# Equation Proof

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## 1 Problem Statement

For every time step of your simulation, the curve parameter u can be updated using the following equation:

$$u_{new} = u_{current} + (\Delta t)\frac{\sqrt{2g(h_{max} - h)}}{||\frac{dp}{du}||}$$

where $\Delta$ is the time step,
$g$ is the gravity constant, $h_{max}$ is the maximum height of the track,
h is the current height of the roller coaster,
$p$ is a function of u that computes the position of the roller coaster at $u = u_{current}$

## 2 Proof

A free fall is a linear motion with constant acceleration. So the physical equation describing the relation between time and distance in a free fall is:

$$\Delta h = \frac{1}{2}gt^2 \quad (i)$$

And the equation describing the relation between time and velocity is:

$$v = gt \quad (ii)$$

According to the two equations above, we have the relation between velocity and the distance:

$$v = \sqrt{2g\Delta h} \quad (iii)$$

So in a free fall, the traveling distance at height $\Delta h$ by time $\Delta t$ is:

$$s = v\Delta t = \Delta t\sqrt{2g\Delta h} \quad (iv)$$

Go back to this problem.The tangent vector at u ($||\frac{dp}{du}||$) refers to the traveling distance while u is increased by 1. So when parameter u is increased by $\Delta u$, the traveling distance is:

$$s = \Delta u||\frac{dp}{du}|| \quad (v)$$

From equation (iv) and (v), we have:

$$\Delta t\sqrt{2g\Delta h} = \Delta u||\frac{dp}{du}||$$

$$\Delta t\sqrt{2g\Delta h} = (u_{new} - u_{current})||\frac{dp}{du}||$$

$$u_{new} = u_{current} + (\Delta t)\frac{\sqrt{2g(h_{max} - h)}}{||\frac{dp}{du}||}$$