Derivative of the Magnitude of a Vector

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Recall $\boldsymbol{v}(t) \cdot \boldsymbol{v}(t) = v(t)^2$. Then,

$$\frac{\mathrm{d}v^2}{\mathrm{d}t} = 2v\frac{\mathrm{d}v}{\mathrm{d}t} \tag{0.1}$$

$$\begin{aligned}
\frac{\mathrm{d}v}{\mathrm{d}t} &= \frac{1}{2v}\frac{\mathrm{d}v^2}{\mathrm{d}t} && (0.2)\\
&= \frac{1}{2v}\frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{v} \cdot \boldsymbol{v}) && (0.3)\\
&= \frac{1}{2v}\left(\boldsymbol{v} \cdot \boldsymbol{v}' + \boldsymbol{v}' \cdot \boldsymbol{v}\right) && (0.4)\\
&= \frac{\boldsymbol{v} \cdot \boldsymbol{v}'}{v} && (0.5)
\end{aligned}$$

$$\boxed{\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\boldsymbol{v} \cdot \boldsymbol{v}'}{v}} \tag{0.6}$$