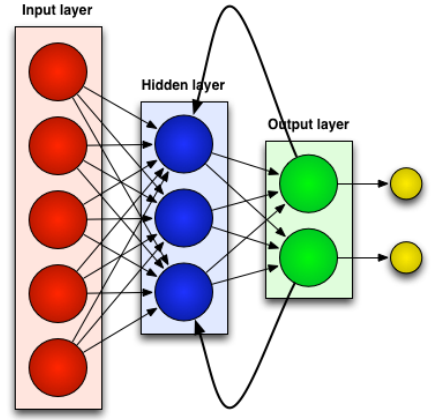
CS 615 – Deep Learning

Recurrent Neural Networks

Time Series Data

- We saw how Markov systems can be used on sequential data, for applications like:
  - Text processing
    - Data can be split into words or characters
  - Market Predictions
  - Video Processing
- Can we somehow tweak Neural Networks for this purpose?!



Recurrent Neural Networks (RNN)

- Recurrent Neural Networks (RNNs) are artificial neural networks that allow for processing sequential data.
- We feed in one sample at a time, in order, to train the network, "storing" gradients from previous observations of the sequence in some context layer.
- Uses of RNNs include:
  - Many to One
  - One to Many
  - Many to Many

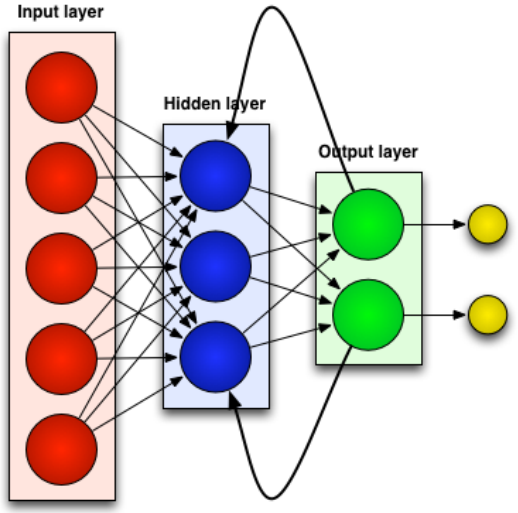
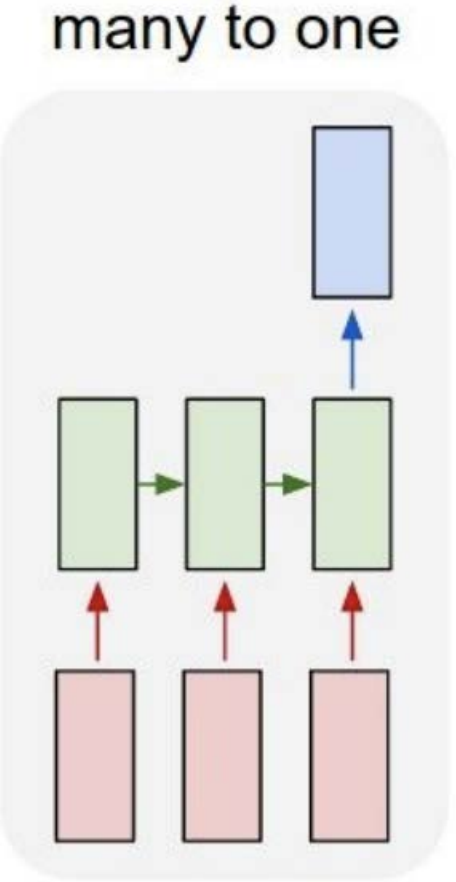


Image from towardsdatascience.com

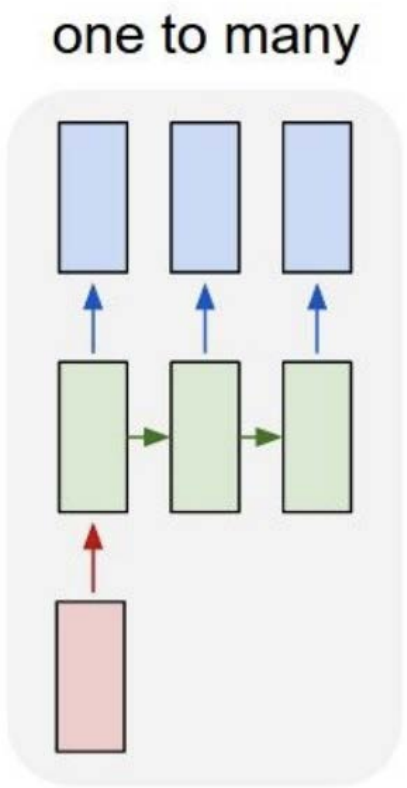
Many to One

- One type of RNN pushes a data sequence through our RNN to obtain a single value.
- Example Application: Video Classification
  - The “many” is the frames
  - The “one” is the classification of the video



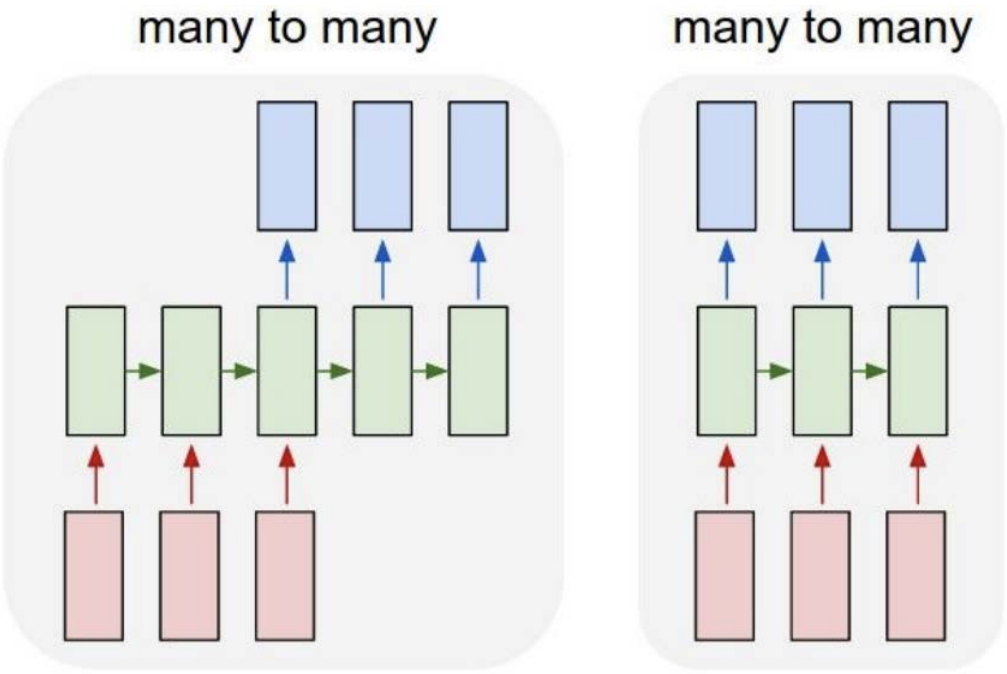
One to Many

- In another type we provide a single “seed” observation to the RNN and let it generate a sequence using the seed and the trained RNN
- Example Application: Generating Synthetic Data Sequence
  - The “one” is some starting value.
  - The “many” is the data sequence (pixels, words, etc..)



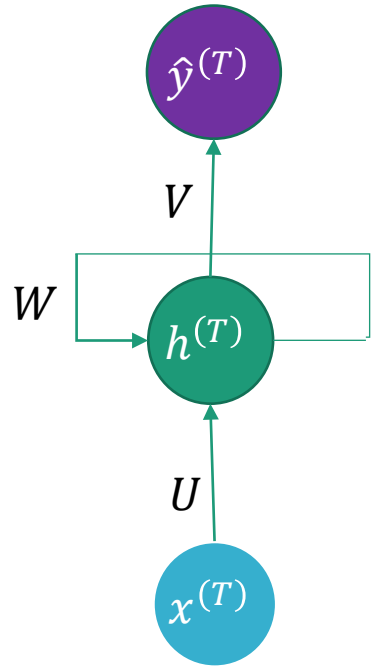
Many to Many

- And finally we might want to feed in a sequence and get out of sequence!
- Example applications: Translation
  - Provide a sequence of words
  - Get back a new sequence of words



