§9.6 Predator-Prey Systems

Two species: prey and predators

Ex: rabbits and wolves
hares and lynxes

Let $R(t)$ be the number of prey (using R for rabbits) at time t.
$W(t)$ — — —— predators (with W for wolves) — —.

In the absence of predators, the ample food supply supports exponential growth of the prey,

$$\frac{dR}{dt} = kR, \quad \text{where } k>0$$

In the absence of prey, assume that the predators decline through mortality at a rate proportional to itself,

$$\frac{dW}{dt} = -rW, \quad \text{where } r>0$$

With both species present, we assume:

1. the prey decline ~~caused~~ by predators;
2. the predator increase by preys;
3. the two species encounter each other at a rate that is proportional to both populations $R \cdot W$

Thus, we have

$$\frac{dR}{dt} = kR - aRW, \quad \frac{dW}{dt} = -rW + bRW, \qquad (1)$$

where $k, r, a,$ and b are positive constants.

(1) is called the predator-prey system, or the Lotka-Volterra system.

Note: 1. We can not find explicit formulas for solutions of (1)

2. We can use graphical methods to analyze (1).

② Ex 1. Suppose that populations of rabbits and wolves are described by the Lotka-Volterra system with $k=0.08$, $a=0.001$, $r=0.02$, and $b=0.00002$.

(a) Find the constant solutions and interpret the answer.

(b) Use the system of differential equations to find dW/dR.

(a)

the Lotka-Volterra system:

$$\begin{cases} \dfrac{dR}{dt} = 0.08R - 0.001RW \\ \dfrac{dW}{dt} = -0.02W + 0.00002R\cdot W \end{cases} \quad (2)$$

Assume R and W are constant solutions of (2). Then

$$\begin{cases} 0 = R' = R(0.08 - 0.001W) = 0 \\ 0 = W' = W(-0.02 + 0.00002R) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} R = 0 \\ W = 0 \end{cases} \text{ or } \begin{cases} R = 1000 \\ W = 80 \end{cases}$$

They are equilibrium solutions of (2).

1000 rabbits are just enough to support 80 wolves.

(b) Use the Chain Rule.

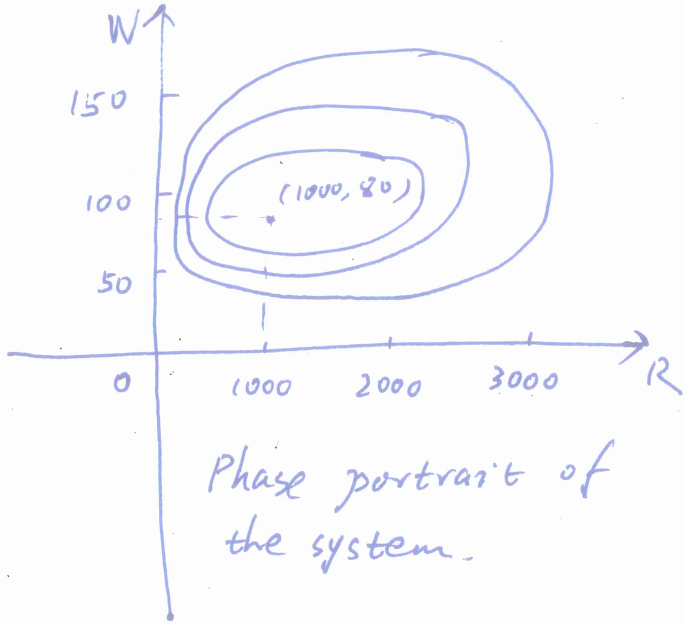
$$\frac{dW}{dt} = \frac{dW}{dR}\cdot\frac{dR}{dt}$$

$$\Rightarrow \frac{dW}{dR} = \frac{\frac{dW}{dt}}{\frac{dR}{dt}} = \frac{-0.02W + 0.00002RW}{0.08R - 0.001RW} \quad (3)$$

(c) Draw a direction field for (3). Then use it to sketch some solution curves

direction field: Use Mathematica "VectorPlot".

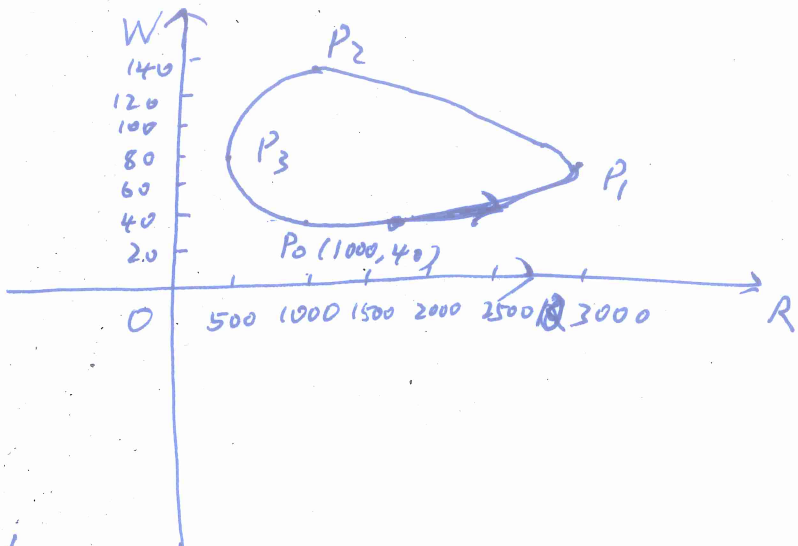
③(c)



Phase portrait of the system.

(1000, 80) is an equilibrium point.
RW-plane is called the phase plane.
the solution curve is called the phase trajectory.

(d) Suppose there are 1000 rabbits and 40 wolves. Draw the corresponding solution curve and use it to describe changes in both population levels



If R = 1000, W = 40, then

$$\frac{dR}{dt} = 0.08(1000) - 0.001(1000)(40)$$
$$= 40 > 0$$

(e) Use (d) to make sketches of R(t) and W(t)

(4) Suppose P_1, P_2 and P_3 are reached at t_1, t_2 and t_3.

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oscillated c~~ave~~ curve