Supplementary Material
Heterogenous firing responses leads to diverse coupling to presynaptic activity in a simplified morphological model of layer V pyramidal neurons

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Contents

1 Heterogeneity in the morphologies

We give here a graphical representation of the two extreme morphologies considered in this study.

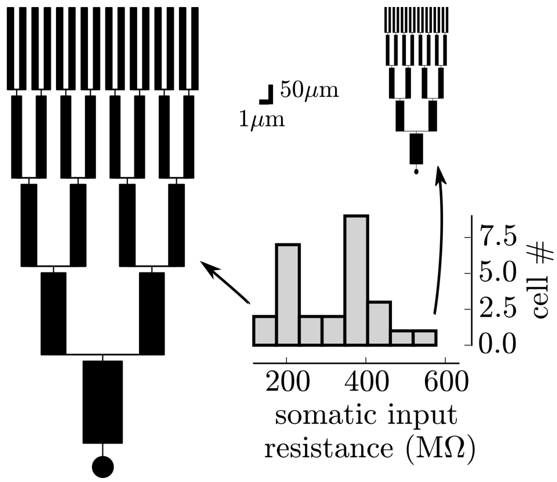

Figure 1: **Representaiton of the biggest and smallest cells.**

2 Reduction to the equivalent cylinder

The key of the derivation relies on having the possibility to reduce the complex morphology to an equivalent cylinder (Rall, 1962). We adapted this procedure to capture the change in integrative properties of the membrane that results from the mean synaptic bombardment during active cortical states, reviewed in Destexhe et al. (2003).

For a set of synaptic stimulation $\{\nu_p^e, \nu_i^p, \nu_e^d, \nu_i^d, s\}$, let's introduce the following stationary densities of conductances:

$$\begin{cases} g_{e0}^p = \pi \, d \, \mathcal{D}_e \, \nu_e^p \, \tau_e^p \, Q_e^p \quad ; \quad g_{i0}^p = \pi \, d \, \mathcal{D}_i \, \nu_i^p \, \tau_i^p \, Q_i^p \\ g_{e0}^d = \pi \, d \, \mathcal{D}_e \, \nu_e^d \, \tau_e^d \, Q_e^d \quad ; \quad g_{i0}^d = \pi \, d \, \mathcal{D}_i \, \nu_i^d \, \tau_i^d \, Q_i^d \end{cases} \tag{1}$$

where $\mathcal{D}_e$ and $\mathcal{D}_i$ are the excitatory and inhibitory synaptic densities.

We introduce two activity-dependent electrotonic constants relative to the proximal and distal part respectively:

$$\lambda^p = \sqrt{\frac{r_m}{r_i(1 + r_m g_{e0}^p + r_m g_{i0}^p)}} \quad \lambda^d = \sqrt{\frac{r_m}{r_i(1 + r_m g_{e0}^d + r_m g_{i0}^d)}}$$

(2)

For a dendritic tree of total length l, whose proximal part ends at l_p and with B evenly spaced generations of branches, we define the space-dependent electrotonic constant:

$$\lambda(x) = \big(\lambda^p + \mathcal{H}(x - l_p)(\lambda^d - \lambda^p)\big) 2^{-\frac{1}{3}\lfloor \frac{B\,x}{l} \rfloor} \tag{3}$$

where $\lfloor . \rfloor$ is the floor function. Note that $\lambda(x)$ is constant on a given generation, but it decreases from generation to generation because of the decreasing diameter along the dendritic tree. It also depends on the synaptic activity and therefore has a discontinuity at $x = l_p$.

Following Rall (1962), we now define a dimensionless length X:

$$X(x) = \int_0^x \frac{dx}{\lambda(x)} \tag{4}$$

We define $L = X(l)$ and $L_p = X(l_p)$, the total length and proximal part length respectively (capital letters design rescaled quantities).

