Twenty Lectures on Algorithmic Game Theory

Lecture 1: Introduction and Examples

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Brief descriptions of the book

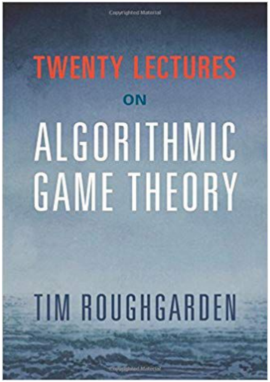

Figure: The cover of the book.

Figure: The author of the book.

Tim Roughgarden is a professor at Stanford University. He won STOC best student paper in 2002, Gödel Prize in 2012. His supervisor is Éva Tardos, who is wife of David Shmoys and ACM/INFORMS/AMS Fellow. His google citation is $16,000^{+}$.

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4. I plan to finish the book within $1.5 \sim 2$ months.

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A scandal: women's double badminton in Olympics 2012

Figure: Women's double badminton.

The basic rule of the match is as follows

- There are four teams in each four groups.
- The tournament has two phases.
  - In the first "round robin" phase, each team plays the other three teams in its group. The top two teams from the group advance to the second phase, and the bottom two teams from each group are eliminated.
  - In the second phase, the remaining eight teams play a standard "knockout" tournament: there are four quarterfinals (with the losers eliminated), then two semifinals (with the losers playing an extra match to decide the bronze model), and then the final (the winner gets the gold, the loser the silver).

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- What does a team want?
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- What does the Olympics committee want?
  They want every team to try their best to win every match.
- Would a team ever want to lose a match?
  Indeed, in the second "knockout" phase of the tournament, where losing leads to instant elimination, it is obvious that winning is better than losing.

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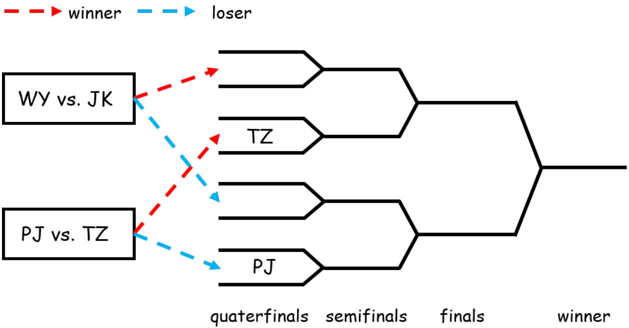

Figure: Women's badminton tournament in Olympics.

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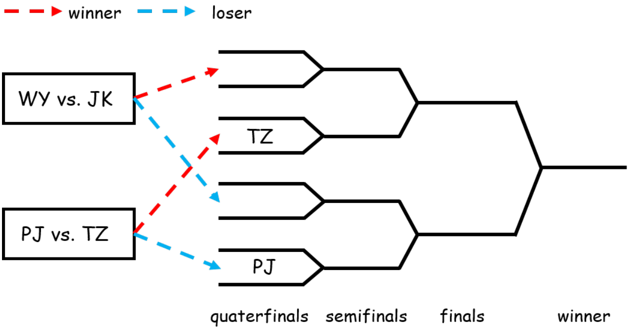

Figure: Women's badminton tournament in Olympics.

- TZ is the best but lose to PJ;

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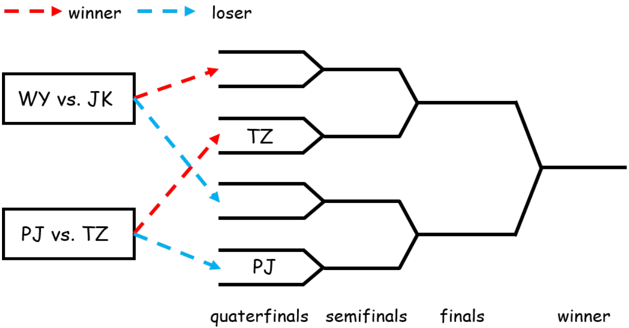