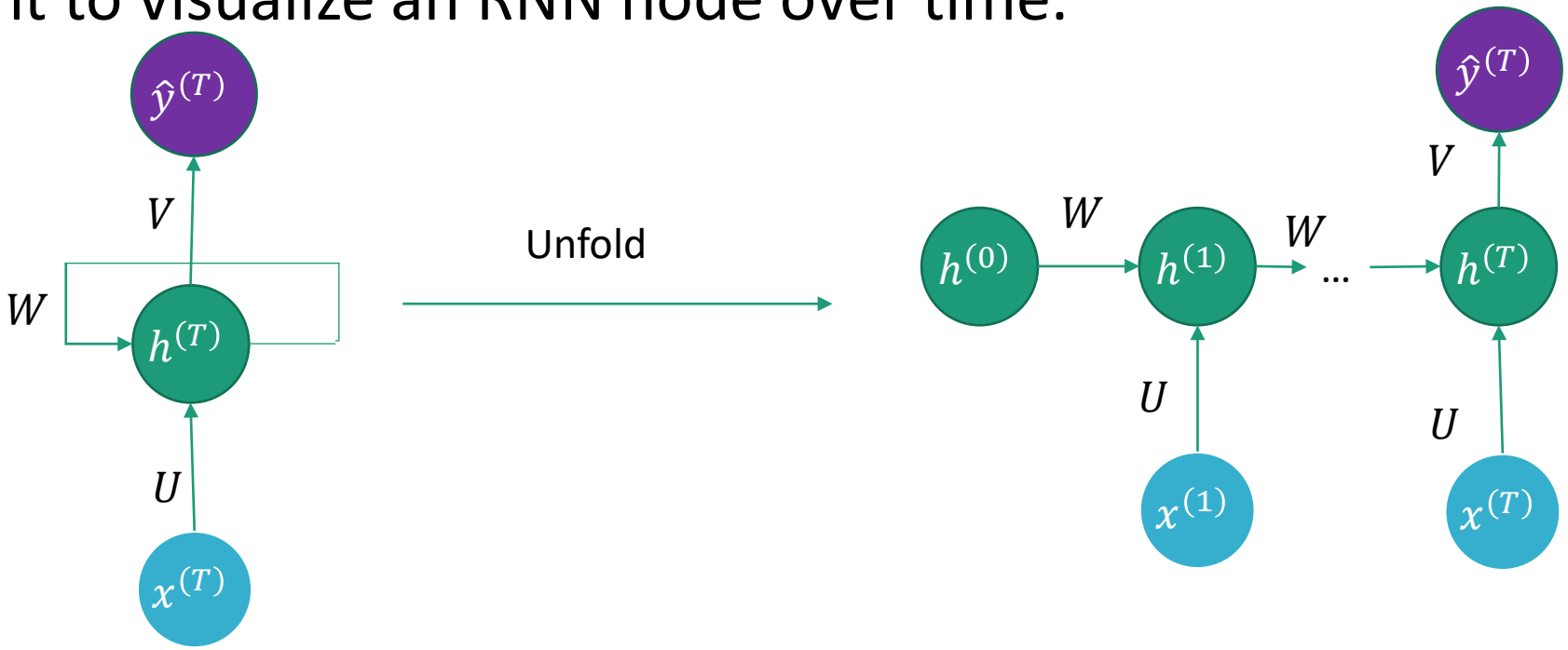
Recurrent Nodes

- A simple RNN has three sets of weights (we'll use different variables, to line up with the provided images).
- Like a traditional shallow ANN we have
  - Weights going from input layer to hidden layer, V
  - Weights going from hidden layer to output layer, U
- With an RNN we also have:
  - Weights going from hidden layer back to hidden layer, $\boldsymbol{W}$



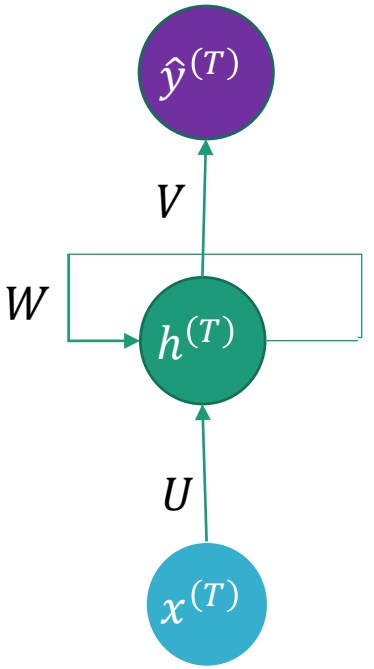
Unrolling

- Although there is no real “sequence” of units in RNNs, for illustrative and comprehension purposes, often you’ll see an RNN “unfolded”.
- This is meant to visualize an RNN node over time.



Notation

- Let
  - $\hat{y}^{(t)}$ be the output at time t
  - $neto^{(t)}$ be the input to the output layer at time t
  - $h^{(t)}$ be the output of the hidden layer at time t
  - $neth^{(t)}$ be the input to the hidden layer at time t
  - $x^{(t)}$ be the input at time t
  - V be the weights going from the hidden layer to the output layer
  - U be the weights going from the input layer to the hidden layer
  - W be the weights going from the hidden layer back to itself.
  - g_o, g_h be the activation functions for the output and hidden layers, respectively.



Forward Propagation

- As per usual, forward propagation is *relatively* easy.
- The only difference is that the **output of the hidden layer involves summing the previous weighed output of the hidden layer with the new weighed input**
- Then then input to the hidden layer at time t is:

$$neth^{(t)} = h^{(t-1)}W + x^{(t)}U$$

Feedback contribution

New contribution

- And the input to the output layer at time t is:

$$neto^{(t)} = h^{(t)}V$$

Counting Example

- As we work through our RNN, let's use a *one-to-many* example where we input 1 and have the RNN output 2 then 3
- So our input sequence is 1
- And our desired output sequence is 2,3
- For training purposes we'll do one-hot encoding:
  - $1 \Rightarrow [1, 0, 0]$
  - $2 \Rightarrow [0, 1, 0]$
  - $3 \Rightarrow [0, 0, 1]$
- Therefore our training set is:

$$x^{(1)} = [1, 0, 0]$$
$$y = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example

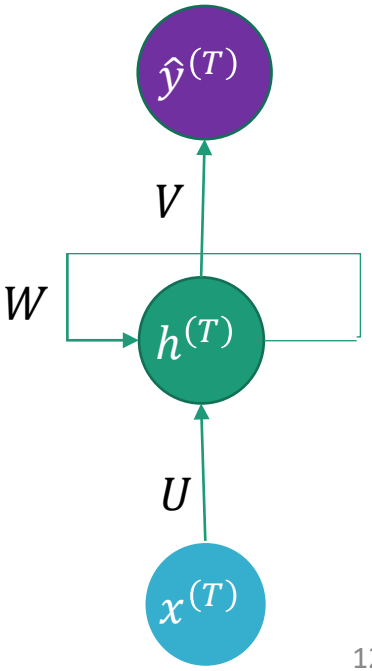
- Let's imagine that our weight matrices are:

$$U = \begin{bmatrix} -0.1565 & 0.9190 \\ 0.8315 & 0.3115 \\ 0.5844 & -0.9286 \end{bmatrix}, V = \begin{bmatrix} 0.6983 & 0.3575 & 0.4863 \\ 0.8680 & 0.5155 & -0.2155 \end{bmatrix}, W = \begin{bmatrix} 0.3110 & 0.4121 \\ -0.6576 & -0.9363 \end{bmatrix}$$

- Forward propagation results in:
  - Time $t = 0$
    - We need to initialize the "previous" h somehow, so let's just set it to zeros.
    - If we choose a hidden layer size of two, then we initialize $h^{(0)}$ to

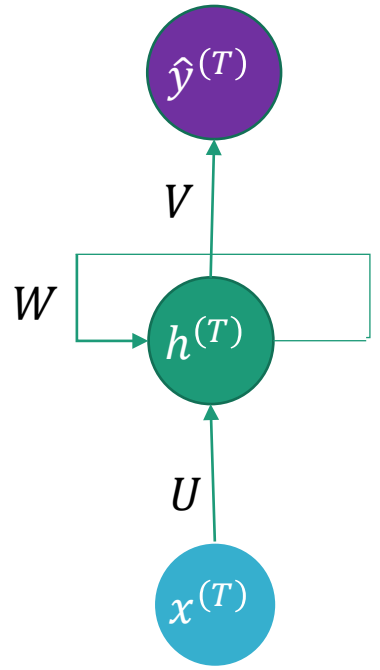
$$h^{(0)} = [0 \quad 0]$$

  - Time $t = 1$
    - $x^{(1)} = [1 \quad 0 \quad 0]$
    - $h^{(1)} = [-0.1565 \quad 0.9190]$
    - $\hat{y}^{(1)} = [0.6884 \quad 0.4178 \quad -0.2742]$



Example

- Forward propagation results in:
  - Time $t = 2$
    - Now we don't have any input x, so let's just set it to $x^{(2)} = [0\ 0\ 0]$
    - $x^{(2)} = [0\ 0\ 0]$
    - $h^{(1)} = [-0.1565 \quad 0.9190]$
    - $h^{(2)} = [-0.6530 \quad -0.9250]$
    - $\hat{y}^{(2)} = [-1.2588 \quad -0.7102 \quad -0.1182]$
- Use the argmax of $\hat{y}^{(t)}$ we get the sequence (1, 3)
- Our desired output was (2,3)
- So let's backpropagate to learn the weights better!



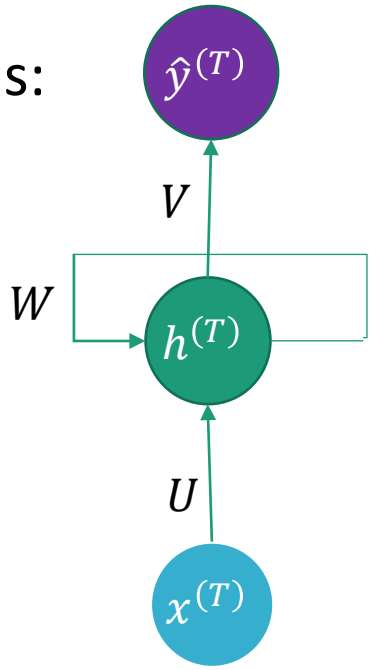
Example: Backpropagate

- For this example, we'll use a relatively simple configuration of a **squared error objective function** and **linear activation functions** at both the hidden and output layers.
- Since we care about the output at every time step, our objective function is:

$$J = \sum_{t=1}^{T} J^{(t)} = \sum_{t=1}^{T} \left(y^{(t)} - \hat{y}^{(t)}\right)^T \left(y^{(t)} - \hat{y}^{(t)}\right)$$

- Now it's time to backpropagate!
- Of course to do so we'll need to determine the gradient rules:

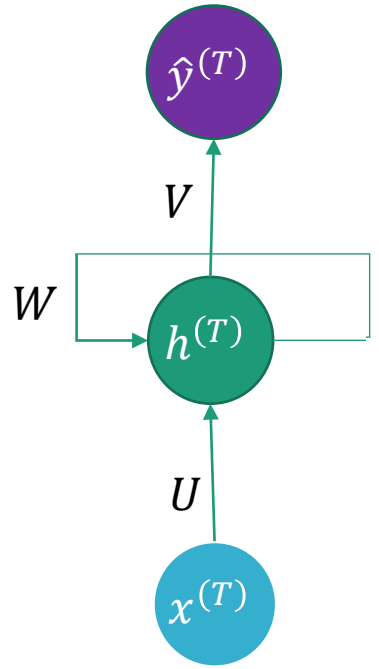
$$\frac{\partial J}{\partial V}, \frac{\partial J}{\partial W}, \frac{\partial J}{\partial U}$$



Single Input, Multiple Output: $\partial J / \partial V$

- As usual, our first gradient rule (the one nearest the output) is the easiest.

$$\frac{\partial J}{\partial V} = \sum_{t=1}^{T} \frac{\partial J^{(t)}}{\partial V} = \sum_{t=1}^{T} \frac{\partial J^{(t)}}{\partial \hat{y}^{(t)}} \cdot \frac{\partial \hat{y}^{(t)}}{\partial neto^{(t)}} \cdot \frac{\partial neto^{(t)}}{\partial V}$$



- And for the purposes of back propagation

$$\delta_o^{(t)} = \frac{\partial J^{(t)}}{\partial \hat{y}^{(t)}} \cdot \frac{\partial \hat{y}^{(t)}}{\partial neto^{(t)}}$$

$$\frac{\partial J}{\partial V} = \sum_{t=1}^{T} (h^{(t)})^T \delta_o^{(t)}$$

Example: $\partial J / \partial V$

- For our configuration:

$$\delta_o^{(t)} = \frac{\partial J^{(t)}}{\partial \hat{y}^{(t)}} \cdot \frac{\partial \hat{y}^{(t)}}{\partial net o^{(t)}} = 2\left(\hat{y}^{(t)} - y^{(t)}\right) \cdot 1$$

$$\frac{\partial J}{\partial V} = \sum_{t=1}^{T} {h^{(t)}}^T \delta_o^{(t)}$$

Example: $\partial J / \partial V$

- Plugging in numbers
  - $\delta_o^{(1)} = 2\left(\hat{y}^{(1)} - y^{(1)}\right) = [1.3768 \quad -1.1644 \quad -0.5483]$
  - $\frac{\partial J^{(1)}}{\partial V} = \left(h^{(1)}\right)^T \cdot \delta_o^{(1)} = \begin{bmatrix} -0.2154 & 0.1822 & 0.0858 \\ 1.2653 & -1.0701 & -0.5039 \end{bmatrix}$
  - $\delta_o^{(2)} = 2\left(\hat{y}^{(2)} - y^{(2)}\right) = [-2.5176 \quad -1.4205 \quad -0.2363]$
  - $\frac{\partial J^{(2)}}{\partial V} = \left(h^{(2)}\right)^T \cdot \delta_o^{(2)} = \begin{bmatrix} 1.6440 & 0.9276 & 1.4603 \\ 2.3287 & 1.3139 & 2.0685 \end{bmatrix}$
- So

$$\frac{\partial J}{\partial V} = \begin{bmatrix} 1.4286 & 1.1098 & 1.5461 \\ 3.5940 & 0.2438 & 1.5646 \end{bmatrix}$$

Single Input, Multiple Output: $\partial J/\partial W$

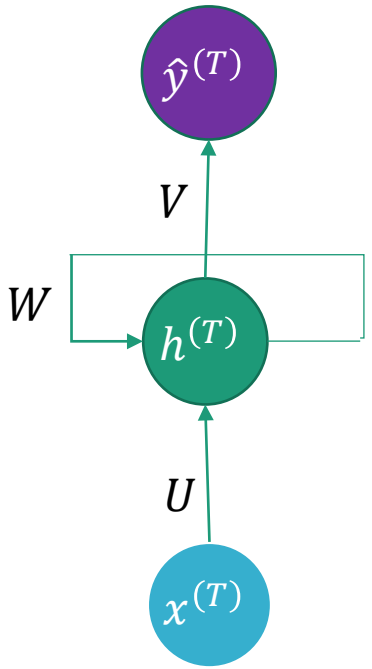
- Next, we have $\frac{\partial J}{\partial W}$

$$\frac{\partial J}{\partial W} = \sum_{t=1}^{T} \left(\frac{\partial \boldsymbol{J^{(t)}}}{\partial \boldsymbol{\hat{y}^{(t)}}} \cdot \frac{\partial \boldsymbol{\hat{y}^{(t)}}}{\partial \boldsymbol{neto^{(t)}}} \cdot \frac{\partial \boldsymbol{neto^{(t)}}}{\partial \boldsymbol{h^{(t)}}} \cdot \frac{\partial \boldsymbol{h^{(t)}}}{\partial \boldsymbol{neth^{(t)}}} \cdot \frac{\partial \boldsymbol{neth^{(t)}}}{\partial \boldsymbol{W}} \right)$$

$$\frac{\partial J}{\partial W} = \sum_{t=1}^{T} \left(\boldsymbol{\delta_o^{(t)}} \cdot \frac{\partial \boldsymbol{neto^{(t)}}}{\partial \boldsymbol{h^{(t)}}} \cdot \frac{\partial \boldsymbol{h^{(t)}}}{\partial \boldsymbol{neth^{(t)}}} \cdot \frac{\partial \boldsymbol{neth^{(t)}}}{\partial \boldsymbol{W}} \right)$$

- Of course $\frac{\partial neto^{(t)}}{\partial h^{(t)}} = V^T$

$$\frac{\partial J}{\partial W} = \sum_{t=1}^{T} \boldsymbol{\delta_o^{(t)}} \cdot \boldsymbol{V^T} \cdot \frac{\partial \boldsymbol{h^{(t)}}}{\partial \boldsymbol{neth^{(t)}}} \cdot \frac{\partial \boldsymbol{neth^{(t)}}}{\partial \boldsymbol{W}}$$



$$h^{(t)} = g\left(neth^{(t)}\right)$$

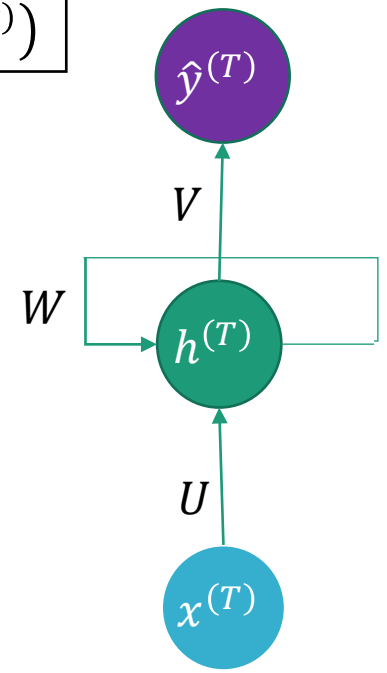
$$neth^{(t)} = h^{(t-1)}W + x^{(t)}U$$

Single Input, Multiple Output: $\partial J/\partial W$

$$neth^{(t)} = h^{(t-1)}W + x^{(t)}U$$

$$h^{(t)} = g\left(neth^{(t)}\right)$$

$$\frac{\partial J}{\partial W} = \sum_{t=1}^{T} \boldsymbol{\delta_o^{(t)}} \cdot \boldsymbol{V^T} \cdot \frac{\boldsymbol{\partial h^{(t)}}}{\boldsymbol{\partial neth^{(t)}}} \cdot \frac{\boldsymbol{\partial neth^{(t)}}}{\boldsymbol{\partial W}}$$



- Upon first inspection $\frac{\partial neth^{(t)}}{\partial W} = \left(h^{(t-1)}\right)^T$.
- However, changing W at time t will affect all previous values of $neth$ for $k = 1, \ldots, t$, which in turn changes $J^{(t)}$

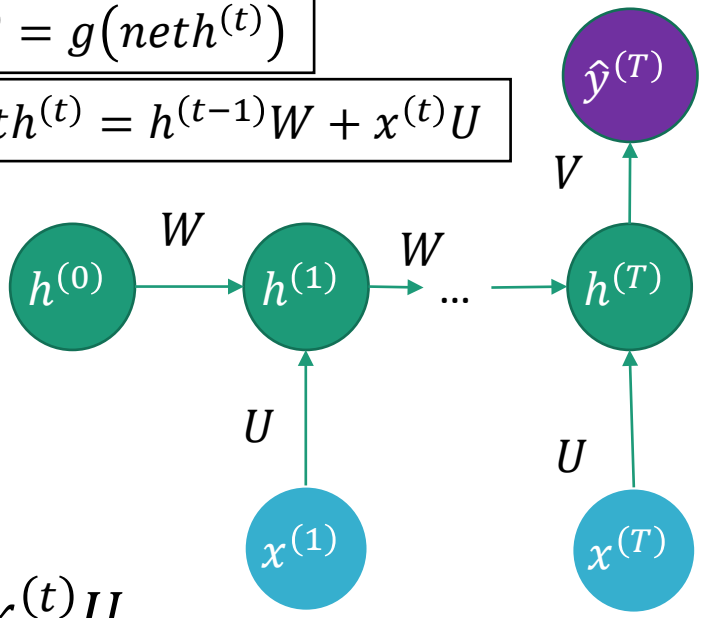
NOTE: The derivations for $\frac{\partial J}{\partial W}, \frac{\partial J}{\partial U}$ will likely be a bit different than in the resources. I think these are the most intuitive, although not necessarily the most efficient.

