

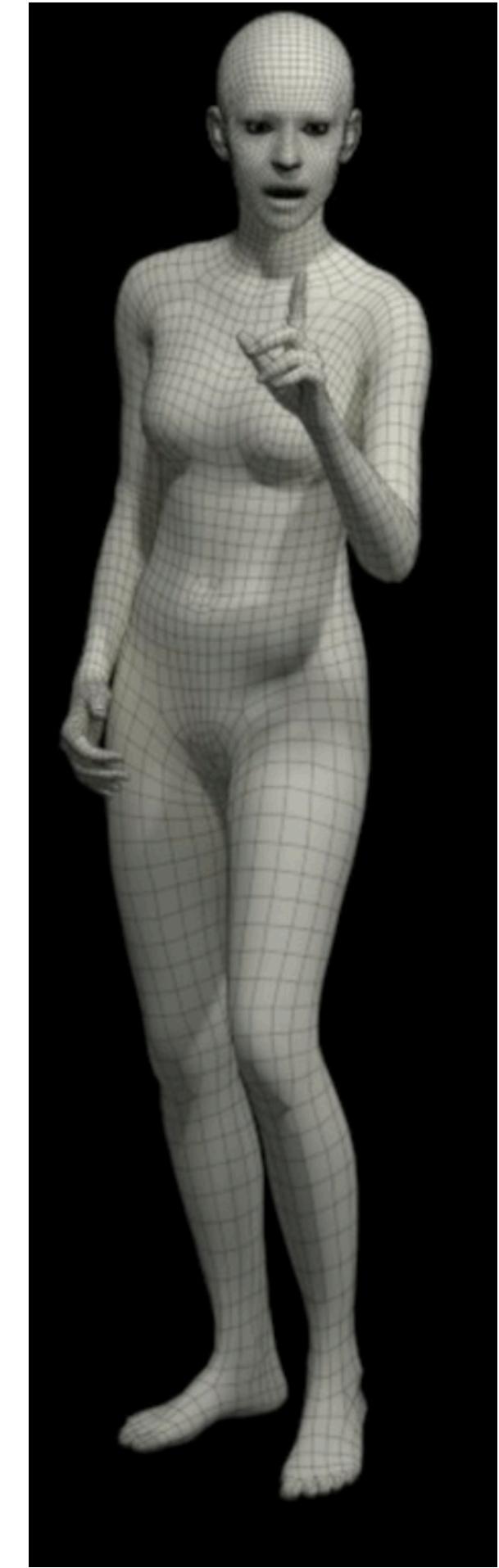
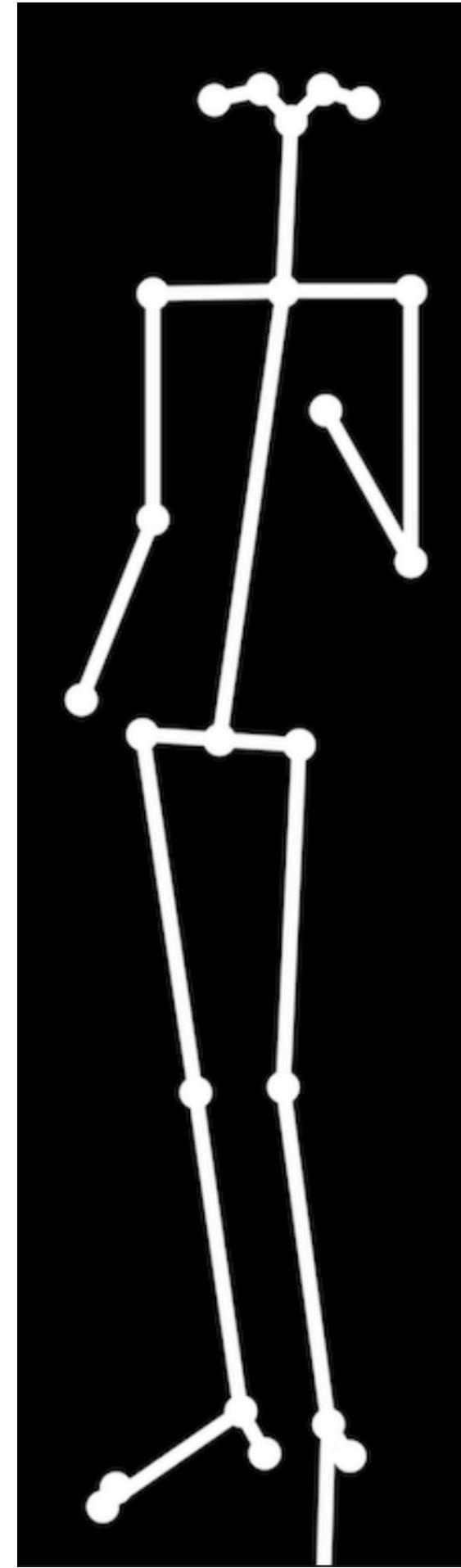
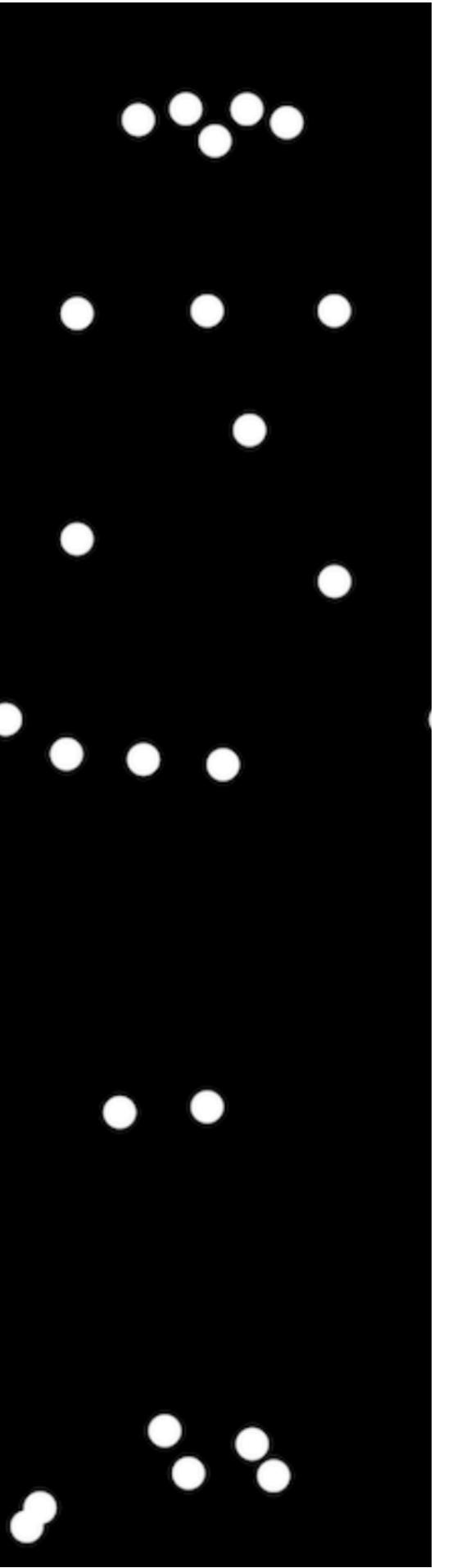
**Lecture 14:**

# **Character Modeling**

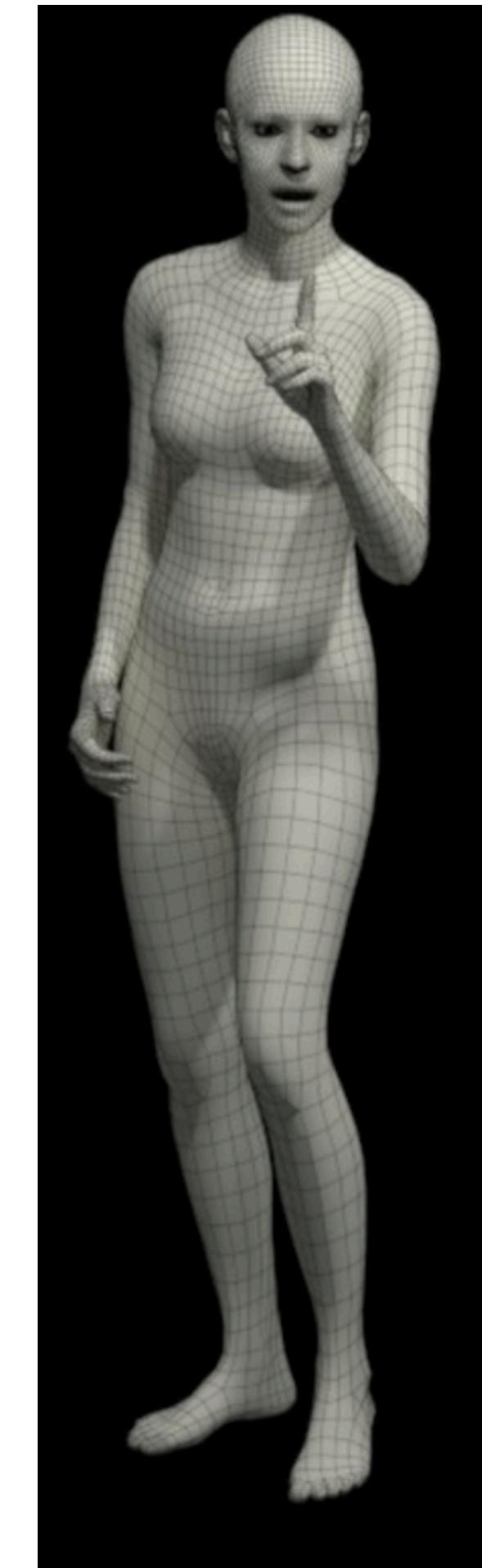
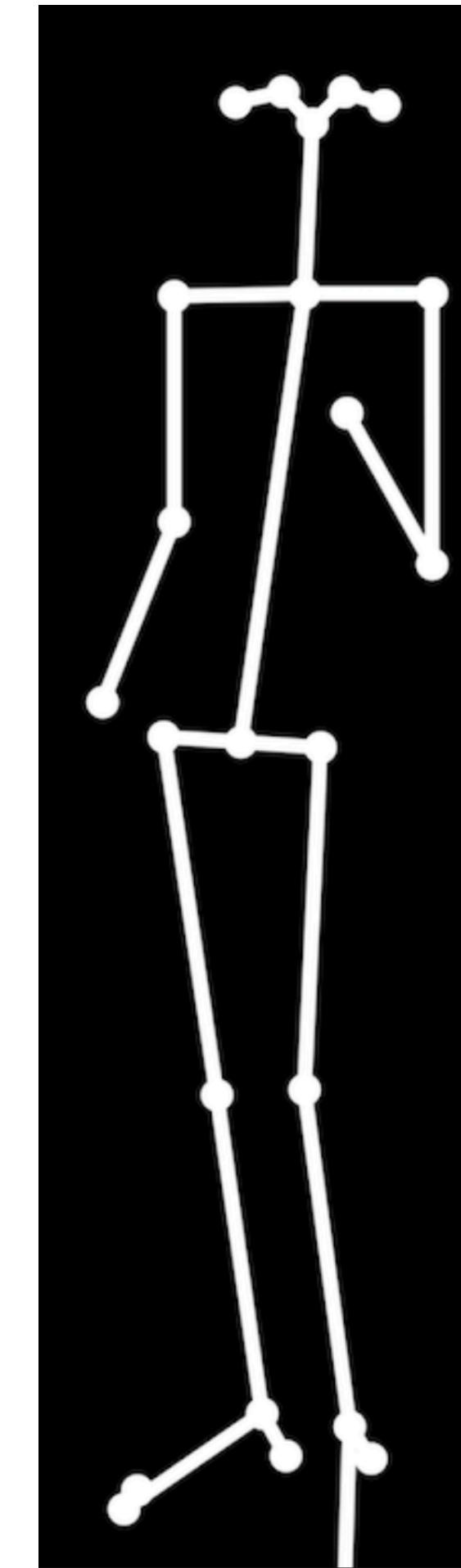
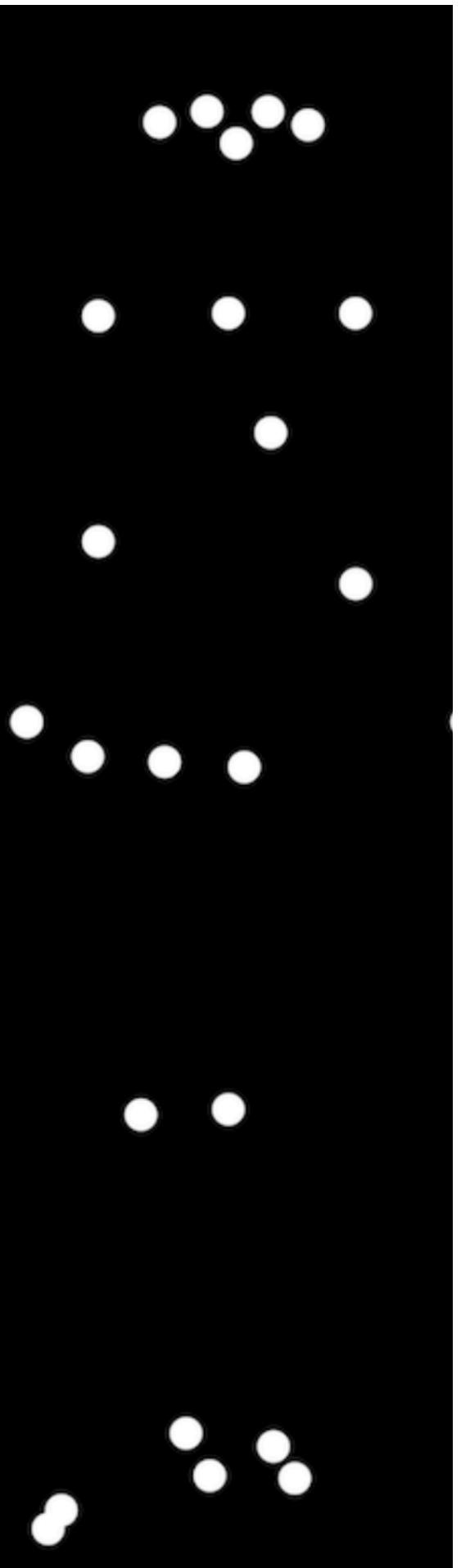
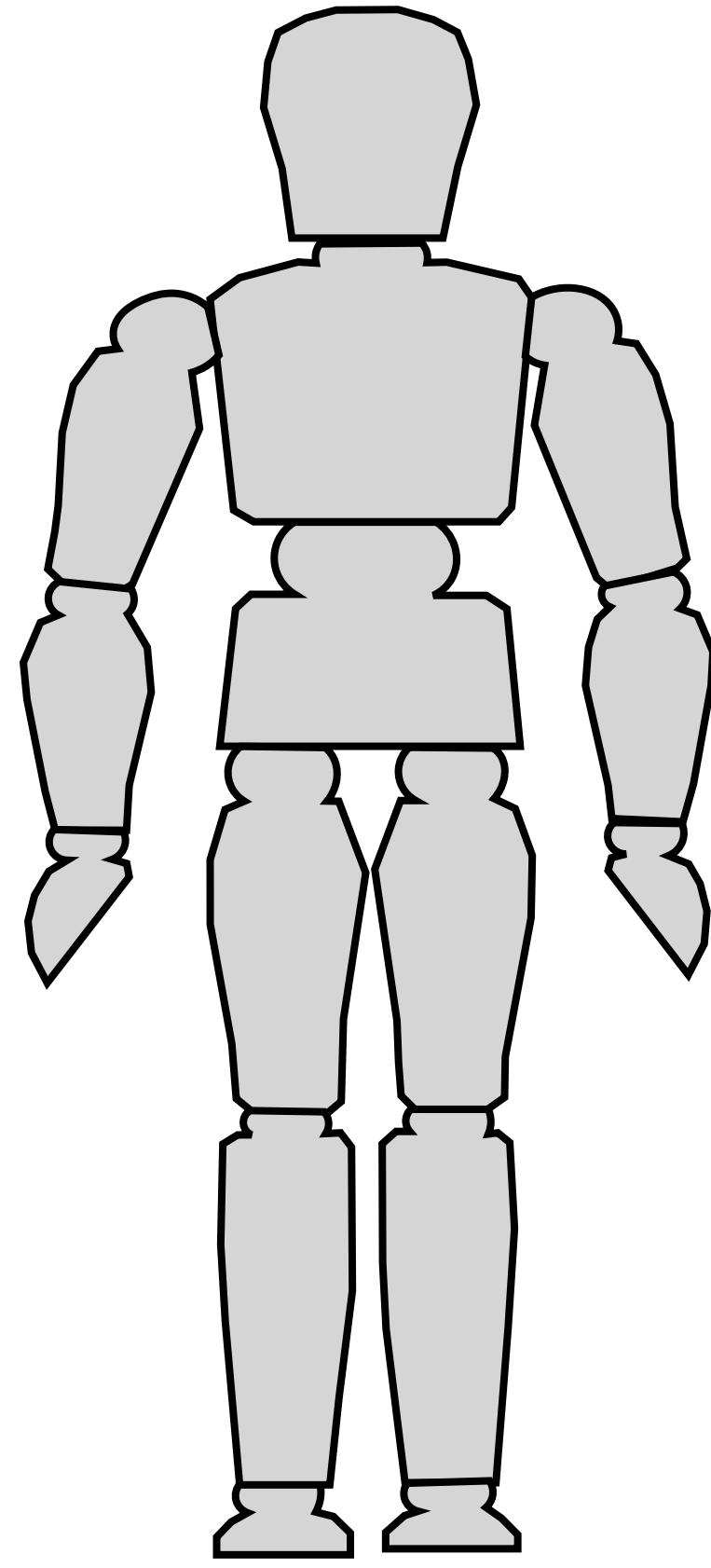
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**FUNDAMENTALS OF COMPUTER GRAPHICS  
Animation & Simulation  
Stanford CS248B, Fall 2022**

# What is a pose?



# What is a pose?



# Modeling with transformations

- From computer graphics, we learn to create models that consist of multiple 3D parts using affine transformations.
- A point in homogeneous coordinates,  $\mathbf{p} = \{x, y, z, 1\}$  can be transformed by:

**Translate**

$$\begin{bmatrix} x + t_x \\ y + t_y \\ z + t_z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

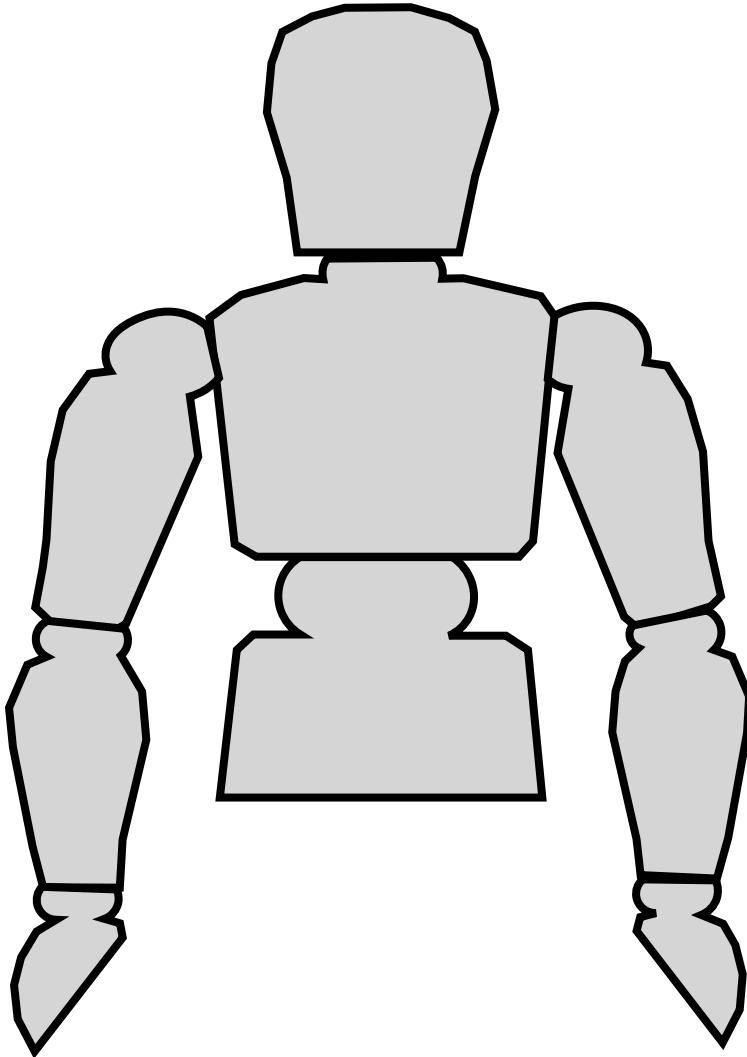
**Rotate (about x axis)**

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

**Scale**

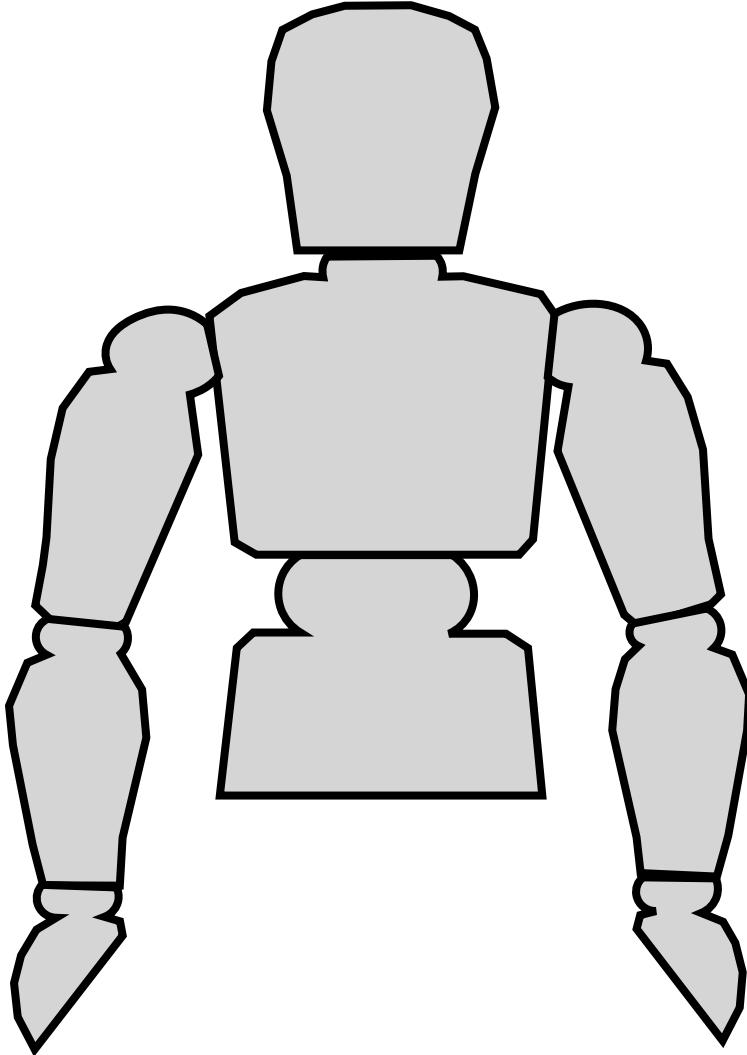
$$\begin{bmatrix} s_x x \\ s_y y \\ s_z z \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# Make a human figure by transforming parts

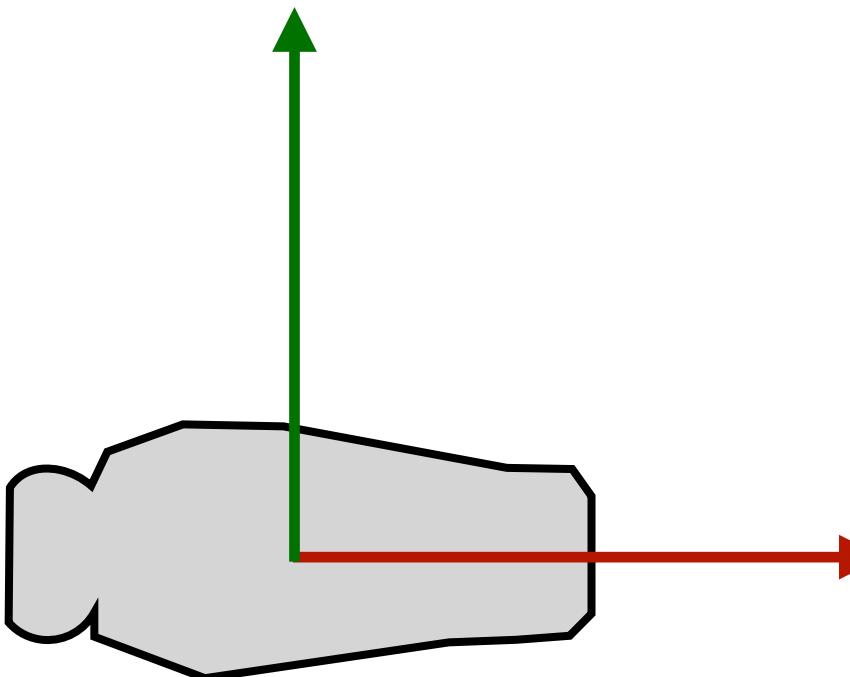


for each part  
**scale** to the right size  
**rotate** to the right orientation  
**translate** to the right position

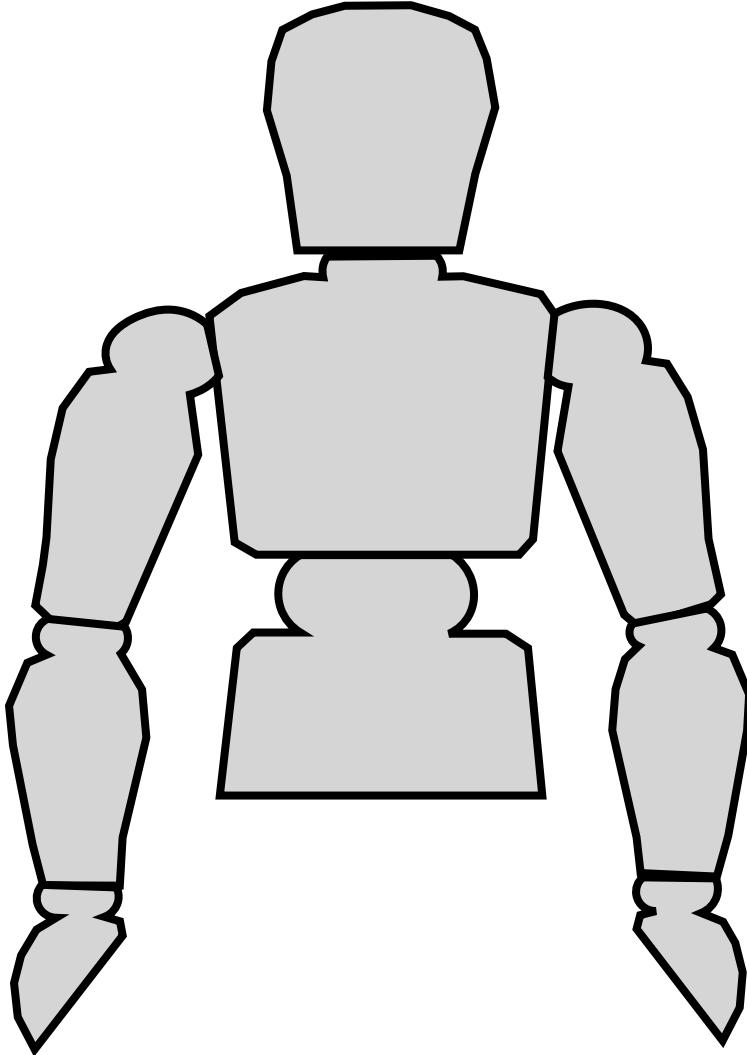
# Make a human figure by transforming parts



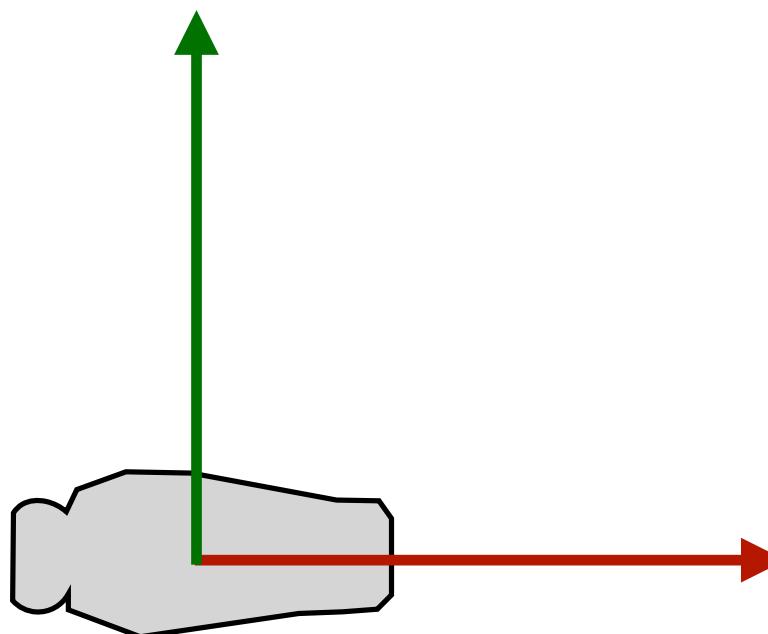
for each part  
**scale** to the right size  
**rotate** to the right orientation  
**translate** to the right position



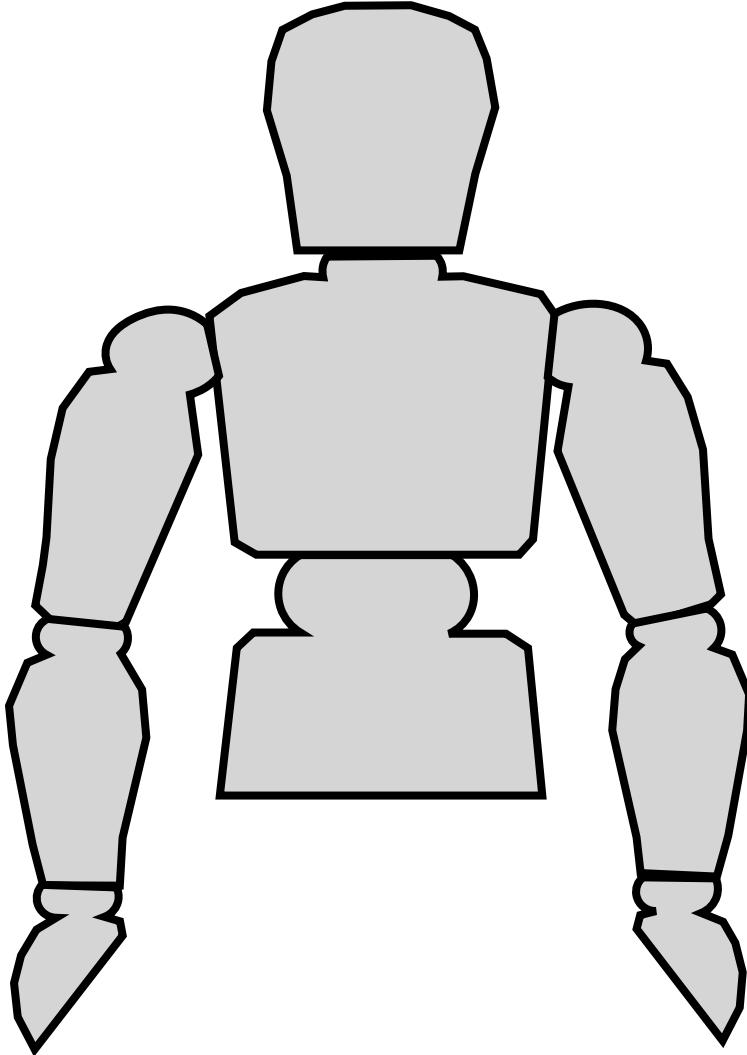
# Make a human figure by transforming parts



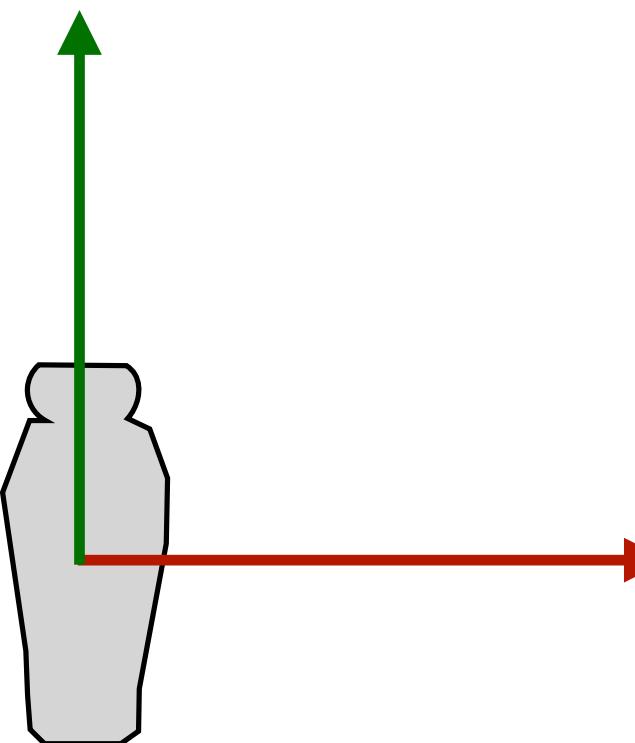
for each part  
**scale** to the right size  
**rotate** to the right orientation  
**translate** to the right position



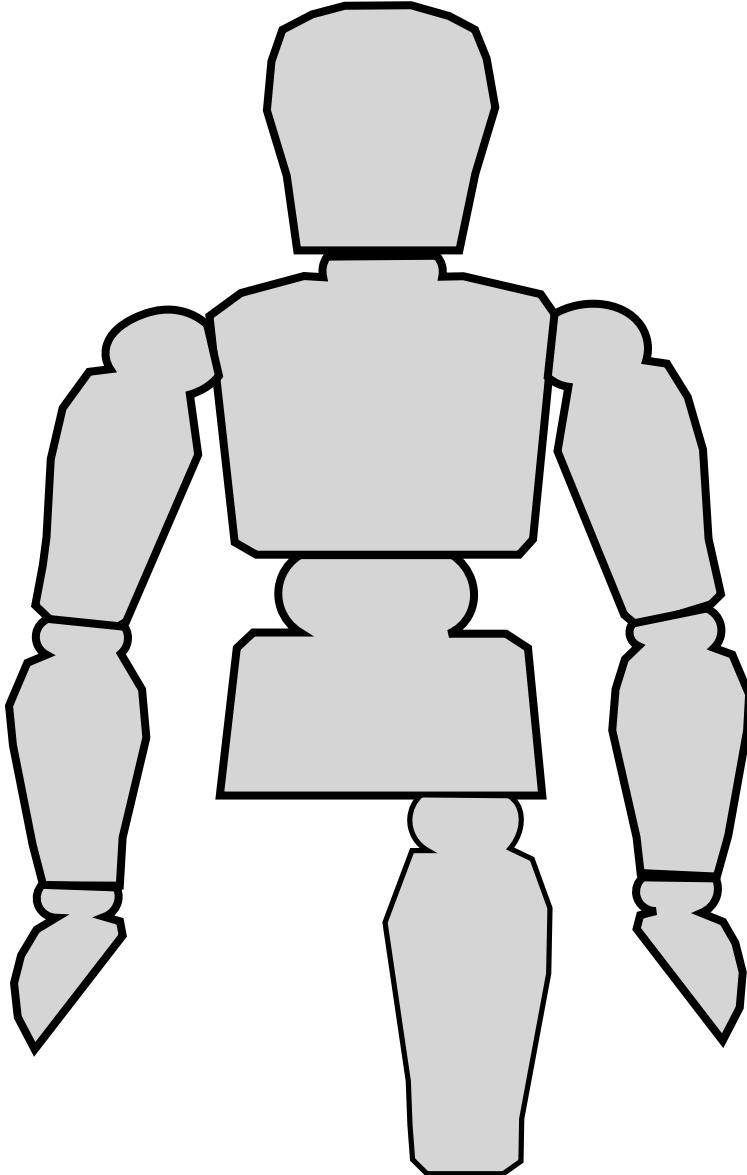
# Make a human figure by transforming parts



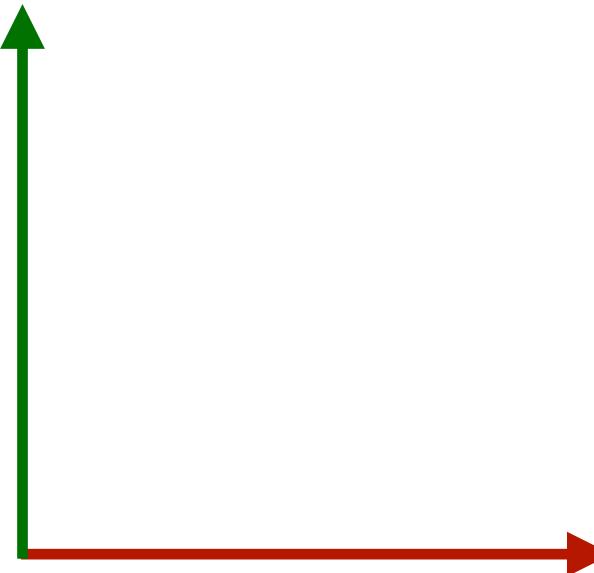
for each part  
**scale** to the right size  
**rotate** to the right orientation  
**translate** to the right position



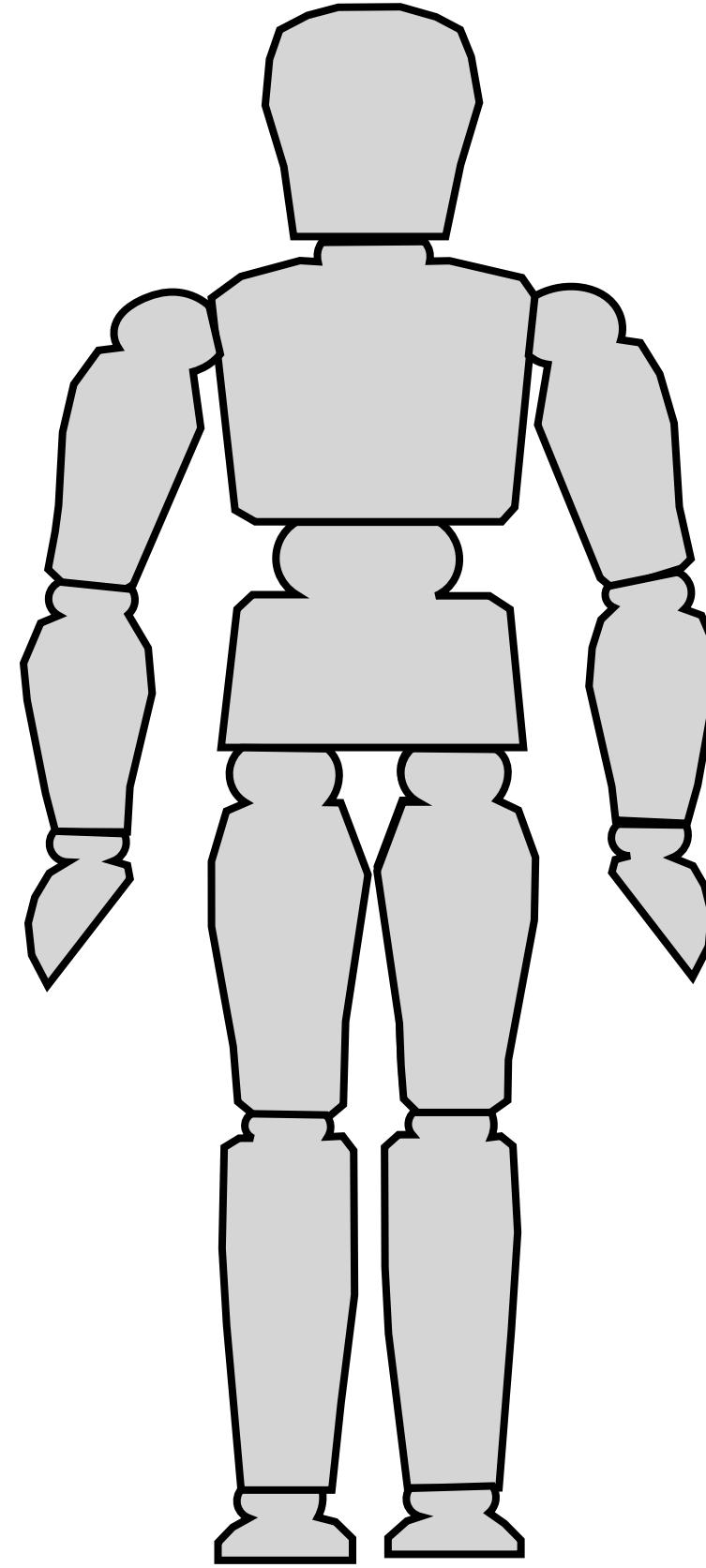
# Make a human figure by transforming parts



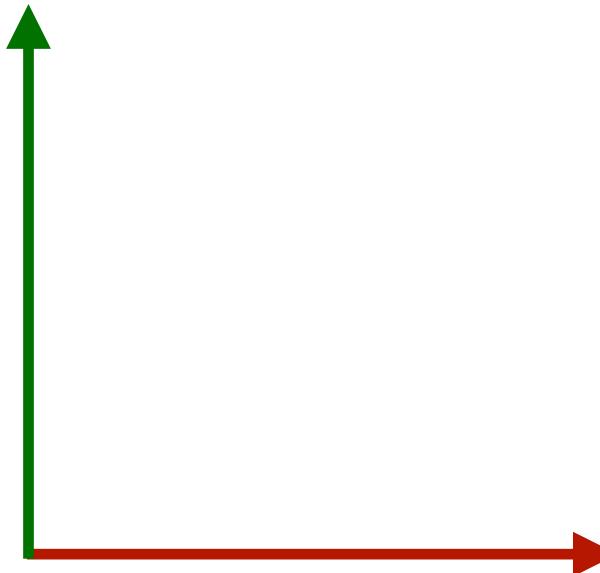
for each part  
**scale** to the right size  
**rotate** to the right orientation  
**translate** to the right position



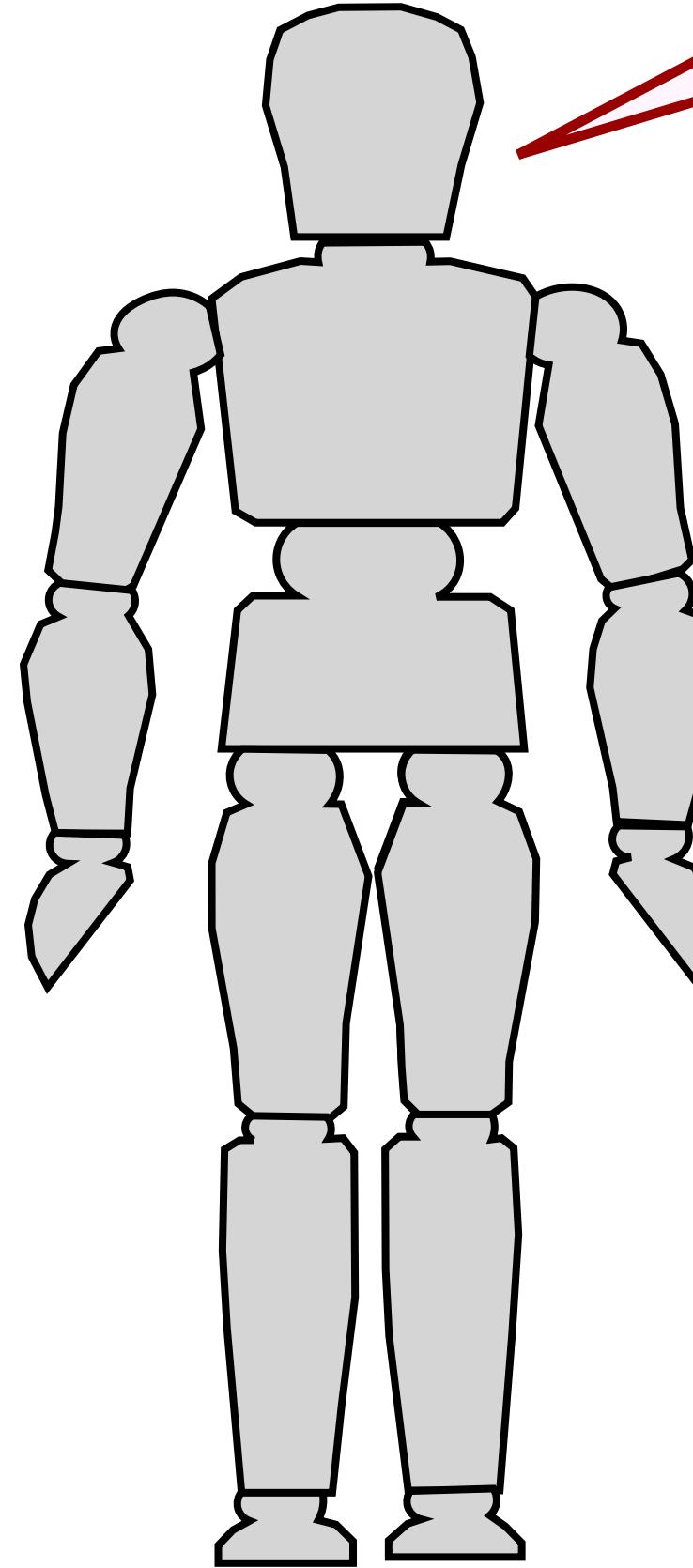
# Make a human figure by transforming parts



for each part  
**scale** to the right size  
**rotate** to the right orientation  
**translate** to the right position



# Make a human figure by transforming parts



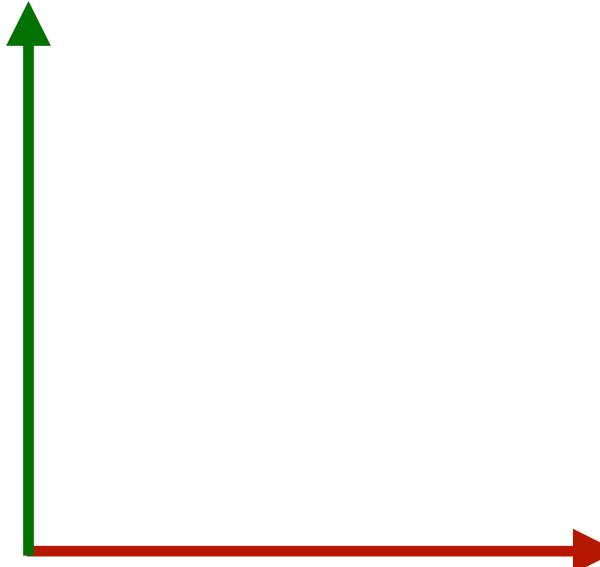
Everything looks great!

for each part

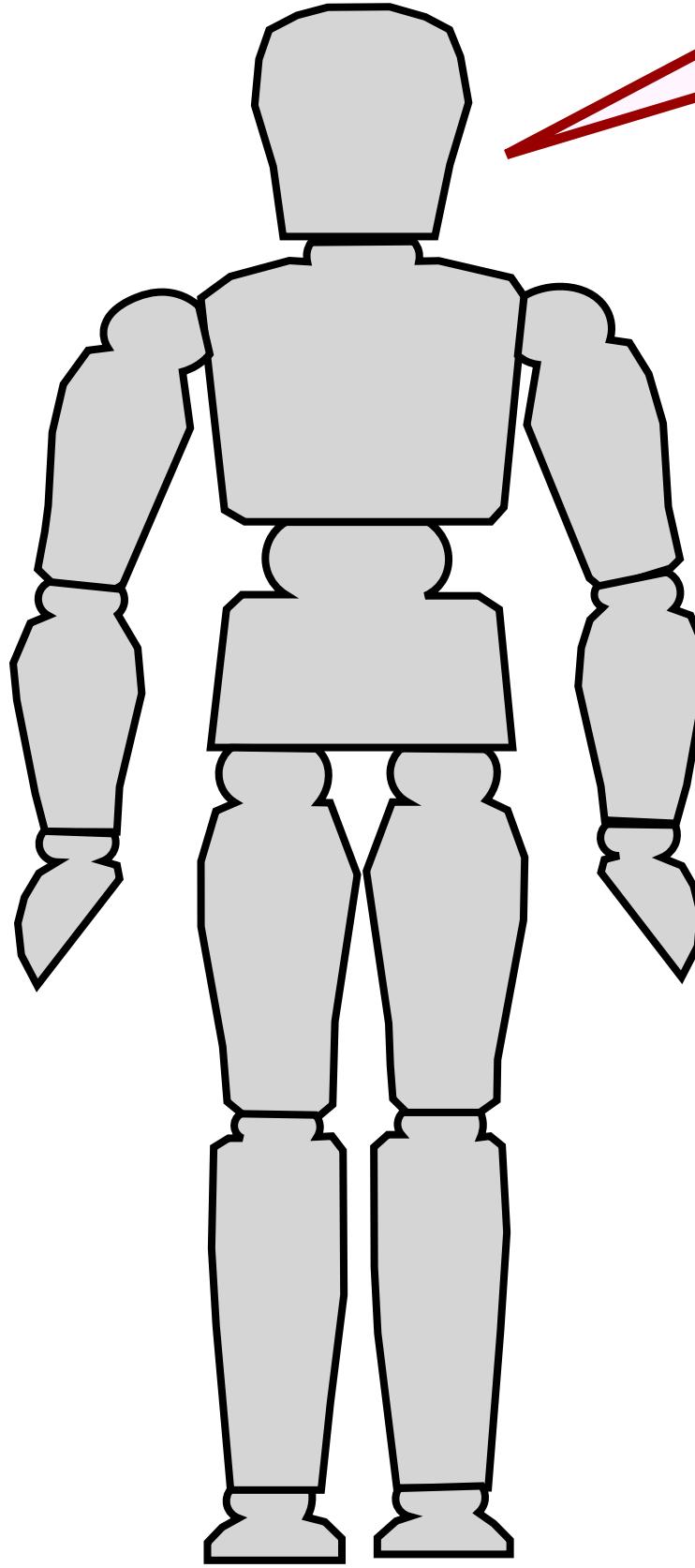
**scale** to the right size

**rotate** to the right orientation

**translate** to the right position

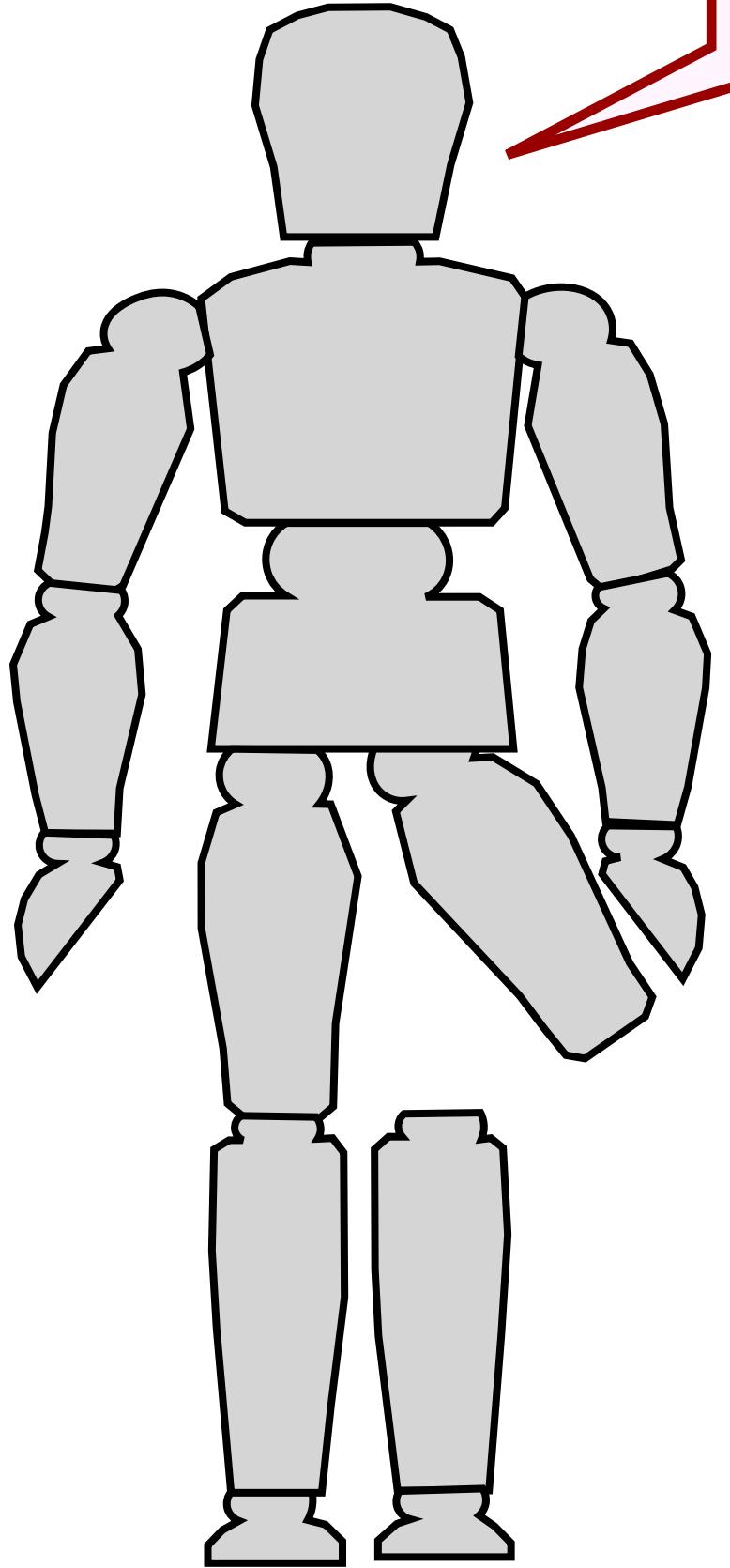


# Now make it move!



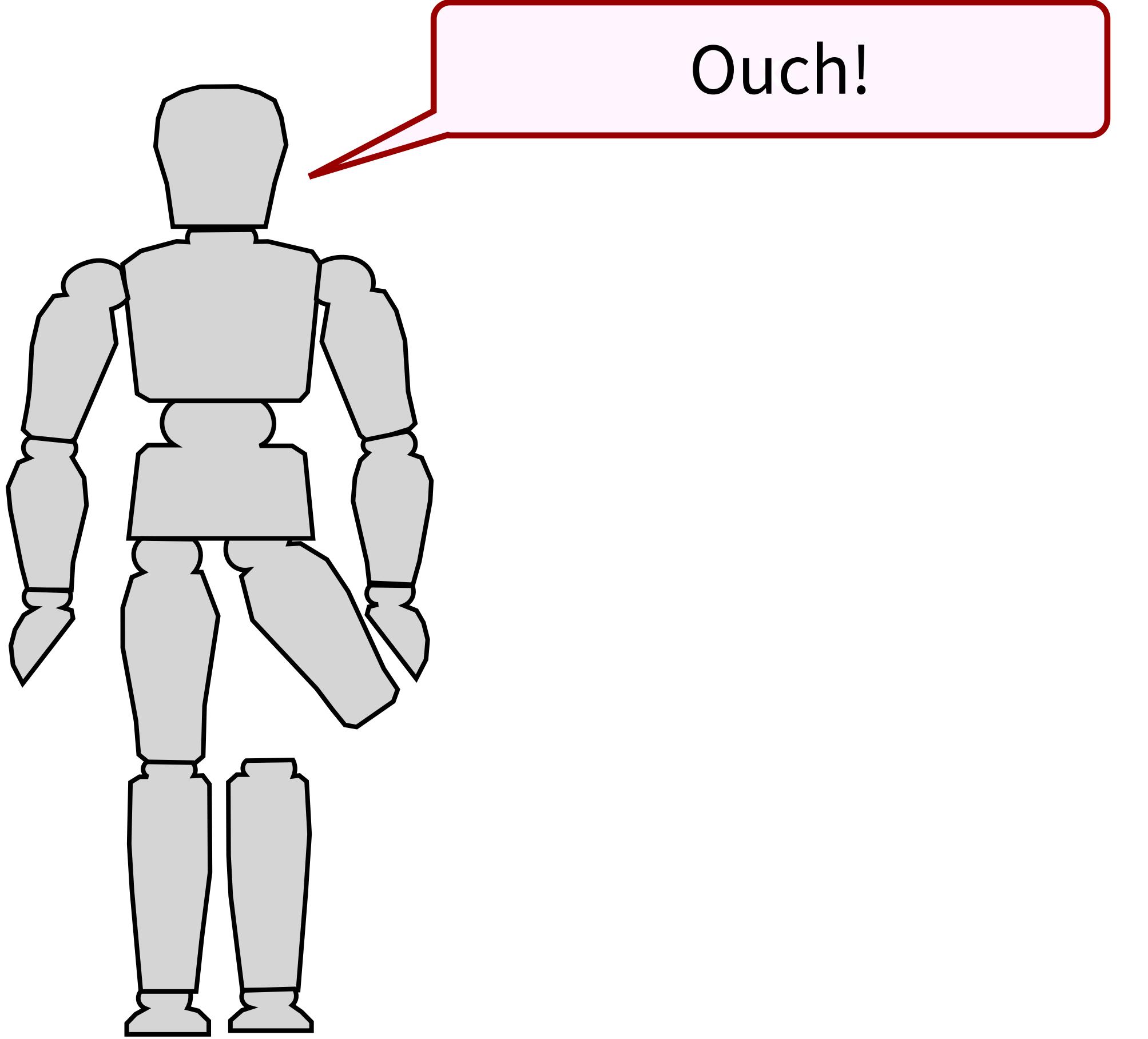
Let's try the left leg!

