# Determining the Discount Rate for Government Projects

## Louise Young

THE TREASURY
Kaitohutohu Kaupapa Rawa

**AUTHOR**

Louise Young
The Treasury, P.O. Box 3724, Wellington, New Zealand

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|---|---|
| Email | Louise.young@treasury.govt.nz |
| Telephone | 64 4 471 5283 |
| Fax | 64 4 47 1151 |

# Abstract

Discount rates are widely used in the public sector to assess policy proposals where costs and benefits accrue over long time periods. Socially optimal policy choices require an appropriate choice of discount rate. This paper assesses the applicability of the two key theoretical approaches to selecting discount rates in the public sector. The two key theoretical approaches considered are the social rate of time preference and the social opportunity cost. Estimation issues in determining the rate using these two approaches are reviewed. The social rate of time preference is considered to be the appropriate approach. When estimates of the social rate of time preference are unavailable or clearly unreliable and the Government is considering financing a project, the social opportunity cost should be used. The social opportunity cost can be used as a proxy for the social rate of time preference. The paper presents an example using the capital asset pricing model in a weighted average cost of capital formula to determine a social opportunity cost.

# Determining the Discount Rate for Government Projects

## 1  Introduction

A discount rate is used to convert flows of costs and benefits over time into a net present value.  There are two key reasons for doing this.  The first is to determine whether a project is worthwhile, that is whether or not it has a positive net present value.  The second reason is to compare two projects that achieve the same objective but have different timeframes.  For example a discount rate can be used to inform the choice between a lease option and a buy option for accommodation if trying to choose the most cost effective approach.

The government is often looking at proposals that have costs and benefits occurring over different time periods.  The proposals cover a range of issues including investment in state owned enterprises, crown entities and departments, and social, environmental, and regulatory policy choices.  The use of discounting enables the different cost and benefit flows to be converted into a single net present value number for decision-making.

When the results of any cost benefit analysis are clear-cut, the choice of discount rate is not material.  In this situation it would not matter if the discount rate were 10% or 5%, the results would still be a positive net present value or a negative net present value. Unfortunately, not all decisions the government makes are clear-cut.   When the government is making judgements, the choice of discount rate matters as it can affect the decision made.

This paper has eight sections.  Section 2 describes the different economic approaches to the setting of discount rates.  Section 3 takes the two main economic approaches and considers how the social rate of time preference and social opportunity cost can be estimated in broad terms.  The fourth section discusses the various models to estimate the social opportunity cost in more detail.  Section 5 outlines the methods currently used in the New Zealand public sector.  Section 6 examines the assumptions required to apply the weighted average cost of capital to estimate a social opportunity cost discount rate. Section 7 outlines a worked example.  It takes the general assumptions in section 6, determines the remaining assumptions for a particular circumstance and works through the relevant calculations.  Finally section 8 concludes.

# 2    Approaches to determining discount rates

There are two main approaches to thinking about discount rates and how they are determined.  A key feature of both approaches is that they have an element of opportunity cost underlying them.  The first approach thinks of a discount rate as the rate of return an investor would expect from different opportunities that have equal risk.   The second approach is to think of a discount rate as the change in the value of consumption in different periods.

This section examines why the choice of approach to the discount rate is important, and considers the options for choosing the discount rate.

It is important to understand why the choice of approach to the discount rate issue is not simple.  The simplest situation is when all markets clear, there is one market interest rate, perfect information and there are no aspects that are not covered by markets.   Under these assumptions the market clears at a rate where the time preference for consumption equates to the opportunity cost of capital.  The time preference for consumption reflects the rate at which people are prepared to trade consumption today for consumption tomorrow.   The opportunity cost of capital is the rate of return the capital must return to investors in order for them to invest.   In this case the market-clearing rate would be the discount rate.  This situation is shown in Diagram 1.

*Diagram 1 The impact of a market imperfection*
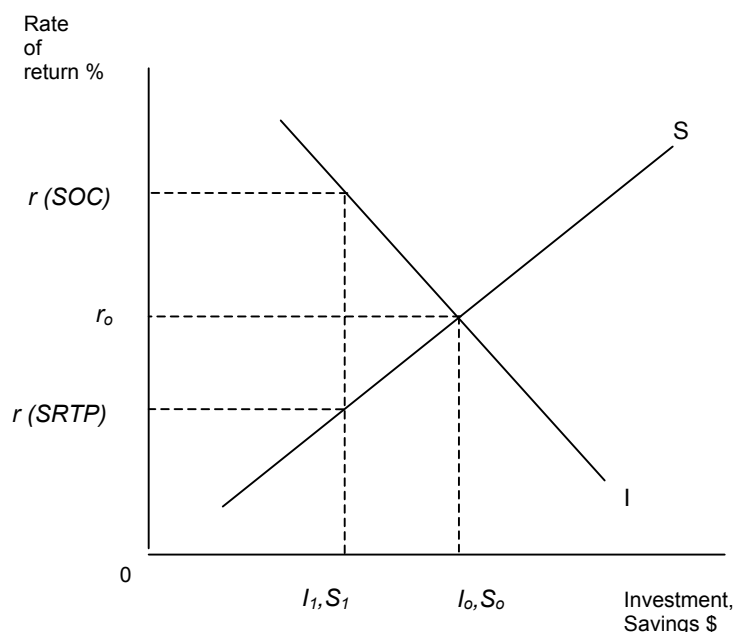


The savings line, S, indicates the value people put on savings when considering whether or not to consume now or in the future.  This is the supply of capital because the amount saved out of current production is available for investment.  People save more if the rate

of return is higher. The savings line can be used to determine the social rate of time preference.

The investment line, I, shows the relationship between how much investors receive for different levels of investment in production. As the rate of return falls, firms find it economic to employ more capital. Therefore this line it the demand for capital and can be used to determine the social opportunity cost of capital.

If all markets were to clear, then there would be a rate of return of $r_o$ where the private rate of substitution between consumption and savings (return to savers) is equal to the rate of transformation for investment (return to investors). At this point savings would be $S_o$ and Investment would be $I_o$. The economically efficient rate of discount in this situation would be $r_o$.

Due to market imperfections the perspective of the individual as an investor and as a consumer do not equate so there is a question about which approach to use. The imperfection can arise because there are monopoly suppliers, there is imperfect information in the market, or there is no market for certain commodities. It may be due to taxation, unemployment or externalities. Diagram 1 shows what can happen with the introduction of a market imperfection, for example a taxation distortion.

In this situation there is a market imperfection of some kind that introduces a gap between the social opportunity cost (SOC), which relates to the return to investors, and the social rate of time preference (SRTP), which relates to the returns to savers. It is unclear which one, if either, should be used (in the first instance) as the discount rate.

The following subsections detail the various alternatives for selecting the appropriate discount rate and what they mean. A range of approaches can be used including social opportunity cost, social rate of time preference, weighted average of the two approaches, and shadow price of capital. The following subsections discuss each of these approaches in turn.

## 2.1   Social Opportunity Cost

The social opportunity cost rate of discount is the rate that reduces the net present value of the best alternative private use of the funds to zero. This means that the social opportunity cost largely reflects the cost in financial market terms. This leads to an approach where the government takes into account what "similar" projects would provide in returns if undertaken in the private sector.

New Zealand is a small open economy where capital is allocated in a global market. This means that there might not be a best alternative use of funds in New Zealand because of the access to overseas capital markets. However, there is also a need to think about the potential impact of projects on financial markets, and the availability of capital more generally. This approach also needs to be in the context of the Government operating with a budget constraint with decisions impacting on debts level and tax rates.

If the public sector uses this discount rate and only invests in positive NPV projects, then public projects would not displace higher value private sector projects. The social opportunity cost rate determines the "efficient" allocation of resources between the public

and private sectors. This is similar to a required rate of return approach except that it relates to the particular investments that would be displaced.

If the government is making decisions on the efficient level of public investment (for example a power station or a new road), on financial grounds then the social opportunity cost approach can be used to provide the appropriate discount rate. This means that valuable public sector projects will be undertaken that may not have been undertaken by the private sector for any number of reasons. The social opportunity cost approach is the most appropriate approach if the government is thinking about investments that could also be undertaken by the private sector, for example if the government is investing in state owned enterprises and service delivery crown entities. Two examples of a service delivery crown entity are Learning Media Limited and Quotable Value New Zealand Limited.

In a large number of circumstances, the government is trying to decide the best way to produce its outputs. This often involves decisions around leasing and buying or whether or not to invest in a new system to produce existing outputs - for example the production of birth, death and marriage certificates. In these cases, the decision is about whether the investment represents value for money, so the social opportunity cost is the appropriate discount rate to use.

The social opportunity cost approach to the discount rate is also the most appropriate approach to consider when undertaking cost recovery of an existing activity or one that could be undertaken in the private sector. This is because it relates to the investment decision being made rather than the service being provided.

## 2.2   Social Rate of Time Preference

The social rate of time preference is equal to the marginal rate of substitution between consumption in one period and the next period. In other words it is the rate of return needed to make society indifferent between consuming x today and $x(1+r)$ in the next period. In an efficient allocation (with no distortions or other market imperfections) all individuals have the same marginal rate of time preference.[1]

Marglin (1963) has suggested that individuals would be better off by undertaking more public investment collectively than is optimal for them as individuals. This would lead to a lower social discount rate relative to the social discount rate when individuals do not take into account the actions of others. This position is supported when individuals are altruistic towards future generations. However, Tullock (1964) has argued against this by raising the question why one generation should be altruistic towards future generations when the future generations are likely to be richer. This would lead to a higher discount rate. If the arguments of both Marglin (1963) and Tullock (1964) were taken into consideration, the starting discount rate would be adjusted upwards and then downwards, potentially ending up at a rate similar to the starting rate.

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[1]   Recent work on individual time preference functions by Ainslie (1992), Laibson (1996) and Laibson et al (2000) suggests these may be hyperbolic rather than exponential in form. The implications of this on social time preferences are not clear. However, since exponential functions approach zero and hyperbolic functions do not, it seems plausible that the hyperbolic function implies a lower long run discount rate than the exponential form. The implications of this work when thinking about social rates of time preference and the evaluation of public sector projects are unclear. This emerging literature should be monitored to assess what if any implications the literature on hyperbolic discount rates has in the public sector context.

Arrow and Lind (1970) argue that there is reduced risk with public investments due to the ability to spread risks amongst all members of the public. This means that the impact of risk is reduced leading to a lower social rate of time preference. This view takes an implicit view that the cost of raising capital from taxes is less than the transaction costs for the private sector to make the same investment. Given that taxation is not costless at the margin, it is not clear that this view would hold in all circumstances.

This assumes that the risks faced in public investment are uncorrelated with individuals' outside investments in the market or other resources; see Bazelon and Smetters (1999). This may or may not hold depending on the circumstances. This means that any social discount rate must reflect the relevant risks and will not necessarily be lower than an individual's own social rate of time preference.

Another reason why the social rate of time preference is different from individuals' marginal rate of substitution between consumption and savings within a period is due to the fact that individuals have a finite lifespan, whereas society is ongoing. The discount rate for public projects may be higher or lower than individual rates of time preference for private projects.