Single Input, Multiple Output: $\partial J/\partial W$

$$\frac{\partial J^{(t)}}{\partial W} = \sum_{t=1}^{T} \boldsymbol{\delta_o^{(t)} \cdot V^T} \cdot \frac{\boldsymbol{\partial h^{(t)}}}{\boldsymbol{\partial neth^{(t)}}} \cdot \frac{\boldsymbol{\partial neth^{(t)}}}{\boldsymbol{\partial W}}$$

$$h^{(t)} = g\left(neth^{(t)}\right)$$

$$neth^{(t)} = h^{(t-1)}W + x^{(t)}U$$



- We can write $neth^{(t)}$ recursively as

$$\begin{aligned} neth^{(t)} &= g_h\left(neth^{(t-1)}\right)W + x^{(t)}U \\ &= g_h\left(h^{(t-2)}W + x^{(t-1)}U\right)W + x^{(t)}U \\ &= g_h\left(g_h(h^{(t-3)}W + x^{(t-2)}U)\, W + x^{(t-1)}U\right)W + x^{(t)}U \\ &= \ \ldots \end{aligned}$$

- Notice how we must also propagate backwards *in time*!
- Therefore, learning $\frac{\partial J}{\partial W}$ and $\frac{\partial J}{\partial U}$ is often called ***back propagation through time (bptt).***

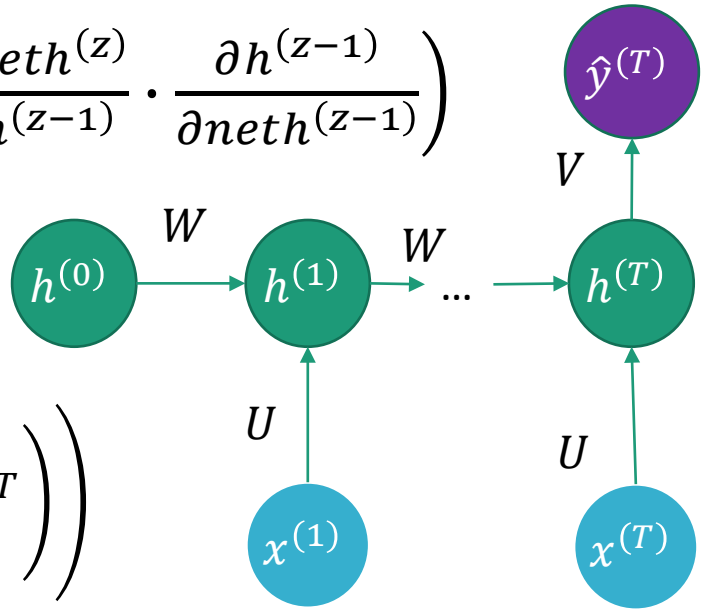
Single Input, Multiple Output: $\partial J/\partial W$

$$neth^{(t)} = g_h\left(g_h\left(h^{(t-3)}W + x^{(t-2)}U\right)W + x^{(t-1)}U\right)W + x^{(t)}U = \cdots$$

- Via the chain rule:

$$\frac{\partial neth^{(t)}}{\partial W} = \frac{\partial neth^{(t)}}{\partial W}\prod_{z=2}^{t}\left(\frac{\partial neth^{(z)}}{\partial neth^{(z-1)}}\right) = \frac{\partial neth^{(t)}}{\partial W}\prod_{z=2}^{t}\left(\frac{\partial neth^{(z)}}{\partial h^{(z-1)}}\cdot\frac{\partial h^{(z-1)}}{\partial neth^{(z-1)}}\right)$$

$$= \left(h^{(t-1)}\right)^T \cdot \prod_{z=2}^{t}\left(\frac{\partial h^{(z-1)}}{\partial neth^{(z-1)}}\cdot W^T\right)$$



- So finally:

$$\frac{\partial J}{\partial W} = \sum_{t=1}^{T}\left(h^{(t-1)^T}\cdot\delta_o^{(t)}\cdot V^T\cdot\prod_{z=2}^{t}\left(\frac{\partial h^{(z-1)}}{\partial neth^{(z-1)}}W^T\right)\right)$$

$$h^{(t)} = g\left(neth^{(t)}\right)$$

$$neth^{(t)} = h^{(t-1)}W + x^{(t)}U$$

Example: $\partial J / \partial W$

$$\frac{\partial J}{\partial W} = \sum_{t=1}^{T} \left(h^{(t-1)^T} \cdot \delta_o^{(t)} \cdot V^T \cdot \prod_{z=2}^{t} \left(\frac{\partial h^{(z-1)}}{\partial neth^{(z-1)}} W^T \right) \right)$$

- For our example…
- $\frac{\partial J^{(1)}}{\partial W} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} [1.3768 \quad -1.1644 \quad -0.5483] \begin{bmatrix} 0.6983 & 0.8680 \\ 0.3575 & 0.5155 \\ 0.4863 & -0.2155 \end{bmatrix} (\quad) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
- $\frac{\partial J^{(2)}}{\partial W} = \begin{bmatrix} -0.1565 \\ 0.9190 \end{bmatrix} [-2.5176 \quad -1.4205 \quad -2.2363] \begin{bmatrix} 0.6983 & 0.8680 \\ 0.3575 & 0.5155 \\ 0.4863 & -0.2155 \end{bmatrix} \left(\begin{bmatrix} 0.3110 & -0.6576 \\ 0.4121 & -0.9363 \end{bmatrix} \right) = \begin{bmatrix} 0.3203 & -0.7019 \\ -1.8805 & 4.1222 \end{bmatrix}$
- So

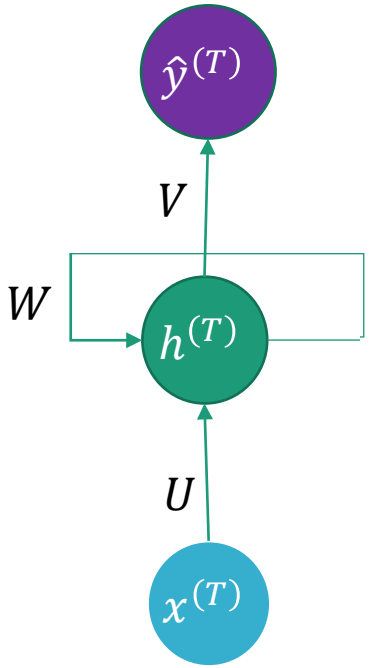
$$\frac{\partial J}{\partial W} = \begin{bmatrix} 0.3203 & -0.7019 \\ -1.8805 & 4.1222 \end{bmatrix}$$

Single Input, Multiple Output: $\partial J/\partial U$

- Next, we have $\frac{\partial J}{\partial U}$
- Changes in U at time t also affects h at previous times and therefore $J^{(t)}$
- Similarly:

$$\frac{\partial J}{\partial U} = \sum_{t=1}^{T} \left(\frac{\partial J^{(t)}}{\partial \hat{y}^{(t)}} \cdot \frac{\partial \hat{y}^{(t)}}{\partial neto^{(t)}} \cdot \frac{\partial neto^{(t)}}{\partial h^{(t)}} \cdot \frac{\partial h^{(t)}}{\partial neth^{(t)}} \cdot \frac{\partial neth^{(t)}}{\partial U} \right)$$

$$= \sum_{t=1}^{T} \delta_o^{(t)} \cdot V^T \cdot \frac{\partial h^{(t)}}{\partial neth^{(t)}} \cdot \frac{\partial neth^{(t)}}{\partial U}$$



$$h^{(t)} = g\left(neth^{(t)}\right)$$

$$neth^{(t)} = h^{(t-1)}W + x^{(t)}U$$

Single Input, Multiple Output: $\partial J/\partial U$

$$neth^{(t)} = g_h\left(g_h\left(h^{(t-3)}W + x^{(t-2)}U\right)W + x^{(t-1)}U\right)W + x^{(t)}U = \cdots$$