# Now make it move!

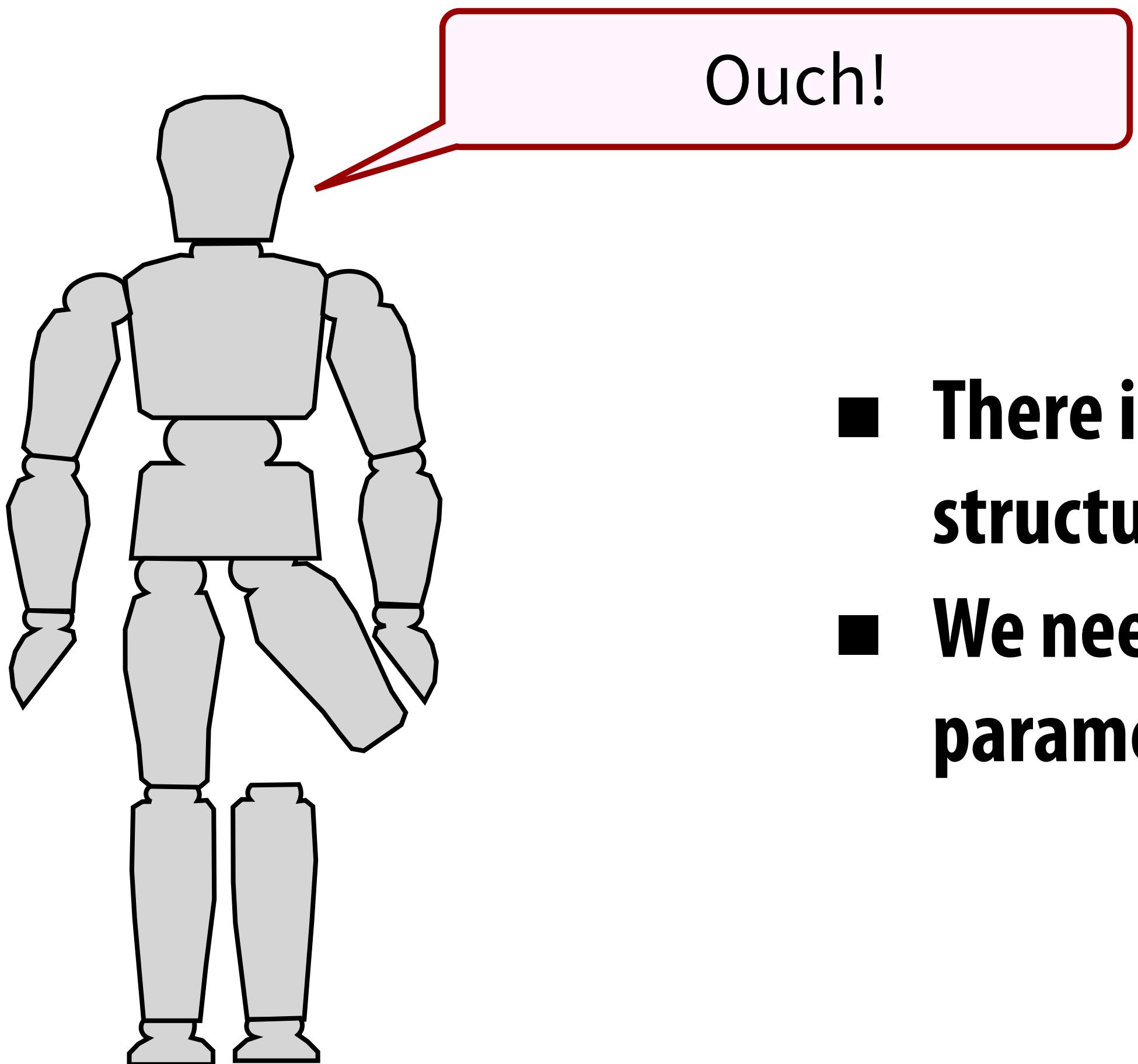


Let's try the left leg!

# Now make it move!

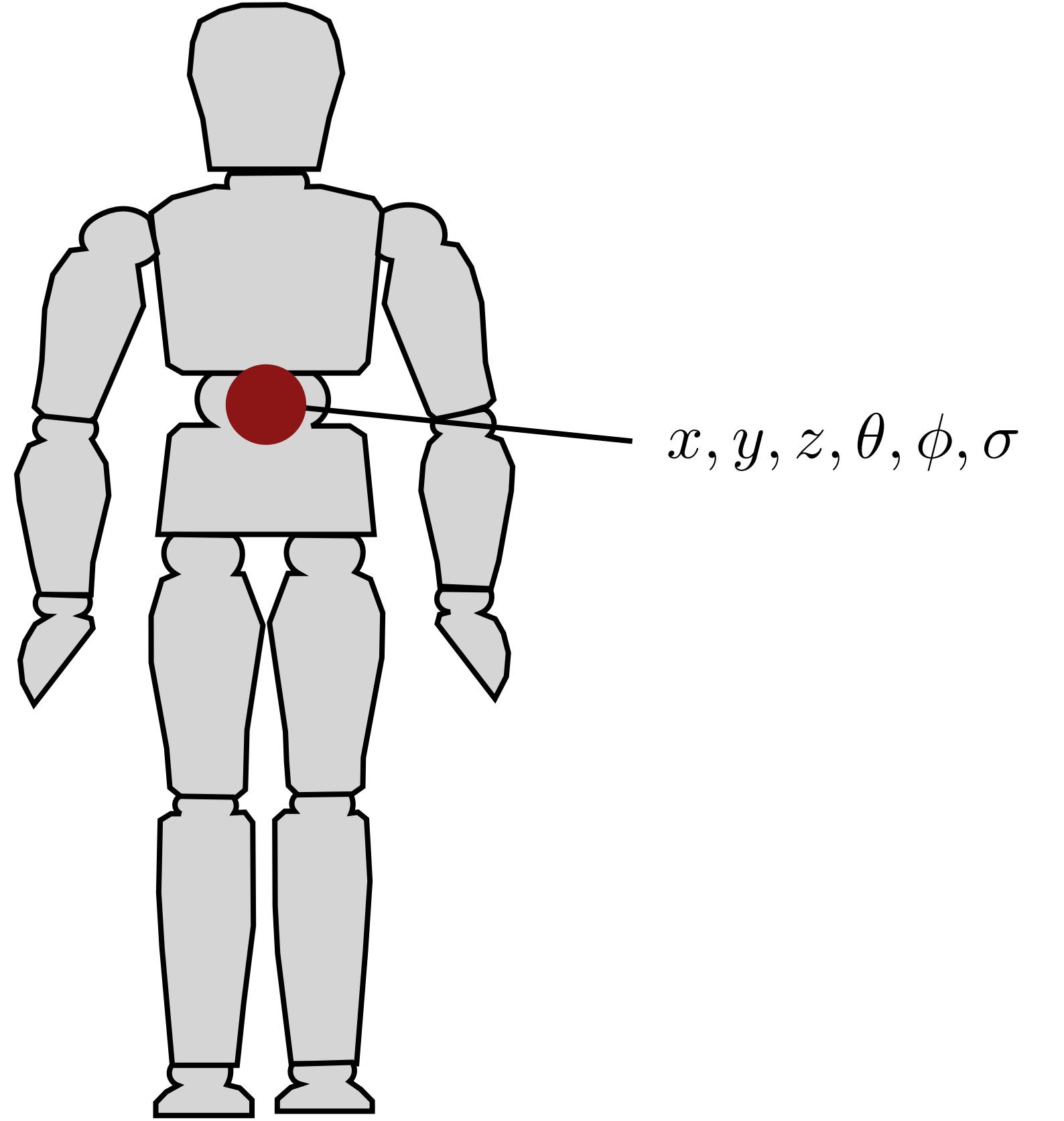


# Now make it move!

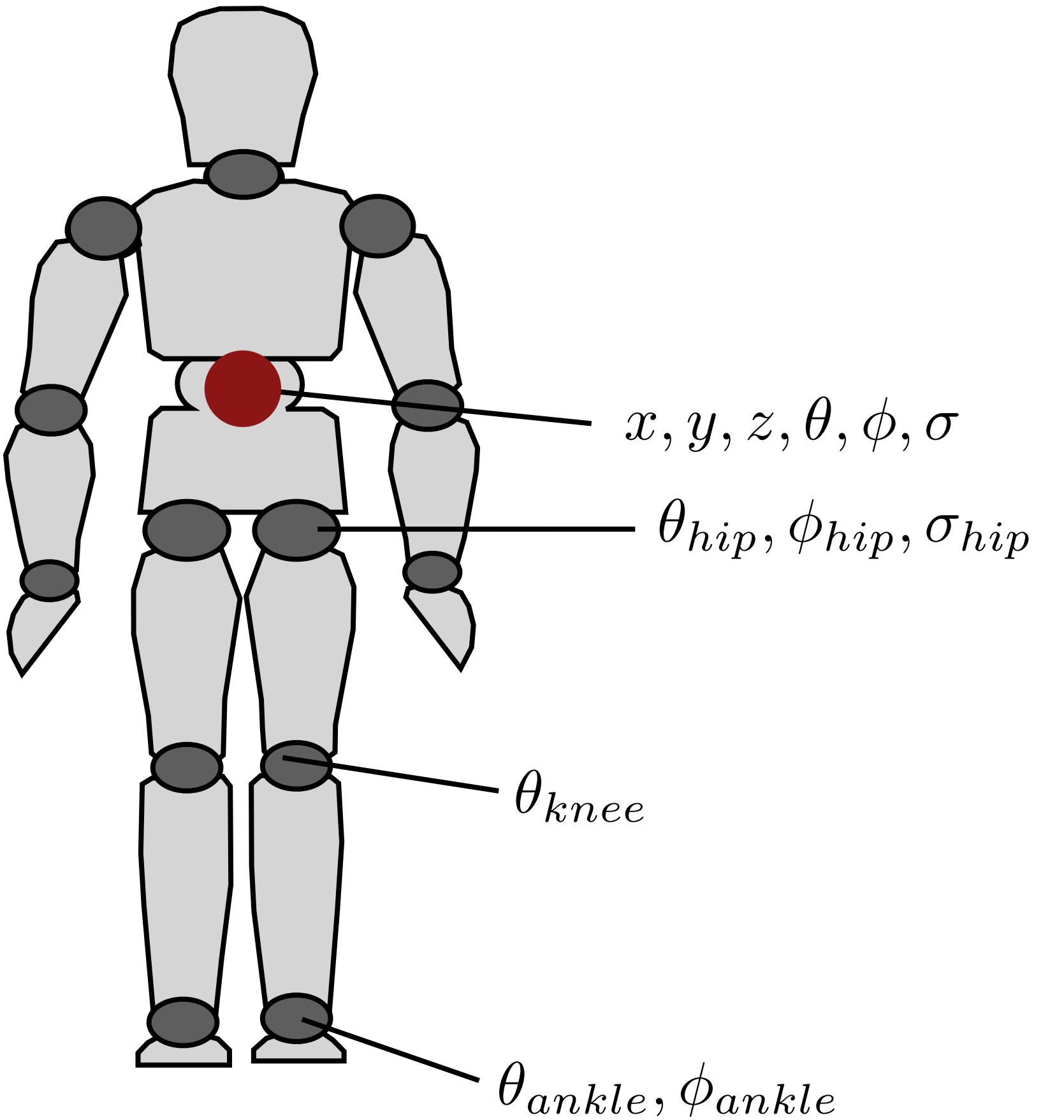


- There is no constraint that enforces the articulated structure to move in the “legal” configuration space.
- We need a hierarchical structure and a compact parameterized space to represent poses.

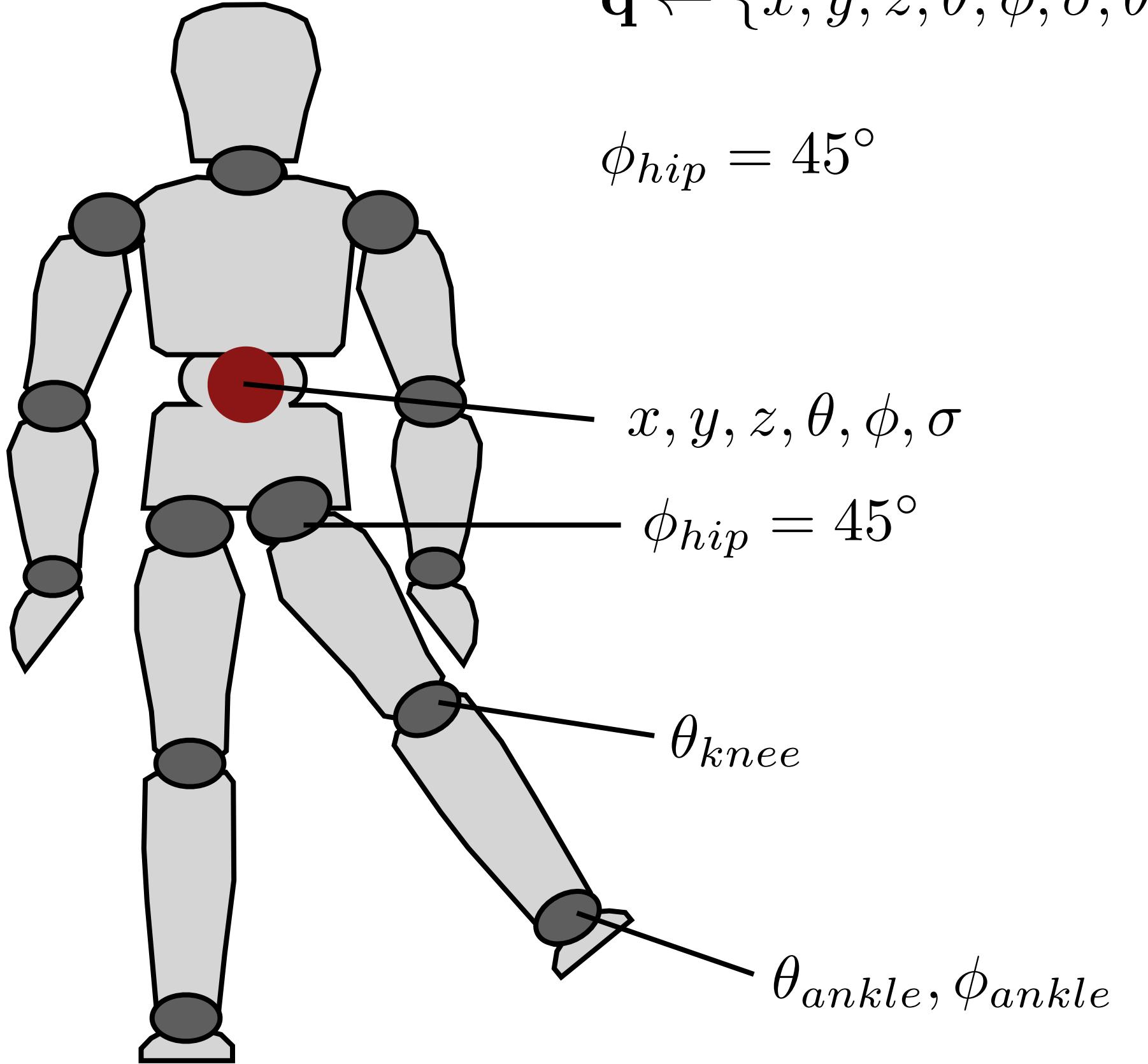
# How to represent a pose?



# How to represent a pose?



# Define hierarchy



$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \dots\}$$

$$\phi_{hip} = 45^\circ$$

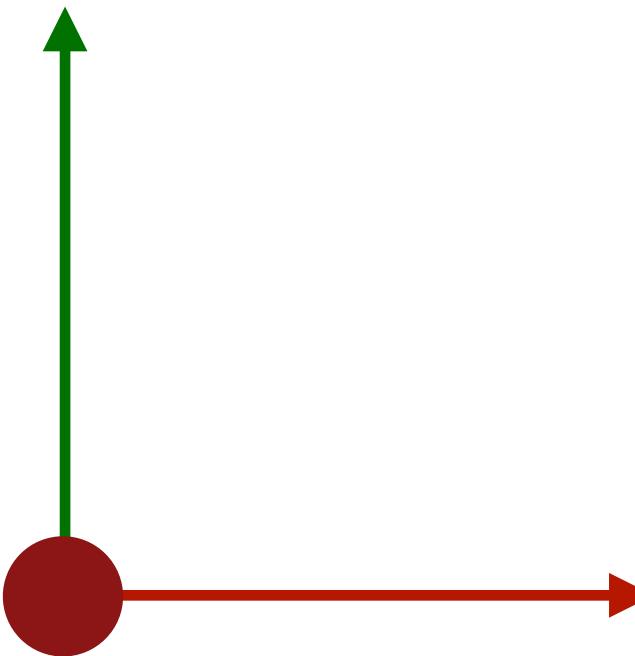
# Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \dots\}$$

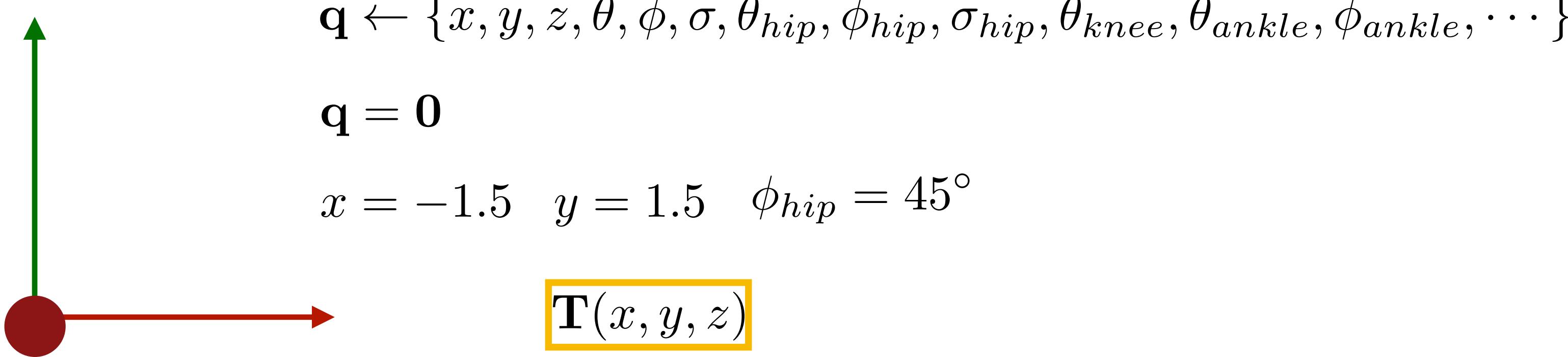
$$\mathbf{q} = \mathbf{0}$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$

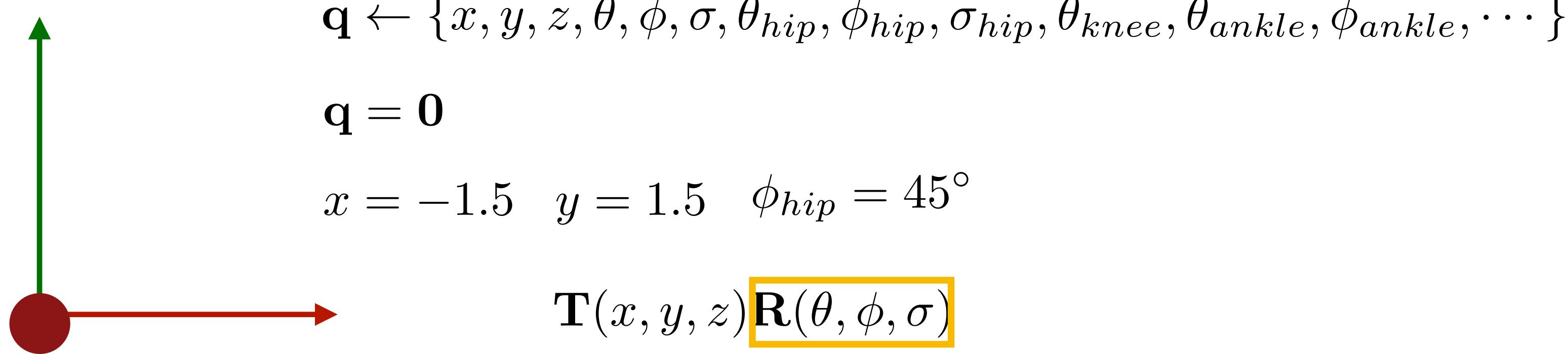
$$\boxed{\mathbf{T}(x, y, z)}$$



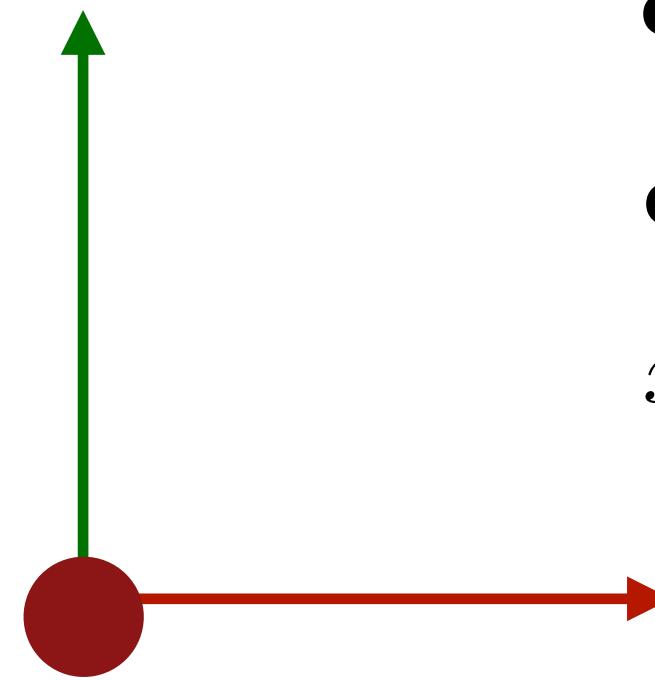
# Hierarchical transformations



# Hierarchical transformations



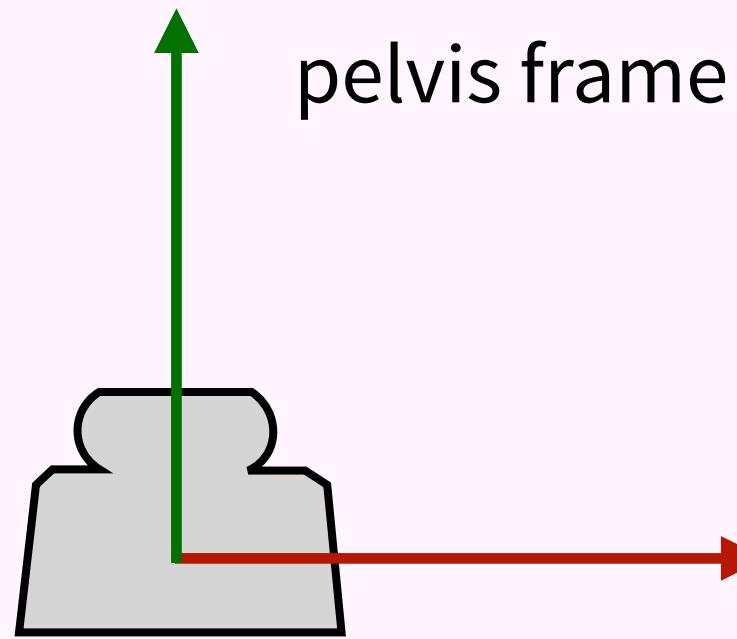
# Hierarchical transformations



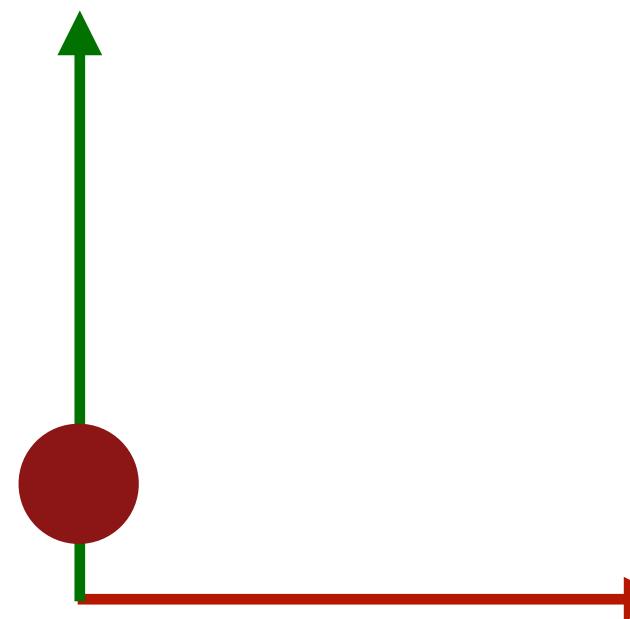
$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \dots\}$$
$$\mathbf{q} = 0$$
$$x = -1.5 \quad y = 1.5$$

$$\mathbf{T}(x, y, z) \mathbf{R}(\theta, \phi, \sigma) \boxed{\mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)}$$

$\mathbf{t}_0, \mathbf{r}_0$ : translation and rotation from the root to the origin of pelvis frame



# Hierarchical transformations

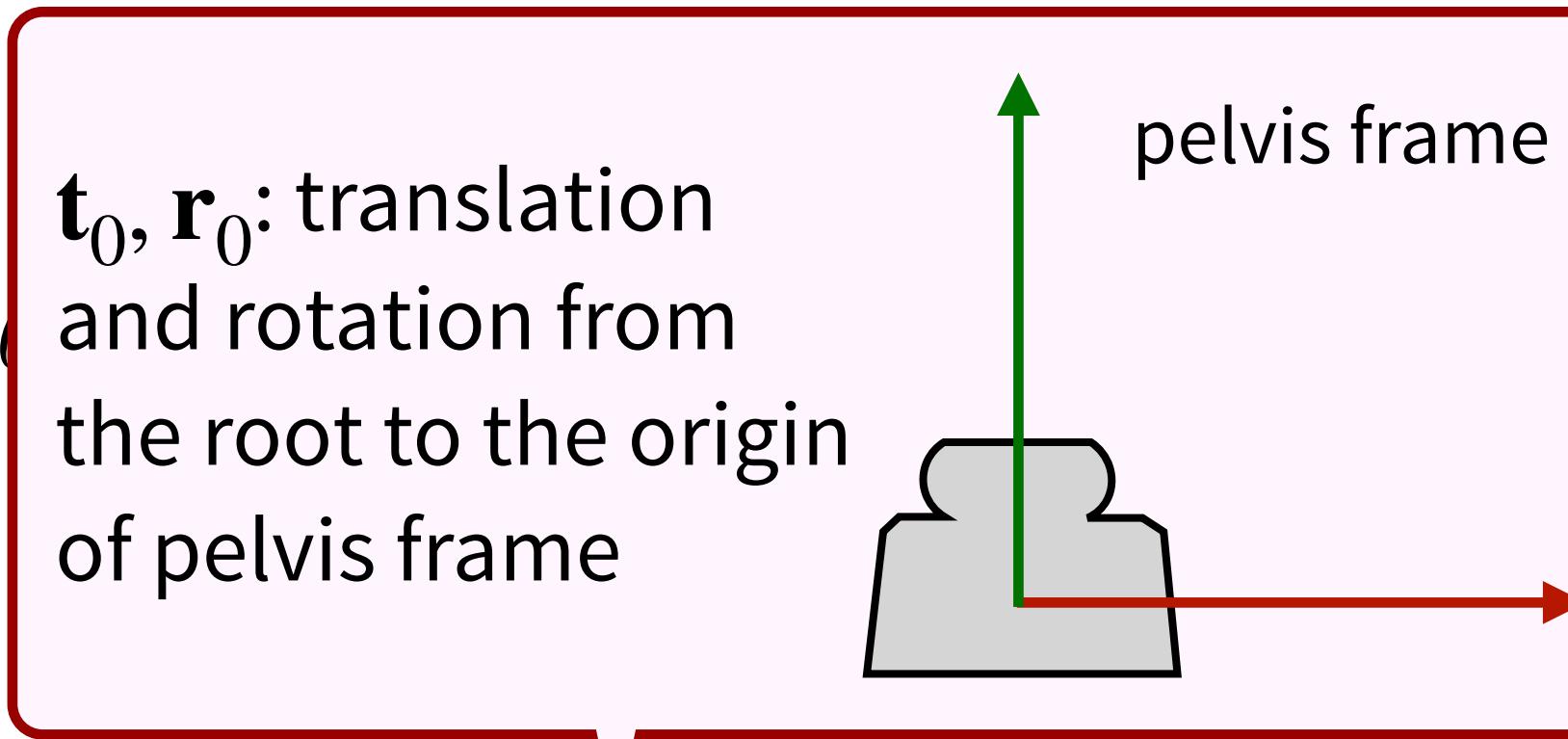


$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \dots\}$$

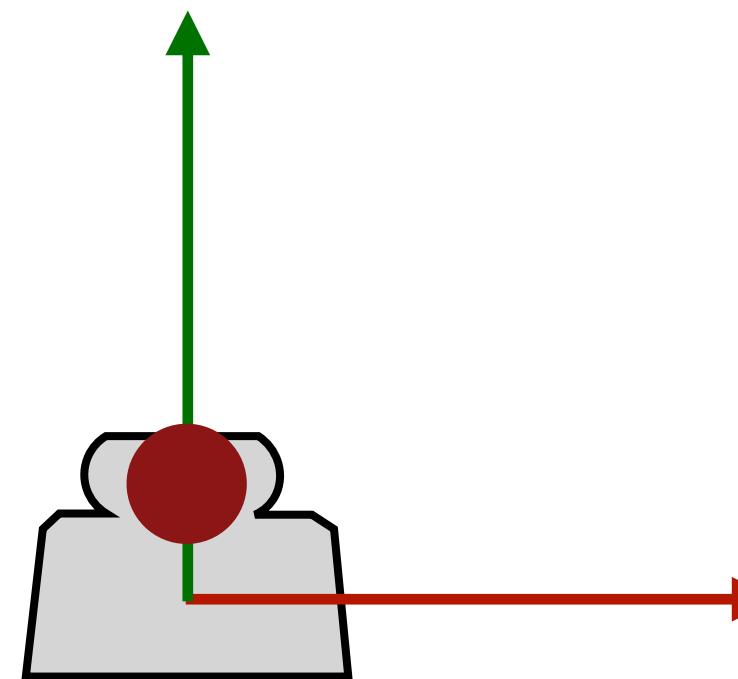
$$\mathbf{q} = 0$$

$$x = -1.5 \quad y = 1.5$$

$$\mathbf{T}(x, y, z) \mathbf{R}(\theta, \phi, \sigma) \boxed{\mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)}$$



# Hierarchical transformations

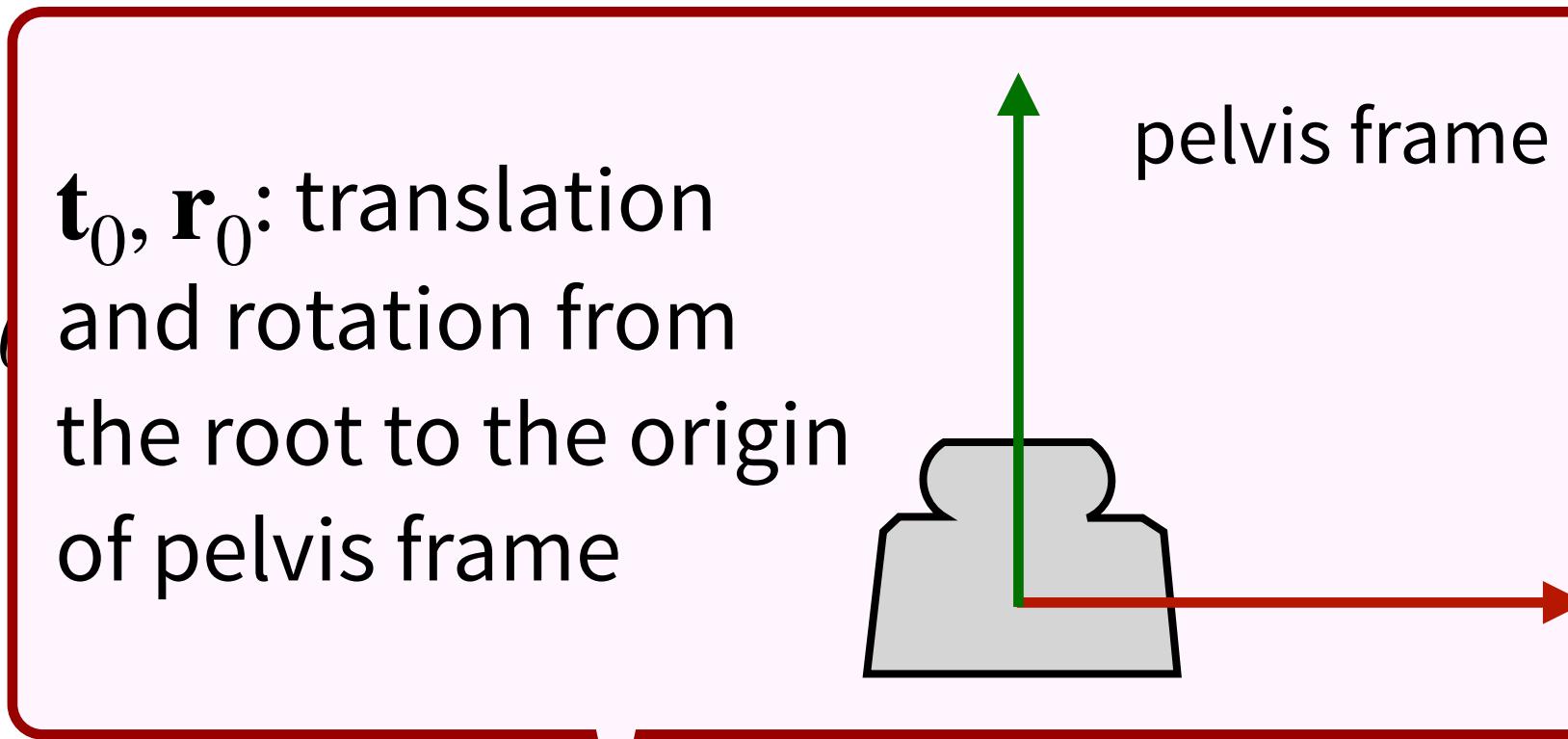


$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \dots\}$$

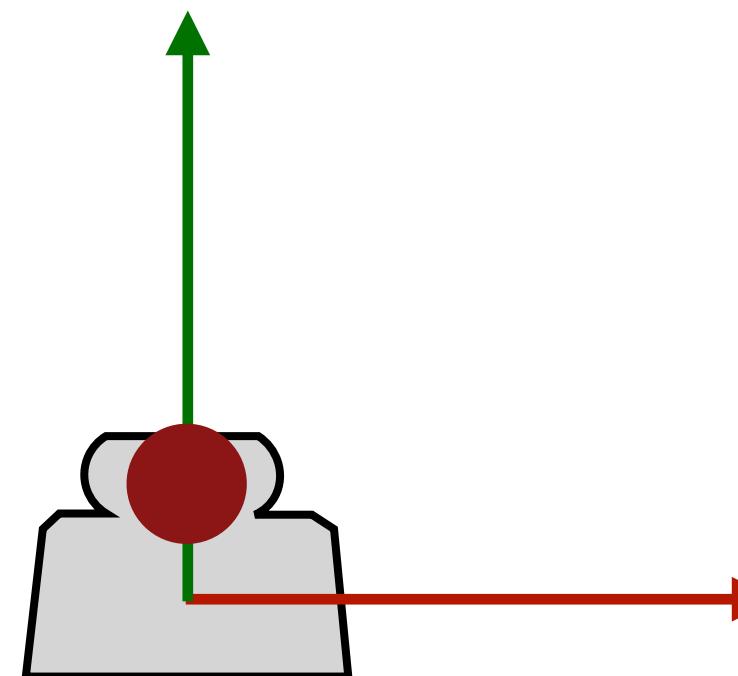
$$\mathbf{q} = 0$$

$$x = -1.5 \quad y = 1.5$$

$$\mathbf{T}(x, y, z) \mathbf{R}(\theta, \phi, \sigma) \mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)$$



# Hierarchical transformations



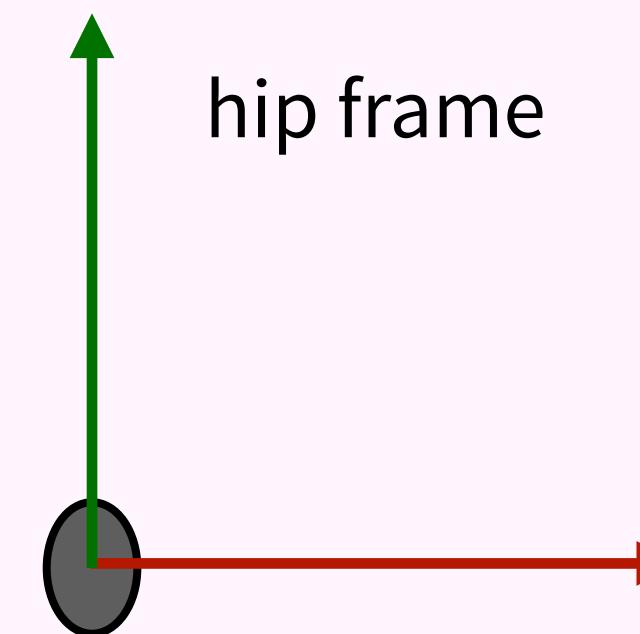
$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}\}$$

$$\mathbf{q} = 0$$

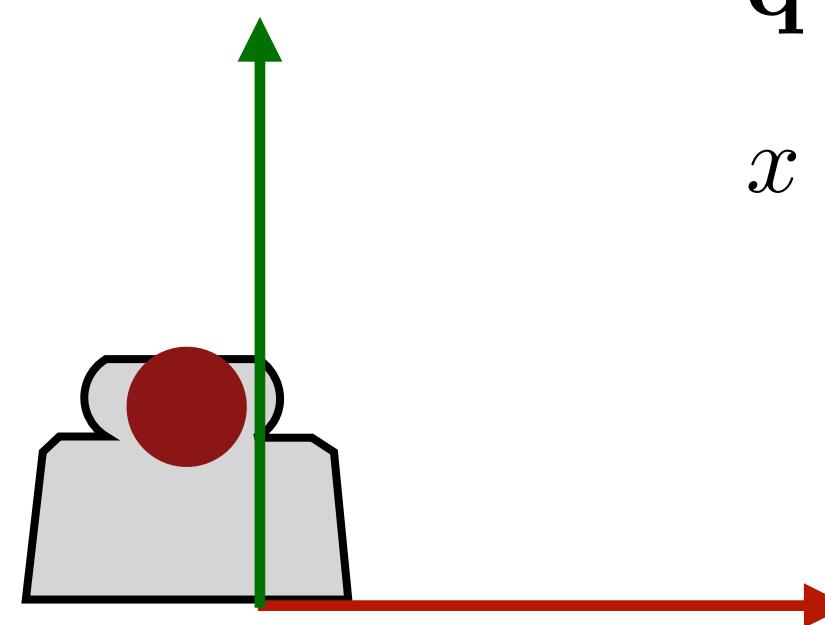
$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$

$$\mathbf{T}(x, y, z) \mathbf{R}(\theta, \phi, \sigma) \mathbf{F}(\mathbf{t}_0, \mathbf{r}_0) \boxed{\mathbf{F}(\mathbf{t}_1, \mathbf{r}_1)}$$

$\mathbf{t}_1$  translates from pelvis to hip joint.  $\mathbf{r}_1$  rotates -90 degree about z-axis.



# Hierarchical transformations



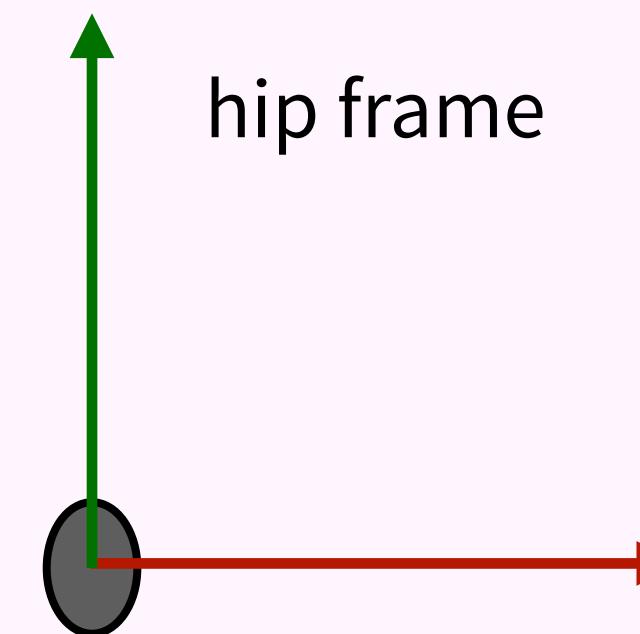
$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}\}$$

$$\mathbf{q} = 0$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$

$$\mathbf{T}(x, y, z) \mathbf{R}(\theta, \phi, \sigma) \mathbf{F}(\mathbf{t}_0, \mathbf{r}_0) \boxed{\mathbf{F}(\mathbf{t}_1, \mathbf{r}_1)}$$

$\mathbf{t}_1$  translates from pelvis to hip joint.  $\mathbf{r}_1$  rotates -90 degree about z-axis.



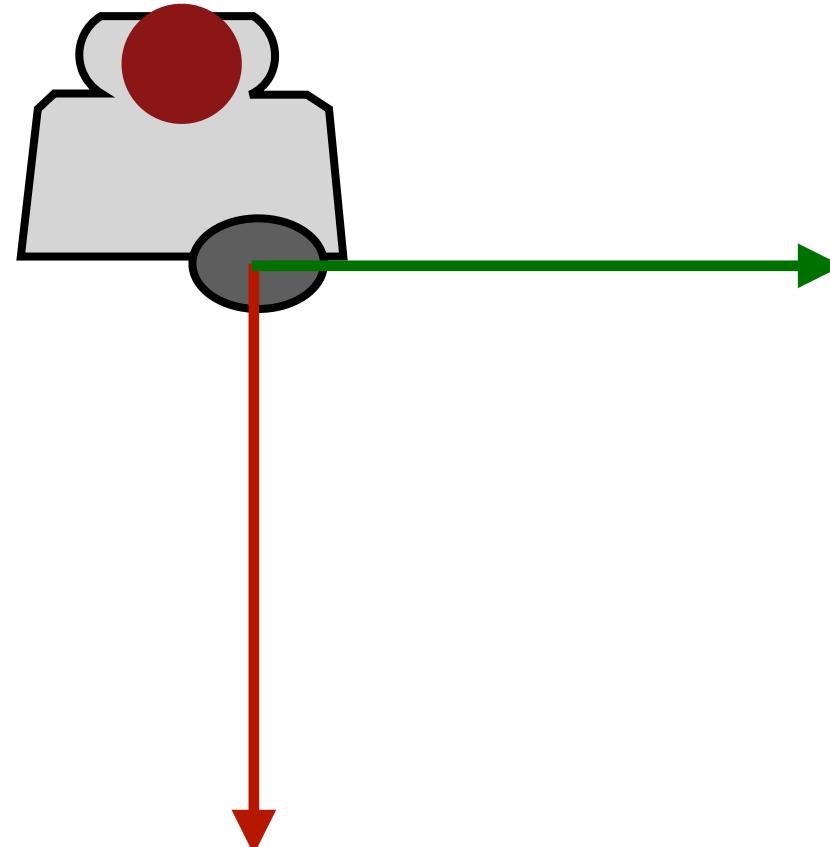
# Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}\}$$

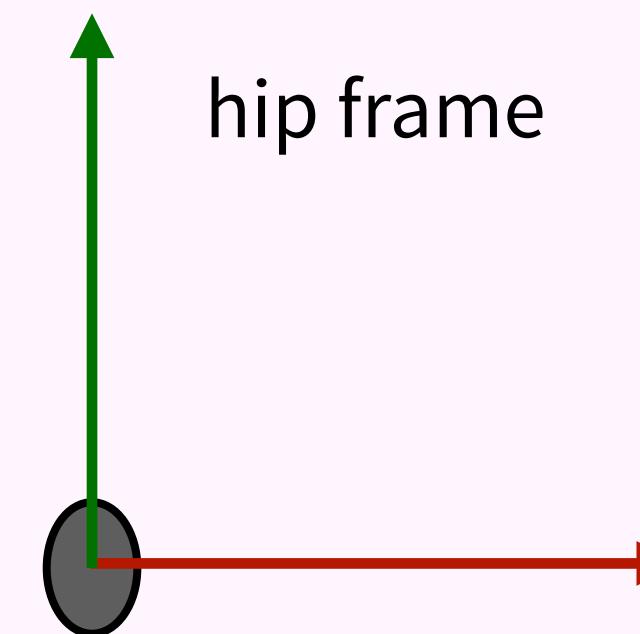
$$\mathbf{q} = 0$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$

$$\mathbf{T}(x, y, z) \mathbf{R}(\theta, \phi, \sigma) \mathbf{F}(\mathbf{t}_0, \mathbf{r}_0) \boxed{\mathbf{F}(\mathbf{t}_1, \mathbf{r}_1)}$$



$\mathbf{t}_1$  translates from pelvis to hip joint.  $\mathbf{r}_1$  rotates -90 degree about z-axis.

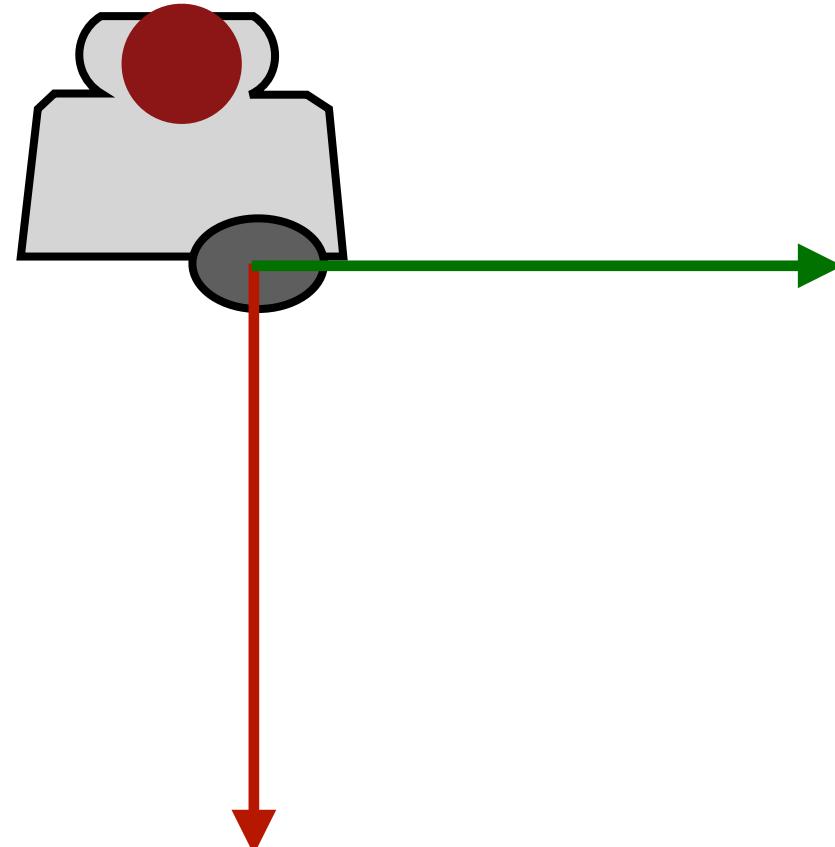


# Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}\}$$

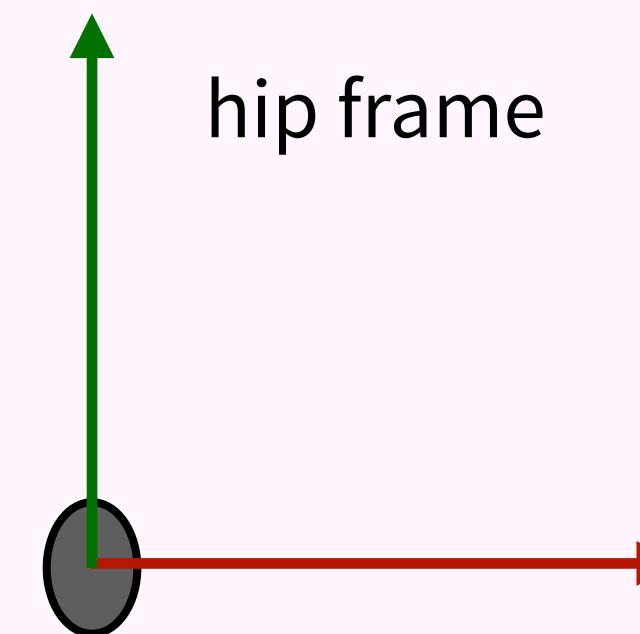
$$\mathbf{q} = 0$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$



$$\mathbf{T}(x, y, z) \mathbf{R}(\theta, \phi, \sigma) \mathbf{F}(\mathbf{t}_0, \mathbf{r}_0) \mathbf{F}(\mathbf{t}_1, \mathbf{r}_1) \boxed{\mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip})}$$

$\mathbf{t}_1$  translates from pelvis to hip joint.  $\mathbf{r}_1$  rotates -90 degree about z-axis.

