

ISOM3360 Assignment: ROC Curve

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| Inst# | Class | Score | Inst# | Class | Score |
|-------|----------|-------|-------|----------|-------|
| 1 | p | .9 | 11 | p | .4 |
| 2 | p | .8 | 12 | n | .39 |
| 3 | n | .7 | 13 | p | .38 |
| 4 | p | .6 | 14 | n | .37 |
| 5 | p | .55 | 15 | n | .36 |
| 6 | p | .54 | 16 | n | .35 |
| 7 | n | .53 | 17 | p | .34 |
| 8 | n | .52 | 18 | n | .33 |
| 9 | p | .51 | 19 | p | .30 |
| 10 | n | .505 | 20 | n | .1 |

1 Answer

By changing the decision threshold t , we can get different pairs of TPR/FPR.

$$t \in (0.9, 1]$$

| | | | |
|--------|---|-----------|----|
| | | Predicted | |
| | | p | n |
| Actual | p | 0 | 10 |
| | n | 0 | 10 |

$$TPR = \frac{0}{0 + 10} = 0.0$$

$$FPR = \frac{0}{0 + 10} = 0.0$$

$$t \in (0.8, 0.9]$$

| | | Predicted | |
|--------|---|-----------|----|
| | | p | n |
| Actual | p | 1 | 9 |
| | n | 0 | 10 |

$$TPR = \frac{1}{1+9} = 0.1$$

$$FPR = \frac{0}{0+10} = 0.0$$

$$t \in (0.7, 0.8]$$

| | | Predicted | |
|--------|---|-----------|----|
| | | p | n |
| Actual | p | 2 | 8 |
| | n | 0 | 10 |

$$TPR = \frac{2}{2+8} = 0.2$$

$$FPR = \frac{0}{0+10} = 0.0$$

$$t \in (0.6, 0.7]$$

| | | Predicted | |
|--------|---|-----------|---|
| | | p | n |
| Actual | p | 2 | 8 |
| | n | 1 | 9 |

$$TPR = \frac{2}{2+8} = 0.2$$

$$FPR = \frac{1}{1+9} = 0.1$$

$$t \in (0.55, 0.6]$$

| | | Predicted | |
|--------|---|-----------|---|
| | | p | n |
| Actual | p | 3 | 7 |
| | n | 1 | 9 |

$$TPR = \frac{3}{3+7} = 0.3$$

$$FPR = \frac{1}{1+9} = 0.1$$

$$t \in (0.54, 0.55]$$

| | | Predicted | |
|--------|---|-----------|---|
| | | p | n |
| Actual | p | 4 | 6 |
| | n | 1 | 9 |

$$TPR = \frac{4}{4+6} = 0.4$$

$$FPR = \frac{1}{1+9} = 0.1$$

$$t \in (0.53, 0.54]$$

| | | Predicted | |
|--------|---|-----------|---|
| | | p | n |
| Actual | p | 5 | 5 |
| | n | 1 | 9 |

$$TPR = \frac{5}{5+5} = 0.5$$

$$FPR = \frac{1}{1+9} = 0.1$$

$$t \in (0.52, 0.53]$$

| | | Predicted | |
|--------|---|-----------|---|
| | | p | n |
| Actual | p | 5 | 5 |
| | n | 2 | 8 |

$$TPR = \frac{5}{5+5} = 0.5$$

$$FPR = \frac{2}{2+8} = 0.2$$

$$t \in (0.51, 0.52]$$

| | | Predicted | |
|--------|---|-----------|---|
| | | p | n |
| Actual | p | 5 | 5 |
| | n | 3 | 7 |

$$TPR = \frac{5}{5+5} = 0.5$$

$$FPR = \frac{3}{3+7} = 0.3$$

$$t \in (0.505, 0.51]$$

| | | Predicted | |
|--------|---|-----------|---|
| | | p | n |
| Actual | p | 6 | 4 |
| | n | 3 | 7 |

$$TPR = \frac{6}{6+4} = 0.6$$

$$FPR = \frac{3}{3+7} = 0.3$$

$$t \in (0.4, 0.505]$$

| | | Predicted | |
|--------|---|-----------|---|
| | | p | n |
| Actual | p | 6 | 4 |
| | n | 4 | 6 |

