

标准模型的拉氏量和 Feynman 规则

余钊焕

中山大学物理学院

<https://yzhxxzxy.github.io>

2024 年 10 月 7 日

目 录

1	约定	2
2	标准模型概述	3
3	量子色动力学	3
4	电弱规范理论	7
4.1	Brout–Englert–Higgs 机制	8
4.2	费米子电弱规范相互作用	16
4.3	电弱规范场的自相互作用	22
5	R_ξ 规范下电弱拉氏量和 Feynman 规则	25
6	内外线一般 Feynman 规则	39
7	常用单位和标准模型参数	41

1 约定

本文采用有理化的自然单位制，推导过程参考文献 [1, 2, 3, 4]，协变导数的约定与 *Review of Particle Physics* [?] 第 9、10、11 章一致。

Minkowski 度规张量

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}. \quad (1)$$

Pauli 矩阵

$$\sigma^1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} & -i \\ i & \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}, \quad (2)$$

$$\sigma^\mu \equiv (1, \boldsymbol{\sigma}), \quad \bar{\sigma}^\mu \equiv (1, -\boldsymbol{\sigma}). \quad (3)$$

Weyl 表象中的 Dirac 矩阵

$$\gamma^\mu = \begin{pmatrix} & \sigma^\mu \\ \bar{\sigma}^\mu & \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -1 & \\ & 1 \end{pmatrix}. \quad (4)$$

左右手投影算符

$$P_L \equiv \frac{1}{2}(1 - \gamma^5) = \begin{pmatrix} 1 & \\ & \mathbf{0} \end{pmatrix}, \quad P_R \equiv \frac{1}{2}(1 + \gamma^5) = \begin{pmatrix} \mathbf{0} & \\ & 1 \end{pmatrix} \quad (5)$$

用于定义左手旋量场 $\psi_L \equiv P_L \psi$ 和右手旋量场 $\psi_R \equiv P_R \psi$ 。Levi-Civita 符号的约定取

$$\varepsilon^{0123} = \varepsilon^{123} = +1. \quad (6)$$

Feynman 规则约定如下。

- 对于指向相互作用顶点的动量 p ，时空导数 ∂_μ 在动量空间 Feynman 规则里贡献一个 $-ip_\mu$ 因子。
- 实线表示费米子，实线上的箭头表示费米子数流动的方向。
- 虚线表示标量玻色子，虚线上的箭头表示玻色子数流动的方向。
- 螺旋线表示胶子；波浪线表示其它规范玻色子，波浪线上的箭头表示玻色子数流动的方向。
- 点线表示鬼粒子，点线上的箭头表示鬼粒子数流动的方向。
- 如果没有额外箭头标记，动量方向与粒子线上的箭头方向一致；否则与额外箭头方向一致。

2 标准模型概述

粒子物理标准模型是一个 $SU(3)_C \times SU(2)_L \times U(1)_Y$ 规范理论。模型中有三代费米子, 包括三代中微子 $\nu_i = \nu_e, \nu_\mu, \nu_\tau$, 三代带电轻子 $\ell_i = e, \mu, \tau$, 三代上型夸克 $u_i = u, c, t$ 和三代下型夸克 $d_i = d, s, b$ ($i = 1, 2, 3$)。规范玻色子传递费米子之间的规范相互作用。

$SU(3)_C$ 部分描述夸克的强相互作用, 称为量子色动力学 (Quantum Chromodynamics, QCD), 相应的规范玻色子是胶子。 $SU(2)_L \times U(1)_Y$ 部分统一描述夸克和轻子的电磁和弱相互作用, 称为电弱规范理论。理论中有一个 Higgs 二重态, 通过 Brout-Englert-Higgs (BEH) 机制引发规范群的自发对称性破缺, 使 $SU(2)_L \times U(1)_Y$ 群破缺为 $U(1)_{EM}$ 群。 $U(1)_{EM}$ 规范理论称为量子电动力学 (Quantum Electrodynamics, QED)。

破缺前, 理论中存在 4 个无质量的规范玻色子和 4 个 Higgs 自由度; 左手费米子和右手费米子都没有质量, 具有不同的量子数。

破缺后, 3 个规范玻色子与 3 个 Higgs 自由度结合, 从而获得质量, 成为 W^\pm 和 Z^0 玻色子, 传递弱相互作用。剩下的 1 个无质量规范玻色子是光子, 即是 $U(1)_{EM}$ 群的规范玻色子, 传递电磁相互作用。剩下的 1 个中性 Higgs 自由度称为 Higgs 玻色子。费米子与 Higgs 二重态的 Yukawa 耦合导致左手费米子和右手费米子获得质量, 组合成 Dirac 费米子。

理论中的中微子没有右手分量, 因而没有获得质量。1998 年实验发现中微子振荡, 证明中微子具有质量, 所以需要扩充标准模型才能正确描述中微子物理。

3 量子色动力学

QCD 的拉氏量表达为

$$\mathcal{L}_{\text{QCD}} = \sum_q \bar{q}(i\gamma^\mu D_\mu - m_q)q - \frac{1}{4}G_{\mu\nu}^a G^{a,\mu\nu}, \quad q = u, d, s, c, b, t, \quad a = 1, \dots, 8, \quad (7)$$

其中

$$D_\mu = \partial_\mu + ig_s G_\mu^a t^a, \quad G_{\mu\nu}^a \equiv \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_s f^{abc} G_\mu^b G_\nu^c. \quad (8)$$

q 为夸克旋量场 $SU(3)_C$ 三重态, $SU(3)_C$ 规范场 G_μ^a 对应于胶子 g , g_s 是 $SU(3)_C$ 规范耦合常数。 $t^a = \lambda^a/2$ 是 $SU(3)_C$ 群基础表示的生成元, 其中 λ^a 为 Gell-Mann 矩阵。 $SU(3)_C$ 生成元对易关系为 $[t^a, t^b] = if^{abc}t^c$, 结构常数 f^{abc} 是全反对称的, 非零分量为

$$f^{123} = 1, \quad f^{147} = f^{246} = f^{257} = f^{345} = f^{516} = f^{637} = \frac{1}{2}, \quad f^{458} = f^{678} = \frac{\sqrt{3}}{2}. \quad (9)$$

由

$$-\frac{1}{4}G_{\mu\nu}^a G^{a,\mu\nu} = -\frac{1}{4}(\partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_s f^{abc} G_\mu^b G_\nu^c)(\partial^\mu G^{a,\nu} - \partial^\nu G^{a,\mu} - g_s f^{ade} G^{d,\mu} G^{e,\nu})$$

$$\begin{aligned}
&= -\frac{1}{2}[(\partial_\mu G_\nu^a)(\partial^\mu G^{a,\nu}) - (\partial_\mu G_\nu^a)(\partial^\nu G^{a,\mu})] + g_s f^{abc}(\partial_\mu G_\nu^a)G^{b,\mu}G^{c,\nu} \\
&\quad - \frac{g_s^2}{4}f^{abc}f^{ade}G_\mu^b G_\nu^c G^{d,\mu}G^{e,\nu},
\end{aligned} \tag{10}$$

推出

$$\begin{aligned}
\mathcal{L}_{\text{QCD}} &= \sum_q [\bar{q}(i\gamma^\mu \partial_\mu - m_q)q - g_s G_\mu^a \bar{q}\gamma^\mu t^a q] + \frac{1}{2}[(\partial_\mu G_\nu^a)(\partial^\nu G^{a,\mu}) - (\partial_\mu G_\nu^a)(\partial^\mu G^{a,\nu})] \\
&\quad + g_s f^{abc}(\partial_\mu G_\nu^a)G^{b,\mu}G^{c,\nu} - \frac{g_s^2}{4}f^{abc}f^{ade}G_\mu^b G_\nu^c G^{d,\mu}G^{e,\nu}.
\end{aligned} \tag{11}$$

设用于固定胶子场规范的函数 $G^a(x) = \partial^\mu G_\mu^a(x) - \omega^a(x)$, 其中 $\omega^a(x)$ 是某个任意函数, 规范固定条件是 $G^a(x) = 0$ 。这是 Lorenz 规范的推广, $\omega^a(x) = 0$ 对应于 Lorenz 规范。在路径积分量子化中, 以中心为 $\omega^a(x) = 0$ 的 Gauss 权重对 $\omega^a(x)$ 作泛函积分, 有

$$\int \mathcal{D}\omega^a \exp \left[-i \int d^4x \frac{1}{2\xi} (\omega^a)^2 \right] \delta(G^a) = \exp \left[-i \int d^4x \frac{1}{2\xi} (\partial^\mu G_\mu^a)^2 \right]. \tag{12}$$

可见, 拉氏量中的规范固定项为

$$\mathcal{L}_{\text{QCD,GF}} = -\frac{1}{2\xi} (\partial^\mu G_\mu^a)^2. \tag{13}$$

ξ 的任何一个取值对应于一种规范。 $\xi = 1$ 称为 Feynman-'t Hooft 规范, $\xi = 0$ 称为 Landau 规范。于是, 胶子传播子相关拉氏量为

$$\begin{aligned}
\mathcal{L}_{\text{QCD,prop}} &= \frac{1}{2} \left[(\partial_\mu G_\nu^a)(\partial^\nu G^{a,\mu}) - (\partial_\mu G_\nu^a)(\partial^\mu G^{a,\nu}) - \frac{1}{\xi} (\partial^\mu G_\mu^a)^2 \right] \\
&\rightarrow \frac{1}{2} G_\mu^a \left[g^{\mu\nu} \partial^2 - \left(1 - \frac{1}{\xi} \right) \partial^\mu \partial^\nu \right] G_\nu^a.
\end{aligned} \tag{14}$$

这里 \rightarrow 代表丢弃一些全散度项。变换到动量空间, 得

$$-g^{\mu\nu} p^2 + \left(1 - \frac{1}{\xi} \right) p^\mu p^\nu, \tag{15}$$

它的逆矩阵是

$$-\frac{1}{p^2} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} (1 - \xi) \right], \tag{16}$$

这是因为

$$\begin{aligned}
&-\frac{1}{p^2} \left[g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2} (1 - \xi) \right] \left[-g^{\mu\nu} p^2 + \left(1 - \frac{1}{\xi} \right) p^\mu p^\nu \right] \\
&= \delta_\rho^\nu - \frac{p_\rho p^\nu}{p^2} \left(1 - \frac{1}{\xi} \right) - \frac{p_\rho p^\nu}{p^2} (1 - \xi) + \frac{p_\rho p^\nu}{p^2} (1 - \xi) \left(1 - \frac{1}{\xi} \right) = \delta_\rho^\nu.
\end{aligned} \tag{17}$$

从而胶子传播子的形式为

$$\frac{-i\delta^{ab}}{p^2 + i\epsilon} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} (1 - \xi) \right]. \quad (18)$$

$SU(3)_C$ 规范变换为

$$q' = Uq, \quad (G_\mu^a t^a)' = U G_\mu^a t^a U^\dagger - \frac{i}{g_s} U \partial_\mu U^\dagger, \quad (19)$$

其中 $U(x) = \exp[i\alpha^a(x)t^a]$ 。胶子场的无穷小规范变换形式是

$$\begin{aligned} (G_\mu^a t^a)' &= (1 + i\alpha^a t^a) G_\mu^b t^b (1 - i\alpha^c t^c) - \frac{i}{g_s} (1 + i\alpha^a t^a) \partial_\mu (1 - i\alpha^c t^c) \\ &= G_\mu^b t^b + i\alpha^c G_\mu^b [t^c, t^b] - \frac{1}{g_s} (\partial_\mu \alpha^c) t^c + \mathcal{O}(\alpha^a \alpha^b) \\ &= G_\mu^a t^a - f^{cba} \alpha^c G_\mu^b t^a - \frac{1}{g_s} (\partial_\mu \alpha^a) t^a + \mathcal{O}(\alpha^a \alpha^b) \\ &= \left(G_\mu^a + f^{abc} G_\mu^b \alpha^c - \frac{1}{g_s} \partial_\mu \alpha^a \right) t^a + \mathcal{O}(\alpha^a \alpha^b), \end{aligned} \quad (20)$$

即 G_μ^a 的无穷小变化为

$$\delta G_\mu^a = (G_\mu^a)' - G_\mu^a = -\frac{1}{g_s} \partial_\mu \alpha^a + f^{abc} G_\mu^b \alpha^c = -\frac{1}{g_s} (D_\mu \alpha)^a = -\frac{1}{g_s} D_\mu^{ac} \alpha^c, \quad (21)$$

其中 $(D_\mu \alpha)^a = \partial_\mu \alpha^a - g_s f^{abc} G_\mu^b \alpha^c$ 是 $SU(3)_C$ 伴随表示中的协变导数, 而

$$D_\mu^{ac} = \delta^{ac} \partial_\mu - g_s f^{abc} G_\mu^b. \quad (22)$$

因此, 规范固定函数 G^a 的无穷小变化为

$$\delta G^a = \partial^\mu \delta G_\mu^a = -\frac{1}{g_s} \partial^\mu D_\mu^{ac} \alpha^c, \quad (23)$$

故

$$\frac{\delta G^a}{\delta \alpha^c} = -\frac{1}{g_s} \partial^\mu D_\mu^{ac} = -\frac{1}{g_s} \delta^{ac} \partial^2 + f^{abc} \partial^\mu G_\mu^b. \quad (24)$$

根据 Grassmann 数的积分式

$$\left(\prod_i \int d\theta_i^* d\theta_i \right) \exp(-\theta_i^* B_{ij} \theta_j) = \det(B), \quad (25)$$

$\delta G^a / \delta \alpha^c$ 的行列式可用 Faddeev-Popov 鬼场 η_g^a 和 $\bar{\eta}_g^a$ 表达为

$$\det \left(\frac{\delta G^a}{\delta \alpha^c} \right) = \det \left(-\frac{1}{g_s} \partial^\mu D_\mu^{ac} \right) = \int \mathcal{D}\eta_g^a \mathcal{D}\bar{\eta}_g^c \exp \left[i \int d^4x \bar{\eta}_g^a (-\partial^\mu D_\mu^{ac}) \eta_g^c \right], \quad (26)$$

这里 $-1/g_s$ 因子被吸收到鬼场 η_g^a 和 $\bar{\eta}_g^a$ 的归一化中。注意到

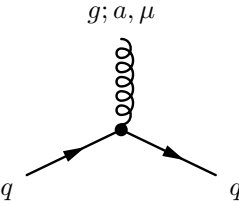
$$-\partial^\mu D_\mu^{ac} = g_s \frac{\delta G^a}{\delta \alpha^c} = -\delta^{ac} \partial^2 + g_s f^{abc} \partial^\mu G_\mu^b, \quad (27)$$

Faddeev-Popov 鬼场在拉氏量中的贡献是

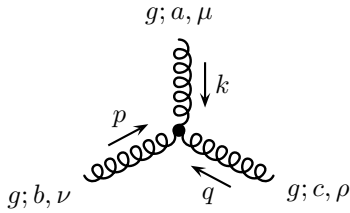
$$\begin{aligned} \mathcal{L}_{\text{QCD,FP}} &= \bar{\eta}_g^a (-\partial^\mu D_\mu^{ac}) \eta_g^c = \bar{\eta}_g^a \left(g_s \frac{\delta G^a}{\delta \alpha^c} \right) \eta_g^c = \bar{\eta}_g^a (-\delta^{ac} \partial^2 + g_s f^{abc} \partial^\mu G_\mu^b) \eta_g^c \\ &\rightarrow -\bar{\eta}_g^a \delta^{ab} \partial^2 \eta_g^b - g_s f^{abc} (\partial^\mu \bar{\eta}_g^a) G_\mu^b \eta_g^c. \end{aligned} \quad (28)$$

下面列出 QCD Feynman 规则。

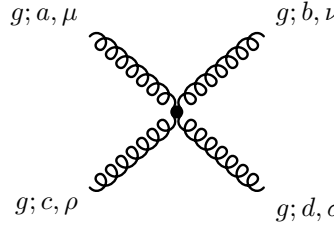
QCD 耦合顶点：



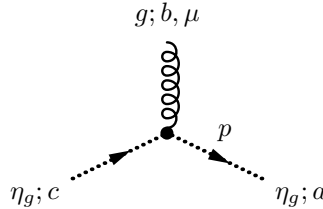
$$= -ig_s \gamma^\mu t^a \quad (29)$$



$$= -g_s f^{abc} [g^{\mu\nu} (k-p)^\rho + g^{\nu\rho} (p-q)^\mu + g^{\rho\mu} (q-k)^\nu] \quad (30)$$