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- The winner of "WY vs. JK" will meat TZ eariler, loser will meet PJ;

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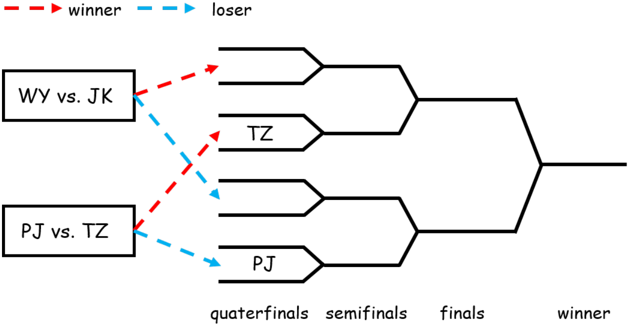

Figure: Women's badminton tournament in Olympics.

- TZ is the best but lose to PJ;
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- Both WY and TZ are Chinese;

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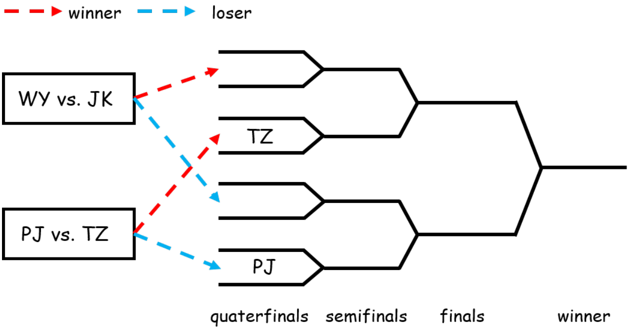

Figure: Women's badminton tournament in Olympics.

- TZ is the best but lose to PJ;
- The winner of "WY vs. JK" will meat TZ eariler, loser will meet PJ;
- Both WY and TZ are Chinese;
- Both WY and JK take efforts to lose the game.

A scandal: women's double badminton in Olympics 2012

- When participants are strategic, the rules matter.
- The designer of the platform should anticipate strategic behavior.

Figure: Killer applications of mechanism design.

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- The goal is to design rules so that strategic behavior by participants leads to a desirable outcome.

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- The designer of the platform should anticipate strategic behavior.
- There is a well-developed science of rule-making, the field of *mechanism design*.
- The goal is to design rules so that strategic behavior by participants leads to a desirable outcome.
- Killer applications of mechanism design include *Internet search auctions*, *wireless specturm auctions*, *the matching between medical residents to hospitals*, and *kidney exchanges*.

Figure: Killer applications of mechanism design.

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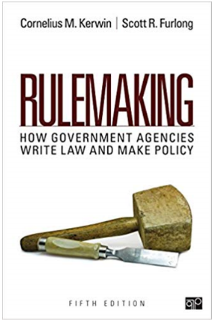
Motivation: Behavior Analysis

Somtimes you do not have the opportunity to design the rules of a game from scratch, and instead want to understand a game that occurs in realistic.



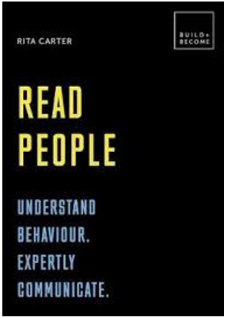
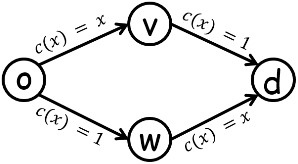


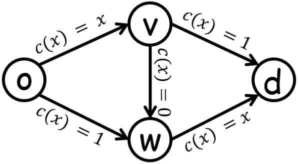
Figure: Rule design vs. Behavior analysis.

Illustration of traffic network

A fixed number of drivers commuting from o to d.



(a) Initial network ($x \in [0,1]$).



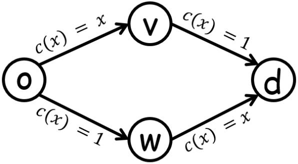
(b) Augmented network ($x \in [0,1]$).