3 Mean membrane potential

We derive the mean membrane potential $\mu_V(x)$ corresponding to the stationary response to constant densities of conductances given by the means of the synaptic stimulation. We obtain the stationary equations by removing temporal derivatives in Equation, the set of equation governing this mean membrane potential in all branches is therefore:

$$\begin{cases} \dfrac{1}{r_i}\dfrac{\partial^2 \mu_v}{\partial x^2} = \dfrac{\mu_v(x) - E_L}{r_m} \\ \qquad - g_{e0}^p\left(\mu_v(x) - E_e\right) - g_{0i}^p\left(\mu_v(x) - E_i\right) \quad \forall x \in [0, l_p] \\ \dfrac{1}{r_i}\dfrac{\partial^2 \mu_v}{\partial x^2} = \dfrac{\mu_v(x) - E_L}{r_m} \\ \qquad - g_{e0}^d\left(\mu_v(x) - E_e\right) - g_{0i}^d\left(\mu_v(x) - E_i\right) \quad \forall x \in [l_p, l] \\ \dfrac{\partial \mu_v}{\partial x}_{|x=0} = r_i\big(\dfrac{\mu_v(0) - E_L}{R_m} + G_{i0}^S\left(\mu_v(0) - E_i\right)\big) \\ \mu_v(l_p^-, t) = \mu_v(l_p^+, t) \\ \dfrac{\partial \mu_v}{\partial x}_{l_p^-} = \dfrac{\partial \mu_v}{\partial x}_{l_p^+} \\ \dfrac{\partial \mu_v}{\partial x}_{x=l} = 0 \end{cases} \tag{5}$$

Because the reduction to the equivalent cylinder conserves the membrane area and the previous equation only depends on density of currents, the equation governing $\mu_v(x)$ in all branches can be transformed into an equation on an equivalent cylinder of length L. We rescale x by $\lambda(x)$ (see Equation 4) and we obtain the equation verified by $\mu_V(X)$:

$$\begin{cases} \dfrac{\partial^2 \mu_v}{\partial X^2} = \mu_v(X) - v_0^p \quad \forall X \in [0, L_p] \\ \dfrac{\partial^2 \mu_v}{\partial X^2} = \mu_v(X) - v_0^d \quad \forall X \in [L_p, L] \\ \dfrac{\partial \mu_v}{\partial X}|_{X=0} = \gamma^p \Big(\mu_v(0) - V_0 \Big) \\ \mu_v(L_p^-) = \mu_v(L_p^+) \\ \dfrac{\partial \mu_v}{\partial X}_{L_p^-} = \dfrac{\lambda^p}{\lambda^d} \dfrac{\partial \mu_v}{\partial X}_{L_p^+} \\ \dfrac{\partial \mu_v}{\partial X}_{X=L} = 0 \end{cases} \tag{6}$$

where:

$$\begin{aligned} v_0^p &= \frac{E_L + r_m g_{e0}^p E_e + r_m g_{i0}^p E_i}{1 + r_m g_{e0}^p + r_m g_{i0}^p} \\ v_0^d &= \frac{E_L + r_m g_{e0}^d E_e + r_m g_{i0}^d E_i}{1 + r_m g_{e0}^d + r_m g_{i0}^d} \\ \gamma^p &= \frac{r_i \lambda^p \left(1 + G_i^0 R_m\right)}{R_m} \\ V_0 &= \frac{E_L + G_i^0 R_m E_i}{1 + + G_i^0 R_m} \end{aligned} \tag{7}$$

We write the solution on the form:

$$\begin{cases} \mu_v(X) = v_0^p + A \cosh(X) + C \sinh(X) \quad \forall\, X \in [0, L_p] \\ \mu_v(X) = v_0^d + B \cosh(X - L) + D \sinh(X - L) \quad \forall\, X \in [L_p L] \end{cases} \tag{8}$$

- Sealed-end boundary condition at cable end implies $D = 0$
- Somatic boudary condition imply: $C = \gamma^p \left(v_0^p - V_0 + A\right)$
- Then v continuity imply : $v_0^p + A \cosh(L_p) + \gamma^p \left(v_0^p - V_0 + A\right) \sinh(L_p) = v_0^d + B \cosh(L_p - L)$
- Then current conservation imply: $A \sinh(L_p) + \gamma^p \left(v_0^p - V_0 + A\right) \cosh(L_p) = \frac{\lambda^p}{\lambda^d} B \sinh(L_p - L)$

We rewrite those condition on a matrix form:

$$\begin{pmatrix} \cosh(L_p) + \gamma^p \sinh(L_p) & -\cosh(L_p - L) \\ \sinh(L_p) + \gamma^p \cosh(L_p) & -\frac{\lambda^p}{\lambda^d} \sinh(L_p - L) \end{pmatrix} \cdot \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} v_0^d - v_0^p - \gamma^p \left(v_0^p - V_0\right) \sinh(L_p) \\ -\gamma^p \left(v_0^p - V_0\right) \cosh(L_p) \end{pmatrix} \tag{9}$$