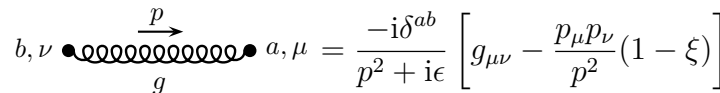


$$\begin{aligned} &= -ig_s^2 [f^{abe} f^{cde} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) \\ &\quad + f^{ace} f^{bde} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}) \\ &\quad + f^{ade} f^{bce} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma})] \end{aligned} \quad (31)$$



$$= g_s f^{abc} p^\mu \quad (32)$$

胶子传播子：



$$= \frac{-i\delta^{ab}}{p^2 + i\epsilon} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} (1 - \xi) \right] \quad (33)$$

鬼粒子传播子：



$$= \frac{i\delta^{ab}}{p^2 + i\epsilon} \quad (34)$$

4 电弱规范理论

电弱规范理论的规范群是 $SU(2)_L \times U(1)_Y$ ，每一代左手旋量场构成 2 个 $SU(2)_L$ 二重态

$$L_{iL} = \begin{pmatrix} P_L \nu_i \\ P_L \ell_i \end{pmatrix} = \begin{pmatrix} \nu_{iL} \\ \ell_{iL} \end{pmatrix}, \quad Q_{iL} = \begin{pmatrix} P_L u'_i \\ P_L d'_i \end{pmatrix} = \begin{pmatrix} u'_{iL} \\ d'_{iL} \end{pmatrix}, \quad i = 1, 2, 3. \quad (35)$$

它们的协变导数是

$$D_\mu = \partial_\mu + igW_\mu^a \tau^a + ig'B_\mu Y, \quad (36)$$

其中 $W_\mu^a(x)$ ($a = 1, 2, 3$) 是 $SU(2)_L$ 规范场, $B_\mu(x)$ 是 $U(1)_Y$ 规范场, g 和 g' 分别是 $SU(2)_L$ 和 $U(1)_Y$ 的规范耦合常数, 取 $g > 0$ 且 $g' > 0$ 。

$$\tau^a = \frac{\sigma^a}{2} \quad (37)$$

是 $SU(2)_L$ 群 2 维表示的生成元, 对应于弱同位旋。生成元 τ^3 的本征值是弱同位旋第 3 分量, 记为 T^3 。 Y 是弱超荷。各代右手旋量场 $\ell_{iR} = P_R \ell_i$ 、 $u'_{iR} = P_R u'_i$ 和 $d'_{iR} = P_R d'_i$ 是 $SU(2)_L$ 单态, 协变导数为

$$D_\mu = \partial_\mu + ig'B_\mu Y. \quad (38)$$

表 1 列出费米子场的电荷 Q 、弱同位旋第 3 分量 T^3 、弱超荷 Y 、重子数 B 和轻子数 $L_e/L_\mu/L_\tau$, 其中电荷 Q 由 T^3 和 Y 定义,

$$Q \equiv T^3 + Y. \quad (39)$$

表 1: 标准模型费米子场的量子数。

统一记号	第一代	第二代	第三代	Q	T^3	Y	B	$L_e/L_\mu/L_\tau$
$L_{iL} = \begin{pmatrix} \nu_{iL} \\ \ell_{iL} \end{pmatrix}$	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	0	1/2	-1/2	0	1
				-1	-1/2	-1/2	0	1
$Q_{iL} = \begin{pmatrix} u'_{iL} \\ d'_{iL} \end{pmatrix}$	$\begin{pmatrix} u'_L \\ d'_L \end{pmatrix}$	$\begin{pmatrix} c'_L \\ s'_L \end{pmatrix}$	$\begin{pmatrix} t'_L \\ b'_L \end{pmatrix}$	2/3	1/2	1/6	1/3	0
				-1/3	-1/2	1/6	1/3	0
ℓ_{iR}	e_R	μ_R	τ_R	-1	0	-1	0	1
u'_{iR}	u'_R	c'_R	t'_R	2/3	0	2/3	1/3	0
d'_{iR}	d'_R	s'_R	b'_R	-1/3	0	-1/3	1/3	0

4.1 Brout-Englert-Higgs 机制

由于左手费米子和右手费米子参与不同的 $SU(2)_L \times U(1)_Y$ 规范相互作用, 耦合左右手费米子场的质量项会破坏规范对称性。另一方面, 规范对称性也禁止规范玻色子具有质量。为了让费米子和弱规范玻色子获得质量, 需要引入 BEH 机制, 使 $SU(2)_L \times U(1)_Y$ 规范对称性自发破缺。因此, 引入 Higgs 标量场

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}, \quad (40)$$

其中 ϕ^+ 和 ϕ^0 都是复标量场。 Φ 是 $SU(2)_L$ 二重态, 弱超荷是

$$Y_H = \frac{1}{2}. \quad (41)$$

Higgs 场的协变动能项和势能项为

$$\mathcal{L}_H = (D^\mu \Phi)^\dagger (D_\mu \Phi) - V_H(\Phi), \quad V_H(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2, \quad (42)$$

其中协变导数为

$$D_\mu \Phi = (\partial_\mu + ig' B_\mu Y_H + ig W_\mu^a \tau^a) \Phi. \quad (43)$$

当 $\lambda > 0$ 且 $\mu^2 > 0$ 时, Higgs 场势能 $V_H(\Phi)$ 呈现出图 1 所示墨西哥草帽状的形式, 势能最小值位于方程

$$\Phi^\dagger \Phi = [\text{Re}(\phi^+)]^2 + [\text{Im}(\phi^+)]^2 + [\text{Re}(\phi^0)]^2 + [\text{Im}(\phi^0)]^2 = \frac{v^2}{2} \quad (44)$$

对应的 3 维球面上, 其中 $v \equiv \sqrt{\mu^2/\lambda}$, 满足

$$\mu^2 = \lambda v^2. \quad (45)$$

Higgs 场的真空期待值位于这个 3 维球面上的某一点, 不失一般性, 可将它取为

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. \quad (46)$$

其它真空期待值可通过 $SU(2)_L \times U(1)_Y$ 整体变换

$$\langle \Phi \rangle \rightarrow \exp(i\alpha^a \tau^a) \exp(i\alpha^Y Y_H) \langle \Phi \rangle \quad (47)$$

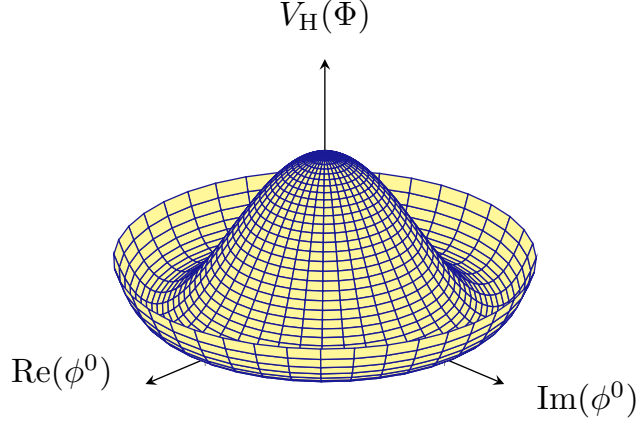


图 1: Higgs 场势能示意图。这里压缩掉 $\text{Re}(\phi^+)$ 和 $\text{Im}(\phi^+)$ 两个维度。

得到, 因为 $\langle \Phi^\dagger \Phi \rangle$ 在这样的变换下保持不变。若 $\alpha^1 = \alpha^2 = 0$ 且 $\alpha^3 = \alpha^Y$, 则

$$\exp(i\alpha^a \tau^a) \exp(i\alpha^Y Y_H) = \exp[i\alpha^3(\sigma^3 + \mathbf{1})/2] = \exp \left[i\alpha^3 \begin{pmatrix} 1 & \\ & 0 \end{pmatrix} \right] = \begin{pmatrix} e^{i\alpha^3} & \\ & 1 \end{pmatrix}, \quad (48)$$

而 $\langle \Phi \rangle$ 在此变换下不变。因此, 有 1 个方向的规范对称性没有受到破坏, 只有 3 个方向的规范对称性发生自发破缺。根据 Goldstone 定理, 破缺后存在 3 个无质量的 Nambu-Goldstone 玻色子。最终, 有 3 个规范玻色子结合 Nambu-Goldstone 玻色子, 通过 BEH 机制获得质量。

以 $\langle \Phi \rangle$ 为基础, 将 Higgs 场参数化为

$$\Phi(x) = \exp \left[-i \frac{\chi^a(x)}{v} \tau^a \right] \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \quad (49)$$

其中 $\chi^a(x)$ 和 $H(x)$ 都是实标量场。 $\exp[-i\chi^a(x)\tau^a/v]$ 因子能够通过 $\text{SU}(2)_L$ 规范变换消去, 因而可将 $\Phi(x)$ 直接取为

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \quad \Phi^\dagger \Phi = \frac{1}{2}(v + H)^2. \quad (50)$$

此时 Higgs 场只剩下一个物理自由度 $H(x)$, 对应于 Higgs 玻色子, 这种取法称为幺正规范。

在幺正规范下, 势能项化为

$$\begin{aligned} -V_H(\Phi) &= \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 = \frac{\mu^2}{2}(v + H)^2 - \frac{\lambda}{4}(v + H)^4 \\ &= \frac{\mu^2}{2}(v^2 + H^2 + 2vH) - \frac{\lambda}{4}(v^4 + 4v^2 H^2 + H^4 + 4v^3 H + 2v^2 H^2 + 4vH^3) \\ &= \frac{1}{4}\mu^2 v^2 + \frac{1}{4}(\mu^2 - \lambda v^2)v^2 + (\mu^2 - \lambda v^2)vH + \frac{1}{2}(\mu^2 - \lambda v^2)H^2 - \lambda v^2 H^2 - \lambda v H^3 - \frac{\lambda}{4} H^4 \end{aligned}$$

$$= \frac{1}{8} m_H^2 v^2 - \frac{1}{2} m_H^2 H^2 - \frac{1}{2} \frac{m_H^2}{v} H^3 - \frac{1}{8} \frac{m_H^2}{v^2} H^4, \quad (51)$$

其中 Higgs 玻色子的质量为

$$m_H \equiv \sqrt{2}\mu, \quad m_H^2 = 2\mu^2 = 2\lambda v^2. \quad (52)$$

由于

$$g' B_\mu Y_H + g W_\mu^a \tau^a = \frac{1}{2} \begin{pmatrix} g' B_\mu + g W_\mu^3 & g(W_\mu^1 - i W_\mu^2) \\ g(W_\mu^1 + i W_\mu^2) & g' B_\mu - g W_\mu^3 \end{pmatrix}, \quad (53)$$

Higgs 场真空期待值 v 对协变导数 $D_\mu \Phi$ 的贡献为

$$\begin{aligned} D_\mu \Phi &\supset i(g' B_\mu Y_H + g W_\mu^a \tau^a) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \\ &= \frac{iv}{2\sqrt{2}} \begin{pmatrix} g' B_\mu + g W_\mu^3 & g(W_\mu^1 - i W_\mu^2) \\ g(W_\mu^1 + i W_\mu^2) & g' B_\mu - g W_\mu^3 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \supset \frac{iv}{2\sqrt{2}} \begin{pmatrix} g(W_\mu^1 - i W_\mu^2) \\ g' B_\mu - g W_\mu^3 \end{pmatrix}, \end{aligned} \quad (54)$$

故协变动能项 $(D^\mu \Phi)^\dagger (D_\mu \Phi)$ 中正比于 v^2 的项是

$$(D^\mu \Phi)^\dagger (D_\mu \Phi) \supset \frac{v^2}{8} [g^2 |W_\mu^1 - i W_\mu^2|^2 + (g' B_\mu - g W_\mu^3)^2] = \frac{v^2}{8} (g^2 W^{a,\mu} W_\mu^a + g'^2 B^\mu B_\mu - 2gg' B^\mu W_\mu^3). \quad (55)$$

这些项是规范玻色子的质量项，重新表达为

$$\mathcal{L}_{\text{GBM}} = \frac{1}{2} m_W^2 (W^{1,\mu} W_\mu^1 + W^{2,\mu} W_\mu^2) + \frac{1}{2} \begin{pmatrix} W^{3,\mu} & B^\mu \end{pmatrix} M_{W^3 B}^2 \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}, \quad (56)$$

其中

$$m_W \equiv \frac{1}{2} g v \quad (57)$$

是 W_μ^1 和 W_μ^2 获得的质量，而 $W^{3\mu}$ 和 B^μ 的质量平方矩阵为

$$M_{W^3 B}^2 \equiv \frac{v^2}{4} \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}. \quad (58)$$

为了使 $M_{W^3 B}^2$ 矩阵对角化，定义

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \equiv \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}, \quad (59)$$

其中

$$s_W \equiv \sin \theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}, \quad c_W \equiv \cos \theta_W = \frac{g}{\sqrt{g^2 + g'^2}}, \quad (60)$$

θ_W 称为 Weinberg 角, 也称为弱混合角。从后面的讨论可以看出 $A_\mu(x)$ 就是电磁场, 对应于光子。 $Z_\mu(x)$ 对应于矢量玻色子 Z 。反过来, 有

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}. \quad (61)$$

由

$$\begin{aligned} M_{W^3 B}^2 \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} &= \frac{(g^2 + g'^2)v^2}{4} \begin{pmatrix} c_W^2 & -s_W c_W \\ -s_W c_W & s_W^2 \end{pmatrix} \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \\ &= \frac{(g^2 + g'^2)v^2}{4} \begin{pmatrix} c_W & 0 \\ -s_W & 0 \end{pmatrix} \end{aligned} \quad (62)$$

得

$$\begin{aligned} \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} M_{W^3 B}^2 \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} &= \frac{(g^2 + g'^2)v^2}{4} \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} c_W & 0 \\ -s_W & 0 \end{pmatrix} \\ &= \frac{(g^2 + g'^2)v^2}{4} \begin{pmatrix} 1 & \\ & 0 \end{pmatrix}, \end{aligned} \quad (63)$$

因此

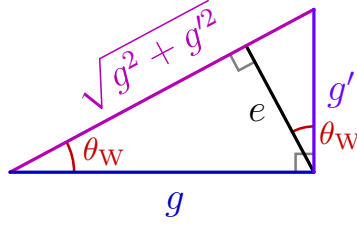
$$\begin{aligned} \frac{1}{2} \begin{pmatrix} W^{3,\mu} & B^\mu \end{pmatrix} M_{W^3 B}^2 \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} &= \frac{1}{2} \begin{pmatrix} Z^\mu & A^\mu \end{pmatrix} \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} M_{W^3 B}^2 \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \\ &= \frac{(g^2 + g'^2)v^2}{8} \begin{pmatrix} Z^\mu & A^\mu \end{pmatrix} \begin{pmatrix} 1 & \\ & 0 \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \frac{1}{2} m_Z^2 Z^\mu Z_\mu, \end{aligned} \quad (64)$$

其中

$$m_Z \equiv \frac{1}{2} \sqrt{g^2 + g'^2} v = \frac{gv}{2c_W} = \frac{m_W}{c_W} \quad (65)$$

是 Z 玻色子的质量, 而光子没有质量。另一方面, 用质量相同的实矢量场 W_μ^1 和 W_μ^2 线性组合出复矢量场

$$W_\mu^+ \equiv \frac{1}{\sqrt{2}} (W_\mu^1 - iW_\mu^2), \quad (66)$$

图 2: g 、 g' 、 e 和 θ_W 的关系。

它的厄米共轭为

$$W_\mu^- \equiv (W_\mu^+)^\dagger = \frac{1}{\sqrt{2}}(W_\mu^1 + iW_\mu^2), \quad (67)$$

则

$$W_\mu^1 = \frac{1}{\sqrt{2}}(W_\mu^+ + W_\mu^-), \quad W_\mu^2 = \frac{i}{\sqrt{2}}(W_\mu^+ - W_\mu^-). \quad (68)$$

从而

$$\begin{aligned} \frac{1}{2}(W^{1,\mu}W_\mu^1 + W^{2,\mu}W_\mu^2) &= \frac{1}{4}[(W^{+,\mu} + W^{-,\mu})(W_\mu^+ + W_\mu^-) - (W^{+,\mu} - W^{-,\mu})(W_\mu^+ - W_\mu^-)] \\ &= W^{+,\mu}W_\mu^-, \end{aligned} \quad (69)$$