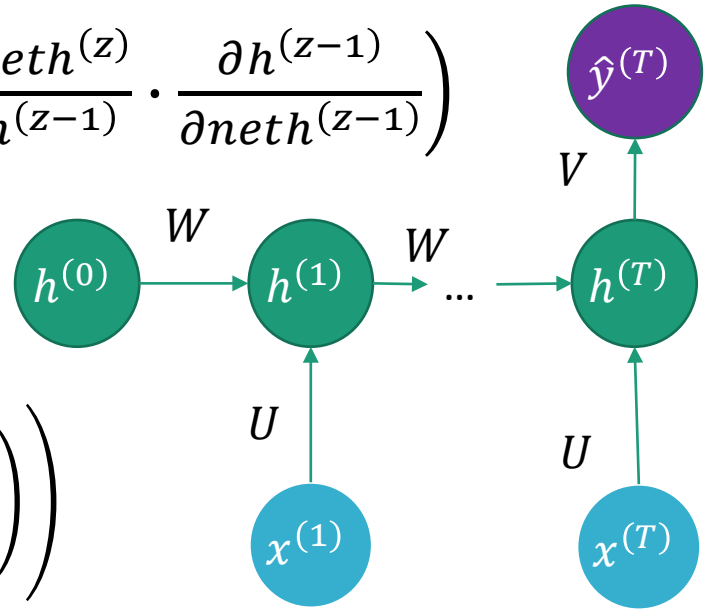
- Via the chain rule:

$$\frac{\partial neth^{(t)}}{\partial U} = \frac{\partial neth^{(t)}}{\partial U}\prod_{z=2}^{t}\left(\frac{\partial neth^{(z)}}{\partial neth^{(z-1)}}\right) = \frac{\partial neth^{(t)}}{\partial U}\prod_{z=2}^{t}\left(\frac{\partial neth^{(z)}}{\partial h^{(z-1)}}\cdot\frac{\partial h^{(z-1)}}{\partial neth^{(z-1)}}\right)$$

$$= \left(x^{(1)}\right)^T \cdot \prod_{z=2}^{t}\left(\frac{\partial h^{(z-1)}}{\partial neth^{(z-1)}}\cdot W^T\right)$$

- So finally:

$$\frac{\partial J}{\partial U} = \sum_{t=1}^{T}\left(x^{(t)^T}\cdot\delta_o^{(t)}\cdot V^T\cdot\prod_{z=2}^{t}\left(\frac{\partial h^{(z-1)}}{\partial neth^{(z-1)}}W^T\right)\right)$$



$$h^{(t)} = g\left(neth^{(t)}\right)$$

$$neth^{(t)} = h^{(t-1)}W + x^{(t)}U$$

Example: $\partial J / \partial U$

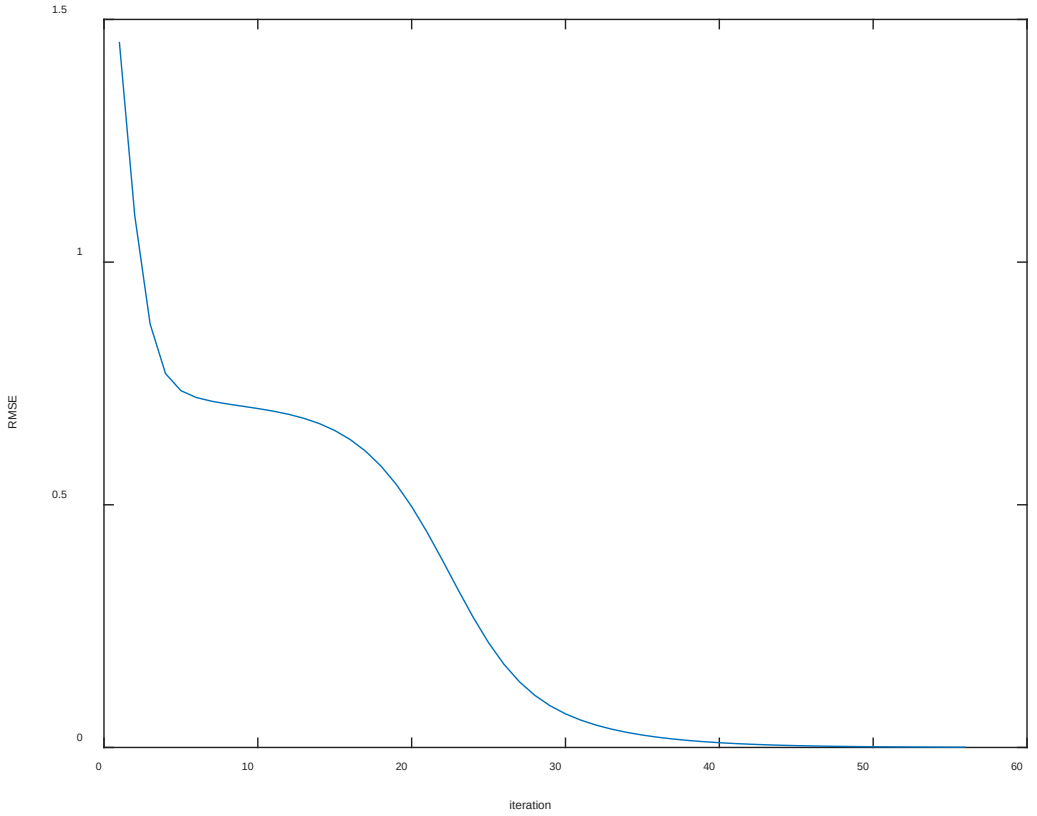
$$\frac{\partial J}{\partial U} = \sum_{t=1}^{T} \left(x^{(t)^T} \cdot \delta_o^{(t)} \cdot V^T \cdot \prod_{z=2}^{t} \left(\frac{\partial h^{(z-1)}}{\partial neth^{(z-1)}} W^T \right) \right)$$

- For our example…
- $\frac{\partial J^{(1)}}{\partial U} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} [1.3768 \quad -1.1644 \quad -0.5483] \begin{bmatrix} 0.6983 & 0.8680 \\ 0.3575 & 0.5155 \\ 0.4863 & -0.2155 \end{bmatrix} (\quad) = \begin{bmatrix} 0.2785 & 0.7130 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$
- $\frac{\partial J^{(2)}}{\partial U} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} [-2.5176 \quad -1.4205 \quad -2.2363] \begin{bmatrix} 0.6983 & 0.8680 \\ 0.3575 & 0.5155 \\ 0.4863 & -0.2155 \end{bmatrix} \left(\begin{bmatrix} 0.3110 & -0.6576 \\ 0.4121 & -0.9363 \end{bmatrix} \right) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
- So

$$\frac{\partial J}{\partial U} = \begin{bmatrix} 0.2785 & 0.7130 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Example

- Terminated if $|RMSE(end) - RMSE(end-1)| < 10^{-4}$
  - Using learning rate $\eta = 0.1$
  - This occurred after 56 iterations.
- Final Model:
  - $V = \begin{bmatrix} 0.0007 & -0.9555 & -0.4747 \\ -0.0001 & 0.3239 & -0.4533 \end{bmatrix}$
  - $W = \begin{bmatrix} 0.2870 & 0.4962 \\ -0.4082 & -1.5386 \end{bmatrix}$
  - $U = \begin{bmatrix} -0.7724 & 0.8089 \\ 0.8315 & 0.3115 \\ 0.5844 & -0.9286 \end{bmatrix}$
- Forward propagating using the learned model
  - $\hat{y}^{(1)} = [-0.0007 \quad 1.0001 \quad -0.0001]$
  - $\hat{y}^{(2)} = [-0.0003 \quad 0.0001 \quad 0.9999]$
- Choosing argmax, we get the sequence $(2, 3)$



Exploding/Vanishing Gradients

- If you noticed, by recurring through time we are often multiplying W by itself several times.

$$\prod_{z=2}^{t}\left(\frac{\partial h^{(z-1)}}{\partial neth^{(z-1)}}W^{T}\right)$$

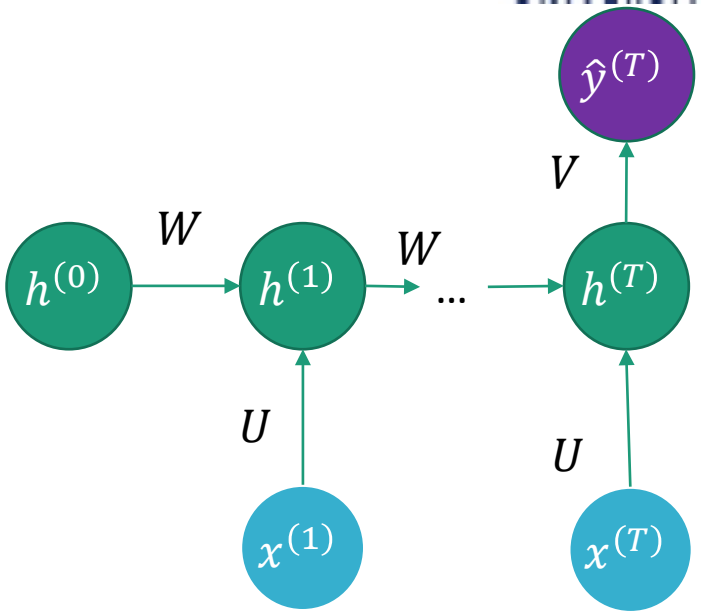
- In fact, t temporal units forward, we'll have to multiply W by itself t times!
- As a result, if $|W|$ is greater than one, we get exponential "blow up".
- Likewise if $|W|$ is very small, then we get exponential decay (vanish).
- There are various creative work-around for this, but we'll leave that up to you to research.
- One idea/fix is to limit how far back you go (don't go all the way back to $t = 1$)

Addition RNN Scenarios

- We just went through the one-to-many scenario.
- Recall that some applications call for a many-to-one situation.
- Here we only evaluate at the end:

$$J = J^{(T)}$$

- However, for all times $x^{(t)}$ we have valid inputs
  - Recall that for the one-to-many situation when $t > 1$ x was just a vector of zeros (no input).
- So we'd need to define/tweak our gradients accordingly.
- Likewise, a many-to-many application would almost be a hybrid of these two.