And we solved this equation with the `solve_linear_system_LU` method of `sympy`

The coefficients A and B are given by:

$$A = \frac{\alpha}{\beta} \qquad B = \frac{\gamma}{\delta} \tag{10}$$

where:

$$\begin{aligned}
\alpha &= V_0\gamma^P\lambda^D \cosh(L_p)\cosh(L-L_p) + V_0\gamma^P\lambda^P \sinh(L_p)\sinh(L-L_p) \\
&\quad - \gamma^P\lambda^D v_0^d \cosh(L_p)\cosh(L-L_p) - \gamma^P\lambda^P v_0^d \sinh(L_p)\sinh(L-L_p) \\
&\quad - \lambda^P v_0^d \sinh(L-L_p) + \lambda^P v_0^p \sinh(L-L_p) \\
\beta &= \gamma^P\lambda^D \cosh(L_p)\cosh(L-L_p) + \gamma^P\lambda^P \sinh(L_p)\sinh(L-L_p) + \\
&\quad \lambda^D \sinh(L_p)\cosh(L-L_p) + \lambda^P \sinh(L-L_p)\cosh(L_p) \\
\gamma &= \lambda^D\big(V_0\gamma^P + \gamma^P v_0^d \cosh(L_p) - \gamma^P v_0^d \\
&\quad - \gamma^P v_0^p \cosh(L_p) + v_0^d \sinh(L_p) - v_0^p \sinh(L_p)\big) \\
\delta &= \gamma^P\lambda^D \cosh(L_p)\cosh(L-L_p) + \gamma^P\lambda^P \sinh(L_p)\sinh(L-L_p) \\
&\quad + \lambda^D \sinh(L_p)\cosh(L-L_p) + \lambda^P \sinh(L-L_p)\cosh(L_p)
\end{aligned} \tag{11}$$

4 Membrane potential response to a synaptic event

We now look for the response to $n_{src} = \lfloor \frac{B\,x_{src}}{l} \rfloor$ synaptic events at position x_{src} on all branches of the generation of x_src, those events have a conductance $g(t)/n_{src}$ and reversal potential E_{rev}. We make the hypothesis that the initial condition correspond to the stationary mean membrane potential $\mu_V(x)$. This potential will also be used to fix the driving force at the synapse to $\mu_v(x_{src}) - E_{rev}$, this linearizes the equation and will allow an analytical treatment. To derive the equation for the response around the mean $\mu_v(x)$, we rewrite Equation 9 in main text with $v(x,t) = \delta v(x,t) + \mu_v(x)$, we obtain the equation for $\delta v(x,t)$:

$$
\begin{cases}
\dfrac{1}{r_i}\dfrac{\partial^2 \delta v}{\partial x^2} = c_m \dfrac{\partial \delta v}{\partial t} + \dfrac{\delta v}{r_m}(1 + r_m\, g_{e0}^p + r_m\, g_{i0}^p) \\
\qquad - \delta(x - x_{src})\left(\mu_v(x_{src}) - E_{rev}\right)\dfrac{g(t)}{n_{src}}, \quad \forall x \in [0, l_p] \\
\dfrac{1}{r_i}\dfrac{\partial^2 \delta v}{\partial x^2} = c_m \dfrac{\partial \delta v}{\partial t} + \dfrac{\delta v}{r_m}(1 + r_m\, g_{e0}^d + r_m\, g_{i0}^d) \\
\qquad - \delta(x - x_{src})\left(\mu_v(x_{src}) - E_{rev}\right)\dfrac{g(t)}{n_{src}}, \quad \forall x \in [l_p, l] \\
\dfrac{1}{r_i}\dfrac{\partial \delta v}{\partial x}_{|x=0} = C_M \dfrac{\partial \delta v}{\partial t}_{|x=0} + \dfrac{\delta v(0,t)}{R_m}(1 + R_m G_{i0}^S) \\
\delta v(l_p^-, t) = \delta v(l_p^+, t) \\
\dfrac{\partial \delta v}{\partial x}_{l_p^-} = \dfrac{\partial \delta v}{\partial x}_{l_p^+} \\
\dfrac{\partial \delta v}{\partial x}_{x=l} = 0
\end{cases}
\tag{12}
$$