(56) 式化为

$$\mathcal{L}_{\text{GBM}} = m_W^2 W^{+,\mu}W_\mu^- + \frac{1}{2}m_Z^2 Z^\mu Z_\mu. \quad (70)$$

复矢量场 W_μ^\pm 描述一对正反矢量玻色子 W^\pm ，质量为 m_W 。可见，BEH 机制使传递弱相互作用的规范玻色子 W^\pm 和 Z 获得了质量。

接下来用质量本征态 W_μ^\pm 、 A_μ 和 Z_μ 表达协变动能项 $(D^\mu\Phi)^\dagger(D_\mu\Phi)$ 。注意到

$$A_\mu = s_W W_\mu^3 + c_W B_\mu, \quad Z_\mu = c_W W_\mu^3 - s_W B_\mu, \quad (71)$$

$$B_\mu = c_W A_\mu - s_W Z_\mu, \quad W_\mu^3 = s_W A_\mu + c_W Z_\mu, \quad (72)$$

有

$$\begin{aligned} g'B_\mu + gW_\mu^3 &= g'(c_W A_\mu - s_W Z_\mu) + g(s_W A_\mu + c_W Z_\mu) = \frac{2gg'}{\sqrt{g^2 + g'^2}} A_\mu + \frac{g^2 - g'^2}{\sqrt{g^2 + g'^2}} Z_\mu \\ &= 2eA_\mu + \frac{g}{c_W}(c_W^2 - s_W^2)Z_\mu, \end{aligned} \quad (73)$$

其中

$$e \equiv \frac{gg'}{\sqrt{g^2 + g'^2}} = gs_W = g'c_W. \quad (74)$$

后面的讨论将表明 e 就是单位电荷量。 g 、 g' 、 e 和 θ_W 的关系如图 2 所示。再利用

$$g'B_\mu - gW_\mu^3 = g'(c_W A_\mu - s_W Z_\mu) - g(s_W A_\mu + c_W Z_\mu) = -\left(\frac{gs_W^2}{c_W} + gc_W\right) Z_\mu = -\frac{g}{c_W} Z_\mu, \quad (75)$$

得

$$\begin{aligned} g'B_\mu Y_H + gW_\mu^a \tau^a &= \frac{1}{2} \begin{pmatrix} g'B_\mu + gW_\mu^3 & g(W_\mu^1 - iW_\mu^2) \\ g(W_\mu^1 + iW_\mu^2) & g'B_\mu - gW_\mu^3 \end{pmatrix} \\ &= \begin{pmatrix} eA_\mu + \frac{g}{2c_W}(c_W^2 - s_W^2)Z_\mu & \frac{g}{\sqrt{2}}W_\mu^+ \\ \frac{g}{\sqrt{2}}W_\mu^- & -\frac{g}{2c_W}Z_\mu \end{pmatrix}. \end{aligned} \quad (76)$$

在么正规范下,

$$\begin{aligned} (D^\mu \Phi)^\dagger (D_\mu \Phi) &= \left| \begin{pmatrix} \partial_\mu + ieA_\mu + \frac{ig}{2c_W}(c_W^2 - s_W^2)Z_\mu & \frac{ig}{\sqrt{2}}W_\mu^+ \\ \frac{ig}{\sqrt{2}}W_\mu^- & \partial_\mu - \frac{ig}{2c_W}Z_\mu \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix} \right|^2 \\ &= \frac{1}{2} \begin{pmatrix} -\frac{ig}{\sqrt{2}}W^{-,\mu}(v + H) & \partial^\mu H + \frac{ig}{2c_W}Z^\mu(v + H) \end{pmatrix} \begin{pmatrix} \frac{ig}{\sqrt{2}}W_\mu^+(v + H) \\ \partial_\mu H - \frac{ig}{2c_W}Z_\mu(v + H) \end{pmatrix} \\ &= \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + (v + H)^2 \left(\frac{g^2}{4}W_\mu^+W^{-,\mu} + \frac{g^2}{8c_W^2}Z_\mu Z^\mu \right) \\ &= \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + m_W^2 W_\mu^+ W^{-,\mu} + \frac{1}{2}m_Z^2 Z_\mu Z^\mu \\ &\quad + gm_W H W_\mu^+ W^{-,\mu} + \frac{gm_Z}{2c_W} H Z_\mu Z^\mu + \frac{g^2}{4} H^2 W_\mu^+ W^{-,\mu} + \frac{g^2}{8c_W^2} H^2 Z_\mu Z^\mu. \end{aligned} \quad (77)$$

除了 W^\pm 和 Z 玻色子的质量项之外, 还出现了 Higgs 玻色子 H 与 W^\pm 、 Z 的三线性和四线性耦合项。

Higgs 场 $\Phi(x)$ 的弱超荷为 $+1/2$ 。引入 $\Phi(x)$ 的共轭态

$$\tilde{\Phi}(x) = i\sigma^2 \Phi^*(x) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{pmatrix} \phi^-(x) \\ \phi^{0*}(x) \end{pmatrix} = \begin{pmatrix} \phi^{0*}(x) \\ -\phi^-(x) \end{pmatrix}, \quad (78)$$

其中 $\phi^- \equiv (\phi^+)^*$, 则 $\tilde{\Phi}(x)$ 是弱超荷为 $-1/2$ 的 $SU(2)_L$ 二重态。在么正规范下, $\tilde{\Phi}(x)$ 化为

$$\tilde{\Phi}(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}. \quad (79)$$

用 $\Phi(x)$ 、 $\tilde{\Phi}(x)$ 和费米子场组成满足 $SU(2)_L \times U(1)_Y$ 规范对称性的 Yukawa 相互作用拉氏量

$$\mathcal{L}_Y = -\tilde{y}_{d,ij} \bar{Q}_{iL} d'_{jR} \Phi - \tilde{y}_{u,ij} \bar{Q}_{iL} u'_{jR} \tilde{\Phi} - y_{\ell_i} \bar{L}_{iL} \ell_{iR} \Phi + \text{H.c.}, \quad (80)$$

其中 H.c. 表示厄米共轭, Yukawa 耦合常数 $\tilde{y}_{d,ij}$ 和 $\tilde{y}_{u,ij}$ 联系着不同代的夸克场, 而 y_{ℓ_i} 只联系同一代的轻子场。在么正规范下, 利用

$$\bar{Q}_{iL} \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \bar{u}'_{iL} & \bar{d}'_{iL} \end{pmatrix} \begin{pmatrix} 0 \\ v + H \end{pmatrix} = \frac{1}{\sqrt{2}} (v + H) \bar{d}'_{iL}, \quad (81)$$

$$\bar{Q}_{iL} \tilde{\Phi} = \frac{1}{\sqrt{2}} \begin{pmatrix} \bar{u}'_{iL} & \bar{d}'_{iL} \end{pmatrix} \begin{pmatrix} v + H \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} (v + H) \bar{u}'_{iL}, \quad (82)$$

$$\bar{L}_{iL} \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \bar{\nu}_{iL} & \bar{\ell}_{iL} \end{pmatrix} \begin{pmatrix} 0 \\ v + H \end{pmatrix} = \frac{1}{\sqrt{2}} (v + H) \bar{\ell}_{iL}, \quad (83)$$

推出

$$\mathcal{L}_Y = -\frac{1}{\sqrt{2}} (v + H) (\tilde{y}_{d,ij} \bar{d}'_{iL} d'_{jR} + \tilde{y}_{u,ij} \bar{u}'_{iL} u'_{jR} + y_{\ell_i} \bar{\ell}_{iL} \ell_{iR} + \text{H.c.}). \quad (84)$$

$\tilde{y}_{d,ij}$ 和 $\tilde{y}_{u,ij}$ 可看作 3×3 矩阵 \tilde{y}_d 和 \tilde{y}_u 的元素。 $\tilde{y}_d \tilde{y}_d^\dagger$ 和 $\tilde{y}_u \tilde{y}_u^\dagger$ 是厄米矩阵, 必定可以分别通过么正矩阵 U_d 和 U_u 对角化成 y_D^2 和 y_U^2 两个对角元为实数的对角矩阵, 满足 $U_d^\dagger \tilde{y}_d \tilde{y}_d^\dagger U_d = y_D^2$ 和 $U_u^\dagger \tilde{y}_u \tilde{y}_u^\dagger U_u = y_U^2$, 即

$$\tilde{y}_d \tilde{y}_d^\dagger = U_d y_D^2 U_d^\dagger, \quad \tilde{y}_u \tilde{y}_u^\dagger = U_u y_U^2 U_u^\dagger. \quad (85)$$

符合这两条式子的 \tilde{y}_d 和 \tilde{y}_u 可以表达为

$$\tilde{y}_d = U_d y_D K_d^\dagger, \quad \tilde{y}_u = U_u y_U K_u^\dagger, \quad (86)$$

其中对角矩阵 y_D 和 y_U 满足 $y_D y_D = y_D^2$ 和 $y_U y_U = y_U^2$, 而 K_d^\dagger 和 K_u^\dagger 是两个么正矩阵。

将 y_D 和 y_U 表示成

$$y_D = \begin{pmatrix} y_{d_1} & & \\ & y_{d_2} & \\ & & y_{d_3} \end{pmatrix} = \begin{pmatrix} y_d & & \\ & y_s & \\ & & y_b \end{pmatrix}, \quad y_U = \begin{pmatrix} y_{u_1} & & \\ & y_{u_2} & \\ & & y_{u_3} \end{pmatrix} = \begin{pmatrix} y_u & & \\ & y_c & \\ & & y_t \end{pmatrix}. \quad (87)$$

通过么正变换定义

$$d_{iL} \equiv (U_d^\dagger)_{ij} d'_{jL}, \quad d_{iR} \equiv (K_d^\dagger)_{ij} d'_{jR}, \quad u_{iL} \equiv (U_u^\dagger)_{ij} u'_{jL}, \quad u_{iR} \equiv (K_u^\dagger)_{ij} u'_{jR}, \quad (88)$$

则 $\bar{d}_{iL} = \bar{d}'_{jL} U_{d,ji}$, $\bar{u}_{iL} = \bar{u}'_{jL} U_{u,ji}$, 从而

$$\tilde{y}_{d,ij} \bar{d}'_{iL} d'_{jR} = \bar{d}_{iL} (U_d y_D K_d^\dagger)_{ij} d'_{jR} = \bar{d}_{iL} U_{d,ik} y_{d_k} (K_d^\dagger)_{kj} d'_{jR} = y_{d_k} \bar{d}_{kL} d_{kR} = y_{d_i} \bar{d}_{iL} d_{iR}, \quad (89)$$

$$\tilde{y}_{u,ij} \bar{u}'_{iL} u'_{jR} = \bar{u}'_{iL} (U_u y_u K_u^\dagger)_{ij} u'_{jR} = y_{u_i} \bar{u}_{iL} u_{iR}, \quad (90)$$

故

$$\begin{aligned} \mathcal{L}_Y &= -\frac{1}{\sqrt{2}}(v + H)(y_{d_i} \bar{d}_{iL} d_{iR} + y_{u_i} \bar{u}_{iL} u_{iR} + y_{\ell_i} \bar{\ell}_{iL} \ell_{iR} + \text{H.c.}) \\ &= -m_{d_i} \bar{d}_i d_i - m_{u_i} \bar{u}_i u_i - m_{\ell_i} \bar{\ell}_i \ell_i - \frac{m_{d_i}}{v} H \bar{d}_i d_i - \frac{m_{u_i}}{v} H \bar{u}_i u_i - \frac{m_{\ell_i}}{v} H \bar{\ell}_i \ell_i, \end{aligned} \quad (91)$$

其中前三项是费米子质量项，后三项是 Higgs 玻色子与费米子的 Yukawa 耦合项。于是，三代夸克和带电轻子获得了质量

$$m_{d_i} \equiv \frac{1}{\sqrt{2}} y_{d_i} v, \quad m_{u_i} \equiv \frac{1}{\sqrt{2}} y_{u_i} v, \quad m_{\ell_i} \equiv \frac{1}{\sqrt{2}} y_{\ell_i} v. \quad (92)$$

d'_{iL} 、 d'_{iR} 、 u'_{iL} 和 u'_{iR} 称为规范本征态（也称为味本征态）， d_{iL} 、 d_{iR} 、 u_{iL} 和 u_{iR} 称为质量本征态。下面给出么正规化下 Higgs 场的顶点 Feynman 规则。

Higgs 玻色子自耦合：

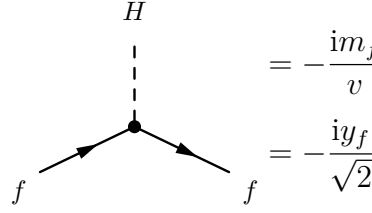
$$\begin{aligned} \text{Higgs 玻色子自耦合} & \quad \begin{array}{c} H \\ | \\ \bullet \\ / \quad \backslash \\ H \quad H \end{array} = -\frac{3im_H^2}{v} = -6i\lambda v \\ & \quad \begin{array}{c} H \quad H \\ \backslash \quad / \\ \bullet \\ / \quad \backslash \\ H \quad H \end{array} = -\frac{3im_H^2}{v^2} = -6i\lambda \end{aligned} \quad (93)$$

Higgs 玻色子与电弱规范玻色子的耦合：

$$\begin{aligned} \text{Higgs 玻色子与电弱规范玻色子的耦合} & \quad \begin{array}{c} H \\ | \\ \bullet \\ / \quad \backslash \\ W; \mu \quad W; \nu \end{array} = igm_W g^{\mu\nu} = \frac{ig^2 v}{2} g^{\mu\nu} \\ & \quad \begin{array}{c} H \\ | \\ \bullet \\ / \quad \backslash \\ Z; \mu \quad Z; \nu \end{array} = \frac{igm_Z}{c_W} g^{\mu\nu} = \frac{ig^2 v}{2c_W^2} g^{\mu\nu} \end{aligned} \quad (94)$$

$$\begin{aligned} & \quad \begin{array}{c} H \quad H \\ \backslash \quad / \\ \bullet \\ / \quad \backslash \\ W; \mu \quad W; \nu \end{array} = \frac{ig^2}{2} g^{\mu\nu} \\ & \quad \begin{array}{c} H \quad H \\ \backslash \quad / \\ \bullet \\ / \quad \backslash \\ Z; \mu \quad Z; \nu \end{array} = \frac{ig^2}{2c_W^2} g^{\mu\nu} \end{aligned} \quad (95)$$

Higgs 玻色子与费米子 $f = d_i, u_i, \ell_i$ 的耦合:



$$\begin{aligned} &= -\frac{im_f}{v} \\ &= -\frac{iy_f}{\sqrt{2}} \end{aligned} \quad (96)$$

4.2 费米子电弱规范相互作用

(88) 式意味着

$$d'_{iL} = U_{d,ij} d_{jL}, \quad d'_{iR} = K_{d,ij} d_{jR}, \quad u'_{iL} = U_{u,ij} u_{jL}, \quad u'_{iR} = K_{u,ij} u_{jR}, \quad (97)$$

从而

$$\bar{d}'_{iL} \gamma^\mu d'_{iL} = \bar{d}_{jL} (U_d^\dagger)_{ji} \gamma^\mu U_{d,ik} d_{kL} = \bar{d}_{jL} \delta_{jk} \gamma^\mu d_{kL} = \bar{d}_{iL} \gamma^\mu d_{iL} \quad (98)$$

同理有

$$\bar{u}'_{iL} \gamma^\mu u'_{iL} = \bar{u}_{iL} \gamma^\mu u_{iL}, \quad \bar{d}'_{iR} \gamma^\mu d'_{iR} = \bar{d}_{iR} \gamma^\mu d_{iR}, \quad \bar{u}'_{iR} \gamma^\mu u'_{iR} = \bar{u}_{iR} \gamma^\mu u_{iR}. \quad (99)$$

另一方面,

$$\bar{u}'_{iL} \gamma^\mu d'_{iL} = \bar{u}_{jL} (U_u^\dagger)_{ji} \gamma^\mu U_{d,ik} d_{kL} = \bar{u}_{iL} \gamma^\mu V_{ij} d_{jL} \quad (100)$$

$$\bar{d}'_{iL} \gamma^\mu u'_{iL} = \bar{d}_{jL} (U_d^\dagger)_{ji} \gamma^\mu U_{u,ik} u_{kL} = \bar{d}_{jL} V_{ji}^\dagger \gamma^\mu u_{iL} \quad (101)$$

其中

$$V \equiv U_u^\dagger U_d \quad (102)$$

称为 Cabibbo-Kobayashi-Maskawa (CKM) 矩阵, 其厄米共轭矩阵为 $V^\dagger = U_d^\dagger U_u$ 。注意, 么正矩阵 U_u 和 U_d 的起源不同, 因而一般来说它们是不相等的, 从而 V 不是恒等矩阵。

