Ordinary Differential Equations – MATH-UA.0262-007

Description of Course

Ordinary Differential Equations, or ODEs for short, is where you start to see the power of calculus: the ability to make predictions. Given the physical laws of the universe (or the markets, or your favorite system), differential equations allow us to forecast how it will evolve. They are indispensable in my field of research, where we work with ODE's more complicated sibling—partial differential equations (which involve multiple dimensions)—to predict the response of climate systems to external forcing, such as greenhouse gas emissions. ODEs are invaluable across the sciences, including physics, chemistry, and biology, and economics.

This course also allows us to ask fundamental questions about equations. Do solutions exist? Are solutions unique? In this, ODEs offer a glimpse into the field of analysis.

The goals of this course are to cover the following topics:

- Methods for solving the few types of first- and second-order equations
- Existence and uniqueness of solutions
- Series solutions for equations
- Systems of linear equations
- Nonlinear dynamical systems and phase plane analysis
- Boundary value problems, separable partial differential equations and Fourier series
- Delta functions and Green's functions

Instructor
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Grader: Arya Lakshmanan (all589@nyu.edu)

Logistics
Class time: Monday and Wednesday | 9:30-10:45 AM | 194 Mercer St Room 307
Office hour: Yi Zhang - Monday and Wednesday | 11:00-12:00 PM | WWH 909
Mingxin Li - Tuesday and Thursday | 2:00-3:00 PM | WWH 1009
Recitation (Mingxin Li): Friday | 8:00-9:15 AM | WWH 201
Final exam: Monday May 12 | 10:00-11:50 am | 194 Mercer St Room 307

Textbook
Braun, Martin, Differential Equations and their Applications, 4th edition, Springer, 1993.
https://link.springer.com/book/10.1007%2F978-1-4612-4360-1 **[free!]**

Grade Policy

- Class and Office Hour Participation: 0%
    - What matters is that you learn—how you do that is up to you!
- Weekly Homework: 30%
    - Homework is due online each Monday (except the Monday after the first week).
    - Collaboration is encouraged, but please note who you worked with on your submission.
    - The two lowest homework grades will be dropped.
    - No late homework will be accepted to ensure fairness to all students.
- Exams: 70% (Midterm 1: 20%; Midterm 2: 20%; Final Exam: 30%)
    - A strict no-electronics policy applies to all exams, including phones, computers, and wearable devices. Please leave only writing tools on the desk and electronic devices inside your bag.

Academic integrity

Any form of cheating will result in punishment procedures per university guidelines, which are beyond my control and leave little room for leniency. Please avoid any actions that might even appear dishonest—for your sake and ours (honestly, it's never easy to report someone and see the consequences :/). As the Chinese saying goes, "Do not adjust your shoes in a melon field or your hat under a plum tree" (瓜田不纳履，李下不正冠).

Tentative Course Schedule

Week	Topics	Readings
1 (1/22)	Introduction, classification of ODE's	1.1,1.2
2 (1/27,1/29)	First order ODES, Separation of variables	1.2, 1.4
3 (2/3, 2/5)	Population models, Exact Solution	1.5,1.9
4 (2/10, 2/12)	Existence and uniqueness Numerical solutions (Euler method)	1.10, 1.13,1.16
5 (2/18,2/19) Legislative Monday	Uniqueness proof via Lipschitz continuity, second order linear ODEs with constant coefficients, characteristic equation Midterm 1 (2/19)	2.2
6 (2/24, 2/26)	Second-order linear ODES, Wronskian, linear dependence of solutions, Abel's theorem, existence of precisely two independent solutions,	2.1
7 (3/3, 3/5)	Inhomogeneous equations, judicious guessing, damped oscillators and resonance Series solutions	2.5, 2.6

8 (3/10, 3/12)	Series solution, Singular points	2.8
9 (3/17, 3/19)	Systems of linear equations, complex eigenvalues, matrix exponential, double root	3.1, 3.8, 3.9, 3.10
10 (3/24-28)	Spring break	
11 (3/31, 4/2)	Predator prey, phase portrait, nonlinear oscillator Midterm 2 (4/2)	4.1-3
12 (4/7, 4/9)	Non-linear systems, stability, phase-plane analysis	4.3-7
13 (4/14, 4/16)	Differential operator analogy in linear algebra, transpose and adjoint, boundary value problem, Fourier series	5.1, 5.4,6.2,6.4
14 (4/21, 4/23)	Separation of variables, heat equation, wave equation, delta function	5.3, 5.5, 5.6, 5.7, 2.12
15 (4/28, 4/30)	Green’s functions BVP and IVP	2.12
16 (5/5)	Review	