Because this synaptic event is concomitant in all branches at distance x_{src}, we can use again the reduction to the equivalent cylinder (note that the event has now a weight multiplied by n_{src} so that its conductance becomes $g(t)$), we obtain:

$$
\begin{cases}
\dfrac{\partial^2 \delta v}{\partial X^2} = \left(\tau_m^p + (\tau_m^d - \tau_m^p)\mathcal{H}(X - L_p)\right)\dfrac{\partial \delta v}{\partial t} + \delta v \\
\qquad - \left(\mu_v(X_{src}) - E_{rev}\right)\delta(X - X_{src}) \times \\
\qquad\qquad \dfrac{g(t)}{c_m}\left(\dfrac{\tau_m^p}{\lambda^p} + (\dfrac{\tau_m^d}{\lambda^d} - \dfrac{\tau_m^p}{\lambda^p})\mathcal{H}(X_{src} - L_p)\right) \\
\dfrac{\partial \delta v}{\partial X}_{|X=0} = \gamma^p\left(\tau_m^S \dfrac{\partial \delta v}{\partial t}_{|X=0} + \delta v(0,t)\right) \\
\delta v(L_p^-, t) = \delta v(L_p^+, t) \\
\dfrac{\partial \delta v}{\partial X}_{L_p^-} = \dfrac{\lambda^p}{\lambda^d}\dfrac{\partial \delta v}{\partial X}_{L_p^+} \\
\dfrac{\partial \delta v}{\partial X}_{X=L} = 0
\end{cases}
\tag{13}
$$

where we have introduced the following time constants:

$$
\begin{aligned}
\tau_m^D &= \frac{r_m\, c_m}{1 + r_m\, g_{e0}^d + r_m\, g_{i0}^d} \\
\tau_m^P &= \frac{r_m\, c_m}{1 + r_m\, g_{e0}^p + r_m\, g_{i0}^p} \\
\tau_m^S &= \frac{R_m\, C_m}{1 + R_m\, G_{i0}^S}
\end{aligned}
\tag{14}
$$

Now used distribution theory (see Appel (2008) for a comprehensive textbook) to translate the synaptic input into boundary conditions at X_{src}, physically this corresponds to: 1) the continuity of the membrane potential and 2) the discontinuity of the current resulting from the synaptic input.

$$\begin{cases} \delta v(X_{src}^-, f) = \delta v(X_{src}^+, f) \\ \dfrac{\partial \delta v}{\partial X}_{X_{src}^+} - \dfrac{\partial \delta v}{\partial X}_{X_{src}^-} = -\big(\mu_v(X_{src}) - E_{rev}\big)\times \\ \qquad\qquad (\dfrac{\tau_m^p}{\lambda^p} + (\dfrac{\tau_m^d}{\lambda^d} - \dfrac{\tau_m^p}{\lambda^p})\mathcal{H}(X_{src} - L_p)\big)\,\dfrac{g(t)}{c_m} \end{cases} \tag{15}$$

We will solve Equation 13 by using Fourier analysis. We take the following convention for the Fourier transform:

$$\hat{F}(f) = \int_{\mathbb{R}} F(t)\, e^{-2i\pi f t}\, dt \tag{16}$$

We Fourier transform the set of Equations 13, we obtain:

$$\begin{cases} \dfrac{\partial^2 \hat{\delta v}}{\partial X^2} = \big(\alpha_f^p + (\alpha_f^d - \alpha_f^p)\mathcal{H}(X - L_p)\big)^2\, \hat{\delta v} \\ \dfrac{\partial \hat{\delta v}}{\partial X}_{|X=0} = \gamma_f^p\, \hat{\delta v}(0, f) \\ \hat{\delta v}(X_{src}^-, f) = \hat{\delta v}(X_{src}^+, f) \\ \dfrac{\partial \hat{\delta v}}{\partial X}_{X_{src}^-} = \dfrac{\partial \hat{\delta v}}{\partial X}_{X_{src}^+} - \big(\mu_v(X_{src}) - E_{rev}\big)\times \\ \qquad\qquad \big(r_f^p + (r_f^d - r_f^p)\mathcal{H}(X_{src} - L_p)\big)\, \hat{g(f)} \\ \hat{\delta v}(L_p^-, f) = \hat{\delta v}(L_p^+, f) \\ \dfrac{\partial \hat{\delta v}}{\partial X}_{L_p^-} = \dfrac{\lambda^p}{\lambda^d}\,\dfrac{\partial \hat{\delta v}}{\partial X}_{L_p^+} \\ \dfrac{\partial \hat{\delta v}}{\partial X}_{X=L} = 0 \end{cases} \tag{17}$$

where

$$\begin{aligned} \alpha_f^p &= \sqrt{1 + 2i\pi f \tau_m^p} \qquad r_f^p = \frac{\tau_m^p}{c_m\, \lambda^p} \\ \alpha_f^d &= \sqrt{1 + 2i\pi f \tau_m^d} \qquad r_f^d = \frac{\tau_m^d}{c_m\, \lambda^d} \\ \gamma_f^p &= \gamma^p\,(1 + 2i\pi f \tau_m^S) \end{aligned} \tag{18}$$

