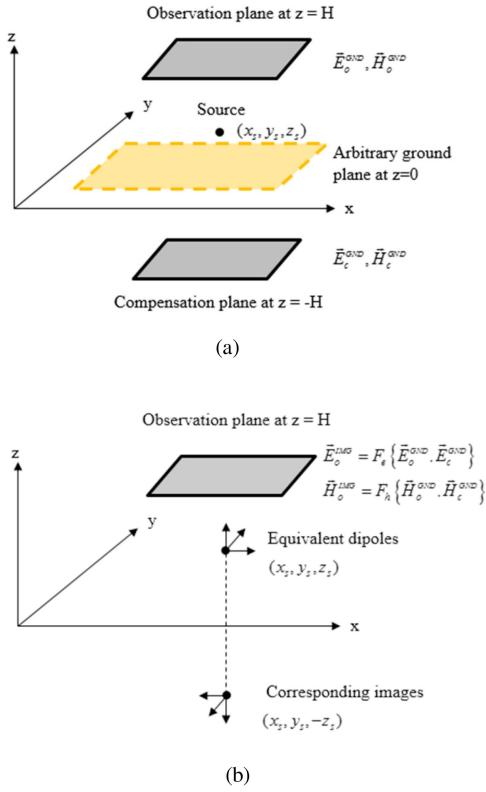


Fig. 5. Illustration of the source reconstruction using compensation method. (a) Obtain the fields at observation and compensation points. (b) Reconstruct the equivalent source with images in free space.

$\tilde{E}_c^{\text{GND}}, \tilde{H}_c^{\text{GND}}$ , respectively. Fig. 5(b) illustrates the model for reconstruction. First, the unknown source is replaced by equivalent dipoles with unknown quantities. Then, removing the ground plane and adding the images of the equivalent dipoles. But the fields at the observation point cannot be directly used for construction because the removed ground plane is not infinitely large. To compensate the arbitrary ground effects, the fields for reconstruction should be calculated from (6). Now, the equivalent dipoles can be accurately quantified by following the conventional reconstruction method.

### III. VALIDATION

For the same finite ground case as shown in Fig. 1(c), a compensation plane was added and the new reconstruction method was applied. The field components  $H_x$  and  $H_y$  on the observation plane have already been shown in Fig. 2(c). The field components on the compensation plane are shown in Fig. 6(a). Combining the fields components at observation and compensation points based on the compensation method, the calculated field components at observation plane is shown in Fig. 6(b). Comparing the calculated field components to those in the ideal ground case as shown in Fig. 2(b), they are almost the same. It was expected because the compensation method eliminates the arbitrary ground effects. The new reconstructed

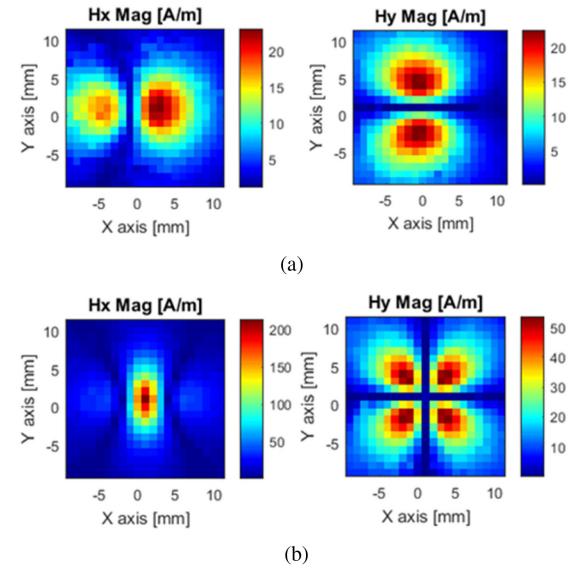


Fig. 6. Magnitudes of field components. (a) Field components on compensation plane. (b) Calculated field components on the observation plane by using the compensation method.

TABLE II  
RECONSTRUCTION RESULT USING THE COMPENSATION METHOD

Magnitude $P: [\text{A}\cdot\text{m}], M: [\text{V}\cdot\text{m}]$	Px	Py	Pz	Mx	My	Mz
Non-ideal ground case	1.1 $\times 10^{-4}$	8.8 $\times 10^{-4}$	2.7 $\times 10^{-5}$	1.0	2.5 $\times 10^{-4}$	2.7 $\times 10^{-3}$

result for the finite ground case is listed in Table II. Magnitude of the reconstructed  $M_x$  dipole is 1.0 V·m and other types of dipole moments are negligible. The new method for the finite ground source reconstruction is as accuracy as the conventional method for the free space or infinite ground reconstruction.

### IV. CONCLUSION

Using the compensation method proposed in this article, the noise source above an arbitrarily shaped ground plane can be correctly reconstructed. It can improve the accuracy of source reconstruction in realistic applications. The reconstructed source will be useful in the RFI analysis: estimating the interference strength, determining the coupling path, and eliminating the unwanted interference.

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