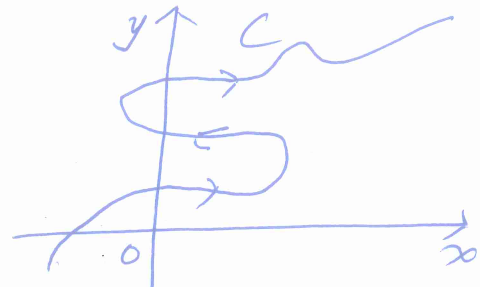
1 Chapter 10 Parametric Equations and Polar Coordinates

§10.1 Curves Defined by Parametric Equations

Imagine that a particle moves along the curve C.

Then C can not be described by

$$y = f(x)$$



Suppose that x and y are both functions of t (parameter):

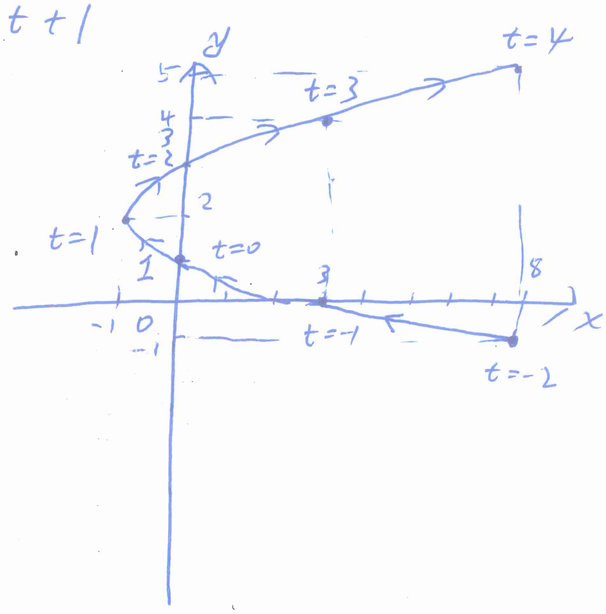
$$x = f(t), \quad y = g(t) \quad \text{(parametric equations)}$$

Then $C = \{ (f(t), g(t)) \mid a \le t \le b \}$ is the corresponding parametric curve.

Ex 1. Sketch and identify the curve defined by

$$x = t^2 - 2t, \quad y = t + 1$$

t	x	y
-2	8	-1
-1	3	0
0	0	1
1	-1	2
2	0	3
3	3	4
4	8	5



$t = y - 1$

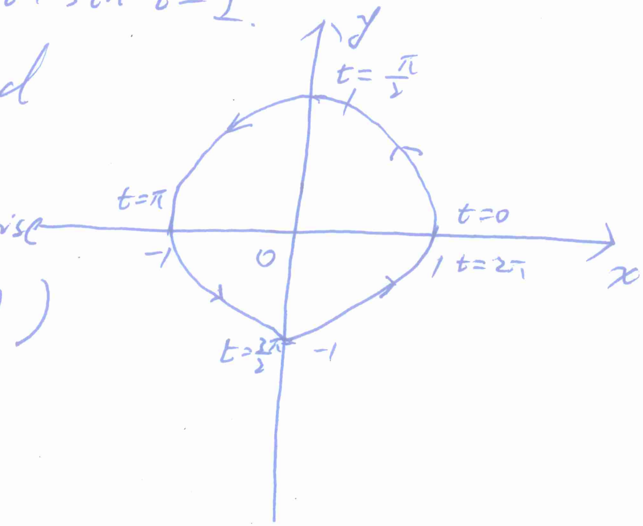
$$\Rightarrow x = t^2 - 2t$$
$$= (y-1)^2 - 2(y-1)$$
$$= y^2 - 4y + 3 \quad \Leftarrow \text{a parabola.}$$

② Ex 2. What curve is represented by the following equation?

$$x = \cos t, \quad y = \sin t, \quad 0 \le t \le 2\pi$$

$$x^2 + y^2 = \cos^2 t + \sin^2 t = 1.$$

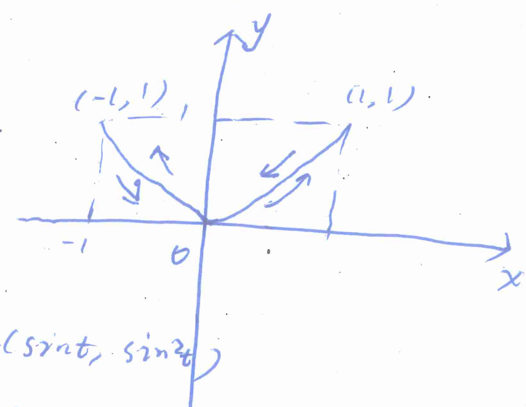
(points moves around the circle ~~from~~ in the counterclockwise direction from $(1,0)$)



Ex 5. Sketch the curve with parametric equations

$$x = \sin t, \quad y = \sin^2 t$$

$$y = (\sin t)^2 = x^2 \quad \text{— a parabola}$$



(the point $(x, y) = (\sin t, \sin^2 t)$ moves back and forth along the parabola from $(-1, 1)$ to $(1, 1)$)

circumference [sər'kʌm fərəns].
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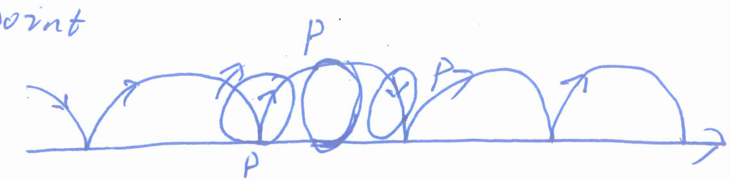
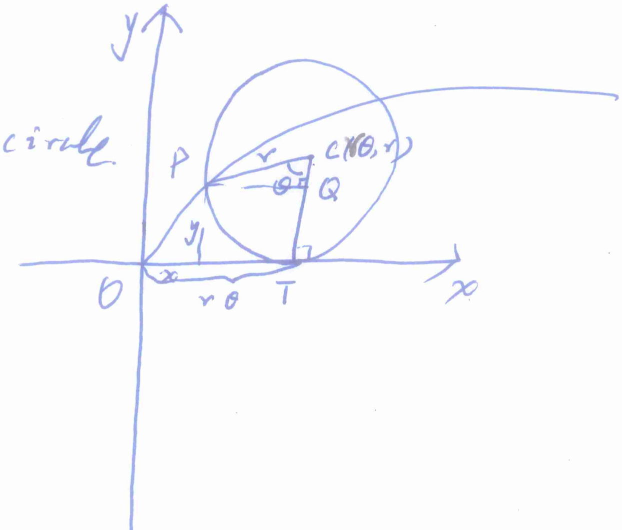
⓪ Ex 7 Cycloid:

The curve traced out by a point P on the circumference of a circle as the circle rolls along a straight line.



③ ~~Assume θ~~

Choose parameter θ to be the angle of rotation of the circle.



When $\theta = 0$, P is at the origin.

Since the circle has been in contact with the line;

$$|OT| = \text{arc} PT = r\theta$$

The center of the circle is $C(r\theta, r)$

Let the coordinates of P be (x, y)

Then

$$x = |OT| - |PQ| = r\theta - r\sin\theta = r(\theta - \sin\theta)$$

$$y = |TC| - |QC| = r - r\cos\theta = r(1 - \cos\theta)$$

Thus, parametric equations of the cycloid are

$$x = r(\theta - \sin\theta), \quad y = r(1 - \cos\theta), \quad \theta \in \mathbb{R}.$$