To obtain the solution, we need to split the solution into two cases:

1. $X_{src} \leq L_p$

Let's write the solution to this equation as the form (already including the boundary conditions at $X = 0$ and $X = L$):

$$\hat{\delta v}(X, X_{src}, f) = \begin{cases} A_f(X_{src})\left(\cosh(\alpha_f^p\, X) + \gamma^p\ \sinh(\alpha_f^p\, X)\right) \\ \qquad\qquad \text{if :} 0 \leq X \leq X_{src} \leq L_p \leq L \\ B_f(X_{src})\ \cosh(\alpha_f^p\,(X - L_p)) + C_f(X_{src})\ \sinh(\alpha_f^p\,(X - L_p)) \\ \qquad\qquad \text{if :} 0 \leq X_{src} \leq X \leq L_p \leq L \\ D_f(X_{src})\ \cosh(\alpha_f^d\,(X - L)) \\ \qquad\qquad \text{if :} 0 \leq X_{src} \leq L_p \leq X \leq L \end{cases} \tag{19}$$

We write the 4 conditions correspondingto the conditions in X_{src} and L_p to get A_f, B_f, C_f, D_f. On a matrix form, this gives:

$$M = \begin{pmatrix} \cosh(\alpha_f^p\, X_{src}) + \gamma_f^p \sinh(\alpha_f^p\, X_{src}) & -\cosh(\alpha_f^p\,(X_{src} - L_p)) & -\sinh(\alpha_f^p\,(X_{src} - L_p)) & 0 \\ \alpha_f^p\big(\sinh(\alpha_f^p\, X_{src}) + \gamma_f^p \cosh(\alpha_f^p\, X_{src})\big) & -\alpha_f^p \sinh(\alpha_f^p\,(X_{src} - L_p)) & -\alpha_f^p \cosh(\alpha_f^p\,(X_{src} - L_p)) & 0 \\ 0 & 1 & 0 & -\cosh(\alpha_f^d\,(L_p - L)) \\ 0 & 0 & \alpha_f^p & -\alpha_f^d\,\frac{\lambda^p}{\lambda^d}\ \sinh(\alpha_f^d\,(L_p - L)) \end{pmatrix} \tag{20}$$

$$M \cdot \begin{pmatrix} A_f \\ B_f \\ C_f \\ D_f \end{pmatrix} = \begin{pmatrix} 0 \\ -r_f^p I_f \\ 0 \\ 0 \end{pmatrix} \tag{21}$$

And we will solve it with the `solve_linear_system_LU` method of `sympy`. For the $A_f(X_{src})$ coefficient, we obtain:

$$A_f(X_{src}) = \frac{a_f^1(X_{src})}{a_f^2(X_{src})} \tag{22}$$

with:

$$
\begin{aligned}
a_f^1(X_{src}) = I_f r_f^P \big(&-\alpha_f^D \lambda^P \cosh\big(L\alpha_f^D - L_p\alpha_f^D - L_p\alpha_f^P + X_s\alpha_f^P\big) \\
&+ \alpha_f^D \lambda^P \cosh\big(L\alpha_f^D - L_p\alpha_f^D + L_p\alpha_f^P - X_s\alpha_f^P\big) \\
&+ \alpha_f^P \lambda^D \cosh\big(L\alpha_f^D - L_p\alpha_f^D - L_p\alpha_f^P + X_s\alpha_f^P\big) \\
&+ \alpha_f^P \lambda^D \cosh\big(L\alpha_f^D - L_p\alpha_f^D + L_p\alpha_f^P - X_s\alpha_f^P\big) \\
a_f^2(X_{src}) = \alpha_f^P \big(&-\alpha_f^D \gamma_f^P \lambda^P \cosh\big(-L\alpha_f^D + L_p\alpha_f^D + L_p\alpha_f^P\big) \\
&+ \alpha_f^D \gamma_f^P \lambda^P \cosh\big(L\alpha_f^D - L_p\alpha_f^D + L_p\alpha_f^P\big) - \\
&\alpha_f^D \lambda^P \sinh\big(-L\alpha_f^D + L_p\alpha_f^D + L_p\alpha_f^P\big) \\
&+ \alpha_f^D \lambda^P \sinh\big(L\alpha_f^D - L_p\alpha_f^D + L_p\alpha_f^P\big) \\
&+ \alpha_f^P \gamma_f^P \lambda^D \cosh\big(-L\alpha_f^D + L_p\alpha_f^D + L_p\alpha_f^P\big) \\
&+ \alpha_f^P \gamma_f^P \lambda^D \cosh\big(L\alpha_f^D - L_p\alpha_f^D + L_p\alpha_f^P\big) \\
&+ \alpha_f^P \lambda^D \sinh\big(-L\alpha_f^D + L_p\alpha_f^D + L_p\alpha_f^P\big) \\
&+ \alpha_f^P \lambda^D \sinh\big(L\alpha_f^D - L_p\alpha_f^D + L_p\alpha_f^P\big)
\end{aligned} \tag{23}
$$