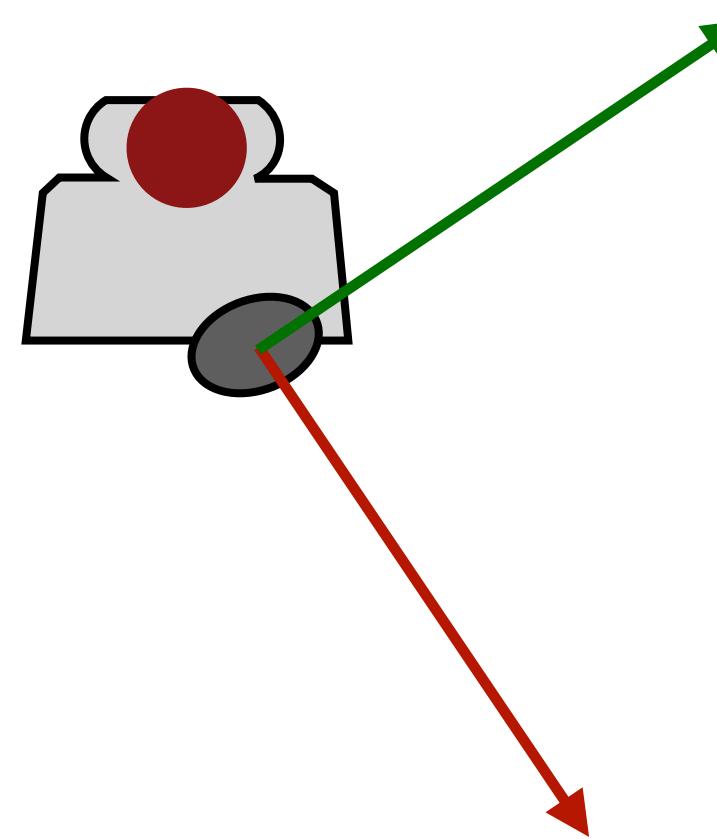


# Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}\}$$

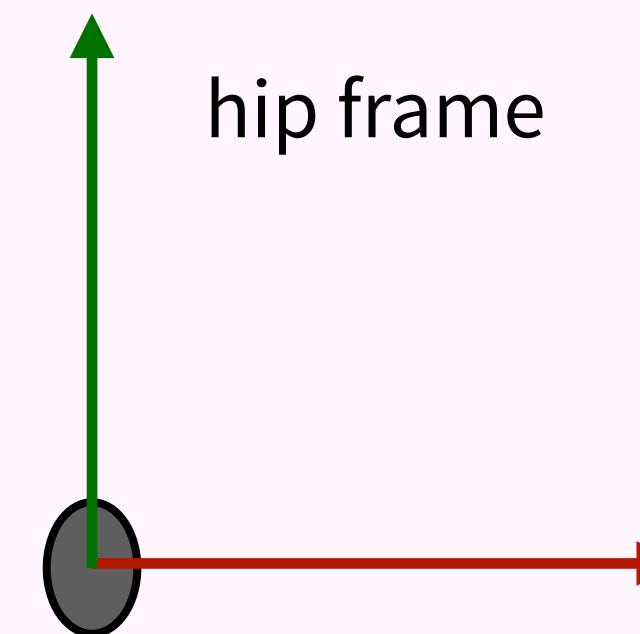
$$\mathbf{q} = 0$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$



$$\mathbf{T}(x, y, z) \mathbf{R}(\theta, \phi, \sigma) \mathbf{F}(\mathbf{t}_0, \mathbf{r}_0) \mathbf{F}(\mathbf{t}_1, \mathbf{r}_1) \boxed{\mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip})}$$

$\mathbf{t}_1$  translates from pelvis to hip joint.  $\mathbf{r}_1$  rotates -90 degree about z-axis.

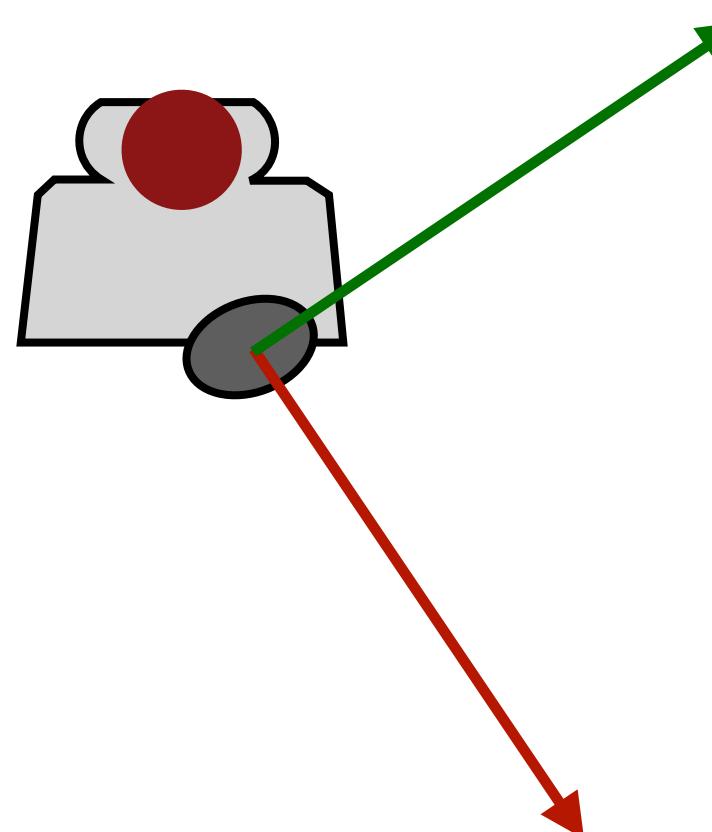


# Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \dots\}$$

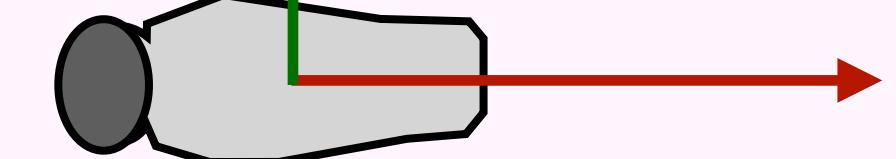
$$\mathbf{q} = 0$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$



$$\mathbf{T}(x, y, z) \mathbf{R}(\theta, \phi, \sigma) \mathbf{F}(\mathbf{t}_0, \mathbf{r}_0) \mathbf{F}(\mathbf{t}_1, \mathbf{r}_1) \mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip}) \boxed{\mathbf{F}(\mathbf{t}_2, \mathbf{r}_2)}$$

$\mathbf{t}_2$  translates from hip joint to thigh.  $\mathbf{r}_2$  rotates 0 degree.



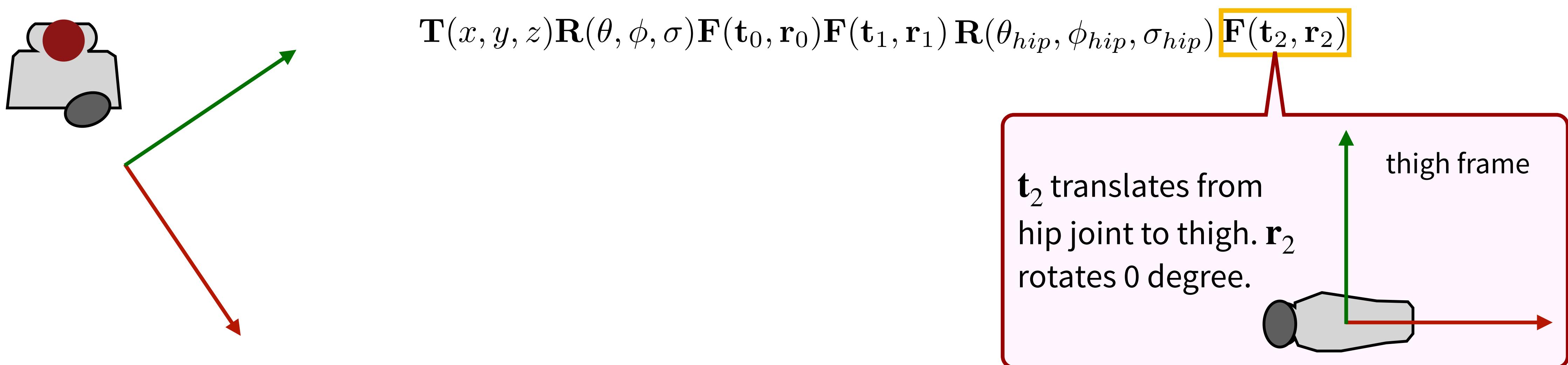
thigh frame

# Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \dots\}$$

$$\mathbf{q} = 0$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$

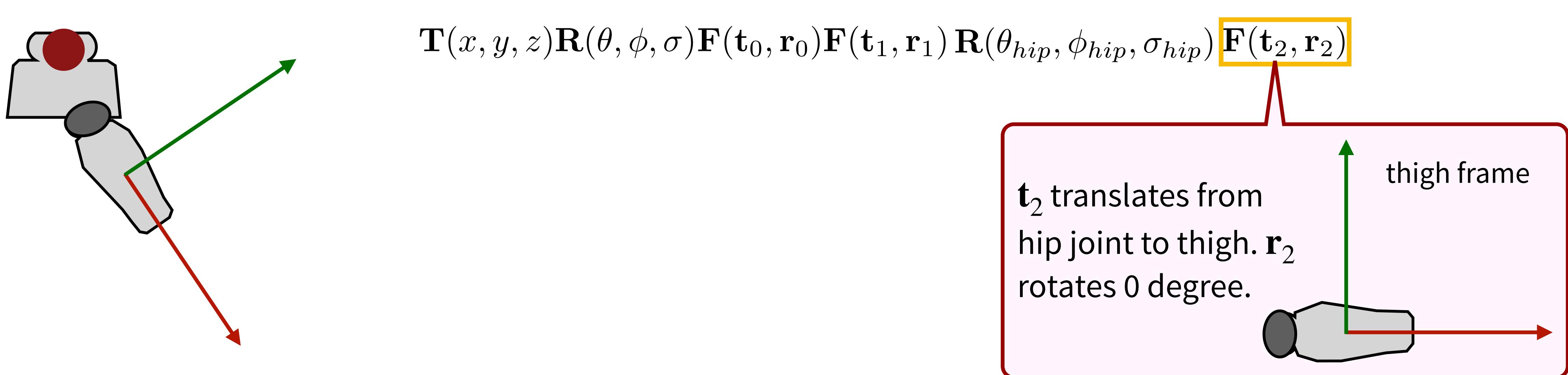


# Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \dots\}$$

$$\mathbf{q} = 0$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$

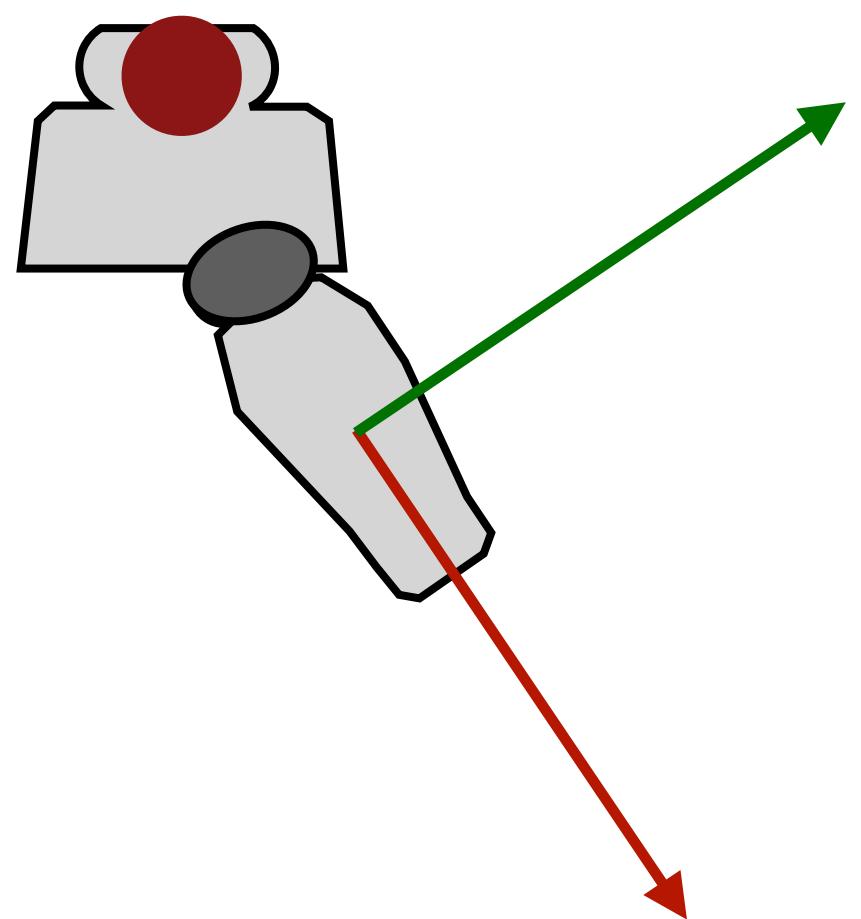


# Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \dots\}$$

$$\mathbf{q} = \mathbf{0}$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$



$$\mathbf{T}(x, y, z) \mathbf{R}(\theta, \phi, \sigma) \mathbf{F}(\mathbf{t}_0, \mathbf{r}_0) \mathbf{F}(\mathbf{t}_1, \mathbf{r}_1) \mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip}) \mathbf{F}(\mathbf{t}_2, \mathbf{r}_2)$$

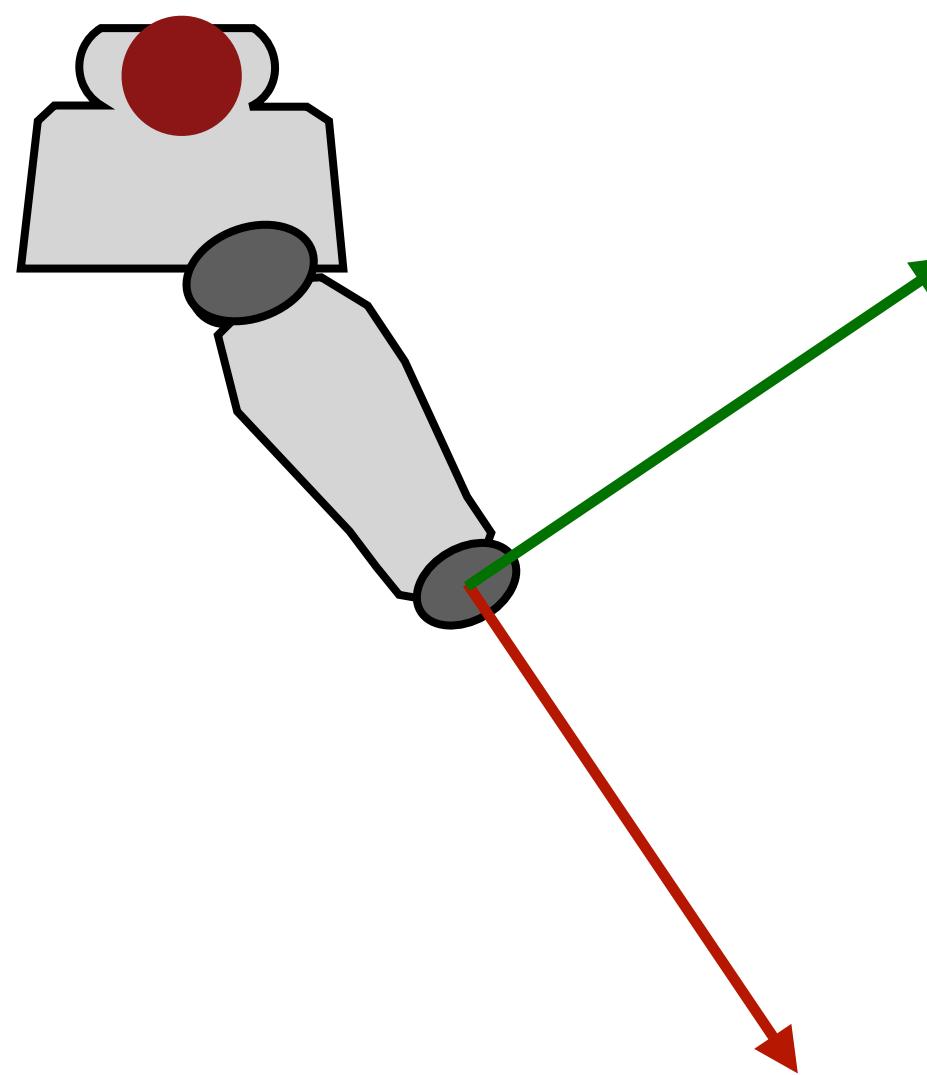
$$\boxed{\mathbf{F}(\mathbf{t}_3, \mathbf{r}_3)}$$

# Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \dots\}$$

$$\mathbf{q} = \mathbf{0}$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$



$$\mathbf{T}(x, y, z) \mathbf{R}(\theta, \phi, \sigma) \mathbf{F}(\mathbf{t}_0, \mathbf{r}_0) \mathbf{F}(\mathbf{t}_1, \mathbf{r}_1) \mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip}) \mathbf{F}(\mathbf{t}_2, \mathbf{r}_2)$$

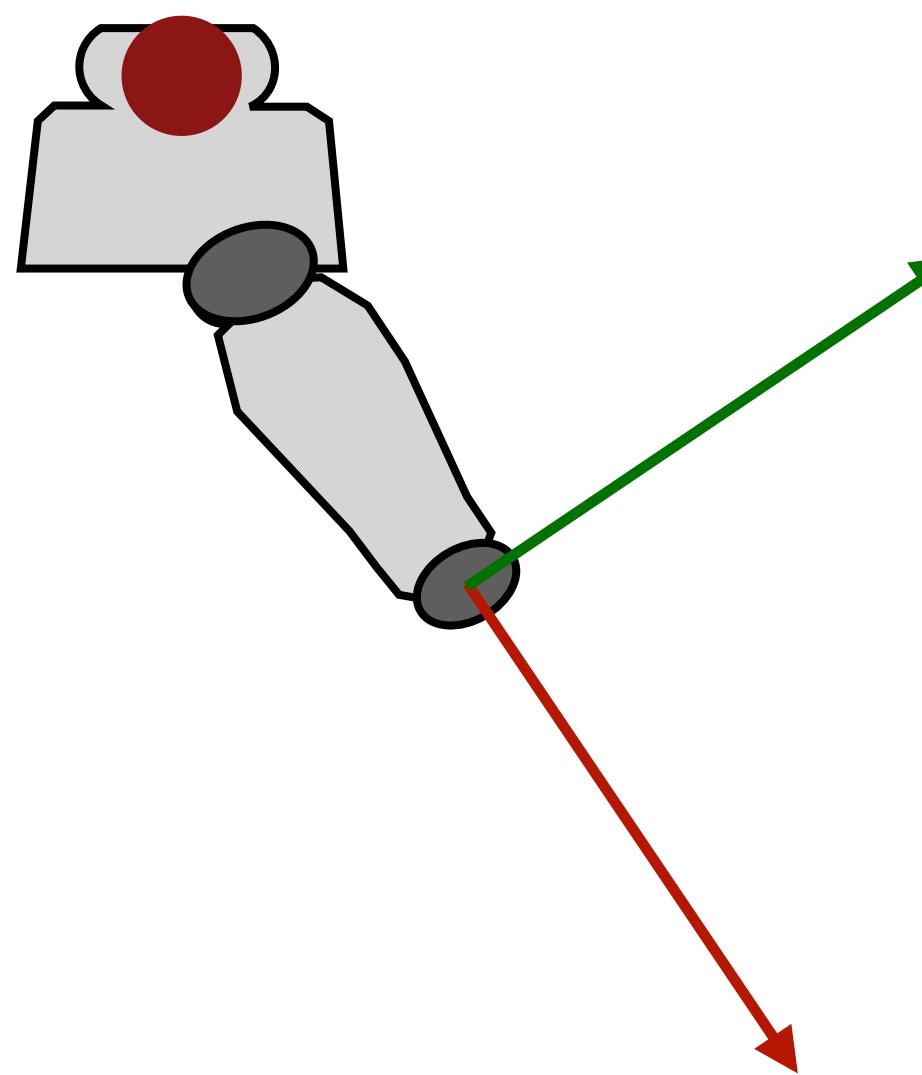
$$\boxed{\mathbf{F}(\mathbf{t}_3, \mathbf{r}_3)}$$

# Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \dots\}$$

$$\mathbf{q} = \mathbf{0}$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$



$$\mathbf{T}(x, y, z) \mathbf{R}(\theta, \phi, \sigma) \mathbf{F}(\mathbf{t}_0, \mathbf{r}_0) \mathbf{F}(\mathbf{t}_1, \mathbf{r}_1) \mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip}) \mathbf{F}(\mathbf{t}_2, \mathbf{r}_2)$$

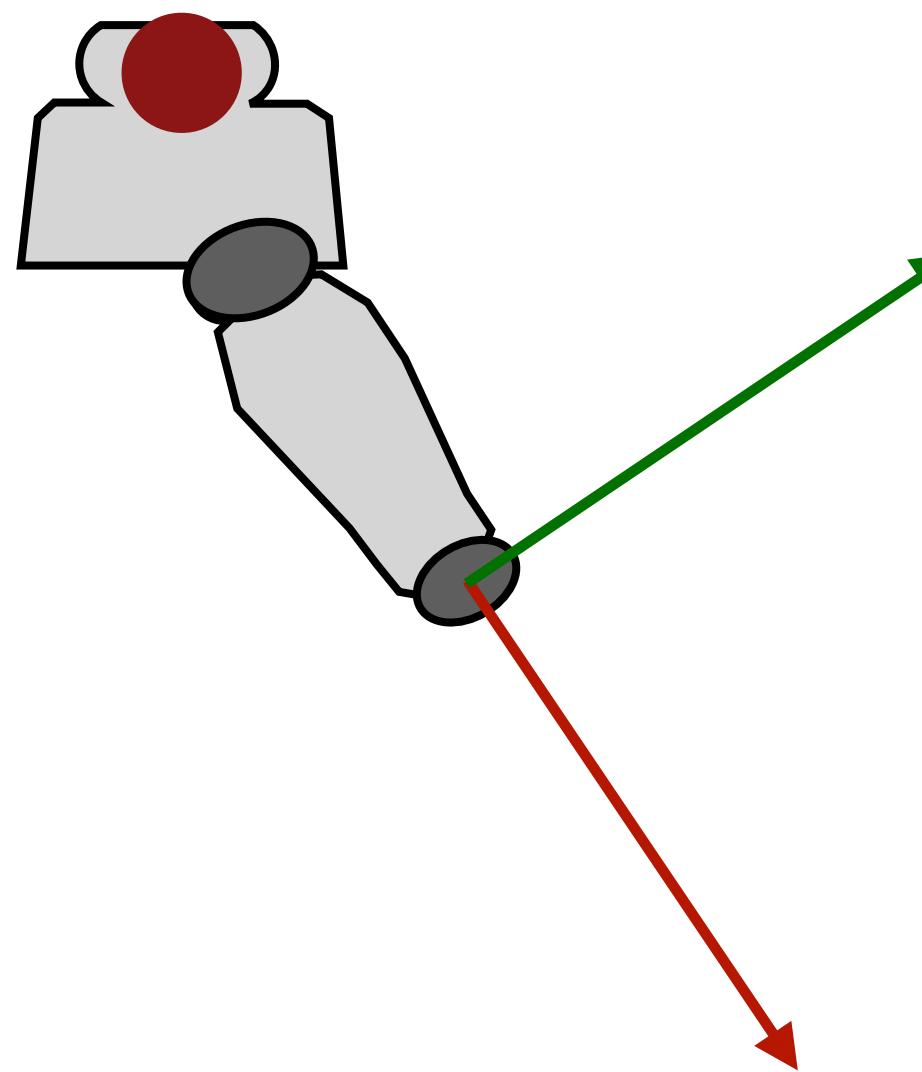
$$\mathbf{F}(\mathbf{t}_3, \mathbf{r}_3) \mathbf{R}(\theta_{knee})$$

# Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \dots\}$$

$$\mathbf{q} = \mathbf{0}$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$



$$\mathbf{T}(x, y, z) \mathbf{R}(\theta, \phi, \sigma) \mathbf{F}(\mathbf{t}_0, \mathbf{r}_0) \mathbf{F}(\mathbf{t}_1, \mathbf{r}_1) \mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip}) \mathbf{F}(\mathbf{t}_2, \mathbf{r}_2)$$

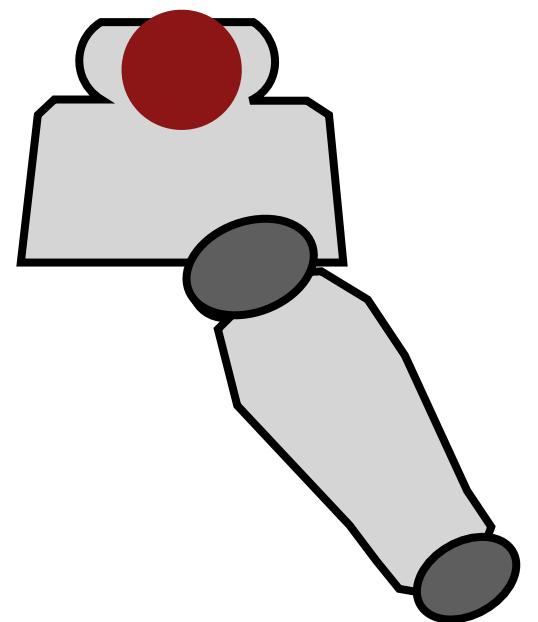
$$\mathbf{F}(\mathbf{t}_3, \mathbf{r}_3) \mathbf{R}(\theta_{knee}) \boxed{\mathbf{F}(\mathbf{t}_4, \mathbf{r}_4)}$$

# Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \dots\}$$

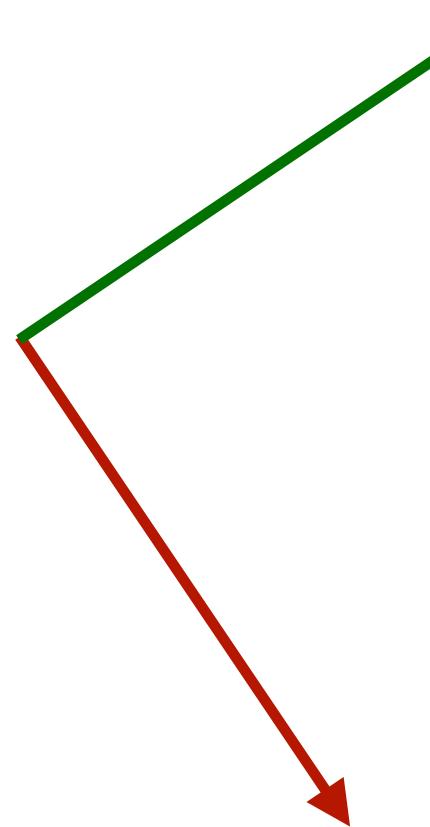
$$\mathbf{q} = \mathbf{0}$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$



$$\mathbf{T}(x, y, z) \mathbf{R}(\theta, \phi, \sigma) \mathbf{F}(\mathbf{t}_0, \mathbf{r}_0) \mathbf{F}(\mathbf{t}_1, \mathbf{r}_1) \mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip}) \mathbf{F}(\mathbf{t}_2, \mathbf{r}_2)$$

$$\mathbf{F}(\mathbf{t}_3, \mathbf{r}_3) \mathbf{R}(\theta_{knee}) \boxed{\mathbf{F}(\mathbf{t}_4, \mathbf{r}_4)}$$

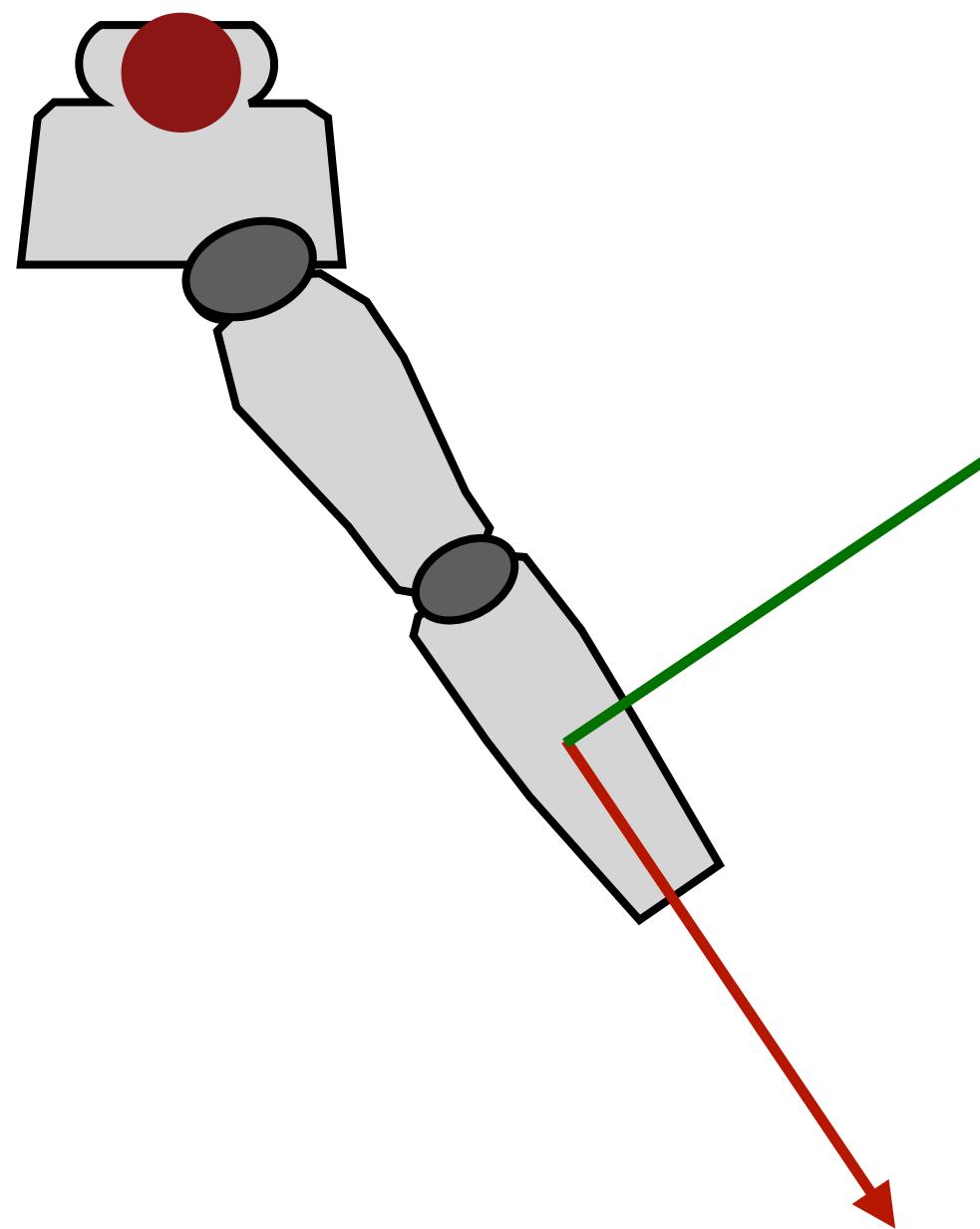


# Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \dots\}$$

$$\mathbf{q} = \mathbf{0}$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$



$$\mathbf{T}(x, y, z) \mathbf{R}(\theta, \phi, \sigma) \mathbf{F}(\mathbf{t}_0, \mathbf{r}_0) \mathbf{F}(\mathbf{t}_1, \mathbf{r}_1) \mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip}) \mathbf{F}(\mathbf{t}_2, \mathbf{r}_2)$$

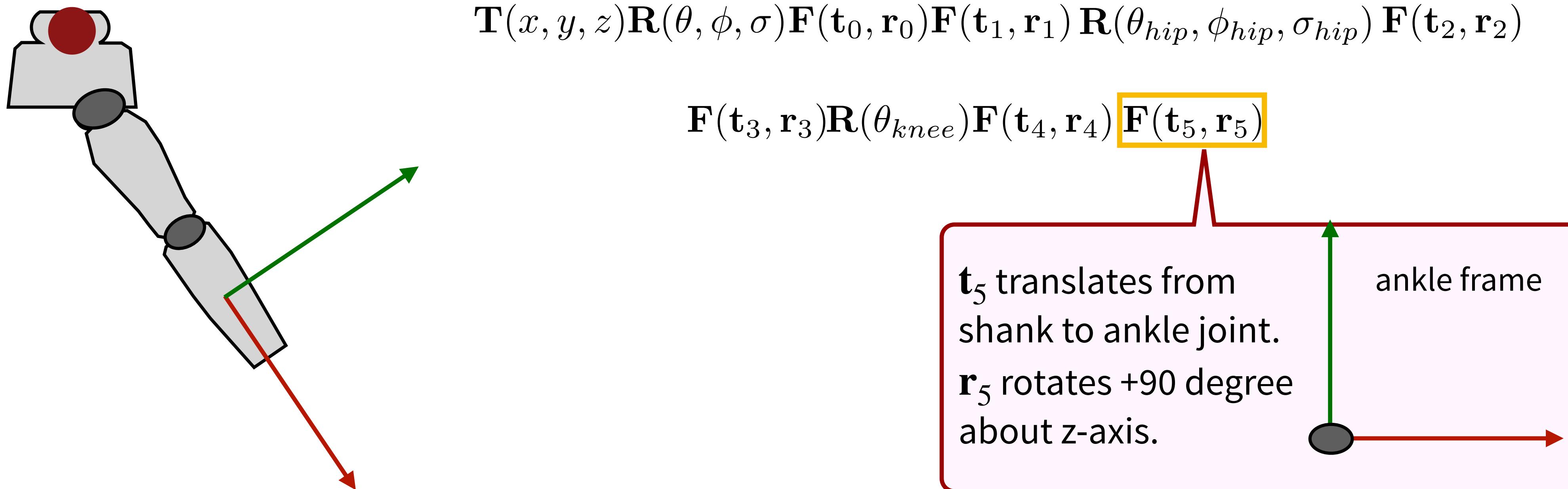
$$\mathbf{F}(\mathbf{t}_3, \mathbf{r}_3) \mathbf{R}(\theta_{knee}) \boxed{\mathbf{F}(\mathbf{t}_4, \mathbf{r}_4)}$$

# Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \dots\}$$

$$\mathbf{q} = \mathbf{0}$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$

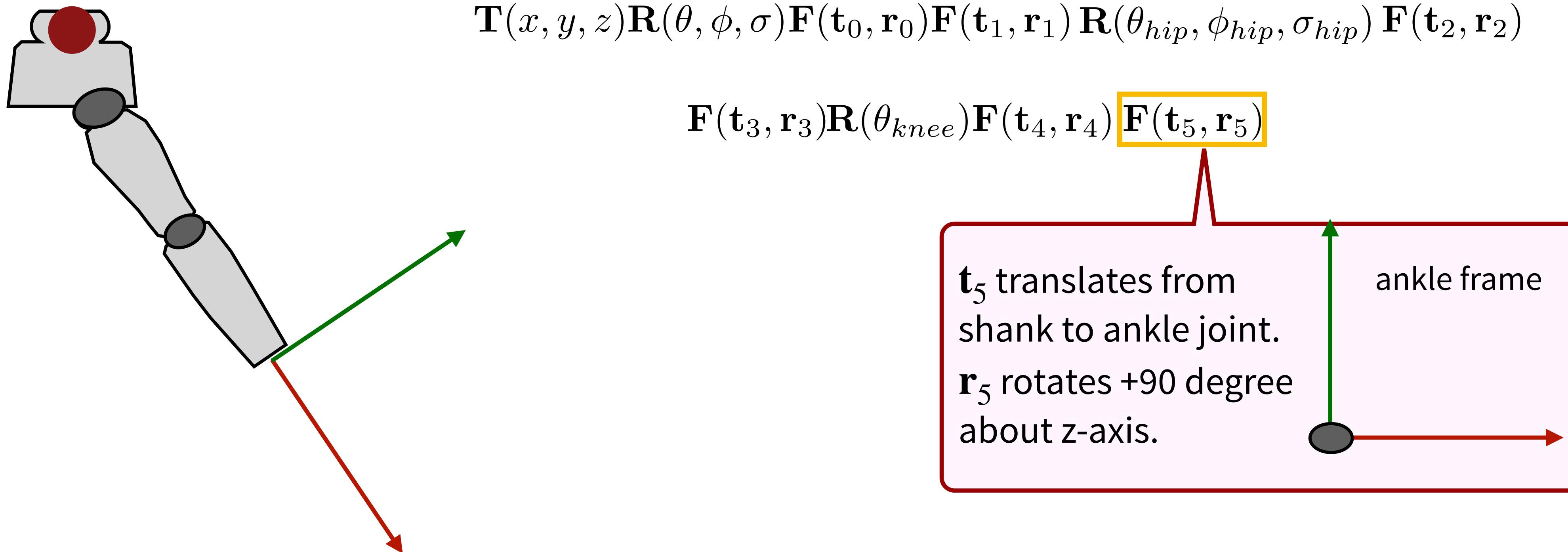


# Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \dots\}$$

$$\mathbf{q} = \mathbf{0}$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$

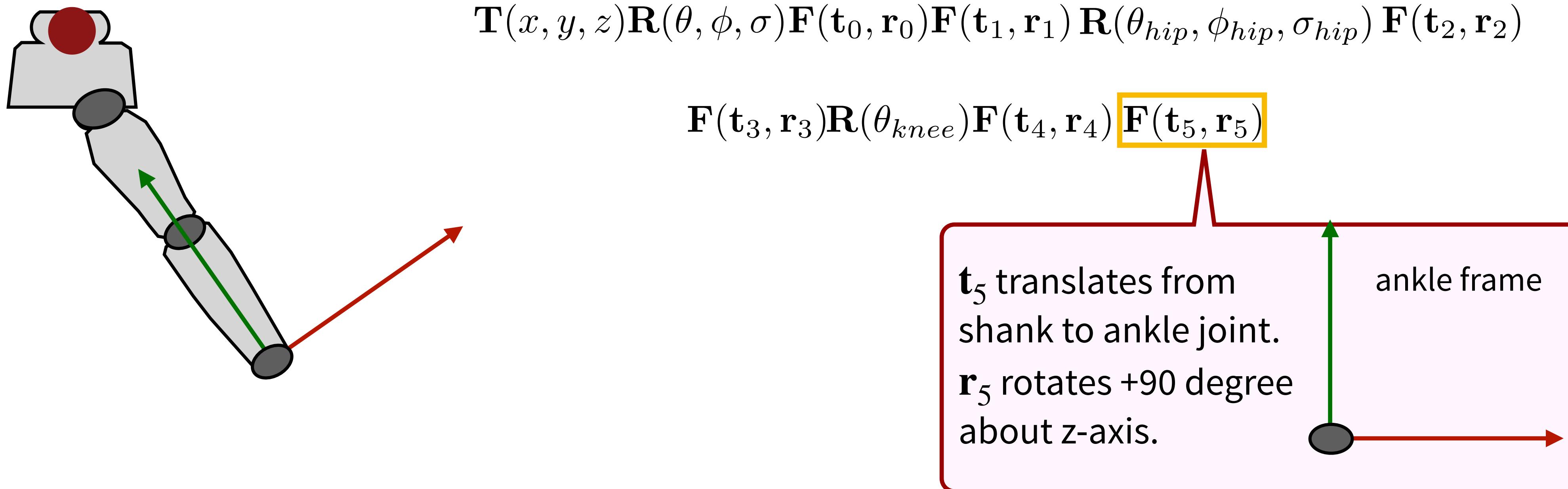


# Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \dots\}$$

$$\mathbf{q} = \mathbf{0}$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$

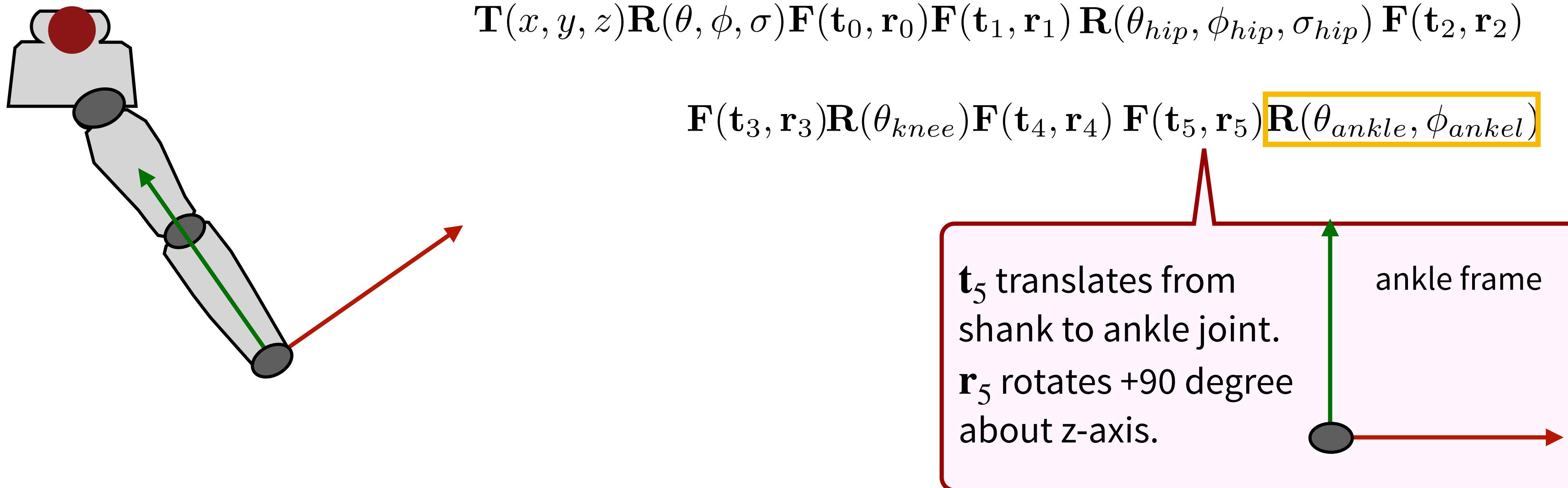


# Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \dots\}$$

$$\mathbf{q} = \mathbf{0}$$

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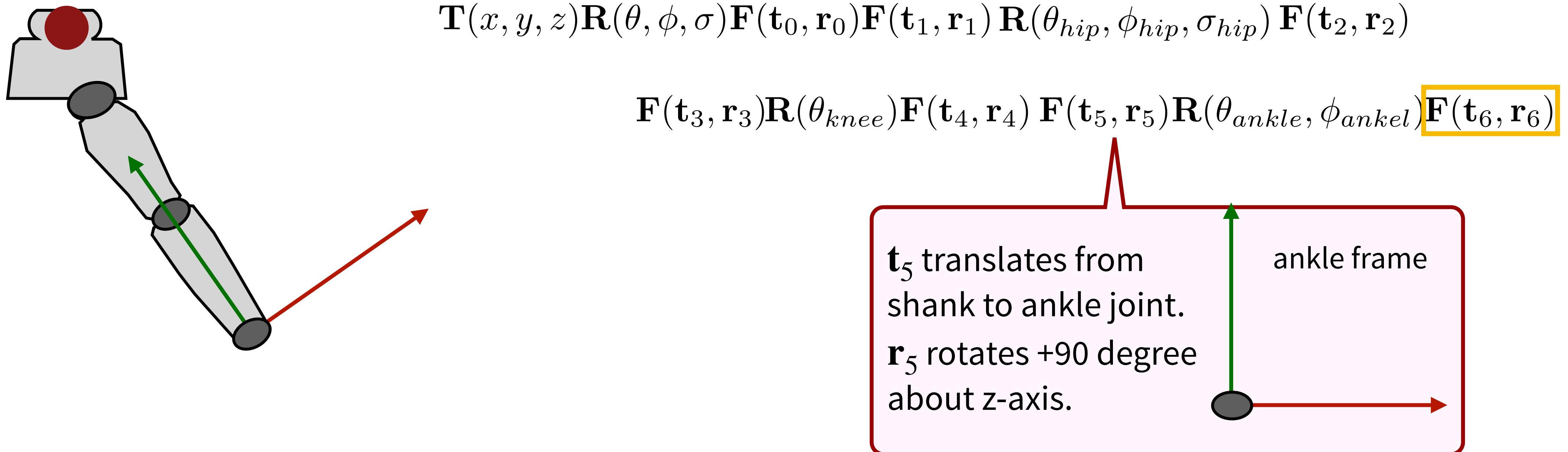


# Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \dots\}$$

$$\mathbf{q} = \mathbf{0}$$

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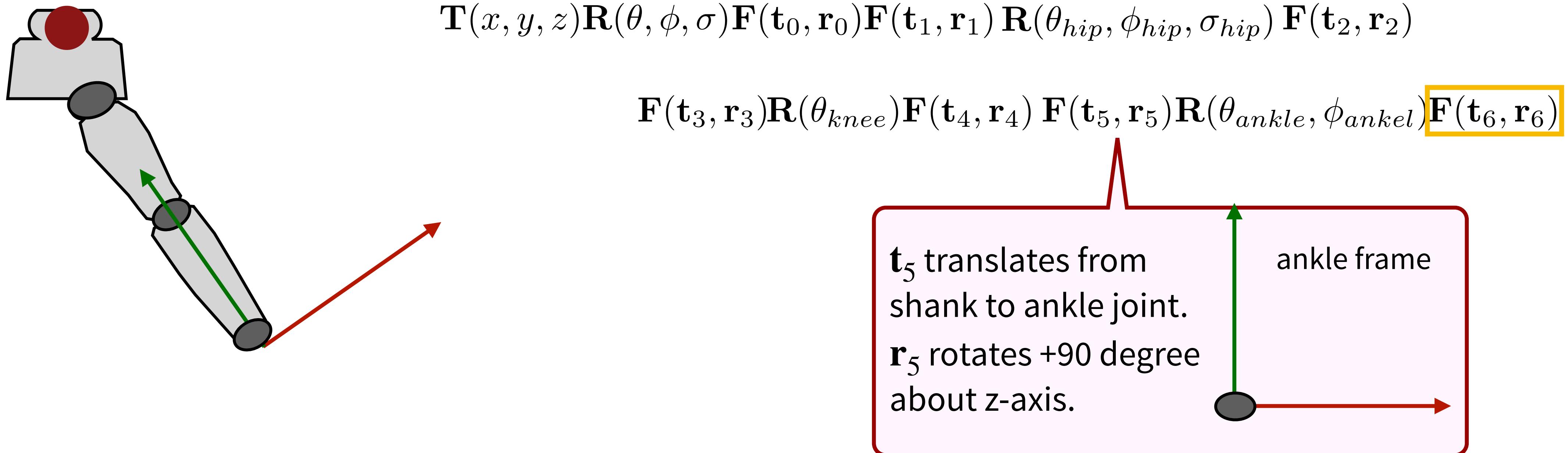


# Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \dots\}$$

$$\mathbf{q} = \mathbf{0}$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$

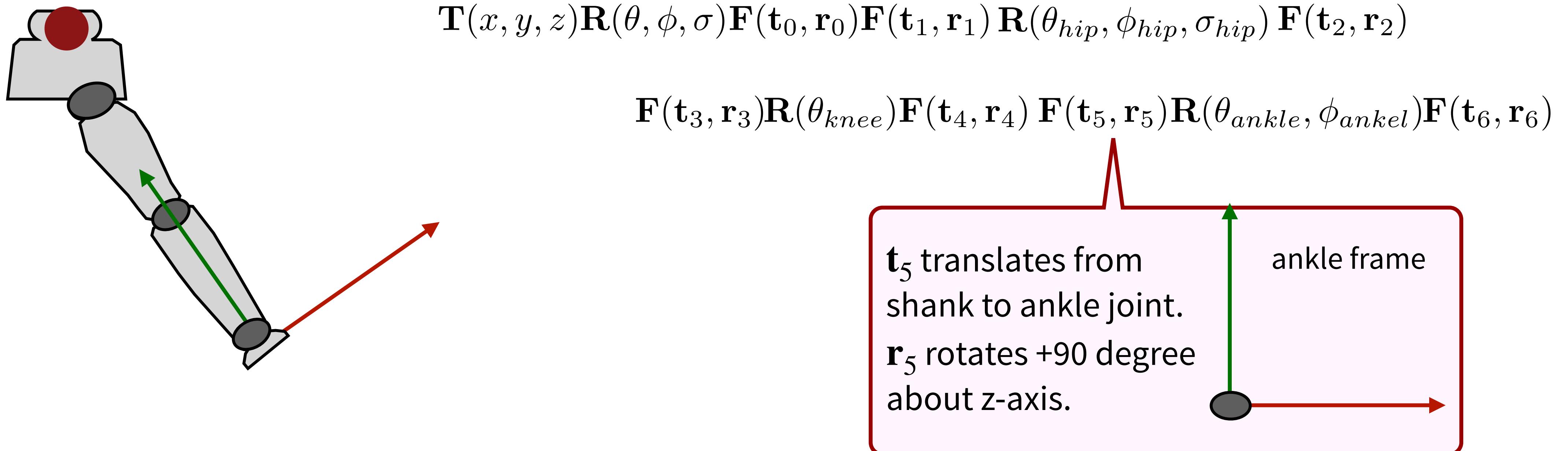


# Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \dots\}$$

$$\mathbf{q} = \mathbf{0}$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$

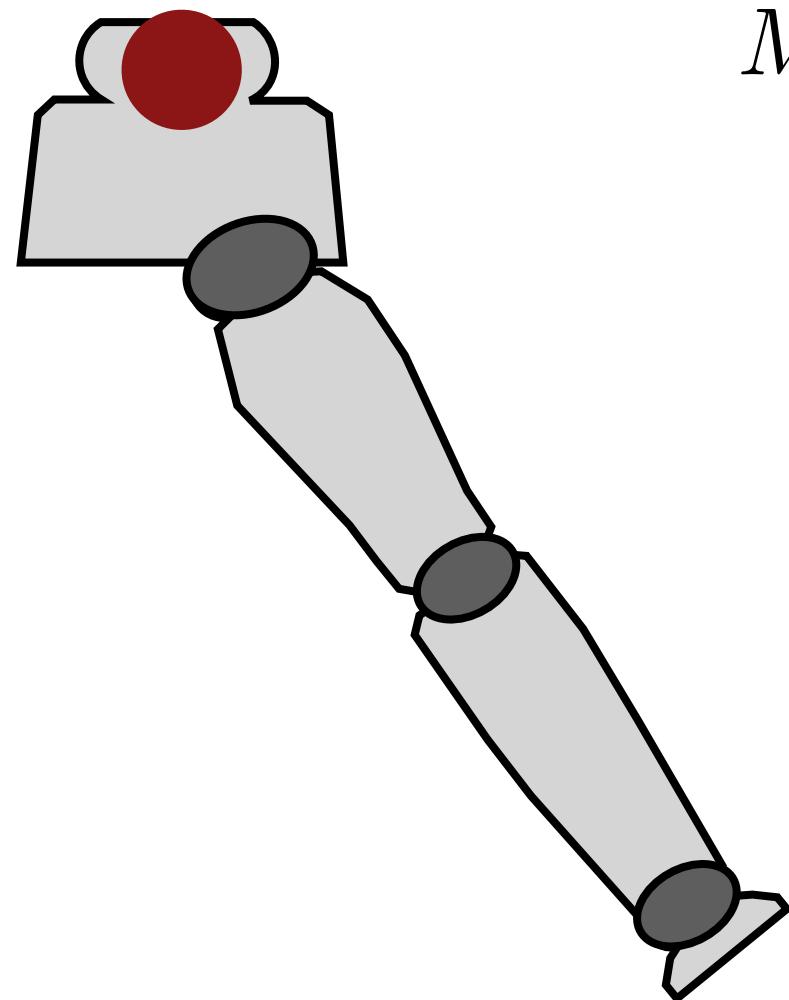


# Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \dots\}$$

$$\mathbf{q} = \mathbf{0}$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$



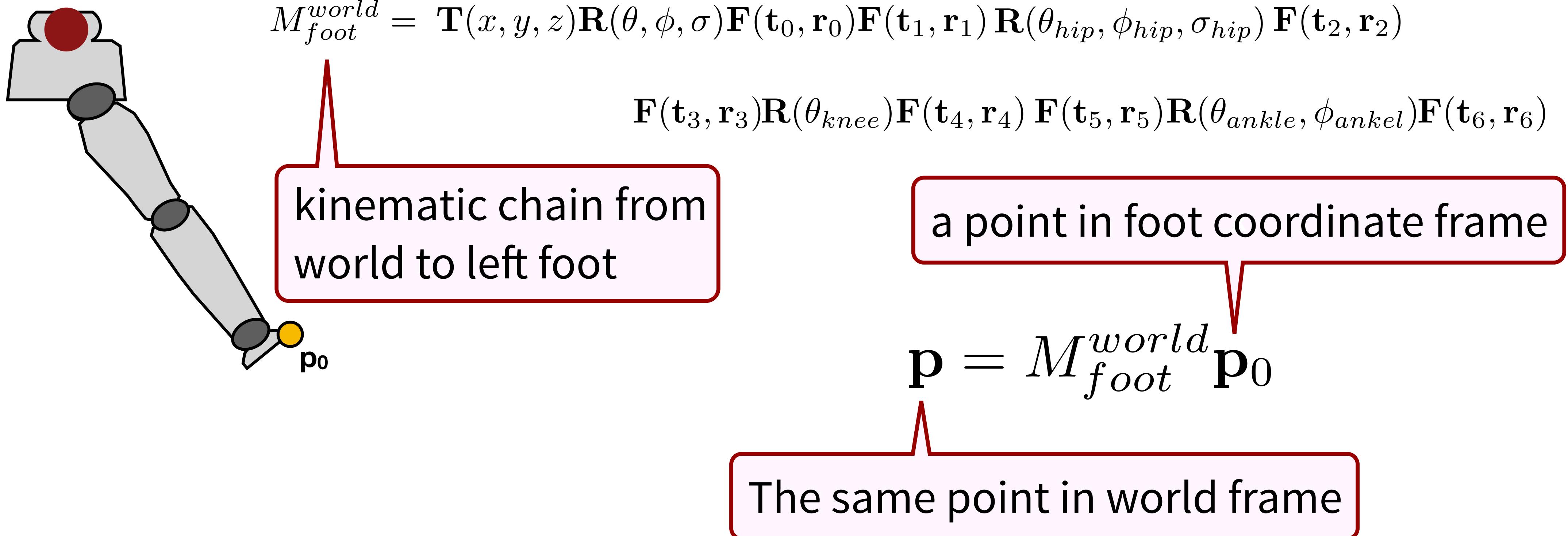
$$M_{foot}^{world} = \mathbf{T}(x, y, z) \mathbf{R}(\theta, \phi, \sigma) \mathbf{F}(\mathbf{t}_0, \mathbf{r}_0) \mathbf{F}(\mathbf{t}_1, \mathbf{r}_1) \mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip}) \mathbf{F}(\mathbf{t}_2, \mathbf{r}_2) \\ \mathbf{F}(\mathbf{t}_3, \mathbf{r}_3) \mathbf{R}(\theta_{knee}) \mathbf{F}(\mathbf{t}_4, \mathbf{r}_4) \mathbf{F}(\mathbf{t}_5, \mathbf{r}_5) \mathbf{R}(\theta_{ankle}, \phi_{ankle}) \mathbf{F}(\mathbf{t}_6, \mathbf{r}_6)$$

# Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \dots\}$$

$$\mathbf{q} = \mathbf{0}$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$



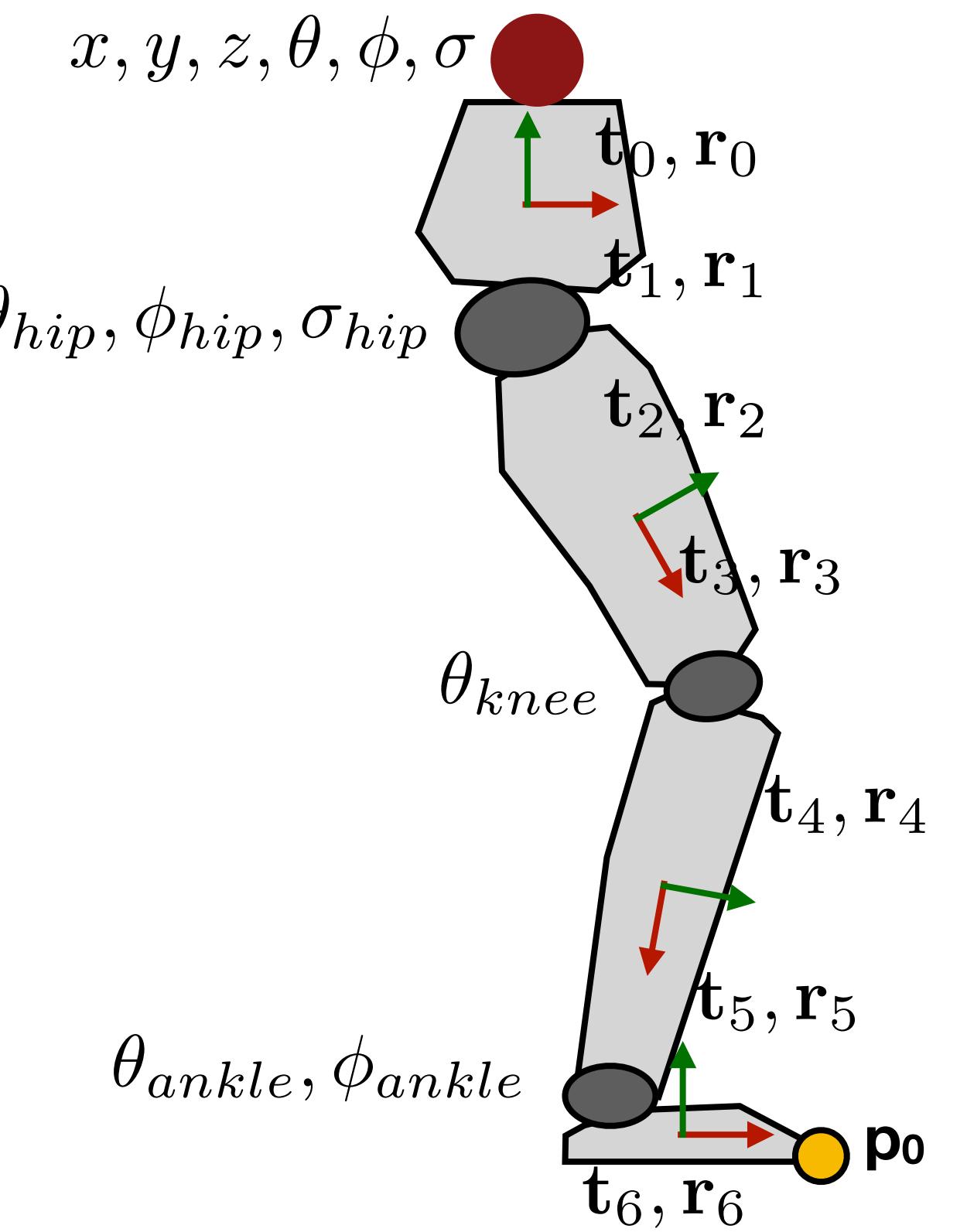
# A point in a local frame

- We now know how to refer a point in its own local frame and in the world frame. For example,

$$\mathbf{p} = M_{foot}^{world} \mathbf{p}_0$$

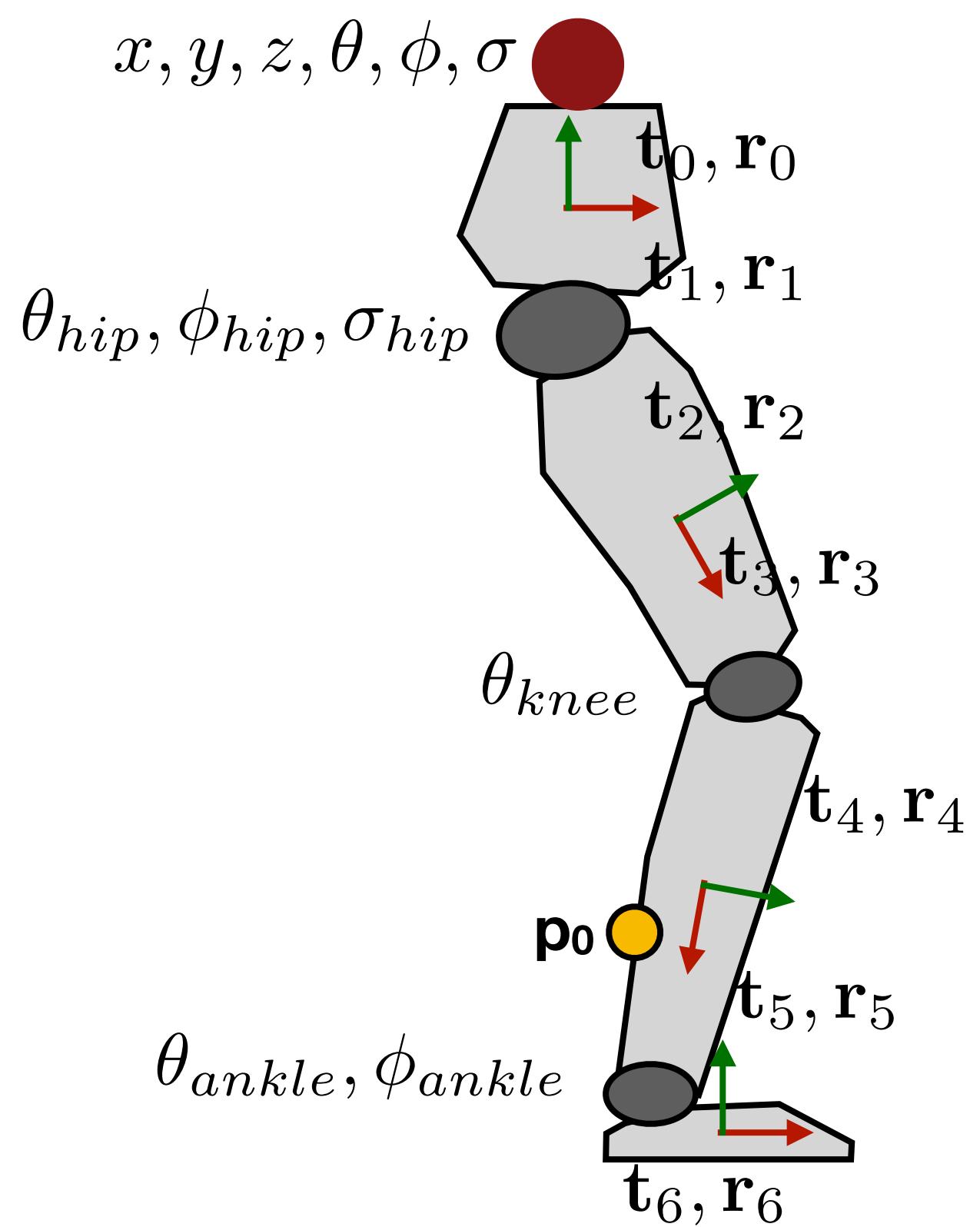
- With the kinematic transformation chain, we can refer to any point in any coordinate frame.
- For example, the toe in the coordinate frame of the thigh is

$$F(t_3, r_3)R(\theta_{knee})F(t_4, r_4)F(t_5, r_5)R(\theta_{ankle})R(\phi_{ankle})F(t_6, r_6)\mathbf{p}_0$$



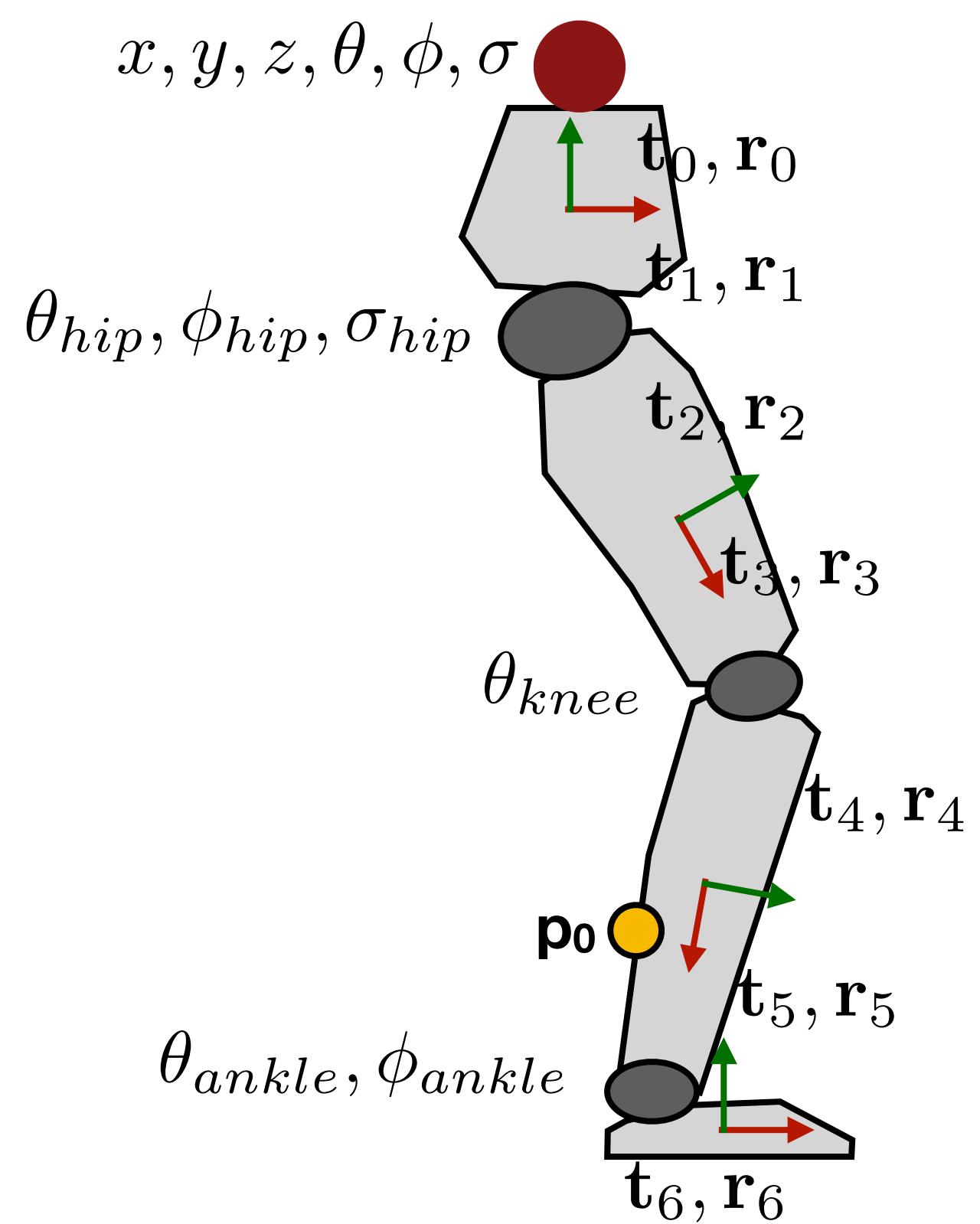
# Quiz

- What is the coordinate of the yellow point in the foot frame?



# Quiz

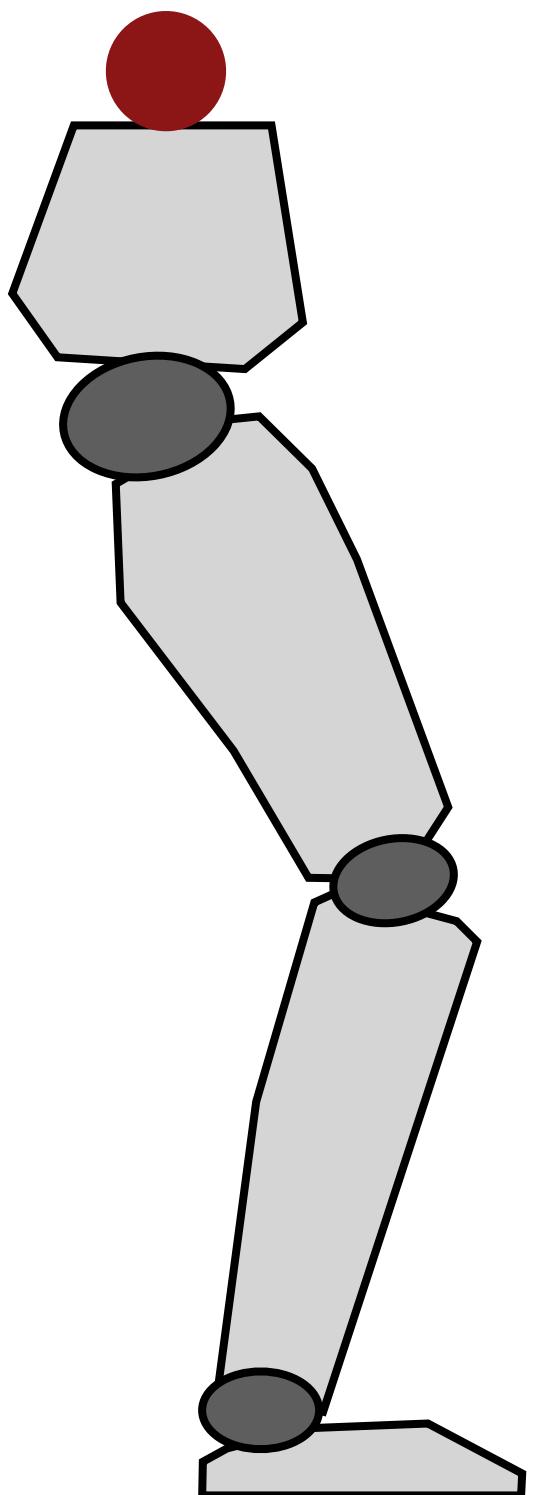
- What is the relative position of the yellow point to the hip joint in the world frame?



# Center of mass

- Center of mass (COM) is an important kinematic concept for simulation and control, such as calculating angular motion or evaluating balance.
- COM is a 3D point that depends on the pose. For example, the COM of the leg is given by the COM of each body segment weighted by its mass.

$$\frac{m_{pelv}M_{pelv}^{world} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + m_{thig}M_{thig}^{world} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + m_{shan}M_{shan}^{world} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + m_{foot}M_{foot}^{world} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}{m_{pelv} + m_{thig} + m_{shan} + m_{foot}}$$



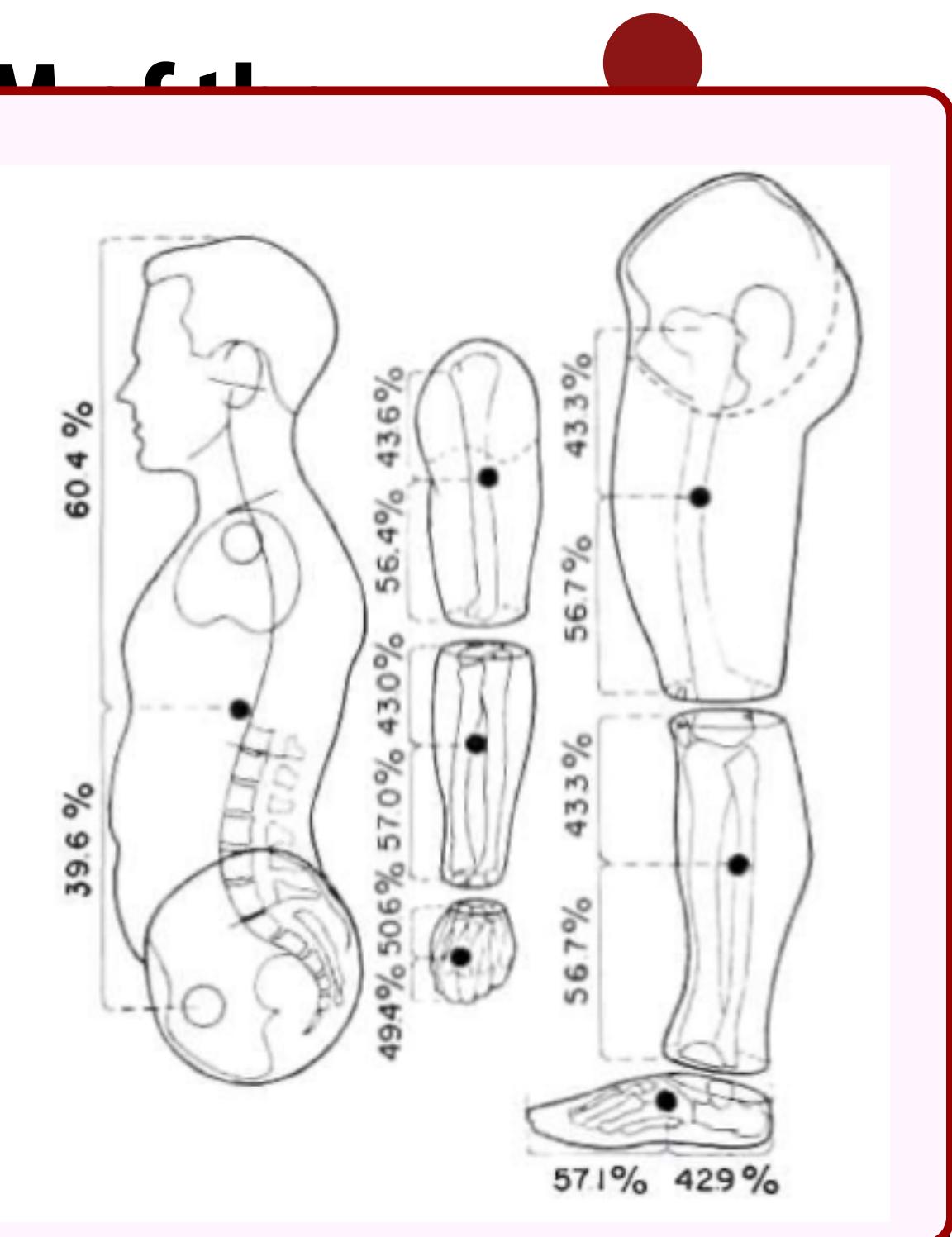
# Center of mass

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$$m_{pelv} M_{pelv}^{\text{world}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + m_{thig} M_{thig}^{\text{world}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + m_{shan} M_{shan}^{\text{world}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + m_{foot} M_{foot}^{\text{world}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

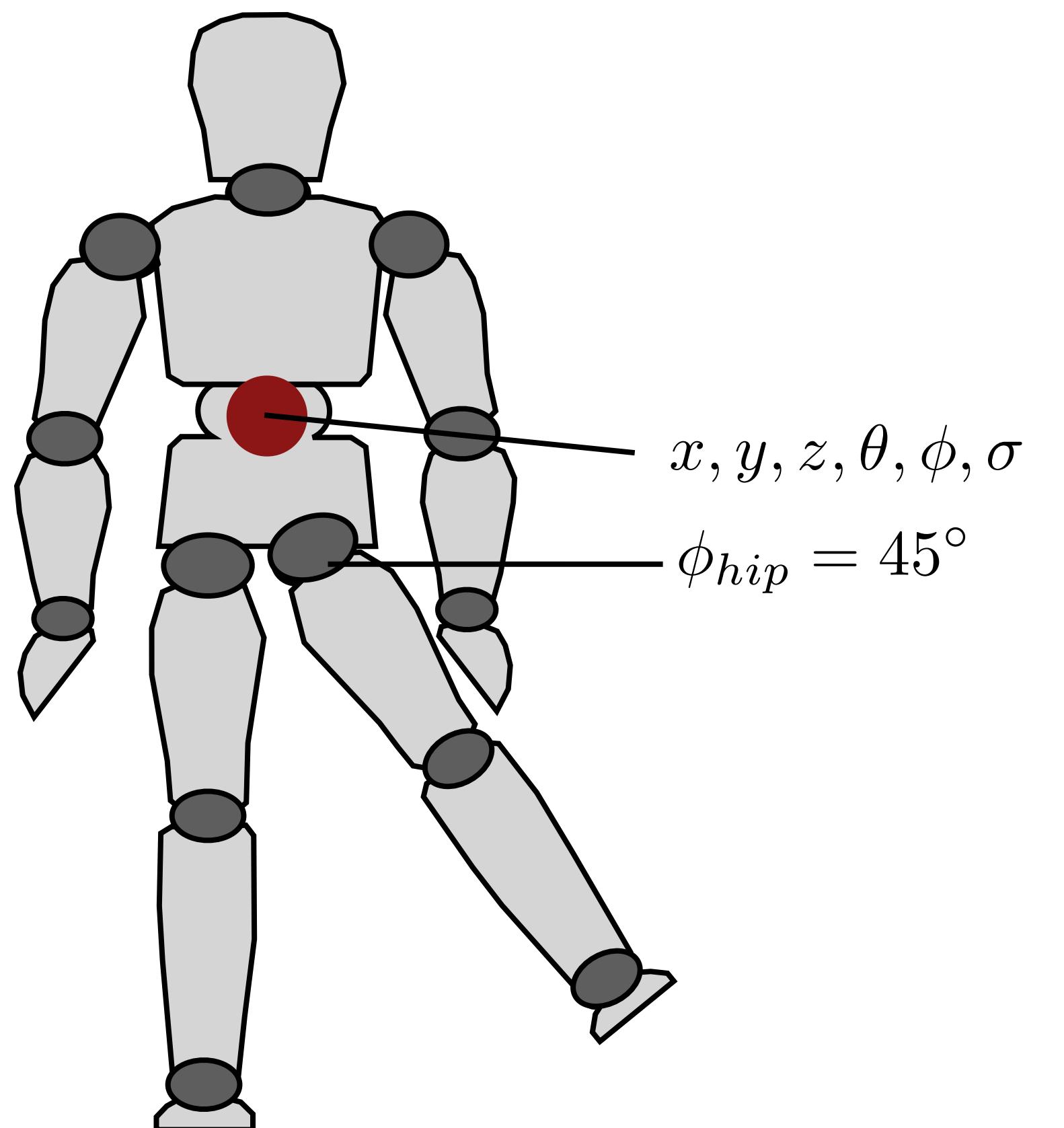
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$$m_{pelv} + m_{thig} + m_{shan} + m_{foot}$$



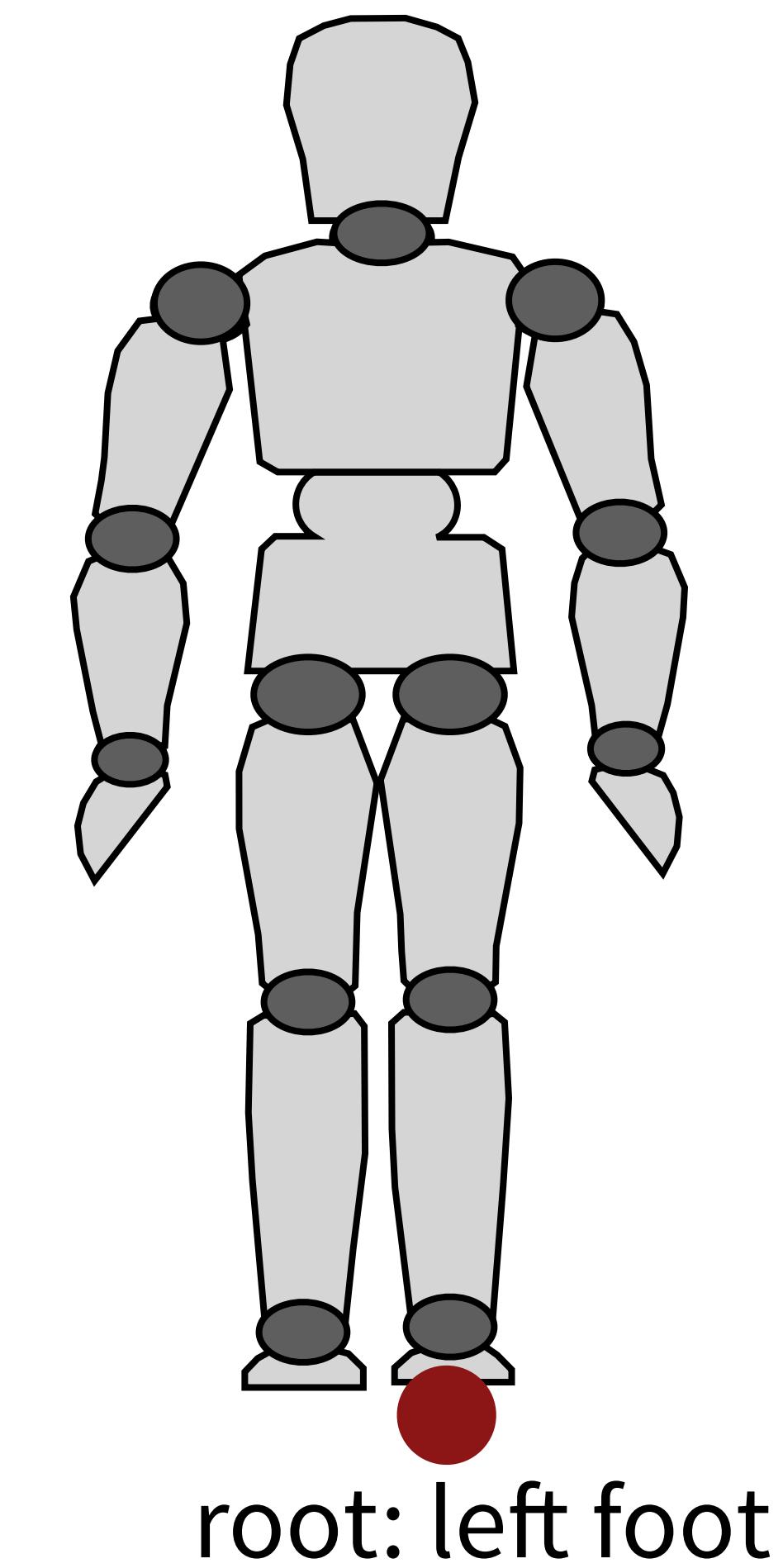
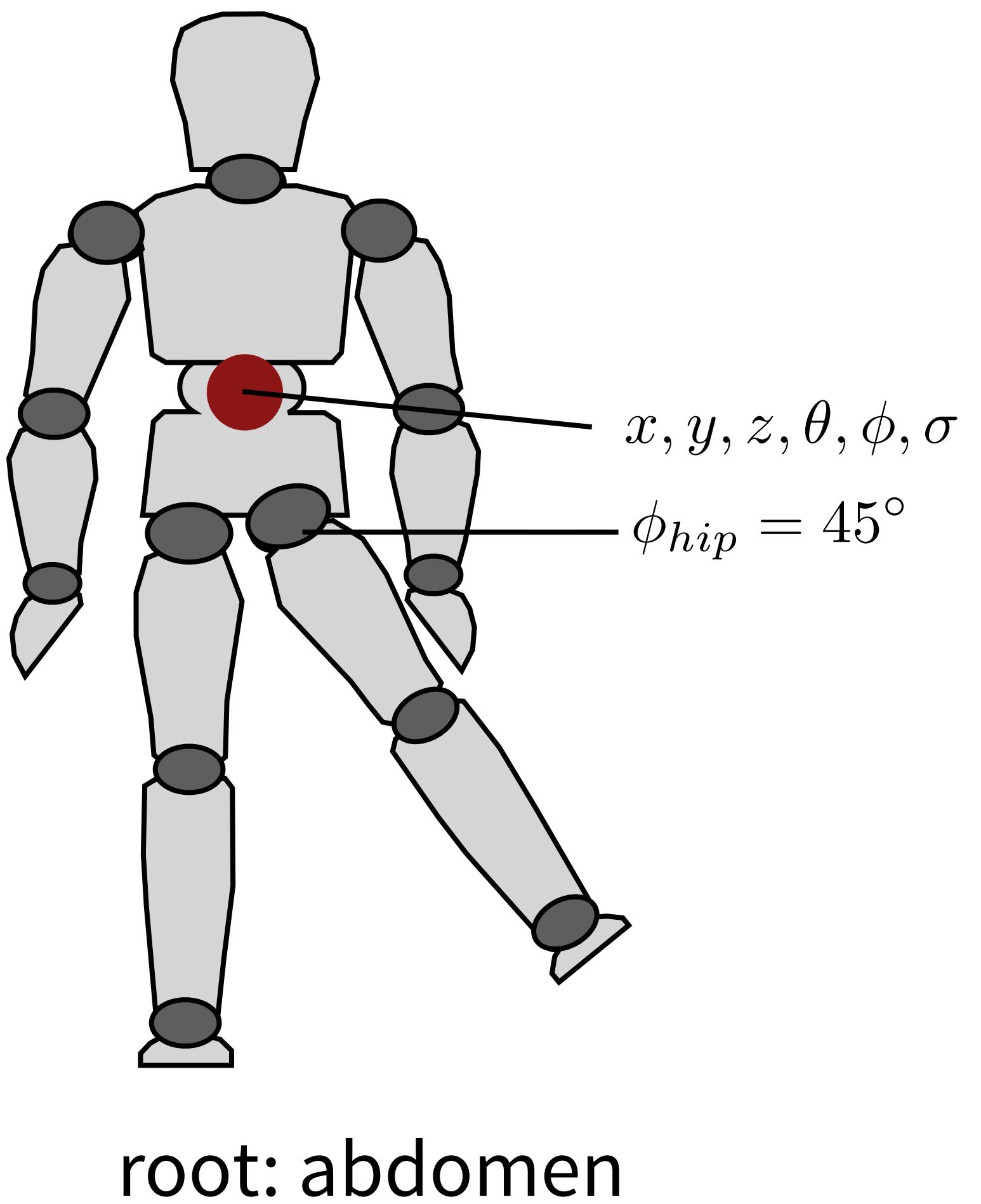
What assumptions are we making here?  
The body has uniform density!

# Root location

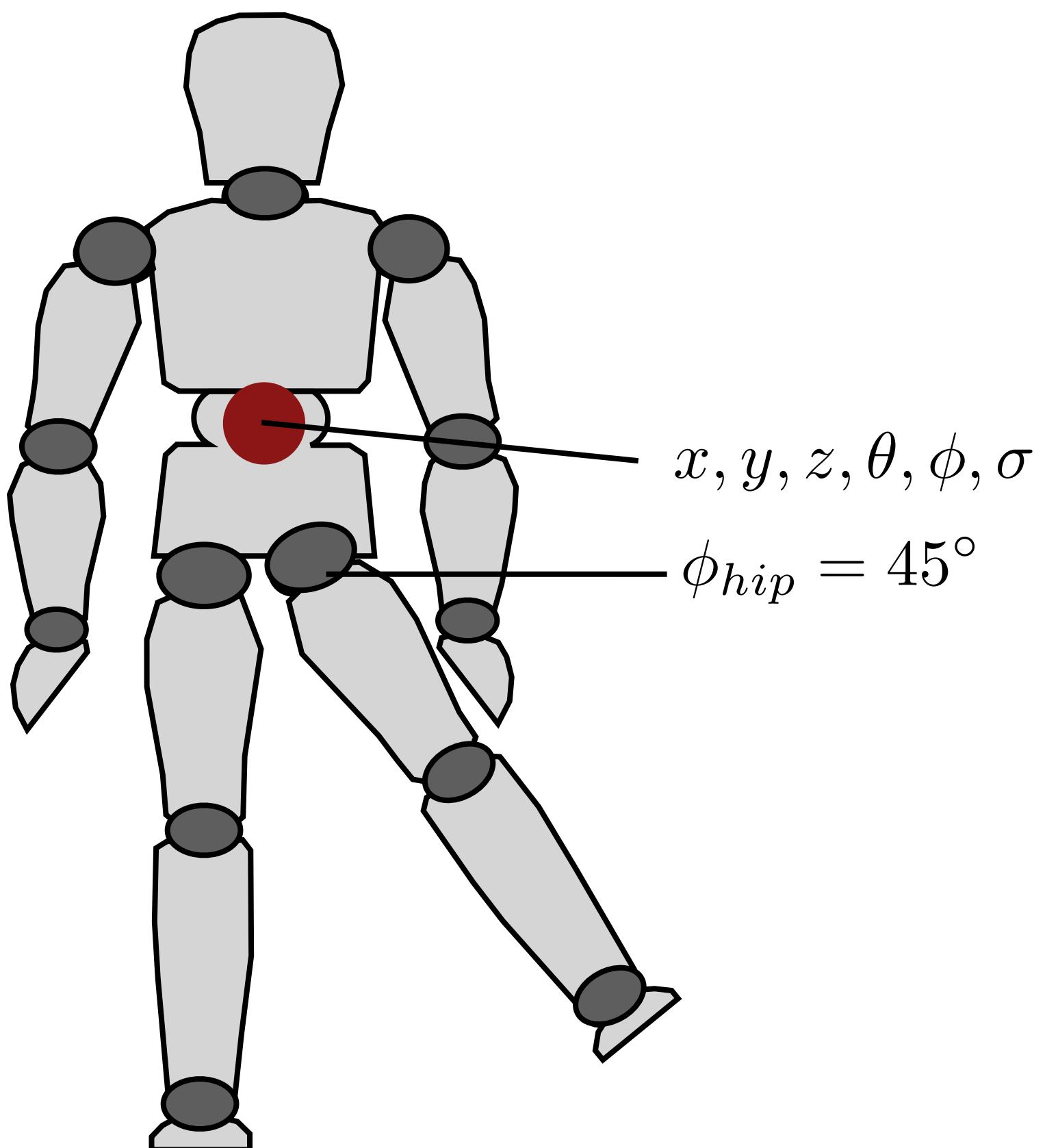


root: abdomen

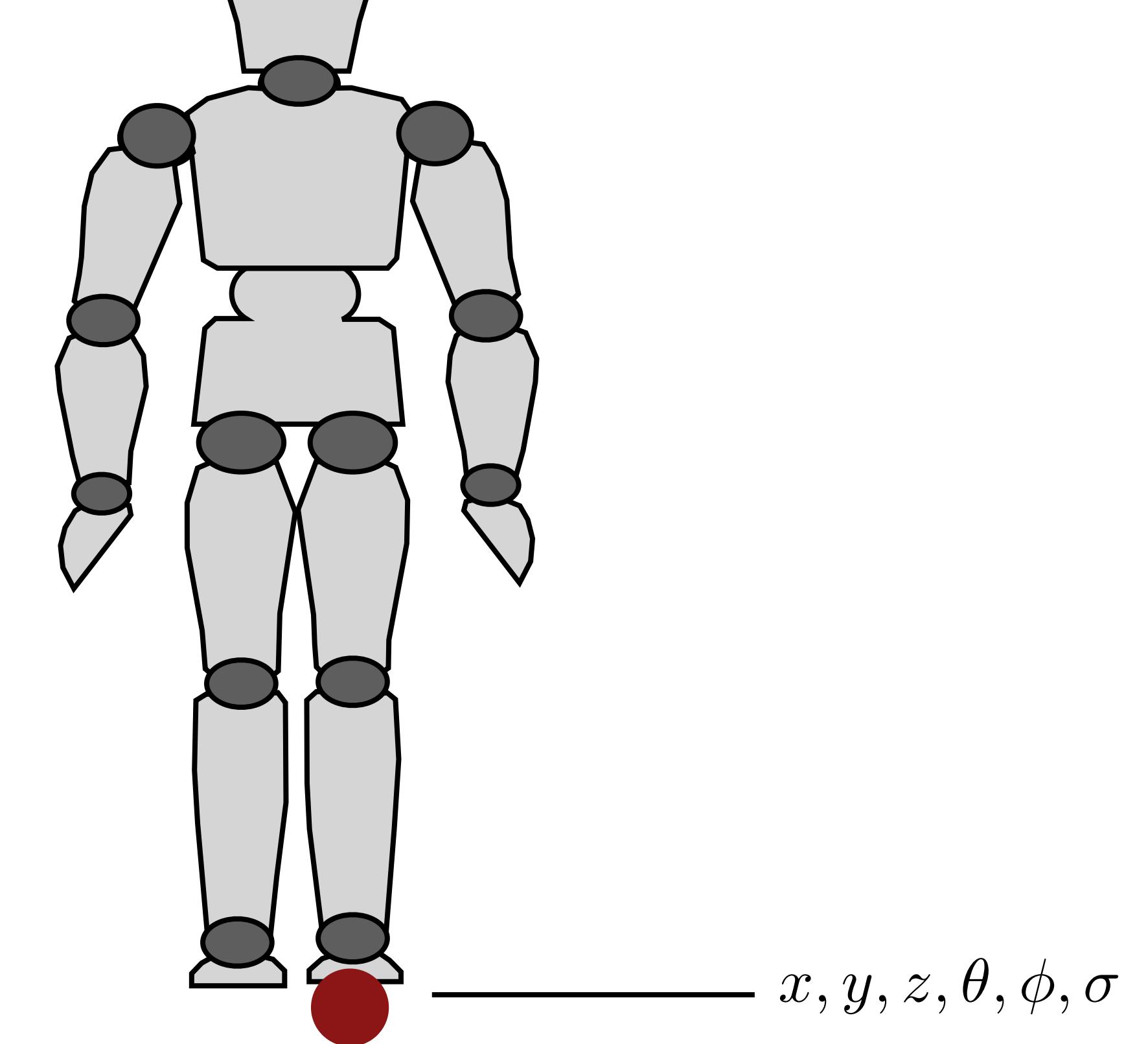
# Root location



# Root location

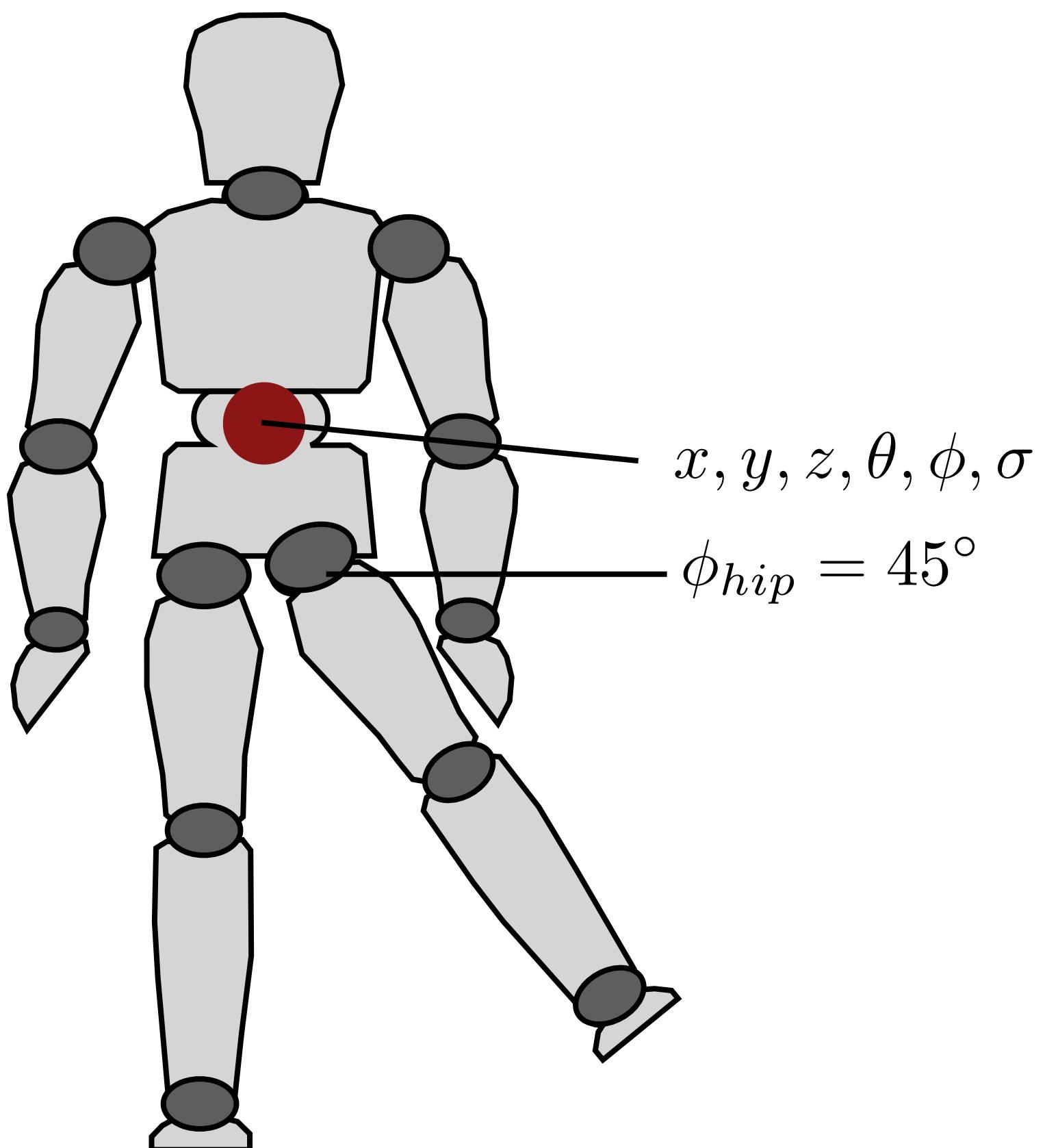


root: abdomen

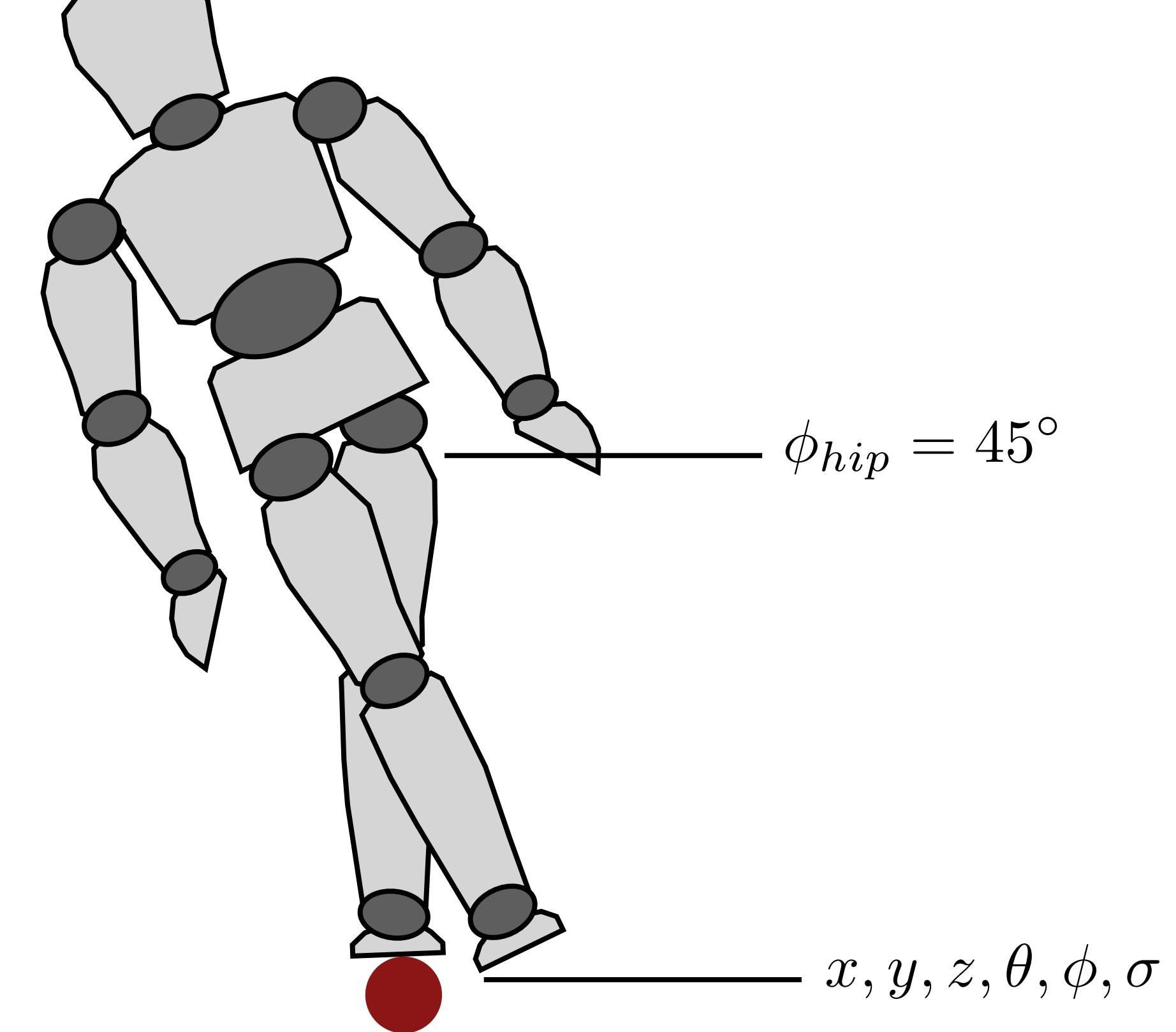


root: left foot

# Root location

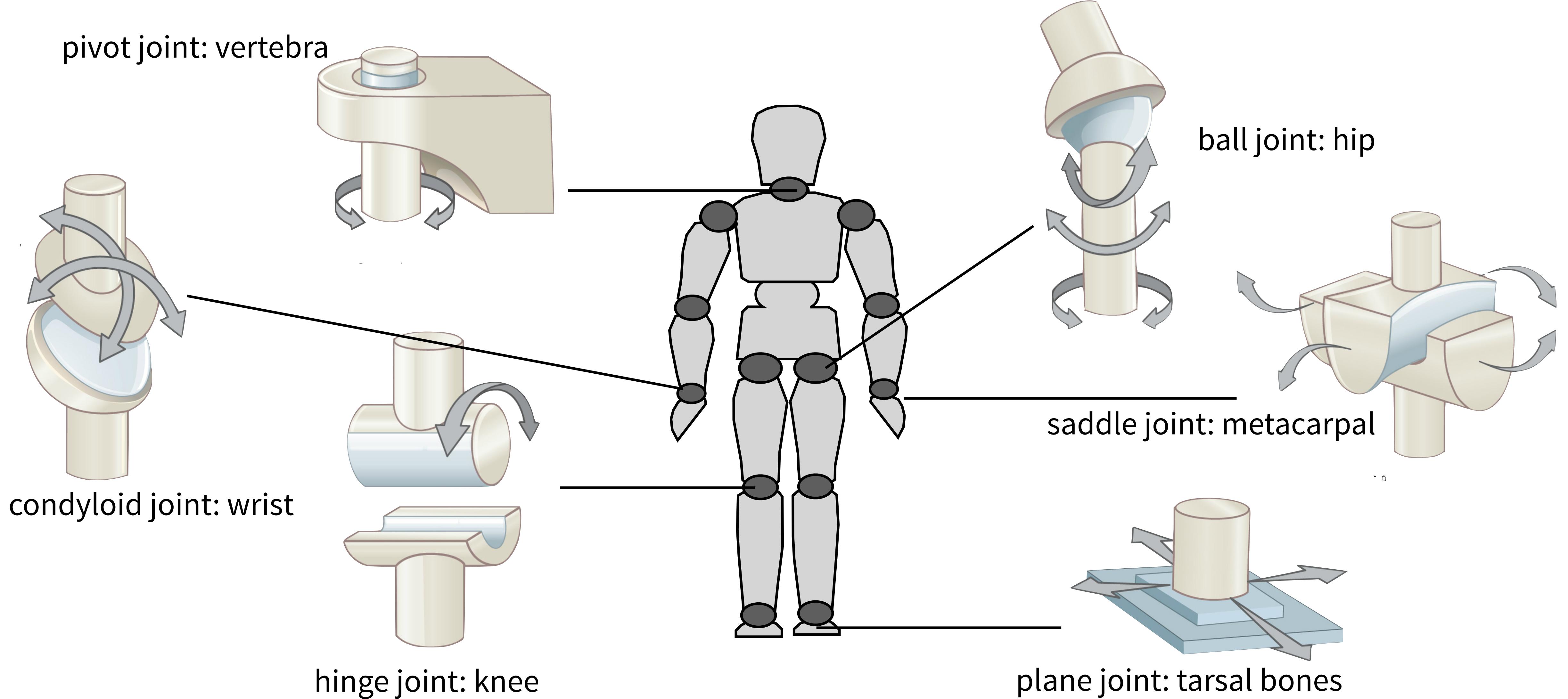


root: abdomen

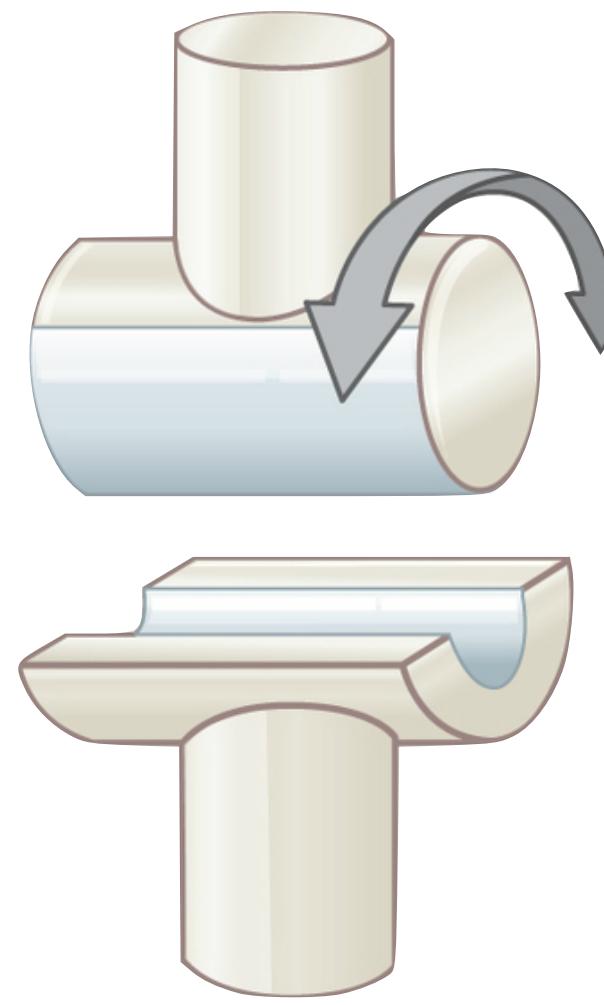
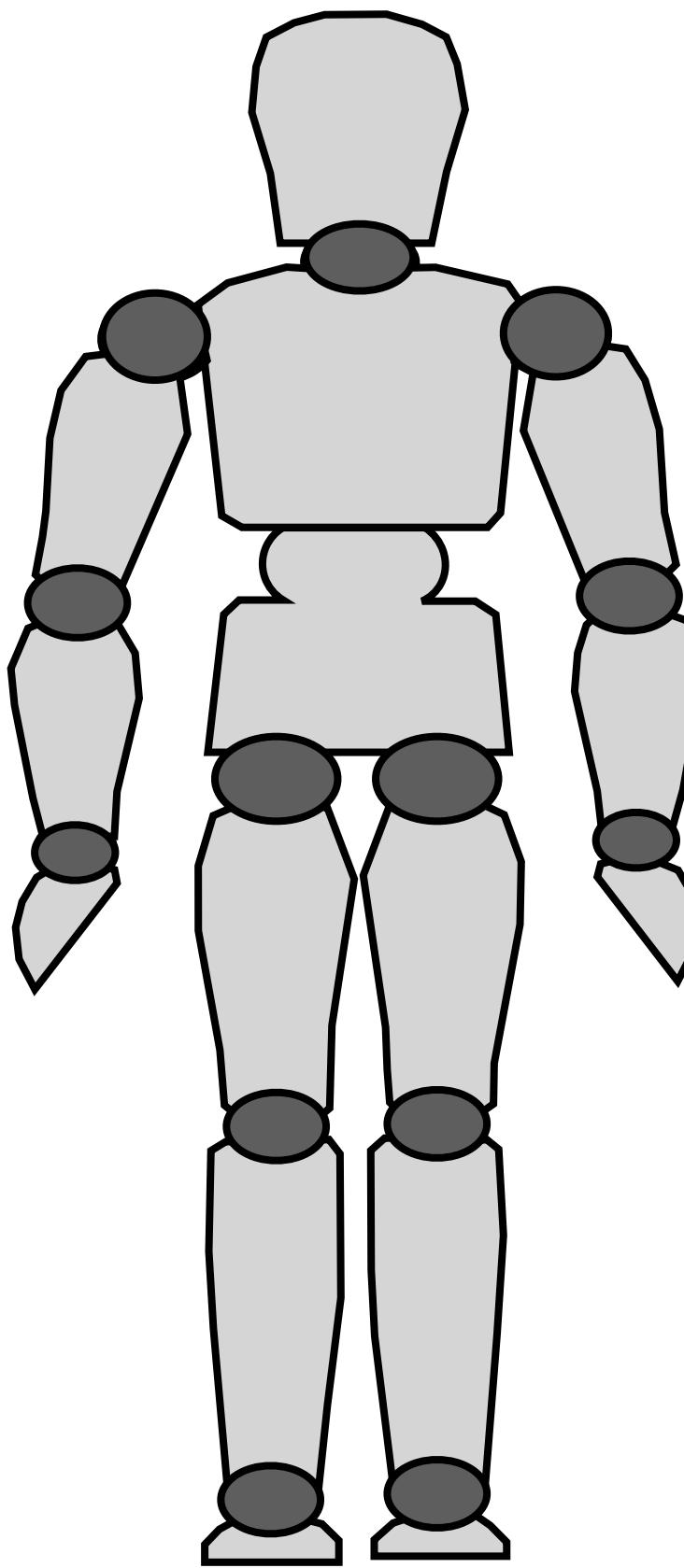


root: left foot

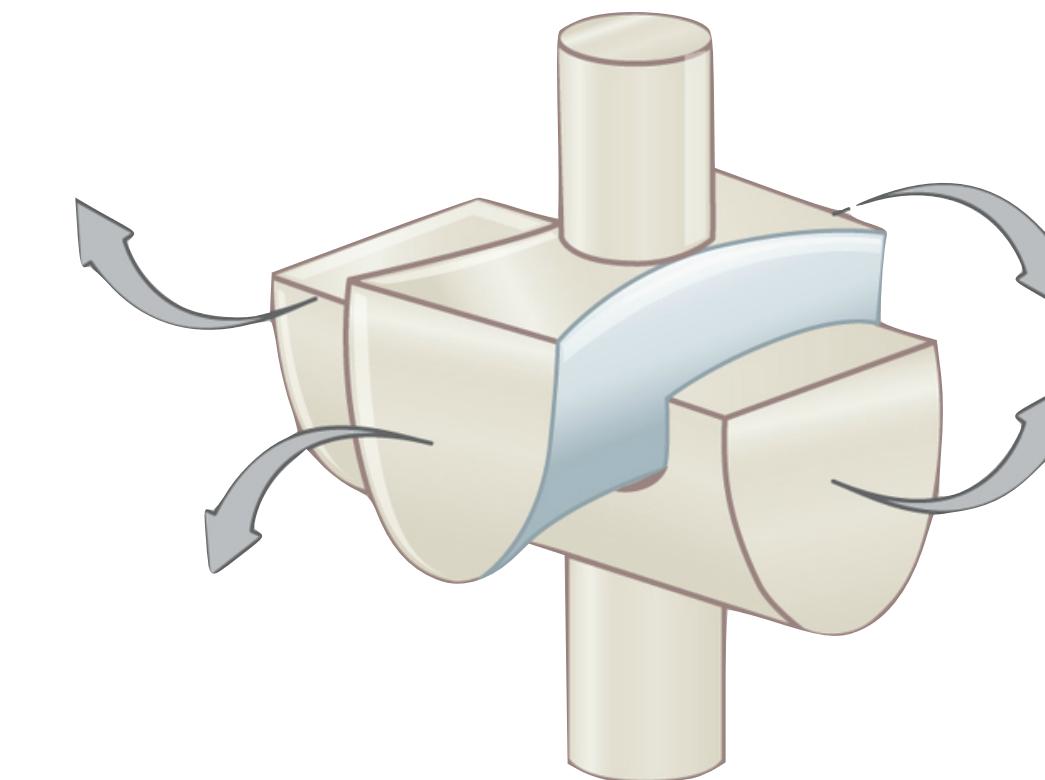
# Parameterizing human model



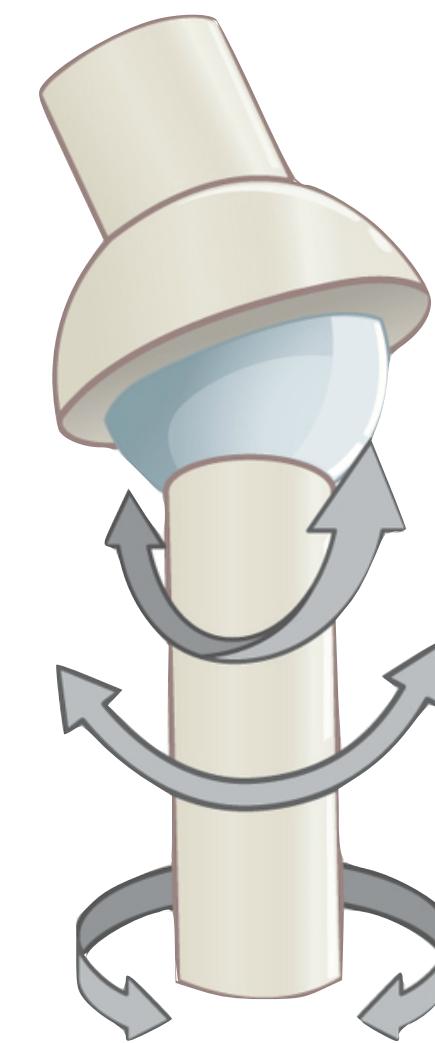
# Parameterizing human model



1-DOF joint

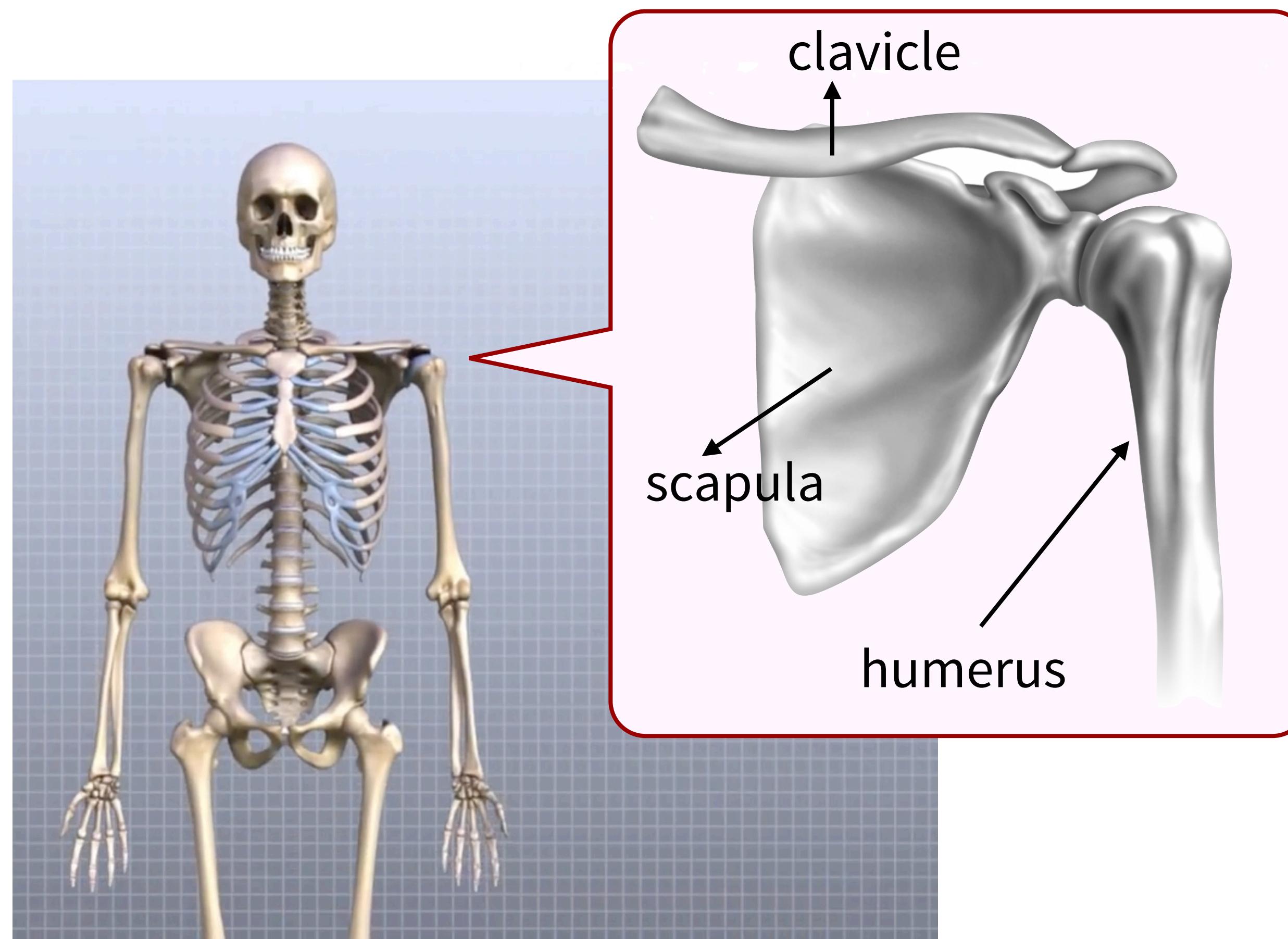


2-DOF joint

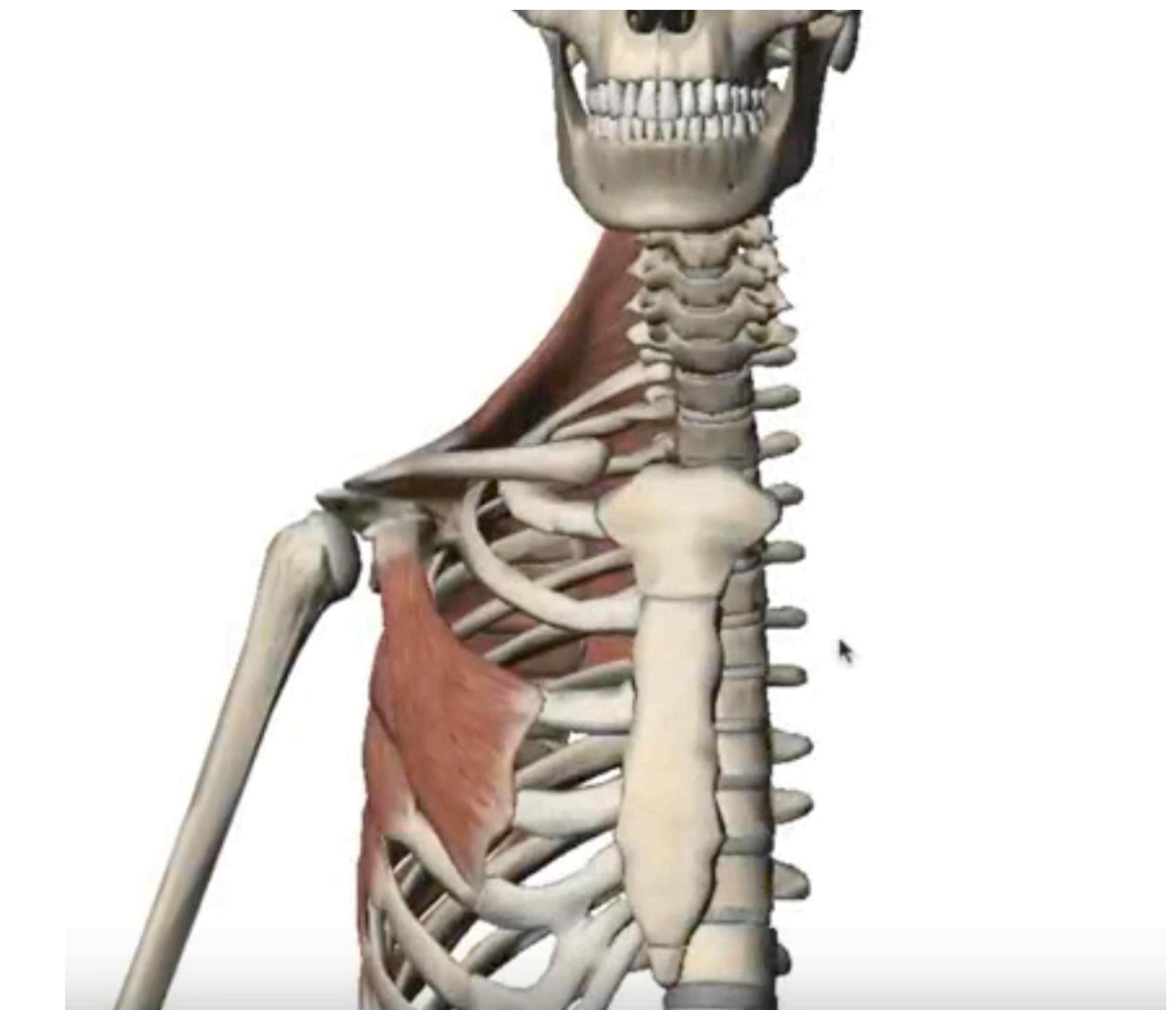
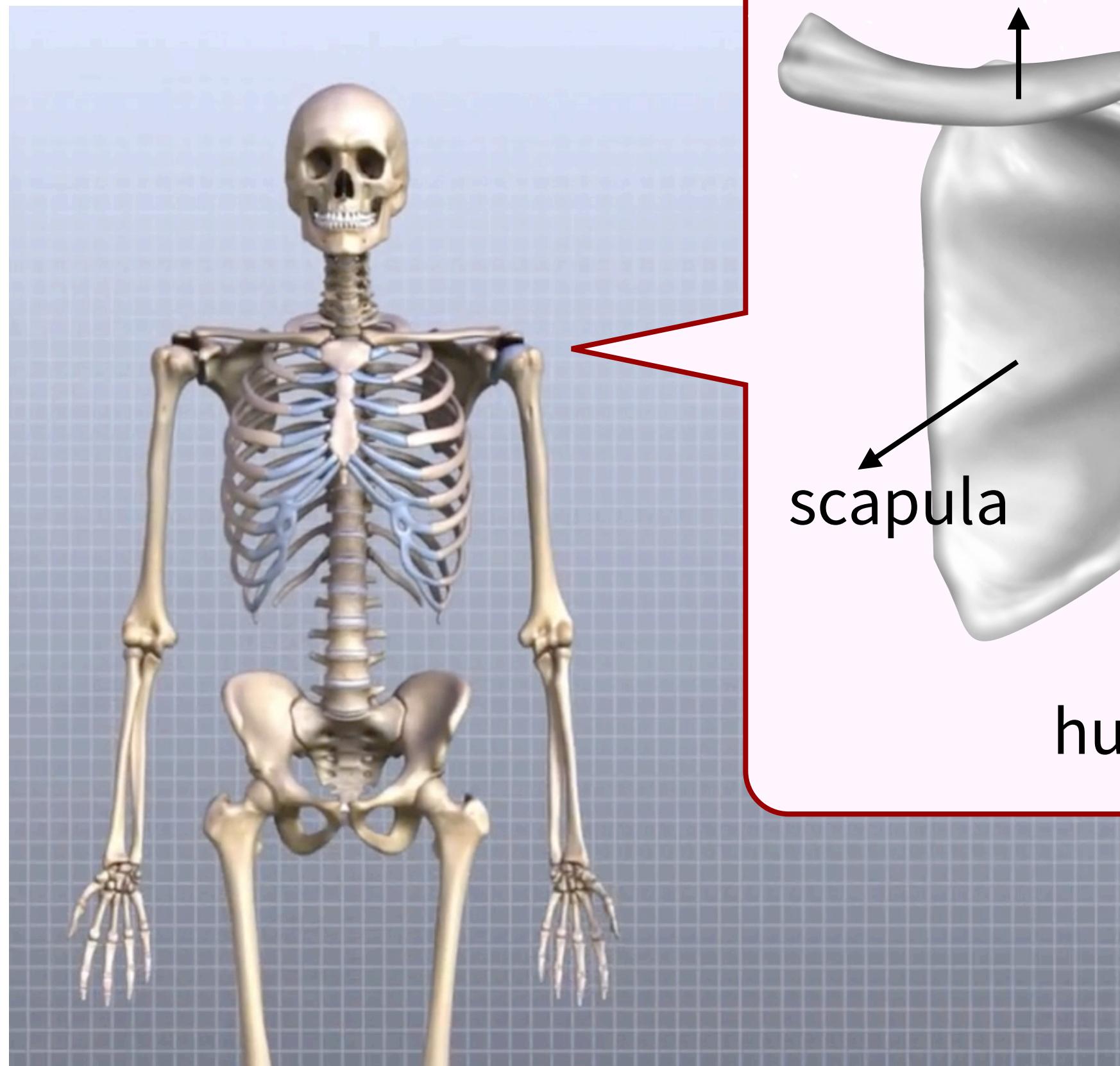


3-DOF joint

# Case study: How to model human shoulder?

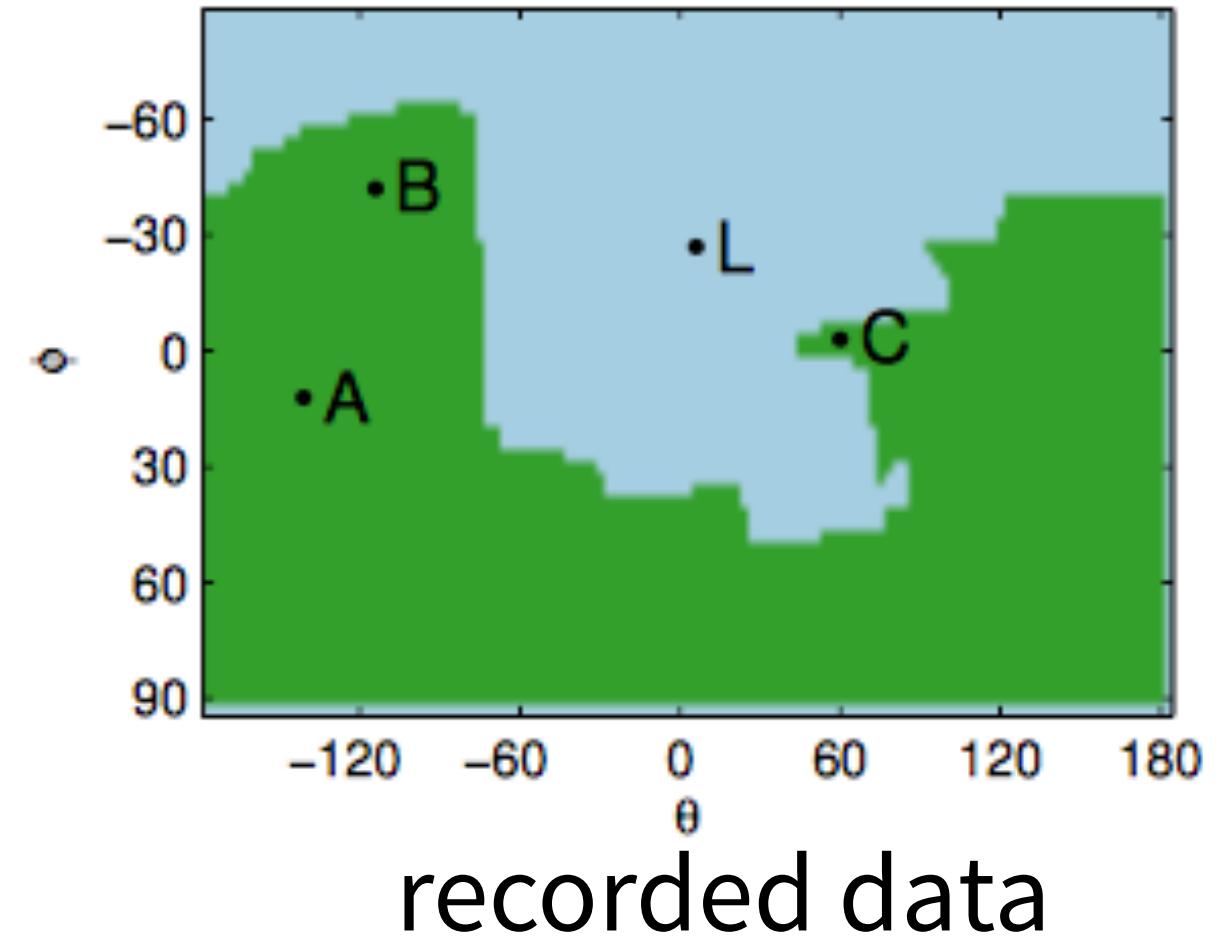


# Case study: How to model human shoulder?



# Joint limits

Valid region of shoulder configurations

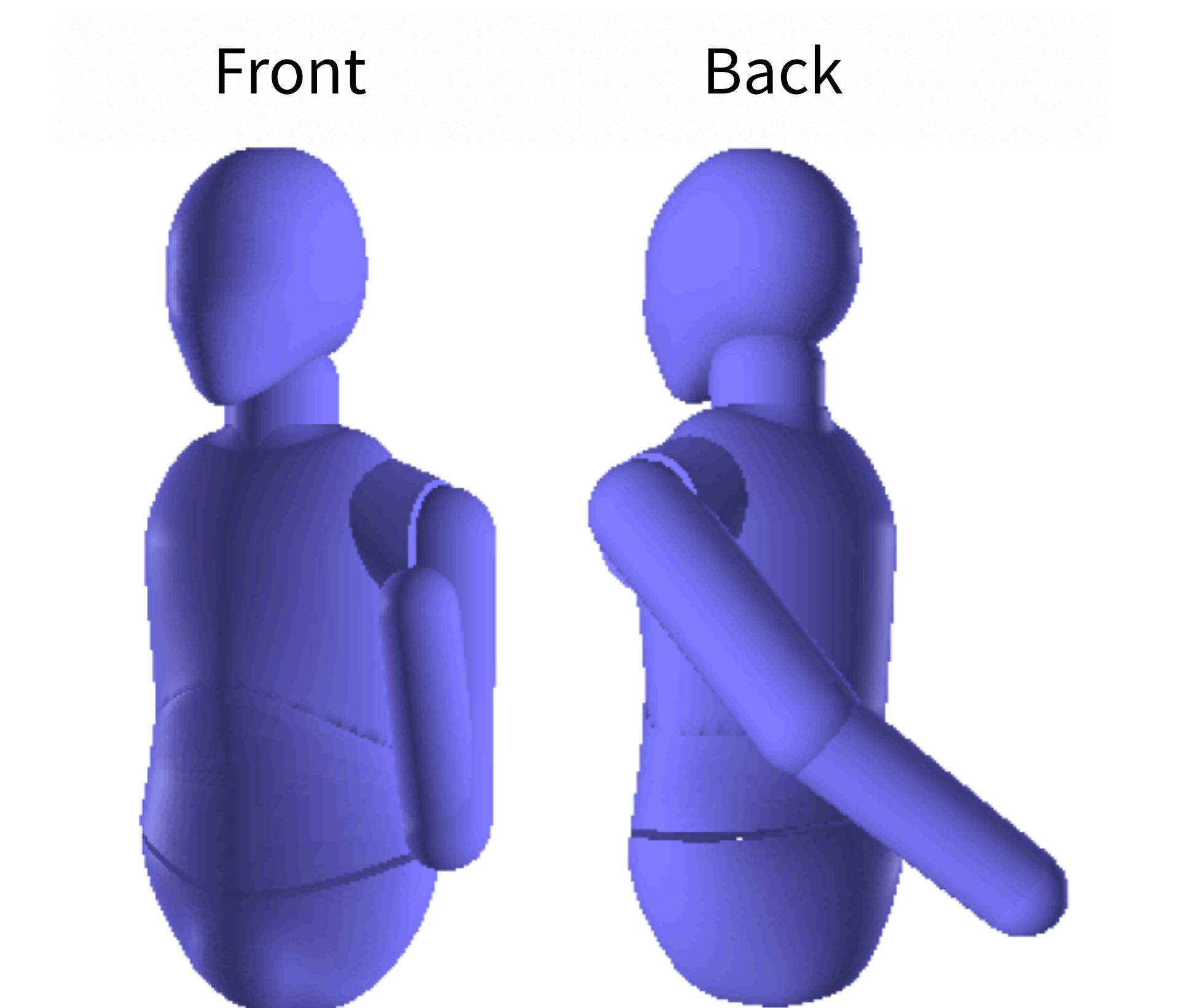


typically modeled

Observation #1

Human joint limit is very complex

# Joint limits



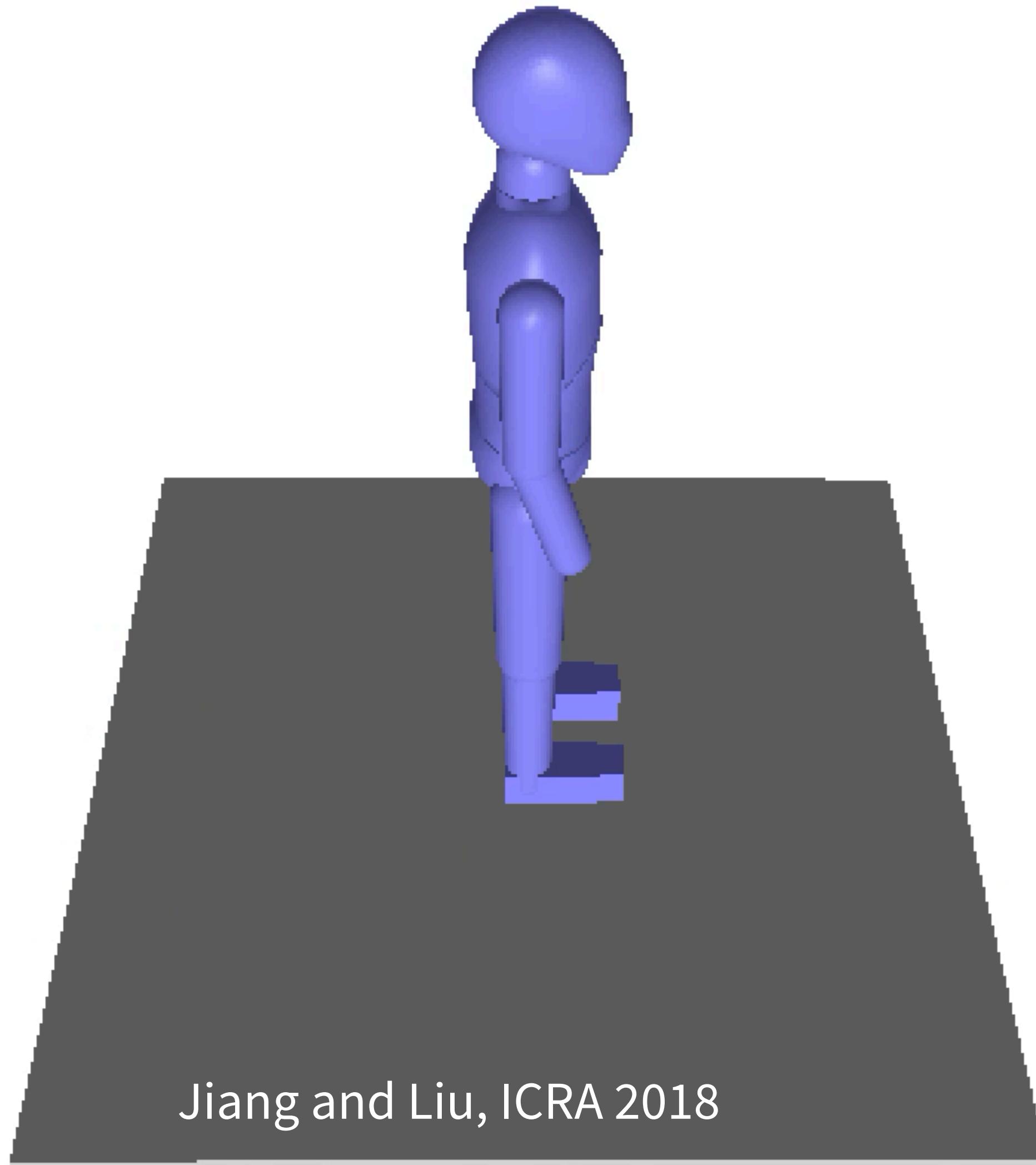
## Observation #1

Human joint limit is very complex

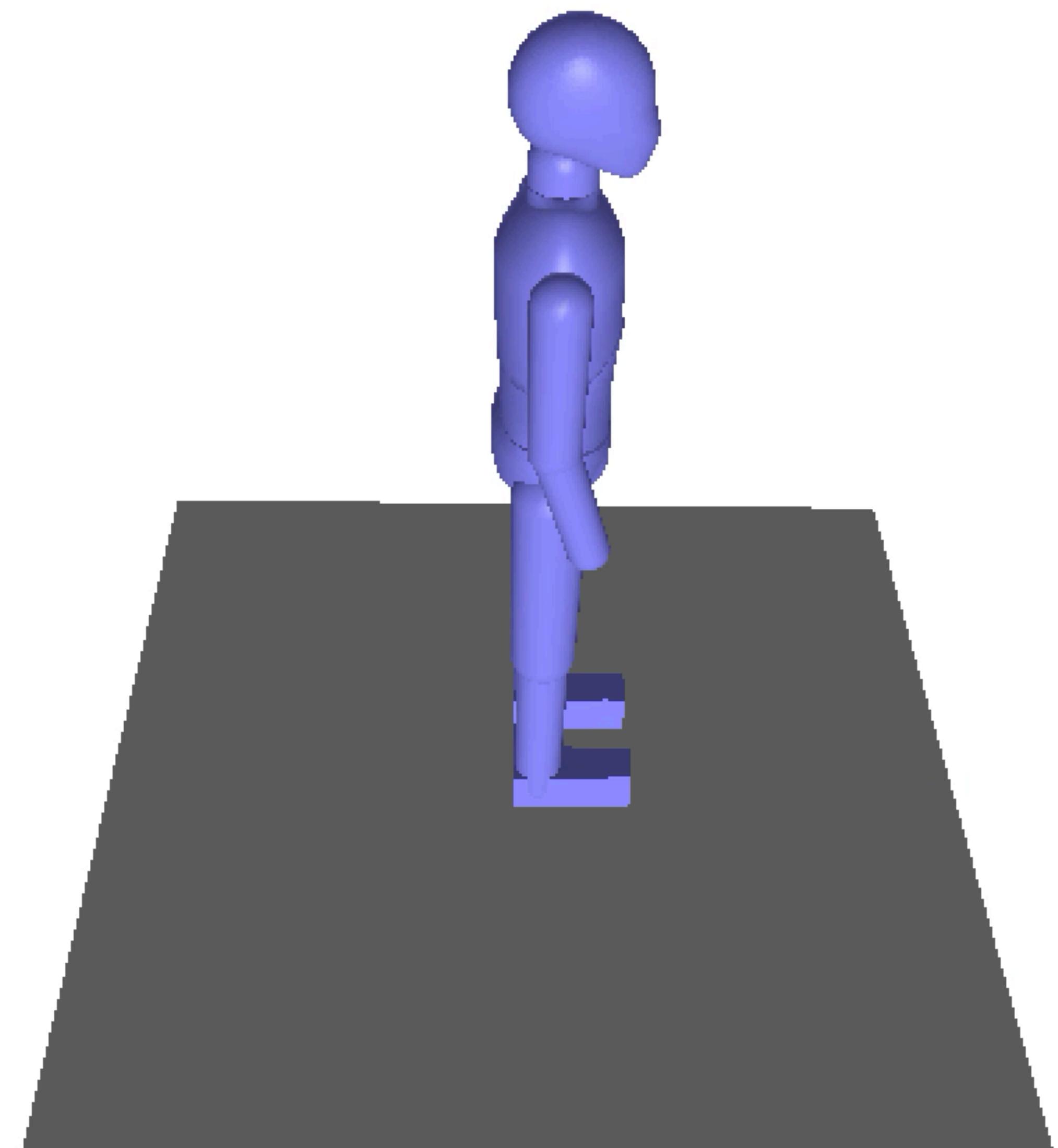
## Observation #2

Human joint limit is pose-dependent

Learned joint limit  $D(\mathbf{q})$

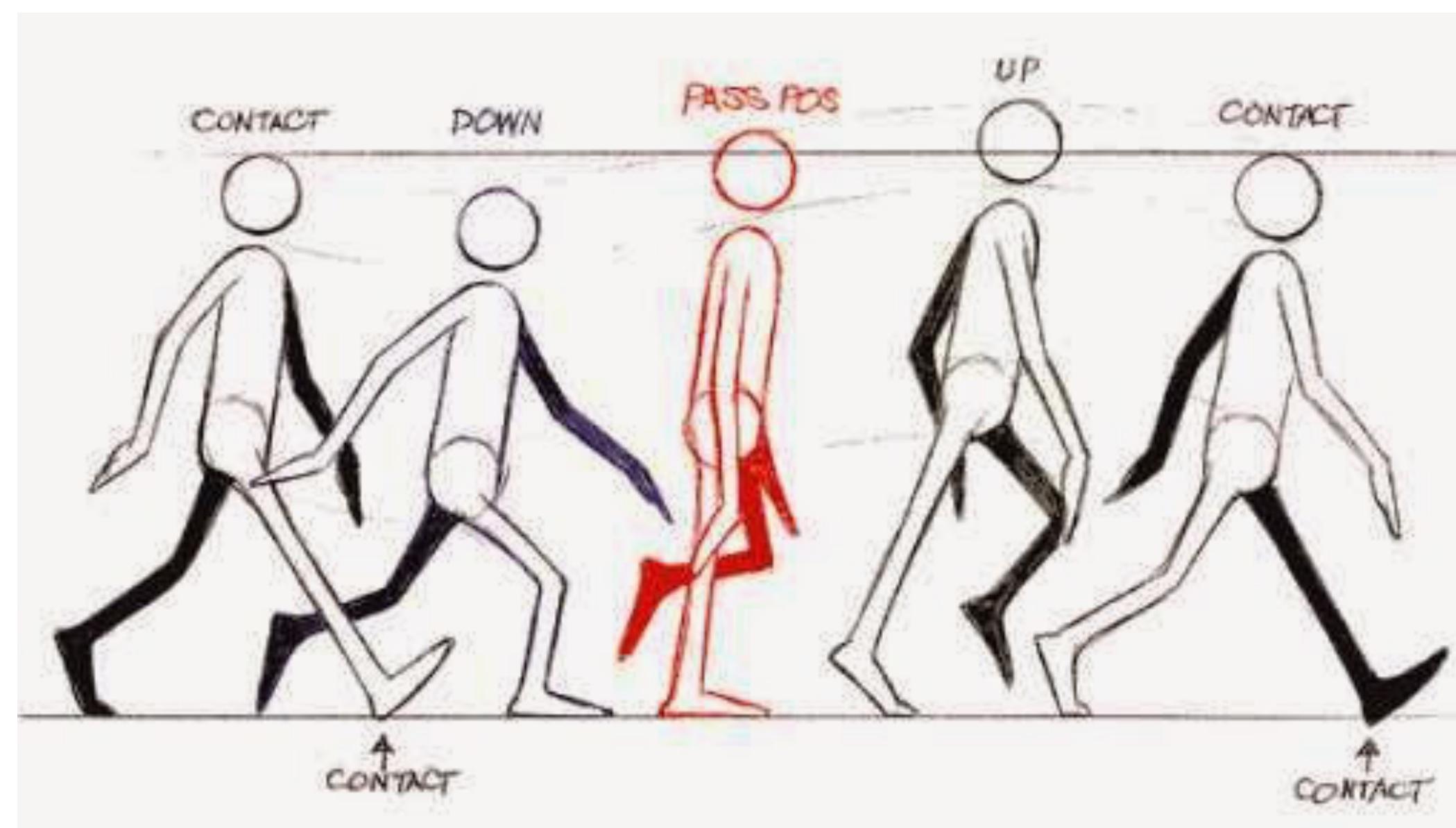


Box Constraints



# Limitations

- Directly controlling joints is not always what we want (e.g. hard to avoid foot-skating when animating human gait).



# FK vs IK

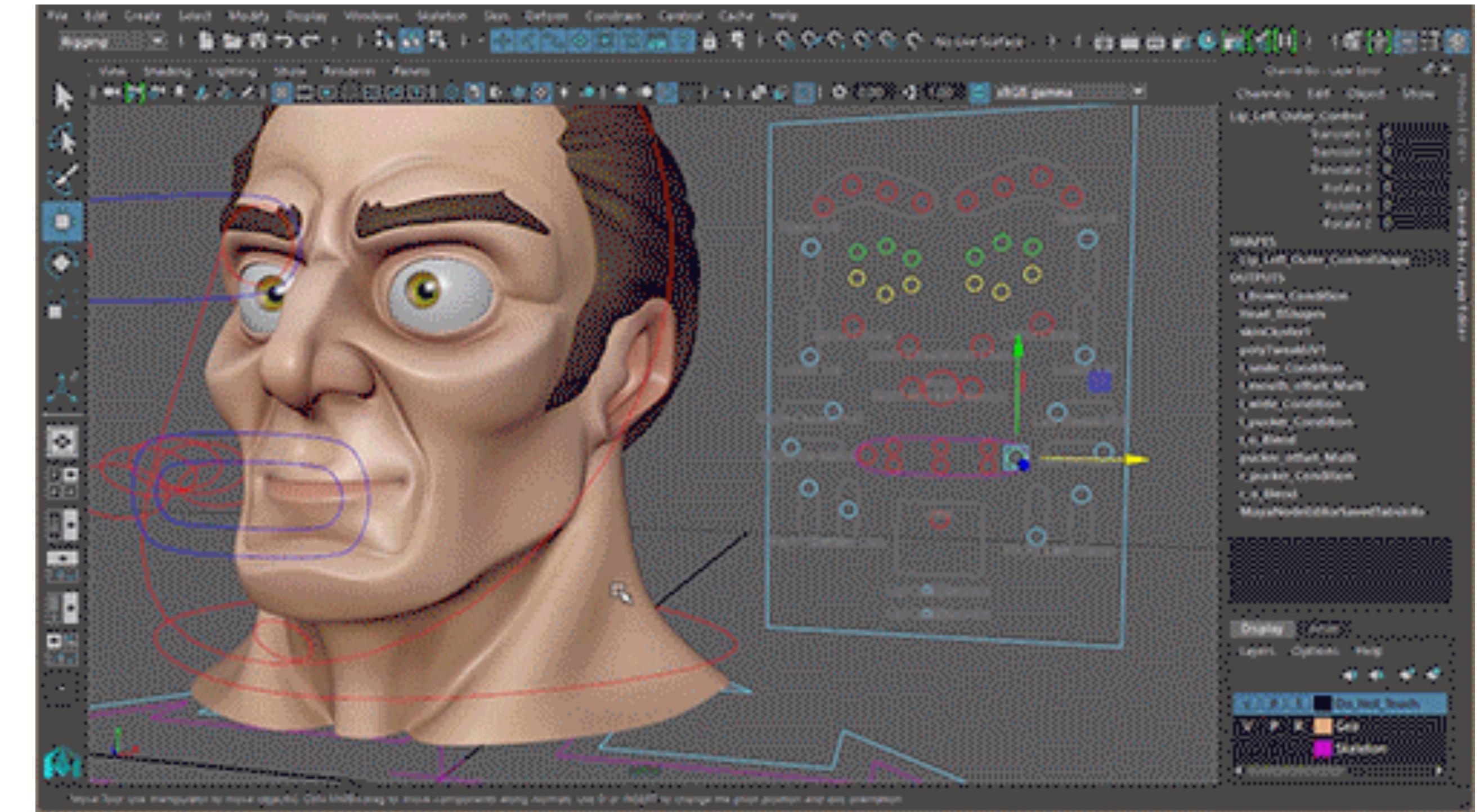
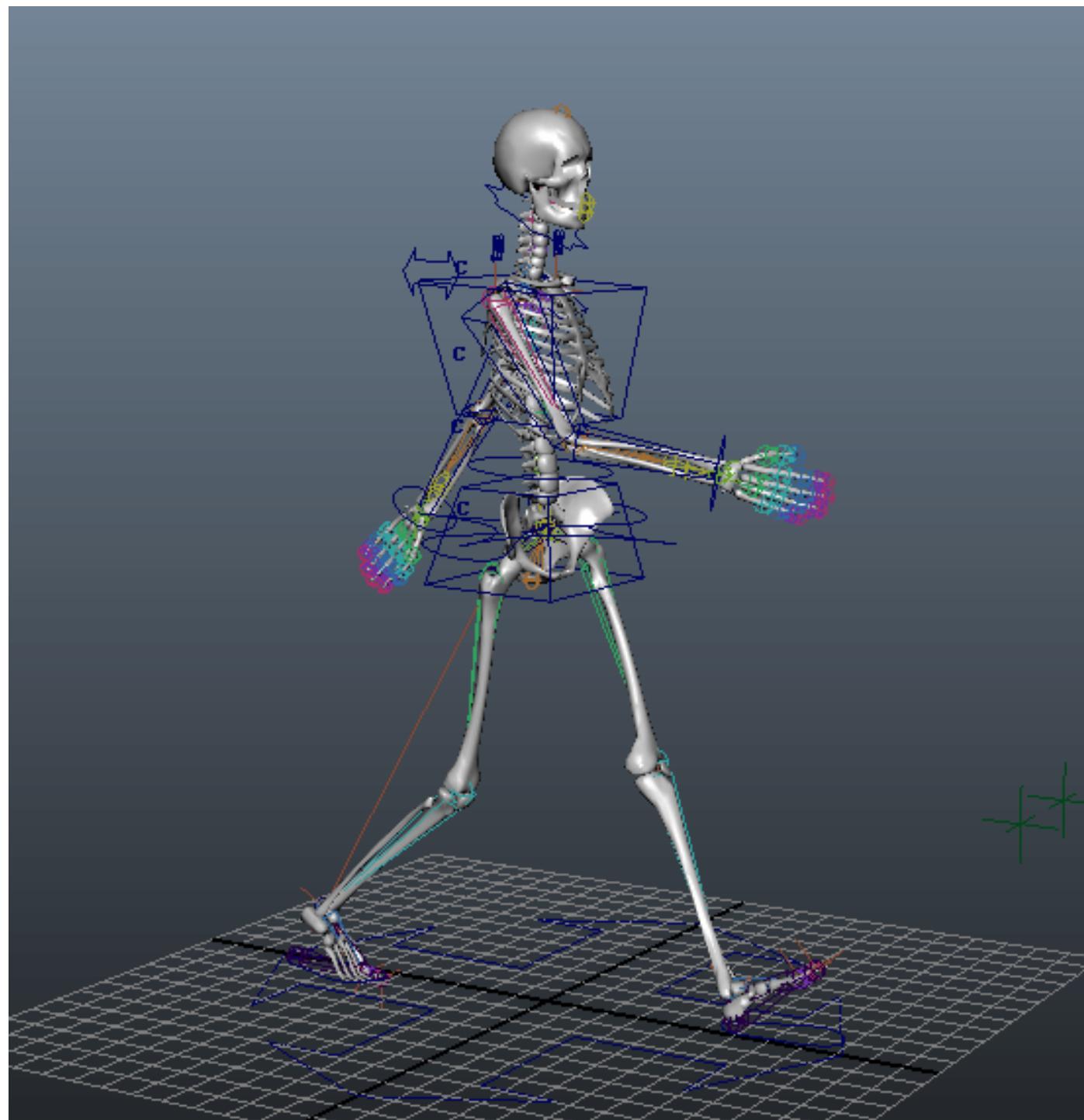
- **Forward kinematics**
  - Given a joint configuration, what is the 3D position of a point on the structure?
- **Inverse kinematics**
  - Given a target position for a point on the structure, what angles do the joints need to be to achieve that target point?

# Limitations

- Directly controlling joints is not always what we want (e.g. hard to avoid foot-skating when animating human gait).
- Assigning the value of each DOF individually is tedious for a complex model.

# Rigging

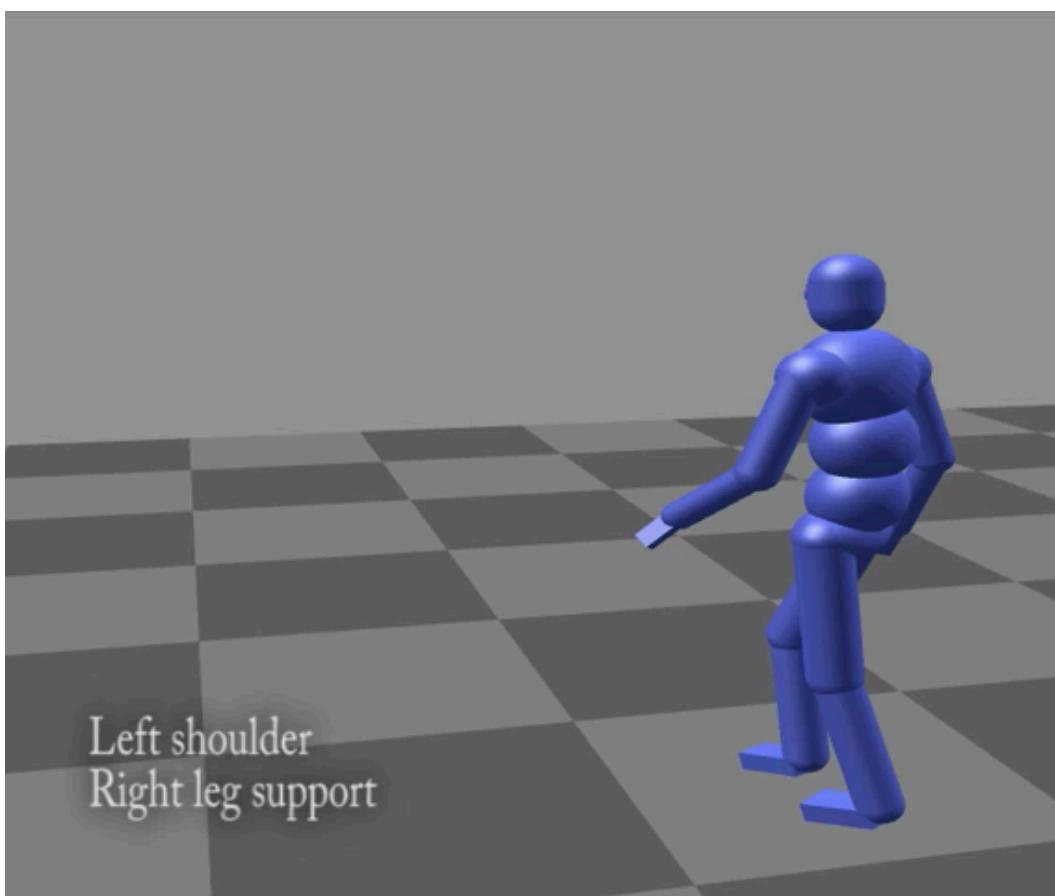
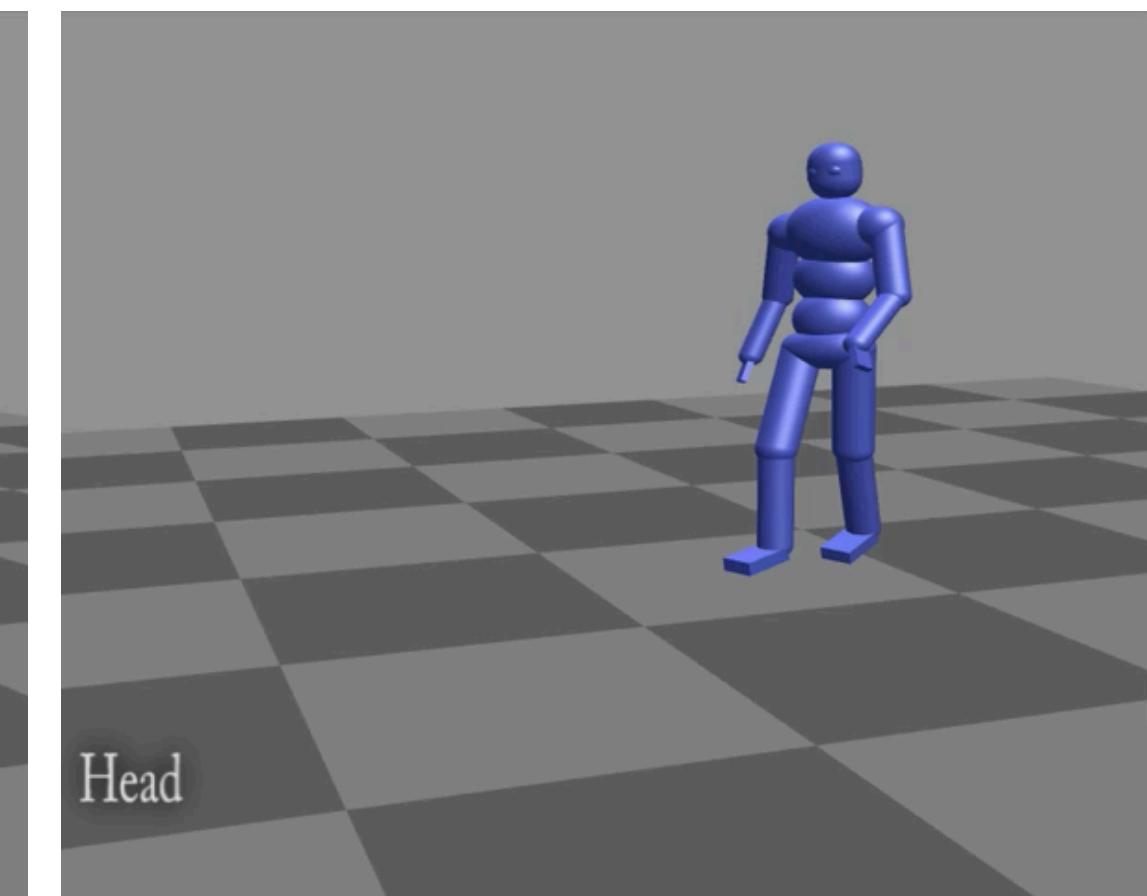
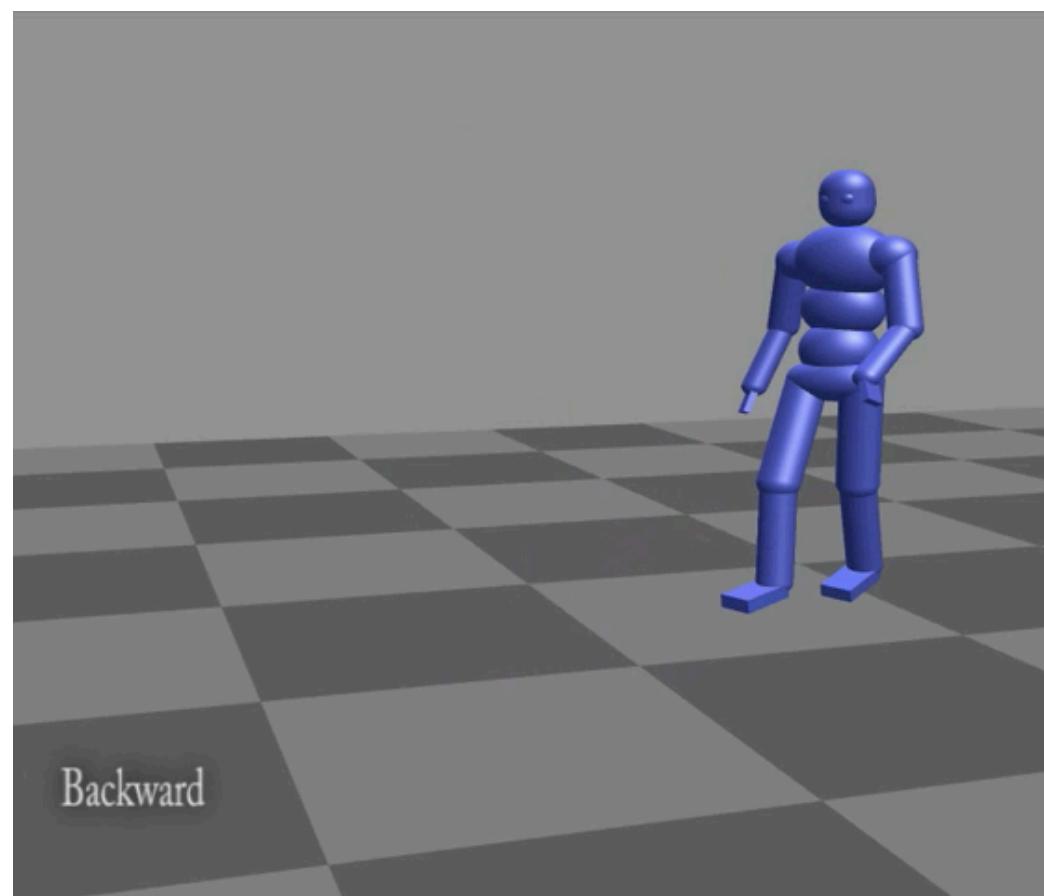
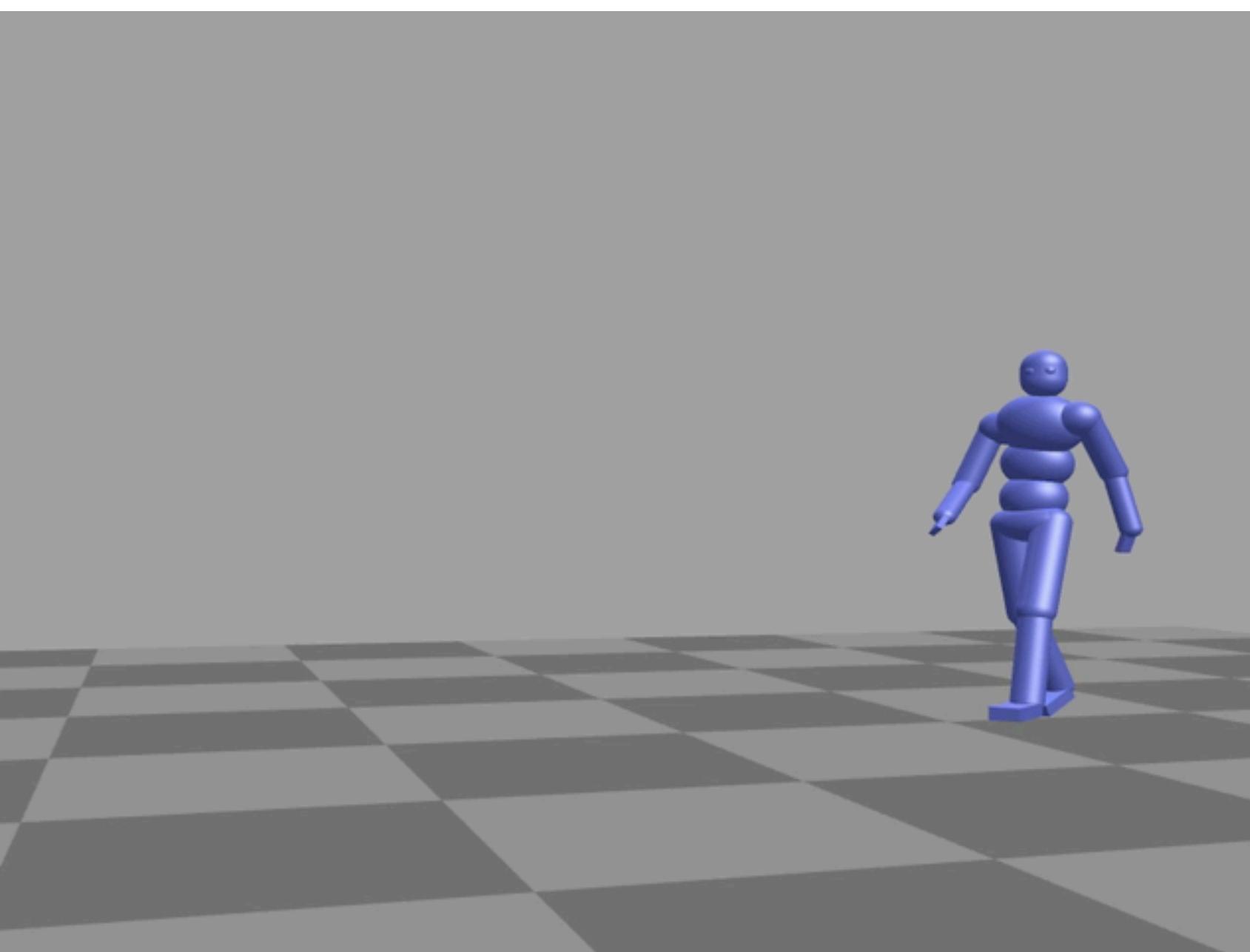
- The process of defining (meta) degrees of freedom on an animatable character.
- Rigging simplifies the animation process and improves production efficiency.