$SU(2)_L \times U(1)_Y$ 规范不变的费米子协变动能项为

$$\mathcal{L}_{\text{EWF}} = \bar{Q}_{iL} i \not{D} Q_{iL} + \bar{u}'_{iR} i \not{D} u'_{iR} + \bar{d}'_{iR} i \not{D} d'_{iR} + \bar{L}_{iL} i \not{D} L_{iL} + \bar{\ell}_{iR} i \not{D} \ell_{iR}. \quad (103)$$

根据 Q 的定义 (39), 有

$$\begin{aligned} g'Y B_\mu + gT^3 W_\mu^3 &= g'Y (c_W A_\mu - s_W Z_\mu) + gT^3 (s_W A_\mu + c_W Z_\mu) \\ &= e(Y + T^3) A_\mu + \left(g c_W T^3 - \frac{g s_W}{c_W} s_W Y \right) Z_\mu = Q e A_\mu + \frac{g}{c_W} (T^3 c_W^2 - Y s_W^2) Z_\mu \\ &= Q e A_\mu + \frac{g}{c_W} (T^3 - Q s_W^2) Z_\mu, \end{aligned} \quad (104)$$

故

$$\begin{aligned}
D_\mu Q_{iL} &= (\partial_\mu + ig' B_\mu Y + ig W_\mu^a \tau^a) Q_{iL} \\
&= \partial_\mu Q_{iL} + i \begin{pmatrix} g' Y B_\mu + g T^3 W_\mu^3 & \frac{g}{2} (W_\mu^1 - i W_\mu^2) \\ \frac{g}{2} (W_\mu^1 + i W_\mu^2) & g' Y B_\mu + g T^3 W_\mu^3 \end{pmatrix} Q_{iL} \\
&= \partial_\mu Q_{iL} + i \begin{pmatrix} QeA_\mu + \frac{g}{c_W} (T^3 - Qs_W^2) Z_\mu & \frac{g}{\sqrt{2}} W_\mu^+ \\ \frac{g}{\sqrt{2}} W_\mu^- & QeA_\mu + \frac{g}{c_W} (T^3 - Qs_W^2) Z_\mu \end{pmatrix} \begin{pmatrix} u'_{iL} \\ d'_{iL} \end{pmatrix} \\
&= \partial_\mu Q_{iL} + i \begin{pmatrix} \left[QeA_\mu + \frac{g}{c_W} (T^3 - Qs_W^2) Z_\mu \right] u'_{iL} + \frac{g}{\sqrt{2}} W_\mu^+ d'_{iL} \\ \frac{g}{\sqrt{2}} W_\mu^- u'_{iL} + \left[QeA_\mu + \frac{g}{c_W} (T^3 - Qs_W^2) Z_\mu \right] d'_{iL} \end{pmatrix}. \tag{105}
\end{aligned}$$

于是

$$\begin{aligned}
\bar{Q}_{iL} i \not{D} Q_{iL} &\supset - \left[QeA_\mu + \frac{g}{c_W} (T^3 - Qs_W^2) Z_\mu \right] \bar{u}'_{iL} \gamma^\mu u'_{iL} - \left[QeA_\mu + \frac{g}{c_W} (T^3 - Qs_W^2) Z_\mu \right] \bar{d}'_{iL} \gamma^\mu d'_{iL} \\
&\quad - \frac{g}{\sqrt{2}} W_\mu^+ \bar{u}'_{iL} \gamma^\mu d'_{iL} - \frac{g}{\sqrt{2}} W_\mu^- \bar{d}'_{iL} \gamma^\mu u'_{iL} \\
&= - \left(QeA_\mu + \frac{g}{c_W} g_L Z_\mu \right) \bar{u}_i \gamma^\mu \frac{1 - \gamma^5}{2} u_i - \left(QeA_\mu + \frac{g}{c_W} g_L Z_\mu \right) \bar{d}_i \gamma^\mu \frac{1 - \gamma^5}{2} d_i \\
&\quad - \frac{g}{\sqrt{2}} W_\mu^+ \bar{u}_i \gamma^\mu P_L V_{ij} d_j - \frac{g}{\sqrt{2}} W_\mu^- \bar{d}_j V_{ji}^\dagger \gamma^\mu P_L u_i, \tag{106}
\end{aligned}$$

其中左手耦合系数

$$g_L \equiv T^3 - Qs_W^2. \tag{107}$$

另一方面,

$$\begin{aligned}
D_\mu d'_{iR} &= (\partial_\mu + ig' B_\mu Y) d'_{iR} = \partial_\mu d'_{iR} + ig' Q (c_W A_\mu - s_W Z_\mu) d'_{iR} \\
&= \partial_\mu d'_{iR} + i QeA_\mu d'_{iR} - \frac{ig}{c_W} Qs_W^2 Z_\mu d'_{iR}, \tag{108}
\end{aligned}$$

则

$$\begin{aligned}
&\bar{u}'_{iR} i \not{D} u'_{iR} + \bar{d}'_{iR} i \not{D} d'_{iR} \\
&\supset - \left(QeA_\mu - \frac{g}{c_W} Qs_W^2 Z_\mu \right) \bar{u}'_{iR} \gamma^\mu u'_{iR} - \left(QeA_\mu - \frac{g}{c_W} Qs_W^2 Z_\mu \right) \bar{d}'_{iR} \gamma^\mu d'_{iR} \\
&= - \left(QeA_\mu + \frac{g}{c_W} g_R Z_\mu \right) \bar{u}_i \gamma^\mu \frac{1 + \gamma^5}{2} u_i - \left(QeA_\mu + \frac{g}{c_W} g_R Z_\mu \right) \bar{d}_i \gamma^\mu \frac{1 + \gamma^5}{2} d_i, \tag{109}
\end{aligned}$$

其中右手耦合系数

$$g_R \equiv -Qs_W^2. \quad (110)$$

引入矢量流和轴矢量流耦合系数

$$g_V \equiv g_L + g_R = T^3 - 2Qs_W^2, \quad g_A \equiv g_L - g_R = T^3, \quad (111)$$

得

$$\begin{aligned} & \bar{Q}_{iL} i \not{D} Q_{iL} + \bar{u}'_{iR} i \not{D} u'_{iR} + \bar{d}'_{iR} i \not{D} d'_{iR} \\ & \supset -Qe\bar{u}_i \gamma^\mu u_i A_\mu - Qe\bar{d}_i \gamma^\mu d_i A_\mu - \frac{g}{2c_W} \bar{u}_i \gamma^\mu (g_V - g_A \gamma^5) u_i Z_\mu - \frac{g}{2c_W} \bar{d}_i \gamma^\mu (g_V - g_A \gamma^5) d_i Z_\mu \\ & - \frac{g}{\sqrt{2}} W_\mu^+ \bar{u}_i \gamma^\mu P_L V_{ij} d_j - \frac{g}{\sqrt{2}} W_\mu^- \bar{d}_j V_{ji}^\dagger \gamma^\mu P_L u_i. \end{aligned} \quad (112)$$

同理, 有

$$\begin{aligned} & \bar{L}_{iL} i \not{D} L_{iL} + \bar{\ell}_{iR} i \not{D} \ell_{iR} \supset -Qe\bar{\ell}_i \gamma^\mu \ell_i A_\mu - \frac{g}{2c_W} \bar{\ell}_i \gamma^\mu (g_V - g_A \gamma^5) \ell_i Z_\mu - \frac{g}{2c_W} \bar{\nu}_i \gamma^\mu (g_V - g_A \gamma^5) \nu_i Z_\mu \\ & - \frac{g}{\sqrt{2}} W_\mu^+ \bar{\nu}_i \gamma^\mu P_L \ell_i - \frac{g}{\sqrt{2}} W_\mu^- \bar{\ell}_i \gamma^\mu P_L \nu_i. \end{aligned} \quad (113)$$

总结起来, 将费米子电弱规范相互作用写成流耦合的形式,

$$\mathcal{L}_{\text{EWF}} \supset -A_\mu J_{\text{EM}}^\mu - Z_\mu J_Z^\mu - W_\mu^+ J_W^{+, \mu} - W_\mu^- J_W^{-, \mu}, \quad (114)$$

其中 f 代表任意费米子场, 电磁流

$$J_{\text{EM}}^\mu \equiv \sum_f Q_f e \bar{f} \gamma^\mu f, \quad (115)$$

弱中性流

$$J_Z^\mu \equiv \frac{g}{2c_W} \sum_f \bar{f} \gamma^\mu (g_V^f - g_A^f \gamma^5) f = \frac{g}{c_W} \sum_f (g_L^f \bar{f}_L \gamma^\mu f_L + g_R^f \bar{f}_R \gamma^\mu f_R), \quad (116)$$

弱带电流

$$J_W^{+, \mu} \equiv \frac{g}{\sqrt{2}} (\bar{u}_i \gamma^\mu V_{ij} P_L d_j + \bar{\nu}_i \gamma^\mu P_L \ell_i), \quad J_W^{-, \mu} \equiv (J_W^{+, \mu})^\dagger = \frac{g}{\sqrt{2}} (\bar{d}_j V_{ji}^\dagger \gamma^\mu P_L u_i + \bar{\ell}_i \gamma^\mu P_L \nu_i). \quad (117)$$

对于各种费米子，相关的系数如下，

$$Q_{u_i} = \frac{2}{3}, \quad Q_{d_i} = -\frac{1}{3}, \quad Q_{\nu_i} = 0, \quad Q_{\ell_i} = -1; \quad (118)$$

$$g_V^{u_i} = \frac{1}{2} - \frac{4}{3}s_W^2, \quad g_A^{u_i} = \frac{1}{2}; \quad g_V^{d_i} = -\frac{1}{2} + \frac{2}{3}s_W^2, \quad g_A^{d_i} = -\frac{1}{2}; \quad (119)$$

$$g_V^{\nu_i} = \frac{1}{2}, \quad g_A^{\nu_i} = \frac{1}{2}; \quad g_V^{\ell_i} = -\frac{1}{2} + 2s_W^2, \quad g_A^{\ell_i} = -\frac{1}{2}; \quad (120)$$

$$g_L^{u_i} = \frac{1}{2} - \frac{2}{3}s_W^2, \quad g_R^{u_i} = -\frac{2}{3}s_W^2; \quad g_L^{d_i} = -\frac{1}{2} + \frac{1}{3}s_W^2, \quad g_R^{d_i} = \frac{1}{3}s_W^2; \quad (121)$$

$$g_L^{\nu_i} = \frac{1}{2}, \quad g_R^{\nu_i} = 0; \quad g_L^{\ell_i} = -\frac{1}{2} + s_W^2, \quad g_R^{\ell_i} = s_W^2. \quad (122)$$

可以看到，电磁流耦合与 QED 耦合完全相同，由此辨认出 A_μ 是电磁场， e 是单位电荷量，由 $Q = T^3 + Y$ 定义的 Q 确实是电荷。为了保持电荷守恒，指定复矢量场 $W_\mu^+(x)$ 携带 $Q = +1$ 的电荷，从而 W^\pm 玻色子的电荷为 ± 1 。不同代夸克之间的相互作用只发生在弱带电流耦合中，源自 CKM 矩阵 V 的非对角元，这是夸克味混合现象。

由于标准模型中没有引入右手中微子场 ν_{iR} ，不能在 (80) 式中写出 $\bar{L}_{iL}\nu_{jR}\tilde{\Phi}$ 形式的相互作用项，因此中微子没有通过 BEH 机制获得质量，从而标准模型里不存在轻子味混合。具体来说，如果将左手中微子场、左手带电轻子场和右手带电轻子场的规范本征态分别记为 ν'_{iL} 、 ℓ'_{iL} 和 ℓ'_{iR} ，将 L_{iL} 改写成

$$L_{iL} = \begin{pmatrix} \nu'_{iL} \\ \ell'_{iL} \end{pmatrix}, \quad (123)$$

而 Yukawa 相互作用项改写为

$$\mathcal{L}_{Y,L\ell\Phi} = -\tilde{y}_{\ell,ij}\bar{L}_{iL}\ell'_{jR}\Phi + \text{H.c.}, \quad (124)$$

那么么正规范给出

$$\mathcal{L}_{Y,L\ell\Phi} \supset -\frac{1}{\sqrt{2}}(v + H)\tilde{y}_{\ell,ij}\bar{\ell}'_{iL}\ell'_{jR} + \text{H.c.} \quad (125)$$

利用么正矩阵 U_ℓ 和 K_ℓ^\dagger 将 Yukawa 耦合矩阵 \tilde{y}_ℓ 表达为 $\tilde{y}_{\ell,ij} = U_\ell y_L K_\ell^\dagger$ ，其中

$$y_L = \begin{pmatrix} y_{\ell_1} & & \\ & y_{\ell_2} & \\ & & y_{\ell_3} \end{pmatrix} = \begin{pmatrix} y_e & & \\ & y_\mu & \\ & & y_\tau \end{pmatrix} \quad (126)$$

是对角矩阵。引入左右手带电轻子场的质量本征态

$$\ell_{iL} \equiv (U_\ell^\dagger)_{ij}\ell'_{jL}, \quad \ell_{iR} \equiv (K_\ell^\dagger)_{ij}\ell'_{jR}, \quad (127)$$

则

$$\tilde{y}_{\ell,ij}\bar{\ell}'_{iL}\ell'_{jR} = \bar{\ell}_{iL}(U_\ell y_L K_\ell^\dagger)_{ij}\ell'_{jR} = y_{\ell_i}\bar{\ell}_{iL}\ell_{iR} \quad (128)$$

给出对角化的带电轻子质量项和 Yukawa 相互作用项, 与 (91) 式一致。此时, 引入左手中微子场的“质量本征态”

$$\nu_{iL} \equiv (U_\ell^\dagger)_{ij} \nu'_{jL}, \quad (129)$$

那么 $\bar{L}_{iL} i \not{D} L_{iL}$ 中的带电流相互作用算符

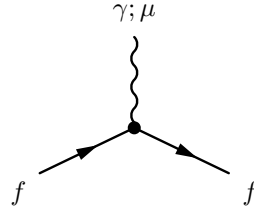
$$W_\mu^+ \bar{\nu}'_{iL} \gamma^\mu \ell'_{iL} = W_\mu^+ \bar{\nu}'_{iL} \gamma^\mu U_{\ell,ij} \ell_{jL} = W_\mu^+ \bar{\nu}_{jL} \gamma^\mu \ell_{jL} = W_\mu^+ \bar{\nu}_i \gamma^\mu P_L \ell_i \quad (130)$$

与 (113) 式中的相应算符是一样的, 没有出现轻子味混合, 在物理上没有任何不同。简单起见, 我们可以让左右手带电轻子场和左手中微子场的质量本征态同时等于它们的规范本征态, 从而之前的讨论都是合理的, (80) 式中只需要让 Yukawa 耦合 y_{ℓ_i} 联系同一代轻子场。

不过, 中微子振荡实验表明中微子具有微小质量, 而且存在味混合, 这是超出标准模型的新物理。仿照夸克味混合的讨论, 需要引入类似于 CKM 矩阵的 Pontecorvo–Maki–Nakagawa–Sakata (PMNS) 矩阵来描述轻子味混合, 但这不在本文的讨论范围之内。

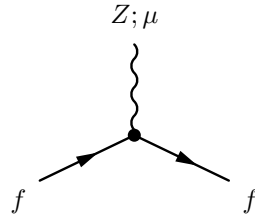
下面给出费米子电弱规范相互作用顶点的 Feynman 规则。

QED 耦合:



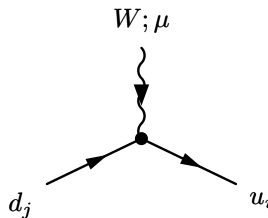
$$= -iQ_f e \gamma^\mu \quad (131)$$

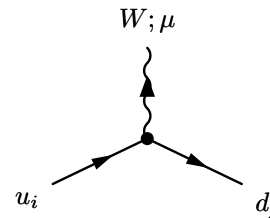
费米子与 Z 玻色子的耦合:



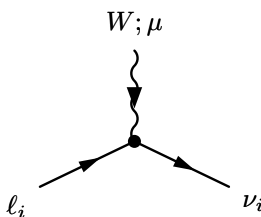
$$= -\frac{ig}{2c_W} \gamma^\mu (g_V^f - g_A^f \gamma^5) \quad (132)$$