2. \$ $\mathrm{L_p} \leq \mathrm{X_{src}}$\$

   Let's write the solution to this equation as the form (already including the boundary conditions at $X = 0$ and $X = L$:

$$
\hat{\delta v}(X, X_{src}, f) =
\begin{cases}
E_f(X_{src})\big(\cosh(\alpha_f^p X) + \gamma^p \sinh(\alpha_f^p X)\big) \\
\qquad \text{if :} 0 \leq X \leq L_p \leq X_{src} \leq L \\
F_f(X_{src}) \cosh(\alpha_f^d (X - L_p)) + G_f(X_{src}) \sinh(\alpha_f^d (X - L_p)) \\
\qquad \text{if :} 0 \leq L_p \leq X \leq X_{src} \leq L \\
H_f(X_{src}) \cosh(\alpha_f^d (X - L)) \\
\qquad \text{if :} 0 \leq L_p \leq X_{src} \leq X \leq L
\end{cases} \tag{24}
$$

   We write the 4 conditions correspondingto the conditions in X_{src} and L_p to get A_f, B_f, C_f, D_f. On a matrix form, this gives:

   We rewrite this condition on a matrix form:

$$
M_2 = \begin{pmatrix}
\cosh(\alpha_f^p L_p) + \gamma_f^p \sinh(\alpha_f^p L_p) & -1 & 0 & 0 & 0 \\
\alpha_f^p\big(\sinh(\alpha_f^p L_p) + \gamma_f^p \cosh(\alpha_f^p L_p)\big) & 0 & -\alpha_f^d \frac{\lambda^p}{\lambda^d} & 0 \\
0 & \cosh(\alpha_f^d (X_{src} - L_p)) & \sinh(\alpha_f^d (X_{src} - L_p)) & -\cosh(\alpha_f^d (X_{src} - L)) \\
0 & \alpha_f^d \sinh(\alpha_f^d (X_{src} - L_p)) & \alpha_f^d \cosh(\alpha_f^d (X_{src} - L_p)) & -\alpha_f^d \sinh(\alpha_f^d (X_{src} - L))
\end{pmatrix} \tag{25}
$$

$$M \cdot \begin{pmatrix} E_f \\ F_f \\ G_f \\ H_f \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -r_f^d I_f \end{pmatrix} \tag{26}$$

And we will solve it with the `solve_linear_system_LU` method of `sympy`. For the $E_f(X_{src})$ coefficient, we obtain:

$$E_f(X_{src}) = \frac{e_f^1(X_{src})}{e_f^2(X_{src})} \tag{27}$$

with:

$$\begin{aligned}
e_f^1(X_{src}) &= 2 I_f \lambda^P r_f^D \cosh\left(\alpha_f^D \left(L - X_s\right)\right) \\
e_f^2(X_{src}) &= -\alpha_f^D \gamma_f^P \lambda^P \cosh\left(-L\alpha_f^D + L_p\alpha_f^D + L_p\alpha_f^P\right) \\
&\quad + \alpha_f^D \gamma_f^P \lambda^P \cosh\left(L\alpha_f^D - L_p\alpha_f^D + L_p\alpha_f^P\right) \\
&\quad - \alpha_f^D \lambda^P \sinh\left(-L\alpha_f^D + L_p\alpha_f^D + L_p\alpha_f^P\right) \\
&\quad + \alpha_f^D \lambda^P \sinh\left(L\alpha_f^D - L_p\alpha_f^D + L_p\alpha_f^P\right) \\
&\quad + \alpha_f^P \gamma_f^P \lambda^D \cosh\left(-L\alpha_f^D + L_p\alpha_f^D + L_p\alpha_f^P\right) \\
&\quad + \alpha_f^P \gamma_f^P \lambda^D \cosh\left(L\alpha_f^D - L_p\alpha_f^D + L_p\alpha_f^P\right) \\
&\quad + \alpha_f^P \lambda^D \sinh\left(-L\alpha_f^D + L_p\alpha_f^D + L_p\alpha_f^P\right) \\
&\quad + \alpha_f^P \lambda^D \sinh\left(L\alpha_f^D - L_p\alpha_f^D + L_p\alpha_f^P\right)
\end{aligned} \tag{28}$$

