ISOM3360 Assignment: ROC Curve

Zhang Yichen

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Inst#	Class	Score	Inst#	Class	Score
1	P	.9	11	p	.4
2	\mathbf{p}	.8	12	\mathbf{n}	.39
3	\mathbf{n}	.7	13	\mathbf{p}	.38
4	\mathbf{p}	.6	14	\mathbf{n}	.37
5	\mathbf{p}	.55	15	\mathbf{n}	.36
6	\mathbf{p}	.54	16	\mathbf{n}	.35
7	\mathbf{n}	.53	17	\mathbf{p}	.34
8	\mathbf{n}	.52	18	\mathbf{n}	.33
9	\mathbf{p}	.51	19	\mathbf{p}	.30
10	\mathbf{n}	.505	20	\mathbf{n}	.1

1 Answer

By changing the decision threshold x, we can get different pairs of TPR/FPR.

$$x \in (0.9, 1]$$

$$TPR = \frac{0}{0+10} = 0.0$$

 $FPR = \frac{0}{0+10} = 0.0$

$$FPR = \frac{0}{0+10} = 0.0$$

$$x \in (0.8, 0.9]$$

$$TPR = \frac{1}{1+9} = 0.1$$

 $FPR = \frac{0}{0+10} = 0.0$

$x \in (0.7, 0.8]$

$$\begin{tabular}{c|c|c} & & Predicted \\ \hline \hline p & p & n \\ \hline p & 2 & 8 \\ \hline p & 0 & 10 \\ \hline \end{tabular}$$

$$TPR = \frac{2}{2+8} = 0.2$$

 $FPR = \frac{0}{0+10} = 0.0$

$x \in (0.6, 0.7]$

$$\begin{array}{c|cccc} & & & & & & \\ & & & & & \\ \hline p & & p & n & \\ \hline p & 2 & 8 & \\ \hline n & 1 & 9 & \\ \end{array}$$

$$TPR = \frac{2}{2+8} = 0.2$$

 $FPR = \frac{1}{1+9} = 0.1$

$x \in (0.55, 6]$

		Predicted				
		p	\mathbf{n}			
tual	p	3	7			
Act	n	1	9			

$$TPR = \frac{3}{3+7} = 0.3$$

$$FPR = \frac{1}{1+9} = 0.1$$

 $x \in (0.54, 0.55]$

		Predicted					
		p	n				
ctual	р	4	6				
Act	n	1	9				

$$TPR = \frac{4}{4+6} = 0.4$$

 $FPR = \frac{1}{1+9} = 0.1$

 $x \in (0.53, 0.54]$

$$TPR = \frac{5}{5+5} = 0.5$$

 $FPR = \frac{1}{1+9} = 0.1$

 $x \in (0.52, 0.53]$

$$\begin{array}{c|cccc} & & \text{Predicted} \\ \hline & p & n \\ \hline p & 5 & 5 \\ \hline & n & 2 & 8 \\ \hline \end{array}$$

$$TPR = \frac{5}{5+5} = 0.5$$
$$FPR = \frac{2}{2+8} = 0.2$$

$$x \in (0.51, 0.52]$$

		Predicted				
		p	n			
ctual	р	5	5			
Act	n	3	7			

$$TPR = \frac{5}{5+5} = 0.5$$

$$FPR = \frac{3}{3+7} = 0.3$$

$x \in (0.505, 0.51]$

$$\begin{array}{c|cccc} & & \text{Predicted} \\ \hline & p & n \\ \hline p & 6 & 4 \\ \hline & n & 3 & 7 \\ \hline \end{array}$$

$$TPR = \frac{6}{6+4} = 0.6$$

 $FPR = \frac{3}{3+7} = 0.3$

$x \in (0.4, 0.505]$

$$\begin{array}{c|cccc} & & \text{Predicted} \\ \hline & p & n \\ \hline & p & 6 & 4 \\ \hline & n & 4 & 6 \\ \hline \end{array}$$

$$TPR = \frac{6}{6+4} = 0.6$$
$$FPR = \frac{4}{4+6} = 0.4$$

$$x \in (0.39, 0.4]$$

$$\begin{array}{c|cccc} & \operatorname{Predicted} \\ & p & n \\ \hline p & 7 & 3 \\ \hline p & n & 4 & 6 \\ \end{array}$$

$$TPR = \frac{7}{7+3} = 0.7$$

 $FPR = \frac{4}{4+6} = 0.4$

 $x \in (0.38, 0.39]$

		Pre	Predicted					
	p n							
ctual	р	7	3					
Act	n	5	5					

$$TPR = \frac{7}{7+3} = 0.7$$

 $FPR = \frac{5}{5+5} = 0.5$

 $x \in (0.37, 0.38]$

$$TPR = \frac{8}{8+2} = 0.8$$

 $FPR = \frac{5}{5+5} = 0.5$

 $x \in (0.36, 0.37]$

$$\begin{array}{c|cccc} & & & & & \\ \hline & p & n \\ \hline p & 8 & 2 \\ \hline & n & 6 & 4 \\ \hline \end{array}$$

$$TPR = \frac{8}{8+2} = 0.8$$

 $FPR = \frac{6}{6+4} = 0.6$

$$x \in (0.35, 0.36]$$

$$TPR = \frac{8}{8+2} = 0.8$$

 $FPR = \frac{7}{7+3} = 0.7$

$x \in (0.34, 0.35]$

$$TPR = \frac{8}{8+2} = 0.8$$

 $FPR = \frac{8}{8+2} = 0.8$

$x \in (0.33, 0.34]$

$$\begin{array}{c|cccc} & & & \text{Predicted} \\ \hline & p & n \\ \hline & p & 9 & 1 \\ \hline & n & 8 & 2 \\ \hline \end{array}$$

$$TPR = \frac{9}{9+1} = 0.9$$

 $FPR = \frac{8}{8+2} = 0.8$

$x \in (0.30, 0.33]$

$$TPR = \frac{9}{9+1} = 0.9$$

 $FPR = \frac{9}{9+1} = 0.9$

$$x\in[0.1,0.30]$$

$$TPR = \frac{10}{10+0} = 1.0$$

 $FPR = \frac{9}{9+1} = 0.9$

$$x \in (0.0, 0.1]$$

$$\begin{array}{c|cccc} & & \text{Predicted} \\ \hline \text{Predicted} & & p & n \\ \hline p & 10 & 0 \\ \hline p & n & 10 & 0 \\ \hline \end{array}$$

$$TPR = \frac{10}{10+0} = 1.0$$

$$FPR = \frac{10}{10+0} = 1.0$$

Therefore, we get the list of the all TPR/FPR pairs:

TPR										
FPR	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.2	0.3	0.3

TPR											
FPR	0.4	0.4	0.5	0.5	0.6	0.7	0.8	0.8	0.9	0.9	1.0

Draw the ROC curve:

