



GPHS-426

**Climatology and
Remote Sensing**

RS Lecture 1

12 December 2023

Remote Sensing (4 X 2-hour classes)

Lecturer: Yizhe Zhan

- **Principles of satellite remote sensing (Wed 13 Dec 2023)**
- Meteorological satellites (Tues 19 Dec 2023)
- Passive remote sensing instruments (Thurs 11 Jan 2024)
- Active remote sensing instruments (Fri 12 Jan 2024)

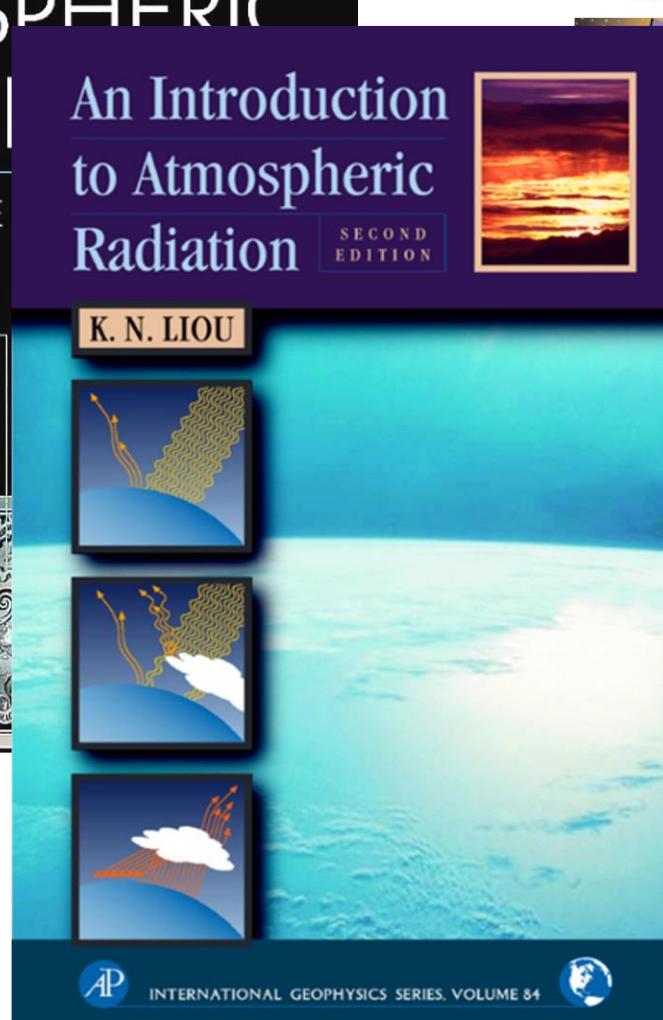
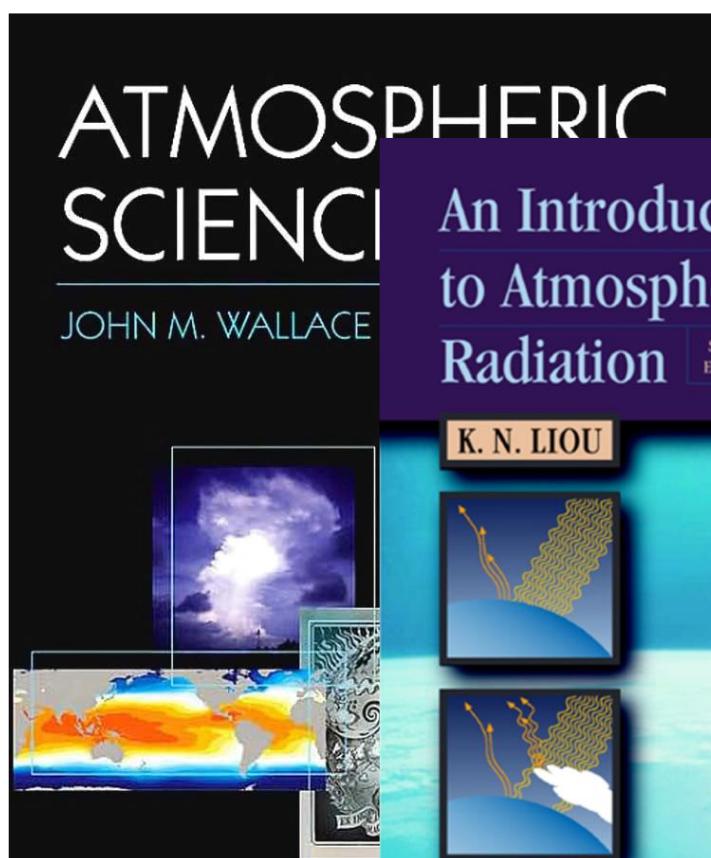
For course material and hands-on session,



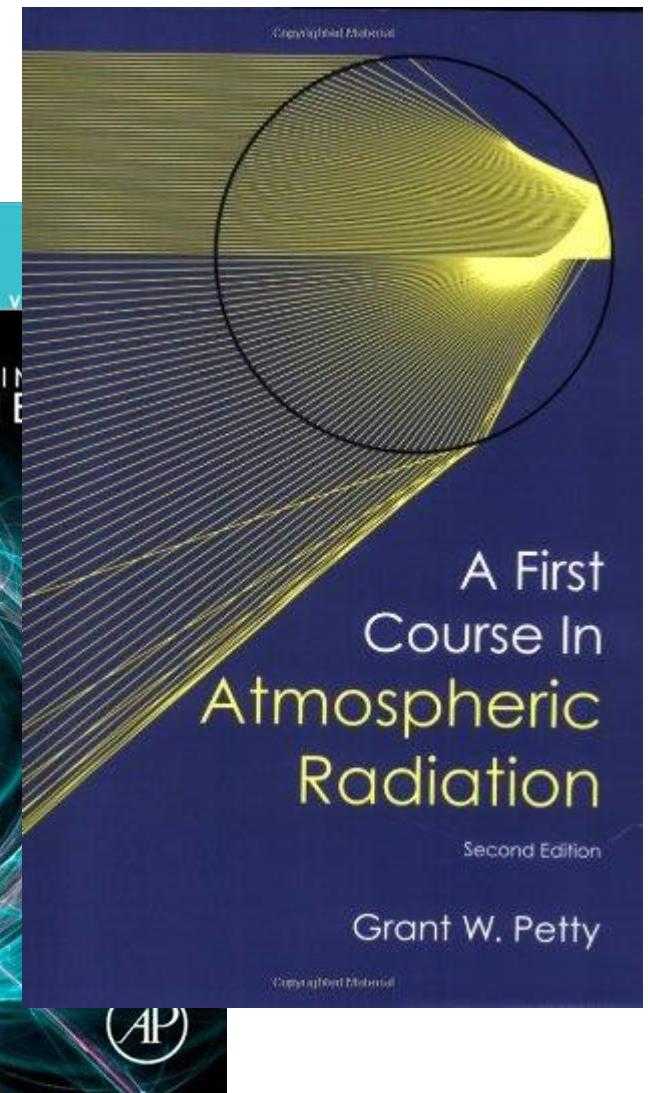
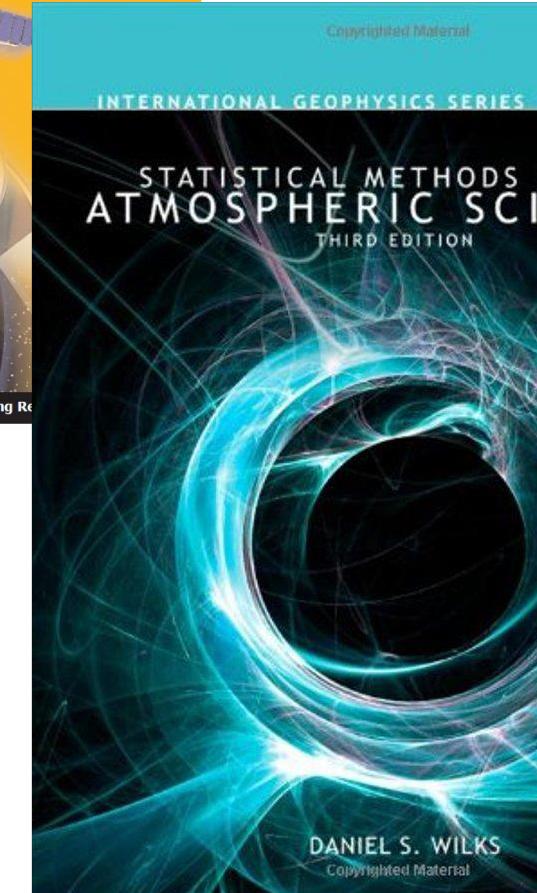
https://github.com/yizhe-met/GPHS-426_RS.git

For reach out for questions, my EDU email is yizhe@illinois.edu.

Recommend readings



Fundamentals
of Remote Sensing



How satellite remote sensing contribute to weather forecast?

Satellite remote sensing plays a crucial role in weather forecast by providing essential observational data that significantly improves the accuracy and reliability of weather forecasts via,

1. Data Assimilation:

Satellite data, encompassing atmospheric parameters like temperature, humidity, and wind patterns, are assimilated into NWP models for accurate initialization. This process corrects model conditions and enhances forecast precision.

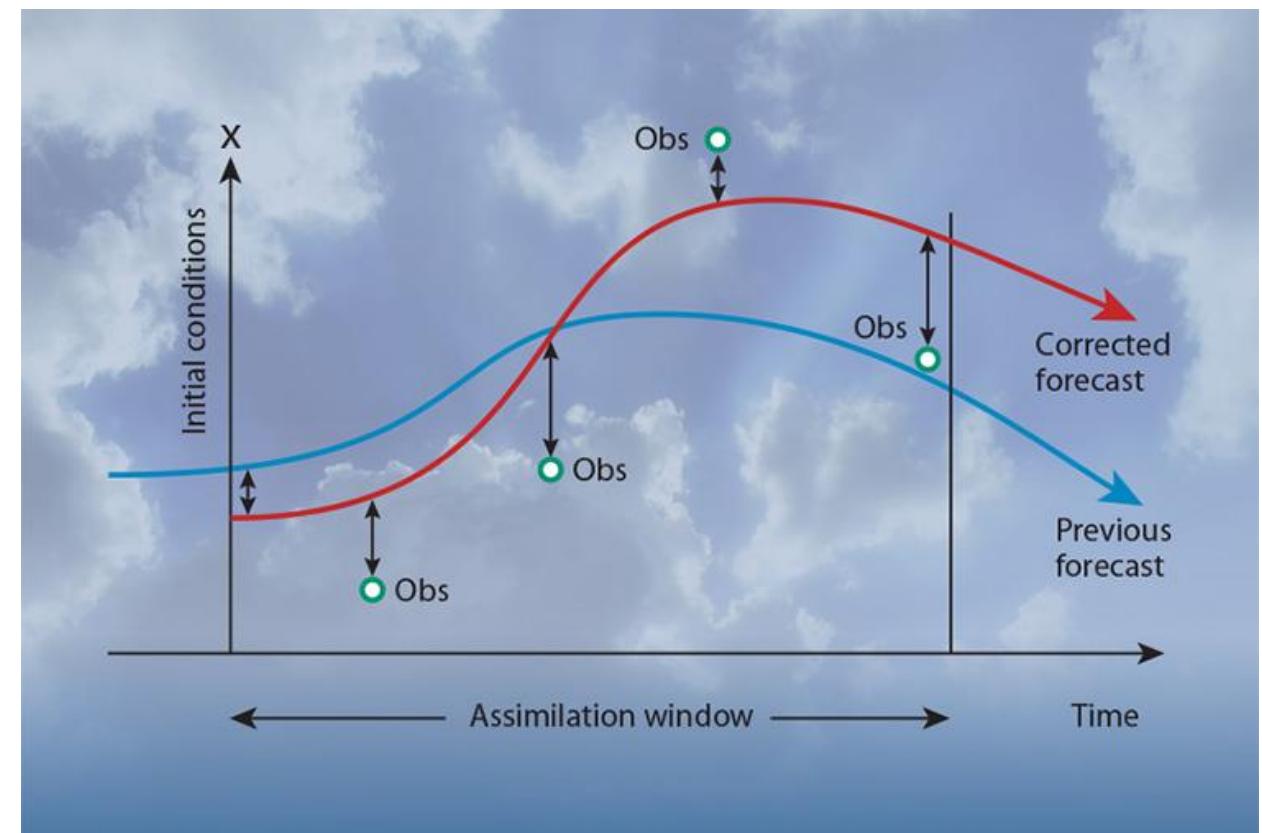
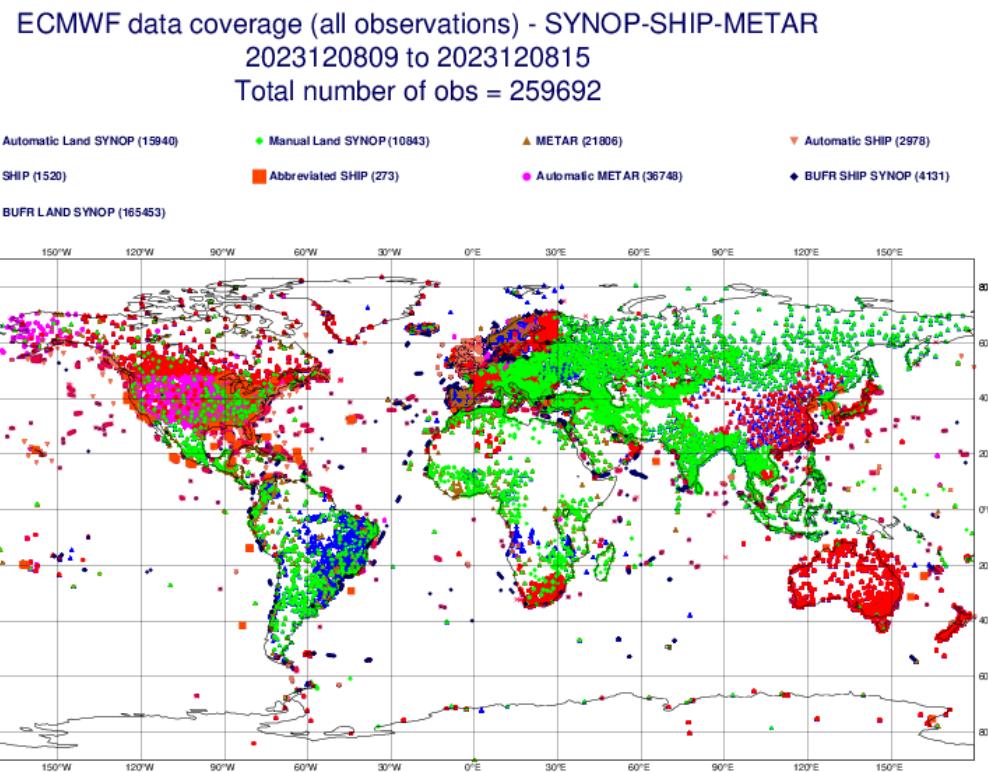
2. Global Coverage and Monitoring:

Satellites offer global coverage, providing essential observational data from remote and inaccessible regions. They enable continuous monitoring and tracking of dynamic weather systems, aiding in the prediction of severe weather events and improving overall forecasting capabilities.

3. Parameterization and Improved Representation:

Satellite observations contribute to improved parameterization and representation of atmospheric processes in NWP models. Data on cloud cover, precipitation, vertical profiling, and surface temperatures enhance the accuracy of simulations, leading to more reliable weather predictions.