3. PSP at the soma

   The main text writes a solution for the PSP at soma of the form:

$$\hat{\delta v}(X=0, X_{src}, f) = K_f(X_{src}) \left(\mu_v(X_{src}) - E_{rev}\right) \hat{g(f)} \tag{29}$$

   The correspondance with the previous calculus is to take a unitary current $I_f = 1$ and $K_f(X_{src})$ given by:

$$K_f(X_{src}) = \begin{cases} A_f(X_{src}) \forall X_{src} \in [0, L_p] \\ E_f(X_{src}) \forall X_{src} \in [L_p, L] \end{cases} \tag{30}$$

5 Variability in the fluctuations properties introduced by the different morphologies

In Figure 2, we investigate what is the variability introduced by the different morphologies for the implemented protocols. We fix ($\nu_{\mathrm{e}}^{\mathrm{p}}$, $\nu_{\mathrm{e}}^{\mathrm{p}}$, s) and μ_{V}, the

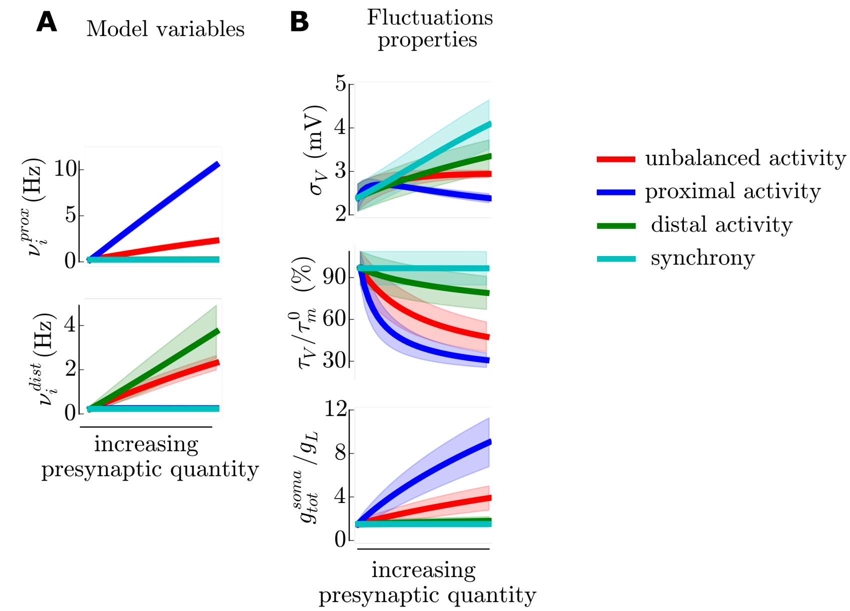

Figure 2: **Variability in the fluctuations properties introduced by the different morphologies.** Showing only the quantities that vary across cells, all other variables (ν_e^p, ν_e^p, s) or μ_V are fixed across cells. **(A)** Model variables. The balance μ_V is adjusted for each cells and the cells have different surfaces, so different number of synapses (and especially different ratio of excitatory to inhibitory numbers) so the inihibitory activity is adjusted differently for each cell. **(B)** Fluctuations properties.

inhibitory frequencies are adjusted adjusted depending on the morphology and the fluctuations properties also depend on the morphology.

We conclude that the impact of the different morphology is weak for those protocols.

6 Residual correlations after accounting for the excitability effects

7 Including cells of very low excitability in the analysis

In this section, we re-include the n=3 cells of very low excitabilities that have been discarded in the analysis presented in the main text.

The global trend is conserved but their very low values of the firing rate render data visualization poorly informative.