$$TPR = \frac{6}{6+4} = 0.6$$

$$FPR = \frac{4}{4+6} = 0.4$$

$$t \in (0.39, 0.4]$$

| | | Predicted | |
|--------|---|-----------|---|
| | | p | n |
| Actual | p | 7 | 3 |
| | n | 4 | 6 |

$$TPR = \frac{7}{7+3} = 0.7$$

$$FPR = \frac{4}{4+6} = 0.4$$

$$t \in (0.38, 0.39]$$

| | | Predicted | |
|--------|---|-----------|---|
| | | p | n |
| Actual | p | 7 | 3 |
| | n | 5 | 5 |

$$TPR = \frac{7}{7+3} = 0.7$$

$$FPR = \frac{5}{5+5} = 0.5$$

$$t \in (0.37, 0.38]$$

| | | Predicted | |
|--------|---|-----------|---|
| | | p | n |
| Actual | p | 8 | 2 |
| | n | 5 | 5 |

$$TPR = \frac{8}{8+2} = 0.8$$

$$FPR = \frac{5}{5+5} = 0.5$$

$$t \in (0.36, 0.37]$$

| | | Predicted | |
|--------|---|-----------|---|
| | | p | n |
| Actual | p | 8 | 2 |
| | n | 6 | 4 |

$$TPR = \frac{8}{8+2} = 0.8$$

$$FPR = \frac{6}{6+4} = 0.6$$

$$t \in (0.35, 0.36]$$

| | | Predicted | |
|--------|---|-----------|---|
| | | p | n |
| Actual | p | 8 | 2 |
| | n | 7 | 3 |

$$TPR = \frac{8}{8+2} = 0.8$$

$$FPR = \frac{7}{7+3} = 0.7$$

$$t \in (0.34, 0.35]$$

| | | Predicted | |
|--------|---|-----------|---|
| | | p | n |
| Actual | p | 8 | 2 |
| | n | 8 | 2 |

$$TPR = \frac{8}{8+2} = 0.8$$

$$FPR = \frac{8}{8+2} = 0.8$$

$$t \in (0.33, 0.34]$$

| | | Predicted | |
|--------|---|-----------|---|
| | | p | n |
| Actual | p | 9 | 1 |
| | n | 8 | 2 |

$$TPR = \frac{9}{9+1} = 0.9$$

$$FPR = \frac{8}{8+2} = 0.8$$

$$t \in (0.30, 0.33]$$

| | | Predicted | |
|--------|---|-----------|---|
| | | p | n |
| Actual | p | 9 | 1 |
| | n | 9 | 1 |

$$TPR = \frac{9}{9+1} = 0.9$$

$$FPR = \frac{9}{9+1} = 0.9$$

$$t \in (0.1, 0.30]$$

| | | Predicted | |
|--------|---|-----------|---|
| | | p | n |
| Actual | p | 10 | 0 |
| | n | 9 | 1 |

$$TPR = \frac{10}{10+0} = 1.0$$

$$FPR = \frac{9}{9+1} = 0.9$$

$$t \in [0.0, 0.1]$$

| | | Predicted | |
|--------|---|-----------|---|
| | | p | n |
| Actual | p | 10 | 0 |
| | n | 10 | 0 |

$$TPR = \frac{10}{10+0} = 1.0$$

$$FPR = \frac{10}{10+0} = 1.0$$

Therefore, we get the list of the all TPR/FPR pairs:

| | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| TPR | 0.0 | 0.1 | 0.2 | 0.2 | 0.3 | 0.4 | 0.5 | 0.5 | 0.5 | 0.6 |
| FPR | 0.0 | 0.0 | 0.0 | 0.1 | 0.1 | 0.1 | 0.1 | 0.2 | 0.3 | 0.3 |

| | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| TPR | 0.6 | 0.7 | 0.7 | 0.8 | 0.8 | 0.8 | 0.8 | 0.9 | 0.9 | 1.0 | 1.0 |
| FPR | 0.4 | 0.4 | 0.5 | 0.5 | 0.6 | 0.7 | 0.8 | 0.8 | 0.9 | 0.9 | 1.0 |

Draw the ROC curve:

