7/Dec/2023

Hypercube model paper:

1) Goal:

Ambulance deployment decision

$\downarrow$

Where should my ambulances be when idle?
If I receive a call for an ambulance, which one to send?

2) Model:

J cities (geographical atoms)

f_j = fraction of calls from city j (workload of atom j)
$j \in \{1, ..., J\}$

τ_{ij} = travel time between cities i and j $i,j \in \{1, ..., J\}$

N ambulances (response vehicles)

L = location matrix with elements
l_{nj} = probability that ambulance n is in city j when idling
$n \in \{1, .., N\}$, $j \in \{1, ..., J\}$.

Calls for ambulances $\sim$ Poisson (λ)
$\Rightarrow$ call for ambulance from city $j \sim$ Poisson (λf_j).

Service time of any ambulance at any city $\sim$ Expo(μ)

$\vec{B}$ = N-dimensional binary vector that represents state of ambulances.

$B_n = \begin{cases} 1 & \text{if ambulance } n \text{ is busy} \\ 0 & \text{ow} \end{cases} = n^{th}$ element of $\vec{B}$

State of system.

* In the paper, state $\vec{B}_j$ is such that the integer with binary representation $\vec{B}_j$ is equal to j.

Eg: $\vec{B}_0 = (0, 0, \dots, 0)$

$\vec{B}_3 = (0, 0, \dots, 0, 1, 1)$

Λ = transition matrix of state vector with elements λ_{ij} ; $i, j \in \{0, \dots, 2^N - 1\}$

needs to be computed

Goal in terms of the model:

Compute stationary distribution of vector $\vec{B}$

Why?

How? Solve balance equations: Paper has a nice representation in terms of enumeration of states as in *

What to do?

1. Compute transition matrix Λ
2. Solve balance equations $\leftarrow$ "easy"

① Transition matrix:

s.6

"Downward transitions" i to j:

If i has $w(B_i)$ nonzero elements, and
j has $w(B_i)-1$ nonzero elements

$$\Rightarrow \quad \lambda_{ij} = \mu\, w(B_i)$$

"Upward transitions" i to j:

If i has $w(B_i) < N$ nonzero elements and
j has $w(B_i)+1$ nonzero elements

λf_k = rate at which a new call from city k occurs. ← known

η_{ik} = # response units that are optimal to send to call from city k when the system is in state i ← need to compute

$$\Rightarrow \quad \lambda_{ij} = \sum_k \frac{\lambda f_k}{\eta_{ik}}$$

$k \in \{$new busy ambulance in set j is one of the optimal ambulances to send in state i with call from city $k\}$

s.7 How to compute the optimal ambulance(s) to dispatch?

$$t_{ij} = \sum_{k=1}^{J} l_{ik} \tau_{kj}$$ = mean travel time of ambulance i to city j when it's available

q_{ij} = identifier of i^{th} closest ambulance to city j when all units are available

$$\Rightarrow \quad q_{1j} \in \underset{a \in [N]}{\operatorname{argmin}}\ t_{aj}, \quad q_{2j} \in \underset{a \in [N] \setminus \{1\}}{\operatorname{argmin}}\ t_{aj}$$, where $[b] = \{1, 2, \dots b\}$.

$$q_{ij} \in \underset{a \in [N] \setminus [i-1]}{\operatorname{argmin}}\ t_{aj}$$

→ q_{ij} is computed by ordering t_{ij} from smallest to largest for each city j

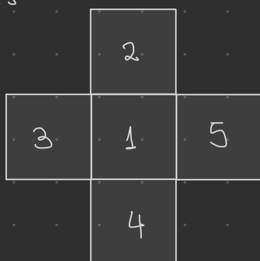
→ The optimal unit to dispatch in state $\vec{B}$ is the one with smallest q_{ij} among the available ones.

→ Efficient computation: compute adjacent vertices one after the other.

Example:

$N = 3$ ambulances

$J = 5$ cities



$\tau_{ij} = 1$ among adjacent squares.

① $\mu = 3$, $\lambda = 9$, $f = \left(\overset{f_1}{\frac{1}{10}}, \overset{f_2}{\frac{2}{10}}, \overset{f_3}{\frac{3}{10}}, \overset{f_4}{\frac{3}{10}}, \overset{f_5}{\frac{1}{10}} \right)$

a) L = uniform distribution $\Rightarrow$ $l_{nj} = 1/5 \quad \forall n, j$

b) All ambulances always wait in $j=1$.

② Repeat with $\mu=1$, $\lambda=3$.

③ $\mu=1$, $\lambda=3$,

a) L uniform
b) L all in $j=1$.
c) Better L?