

# 标准模型的拉氏量和 Feynman 规则

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## 1 约定

本文采用有理化的自然单位制，推导过程参考文献 [1, 2, 3, 4]，协变导数的约定与 *Review of Particle Physics* [5] 第 9、10、11 章一致。

Minkowski 度规张量

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}. \quad (1)$$

Pauli 矩阵

$$\sigma^1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} & -i \\ i & \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}, \quad (2)$$

$$\sigma^\mu \equiv (1, \boldsymbol{\sigma}), \quad \bar{\sigma}^\mu \equiv (1, -\boldsymbol{\sigma}). \quad (3)$$

Weyl 表象中的 Dirac 矩阵

$$\gamma^\mu = \begin{pmatrix} & \sigma^\mu \\ \bar{\sigma}^\mu & \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -1 & \\ & 1 \end{pmatrix}. \quad (4)$$

左右手投影算符

$$P_L \equiv \frac{1}{2}(1 - \gamma^5) = \begin{pmatrix} 1 & \\ & \mathbf{0} \end{pmatrix}, \quad P_R \equiv \frac{1}{2}(1 + \gamma^5) = \begin{pmatrix} \mathbf{0} & \\ & 1 \end{pmatrix} \quad (5)$$

用于定义左手旋量场  $\psi_L \equiv P_L \psi$  和右手旋量场  $\psi_R \equiv P_R \psi$ 。Levi-Civita 符号的约定取

$$\varepsilon^{0123} = \varepsilon^{123} = +1. \quad (6)$$

Feynman 规则约定如下。

- 对于指向相互作用顶点的动量  $p$ ，时空导数  $\partial_\mu$  在动量空间 Feynman 规则里贡献一个  $-ip_\mu$  因子。
- 实线表示费米子，实线上的箭头表示费米子数流动的方向。
- 虚线表示标量玻色子，虚线上的箭头表示玻色子数流动的方向。
- 螺旋线表示胶子；波浪线表示其它规范玻色子，波浪线上的箭头表示玻色子数流动的方向。
- 点线表示鬼粒子，点线上的箭头表示鬼粒子数流动的方向。
- 如果没有额外箭头标记，动量方向与粒子线上的箭头方向一致；否则与额外箭头方向一致。

## 2 标准模型概述

粒子物理标准模型是一个  $SU(3)_C \times SU(2)_L \times U(1)_Y$  规范理论。模型中有三代费米子, 包括三代中微子  $\nu_i = \nu_e, \nu_\mu, \nu_\tau$ , 三代带电轻子  $\ell_i = e, \mu, \tau$ , 三代上型夸克  $u_i = u, c, t$  和三代下型夸克  $d_i = d, s, b$  ( $i = 1, 2, 3$ )。规范玻色子传递费米子之间的规范相互作用。

$SU(3)_C$  部分描述夸克的强相互作用, 称为量子色动力学 (Quantum Chromodynamics, QCD), 相应的规范玻色子是胶子。 $SU(2)_L \times U(1)_Y$  部分统一描述夸克和轻子的电磁和弱相互作用, 称为电弱规范理论。理论中有一个 Higgs 二重态, 通过 Brout-Englert-Higgs (BEH) 机制引发规范群的自发对称性破缺, 使  $SU(2)_L \times U(1)_Y$  群破缺为  $U(1)_{EM}$  群。 $U(1)_{EM}$  规范理论称为量子电动力学 (Quantum Electrodynamics, QED)。

破缺前, 理论中存在 4 个无质量的规范玻色子和 4 个 Higgs 自由度; 左手费米子和右手费米子都没有质量, 具有不同的量子数。

破缺后, 3 个规范玻色子与 3 个 Higgs 自由度结合, 从而获得质量, 成为  $W^\pm$  和  $Z^0$  玻色子, 传递弱相互作用。剩下的 1 个无质量规范玻色子是光子, 即是  $U(1)_{EM}$  群的规范玻色子, 传递电磁相互作用。剩下的 1 个中性 Higgs 自由度称为 Higgs 玻色子。费米子与 Higgs 二重态的 Yukawa 耦合导致左手费米子和右手费米子获得质量, 组合成 Dirac 费米子。

理论中的中微子没有右手分量, 因而没有获得质量。1998 年实验发现中微子振荡, 证明中微子具有质量, 所以需要扩充标准模型才能正确描述中微子物理。

## 3 量子色动力学

QCD 的拉氏量表达为

$$\mathcal{L}_{QCD} = \sum_q \bar{q}(i\gamma^\mu D_\mu - m_q)q - \frac{1}{4}G_{\mu\nu}^a G^{a,\mu\nu}, \quad q = u, d, s, c, b, t, \quad a = 1, \dots, 8, \quad (7)$$

其中

$$D_\mu = \partial_\mu + ig_s G_\mu^a t^a, \quad G_{\mu\nu}^a \equiv \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_s f^{abc} G_\mu^b G_\nu^c. \quad (8)$$

$q$  为夸克旋量场  $SU(3)_C$  三重态,  $SU(3)_C$  规范场  $G_\mu^a$  对应于胶子  $g$ ,  $g_s$  是  $SU(3)_C$  规范耦合常数。 $t^a = \lambda^a/2$  是  $SU(3)_C$  群基础表示的生成元, 其中  $\lambda^a$  为 Gell-Mann 矩阵。 $SU(3)_C$  生成元对易关系为  $[t^a, t^b] = if^{abc}t^c$ , 结构常数  $f^{abc}$  是全反对称的, 非零分量为

$$f^{123} = 1, \quad f^{147} = f^{246} = f^{257} = f^{345} = f^{516} = f^{637} = \frac{1}{2}, \quad f^{458} = f^{678} = \frac{\sqrt{3}}{2}. \quad (9)$$

由

$$\begin{aligned} -\frac{1}{4}G_{\mu\nu}^a G^{a,\mu\nu} &= -\frac{1}{4}(\partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g_s f^{abc} G_\mu^b G_\nu^c)(\partial^\mu G^{a,\nu} - \partial^\nu G^{a,\mu} + g_s f^{ade} G^{d,\mu} G^{e,\nu}) \\ &= -\frac{1}{2}[(\partial_\mu G_\nu^a)(\partial^\mu G^{a,\nu}) - (\partial_\mu G_\nu^a)(\partial^\nu G^{a,\mu})] + g_s f^{abc}(\partial_\mu G_\nu^a)G^{b,\mu}G^{c,\nu} \end{aligned}$$

$$-\frac{g_s^2}{4}f^{abc}f^{ade}G_\mu^bG_\nu^cG^{d,\mu}G^{e,\nu}, \quad (10)$$

推出

$$\begin{aligned} \mathcal{L}_{\text{QCD}} = & \sum_q [\bar{q}(i\gamma^\mu\partial_\mu - m_q)q - g_s G_\mu^a \bar{q}\gamma^\mu t^a q] + \frac{1}{2}[(\partial_\mu G_\nu^a)(\partial^\nu G^{a,\mu}) - (\partial_\mu G_\nu^a)(\partial^\mu G^{a,\nu})] \\ & + g_s f^{abc}(\partial_\mu G_\nu^a)G^{b,\mu}G^{c,\nu} - \frac{g_s^2}{4}f^{abc}f^{ade}G_\mu^bG_\nu^cG^{d,\mu}G^{e,\nu}. \end{aligned} \quad (11)$$

设用于固定胶子场规范的函数  $G^a(x) = \partial^\mu G_\mu^a(x) - \omega^a(x)$ , 其中  $\omega^a(x)$  是某个任意函数, 规范固定条件是  $G^a(x) = 0$ 。这是 Lorenz 规范的推广,  $\omega^a(x) = 0$  对应于 Lorenz 规范。在路径积分量子化中, 以中心为  $\omega^a(x) = 0$  的 Gauss 权重对  $\omega^a(x)$  作泛函积分, 有

$$\int \mathcal{D}\omega^a \exp \left[ -i \int d^4x \frac{1}{2\xi} (\omega^a)^2 \right] \delta(G^a) = \exp \left[ -i \int d^4x \frac{1}{2\xi} (\partial^\mu G_\mu^a)^2 \right]. \quad (12)$$

可见, 拉氏量中的规范固定项为

$$\mathcal{L}_{\text{QCD,GF}} = -\frac{1}{2\xi} (\partial^\mu G_\mu^a)^2. \quad (13)$$

$\xi$  的任何一个取值对应于一种规范。 $\xi = 1$  称为 Feynman-'t Hooft 规范,  $\xi = 0$  称为 Landau 规范。于是, 胶子传播子相关拉氏量为

$$\begin{aligned} \mathcal{L}_{\text{QCD,prop}} = & \frac{1}{2} \left[ (\partial_\mu G_\nu^a)(\partial^\nu G^{a,\mu}) - (\partial_\mu G_\nu^a)(\partial^\mu G^{a,\nu}) - \frac{1}{\xi} (\partial^\mu G_\mu^a)^2 \right] \\ \rightarrow & \frac{1}{2} G_\mu^a \left[ g^{\mu\nu} \partial^2 - \left( 1 - \frac{1}{\xi} \right) \partial^\mu \partial^\nu \right] G_\nu^a. \end{aligned} \quad (14)$$

这里  $\rightarrow$  代表丢弃一些全散度项。变换到动量空间, 得

$$-g^{\mu\nu} p^2 + \left( 1 - \frac{1}{\xi} \right) p^\mu p^\nu, \quad (15)$$

它的逆矩阵是

$$-\frac{1}{p^2} \left[ g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} (1 - \xi) \right], \quad (16)$$

这是因为

$$\begin{aligned} & -\frac{1}{p^2} \left[ g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2} (1 - \xi) \right] \left[ -g^{\mu\nu} p^2 + \left( 1 - \frac{1}{\xi} \right) p^\mu p^\nu \right] \\ = & \delta_\rho^\nu - \frac{p_\rho p^\nu}{p^2} \left( 1 - \frac{1}{\xi} \right) - \frac{p_\rho p^\nu}{p^2} (1 - \xi) + \frac{p_\rho p^\nu}{p^2} (1 - \xi) \left( 1 - \frac{1}{\xi} \right) = \delta_\rho^\nu. \end{aligned} \quad (17)$$

从而胶子传播子的形式为

$$\frac{-i\delta^{ab}}{p^2 + i\epsilon} \left[ g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} (1 - \xi) \right]. \quad (18)$$

$SU(3)_C$  规范变换为

$$q' = Uq, \quad (G_\mu^a t^a)' = U G_\mu^a t^a U^\dagger - \frac{i}{g_s} U \partial_\mu U^\dagger, \quad (19)$$

其中  $U(x) = \exp[i\alpha^a(x)t^a]$ 。胶子场的无穷小规范变换形式是

$$\begin{aligned} (G_\mu^a t^a)' &= (1 + i\alpha^a t^a) G_\mu^b t^b (1 - i\alpha^c t^c) - \frac{i}{g_s} (1 + i\alpha^a t^a) \partial_\mu (1 - i\alpha^c t^c) \\ &= G_\mu^b t^b + i\alpha^c G_\mu^b [t^c, t^b] - \frac{1}{g_s} (\partial_\mu \alpha^c) t^c + \mathcal{O}(\alpha^a \alpha^b) \\ &= G_\mu^a t^a - f^{cba} \alpha^c G_\mu^b t^a - \frac{1}{g_s} (\partial_\mu \alpha^a) t^a + \mathcal{O}(\alpha^a \alpha^b) \\ &= \left( G_\mu^a + f^{abc} G_\mu^b \alpha^c - \frac{1}{g_s} \partial_\mu \alpha^a \right) t^a + \mathcal{O}(\alpha^a \alpha^b), \end{aligned} \quad (20)$$

即  $G_\mu^a$  的无穷小变化为

$$\delta G_\mu^a = (G_\mu^a)' - G_\mu^a = -\frac{1}{g_s} \partial_\mu \alpha^a + f^{abc} G_\mu^b \alpha^c = -\frac{1}{g_s} (D_\mu \alpha)^a = -\frac{1}{g_s} D_\mu^{ac} \alpha^c, \quad (21)$$

其中  $(D_\mu \alpha)^a = \partial_\mu \alpha^a - g_s f^{abc} G_\mu^b \alpha^c$  是  $SU(3)_C$  伴随表示中的协变导数，而

$$D_\mu^{ac} = \delta^{ac} \partial_\mu - g_s f^{abc} G_\mu^b. \quad (22)$$

因此，规范固定函数  $G^a$  的无穷小变化为

$$\delta G^a = \partial^\mu \delta G_\mu^a = -\frac{1}{g_s} \partial^\mu D_\mu^{ac} \alpha^c, \quad (23)$$

故

$$\frac{\delta G^a}{\delta \alpha^c} = -\frac{1}{g_s} \partial^\mu D_\mu^{ac} = -\frac{1}{g_s} \delta^{ac} \partial^2 + f^{abc} \partial^\mu G_\mu^b. \quad (24)$$

根据 Grassmann 数的积分式

$$\left( \prod_i \int d\theta_i^* d\theta_i \right) \exp(-\theta_i^* B_{ij} \theta_j) = \det(B), \quad (25)$$

$\delta G^a / \delta \alpha^c$  的行列式可用 Faddeev-Popov 鬼场  $\eta_g^a$  和  $\bar{\eta}_g^a$  表达为

$$\det \left( \frac{\delta G^a}{\delta \alpha^c} \right) = \det \left( -\frac{1}{g_s} \partial^\mu D_\mu^{ac} \right) = \int \mathcal{D}\eta_g^a \mathcal{D}\bar{\eta}_g^c \exp \left[ i \int d^4x \bar{\eta}_g^a (-\partial^\mu D_\mu^{ac}) \eta_g^c \right], \quad (26)$$

这里  $-1/g_s$  因子被吸收到鬼场  $\eta_g^a$  和  $\bar{\eta}_g^a$  的归一化中。注意到

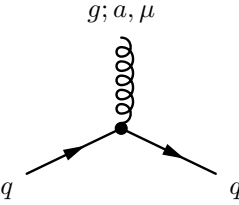
$$-\partial^\mu D_\mu^{ac} = g_s \frac{\delta G^a}{\delta \alpha^c} = -\delta^{ac} \partial^2 + g_s f^{abc} \partial^\mu G_\mu^b, \quad (27)$$

Faddeev-Popov 鬼场在拉氏量中的贡献是

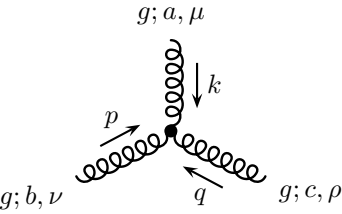
$$\begin{aligned} \mathcal{L}_{\text{QCD,FP}} &= \bar{\eta}_g^a (-\partial^\mu D_\mu^{ac}) \eta_g^c = \bar{\eta}_g^a \left( g_s \frac{\delta G^a}{\delta \alpha^c} \right) \eta_g^c = \bar{\eta}_g^a (-\delta^{ac} \partial^2 + g_s f^{abc} \partial^\mu G_\mu^b) \eta_g^c \\ &\rightarrow -\bar{\eta}_g^a \delta^{ab} \partial^2 \eta_g^b - g_s f^{abc} (\partial^\mu \bar{\eta}_g^a) G_\mu^b \eta_g^c. \end{aligned} \quad (28)$$

下面列出 QCD Feynman 规则。

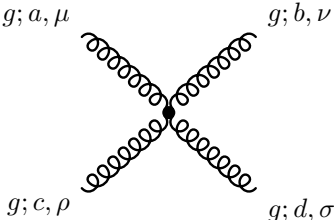
QCD 耦合顶点：



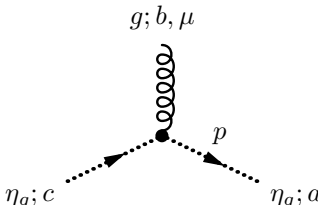
$$= -ig_s \gamma^\mu t^a \quad (29)$$



$$= -g_s f^{abc} [g^{\mu\nu} (k-p)^\rho + g^{\nu\rho} (p-q)^\mu + g^{\rho\mu} (q-k)^\nu] \quad (30)$$

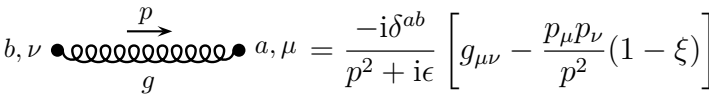


$$\begin{aligned} &= -ig_s^2 [f^{abe} f^{cde} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) \\ &\quad + f^{ace} f^{bde} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}) \\ &\quad + f^{ade} f^{bce} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma})] \end{aligned} \quad (31)$$




$$= g_s f^{abc} p^\mu \quad (32)$$

胶子传播子：



$$= \frac{-i\delta^{ab}}{p^2 + i\epsilon} \left[ g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} (1 - \xi) \right] \quad (33)$$

鬼粒子传播子：



$$= \frac{i\delta^{ab}}{p^2 + i\epsilon} \quad (34)$$

## 4 电弱规范理论

电弱规范理论的规范群是  $SU(2)_L \times U(1)_Y$ ，每一代左手旋量场构成 2 个  $SU(2)_L$  二重态

$$L_{iL} = \begin{pmatrix} P_L \nu_i \\ P_L \ell_i \end{pmatrix} = \begin{pmatrix} \nu_{iL} \\ \ell_{iL} \end{pmatrix}, \quad Q_{iL} = \begin{pmatrix} P_L u'_i \\ P_L d'_i \end{pmatrix} = \begin{pmatrix} u'_{iL} \\ d'_{iL} \end{pmatrix}, \quad i = 1, 2, 3. \quad (35)$$

它们的协变导数是

$$D_\mu = \partial_\mu + igW_\mu^a \tau^a + ig'B_\mu Y, \quad (36)$$

其中  $W_\mu^a(x)$  ( $a = 1, 2, 3$ ) 是  $SU(2)_L$  规范场,  $B_\mu(x)$  是  $U(1)_Y$  规范场,  $g$  和  $g'$  分别是  $SU(2)_L$  和  $U(1)_Y$  的规范耦合常数, 取  $g > 0$  且  $g' > 0$ 。

$$\tau^a = \frac{\sigma^a}{2} \quad (37)$$

是  $SU(2)_L$  群 2 维表示的生成元, 对应于弱同位旋。生成元  $\tau^3$  的本征值是弱同位旋第 3 分量, 记为  $T^3$ 。 $Y$  是弱超荷。各代右手旋量场  $\ell_{iR} = P_R \ell_i$ 、 $u'_{iR} = P_R u'_i$  和  $d'_{iR} = P_R d'_i$  是  $SU(2)_L$  单态, 协变导数为

$$D_\mu = \partial_\mu + ig'B_\mu Y. \quad (38)$$

表 1 列出费米子场的电荷  $Q$ 、弱同位旋第 3 分量  $T^3$ 、弱超荷  $Y$ 、重子数  $B$  和轻子数  $L_e/L_\mu/L_\tau$ , 其中电荷  $Q$  由  $T^3$  和  $Y$  定义,

$$Q \equiv T^3 + Y. \quad (39)$$

表 1: 标准模型费米子场的量子数。

统一记号	第一代	第二代	第三代	$Q$	$T^3$	$Y$	$B$	$L_e/L_\mu/L_\tau$
$L_{iL} = \begin{pmatrix} \nu_{iL} \\ \ell_{iL} \end{pmatrix}$	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	0	1/2	-1/2	0	1
				-1	-1/2	-1/2	0	1
$Q_{iL} = \begin{pmatrix} u'_{iL} \\ d'_{iL} \end{pmatrix}$	$\begin{pmatrix} u'_L \\ d'_L \end{pmatrix}$	$\begin{pmatrix} c'_L \\ s'_L \end{pmatrix}$	$\begin{pmatrix} t'_L \\ b'_L \end{pmatrix}$	2/3	1/2	1/6	1/3	0
				-1/3	-1/2	1/6	1/3	0
$\ell_{iR}$	$e_R$	$\mu_R$	$\tau_R$	-1	0	-1	0	1
$u'_{iR}$	$u'_R$	$c'_R$	$t'_R$	2/3	0	2/3	1/3	0
$d'_{iR}$	$d'_R$	$s'_R$	$b'_R$	-1/3	0	-1/3	1/3	0