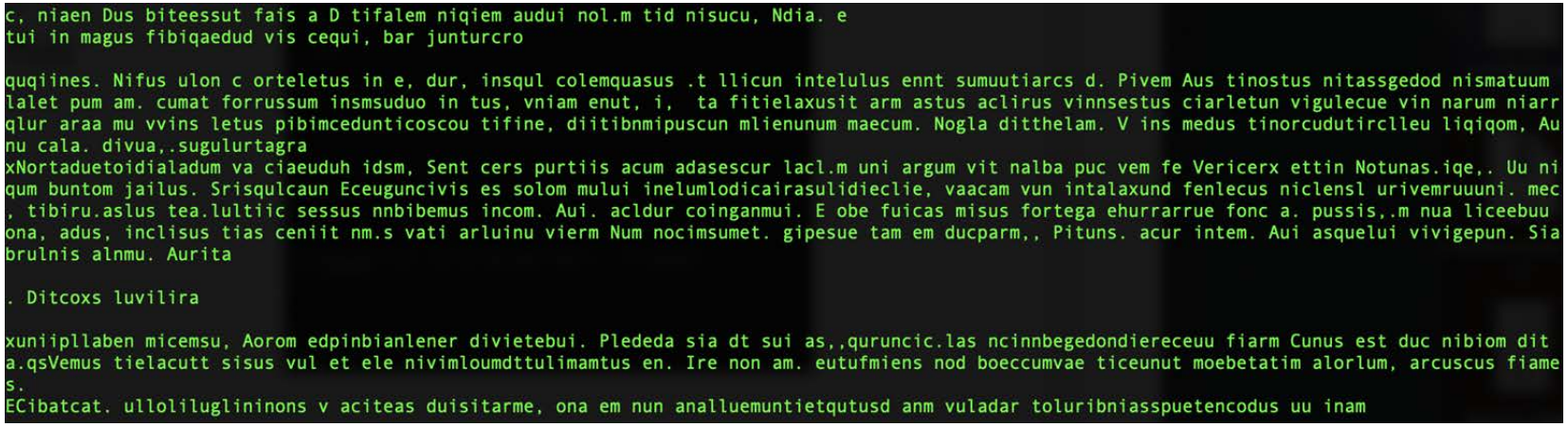


Multiple Sequences

- What if we wanted to train multiple sequences?
- Each would be a sequence to pass through the training process.
- Maybe even batch the gradients across sequences.
- The next example shows the result of passing many sequences of words (text) into an RNN to train it to generate text using a “seed word”.
  - This would again be an example of one-to-many.

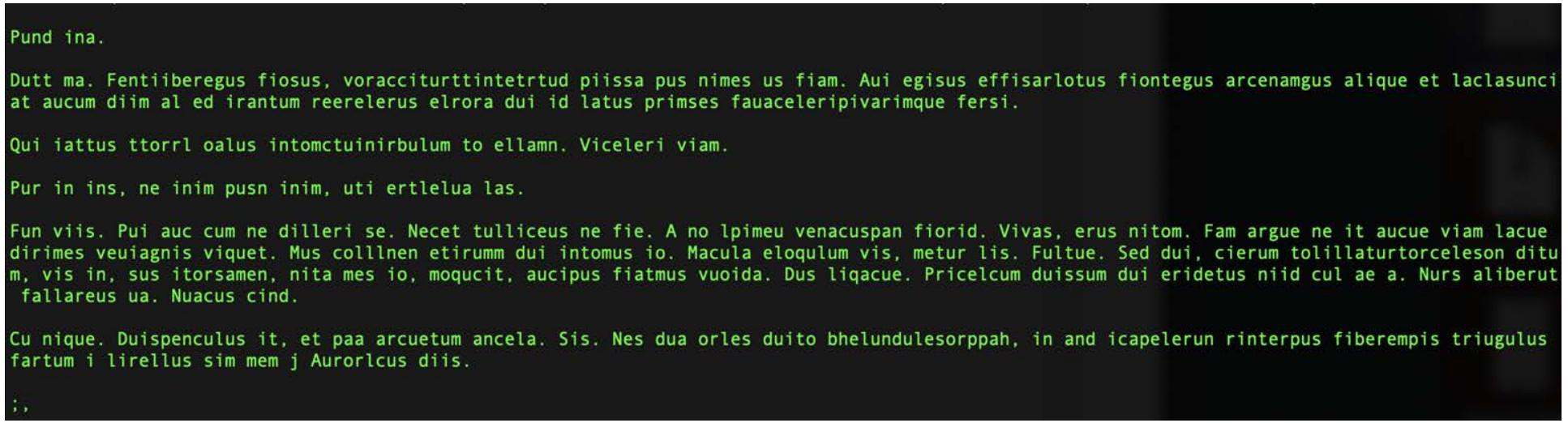
10 Iterations

- Has some newlines
- Odd usage of commas
- No real sentence structure



100 Iterations

- Some sentence structure
- Better punctuation



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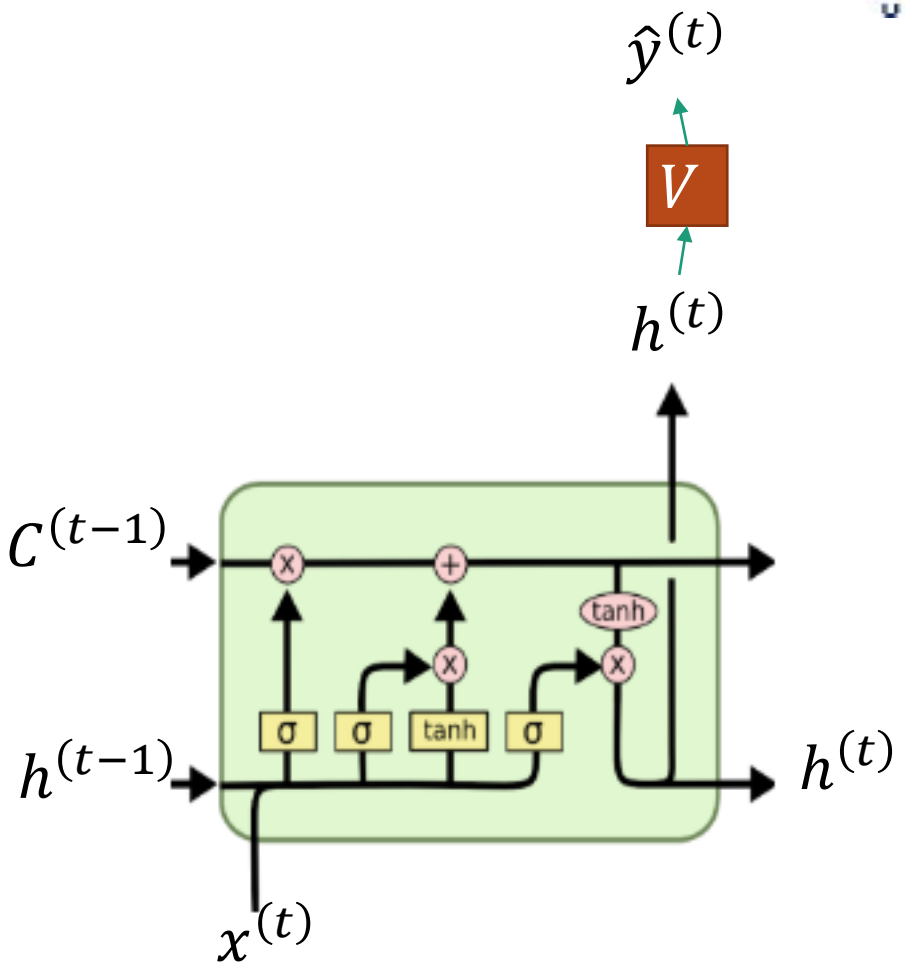
Recurrent Neural Networks Shortcomings

- Issues with training
  - Model can diverge for odd reasons like random weights starting at wrong values
- Vanishing and Exploding Gradients
  - Multiplying by the same weights repeatedly causes value to grow or shrink more than needed
- Close to what we want
  - Makes good predictions when trained correctly
  - Models taking steps through time well

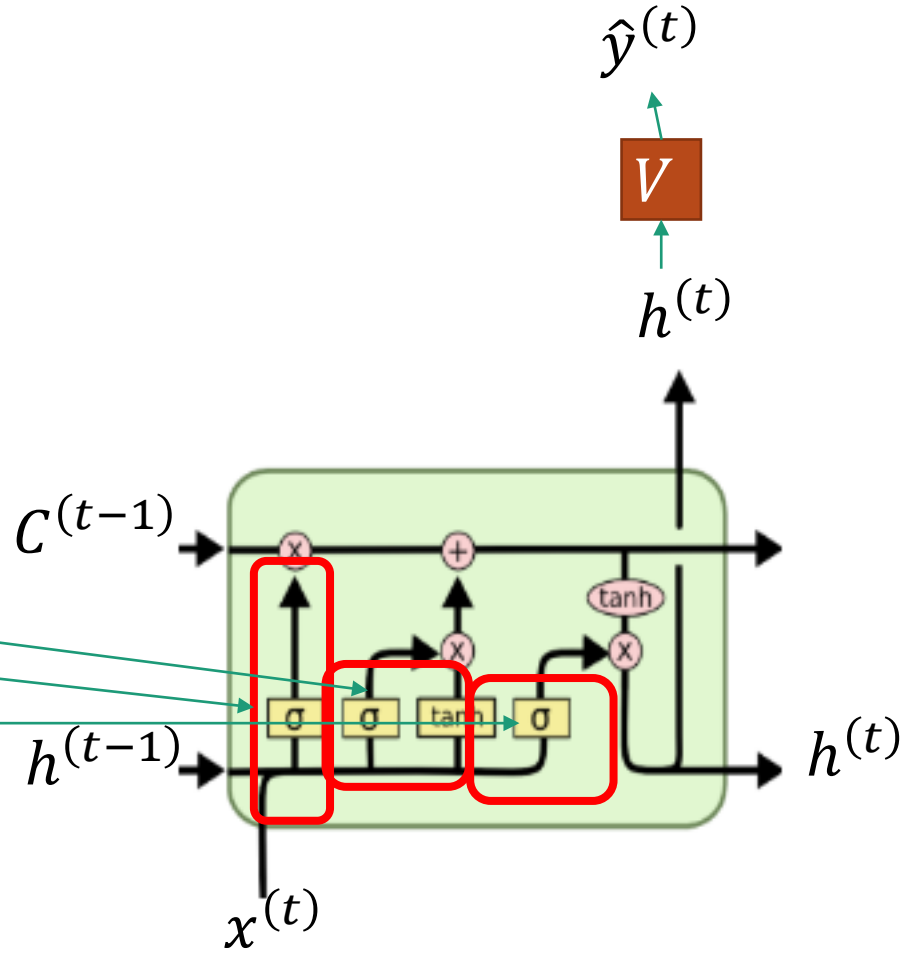
LSTM Node Overview

- LSTMs have a different internal structure.
- Whereas the RNN had three sets of weights to learn (U, V, W), LSTMs have five!
- Below is a LSTM node in green
- Note: All the operators shown are element-wise operators.



