

# Relational Database Design Theory

## Part II

CPS 196.3  
Introduction to Database Systems

---

---

---

---

---

---

---

---

### Announcement

2

- ❖ Project proposal/progress report due today
- ❖ Midterm next Thursday in class
  - Everything up to today's lecture, with a focus on the materials covered by the first two homework assignments
  - Open book, open notes
- ❖ Will assign an optional problem set tonight as a study guide for midterm
  - Entirely optional
  - If you turn it in on Tuesday in class, you can use its grade to replace your lowest homework grade so far
  - Solution will be posted on Tuesday midnight
- ❖ Graded Homework #2 will be available on Tuesday

---

---

---

---

---

---

---

---

### Review

3

- ❖ Functional dependencies
  - $X \rightarrow Y$ : If two rows agree on  $X$ , they must agree on  $Y$
  - ☞ A generalization of the key concept
- ❖ Non-key functional dependencies: a source of redundancy
  - No trivial  $X \rightarrow Y$  where  $X$  is not a superkey
  - ☞ Called a BCNF violation
- ❖ BCNF decomposition: a method for removing redundancies
  - Given  $R(X, Y, Z)$  and a BCNF violation  $X \rightarrow Y$ , decompose  $R$  into  $R_1(X, Y)$  and  $R_2(X, Z)$
  - ☞ A lossless join decomposition
- ❖ Schema in BCNF has no redundancy due to FD's

---

---

---

---

---

---

---

---

## Next

4

- ❖ 3NF (BCNF is too much)
- ❖ Multivalued dependencies: another source of redundancy
- ❖ 4NF (BCNF is not enough)

---

---

---

---

---

---

---

---

## Motivation for 3NF

5

- ❖ *Address* (*street\_address*, *city*, *state*, *zip*)
  - $\text{street\_address, city, state} \rightarrow \text{zip}$
  - $\text{zip} \rightarrow \text{city, state}$
- ❖ Keys
- ❖ BCNF?

---

---

---

---

---

---

---

---

## To decompose or not to decompose

6

*Address*<sub>1</sub>

*Address*<sub>2</sub>

- ❖ FD's in *Address*<sub>1</sub>
- ❖ FD's in *Address*<sub>2</sub>
- ❖ Hey, where is  $\text{street\_address, city, state} \rightarrow \text{zip}$ ?
  - Cannot check without joining *Address*<sub>1</sub> and *Address*<sub>2</sub> back together
- ❖ Problem: Some lossless join decomposition is not dependency-preserving
- ❖ Dilemma: Should we get rid of redundancy at the expense of making constraints harder to enforce?

---

---

---

---

---

---

---

---

## 3NF

7

- ❖  $R$  is in Third Normal Form (3NF) if for every non-trivial FD  $X \rightarrow A$  (where  $A$  is single attribute), either
  - $X$  is a superkey of  $R$ , or
  - $A$  is a member of at least one key of  $R$
- ☞ Intuitively, BCNF decomposition on  $X \rightarrow A$  would “break” the key containing  $A$
- ❖ So *Address* is already in 3NF
- ❖ Tradeoff:
  - Can enforce all original FD's on individual decomposed relations
  - Might have some redundancy due to FD's

---

---

---

---

---

---

---

---

## BCNF = no redundancy?

8

- ❖ *Student* (*SID*, *CID*, *club*)
  - Suppose your classes have nothing to do with the clubs you join
  - FD's?
  - BCNF?
  - Redundancies?

| <i>SID</i> | <i>CID</i> | <i>club</i> |
|------------|------------|-------------|
| 142        | CPS196     | ballet      |
| 142        | CPS196     | sumo        |
| 142        | CPS114     | ballet      |
| 142        | CPS114     | sumo        |
| 123        | CPS196     | chess       |
| 123        | CPS196     | golf        |
| ...        | ...        | ...         |

---

---

---

---

---

---

---

---

## Multivalued dependencies

9

- ❖ A multivalued dependency (MVD) has the form  $X \twoheadrightarrow Y$ , where  $X$  and  $Y$  are sets of attributes in a relation  $R$
- ❖  $X \twoheadrightarrow Y$  means that whenever two rows in  $R$  agree on all the attributes of  $X$ , then we can swap their  $Y$  components and get two new rows that are also in  $R$

| $X$ | $Y$  | $Z$  |
|-----|------|------|
| $a$ | $b1$ | $c1$ |
| $a$ | $b2$ | $c2$ |
| $a$ | $b1$ | $c2$ |
| $a$ | $b2$ | $c1$ |
| ... | ...  | ...  |

} Must be in  $R$  too

---

---

---

---

---

---

---

---

## MVD examples

10

*Student* (*SID*, *CID*, *club*)

