Gravitational Waves from Topological Defects in the Early Universe

Zhao-Huan Yu（余钊焕）

School of Physics, Sun Yat-Sen University

https://yzhxxzxy.github.io

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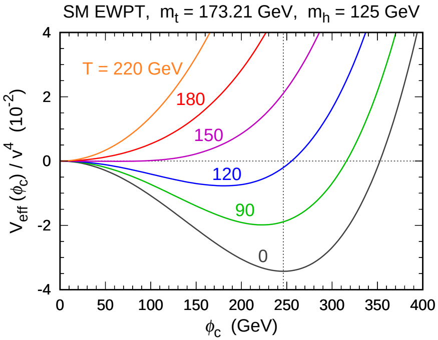
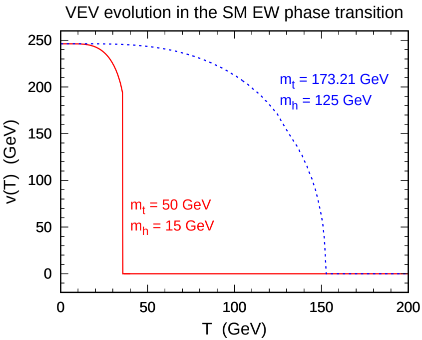
Cosmological Phase Transition

🔥 **Spontaneously broken symmetries** in field theories can be **restored** at **sufficiently high temperatures** due to **thermal corrections** to the **effective potential**

🌨️ In the history of the Universe, **spontaneous symmetry breaking** manifests itself as a **cosmological phase transition**





Topological Defects

✂️ Consider that **some scalar fields** acquire nonzero **vacuum expectation values** (VEVs), which **break** a **symmetry group** G to a **subgroup** H

⚖️ The **manifold** consisting of all **degenerate vacua** is the **coset space** G/H

🧵 The **topology** of the **vacuum manifold** G/H can be characterized by its n**-th homotopy group** $\pi_n(G/H)$, which are formed by the homotopy classes of the mappings from an n**-dimensional sphere** S^n into G/H

🪢 A **nontrivial** $\pi_n(G/H)$ leads **topological defects** [Kibble, J. Phys. A9 (1976) 1387]

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🟥 **Nontrivial** $\pi_0(G/H)$: two or more disconnected components

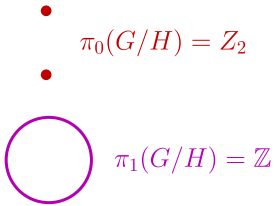
- 👉 **Domain walls** (2-dim topological defects)

♑ **Nontrivial** $\pi_1(G/H)$: incontractable closed paths

- 👉 **Cosmic strings** (1-dim topological defects)

🟠 **Nontrivial** $\pi_2(G/H)$: incontractable spheres

- 👉 **Monopoles** (0-dim topological defects)

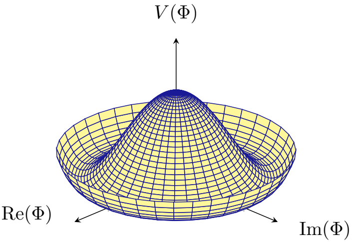


Cosmic Strings from U(1) Gauge Symmetry Breaking

- Consider the **Abelian Higgs model** with a **complex scalar field** Φ

$$\mathcal{L} = (D^\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi) - \frac{1}{4} X^{\mu\nu} X_{\mu\nu}, \quad V(\Phi) = -\mu_\phi^2 |\Phi|^2 + \frac{\lambda_\Phi}{2} |\Phi|^4$$

- The covariant derivative of Φ is $D_\mu \Phi = (\partial_\mu - \mathrm{i} q_\Phi g_X X_\mu)\Phi$
- The field strength tensor of the $\mathrm{U}(1)_\mathrm{X}$ **gauge field** X^μ is $X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$
- Assume a **Mexican-hat potential** $V(\Phi)$ with **degenerate vacua** $\langle \Phi \rangle = v_\Phi \mathrm{e}^{\mathrm{i}\varphi}/\sqrt{2}$



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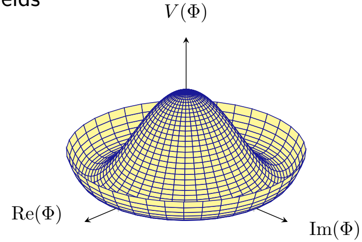
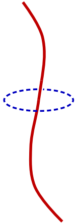
Assume a **Mexican-hat potential** $V(\Phi)$ with **degenerate vacua** $\langle \Phi \rangle = v_\Phi \mathrm{e}^{\mathrm{i}\varphi}/\sqrt{2}$

The **spontaneous breaking** of the $\mathrm{U}(1)_\mathrm{X}$ **gauge symmetry** in the early Universe would induce **cosmic strings**, which are concentrated with energies of the scalar and gauge fields





Degenerate vacua

$v_\Phi \mathrm{e}^{\mathrm{i}\varphi}/\sqrt{2}$

$\varphi = \varphi + 2\pi n$

$n \neq 0$ leads to

cosmic stirngs

Cosmic String Tension

A **network** of **cosmic strings** would be formed in the early universe after the spontaneous breaking of the $\mathrm{U}(1)_\mathrm{X}$ gauge symmetry

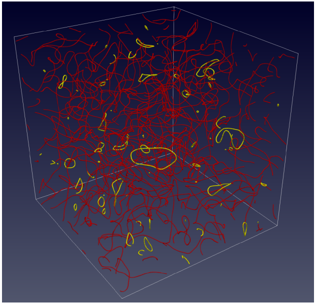
The **tension** of **cosmic string** μ (energy per unit length) can be estimated as

$$\mu \simeq \begin{cases} 1.19\pi v_\Phi^2 b^{-0.195}, & 0.01 < b < 100, \\ \dfrac{2.4\pi v_\Phi^2}{\ln b}, & b > 100, \end{cases} \qquad b \equiv \frac{2q_\Phi^2 g_X^2}{\lambda_\Phi}$$

[Hill, Hodges, Turner, PRD **37**, 263 (1988)]

As $\mu \propto v_\Phi^2$, a **high symmetry-breaking scale** v_Φ would lead to cosmic strings with **high tension**

Denoting G as the **Newtonian constant of gravitation**, the **dimensionless quantity** $G\mu$ is commonly used to describe the **tension** of cosmic strings



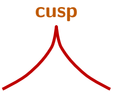
[Kitajima, Nakayama, 2212.13573, JHEP]

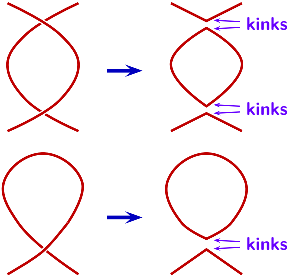
Gravitational Waves from Cosmic Strings

- According to the analysis of string dynamics, the **intersections** of **long strings** could produce **closed loops**, whose size is smaller than the Hubble radius

- **Cosmic string loops** could further fragment into **smaller loops** or reconnect to **long strings**

- Loops typically have localized features called **"cusps"** and **"kinks"**





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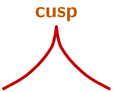
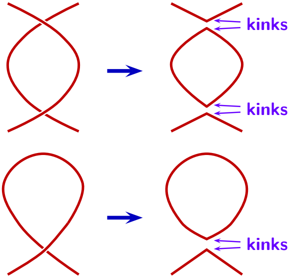
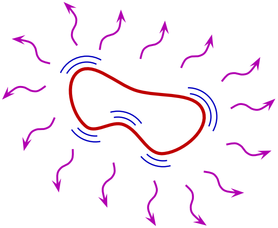




🔊 The **relativistic oscillations** of the **loops** due to their **tension** emit **Gravitational Waves (GWs)**, and the loops would **shrink** because of **energy loss**



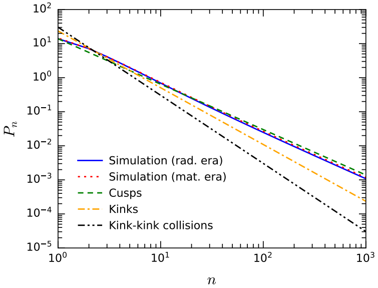
🔔 Moreover, the **cusps** and **kinks** propagating along the loops could produce **GW bursts** [Damour & Vilenkin, gr-qc/0004075, PRL]

Power of Gravitational Radiation

At the **emission time** t_e, a **cosmic string loop** of **length** L emits GWs with **frequencies** $f_\mathrm{e} = \dfrac{2n}{L}$

$n = 1, 2, 3, \cdots$ denotes the **harmonic modes** of the loop oscillation

Denoting P_n as the **power** of **gravitational radiation** for the harmonic mode n in units of $G\mu^2$, the total power is given by $P = G\mu^2 \sum_n P_n$



According to the **simulation** of **smoothed cosmic string loops** [Blanco-Pillado & Olum, 1709.02693, PRD], P_n for loops in the **radiation** and **matter** eras are obtained

The **total dimensionless power** $\Gamma = \sum_n P_n$ is estimated to be ~ 50

For comparison, analytic studies show that $P_n \simeq \dfrac{\Gamma}{\zeta(q) n^q}$ with $q = \dfrac{4}{3}, \dfrac{5}{3}, 2$ for **cusps**, **kinks**, and **kink-kink collisions**

Stochastic GW Background Induced by Cosmic Strings

🔌 The **energy** of **cosmic strings** is converted into the **energy** of **GWs**, and an **stochastic GW background (SGWB)** is formed due to **incoherent superposition**

🏮 The **SGWB energy density** ρ_{GW} per unit frequency at the present is

$$\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}f} = G\mu^2 \int_0^{z_*} \frac{1}{H(z)(1+z)^6} \sum_n \frac{2nP_n}{f^2}\, \mathsf{n}\left(\frac{2n}{f(1+z)}, t(z)\right) \mathrm{d}z$$

🪔 $\mathsf{n}(L,t)\,\mathrm{d}L$ is the **number density** of **cosmic string loops** at cosmic time t in length interval $\mathrm{d}L$

🕯️ $H(z)$ is the Hubble rate and z_* is the redshift where the GW emissions start

💡 The **SGWB spectrum** is often represented by

$$\Omega_{\mathrm{GW}}(f) = \frac{1}{\rho_{\mathrm{c}}} \frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}\ln f} = \frac{f}{\rho_{\mathrm{c}}} \frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}f}$$

🔦 $\rho_{\mathrm{c}} = \dfrac{3H_0^2}{8\pi G}$ is the critical density

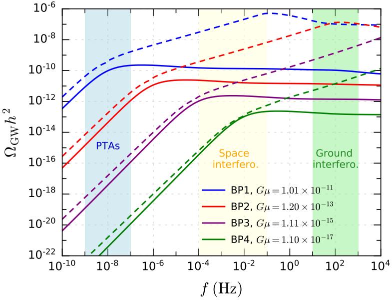
Loop Number Density: BOS model

- There are various approaches for modeling the **loop number density** $\mathsf{n}(L,t)$
- The **BOS model** [Blanco-Pillado, Olum & Shlaer, 1309.6637, PRD] extrapolates the loop production function found in simulations of Nambu-Goto strings
- The loop number densities produced in the **radiation** and **matter** era, and that **produced in the radiation era and still surviving in the matter era** are given by

$$\mathsf{n}_\mathrm{r}(L,t) \simeq \frac{0.18\,\theta(0.1t-L)}{t^4(\gamma+\gamma_\mathrm{d})^{5/2}}$$

$$\mathsf{n}_\mathrm{m}(L,t) \simeq \frac{(0.27-0.45\gamma^{0.31})\,\theta(0.18t-L)}{t^4(\gamma+\gamma_\mathrm{d})^{2}}$$

$$\mathsf{n}_{\mathrm{r}\to\mathrm{m}}(L,t) \simeq \frac{0.18t_\mathrm{eq}^{1/2}\,\theta(0.09t_\mathrm{eq}-\gamma_\mathrm{d}t-L)}{t^{9/2}(\gamma+\gamma_\mathrm{d})^{5/2}}$$



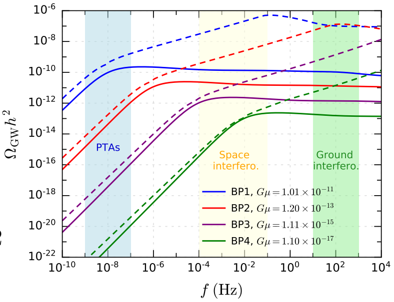
BOS model: solid lines

- $\gamma \equiv \dfrac{L}{t}$ is a **dimensionless variable**
- $\gamma_\mathrm{d} = -\dfrac{\mathrm{d}L}{\mathrm{d}t} \simeq \Gamma G\mu$ is the **loop shrinking rate**
- $t_\mathrm{eq} = 51.1 \pm 0.8\ \mathrm{kyr}$ is the cosmic time at the **matter-radiation equality**

Loop Number Density: LRS model

The **LRS model** [Lorenz, Ringeval & Sakellariadou, 1006.0931, JCAP] takes into account the **gravitational backreaction effect**, which prevents loop production below a certain scale $\gamma_\mathrm{c} \simeq 20(G\mu)^{1+2\chi}$ [Polchinski & Rocha, gr-qc/0702055, PRD]

$$\mathsf{n}(L,t) \simeq \begin{cases} \dfrac{C}{t^4(\gamma+\gamma_\mathrm{d})^{3-2\chi}}, & \gamma_\mathrm{d} < \gamma \\ \dfrac{(3\nu-2\chi-1)C}{2t^4(1-\chi)\gamma_\mathrm{d}\gamma^{2(1-\chi)}}, & \gamma_\mathrm{c} < \gamma < \gamma_\mathrm{d} \\ \dfrac{(3\nu-2\chi-1)C}{2t^4(1-\chi)\gamma_\mathrm{d}\gamma_\mathrm{c}^{2(1-\chi)}}, & \gamma < \gamma_\mathrm{c} \end{cases}$$



LRS model: dashed lines

Radiation era: $\nu = 1/2$, $C \simeq 0.0796$, $\chi \simeq 0.2$

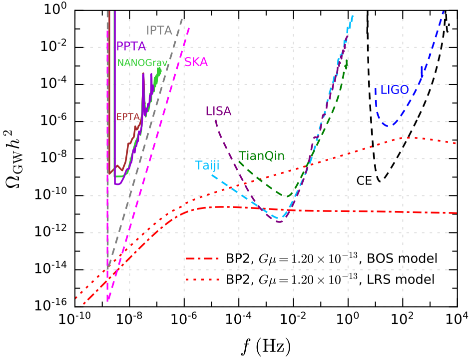
Matter era: $\nu = 3/2$, $C \simeq 0.0157$, $\chi \simeq 0.295$

Smaller $G\mu$ means smaller GW emission power, and loops could survive longer, leading to **more smaller loops** radiating at **higher f**

The **LRS model** gives a **very high number density** of **small loops** in the $\gamma < \gamma_c$ regime, which significantly contribute to **high frequency GWs**

GW Experiments

- The **SGWB** originating from **cosmic strings** covers an **extremely broad range** of **GW frequencies**
- It is an interesting target for various types of **GW experiments**
- **Pulsar timing arrays (PTAs)** in 10^{-9}–10^{-7} Hz: **NANOGrav**, **PPTA**, **EPTA**, **CPTA**, **IPTA**, **SKA**, $\cdots$
- **Ground-based interferometers** in 10–10^{3} Hz: **LIGO**, **Virgo**, **KAGRA**, **CE**, **ET**, $\cdots$
- **Space-borne interferometers** in 10^{-4}–10^{-1} Hz: **LISA**, **TianQin**, **Taiji**, **BBO**, **DECIGO**, $\cdots$



Constraints and Sensitivity of GW Experiments

🦁 We study the **SGWB** from **cosmic strings** generated in a UV-complete model for **pNGB dark matter** (DM) with a **spontaneously broken** $\mathrm{U}(1)_\mathrm{X}$ **gauge symmetry** [DY Liu, CF Cai, XM Jiang, **ZHY**, HH Zhang, 2208.06653, JHEP]

🐯 The DM candidate in this model can **naturally evade direct detection bounds**

🐮 The **bound** on the **DM lifetime** implies a symmetry-breaking scale $v_\Phi > 10^9$ GeV