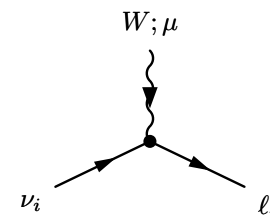
费米子与 W^\pm 玻色子的耦合:



$$= -\frac{ig}{\sqrt{2}} V_{ij} \gamma^\mu P_L$$


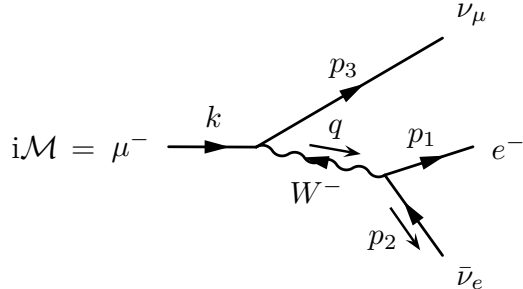
$$= -\frac{ig}{\sqrt{2}} V_{ji}^\dagger \gamma^\mu P_L \quad (133)$$



$$= -\frac{ig}{\sqrt{2}} \gamma^\mu P_L$$


$$= -\frac{ig}{\sqrt{2}} \gamma^\mu P_L \quad (134)$$

考虑 μ 子衰变的最主要过程 $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$ ，相应的领头阶不变振幅为



$$\begin{aligned}
 i\mathcal{M} &= \mu^- \rightarrow \mu^- \rightarrow \nu_\mu + W^- \rightarrow e^- + \bar{\nu}_e \\
 &= \left(\frac{-ig}{\sqrt{2}} \right)^2 \bar{u}(p_3) \gamma^\mu P_L u(k) \frac{-i(g_{\mu\nu} - q_\mu q_\nu / m_W^2)}{q^2 - m_W^2} \bar{u}(p_1) \gamma^\nu P_L v(p_2) \\
 &= \frac{ig^2(g_{\mu\nu} - q_\mu q_\nu / m_W^2)}{8(q^2 - m_W^2)} \bar{u}(p_3) \gamma^\mu (1 - \gamma^5) u(k) \bar{u}(p_1) \gamma^\nu (1 - \gamma^5) v(p_2) \quad (135)
 \end{aligned}$$

由于 $m_\mu \ll m_W$ ， W 传播子的四维动量 q^μ 满足 $q^2 \ll m_W^2$ 。因此，可在低能近似下忽略 q^μ 和 q^2 ，将不变振幅化为

$$i\mathcal{M} \simeq -\frac{ig^2}{8m_W^2} \bar{u}(p_3) \gamma^\mu (1 - \gamma^5) u(k) \bar{u}(p_1) \gamma_\mu (1 - \gamma^5) v(p_2). \quad (136)$$

可以将这样的振幅看作有效拉氏量

$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \bar{\nu}_\mu \gamma^\mu (1 - \gamma^5) \mu \bar{e} \gamma_\mu (1 - \gamma^5) \nu_e + \text{H.c.} \quad (137)$$

的结果，其中 Fermi 常数 G_F 定义为

$$\frac{G_F}{\sqrt{2}} \equiv \frac{g^2}{8m_W^2}.$$

(138)

可以进一步将 \mathcal{L}_{eff} 推广到其它参与弱相互作用的标准模型费米子，而耦合常数 G_F 是普适的，对所有费米子都适用，这样得到的理论称为四费米子相互作用理论。为了解释 β 衰变，Enrico Fermi 于 1933 年首次提出这个理论。现在认为它是标准模型弱相互作用的低能有效理论。

忽略电子质量，由以上振幅推出 μ 子寿命为

$$\tau_\mu = \frac{1}{\Gamma_\mu} \simeq \frac{192\pi^3}{G_F^2 m_\mu^5}. \quad (139)$$

于是，通过测量 μ 子寿命可以得到 Fermi 常数的观测值。根据 (57) 式，Fermi 常数 G_F 与 Higgs 场真空期待值 v 的关系为

$$G_F = \frac{\sqrt{2}g^2}{8m_W^2} = \frac{\sqrt{2}g^2}{2g^2 v^2} = \frac{1}{\sqrt{2}v^2}, \quad v = (\sqrt{2}G_F)^{-1/2}. \quad (140)$$

4.3 电弱规范场的自相互作用

电弱规范场自相互作用的拉氏量是

$$\mathcal{L}_{\text{EWG}} = -\frac{1}{4} W_{\mu\nu}^a W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}, \quad (141)$$

其中

$$W_{\mu\nu}^a \equiv \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - g\varepsilon^{abc} W_\mu^b W_\nu^c, \quad B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu. \quad (142)$$

利用 (68) 和 (72) 式, 推出

$$\begin{aligned} W_\mu^2 W_\nu^3 - W_\mu^3 W_\nu^2 &= \frac{i}{\sqrt{2}} [(W_\mu^+ - W_\mu^-)(s_W A_\nu + c_W Z_\nu) - (s_W A_\mu + c_W Z_\mu)(W_\nu^+ - W_\nu^-)] \\ &= \frac{i}{\sqrt{2}} [s_W(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+) \\ &\quad - s_W(W_\mu^- A_\nu - A_\mu W_\nu^-) - c_W(W_\mu^- Z_\nu - Z_\mu W_\nu^-)], \end{aligned} \quad (143)$$

$$\begin{aligned} W_\mu^3 W_\nu^1 - W_\mu^1 W_\nu^3 &= \frac{1}{\sqrt{2}} [(s_W A_\mu + c_W Z_\mu)(W_\nu^+ + W_\nu^-) - (W_\mu^+ + W_\mu^-)(s_W A_\nu + c_W Z_\nu)] \\ &= -\frac{1}{\sqrt{2}} [s_W(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+) \\ &\quad + s_W(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_W(W_\mu^- Z_\nu - Z_\mu W_\nu^-)]. \end{aligned} \quad (144)$$

从而

$$\begin{aligned} W_{\mu\nu}^1 &= \partial_\mu W_\nu^1 - \partial_\nu W_\mu^1 - g\varepsilon^{1bc} W_\mu^b W_\nu^c = \partial_\mu W_\nu^1 - \partial_\nu W_\mu^1 - g(W_\mu^2 W_\nu^3 - W_\mu^3 W_\nu^2) \\ &= \frac{1}{\sqrt{2}} (\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) + \frac{1}{\sqrt{2}} (\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) - g(W_\mu^2 W_\nu^3 - g W_\mu^3 W_\nu^2) \\ &= \frac{1}{\sqrt{2}} \{ \partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ - ig[s_W(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)] \} \\ &\quad + \frac{1}{\sqrt{2}} \{ \partial_\mu W_\nu^- - \partial_\nu W_\mu^- + ig[s_W(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_W(W_\mu^- Z_\nu - Z_\mu W_\nu^-)] \} \\ &= \frac{1}{\sqrt{2}} (F_{\mu\nu}^+ + F_{\mu\nu}^-), \end{aligned} \quad (145)$$

其中,

$$F_{\mu\nu}^+ \equiv \partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ - ie(W_\mu^+ A_\nu - A_\mu W_\nu^+) - igc_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+), \quad (146)$$

$$F_{\mu\nu}^- \equiv \partial_\mu W_\nu^- - \partial_\nu W_\mu^- + ie(W_\mu^- A_\nu - A_\mu W_\nu^-) + igc_W(W_\mu^- Z_\nu - Z_\mu W_\nu^-). \quad (147)$$

另一方面,

$$\begin{aligned} W_{\mu\nu}^2 &= \partial_\mu W_\nu^2 - \partial_\nu W_\mu^2 - g\varepsilon^{2bc} W_\mu^b W_\nu^c = \partial_\mu W_\nu^2 - \partial_\nu W_\mu^2 - g(W_\mu^3 W_\nu^1 - W_\mu^1 W_\nu^3) \\ &= \frac{i}{\sqrt{2}} (\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) - \frac{i}{\sqrt{2}} (\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) - g(W_\mu^3 W_\nu^1 - W_\mu^1 W_\nu^3) \end{aligned}$$

$$\begin{aligned}
&= \frac{i}{\sqrt{2}} \{ \partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ - ig[s_W(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)] \} \\
&\quad - \frac{i}{\sqrt{2}} \{ \partial_\mu W_\nu^- - \partial_\nu W_\mu^- + ig[s_W(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_W(W_\mu^- Z_\nu - Z_\mu W_\nu^-)] \} \\
&= \frac{i}{\sqrt{2}} (F_{\mu\nu}^+ - F_{\mu\nu}^-).
\end{aligned} \tag{148}$$

因此

$$\begin{aligned}
&-\frac{1}{4} W_{\mu\nu}^1 W^{1,\mu\nu} - \frac{1}{4} W_{\mu\nu}^2 W^{2,\mu\nu} \\
&= -\frac{1}{8} (F_{\mu\nu}^+ + F_{\mu\nu}^-) (F^{+,\mu\nu} + F^{-,\mu\nu}) + \frac{1}{8} (F_{\mu\nu}^+ - F_{\mu\nu}^-) (F^{+,\mu\nu} - F^{-,\mu\nu}) = -\frac{1}{2} F_{\mu\nu}^+ F^{-,\mu\nu} \\
&= -\frac{1}{2} [\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ - ie(W_\mu^+ A_\nu - A_\mu W_\nu^+) - igc_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)] \\
&\quad \times [\partial^\mu W^{-,\nu} - \partial^\nu W^{-,\mu} + ie(W^{-,\mu} A^\nu - A^\mu W^{-,\nu}) + igc_W(W^{-,\mu} Z^\nu - Z^\mu W^{-,\nu})] \\
&= -(\partial_\mu W_\nu^+) (\partial^\mu W^{-,\nu}) + (\partial_\mu W_\nu^+) (\partial^\nu W^{-,\mu}) \\
&\quad - ie[(\partial_\mu W_\nu^+) W^{-,\mu} A^\nu - (\partial_\mu W_\nu^+) W^{-,\nu} A^\mu - W_\mu^+ (\partial^\mu W^{-,\nu}) A_\nu + W_\nu^+ (\partial^\mu W^{-,\nu}) A_\mu] \\
&\quad - igc_W[(\partial_\mu W_\nu^+) W^{-,\mu} Z^\nu - (\partial_\mu W_\nu^+) W^{-,\nu} Z^\mu - W_\mu^+ (\partial^\mu W^{-,\nu}) Z_\nu + W_\nu^+ (\partial^\mu W^{-,\nu}) Z_\mu] \\
&\quad + e^2 (W_\mu^+ W^{-,\nu} A_\nu A^\mu - W_\mu^+ W^{-,\mu} A_\nu A^\nu) + g^2 c_W^2 (W_\mu^+ W^{-,\nu} Z_\nu Z^\mu - W_\mu^+ W^{-,\mu} Z_\nu Z^\nu) \\
&\quad + egc_W (W_\mu^+ W^{-,\nu} A_\nu Z^\mu + W_\mu^+ W^{-,\nu} A^\mu Z_\nu - 2W_\mu^+ W^{-,\mu} A_\nu Z^\nu).
\end{aligned} \tag{149}$$

由

$$\begin{aligned}
W_\mu^1 W_\nu^2 - W_\mu^2 W_\nu^1 &= \frac{i}{2} (W_\mu^+ + W_\mu^-) (W_\nu^+ - W_\nu^-) - \frac{i}{2} (W_\mu^+ - W_\mu^-) (W_\nu^+ + W_\nu^-) \\
&= -i(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+),
\end{aligned} \tag{150}$$

得到

$$\begin{aligned}
W_{\mu\nu}^3 &= \partial_\mu W_\nu^3 - \partial_\nu W_\mu^3 - g\epsilon^{3bc} W_\mu^b W_\nu^c = \partial_\mu W_\nu^3 - \partial_\nu W_\mu^3 - g(W_\mu^1 W_\nu^2 - W_\mu^2 W_\nu^1) \\
&= s_W \partial_\mu A_\nu - c_W \partial_\mu Z_\nu - s_W \partial_\nu A_\mu - c_W \partial_\nu Z_\mu - g(W_\mu^1 W_\nu^2 - W_\mu^2 W_\nu^1) \\
&= s_W (\partial_\mu A_\nu - \partial_\nu A_\mu) + c_W (\partial_\mu Z_\nu - \partial_\nu Z_\mu) + ig(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+),
\end{aligned} \tag{151}$$

$$\begin{aligned}
B_{\mu\nu} &= \partial_\mu (c_W A_\nu - s_W Z_\nu) - \partial_\nu (c_W A_\mu - s_W Z_\mu) \\
&= c_W (\partial_\mu A_\nu - \partial_\nu A_\mu) - s_W (\partial_\mu Z_\nu - \partial_\nu Z_\mu).
\end{aligned} \tag{152}$$

于是

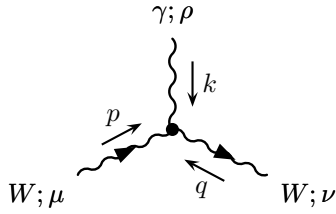
$$\begin{aligned}
&-\frac{1}{4} W_{\mu\nu}^3 W^{3,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\
&= -\frac{1}{2} [(\partial_\mu A_\nu)(\partial^\mu A^\nu) - (\partial_\mu A_\nu)(\partial^\nu A^\mu)] - \frac{1}{2} [(\partial_\mu Z_\nu)(\partial^\mu Z^\nu) - (\partial_\mu Z_\nu)(\partial^\nu Z^\mu)] \\
&\quad - ie[W^{+,\mu} W^{-,\nu} (\partial_\mu A_\nu) - W^{+,\nu} W^{-,\mu} (\partial_\mu A_\nu)] - igc_W[W^{+,\mu} W^{-,\nu} (\partial_\mu Z_\nu) - W^{+,\nu} W^{-,\mu} (\partial_\mu Z_\nu)]
\end{aligned}$$

$$+\frac{g^2}{2}(W_\mu^+W^{+,\mu}W_\nu^-W^{-,\nu}-W_\mu^+W^{+,\nu}W_\nu^-W^{-,\mu}). \quad (153)$$

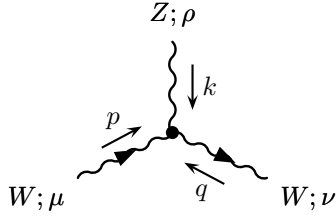
综合起来, 有

$$\begin{aligned} \mathcal{L}_{\text{EWG}} = & \frac{1}{2}[(\partial_\mu A_\nu)(\partial^\nu A^\mu) - (\partial_\mu A_\nu)(\partial^\mu A^\nu)] + \frac{1}{2}[(\partial_\mu Z_\nu)(\partial^\nu Z^\mu) - (\partial_\mu Z_\nu)(\partial^\mu Z^\nu)] \\ & + (\partial_\mu W_\nu^+)(\partial^\nu W^{-,\mu}) - (\partial_\mu W_\nu^+)(\partial^\mu W^{-,\nu}) + \frac{g^2}{2}(W_\mu^+W^{+,\mu}W_\nu^-W^{-,\nu} - W_\mu^+W^{+,\nu}W_\nu^-W^{-,\mu}) \\ & - ie[(\partial_\mu W_\nu^+)W^{-,\mu}A^\nu - (\partial_\mu W_\nu^+)W^{-,\nu}A^\mu - W^{+,\mu}(\partial_\mu W_\nu^-)A^\nu + W^{+,\nu}(\partial_\mu W_\nu^-)A^\mu \\ & \quad + W^{+,\mu}W^{-,\nu}(\partial_\mu A_\nu) - W^{+,\nu}W^{-,\mu}(\partial_\mu A_\nu)] \\ & - igc_W[(\partial_\mu W_\nu^+)W^{-,\mu}Z^\nu - (\partial_\mu W_\nu^+)W^{-,\nu}Z^\mu - W^{+,\mu}(\partial_\mu W_\nu^-)Z^\nu + W^{+,\nu}(\partial_\mu W_\nu^-)Z^\mu \\ & \quad + W^{+,\mu}W^{-,\nu}(\partial_\mu Z_\nu) - W^{+,\nu}W^{-,\mu}(\partial_\mu Z_\nu)] \\ & + e^2(W_\mu^+W^{-,\nu}A_\nu A^\mu - W_\mu^+W^{-,\mu}A_\nu A^\nu) + g^2c_W^2(W_\mu^+W^{-,\nu}Z_\nu Z^\mu - W_\mu^+W^{-,\mu}Z_\nu Z^\nu) \\ & + egc_W(W_\mu^+W^{-,\nu}A_\nu Z^\mu + W_\mu^+W^{-,\nu}A^\mu Z_\nu - 2W_\mu^+W^{-,\mu}A_\nu Z^\nu). \end{aligned} \quad (154)$$