# Limitations

- Directly controlling joints is not always what we want (e.g. hard to avoid foot-skating when animating human gait).
- Assigning the value of each DOF individually is tedious for a complex model.
- Character is not affected by physics (e.g. going through wall).

# Physics

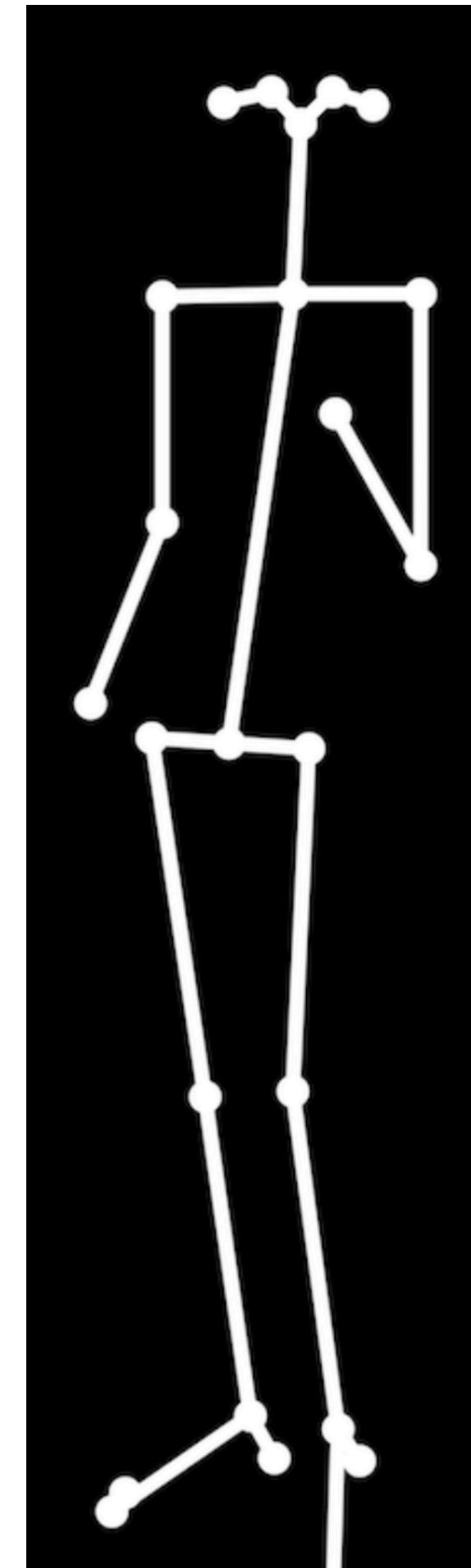
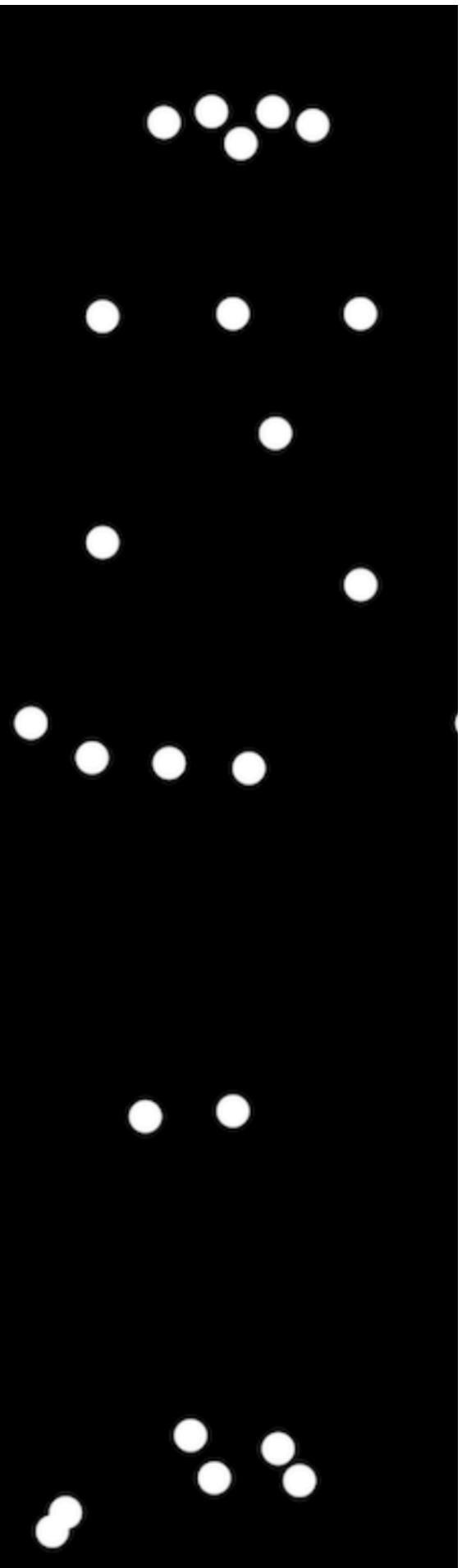
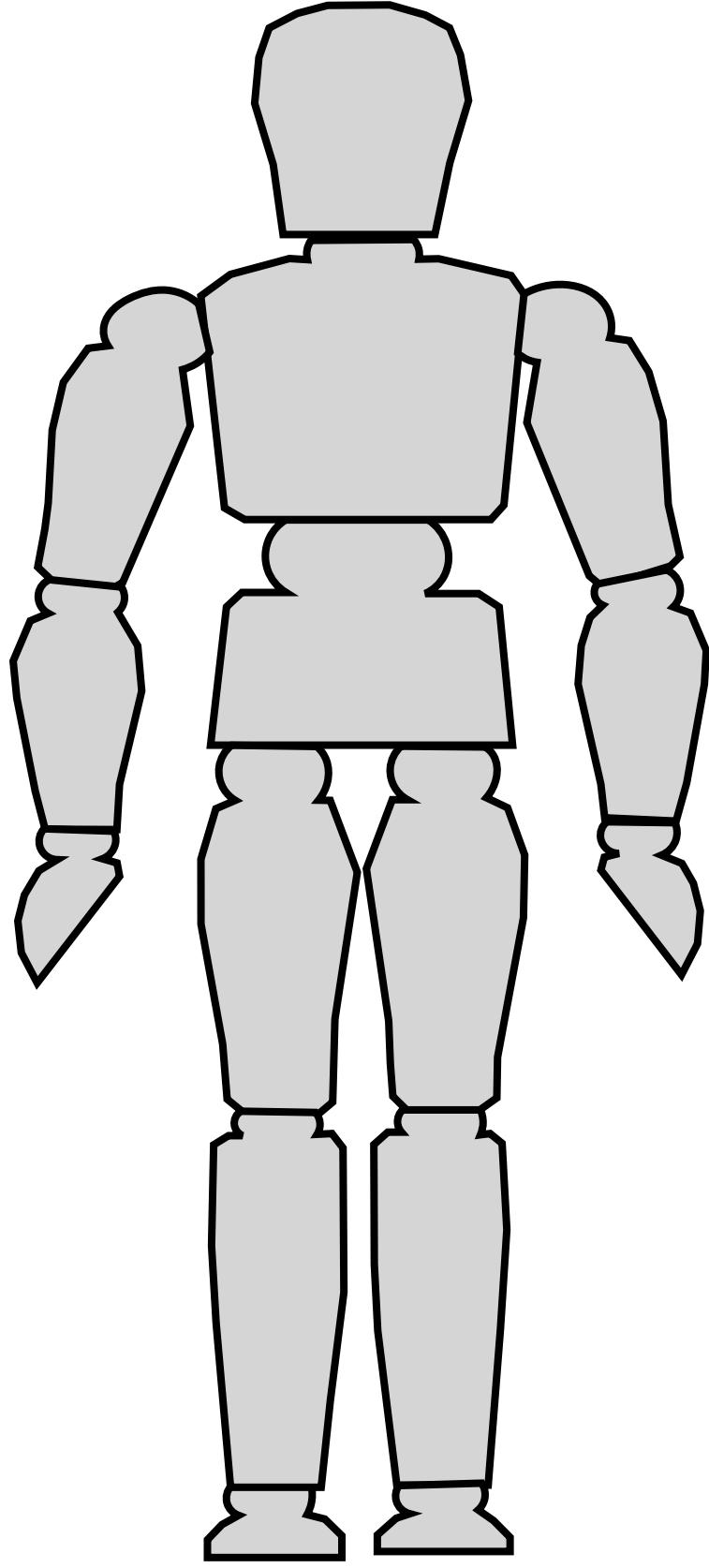


Character responding to various external pushes

# Limitations

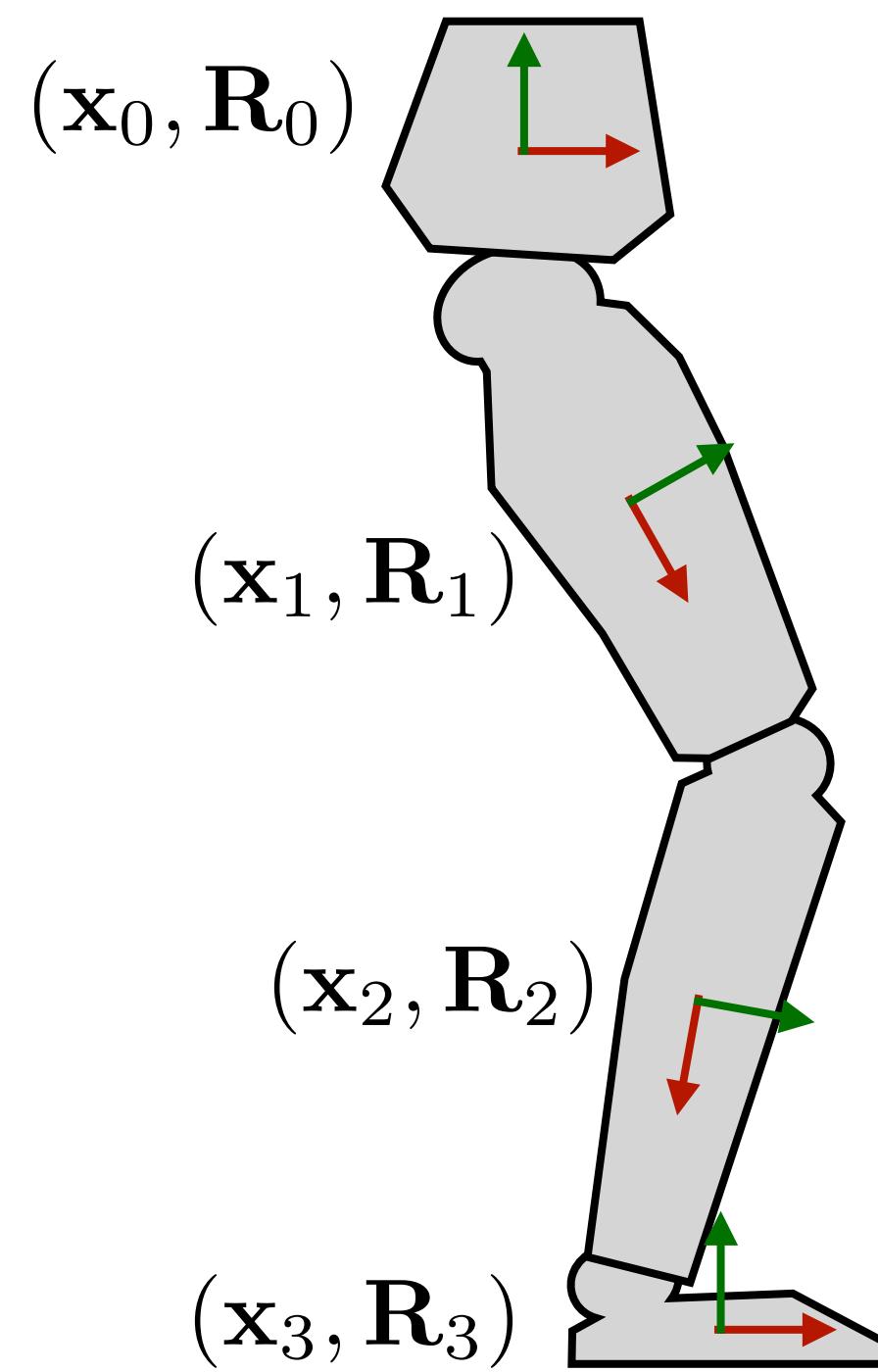
- Directly controlling joints is not always what we want (e.g. hard to avoid foot-skating when animating human gait).
- Assigning the value of each DOF individually is tedious for a complex model.
- Character is not affected by physics (e.g. going through wall).
- It's difficult to change the hierarchical structure on-the-fly. For example, dismembering the human figure (usually in video games).

# How to model a pose?



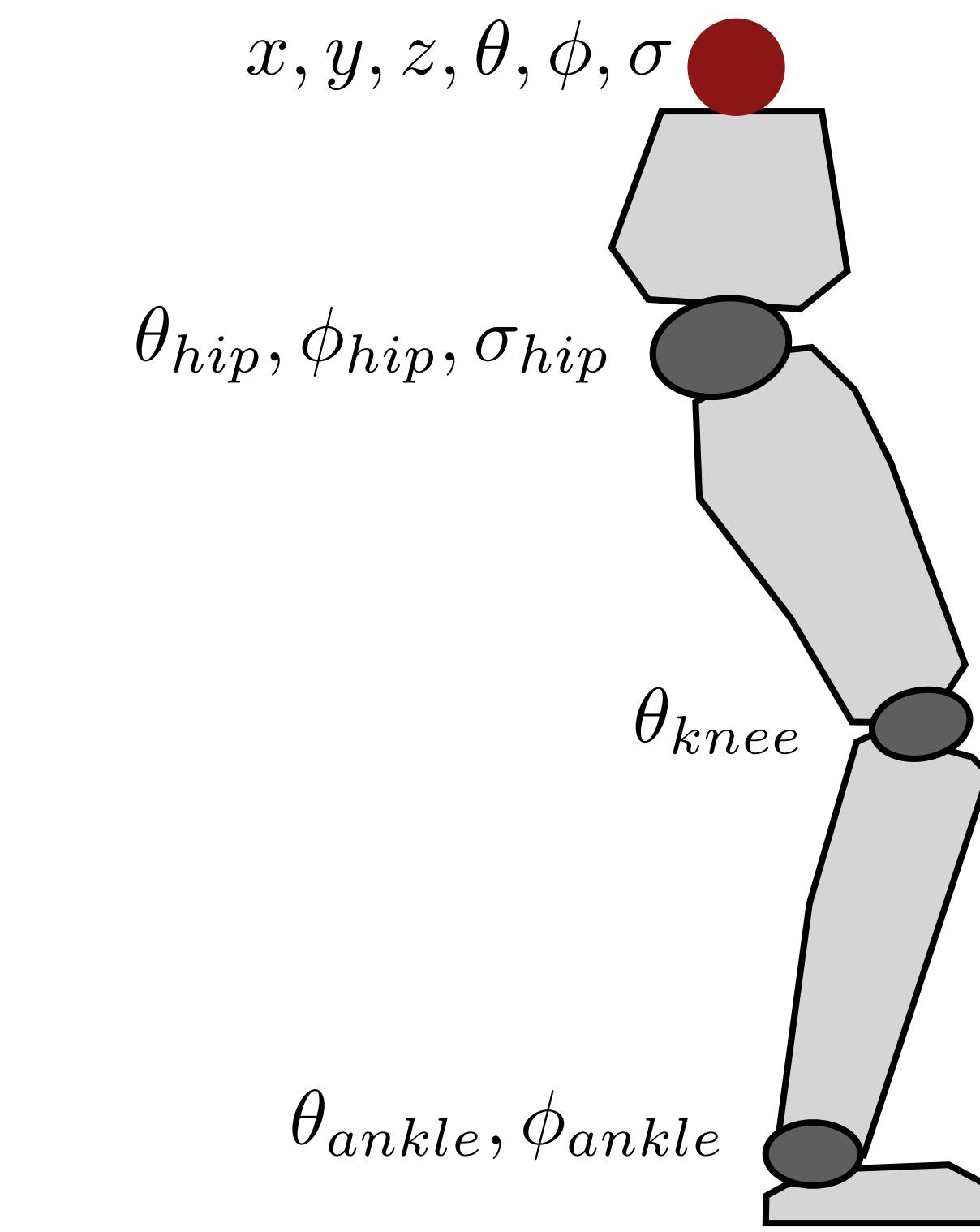
# Maximum vs generalized coordinates

Maximum coordinates



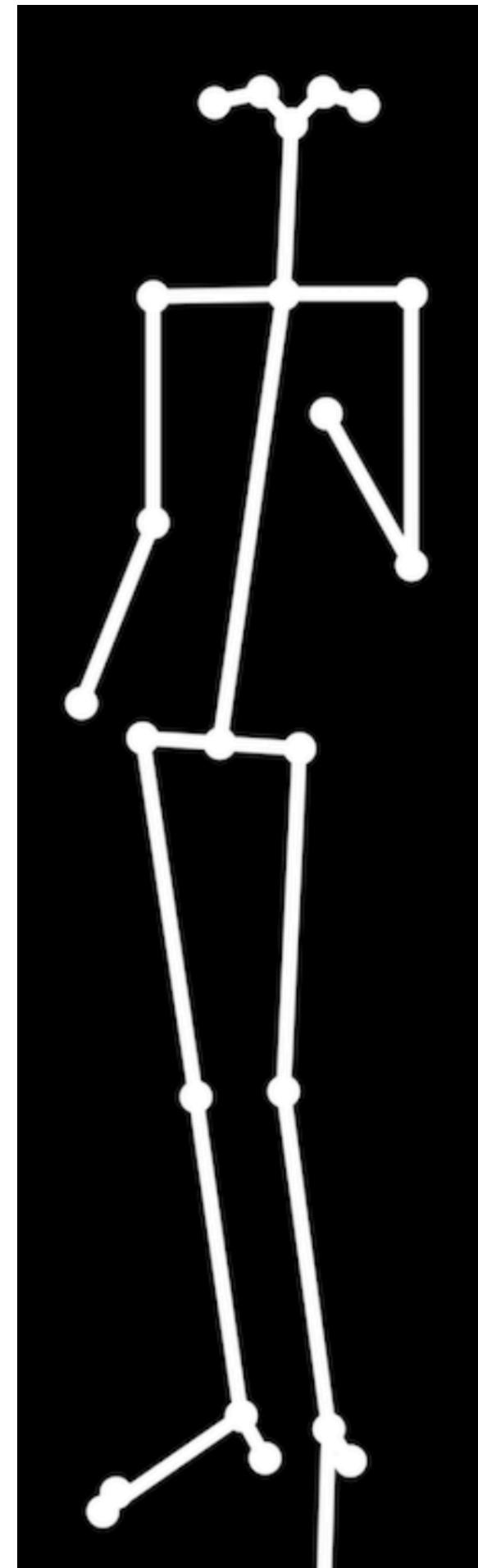
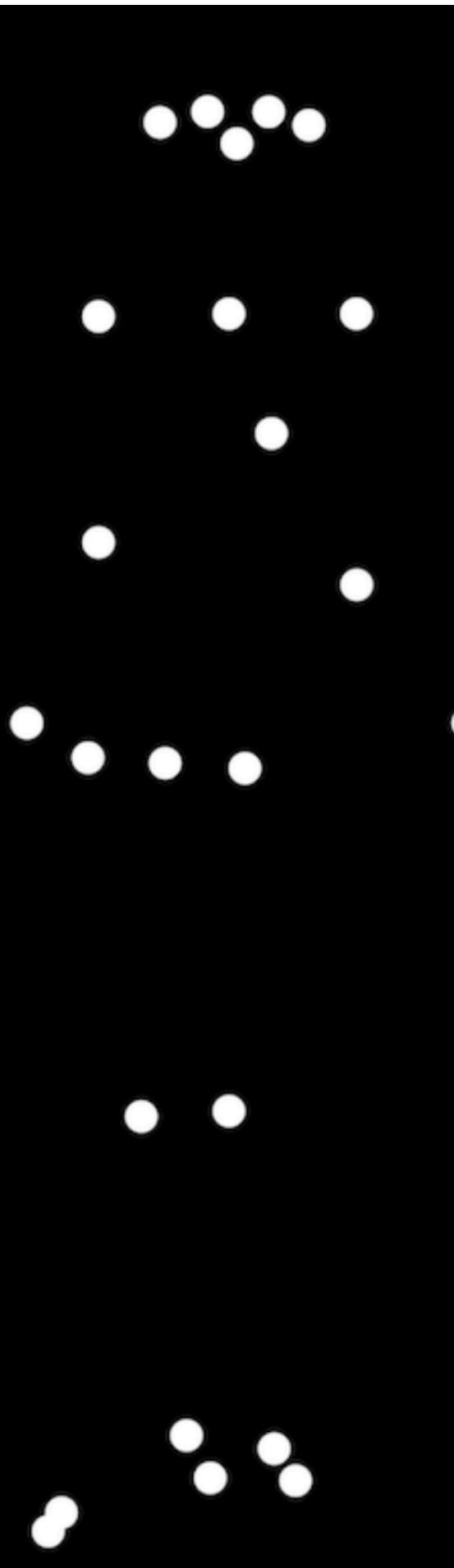
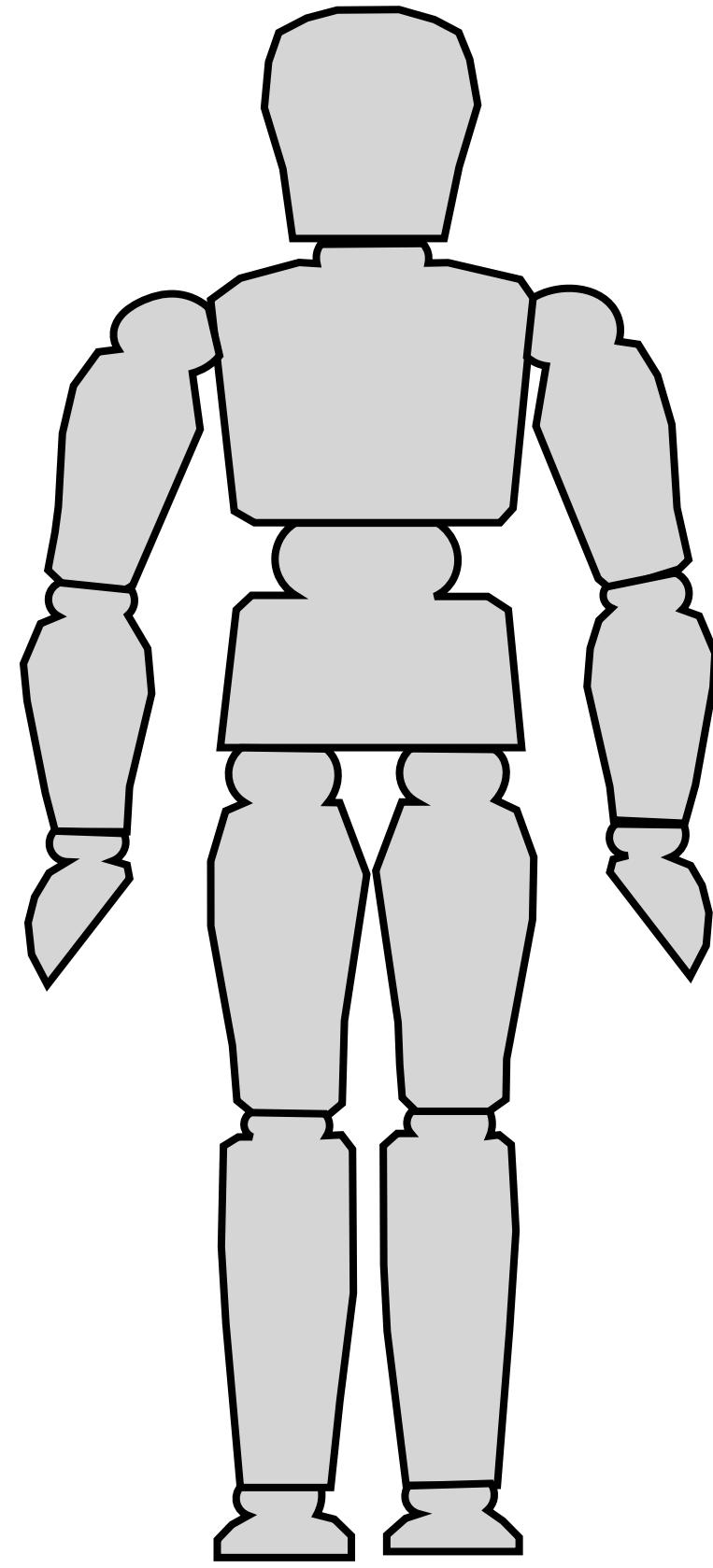
degree of freedom: 24

Generalized (reduced) coordinates

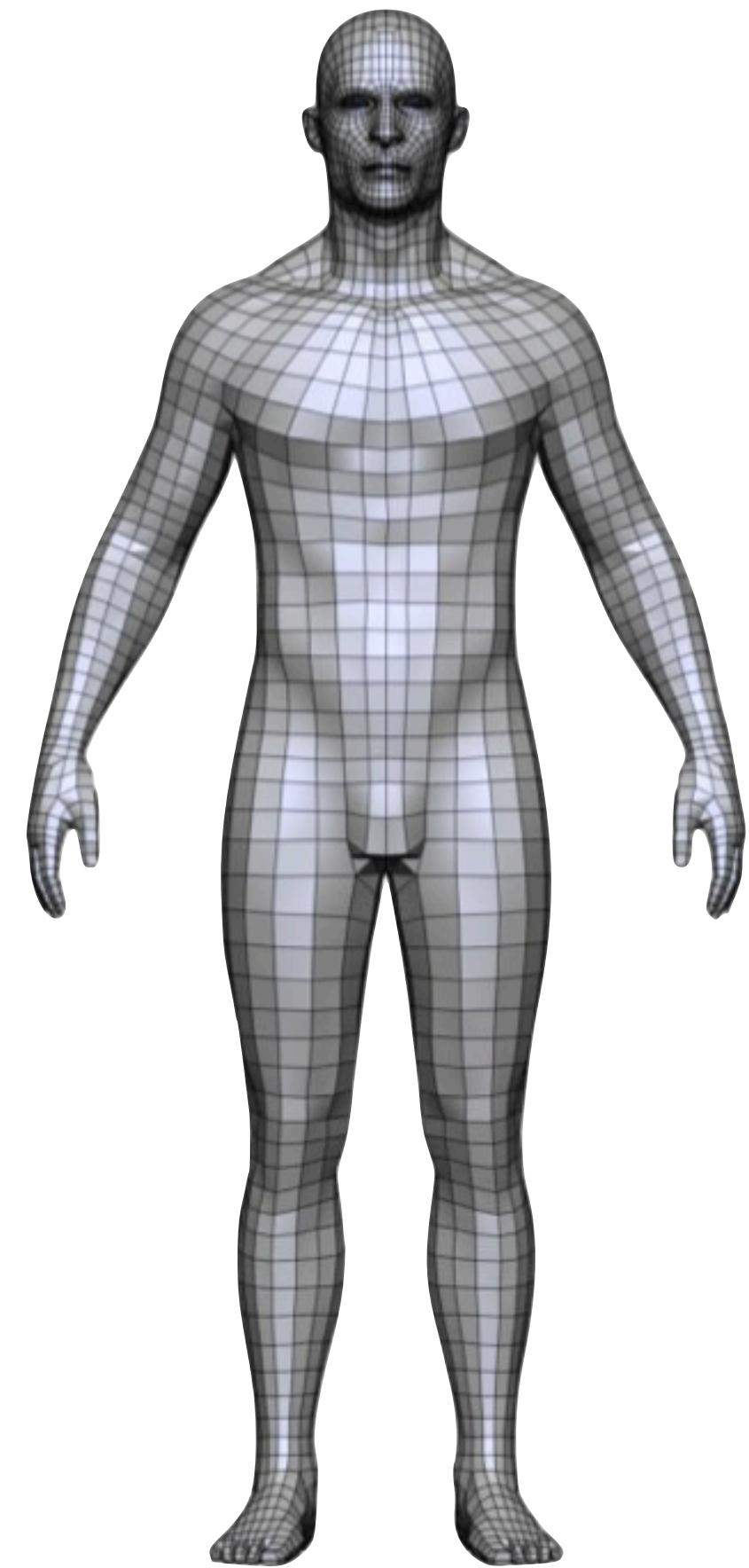
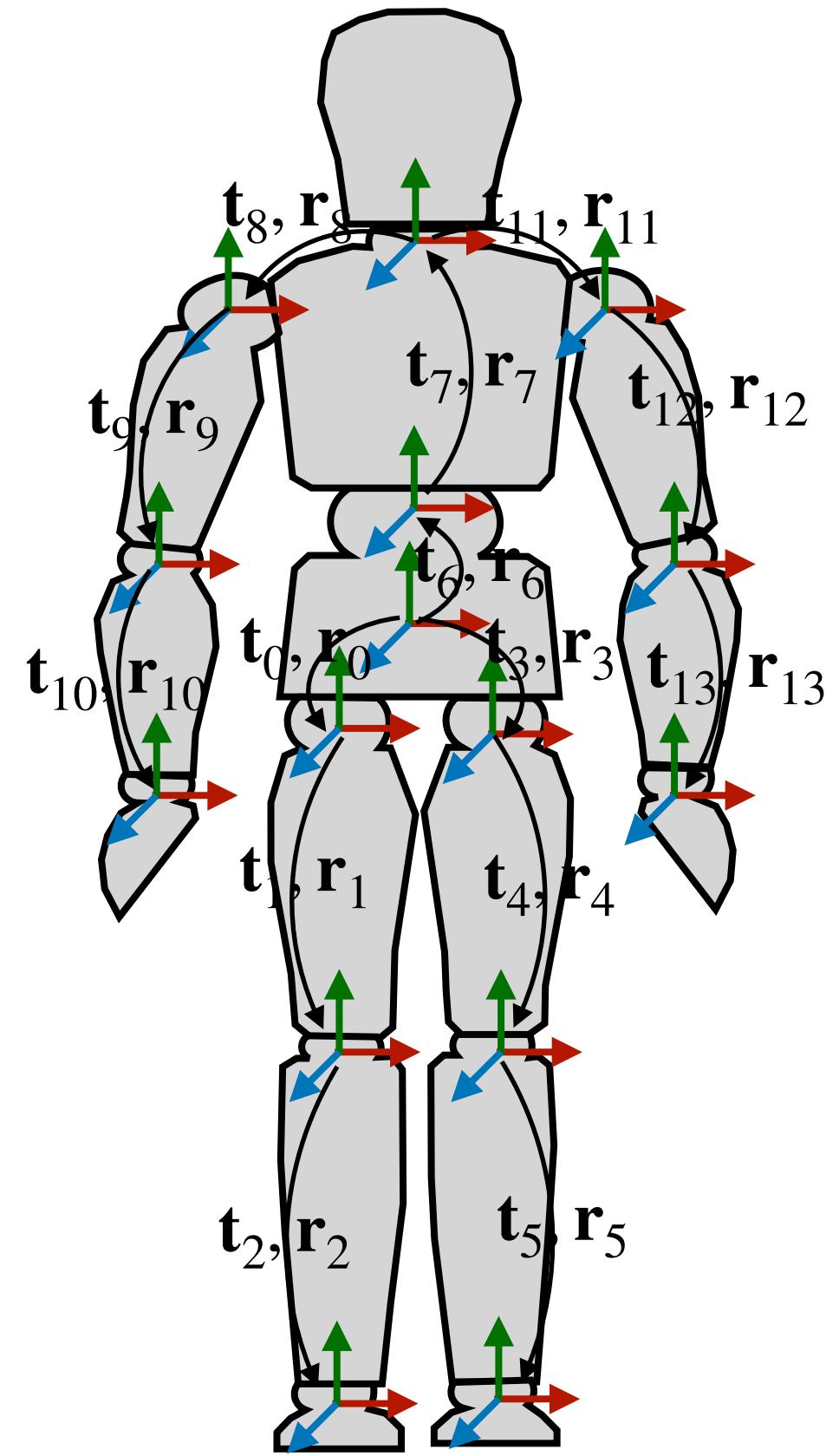


degree of freedom: 12

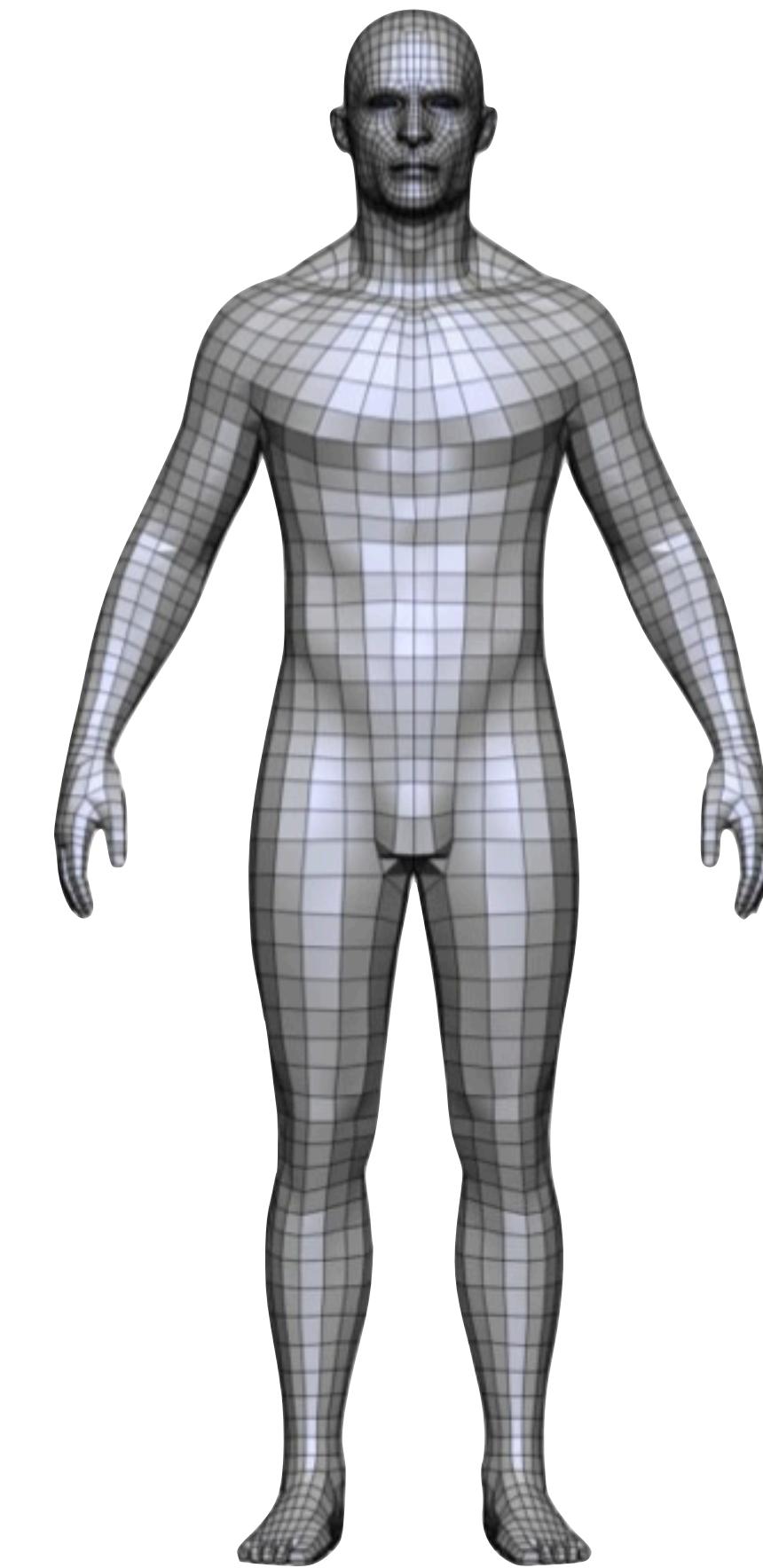
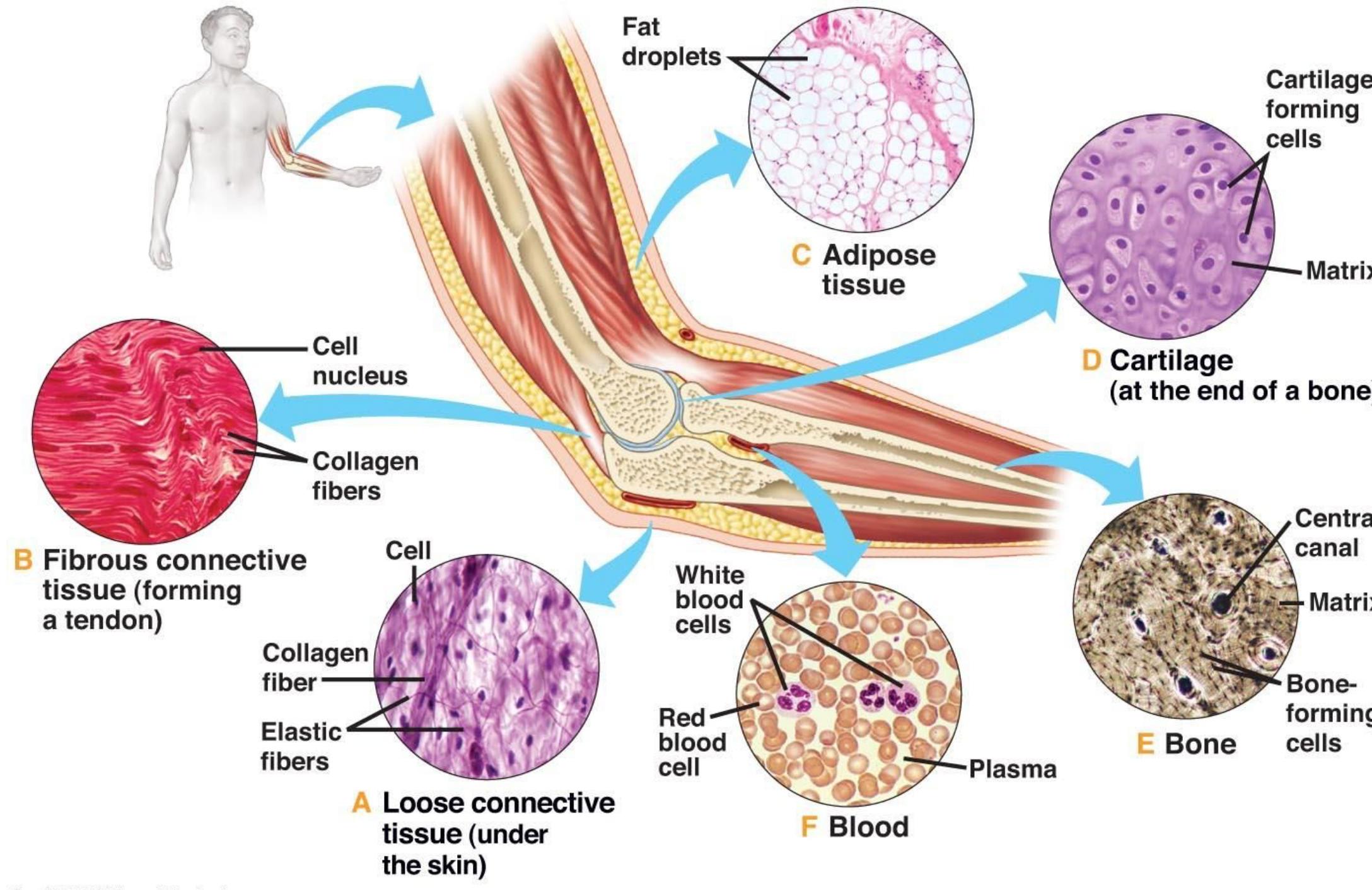
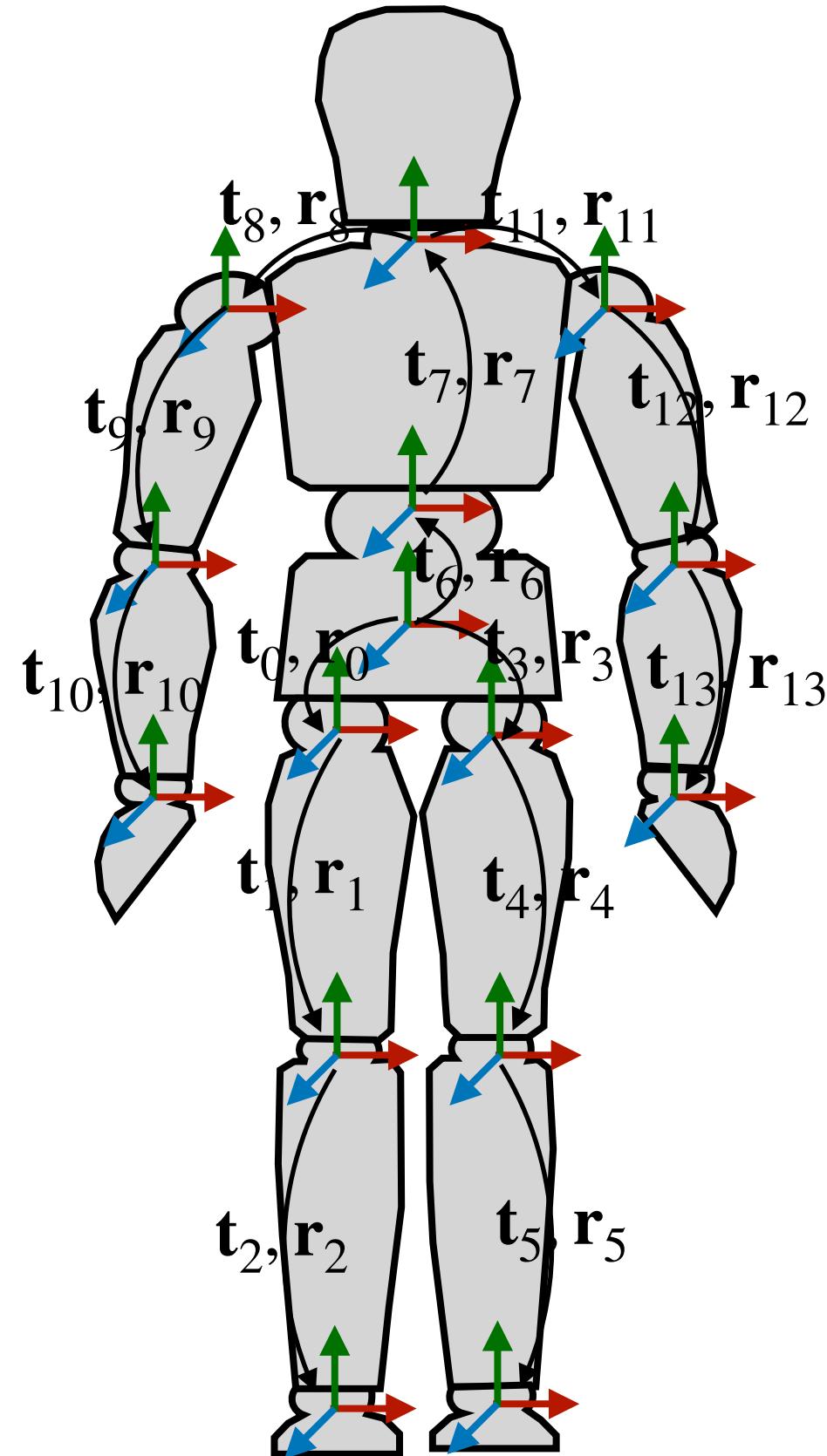
# How to model body shapes?



