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Chapter 12 Vectors and the Geometry of Space

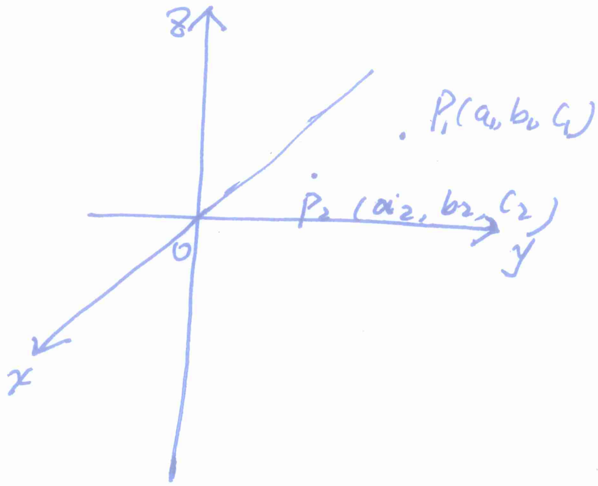
§12.1 3-D Coordinate Systems

The distance $|P_1P_2| =$

$$\sqrt{(a_2-a_1)^2+(b_2-b_1)^2+(c_2-c_1)^2}$$

~~(P_1P_2 is a the diagonal of~~

(P_1P_2 is a diagonal of a rectangular box)



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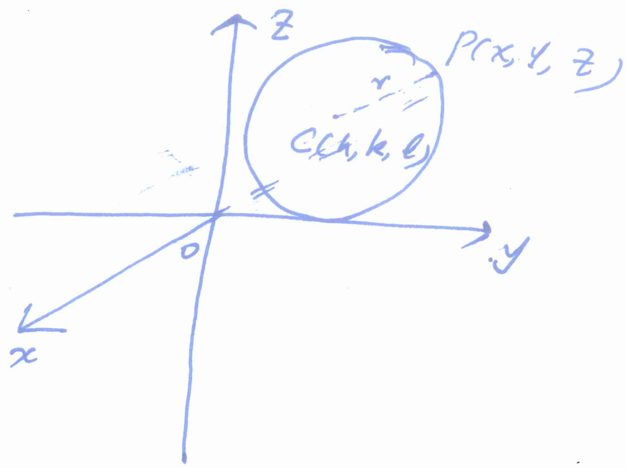
Equation of a Sphere

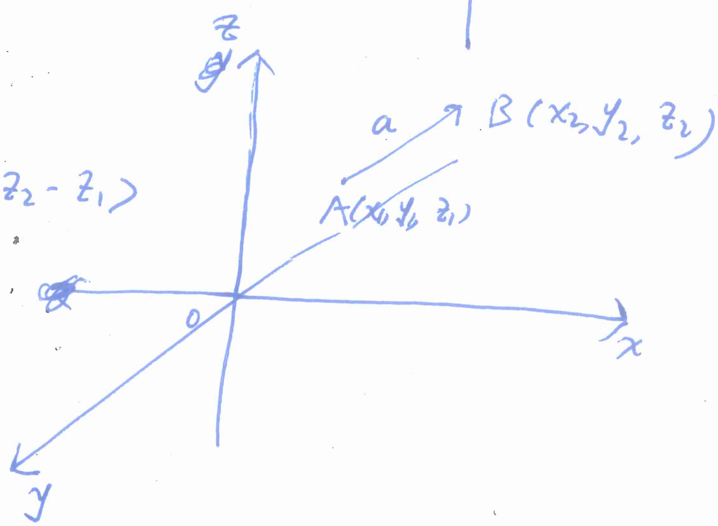
$$(x-h)^2+(y-k)^2+(z-l)^2=r^2$$



§12.2 Vectors

$a = \overrightarrow{AB}$

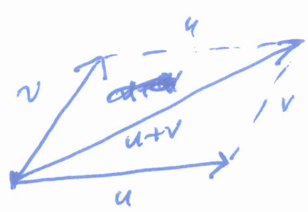
$= \langle x_2-x_1, y_2-y_1, z_2-z_1\rangle$



Vector Addition

If $u=\langle a_1, b_1\rangle$, $v=\langle a_2, b_2\rangle$,

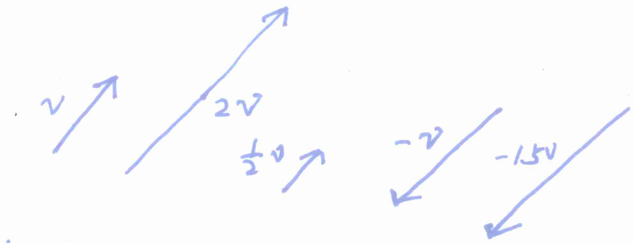
the $u+v=\langle a_1+a_2, b_1+b_2\rangle$



Scalar Multiplication

$v = \langle a_1, b_1 \rangle$,

$c \cdot v = \langle ca_1, cb_1 \rangle$



The length of a vector

$v = \langle a_1, b_1 \rangle$

$|v| = \sqrt{a_1^2 + b_1^2}$

Chapter 7 Techniques of Integration.

§7.1 ~ 7.5 Indefinite Integral

Given $f(x)$, Find $F(x)$ such that $\frac{dF}{dx} = f$

We denote F by $\int \cancel{d}f(x)\,dx$

0. Memorize basic formulas.

Ex: $\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$

$\int \frac{dx}{x} = \ln|x|$, $\int e^x\,dx = e^x$,

$\int \sin x\,dx = -\cos x$.