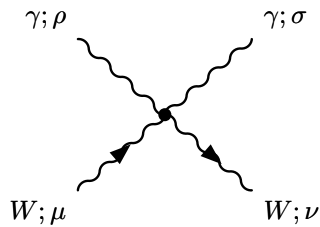
下面是电弱规范玻色子自耦合的 Feynman 规则:



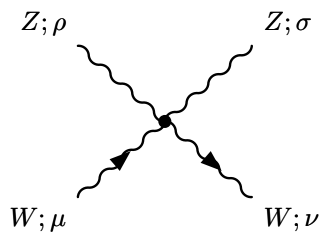
$$= ie[g^{\mu\nu}(p-q)^\rho + g^{\nu\rho}(q-k)^\mu + g^{\rho\mu}(k-p)^\nu] \quad (155)$$




$$= igc_W[g^{\mu\nu}(p-q)^\rho + g^{\nu\rho}(q-k)^\mu + g^{\rho\mu}(k-p)^\nu] \quad (156)$$



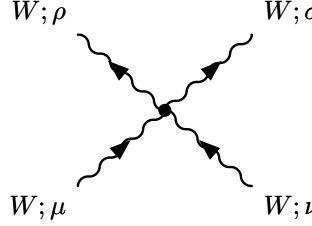
$$= ie^2(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma}) \quad (157)$$



$$= ig^2c_W^2(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma}) \quad (158)$$



$$= i e g c_W (g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho} - 2 g^{\mu\nu} g^{\rho\sigma}) \quad (159)$$



$$= -i g^2 (g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho} - 2 g^{\mu\nu} g^{\rho\sigma}) \quad (160)$$

5 R_ξ 规范下电弱拉氏量和 Feynman 规则

在 R_ξ 规范下, 将 Higgs 场参数化为

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}[v + H(x) + i\chi(x)] \end{pmatrix}, \quad (161)$$

其中 ϕ^+ 和 χ 是 Nambu-Goldstone 标量场。那么, $\tilde{\Phi}(x)$ 的形式是

$$\tilde{\Phi}(x) = \begin{pmatrix} \phi^{0*}(x) \\ -\phi^-(x) \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}[v + H(x) - i\chi(x)] \\ -\phi^-(x) \end{pmatrix}. \quad (162)$$

利用

$$\Phi^\dagger \Phi = \frac{1}{2}(v^2 + H^2 + 2vH + \chi^2) + |\phi^+|^2, \quad (163)$$

$$(\Phi^\dagger \Phi)^2 = \frac{1}{4}(v^2 + H^2 + 2vH + \chi^2)^2 + |\phi^+|^4 + |\phi^+|^2(v^2 + H^2 + 2vH + \chi^2), \quad (164)$$

推出 Higgs 场势能项

$$\begin{aligned} -V_H(\Phi) &= \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 \\ &= \frac{\mu^2}{2}(v^2 + H^2 + 2vH + \chi^2) + \mu^2 |\phi^+|^2 - \frac{\lambda}{4}(v^2 + H^2 + 2vH + \chi^2)^2 - \lambda |\phi^+|^4 \\ &\quad - \lambda |\phi^+|^2(v^2 + H^2 + 2vH + \chi^2) \\ &= \frac{1}{2} \left(\mu^2 - \frac{\lambda}{2} v^2 \right) v^2 + \frac{1}{2}(\mu^2 - 3\lambda v^2) H^2 + (\mu^2 - \lambda v^2) v H + \frac{1}{2}(\mu^2 - \lambda v^2) \chi^2 - \frac{\lambda}{4} H^4 - \frac{\lambda}{4} \chi^4 \\ &\quad - \lambda v H^3 - \frac{\lambda}{2} H^2 \chi^2 - \lambda v H \chi^2 + (\mu^2 - \lambda v^2) |\phi^+|^2 - \lambda |\phi^+|^4 - \lambda |\phi^+|^2 (H^2 + 2vH + \chi^2) \\ &= \frac{\lambda}{4} v^4 - \lambda v^2 H^2 - \frac{\lambda}{4} H^4 - \frac{\lambda}{4} \chi^4 - \lambda v H^3 - \frac{\lambda}{2} H^2 \chi^2 - \lambda v H \chi^2 \end{aligned}$$

$$\begin{aligned}
& -\lambda\phi^+\phi^-(\phi^+\phi^- + H^2 + 2vH + \chi^2) \\
& = \frac{1}{8}m_H^2v^2 - \frac{1}{2}m_H^2H^2 - \frac{m_H^2}{2v}H^3 - \frac{m_H^2}{8v^2}H^4 - \frac{m_H^2}{2v}H\chi^2 - \frac{m_H^2}{4v^2}H^2\chi^2 - \frac{m_H^2}{8v^2}\chi^4 \\
& \quad - \frac{m_H^2}{2v^2}\phi^+\phi^-(\phi^+\phi^- + H^2 + 2vH + \chi^2).
\end{aligned} \tag{165}$$

由 $V = U_u^\dagger U_d$ 得到 $V^\dagger = U_d^\dagger U_u$ 和 $U_d = U_u V$, $U_u = U_d V^\dagger$, 则

$$\tilde{y}_d = U_d y_D K_d^\dagger = U_u V y_D K_d^\dagger, \quad \tilde{y}_u = U_u y_U K_u^\dagger = U_d V^\dagger y_U K_u^\dagger, \tag{166}$$

故

$$\tilde{y}_{d,ij} \bar{u}'_{iL} d'_{jR} = \bar{u}'_{iL} (U_u V y_D K_d^\dagger)_{ij} d'_{jR} = \bar{u}'_{iL} U_{u,ik} V_{kl} y_{d_l} (K_d^\dagger)_{lj} d'_{jR} = y_{d_j} \bar{u}_{iL} V_{ij} d_{jR}, \tag{167}$$

$$\tilde{y}_{u,ij} \bar{d}'_{iL} u'_{jR} = \bar{d}'_{iL} (U_d V^\dagger y_U K_u^\dagger)_{ij} u'_{jR} = \bar{d}'_{iL} U_{d,ik} V_{kl}^\dagger y_{u_l} (K_u^\dagger)_{lj} u'_{jR} = y_{u_i} \bar{d}_{jL} V_{ji}^\dagger u_{iR}. \tag{168}$$

结合 (89) 和 (90) 式, 得

$$\begin{aligned}
\tilde{y}_{d,ij} \bar{Q}_{iL} d'_{jR} \Phi & = \tilde{y}_{d,ij} \left[\bar{u}'_{iL} d'_{jR} \phi^+ + \frac{1}{\sqrt{2}} \bar{d}'_{iL} d'_{jR} (v + H + i\chi) \right] \\
& = y_{d_j} \bar{u}_{iL} V_{ij} d_{jR} \phi^+ + \frac{y_{d_i}}{\sqrt{2}} \bar{d}_{iL} d_{iR} (v + H + i\chi),
\end{aligned} \tag{169}$$

$$\begin{aligned}
\tilde{y}_{u,ij} \bar{Q}_{iL} u'_{jR} \tilde{\Phi} & = \tilde{y}_{u,ij} \left[\frac{1}{\sqrt{2}} \bar{u}'_{iL} u'_{jR} (v + H - i\chi) - \bar{d}'_{iL} u'_{jR} \phi^- \right] \\
& = \frac{y_{u_i}}{\sqrt{2}} \bar{u}_{iL} u_{iR} (v + H - i\chi) - y_{u_i} \bar{d}_{jL} V_{ji}^\dagger u_{iR} \phi^-.
\end{aligned} \tag{170}$$

从而, Yukawa 相互作用拉氏量化为

$$\begin{aligned}
\mathcal{L}_Y & = -\tilde{y}_{d,ij} \bar{Q}_{iL} d'_{jR} \Phi - \tilde{y}_{u,ij} \bar{Q}_{iL} u'_{jR} \tilde{\Phi} - y_{\ell_i} \bar{L}_{iL} \ell_{iR} \Phi + \text{H.c.} \\
& = -y_{d_j} \bar{u}_{iL} V_{ij} d_{jR} \phi^+ - \frac{y_{d_i}}{\sqrt{2}} \bar{d}_{iL} d_{iR} (v + H + i\chi) - \frac{y_{u_i}}{\sqrt{2}} \bar{u}_{iL} u_{iR} (v + H - i\chi) + y_{u_i} \bar{d}_{jL} V_{ji}^\dagger u_{iR} \phi^- \\
& \quad - y_{\ell_i} \bar{\nu}_{iL} \ell_{iR} \phi^+ - \frac{y_{\ell_i}}{\sqrt{2}} \bar{\ell}_{iL} \ell_{iR} (v + H + i\chi) + \text{H.c.} \\
& = -m_{d_i} \bar{d}_{iL} d_{iR} - m_{u_i} \bar{u}_{iL} u_{iR} - m_{\ell_i} \bar{\ell}_{iL} \ell_{iR} - \frac{m_{d_i}}{v} \bar{d}_{iL} d_{iR} (H + i\chi) - \frac{m_{u_i}}{v} \bar{u}_{iL} u_{iR} (H - i\chi) \\
& \quad - \frac{m_{\ell_i}}{v} \bar{\ell}_{iL} \ell_{iR} (H + i\chi) - \frac{\sqrt{2}m_{d_j}}{v} \bar{u}_{iL} V_{ij} d_{jR} \phi^+ + \frac{\sqrt{2}m_{u_i}}{v} \bar{d}_{jL} V_{ji}^\dagger u_{iR} \phi^- - \frac{\sqrt{2}m_{\ell_i}}{v} \bar{\nu}_{iL} \ell_{iR} \phi^+ + \text{H.c.} \\
& = -m_{d_i} \bar{d}_i d_i - m_{u_i} \bar{u}_i u_i - m_{\ell_i} \bar{\ell}_i \ell_i - \frac{m_{d_i}}{v} H \bar{d}_i d_i - \frac{m_{u_i}}{v} H \bar{u}_i u_i - \frac{m_{\ell_i}}{v} H \bar{\ell}_i \ell_i \\
& \quad - \frac{m_{d_i}}{v} \chi \bar{d}_i i\gamma^5 d_i + \frac{m_{u_i}}{v} \chi \bar{u}_i i\gamma^5 u_i - \frac{m_{\ell_i}}{v} \chi \bar{\ell}_i i\gamma^5 \ell_i + \frac{\sqrt{2}V_{ij}}{v} \phi^+ \bar{u}_i (m_{u_i} P_L - m_{d_j} P_R) d_j \\
& \quad - \frac{\sqrt{2}V_{ji}^\dagger}{v} \phi^- \bar{d}_j (m_{d_j} P_L - m_{u_i} P_R) u_i - \frac{\sqrt{2}m_{\ell_i}}{v} (\phi^+ \bar{\nu}_i P_R \ell_i + \phi^- \bar{\ell}_i P_L \nu_i).
\end{aligned} \tag{171}$$

利用

$$\begin{aligned}
 D_\mu \Phi &= \begin{pmatrix} \partial_\mu + ieA_\mu + \frac{ig}{2c_W}(c_W^2 - s_W^2)Z_\mu & \frac{ig}{\sqrt{2}}W_\mu^+ \\ \frac{ig}{\sqrt{2}}W_\mu^- & \partial_\mu - \frac{ig}{2c_W}Z_\mu \end{pmatrix} \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\chi) \end{pmatrix} \\
 &= \begin{pmatrix} \partial_\mu \phi^+ + i \left[eA_\mu + \frac{g(c_W^2 - s_W^2)}{2c_W} Z_\mu \right] \phi^+ + \frac{ig}{2} W_\mu^+ (H + i\chi) + im_W W_\mu^+ \\ \frac{1}{\sqrt{2}} \left[\partial_\mu (H + i\chi) + igW_\mu^- \phi^+ - \frac{ig}{2c_W} Z_\mu (H + i\chi) - im_Z Z_\mu \right] \end{pmatrix}, \quad (172)
 \end{aligned}$$

将 Higgs 场协变动能项化为

$$\begin{aligned}
 &(D^\mu \Phi)^\dagger D_\mu \Phi \\
 &= \left| \partial_\mu \phi^+ + i \left[eA_\mu + \frac{g(c_W^2 - s_W^2)}{2c_W} Z_\mu \right] \phi^+ + \frac{ig}{2} W_\mu^+ (H + i\chi) + im_W W_\mu^+ \right|^2 \\
 &\quad + \frac{1}{2} \left| \partial_\mu (H + i\chi) + igW_\mu^- \phi^+ - \frac{ig}{2c_W} Z_\mu (H + i\chi) - im_Z Z_\mu \right|^2 \\
 &= (\partial^\mu \phi^+) (\partial_\mu \phi^-) + \frac{1}{2} (\partial^\mu H) (\partial_\mu H) + \frac{1}{2} (\partial^\mu \chi) (\partial_\mu \chi) \\
 &\quad + \left(i \partial^\mu \phi^- \left\{ \left[eA_\mu + \frac{g(c_W^2 - s_W^2)}{2c_W} Z_\mu \right] \phi^+ + \frac{g}{2} W_\mu^+ (H + i\chi) + m_W W_\mu^+ \right\} + \text{H.c.} \right) \\
 &\quad + \left\{ \frac{i}{2} \partial^\mu (H - i\chi) \left[gW_\mu^- \phi^+ - \frac{g}{2c_W} Z_\mu (H + i\chi) - m_Z Z_\mu \right] + \text{H.c.} \right\} \\
 &\quad + \left| \left[eA_\mu + \frac{g(c_W^2 - s_W^2)}{2c_W} Z_\mu \right] \phi^+ + \frac{g}{2} W_\mu^+ (H + i\chi) + m_W W_\mu^+ \right|^2 \\
 &\quad + \frac{1}{2} \left| gW_\mu^- \phi^+ - \frac{g}{2c_W} Z_\mu (H + i\chi) - m_Z Z_\mu \right|^2 \\
 &= (\partial^\mu \phi^+) (\partial_\mu \phi^-) + \frac{1}{2} (\partial^\mu H) (\partial_\mu H) + \frac{1}{2} (\partial^\mu \chi) (\partial_\mu \chi) \\
 &\quad + m_W^2 W_\mu^+ W^{-,\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu + gm_W H W_\mu^+ W^{-,\mu} + \frac{gm_Z}{2c_W} H Z_\mu Z^\mu \\
 &\quad - \frac{g}{2} [iW_\mu^+ \phi^- \overleftrightarrow{\partial}^\mu (H + i\chi) + \text{H.c.}] - ieA_\mu \phi^- \overleftrightarrow{\partial}^\mu \phi^+ - \frac{g}{2c_W} Z_\mu [-\chi \overleftrightarrow{\partial}^\mu H + i(c_W^2 - s_W^2) \phi^- \overleftrightarrow{\partial}^\mu \phi^+] \\
 &\quad + \frac{g^2}{4} W_\mu^+ W^{-,\mu} (2\phi^+ \phi^- + H^2 + \chi^2) + e^2 A_\mu A^\mu \phi^+ \phi^- \\
 &\quad + \frac{g^2}{4c_W^2} Z_\mu Z^\mu \left[(c_W^2 - s_W^2)^2 \phi^+ \phi^- + \frac{1}{2} H^2 + \frac{1}{2} \chi^2 \right] \\
 &\quad + \left[\frac{eg}{2} W_\mu^+ A^\mu \phi^- (H + i\chi) - \frac{g^2 s_W^2}{2c_W} W_\mu^+ Z^\mu \phi^- (H + i\chi) + \text{H.c.} \right] + \frac{eg(c_W^2 - s_W^2)}{c_W} A_\mu Z^\mu \phi^+ \phi^- \\
 &\quad + (em_W A^\mu \phi^+ W_\mu^- - gs_W^2 m_Z Z^\mu \phi^+ W_\mu^- + \text{H.c.}) + \mathcal{L}_{\text{b1}}, \quad (173)
 \end{aligned}$$

其中

$$\mathcal{L}_{b1} = im_W(\partial^\mu \phi^-)W_\mu^+ - im_W(\partial^\mu \phi^+)W_\mu^- - m_Z(\partial^\mu \chi)Z_\mu. \quad (174)$$

在 R_ξ 规范下, 将规范固定函数设为

$$G^\pm = \frac{1}{\sqrt{\xi_W}}(\partial^\mu W_\mu^\pm \pm i\xi_W m_W \phi^\pm), \quad G^Z = \frac{1}{\sqrt{\xi_Z}}(\partial^\mu Z_\mu + \xi_Z m_Z \chi), \quad G^\gamma = \frac{1}{\sqrt{\xi_\gamma}} \partial^\mu A_\mu, \quad (175)$$