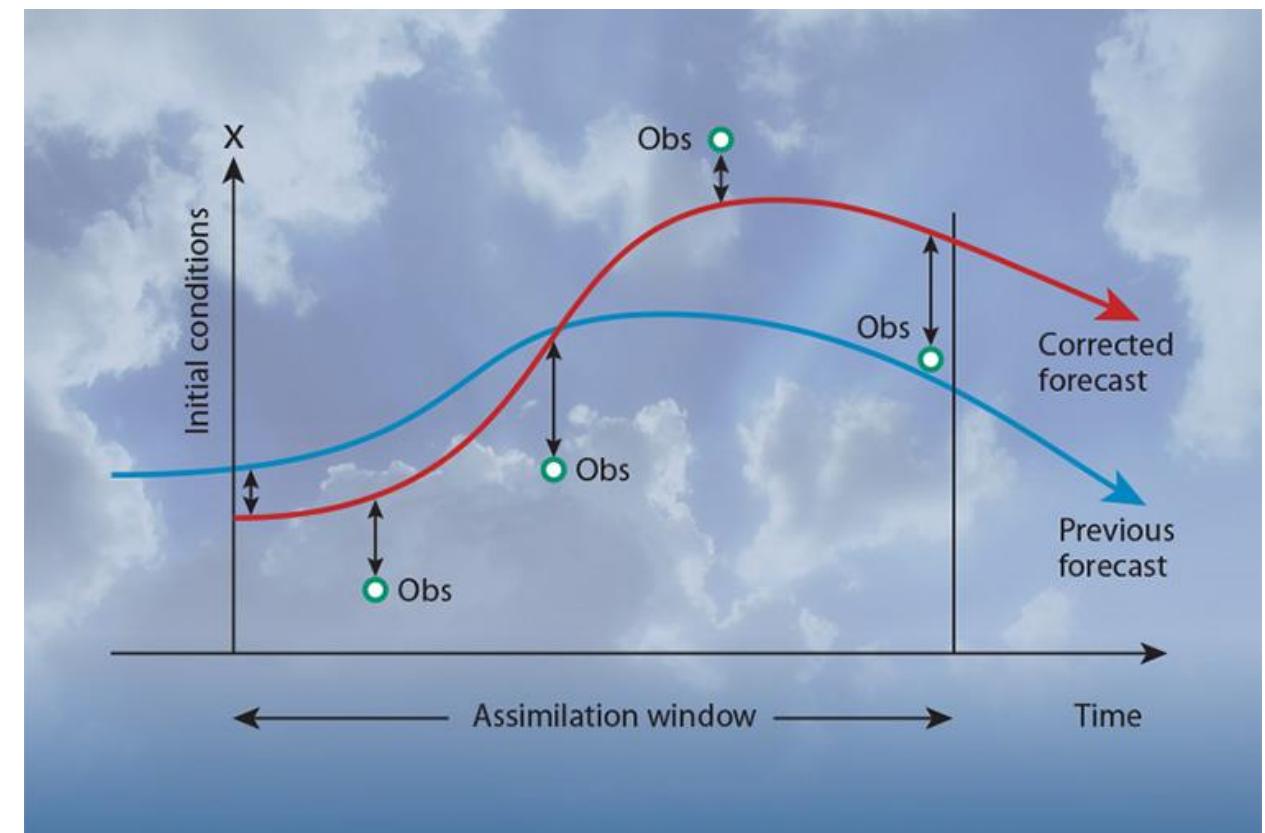
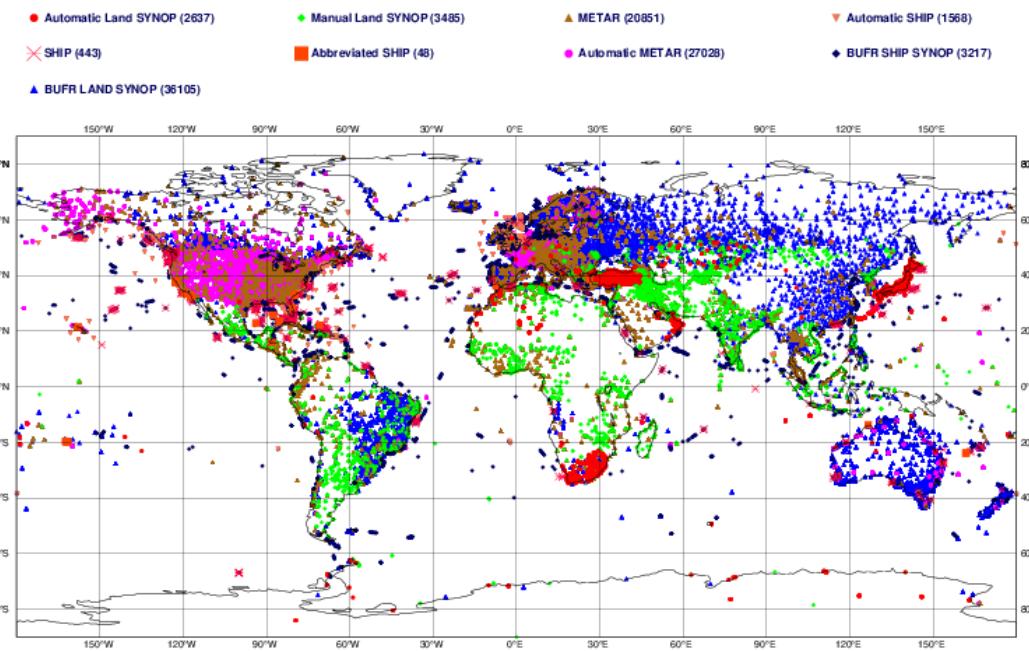
Data Assimilation



<https://charts.ecmwf.int/catalogue/packages/monitoring/products/dcover>

Data Assimilation

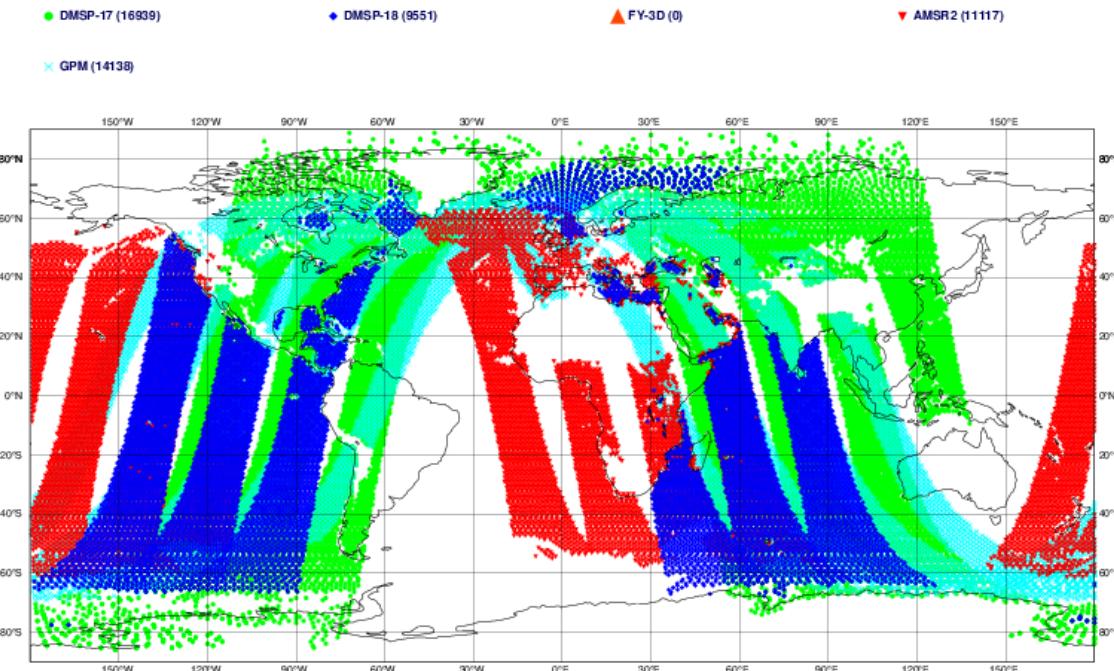
ECMWF data coverage (used observations) - SYNOP-SHIP-METAR
2023120809 to 2023120815
Total number of obs = 95382



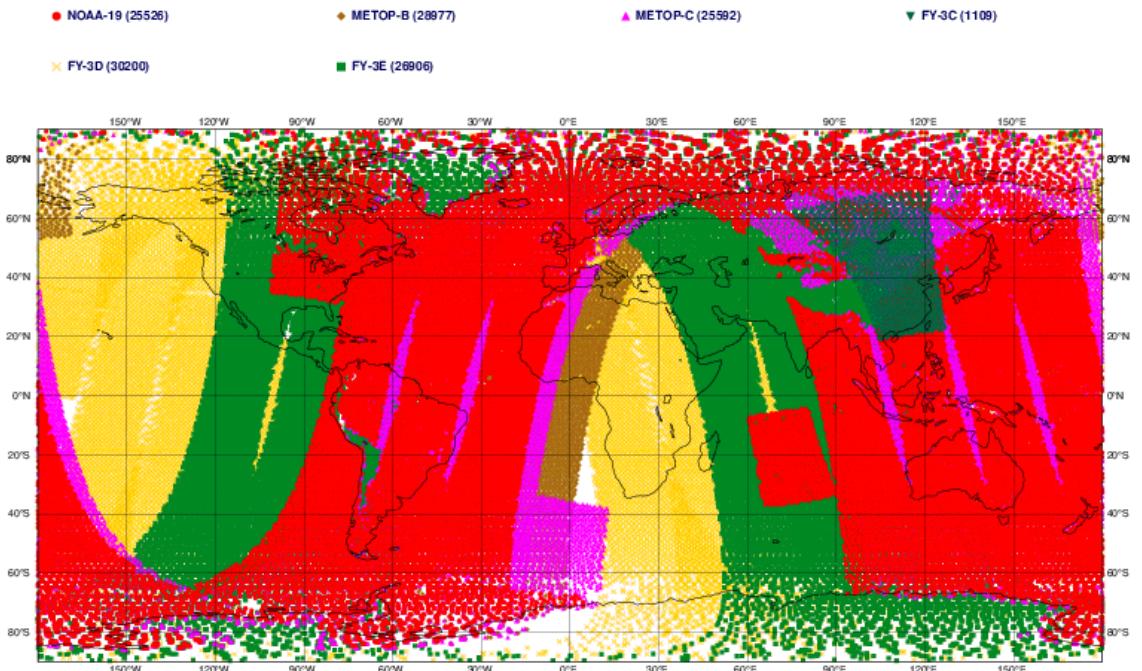
<https://charts.ecmwf.int/catalogue/packages/monitoring/products/dcover>

Data Assimilation

ECMWF data coverage (used observations) - MICROWAVE HUMIDITY IMAGERS
2023120809 to 2023120815
Total number of obs = 51745



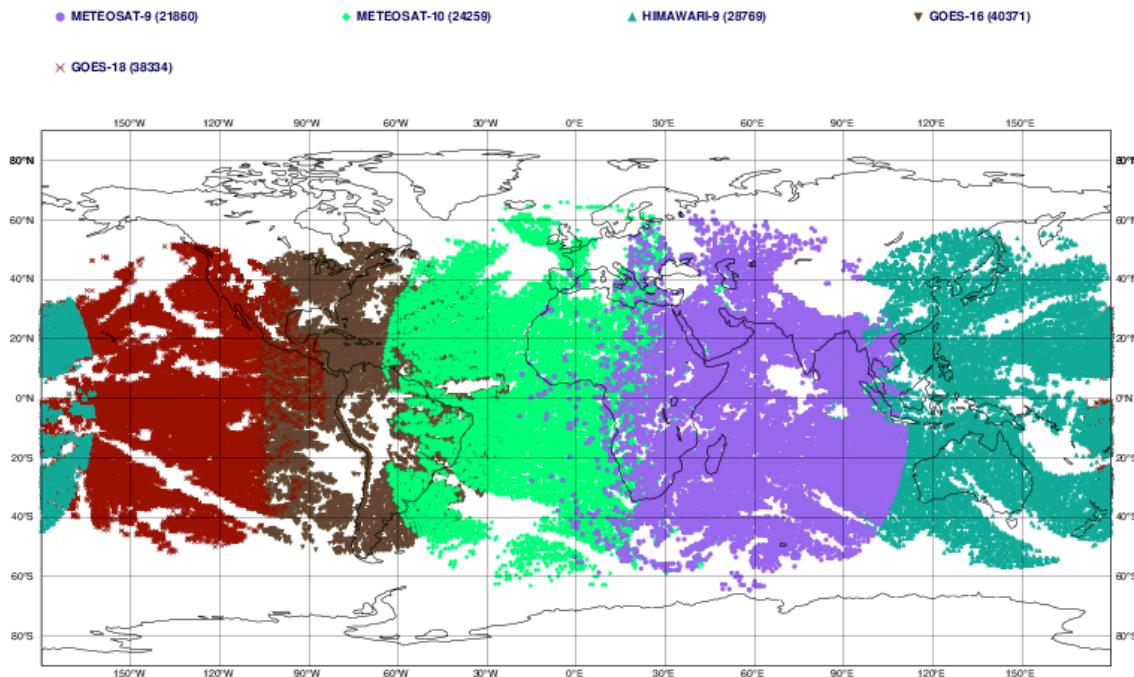
ECMWF data coverage (used observations) - MICROWAVE HUMIDITY SOUNDERs
2023120809 to 2023120815
Total number of obs = 138310



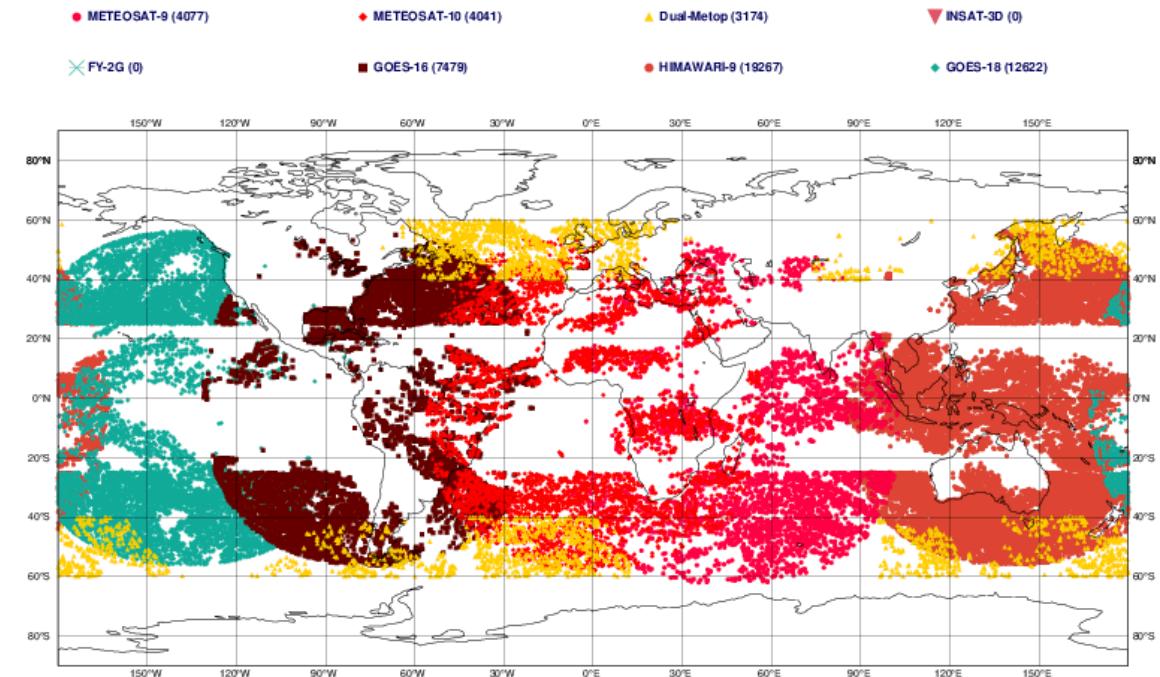
<https://charts.ecmwf.int/catalogue/packages/monitoring/products/dcover>

Data Assimilation

ECMWF data coverage (used observations) - GEOSTATIONARY RADIANCES
2023120809 to 2023120815
Total number of obs = 153593

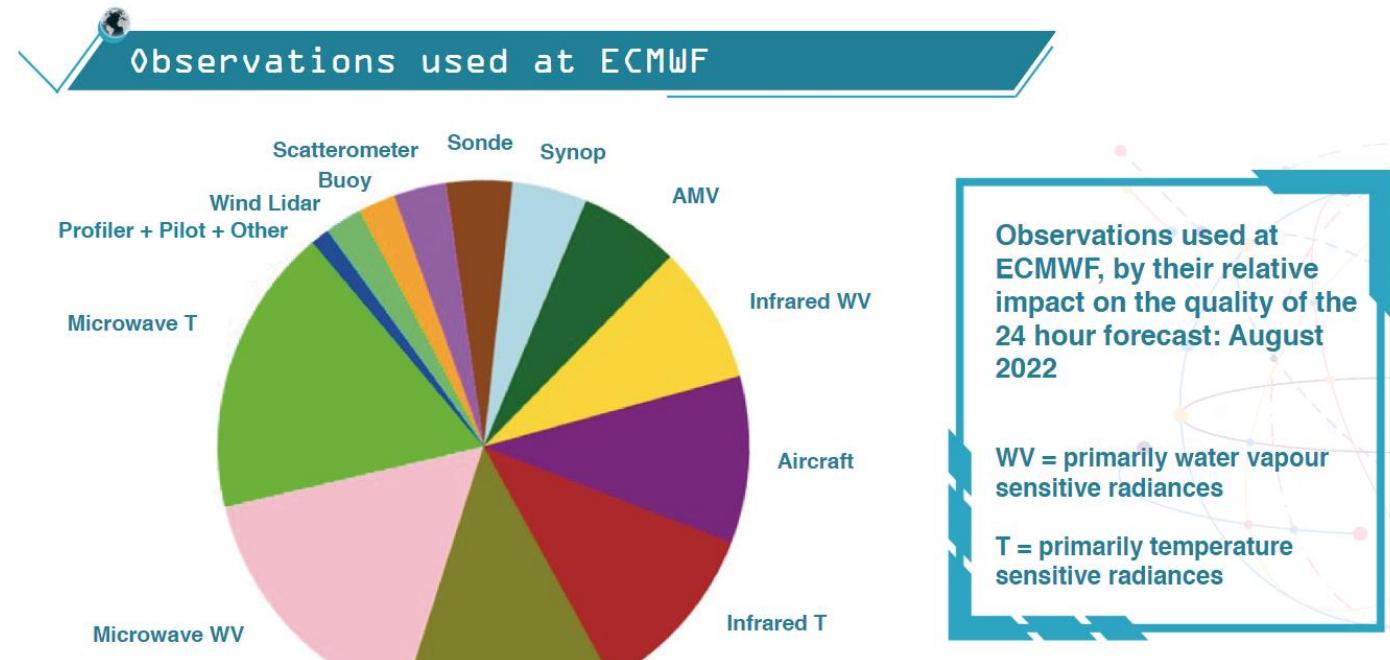


ECMWF data coverage (used observations) - AMV IR
2023120809 to 2023120815
Total number of obs = 50660