1. Simplify the integrand if possible.

Ex: $\int (\sin x + \cos x)^2\,dx = \int (\sin^2 x + 2\sin x \cos x + \cos^2 x)\,dx$

$= \int (1 + 2\sin x \cos x)\,dx$

2. Look for an Obvious Substitution.

Ex: $\int \frac{x}{x^2 - 3}\,dx$. Set $u = x^2 - 3$, $du = 2x$

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$$\int \frac{x}{x^2-3}\,dx = \frac{1}{2}\int \frac{du}{u}$$

3. Classify the ~~Int~~ integrand according to its form.

(a) Trigonometric functions. ①: Using trigonometric identities.

$$\int \cos^3 x\,dx = \int (1-\sin^2 x)\cos x\,dx$$

$$\overset{u=\sin x}{=\!=} \int (1-u^2)\,du$$

(b) Rational Functions.

$$\int \frac{P(x)}{Q(x)}\,dx$$

①: Step 1: if $\deg(P) \geq \deg(Q)$, then

$$\frac{P}{Q} = \frac{SQ+R}{Q} = S + \frac{R}{Q} \leftarrow \text{proper}$$

Step 2: Factor Q into products of $(ax+b)$'s and (ax^2+bx+c)'s

Step 3. Express R/Q as a sum of

$$\frac{A}{(ax+b)^i} \quad \text{or} \quad \frac{Ax+B}{(ax^2+bx+c)^j}$$

Ex: $\int \frac{dx}{x^2(x-1)}$

$$\frac{1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$$

Determine A, B, C by solving linear equations.

(c) Integration by parts.

Ex: $\int x\sin x\,dx \overset{u=-\cos x}{=} \int x\,d(-\cos x) = -x\cos x - \int (-\cos x)\,dx$

④(d) Radicals

(i) $\sqrt{\pm x^2 \pm a^2}$ ⟵ trigonometric substition

(ii) $\sqrt[n]{ax+b}$ ⟵ $u=\sqrt[n]{ax+b}$

§7.8 Improper Integrals

Type 1. Infinite Intervals

$$A=\int_a^{\infty} f(x)\,dx = \lim_{t\to\infty}\int_a^t f(x)\,dx \qquad \text{Ex: } \int_1^{+\infty}\frac{dx}{x}$$

A is convergent if the limit is a finite number.

Typer 2: Discontinuous Integrands

f is continous on $[a,b)$, discontinuous at b

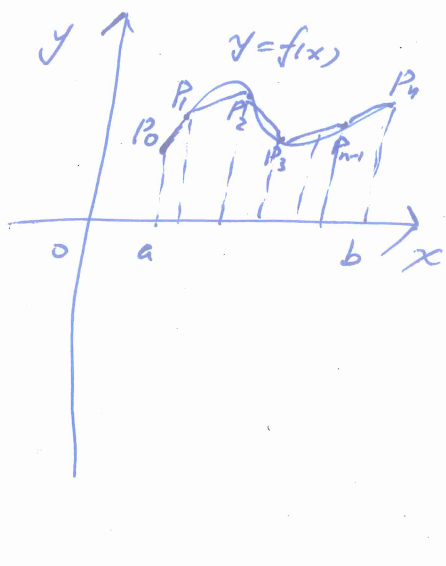
$$B=\int_a^b f(x)\,dx = \lim_{t\to b^-}\int_a^t f(x)\,dx \qquad \text{Ex: } \int_2^5 \frac{dx}{\sqrt{x-2}}$$

B is convergent ________________________

Chapter 8. Further Applications of Integration.

§8.1 Arc Length

C: $y=f(x)$, $a\le x\le b$



The length of C is

$$L=\lim_{n\to\infty}\sum_{i=1}^{n}|P_{i-1}P_i|$$

$$=\int_a^b \sqrt{1+[f'(x)]^2}\,dx$$

$$=\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \quad \text{The Arc Length Formula.}$$

Set $s(x)=\int_a^x \sqrt{1+\left(\frac{dy}{dt}\right)^2}\,dt$ The Arc Length Function.

⑤ $\boxed{(ds)^2 = (dx)^2 + (dy)^2}$

$L = \int ds$

$x = g(y), \quad c \le y \le d$

$ds = \sqrt{(dx)^2 + (dy)^2}$

$= \sqrt{1 + (\frac{dx}{dy})^2}\, dy$

$L = \int ds$

$= \int_c^d \sqrt{1 + (\frac{dx}{dy})^2}\, dy$

§8.2 Area of a Surface of Revolution

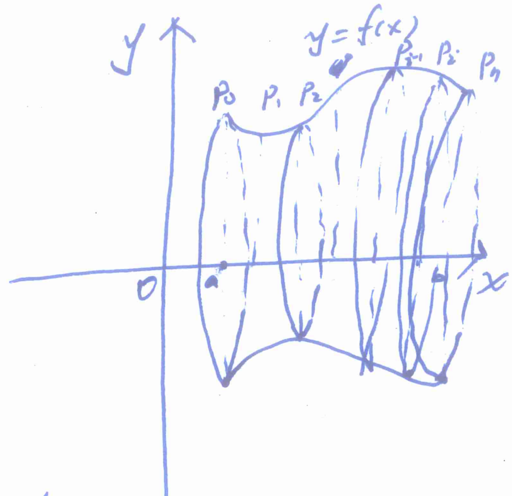
The surface area of the curve

$y = f(x), \ a \le x \le b$ about x-axis

is

$$S = \int_a^b 2\pi f(x) \sqrt{1 + [f'(x)]^2}\, dx$$

$$= \int 2\pi y\, ds$$



For rotation about the y-axis, the surface area formula

$$S = \int 2\pi x\, ds$$