它们在路径积分量子化中的泛函积分形式为

$$\begin{aligned} & \int \mathcal{D}\omega^+ \int \mathcal{D}\omega^- \int \mathcal{D}\omega^Z \int \mathcal{D}\omega^\gamma \exp \left[-i \int d^4x \left(\omega^+ \omega^- + \frac{1}{2} \omega^Z \omega^Z + \frac{1}{2} \omega^\gamma \omega^\gamma \right) \right] \\ & \quad \times \delta(G^+ - \omega^+) \delta(G^- - \omega^-) \delta(G^Z - \omega^Z) \delta(G^\gamma - \omega^\gamma) \\ & = \exp \left[-i \int d^4x \left(G^+ G^- + \frac{1}{2} G^Z G^Z + \frac{1}{2} G^\gamma G^\gamma \right) \right]. \end{aligned} \quad (176)$$

由此得到拉氏量中的规范固定项

$$\begin{aligned} \mathcal{L}_{EW,GF} &= -G^+ G^- - \frac{1}{2}(G^Z)^2 - \frac{1}{2}(G^\gamma)^2 \\ &= -\frac{1}{\xi_W}(\partial^\mu W_\mu^+ + i\xi_W m_W \phi^+)(\partial^\nu W_\nu^- - i\xi_W m_W \phi^-) - \frac{1}{2\xi_Z}(\partial^\mu Z_\mu + \xi_Z m_Z \chi)^2 - \frac{1}{2\xi_\gamma}(\partial^\mu A_\mu)^2 \\ &= -\frac{1}{\xi_W}(\partial^\mu W_\mu^+)(\partial^\nu W_\nu^-) - \frac{1}{2\xi_Z}(\partial^\mu Z_\mu)^2 - \frac{1}{2\xi_\gamma}(\partial^\mu A_\mu)^2 - \xi_W m_W^2 \phi^+ \phi^- - \frac{1}{2} \xi_Z m_Z^2 \chi^2 + \mathcal{L}_{b2}. \end{aligned} \quad (177)$$

可见, Nambu-Goldstone 玻色子 ϕ^\pm 和 χ 在 R_ξ 规范下具有依赖于 ξ_W 和 ξ_Z 的非物理质量,

$$m_\phi = \sqrt{\xi_W} m_W, \quad m_\chi = \sqrt{\xi_Z} m_Z. \quad (178)$$

这里

$$\mathcal{L}_{b2} = im_W \phi^- (\partial^\mu W_\mu^+) - im_W \phi^+ \partial^\mu W_\mu^- - m_Z \chi \partial^\mu Z_\mu. \quad (179)$$

\mathcal{L}_{b1} 与 \mathcal{L}_{b2} 之和

$$\mathcal{L}_{b1} + \mathcal{L}_{b2} = im_W \partial^\mu (\phi^- W_\mu^+) - im_W \partial^\mu (\phi^+ W_\mu^-) - m_Z \partial^\mu (\chi Z_\mu) \quad (180)$$

是全散度, 不会有物理效应。可见, 协变动能项中规范场与 Nambu-Goldstone 标量场之间的双线性耦合项 \mathcal{L}_{b1} 被规范固定项中的 \mathcal{L}_{b2} 抵消掉, 这就是如此选取规范固定函数的目的。

这样一来, 电弱规范场传播子相关拉氏量变成

$$\begin{aligned} \mathcal{L}_{EW,prop} &= (\partial_\mu W_\nu^+)(\partial^\nu W^{-,\mu}) - (\partial_\mu W_\nu^+)(\partial^\mu W^{-,\nu}) - \frac{1}{\xi_W}(\partial^\mu W_\mu^+)(\partial^\nu W_\nu^-) + m_W^2 W_\mu^+ W^{-,\mu} \\ & \quad + \frac{1}{2} \left[(\partial_\mu Z_\nu)(\partial^\nu Z^\mu) - (\partial_\mu Z_\nu)(\partial^\mu Z^\nu) - \frac{1}{\xi_Z}(\partial^\mu Z_\mu)^2 + m_Z^2 Z_\mu Z^\mu \right] \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2} \left[(\partial_\mu A_\nu)(\partial^\nu A^\mu) - (\partial_\mu A_\nu)(\partial^\mu A^\nu) - \frac{1}{\xi_\gamma} (\partial^\mu A_\mu)^2 \right] \\
& \rightarrow W_\mu^+ \left[g^{\mu\nu} (\partial^2 + m_W^2) - \left(1 - \frac{1}{\xi_W} \right) \partial^\mu \partial^\nu \right] W_\nu^- \\
& + \frac{1}{2} Z_\mu \left[g^{\mu\nu} (\partial^2 + m_Z^2) - \left(1 - \frac{1}{\xi_Z} \right) \partial^\mu \partial^\nu \right] Z_\nu \\
& + \frac{1}{2} A_\mu \left[g^{\mu\nu} \partial^2 - \left(1 - \frac{1}{\xi_\gamma} \right) \partial^\mu \partial^\nu \right] A_\nu.
\end{aligned} \tag{181}$$

于是, 光子的传播子与胶子形式类似, 为

$$\frac{-i}{p^2 + i\epsilon} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} (1 - \xi_\gamma) \right]. \tag{182}$$

将 W^\pm 传播子相关拉氏量变换到动量空间, 得

$$-g^{\mu\nu} (p^2 - m_W^2) + \left(1 - \frac{1}{\xi_W} \right) p^\mu p^\nu = - \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) (p^2 - m_W^2) - \frac{p^\mu p^\nu}{p^2} \frac{p^2 - \xi_W m_W^2}{\xi_W}, \tag{183}$$

它的逆矩阵是

$$-\frac{1}{p^2 - m_W^2} \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) - \frac{\xi_W}{p^2 - \xi_W m_W^2} \frac{p_\mu p_\nu}{p^2} = -\frac{1}{p^2 - m_W^2} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_W m_W^2} (1 - \xi_W) \right], \tag{184}$$

这是因为由

$$\left(g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2} \right) \frac{p^\mu p^\nu}{p^2} = \frac{p_\rho p^\nu}{p^2} - \frac{p_\rho p^\nu}{p^2} = 0, \quad \left(g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2} \right) \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) = \delta_\rho^\nu - \frac{p_\rho p^\nu}{p^2} \tag{185}$$

得

$$\begin{aligned}
& \left[-\frac{1}{p^2 - m_W^2} \left(g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2} \right) - \frac{\xi_W}{p^2 - \xi_W m_W^2} \frac{p_\rho p_\mu}{p^2} \right] \\
& \times \left[- \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) (p^2 - m_W^2) - \frac{p^\mu p^\nu}{p^2} \frac{p^2 - \xi_W m_W^2}{\xi_W} \right] \\
& = \delta_\rho^\nu - \frac{p_\rho p^\nu}{p^2} + \frac{p_\rho p^\nu}{p^2} = \delta_\rho^\nu.
\end{aligned} \tag{186}$$

从而, W^\pm 传播子的形式为

$$\frac{-i}{p^2 - m_W^2 + i\epsilon} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_W m_W^2} (1 - \xi_W) \right]. \tag{187}$$

同理, Z 传播子的形式为

$$\frac{-i}{p^2 - m_Z^2 + i\epsilon} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_Z m_Z^2} (1 - \xi_Z) \right]. \tag{188}$$

电弱规范场的无穷小规范变换形式是

$$\delta W_\mu^a = -\frac{1}{g} \partial_\mu \alpha^a + \varepsilon^{abc} W_\mu^b \alpha^c, \quad \delta B_\mu = -\frac{1}{g'} \partial_\mu \alpha^Y. \quad (189)$$

定义

$$\alpha^\pm \equiv \frac{1}{\sqrt{2}}(\alpha^1 \mp i\alpha^2), \quad \alpha^Z \equiv \alpha^3 - \alpha^Y, \quad \alpha^\gamma \equiv s_W^2 \alpha^3 + c_W^2 \alpha^Y, \quad (190)$$

利用

$$\varepsilon^{1bc} W_\mu^b \alpha^c = W_\mu^2 \alpha^3 - W_\mu^3 \alpha^2, \quad \varepsilon^{2bc} W_\mu^b \alpha^c = -W_\mu^1 \alpha^3 + W_\mu^3 \alpha^1, \quad (191)$$

$$\pm i\sqrt{2} \alpha^\pm = \pm i\alpha^1 + \alpha^2, \quad \pm i\sqrt{2} W_\mu^\pm = \pm iW_\mu^1 + W_\mu^2, \quad (192)$$

有

$$\begin{aligned} \varepsilon^{1bc} W_\mu^b \alpha^c \mp i\varepsilon^{2bc} W_\mu^b \alpha^c &= (W_\mu^2 \alpha^3 - W_\mu^3 \alpha^2) \mp i(-W_\mu^1 \alpha^3 + W_\mu^3 \alpha^1) \\ &= (W_\mu^2 \pm iW_\mu^1) \alpha^3 - W_\mu^3 (\alpha^2 \pm i\alpha^1) \\ &= \pm i\sqrt{2} W_\mu^\pm (c_W^2 \alpha^Z + \alpha^\gamma) \mp i\sqrt{2} (s_W A_\mu + c_W Z_\mu) \alpha^\pm, \end{aligned} \quad (193)$$

$$\begin{aligned} \varepsilon^{3bc} W_\mu^b \alpha^c &= W_\mu^1 \alpha^2 - W_\mu^2 \alpha^1 = \frac{1}{\sqrt{2}} (W_\mu^+ + W_\mu^-) \frac{i}{\sqrt{2}} (\alpha^+ - \alpha^-) - \frac{i}{\sqrt{2}} (W_\mu^+ - W_\mu^-) \frac{1}{\sqrt{2}} (\alpha^+ + \alpha^-) \\ &= -i(W_\mu^+ \alpha^- - W_\mu^- \alpha^+). \end{aligned} \quad (194)$$

因此,

$$\begin{aligned} \delta W_\mu^+ &= \frac{1}{\sqrt{2}} (\delta W_\mu^1 - i\delta W_\mu^2) = -\frac{1}{\sqrt{2}g} \partial_\mu (\alpha^1 - i\alpha^2) + \frac{1}{\sqrt{2}} (\varepsilon^{1bc} W_\mu^b \alpha^c - i\varepsilon^{2bc} W_\mu^b \alpha^c) \\ &= -\frac{1}{g} \partial_\mu \alpha^+ - i(s_W A_\mu + c_W Z_\mu) \alpha^+ + iW_\mu^+ (c_W^2 \alpha^Z + \alpha^\gamma), \end{aligned} \quad (195)$$

$$\delta W_\mu^- = (\delta W_\mu^+)^{\dagger} = -\frac{1}{g} \partial_\mu \alpha^- + i(s_W A_\mu + c_W Z_\mu) \alpha^- - iW_\mu^- (c_W^2 \alpha^Z + \alpha^\gamma), \quad (196)$$

$$\begin{aligned} \delta Z_\mu^a &= c_W \delta W_\mu^3 - s_W \delta B_\mu = -\frac{c_W}{g} \partial_\mu \alpha^3 + c_W \varepsilon^{3bc} W_\mu^b \alpha^c + \frac{s_W}{g'} \partial_\mu \alpha^Y \\ &= -\frac{c_W}{g} \partial_\mu \alpha^Z - ic_W (W_\mu^+ \alpha^- - W_\mu^- \alpha^+), \end{aligned} \quad (197)$$

$$\begin{aligned} \delta A_\mu &= s_W \delta W_\mu^3 + c_W \delta B_\mu = -\frac{s_W}{g} \partial_\mu \alpha^3 + s_W \varepsilon^{3bc} W_\mu^b \alpha^c - \frac{c_W}{g'} \partial_\mu \alpha^Y \\ &= -\frac{1}{e} \partial_\mu \alpha^\gamma - is_W (W_\mu^+ \alpha^- - W_\mu^- \alpha^+). \end{aligned} \quad (198)$$

另一方面, 根据

$$\alpha^a T^a + \alpha^Y Y_H = \frac{1}{2} (\alpha^a \sigma^a + \alpha^Y) = \frac{1}{2} \begin{pmatrix} \alpha^3 + \alpha^Y & \alpha^1 - i\alpha^2 \\ \alpha^1 + i\alpha^2 & -\alpha^3 + \alpha^Y \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} 2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z & \sqrt{2}\alpha^+ \\ \sqrt{2}\alpha^- & -\alpha^Z \end{pmatrix}, \quad (199)$$

可知 Higgs 场的无穷小规范变换形式为

$$\begin{aligned} \delta\Phi &= i(\alpha^a T^a + \alpha^Y Y_H)\Phi = \frac{i}{2} \begin{pmatrix} 2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z & \sqrt{2}\alpha^+ \\ \sqrt{2}\alpha^- & -\alpha^Z \end{pmatrix} \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\chi) \end{pmatrix} \\ &= \begin{pmatrix} \frac{i}{2}\{\phi^+[2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z] + (v + H + i\chi)\alpha^+\} \\ \frac{1}{\sqrt{2}}\left[i\phi^+\alpha^- - \frac{1}{2}(iv + iH - \chi)\alpha^Z\right] \end{pmatrix} = \begin{pmatrix} \delta\phi^+ \\ \frac{1}{\sqrt{2}}(\delta H + i\delta\chi) \end{pmatrix}. \end{aligned} \quad (200)$$

利用

$$\text{Re}(\phi^+\alpha^-) = \frac{1}{2}(\phi^+\alpha^- + \phi^-\alpha^+), \quad \text{Im}(\phi^+\alpha^-) = -\frac{i}{2}(\phi^+\alpha^- - \phi^-\alpha^+), \quad (201)$$

推出

$$\delta\phi^+ = \frac{i}{2}\{\phi^+[2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z] + (v + H + i\chi)\alpha^+\}, \quad (202)$$

$$\delta\phi^- = -\frac{i}{2}\{\phi^-[2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z] + (v + H - i\chi)\alpha^-\}, \quad (203)$$

$$\delta H = \frac{1}{2}[i(\phi^+\alpha^- - \phi^-\alpha^+) + \chi\alpha^Z], \quad \delta\chi = \frac{1}{2}[\phi^+\alpha^- + \phi^-\alpha^+ - (v + H)\alpha^Z]. \quad (204)$$

于是, 规范固定函数的无穷小规范变换为

$$\begin{aligned} \sqrt{\xi_W} \delta G^+ &= \partial^\mu \delta W_\mu^+ + i\xi_W m_W \delta\phi^+ \\ &= \partial^\mu \left[-\frac{1}{g} \partial_\mu \alpha^+ - i(s_W A_\mu + c_W Z_\mu) \alpha^+ + iW_\mu^+ (c_W^2 \alpha^Z + \alpha^\gamma) \right] \\ &\quad - \frac{1}{2} \xi_W m_W \{ \phi^+[2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z] + (v + H + i\chi)\alpha^+ \}, \end{aligned} \quad (205)$$

$$\begin{aligned} \sqrt{\xi_W} \delta G^- &= \partial^\mu \delta W_\mu^- - i\xi_W m_W \delta\phi^- \\ &= \partial^\mu \left[-\frac{1}{g} \partial_\mu \alpha^- + i(s_W A_\mu + c_W Z_\mu) \alpha^- - iW_\mu^- (c_W^2 \alpha^Z + \alpha^\gamma) \right] \\ &\quad - \frac{1}{2} \xi_W m_W \{ \phi^-[2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z] + (v + H - i\chi)\alpha^- \}, \end{aligned} \quad (206)$$

$$\begin{aligned} \sqrt{\xi_Z} \delta G^Z &= \partial^\mu \delta Z_\mu + \xi_Z m_Z \delta\chi \\ &= \partial^\mu \left[-\frac{c_W}{g} \partial_\mu \alpha^Z - ic_W (W_\mu^+ \alpha^- - W_\mu^- \alpha^+) \right] \\ &\quad + \frac{1}{2} \xi_Z m_Z [\phi^+\alpha^- + \phi^-\alpha^+ - (v + H)\alpha^Z], \end{aligned} \quad (207)$$

$$\sqrt{\xi_\gamma} \delta G^\gamma = \partial^\mu \delta A_\mu = \partial^\mu \left[-\frac{1}{e} \partial_\mu \alpha^\gamma - is_W (W_\mu^+ \alpha^- - W_\mu^- \alpha^+) \right]. \quad (208)$$