❖  $SID \twoheadrightarrow CID$

---

---

---

---

---

---

---

---

## Complete MVD + FD rules

11

- ❖ FD reflexivity, augmentation, and transitivity
- ❖ MVD complementation:  
If  $X \twoheadrightarrow Y$ , then  $X \twoheadrightarrow \text{attrs}(R) - X - Y$
- ❖ MVD augmentation:  
If  $X \twoheadrightarrow Y$  and  $V \subseteq W$ , then  $XW \twoheadrightarrow YV$
- ❖ MVD transitivity:  
If  $X \twoheadrightarrow Y$  and  $Y \twoheadrightarrow Z$ , then  $X \twoheadrightarrow Z$
- ❖ Replication (FD is MVD):  
If  $X \rightarrow Y$ , then  $X \twoheadrightarrow Y$  Try proving things using these!
- ❖ Coalescence:  
If  $X \twoheadrightarrow Y$  and  $Z \subseteq Y$  and there is some  $W$  disjoint from  $Y$  such that  $W \rightarrow Z$ , then  $X \rightarrow Z$

---

---

---

---

---

---

---

---

## An elegant solution: chase

12

- ❖ Given a set of FD's and MVD's  $\mathcal{D}$ , does another dependency  $d$  (FD or MVD) follow from  $\mathcal{D}$ ?
- ❖ Procedure
  - Start with the hypotheses of  $d$ , and treat them as “seed” tuples in a relation
  - Apply the given dependencies in  $\mathcal{D}$  repeatedly
    - If we apply an FD, we infer equality of two symbols
    - If we apply an MVD, we infer more tuples
  - If we infer the conclusion of  $d$ , we have a proof
  - Otherwise, if nothing more can be inferred, we have a counterexample

---

---

---

---

---

---

---

---

## Proof by chase

13

❖ In  $R(A, B, C, D)$ , does  $A \twoheadrightarrow B$  and  $B \twoheadrightarrow C$  imply that  $A \twoheadrightarrow C$ ?

Have

| A | B  | C  | D  |
|---|----|----|----|
| a | b1 | c1 | d1 |
| a | b2 | c2 | d2 |

Need

| A | B  | C  | D  |
|---|----|----|----|
| a | b1 | c2 | d1 |
| a | b2 | c1 | d2 |

## Another proof by chase

14

❖ In  $R(A, B, C, D)$ , does  $A \rightarrow B$  and  $B \rightarrow C$  imply that  $A \rightarrow C$ ?

Have

| A | B  | C  | D  |
|---|----|----|----|
| a | b1 | c1 | d1 |
| a | b2 | c2 | d2 |

Need

$c1 = c2$

In general, both new tuples and new equalities may be generated

## Counterexample by chase

15

❖ In  $R(A, B, C, D)$ , does  $A \twoheadrightarrow BC$  and  $CD \rightarrow B$  imply that  $A \rightarrow B$ ?

Have

| A | B  | C  | D  |
|---|----|----|----|
| a | b1 | c1 | d1 |
| a | b2 | c2 | d2 |

Need

$b1 = b2$

## 4NF

16

- ❖ A relation  $R$  is in Fourth Normal Form (4NF) if
  - For every non-trivial MVD  $X \twoheadrightarrow Y$  in  $R$ ,  $X$  is a superkey
  - That is, all FD's and MVD's follow from "key  $\rightarrow$  other attributes" (i.e., no MVD's and no FD's besides key functional dependencies)
- ❖ 4NF is stronger than BCNF
  - Because every FD is also a MVD

---

---

---

---

---

---

---

---

## 4NF decomposition algorithm

17

- ❖ Find a 4NF violation
  - A non-trivial MVD  $X \twoheadrightarrow Y$  in  $R$  where  $X$  is not a superkey
- ❖ Decompose  $R$  into  $R_1$  and  $R_2$ , where
  - $R_1$  has attributes  $X \cup Y$
  - $R_2$  has attributes  $X \cup Z$  ( $Z$  contains attributes not in  $X$  or  $Y$ )
- ❖ Repeat until all relations are in 4NF
- ❖ Almost identical to BCNF decomposition algorithm
- ❖ Any decomposition on a 4NF violation is lossless

---

---

---

---

---

---

---

---

## 4NF decomposition example

18

*Student (SID, CID, club)*

| SID | CID    | club   |
|-----|--------|--------|
| 142 | CPS196 | ballet |
| 142 | CPS196 | sumo   |
| 142 | CPS114 | ballet |
| 142 | CPS114 | sumo   |
| 123 | CPS196 | chess  |
| 123 | CPS196 | golf   |
| ... | ...    | ...    |

---

---

---

---

---

---

---

---

## 3NF, BCNF, 4NF, and beyond

19

| Anomaly/normal form     | 3NF      | BCNF     | 4NF      |
|-------------------------|----------|----------|----------|
| Lose FD's?              | No       | Possible | Possible |
| Redundancy due to FD's  | Possible | No       | No       |
| Redundancy due to MVD's | Possible | Possible | No       |

### ❖ Of historical interests

- 1NF: All column values must be atomic
- 2NF: There is no partial functional dependency (a non-trivial FD  $X \rightarrow A$  where  $X$  is a proper subset of some key)

---

---

---

---

---

---

---

## Summary

20

- ❖ Philosophy behind BCNF, 4NF:  
Data should depend on the key, the whole key, and nothing but the key!
- ❖ Philosophy behind 3NF:  
... But not at the expense of more expensive constraint enforcement!

---

---

---

---

---

---

---