LSTM Node Overview

- If you recall, the sigmoid function returns a value between zero and one.
- Therefore, the sigmoid functions that are followed by an element-wise multiplication, are often referred to as *gates,* since they control how much "content" gets through.
- These are referred to as the *forget* gate, the *input gate*, and the *output gate*.

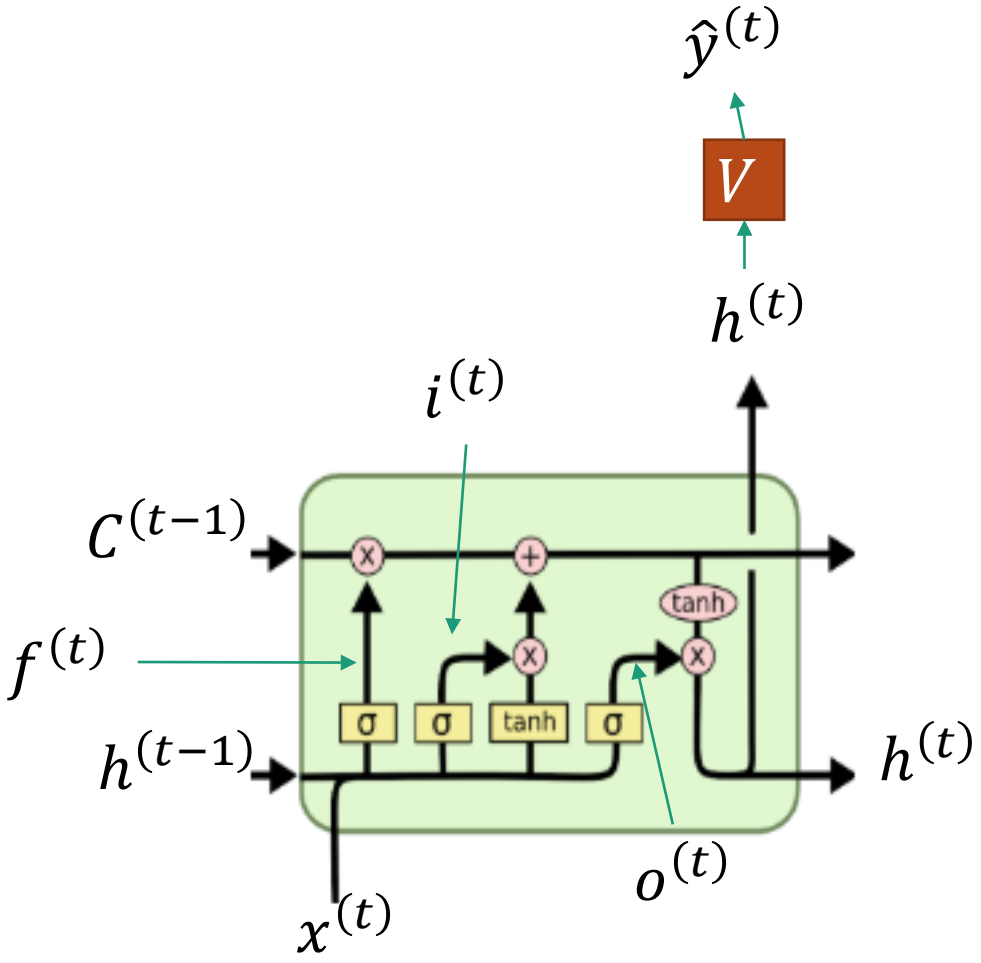


LSTM Gates

- Like regular RNNs, LSTMs combine the current input and the previous hidden info to form the new input:

$$\left[x^{(t)}, h^{(t-1)}\right]$$

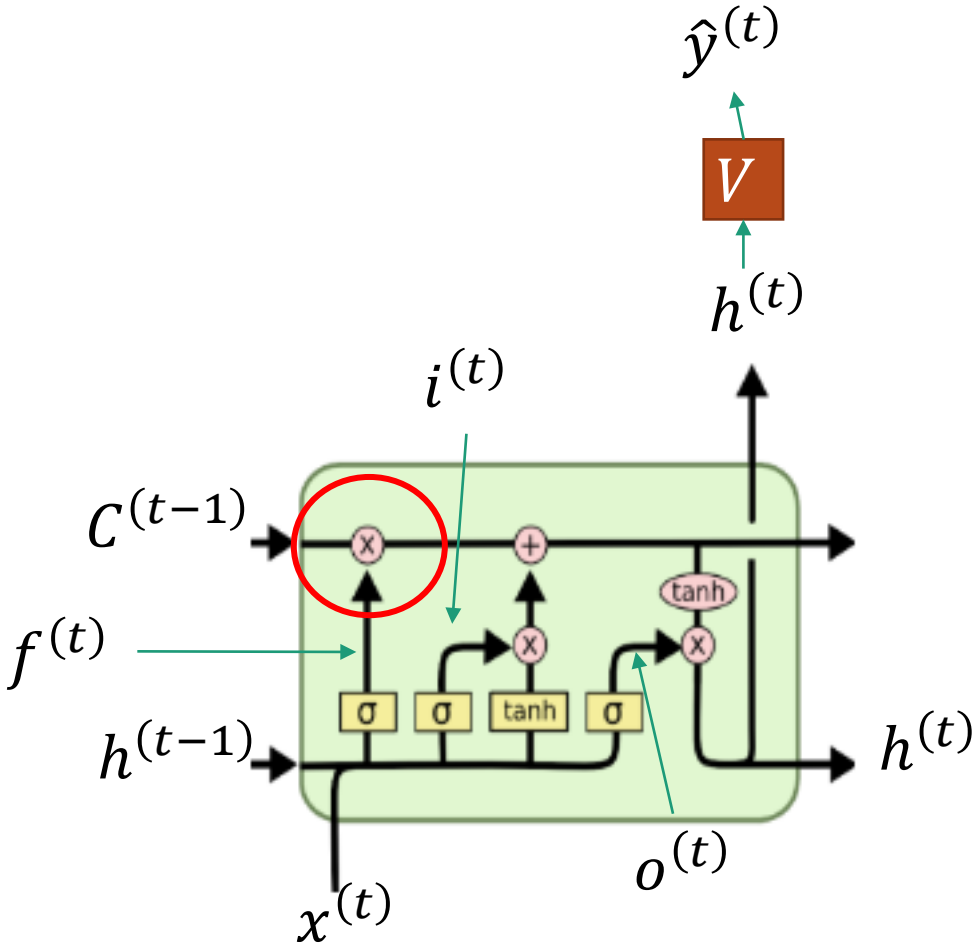
- The outputs of the gates can then be written as:
  - Forget Gate: $f^{(t)} = \sigma\left(\left[x^{(t)}, h^{(t-1)}\right]W_f\right)$
  - Input Gate: $i^{(t)} = \sigma\left(\left[x^{(t)}, h^{(t-1)}\right]W_i\right)$
  - Output Gate: $o^{(t)} = \sigma\left(\left[x^{(t)}, h^{(t-1)}\right]W_o\right)$



