Signal Processing Optimization for Federated Learning over Multi-User MIMO Uplink Channel

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***Abstract*—In federated learning, remote mobile devices, which are equipped with local datasets, collaborate through a parameter server (PS) in order to train a machine learning model. An advantage of the federated learning is its effectiveness of preserving the privacy of local raw data. However, it is challenging to meet the demands on latency of exchanging data on wireless multiple access channel (MAC) with limited bandwidth. Over-the-air computation (AirComp) is a potential solution to this problem, which leverages the superposition property of MAC channel. This work addresses the signal processing optimization of both digital federated learning and AirComp schemes under multi-user MIMO uplink system. For either system, a mathematical optimization problem is formulated and tackled by deriving an iterative algorithm. Via numerical results, the mean squared error (MSE) performance of the digital and AirComp schemes is compared.**

***Index Terms*—Multi-user uplink, federated learning, over-the-air computation.**

I. Introduction

Federated learning is a distributed learning technique in which remote mobile devices collaborate through a server to train a machine learning model. One of the advantages of federated learning is that it is effective to preserve the privacy of local data [1], [2]. It is challenging to satisfy the latency and bandwidth constraints of federated learning systems particularly when many devices are involved. Over-the-air computation (AirComp) can be a potential solution to that problem. A key idea of AirComp is to leverage the superposition property of wireless multiple access channel (MAC) from mobile worker devices to a parameter server (PS) [3]. The design and analysis of the AirComp technique were studied in [4]–[6] in various aspects. However, the comparison between digital federated learning and AirComp systems has not been well studied in the literature.

In this work, we discuss the optimization of baseband signal processing strategies both for federated learning and AirComp schemes under multi-user MIMO uplink systems. For either system, we formulate the problem of minimizing the mean squared error (MSE) of parameter vector while satisfying the transmit power constraints at mobile workers. The problems, which are non-convex, are tackled by deriving iterative algorithms based on the block coordinated descent approach. Some numerical results are presented that compare the performance of the optimized digital and AirComp schemes with various system parameters. The numerical results show that the advantage of optimizing precoding matrices of AirComp scheme becomes more pronounced as the workers use a larger number of antennas and that the AirComp scheme significantly outperforms the digital federated learning scheme when many workers are involved thanks to the exploitation of superposition property of MAC channel.

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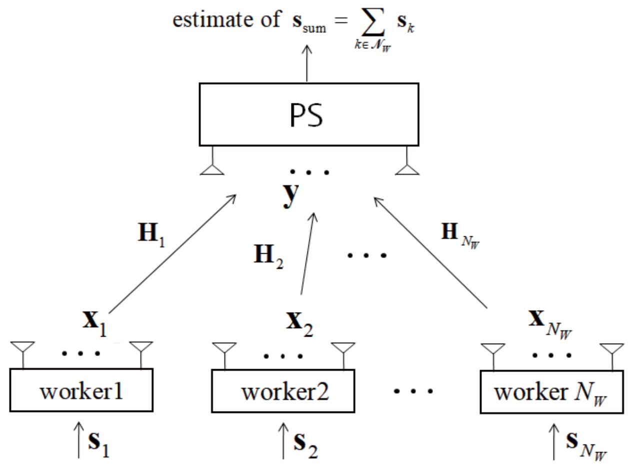

Fig. 1. Federated learning model over multi-user MIMO uplink channel.

II. System Model

As shown in Fig. 1, we consider federated learning in a multi-user uplink system, in which N_W worker devices, each equipped with n_W transmit antennas, report locally updated models, or gradients, with a PS equipped with n_S receive antennas. We define the set $\mathcal{N}_\mathcal{W} \triangleq \{1, 2, \ldots, N_W\}$ of workers' indices. The local model of device k is denoted by $\bar{\mathrm{s}}_k \in \mathbb{C}^{d\times 1}$, where the dimension d of $\bar{\mathrm{s}}_k$ is assumed to be sufficiently large. To report the local models, uplink communication from workers to the PS is performed over $\tau = \lceil d/n_W \rceil$ symbol periods (or time slots). At the tth symbol ($t \in \mathcal{T} \triangleq \{1, 2, \ldots, \tau\}$), each worker k sends the information of the tth subvector of $\bar{\mathrm{s}}_k$ of dimension n_W.