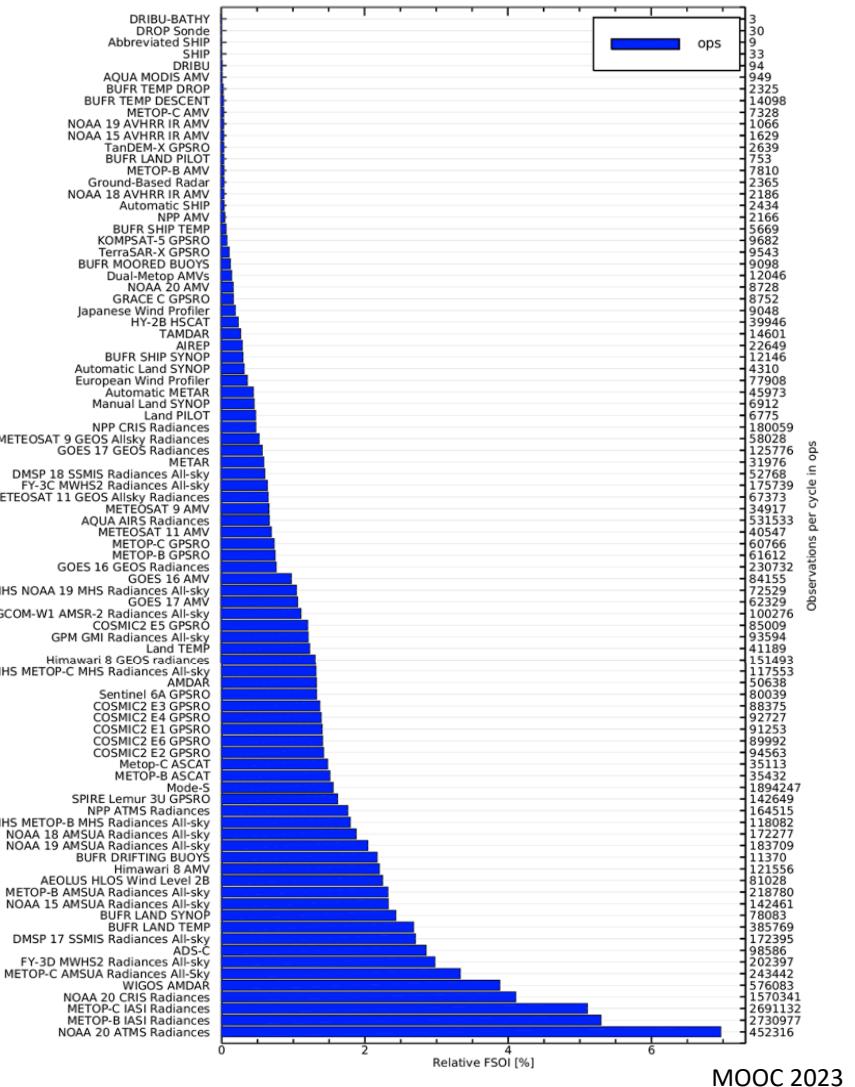


<https://charts.ecmwf.int/catalogue/packages/monitoring/products/dcover>

Data Assimilation



FSOI quantifies impact of all assimilated observations on a selected forecast metrics ... shows if any observation decreases or increase forecast error (Langland and Baker 2004).



Data Assimilation

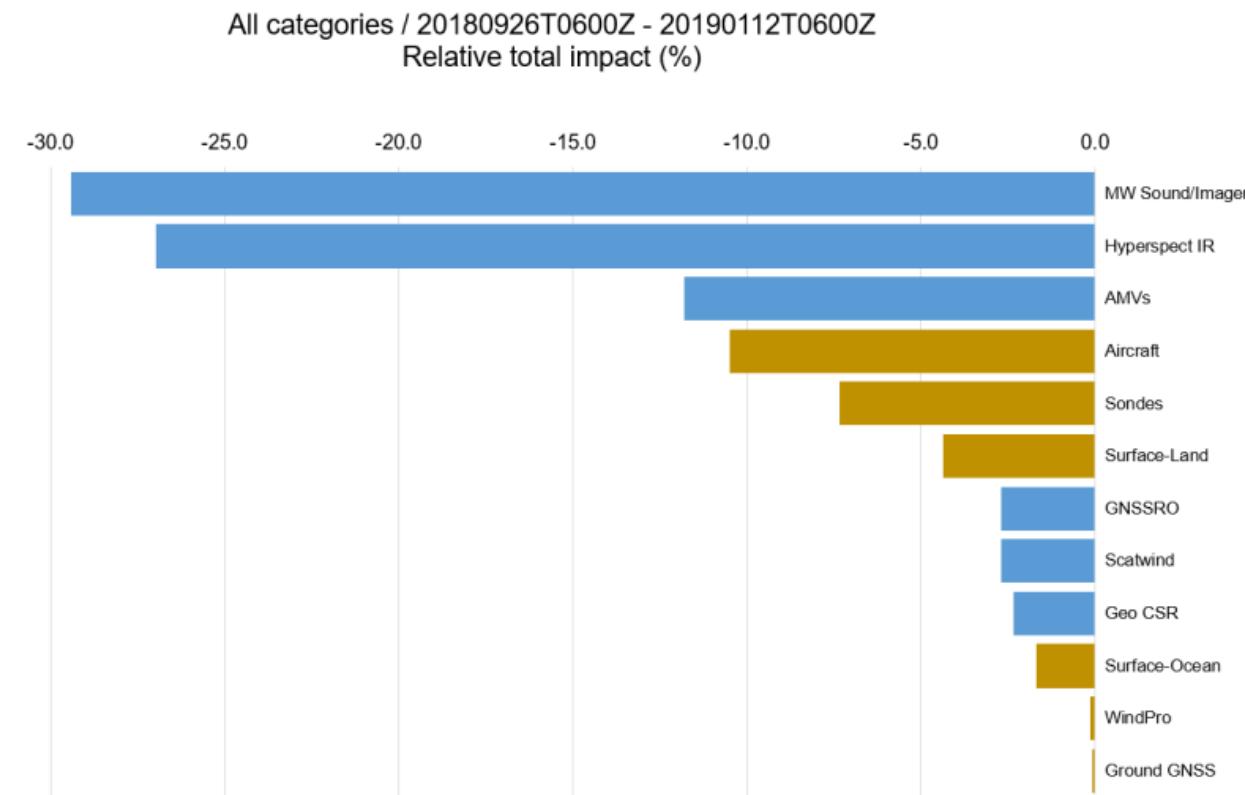


Figure 5: Relative FSOI for all observation types assimilated in the Met Office global NWP system. The impact is expressed as the percentage of the total impact on 24-hour forecast error. A negative value means a reduction in forecast error. Space-based observations are colored blue, while surface-based observations are gold. Annex A provides an explanation of the observation types.

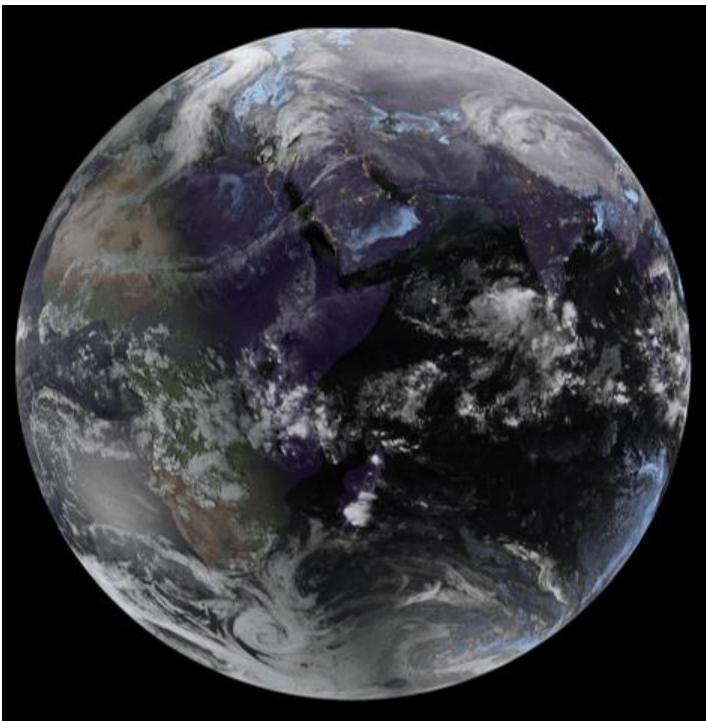
Global coverage



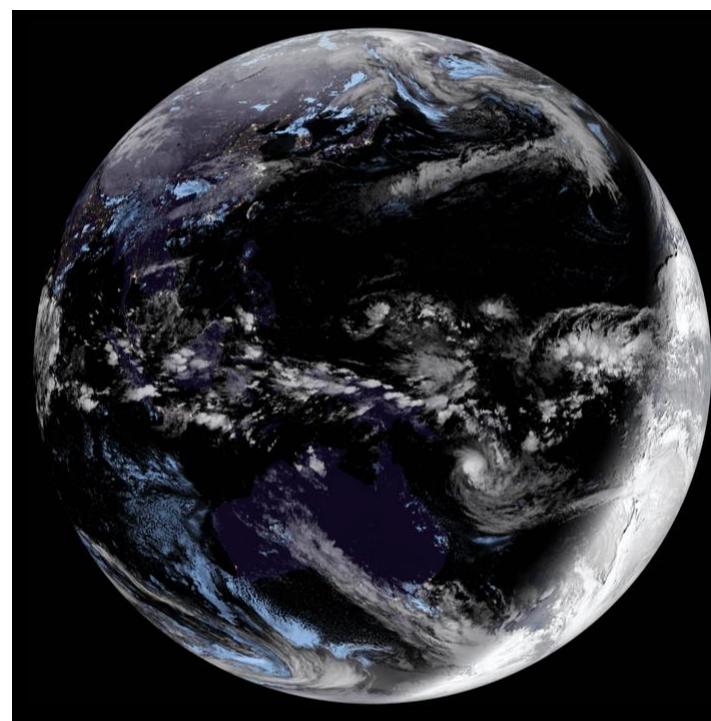
image courtesy of NOAA

Global coverage

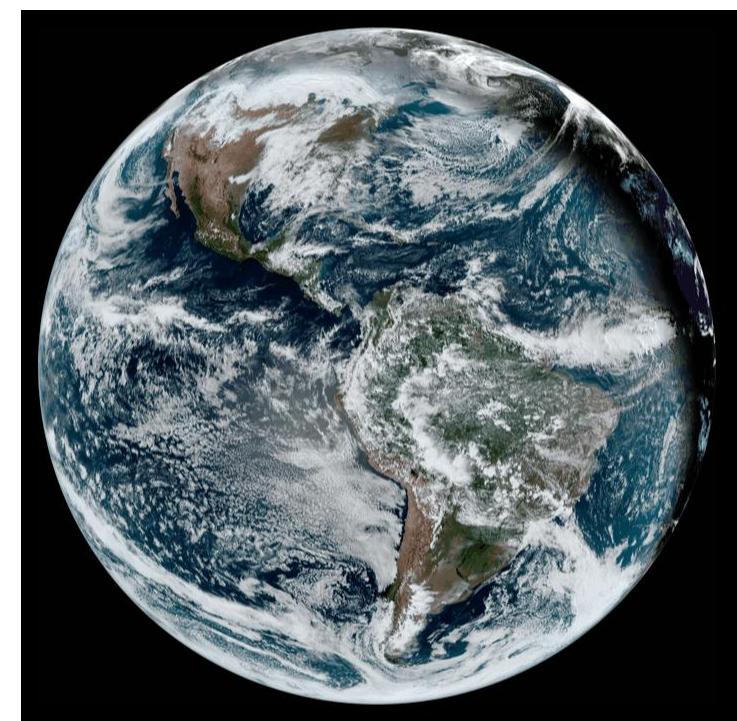
Meteosat-9 (45.5 E)



Himawari-9 (140.7 E)



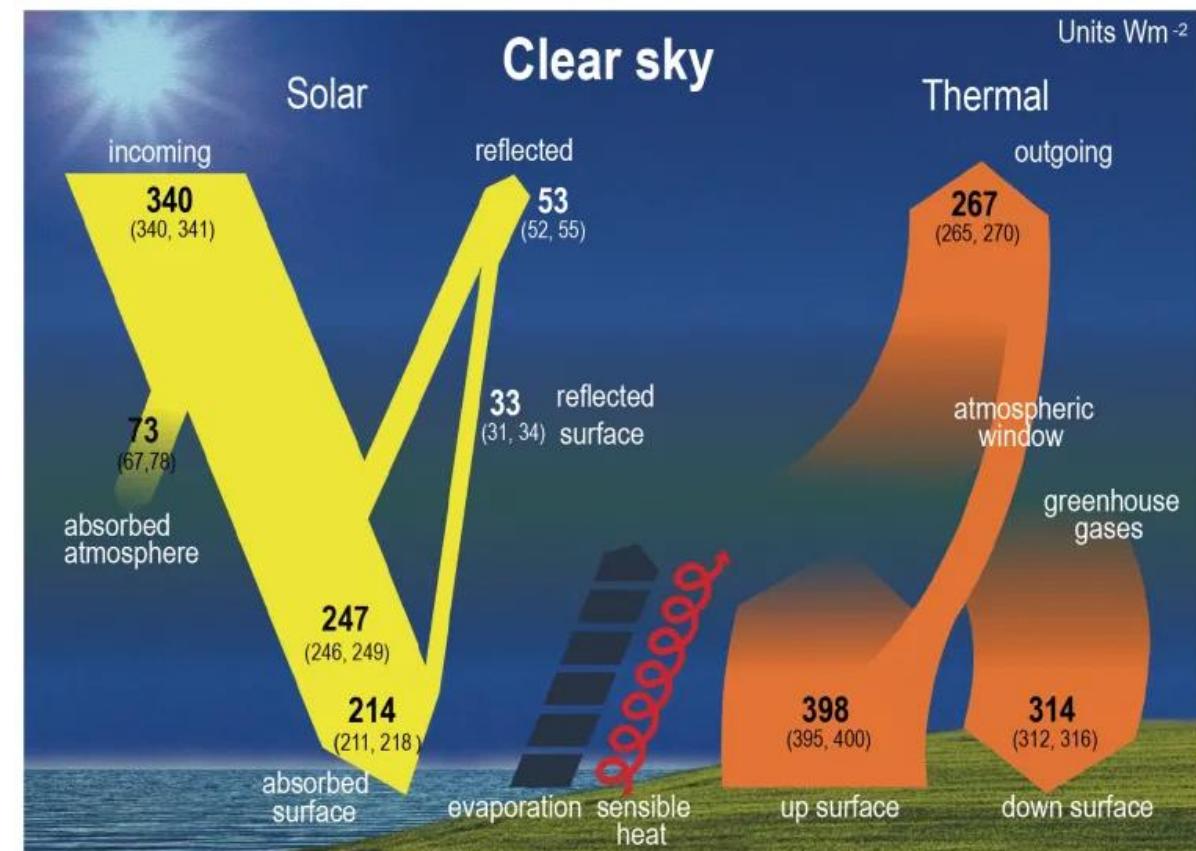
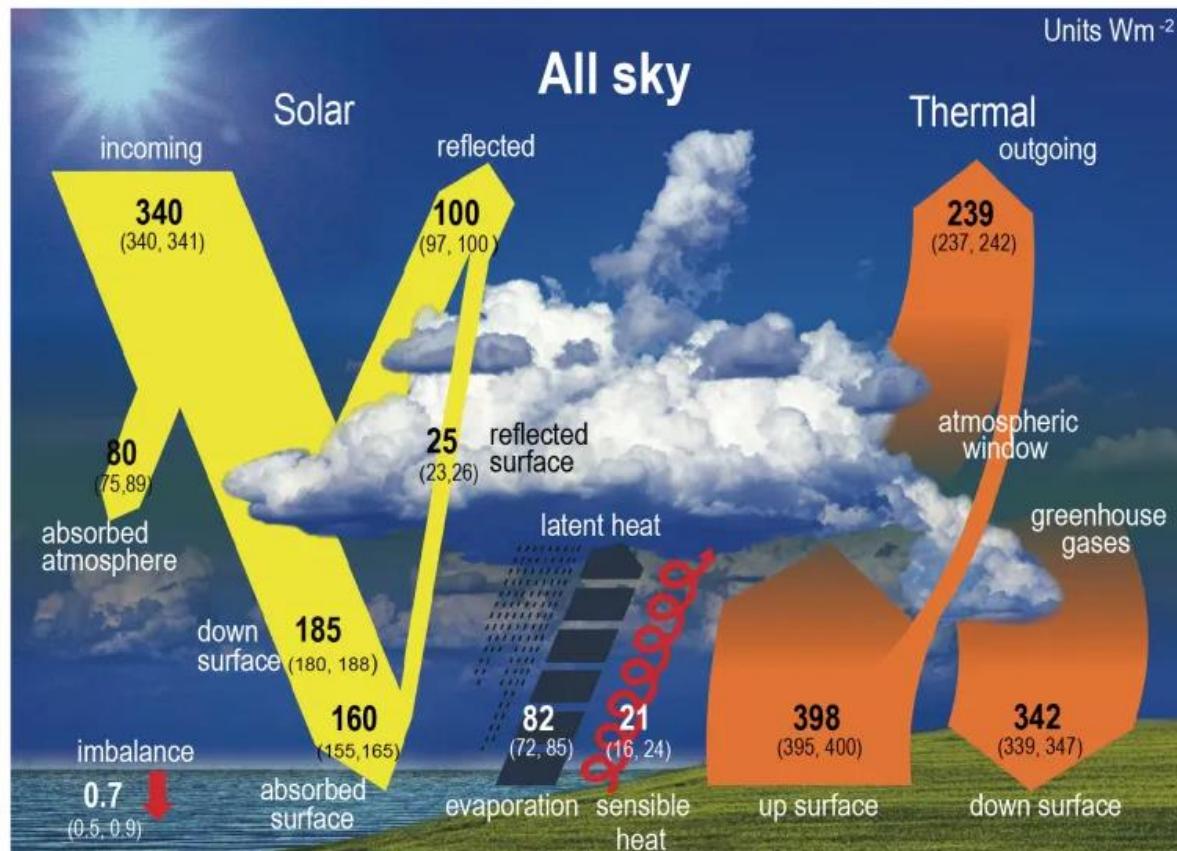
GOES-16 (75.2 W)