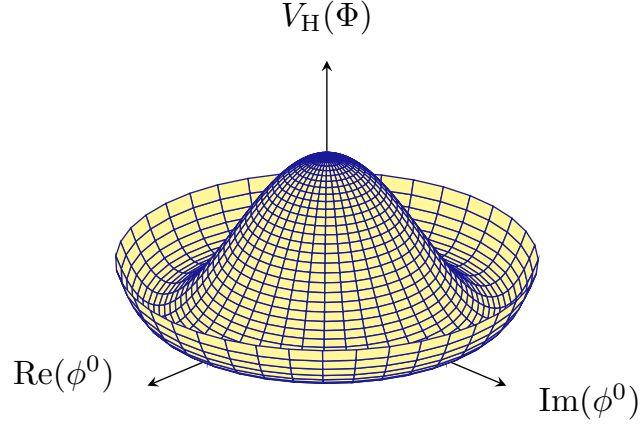


图 1: Higgs 场势能示意图。这里压缩掉  $\text{Re}(\phi^+)$  和  $\text{Im}(\phi^+)$  两个维度。

#### 4.1 Brout-Englert-Higgs 机制

由于左手费米子和右手费米子参与不同的  $\text{SU}(2)_L \times \text{U}(1)_Y$  规范相互作用，耦合左右手费米子场的质量项会破坏规范对称性。另一方面，规范对称性也禁止规范玻色子具有质量。为了让费米子和弱规范玻色子获得质量，需要引入 BEH 机制，使  $\text{SU}(2)_L \times \text{U}(1)_Y$  规范对称性自发破缺。因此，引入 Higgs 标量场

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}, \quad (40)$$

其中  $\phi^+$  和  $\phi^0$  都是复标量场。 $\Phi$  是  $\text{SU}(2)_L$  二重态，弱超荷是

$$Y_H = \frac{1}{2}. \quad (41)$$

Higgs 场的协变动能项和势能项为

$$\mathcal{L}_H = (D^\mu \Phi)^\dagger (D_\mu \Phi) - V_H(\Phi), \quad V_H(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2, \quad (42)$$

其中协变导数为

$$D_\mu \Phi = (\partial_\mu + ig' B_\mu Y_H + ig W_\mu^a \tau^a) \Phi. \quad (43)$$

当  $\lambda > 0$  且  $\mu^2 > 0$  时，Higgs 场势能  $V_H(\Phi)$  呈现出图 1 所示墨西哥草帽状的形式，势能最小值位于方程

$$\Phi^\dagger \Phi = [\text{Re}(\phi^+)]^2 + [\text{Im}(\phi^+)]^2 + [\text{Re}(\phi^0)]^2 + [\text{Im}(\phi^0)]^2 = \frac{v^2}{2} \quad (44)$$

对应的 3 维球面上，其中  $v \equiv \sqrt{\mu^2/\lambda}$ ，满足

$$\mu^2 = \lambda v^2. \quad (45)$$



Higgs 场的真空期待值位于这个 3 维球面上的某一点, 不失一般性, 可将它取为

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. \quad (46)$$

其它真空期待值可通过  $SU(2)_L \times U(1)_Y$  整体变换

$$\langle \Phi \rangle \rightarrow \exp(i\alpha^a \tau^a) \exp(i\alpha^Y Y_H) \langle \Phi \rangle \quad (47)$$

得到, 因为  $\langle \Phi^\dagger \Phi \rangle$  在这样的变换下保持不变。若  $\alpha^1 = \alpha^2 = 0$  且  $\alpha^3 = \alpha^Y$ , 则

$$\exp(i\alpha^a \tau^a) \exp(i\alpha^Y Y_H) = \exp[i\alpha^3(\sigma^3 + \mathbf{1})/2] = \exp \left[ i\alpha^3 \begin{pmatrix} 1 & \\ & 0 \end{pmatrix} \right] = \begin{pmatrix} e^{i\alpha^3} & \\ & 1 \end{pmatrix}, \quad (48)$$

而  $\langle \Phi \rangle$  在此变换下不变。因此, 有 1 个方向的规范对称性没有受到破坏, 只有 3 个方向的规范对称性发生自发破缺。根据 Goldstone 定理, 破缺后存在 3 个无质量的 Nambu-Goldstone 玻色子。最终, 有 3 个规范玻色子结合 Nambu-Goldstone 玻色子, 通过 BEH 机制获得质量。

以  $\langle \Phi \rangle$  为基础, 将 Higgs 场参数化为

$$\Phi(x) = \exp \left[ -i \frac{\chi^a(x)}{v} \tau^a \right] \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \quad (49)$$

其中  $\chi^a(x)$  和  $H(x)$  都是实标量场。  $\exp[-i\chi^a(x)\tau^a/v]$  因子能够通过  $SU(2)_L$  规范变换消去, 因而可将  $\Phi(x)$  直接取为

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \quad \Phi^\dagger \Phi = \frac{1}{2}(v + H)^2. \quad (50)$$

此时 Higgs 场只剩下一个物理自由度  $H(x)$ , 对应于 Higgs 玻色子, 这种取法称为么正规范。

在么正规范下, 势能项化为

$$\begin{aligned} -V_H(\Phi) &= \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 = \frac{\mu^2}{2}(v + H)^2 - \frac{\lambda}{4}(v + H)^4 \\ &= \frac{\mu^2}{2}(v^2 + H^2 + 2vH) - \frac{\lambda}{4}(v^4 + 4v^2 H^2 + H^4 + 4v^3 H + 2v^2 H^2 + 4vH^3) \\ &= \frac{1}{4}\mu^2 v^2 + \frac{1}{4}(\mu^2 - \lambda v^2)v^2 + (\mu^2 - \lambda v^2)vH + \frac{1}{2}(\mu^2 - \lambda v^2)H^2 - \lambda v^2 H^2 - \lambda v H^3 - \frac{\lambda}{4} H^4 \\ &= \frac{1}{8}m_H^2 v^2 - \frac{1}{2}m_H^2 H^2 - \frac{1}{2}\frac{m_H^2}{v} H^3 - \frac{1}{8}\frac{m_H^2}{v^2} H^4, \end{aligned} \quad (51)$$

其中 Higgs 玻色子的质量为

$$m_H \equiv \sqrt{2}\mu, \quad m_H^2 = 2\mu^2 = 2\lambda v^2. \quad (52)$$

由于

$$g'B_\mu Y_H + gW_\mu^a \tau^a = \frac{1}{2} \begin{pmatrix} g'B_\mu + gW_\mu^3 & g(W_\mu^1 - iW_\mu^2) \\ g(W_\mu^1 + iW_\mu^2) & g'B_\mu - gW_\mu^3 \end{pmatrix}, \quad (53)$$

Higgs 场真空期待值  $v$  对协变导数  $D_\mu \Phi$  的贡献为

$$\begin{aligned} D_\mu \Phi &\supset i(g'B_\mu Y_H + gW_\mu^a \tau^a) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \\ &= \frac{iv}{2\sqrt{2}} \begin{pmatrix} g'B_\mu + gW_\mu^3 & g(W_\mu^1 - iW_\mu^2) \\ g(W_\mu^1 + iW_\mu^2) & g'B_\mu - gW_\mu^3 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \supset \frac{iv}{2\sqrt{2}} \begin{pmatrix} g(W_\mu^1 - iW_\mu^2) \\ g'B_\mu - gW_\mu^3 \end{pmatrix}, \end{aligned} \quad (54)$$

故协变动能项  $(D^\mu \Phi)^\dagger (D_\mu \Phi)$  中正比于  $v^2$  的项是

$$(D^\mu \Phi)^\dagger (D_\mu \Phi) \supset \frac{v^2}{8} [g^2 |W_\mu^1 - iW_\mu^2|^2 + (g'B_\mu - gW_\mu^3)^2] = \frac{v^2}{8} (g^2 W^{a,\mu} W_\mu^a + g'^2 B^\mu B_\mu - 2gg' B^\mu W_\mu^3). \quad (55)$$

这些项是规范玻色子的质量项，重新表达为

$$\mathcal{L}_{\text{GBM}} = \frac{1}{2} m_W^2 (W^{1,\mu} W_\mu^1 + W^{2,\mu} W_\mu^2) + \frac{1}{2} \begin{pmatrix} W^{3,\mu} & B^\mu \end{pmatrix} M_{W^3 B}^2 \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}, \quad (56)$$

其中

$$m_W \equiv \frac{1}{2} gv \quad (57)$$

是  $W_\mu^1$  和  $W_\mu^2$  获得的质量，而  $W^{3\mu}$  和  $B^\mu$  的质量平方矩阵为

$$M_{W^3 B}^2 \equiv \frac{v^2}{4} \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}. \quad (58)$$

为了使  $M_{W^3 B}^2$  矩阵对角化，定义

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \equiv \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}, \quad (59)$$

其中

$$s_W \equiv \sin \theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}, \quad c_W \equiv \cos \theta_W = \frac{g}{\sqrt{g^2 + g'^2}}, \quad (60)$$

$\theta_W$  称为 Weinberg 角，也称为弱混合角。从后面的讨论可以看出  $A_\mu(x)$  就是电磁场，对应于光子。 $Z_\mu(x)$  对应于矢量玻色子  $Z$ 。反过来，有

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}. \quad (61)$$

由

$$\begin{aligned} M_{W^3B}^2 \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} &= \frac{(g^2 + g'^2)v^2}{4} \begin{pmatrix} c_W^2 & -s_W c_W \\ -s_W c_W & s_W^2 \end{pmatrix} \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \\ &= \frac{(g^2 + g'^2)v^2}{4} \begin{pmatrix} c_W & 0 \\ -s_W & 0 \end{pmatrix} \end{aligned} \quad (62)$$

得

$$\begin{aligned} \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} M_{W^3B}^2 \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} &= \frac{(g^2 + g'^2)v^2}{4} \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} c_W & 0 \\ -s_W & 0 \end{pmatrix} \\ &= \frac{(g^2 + g'^2)v^2}{4} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \end{aligned} \quad (63)$$

因此

$$\begin{aligned} \frac{1}{2} \begin{pmatrix} W^{3,\mu} & B^\mu \end{pmatrix} M_{W^3B}^2 \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} &= \frac{1}{2} \begin{pmatrix} Z^\mu & A^\mu \end{pmatrix} \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} M_{W^3B}^2 \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \\ &= \frac{(g^2 + g'^2)v^2}{8} \begin{pmatrix} Z^\mu & A^\mu \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \frac{1}{2} m_Z^2 Z^\mu Z_\mu, \end{aligned} \quad (64)$$

其中

$$m_Z \equiv \frac{1}{2} \sqrt{g^2 + g'^2} v = \frac{gv}{2c_W} = \frac{m_W}{c_W} \quad (65)$$

是  $Z$  玻色子的质量，而光子没有质量。另一方面，用质量相同的实矢量场  $W_\mu^1$  和  $W_\mu^2$  线性组合出复矢量场

$$W_\mu^+ \equiv \frac{1}{\sqrt{2}}(W_\mu^1 - iW_\mu^2), \quad (66)$$

它的厄米共轭为

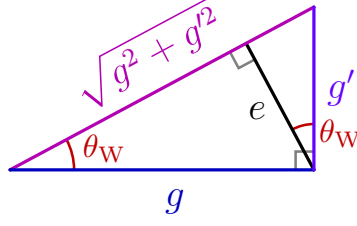
$$W_\mu^- \equiv (W_\mu^+)^{\dagger} = \frac{1}{\sqrt{2}}(W_\mu^1 + iW_\mu^2), \quad (67)$$

则

$$W_\mu^1 = \frac{1}{\sqrt{2}}(W_\mu^+ + W_\mu^-), \quad W_\mu^2 = \frac{i}{\sqrt{2}}(W_\mu^+ - W_\mu^-). \quad (68)$$

从而

$$\begin{aligned} \frac{1}{2}(W^{1,\mu}W_\mu^1 + W^{2,\mu}W_\mu^2) &= \frac{1}{4}[(W^{+,\mu} + W^{-,\mu})(W_\mu^+ + W_\mu^-) - (W^{+,\mu} - W^{-,\mu})(W_\mu^+ - W_\mu^-)] \\ &= W^{+,\mu}W_\mu^-, \end{aligned} \quad (69)$$

图 2:  $g$ 、 $g'$ 、 $e$  和  $\theta_W$  的关系。

(56) 式化为

$$\mathcal{L}_{\text{GBM}} = m_W^2 W^{+, \mu} W_\mu^- + \frac{1}{2} m_Z^2 Z^\mu Z_\mu. \quad (70)$$

复矢量场  $W_\mu^\pm$  描述一对正反矢量玻色子  $W^\pm$ ，质量为  $m_W$ 。可见，BEH 机制使传递弱相互作用的规范玻色子  $W^\pm$  和  $Z$  获得了质量。

接下来用质量本征态  $W_\mu^\pm$  和  $Z_\mu$  表达协变动能项  $(D^\mu \Phi)^\dagger (D_\mu \Phi)$ 。注意到

$$A_\mu = s_W W_\mu^3 + c_W B_\mu, \quad Z_\mu = c_W W_\mu^3 - s_W B_\mu, \quad (71)$$

$$B_\mu = c_W A_\mu - s_W Z_\mu, \quad W_\mu^3 = s_W A_\mu + c_W Z_\mu, \quad (72)$$

有

$$\begin{aligned} g' B_\mu + g W_\mu^3 &= g' (c_W A_\mu - s_W Z_\mu) + g (s_W A_\mu + c_W Z_\mu) = \frac{2gg'}{\sqrt{g^2 + g'^2}} A_\mu + \frac{g^2 - g'^2}{\sqrt{g^2 + g'^2}} Z_\mu \\ &= 2e A_\mu + \frac{g}{c_W} (c_W^2 - s_W^2) Z_\mu, \end{aligned} \quad (73)$$

其中

$$e \equiv \frac{gg'}{\sqrt{g^2 + g'^2}} = g s_W = g' c_W. \quad (74)$$

后面的讨论将表明  $e$  就是单位电荷量。 $g$ 、 $g'$ 、 $e$  和  $\theta_W$  的关系如图 2 所示。再利用

$$g' B_\mu - g W_\mu^3 = g' (c_W A_\mu - s_W Z_\mu) - g (s_W A_\mu + c_W Z_\mu) = - \left( \frac{g s_W^2}{c_W} + g c_W \right) Z_\mu = - \frac{g}{c_W} Z_\mu, \quad (75)$$

得

$$\begin{aligned} g' B_\mu Y_H + g W_\mu^a \tau^a &= \frac{1}{2} \begin{pmatrix} g' B_\mu + g W_\mu^3 & g(W_\mu^1 - i W_\mu^2) \\ g(W_\mu^1 + i W_\mu^2) & g' B_\mu - g W_\mu^3 \end{pmatrix} \\ &= \begin{pmatrix} e A_\mu + \frac{g}{2c_W} (c_W^2 - s_W^2) Z_\mu & \frac{g}{\sqrt{2}} W_\mu^+ \\ \frac{g}{\sqrt{2}} W_\mu^- & - \frac{g}{2c_W} Z_\mu \end{pmatrix}. \end{aligned} \quad (76)$$

在么正规范下,

$$\begin{aligned}
(D^\mu \Phi)^\dagger (D_\mu \Phi) &= \left| \begin{pmatrix} \partial_\mu + ieA_\mu + \frac{ig}{2c_W}(c_W^2 - s_W^2)Z_\mu & \frac{ig}{\sqrt{2}}W_\mu^+ \\ \frac{ig}{\sqrt{2}}W_\mu^- & \partial_\mu - \frac{ig}{2c_W}Z_\mu \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+H \end{pmatrix} \right|^2 \\
&= \frac{1}{2} \begin{pmatrix} -\frac{ig}{\sqrt{2}}W^{-,\mu}(v+H) & \partial^\mu H + \frac{ig}{2c_W}Z^\mu(v+H) \end{pmatrix} \begin{pmatrix} \frac{ig}{\sqrt{2}}W_\mu^+(v+H) \\ \partial_\mu H - \frac{ig}{2c_W}Z_\mu(v+H) \end{pmatrix} \\
&= \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + (v+H)^2 \left( \frac{g^2}{4}W_\mu^+W^{-,\mu} + \frac{g^2}{8c_W^2}Z_\mu Z^\mu \right) \\
&= \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + m_W^2 W_\mu^+ W^{-,\mu} + \frac{1}{2}m_Z^2 Z_\mu Z^\mu \\
&\quad + gm_W H W_\mu^+ W^{-,\mu} + \frac{gm_Z}{2c_W} H Z_\mu Z^\mu + \frac{g^2}{4} H^2 W_\mu^+ W^{-,\mu} + \frac{g^2}{8c_W^2} H^2 Z_\mu Z^\mu. \quad (77)
\end{aligned}$$

除了  $W^\pm$  和  $Z$  玻色子的质量项之外, 还出现了 Higgs 玻色子  $H$  与  $W^\pm$ 、 $Z$  的三线性和四线性耦合项。

Higgs 场  $\Phi(x)$  的弱超荷为  $+1/2$ 。引入  $\Phi(x)$  的共轭态

$$\tilde{\Phi}(x) = i\sigma^2 \Phi^*(x) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{pmatrix} \phi^-(x) \\ \phi^{0*}(x) \end{pmatrix} = \begin{pmatrix} \phi^{0*}(x) \\ -\phi^-(x) \end{pmatrix}, \quad (78)$$

其中  $\phi^- \equiv (\phi^+)^*$ , 则  $\tilde{\Phi}(x)$  是弱超荷为  $-1/2$  的  $SU(2)_L$  二重态。在么正规范下,  $\tilde{\Phi}(x)$  化为

$$\tilde{\Phi}(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{pmatrix} 0 \\ v+H(x) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} v+H(x) \\ 0 \end{pmatrix}. \quad (79)$$

用  $\Phi(x)$ 、 $\tilde{\Phi}(x)$  和费米子场组成满足  $SU(2)_L \times U(1)_Y$  规范对称性的 Yukawa 相互作用拉氏量

$$\mathcal{L}_Y = -\tilde{y}_{d,ij} \bar{Q}_{iL} d'_{jR} \Phi - \tilde{y}_{u,ij} \bar{Q}_{iL} u'_{jR} \tilde{\Phi} - y_{\ell_i} \bar{L}_{iL} \ell_{iR} \Phi + \text{H.c.}, \quad (80)$$

其中 H.c. 表示厄米共轭, Yukawa 耦合常数  $\tilde{y}_{d,ij}$  和  $\tilde{y}_{u,ij}$  联系着不同代的夸克场, 而  $y_{\ell_i}$  只联系同一代的轻子场。在么正规范下, 利用

$$\bar{Q}_{iL} \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \bar{u}'_{iL} & \bar{d}'_{iL} \end{pmatrix} \begin{pmatrix} 0 \\ v+H \end{pmatrix} = \frac{1}{\sqrt{2}}(v+H)\bar{d}'_{iL}, \quad (81)$$

$$\bar{Q}_{iL} \tilde{\Phi} = \frac{1}{\sqrt{2}} \begin{pmatrix} \bar{u}'_{iL} & \bar{d}'_{iL} \end{pmatrix} \begin{pmatrix} v+H \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}(v+H)\bar{u}'_{iL}, \quad (82)$$

$$\bar{L}_{iL}\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \bar{\nu}_{iL} & \bar{\ell}_{iL} \end{pmatrix} \begin{pmatrix} 0 \\ v + H \end{pmatrix} = \frac{1}{\sqrt{2}}(v + H)\bar{\ell}_{iL}, \quad (83)$$

推出

$$\mathcal{L}_Y = -\frac{1}{\sqrt{2}}(v + H)(\tilde{y}_{d,ij}\bar{d}'_{iL}d'_{jR} + \tilde{y}_{u,ij}\bar{u}'_{iL}u'_{jR} + y_{\ell_i}\bar{\ell}_{iL}\ell_{iR} + \text{H.c.}). \quad (84)$$

$\tilde{y}_{d,ij}$  和  $\tilde{y}_{u,ij}$  可看作  $3 \times 3$  矩阵  $\tilde{y}_d$  和  $\tilde{y}_u$  的元素。 $\tilde{y}_d\tilde{y}_d^\dagger$  和  $\tilde{y}_u\tilde{y}_u^\dagger$  是厄米矩阵，必定可以分别通过么正矩阵  $U_d$  和  $U_u$  对角化成  $y_D^2$  和  $y_U^2$  两个对角元为实数的对角矩阵，满足  $U_d^\dagger\tilde{y}_d\tilde{y}_d^\dagger U_d = y_D^2$  和  $U_u^\dagger\tilde{y}_u\tilde{y}_u^\dagger U_u = y_U^2$ ，即

$$\tilde{y}_d\tilde{y}_d^\dagger = U_dy_D^2U_d^\dagger, \quad \tilde{y}_u\tilde{y}_u^\dagger = U_uy_U^2U_u^\dagger. \quad (85)$$

符合这两条式子的  $\tilde{y}_d$  和  $\tilde{y}_u$  可以表达为

$$\tilde{y}_d = U_dy_DK_d^\dagger, \quad \tilde{y}_u = U_uy_UK_u^\dagger, \quad (86)$$

其中对角矩阵  $y_D$  和  $y_U$  满足  $y_Dy_D = y_D^2$  和  $y_Uy_U = y_U^2$ ，而  $K_d^\dagger$  和  $K_u^\dagger$  是两个么正矩阵。

将  $y_D$  和  $y_U$  表示成

$$y_D = \begin{pmatrix} y_{d1} & & \\ & y_{d2} & \\ & & y_{d3} \end{pmatrix} = \begin{pmatrix} y_d & & \\ & y_s & \\ & & y_b \end{pmatrix}, \quad y_U = \begin{pmatrix} y_{u1} & & \\ & y_{u2} & \\ & & y_{u3} \end{pmatrix} = \begin{pmatrix} y_u & & \\ & y_c & \\ & & y_t \end{pmatrix}. \quad (87)$$

通过么正变换定义

$$d_{iL} \equiv (U_d^\dagger)_{ij}d'_{jL}, \quad d_{iR} \equiv (K_d^\dagger)_{ij}d'_{jR}, \quad u_{iL} \equiv (U_u^\dagger)_{ij}u'_{jL}, \quad u_{iR} \equiv (K_u^\dagger)_{ij}u'_{jR}, \quad (88)$$

则  $\bar{d}_{iL} = \bar{d}'_{jL}U_{d,ji}$ ， $\bar{u}_{iL} = \bar{u}'_{jL}U_{u,ji}$ ，从而

$$\tilde{y}_{d,ij}\bar{d}'_{iL}d'_{jR} = \bar{d}_{iL}(U_dy_dK_d^\dagger)_{ij}d'_{jR} = \bar{d}_{iL}U_{d,ik}y_{dk}(K_d^\dagger)_{kj}d'_{jR} = y_{dk}\bar{d}_{iL}d_{kR} = y_{d_i}\bar{d}_{iL}d_{iR}, \quad (89)$$

$$\tilde{y}_{u,ij}\bar{u}'_{iL}u'_{jR} = \bar{u}_{iL}(U_uy_uK_u^\dagger)_{ij}u'_{jR} = y_{u_i}\bar{u}_{iL}u_{iR}, \quad (90)$$

故

$$\begin{aligned} \mathcal{L}_Y &= -\frac{1}{\sqrt{2}}(v + H)(y_{d_i}\bar{d}_{iL}d_{iR} + y_{u_i}\bar{u}_{iL}u_{iR} + y_{\ell_i}\bar{\ell}_{iL}\ell_{iR} + \text{H.c.}) \\ &= -m_{d_i}\bar{d}_id_i - m_{u_i}\bar{u}_iu_i - m_{\ell_i}\bar{\ell}_i\ell_i - \frac{m_{d_i}}{v}H\bar{d}_id_i - \frac{m_{u_i}}{v}H\bar{u}_iu_i - \frac{m_{\ell_i}}{v}H\bar{\ell}_i\ell_i, \end{aligned} \quad (91)$$

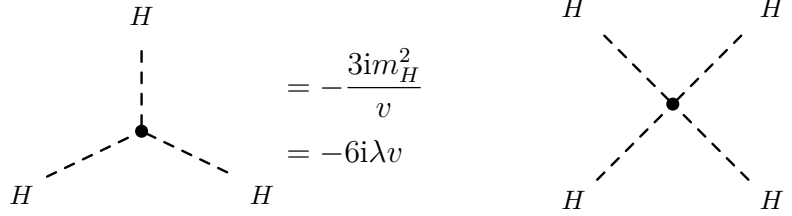
其中前三项是费米子质量项，后三项是 Higgs 玻色子与费米子的 Yukawa 耦合项。于是，三代夸克和带电轻子获得了质量

$$m_{d_i} \equiv \frac{1}{\sqrt{2}}y_{d_i}v, \quad m_{u_i} \equiv \frac{1}{\sqrt{2}}y_{u_i}v, \quad m_{\ell_i} \equiv \frac{1}{\sqrt{2}}y_{\ell_i}v. \quad (92)$$

$d'_{iL}$ 、 $d'_{iR}$ 、 $u'_{iL}$  和  $u'_{iR}$  称为规范本征态,  $d_{iL}$ 、 $d_{iR}$ 、 $u_{iL}$  和  $u_{iR}$  称为质量本征态。

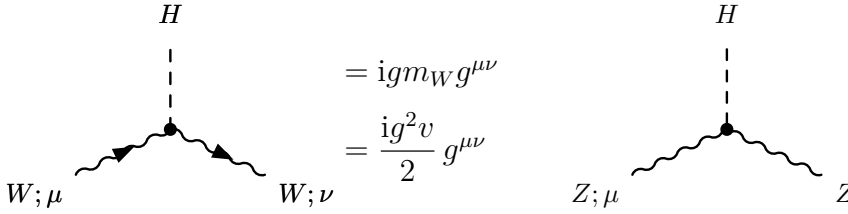
下面给出么正规化下 Higgs 场的顶点 Feynman 规则。

Higgs 玻色子自耦合:

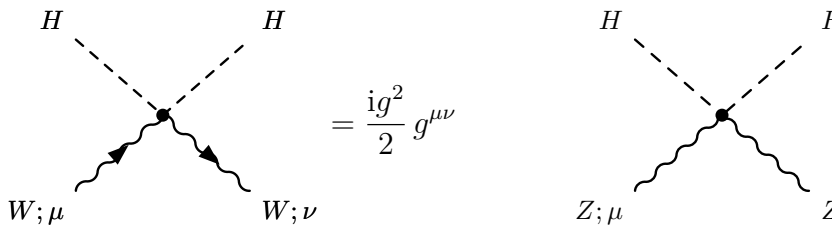


$$\begin{aligned}
 & \text{Three-point vertex: } = -\frac{3im_H^2}{v} = -6i\lambda v \\
 & \text{Four-point vertex: } = -\frac{3im_H^2}{v^2} = -6i\lambda
 \end{aligned} \tag{93}$$

Higgs 玻色子与电弱规范玻色子的耦合:

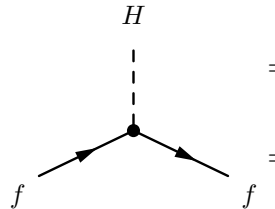


$$\begin{aligned}
 & \text{Higgs-W-W vertex: } = igm_W g^{\mu\nu} = \frac{ig^2 v}{2} g^{\mu\nu} \\
 & \text{Higgs-Z-Z vertex: } = \frac{igm_Z}{c_W} g^{\mu\nu} = \frac{ig^2 v}{2c_W^2} g^{\mu\nu}
 \end{aligned} \tag{94}$$



$$\begin{aligned}
 & \text{Higgs-W-W four-point vertex: } = \frac{ig^2}{2} g^{\mu\nu} \\
 & \text{Higgs-Z-Z four-point vertex: } = \frac{ig^2}{2c_W^2} g^{\mu\nu}
 \end{aligned} \tag{95}$$

Higgs 玻色子与费米子  $f = d_i, u_i, \ell_i$  的耦合:



$$\begin{aligned}
 & \text{Higgs-fermion vertex: } = -\frac{im_f}{v} \\
 & \text{Higgs-fermion vertex: } = -\frac{iy_f}{\sqrt{2}}
 \end{aligned} \tag{96}$$

## 4.2 费米子电弱规范相互作用

(88) 式意味着

$$d'_{iL} = U_{d,ij} d_{jL}, \quad d'_{iR} = K_{d,ij} d_{jR}, \quad u'_{iL} = U_{u,ij} u_{jL}, \quad u'_{iR} = K_{u,ij} u_{jR}, \tag{97}$$

从而

$$\bar{d}'_{iL} \gamma^\mu d'_{iL} = \bar{d}_{jL} (U_d^\dagger)_{ji} \gamma^\mu U_{d,ik} d_{kL} = \bar{d}_{jL} \delta_{jk} \gamma^\mu d_{kL} = \bar{d}_{iL} \gamma^\mu d_{iL} \tag{98}$$

同理有

$$\bar{u}'_{iL} \gamma^\mu u'_{iL} = \bar{u}_{iL} \gamma^\mu u_{iL}, \quad \bar{d}'_{iR} \gamma^\mu d'_{iR} = \bar{d}_{iR} \gamma^\mu d_{iR}, \quad \bar{u}'_{iR} \gamma^\mu u'_{iR} = \bar{u}_{iR} \gamma^\mu u_{iR}. \tag{99}$$

另一方面,

$$\bar{u}'_{iL}\gamma^\mu d'_{iL} = \bar{u}_{jL}(U_u^\dagger)_{ji}\gamma^\mu U_{d,ik}d_{kL} = \bar{u}_{iL}\gamma^\mu V_{ij}d_{jL} \quad (100)$$

$$\bar{d}'_{iL}\gamma^\mu u'_{iL} = \bar{d}_{jL}(U_d^\dagger)_{ji}\gamma^\mu U_{u,ik}u_{kL} = \bar{d}_{jL}V_{ji}^\dagger\gamma^\mu u_{iL} \quad (101)$$

其中

$$V \equiv U_u^\dagger U_d \quad (102)$$

称为 Cabibbo-Kobayashi-Maskawa (CKM) 矩阵, 其厄米共轭矩阵为  $V^\dagger = U_d^\dagger U_u$ 。

$SU(2)_L \times U(1)_Y$  规范不变的费米子协变动能项为

$$\mathcal{L}_{\text{EWF}} = \bar{Q}_{iL}i\not{D}Q_{iL} + \bar{u}'_{iR}i\not{D}u'_{iR} + \bar{d}'_{iR}i\not{D}d'_{iR} + \bar{L}_{iL}i\not{D}L_{iL} + \bar{\ell}_{iR}i\not{D}\ell_{iR}. \quad (103)$$

根据  $Q$  的定义 (39), 有

$$\begin{aligned} g'Y B_\mu + gT^3 W_\mu^3 &= g'Y(c_W A_\mu - s_W Z_\mu) + gT^3(s_W A_\mu + c_W Z_\mu) \\ &= e(Y + T^3)A_\mu + \left(gc_W T^3 - \frac{gs_W}{c_W} s_W Y\right) Z_\mu = QeA_\mu + \frac{g}{c_W}(T^3 c_W^2 - Y s_W^2)Z_\mu \\ &= QeA_\mu + \frac{g}{c_W}(T^3 - Qs_W^2)Z_\mu, \end{aligned} \quad (104)$$

故

$$\begin{aligned} D_\mu Q_{iL} &= (\partial_\mu + ig'B_\mu Y + igW_\mu^a \tau^a)Q_{iL} \\ &= \partial_\mu Q_{iL} + i \begin{pmatrix} g'Y B_\mu + gT^3 W_\mu^3 & \frac{g}{2}(W_\mu^1 - iW_\mu^2) \\ \frac{g}{2}(W_\mu^1 + iW_\mu^2) & g'Y B_\mu + gT^3 W_\mu^3 \end{pmatrix} Q_{iL} \\ &= \partial_\mu Q_{iL} + i \begin{pmatrix} QeA_\mu + \frac{g}{c_W}(T^3 - Qs_W^2)Z_\mu & \frac{g}{\sqrt{2}}W_\mu^+ \\ \frac{g}{\sqrt{2}}W_\mu^- & QeA_\mu + \frac{g}{c_W}(T^3 - Qs_W^2)Z_\mu \end{pmatrix} \begin{pmatrix} u'_{iL} \\ d'_{iL} \end{pmatrix} \\ &= \partial_\mu Q_{iL} + i \begin{pmatrix} \left[QeA_\mu + \frac{g}{c_W}(T^3 - Qs_W^2)Z_\mu\right] u'_{iL} + \frac{g}{\sqrt{2}}W_\mu^+ d'_{iL} \\ \frac{g}{\sqrt{2}}W_\mu^- u'_{iL} + \left[QeA_\mu + \frac{g}{c_W}(T^3 - Qs_W^2)Z_\mu\right] d'_{iL} \end{pmatrix}. \end{aligned} \quad (105)$$

于是

$$\begin{aligned} \bar{Q}_{iL}i\not{D}Q_{iL} &\supset - \left[QeA_\mu + \frac{g}{c_W}(T^3 - Qs_W^2)Z_\mu\right] \bar{u}'_{iL}\gamma^\mu u'_{iL} - \left[QeA_\mu + \frac{g}{c_W}(T^3 - Qs_W^2)Z_\mu\right] \bar{d}'_{iL}\gamma^\mu d'_{iL} \\ &\quad - \frac{g}{\sqrt{2}}W_\mu^+ \bar{u}'_{iL}\gamma^\mu d'_{iL} - \frac{g}{\sqrt{2}}W_\mu^- \bar{d}'_{iL}\gamma^\mu u'_{iL} \\ &= - \left(QeA_\mu + \frac{g}{c_W}g_L Z_\mu\right) \bar{u}_i\gamma^\mu \frac{1-\gamma^5}{2} u_i - \left(QeA_\mu + \frac{g}{c_W}g_L Z_\mu\right) \bar{d}_i\gamma^\mu \frac{1-\gamma^5}{2} d_i \end{aligned}$$



$$-\frac{g}{\sqrt{2}} W_{\mu}^{+} \bar{u}_i \gamma^{\mu} P_L V_{ij} d_j - \frac{g}{\sqrt{2}} W_{\mu}^{-} \bar{d}_j V_{ji}^{\dagger} \gamma^{\mu} P_L u_i, \quad (106)$$

其中左手耦合系数

$$g_L \equiv T^3 - Q s_W^2. \quad (107)$$

另一方面,

$$\begin{aligned} D_{\mu} d'_{iR} &= (\partial_{\mu} + i g' B_{\mu} Y) d'_{iR} = \partial_{\mu} d'_{iR} + i g' Q (c_W A_{\mu} - s_W Z_{\mu}) d'_{iR} \\ &= \partial_{\mu} d'_{iR} + i Q e A_{\mu} d'_{iR} - \frac{i g}{c_W} Q s_W^2 Z_{\mu} d'_{iR}, \end{aligned} \quad (108)$$

则

$$\begin{aligned} &\bar{u}'_{iR} i \not{D} u'_{iR} + \bar{d}'_{iR} i \not{D} d'_{iR} \\ &\supset - \left( Q e A_{\mu} - \frac{g}{c_W} Q s_W^2 Z_{\mu} \right) \bar{u}'_{iR} \gamma^{\mu} u'_{iR} - \left( Q e A_{\mu} - \frac{g}{c_W} Q s_W^2 Z_{\mu} \right) \bar{d}'_{iR} \gamma^{\mu} d'_{iR} \\ &= - \left( Q e A_{\mu} + \frac{g}{c_W} g_R Z_{\mu} \right) \bar{u}_i \gamma^{\mu} \frac{1 + \gamma^5}{2} u_i - \left( Q e A_{\mu} + \frac{g}{c_W} g_R Z_{\mu} \right) \bar{d}_i \gamma^{\mu} \frac{1 + \gamma^5}{2} d_i, \end{aligned} \quad (109)$$

其中右手耦合系数

$$g_R \equiv -Q s_W^2. \quad (110)$$

引入矢量流和轴矢量流耦合系数

$$g_V \equiv g_L + g_R = T^3 - 2Q s_W^2, \quad g_A \equiv g_L - g_R = T^3, \quad (111)$$

得

$$\begin{aligned} &\bar{Q}_{iL} i \not{D} Q_{iL} + \bar{u}'_{iR} i \not{D} u'_{iR} + \bar{d}'_{iR} i \not{D} d'_{iR} \\ &\supset -Q e \bar{u}_i \gamma^{\mu} u_i A_{\mu} - Q e \bar{d}_i \gamma^{\mu} d_i A_{\mu} - \frac{g}{2c_W} \bar{u}_i \gamma^{\mu} (g_V - g_A \gamma^5) u_i Z_{\mu} - \frac{g}{2c_W} \bar{d}_i \gamma^{\mu} (g_V - g_A \gamma^5) d_i Z_{\mu} \\ &\quad - \frac{g}{\sqrt{2}} W_{\mu}^{+} \bar{u}_i \gamma^{\mu} P_L V_{ij} d_j - \frac{g}{\sqrt{2}} W_{\mu}^{-} \bar{d}_j V_{ji}^{\dagger} \gamma^{\mu} P_L u_i. \end{aligned} \quad (112)$$

同理, 有

$$\begin{aligned} \bar{L}_{iL} i \not{D} L_{iL} + \bar{\ell}_{iR} i \not{D} \ell_{iR} &\supset -Q e \bar{\ell}_i \gamma^{\mu} \ell_i A_{\mu} - \frac{g}{2c_W} \bar{\ell}_i \gamma^{\mu} (g_V - g_A \gamma^5) \ell_i Z_{\mu} - \frac{g}{2c_W} \bar{\nu}_i \gamma^{\mu} (g_V - g_A \gamma^5) \nu_i Z_{\mu} \\ &\quad - \frac{g}{\sqrt{2}} W_{\mu}^{+} \bar{\nu}_i \gamma^{\mu} P_L \ell_i - \frac{g}{\sqrt{2}} W_{\mu}^{-} \bar{\ell}_i \gamma^{\mu} P_L \nu_i. \end{aligned} \quad (113)$$

总结起来, 将费米子电弱规范相互作用写成流耦合的形式,

$$\mathcal{L}_{\text{EWF}} \supset -A_{\mu} J_{\text{EM}}^{\mu} - Z_{\mu} J_Z^{\mu} - W_{\mu}^{+} J_W^{+, \mu} - W_{\mu}^{-} J_W^{-, \mu}, \quad (114)$$

其中  $f$  代表任意费米子场, 电磁流

$$J_{\text{EM}}^\mu \equiv \sum_f Q_f e \bar{f} \gamma^\mu f, \quad (115)$$

弱中性流

$$J_Z^\mu \equiv \frac{g}{2c_W} \sum_f \bar{f} \gamma^\mu (g_V^f - g_A^f \gamma^5) f = \frac{g}{c_W} \sum_f (g_L^f \bar{f}_L \gamma^\mu f_L + g_R^f \bar{f}_R \gamma^\mu f_R), \quad (116)$$

弱带电流

$$J_W^{+, \mu} \equiv \frac{g}{\sqrt{2}} (\bar{u}_i \gamma^\mu V_{ij} P_L d_j + \bar{\nu}_i \gamma^\mu P_L \ell_i), \quad J_W^{-, \mu} \equiv (J_W^{+, \mu})^\dagger = \frac{g}{\sqrt{2}} (\bar{d}_j V_{ji}^\dagger \gamma^\mu P_L u_i + \bar{\ell}_i \gamma^\mu P_L \nu_i). \quad (117)$$

对于各种费米子, 相关的系数如下,

$$Q_{u_i} = \frac{2}{3}, \quad Q_{d_i} = -\frac{1}{3}, \quad Q_{\nu_i} = 0, \quad Q_{\ell_i} = -1; \quad (118)$$

$$g_V^{u_i} = \frac{1}{2} - \frac{4}{3} s_W^2, \quad g_A^{u_i} = \frac{1}{2}; \quad g_V^{d_i} = -\frac{1}{2} + \frac{2}{3} s_W^2, \quad g_A^{d_i} = -\frac{1}{2}; \quad (119)$$

$$g_V^{\nu_i} = \frac{1}{2}, \quad g_A^{\nu_i} = \frac{1}{2}; \quad g_V^{\ell_i} = -\frac{1}{2} + 2s_W^2, \quad g_A^{\ell_i} = -\frac{1}{2}; \quad (120)$$

$$g_L^{u_i} = \frac{1}{2} - \frac{2}{3} s_W^2, \quad g_R^{u_i} = -\frac{2}{3} s_W^2; \quad g_L^{d_i} = -\frac{1}{2} + \frac{1}{3} s_W^2, \quad g_R^{d_i} = \frac{1}{3} s_W^2; \quad (121)$$

$$g_L^{\nu_i} = \frac{1}{2}, \quad g_R^{\nu_i} = 0; \quad g_L^{\ell_i} = -\frac{1}{2} + s_W^2, \quad g_R^{\ell_i} = s_W^2. \quad (122)$$

可以看到, 电磁流耦合与 QED 耦合完全相同, 由此辩认出  $A_\mu$  是电磁场,  $e$  是单位电荷量, 由  $Q = T^3 + Y$  定义的  $Q$  确实是电荷。为了保持电荷守恒, 指定复矢量场  $W_\mu^+(x)$  携带  $Q = +1$  的电荷, 从而  $W^\pm$  玻色子的电荷为  $\pm 1$ 。不同代夸克之间的相互作用只发生在弱带电流耦合中, 源自 CKM 矩阵  $V$  的非对角元。

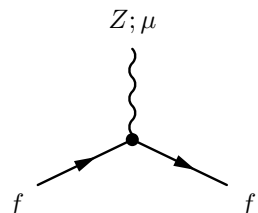
下面给出费米子电弱规范相互作用顶点的 Feynman 规则。

QED 耦合:



$$= -iQ_f e \gamma^\mu \quad (123)$$

费米子与  $Z$  玻色子的耦合:



$$= -\frac{ig}{2c_W} \gamma^\mu (g_V^f - g_A^f \gamma^5) \quad (124)$$

费米子与  $W^\pm$  玻色子的耦合:

$$\begin{aligned}
 \begin{array}{c} W; \mu \\ \downarrow \\ d_j \rightarrow \bullet \rightarrow u_i \end{array} &= -\frac{ig}{\sqrt{2}} V_{ij} \gamma^\mu P_L & \begin{array}{c} W; \mu \\ \downarrow \\ u_i \rightarrow \bullet \rightarrow d_j \end{array} &= -\frac{ig}{\sqrt{2}} V_{ji}^\dagger \gamma^\mu P_L & (125)
 \end{aligned}$$

$$\begin{aligned}
 \begin{array}{c} W; \mu \\ \downarrow \\ \ell_i \rightarrow \bullet \rightarrow \nu_i \end{array} &= -\frac{ig}{\sqrt{2}} \gamma^\mu P_L & \begin{array}{c} W; \mu \\ \downarrow \\ \nu_i \rightarrow \bullet \rightarrow \ell_i \end{array} &= -\frac{ig}{\sqrt{2}} \gamma^\mu P_L & (126)
 \end{aligned}$$

考虑  $\mu$  子衰变的最主要过程  $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$ , 相应的领头阶不变振幅为

$$\begin{aligned}
 i\mathcal{M} &= \mu^- \xrightarrow{k} \begin{array}{c} \nearrow p_3 \nu_\mu \\ \searrow p_1 e^- \\ \searrow p_2 \bar{\nu}_e \end{array} \xleftarrow{q} W^- \\
 &= \left( \frac{-ig}{\sqrt{2}} \right)^2 \bar{u}(p_3) \gamma^\mu P_L u(k) \frac{-i(g_{\mu\nu} - q_\mu q_\nu / m_W^2)}{q^2 - m_W^2} \bar{u}(p_1) \gamma^\nu P_L v(p_2) \\
 &= \frac{ig^2(g_{\mu\nu} - q_\mu q_\nu / m_W^2)}{8(q^2 - m_W^2)} \bar{u}(p_3) \gamma^\mu (1 - \gamma^5) u(k) \bar{u}(p_1) \gamma^\nu (1 - \gamma^5) v(p_2) & (127)
 \end{aligned}$$

由于  $m_\mu \ll m_W$ ,  $W$  传播子的四维动量  $q^\mu$  满足  $q^2 \ll m_W^2$ 。因此, 可在低能近似下忽略  $q^\mu$  和  $q^2$ , 将不变振幅化为

$$i\mathcal{M} \simeq -\frac{ig^2}{8m_W^2} \bar{u}(p_3) \gamma^\mu (1 - \gamma^5) u(k) \bar{u}(p_1) \gamma_\mu (1 - \gamma^5) v(p_2). \quad (128)$$

可以将这样的振幅看作有效拉氏量

$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \bar{\nu}_\mu \gamma^\mu (1 - \gamma^5) \mu \bar{e} \gamma_\mu (1 - \gamma^5) \nu_e + \text{H.c.} \quad (129)$$

的结果, 其中 Fermi 常数  $G_F$  定义为

$$\frac{G_F}{\sqrt{2}} \equiv \frac{g^2}{8m_W^2}. \quad (130)$$

可以进一步将  $\mathcal{L}_{\text{eff}}$  推广到其它参与弱相互作用的标准模型费米子, 而耦合常数  $G_F$  是普适的,

对所有费米子都适用，这样得到的理论称为四费米子相互作用理论。为了解释  $\beta$  衰变，Enrico Fermi 于 1933 年首次提出这个理论。现在认为它是标准模型弱相互作用的低能有效理论。

忽略电子质量，由以上振幅推出  $\mu$  子寿命为

$$\tau_\mu = \frac{1}{\Gamma_\mu} \simeq \frac{192\pi^3}{G_F^2 m_\mu^5}. \quad (131)$$

于是，通过测量  $\mu$  子寿命可以得到 Fermi 常数的观测值。根据 (57) 式，Fermi 常数  $G_F$  与 Higgs 场真空期待值  $v$  的关系为

$$G_F = \frac{\sqrt{2}g^2}{8m_W^2} = \frac{\sqrt{2}g^2}{2g^2v^2} = \frac{1}{\sqrt{2}v^2}, \quad v = (\sqrt{2}G_F)^{-1/2}. \quad (132)$$

### 4.3 电弱规范场的自相互作用

电弱规范场自相互作用的拉氏量是

$$\mathcal{L}_{\text{EWG}} = -\frac{1}{4} W_{\mu\nu}^a W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}, \quad (133)$$

其中

$$W_{\mu\nu}^a \equiv \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - g\varepsilon^{abc} W_\mu^b W_\nu^c, \quad B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu. \quad (134)$$

利用 (68) 和 (72) 式，推出

$$\begin{aligned} W_\mu^2 W_\nu^3 - W_\mu^3 W_\nu^2 &= \frac{i}{\sqrt{2}} [(W_\mu^+ - W_\mu^-)(s_W A_\nu + c_W Z_\nu) - (s_W A_\mu + c_W Z_\mu)(W_\nu^+ - W_\nu^-)] \\ &= \frac{i}{\sqrt{2}} [s_W(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+) \\ &\quad - s_W(W_\mu^- A_\nu - A_\mu W_\nu^-) - c_W(W_\mu^- Z_\nu - Z_\mu W_\nu^-)], \end{aligned} \quad (135)$$

$$\begin{aligned} W_\mu^3 W_\nu^1 - W_\mu^1 W_\nu^3 &= \frac{1}{\sqrt{2}} [(s_W A_\mu + c_W Z_\mu)(W_\nu^+ + W_\nu^-) - (W_\mu^+ + W_\mu^-)(s_W A_\nu + c_W Z_\nu)] \\ &= -\frac{1}{\sqrt{2}} [s_W(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+) \\ &\quad + s_W(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_W(W_\mu^- Z_\nu - Z_\mu W_\nu^-)]. \end{aligned} \quad (136)$$

从而

$$\begin{aligned} W_{\mu\nu}^1 &= \partial_\mu W_\nu^1 - \partial_\nu W_\mu^1 - g\varepsilon^{1bc} W_\mu^b W_\nu^c = \partial_\mu W_\nu^1 - \partial_\nu W_\mu^1 - g(W_\mu^2 W_\nu^3 - W_\mu^3 W_\nu^2) \\ &= \frac{1}{\sqrt{2}} (\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) + \frac{1}{\sqrt{2}} (\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) - g(W_\mu^2 W_\nu^3 - gW_\mu^3 W_\nu^2) \\ &= \frac{1}{\sqrt{2}} \{ \partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ - ig[s_W(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)] \} \\ &\quad + \frac{1}{\sqrt{2}} \{ \partial_\mu W_\nu^- - \partial_\nu W_\mu^- + ig[s_W(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_W(W_\mu^- Z_\nu - Z_\mu W_\nu^-)] \} \end{aligned}$$

$$= \frac{1}{\sqrt{2}}(F_{\mu\nu}^+ + F_{\mu\nu}^-), \quad (137)$$

其中,

$$F_{\mu\nu}^+ \equiv \partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ - ie(W_\mu^+ A_\nu - A_\mu W_\nu^+) - igc_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+), \quad (138)$$

$$F_{\mu\nu}^- \equiv \partial_\mu W_\nu^- - \partial_\nu W_\mu^- + ie(W_\mu^- A_\nu - A_\mu W_\nu^-) + igc_W(W_\mu^- Z_\nu - Z_\mu W_\nu^-). \quad (139)$$

另一方面,

$$\begin{aligned} W_{\mu\nu}^2 &= \partial_\mu W_\nu^2 - \partial_\nu W_\mu^2 - g\varepsilon^{2bc}W_\mu^b W_\nu^c = \partial_\mu W_\nu^2 - \partial_\nu W_\mu^2 - g(W_\mu^3 W_\nu^1 - W_\mu^1 W_\nu^3) \\ &= \frac{i}{\sqrt{2}}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) - \frac{i}{\sqrt{2}}(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) - g(W_\mu^3 W_\nu^1 - W_\mu^1 W_\nu^3) \\ &= \frac{i}{\sqrt{2}}\{\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ - ig[s_W(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)]\} \\ &\quad - \frac{i}{\sqrt{2}}\{\partial_\mu W_\nu^- - \partial_\nu W_\mu^- + ig[s_W(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_W(W_\mu^- Z_\nu - Z_\mu W_\nu^-)]\} \\ &= \frac{i}{\sqrt{2}}(F_{\mu\nu}^+ - F_{\mu\nu}^-). \end{aligned} \quad (140)$$

因此

$$\begin{aligned} & -\frac{1}{4}W_{\mu\nu}^1 W^{1,\mu\nu} - \frac{1}{4}W_{\mu\nu}^2 W^{2,\mu\nu} \\ &= -\frac{1}{8}(F_{\mu\nu}^+ + F_{\mu\nu}^-)(F^{+,\mu\nu} + F^{-,\mu\nu}) + \frac{1}{8}(F_{\mu\nu}^+ - F_{\mu\nu}^-)(F^{+,\mu\nu} - F^{-,\mu\nu}) = -\frac{1}{2}F_{\mu\nu}^+ F^{-,\mu\nu} \\ &= -\frac{1}{2}[\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ - ie(W_\mu^+ A_\nu - A_\mu W_\nu^+) - igc_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)] \\ &\quad \times [\partial^\mu W^{-,\nu} - \partial^\nu W^{-,\mu} + ie(W^{-,\mu} A^\nu - A^\mu W^{-,\nu}) + igc_W(W^{-,\mu} Z^\nu - Z^\mu W^{-,\nu})] \\ &= -(\partial_\mu W_\nu^+)(\partial^\mu W^{-,\nu}) + (\partial_\mu W_\nu^+)(\partial^\nu W^{-,\mu}) \\ &\quad - ie[(\partial_\mu W_\nu^+)W^{-,\mu} A^\nu - (\partial_\mu W_\nu^+)W^{-,\nu} A^\mu - W_\mu^+(\partial^\mu W^{-,\nu})A_\nu + W_\nu^+(\partial^\mu W^{-,\nu})A_\mu] \\ &\quad - igc_W[(\partial_\mu W_\nu^+)W^{-,\mu} Z^\nu - (\partial_\mu W_\nu^+)W^{-,\nu} Z^\mu - W_\mu^+(\partial^\mu W^{-,\nu})Z_\nu + W_\nu^+(\partial^\mu W^{-,\nu})Z_\mu] \\ &\quad + e^2(W_\mu^+ W^{-,\nu} A_\nu A^\mu - W_\mu^+ W^{-,\mu} A_\nu A^\nu) + g^2 c_W^2(W_\mu^+ W^{-,\nu} Z_\nu Z^\mu - W_\mu^+ W^{-,\mu} Z_\nu Z^\nu) \\ &\quad + egc_W(W_\mu^+ W^{-,\nu} A_\nu Z^\mu + W_\mu^+ W^{-,\nu} A^\mu Z_\nu - 2W_\mu^+ W^{-,\mu} A_\nu Z^\nu). \end{aligned} \quad (141)$$

由

$$\begin{aligned} W_\mu^1 W_\nu^2 - W_\mu^2 W_\nu^1 &= \frac{i}{2}(W_\mu^+ + W_\mu^-)(W_\nu^+ - W_\nu^-) - \frac{i}{2}(W_\mu^+ - W_\mu^-)(W_\nu^+ + W_\nu^-) \\ &= -i(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+), \end{aligned} \quad (142)$$

得到

$$\begin{aligned}
 W_{\mu\nu}^3 &= \partial_\mu W_\nu^3 - \partial_\nu W_\mu^3 - g\epsilon^{3bc}W_\mu^b W_\nu^c = \partial_\mu W_\nu^3 - \partial_\nu W_\mu^3 - g(W_\mu^1 W_\nu^2 - W_\mu^2 W_\nu^1) \\
 &= s_W \partial_\mu A_\nu + c_W \partial_\mu Z_\nu - s_W \partial_\nu A_\mu + c_W \partial_\nu Z_\mu - g(W_\mu^1 W_\nu^2 - W_\mu^2 W_\nu^1) \\
 &= s_W (\partial_\mu A_\nu - \partial_\nu A_\mu) + c_W (\partial_\mu Z_\nu - \partial_\nu Z_\mu) + ig(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+), \tag{143}
 \end{aligned}$$

$$\begin{aligned}
 B_{\mu\nu} &= \partial_\mu (c_W A_\nu - s_W Z_\nu) - \partial_\nu (c_W A_\mu - s_W Z_\mu) \\
 &= c_W (\partial_\mu A_\nu - \partial_\nu A_\mu) - s_W (\partial_\mu Z_\nu - \partial_\nu Z_\mu). \tag{144}
 \end{aligned}$$

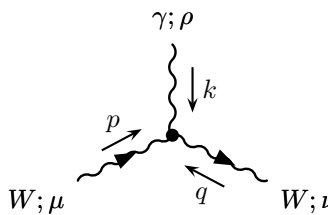
于是

$$\begin{aligned}
 &-\frac{1}{4} W_{\mu\nu}^3 W^{3,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\
 &= -\frac{1}{2} [(\partial_\mu A_\nu)(\partial^\mu A^\nu) - (\partial_\mu A_\nu)(\partial^\nu A^\mu)] - \frac{1}{2} [(\partial_\mu Z_\nu)(\partial^\mu Z^\nu) - (\partial_\mu Z_\nu)(\partial^\nu Z^\mu)] \\
 &\quad -ie[W^{+,\mu} W^{-,\nu}(\partial_\mu A_\nu) - W^{+,\nu} W^{-,\mu}(\partial_\mu A_\nu)] - igc_W[W^{+,\mu} W^{-,\nu}(\partial_\mu Z_\nu) - W^{+,\nu} W^{-,\mu}(\partial_\mu Z_\nu)] \\
 &\quad + \frac{g^2}{2} (W_\mu^+ W^{+,\mu} W_\nu^- W^{-,\nu} - W_\mu^+ W^{+,\nu} W_\nu^- W^{-,\mu}). \tag{145}
 \end{aligned}$$

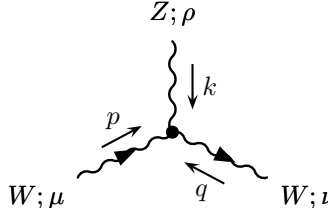
综合起来, 有

$$\begin{aligned}
 \mathcal{L}_{\text{EWG}} &= \frac{1}{2} [(\partial_\mu A_\nu)(\partial^\nu A^\mu) - (\partial_\mu A_\nu)(\partial^\mu A^\nu)] + \frac{1}{2} [(\partial_\mu Z_\nu)(\partial^\nu Z^\mu) - (\partial_\mu Z_\nu)(\partial^\mu Z^\nu)] \\
 &\quad + (\partial_\mu W_\nu^+)(\partial^\nu W^{-,\mu}) - (\partial_\mu W_\nu^+)(\partial^\mu W^{-,\nu}) + \frac{g^2}{2} (W_\mu^+ W^{+,\mu} W_\nu^- W^{-,\nu} - W_\mu^+ W^{+,\nu} W_\nu^- W^{-,\mu}) \\
 &\quad -ie[(\partial_\mu W_\nu^+) W^{-,\mu} A^\nu - (\partial_\mu W_\nu^+) W^{-,\nu} A^\mu - W^{+,\mu}(\partial_\mu W_\nu^-) A^\nu + W^{+,\nu}(\partial_\mu W_\nu^-) A^\mu \\
 &\quad + W^{+,\mu} W^{-,\nu}(\partial_\mu A_\nu) - W^{+,\nu} W^{-,\mu}(\partial_\mu A_\nu)] \\
 &\quad -igc_W[(\partial_\mu W_\nu^+) W^{-,\mu} Z^\nu - (\partial_\mu W_\nu^+) W^{-,\nu} Z^\mu - W^{+,\mu}(\partial_\mu W_\nu^-) Z^\nu + W^{+,\nu}(\partial_\mu W_\nu^-) Z^\mu \\
 &\quad + W^{+,\mu} W^{-,\nu}(\partial_\mu Z_\nu) - W^{+,\nu} W^{-,\mu}(\partial_\mu Z_\nu)] \\
 &\quad + e^2 (W_\mu^+ W^{-,\nu} A_\nu A^\mu - W_\mu^+ W^{-,\mu} A_\nu A^\nu) + g^2 c_W^2 (W_\mu^+ W^{-,\nu} Z_\nu Z^\mu - W_\mu^+ W^{-,\mu} Z_\nu Z^\nu) \\
 &\quad + egc_W (W_\mu^+ W^{-,\nu} A_\nu Z^\mu + W_\mu^+ W^{-,\nu} A^\mu Z_\nu - 2W_\mu^+ W^{-,\mu} A_\nu Z^\nu). \tag{146}
 \end{aligned}$$

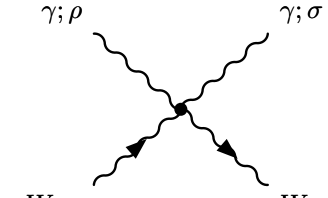
下面是电弱规范玻色子自耦合的 Feynman 规则:



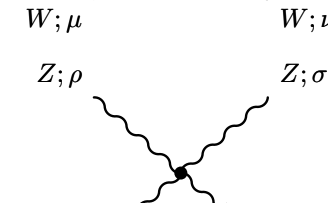
$$= ie[g^{\mu\nu}(p-q)^\rho + g^{\nu\rho}(q-k)^\mu + g^{\rho\mu}(k-p)^\nu] \tag{147}$$



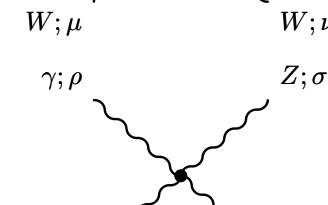
$$= ig_{cW}[g^{\mu\nu}(p-q)^\rho + g^{\nu\rho}(q-k)^\mu + g^{\rho\mu}(k-p)^\nu] \quad (148)$$



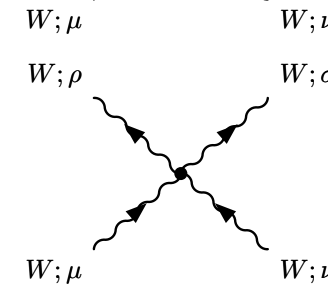
$$= ie^2(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma}) \quad (149)$$



$$= ig^2 c_W^2 (g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma}) \quad (150)$$



$$= ieg_{cW}(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma}) \quad (151)$$



$$= -ig^2(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma}) \quad (152)$$

## 5 $R_\xi$ 规范下电弱拉氏量和 Feynman 规则

在  $R_\xi$  规范下, 将 Higgs 场参数化为

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}[v + H(x) + i\chi(x)] \end{pmatrix}, \quad (153)$$

其中  $\phi^+$  和  $\chi$  是 Nambu-Goldstone 标量场。那么,  $\tilde{\Phi}(x)$  的形式是

$$\tilde{\Phi}(x) = \begin{pmatrix} \phi^{0*}(x) \\ -\phi^-(x) \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}[v + H(x) - i\chi(x)] \\ -\phi^-(x) \end{pmatrix}. \quad (154)$$

利用

$$\Phi^\dagger \Phi = \frac{1}{2}(v^2 + H^2 + 2vH + \chi^2) + |\phi^+|^2, \quad (155)$$

$$(\Phi^\dagger \Phi)^2 = \frac{1}{4}(v^2 + H^2 + 2vH + \chi^2)^2 + |\phi^+|^4 + |\phi^+|^2(v^2 + H^2 + 2vH + \chi^2), \quad (156)$$

推出 Higgs 场势能项

$$\begin{aligned} -V_H(\Phi) &= \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 \\ &= \frac{\mu^2}{2}(v^2 + H^2 + 2vH + \chi^2) + \mu^2 |\phi^+|^2 - \frac{\lambda}{4}(v^2 + H^2 + 2vH + \chi^2)^2 - \lambda |\phi^+|^4 \\ &\quad - \lambda |\phi^+|^2(v^2 + H^2 + 2vH + \chi^2) \\ &= \frac{1}{2} \left( \mu^2 - \frac{\lambda}{2} v^2 \right) v^2 + \frac{1}{2} (\mu^2 - 3\lambda v^2) H^2 + (\mu^2 - \lambda v^2) vH + \frac{1}{2} (\mu^2 - \lambda v^2) \chi^2 - \frac{\lambda}{4} H^4 - \frac{\lambda}{4} \chi^4 \\ &\quad - \lambda v H^3 - \frac{\lambda}{2} H^2 \chi^2 - \lambda v H \chi^2 + (\mu^2 - \lambda v^2) |\phi^+|^2 - \lambda |\phi^+|^4 - \lambda |\phi^+|^2 (H^2 + 2vH + \chi^2) \\ &= \frac{\lambda}{4} v^4 - \lambda v^2 H^2 - \frac{\lambda}{4} H^4 - \frac{\lambda}{4} \chi^4 - \lambda v H^3 - \frac{\lambda}{2} H^2 \chi^2 - \lambda v H \chi^2 \\ &\quad - \lambda \phi^+ \phi^- (\phi^+ \phi^- + H^2 + 2vH + \chi^2) \\ &= \frac{1}{8} m_H^2 v^2 - \frac{1}{2} m_H^2 H^2 - \frac{m_H^2}{2v} H^3 - \frac{m_H^2}{8v^2} H^4 - \frac{m_H^2}{2v} H \chi^2 - \frac{m_H^2}{4v^2} H^2 \chi^2 - \frac{m_H^2}{8v^2} \chi^4 \\ &\quad - \frac{m_H^2}{2v^2} \phi^+ \phi^- (\phi^+ \phi^- + H^2 + 2vH + \chi^2). \end{aligned} \quad (157)$$

由  $V = U_u^\dagger U_d$  得到  $V^\dagger = U_d^\dagger U_u$  和  $U_d = U_u V$ ,  $U_u = U_d V^\dagger$ , 则

$$\tilde{y}_d = U_d y_D K_d^\dagger = U_u V y_D K_d^\dagger, \quad \tilde{y}_u = U_u y_U K_u^\dagger = U_d V^\dagger y_U K_u^\dagger, \quad (158)$$

故

$$\tilde{y}_{d,ij} \bar{u}'_{iL} d'_{jR} = \bar{u}'_{iL} (U_u V y_D K_d^\dagger)_{ij} d'_{jR} = \bar{u}'_{iL} U_{u,ik} V_{kl} y_{d_l} (K_d^\dagger)_{lj} d'_{mR} = y_{d_j} \bar{u}_{iL} V_{ij} d_{jR}, \quad (159)$$

$$\tilde{y}_{u,ij} \bar{d}'_{iL} u'_{jR} = \bar{d}'_{iL} (U_d V^\dagger y_U K_u^\dagger)_{ij} u'_{jR} = \bar{d}'_{iL} U_{d,ik} V_{kl}^\dagger y_{u_l} (K_u^\dagger)_{lj} u'_{jR} = y_{u_i} \bar{d}_{jL} V_{ji}^\dagger u_{iR}. \quad (160)$$

结合 (89) 和 (90) 式, 得

$$\begin{aligned} \tilde{y}_{d,ij} \bar{Q}_{iL} d'_{jR} \Phi &= \tilde{y}_{d,ij} \left[ \bar{u}'_{iL} d'_{jR} \phi^+ + \frac{1}{\sqrt{2}} \bar{d}'_{iL} d'_{jR} (v + H + i\chi) \right] \\ &= y_{d_j} \bar{u}_{iL} V_{ij} d_{jR} \phi^+ + \frac{y_{d_i}}{\sqrt{2}} \bar{d}_{iL} d_{iR} (v + H + i\chi), \end{aligned} \quad (161)$$

$$\begin{aligned} \tilde{y}_{u,ij} \bar{Q}_{iL} u'_{jR} \tilde{\Phi} &= \tilde{y}_{u,ij} \left[ \frac{1}{\sqrt{2}} \bar{u}'_{iL} u'_{jR} (v + H - i\chi) - \bar{d}'_{iL} u'_{jR} \phi^- \right] \\ &= \frac{y_{u_i}}{\sqrt{2}} \bar{u}_{iL} u_{jR} (v + H - i\chi) - y_{u_i} \bar{d}_{jL} V_{ji}^\dagger u_{iR} \phi^-. \end{aligned} \quad (162)$$



从而, Yukawa 相互作用拉氏量化为

$$\begin{aligned}
\mathcal{L}_Y &= -\tilde{y}_{d,ij} \bar{Q}_{iL} d'_{jR} \Phi - \tilde{y}_{u,ij} \bar{Q}_{iL} u'_{jR} \tilde{\Phi} - y_{\ell_i} \bar{L}_{iL} \ell_{iR} \Phi + \text{H.c.} \\
&= -y_{d_j} \bar{u}_{iL} V_{ij} d_{jR} \phi^+ - \frac{y_{d_i}}{\sqrt{2}} \bar{d}_{iL} d_{iR} (v + H + i\chi) - \frac{y_{u_i}}{\sqrt{2}} \bar{u}_{iL} u_{iR} (v + H - i\chi) + y_{u_i} \bar{d}_{jL} V_{ji}^\dagger u_{iR} \phi^- \\
&\quad - y_{\ell_i} \bar{\nu}_{iL} \ell_{iR} \phi^+ - \frac{y_{\ell_i}}{\sqrt{2}} \bar{\ell}_{iL} \ell_{iR} (v + H + i\chi) + \text{H.c.} \\
&= -m_{d_i} \bar{d}_{iL} d_{iR} - m_{u_i} \bar{u}_{iL} u_{iR} - m_{\ell_i} \bar{\ell}_{iL} \ell_{iR} - \frac{m_{d_i}}{v} \bar{d}_{iL} d_{iR} (H + i\chi) - \frac{m_{u_i}}{v} \bar{u}_{iL} u_{iR} (H - i\chi) \\
&\quad - \frac{m_{\ell_i}}{v} \bar{\ell}_{iL} \ell_{iR} (H + i\chi) - \frac{\sqrt{2} m_{d_j}}{v} \bar{u}_{iL} V_{ij} d_{jR} \phi^+ + \frac{\sqrt{2} m_{u_i}}{v} \bar{d}_{jL} V_{ji}^\dagger u_{iR} \phi^- - \frac{\sqrt{2} m_{\ell_i}}{v} \bar{\nu}_{iL} \ell_{iR} \phi^+ + \text{H.c.} \\
&= -m_{d_i} \bar{d}_i d_i - m_{u_i} \bar{u}_i u_i - m_{\ell_i} \bar{\ell}_i \ell_i - \frac{m_{d_i}}{v} H \bar{d}_i d_i - \frac{m_{u_i}}{v} H \bar{u}_i u_i - \frac{m_{\ell_i}}{v} H \bar{\ell}_i \ell_i \\
&\quad - \frac{m_{d_i}}{v} \chi \bar{d}_i i\gamma^5 d_i + \frac{m_{u_i}}{v} \chi \bar{u}_i i\gamma^5 u_i - \frac{m_{\ell_i}}{v} \chi \bar{\ell}_i i\gamma^5 \ell_i + \frac{\sqrt{2} V_{ij}}{v} \phi^+ \bar{u}_i (m_{u_i} P_L - m_{d_j} P_R) d_j \\
&\quad - \frac{\sqrt{2} V_{ji}^\dagger}{v} \phi^- \bar{d}_j (m_{d_j} P_L - m_{u_i} P_R) u_i - \frac{\sqrt{2} m_{\ell_i}}{v} (\phi^+ \bar{\nu}_i P_R \ell_i + \phi^- \bar{\ell}_i P_L \nu_i). \tag{163}
\end{aligned}$$

利用

$$\begin{aligned}
D_\mu \Phi &= \begin{pmatrix} \partial_\mu + ieA_\mu + \frac{ig}{2c_W} (c_W^2 - s_W^2) Z_\mu & \frac{ig}{\sqrt{2}} W_\mu^+ \\ \frac{ig}{\sqrt{2}} W_\mu^- & \partial_\mu - \frac{ig}{2c_W} Z_\mu \end{pmatrix} \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}} (v + H + i\chi) \end{pmatrix} \\
&= \begin{pmatrix} \partial_\mu \phi^+ + i \left[ eA_\mu + \frac{g(c_W^2 - s_W^2)}{2c_W} Z_\mu \right] \phi^+ + \frac{ig}{2} W_\mu^+ (H + i\chi) + im_W W_\mu^+ \\ \frac{1}{\sqrt{2}} \left[ \partial_\mu (H + i\chi) + igW_\mu^- \phi^+ - \frac{ig}{2c_W} Z_\mu (H + i\chi) - im_Z Z_\mu \right] \end{pmatrix}, \tag{164}
\end{aligned}$$

将 Higgs 场协变动能项化为

$$\begin{aligned}
&(D^\mu \Phi)^\dagger D_\mu \Phi \\
&= \left| \partial_\mu \phi^+ + i \left[ eA_\mu + \frac{g(c_W^2 - s_W^2)}{2c_W} Z_\mu \right] \phi^+ + \frac{ig}{2} W_\mu^+ (H + i\chi) + im_W W_\mu^+ \right|^2 \\
&\quad + \frac{1}{2} \left| \partial_\mu (H + i\chi) + igW_\mu^- \phi^+ - \frac{ig}{2c_W} Z_\mu (H + i\chi) - im_Z Z_\mu \right|^2 \\
&= (\partial^\mu \phi^+) (\partial_\mu \phi^-) + \frac{1}{2} (\partial^\mu H) (\partial_\mu H) + \frac{1}{2} (\partial^\mu \chi) (\partial_\mu \chi) \\
&\quad + \left( i\partial^\mu \phi^- \left\{ \left[ eA_\mu + \frac{g(c_W^2 - s_W^2)}{2c_W} Z_\mu \right] \phi^+ + \frac{g}{2} W_\mu^+ (H + i\chi) + m_W W_\mu^+ \right\} + \text{H.c.} \right) \\
&\quad + \left\{ \frac{i}{2} \partial^\mu (H - i\chi) \left[ gW_\mu^- \phi^+ - \frac{g}{2c_W} Z_\mu (H + i\chi) - m_Z Z_\mu \right] + \text{H.c.} \right\} \\
&\quad + \left| \left[ eA_\mu + \frac{g(c_W^2 - s_W^2)}{2c_W} Z_\mu \right] \phi^+ + \frac{g}{2} W_\mu^+ (H + i\chi) + m_W W_\mu^+ \right|^2
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2} \left| g W_\mu^- \phi^+ - \frac{g}{2c_W} Z_\mu (H + i\chi) - m_Z Z_\mu \right|^2 \\
& = (\partial^\mu \phi^+) (\partial_\mu \phi^-) + \frac{1}{2} (\partial^\mu H) (\partial_\mu H) + \frac{1}{2} (\partial^\mu \chi) (\partial_\mu \chi) \\
& + m_W^2 W_\mu^+ W^{-,\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu + g m_W H W_\mu^+ W^{-,\mu} + \frac{g m_Z}{2c_W} H Z_\mu Z^\mu \\
& - \frac{g}{2} [i W_\mu^+ \phi^- \overleftrightarrow{\partial}^\mu (H + i\chi) + \text{H.c.}] - i e A_\mu \phi^- \overleftrightarrow{\partial}^\mu \phi^+ - \frac{g}{2c_W} Z_\mu [-\chi \overleftrightarrow{\partial}^\mu H + i(c_W^2 - s_W^2) \phi^- \overleftrightarrow{\partial}^\mu \phi^+] \\
& + \frac{g^2}{4} W_\mu^+ W^{-,\mu} (2\phi^+ \phi^- + H^2 + \chi^2) + e^2 A_\mu A^\mu \phi^+ \phi^- \\
& + \frac{g^2}{4c_W^2} Z_\mu Z^\mu \left[ (c_W^2 - s_W^2)^2 \phi^+ \phi^- + \frac{1}{2} H^2 + \frac{1}{2} \chi^2 \right] \\
& + \left[ \frac{eg}{2} W_\mu^+ A^\mu \phi^- (H + i\chi) - \frac{g^2 s_W^2}{2c_W} W_\mu^+ Z^\mu \phi^- (H + i\chi) + \text{H.c.} \right] + \frac{eg(c_W^2 - s_W^2)}{c_W} A_\mu Z^\mu \phi^+ \phi^- \\
& + (em_W A^\mu \phi^+ W_\mu^- - g s_W^2 m_Z Z^\mu \phi^+ W_\mu^- + \text{H.c.}) + \mathcal{L}_{b1}, \tag{165}
\end{aligned}$$

其中

$$\mathcal{L}_{b1} = i m_W (\partial^\mu \phi^-) W_\mu^+ - i m_W (\partial^\mu \phi^+) W_\mu^- - m_Z (\partial^\mu \chi) Z_\mu. \tag{166}$$

在  $R_\xi$  规范下, 将规范固定函数设为

$$G^\pm = \frac{1}{\sqrt{\xi_W}} (\partial^\mu W_\mu^\pm \pm i \xi_W m_W \phi^\pm), \quad G^Z = \frac{1}{\sqrt{\xi_Z}} (\partial^\mu Z_\mu + \xi_Z m_Z \chi), \quad G^\gamma = \frac{1}{\sqrt{\xi_\gamma}} \partial^\mu A_\mu, \tag{167}$$

它们在路径积分量子化中的泛函积分形式为

$$\begin{aligned}
& \int \mathcal{D}\omega^+ \int \mathcal{D}\omega^- \int \mathcal{D}\omega^Z \int \mathcal{D}\omega^\gamma \exp \left[ -i \int d^4x \left( \omega^+ \omega^- + \frac{1}{2} \omega^Z \omega^Z + \frac{1}{2} \omega^\gamma \omega^\gamma \right) \right] \\
& \quad \times \delta(G^+ - \omega^+) \delta(G^- - \omega^-) \delta(G^Z - \omega^Z) \delta(G^\gamma - \omega^\gamma) \\
& = \exp \left[ -i \int d^4x \left( G^+ G^- + \frac{1}{2} G^Z G^Z + \frac{1}{2} G^\gamma G^\gamma \right) \right]. \tag{168}
\end{aligned}$$

由此得到拉氏量中的规范固定项

$$\begin{aligned}
\mathcal{L}_{\text{EW,GF}} & = -G^+ G^- - \frac{1}{2} (G^Z)^2 - \frac{1}{2} (G^\gamma)^2 \\
& = -\frac{1}{\xi_W} (\partial^\mu W_\mu^+ + i \xi_W m_W \phi^+) (\partial^\nu W_\nu^- - i \xi_W m_W \phi^-) - \frac{1}{2\xi_Z} (\partial^\mu Z_\mu + \xi_Z m_Z \chi)^2 - \frac{1}{2\xi_\gamma} (\partial^\mu A_\mu)^2 \\
& = -\frac{1}{\xi_W} (\partial^\mu W_\mu^+) (\partial^\nu W_\nu^-) - \frac{1}{2\xi_Z} (\partial^\mu Z_\mu)^2 - \frac{1}{2\xi_\gamma} (\partial^\mu A_\mu)^2 - \xi_W m_W^2 \phi^+ \phi^- - \frac{1}{2} \xi_Z m_Z^2 \chi^2 + \mathcal{L}_{b2}. \tag{169}
\end{aligned}$$

可见, Nambu-Goldstone 玻色子  $\phi^\pm$  和  $\chi$  在  $R_\xi$  规范下具有依赖于  $\xi_W$  和  $\xi_Z$  的非物理质量,

$$m_\phi = \sqrt{\xi_W} m_W, \quad m_\chi = \sqrt{\xi_Z} m_Z. \tag{170}$$

这里

$$\mathcal{L}_{b2} = im_W \phi^- (\partial^\mu W_\mu^+) - im_W \phi^+ \partial^\mu W_\mu^- - m_Z \chi \partial^\mu Z_\mu. \quad (171)$$

$\mathcal{L}_{b1}$  与  $\mathcal{L}_{b2}$  之和

$$\mathcal{L}_{b1} + \mathcal{L}_{b2} = im_W \partial^\mu (\phi^- W_\mu^+) - im_W \partial^\mu (\phi^+ W_\mu^-) - m_Z \partial^\mu (\chi Z_\mu) \quad (172)$$

是全散度，不会有物理效应。可见，协变动能项中规范场与 Nambu-Goldstone 标量场之间的双线性耦合项  $\mathcal{L}_{b1}$  被规范固定项中的  $\mathcal{L}_{b2}$  抵消掉，这就是如此选取规范固定函数的目的。

这样一来，电弱规范场传播子相关拉氏量变成

$$\begin{aligned} \mathcal{L}_{EW,prop} &= (\partial_\mu W_\nu^+) (\partial^\nu W^{-,\mu}) - (\partial_\mu W_\nu^+) (\partial^\mu W^{-,\nu}) - \frac{1}{\xi_W} (\partial^\mu W_\mu^+) (\partial^\nu W_\nu^-) + m_W^2 W_\mu^+ W^{-,\mu} \\ &\quad + \frac{1}{2} \left[ (\partial_\mu Z_\nu) (\partial^\nu Z^\mu) - (\partial_\mu Z_\nu) (\partial^\mu Z^\nu) - \frac{1}{\xi_Z} (\partial^\mu Z_\mu)^2 + m_Z^2 Z_\mu Z^\mu \right] \\ &\quad + \frac{1}{2} \left[ (\partial_\mu A_\nu) (\partial^\nu A^\mu) - (\partial_\mu A_\nu) (\partial^\mu A^\nu) - \frac{1}{\xi_\gamma} (\partial^\mu A_\mu)^2 \right] \\ &\rightarrow W_\mu^+ \left[ g^{\mu\nu} (\partial^2 + m_W^2) - \left( 1 - \frac{1}{\xi_W} \right) \partial^\mu \partial^\nu \right] W_\nu^- \\ &\quad + \frac{1}{2} Z_\mu \left[ g^{\mu\nu} (\partial^2 + m_Z^2) - \left( 1 - \frac{1}{\xi_Z} \right) \partial^\mu \partial^\nu \right] Z_\nu \\ &\quad + \frac{1}{2} A_\mu \left[ g^{\mu\nu} \partial^2 - \left( 1 - \frac{1}{\xi_\gamma} \right) \partial^\mu \partial^\nu \right] A_\nu. \end{aligned} \quad (173)$$

于是，光子的传播子与胶子形式类似，为

$$\frac{-i}{p^2 + i\epsilon} \left[ g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} (1 - \xi_\gamma) \right]. \quad (174)$$

将  $W^\pm$  传播子相关拉氏量变换到动量空间，得

$$-g^{\mu\nu} (p^2 - m_W^2) + \left( 1 - \frac{1}{\xi_W} \right) p^\mu p^\nu = - \left( g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) (p^2 - m_W^2) - \frac{p^\mu p^\nu}{p^2} \frac{p^2 - \xi_W m_W^2}{\xi_W}, \quad (175)$$

它的逆矩阵是

$$-\frac{1}{p^2 - m_W^2} \left( g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) - \frac{\xi_W}{p^2 - \xi_W m_W^2} \frac{p_\mu p_\nu}{p^2} = -\frac{1}{p^2 - m_W^2} \left[ g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_W m_W^2} (1 - \xi_W) \right], \quad (176)$$

这是因为由

$$\left( g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2} \right) \frac{p^\mu p^\nu}{p^2} = \frac{p_\rho p^\nu}{p^2} - \frac{p_\rho p^\nu}{p^2} = 0, \quad \left( g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2} \right) \left( g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) = \delta_\rho^\nu - \frac{p_\rho p^\nu}{p^2} \quad (177)$$

得

$$\begin{aligned}
& \left[ -\frac{1}{p^2 - m_W^2} \left( g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2} \right) - \frac{\xi_W}{p^2 - \xi_W m_W^2} \frac{p_\rho p_\mu}{p^2} \right] \\
& \times \left[ -\left( g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) (p^2 - m_W^2) - \frac{p^\mu p^\nu}{p^2} \frac{p^2 - \xi_W m_W^2}{\xi} \right] \\
& = \delta_\rho^\nu - \frac{p_\rho p^\nu}{p^2} + \frac{p_\rho p^\nu}{p^2} = \delta_\rho^\nu.
\end{aligned} \tag{178}$$

从而,  $W^\pm$  传播子的形式为

$$\frac{-i}{p^2 - m_W^2 + i\epsilon} \left[ g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_W m_W^2} (1 - \xi_W) \right]. \tag{179}$$

同理,  $Z$  传播子的形式为

$$\frac{-i}{p^2 - m_Z^2 + i\epsilon} \left[ g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_Z m_Z^2} (1 - \xi_Z) \right]. \tag{180}$$

电弱规范场的无穷小规范变换形式是

$$\delta W_\mu^a = -\frac{1}{g} \partial_\mu \alpha^a + \varepsilon^{abc} W_\mu^b \alpha^c, \quad \delta B_\mu = -\frac{1}{g'} \partial_\mu \alpha^Y. \tag{181}$$

定义

$$\alpha^\pm \equiv \frac{1}{\sqrt{2}}(\alpha^1 \mp i\alpha^2), \quad \alpha^Z \equiv \alpha^3 - \alpha^Y, \quad \alpha^\gamma \equiv s_W^2 \alpha^3 + c_W^2 \alpha^Y, \tag{182}$$

利用

$$\varepsilon^{1bc} W_\mu^b \alpha^c = W_\mu^2 \alpha^3 - W_\mu^3 \alpha^2, \quad \varepsilon^{2bc} W_\mu^b \alpha^c = -W_\mu^1 \alpha^3 + W_\mu^3 \alpha^1, \tag{183}$$

$$\pm i\sqrt{2} \alpha^\pm = \pm i\alpha^1 + \alpha^2, \quad \pm i\sqrt{2} W_\mu^\pm = \pm iW_\mu^1 + W_\mu^2, \tag{184}$$

有

$$\begin{aligned}
\varepsilon^{1bc} W_\mu^b \alpha^c \mp i\varepsilon^{2bc} W_\mu^b \alpha^c &= (W_\mu^2 \alpha^3 - W_\mu^3 \alpha^2) \mp i(-W_\mu^1 \alpha^3 + W_\mu^3 \alpha^1) \\
&= (W_\mu^2 \pm iW_\mu^1) \alpha^3 - W_\mu^3 (\alpha^2 \pm i\alpha^1) \\
&= \pm i\sqrt{2} W_\mu^\pm (c_W^2 \alpha^Z + \alpha^\gamma) \mp i\sqrt{2} (s_W A_\mu + c_W Z_\mu) \alpha^\pm,
\end{aligned} \tag{185}$$

$$\begin{aligned}
\varepsilon^{3bc} W_\mu^b \alpha^c &= W_\mu^1 \alpha^2 - W_\mu^2 \alpha^1 = \frac{1}{\sqrt{2}} (W_\mu^+ + W_\mu^-) \frac{i}{\sqrt{2}} (\alpha^+ - \alpha^-) - \frac{i}{\sqrt{2}} (W_\mu^+ - W_\mu^-) \frac{1}{\sqrt{2}} (\alpha^+ + \alpha^-) \\
&= -i(W_\mu^+ \alpha^- - W_\mu^- \alpha^+).
\end{aligned} \tag{186}$$

因此,

$$\begin{aligned}\delta W_\mu^+ &= \frac{1}{\sqrt{2}}(\delta W_\mu^1 - i\delta W_\mu^2) = -\frac{1}{\sqrt{2}g} \partial_\mu(\alpha^1 - i\alpha^2) + \frac{1}{\sqrt{2}}(\varepsilon^{1bc}W_\mu^b\alpha^c - i\varepsilon^{2bc}W_\mu^b\alpha^c) \\ &= -\frac{1}{g} \partial_\mu\alpha^+ - i(s_W A_\mu + c_W Z_\mu)\alpha^+ + iW_\mu^+(c_W^2\alpha^Z + \alpha^\gamma),\end{aligned}\quad (187)$$

$$\delta W_\mu^- = (\delta W_\mu^+)^{\dagger} = -\frac{1}{g} \partial_\mu\alpha^- + i(s_W A_\mu + c_W Z_\mu)\alpha^- - iW_\mu^-(c_W^2\alpha^Z + \alpha^\gamma),\quad (188)$$

$$\begin{aligned}\delta Z_\mu^a &= c_W\delta W_\mu^3 - s_W\delta B_\mu = -\frac{c_W}{g} \partial_\mu\alpha^3 + c_W\varepsilon^{3bc}W_\mu^b\alpha^c + \frac{s_W}{g'} \partial_\mu\alpha^Y \\ &= -\frac{c_W}{g} \partial_\mu\alpha^Z - ic_W(W_\mu^+\alpha^- - W_\mu^-\alpha^+),\end{aligned}\quad (189)$$

$$\begin{aligned}\delta A_\mu &= s_W\delta W_\mu^3 + c_W\delta B_\mu = -\frac{s_W}{g} \partial_\mu\alpha^3 + s_W\varepsilon^{3bc}W_\mu^b\alpha^c - \frac{c_W}{g'} \partial_\mu\alpha^Y \\ &= -\frac{1}{e} \partial_\mu\alpha^\gamma - is_W(W_\mu^+\alpha^- - W_\mu^-\alpha^+).\end{aligned}\quad (190)$$

另一方面, 根据

$$\begin{aligned}\alpha^a T^a + \alpha^Y Y_H &= \frac{1}{2}(\alpha^a \sigma^a + \alpha^Y) = \frac{1}{2} \begin{pmatrix} \alpha^3 + \alpha^Y & \alpha^1 - i\alpha^2 \\ \alpha^1 + i\alpha^2 & -\alpha^3 + \alpha^Y \end{pmatrix} \\ &= \frac{1}{2} \begin{pmatrix} 2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z & \sqrt{2}\alpha^+ \\ \sqrt{2}\alpha^- & -\alpha^Z \end{pmatrix},\end{aligned}\quad (191)$$

可知 Higgs 场的无穷小规范变换形式为

$$\begin{aligned}\delta\Phi &= i(\alpha^a T^a + \alpha^Y Y_H)\Phi = \frac{i}{2} \begin{pmatrix} 2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z & \sqrt{2}\alpha^+ \\ \sqrt{2}\alpha^- & -\alpha^Z \end{pmatrix} \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\chi) \end{pmatrix} \\ &= \begin{pmatrix} \frac{i}{2}\{\phi^+[2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z] + (v + H + i\chi)\alpha^+\} \\ \frac{1}{\sqrt{2}}\left[i\phi^+\alpha^- - \frac{1}{2}(iv + iH - \chi)\alpha^Z\right] \end{pmatrix} = \begin{pmatrix} \delta\phi^+ \\ \frac{1}{\sqrt{2}}(\delta H + i\delta\chi) \end{pmatrix}.\end{aligned}\quad (192)$$

利用

$$\text{Re}(\phi^+\alpha^-) = \frac{1}{2}(\phi^+\alpha^- + \phi^-\alpha^+), \quad \text{Im}(\phi^+\alpha^-) = -\frac{i}{2}(\phi^+\alpha^- - \phi^-\alpha^+),\quad (193)$$

推出

$$\delta\phi^+ = \frac{i}{2}\{\phi^+[2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z] + (v + H + i\chi)\alpha^+\},\quad (194)$$

$$\delta\phi^- = -\frac{i}{2}\{\phi^-[2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z] + (v + H - i\chi)\alpha^-\},\quad (195)$$

$$\delta H = \frac{1}{2}[i(\phi^+\alpha^- - \phi^-\alpha^+) + \chi\alpha^Z], \quad \delta\chi = \frac{1}{2}[\phi^+\alpha^- + \phi^-\alpha^+ - (v + H)\alpha^Z].\quad (196)$$

于是, 规范固定函数的无穷小规范变换为

$$\begin{aligned}\sqrt{\xi_W} \delta G^+ &= \partial^\mu \delta W_\mu^+ + i\xi_W m_W \delta \phi^+ \\ &= \partial^\mu \left[ -\frac{1}{g} \partial_\mu \alpha^+ - i(s_W A_\mu + c_W Z_\mu) \alpha^+ + iW_\mu^+ (c_W^2 \alpha^Z + \alpha^\gamma) \right] \\ &\quad - \frac{1}{2} \xi_W m_W \{ \phi^+ [2\alpha^\gamma + (c_W^2 - s_W^2) \alpha^Z] + (v + H + i\chi) \alpha^+ \},\end{aligned}\quad (197)$$

$$\begin{aligned}\sqrt{\xi_W} \delta G^- &= \partial^\mu \delta W_\mu^- - i\xi_W m_W \delta \phi^- \\ &= \partial^\mu \left[ -\frac{1}{g} \partial_\mu \alpha^- + i(s_W A_\mu + c_W Z_\mu) \alpha^- - iW_\mu^- (c_W^2 \alpha^Z + \alpha^\gamma) \right] \\ &\quad - \frac{1}{2} \xi_W m_W \{ \phi^- [2\alpha^\gamma + (c_W^2 - s_W^2) \alpha^Z] + (v + H - i\chi) \alpha^- \},\end{aligned}\quad (198)$$

$$\begin{aligned}\sqrt{\xi_Z} \delta G^Z &= \partial^\mu \delta Z_\mu + \xi_Z m_Z \delta \chi \\ &= \partial^\mu \left[ -\frac{c_W}{g} \partial_\mu \alpha^Z - i c_W (W_\mu^+ \alpha^- - W_\mu^- \alpha^+) \right] \\ &\quad + \frac{1}{2} \xi_Z m_Z [\phi^+ \alpha^- + \phi^- \alpha^+ - (v + H) \alpha^Z],\end{aligned}\quad (199)$$

$$\sqrt{\xi_\gamma} \delta G^\gamma = \partial^\mu \delta A_\mu = \partial^\mu \left[ -\frac{1}{e} \partial_\mu \alpha^\gamma - i s_W (W_\mu^+ \alpha^- - W_\mu^- \alpha^+) \right]. \quad (200)$$

因此,

$$\sqrt{\xi_W} g \frac{\delta G^+}{\delta \alpha^+} = -\partial^2 - \xi_W m_W^2 - i e \partial^\mu A_\mu - i g c_W \partial^\mu Z_\mu - \frac{1}{2} g \xi_W m_W (H + i\chi), \quad (201)$$

$$\frac{\sqrt{\xi_W} g}{c_W} \frac{\delta G^+}{\delta \alpha^Z} = i g c_W \partial^\mu W_\mu^+ - \frac{g(c_W^2 - s_W^2) \xi_W m_W}{2 c_W} \phi^+, \quad (202)$$

$$\sqrt{\xi_W} e \frac{\delta G^+}{\delta \alpha^\gamma} = i e \partial^\mu W_\mu^+ - e \xi_W m_W \phi^+, \quad (203)$$

$$\sqrt{\xi_W} g \frac{\delta G^-}{\delta \alpha^-} = -\partial^2 - \xi_W m_W^2 + i e \partial^\mu A_\mu + i g c_W \partial^\mu Z_\mu - \frac{1}{2} g \xi_W m_W (H - i\chi), \quad (204)$$

$$\frac{\sqrt{\xi_W} g}{c_W} \frac{\delta G^-}{\delta \alpha^Z} = -i g c_W \partial^\mu W_\mu^- - \frac{g(c_W^2 - s_W^2) \xi_W m_W}{2 c_W} \phi^-, \quad (205)$$

$$\sqrt{\xi_W} e \frac{\delta G^-}{\delta \alpha^\gamma} = -i e \partial^\mu W_\mu^- - e \xi_W m_W \phi^-, \quad (206)$$

$$\sqrt{\xi_Z} g \frac{\delta G^Z}{\delta \alpha^+} = i g c_W \partial^\mu W_\mu^- + \frac{1}{2} g \xi_Z m_Z \phi^-, \quad (207)$$

$$\sqrt{\xi_Z} g \frac{\delta G^Z}{\delta \alpha^-} = -i g c_W \partial^\mu W_\mu^+ + \frac{1}{2} g \xi_Z m_Z \phi^+, \quad (208)$$

$$\frac{\sqrt{\xi_Z} g}{c_W} \frac{\delta G^Z}{\delta \alpha^Z} = -\partial^2 - \xi_Z m_Z^2 - \frac{g \xi_Z m_Z}{2 c_W} H, \quad (209)$$

$$\sqrt{\xi_\gamma} g \frac{\delta G^\gamma}{\delta \alpha^+} = i e \partial^\mu W_\mu^-, \quad \sqrt{\xi_\gamma} g \frac{\delta G^\gamma}{\delta \alpha^-} = -i e \partial^\mu W_\mu^+, \quad \sqrt{\xi_\gamma} e \frac{\delta G^\gamma}{\delta \alpha^\gamma} = -\partial^2. \quad (210)$$

最后, 得到以下 Faddeev-Popov 鬼场拉氏量,

$$\begin{aligned}
\mathcal{L}_{\text{EWG,FP}} = & \bar{\eta}^+ \left( \sqrt{\xi_W} g \frac{\delta G^+}{\delta \alpha^+} \right) \eta^+ + \bar{\eta}^Z \left( \sqrt{\xi_Z} g \frac{\delta G^Z}{\delta \alpha^+} \right) \eta^+ + \bar{\eta}^\gamma \left( \sqrt{\xi_\gamma} g \frac{\delta G^\gamma}{\delta \alpha^+} \right) \eta^+ \\
& + \bar{\eta}^- \left( \sqrt{\xi_W} g \frac{\delta G^-}{\delta \alpha^-} \right) \eta^- + \bar{\eta}^Z \left( \sqrt{\xi_Z} g \frac{\delta G^Z}{\delta \alpha^-} \right) \eta^- + \bar{\eta}^\gamma \left( \sqrt{\xi_\gamma} g \frac{\delta G^\gamma}{\delta \alpha^-} \right) \eta^- \\
& + \bar{\eta}^Z \left( \frac{\sqrt{\xi_Z} g}{c_W} \frac{\delta G^Z}{\delta \alpha^Z} \right) \eta^Z + \bar{\eta}^+ \left( \frac{\sqrt{\xi_W} g}{c_W} \frac{\delta G^+}{\delta \alpha^Z} \right) \eta^Z + \bar{\eta}^- \left( \frac{\sqrt{\xi_W} g}{c_W} \frac{\delta G^-}{\delta \alpha^Z} \right) \eta^Z \\
& + \bar{\eta}^\gamma \left( \sqrt{\xi_\gamma} e \frac{\delta G^\gamma}{\delta \alpha^\gamma} \right) \eta^\gamma + \bar{\eta}^+ \left( \sqrt{\xi_W} e \frac{\delta G^+}{\delta \alpha^\gamma} \right) \eta^\gamma + \bar{\eta}^- \left( \sqrt{\xi_W} e \frac{\delta G^-}{\delta \alpha^\gamma} \right) \eta^\gamma \\
\rightarrow & \bar{\eta}^+ \left[ -\partial^2 - \xi_W m_W^2 + i e \overleftarrow{\partial}^\mu A_\mu + i g c_W \overleftarrow{\partial}^\mu Z_\mu - \frac{1}{2} g \xi_W m_W (H + i\chi) \right] \eta^+ \\
& + \bar{\eta}^Z \left( -i g c_W \overleftarrow{\partial}^\mu W_\mu^- + \frac{1}{2} g \xi_Z m_Z \phi^- \right) \eta^+ - i e (\partial^\mu \bar{\eta}^\gamma) W_\mu^- \eta^+ \\
& + \bar{\eta}^- \left[ -\partial^2 - \xi_W m_W^2 - i e \overleftarrow{\partial}^\mu A_\mu - i g c_W \overleftarrow{\partial}^\mu Z_\mu - \frac{1}{2} g \xi_W m_W (H - i\chi) \right] \eta^- \\
& + \bar{\eta}^Z \left( i g c_W \overleftarrow{\partial}^\mu W_\mu^+ + \frac{1}{2} g \xi_Z m_Z \phi^+ \right) \eta^- + i e (\partial^\mu \bar{\eta}^\gamma) W_\mu^+ \eta^- \\
& + \bar{\eta}^Z \left( -\partial^2 - \xi_Z m_Z^2 - \frac{g \xi_Z m_Z}{2 c_W} H \right) \eta^Z \\
& + \bar{\eta}^+ \left( -i g c_W \overleftarrow{\partial}^\mu W_\mu^+ - \frac{g(c_W^2 - s_W^2) \xi_W m_W}{2 c_W} \phi^+ \right) \eta^Z \\
& + \bar{\eta}^- \left( i g c_W \overleftarrow{\partial}^\mu W_\mu^- - \frac{g(c_W^2 - s_W^2) \xi_W m_W}{2 c_W} \phi^- \right) \eta^Z \\
& - \bar{\eta}^\gamma \partial^2 \eta^\gamma + \bar{\eta}^+ (-i e \overleftarrow{\partial}^\mu W_\mu^+ - e \xi_W m_W \phi^+) \eta^\gamma + \bar{\eta}^- (i e \overleftarrow{\partial}^\mu W_\mu^- - e \xi_W m_W \phi^-) \eta^\gamma. \quad (211)
\end{aligned}$$

可以认为这里通过鬼场  $\eta^\pm$ 、 $\eta^Z$ 、 $\eta^\gamma$  的归一化吸收了  $-1/g$ 、 $-c_W/g$ 、 $-1/e$  因子, 通过鬼场  $\bar{\eta}^\pm$ 、 $\bar{\eta}^Z$ 、 $\bar{\eta}^\gamma$  的归一化吸收了  $1/\sqrt{\xi_W}$ 、 $1/\sqrt{\xi_Z}$ 、 $1/\sqrt{\xi_\gamma}$  因子。鬼粒子的质量为

$$m_{\eta^+} = m_{\eta^-} = \sqrt{\xi_W} m_W, \quad m_{\eta^Z} = \sqrt{\xi_Z} m_Z, \quad m_{\eta^\gamma} = 0. \quad (212)$$

下面给出  $R_\xi$  规范下的 Feynman 规则。  $\xi_i = 1$  对应 Feynman-'t Hooft 规范,  $\xi_i = 0$  对应 Landau 规范,  $\xi_W, \xi_Z \rightarrow \infty$  对应么正规规范。在树图计算中, 常取  $\xi_\gamma = 1$  和  $\xi_W, \xi_Z \rightarrow \infty$ 。在圈图计算中, 常取  $\xi_\gamma = \xi_W = \xi_Z = 1$ 。

传播子:

$$\bullet \text{---} \frac{\overrightarrow{p}}{H} \text{---} \bullet = \frac{i}{p^2 - m_H^2 + i\epsilon} \quad (213)$$

$$\bullet \text{---} \frac{\overrightarrow{p}}{\chi} \text{---} \bullet = \frac{i}{p^2 - \xi_Z m_Z^2 + i\epsilon} \quad (214)$$

$$\bullet \text{---} \frac{\overrightarrow{p}}{\phi} \text{---} \bullet = \frac{i}{p^2 - \xi_W m_W^2 + i\epsilon} \quad (215)$$

$$\nu \text{---} \overset{p}{\gamma} \text{---} \mu = \frac{-i}{p^2 + i\epsilon} \left[ g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} (1 - \xi_\gamma) \right] \quad (216)$$

$$\nu \text{---} \overset{p}{Z} \text{---} \mu = \frac{-i}{p^2 - m_Z^2 + i\epsilon} \left[ g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_Z m_Z^2} (1 - \xi_Z) \right] \quad (217)$$

$$\nu \text{---} \overset{p}{W} \text{---} \mu = \frac{-i}{p^2 - m_W^2 + i\epsilon} \left[ g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_W m_W^2} (1 - \xi_W) \right] \quad (218)$$

$$\bullet \text{---} \overset{p}{\eta^\gamma} \text{---} \bullet = \frac{i}{p^2 + i\epsilon} \quad (219)$$

$$\bullet \text{---} \overset{p}{\eta^Z} \text{---} \bullet = \frac{i}{p^2 - \xi_Z m_Z^2 + i\epsilon} \quad (220)$$

$$\bullet \text{---} \overset{p}{\eta^\pm} \text{---} \bullet = \frac{i}{p^2 - \xi_W m_W^2 + i\epsilon} \quad (221)$$

标量玻色子三线耦合:

$$\begin{array}{ccc} \begin{array}{c} H \\ | \\ \bullet \\ / \quad \backslash \\ H \quad H \end{array} & = -\frac{3im_H^2}{v} \\ & = -6i\lambda v \\ \begin{array}{c} H \\ | \\ \bullet \\ / \quad \backslash \\ \chi \quad \chi \end{array} & = -\frac{im_H^2}{v} \\ & = -2i\lambda v \\ \begin{array}{c} H \\ | \\ \bullet \\ / \quad \backslash \\ \phi \quad \phi \end{array} & = -\frac{im_H^2}{v} \\ & = -2i\lambda v \end{array} \quad (222)$$

标量玻色子四线耦合:

$$\begin{array}{ccc} \begin{array}{c} H \quad H \\ \backslash \quad / \\ \bullet \\ / \quad \backslash \\ H \quad H \end{array} & = -\frac{3im_H^2}{v^2} \\ & = -6i\lambda \\ \begin{array}{c} H \quad H \\ \backslash \quad / \\ \bullet \\ / \quad \backslash \\ \phi \quad \phi \end{array} & = -\frac{im_H^2}{v^2} \\ & = -2i\lambda \end{array} \quad (223)$$

$$\begin{array}{ccc} \begin{array}{c} \chi \quad \chi \\ \backslash \quad / \\ \bullet \\ / \quad \backslash \\ \chi \quad \chi \end{array} & = -\frac{3im_H^2}{v^2} \\ & = -6i\lambda \\ \begin{array}{c} \chi \quad \chi \\ \backslash \quad / \\ \bullet \\ / \quad \backslash \\ \phi \quad \phi \end{array} & = -\frac{2im_H^2}{v^2} \\ & = -4i\lambda \end{array} \quad (224)$$

Yukawa 耦合:

$$\begin{array}{ccc} \begin{array}{c} H \\ | \\ \bullet \\ / \quad \backslash \\ f \quad f \end{array} & = -\frac{im_f}{v} \\ & = -\frac{iy_f}{\sqrt{2}} \\ \begin{array}{c} \chi \\ | \\ \bullet \\ / \quad \backslash \\ \ell_i \quad \ell_i \end{array} & = \frac{m_{\ell_i}}{v} \gamma^5 \\ & = \frac{y_{\ell_i}}{\sqrt{2}} \gamma^5 \end{array} \quad (225)$$



$$\begin{aligned}
\begin{array}{c} \chi \\ | \\ \bullet \\ / \quad \backslash \\ u_i \quad u_i \end{array} &= -\frac{m_{u_i}}{v} \gamma^5 \\
&= -\frac{y_{u_i}}{\sqrt{2}} \gamma^5
\end{aligned}
\quad
\begin{aligned}
\begin{array}{c} \chi \\ | \\ \bullet \\ / \quad \backslash \\ d_i \quad d_i \end{array} &= \frac{m_{d_i}}{v} \gamma^5 \\
&= \frac{y_{d_i}}{\sqrt{2}} \gamma^5
\end{aligned}
\quad (226)$$

$$\begin{aligned}
\begin{array}{c} \phi \\ | \\ \bullet \\ / \quad \backslash \\ \ell_i \quad \nu_i \end{array} &= -\frac{i\sqrt{2}m_{\ell_i}}{v} P_R \\
&= -iy_{\ell_i} P_R
\end{aligned}
\quad
\begin{aligned}
\begin{array}{c} \phi \\ | \\ \bullet \\ / \quad \backslash \\ \nu_i \quad \ell_j \end{array} &= -\frac{i\sqrt{2}m_{\ell_i}}{v} P_L \\
&= -iy_{\ell_i} P_L
\end{aligned}
\quad (227)$$

$$\begin{aligned}
\begin{array}{c} \phi \\ | \\ \bullet \\ / \quad \backslash \\ d_j \quad u_i \end{array} &= \frac{i\sqrt{2}V_{ij}}{v} (m_{u_i} P_L - m_{d_j} P_R) \\
&= iV_{ij} (y_{u_i} P_L - y_{d_j} P_R)
\end{aligned}
\quad
\begin{aligned}
\begin{array}{c} \phi \\ | \\ \bullet \\ / \quad \backslash \\ u_i \quad d_j \end{array} &= -\frac{i\sqrt{2}V_{ji}^\dagger}{v} (m_{d_j} P_L - m_{u_i} P_R) \\
&= -iV_{ji}^\dagger (y_{d_j} P_L - y_{u_i} P_R)
\end{aligned}
\quad (228)$$

标量玻色子与电弱规范玻色子的三线耦合：

$$\begin{aligned}
\begin{array}{c} H \\ | \\ \bullet \\ / \quad \backslash \\ Z; \mu \quad Z; \nu \end{array} &= \frac{igm_Z}{c_W} g^{\mu\nu} \\
&= \frac{ig^2 v}{2c_W^2} g^{\mu\nu}
\end{aligned}
\quad
\begin{aligned}
\begin{array}{c} H \\ | \\ \bullet \\ / \quad \backslash \\ W; \mu \quad W; \nu \end{array} &= igm_W g^{\mu\nu} \\
&= \frac{ig^2 v}{2} g^{\mu\nu}
\end{aligned}
\quad (229)$$

$$\begin{aligned}
\begin{array}{c} \phi \\ | \\ \bullet \\ / \quad \backslash \\ W; \mu \quad \gamma; \nu \end{array} &= iem_W g^{\mu\nu} \\
&= \frac{ieg v}{2} g^{\mu\nu}
\end{aligned}
\quad
\begin{aligned}
\begin{array}{c} \phi \\ | \\ \bullet \\ / \quad \backslash \\ W; \mu \quad \gamma; \nu \end{array} &= iem_W g^{\mu\nu} \\
&= \frac{ieg v}{2} g^{\mu\nu}
\end{aligned}
\quad (230)$$

$$\begin{aligned}
\begin{array}{c} \phi \\ | \\ \bullet \\ / \quad \backslash \\ W; \mu \quad Z; \nu \end{array} &= -igs_W^2 m_Z g^{\mu\nu} \\
&= -\frac{ig^2 s_W^2 v}{2c_W} g^{\mu\nu}
\end{aligned}
\quad
\begin{aligned}
\begin{array}{c} \phi \\ | \\ \bullet \\ / \quad \backslash \\ W; \mu \quad Z; \nu \end{array} &= -igs_W^2 m_Z g^{\mu\nu} \\
&= -\frac{ig^2 s_W^2 v}{2c_W} g^{\mu\nu}
\end{aligned}
\quad (231)$$

$$\begin{array}{c} \gamma; \mu \\ | \\ \bullet \\ / \quad \backslash \\ \phi \quad \phi \end{array}
\quad
\begin{array}{c} p \\ \nearrow \\ \bullet \\ \searrow \\ q \end{array}
= -ie(p+q)^\mu
\quad (232)$$

$$\begin{aligned}
 & \text{Diagram: } Z; \mu \text{ (wavy), } \phi \text{ (dashed, } p \text{), } \phi \text{ (dashed, } q \text{)} \\
 & = -\frac{ig(c_W^2 - s_W^2)}{2c_W} (p+q)^\mu = \frac{g}{2c_W} (p+q)^\mu \quad (233)
 \end{aligned}$$

$$\begin{aligned}
 & \text{Diagram: } W; \mu \text{ (wavy), } H \text{ (dashed, } p \text{), } \phi \text{ (dashed, } q \text{)} \\
 & = -\frac{ig}{2} (p+q)^\mu \\
 & \text{Diagram: } W; \mu \text{ (wavy), } \phi \text{ (dashed, } p \text{), } H \text{ (dashed, } q \text{)} \\
 & = -\frac{ig}{2} (p+q)^\mu \quad (234)
 \end{aligned}$$

$$\begin{aligned}
 & \text{Diagram: } W; \mu \text{ (wavy), } \chi \text{ (dashed, } p \text{), } \phi \text{ (dashed, } q \text{)} \\
 & = \frac{g}{2} (p+q)^\mu \\
 & \text{Diagram: } W; \mu \text{ (wavy), } \phi \text{ (dashed, } p \text{), } \chi \text{ (dashed, } q \text{)} \\
 & = -\frac{g}{2} (p+q)^\mu \quad (235)
 \end{aligned}$$

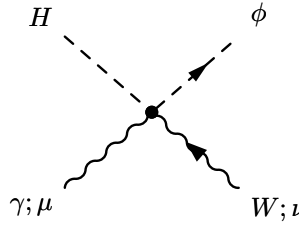
标量玻色子与电弱规范玻色子的四线性耦合:

$$\begin{aligned}
 & \text{Diagram: } H \text{ (dashed), } H \text{ (dashed), } Z; \mu \text{ (wavy), } Z; \nu \text{ (wavy)} \\
 & = \frac{ig^2}{2c_W^2} g^{\mu\nu} \\
 & \text{Diagram: } H \text{ (dashed), } H \text{ (dashed), } W; \mu \text{ (wavy), } W; \nu \text{ (wavy)} \\
 & = \frac{ig^2}{2} g^{\mu\nu} \quad (236)
 \end{aligned}$$

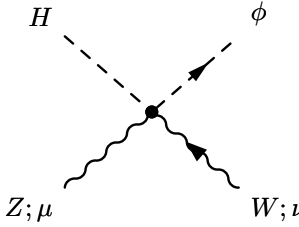
$$\begin{aligned}
 & \text{Diagram: } H \text{ (dashed), } H \text{ (dashed), } Z; \mu \text{ (wavy), } Z; \nu \text{ (wavy)} \\
 & = \frac{ig^2}{2c_W^2} g^{\mu\nu} \\
 & \text{Diagram: } H \text{ (dashed), } H \text{ (dashed), } W; \mu \text{ (wavy), } W; \nu \text{ (wavy)} \\
 & = \frac{ig^2}{2} g^{\mu\nu} \quad (237)
 \end{aligned}$$

$$\begin{aligned}
 & \text{Diagram: } H \text{ (dashed), } H \text{ (dashed), } \gamma; \mu \text{ (wavy), } \gamma; \nu \text{ (wavy)} \\
 & = 2ie^2 g^{\mu\nu} \\
 & \text{Diagram: } H \text{ (dashed), } H \text{ (dashed), } \gamma; \mu \text{ (wavy), } Z; \nu \text{ (wavy)} \\
 & = \frac{ig(c_W^2 - s_W^2)}{c_W} g^{\mu\nu} \quad (238)
 \end{aligned}$$

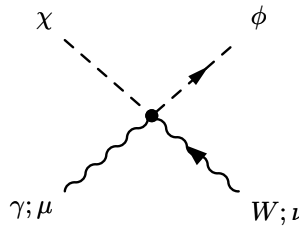
$$\begin{aligned}
 & \text{Diagram: } H \text{ (dashed), } H \text{ (dashed), } Z; \mu \text{ (wavy), } Z; \nu \text{ (wavy)} \\
 & = \frac{ig^2(c_W^2 - s_W^2)^2}{2c_W^2} g^{\mu\nu} \\
 & \text{Diagram: } H \text{ (dashed), } H \text{ (dashed), } W; \mu \text{ (wavy), } W; \nu \text{ (wavy)} \\
 & = \frac{ig^2}{2} g^{\mu\nu} \quad (239)
 \end{aligned}$$



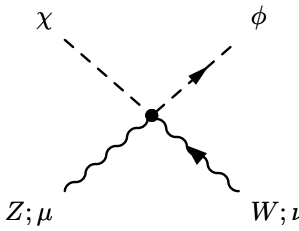
$$= \frac{ieg}{2} g^{\mu\nu} \quad (240)$$



$$= -\frac{ig^2 s_W^2}{2c_W} g^{\mu\nu} \quad (241)$$

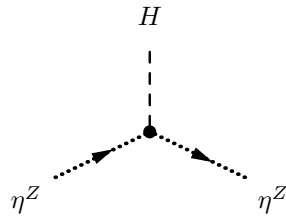


$$= -\frac{eg}{2} g^{\mu\nu} \quad (242)$$

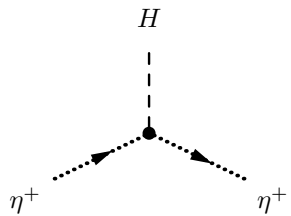


$$= \frac{g^2 s_W^2}{2c_W} g^{\mu\nu} \quad (243)$$

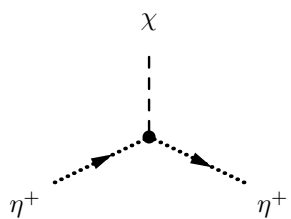
鬼粒子与标量玻色子的耦合:

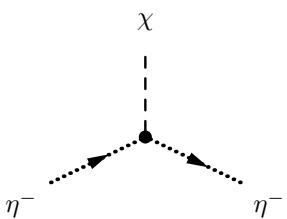


$$= -\frac{ig\xi_Z m_Z}{2c_W} = -\frac{ig^2 \xi_Z v}{4c_W^2} \quad (244)$$



$$= -\frac{ig\xi_W m_W}{2} = -\frac{ig^2 \xi_W v}{4} \quad (245)$$



$$= \frac{g\xi_W m_W}{2} = \frac{g^2 \xi_W v}{4}$$


$$= -\frac{g\xi_W m_W}{2} = -\frac{g^2 \xi_W v}{4} \quad (246)$$

$$\begin{aligned}
\begin{array}{c} \phi \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^\gamma \quad \eta^+ \end{array} &= -ie\xi_W m_W \\
&= -\frac{ieg\xi_W v}{2}
\end{aligned}
\quad
\begin{aligned}
\begin{array}{c} \phi \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^\gamma \quad \eta^- \end{array} &= -ie\xi_W m_W \\
&= -\frac{ieg\xi_W v}{2}
\end{aligned} \quad (247)$$

$$\begin{aligned}
\begin{array}{c} \phi \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^Z \quad \eta^+ \end{array} &= -\frac{ig(c_W^2 - s_W^2)\xi_W m_W}{2c_W} \\
&= -\frac{ig^2(c_W^2 - s_W^2)\xi_W v}{4c_W}
\end{aligned}
\quad
\begin{aligned}
\begin{array}{c} \phi \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^+ \quad \eta^Z \end{array} &= \frac{ig\xi_Z m_Z}{2} \\
&= \frac{ig^2\xi_Z v}{4c_W}
\end{aligned} \quad (248)$$

$$\begin{aligned}
\begin{array}{c} \phi \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^- \quad \eta^Z \end{array} &= \frac{ig\xi_Z m_Z}{2} \\
&= \frac{ig^2\xi_Z v}{4c_W}
\end{aligned}
\quad
\begin{aligned}
\begin{array}{c} \phi \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^Z \quad \eta^- \end{array} &= -\frac{ig(c_W^2 - s_W^2)\xi_W m_W}{2c_W} \\
&= -\frac{ig^2(c_W^2 - s_W^2)\xi_W v}{4c_W}
\end{aligned} \quad (249)$$

鬼粒子与电弱规范玻色子的耦合：

$$\begin{aligned}
\begin{array}{c} \gamma; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^+ \quad \eta^+ \end{array} &= -iep^\mu \\
\begin{array}{c} \gamma; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^- \quad \eta^- \end{array} &= iep^\mu
\end{aligned} \quad (250)$$

$$\begin{aligned}
\begin{array}{c} Z; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^+ \quad \eta^+ \end{array} &= -igc_W p^\mu \\
\begin{array}{c} Z; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^- \quad \eta^- \end{array} &= igc_W p^\mu
\end{aligned} \quad (251)$$

$$\begin{aligned}
\begin{array}{c} W; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^\gamma \quad \eta^+ \end{array} &= iep^\mu \\
\begin{array}{c} W; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^+ \quad \eta^\gamma \end{array} &= iep^\mu
\end{aligned} \quad (252)$$

$$\begin{aligned}
\begin{array}{c} W; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^- \quad \eta^\gamma \end{array} &= -iep^\mu \\
\begin{array}{c} W; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^\gamma \quad \eta^- \end{array} &= -iep^\mu
\end{aligned} \quad (253)$$

$$\begin{array}{ccc}
\begin{array}{c} W; \mu \\ \downarrow \\ \eta^Z \quad \quad \eta^+ \\ \nearrow \quad \searrow \\ p \end{array} & = ig_{CW} p^\mu & \begin{array}{c} W; \mu \\ \downarrow \\ \eta^+ \quad \quad \eta^Z \\ \nearrow \quad \searrow \\ p \end{array} = ig_{CW} p^\mu
\end{array} \quad (254)$$

$$\begin{array}{ccc}
\begin{array}{c} W; \mu \\ \downarrow \\ \eta^- \quad \quad \eta^Z \\ \nearrow \quad \searrow \\ p \end{array} & = -ig_{CW} p^\mu & \begin{array}{c} W; \mu \\ \downarrow \\ \eta^Z \quad \quad \eta^- \\ \nearrow \quad \searrow \\ p \end{array} = -ig_{CW} p^\mu
\end{array} \quad (255)$$

## 6 内外线一般 Feynman 规则

本节列出一些通用的内外线 Feynman 规则。

内线 Feynman 规则如下。

• 实标量玻色子传播子:  $\bullet \dashrightarrow^p \bullet = \frac{i}{p^2 - m^2 + i\epsilon}$

• 复标量玻色子传播子:  $\bullet \dashrightarrow^p \bullet = \frac{i}{p^2 - m^2 + i\epsilon}$

• Dirac 费米子传播子:  $\bullet \xrightarrow{p} \bullet = \frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon}$

• 无质量实矢量玻色子传播子:

$$\nu \xrightarrow{p} \mu = \frac{-ig^{\mu\nu}}{p^2 + i\epsilon} \quad (\text{Feynman 规范})$$

$$\nu \xrightarrow{p} \mu = \frac{-i(g^{\mu\nu} - p^\mu p^\nu / p^2)}{p^2 + i\epsilon} \quad (\text{Landau 规范})$$

• 有质量实矢量玻色子传播子:

$$\nu \xrightarrow{p} \mu = \frac{-i(g^{\mu\nu} - p^\mu p^\nu / m^2)}{p^2 - m^2 + i\epsilon} \quad (\text{幺正规范})$$

$$\nu \xrightarrow{p} \mu = \frac{-ig^{\mu\nu}}{p^2 - m^2 + i\epsilon} \quad (\text{Feynman 规范})$$

• 有质量复矢量玻色子传播子:

$$\nu \xrightarrow{p} \mu = \frac{-i(g^{\mu\nu} - p^\mu p^\nu / m^2)}{p^2 - m^2 + i\epsilon} \quad (\text{幺正规范})$$

$$\nu \xrightarrow{p} \mu = \frac{-ig^{\mu\nu}}{p^2 - m^2 + i\epsilon} \quad (\text{Feynman 规范})$$

实标量场外线 Feynman 规则如下。

- 实标量玻色子入射外线:  $---\xrightarrow{p}\bullet = 1$
- 实标量玻色子出射外线:  $\bullet---\xrightarrow{p} = 1$

复标量场外线 Feynman 规则如下。

- 正标量玻色子入射外线:  $---\blacktriangleright\xrightarrow{p}\bullet = 1$
- 反标量玻色子入射外线:  $---\blacktriangleleft\xrightarrow{p}\bullet = 1$
- 正标量玻色子出射外线:  $\bullet---\blacktriangleright\xrightarrow{p} = 1$
- 反标量玻色子出射外线:  $\bullet---\blacktriangleleft\xrightarrow{p} = 1$

以  $\lambda$  代表自旋极化指标 (如螺旋度), Dirac 旋量场外线 Feynman 规则如下。

- Dirac 正费米子入射外线:  $\lambda \longrightarrow\xrightarrow{p}\bullet = u(\mathbf{p}, \lambda)$
- Dirac 反费米子入射外线:  $\lambda \longleftarrow\xrightarrow{p}\bullet = \bar{v}(\mathbf{p}, \lambda)$
- Dirac 正费米子出射外线:  $\bullet\longrightarrow\xrightarrow{p}\lambda = \bar{u}(\mathbf{p}, \lambda)$
- Dirac 反费米子出射外线:  $\bullet\longleftarrow\xrightarrow{p}\lambda = v(\mathbf{p}, \lambda)$

在计算非极化振幅模方时, 可利用自旋求和关系

$$\sum_{\lambda} u(\mathbf{p}, \lambda) \bar{u}(\mathbf{p}, \lambda) = \not{p} + m, \quad \sum_{\lambda} v(\mathbf{p}, \lambda) \bar{v}(\mathbf{p}, \lambda) = \not{p} - m. \quad (256)$$

以  $\lambda$  代表自旋极化指标, 实矢量场外线 Feynman 规则如下。

- 实矢量玻色子入射外线:  $\lambda; \mu \sim\xrightarrow{p}\bullet = \varepsilon^{\mu}(\mathbf{p}, \lambda)$
- 实矢量玻色子出射外线:  $\bullet\sim\xrightarrow{p}\lambda; \mu = \varepsilon^{\mu*}(\mathbf{p}, \lambda)$

复矢量场外线 Feynman 规则如下。

- 正矢量玻色子入射外线:  $\lambda; \mu \sim \overset{p}{\rightarrow} = \varepsilon^\mu(\mathbf{p}, \lambda)$
- 反矢量玻色子入射外线:  $\lambda; \mu \sim \overset{p}{\leftarrow} = \varepsilon^\mu(\mathbf{p}, \lambda)$
- 正矢量玻色子出射外线:  $\bullet \sim \overset{p}{\rightarrow} \lambda; \mu = \varepsilon^{\mu*}(\mathbf{p}, \lambda)$
- 反矢量玻色子出射外线:  $\bullet \sim \overset{p}{\leftarrow} \lambda; \mu = \varepsilon^{\mu*}(\mathbf{p}, \lambda)$

在计算非极化振幅模方时, 若包含无质量矢量玻色子外线, 可利用极化求和替换关系

$$\sum_{\lambda} \varepsilon_{\mu}^*(\mathbf{p}, \lambda) \varepsilon_{\nu}(\mathbf{p}, \lambda) \rightarrow -g_{\mu\nu}; \quad (257)$$

若包含有质量矢量玻色子外线, 可利用极化求和关系

$$\sum_{\lambda} \varepsilon_{\mu}^*(\mathbf{p}, \lambda) \varepsilon_{\nu}(\mathbf{p}, \lambda) = -g_{\mu\nu} + \frac{p_{\mu} p_{\nu}}{m^2}. \quad (258)$$

## 7 常用单位和标准模型参数

本节数据来自 Particle Data Group 发布的 2022 版 *Review of Particle Physics* [5]。

在有理化的自然单位制中, 光速、约化 Planck 常数和真空介电常数均取为 1, 即  $c = \hbar = \varepsilon_0 = 1$ 。从而, 速度没有量纲 (dimension); 长度量纲与时间量纲相同, 是能量量纲的倒数; 能量、质量和动量具有相同的量纲; 精细结构常数表达为

$$\alpha = \frac{e^2}{4\pi}, \quad (259)$$

而单位电荷量  $e = \sqrt{4\pi\alpha}$  是没有量纲的。可以将能量单位电子伏特 (eV) 视作上述有量纲物理量的基本单位。

单位间转换关系为

$$1 = c = 2.99792458 \times 10^{10} \text{ cm} \cdot \text{s}^{-1}, \quad (260)$$

$$1 = \hbar = 6.582119569 \times 10^{-25} \text{ GeV} \cdot \text{s}, \quad (261)$$

$$1 = \hbar c = 1.973269804 \times 10^{-14} \text{ GeV} \cdot \text{cm}, \quad (262)$$

$$1 = (\hbar c)^2 = 3.893793721 \times 10^8 \text{ GeV}^2 \cdot \text{pb}, \quad (263)$$

由此得到

$$1 \text{ s} = 2.997925 \times 10^{10} \text{ cm}, \quad 1 \text{ cm} = 3.335641 \times 10^{-11} \text{ s}, \quad (264)$$

$$1 \text{ s} = 1.519267 \times 10^{24} \text{ GeV}^{-1}, \quad 1 \text{ GeV}^{-1} = 6.582120 \times 10^{-25} \text{ s}, \quad (265)$$

$$1 \text{ cm} = 5.067731 \times 10^{13} \text{ GeV}^{-1}, \quad 1 \text{ GeV}^{-1} = 1.973270 \times 10^{-14} \text{ cm}, \quad (266)$$

$$1 \text{ cm}^2 = 2.568189 \times 10^{27} \text{ GeV}^{-2}, \quad 1 \text{ GeV}^{-2} = 3.893794 \times 10^{-28} \text{ cm}^2, \quad (267)$$

$$1 \text{ cm}^3 \cdot \text{s}^{-1} = 8.566558 \times 10^{16} \text{ GeV}^{-2}, \quad 1 \text{ GeV}^{-2} = 1.167330 \times 10^{-17} \text{ cm}^3 \cdot \text{s}^{-1}. \quad (268)$$

靶 (barn) 是散射截面的常用单位, 记作 b, 满足

$$1 \text{ b} = 10^{-24} \text{ cm}^2 = 10^9 \text{ nb} = 10^{12} \text{ pb} = 10^{15} \text{ fb} = 10^{18} \text{ ab}, \quad (269)$$

$$1 \text{ pb} = 10^{-36} \text{ cm}^2 = 2.568189 \times 10^{-9} \text{ GeV}^{-2}, \quad 1 \text{ GeV}^{-2} = 3.893794 \times 10^8 \text{ pb}. \quad (270)$$

Fermi 耦合常数是

$$G_F = 1.1663788(6) \times 10^{-5} \text{ GeV}^{-2}, \quad (271)$$

括号内数字代表测量值的  $1\sigma$  不确定度, 由树图阶关系式

$$\frac{G_F}{\sqrt{2}} = \frac{1}{2v^2} = \frac{g^2}{8m_W^2}, \quad (272)$$

得到 Higgs 场真空期待值为

$$v = (\sqrt{2}G_F)^{-1/2} = 246.2196 \text{ GeV}. \quad (273)$$

在低能标 (Thomson 极限) 处, 精细结构常数为

$$\alpha = \frac{1}{137.035999180(10)}; \quad (274)$$

在  $\overline{\text{MS}}$  重整化方案 (以  $\wedge$  为标志) 中,  $\alpha^{-1}$  跑到  $\mu = m_Z$  能标处的数值是

$$\hat{\alpha}^{-1}(m_Z) = 127.951 \pm 0.009. \quad (275)$$

在  $\overline{\text{MS}}$  方案中,  $\mu = m_Z$  能标处强耦合常数  $\alpha_s = g_s^2/(4\pi)$  的数值为

$$\hat{\alpha}_s(m_Z) = 0.1179 \pm 0.0009, \quad (276)$$

Weinberg 角  $\theta_W$  的数值对应于

$$\hat{s}_W^2 = \sin^2 \hat{\theta}_W(m_Z) = 0.23122 \pm 0.00004. \quad (277)$$

在标准模型中, 光子、胶子和中微子没有质量, 其它基本粒子的质量为

$$m_W = 80.377 \pm 0.012 \text{ GeV}, \quad m_Z = 91.1876 \pm 0.0021 \text{ GeV}, \quad (278)$$



$$m_H = 125.25 \pm 0.17 \text{ GeV}, \quad m_e = 0.51099895000(15) \text{ MeV}, \quad (279)$$

$$m_\mu = 105.6583755(23) \text{ MeV}, \quad m_\tau = 1776.86 \pm 0.12 \text{ MeV}, \quad (280)$$

$$m_u = 2.16^{+0.49}_{-0.26} \text{ MeV}, \quad m_d = 4.67^{+0.48}_{-0.17} \text{ MeV}, \quad (281)$$

$$m_s = 93.4^{+8.6}_{-3.4} \text{ MeV}, \quad m_c = 1.27 \pm 0.02 \text{ GeV}, \quad (282)$$

$$m_b = 4.18^{+0.03}_{-0.02} \text{ GeV}, \quad m_t = 172.69 \pm 0.30 \text{ GeV}. \quad (283)$$

这里,  $u$ 、 $d$ 、 $s$  夸克的质量是  $\mu = 2 \text{ GeV}$  能标处的  $\overline{\text{MS}}$  质量,  $c$  和  $b$  夸克的质量分别是  $\mu = m_c$  和  $\mu = m_b$  能标处的  $\overline{\text{MS}}$  质量, 其余粒子的质量均为极点质量 (pole mass)。相应地, 计算出来的  $c$ 、 $b$  夸克极点质量为

$$m_c^{\text{pole}} = 1.67 \pm 0.07 \text{ GeV}, \quad m_b^{\text{pole}} = 4.78 \pm 0.06 \text{ GeV}. \quad (284)$$

质子和中子的质量为

$$m_p = 938.27208816(29) \text{ MeV}, \quad m_n = 939.56542052(54) \text{ MeV}. \quad (285)$$

在电弱能标附近作领头阶计算时, 可将单位电荷量取为

$$e = \sqrt{4\pi\hat{\alpha}(m_Z)} = 0.3133885, \quad (286)$$

将强耦合常数取为

$$g_s = \sqrt{4\pi\hat{\alpha}_s(m_Z)} = 1.217200. \quad (287)$$

从树图阶关系计算 Higgs 场四线性耦合常数  $\lambda$  和 Yukawa 耦合常数  $y_t$ 、 $y_b$ 、 $y_\tau$ 、 $y_c$ , 得

$$\lambda = \frac{m_H^2}{2v^2} = 0.1293839, \quad y_t = \frac{\sqrt{2}m_t}{v} = 0.9918808, \quad y_b = \frac{\sqrt{2}m_b}{v} = 2.400870 \times 10^{-2}, \quad (288)$$

$$y_\tau = \frac{\sqrt{2}m_\tau}{v} = 1.020576 \times 10^{-2}, \quad y_c = \frac{\sqrt{2}m_c}{v} = 7.294508 \times 10^{-3}. \quad (289)$$

耦合常数  $g$  和  $g'$  有以下两种取值方式。

1. 根据树图阶关系  $\sin^2 \theta_W = 1 - m_W^2/m_Z^2$  计算 Weinberg 角, 得

$$s_W^2 = 1 - \frac{m_W^2}{m_Z^2} = 0.2230519, \quad c_W^2 = 1 - s_W^2 = 0.7769481, \quad (290)$$

$$s_W = \sqrt{s_W^2} = 0.4722837, \quad c_W = \sqrt{c_W^2} = 0.8814466, \quad (291)$$

故

$$g = \frac{e}{s_W} = 0.6635599, \quad g' = \frac{e}{c_W} = 0.3555389. \quad (292)$$

2. 根据  $\overline{\text{MS}}$  方案中 Weinberg 角的数值 (277) 计算  $g$  和  $g'$ , 得

$$c_W^2 = 1 - \hat{s}_W^2 = 0.76878, \quad s_W = \sqrt{\hat{s}_W^2} = 0.4808534, \quad c_W = \sqrt{c_W^2} = 0.8768010, \quad (293)$$

$$g = \frac{e}{s_W} = 0.6517340, \quad g' = \frac{e}{c_W} = 0.3574226. \quad (294)$$

将 CKM 矩阵参数化为

$$V = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & s_{13}e^{-i\delta} & \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}, \quad (295)$$

其中  $s_{ij} \equiv \sin \theta_{ij}$ ,  $c_{ij} \equiv \cos \theta_{ij}$ 。实验拟合值为

$$s_{12} = 0.22500 \pm 0.00067, \quad s_{13} = 0.00369 \pm 0.00011, \quad (296)$$

$$s_{23} = 0.04182^{+0.00085}_{-0.00074}, \quad \delta = 1.144 \pm 0.027. \quad (297)$$

如果只讨论第一、二代夸克的混合, 可利用 Cabibbo 转动角  $\theta_C$  将 CKM 矩阵近似地表达为

$$V \simeq \begin{pmatrix} \cos \theta_C & \sin \theta_C & \\ -\sin \theta_C & \cos \theta_C & \\ & & 1 \end{pmatrix}, \quad \sin \theta_C = s_{12} = 0.225. \quad (298)$$

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