Figure: Braess's paradox. Each edge is labeled with a fucntion that describes the travel time as a function of the fraction of the traffic that uses the edge.

Example #1: Braess's Paradox

Illustration of traffic network

A fixed number of drivers commuting from o to d. Assume that there are two non-interfering routes from o to d, each comprising one *long wide* road and one *short narrow* road.



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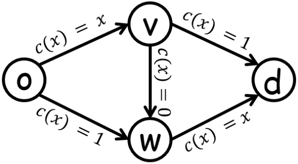

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A fixed number of drivers commuting from o to d. Assume that there are two non-interfering routes from o to d, each comprising one *long wide* road and one *short narrow* road. The travel time is denoted as function $c(x)$ and the travel time for each route is $1 + x$, where x is the fraction of the traffic using this route.

(a) Initial network ($x \in [0, 1]$). (b) Augmented network ($x \in [0, 1]$).

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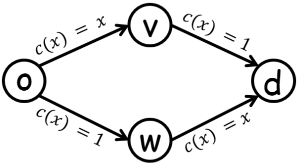
(a) Initial network ($x \in [0, 1]$).

(b) Augmented network ($x \in [0, 1]$).

Figure: Braess's paradox. Each edge is labeled with a fucntion that describes the travel time as a function of the fraction of the traffic that uses the edge.

Question

Suppose we try to improve commute times by installing a transportation device that allows drivers to travel instantly from v to w (*i.e.*, Fig.(b)). How will the drivers react?



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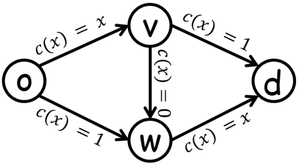

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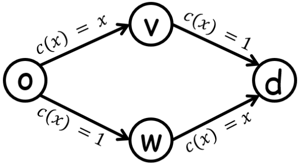
Figure: Braess's paradox. Each edge is labeled with a fucntion that describes the travel time as a function of the fraction of the traffic that uses the edge.

Illustration of Braess's paradox

- The travel time along the new route $o \rightarrow v \rightarrow w \rightarrow d$ is never worse than along the two original routes (since $2x \leq 1 + x$).



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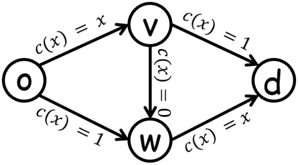

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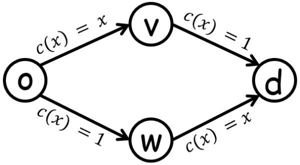

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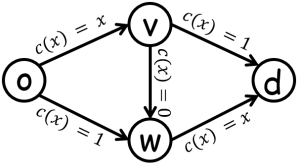

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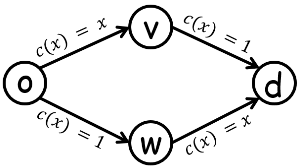

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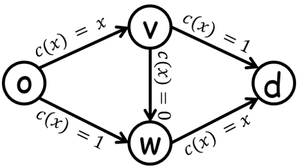

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- Hence the drivers are expected to deviate to the new route.
- As a result, they would expend two hours travelling from o to d.
- Braess's paradox shows that the intuitively helpful action of adding a superfast link can negatively impact all of the traffic.

(a) Initial network. (b) Augmented network.

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Example #1: Braess's Paradox

Concepts in Selfish routing

- Braess's paradox shows that *selfish routing* does not minimize the commute time of drivers, an altrustic dictator could improve everyone's commute time by 25%.

(a) Initial network. (b) Augmented network.

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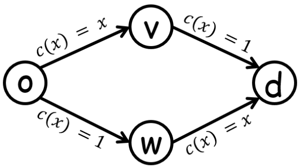
- Braess's paradox shows that *selfish routing* does not minimize the commute time of drivers, an altrustic dictator could improve everyone's commute time by 25%.
- *Price of anarchy (POA)*: the ratio between the system performance with strategic players and the best-possible system performance.



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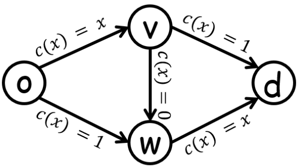

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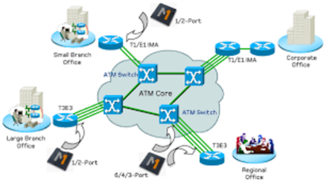
- *Price of anarchy (POA)*: the ratio between the system performance with strategic players and the best-possible system performance.
- For the network below, the POA is $\frac{2}{3/2} = \frac{4}{3}$.

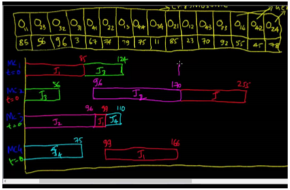
(a) Initial network. (b) Augmented network.

Figure: Braess's paradox. Each edge is labeled with a fucntion that describes the travel time as a function of the fraction of the traffic that uses the edge.

Selfish Behavior

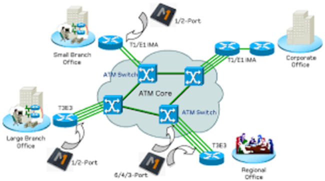
- The POA is close to 1 under reasonable conditions in a wide range of applications domains, including *network routing*, *scheduling*, *resource allocation*, and *auctions*.

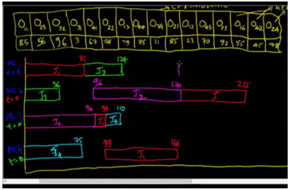




Selfish Behavior

- The POA is close to 1 under reasonable conditions in a wide range of applications domains, including *network routing*, *scheduling*, *resource allocation*, and *auctions*.
- In such cases, selfish behavior leads to a near-optimal outcome.





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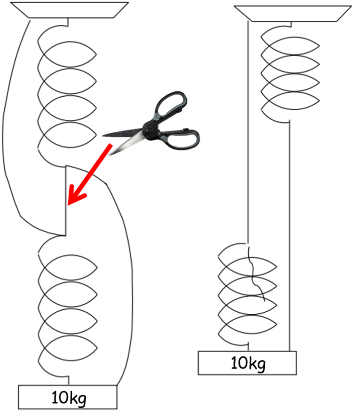
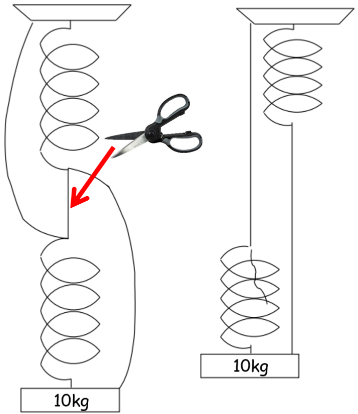


Figure: Strings and springs. Severing a taut string lifts a heavy weight.

- A spring is attached to a fixed sport and the end of a string.

Example #2: Braess's paradox



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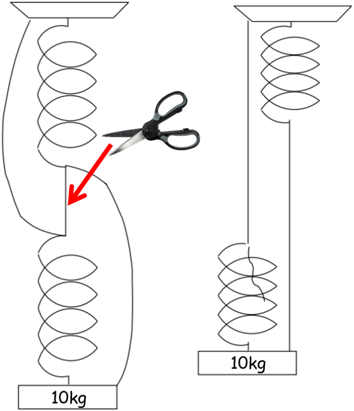

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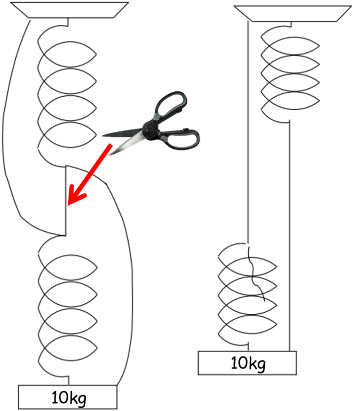