We focus on the communication at a specific time slot t, in which each worker k reports the tth subvector of $\bar{\mathrm{s}}_k$, which is denoted by $\mathrm{s}_k(t) \in \mathbb{C}^{n_W\times 1}$. For the brevity of notation, we omit the time index t in the equations. Also, we denote the covariance matrix of s_k by $\mathbf{S}_k \triangleq \mathtt{E}[\mathrm{s}_k\mathrm{s}_k^H] \succeq \mathbf{0}$, and assume that the model vectors s_k are independent across the device index k. The goal of the PS is to estimate the sum $\mathbf{s}_{\text{sum}} \triangleq \sum_{k\in\mathcal{N}_W} \mathbf{s}_k$ of the parameter vectors transmitted by the workers.

Under flat-fading channel model, the received signal $\mathbf{y}$ of the PS can be written as

$$\mathbf{y} = \sum_{k\in\mathcal{N}_W} \mathbf{H}_k \mathbf{x}_k + \mathbf{z}, \tag{1}$$

where $\mathbf{x}_k \in \mathbb{C}^{n_W \times 1}$ denotes the transmit signal vector of worker k; $\mathbf{H}_k \in \mathbb{C}^{n_S \times n_W}$ represents the channel matrix from worker k to the PS; and $\mathbf{z} \in \mathbb{C}^{n_S \times 1}$ is the additive noise vector with $\mathbf{z} \sim \mathcal{CN}(\mathbf{0}, \sigma_z^2 \mathbf{I})$. The transmit signal vector $\mathbf{x}_k$ is subject to the power constraint $\mathtt{E}[||\mathbf{x}_k||^2] \le P_W$.

III. Digital Federated Learning

Let $\mathbf{d}_k \in \mathbb{C}^{n_W \times 1}$ denote an encoded baseband signal generated by worker k. We assume Gaussian channel codebook so that $\mathbf{d}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$. Worker k linearly precodes the data signal $\mathbf{d}_k$ obtaining its transmit signal vector $\mathbf{x}_k$ as $\mathbf{x}_k = \mathbf{V}_k \mathbf{d}_k$ with the precoding matrix $\mathbf{V}_k \in \mathbb{C}^{n_W \times n_W}$. Under this precoding model, the transmit power constraint of worker k can be restated as

$$\text{tr}\left(\mathbf{V}_k \mathbf{V}_k^H\right) \le P_W,\, k \in \mathcal{N}_W. \tag{2}$$

We assume that the PS decodes the data signals $\{\mathbf{d}_k\}_{k\in\mathcal{N}_W}$ with the successive interference cancellation (SIC) decoding with a given order $\mathbf{d}_{\pi(1)} \to \mathbf{d}_{\pi(2)} \to \ldots \to \mathbf{d}_{\pi(N_W)}$. Then, the maximum achievable data rate $R_{\pi(k)}$ [bit/symbol], at which worker $\pi(k)$ can communicate with the PS, is given as

$$\begin{aligned} R_{\pi(k)} &= f_k(\pi, \mathbf{V}) = I\left(\mathbf{d}_{\pi(k)}; \mathbf{y} | \{\mathbf{d}_{\pi(l)}\}_{l=1}^{k-1}\right) \\ &= \log_2 \det\left(\sigma_z^2 \mathbf{I} + \sum_{l=k}^{N_W} \mathbf{H}_{\pi(l)} \mathbf{V}_{\pi(l)} \mathbf{V}_{\pi(l)}^H \mathbf{H}_{\pi(l)}^H\right) \\ &\quad - \log_2 \det\left(\sigma_z^2 \mathbf{I} + \sum_{l=k+1}^{N_W} \mathbf{H}_{\pi(l)} \mathbf{V}_{\pi(l)} \mathbf{V}_{\pi(l)}^H \mathbf{H}_{\pi(l)}^H\right), \end{aligned} \tag{3}$$

with the notation $\mathbf{V} \triangleq \{\mathbf{V}_k\}_{k\in\mathcal{N}_W}$.