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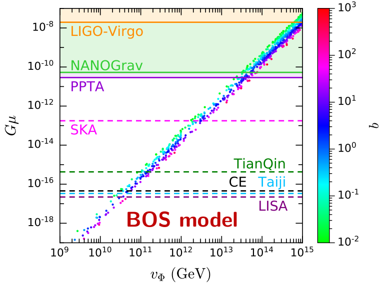
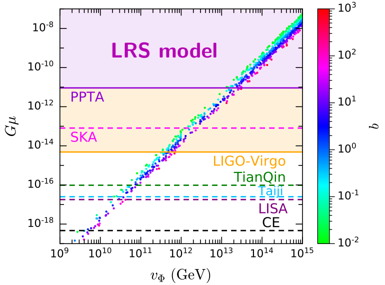
- The DM candidate in this model can **naturally evade direct detection bounds**
- The **bound** on the **DM lifetime** implies a symmetry-breaking scale $v_\Phi > 10^9$ GeV
- Constraints from **LIGO-Virgo**, **NANOGrav**, and **PPTA** have excluded the parameter points with $v_\Phi \gtrsim 5 \times 10^{13}$ (7×10^{11}) GeV
- The future experiment **LISA** (**CE**) can probe v_Φ down to $\sim 2 \times 10^{10}$ (5×10^9) GeV assuming the **BOS** (**LRS**) model for loop production

[ZY Qiu, **ZHY**, 2304.02506, CPC]





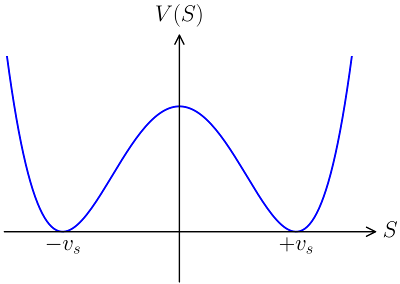
Domain Walls

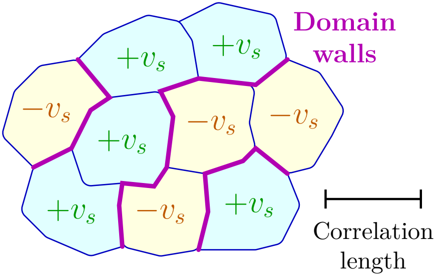
☯ **Domain walls (DWs)** are **two-dimensional topological defects** which could be formed when a **discrete symmetry** of the **scalar potential** is **spontaneously broken** in the early universe

♊ They are **boundaries** separating spatial regions with different **degenerate vacua**

🚫 **Stable DWs** are thought to be a **cosmological problem** [Zeldovich, Kobzarev, Okun, Zh.Eksp.Teor.Fiz. **67** (1974) 3]

⚠ As the universe expands, the **DW energy density** decreases **slower** than radiation and matter, and would soon **dominate** the total energy density



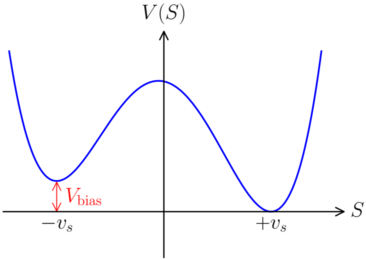


Collapsing Domain Walls

It is **allowed** if **DWs collapse** at a very early epoch [Vilenkin, PRD **23** (1981) 852; Gelmini, Gleiser, Kolb, PRD **39** (1989) 1558; Larsson, Sarkar, White, hep-ph/9608319, PRD]

Such **unstable DWs** can be realized if the **discrete symmetry** is **explicitly broken** by a **small potential term** that gives an **energy bias** among the minima of the potential

The bias induces a **volume pressure force** acting on the DWs that leads to their collapse



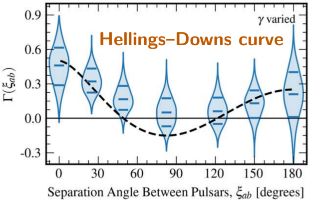
Collapsing DWs significantly produce **GWs** [Preskill *et al.*, NPB 363 (1991) 207; Gleiser, Roberts, astro-ph/9807260, PRL; Hiramatsu, Kawasaki, Saikawa, 1002.1555, JCAP]

A **SGWB** would be formed and remain to the present time

It could be the one probed by **recent PTA experiments**

Strong Evidence for a nHz SGWB from PTAs

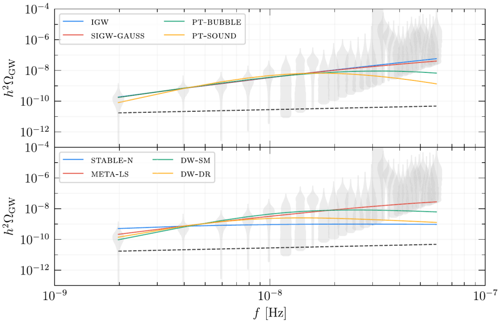
On June 29, four **pulsar timing array (PTA)** collaborations **NANOGrav** [2306.16213, 2306.16219, ApJL], **CPTA** [2306.16216, RAA], **PPTA** [2306.16215, ApJL], and **EPTA** [2306.16214, 2306.16227] reported **strong evidence** for a nHz **stochastic gravitational wave background (SGWB)** with expected **Hellings–Downs correlations**



[NANOGrav Coll., 2306.16213]

Potential **gravitational wave (GW) sources** include

- **Supermassive black hole binaries**
- **Inflation**
- **Scalar-induced GWs**
- **First-order phase transitions**
- **Cosmic strings**
- **Collapsing domain walls**



[NANOGrav Coll., 2306.16219]

Spontaneously Broken Z_2 Symmetry

🌈 We consider a **real scalar field** S with a **spontaneously broken** Z_2**-symmetric potential** as the **origin** of **DWs** [Zhang, Cai, Su, Wang, **ZHY**, Zhang, 2307.11495, PRD]

⛱ The **Lagrangian** is $\mathcal{L} = \frac{1}{2}(\partial_\mu S)\partial^\mu S + (D_\mu H)^\dagger D^\mu H - V_{Z_2}$ with a Z_2**-conserving potential** $V_{Z_2} = -\frac{1}{2}\mu_S^2 S^2 + \mu_H^2 |H|^2 + \frac{1}{4}\lambda_S S^4 + \lambda_H |H|^4 + \frac{1}{2}\lambda_{HS}|H|^2 S^2$

💧 H is the **standard model (SM) Higgs field** and S is a **SM gauge singlet**

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💧 H is the **standard model (SM) Higgs field** and S is a **SM gauge singlet**

🌀 $\mathcal{L}$ respects a Z_2 **symmetry** $S \to -S$, which is **spontaneously broken** as S gains nonzero **vacuum expectation values (VEVs)** $\langle S \rangle = \pm v_s$ with $v_s \gg v$ for $\mu_S^2 > 0$

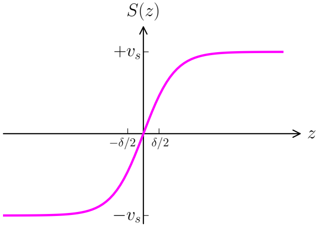
❄️ Assuming $\mu_H^2 > 0$ and $\lambda_{HS} < 0$, the effective quadratic parameter for H becomes $\mu_H^2 + \lambda_{HS} v_s^2/2 < 0$, resulting in a nonzero Higgs VEV $\langle H \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$ and the **spontaneous breaking** of the **electroweak symmetry**

🔥 The **electroweak** and Z_2 **symmetries** would be **restored** at **sufficiently high temperatures** due to **thermal corrections** to the scalar potential

Kink Solution

A **DW** corresponds to a **kink solution** of the equation of motion for S given by

$$S(z) = v_s \tanh\frac{z}{\delta}, \quad \delta \equiv \left(\sqrt{\frac{\lambda_S}{2}}\, v_s\right)^{-1}$$



$S(z)$ approaches the **VEVs** $\pm v_s$ for $z \to \pm\infty$

The **DW** locates at $z = 0$ with a **thickness** δ, separating **two domains** with $S(z) > 0$ and $S(z) < 0$

The **DW tension** (**surface energy density**) is $\sigma = \dfrac{4}{3}\sqrt{\dfrac{\lambda_S}{2}}\, v_s^3$

