§9.2 Direction Fields and Euler's Method

Consider

$$y' = f(x, y) \quad (1)$$

Usually, it is impossible to find explicit solution of (1)

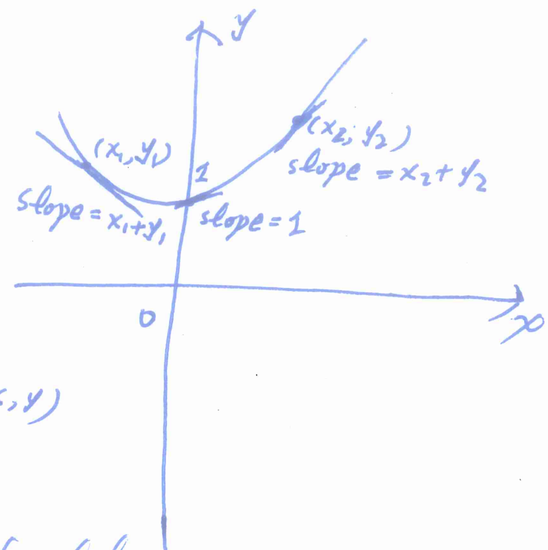
Ex for explicit solution: $y' = x^3 \Rightarrow y = \frac{x^3}{4} + C$

Graphical approach: Direction Fields

Question: sketch the graph of the solution of the initial-value problem

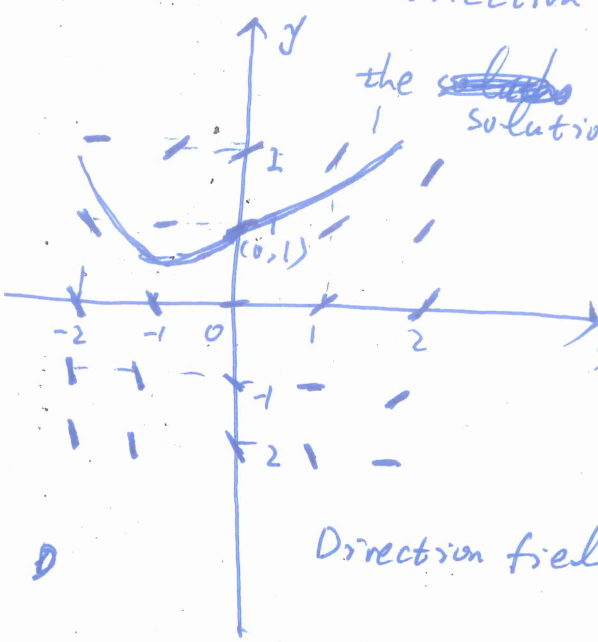
$$y' = x + y, \quad y(0) = 1 \quad (2)$$

Slope at $(0,1)$: $y'(0) = 0 + 1 = 1$



Idea: To sketch the solution curve of (2), we draw short line segments at a number of points (x, y) with slope $x + y$.

The result is called a direction field.



the ~~solution~~ solution through $(0,1)$

Mathematica "VectorPlot".

Direction field for $y' = x + y$

Consider $y' = F(x, y)$ (1)

The slope of a solution curve at (x, y) ~~on the~~ is $F(x, y)$

② If we draw line segments with slope $F(x, y)$ at several points, the result is ~~a~~ a direction ~~fiele~~ field (slope field)

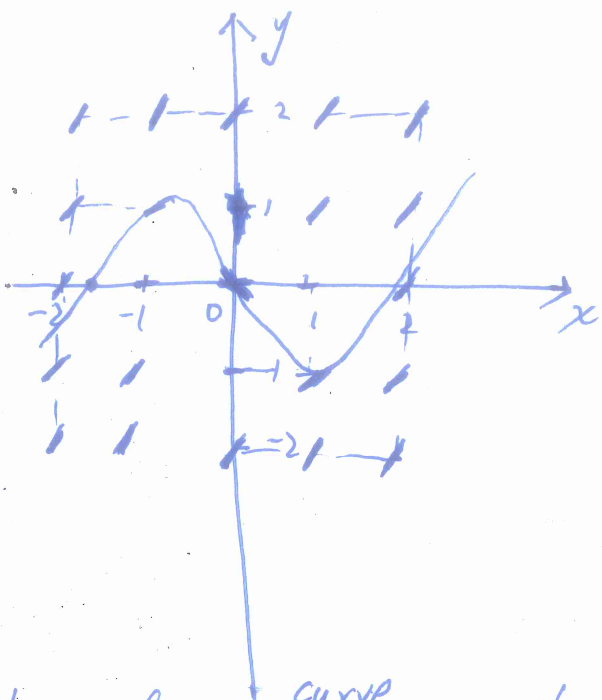
Ex 1. (Consider

$$y' = x^2 + y^2 - 1 \quad (3)$$

(a) Sketch the ~~id~~ direction field of (3)

(b) Sketch the solution curve that passes through (0,0).

(a)



(b). Draw the solution ~~curve~~ curve so that it moves parallel to the nearby line segments