Worker k quantizes and compresses its local model $\mathbf{s}_k$ to express it with a compression rate R_k bit/symbol. If we adopt the Gaussian test channel without claim of optimality, the quantized model vector $\hat{\mathbf{s}}_k \in \mathbb{C}^{n_W \times 1}$ can be modeled as

$$\hat{\mathbf{s}}_k = \mathbf{s}_k + \mathbf{q}_k, \tag{4}$$

where $\mathbf{q}_k$ denotes the quantization noise signal distributed as $\mathbf{q}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Omega}_k)$ with the covariance matrix $\mathbf{\Omega}_k \succeq \mathbf{0}$. According to the standard rate-distortion theoretic argument, the covariance matrix $\mathbf{\Omega}_k$ of $\mathbf{q}_k$ should satisfy the constraint:

$$\begin{aligned} &g_k(\mathbf{\Omega}_k) \triangleq I(\mathbf{s}_k; \hat{\mathbf{s}}_k) \\ &= \log_2 \det(\mathbf{S}_k + \mathbf{\Omega}_k) - \log_2 \det(\mathbf{\Omega}_k) \le R_k. \end{aligned} \tag{5}$$

If the workers have training datasets of different sizes, the goal would change to an estimation of weighted sum of local models.

The PS can recover the quantized signal $\hat{\mathbf{s}}_k$ from the corresponding data signal $\mathbf{d}_k$. The PS estimate the target vector $\mathbf{s}_{\text{sum}}$ as

$$\hat{\mathbf{s}}_{\text{sum}} = \sum_{k\in\mathcal{N}_W} \mathbf{L}_k^H \hat{\mathbf{s}}_k, \tag{6}$$

with combining matrices $\mathbf{L}_k \in \mathbb{C}^{n_W \times n_W}$. The mean squared error (MSE) of this estimation is given as

$$\begin{aligned} e_{\text{digital}}(\mathbf{V}, \mathbf{\Omega}, \mathbf{L}) &\triangleq \mathtt{E}\left[||\hat{\mathbf{s}}_{\text{sum}} - \mathbf{s}_{\text{sum}}||^2\right] \\ &= \sum_{k\in\mathcal{N}_W} \mathtt{E}\left[||\mathbf{L}_k^H \hat{\mathbf{s}}_k - \mathbf{s}_k||^2\right] \\ &= \sum_{k\in\mathcal{N}_W} \text{tr}\left(\left(\mathbf{L}_k^H - \mathbf{I}\right)\mathbf{S}_k\left(\mathbf{L}_k^H - \mathbf{I}\right)^H\right) \\ &\quad + \sum_{k\in\mathcal{N}_W} \text{tr}\left(\mathbf{L}_k^H \mathbf{\Omega}_k \mathbf{L}_k\right), \end{aligned} \tag{7}$$

with the notations $\mathbf{\Omega} \triangleq \{\mathbf{\Omega}_k\}_{k\in\mathcal{N}_W}$ and $\mathbf{L} \triangleq \{\mathbf{L}_k\}_{k\in\mathcal{N}_W}$.

We aim at jointly optimizing $\mathbf{V}$ and $\mathbf{\Omega}$ with the goal of minimizing the MSE (7) while satisfying the constraints (2), (3) and (5). We can formulate the problem as

$$\underset{\pi, \mathbf{V}, \mathbf{\Omega}, \mathbf{L}\mathbf{R}}{\text{minimize}} \;\; e_{\text{digital}}(\mathbf{V}, \mathbf{\Omega}, \mathbf{L}) \tag{8a}$$

$$\text{s.t.} \;\; R_{\pi(k)} \le f_k(\pi, \mathbf{V}),\, k \in \mathcal{N}_W, \tag{8b}$$

$$g_k(\mathbf{\Omega}_k) \le R_k,\, k \in \mathcal{N}_W, \tag{8c}$$

$$\text{tr}\left(\mathbf{V}_k \mathbf{V}_k^H\right) \le P_W,\, k \in \mathcal{N}_W, \tag{8d}$$

where we have defined $\mathbf{R} \triangleq \{R_k\}_{k\in\mathcal{N}_W}$.

It is difficult to solve the problem (8), since an exhaustive search is required for π over N_W possible permutations, and the constraints (8b) and (8c) are not convex constraints. To obtain a tractable solution, we fix π as an arbitrary permutation with some loss of optimality, and propose a suboptimal solution of $\mathbf{V}$ and $\mathbf{\Omega}$ for fixed π. To this end, we adopt the weighted minimum MSE (WMMSE) approach (see, e.g., [7], [8]).