The social rate of time preference reflects social preferences and not just financial sector considerations. The benefits or costs to society would be included in the costs and benefits to be discounted by the rate used. Even without using a social discount rate, the net present value calculation can take into account social considerations by discounting society's costs and benefits.

In addition, when the government is deciding whether or not to undertake a new government service or activity, the social rate of time preference is an appropriate discount rate. For example the social rate of time preference is appropriate if deciding whether or not to introduce a new social service or environmental regulation. This will take into account social preferences for whether investments are undertaken by government for public consumption. This is the appropriate rate for deciding whether or not the government should be involved in various social activities.

In a number of circumstances, whichever social discount rate used (social opportunity cost or social rate of time preference) the resulting decision will be the same (see the diagram below). However, this is not always true and the choice at the margin is important.

Diagram 2: A situation of mixed results
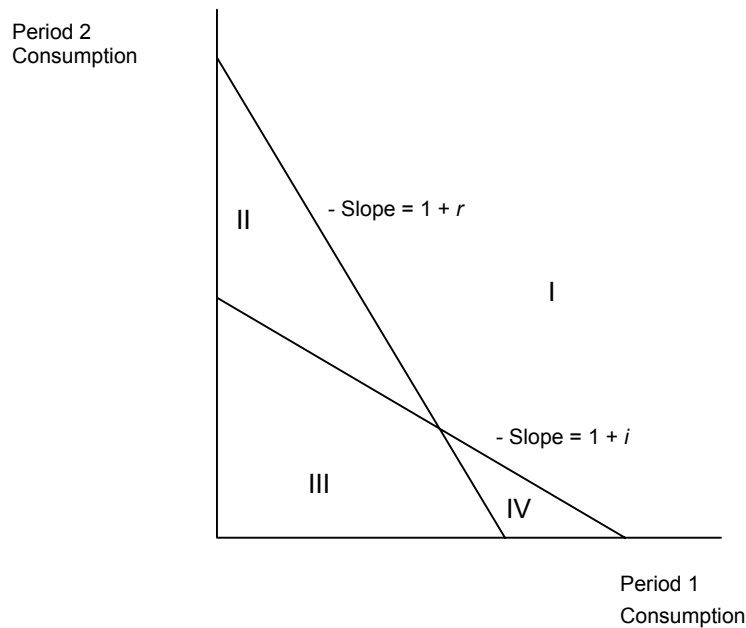


This point is demonstrated in Diagram 2, which is a stylised version of the situation shown in Stiglitz (1994). In Diagram 2 "r" stands for the social opportunity cost discount rate and "*i*" stands for the social rate of time preference discount rate. In areas I and III, the use of either approach for the discount rate leads to the same result. This means that either rate could be a proxy for the other rate depending on which can be estimated. In areas II and IV, the conclusions of an NPV analysis would differ depending on the approach taken to determining the discount rate, so these are the areas of interest in this discussion.


## 2.3   Weighted Average

Several Economists, including Broadman et al (1996), and Sadmo and Dreze (1971), have suggested that the social discount rate should be calculated in terms of the source of the resources used in a particular project. This would be a weighted average cost of the above two approaches. The social rate of time preference would be used to reflect the cost of forgone consumption, while the social opportunity cost would be used to represent the loss in private investment. In the extreme cases, the result would be the same as either the social rate of time preference or the social opportunity cost. This is represented as:

$$\text{Social discount rate} = (\alpha)\text{SOC} + (1- \alpha)\text{SRTP} \qquad (1)$$

where α is the proportion of resources or costs displacing private investment and (1- α) equals the proportion of resources or costs displacing current consumption. There is an issue about setting α, which is project dependent. It may not be clear what the impact will be on private investment and consumption levels.

## 2.4    Shadow price of capital

An alternative is to use the shadow price of capital.  This involves converting all costs and benefits into their corresponding changes in consumption.  Then the social rate of time preference is used as the discount rate.  The conversion of costs to changes in consumption, which may reduce either consumption or private investment in the first instance, requires finding the shadow price of capital.  The shadow price of capital is not observable directly in the market.  The calculations used to derive an estimate are complicated and involve a certain about of subjectivity.


# 3    Calculating a social discount rate

In terms of the rates that can be estimated, the social opportunity cost is the easier of the social rate of time preference and social opportunity cost to estimate.  Section 3.1 discusses estimation of the social rate of time preference.  Section 3.2 discusses the social opportunity cost and its estimation.


## 3.1    Social rate of time preference

There are several approaches to estimating the social rate of time preference.  Factors to take into account are morbidity, uncertainty, and prospects for economic growth with diminishing marginal utility of consumption.

One approach is to use the after-tax market interest rate as all individuals equate their discount rate to the market.  This is reasonable if there are perfect current and future markets.  If everyone has access to the market and takes market interest rates into account then in simple capital markets with a tax distortion, the after-tax rate of interest is a potential estimate of the social rate of time preference.  Unfortunately, this assumption does not hold.  Warner and Pleeter (2201) have observed individual discount rates that were higher than the after tax market interest rate.

A key consideration for other methods of estimating the social rate of time preference is the fact that growth in consumption over time is expected and needs to be taken into account in the discount rate.  Boscolo et al (1998 p. 1) outline the social discount rate as being the sum of two components.  One is the "pure" rate of time preference based on consumption now or later.  The other component indicates, "*how changes in consumption affect the marginal utility of consumption*" (Boscolo et al p. 1).  This may be represented as:

$$\text{Social rate of time preference} = r + \mu g \tag{2}$$

Where $r$ is the "pure" rate of time preference, $g$ is the expected growth rate in per capital consumption and $\mu$ is the negative elasticity of marginal utility with respect to consumption.  In addition, Henderson (1968) took into account consumption growth and the shape of the utility function to derive a social rate of time preference.

Another similar approach is to determine the social rate of time preference based on the optimal rate of growth.  Marglin (1963) suggests working backwards from the rate of optimal rate of growth to the discount rate that ensures a level of investment that achieves

this optimal level of growth. Unfortunately, this approach requires this rate of growth to be known, as well as a number of other assumptions, making it rather difficult to estimate.

None of these methods is without problems. This means that when estimating the social rate of time preference, the results should be used with caution. Diagram 2 indicates that the social rate of time preference and the social opportunity cost lead to the same result in a number of circumstances. When using the after-tax market interest rate approach to estimate the social rate of time preference, the social opportunity cost is a good proxy.

When thinking about the relationship between the social opportunity cost and social rate of time preference, it is useful to determine the relative values. Using diagram 1 the social opportunity cost is greater than the social rate of time preference when on the left hand side of the market-clearing equilibrium because of the distortion. This is likely when there is a tax or other distortion creating a wedge between what savers receive and what investments must return.

Given that there are estimation difficulties with the social rate of time preference, it may be preferable to use the social opportunity cost. This means that when using the social opportunity cost as a proxy and a positive net present value results, there should be a positive net present value from using the social rate of time preference. This is satisfactory for clear-cut results, but could lead to under-investment if net present value results close to zero are not investigated further. Any omission or underestimation of costs would offset this potential issue.

It is important to undertake sensitivity analysis, given that there are only estimates of the social rate of time preference available and not an actual value. Any dramatic changes in the conclusions from small changes in the discount rate should be investigated further.

## 3.2   Social opportunity cost

The social opportunity cost discount rate can be estimated using a number of different models. The models aim to work out what the market would expect to receive for a particular project. This is the rate of return to balance the social opportunity cost of undertaking the project in the public sector versus the next best alternative in the private sector where rates are observable.

The calculation needs to take into account whether the project it is replacing would have received a subsidy, which would lower the rate, or would have been taxed, thereby raising the rate. The calculation also needs to take into account risk as well as any social costs or benefits (externalities). If the project were to replace private sector projects with negative externalities then the rate would be lower. Alternatively, if the private sector project delivers positive externalities then the discount rate would need to be higher.

The discussion does not assume that the exact private sector project displaced is known. In the New Zealand system, there are few subsidies and few tax concessions to take into account, so looking at the general case is a good approximation. In general, the externalities generated by the private sector project will be the same as for the public sector project. Risk is dealt with explicitly in the model used.

The choice of underlying model can significantly alter the result obtained. The main models to choose from include the Capital Asset Pricing Model (CAPM), the Arbitrage

Pricing Theory (APT), and Fama and French's multi-factor model (1993). The various models are briefly described in section 4 below.

The results from these models are then used in the standard weighted average cost of capital (WACC) formula to get a discount rate. The discount rate would be the weighted average cost of capital. The formula is:[2]

$$\text{WACC} = (1-T_c)\, k_b\, D/(D+E) + k_e\, E/(D+E) \tag{3}$$

where $T_c$ is the corporate tax rate, $k_b$ is the return on debt calculated using CAPM, $k_e$ is the return on equity calculated using CAPM, D is bonds or debt and E is equity (also called stock).

This formula needs to be adjusted to reflect that the government does not pay tax or get a tax break on paying interest. This ensures the rate reflects the tax situation for a public sector project. This requires the formula to be divided by $(1-T_c)$, such that:

$$\text{WACC} = k_b\, D/(D+E) + K_e\, E/(D+E) \tag{4}$$

where $K_e$ is the return on equity calculated using the CAPM adjusted for the fact that the government does not pay corporate tax or algebraically as:

$$K_e = k_e / (1 - T_c). \tag{5}$$

As suggested above, it is important to undertake sensitivity analysis, as any method of calculation of the discount rate will only provide an estimate and not the actual value.

# 4    Models for calculating the social opportunity cost

While CAPM currently dominates the other models, the other models are continually challenging this dominance. Lally (2000 p. 47) concluded that:

> "All versions of the CAPM along with APT suffer from considerable ambiguity in empirical testing. However, parameter estimation problems appear to be considerably less for the CAPM than for APT, multi-factor models (such as Fama-French) and the dividend growth model. These considerations do not favour any alternative to the CAPM, and this is consistent with the CAPM's dominance in practice."

In addition, it is useful to understand why the CAPM is the preferred choice. Therefore this section briefly covers the alternative models.[3]

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[2]    Tax only enters this formula once as interest is a deductible expense and dividends are not.

[3]    The capital asset pricing model and the arbitrage pricing theory models are covered in more detail in a number of finance textbook including Copeland and Weston (1992)

## 4.1   Capital Asset Pricing Model

CAPM is concerned with the way different investments move in relation to the market. The expected return derived using CAPM assumes all risks that can be removed by diversification are done so.  This means the resulting expected return include only an allowance for the risk that cannot be removed by diversification.

The CAPM approach gives an expected return equal to the risk-free return (tax adjusted) plus a market related risk premium.  This risk premium is based on how the security or investment moves in relation to the market.  The way the security or investment moves in relation to the market is the $\beta_e$.  The difference between the expected return on the market ($R_m$) and the after tax risk free rate ($R_f(1-T_c)$) is the after tax market risk premium ($R_m - (1-T_c)R_f$).  The Equity beta ($\beta_e$) and the market risk premium are multiplied together to get the additional return for systematic risk.  $T_c$ is the corporate tax rate.   The variables used in the capital asset pricing model are explained in more detail in section 6.  As a formula this is:

$$k_e = R_f (1 - T_c) + [R_m - R_f(1-T_c)]\ \beta_e \tag{6}$$

When taking into account that the government does not pay corporate income tax, the formula becomes the following:

$$K_e = [R_f (1 - T_c) + (R_m - R_f(1-T_c))\ \beta_e] / (1 - T_c) \tag{7}$$

The Capital Asset Pricing Model is widely used in the private and public sector.