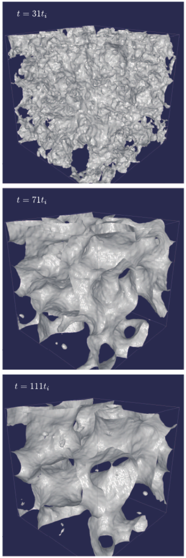
Inside each domain with $S \sim S(\pm\infty) \approx \pm v_s$, we can parametrize H and S as

$$H(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}, \quad S(x) = \pm v_s + s(x)$$

Assuming $v_s \gg v$ and $\lambda_{HS}^2 \ll \lambda_H \lambda_S$, the masses squared of the **scalar bosons** h and s are given by $m_h^2 \approx 2\lambda_H v^2$ and $m_s^2 \approx 2\lambda_S v_s^2$

Evolution of Domain Walls

- After DWs are created, their **tension** σ acts to **stretch** them up to the **horizon size** if the **friction** is **negligible**, and they would enter the **scaling regime** with **energy density** $\rho_{\mathrm{DW}} = \dfrac{\mathcal{A}\sigma}{t}$
- $\mathcal{A} \approx 0.8 \pm 0.1$ is a numerical factor given by lattice simulation
- $\rho_{\mathrm{DW}} \propto t^{-1}$ implies that DWs are **diluted more slowly** than **radiation** and **matter**
- If DWs are **stable**, they would soon **dominate** the evolution of the universe, **conflicting** with cosmological observations



[Hiramatsu *et al.*, 1002.1555]

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This can be evaded by an **explicit Z_2-violating potential**

$$V_{\mathrm{vio}} = \kappa_1 S + \frac{\kappa_3}{6} S^3$$

V_{vio} generates a **small energy bias** between the two minima

It leads to a **volume pressure force** acting on the DWs, making the **DWs collapse** and the **false vacuum domains shrink**

[Hiramatsu *et al.*, 1002.1555]

Energy Bias and Annihilation Temperature

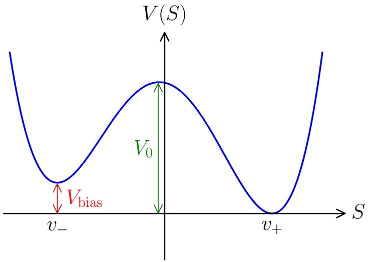
With the Z_2**-violating potential** V_{vio}, the **two minima** are shifted to

$$v_\pm \approx \pm v_s - \delta, \text{ with } \delta \approx \frac{2\kappa_1 + \kappa_3 v_s^2}{4\lambda_S v_s^2}$$

The **energy bias** between **the minima** is

$$V_{\text{bias}} = V(v_-) - V(v_+) = \frac{4}{3}\epsilon v_s^4$$

$$\epsilon = -\frac{6\kappa_1 + \kappa_3 v_s^2}{4v_s^3}$$



DWs collapse when the **pressure force** becomes **larger** than the **tension force**

Consequently, the **annihilation temperature** of DWs can be estimated as

$$T_{\text{ann}} = 34.1 \text{ MeV } \mathcal{A}^{-1/2} \left[\frac{g_*(T_{\text{ann}})}{10}\right]^{-1/4} \left(\frac{\sigma}{\text{TeV}^3}\right)^{-1/2} \left(\frac{V_{\text{bias}}}{\text{MeV}^4}\right)^{1/2}$$

$$= 76.3 \text{ MeV } \mathcal{A}^{-1/2} \left[\frac{g_*(T_{\text{ann}})}{10}\right]^{-1/4} \left(\frac{0.2}{\lambda_S}\frac{m_s}{10^5 \text{ GeV}}\frac{\epsilon}{10^{-26}}\right)^{1/2}$$

SGWB Spectrum from Collapsing DWs

- The **SGWB spectrum** is commonly characterized by $\Omega_{\mathrm{GW}}(f) = \dfrac{f}{\rho_{\mathrm{c}}} \dfrac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}f}$
- ρ_{GW} is the **GW energy density**, and ρ_{c} is the critical energy density
- The SGWB from **collapsing DWs** can be estimated by **numerical simulations**

  [Hiramatsu, Kawasaki, Saikawa, 1002.1555, 1309.5001, JCAP]

- The **present SGWB spectrum** induced by collapsing DWs can be evaluated by

$$\Omega_{\mathrm{GW}}(f)h^2 = \Omega_{\mathrm{GW}}^{\mathrm{peak}} h^2 \times \begin{cases} \left(\dfrac{f}{f_{\mathrm{peak}}}\right)^3, & f < f_{\mathrm{peak}} \\ \dfrac{f_{\mathrm{peak}}}{f}, & f > f_{\mathrm{peak}} \end{cases}$$

$$\Omega_{\mathrm{GW}}^{\mathrm{peak}} h^2 = 7.2 \times 10^{-18}\ \tilde{\epsilon}_{\mathrm{GW}} \mathcal{A}^2 \left[\frac{g_{*s}(T_{\mathrm{ann}})}{10}\right]^{-4/3} \left(\frac{\sigma}{1\ \mathrm{TeV}^3}\right)^2 \left(\frac{T_{\mathrm{ann}}}{10\ \mathrm{MeV}}\right)^{-4}$$

$$f_{\mathrm{peak}} = 1.1 \times 10^{-9}\ \mathrm{Hz} \left[\frac{g_*(T_{\mathrm{ann}})}{10}\right]^{1/2} \left[\frac{g_{*s}(T_{\mathrm{ann}})}{10}\right]^{-1/3} \frac{T_{\mathrm{ann}}}{10\ \mathrm{MeV}}$$

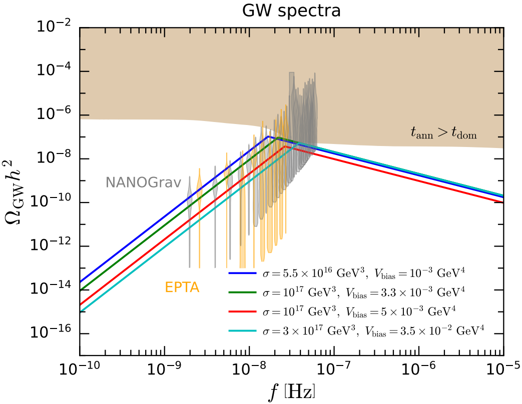
- $\tilde{\epsilon}_{\mathrm{GW}} = 0.7 \pm 0.4$ is derived from numerical simulation

Comparison [Z Zhang, CF Cai, YH Su, SY Wang, **ZHY**, HH Zhang, 2307.11495, PRD]

🎨 Comparing with the **reconstructed posterior distributions** for the **NANOGrav** and **EPTA** nHz GW signals, we find that the **GW spectra** from **collapsing DWs** with $\sigma \sim \mathcal{O}(10^{17})\ \mathrm{GeV}^3$ and $V_\mathrm{bias} \sim \mathcal{O}(10^{-3})\ \mathrm{GeV}^4$ can explain the **PTA observations**

🖼 The **brown region** is **excluded** by the requirement that **DWs** should **annihilate before** they **dominate** the universe

$$\sigma = 10^{17}\ \mathrm{GeV}^3$$
$$V_\mathrm{bias} = 3.3 \times 10^{-3}\ \mathrm{GeV}^4$$
$$\lambda_S = 0.2$$
$$v_s = 6.2 \times 10^5\ \mathrm{GeV}$$
$$m_s = 3.9 \times 10^5\ \mathrm{GeV}$$
$$\epsilon = 3.6 \times 10^{-26}$$
$$T_\mathrm{ann} = 163\ \mathrm{MeV}$$
$$\Omega_\mathrm{GW}^\mathrm{peak} h^2 = 9.4 \times 10^{-8}$$
$$f_\mathrm{peak} = 2.2 \times 10^{-8}\ \mathrm{Hz}$$