# Visualization of kinematics

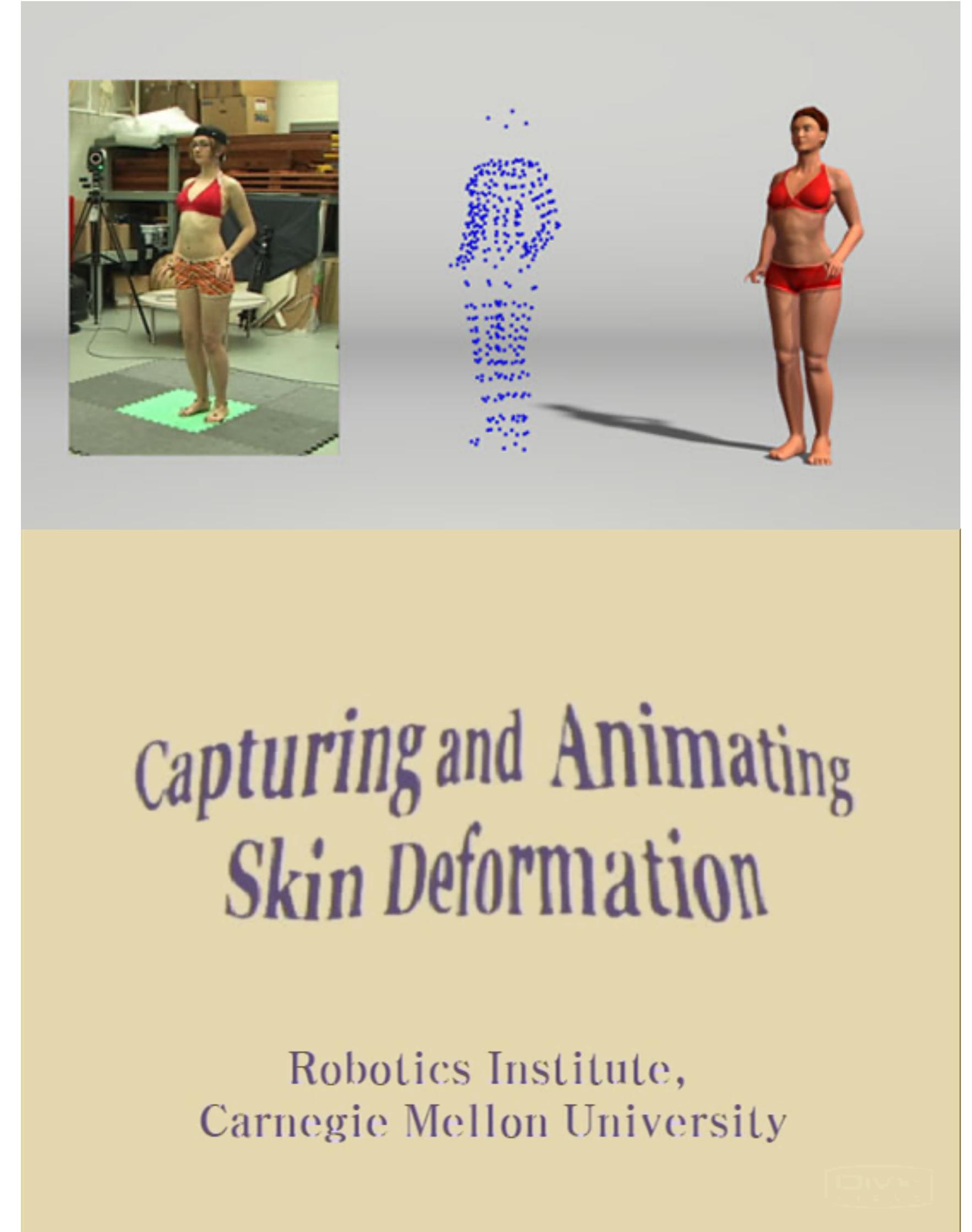


# Physics approach



# Real-data approach

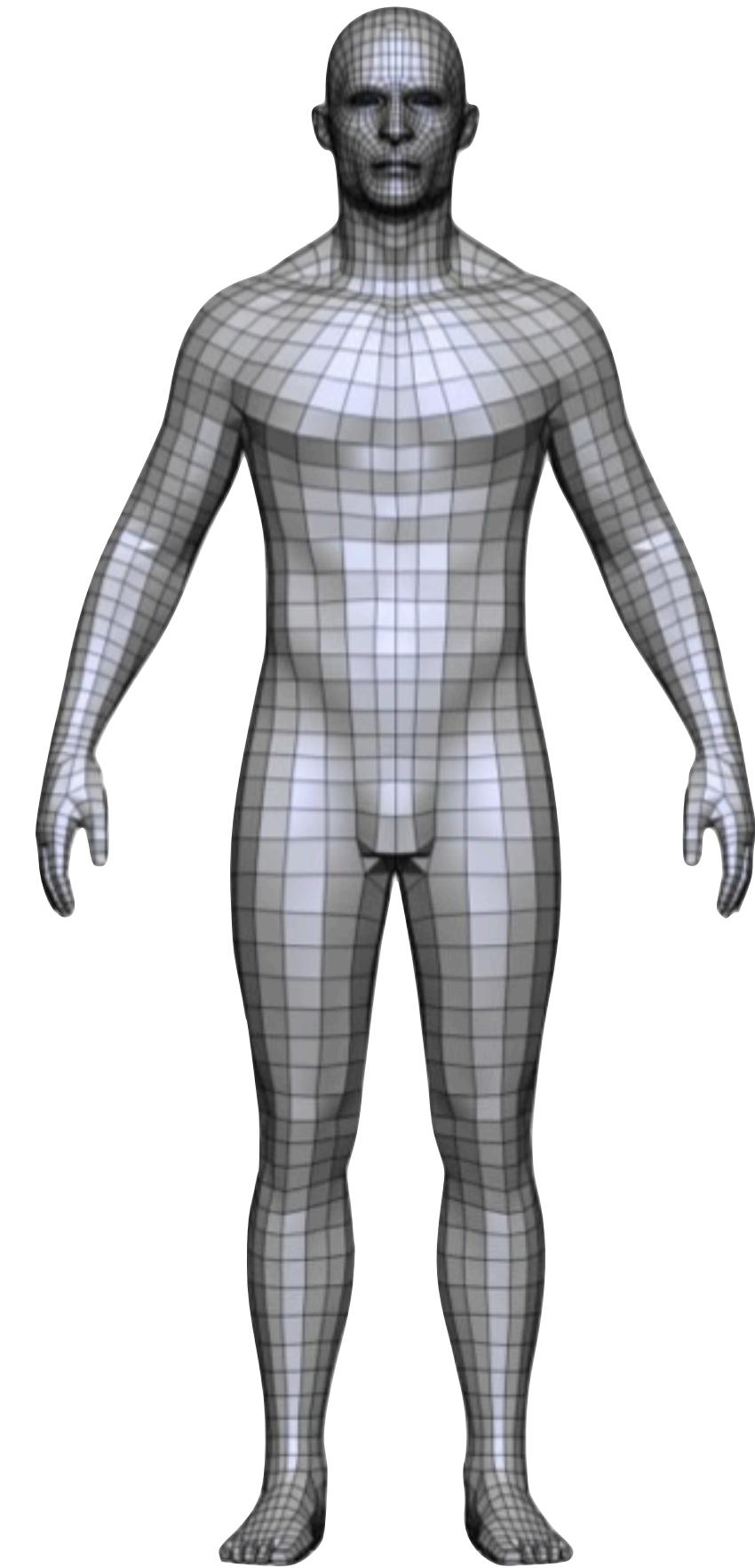
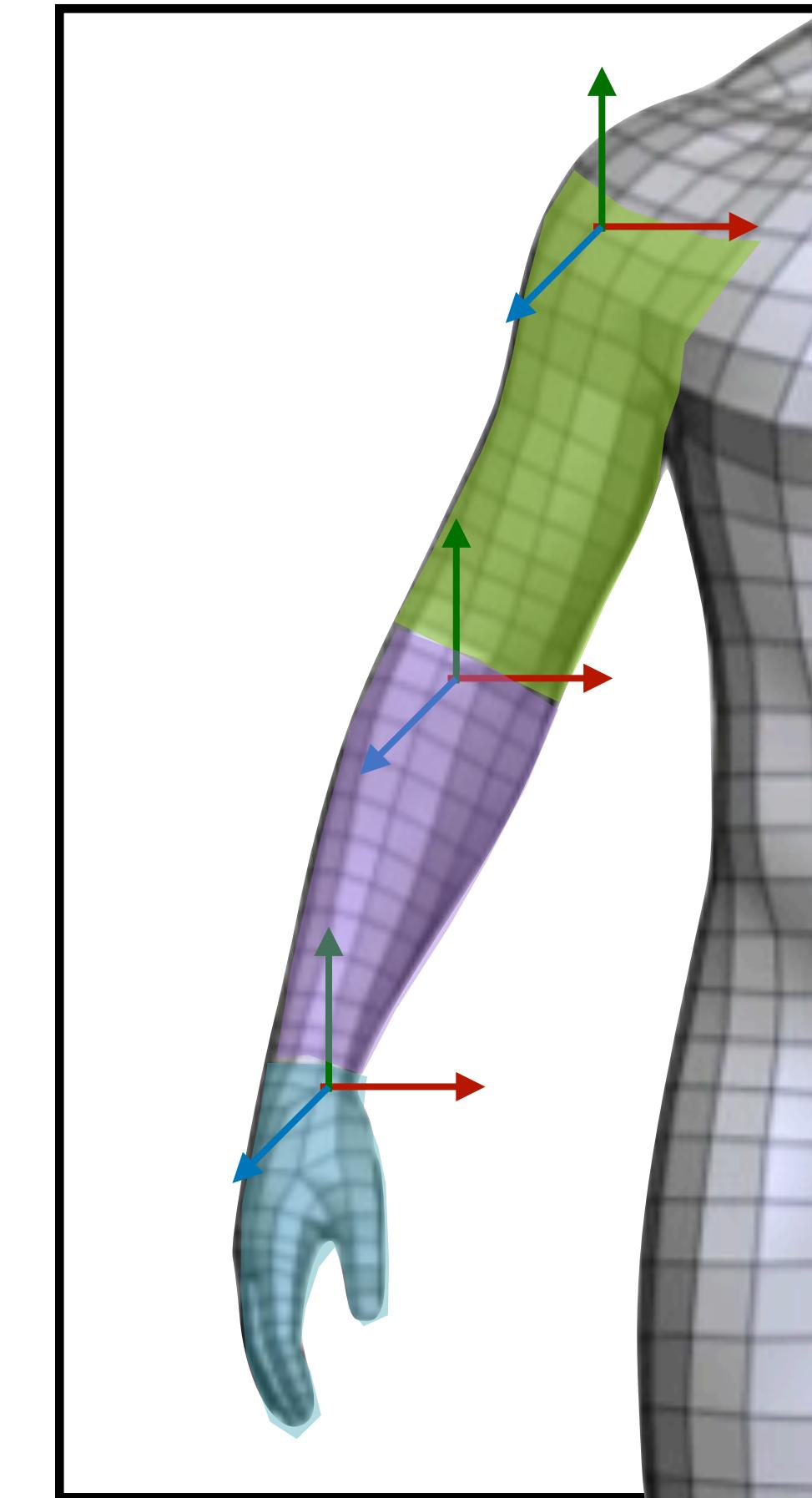
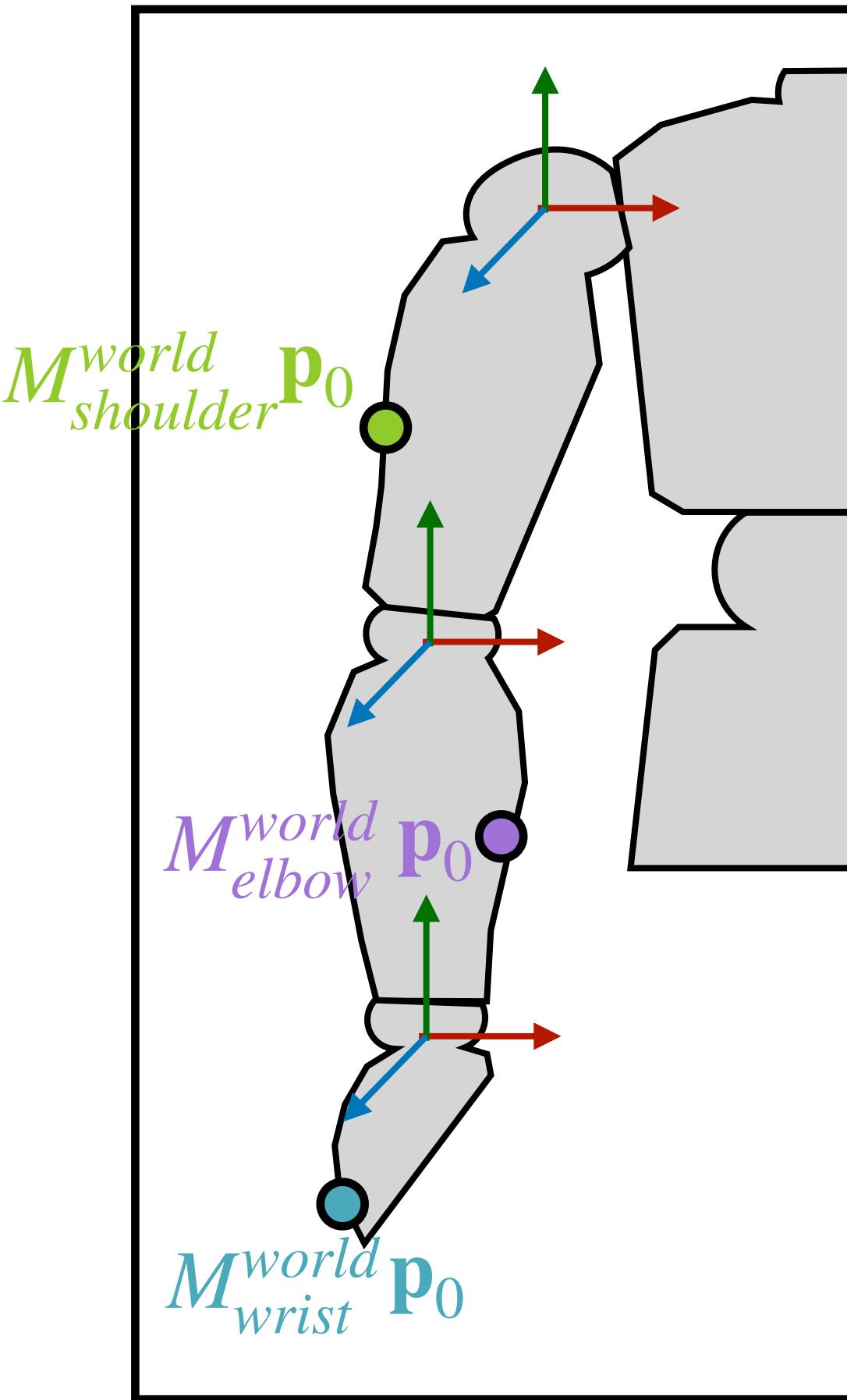
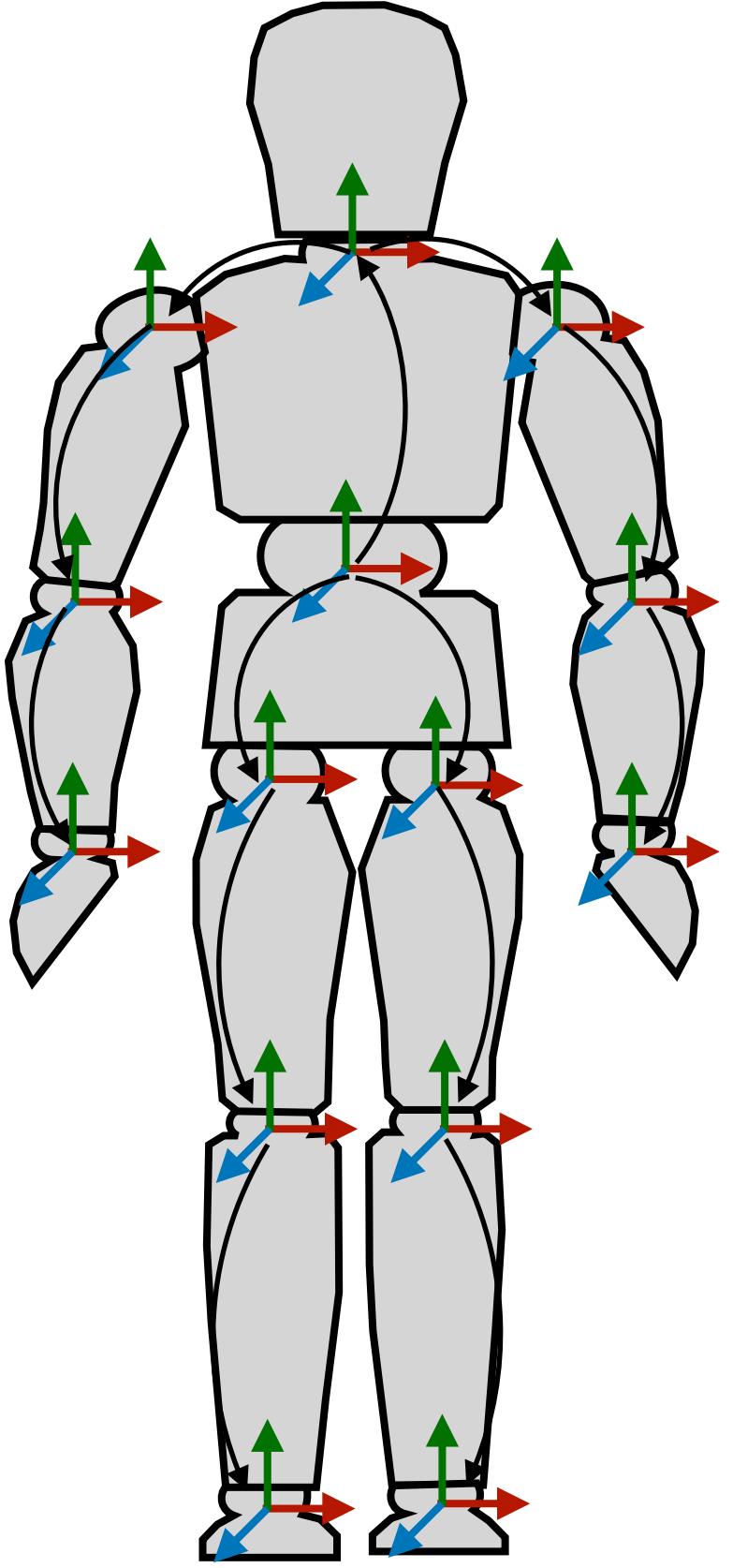
- Use a commercial motion capture system with approximately 450 markers.
- Still need to build an actor-specific surface model using rigid transformation plus residual deformation.
- Can model muscles budging, jiggling, stretching and high-frequency motion



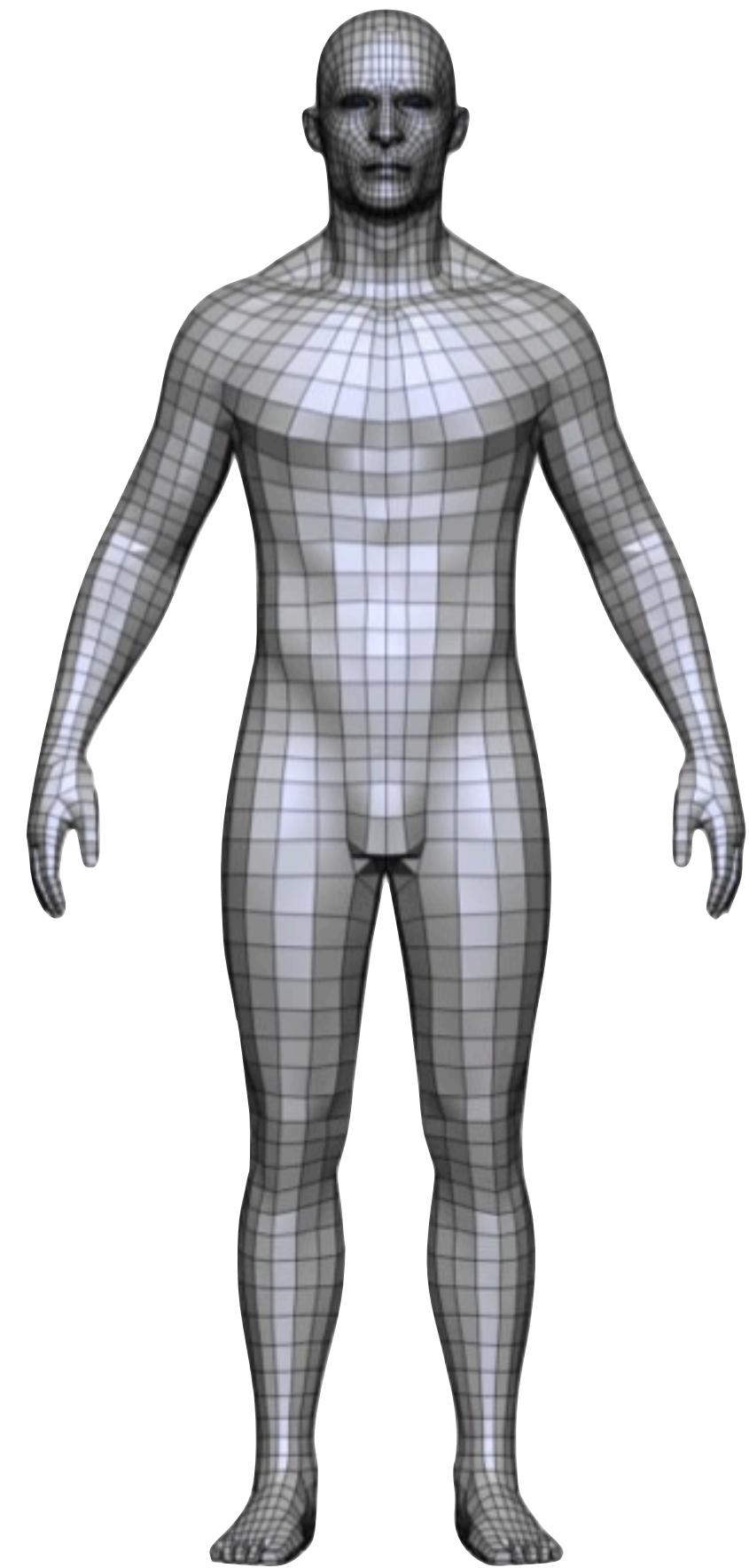
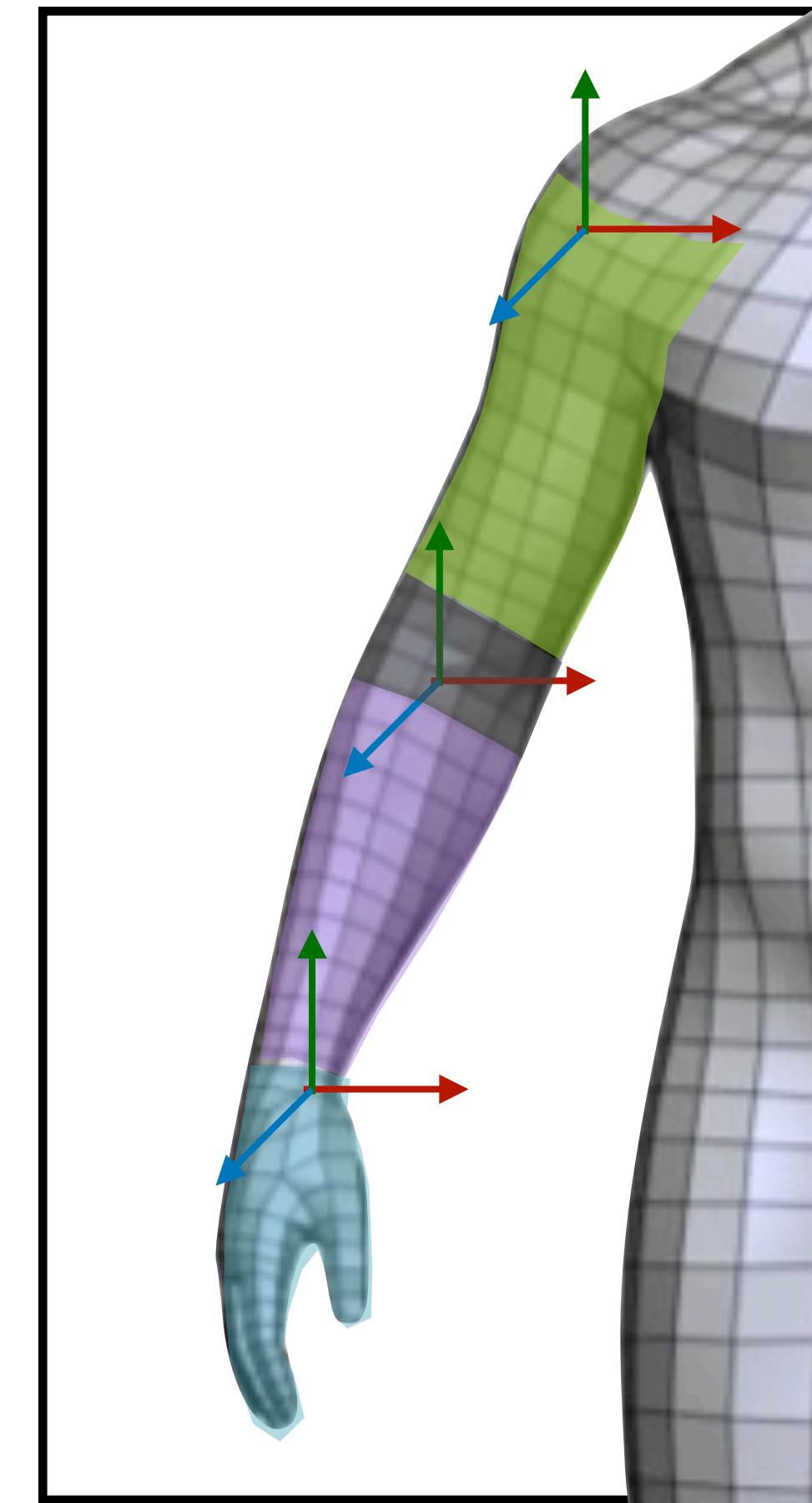
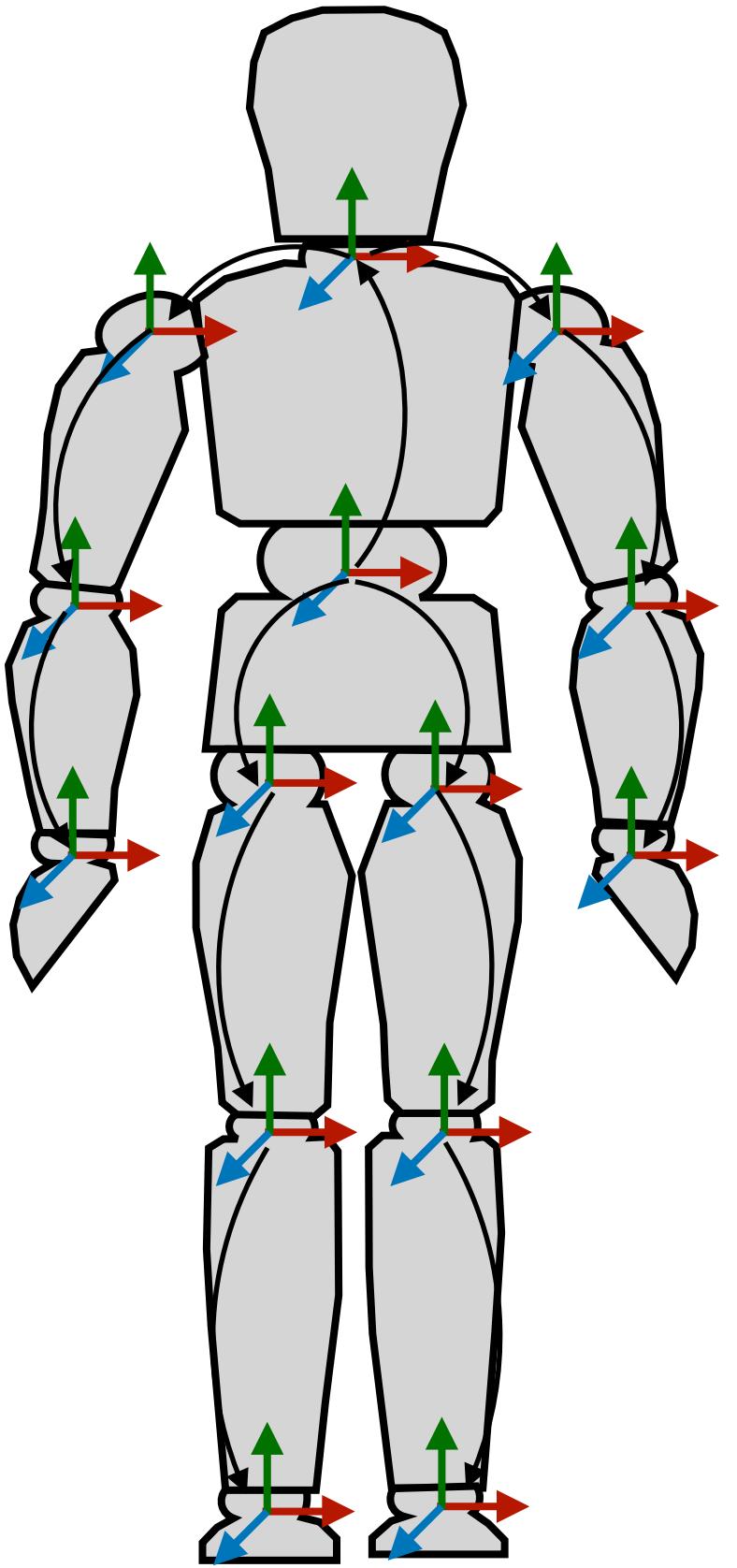
Park et al SIGGRAPH 2006



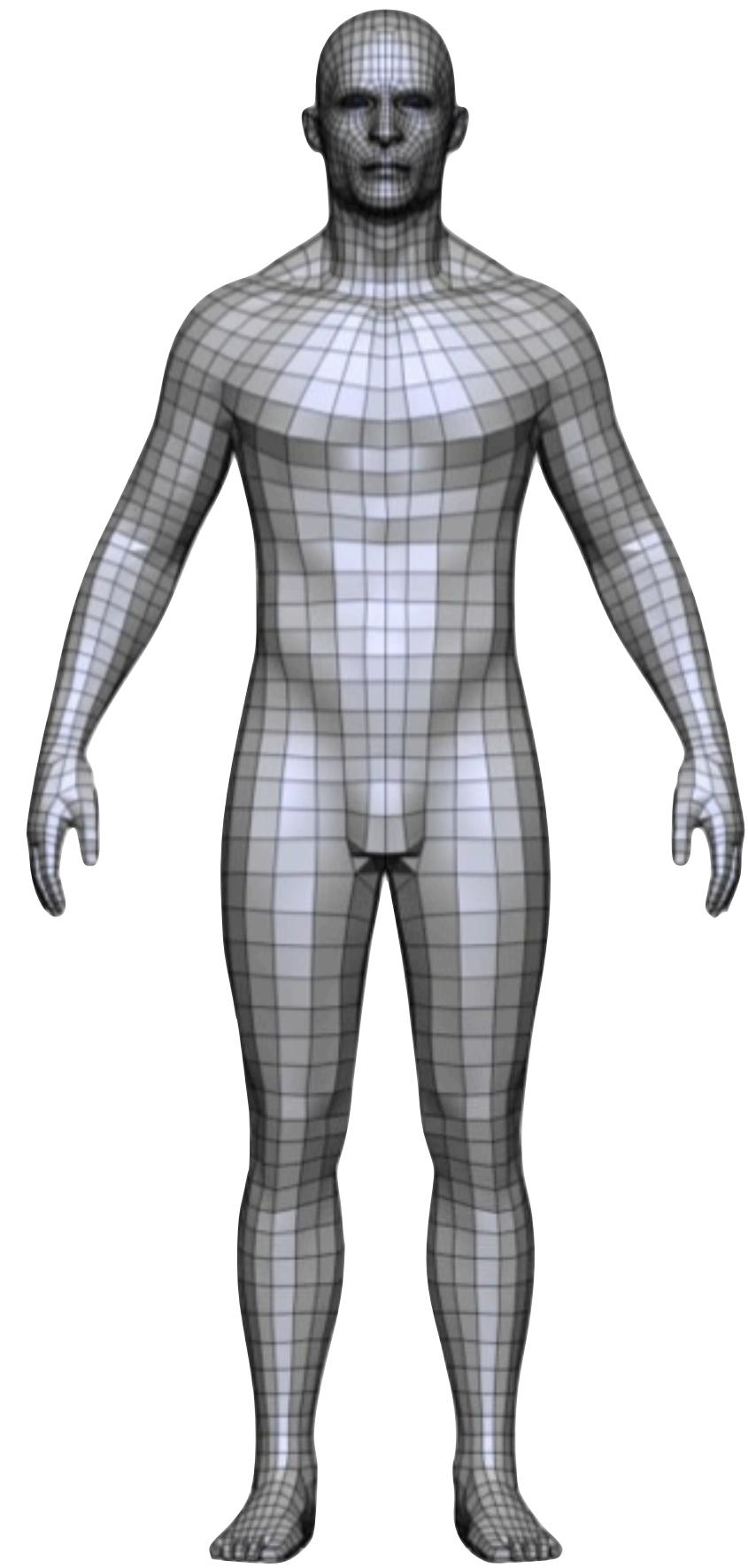
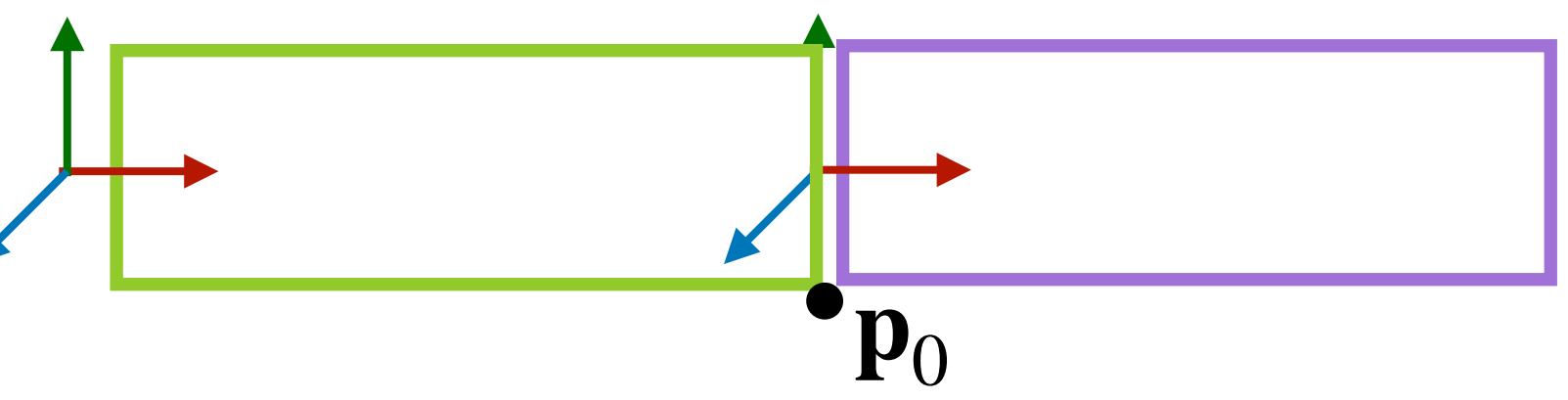
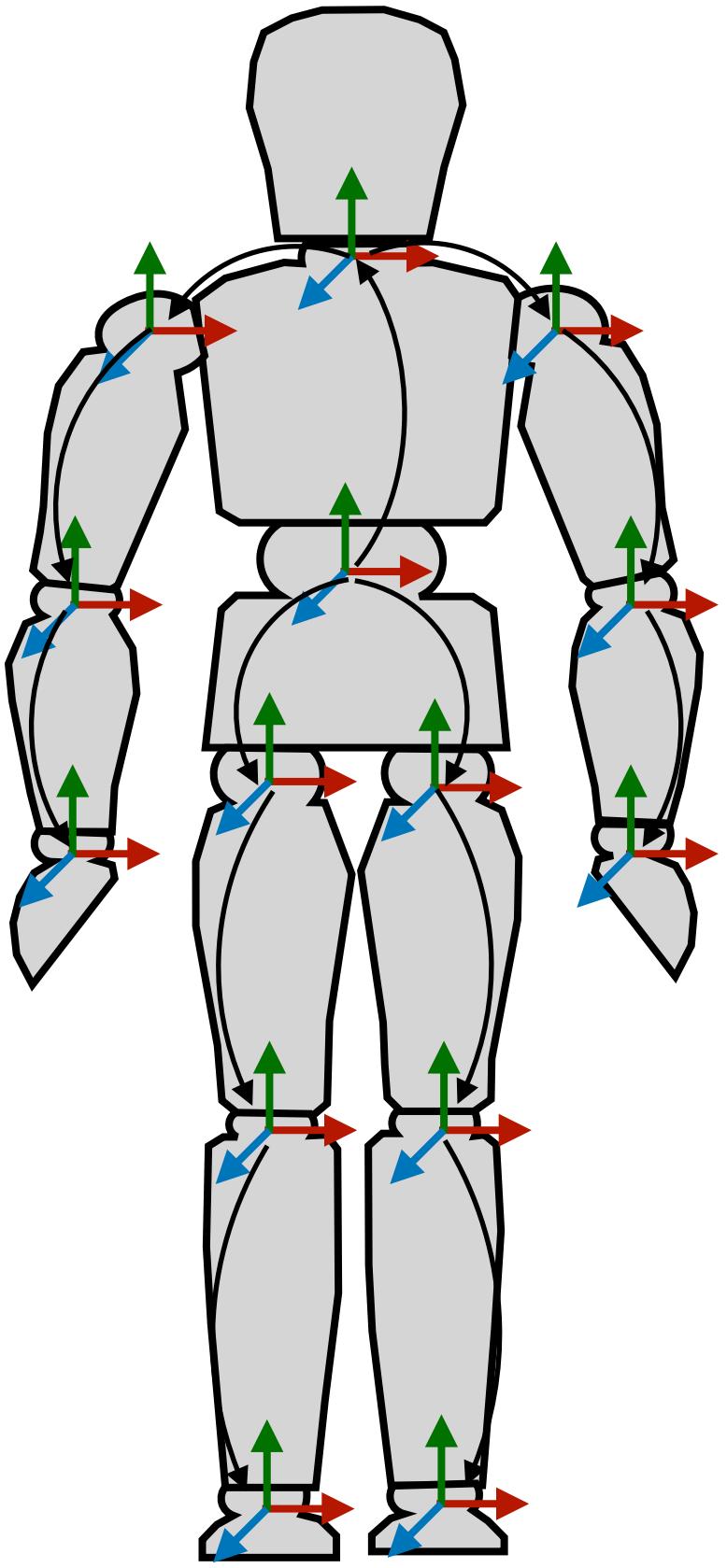
# Skinning approach



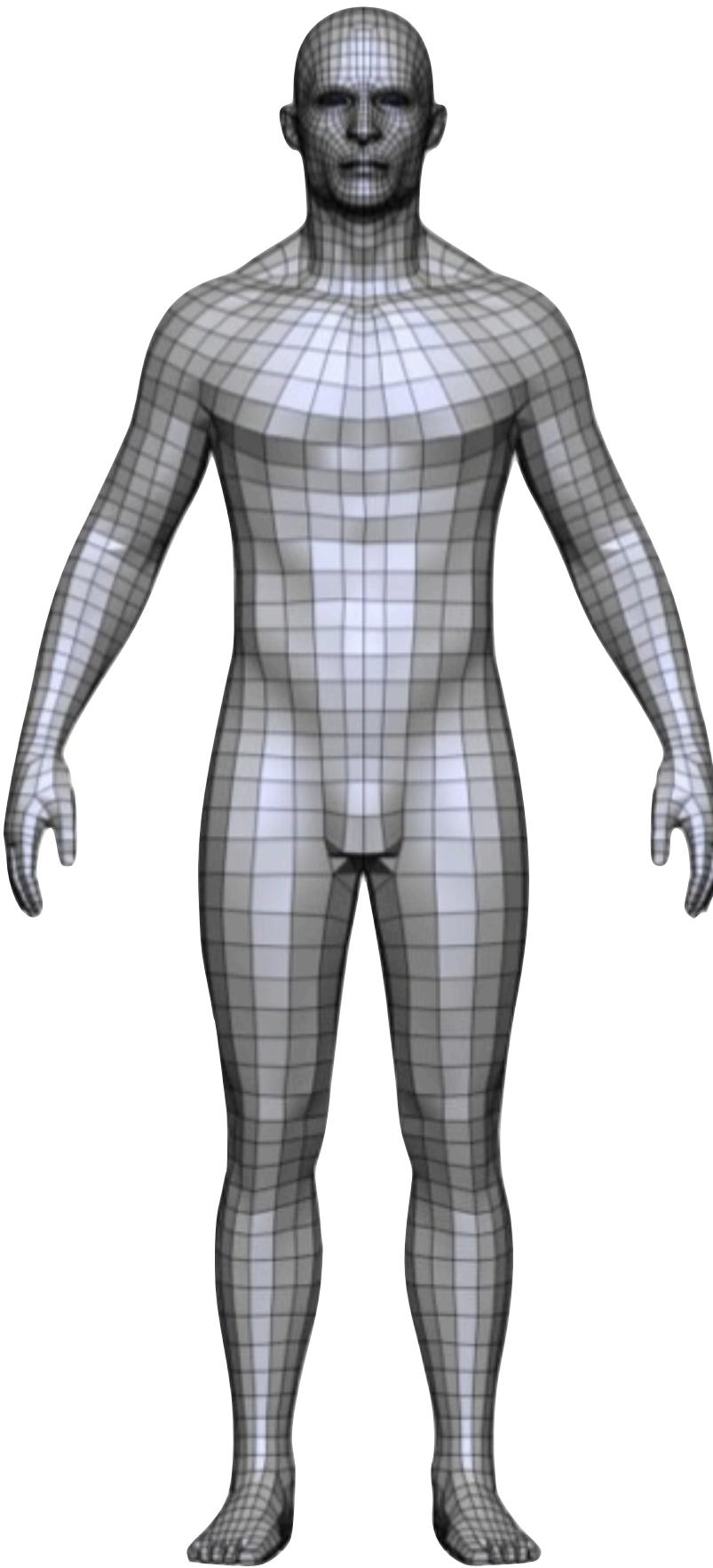
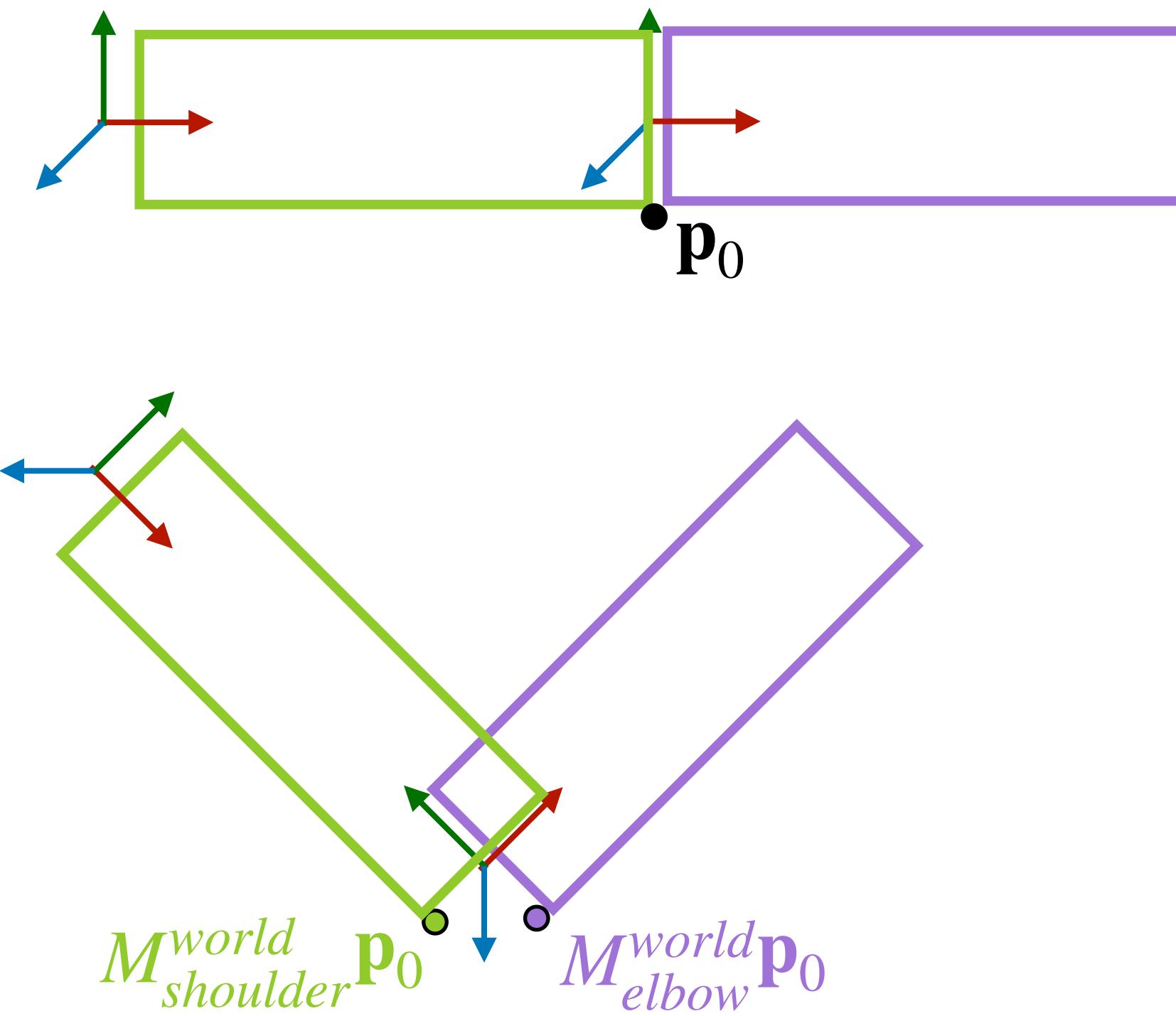
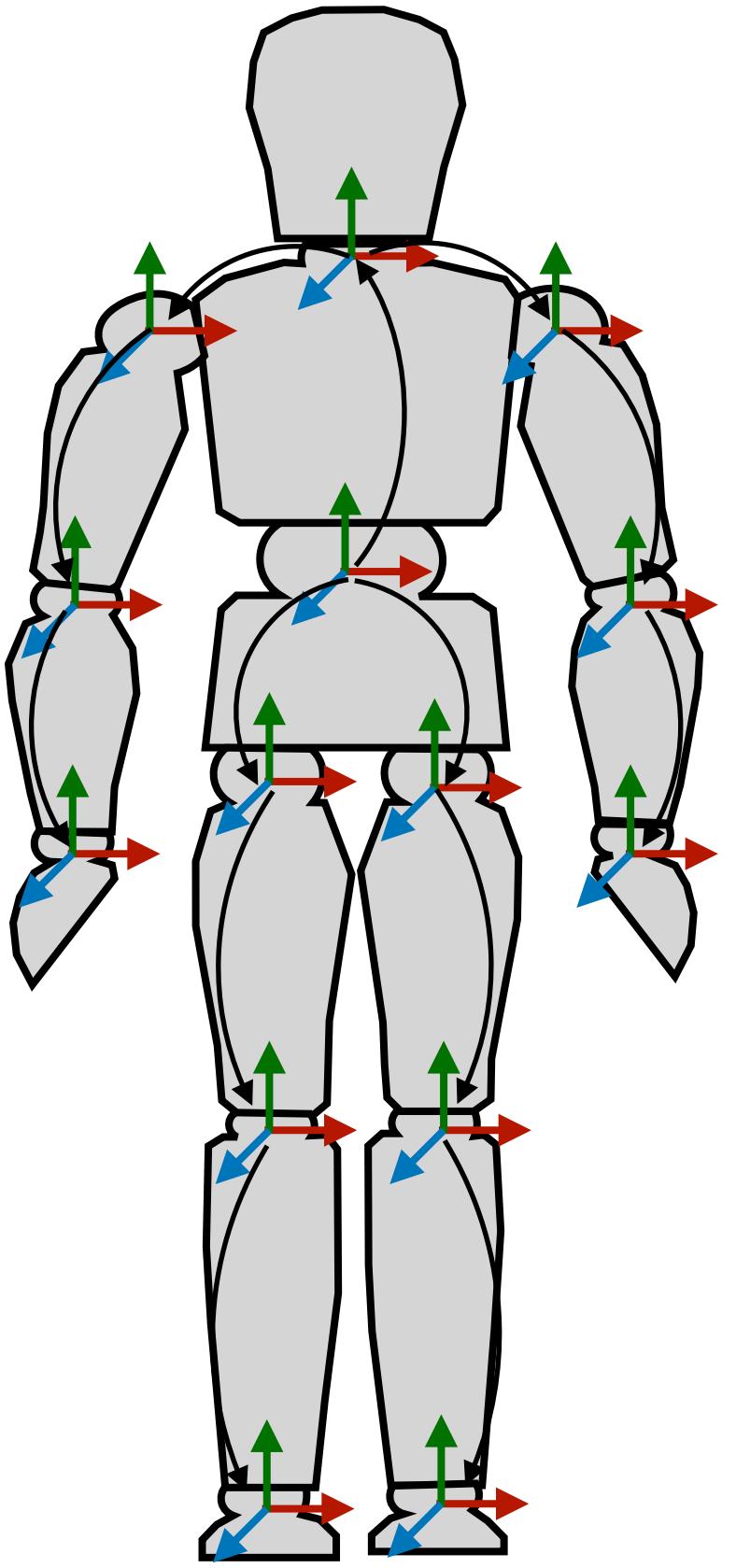
# Linear blend skinning



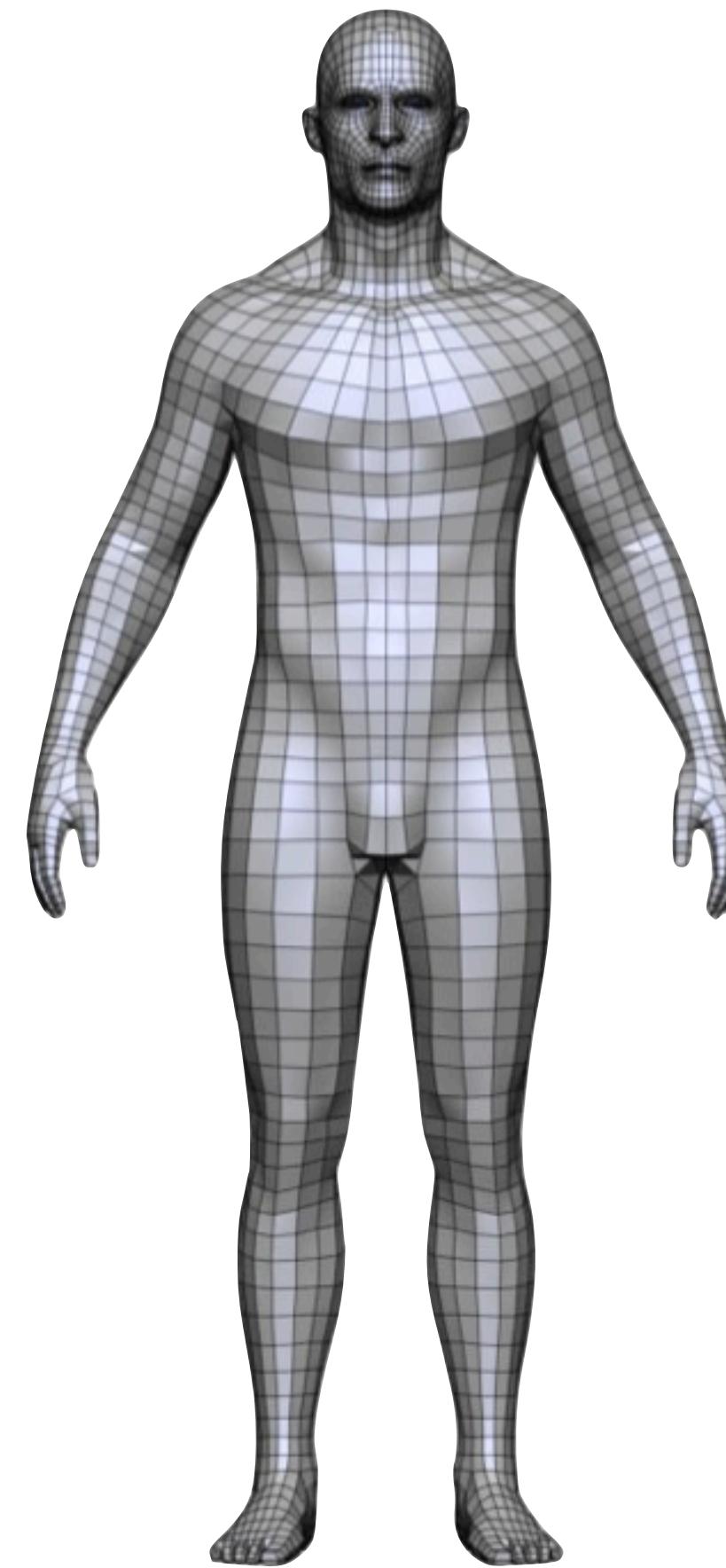
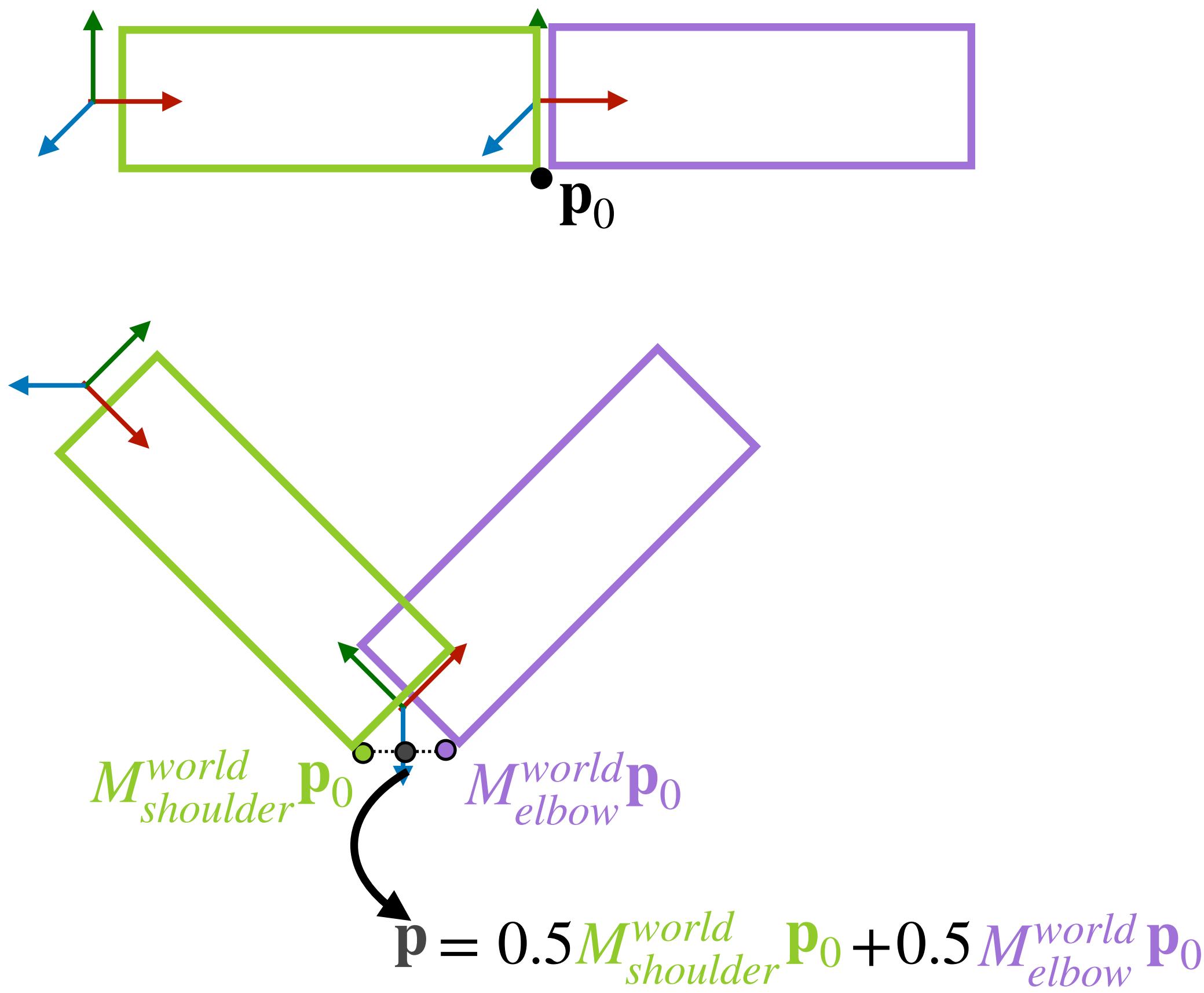
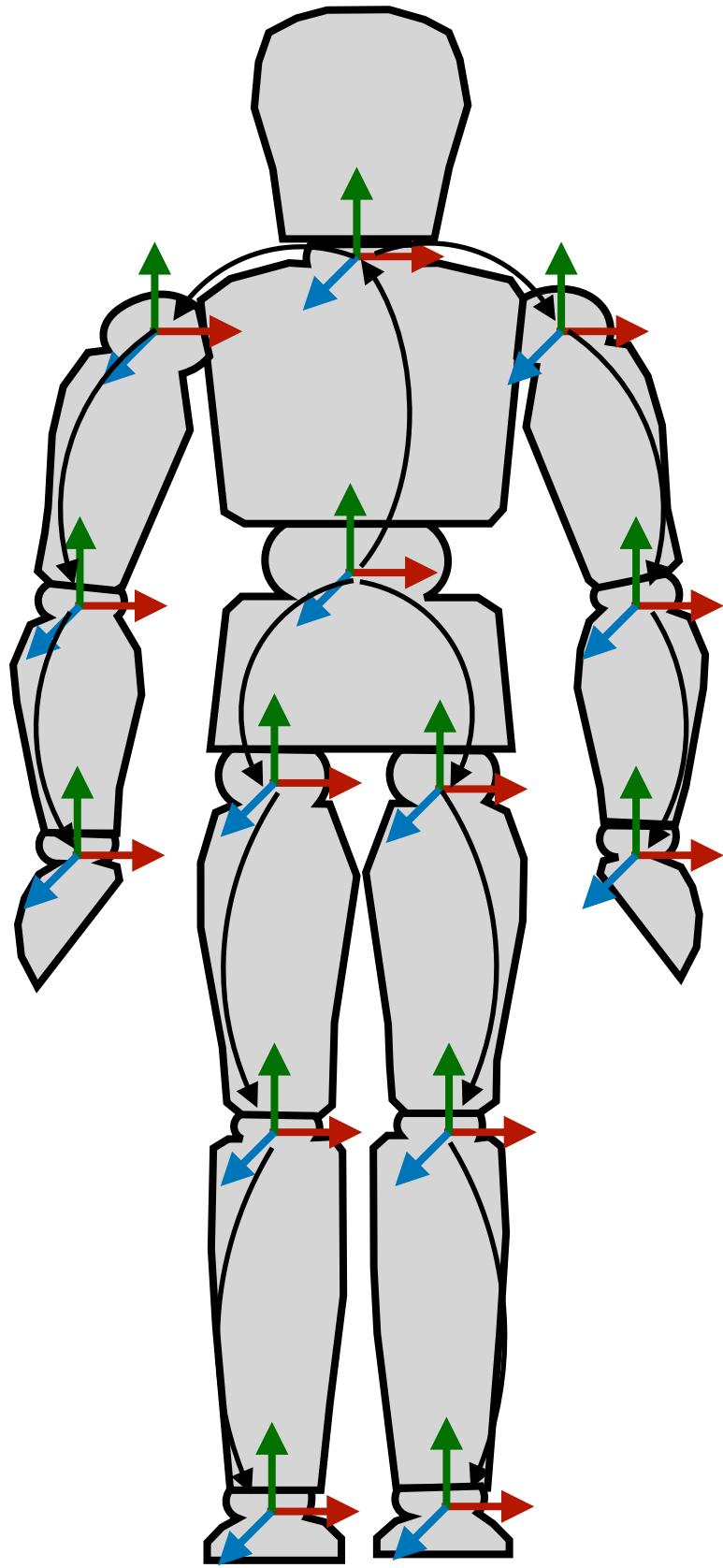
# Linear blend skinning