This model is based on a choice of securities from the efficient set.  This means that if there are two securities with the same expected return but different standard deviations, only the one with the smaller standard deviation is considered because the variance is taken into account in the market beta used.

The Roll (1977) critique implies that caution should be used in interpreting the results from testing CAPM rather than that the theory is invalid.  Copeland and Weston (1992 p. 219) comment that:

> "In fact, the only way to test the CAPM directly is to see whether or not the true market portfolio is *ex post* efficient.  Unfortunately, because the market portfolio contains all assets (marketable and nonmarketable, e.g., human capital, coins, houses, bonds, stocks, options, land, etc.), it is impossible to observe."

There is going to be some bias or error in the estimation but the direction is unclear.  The estimation of the various variables in the calculations adds to any problems there might be in the underlying model.  For this reason, it is important to use sensitivity analysis when working through any analysis.

## 4.2   Arbitrage Pricing Theory

Arbitrage Pricing Theory, developed by Ross (1976), is an alternative model to CAPM. This is an equilibrium-pricing model.  The APT assumes that returns on securities are

generated by a number of industry-wide and market-wide factors. The APT is a factor model enabling the use of multiple factors to explain or determine the expected return.

The APT theory does not specify exactly which factors determine the expected return or how many should be used. The application of the theory could include the use of the GDP or GNP, inflation or interest rates.

CAPM is a special case of the APT where the only factor is the market risk premium and how the security or investment moves in relation to the market.

The Arbitrage Pricing Theory could be of help, but it is unclear what factors would be appropriate to use in this case. This means that any results from using this approach are open to question more than by using CAPM.

## 4.3    Fama and French's multi-factor models

Fama and French (1993) have developed several multifactor models designed to predict the expected return of particular market investments. They are like the APT in that they are factor models. However, they are more specific about which factors to use. The Fama and French (1993) multi-factor model uses five factors to explain average market performance of particular stocks. There are three stock market related factors (overall market performance, firm size, and book-to-market equity) and two-bond market related factors (default risk and affect of unexpected changes in interest rates).

However, the present situation is not concerned with a true market investment, so the model is not appropriate. For example it is impossible to derive a book value of equity relative to the share value of equity for the government, as government equities (share values) are not listed in the market (except Air New Zealand).

The benefit of a tax system with few investment distortions in particular industries, if any, is the ability to assume that the return on this project is the same as the return of the project being displaced.

Overall using the capital asset pricing model and the weighted average cost of capital is a robust method for determining a social opportunity cost. This means that even though the social rate of time preference is the preferable discount rate in certain circumstances, the lack of a robust estimate means that at a minimum the social opportunity cost should be used in all cases discussed.

In addition, the use of the risk-free interest rate in the capital asset pricing model brings in an element of the social rate of time preference to the estimation of the social opportunity cost. The social rate of time preference and social opportunity cost approaches could yield similar results if a 100% debt-funding scenario were considered.

# 5    Methods used in the public sector

After broadly examining the options above, it is useful to examine the methods currently used in government. In practice the use of the capital asset pricing model (CAPM)

dominates in determining a cost of capital, and other models are not considered superior. The use of CAPM reflects an opportunity cost of capital approach.

The CAPM formula is used to calculate the expected return on equity for government departments. This is then used in a weighted average cost of capital (WACC) formula to take account of the assumed debt equity structure in government departments. The departmental capital charge rate, 9.0% for 2001/02, reduced to 8.5% for 2002/03, represents the average business risk across all departments. It is a real rate. The capital charge system gives departments an incentive to use the taxpayers' investment in the department in the best possible way. It is set to be comparable to investments of a similar risk in the private sector. As the government is looking at it from a whole of government perspective this is a social opportunity cost, although it is making some assumption about the next best option having the same characteristics as public investment. The government wants to ensure the best use of existing resources within the public sector.

The departmental capital charge rate is often used as a de facto discount rate by departments for calculating the net present value of an investment or alternative policy options. This is likely to be because it is the opportunity cost of their existing capital. If they seek capital contributions these have the same capital charge rate as their existing capital.

The government also uses the CAPM and WACC formulas to calculate the expected rate of return for State Owned Enterprises.

In addition, the New Zealand Treasury uses a 10% real rate whenever there is no other agreed sector discount rate for costing policy proposals.[4] Wilkinson (1982) outlines the long history of the 10% discount rate. Cabinet set this rate in 1971 and the Minister of Finance reaffirmed it in the 1982 Budget. The rate was based on the opportunity cost of capital based on the expected return of a low risk private sector investment at the time. Current Cabinet Office circulars do not refer to the 10% real discount rate as a standard rate. The Cabinet Office Circular (CO (00) 12 Annex One) requires that business cases should identify and detail "the discount rate used, and its derivation" amongst other things.

It is useful to contrast the New Zealand experience with overseas experience. For instance, in the context of health research Wright (1998 p.12) comments that:

> "The current preferred proxy for the society's rate of pure time preference being the
> real interest-rate on a risk-free long-term investment. Using this approach, the U.S.
> Panel on Cost Effectiveness in Health and Medicine recommended using a rate of
> 3%."