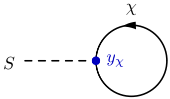


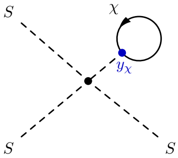
Loop-induced Z_2-violating Potential

- The **PTA GW signals** require a **very small** $V_{\text{bias}} = \frac{4}{3}\epsilon v_s^4$ with $\epsilon \sim \mathcal{O}(10^{-26})$
- We consider V_{bias} to be generated by **loops** of **fermionic dark matter** through a **feeble Yukawa interaction** with the **scalar field** S
- Assume a Lagrangian with a **Dirac fermion field** χ: $\mathcal{L}_\chi = \bar{\chi}(i\partial\!\!\!/ - m_\chi)\chi + y_\chi S\bar{\chi}\chi$
- y_χ is the **Yukawa coupling constant**
- When S acquires the VEV $\langle S\rangle \approx \pm v_s$, the χ mass becomes $m_\chi^{(\pm)} \approx m_\chi \mp y_\chi v_s$
- We assume that $m_\chi \gg y_\chi v_s$, so $m_\chi^{(\pm)} \approx m_\chi$ holds
- The $S\bar{\chi}\chi$ **coupling explicitly breaks** the Z_2 **symmetry** even if the **tree-level** Z_2**-violating potential** is **absent**
- The ϵ **value** at the m_s scale induced by χ **loops** is

$$\epsilon(m_s) \approx \frac{3\lambda_S^{3/2} y_\chi}{\sqrt{2}\pi^2}\left(\frac{m_\chi}{m_s}\right)^3 \ln\frac{\Lambda_{\text{UV}}}{m_s}$$

- Here, $\epsilon = 0$ at a **UV scale** Λ_{UV} is assumed



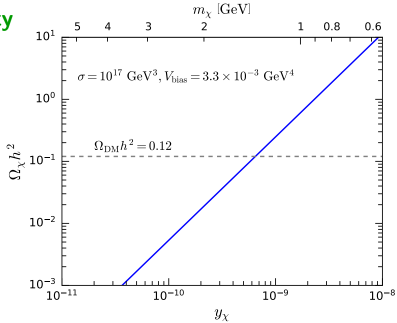


Freeze-in Dark Matter

- After reheating, s **bosons** are in **thermal equilibrium** with the SM particles, while χ **fermions** would be **out of equilibrium** with $n_\chi \approx 0$ for a **feeble coupling** y_χ
- In this case, χ **fermions** could be **produced** via the s **decay** $s \to \chi\bar{\chi}$, but **never reach thermal equilibrium** if y_χ is **extremely small**, say, $y_\chi \sim \mathcal{O}(10^{-10})$
- This is the **freeze-in mechanism** of DM production [Hall *et al.*, 0911.1120, JHEP]
- χ acts as a **DM candidate** with a **relic density**

$$\Omega_\chi h^2 \approx 8.13 \times 10^{22}\ \frac{y_\chi^2 m_\chi}{m_s}$$

- Both the **extremely tiny** $\epsilon \sim \mathcal{O}(10^{-26})$ and the **observed DM relic density** $\Omega_{\mathrm{DM}} h^2 = 0.1200 \pm 0.0012$ can be **naturally explained** by the **feeble Yukawa coupling** $y_\chi \sim \mathcal{O}(10^{-10})$

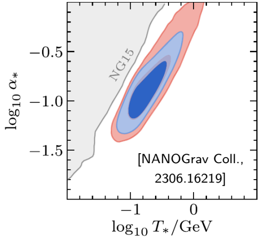


Favored Parameter Regions

The **NANOGrav collaboration** has reconstructed the posterior distributions of $(T_{\mathrm{ann}}, \alpha_*)$ accounting for the **observed nHz GW signal**, where

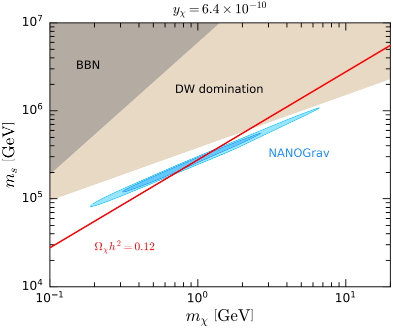
$$\alpha_* \equiv \left.\frac{\rho_{\mathrm{DW}}}{\rho_{\mathrm{rad}}}\right|_{T=T_{\mathrm{ann}}}$$

$$= 0.035 \left[\frac{10}{g_*(T_{\mathrm{ann}})}\right]^{1/2} \frac{\mathcal{A}}{0.8} \frac{0.2}{\lambda_S} \left(\frac{m_s}{10^5\ \mathrm{GeV}}\right)^3 \left(\frac{100\ \mathrm{MeV}}{T_{\mathrm{ann}}}\right)^2$$



We apply this result to our model and find the **favored parameter regions**

- **Deep** and **light** **blue regions** corresponds to the **68%** and **95%** **Bayesian credible regions** favored by the **NANOGrav data**, respectively
- **Brown** and **gray** regions are excluded because DWs would **dominate the universe** and would inject energetic particles to **affect the Big Bang Nucleosynthesis**, respectively



Summary

- In the early Universe, the **spontaneous breaking** of **symmetries** could leads to **topological defects**, such as **monopoles**, **cosmic strings**, and **domain walls**
- **Cosmic strings** or **collapsing domain walls** may results in a **stochastic GW background**, which could be probed in GW experiments
- We have studied the possible links to **dark matter** and to the recent observations of a **nHz SGWB** by **PTA collaborations NANOGrav, EPTA, CPTA, and PPTA**

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Thanks for your attention!

Original pNGB Dark Matter [Gross, Lebedev, Toma, 1708.02253, PRL]

- **Standard model (SM) Higgs doublet H, complex scalar S** (SM singlet)
- Scalar potential respects a **softly broken global $\mathrm{U}(1)$ symmetry** $S \to e^{i\alpha} S$
- $\mathrm{U}(1)$ **symmetric:** $V_0 = -\dfrac{\mu_H^2}{2}|H|^2 - \dfrac{\mu_S^2}{2}|S|^2 + \dfrac{\lambda_H}{2}|H|^4 + \dfrac{\lambda_S}{2}|S|^4 + \lambda_{HS}|H|^2|S|^2$
- **Soft breaking:** $V_{\mathrm{soft}} = -\dfrac{\mu_S'^2}{4} S^2 + \mathrm{H.c.}$

Approximate global $\mathrm{U}(1)$

⬇ v_S

- H and S develop **vacuum expectation values (VEVs)**

$$H \to \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h \end{pmatrix}, \quad S = \frac{1}{\sqrt{2}}(v_s + s + i\chi)$$

Z_2 symmetry

- The **soft breaking term V_{soft}** give a mass to χ: $m_\chi = \mu_S'$
- A **Z_2 symmetry $\chi \to -\chi$** remains after **$\mathrm{U}(1)$ spontaneous symmetry breaking**
- The **DM candidate χ** is a **stable pseudo-Nambu-Goldstone boson (pNGB)**
- Rotate **CP-even Higgs bosons h** and s to **mass eigenstates h_1** and h_2

$$\begin{pmatrix} h \\ s \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}, \quad m_{h_1,h_2}^2 = \frac{1}{2}\left(\lambda_H v^2 + \lambda_S v_s^2 \mp \frac{\lambda_S v_s^2 - \lambda_H v^2}{\cos 2\theta}\right)$$

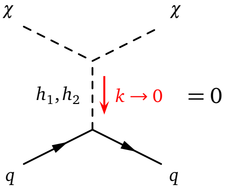
DM-nucleon Scattering [Gross, Lebedev, Toma, 1708.02253, PRL]

DM-quark interactions induce **DM-nucleon** scattering in direct detection

DM-quark scattering amplitude from Higgs portal interactions

$$\mathcal{M}(\chi q \to \chi q) \propto \frac{m_q s_\theta c_\theta}{v v_s} \left(\frac{m_{h_1}^2}{t - m_{h_1}^2} - \frac{m_{h_2}^2}{t - m_{h_2}^2} \right)$$

$$= \frac{m_q s_\theta c_\theta}{v v_s} \frac{t(m_{h_1}^2 - m_{h_2}^2)}{(t - m_{h_1}^2)(t - m_{h_2}^2)}$$



Zero momentum transfer limit $t = k^2 \to 0, \quad \mathcal{M}(\chi q \to \chi q) \to 0$

DM-nucleon scattering cross section **vanishes** at tree level

Tree-level interactions of a **pNGB** are generally **momentum-suppressed**

One-loop corrections typically lead to $\sigma_{\chi N}^{\mathrm{SI}} \lesssim \mathcal{O}(10^{-50})$ cm^2

[Azevedo *et al.*, 1810.06105, JHEP; Ishiwata & Toma, 1810.08139, JHEP]

Beyond capability of current and near future direct detection experiments

UV Completion of pNGB DM

🔒 In the **original pNGB DM model**, the term $V_{\text{soft}} = -\frac{\mu_S'^2}{4}(S^2 + S^{\dagger 2})$, which **softly breaks** the $\mathrm{U}(1)$ **global symmetry** $S \to e^{i\alpha} S$ into a Z_2 symmetry, is ***ad hoc***

🔒 Other soft breaking terms, such as a trilinear term $\propto S^3 + S^{\dagger 3}$, would **spoil** the **vanishing scattering amplitude**

🔑 It demands an appropriate **ultraviolet (UV) completion** to **realize only** V_{soft}

🔨 A possible UV completion is to **gauge the** $\mathrm{U}(1)$ **symmetry** with $B - L$ **charges**

[Abe, Toma & Tsumura, 2001.03954, JHEP; Okada, Raut & Shafi, 2001.05910, PRD]

🪓 We consider another option that pNGB DM arises from a **hidden** $\mathrm{U}(1)_\mathrm{X}$ **gauge symmetry**, where all the SM fields **do not** carry $\mathrm{U}(1)_\mathrm{X}$ charges

[DY Liu, CF Cai, XM Jiang, **ZHY**, HH Zhang, 2208.06653, JHEP]

⚔️ The **gauge anomalies** are **canceled** **without** introducing **right-handed neutrinos**, so **less fields** are involved in this setup

UV Completion with a Hidden $\mathrm{U}(1)_\mathrm{X}$ Gauge Symmetry

We introduce two **complex scalar fields** S and Φ carrying $\mathrm{U}(1)_\mathrm{X}$ **charges** 1 and 2

$$D_\mu S = (\partial_\mu - \mathrm{i} g_X X_\mu) S, \quad D_\mu \Phi = (\partial_\mu - 2\mathrm{i} g_X X_\mu)\Phi$$

$$\mathcal{L} \supset (D^\mu H)^\dagger (D_\mu H) + (D^\mu S)^\dagger (D_\mu S) + (D^\mu \Phi)^\dagger (D_\mu \Phi) - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} - \frac{1}{4} X^{\mu\nu} X_{\mu\nu}$$
$$- \frac{s_\varepsilon}{2} B^{\mu\nu} X_{\mu\nu} + \mu_H^2 |H|^2 + \mu_S^2 |S|^2 + \mu_\Phi^2 |\Phi|^2 - \frac{\lambda_H}{2}|H|^4 - \frac{\lambda_S}{2}|S|^4 - \frac{\lambda_\Phi}{2}|\Phi|^4$$
$$- \lambda_{HS}|H|^2|S|^2 - \lambda_{H\Phi}|H|^2|\Phi|^2 - \lambda_{S\Phi}|S|^2|\Phi|^2 + \frac{\mu_{S\Phi}}{\sqrt{2}}(\Phi^\dagger S^2 + \Phi S^{\dagger 2})$$

The $B^{\mu\nu} X_{\mu\nu}$ term implies a **kinetic mixing** between the $\mathrm{U}(1)_\mathrm{Y}$ gauge field B^μ and the $\mathrm{U}(1)_\mathrm{X}$ **gauge field** X^μ with a mixing parameter $s_\varepsilon \equiv \sin\varepsilon \in (-1, 1)$

S and Φ develop **nonzero VEVs** v_S and v_Φ with a **hierarchy** $v_S \sim v \ll v_\Phi$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}, \quad S = \frac{1}{\sqrt{2}}(v_S + s + \mathrm{i}\eta_S), \quad \Phi = \frac{1}{\sqrt{2}}(v_\Phi + \phi + \mathrm{i}\eta_\Phi)$$

The v_Φ **contribution** to the $\Phi^\dagger S^2$ **term** leads to the **desired soft breaking term**

$$V_\mathrm{soft} = -\frac{\mu_S'^2}{4}(S^2 + S^{\dagger 2}) \text{ with } \mu_S'^2 = 2\mu_{S\Phi} v_\Phi$$

Physical Scalars

Rotate the scalars from the interaction bases to the mass bases

$$\begin{pmatrix} h \\ s \\ \phi \end{pmatrix} = U \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}, \quad \begin{pmatrix} \eta_S \\ \eta_\Phi \end{pmatrix} = V \begin{pmatrix} \chi \\ \tilde{\chi} \end{pmatrix}$$

h_1 (SM-like), h_2, and h_3 are CP**-even Higgs bosons**, and $\tilde{\chi}$ is a **massless Nambu-Goldstone boson** associated with the $\mathrm{U}(1)_\mathrm{X}$ **gauge symmetry breaking**

χ is a **pNGB DM candidate** with a mass squared of $m_\chi^2 = \frac{\mu_{S\Phi}}{2v_\Phi}(v_S^2 + 4v_\Phi^2)$

v_Φ represents a **UV scale** that breaks the $\mathrm{U}(1)_\mathrm{X}$ **gauge symmetry** into an **approximate** $\mathrm{U}(1)_\mathrm{X}$ **global symmetry**

Below the **lower scale** v_S, the **global** $\mathrm{U}(1)_\mathrm{X}$ is spontaneously broken, resulting in **pNGB DM**

In the **limit** $v_\Phi \to \infty$ and $\mu_{S\Phi} \to 0$ with **finite** $\mu_S'^2$, the **original pNGB DM model** is recovered

Gauge $\mathrm{U}(1)_\mathrm{X}$

UV scale ⬇️ v_Φ

Approximate global $\mathrm{U}(1)_\mathrm{X}$

Lower scale ⬇️ v_S

Approximate Z_2

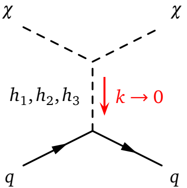
Direct Detection

🦊 The **UV completion** gives $\mu_S'^2$ a **dynamical origin**, but inevitably introduces the χ-χ-ϕ **coupling**, leading to a **nonvanishing** χ-nucleon scattering amplitude

🐷 χN **scattering cross section** is **highly suppressed by** v_Φ^{-4}

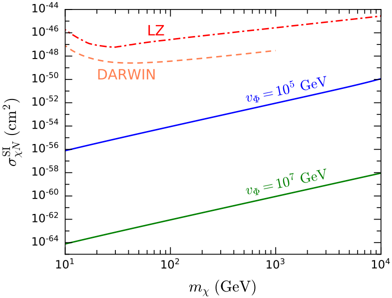
$$\sigma_{\chi N}^{\mathrm{SI}} \simeq \frac{\tilde{\lambda}^2 m_N^4 m_\chi^4 [2 + 7(f_u^N + f_d^N + f_s^N)]^2}{1296\pi (m_N + m_\chi)^2 v^4 v_\Phi^4} + \mathcal{O}(v_\Phi^{-6})$$

$$\tilde{\lambda} = \frac{\lambda_{H\Phi}\lambda_{S\Phi} - \lambda_\Phi \lambda_{HS} + 2\lambda_{HS}\lambda_{S\Phi} - 2\lambda_S \lambda_{H\Phi}}{\lambda_H \lambda_S \lambda_\Phi + 2\lambda_{HS}\lambda_{H\Phi}\lambda_{S\Phi} - \lambda_S \lambda_{H\Phi}^2 - \lambda_\Phi \lambda_{HS}^2 - \lambda_H \lambda_{S\Phi}^2}$$



🐶 $v_\Phi = 10^5$ GeV can result in $\sigma_{\chi N}^{\mathrm{SI}}$ **much smaller** than 90% C.L. upper limits from the **LZ experiment** [2207.03764], and even **beyond the reach** of the future **DARWIN experiment** with a 200 t $\cdot$ yr exposure [1606.07001, JCAP]

$$v_S = 1 \text{ TeV}, \quad m_{h_2} = 300 \text{ GeV}, \quad m_{h_3} = 0.1 v_\Phi$$
$$\lambda_{HS} = 0.03, \quad \lambda_{H\Phi} = \lambda_{S\Phi} = 0.01$$