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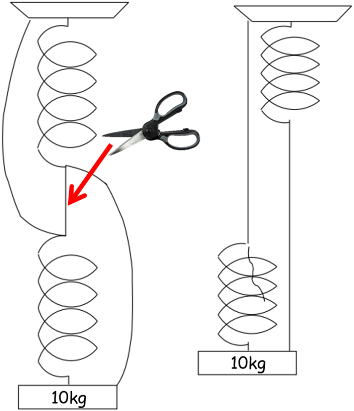
- A spring is attached to a fixed sport and the end of a string.
- Another spring is hung from the end of another string and carries a heavy weight.
- Two strings are connected, with a slack.
- Assume the stretch lengths of a spring is a linear function of the weight.
- Similar to traffic network, weight corresponds to traffic, and length corresponds to travel time.

Example #2: Braess's paradox



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- The *equilibrium* position of this mechanics is as shown in Fig.(a).

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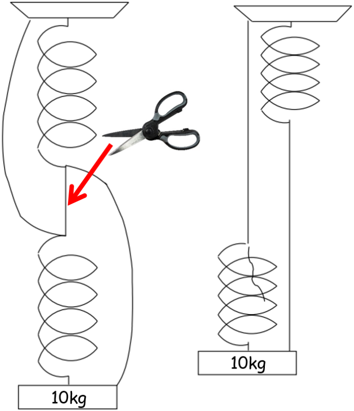

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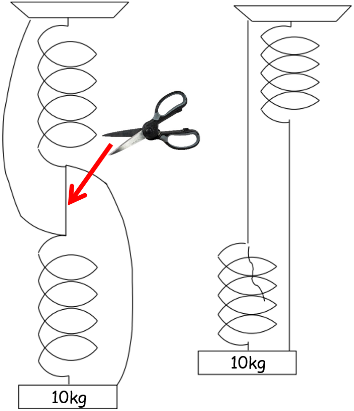

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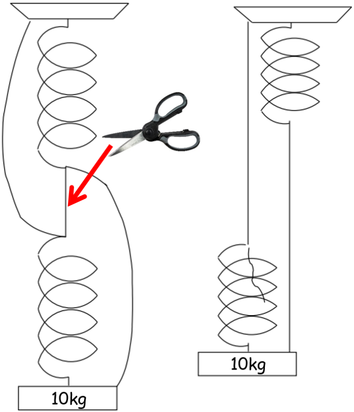

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- Each spring carries full weight in Fig.(a) and half of the weight in Fig.(b).

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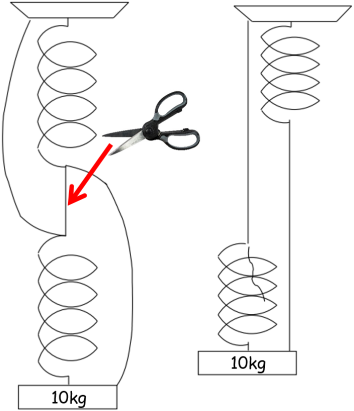

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- Initially, each spring bears the full weight. After cutting off, they both carry the weight in parallel.
- Each spring carries full weight in Fig.(a) and half of the weight in Fig.(b).
- Thus, the weight rises, which is kind of Braess's paradox.

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Example: Rock-Paper-Scissors

In most games, the best action to play depends on what others do.

	Rock	Paper	Scissors
Rock	0,0	-1,1	1,-1
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- Informally, an *equilibrium* is a steady state of a system, where each participant, assuming everything else stays the same, wants to remain as is.
- There is no "*deterministic equilibrium*" in the Rock-Paper-Scissors game: whatever the current state, at least one player can benefit from a unilateral deviation.

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- If both players randomize uniformly in this game, then neither player can increase the expected payoff via a unilateral deviation.
- A pair of probability distributions with this property is *(mixed-strategy) Nash equilibrium*.
- Nash's Theorem: *Every finite two-player game has a Nash equilibrium*.