Wright (1998 p. 12) also comments, "For some years, the convention in the health economics literature has been to use a 5% discount rate for cost-effectiveness and cost-utility analyses".[5] Her paper details that the U.S. Panel then recommend the use of both 3% and 5% for base-case analysis and critical sensitivity analyses. This recommendation would reflect the US situation and not the New Zealand situation.

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[4]  The Treasury Office Minute 1999/B41 Guidelines for Costing Policy Proposals 21 December 1999, page 13.

[5]  Wright (1998) does not detail the basis for the 5% being used.

"The Green Book" [6] on appraisal and evaluation in central government issued by HM Treasury (1997 p. 24) details that " for most applications in central government the real discount rate is 6 per cent. … Exceptions include industrial assistance proposals and projects under the overseas aid programme." This rate has been in effect since 1989. Parsonage and Neuburger (1992) state that this standard rate was widely used and had almost invariably been applied to costs and benefits.

Wright (1998) suggests that researchers in New Zealand may want to use overseas rates to check their results, for example in the health sector where considerable research is undertaken overseas. This does not mean that for decision-making purposes that the same discount rate should be used in government decision making if it does not reflect the discount rate applicable to that situation in New Zealand. For example it is likely that the discount rate would be different in New Zealand. The New Zealand government bond rate would be used in New Zealand as the risk free rate and this is higher than the US bond rate.

# 6    Assumptions required

This section works through the general assumptions required before the weighted average cost of capital can be estimated. There are several components required to calculate the return on equity calculated using CAPM. These are the risk-free rate (including inflation if a nominal rate is required), beta or the adjustment for risk and the market risk premium. The risk-free rate and debt premium are required for calculating the return on debt.

## 6.1    Risk-free rate

The risk-free rate reflects the rate of return that a person can expect on a completely risk less asset. Most people normally use a government bond rate as the risk-free rate. This is because relative to the other investments in the market, the government bond has very little if any risk.[7] The government bond rate can be thought of representing the time value of money. To be consistent with how other people use CAPM, the government bond rate is considered to be the risk free rate.

The time frame of the government bond rate used should be consistent with the project timeframe. So a five-year project would use a five-year government bond rate and a ten-year project would use a ten-year bond rate or the closest approximation.

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[6]    The HM Treasury has recently issued a consultation paper and revised draft of "The Green Book", this calculates the social rate of time preference to be 3.5%.

[7]    This is not always true as sometimes highly indebted governments have had to renegotiate the repayment of debt to avoid default on their debt, for example Mexico and Argentina. Government bond rates include a debt premium for country risk or this Government default risk. The level of this premium would be affected by the ratings made by agencies such as Standard and Poors and Moody's rating agencies. In addition, there is an inflation risk involved in all bonds issued on a nominal basis.

## 6.2  Debt premium

In the case of the government the risk-free rate is also its own bond rate. The government guarantee associated with government borrowing means the government does not pay a debt premium for the risk of default relative to other types of borrowers in New Zealand. The debt premium is related to the quality of the debt on issue. The debt risk premiums observed in the New Zealand bond market for high quality debt without a government guarantee are: Transpower (0.9%), Housing New Zealand (1%), Telecom (1%) and Auckland Airport (0.9%)[8]. Based on this data the departmental capital charge formula uses a debt premium of 1% on the risk free rate to determine the bond rate for the WACC formula.

The appropriate rate of return on debt for the WACC formula is a long-term government bond rate with a debt premium and the alternative uses of those funds. This takes into account the impact on the market of the government raising the debt. The relevant debt premium would be 1% based on the evidence above. This is also consistent with the debt premium used elsewhere in the government sector.

The debt premium is not required to calculate the return on equity using CAPM.

## 6.3  Inflation Adjustment

The historic government bond rates include inflation. Some people forecast the real interest rate while others forecast the nominal interest rate. There should be consistency in the use of nominal and real figures throughout the calculations. In addition, the final discount rate should be real or nominal depending on the flows it is being used to discount.

The relationship between real and nominal data is multiplicative:

$$\text{Real rate} = [(1+ \text{nominal rate}) / (1 + \text{inflation})] - 1 \tag{8}$$

This adjustment is used to convert a nominal capital charge rate into a real capital charge rate. This formula is consistent with the capital charge formula.

When including a debt premium, a real rate needs to be converted to a nominal rate before adding the debt premium. The result would then be converted back to a real figure if the cash flows to be discounted are real. This ensures consistency in the use of a real discount rate to convert the real cash flows to a net present value calculation.

When calculating an equity return a nominal risk free rate is required to be consistent with the use of a nominal market risk premium in the CAPM formula.

To determine what inflation rate should be used, it is important to choose a rate that is consistent with the time frame of the project and the interest rate used as the risk-free rate of return. If using a 10-year nominal risk-free rate of return, this should be adjusted by a 10-year inflation figure for consistency.

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[8]     Data from the National Bank of New Zealand as set out in Lally (1999)

## 6.4   Market Risk Premium

The market risk premium depends on the way the market moves relative to the risk-free asset or $R_m - R_f(1-T_c)$.  Where $R_m$ is the market return, $R_f$ is the risk free rate and $T_c$ the corporate tax rate.  It reflects the extra return an investor expects for investing in the market over a risk-free asset.  It reflects the systematic risk present in a market that cannot be diversified away.  The proportion of extra return taken into account reflects the equity beta. This is discussed below.