2023/12/08 18 UTC ~ 2023/12/08 23 UTC

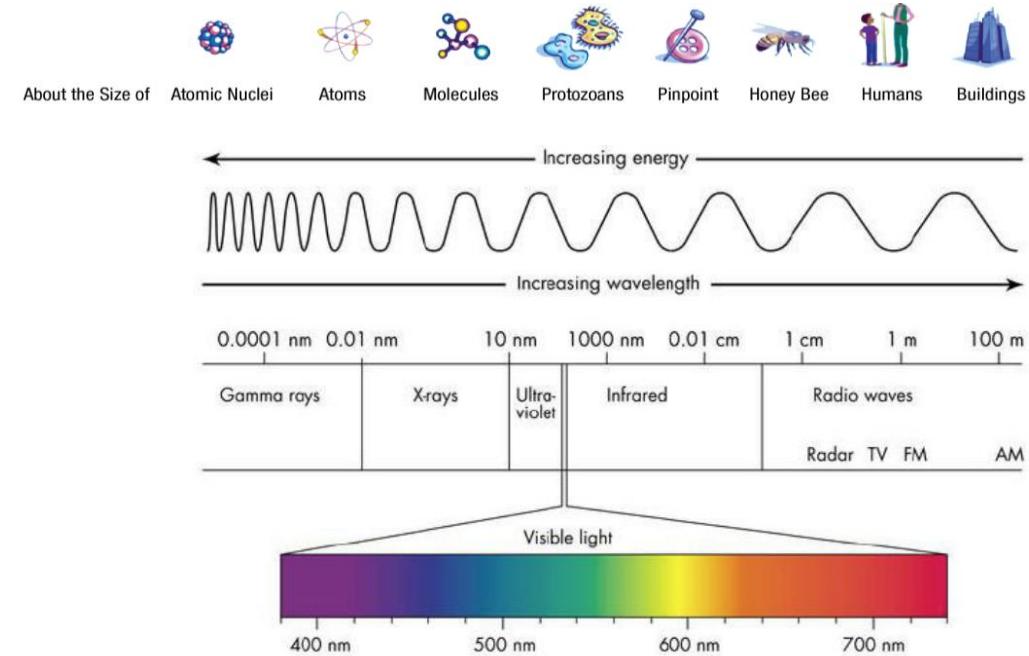
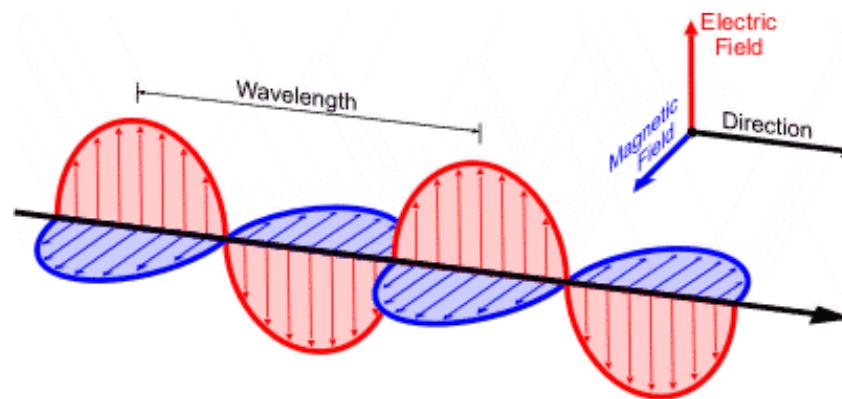
image courtesy of CIRA

Earth's Energy Budget



Scientists have been able to document global incoming and outgoing radiation averages, which provide an understanding of how energy is absorbed, reflected, and released by Earth's atmosphere, clouds, and surface. The numbers in parentheses represent the uncertainty range, or variability, associated with these averages.

Electromagnetic spectrum

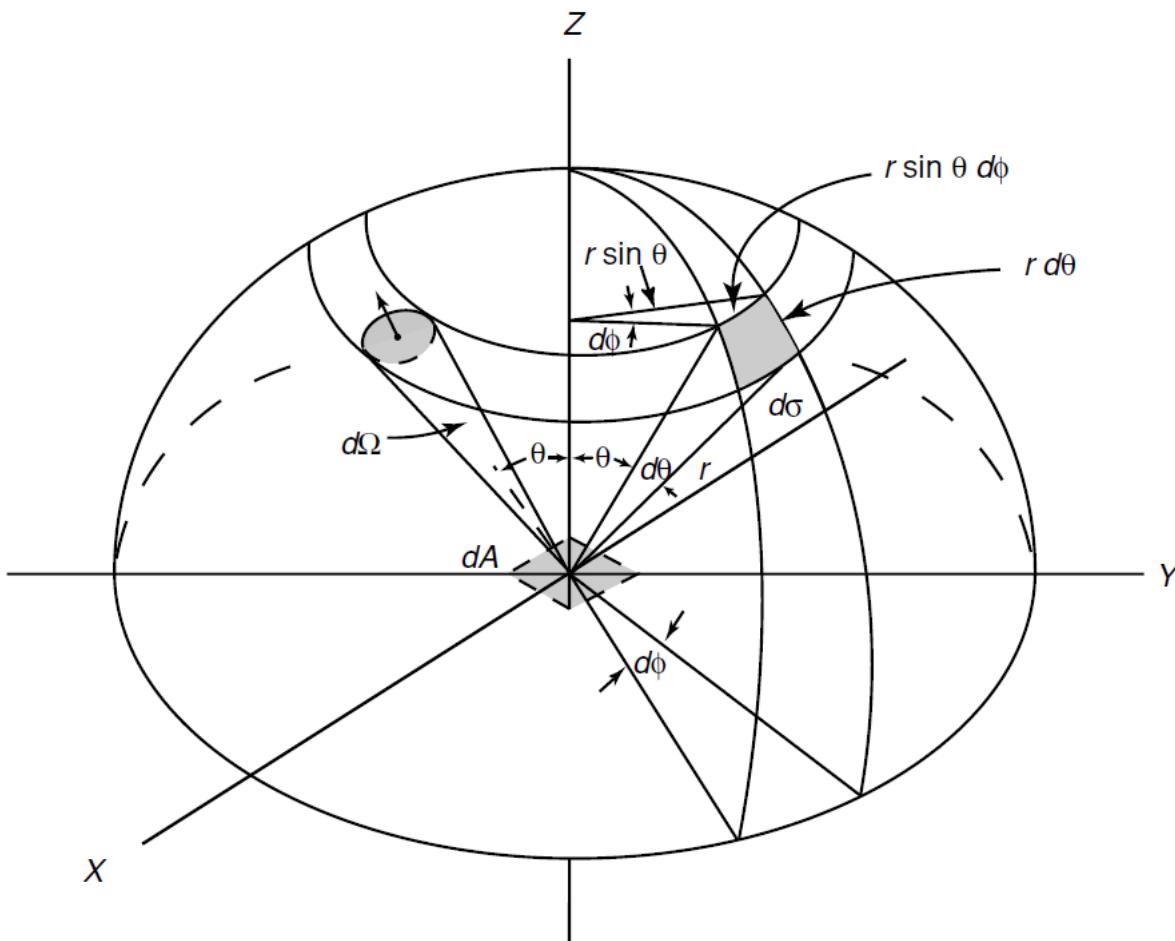


$$\lambda = \frac{c}{v}$$

where:

- λ = wavelength (m)
- v = frequency (Hz)
- c = speed of light (3×10^8 m/s)

Solid angle



$$\Omega = \frac{\sigma}{r^2} \quad \text{unit: sr (steradian)}$$

The differential surface area in spherical coordinates is

$$d\sigma = (rd\vartheta)(rsin\vartheta d\varphi)$$

The differential solid angle is then

$$d\Omega = \frac{d\sigma}{r^2} = \sin\vartheta d\vartheta d\varphi$$

where ϑ and φ are *zenith angle* and *azimuthal angle* in the polar coordinates, respectively.

Radiance versus Irradiance

The monochromatic intensity (or radiance, unit: $\text{W} \cdot \text{m}^{-2} \cdot \text{sr}^{-1}$) can be defined as

$$I_\lambda = \frac{dE_\lambda}{\cos\theta dA d\Omega d\lambda dt}$$

where E_λ is the monochromatic energy (units: J), θ is the angle of the differential solid angle $d\Omega$ from the normal to the surface area dA . Thus $\cos\theta dA$ denotes the effective area at which the energy is being intercepted.

The monochromatic flux density (or irradiance, unit: $\text{W} \cdot \text{m}^{-2}$) is the integral of solid angle over the hemisphere. In the polar coordinates,

$$F_\lambda = \int_{\Omega} I_\lambda \cos\theta d\Omega = \int_0^{2\pi} \int_0^{\pi/2} I_\lambda(\theta, \phi) \cos\theta \sin\theta d\theta d\phi$$

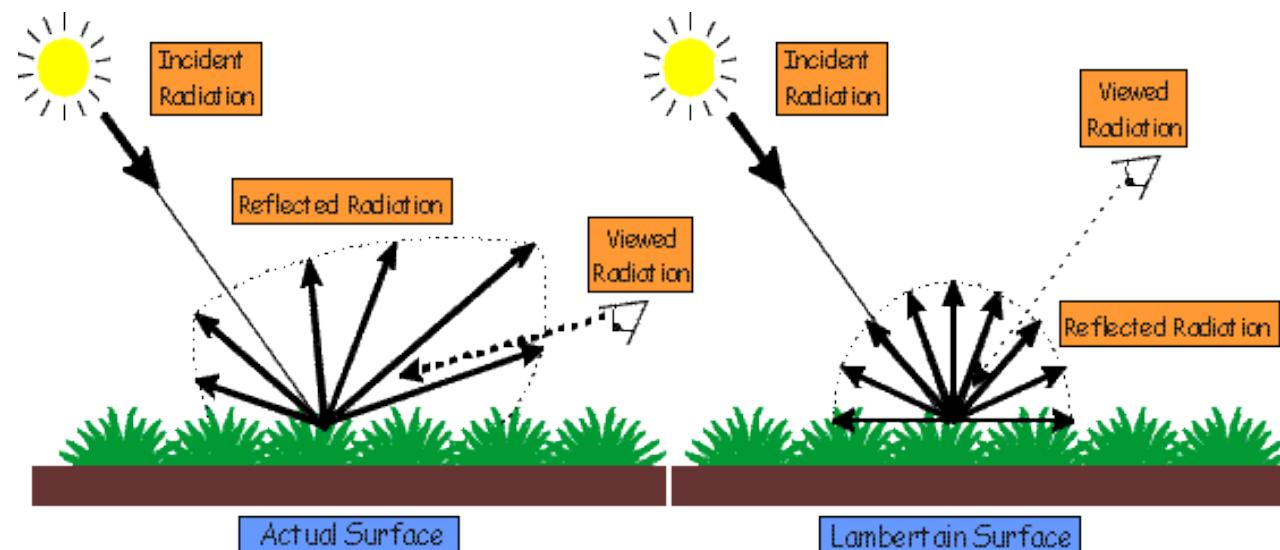
For isotropic radiation, the monochromatic flux density can be calculated by,

$$F_\lambda = \pi \cdot I_\lambda$$

When the irradiance is from an emitting surface, the quantity is called the *emittance*. When it is the reflected radiation, the previous plot can also be used as the scattering geometry of a scene that can be described by the *BRDF*.

BRDF

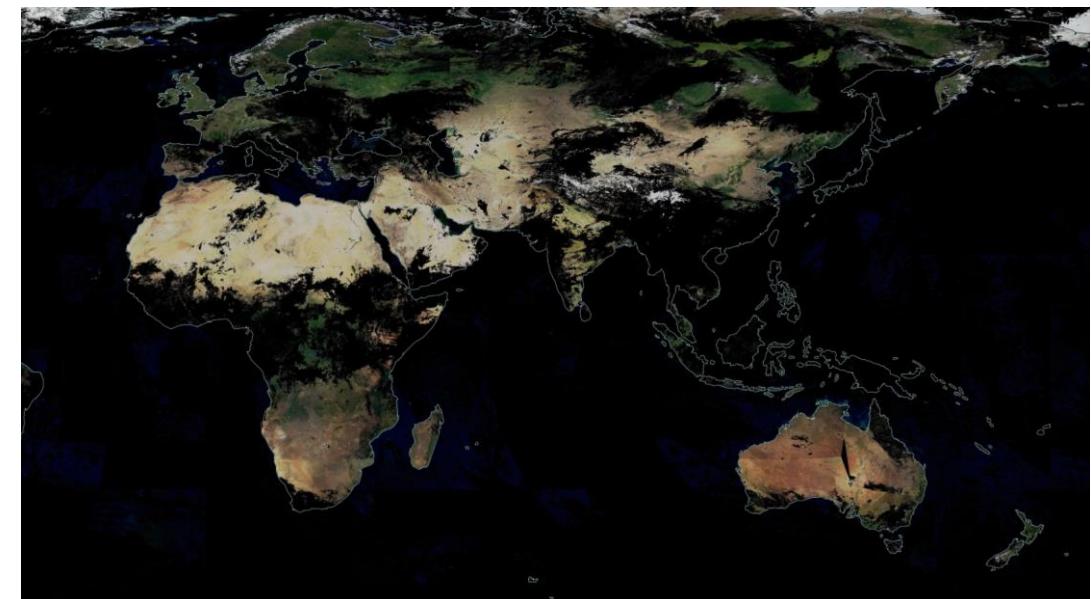
- BRDF (bidirectional reflectance distribution function, $f_r(\omega_i, \omega_r)$, unit: sr) - a function of four real variables that defines how light is reflected at an opaque surface.
- In general, the reflectance factor depends on the view angle and the solar angle. Since two directions are involved, the term bidirectional reflectance factor (BRDF, f_r) is used.
- $f_r(\theta_i, \phi_i; \theta_r, \phi_r)$ takes an incoming light direction, (θ_i, ϕ_i) , and outgoing direction, (θ_r, ϕ_r) , and return the ratio of reflected radiance along (θ_r, ϕ_r) of an ideal Lambertian surface to the irradiance incident on the surface.