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Question: Rock-Paper-Scissors

- Can a Nash equilibrium be computed efficiently, either by an algorithm or by strategic players?
- In *zero-sum* games like Rock-Paper-Scissors, where the payoff in each entry sums to zero, this can be done via linear programming, iterative learning, etc.
- In *non-zero-sum* games, recent results indicate that there is no computationally efficient algorithm for computing a Nash equilibrium (*i.e.*, not *NP-hardness*, called "*PPAD-hard*").

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Interpretations: Nash Equilibrium

- On a conceptual level, many interpretations of an equilibrium concept involve someone (the participants or a designer) determining an equilibrium.
- If all parties are boundedly rational, then an equilibrium can be interpreted as a credible prediction only if it can be computed with reasonable effort.
- This rigorous intractability results provides novel motivation for our study of "*easier*" equilibrium concepts, like *correlated equilibria* and *coarse correlated equilibria*.

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Summary of Lecture #1

- *Mechanism Design*: It is similar to rule making.
- *Behavior Analysis*: Selfish behaviors might lead to near optimal performance.
- *Nash Equilibrium*: Both positive results and negative results exist.

Future Plan

Presentation and Discussion

14 Lectures: #1-#7, #10, #11, #13-#17

Self Study and Disucssion

6 Lectures: #8, #9, #12, #18, #19, #20.

Recent Arrangements

- On every Friday evening, from 7:30PM to 9:50PM, Room G1139.
- March 15th: #1 (Yuxiang) and #2 (Jingzhi)
- March 22th: #3 (Yuxiang)
- March 29th: #4 (Jingzhi)

Future Plan

Basic Requirements

- We will use most of the time to discuss the lectures.
- Beyond the lectures, we will arrange 30min-40min for paper readings.
- As usual, we will select some papers (at least one paper each participant) to share the *what did they do*, *why did they do* and *how did they do* (their basic idea is okay).

Refered Books

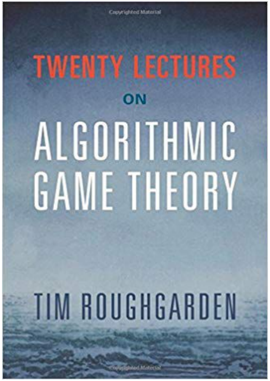

Figure: Twenty Lectures.

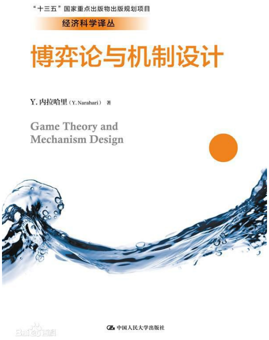

Figure: Orange Book.

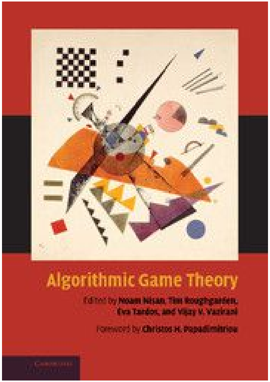

Figure: Algorithmic Game Theory.

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Paper List

Discussion on March 22th

- Zeng Yuxiang: Wu Fan and Chen Guihai et al. in AAAI'14.
- Fang Jingzhi: Wu Fan and Chen Guihai et al. in AAMAS'18.
- Li Jiangneng: Tang Pingzhong et al. in AAAI'17.
- Tao Qian: Zheng Libin and Cheng Peng et al. in ICDE'19.
- Zhen He: Mohammad Asghari and Cyrus Shahabi et al. in GIS'16.
- Cheng Hao: Tang Pingzhong et al. in IJCAI'16.
- Li Shuyuan: CACM'17, Inference and Auction Design in Online Advertising.
- Liu Wei: CACM'03, Living and Bidding in an Auction Economy.
- Pan Xuchen: CACM'12, incentive auctions.