The market risk premium can be obtained from elsewhere and does not need to be calculated.  For example the SOE and CE cost of capital formula uses a tax adjusted market risk premium of 9%, as does the capital charge rate for government departments (Young, 2000).  In March 2000 Pricewaterhouse Coopers New Zealand produced a paper on the New Zealand Equity Market Risk Premium gives a tax adjusted market risk premium in the range of 8%-9%.  Pricewaterhouse Coopers New Zealand (August 2002) has updated this and now use a 7.5% tax adjusted market risk premium.  The conclusion in their March 2000 paper was driven by low rates of return in the late 1920's and 1930's when the financial markets were very different from today.  There was the depression in the 1930's and there were fewer financial market instruments available.  Their conclusion was also based on overseas expectations that future rates will be lower than historical rates.

This variable is only required when the case you are examining includes an equity component in the weighted average cost of capital.  If this is the case then, use 9% as the most reliable estimate and it also provides for consistency with the departmental capital charge formula.

## 6.5   Beta or adjustment for risk

In CAPM this risk adjustment is for the systematic risk that cannot be diversified away.  It is related to how the particular investment varies with the market.  As with the market risk premium above, this is only relevant if there is an equity component in the weighted average cost of capital.  An equity beta of 1 would mean that a security moves in step with the market as a whole.  If the market return is increasing, the return on the security will be increasing.  If the market return is decreasing the return on the security is decreasing.

The equity beta is a multiplier on the market premium.  A positive value less than 1 means the security moves in the same direct as the market but not with the same magnitude.  A value of more than 1 means that the security moves in the same direct as the market but with a greater magnitude.  A negative value means that the security moves in the opposite direction to the market.

The equity beta used is dependent on the situation.  There is a general equity beta that is used in the departmental capital charge rate and it takes into account the overall nature of government and its (assumed) debt/equity structure.  This reflects the general regulated nature of government.

The equity beta to be used can be derived from an asset beta and debt/equity structure or may be obtained from comparator equity betas. Asset betas can be converted to equity betas using the following formula, assuming tax neutrality[9]:

$$\beta_e = \beta_a \, [1+(D/E)] \tag{9}$$

An optimal debt/equity structure ensures that given the asset beta the balance sheet ensures that the equity beta is 1. Using the optimal structure would divorce the situation from reality, as most organisations do not operate with an optimal debt equity structure. It is also debatable about whether moving perfectly in line with the market is optimal from the point of view of the government or the taxpayer.

The beta that applies in a particular case would need to be estimated separately, unless the project has average government risk. In which case the equity beta used in the departmental capital charge formula would be appropriate. The equity beta used in the departmental capital charge formula is 0.6, and the asset beta underlying this is 0.3.

If the project does not have average government risk then the nature of the situation needs to be clearly defined. For example, a service delivery agency is undertaking a major information technology project with leading edge technology. In this case the project would be more risky than its normal business activities.

Potential questions to ask when thinking about choosing a beta include the following: how stable is the sector? how regulated is the sector? is it a monopoly supplier? is it part of their business as usual activities? what aspects are risky? how risky is the technology? how variable are the cash flows? Once the characteristics of the situation are defined, they can then be compared to those of listed companies and particular sectors. Having defined a match, it is a matter of obtaining a suitable beta.

# 7    An example

This section of the paper applies the weighted average cost of capital to determine the appropriate social opportunity cost discount rate to apply to the long term cost recovery situation where the costs to be incurred are known with a high degree of certainty. This example also assumes that the costs will be recovered prior to any expenditure being incurred. This covers any circumstance where there is an ability of the Government to set the price to ensure cost recovery and there is no cross subsidisation between the users of the services and other taxpayers. The costs and benefits are occurring over a number of years. The example below estimates a real discount rate.

## 7.1    Weighted Average Cost of Capital

The weights for the debt and equity components should follow the particular circumstances of the situation. This may depend on the type of agency within government that is undertaking the activity. For example, if the government is undertaking something

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[9] In a tax neutral situation, the tax term (T) drops out of Hamada's formula: $\beta_e = \beta_a \, [1+(D/E)(1-T)]$

directly through taxation or a subsidy, versus through a department or SOE. In addition, it depends on the type of market the agency is operating within.

The application of the WACC uses the results from applying CAPM. This full cost recovery situation example is, however, a special case. This is because it is assumes there is certain that the Government will fully cost recover and the Government is not trying to cross subsidise between users. This situation would not have general taxpayers subsidising the users of the service. In this case, it looks very much like a debt smoothing or financing operation rather than an investment situation. Under this view, there would be 100% bond financing and no equity financing.

The WACC calculation would then become one of 100% times the return on debt. This is a rather simpler calculation. Later subsection 7.3 covers the determination of the return on debt.

A hypothetical situation would have the optimal level of debt and equity determined by the asset beta so that the equity beta is 1 and moves exactly in line with the market. For example if the asset beta has a value of 0.6, the level of equity would be 60%, giving a unity equity beta so the stock value would move in line with the market. In this situation, the amount of borrowings should be inversely proportional to the asset beta. The use of equity and debt financing could be seen as using general tax revenues as well as any fees or charges to smooth the expenditure flows.

Overall, the alternative approach in this case is not superior to the 100% debt situation of debt smoothing so the example in section 7.3 will use a 100% debt weighting in the weighted average cost of capital formula.

If this were not a special case, then the proportion of equity and debt would need to be determined. The optimal debt equity level given the asset beta is one approach. Debt funding can be seen as financial smoothing for user charging. Equity funding can be though of as first order tax funding of the proposal. If the proposal is to be funded by an increase in general taxation then 100% equity may be appropriate.

As an aside, the weighted average will not always be the same as the 50% debt/ 50% equity used in the departmental capital charge formula. For example, if a new project is particularly risky and does not reflect the existing business then there is no reason why the same discount rate should be used.