BRDF

The BRDF is a fundamental radiometric concept, NASA uses a BRDF model to characterise surface reflectance anisotropy. For a given land area (or scene type), the BRDF is established based on selected multi-angular observations of surface (scene) reflectance.

- While single observations depend on view geometry and solar angle, the MODIS BRDF/Surface-Albedo product describes intrinsic surface properties in several spectral bands, at a resolution of 500 meters. The BRDF/Albedo product can be used to model surface albedo depending on atmospheric scattering.
- Correspondingly, the CERES Angular Distribution Models (ADMs, <https://ceres.larc.nasa.gov/data/angular-distribution-models/>) describe the unique anisotropic characteristics of different scene types at broadband SW and LW. Its irradiance product is widely used in climate and energy applications to provide most accurate estimate of global/regional radiation budget.



MAIAC BRDF-adjusted reflectance (Jun 14, 2023)

SW cloudy ocean

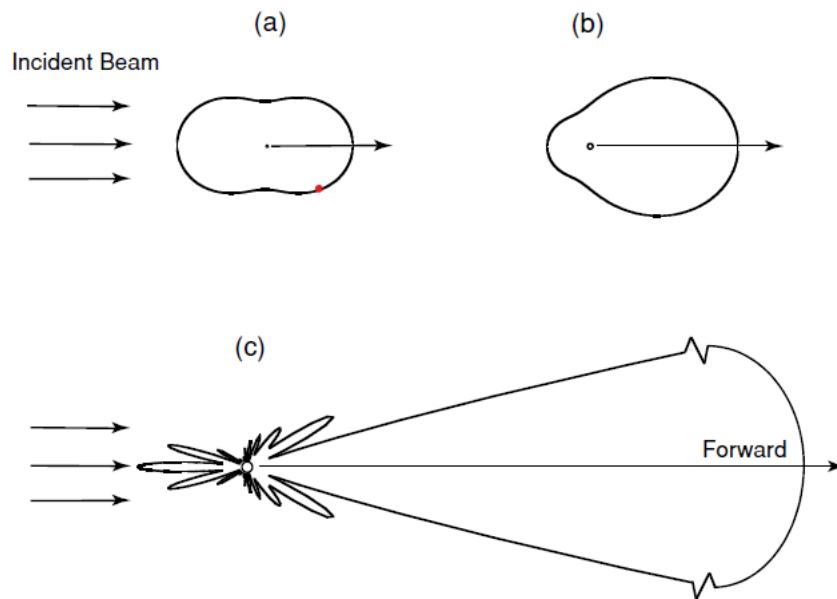
Non-glint	Glint	Definition	Comments
80	680	Clouds over ocean; $1.00 < \text{phase} \leq 1.01$	Liquid
81	681	Clouds over ocean; $1.01 < \text{phase} \leq 1.75$	Mixed
82	682	Clouds over ocean; $\text{phase} > 1.75$	Ice
99	699	Cloudy mostly water + land/desert/snow	

CERES Edition 4 ADMs for SW cloudy ocean scene type

Scattering

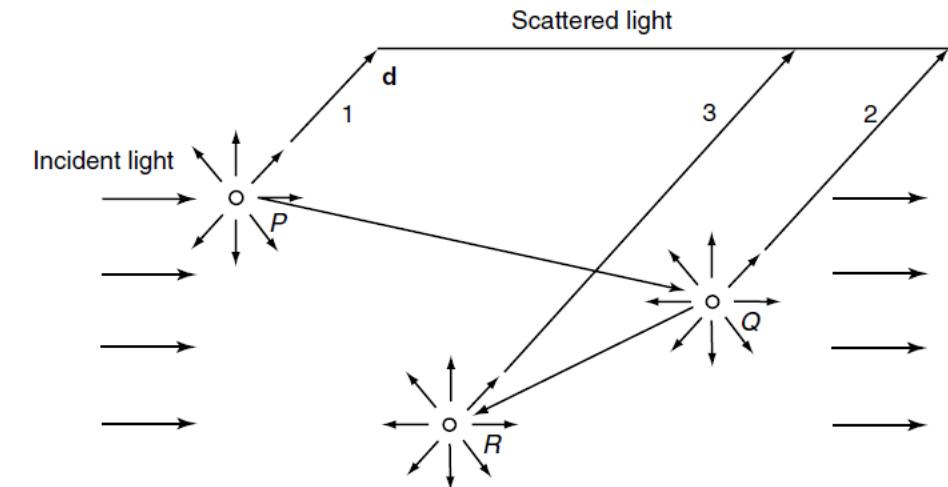
Scattering is a physical process by which a particle in the path of an electromagnetic wave continuously abstracts energy from the incident wave and reradiates that energy in all directions.

$$\text{size parameter } (x) = 2\pi a/\lambda$$



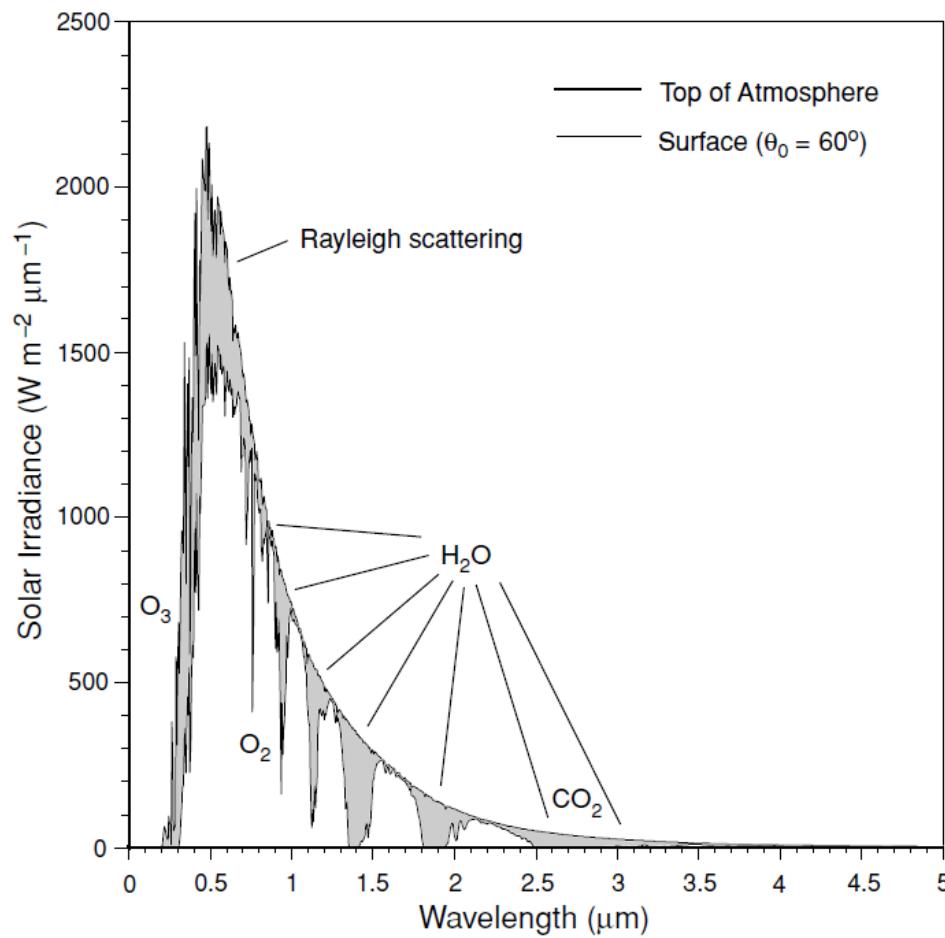
Demonstrative angular patterns of the scattered intensity from spherical aerosols of three sizes illuminated by the visible light ($0.5 \mu\text{m}$):

- (a) Aerosol size of $10^{-4} \mu\text{m}$;
- (b) Aerosol size of $0.1 \mu\text{m}$;
- (c) Aerosol size of $1 \mu\text{m}$.



Multiple scattering process involving first (P), second (Q), and third (R) order scattering in the direction denoted by \mathbf{d} .

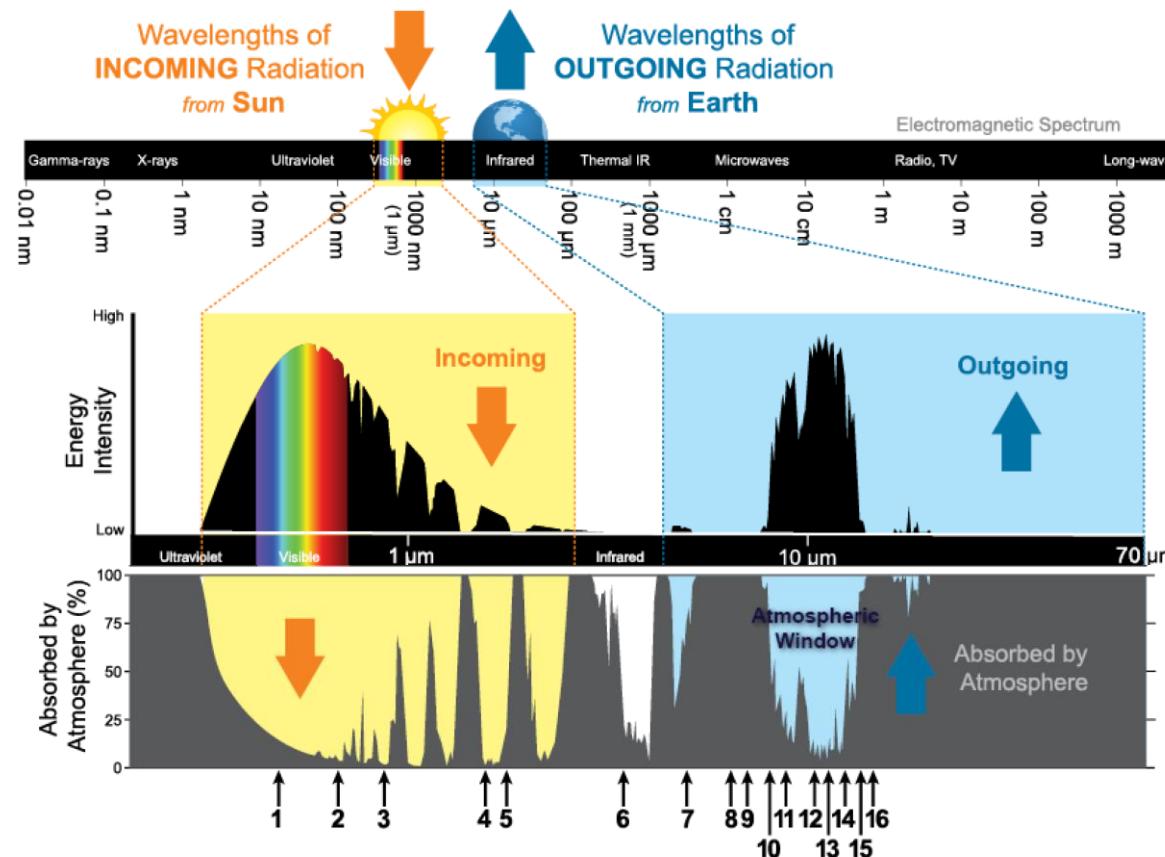
Absorption



Solar Absorption Bands of Atmospheric Gases			
Species	Band (μm)	$\Delta\nu (\text{cm}^{-1})$	Major transitions
H_2O	3.2, 2.7	2500–4500	020, 001, 100
	1.87	4800–6200	110, 011
	1.38	6400–7600	021, 200, 101
	1.1	8200–9400	111
	0.94	10,100–11,300	121, 201, 003
	0.82	11,700–12,700	211
	0.72	13,400–14,600	221, 202, 301
	Visible	15,000–22,600	500, 203
	4.3	2000–2400	00 ⁰ 1
	2.7	3400–3850	10 ⁰ 1
CO_2	2.0	4700–5200	20 ⁰ 1
	1.6	6100–6450	30 ⁰ 1
	1.4	6850–7000	00 ⁰ 3
	4.74	2000–2300	101
	3.3	3000–3100	003
O_3	Visible	10,600–22,600	—
	UV	29,000–50,000	—
	1.58	6300–6350	a \leftarrow X (0 \leftarrow 1)
O_2	1.27	7700–8050	a \leftarrow X (0 \leftarrow 0)
	1.06	9350–9400	a \leftarrow X (1 \leftarrow 0)
	0.76(A)	12,850–13,200	b \leftarrow X (0 \leftarrow 0)
$\text{O}_2 \cdot \text{O}_2$	0.69(B)	14,300–14,600	b \leftarrow X (1 \leftarrow 0)
	0.63(γ)	14,750–15,900	b \leftarrow X (2 \leftarrow 0)
	Visible	7600–30,000	—
$\text{O}_2 \cdot \text{N}_2$	1.26	7600–8300	—
	4.5	2100–2300	00 ⁰ 1
	4.06, 3.9	2100–2800	12 ⁰ 0, 20 ⁰ 0
N_2O	2.97, 2.87	3300–3500	02 ⁰ 1, 10 ⁰ 1
	3.83, 3.53	—	—
	3.31, 3.26	2500–3200	0002, 0101, 0200
CH_4	2.37, 2.30	—	—
	2.20	4000–4600	1001, 0011, 0110
	1.66	5850–6100	0020
CO	4.67	2000–2300	1
	2.34	4150–4350	2
NO_2	Visible	14,400–50,000	—

K. N. Liou 2002

Absorption



- Most of the Sun's energy comes from visible light and the near infrared portion of the electromagnetic spectrum.
- All the outgoing energy emitted by the Earth is infrared.
- The dips in the incoming and outgoing energy are where the atmosphere absorbs energy.
- Some of the incoming energy is absorbed by the atmosphere, whereas most of the infrared energy emitted by the Earth is absorbed.

Blackbody Radiation Laws

- The laws of blackbody radiation are basic to an understanding of the absorption and emission processes.
- A *blackbody* is a perfect emitter and absorber of radiation.
- The monochromatic intensity (B_λ , units: $W m^{-2} sr^{-1} \mu m^{-1}$) emitted by a blackbody is described by the **Planck function**,

$$B_\lambda = \frac{2hc^2}{\lambda^5 [\exp\left(\frac{hc}{k\lambda T}\right) - 1]}$$

where k is the Boltzmann constant, h is the Planck's constant, and c is the speed of light in a vacuum.

- Because bb radiation is isotropic, and we know that for isotropic radiation, flux is simply the intensity times π , therefore the

monochromatic flux emitted by a bb is $F_\lambda = \frac{2\pi hc^2}{\lambda^5 [\exp\left(\frac{hc}{k\lambda T}\right) - 1]}$

- If we define two constants, $c_1 = 2\pi hc^2$ and $c_2 = hc/k$, then Planck's law can be written as

$$F_\lambda = \frac{c_1}{\lambda^5 [\exp\left(\frac{c_2}{\lambda T}\right) - 1]}$$

Blackbody Radiation Laws

- To get the wavelength of maximum emission for a blackbody, we can use *Wien's displacement law*.
 - The law is found by setting

$$\frac{\partial F_{\lambda}^{BB}}{\partial \lambda} = 0$$

- and solving for λ to get

$$\lambda_{max}^{BB} = \frac{2897 \mu m K}{T}$$

- If we do integration over the wavelength, we then get the *Stefan-Boltzmann law* that gives the total flux emitted from a blackbody,

$$F^{BB} = \int_0^{\infty} F_{\lambda}^{BB} d\lambda = \sigma T^4$$

where σ is the Stefan-Boltzmann constant, $5.67 \times 10^{-8} W m^{-2} K^{-4}$.

Absorptivity and emissivity

- Non-blackbodies emit less radiation than do blackbodies.
- The ratio of emitted monochromatic flux to blackbody monochromatic flux is known as the monochromatic emissivity, ε_λ (0-1)

$$\varepsilon_\lambda \equiv F_\lambda / F_\lambda^{\text{BB}}$$

- Similarly, the ratio of total emitted flux to blackbody total flux is known as the *gray-body emissivity*, ε .
- Applying the *Stefan-Boltzmann law* to a gray body, the emitted flux is then

$$F = \varepsilon F^{\text{BB}} = \varepsilon \sigma T^4$$

- Correspondingly, the ratio of absorbed flux to blackbody absorbed flux is known as the gray-body *absorptivity*, a .
- According to the **Kirchoff's law**, an object's emissivity equals to its absorptivity.