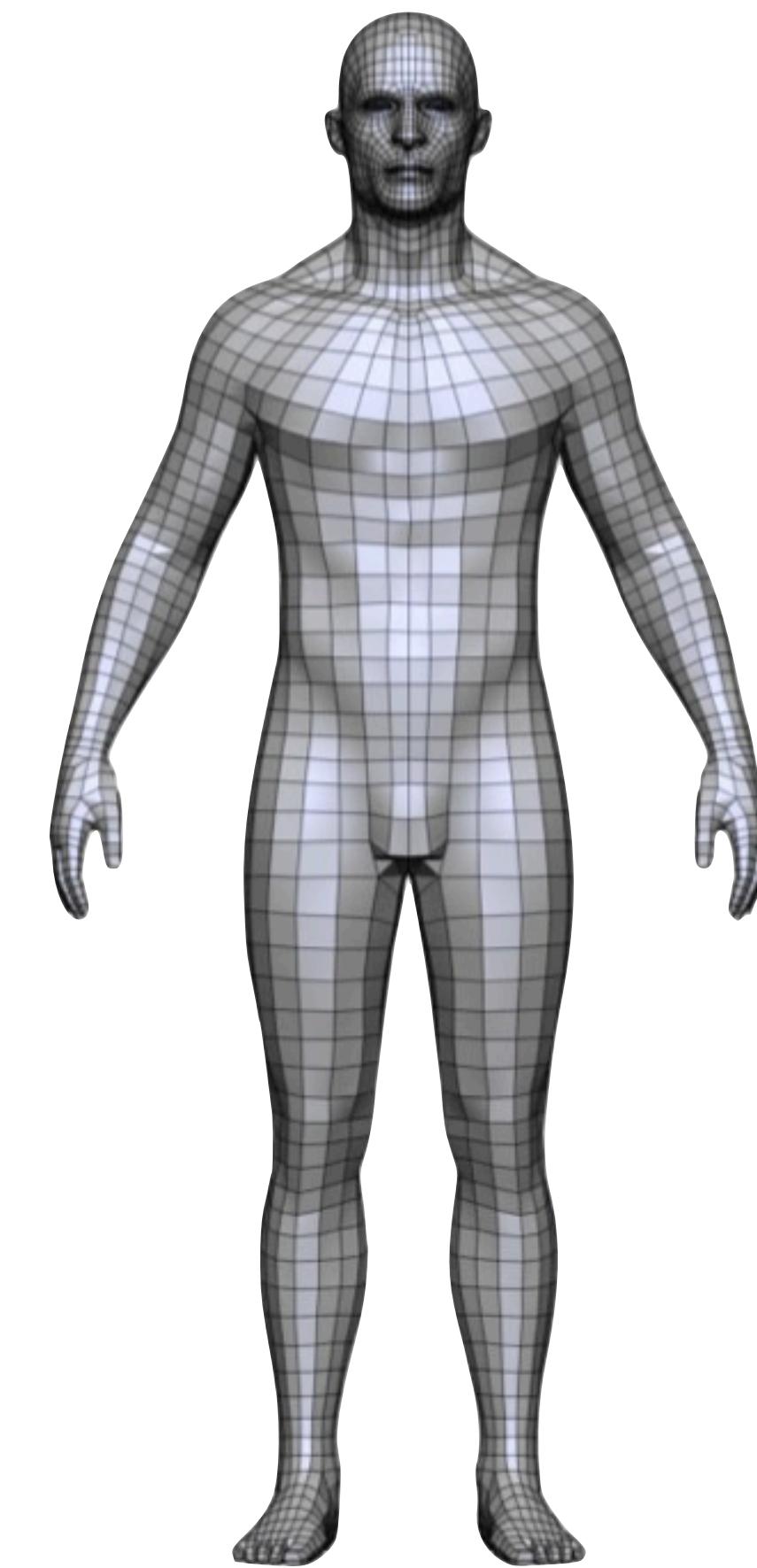
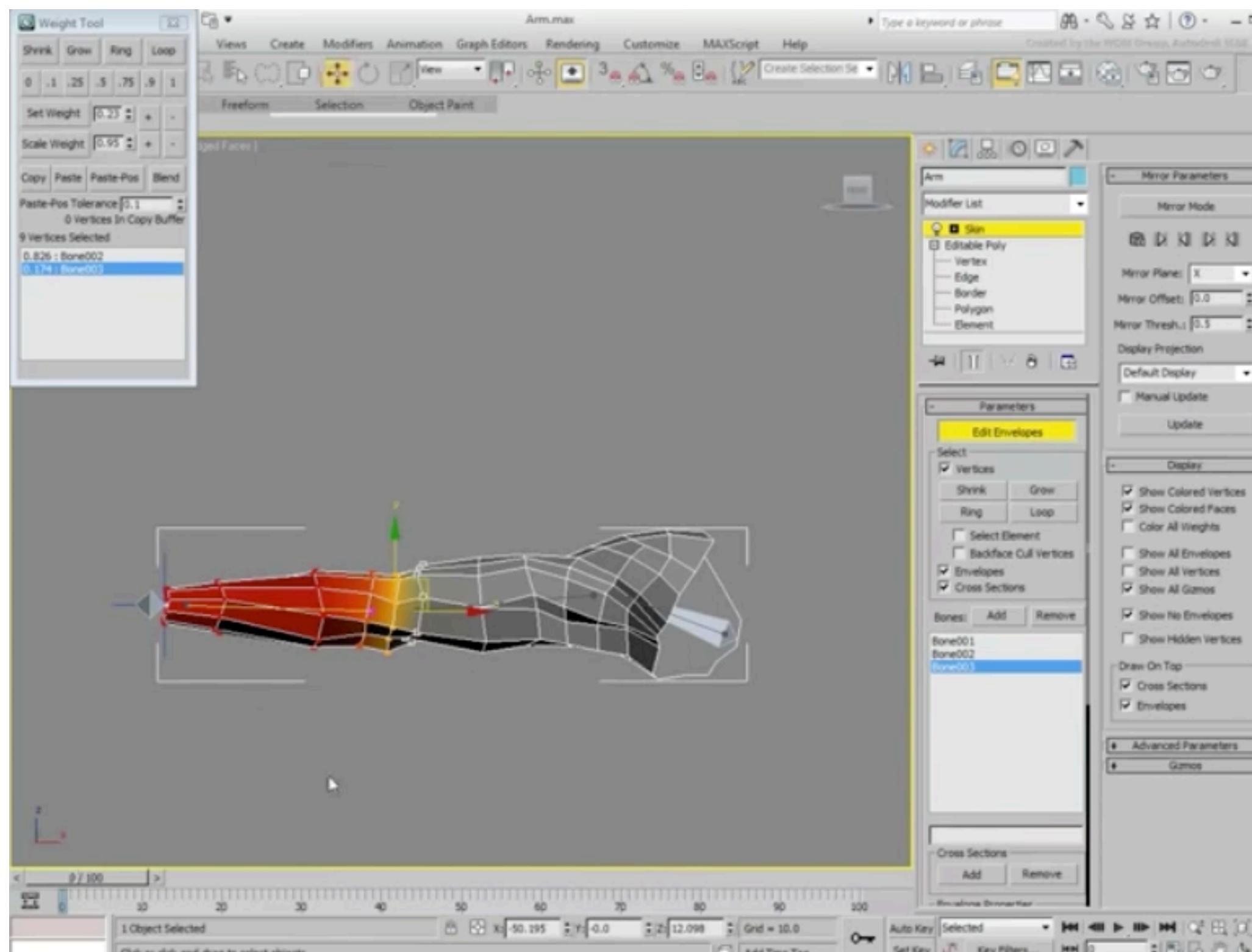
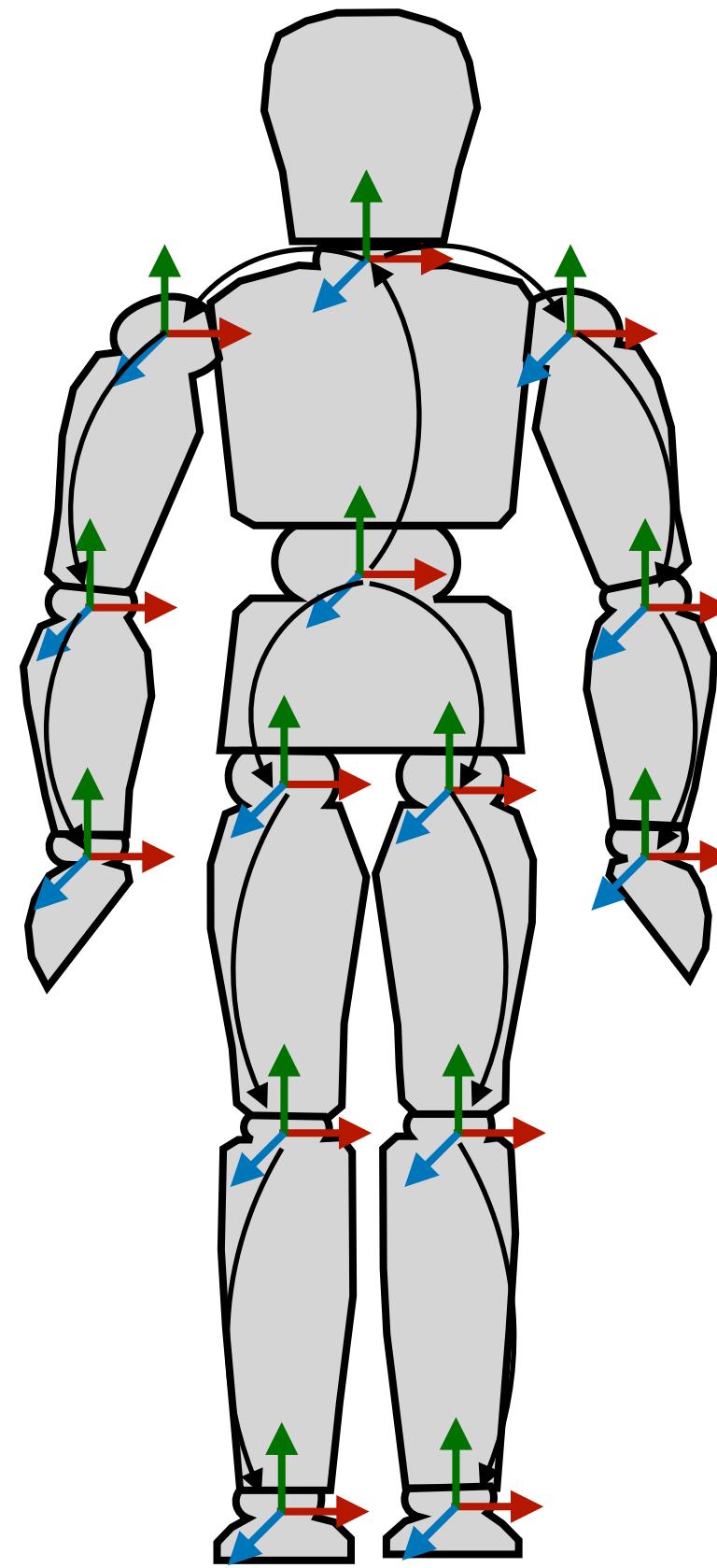
# Linear blend skinning



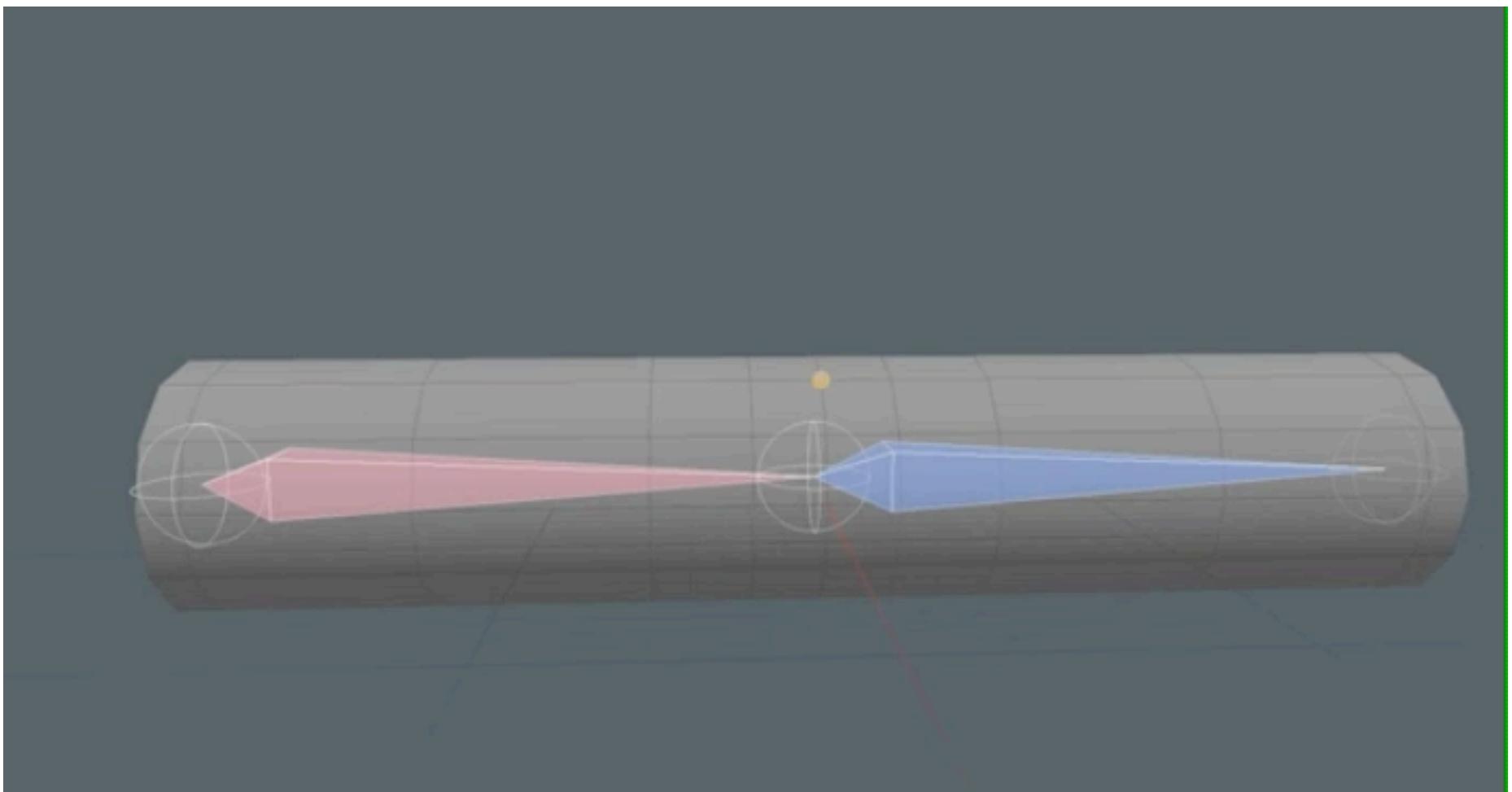
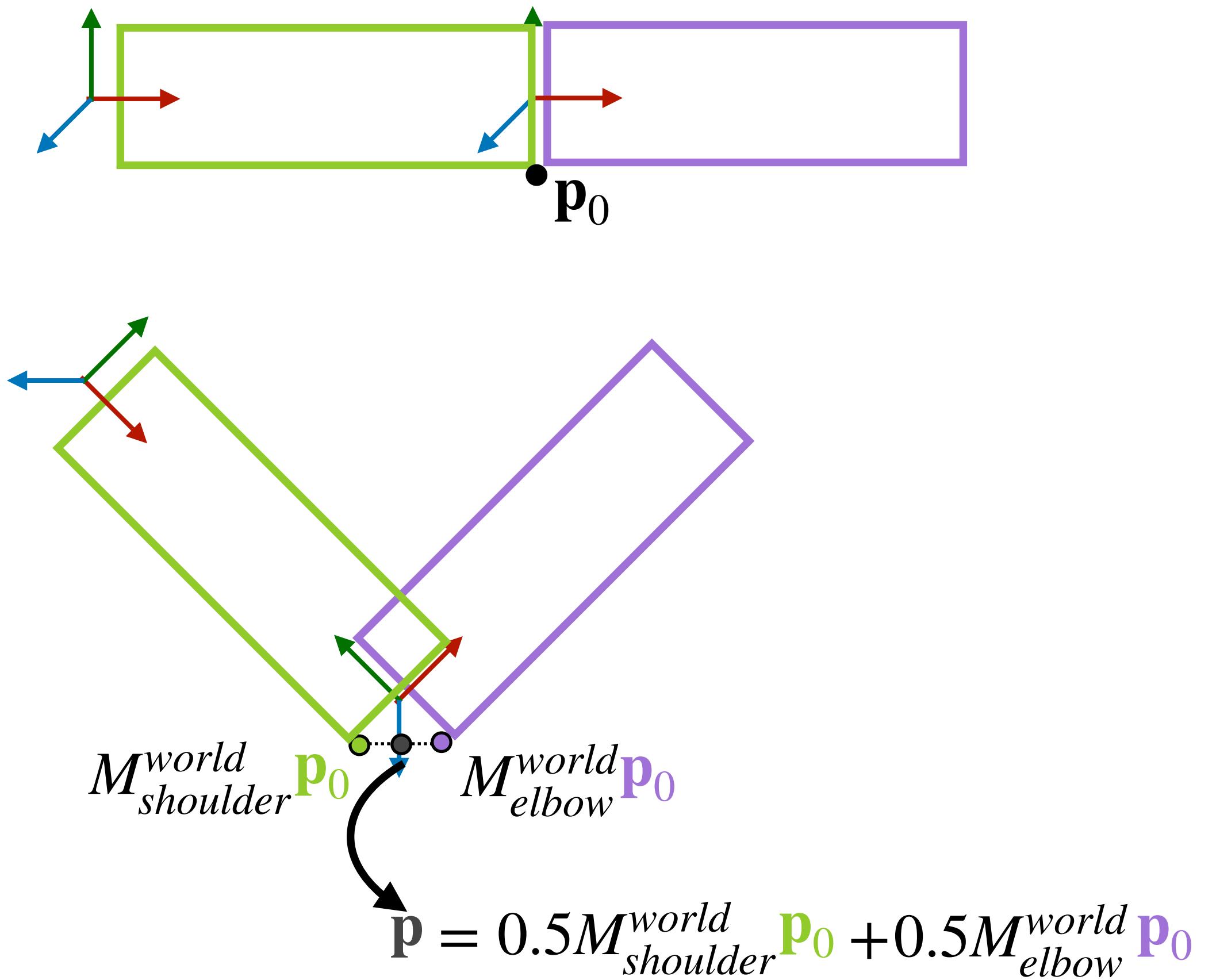
# Linear blend skinning



# Linear blend skinning

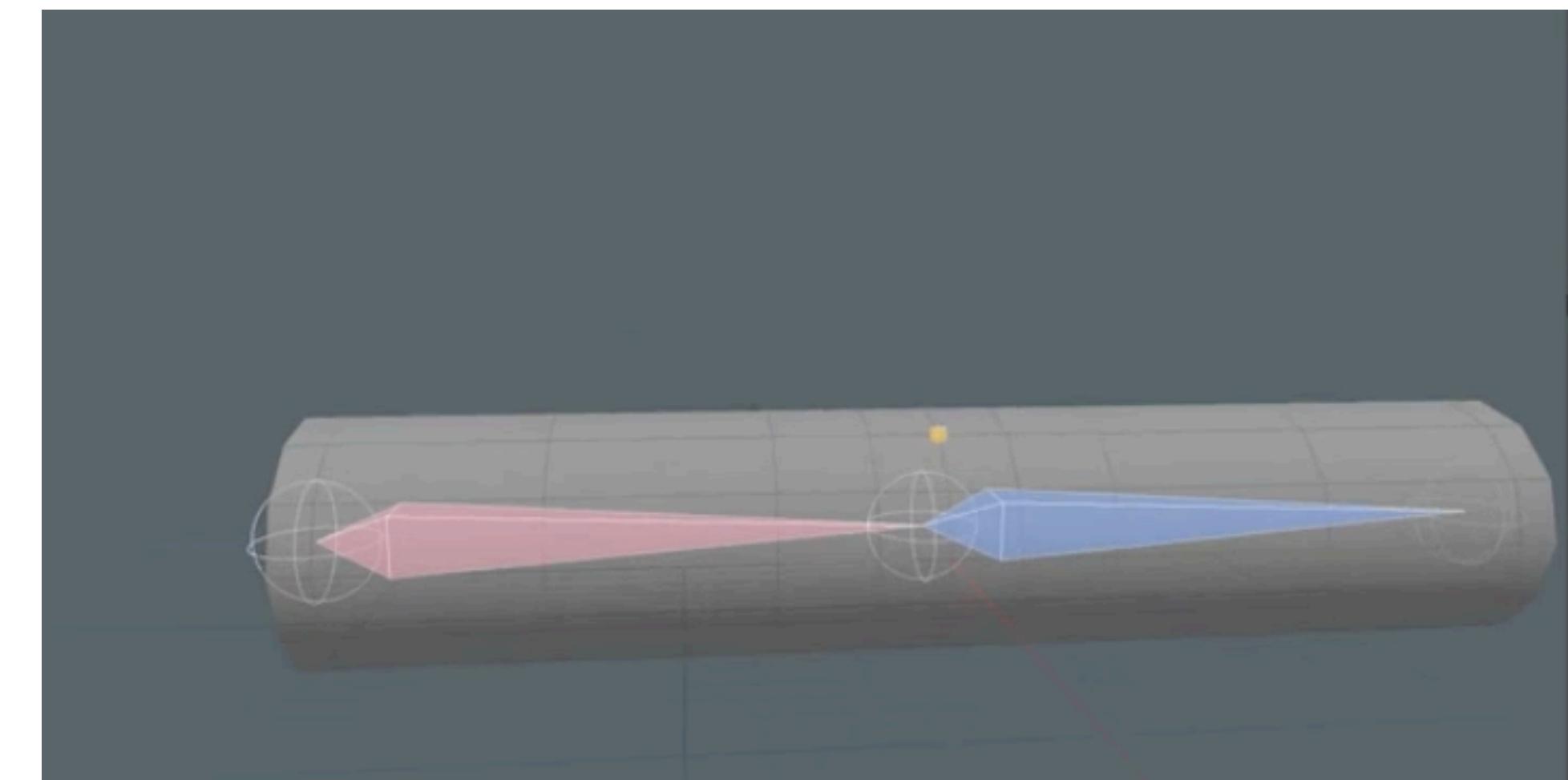
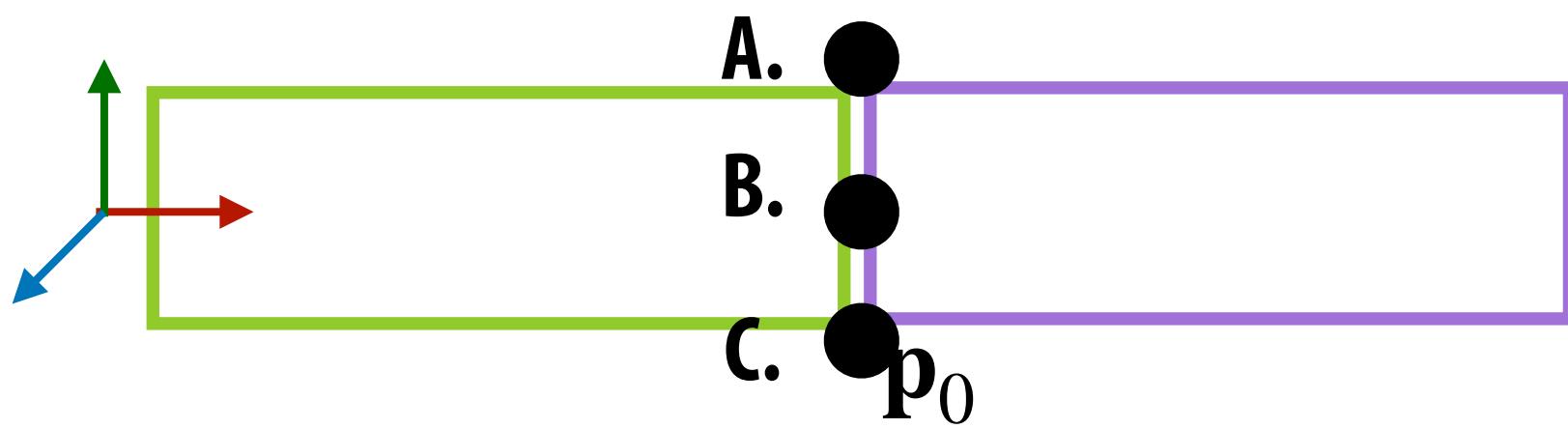
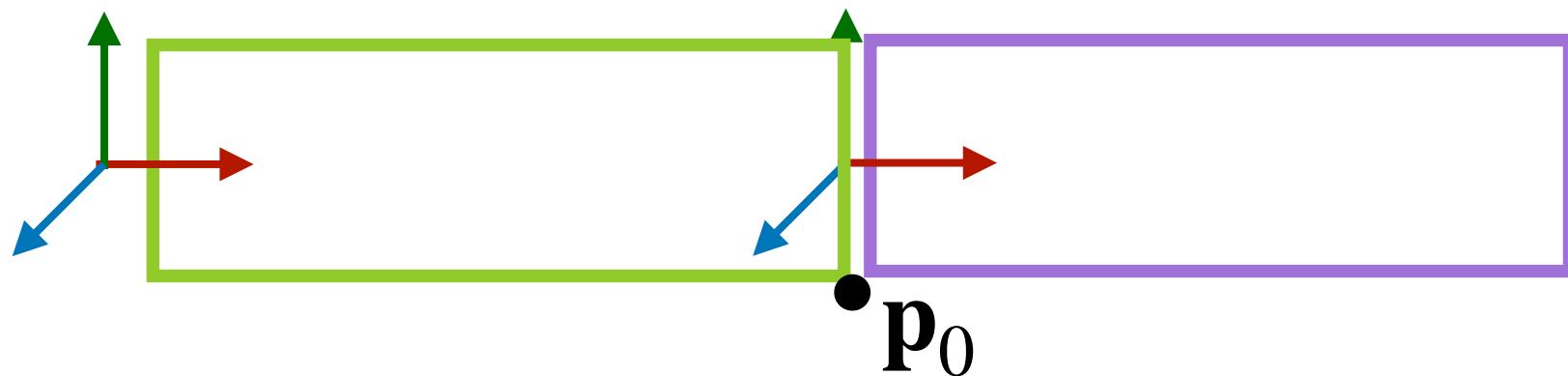


# What can go wrong?



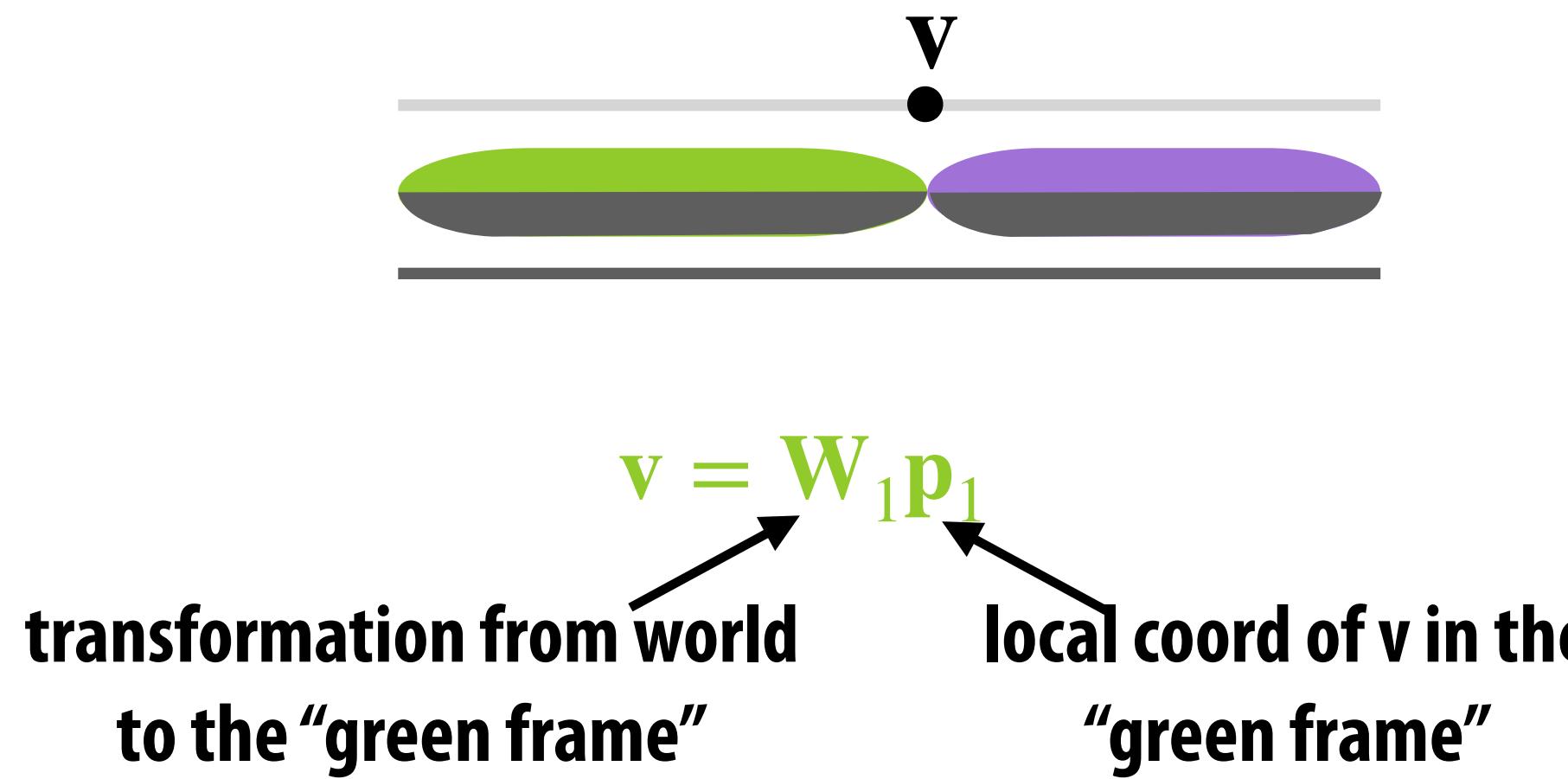
# Quiz

- If  $p_0$  is assigned to both green and purple bones with 50/50 weighting, where would  $p_0$  be if the elbow twists for 180-degree?



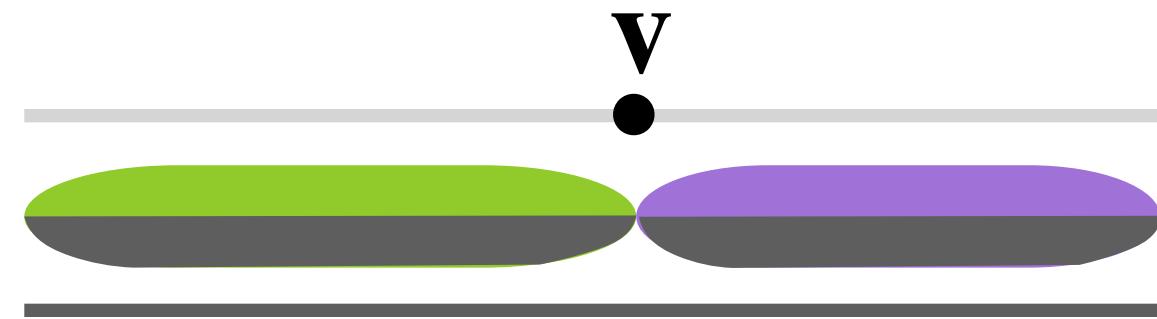
# Solution: Dual quaternion

- The problem is that we linearly interpolate rigid transformations



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- The problem is that we linearly interpolate rigid transformations

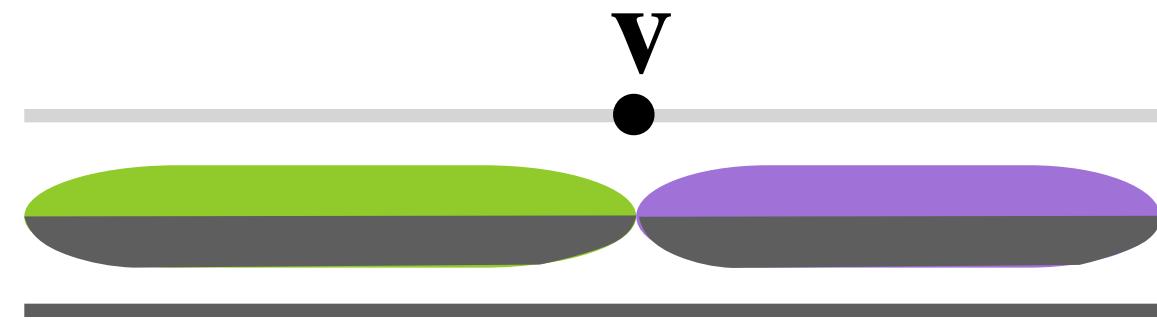


$$v = W_1 p_1 = I p_1$$

$$p_1 = v$$

# Solution: Dual quaternion

- The problem is that we linearly interpolate rigid transformations



$$v = W_1 p_1 = I p_1$$

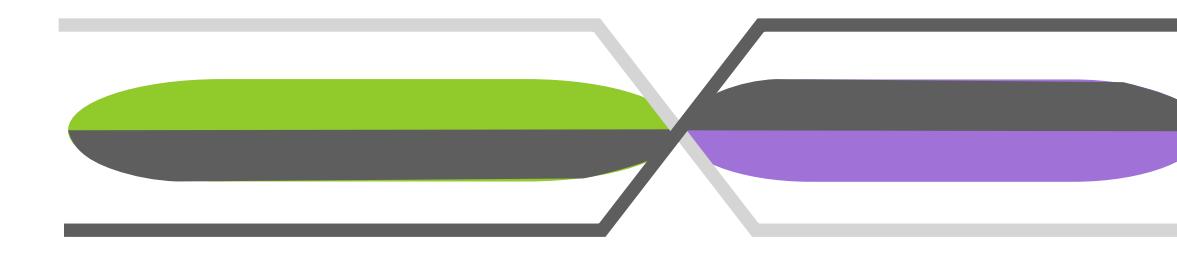
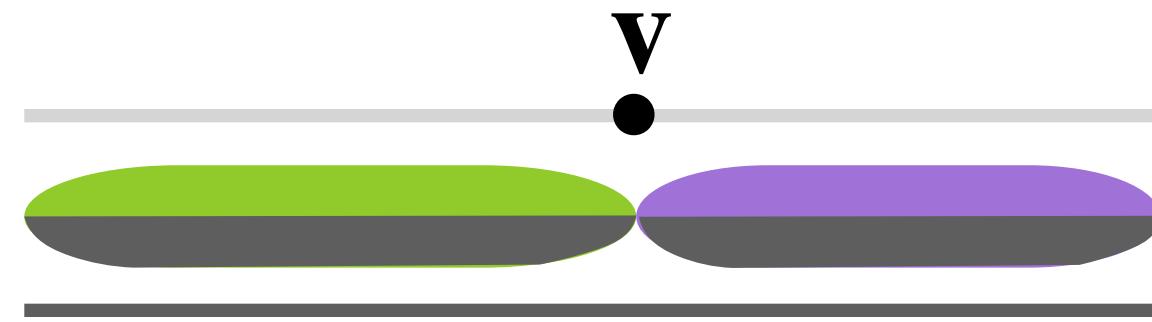
$$p_1 = v$$

$$v = W_2 p_2 = I T_{12} I p_2$$

$$p_2 = T_{12}^{-1} v$$

# Solution: Dual quaternion

- The problem is that we linearly interpolate rigid transformations



$$v = W_1 p_1 = I p_1$$

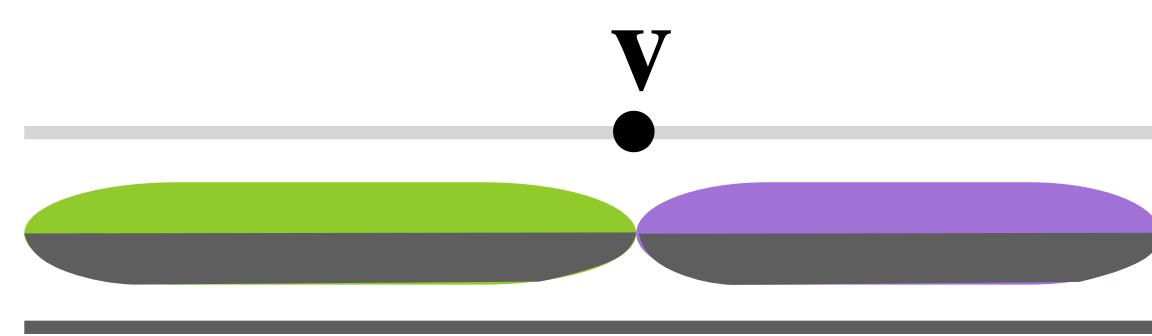
$$p_1 = v$$

$$v = W_2 p_2 = I T_{12} I p_2$$

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# Solution: Dual quaternion

- The problem is that we linearly interpolate rigid transformations

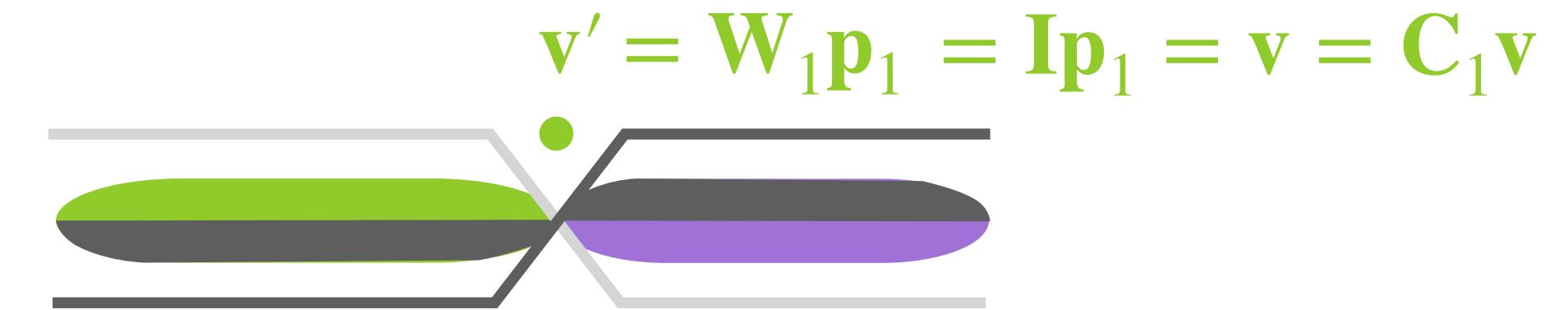


$$v = W_1 p_1 = I p_1$$

$$p_1 = v$$

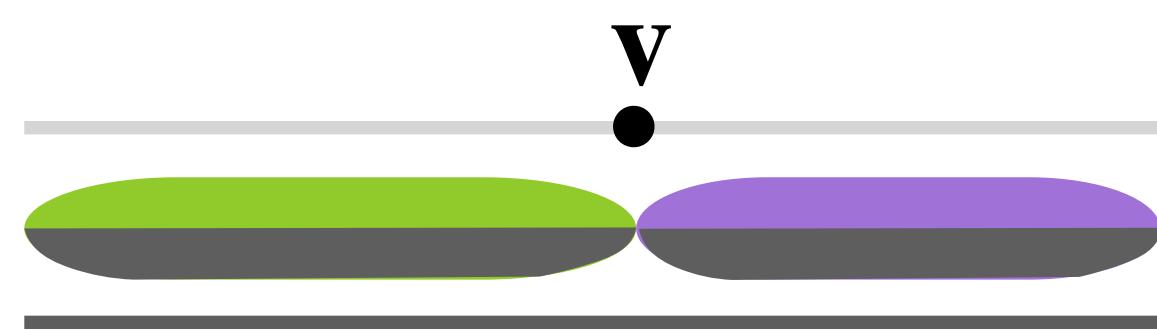
$$v = W_2 p_2 = I T_{12} I p_2$$

$$p_2 = T_{12}^{-1} v$$



# Solution: Dual quaternion

- The problem is that we linearly interpolate rigid transformations

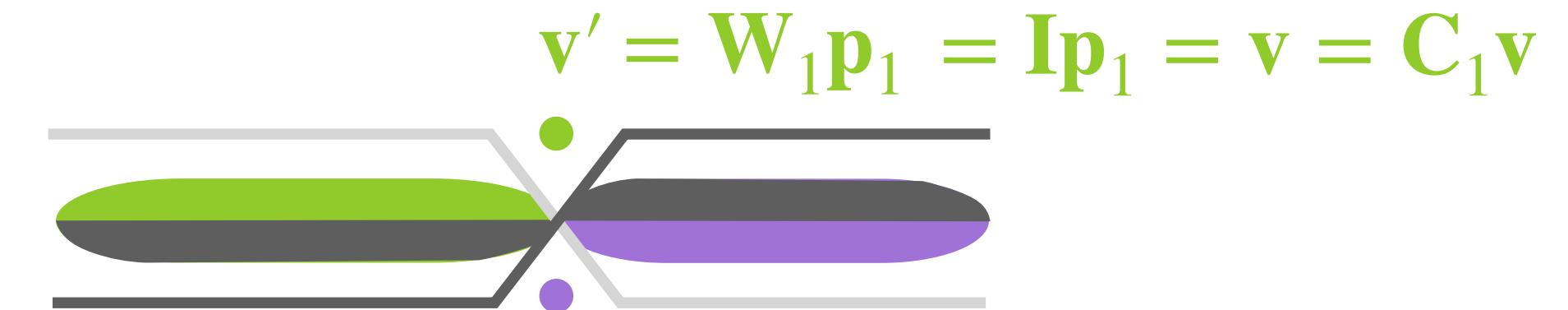


$$v = W_1 p_1 = I p_1$$

$$p_1 = v$$

$$v = W_2 p_2 = I T_{12} I p_2$$

$$p_2 = T_{12}^{-1} v$$



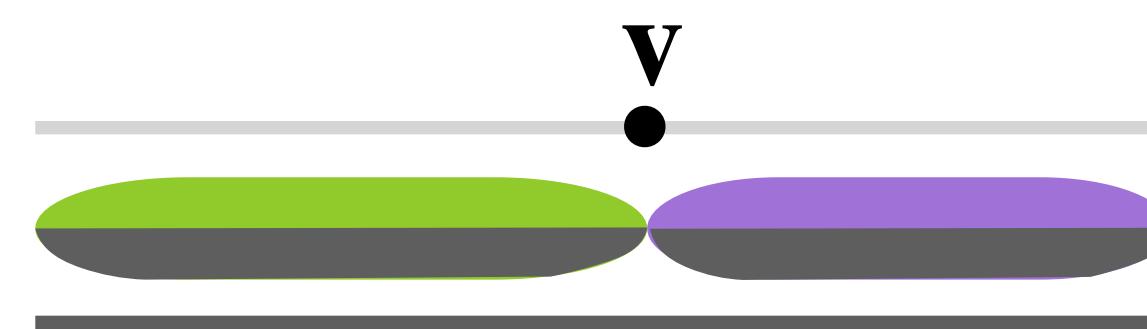
$$v' = W_1 p_1 = I p_1 = v = C_1 v$$

$$v' = W_2 p_2 = I T_{12} R(\pi) p_2$$

$$= T_{12} R(\pi) T_{12}^{-1} v = C_2 v$$

# Solution: Dual quaternion

- The problem is that we linearly interpolate rigid transformations

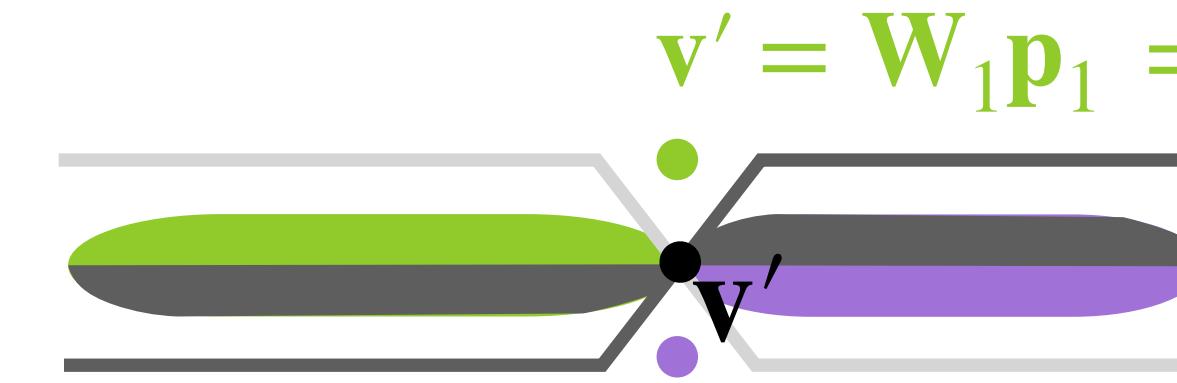


$$\mathbf{v} = \mathbf{W}_1 \mathbf{p}_1 = \mathbf{I} \mathbf{p}_1$$

$$\mathbf{p}_1 = \mathbf{v}$$

$$\mathbf{v} = \mathbf{W}_2 \mathbf{p}_2 = \mathbf{I} \mathbf{T}_{12} \mathbf{I} \mathbf{p}_2$$

$$\mathbf{p}_2 = \mathbf{T}_{12}^{-1} \mathbf{v}$$



$$\mathbf{v}' = \mathbf{W}_1 \mathbf{p}_1 = \mathbf{I} \mathbf{p}_1 = \mathbf{v} = \mathbf{C}_1 \mathbf{v}$$

$$\mathbf{v}' = \mathbf{W}_2 \mathbf{p}_2 = \mathbf{I} \mathbf{T}_{12} \mathbf{R}(\pi) \mathbf{p}_2$$

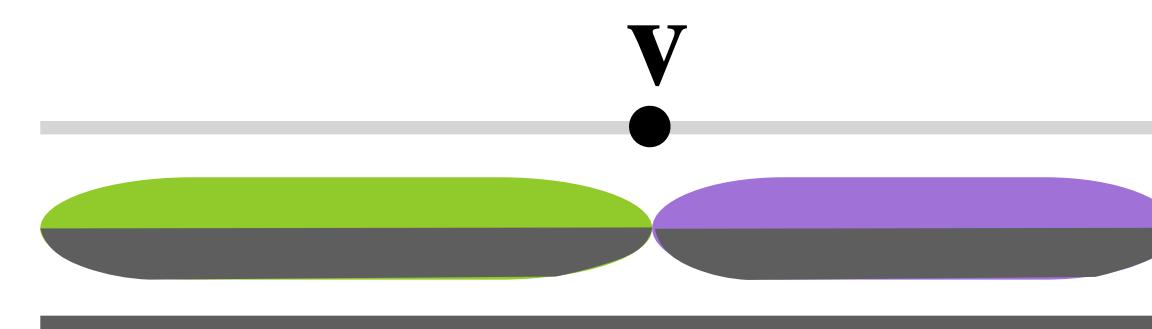
$$= \mathbf{T}_{12} \mathbf{R}(\pi) \mathbf{T}_{12}^{-1} \mathbf{v} = \mathbf{C}_2 \mathbf{v}$$

$$\mathbf{v}' = 0.5 \mathbf{C}_1 \mathbf{v} + 0.5 \mathbf{C}_2 \mathbf{v} = (0.5 \mathbf{C}_1 + 0.5 \mathbf{C}_2) \mathbf{v}$$

Bad idea: linear interpolating two matrices!

# Solution: Dual quaternion

- The problem is that we linearly interpolate rigid transformations



$$\mathbf{v} = \mathbf{W}_1 \mathbf{p}_1 = \mathbf{I} \mathbf{p}_1$$

$$\mathbf{p}_1 = \mathbf{v}$$

$$\mathbf{v} = \mathbf{W}_2 \mathbf{p}_2 = \mathbf{I} \mathbf{T}_{12} \mathbf{I} \mathbf{p}_2$$

$$\mathbf{p}_2 = \mathbf{T}_{12}^{-1} \mathbf{v}$$



$$\mathbf{v}' = \mathbf{W}_2 \mathbf{p}_2 = \mathbf{I} \mathbf{T}_{12} \mathbf{I} \mathbf{p}_2$$

$$= \mathbf{T}_{12} \mathbf{R}(\pi) \mathbf{T}_{12}^{-1} \mathbf{v} = \mathbf{C}_2 \mathbf{v}$$

$$\mathbf{v}' = 0.5 \mathbf{C}_1 \mathbf{v} + 0.5 \mathbf{C}_2 \mathbf{v} = (0.5 \mathbf{C}_1 + 0.5 \mathbf{C}_2) \mathbf{v}$$

Bad idea: linear interpolating two matrices!

- We need a better interpolation method that properly blends centers of rotation and the 3D rotations

# Solution: Dual quaternion

- Dual quaternion definition:

$$\hat{\mathbf{q}} = \begin{bmatrix} \hat{w} \\ \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} = \begin{bmatrix} w_0 + \epsilon w_\epsilon \\ x_0 + \epsilon x_\epsilon \\ y_0 + \epsilon y_\epsilon \\ z_0 + \epsilon z_\epsilon \end{bmatrix} = \mathbf{q}_0 + \epsilon \mathbf{q}_\epsilon$$

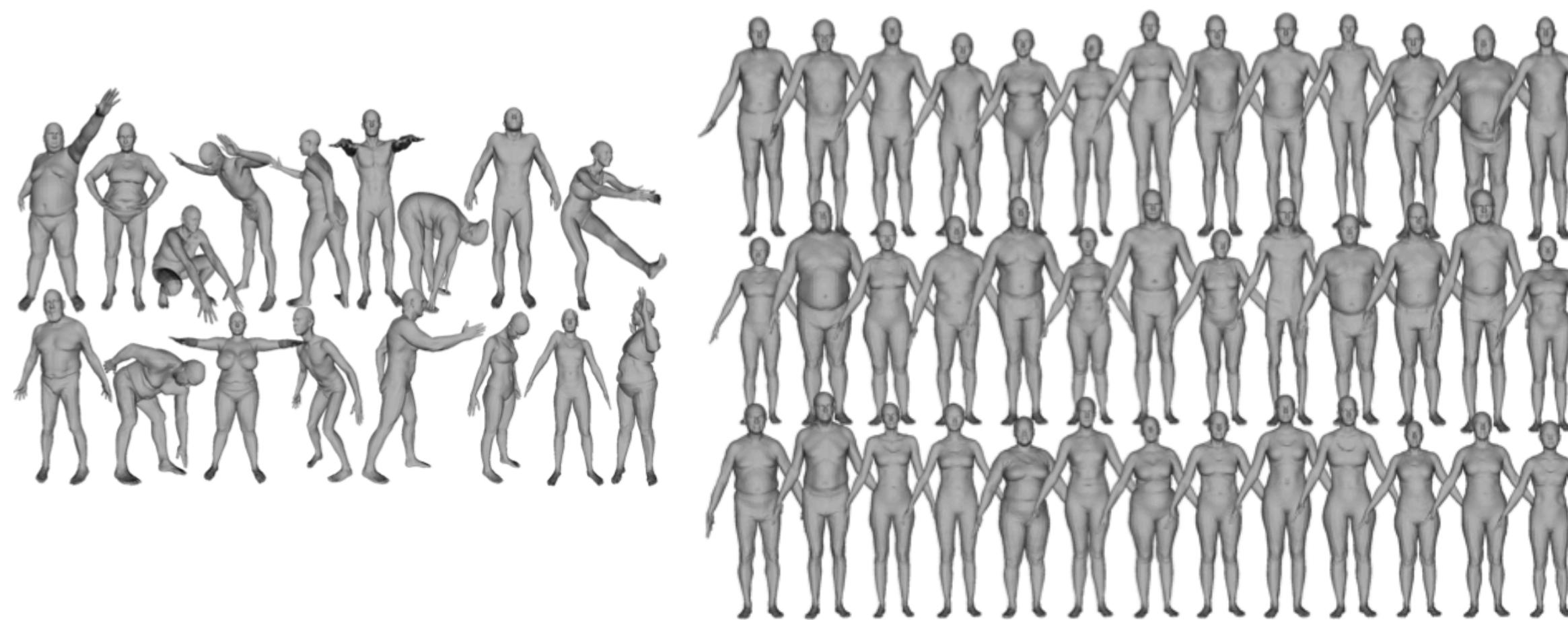
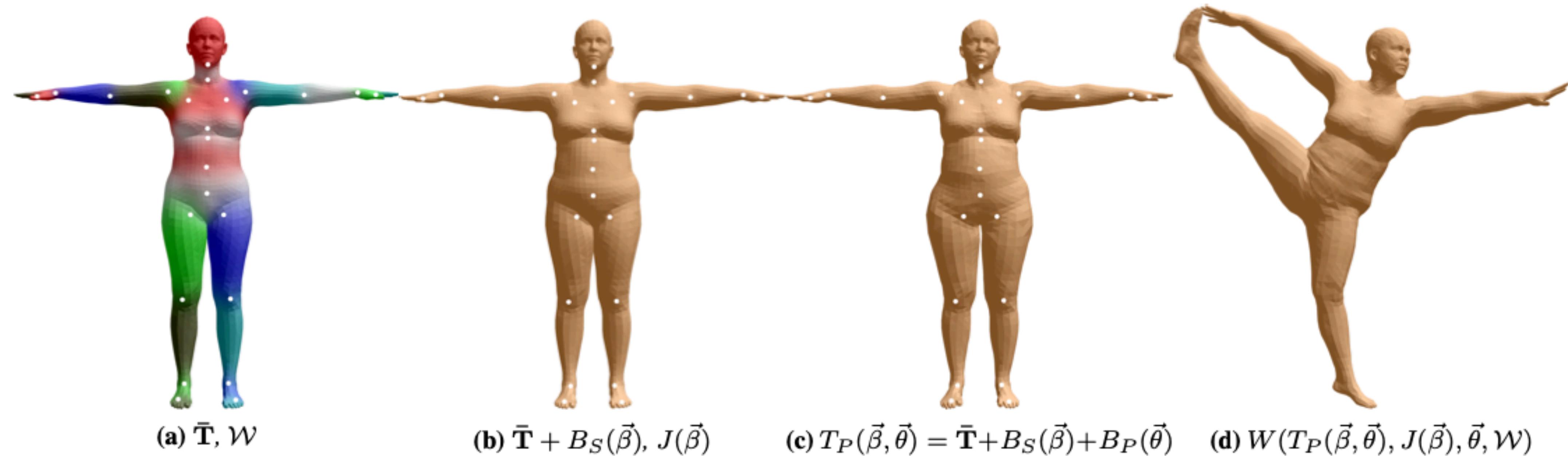
- Given a 3D rotation  $\mathbf{q}_0$  and a 3D translation  $(t_x, t_y, t_z)$ , the rigid transformation can be expressed as a dual quaternion:

$$\hat{\mathbf{q}} = \mathbf{q}_0 + \begin{bmatrix} 1 \\ \frac{\epsilon}{2}t_x \\ \frac{\epsilon}{2}t_y \\ \frac{\epsilon}{2}t_z \end{bmatrix} \mathbf{q}_0$$

- Using dual quaternion algebra, we can easily interpolate two rigid transformations:

$$\hat{\mathbf{q}}(t) = \frac{(1-t)\hat{\mathbf{q}}_1 + t\hat{\mathbf{q}}_2}{\|(1-t)\hat{\mathbf{q}}_1 + t\hat{\mathbf{q}}_2\|}$$

# Learning-based skinning: SMPL representation



$$M(\vec{\beta}, \vec{\theta}; \Phi) = W \left( T_P(\vec{\beta}, \vec{\theta}; \bar{\mathbf{T}}, \mathcal{S}, \mathcal{P}), J(\vec{\beta}; \mathcal{J}, \bar{\mathbf{T}}, \mathcal{S}), \vec{\theta}, \mathcal{W} \right)$$

# References

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- Nancy Pollard: <http://graphics.cs.cmu.edu/nsp/course/15-462/Spring04/slides/05-hierarchy.pdf>
- Radiopaedia: <https://radiopaedia.org/articles/joints-1?lang=us>
- Physiopedia: [https://www.youtube.com/watch?v=l7h2FJnSXyw&feature=emb\\_logo](https://www.youtube.com/watch?v=l7h2FJnSXyw&feature=emb_logo)
- Randale Sechrest: <https://www.youtube.com/watch?v=D3GVKjeY1FM>
- Geometric Skinning with Approximate Dual Quaternion Blending, Kavan et al. ACM TOG 2008
- SMPL: A Skinned Multi-Person Linear Model, Loper et al. ACM TOG 2015