## 7.2    Capital Asset Pricing Model

There are several components to the return calculated using CAPM. The key components are the risk-free rate including inflation if a nominal rate is required, beta or the adjustment for risk, and market risk premium

This cost recovery-focused situation is often very long-term in nature so the term of the risk-free rate should match this. Therefore, the appropriate government bond rate is a long term one. In this situation the 10-year bond rate is therefore the most appropriate rate as it is the longest rate available for which there are forecasts or robust data.

The Treasury's long-term fiscal model assumes that the interest rate forecast to occur in the last period of the forecasts continues on for the timeframe of the long-term fiscal

model. In the 2002 Budget Economic and Fiscal Update the nominal 10-year government bond rate in the long-term fiscal model was 6.2%.

The other alternative is to use the longest government stock currently available, that there is sufficient market information on. This is a government bond maturing April 2013 with an interest rate of 6.35% on 5 August 2002.[10]

This example uses the 10-year nominal interest rate from the long-term fiscal model as this rate reflect the long term nature of the example and takes a neutral position on the output gap. This rate is therefore effectively a steady state rate.

The Reserve Bank Act sets a band within which the Reserve Bank Governor needs to maintain the inflation rate. The band for the inflation rate was 0-3%. The long-term fiscal model used by the Treasury in the 2002 Budget assumed that the inflation rate would be 1.5% or in the middle of the inflation rate band.

In the short term, the Treasury forecasts the inflation rate explicitly. This can be used to derive real short-term interest rates if required. As this case is long-term in nature, if an inflation rate is required for converting rates from real to nominal or vice versa, 1.5% will be used to match the timeframe of the project in this example.

The market risk premium used should be consistent with the market risk premium used elsewhere in the Treasury. The market risk premium used in the departmental capital charge formula is 9%. This is not required in this case. The government is guaranteed to receive the revenue in this example so the correlation of the return with the market is zero. This means that the market related term drops out. This example then reverts to a 100% debt related situation so the return on equity does not have to be considered further.

The nominal rate of return on the forecast 10 year government bond rate is:

$$\text{nominal 10 year government bond rate} + \text{debt premium} = \text{nominal return} \qquad (10)$$

$$k_b = 0.062 + 0.010 = 0.072 \qquad (11)$$

## 7.3 Numerical Results

Using the formula in equation 4 above:

$$\text{WACC} = k_b\, D/(D+E) + K_e\, E/(D+E) \qquad (12)$$

This formula uses $K_e$ as the tax adjusted return on equity and is defined in equation 5 above. However, in this case there is a 100% debt situation so the equation becomes:

$$\text{WACC} = k_b\, D/(D) \qquad (13)$$

So:

$$\text{WACC} = k_b = 0.072 \qquad (14)$$

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[10] Information provided by The Treasury- New Zealand Debt Management Office.

After converting this back to a real WACC figure using equation 8 above the real discount rate under the 100% debt/bond scenario is: 0.056 or 5.6%.

The above leads to a 5.6% real discount rate for this example of a cost recovery situation. This is based on a scenario of receiving revenue in advance of incurring costs making a 100% debt-financing situation appropriate. The result depends on the assumptions used in the long-term fiscal model, a 1% debt premium to reflect the opportunity cost of capital, and the application of the capital asset pricing model and the weighted average cost of capital formula when there is 100% debt financing. A different interest or inflation rate would give a different real discount rate.

When undertaking calculations using the estimated discount rate, it is important to undertake sensitivity analysis. This analysis could be done by repeating the net present value calculations with different discount rates. If small changes in the discount rate impact markedly on the result then the results should be used with caution.

The discount rate for other investment decisions would vary, depending on the market-related factors associated with the particular investment.

# 8    Conclusions

The paper has examined the two main perspectives relating to the question of discounting, and the resulting approaches of the social rate of time preference and the social opportunity cost. The paper then considers each of these approaches and their estimation. An example is provided to detail some of the estimation issues to be considered.

Overall, the paper concludes that the social rate of time preference is appropriate (subject to estimation considerations) when the government is considering new government activities or ceasing existing government activities[11], because society's preferences are important. In a significant number of cases the results from using the social opportunity cost and the social rate of time preference are the same.[12] However, the rate is important in a number of circumstances such as when the net present value is close to zero.

The use of the social opportunity cost is a good proxy for the social rate of time preference in the majority of circumstances. This is because one of the methods of estimating the social rate of time preference is similar to the method for estimating the social opportunity cost. In addition, distortions mean that the social opportunity cost is likely to be higher than the underlying social rate of time preference. Hence any positive net present value achieved using a social opportunity cost discount rate, should lead to the same result when using a social rate of time preference discount rate.

Given difficulties in estimating the social rate of time preference, new or significant social projects should be assessed using the social opportunity cost. All public sector

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[11] For example, starting a new social service, ceasing an existing benefit, introducing new environmental regulation etc.

[12] The empirical work on hyperbolic discounting may impact on the way net present value calculations are undertaken when using a social rate of time preference discount rate. This area would need to be examined in further detail before there is a move away from our current approach to calculating net present values.

investment projects should be assessed using the social opportunity cost. The social opportunity cost can be estimated using the capital asset pricing model and the weighted average cost of capital formula. Where a reasonable estimate of the social rate of time preference is available this should be used as well. This approach would ensure the best allocation of resources as well as meeting society's preferences.

The social opportunity cost is appropriate for determining how to undertake government activities once there is a decision to have an intervention. In addition, when there is no change in the social preference for undertaking an activity, and the government is revisiting how it is provided, the social opportunity cost is the appropriate discount rate.

In all cases, sensitivity analysis should be undertaken, as the "true underlying" social rate of time preference or social opportunity cost cannot be exactly determined.

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