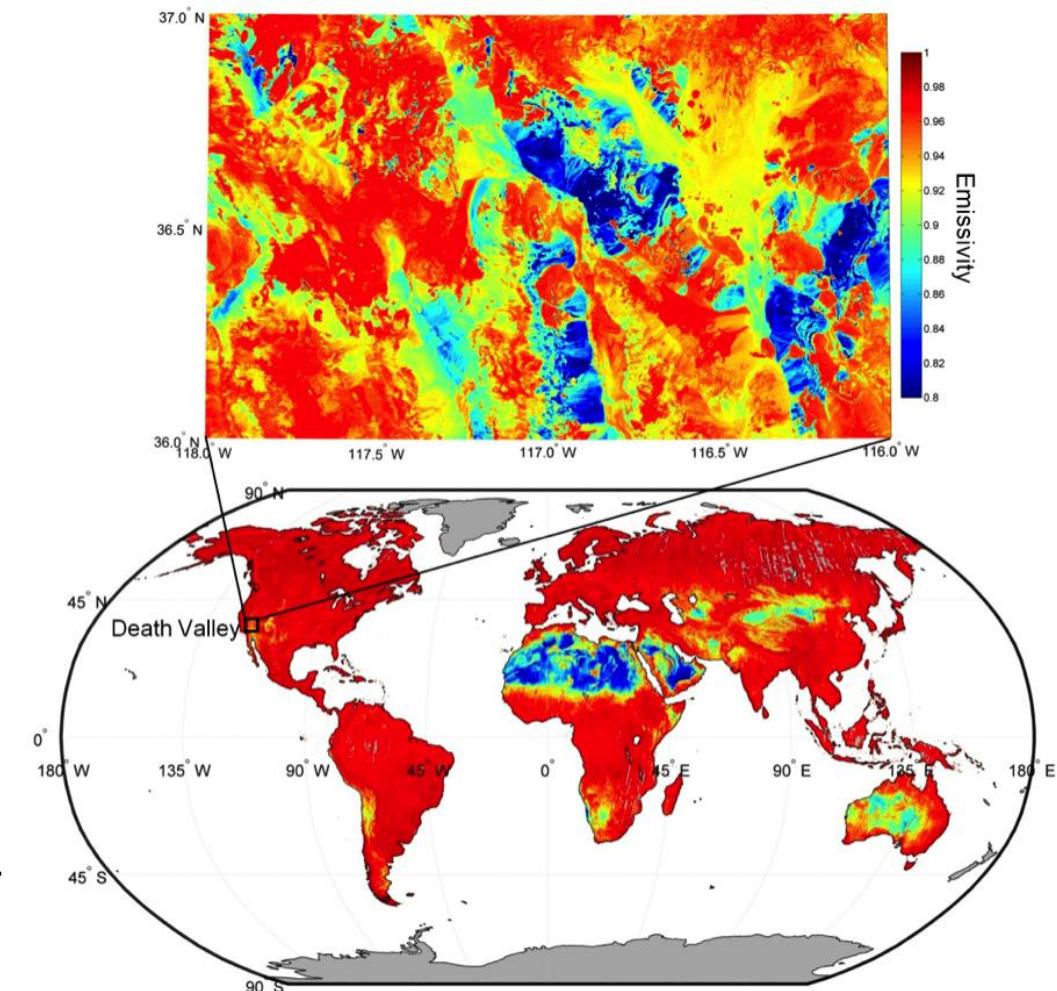


Image Credit: NASA/GSFC/METI/ERSDAC/JAROS, and
U.S./Japan ASTER Science Team

Extinction

- ***extinction*** is a result of scattering plus absorption – scattering will be the sole process of extinction in a nonabsorbing medium.
- ***cross section*** is often used to denote the amount of energy removed from the original beam by the particle.

- when it is associated with a particle dimension, called *volume extinction cross section* (β_λ), units are cm^2 ;

$$\beta_\lambda = \beta_\lambda^a + \beta_\lambda^s$$

- when it is referenced to unit mass, called *mass extinction cross section* (k_λ), units are cm^2g^{-1} .

$$k_\lambda = k_\lambda^a + k_\lambda^s$$

- The *extinction coefficient* (units: cm^{-1}) equals to β_λ times particle number density (cm^{-3}) or, k_λ times the particle density (g cm^{-3}).
- In the infrared radiative transfer, k_λ^a is simply referred to as the *absorption coefficient*.

- ***single scattering albedo*** ($\tilde{\omega}$) is the ratio of scattering efficiency to total extinction efficiency, defined as,

$$\tilde{\omega} = \frac{\beta_\lambda^s}{\beta_\lambda} = \frac{k_\lambda^s}{k_\lambda}$$

- ***optical depth*** (τ) defines the opaqueness of two points (s , and s_1) for radiation passing through it, defined as,

$$\tau_\lambda(s_1, s) = \int_s^{s_1} k_\lambda \rho ds'$$

Basic radiative transfer equation

When radiation traversing a medium, its intensity will be weakened by its interaction with matter (scattering and absorption),

$$dI_\lambda = -k_\lambda \rho I_\lambda ds$$

At the same time, the radiation's intensity could be strengthened by emission from the material plus multiple scattering from all other directions,

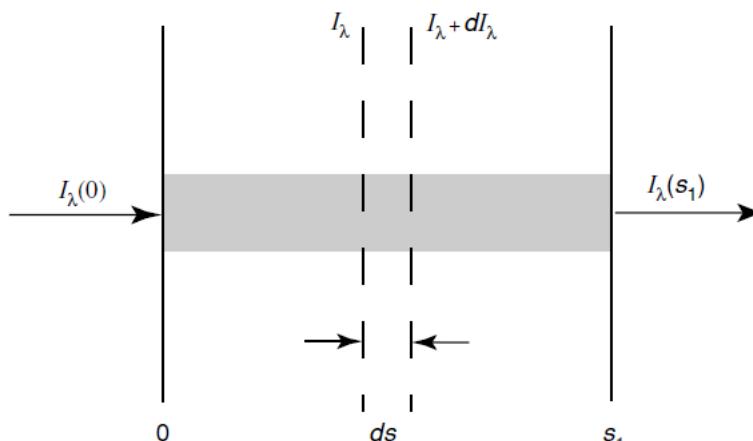
$$dI_\lambda = j_\lambda \rho ds$$

j_λ is the **source function coefficient** such that the increase in intensity due to emission and multiple scattering. Combining two components we have,

$$dI_\lambda = -k_\lambda \rho I_\lambda ds + j_\lambda \rho ds$$

For simplicity, it is convenient to define the **source function** J_λ to have the units of radiant intensity, $J_\lambda \equiv j_\lambda/k_\lambda$ such that we derive the **general radiative transfer equation** without attaching to any coordinate system,

$$\frac{dI_\lambda}{k_\lambda \rho ds} = -I_\lambda + J_\lambda \quad (1)$$



RTE with no Scattering or Emission

For a light beam travels through a homogeneous medium where the contribution to the beam by scattering and emission is negligible (J_λ),

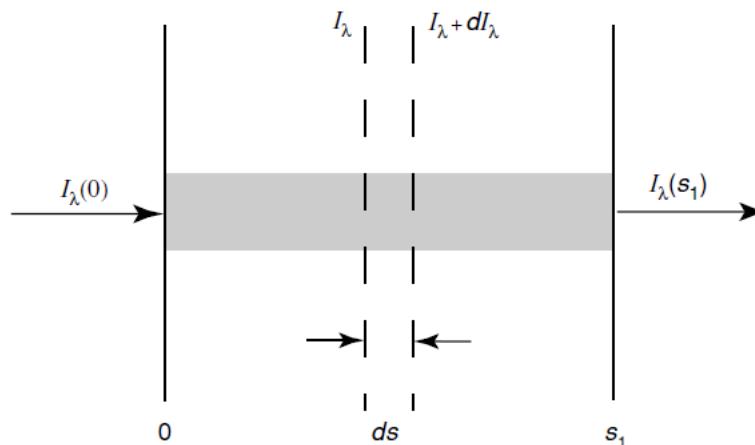
$$\frac{dI_\lambda}{k_\lambda \rho ds} = -I_\lambda$$

Let the incident intensity at $s = 0$ be $I_\lambda(0)$. Then the emergent intensity at point s_1 can be obtained by integrating the distance,

$$I_\lambda(s_1) = I_\lambda(0) \exp\left(-\int_0^{s_1} k_\lambda \rho ds\right)$$

Because the media is homogeneous, so k_λ is independent of the distance s , let the **path length** $u = \int_0^{s_1} \rho ds$,

$$I_\lambda(s_1) = I_\lambda(0) e^{-k_\lambda u} \quad (2)$$



The above equation is referred to as *Beer's law or Bouguer's law or Lambert's law*, which states that the decrease of the radiant intensity traversing a homogeneous extinction medium is in accord with the simple exponential function. $e^{-k_\lambda u}$ is also known as the monochromatic transmissivity, T_λ .

RTE with no Scattering

Next, let's consider a non-scattering medium which acts as a blackbody and is in local thermodynamic equilibrium. A beam passing through it will undergo the absorption and emission processes, in this case,

$$J_\lambda = B_\lambda(T)$$

and the RTE can be written as,

$$\frac{dI_\lambda}{k_\lambda \rho ds} = -I_\lambda + B_\lambda(T)$$

Recall the definition of optical depth (τ_λ), we can substitute $k_\lambda \rho ds$ with $-d\tau(s_1, s)$,

$$-\frac{dI_\lambda(s)}{d\tau(s_1, s)} = -I_\lambda(s) + B_\lambda[T(s)]$$

Rearranging this equation and integrating the thickness ds from 0 to s_1 , we can obtain,

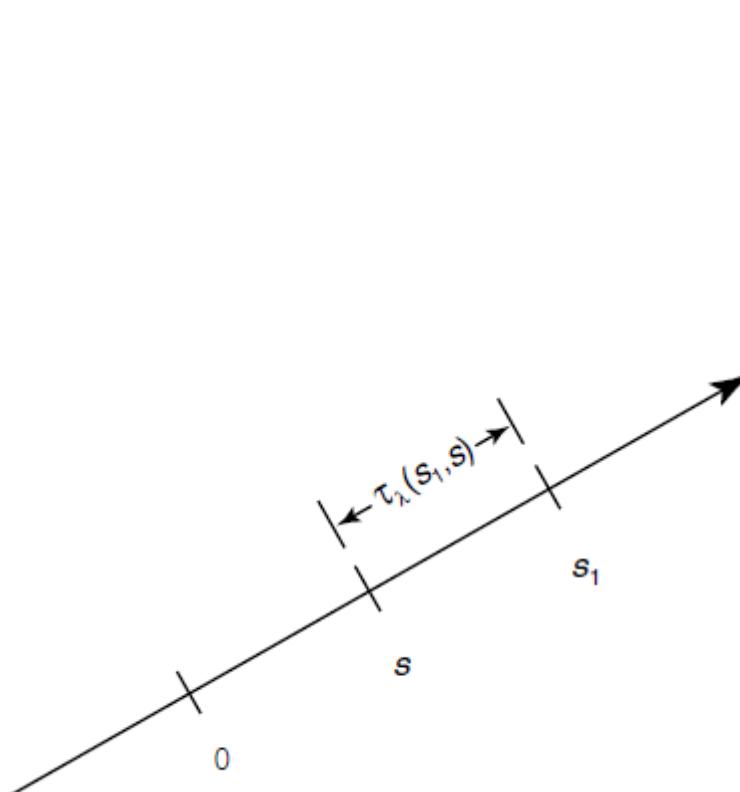
$$I_\lambda(s_1) = I_\lambda(0)e^{-\tau_\lambda(s_1, 0)} + \int_0^{s_1} B_\lambda[T(s)]e^{-\tau_\lambda(s_1, s)} k_\lambda \rho ds \quad (3)$$

Configuration of the optical thickness τ_λ

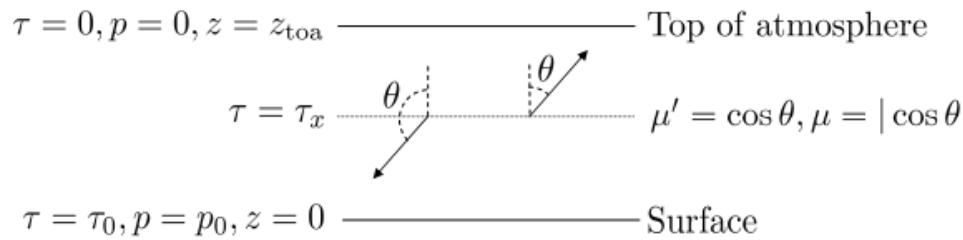
Absorption attenuation term

Schwarzschild's equation

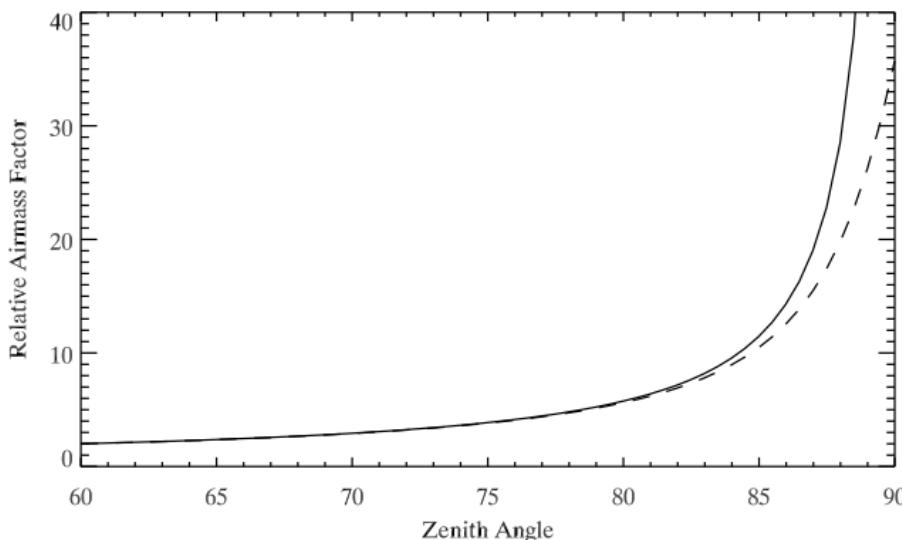
Emission contribution term



RTE for Plane-Parallel atmospheres



Plane-parallel coordinates



A plane-parallel atmosphere assumes,

- Curvature associated with sphericity of the Earth is ignored.
- The medium is regarded as horizontally homogeneous and the radiation field horizontally isotropic.

In this case, the RTE can be rewritten as,

$$\cos \theta \frac{dI_\lambda(z; \theta, \phi)}{k_\lambda \rho dz} = -I_\lambda(z; \theta, \phi) + J_\lambda(z; \theta, \phi)$$

where θ denotes the inclination to the upward normal, ϕ the azimuthal angle. Introducing the normal optical depth by measuring downward from the outer boundary (i.e., TOA),

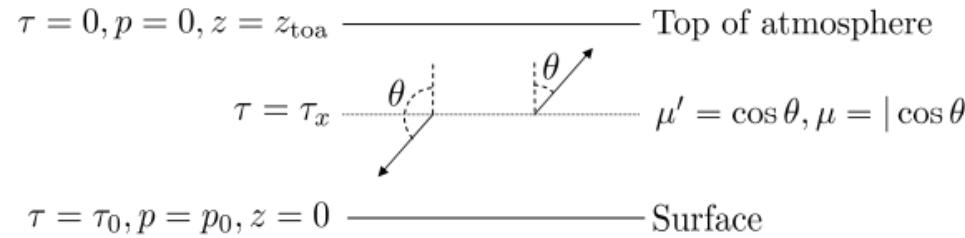
$$\tau = \int_z^\infty k_\lambda \rho dz'$$

we show the basic equation for the problem of multiple scattering in plane-parallel atmospheres (τ -level),

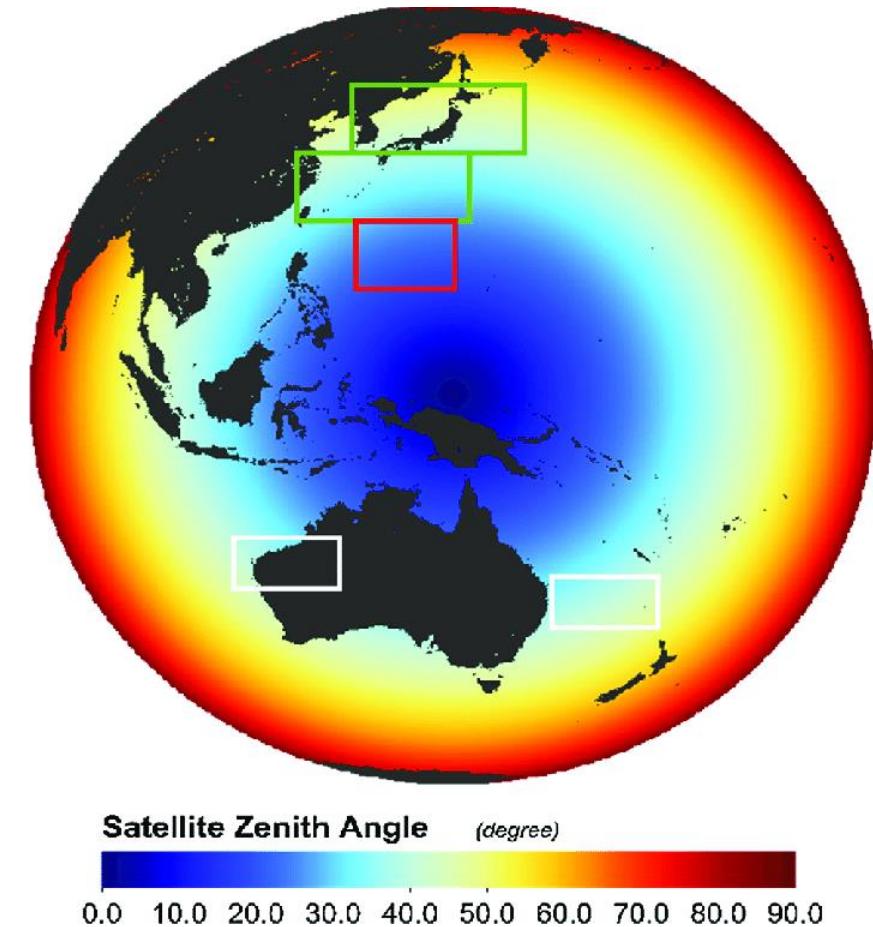
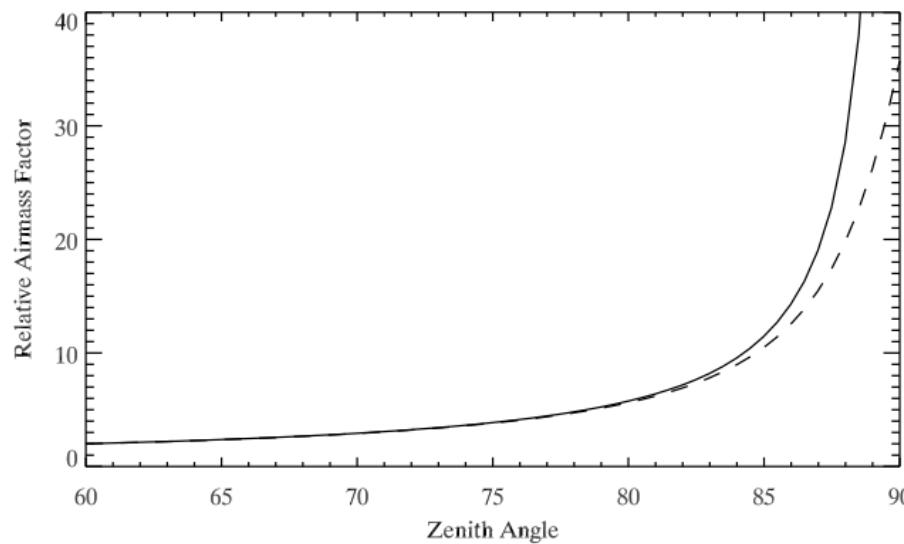
$$\mu \frac{dI_\lambda(\tau; \theta, \phi)}{d\tau} = I_\lambda(\tau; \mu, \phi) - J_\lambda(\tau; \mu, \phi) \quad (4)$$

where $\mu = \cos \theta$.

RTE for Plane-Parallel atmospheres

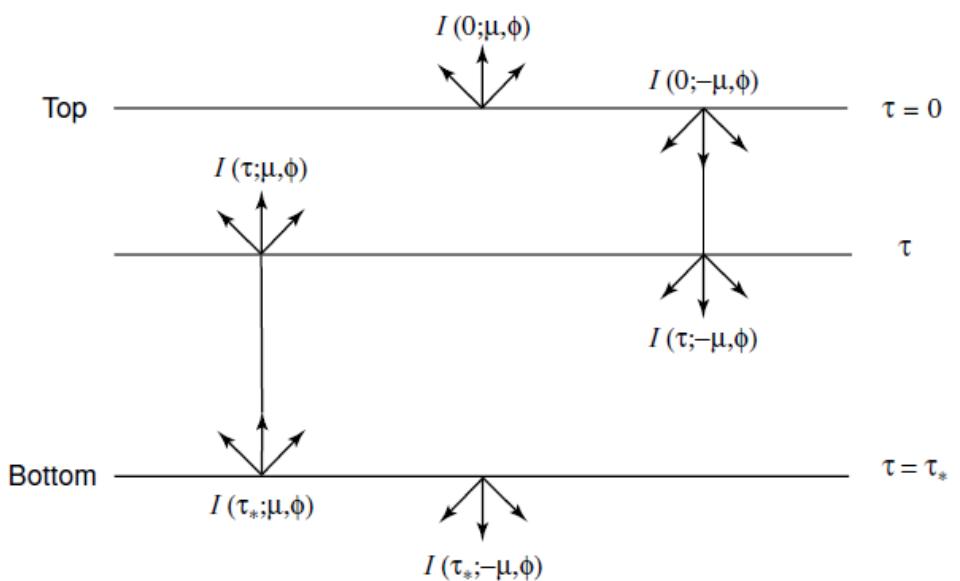


Plane-parallel coordinates



Full disk view of earth as seen from Himawari-8

RTE for Plane-Parallel atmospheres



By applying the Schwarzschild's equation, $\mu \frac{dI_\lambda(\tau; \theta, \phi)}{d\tau} = I_\lambda(\tau; \mu, \phi) + J_\lambda(\tau; \mu, \phi)$ can be solved to give the upward and downward intensities for a finite atmosphere that is bounded on two sides at $\tau = 0$ and $\tau = \tau_*$.

To get the upward ($\mu > 0$) radiance at level τ , multiply by $e^{-\tau/\mu}$ and integrate from τ to $\tau = \tau_*$,

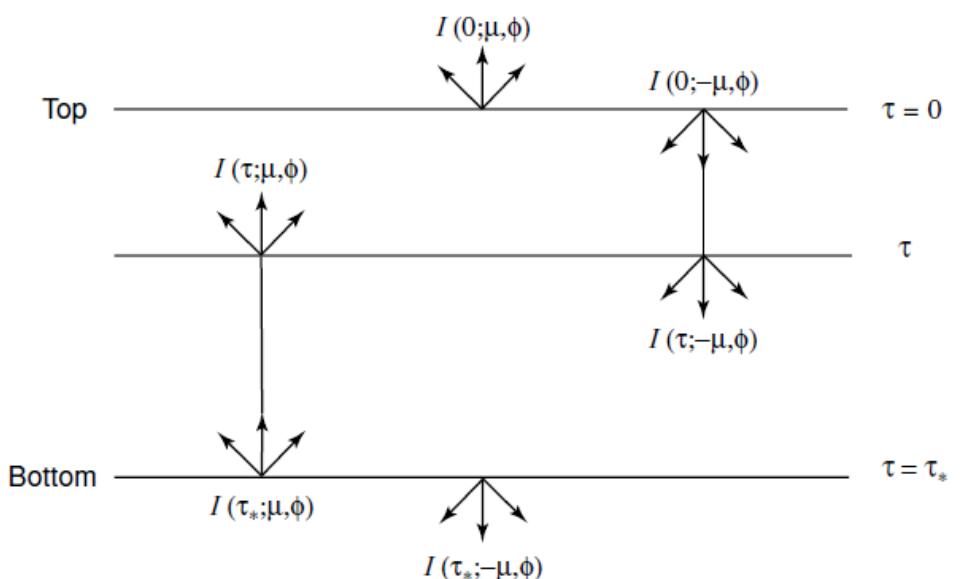
$$I_\lambda(\tau; \mu, \phi) = I_\lambda(\tau_*; \mu, \phi) e^{-(\tau_* - \tau)/\mu} + \frac{1}{\mu} \int_{\tau}^{\tau_*} J_\lambda(\tau'; \mu, \phi) e^{-(\tau' - \tau)/\mu} d\tau'$$

To get the downward ($\mu < 0$) radiance at level τ , multiply by $e^{\tau/\mu}$ where μ is replaced by $-\mu$, then integrate from $\tau = 0$ to τ ,

$$I_\lambda(\tau; -\mu, \phi) = I_\lambda(0; -\mu, \phi) e^{-\tau/\mu} + \frac{1}{\mu} \int_0^{\tau} J_\lambda(\tau'; -\mu, \phi) e^{-(\tau - \tau')/\mu} d\tau'$$

The $I_\lambda(\tau_*; \mu, \phi)$ and $I_\lambda(0; -\mu, \phi)$ represent the inward source radiance intensities at the bottom and top surfaces, respectively.

RTE for Plane-Parallel atmospheres



By applying the Schwarzschild's equation, $\mu \frac{dI_\lambda(\tau; \theta, \phi)}{d\tau} = I_\lambda(\tau; \mu, \phi) + J_\lambda(\tau; \mu, \phi)$ can be solved to give the upward and downward intensities for a finite atmosphere that is bounded on two sides at $\tau = 0$ and $\tau = \tau_*$.

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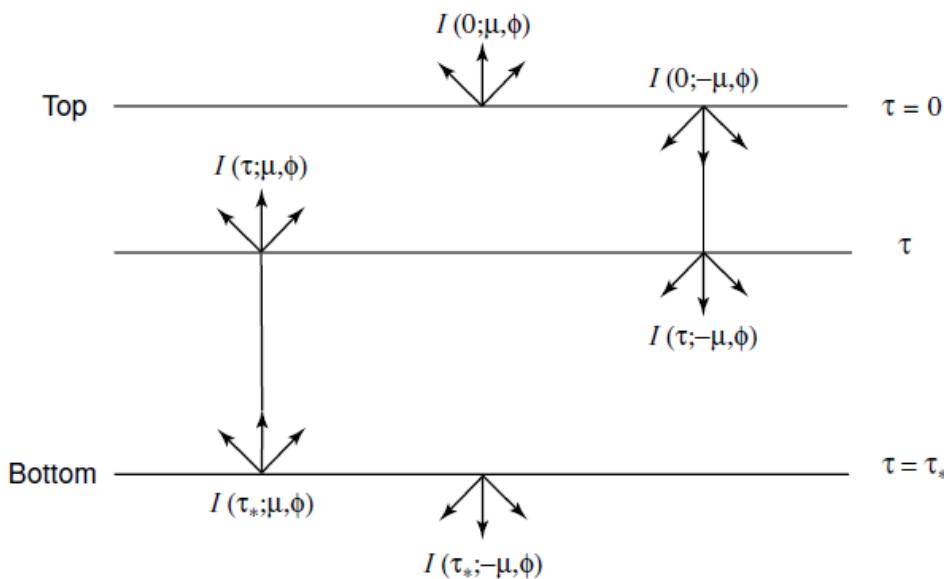
$$I_\lambda(\tau; \mu, \phi) = I_\lambda(\tau_*; \mu, \phi) e^{-(\tau_* - \tau)/\mu} + \frac{1}{\mu} \int_{\tau}^{\tau_*} J_\lambda(\tau'; \mu, \phi) e^{-(\tau' - \tau)/\mu} d\tau'$$

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$$I_\lambda(\tau; -\mu, \phi) = I_\lambda(0; -\mu, \phi) e^{-\tau/\mu} + \frac{1}{\mu} \int_0^{\tau} J_\lambda(\tau'; -\mu, \phi) e^{-(\tau - \tau')/\mu} d\tau'$$

The $I_\lambda(\tau_*; \mu, \phi)$ and $I_\lambda(0; -\mu, \phi)$ represent the inward source radiance intensities at the bottom and top surfaces, respectively.

RTE for Plane-Parallel atmospheres



There are two special cases of the above equations that are most often used:

- ❖ Calculating the upward radiance at the top-of-atmosphere ($\tau = 0$).

$$I_\lambda(0; \mu, \phi) = I_\lambda(\tau_*; \mu, \phi) e^{-\tau_*/\mu} + \frac{1}{\mu} \int_{\tau}^{\tau_*} J_\lambda(\tau'; \mu, \phi) e^{-(\tau' - \tau)/\mu} d\tau' \quad (5)$$

- ❖ Calculating the downward radiance at the bottom of the atmosphere ($\tau = \tau_*$).

$$I_\lambda(\tau_*; -\mu, \phi) = I_\lambda(0; -\mu, \phi) e^{-\tau_*/\mu} + \frac{1}{\mu} \int_0^{\tau_*} J_\lambda(\tau'; -\mu, \phi) e^{-(\tau_* - \tau')/\mu} d\tau' \quad (6)$$

RTE for Plane-Parallel atmospheres

One of the key components in these two equations is the source function (J_λ), which consists of three components for Earth's atmosphere, diffuse field source ($J_\lambda^{\text{diffuse}}$), solar source (J_λ^{solar}), and internal thermal source ($J_\lambda^{\text{thermal}}$). The overall source function is then,

$$J_\lambda = J_\lambda^{\text{diffuse}} + J_\lambda^{\text{solar}} + J_\lambda^{\text{thermal}}$$

Using angular vector ω to replace the geometry components (θ, ϕ), let $F_\odot = E_\lambda^0(\omega_0)$ that is the incoming solar radiation at the TOA, these three source functions are defined as,

$$J_\lambda^{\text{diffuse}}(\tau, \omega) = \frac{\tilde{\omega}}{4\pi} \int_0^{4\pi} I_\lambda(\tau, \omega') P(\omega', \omega) d\omega'$$

$$J_\lambda^{\text{solar}}(\tau, \omega) = \frac{\tilde{\omega}}{4\pi} F_\odot P(\tau, \omega_0, \omega) e^{-\tau/\mu_0}$$

$$J_\lambda^{\text{thermal}}(\tau, \omega, T) = (1 - \tilde{\omega}) B_\lambda[T(\tau)]$$

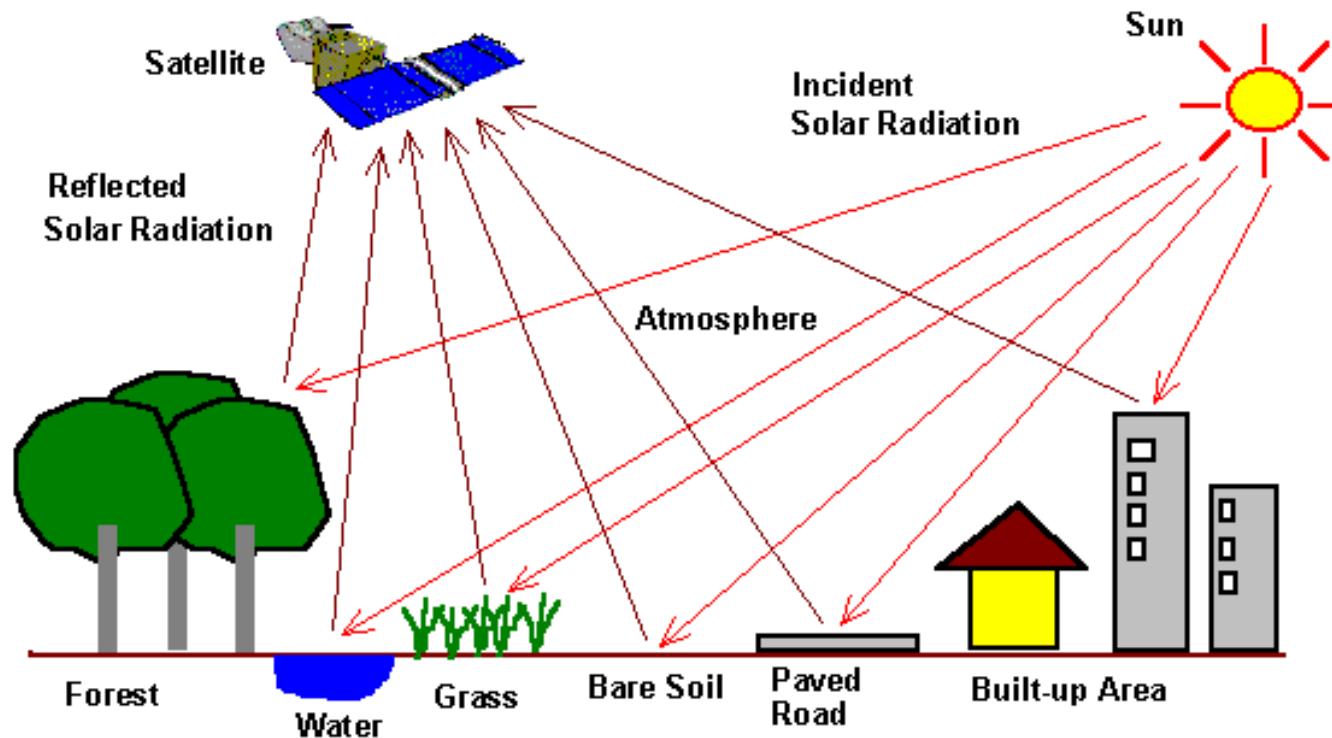
where $\tilde{\omega}$ is called ***single-scattering albedo***, which is ratio of the scattering coefficient to the extinction coefficient. P is the called ***phase function***, which represents the angular distribution of the scattered energy as a function of the scattering angle (similar to the concept of BRDF but for particles).