The achievable rate $R_{\pi(k)}$ in (3) is lower bounded as

$$\begin{aligned} f_k(\pi, \mathbf{V}) &\ge \tilde{f}_k\left(\pi, \mathbf{V}, \mathbf{U}_{\pi(k)}, \mathbf{W}_{\pi(k)}\right) \\ &\triangleq \log_2 \det\left(\mathbf{W}_{\pi(k)}\right) - \frac{1}{\ln 2} \text{tr}\left(\mathbf{W}_{\pi(k)} \mathbf{E}_{\pi(k)}\right) \\ &\quad + \frac{n_W}{\ln 2}, \end{aligned} \tag{9}$$

for any $\mathbf{U}_{\pi(k)} \in \mathbb{C}^{n_S \times n_W}$ and $\mathbf{W}_{\pi(k)} \in \mathbb{C}^{n_W \times n_W} \succeq \mathbf{0}$, where we have defined $\mathbf{E}_{\pi(k)}$ as

$$\begin{aligned} \mathbf{E}_{\pi(k)} &= \mathtt{E}\left[\left(\mathbf{d}_{\pi(k)} - \mathbf{U}_{\pi(k)}^H \tilde{\mathbf{y}}_{\pi(k)}\right)\left(\mathbf{d}_{\pi(k)} - \mathbf{U}_{\pi(k)}^H \tilde{\mathbf{y}}\right)^H\right] \\ &= \left(\mathbf{I} - \mathbf{U}_{\pi(k)}^H \mathbf{H}_{\pi(k)} \mathbf{V}_{\pi(k)}\right)\left(\mathbf{I} - \mathbf{U}_{\pi(k)}^H \mathbf{H}_{\pi(k)} \mathbf{V}_{\pi(k)}\right)^H \\ &\quad + \sum_{l=k+1}^{N_W} \mathbf{U}_{\pi(k)}^H \mathbf{H}_{\pi(l)} \mathbf{V}_{\pi(l)} \mathbf{V}_{\pi(l)}^H \mathbf{H}_{\pi(l)}^H \mathbf{U}_{\pi(k)} \\ &\quad + \sigma_z^2 \mathbf{U}_{\pi(k)}^H \mathbf{U}_{\pi(k)}, \end{aligned} \tag{10}$$

where $\tilde{\mathbf{y}}_{\pi(k)} = \mathbf{y} - \sum_{l=1}^{k-1} \mathbf{H}_{\pi(l)} \mathbf{V}_{\pi(l)} \mathbf{d}_{\pi(l)}$.

Algorithm 1 Proposed algorithm for digital federated learning

1. Initialize $\mathbf{V}$ and $\mathbf{\Omega}$ as arbitrary matrices that satisfy the constraints (8c)-(8d).
2. Update $\mathbf{U}$, $\mathbf{W}$, $\mathbf{\Sigma}$ and $\mathbf{L}$ as (11), (12), (14) and (15).
3. Update $\mathbf{V}$ and $\mathbf{\Omega}$ as a solution of the convex problem obtained by fixing π, $\mathbf{U}$, $\mathbf{W}$, $\mathbf{\Sigma}$ and $\mathbf{L}$ in problem (16).
4. If the cost function $e_{\text{digital}}(\mathbf{V}, \mathbf{\Omega}, \mathbf{L})$ has converged, stop. Otherwise, go back to Step 2.

The constraint (9) is satisfied with equality if we set $\mathbf{U}_{\pi(k)}$ and $\mathbf{W}_{\pi(k)}$ to

$$\mathbf{U}_{\pi(k)} = \begin{pmatrix} \sum_{l=k}^{N_W} \mathbf{H}_{\pi(l)}\mathbf{V}_{\pi(l)}\mathbf{V}_{\pi(l)}^H\mathbf{H}_{\pi(l)}^H \\ +\sigma_z^2\mathbf{I} \end{pmatrix}^{-1} \mathbf{H}_{\pi(k)}\mathbf{V}_{\pi(k)}, \tag{11}$$

$$\mathbf{W}_{\pi(k)} = \mathbf{E}_{\pi(k)}^{-1}. \tag{12}$$

To handle the other non-convex constraint (8c), we use the result in [9]. The function $g_k(\mathbf{\Omega}_k)$ in (8c) is upper bounded for any $\mathbf{\Sigma}_k \in \mathbb{C}^{n_W \times n_W} \succeq \mathbf{0}$ as

$$\begin{aligned} g_k(\mathbf{\Omega}_k) &\leq \tilde{g}_k\left(\mathbf{\Omega}_k, \mathbf{\Sigma}_k\right) \\ &= \log_2 \det\left(\mathbf{\Sigma}_k\right) + \frac{1}{\ln 2}\text{tr}\left(\mathbf{\Sigma}_k^{-1}\left(\mathbf{S}_k + \mathbf{\Omega}_k\right)\right) \\ &- \frac{n_W}{\ln 2} - \log_2 \det\left(\mathbf{\Omega}_k\right). \end{aligned} \tag{13}$$

Also, the constraint (13) is tight if $\mathbf{\Sigma}_k$ is equal to

$$\mathbf{\Sigma}_k = \mathbf{S}_k + \mathbf{\Omega}_k. \tag{14}$$

Lastly, we observe that the optimal combining matrix $\mathbf{L}_k$ that minimizes the MSE (7) for fixed $\mathbf{\Omega}$ is simply given as

$$\mathbf{L}_k = \left(\mathbf{S}_k + \mathbf{\Omega}_k\right)^{-1}\mathbf{S}_k. \tag{15}$$

Based on the lower and upper bounds in (9) and (13), we restate the problem (8) as

$$\begin{aligned} \underset{\substack{\pi,\mathbf{V},\mathbf{\Omega},\mathbf{L},\\ \mathbf{R},\mathbf{U},\mathbf{W},\mathbf{\Sigma}}}{\text{minimize}} \;\; & e_{\text{digital}}\left(\mathbf{V}, \mathbf{\Omega}, \mathbf{L}\right) \\ \text{s.t.} \;\; & R_{\pi(k)} \leq \tilde{f}_k\left(\pi, \mathbf{V}, \mathbf{U}_{\pi(k)}, \mathbf{W}_{\pi(k)}\right), \, k \in \mathcal{N}_W, \\ & \tilde{g}_k\left(\mathbf{\Omega}_k, \mathbf{\Sigma}_k\right) \leq R_k, \, k \in \mathcal{N}_W, \\ & \text{tr}\left(\mathbf{V}_k\mathbf{V}_k^H\right) \leq P_W, \, k \in \mathcal{N}_W, \end{aligned} \tag{16}$$

where $\mathbf{U} = \{\mathbf{U}_k\}_{k\in\mathcal{N}_W}$, $\mathbf{W} = \{\mathbf{W}_k\}_{k\in\mathcal{N}_W}$ and $\mathbf{\Sigma} = \{\mathbf{\Sigma}_k\}_{k\in\mathcal{N}_W}$.

If we fix π, $\mathbf{U}$, $\mathbf{W}$, $\mathbf{\Sigma}$ and $\mathbf{L}$ in (16), a convex problem is obtained. The optimal $\mathbf{U}$, $\mathbf{W}$, $\mathbf{\Sigma}$ and $\mathbf{L}$ for fixed $\mathbf{V}$ and $\mathbf{\Omega}$ are obtained in closed form as (11), (12), (14) and (15). Thus, we can derive an iterative algorithm which alternately optimizes the variable sets $\{\mathbf{V}, \mathbf{\Omega}\}$, $\mathbf{U}$, $\mathbf{W}$, $\mathbf{\Sigma}$ and $\mathbf{L}$ to guarantee a sequence of monotonically decreasing cost function values. We describe the detailed algorithm in Algorithm 1.

Algorithm 2 Proposed algorithm for AirComp

1. Initialize $\mathbf{F}$ as arbitrary matrices that satisfy the constraints (22b).
2. Update $\mathbf{G}$ as

$$\mathbf{G} = \left(\sum_{k\in\mathcal{N}_W} \mathbf{H}_k\mathbf{F}_k\mathbf{S}_k\mathbf{F}_k^H\mathbf{H}_k^H + \sigma_z^2\mathbf{I}\right)^{-1} \sum_{k\in\mathcal{N}_W} \mathbf{H}_k\mathbf{F}_k\mathbf{S}_k.$$

3. Update $\mathbf{F}$ as a solution of the convex problem obtained by fixing $\mathbf{G}$ in problem (22).
4. Repeat Step 2.
5. If the cost function $e_{\text{AirComp}}(\mathbf{F}, \mathbf{G})$ has converged, stop. Otherwise, go back to Step 2.

IV. Over-the-Air Computation

In this section, as in [4], we consider an over-the-air computation (AirComp) scheme, in which worker device k transmits the signal $\mathbf{x}_k$ given as

$$\mathbf{x}_k = \mathbf{F}_k\mathbf{s}_k, \tag{17}$$

where $\mathbf{F}_k \in \mathbb{C}^{n_W \times n_W}$ denotes the precoding matrix for $\mathbf{s}_k$. The matrix $\mathbf{F}_k$ should satisfy the power constraint:

$$\text{tr}\left(\mathbf{F}_k\mathbf{S}_k\mathbf{F}_k^H\right) \leq P_W. \tag{18}$$

Under the transmission model (17), the received signal $\mathbf{y}$ of the PS can be written as

$$\mathbf{y} = \sum_{k\in\mathcal{N}_W} \mathbf{H}_k\mathbf{F}_k\mathbf{s}_k + \mathbf{z}. \tag{19}$$

From the received signal in (19), the PS aims at estimating the target parameter vector $\mathbf{s}_{\text{sum}}$. We assume a linear estimator, where an estimate $\hat{\mathbf{s}}_{\text{sum}}$ of $\mathbf{s}_{\text{sum}}$ is given as

$$\hat{\mathbf{s}}_{\text{sum}} = \mathbf{G}^H\mathbf{y}, \tag{20}$$

with $\mathbf{G} \in \mathbb{C}^{n_S \times n_W}$ denoting the estimation matrix.

For given $\mathbf{F} \triangleq \{\mathbf{F}_k\}_{k\in\mathcal{N}_W}$ and $\mathbf{G}$, the MSE is given as

$$\begin{aligned} & e_{\text{AirComp}}(\mathbf{F}, \mathbf{G}) \triangleq \mathrm{E}\left[||\hat{\mathbf{s}}_{\text{sum}} - \mathbf{s}_{\text{sum}}||^2\right] \qquad (21) \\ &= \sum_{k\in\mathcal{N}_W} \text{tr}\left(\left(\mathbf{I} - \mathbf{G}^H\mathbf{H}_k\mathbf{F}_k\right)\mathbf{S}_k\left(\mathbf{I} - \mathbf{G}^H\mathbf{H}_k\mathbf{F}_k\right)^H\right) \\ &+ \sigma_z^2 \,\text{tr}\left(\mathbf{G}^H\mathbf{G}\right). \end{aligned}$$

Our goal is to jointly optimize $\mathbf{F}$ and $\mathbf{G}$ with the criterion of minimizing the MSE function $e_{\text{AirComp}}(\mathbf{F}, \mathbf{G})$. We can formulate the problem as

$$\underset{\mathbf{F},\mathbf{G}}{\text{minimize}} \;\; e_{\text{AirComp}}(\mathbf{F}, \mathbf{G}) \tag{22a}$$

$$\text{s.t.} \;\; \text{tr}\left(\mathbf{F}_k\mathbf{S}_k\mathbf{F}_k^H\right) \leq P_W, \, k \in \mathcal{N}_W. \tag{22b}$$

The problem (22) is bi-convex with respect to $\mathbf{F}$ and $\mathbf{G}$. That is, the problem is convex if we fix one of $\mathbf{F}$ and $\mathbf{G}$. Using this property, we tackle the problem (22) by alternately optimizing one of the variables $\mathbf{F}$ and $\mathbf{G}$ for fixed other repeatedly until convergence. The detailed algorithm is summarized in Algorithm 2.

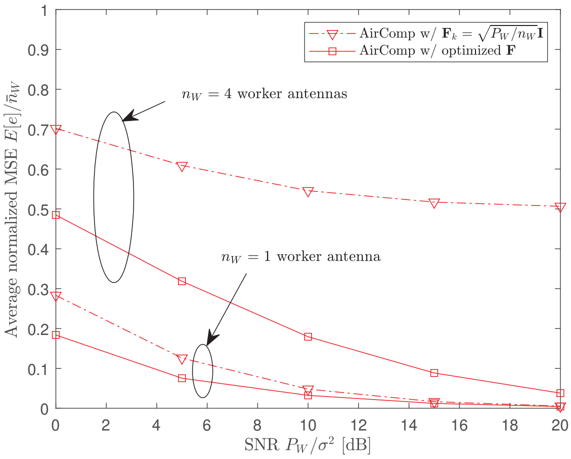

Fig. 2. Average normalized MSE $\mathtt{E}[e]/\bar{n}_W$ versus the SNR for $N_W = 2$, $n_W \in \{1, 4\}$ and $n_S = 4$.

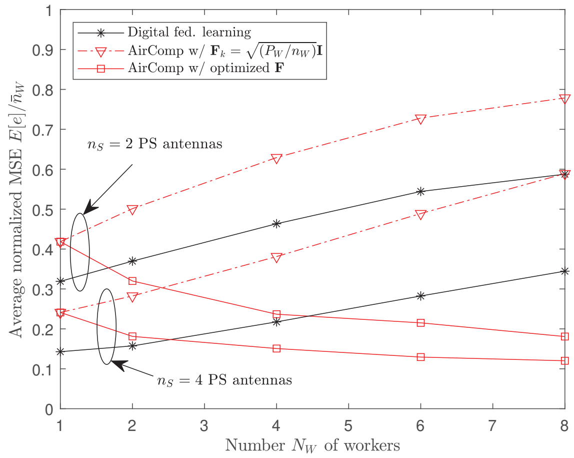

Fig. 3. Average normalized MSE $\mathtt{E}[e]/\bar{n}_W$ versus the number N_W of workers for $n_W = 1$, $n_S \in \{2, 4\}$ and 0 dB SNR.

V. Numerical Results

In this section, we validate the effectiveness of the proposed algorithms for digital federated learning and AirComp systems via numerical results. We assume that the elements of $\mathbf{H}_k$ are independent and identically distributed (i.i.d.) as $\mathcal{CN}(0, 1)$, and the parameter vector $\mathbf{s}_k$ has a covariance $\mathbf{S}_k = \mathbf{I}$. The uplink signal-to-noise ratio (SNR) is defined as P_W/σ^2. We evaluate the average normalized MSE which is defined as the MSEs in (7) and (21) divided by the upper bound $\sum_{k\in\mathcal{N}_W} \mathtt{E}[||\mathbf{s}_k||^2] = \bar{n}_W$ with $\bar{n}_W = N_W n_W$.

In Fig. 2, we plot the average normalized MSE of the AirComp scheme studied in Sec. IV versus the SNR P_W/σ^2 for a multi-user uplink system with $N_W = 2$, $n_W \in \{1, 4\}$ and $n_S = 4$. To check the importance of the proposed optimization algorithm described in Algorithm 2, we compare the performance of the proposed optimized AirComp scheme with that of a baseline AirComp scheme, in which the precoding matrices $\mathbf{F}_k$, $k \in \mathcal{N}_W$, are set to a scaled identity matrix such that the power constraint (18) is satisfied with equality, i.e., $\mathbf{F}_k = \sqrt{P_W/n_W}\mathbf{I}$. By comparing the cases with $n_W = 1$ and $n_W = 4$, we can see that the impact of optimizing the precoding matrices $\mathbf{F}$ becomes more significant as the number n_W of worker antennas increases. This is because, when $n_W = 1$, the workers can perform only power control, while with $n_W \geq 2$, the precoding matrices $\mathbf{F}$ can adjust both the power and direction of the transmitted signals so that the parameter vectors $\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_{N_W}$ are aligned at the received signal of the PS.

In Fig. 3, we plot the average normalized MSE of the digital federated learning and AirComp schemes with respect to the number N_W of workers for a multi-user system with $n_W = 1$, $n_S \in \{2, 4\}$ and $P_W/\sigma^2 = 0$ dB. When there are only a few workers (i.e., N_W is small), the digital federated learning scheme outperforms the AirComp scheme thanks to the robustness of digital processing to noise impairments. However, as N_W increases, the AirComp scheme with optimized precoding matrices $\mathbf{F}$ shows significantly improved performance than the digital scheme by exploiting the superposition nature of the multiple access channel. Moreover, the figure confirms the importance of optimizing the precoding matrices $\mathbf{F}$ for the AirComp scheme.

VI. Conclusion

We have studied the signal processing optimization of digital federated learning and AirComp schemes for a multi-user MIMO uplink system. We formulated the problem of minimizing the MSE of the parameter vector subject to per-worker transmit power constraints. Iterative algorithms have been derived which guarantee monotonically decreasing MSEs with respect to the number of iterations. We have evaluated the MSE performance of the digital learning and AirComp schemes for multi-user system with various parameters.

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