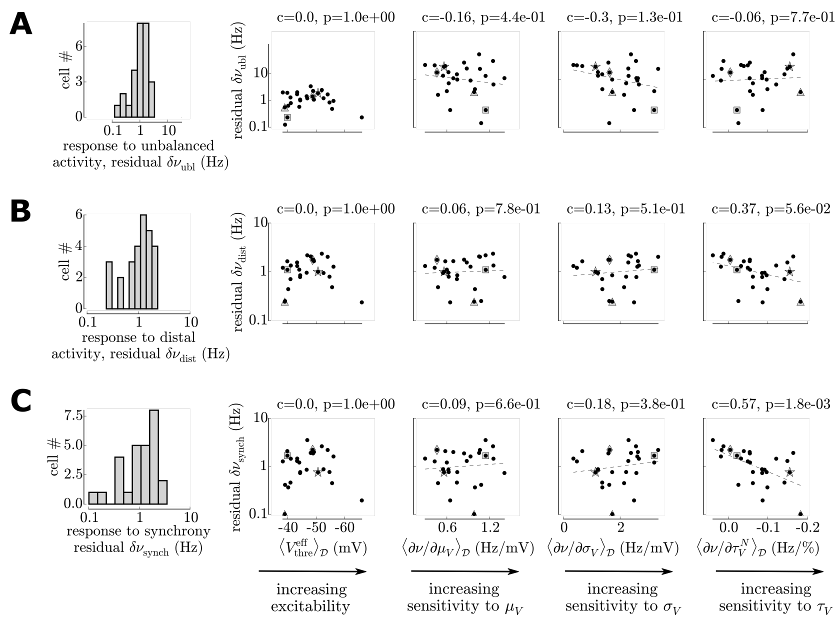

Figure 3: **Residual correaltions after removing the dependency on the excitability (see null correlation with excitability).**

8 References

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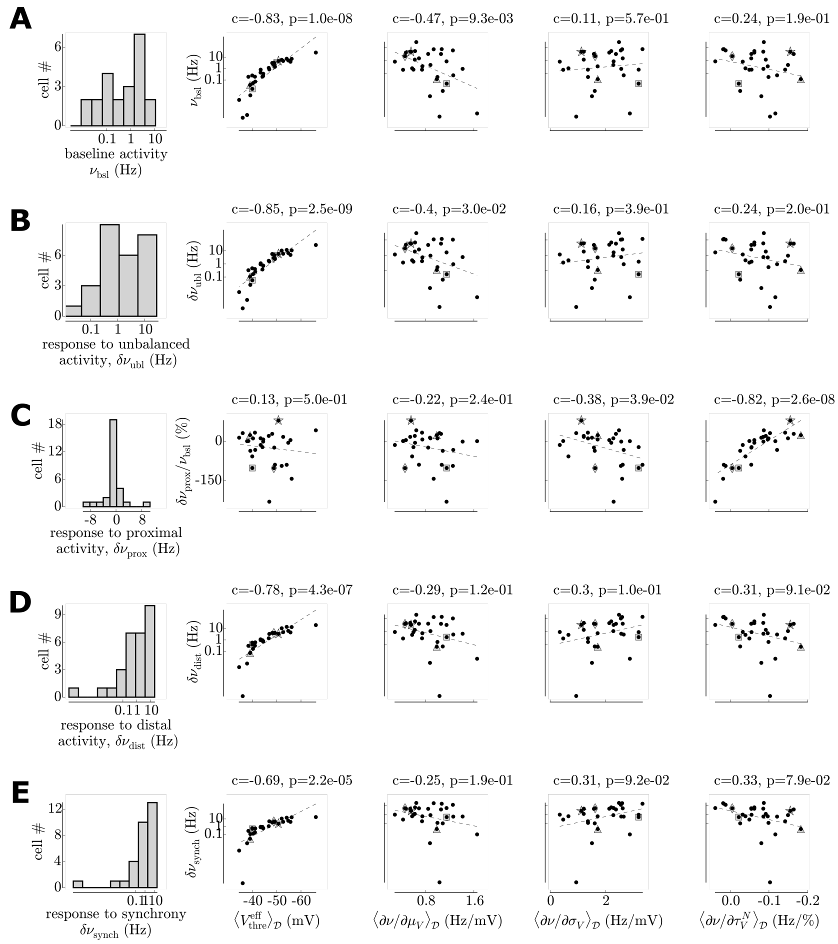

Figure 4: **Including all cells in the correlating responses with biophysical features**. (i.e. the very low firing rates: ν<10^{-4})