Despite the RTE has been developed including both scattering and emission processes. It is generally not necessary to consider all radiation sources.

RTE for Plane-Parallel atmospheres with a Solar Source

For wavelengths $\lambda \leq 3.5 \mu m$, the atmosphere is illuminated by the Sun and thermal sources are neglected such that,

$$J_\lambda = J_\lambda^{\text{diffuse}} + J_\lambda^{\text{solar}}$$



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Satellite view of cloud thickness



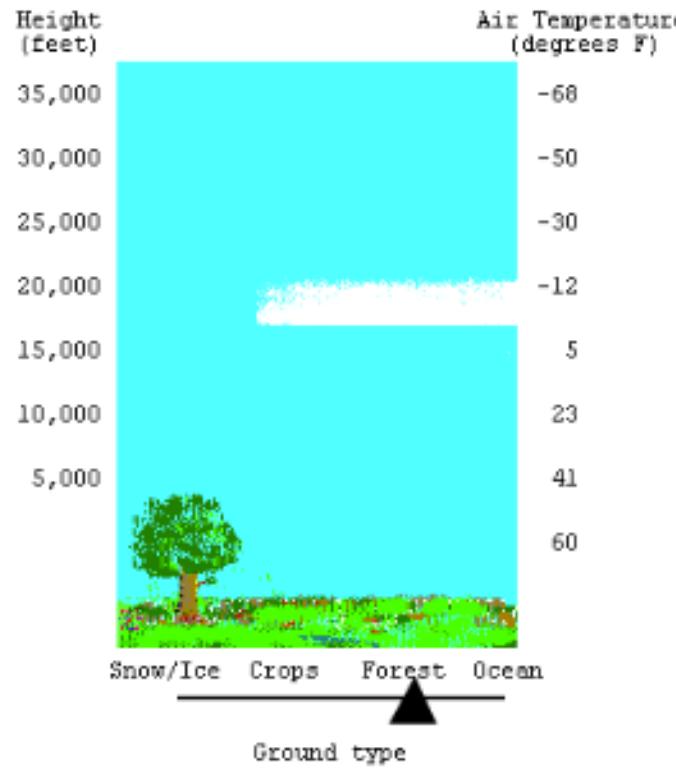
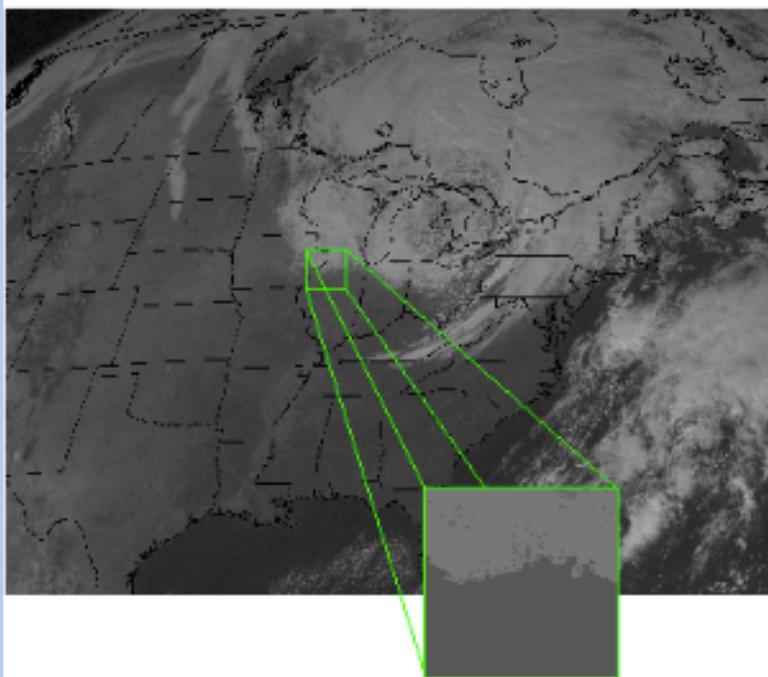
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How does the thickness of a cloud change the way it looks from the satellite?

This image is in the visible part of the spectrum, and the radiant energy is a function of not just temperature, as was the case in the IR example.

Dragging on the cloud changes its thickness and its effective brightness.

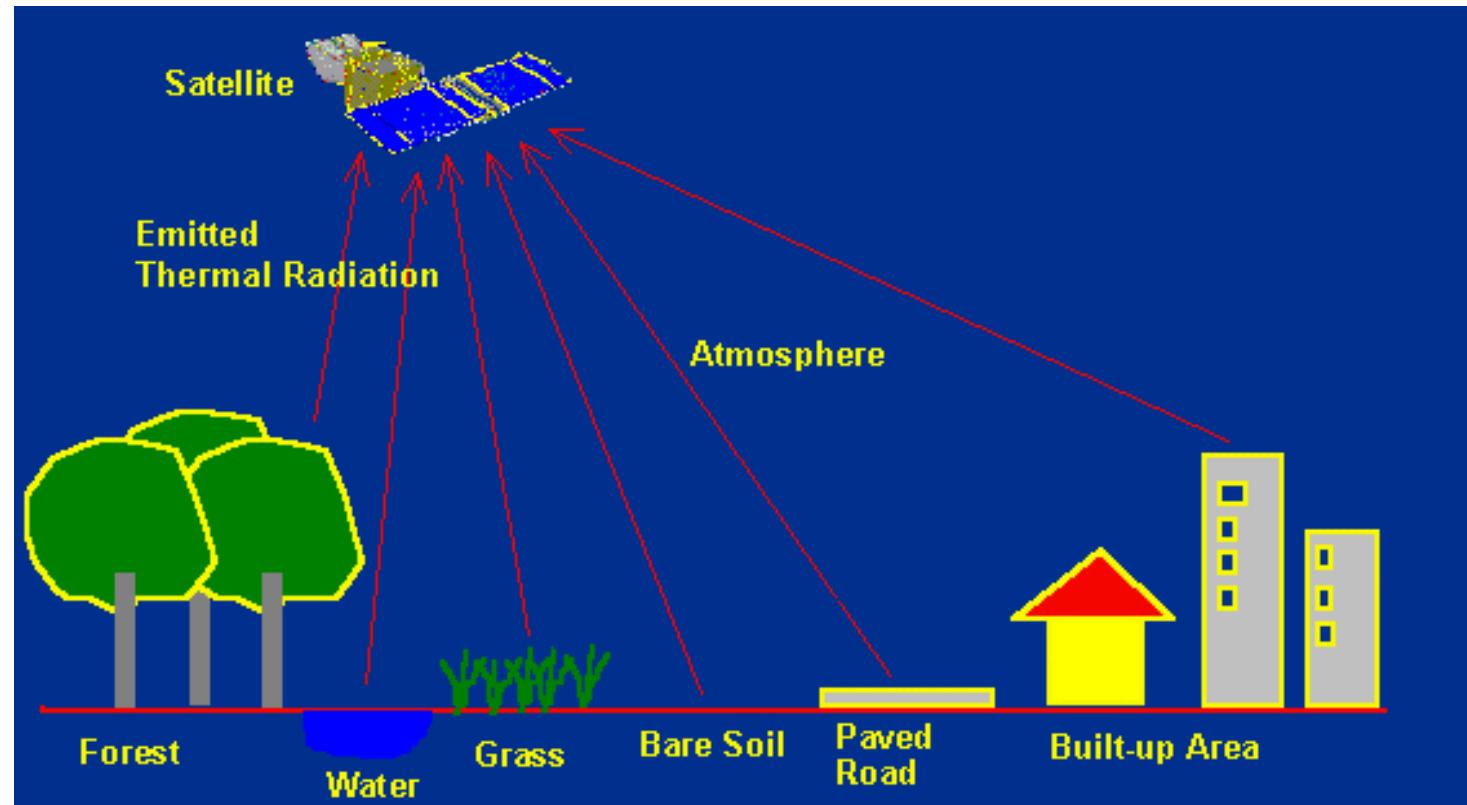
You can modify the surface temperature using the black triangle on the horizontal sliding scale. This will, in turn, alter the brightness of the scene.



RTE for Plane-Parallel atmospheres with a Thermal Source

For wavelengths $\lambda \geq 8.9 \mu m$, the contribution to the radiance field by sunlight can be ignored, such that

$$J_\lambda = J_\lambda^{\text{thermal}}$$



Explore Infrared Satellite Imagery

What are those cloud images on TV?



THE UNIVERSITY
of
WISCONSIN
MADISON



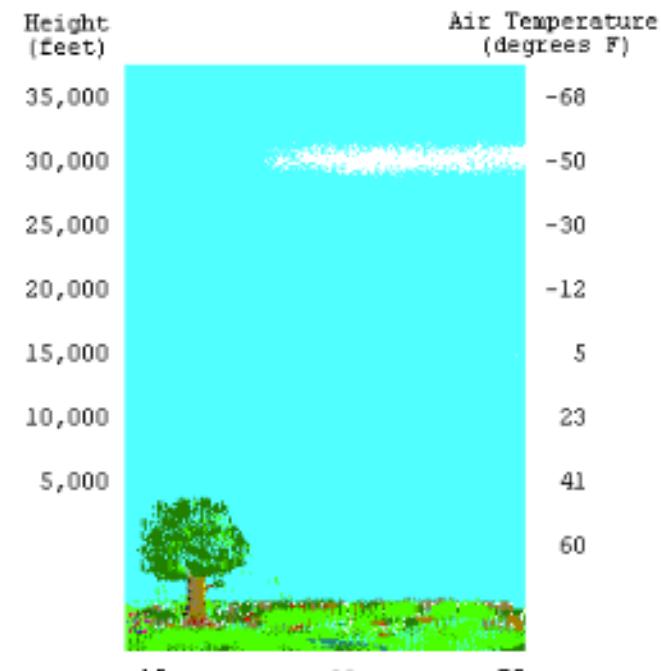
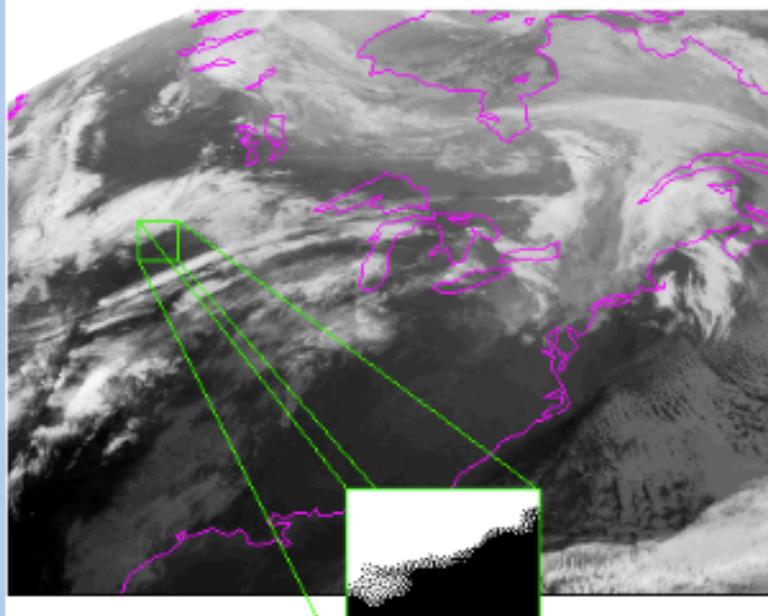
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What does the brightness of a cloud mean on the TV weather shows?

The image to the left is an example of an infrared (IR) image made from a radiometer flown on the GOES satellites. This is very typical of the images you see on TV weather shows. A piece of this image has been extracted and appears as a square below the larger satellite image. This portion of the image contains a cloud and some clear sky areas. You can change the appearance of this extracted image by changing the cloud altitude (and its associated ambient air temperature) or the surface temperature. You do this by moving the sliding scales (scroll bars) accompanying the picture on the right.

Drag the cloud up and down to change its altitude and its temperature. The numbers on each side of figure show the altitude and corresponding temperature, assuming standard atmospheric conditions.

You can modify the surface temperature with the horizontal sliding scale.



Approximations of Radiative Transfer

To resolve the RT of solar radiation numerically, here are two well-known useful approximations.

Single-scattering approximation for a plane-parallel atmosphere

- In a domain where the optical depth is small (e.g., $\tau < 0.1$), a large portion of scattering events is dominated by single scattering of the direct solar beam. It happens in optically thin cirrus and aerosol atmospheres.
- Most important term in the source function

$$J_\lambda \cong J_\lambda^{\text{solar}} = \frac{\tilde{\omega}}{4\pi} F_\odot P(\mu, \phi; -\mu_0, \phi_0) e^{-\tau/\mu_0}$$

- Consider a blackbody surface $I_\lambda(\tau_*; \mu, \phi) = 0$ where τ_* is the total atmospheric optical depth, when τ_* is small, we have

$$I_\lambda(0; \mu, \phi) = \frac{1}{\mu} \int_0^{\tau_*} J_\lambda(\tau'; \mu, \phi) e^{-\tau'/\mu} d\tau' = \frac{\mu_0 F_\odot}{\pi} \frac{\tilde{\omega}}{4(\mu + \mu_0)} P(\mu, \phi; -\mu_0, \phi_0) \{1 - \exp[-\tau_* (\frac{1}{\mu} + \frac{1}{\mu_0})]\}$$

- For a small τ_* , we have

$$R_\lambda(\mu, \phi; \mu_0, \phi_0) = \frac{\pi I_\lambda(0; \mu, \phi)}{\mu_0 F_\odot} = \tau_* \frac{\tilde{\omega}}{4\mu\mu_0} P(\mu, \phi; -\mu_0, \phi_0) \quad (7)$$

- The term $R_\lambda(\mu, \phi; \mu_0, \phi_0)$ is referred to as the *bidirectional reflectance (BRF)*. This equation establishes the foundation for the retrieval of the optical depth of aerosols from satellites.

Approximations of Radiative Transfer

Diffusion approximation

- In a domain where the directional dependence of multiple scattering events is largely lost. In this case, it is appropriate to consider the transfer of hemispheric upward and downward flux densities defined by

$$F^{\uparrow\downarrow}(\tau) = \int_0^{2\pi} \int_0^{\pm 1} I(\tau; \mu, \phi) \mu d\mu d\phi$$

- We may formulate the transfer problem based on the physical reasoning that the differential changes of the upward and downward flux densities must be related to these fluxes as well as to the direct downward flux from the sun, such that

$$\frac{dF^{\uparrow}}{d\tau} = \gamma_1 F^{\uparrow} - \gamma_2 F^{\downarrow} - \gamma_3 \tilde{\omega} F_{\odot} e^{-\tau/\mu_0},$$

$$\frac{dF^{\downarrow}}{d\tau} = \gamma_2 F^{\uparrow} - \gamma_1 F^{\downarrow} + (1 - \gamma_3) \tilde{\omega} F_{\odot} e^{-\tau/\mu_0},$$

where γ_1 , γ_2 , and γ_3 are appropriate weighting coefficients related to multiple scattering events.

- Solutions for the upward and downward fluxes can be derived by setting $F_{\text{dif}} = F^{\downarrow} - F^{\uparrow}$, and $F_{\text{sum}} = F^{\downarrow} + F^{\uparrow}$. In this manner the *diffusion equation for radiative transfer* can be expressed as,

$$\frac{d^2 F_{\text{dif}}}{d\tau^2} = k^2 F_{\text{dif}} + \chi e^{-\tau/\mu_0}$$

where $k^2 = \gamma_1^2 - \gamma_2^2$ are the eigenvalues and χ is a certain coefficient. The general solution for this second-order nonhomogeneous differential equation is given by

$$F_{\text{dif}} = c_1 e^{-k\tau} + c_2 e^{+k\tau} + \chi \left(\frac{1}{\mu_0^2} - k^2 \right) e^{-\frac{\tau}{\mu_0}} \quad (8)$$

where $c_{1,2}$ certain coefficients. Likewise, we can also derive a solution for F_{sum} which, together with F_{dif} , can be used to determine the analytic solutions for upward and downward flux densities (irradiances).