LSTM Cell State

- In addition to combining the new input and the previous hidden output, LSTMs also carry along additional old info.
- LSTMs call this the *cell* state C and it "*remembers*" some amount of info from the past
  - How much, is based on the gates
  - The amount remembered is

$$f^{(t)} \circ C^{(t-1)}$$



LSTM Element-Wise Operators

- To update the Cell State at time t we take the "some amount of old" and add to it "some amount of new".
- The new stuff is computed as:

$$\tilde{C}^{(t)} = \tanh\left(\left[x^{(t)}, h^{(t-1)}\right]W_C\right)$$

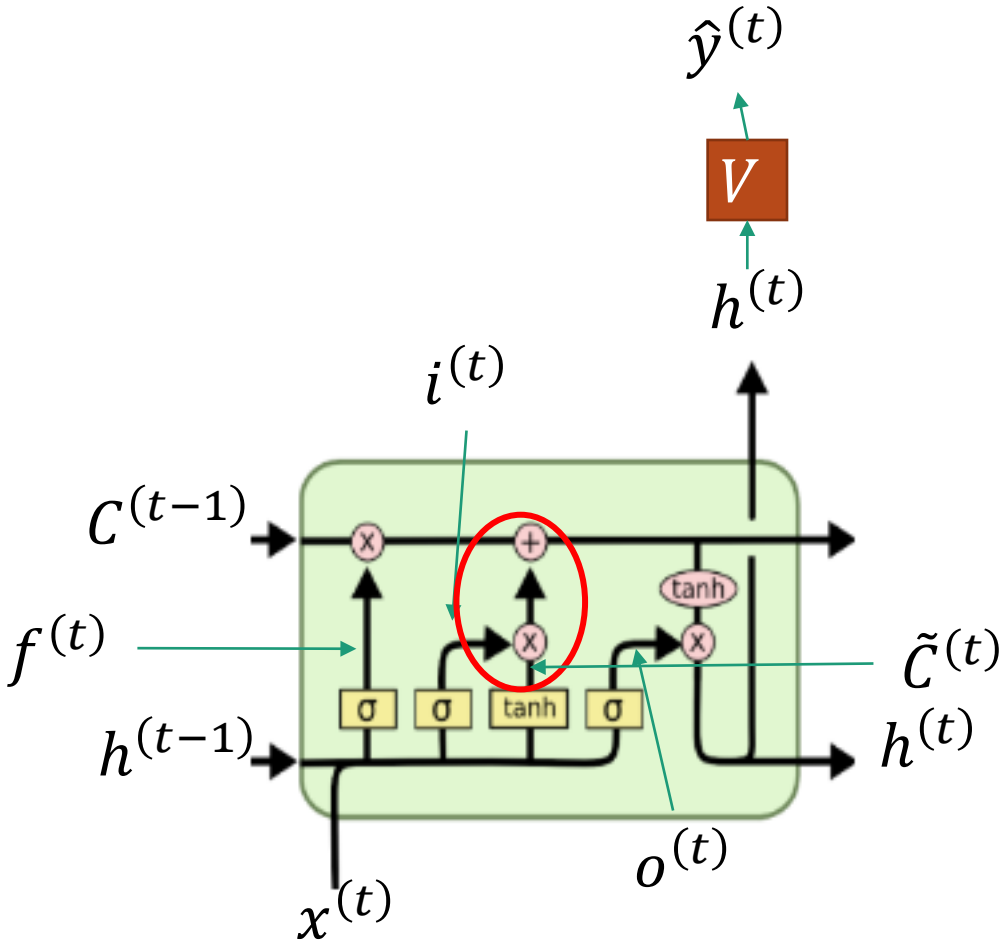
- This is then gated and added to the old stuff:

$$C^{(t)} = \left(f^{(t)} \circ C^{(t-1)}\right) + \left(i^{(t)} \circ \tilde{C}^{(t)}\right)$$

Gates

Old stuff

New stuff



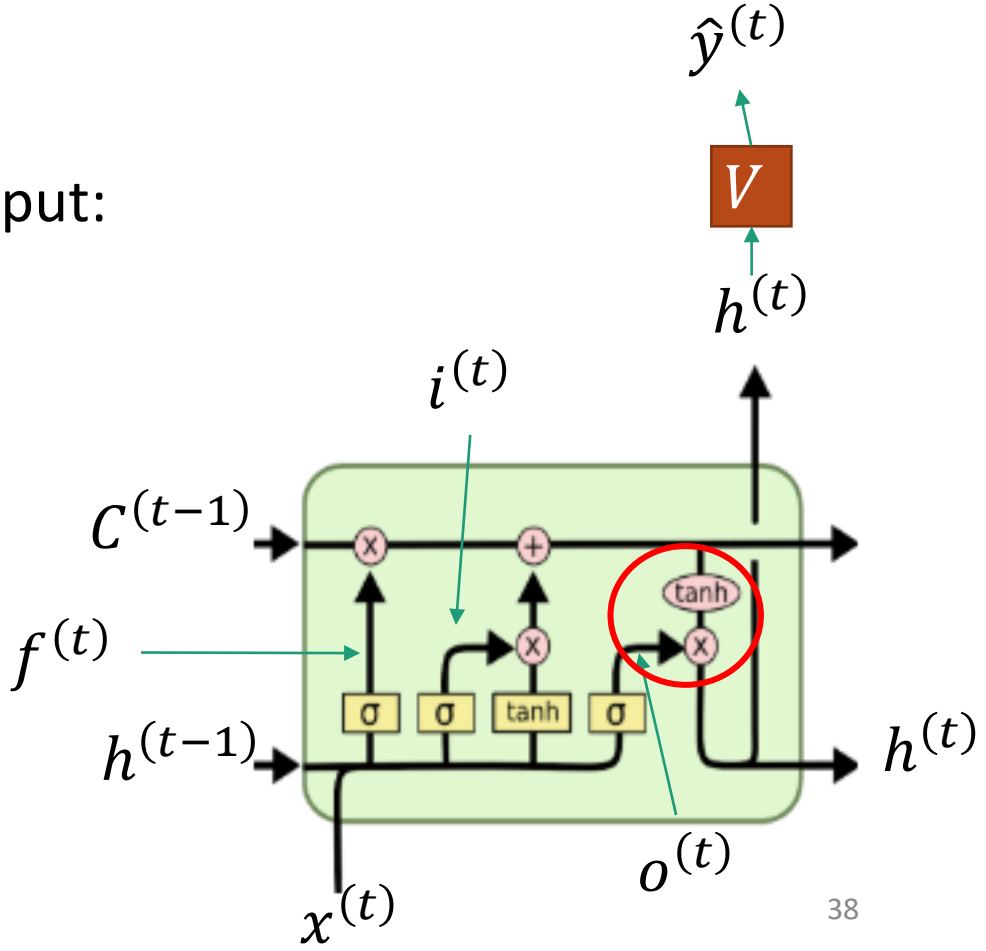
LSTM Element-Wise Operators

- Next our new/current hidden output is computed by gating the updated Cell State:

$$h^{(t)} = o^{(t)} \circ tanh(C^{(t)})$$

- And this can be used to compute the current output:

$$\hat{y}^{(t)} = g_y(h^{(t)}V)$$

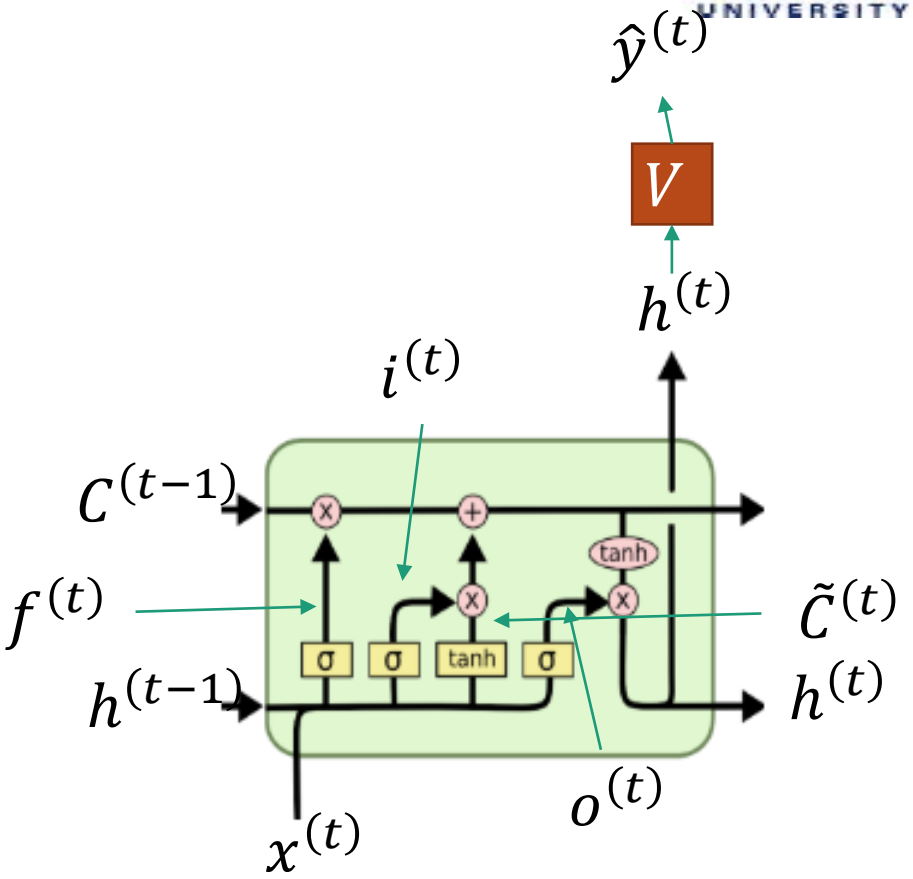


LSTM Gradients

- Now we get to learn:

$$\frac{\partial J}{\partial V}, \frac{\partial J}{\partial W_f}, \frac{\partial J}{\partial W_i}, \frac{\partial J}{\partial W_c}, \frac{\partial J}{\partial W_o}$$

- In the interest of time, we'll leave the derivations of these gradients to your own reading and exploration.
  - The resources provided to you at the end of this and in Bblearn should be a good starting point!



LSTM Benefits

- Why do LSTMs almost always outperform RNNs?
- First off, most operations are element-wise instead of true matrix multiplications.
  - More efficient.
- Secondly, the σ and $tanh$ functions force data to be in the $[0,1]$ or $[-1,+1]$ range, which helps avoid vanishing and/or exploding gradients.

Sources

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