Neutral Gauge Boson Mixing

Transform the **gauge basis** (B_μ, W^3_μ, X_μ) to the **mass basis** (A_μ, Z_μ, Z'_μ)

$$\begin{pmatrix} B_\mu \\ W^3_\mu \\ X_\mu \end{pmatrix} = V_{\mathrm{K}}(\varepsilon)\, R_3(\hat{\theta}_W)\, R_1(\xi) \begin{pmatrix} A_\mu \\ Z_\mu \\ Z'_\mu \end{pmatrix}$$

$$V_{\mathrm{K}}(\varepsilon) = \begin{pmatrix} 1 & & -t_\varepsilon \\ & 1 & \\ 0 & & 1/c_\varepsilon \end{pmatrix}, \quad R_3(\hat{\theta}_W) = \begin{pmatrix} \hat{c}_{\mathrm{W}} & -\hat{s}_{\mathrm{W}} & \\ \hat{s}_{\mathrm{W}} & \hat{c}_{\mathrm{W}} & \\ & & 1 \end{pmatrix}, \quad R_1(\xi) = \begin{pmatrix} 1 & & \\ & c_\xi & -s_\xi \\ & s_\xi & c_\xi \end{pmatrix}$$

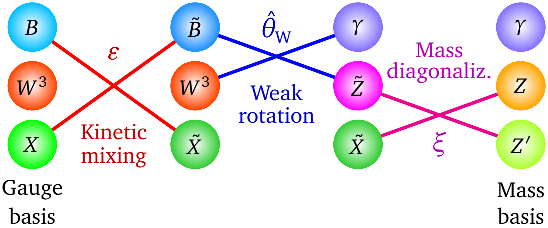
[Babu, Kolda, March-Russell, hep-ph/9710441, PRD]

$$t_\varepsilon \equiv \tan\varepsilon, \quad c_\varepsilon \equiv \cos\varepsilon$$

$$\hat{s}_{\mathrm{W}} \equiv \sin\hat{\theta}_{\mathrm{W}}, \quad \hat{c}_{\mathrm{W}} \equiv \cos\hat{\theta}_{\mathrm{W}}$$

$$\hat{\theta}_{\mathrm{W}} \equiv \tan^{-1}\frac{g'}{g}$$

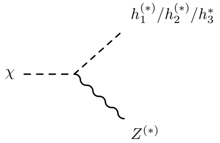
$$s_\xi \equiv \sin\xi, \quad c_\xi \equiv \cos\xi$$



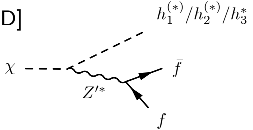
The **hierarchy** $v \sim v_S \ll v_\Phi$ implies a **mass hierarchy** $m_{h_1} \sim m_{h_2} \ll m_{h_3} \sim m_{Z'}$

DM Lifetime [DY Liu, CF Cai, XM Jiang, **ZHY**, HH Zhang, 2208.06653, JHEP]

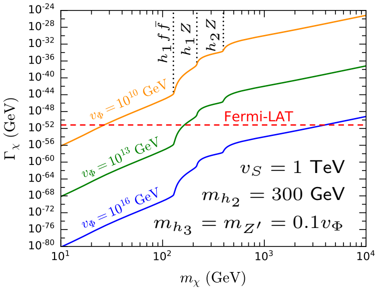
- For **finite** v_Φ, the Z-χ-h_i and Z'-χ-h_i **couplings** from gauge interactions **break** the Z_2 **symmetry** $\chi \to -\chi$, inducing χ **decay processes** $\chi \to h_i^{(*)} Z^{(*)}$ and $\chi \to h_i^{(*)} Z'^{*}$ for $m_\chi \ll m_{Z'} \sim m_{h_3}$

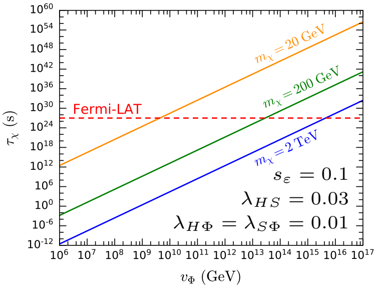


- **Fermi-LAT** γ-ray observations of dwarf galaxies imply a **bound** on the **DM lifetime**, $\tau_\chi \gtrsim 10^{27}$ s [Baring et al., 1510.00389, PRD]



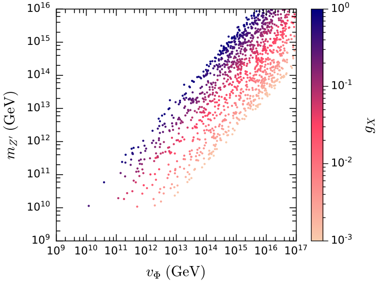
- This corresponds to $\Gamma_\chi \equiv 1/\tau_\chi \lesssim 6.6 \times 10^{-52}$ GeV, which will give a **lower bound** on the **UV scale** v_Φ

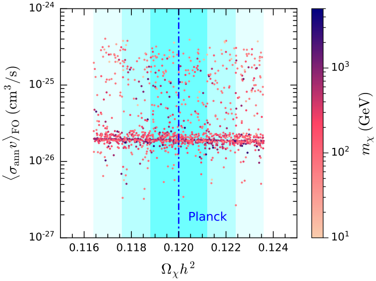


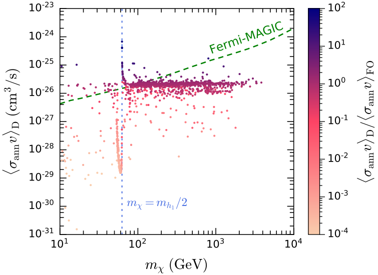


Parameter Scan [DY Liu, CF Cai, XM Jiang, **ZHY**, HH Zhang, 2208.06653, JHEP]

- We perform a **random scan** in 10-dimensional parameter space of $(v_S,\ v_\Phi,\ m_\chi,\ m_{h_2},\ m_{h_3},\ m_{Z'},\ \lambda_{HS},\ \lambda_{H\Phi},\ \lambda_{S\Phi},\ s_\varepsilon)$, taking into account the constraints from the **DM lifetime**, the **LHC Higgs measurements**, and the **relic abundance**
- We find that the **lower bound** on the **UV scale** v_Φ is down to $\sim 10^9$ GeV, given by the **Fermi-LAT constraint on** τ_χ







Higgs Physics [DY Liu, CF Cai, XM Jiang, **ZHY**, HH Zhang, 2208.06653, JHEP]

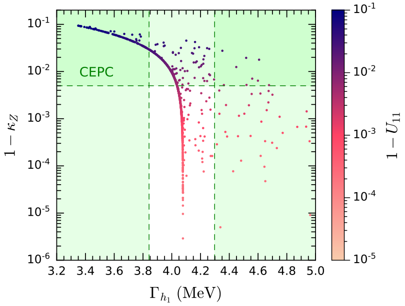
- Couplings of the **SM-like Higgs boson** h_1 to SM particles can be parametrized as

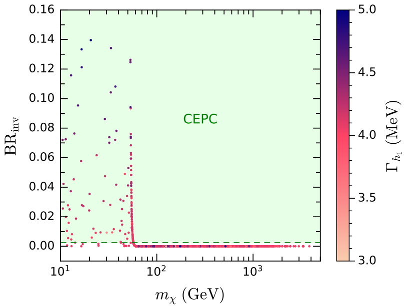
$$\mathcal{L}_{h_1} = \kappa_W \frac{2m_W^2}{v} h_1 W_\mu^+ W^{-,\mu} + \kappa_Z \frac{m_Z^2}{v} h_1 Z_\mu Z^\mu - \sum_f \kappa_f \frac{m_f}{v} h_1 \bar{f} f$$

- The **SM** corresponds to $\kappa_W = \kappa_Z = \kappa_f = 1$, while this model gives

$$\kappa_W = \kappa_f = U_{11},, \quad \kappa_Z = U_{11} c_\xi^2 (1 + \hat{s}_{\mathrm{W}} t_\varepsilon t_\xi) + \frac{s_\xi^2 g_X^2 v}{c_\varepsilon^2 m_Z^2} (U_{21} v_S + 4 U_{31} v_\Phi)$$

- **Exotic h_1 decay channels** may include $h_1 \to \chi\chi$, $h_1 \to \chi Z$, and $h_1 \to h_2 h_2$





Parameter Point Selection [ZY Qiu, **ZHY**, 2304.02506, CPC]

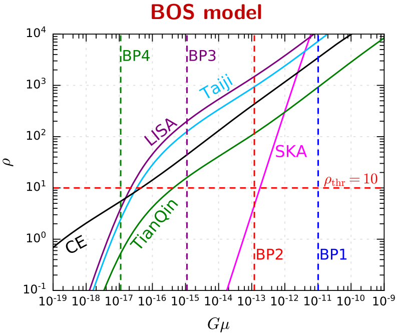
The following criteria are used to select the parameter points

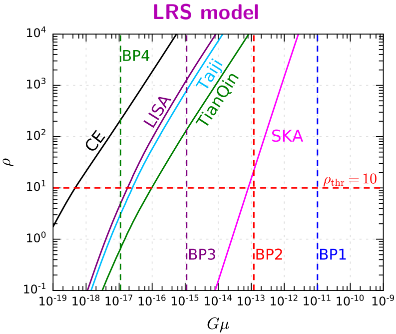
1. In order to guarantee the **vacuum stability**, the scalar potential should satisfy the **copositivity criteria**
2. The **lifetime** of the **pNGB DM particle** χ should satisfy the **Fermi-LAT bound** $\tau_\chi \gtrsim 10^{27}$ s
3. The **DM relic abundance** $\Omega_\chi h^2$ calculated by `micrOMEGAs` should be in the 3σ range of the **Planck value** $\Omega_{\mathrm{DM}} h^2 = 0.1200 \pm 0.0012$
4. The **total** $\chi\chi$ **annihilation cross section** $\langle \sigma_{\mathrm{ann}} v \rangle$ should not be excluded by the upper limits at the 95% C.L. given by the combined **Fermi-LAT** and **MAGIC** γ**-ray observations** of dwarf spheroidal galaxies in the $b\bar{b}$ channel
5. The signal strengths of the **SM-like Higgs boson** h_1 should be consistent with the **LHC Higgs measurements** at 95% C.L. based on the `HiggsSignals` calculation
6. The **exotic Higgs boson** h_2 should not be excluded at 95% C.L. by the **direct searches** at the **LHC** and the **Tevatron** according to `HiggsBounds`

Benchmark Points [ZY Qiu, **ZHY**, 2304.02506, CPC]

	BP1	BP2	BP3	BP4
v_S (GeV)	1953	2101	548.5	1388
v_Φ (GeV)	1.335×10^{13}	1.939×10^{12}	1.969×10^{11}	3.179×10^{10}
m_χ (GeV)	199.8	56.26	98.16	123.1
m_{h_2} (GeV)	986.7	627.7	484.3	362.6
m_{h_3} (GeV)	8.403×10^{12}	1.469×10^{12}	1.893×10^{11}	8.312×10^{9}
$m_{Z'}$ (GeV)	7.255×10^{11}	5.929×10^{11}	9.661×10^{10}	4.979×10^{10}
$\lambda_{H\Phi}$	-6.330×10^{-2}	-3.786×10^{-1}	-1.278×10^{-2}	-6.114×10^{-2}
$\lambda_{S\Phi}$	-2.870×10^{-1}	-5.416×10^{-2}	2.813×10^{-1}	3.188×10^{-2}
λ_{HS}	3.259×10^{-1}	1.189×10^{-1}	-1.750×10^{-1}	1.819×10^{-2}
s_ε	4.840×10^{-3}	3.222×10^{-1}	7.161×10^{-2}	1.929×10^{-3}
$G\mu$	1.01×10^{-11}	1.20×10^{-13}	1.11×10^{-15}	1.10×10^{-17}
$\Omega_\chi h^2$	0.118	0.121	0.120	0.119
$\sigma_{\chi N}^{\mathrm{SI}}$ (cm^2)	1.38×10^{-86}	1.62×10^{-86}	1.59×10^{-82}	8.45×10^{-77}
$\langle \sigma_{\mathrm{ann}} v \rangle$ (cm^3/s)	2.00×10^{-26}	2.87×10^{-29}	2.01×10^{-26}	1.71×10^{-26}
ρ_{LISA} (BOS)	1.15×10^{4}	1.48×10^{3}	2.00×10^{2}	3.97
ρ_{Taiji} (BOS)	7.26×10^{3}	9.37×10^{2}	1.26×10^{2}	2.45
ρ_{TianQin} (BOS)	9.25×10^{2}	1.15×10^{2}	1.59×10^{1}	5.28×10^{-1}
ρ_{CE} (BOS)	3.49×10^{3}	4.33×10^{2}	4.42×10^{1}	5.48
ρ_{LISA} (LRS)	1.15×10^{7}	1.38×10^{5}	1.28×10^{3}	4.93
ρ_{Taiji} (LRS)	7.19×10^{6}	8.57×10^{4}	7.95×10^{2}	3.05
ρ_{TianQin} (LRS)	1.20×10^{6}	1.42×10^{4}	1.36×10^{2}	6.48×10^{-1}
ρ_{CE} (LRS)	4.36×10^{6}	2.18×10^{6}	2.02×10^{4}	2.11×10^{2}

Sensitivity of Future GW Experiments [ZY Qiu, **ZHY**, 2304.02506, CPC]





🍹 **Expected upper limits** on $G\mu$ corresponding to the signal-to-noise ratio $\rho_{\rm thr} = 10$

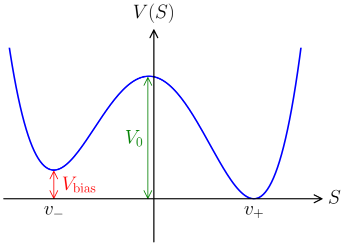
	LISA	Taiji	TianQin	CE	SKA
BOS	2.21×10^{-17}	3.34×10^{-17}	4.28×10^{-16}	4.54×10^{-17}	1.77×10^{-13}
LRS	1.79×10^{-17}	2.51×10^{-17}	9.67×10^{-17}	4.66×10^{-19}	8.09×10^{-14}

Upper and Lower Bounds on $V_{\rm bias}$

🧀 If $V_{\rm bias}$ is **too large**, DWs **cannot** be created from the beginning

🌭 According to **percolation theory**, **large-scale DWs** can be **formed** only if $V_{\rm bias} < 0.795 V_0$

🍞 Requiring DWs should **collapse before** they **dominate** the universe leads to



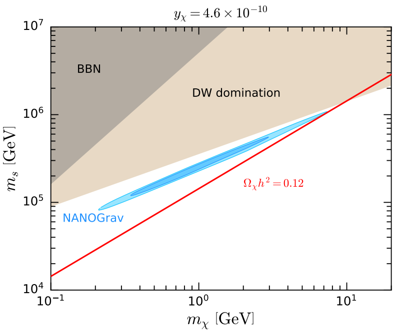
$$V_{\rm bias}^{1/4} > 0.0218 \text{ MeV } \mathcal{A}^{1/2} \left(\frac{\sigma}{\text{TeV}^3} \right)^{1/2}$$

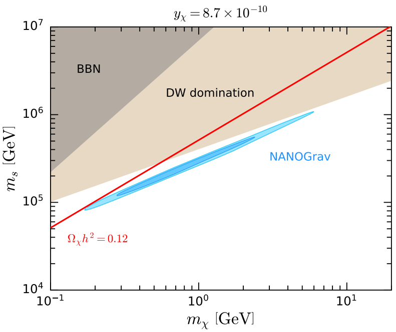
🍔 Moreover, the **energetic particles** produced from **DW collapse** could **destroy** the **light elements** generated in the **Big Bang Nucleosynthesis (BBN)**

🌮 Thus, we should require that **DWs annihilate before the BBN epoch**

🌯 This leads to $V_{\rm bias}^{1/4} > 0.507 \text{ MeV } \mathcal{A}^{1/4} \left(\frac{\sigma}{\text{TeV}^3} \right)^{1/4}$

Viable Parameter Ranges [Zhang, Cai, Su, Wang, **ZHY**, Zhang, 2307.11495, PRD]





The **intersection** of the $\Omega_\chi h^2 = 0.12$ **line** and the **NANOGrav favored regions sensitively depends** on the y_χ **value**

For $\lambda_S = 0.2$, the parameter ranges where our model can **simultaneously explain** the **NANOGrav GW signal** and the **DM relic density** are

$$4.6 \times 10^{-10} \lesssim y_\chi \lesssim 8.7 \times 10^{-10}$$

$$0.17\ \text{GeV} \lesssim m_\chi \lesssim 7.5\ \text{GeV}, \quad 8.1 \times 10^4\ \text{GeV} \lesssim m_s \lesssim 10^6\ \text{GeV}$$