因此,

$$\sqrt{\xi_W} g \frac{\delta G^+}{\delta \alpha^+} = -\partial^2 - \xi_W m_W^2 - ie\partial^\mu A_\mu - igc_W \partial^\mu Z_\mu - \frac{1}{2} g\xi_W m_W (H + i\chi), \quad (209)$$

$$\frac{\sqrt{\xi_W} g}{c_W} \frac{\delta G^+}{\delta \alpha^Z} = igc_W \partial^\mu W_\mu^+ - \frac{g(c_W^2 - s_W^2)\xi_W m_W}{2c_W} \phi^+, \quad (210)$$

$$\sqrt{\xi_W} e \frac{\delta G^+}{\delta \alpha^\gamma} = ie\partial^\mu W_\mu^+ - e\xi_W m_W \phi^+, \quad (211)$$

$$\sqrt{\xi_W} g \frac{\delta G^-}{\delta \alpha^-} = -\partial^2 - \xi_W m_W^2 + ie\partial^\mu A_\mu + igc_W \partial^\mu Z_\mu - \frac{1}{2} g\xi_W m_W (H - i\chi), \quad (212)$$

$$\frac{\sqrt{\xi_W} g}{c_W} \frac{\delta G^-}{\delta \alpha^Z} = -igc_W \partial^\mu W_\mu^- - \frac{g(c_W^2 - s_W^2)\xi_W m_W}{2c_W} \phi^-, \quad (213)$$

$$\sqrt{\xi_W} e \frac{\delta G^-}{\delta \alpha^\gamma} = -ie\partial^\mu W_\mu^- - e\xi_W m_W \phi^-, \quad (214)$$

$$\sqrt{\xi_Z} g \frac{\delta G^Z}{\delta \alpha^+} = igc_W \partial^\mu W_\mu^- + \frac{1}{2} g\xi_Z m_Z \phi^-, \quad (215)$$

$$\sqrt{\xi_Z} g \frac{\delta G^Z}{\delta \alpha^-} = -igc_W \partial^\mu W_\mu^+ + \frac{1}{2} g\xi_Z m_Z \phi^+, \quad (216)$$

$$\frac{\sqrt{\xi_Z} g}{c_W} \frac{\delta G^Z}{\delta \alpha^Z} = -\partial^2 - \xi_Z m_Z^2 - \frac{g\xi_Z m_Z}{2c_W} H, \quad (217)$$

$$\sqrt{\xi_\gamma} g \frac{\delta G^\gamma}{\delta \alpha^+} = ie\partial^\mu W_\mu^-, \quad \sqrt{\xi_\gamma} g \frac{\delta G^\gamma}{\delta \alpha^-} = -ie\partial^\mu W_\mu^+, \quad \sqrt{\xi_\gamma} e \frac{\delta G^\gamma}{\delta \alpha^\gamma} = -\partial^2. \quad (218)$$

最后, 得到以下 Faddeev-Popov 鬼场拉氏量,

$$\begin{aligned} \mathcal{L}_{\text{EWG,FP}} = & \bar{\eta}^+ \left(\sqrt{\xi_W} g \frac{\delta G^+}{\delta \alpha^+} \right) \eta^+ + \bar{\eta}^Z \left(\sqrt{\xi_Z} g \frac{\delta G^Z}{\delta \alpha^+} \right) \eta^+ + \bar{\eta}^\gamma \left(\sqrt{\xi_\gamma} g \frac{\delta G^\gamma}{\delta \alpha^+} \right) \eta^+ \\ & + \bar{\eta}^- \left(\sqrt{\xi_W} g \frac{\delta G^-}{\delta \alpha^-} \right) \eta^- + \bar{\eta}^Z \left(\sqrt{\xi_Z} g \frac{\delta G^Z}{\delta \alpha^-} \right) \eta^- + \bar{\eta}^\gamma \left(\sqrt{\xi_\gamma} g \frac{\delta G^\gamma}{\delta \alpha^-} \right) \eta^- \\ & + \bar{\eta}^Z \left(\frac{\sqrt{\xi_Z} g}{c_W} \frac{\delta G^Z}{\delta \alpha^Z} \right) \eta^Z + \bar{\eta}^+ \left(\frac{\sqrt{\xi_W} g}{c_W} \frac{\delta G^+}{\delta \alpha^Z} \right) \eta^Z + \bar{\eta}^- \left(\frac{\sqrt{\xi_W} g}{c_W} \frac{\delta G^-}{\delta \alpha^Z} \right) \eta^Z \\ & + \bar{\eta}^\gamma \left(\sqrt{\xi_\gamma} e \frac{\delta G^\gamma}{\delta \alpha^\gamma} \right) \eta^\gamma + \bar{\eta}^+ \left(\sqrt{\xi_W} e \frac{\delta G^+}{\delta \alpha^\gamma} \right) \eta^\gamma + \bar{\eta}^- \left(\sqrt{\xi_W} e \frac{\delta G^-}{\delta \alpha^\gamma} \right) \eta^\gamma \\ \rightarrow & \bar{\eta}^+ \left[-\partial^2 - \xi_W m_W^2 + ie\overleftarrow{\partial}^\mu A_\mu + igc_W \overleftarrow{\partial}^\mu Z_\mu - \frac{1}{2} g\xi_W m_W (H + i\chi) \right] \eta^+ \\ & + \bar{\eta}^Z \left(-igc_W \overleftarrow{\partial}^\mu W_\mu^- + \frac{1}{2} g\xi_Z m_Z \phi^- \right) \eta^+ - ie(\partial^\mu \bar{\eta}^\gamma) W_\mu^- \eta^+ \\ & + \bar{\eta}^- \left[-\partial^2 - \xi_W m_W^2 - ie\overleftarrow{\partial}^\mu A_\mu - igc_W \overleftarrow{\partial}^\mu Z_\mu - \frac{1}{2} g\xi_W m_W (H - i\chi) \right] \eta^- \\ & + \bar{\eta}^Z \left(igc_W \overleftarrow{\partial}^\mu W_\mu^+ + \frac{1}{2} g\xi_Z m_Z \phi^+ \right) \eta^- + ie(\partial^\mu \bar{\eta}^\gamma) W_\mu^+ \eta^- \\ & + \bar{\eta}^Z \left(-\partial^2 - \xi_Z m_Z^2 - \frac{g\xi_Z m_Z}{2c_W} H \right) \eta^Z \\ & + \bar{\eta}^+ \left(-igc_W \overleftarrow{\partial}^\mu W_\mu^+ - \frac{g(c_W^2 - s_W^2)\xi_W m_W}{2c_W} \phi^+ \right) \eta^Z \end{aligned}$$

$$\begin{aligned}
& + \bar{\eta}^- \left(ig_{cW} \overleftarrow{\partial}^\mu W_\mu^- - \frac{g(c_W^2 - s_W^2) \xi_W m_W}{2c_W} \phi^- \right) \eta^Z \\
& - \bar{\eta}^\gamma \partial^2 \eta^\gamma + \bar{\eta}^+ (-ie \overleftarrow{\partial}^\mu W_\mu^+ - e \xi_W m_W \phi^+) \eta^\gamma + \bar{\eta}^- (ie \overleftarrow{\partial}^\mu W_\mu^- - e \xi_W m_W \phi^-) \eta^\gamma. \quad (219)
\end{aligned}$$

可以认为这里通过鬼场 η^\pm 、 η^Z 、 η^γ 的归一化吸收了 $-1/g$ 、 $-c_W/g$ 、 $-1/e$ 因子, 通过鬼场 $\bar{\eta}^\pm$ 、 $\bar{\eta}^Z$ 、 $\bar{\eta}^\gamma$ 的归一化吸收了 $1/\sqrt{\xi_W}$ 、 $1/\sqrt{\xi_Z}$ 、 $1/\sqrt{\xi_\gamma}$ 因子。鬼粒子的质量为

$$m_{\eta^+} = m_{\eta^-} = \sqrt{\xi_W} m_W, \quad m_{\eta^Z} = \sqrt{\xi_Z} m_Z, \quad m_{\eta^\gamma} = 0. \quad (220)$$

下面给出 R_ξ 规范下的 Feynman 规则。 $\xi_i = 1$ 对应 Feynman-'t Hooft 规范, $\xi_i = 0$ 对应 Landau 规范, $\xi_W, \xi_Z \rightarrow \infty$ 对应么正规范。在树图计算中, 常取 $\xi_\gamma = 1$ 和 $\xi_W, \xi_Z \rightarrow \infty$ 。在圈图计算中, 常取 $\xi_\gamma = \xi_W = \xi_Z = 1$ 。

传播子:

$$\bullet \text{---} \frac{p}{H} \text{---} \bullet = \frac{i}{p^2 - m_H^2 + i\epsilon} \quad (221)$$

$$\bullet \text{---} \frac{p}{\chi} \text{---} \bullet = \frac{i}{p^2 - \xi_Z m_Z^2 + i\epsilon} \quad (222)$$

$$\bullet \text{---} \frac{p}{\phi} \text{---} \bullet = \frac{i}{p^2 - \xi_W m_W^2 + i\epsilon} \quad (223)$$

$$\nu \text{---} \frac{p}{\gamma} \text{---} \mu = \frac{-i}{p^2 + i\epsilon} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} (1 - \xi_\gamma) \right] \quad (224)$$

$$\nu \text{---} \frac{p}{Z} \text{---} \mu = \frac{-i}{p^2 - m_Z^2 + i\epsilon} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_Z m_Z^2} (1 - \xi_Z) \right] \quad (225)$$

$$\nu \text{---} \frac{p}{W} \text{---} \mu = \frac{-i}{p^2 - m_W^2 + i\epsilon} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_W m_W^2} (1 - \xi_W) \right] \quad (226)$$

$$\bullet \text{---} \frac{p}{\eta^\gamma} \text{---} \bullet = \frac{i}{p^2 + i\epsilon} \quad (227)$$

$$\bullet \text{---} \frac{p}{\eta^Z} \text{---} \bullet = \frac{i}{p^2 - \xi_Z m_Z^2 + i\epsilon} \quad (228)$$

$$\bullet \text{---} \frac{p}{\eta^\pm} \text{---} \bullet = \frac{i}{p^2 - \xi_W m_W^2 + i\epsilon} \quad (229)$$

标量玻色子三线性耦合:

$$\begin{aligned}
& \begin{array}{c} H \\ | \\ \bullet \\ / \quad \backslash \\ H \quad H \end{array} = -\frac{3im_H^2}{v} = -6i\lambda v \\
& \begin{array}{c} H \\ | \\ \bullet \\ / \quad \backslash \\ \chi \quad \chi \end{array} = -\frac{im_H^2}{v} = -2i\lambda v \\
& \begin{array}{c} H \\ | \\ \bullet \\ / \quad \backslash \\ \phi \quad \phi \end{array} = -\frac{im_H^2}{v} = -2i\lambda v
\end{aligned} \quad (230)$$

标量玻色子四线性耦合:

$$\begin{array}{ccc}
 \begin{array}{c} H \\ \diagdown \\ \bullet \\ \diagup \\ H \end{array} & = -\frac{3im_H^2}{v^2} \\
 \begin{array}{c} H \\ \diagup \\ \bullet \\ \diagdown \\ H \end{array} & = -6i\lambda \\
 \begin{array}{c} H \\ \diagdown \\ \bullet \\ \diagup \\ H \end{array} & = -\frac{im_H^2}{v^2} \\
 \begin{array}{c} \phi \\ \diagup \\ \bullet \\ \diagdown \\ \phi \end{array} & = -2i\lambda
 \end{array}
 \quad
 \begin{array}{ccc}
 \begin{array}{c} H \\ \diagdown \\ \bullet \\ \diagup \\ H \end{array} & = -\frac{im_H^2}{v^2} \\
 \begin{array}{c} \chi \\ \diagup \\ \bullet \\ \diagdown \\ \chi \end{array} & = -2i\lambda \\
 \begin{array}{c} \chi \\ \diagdown \\ \bullet \\ \diagup \\ \chi \end{array} & = -\frac{im_H^2}{v^2} \\
 \begin{array}{c} \phi \\ \diagup \\ \bullet \\ \diagdown \\ \phi \end{array} & = -2i\lambda
 \end{array}
 \quad
 \begin{array}{ccc}
 \begin{array}{c} \chi \\ \diagdown \\ \bullet \\ \diagup \\ \chi \end{array} & = -\frac{3im_H^2}{v^2} \\
 \begin{array}{c} \chi \\ \diagup \\ \bullet \\ \diagdown \\ \chi \end{array} & = -6i\lambda \\
 \begin{array}{c} \phi \\ \diagdown \\ \bullet \\ \diagup \\ \phi \end{array} & = -\frac{2im_H^2}{v^2} \\
 \begin{array}{c} \phi \\ \diagup \\ \bullet \\ \diagdown \\ \phi \end{array} & = -4i\lambda
 \end{array}
 \quad (231)$$

Yukawa 耦合:

$$\begin{array}{ccc}
 \begin{array}{c} H \\ \vdots \\ \bullet \\ \swarrow \searrow \\ f \quad f \end{array} & = -\frac{im_f}{v} \\
 & = -\frac{iy_f}{\sqrt{2}}
 \end{array}
 \quad
 \begin{array}{ccc}
 \begin{array}{c} \chi \\ \vdots \\ \bullet \\ \swarrow \searrow \\ \ell_i \quad \ell_i \end{array} & = \frac{m_{\ell_i}}{v} \gamma^5 \\
 & = \frac{y_{\ell_i}}{\sqrt{2}} \gamma^5
 \end{array}
 \quad (233)$$

$$\begin{array}{ccc}
 \begin{array}{c} \chi \\ \vdots \\ \bullet \\ \swarrow \searrow \\ u_i \quad u_i \end{array} & = -\frac{m_{u_i}}{v} \gamma^5 \\
 & = -\frac{y_{u_i}}{\sqrt{2}} \gamma^5
 \end{array}
 \quad
 \begin{array}{ccc}
 \begin{array}{c} \chi \\ \vdots \\ \bullet \\ \swarrow \searrow \\ d_i \quad d_i \end{array} & = \frac{m_{d_i}}{v} \gamma^5 \\
 & = \frac{y_{d_i}}{\sqrt{2}} \gamma^5
 \end{array}
 \quad (234)$$

$$\begin{array}{ccc}
 \begin{array}{c} \phi \\ \vdots \\ \bullet \\ \swarrow \searrow \\ \ell_i \quad \nu_i \end{array} & = -\frac{i\sqrt{2}m_{\ell_i}}{v} P_R \\
 & = -iy_{\ell_i} P_R
 \end{array}
 \quad
 \begin{array}{ccc}
 \begin{array}{c} \phi \\ \vdots \\ \bullet \\ \swarrow \searrow \\ \nu_i \quad \ell_j \end{array} & = -\frac{i\sqrt{2}m_{\ell_i}}{v} P_L \\
 & = -iy_{\ell_i} P_L
 \end{array}
 \quad (235)$$

$$\begin{array}{ccc}
 \begin{array}{c} \phi \\ \vdots \\ \bullet \\ \swarrow \searrow \\ d_j \quad u_i \end{array} & = \frac{i\sqrt{2}V_{ij}}{v} (m_{u_i} P_L - m_{d_j} P_R) \\
 & = iV_{ij} (y_{u_i} P_L - y_{d_j} P_R)
 \end{array}
 \quad
 \begin{array}{ccc}
 \begin{array}{c} \phi \\ \vdots \\ \bullet \\ \swarrow \searrow \\ u_i \quad d_j \end{array} & = -\frac{i\sqrt{2}V_{ji}^\dagger}{v} (m_{d_j} P_L - m_{u_i} P_R) \\
 & = -iV_{ji}^\dagger (y_{d_j} P_L - y_{u_i} P_R)
 \end{array}
 \quad (236)$$

标量玻色子与电弱规范玻色子的三线耦合：

$$\begin{aligned}
 \begin{array}{c} H \\ | \\ \bullet \\ / \quad \backslash \\ Z; \mu \quad Z; \nu \end{array} &= \frac{igm_Z}{c_W} g^{\mu\nu} = \frac{ig^2 v}{2c_W^2} g^{\mu\nu} \\
 \begin{array}{c} H \\ | \\ \bullet \\ / \quad \backslash \\ W; \mu \quad W; \nu \end{array} &= igm_W g^{\mu\nu} = \frac{ig^2 v}{2} g^{\mu\nu}
 \end{aligned} \quad (237)$$

$$\begin{aligned}
 \begin{array}{c} \phi \\ | \\ \bullet \\ / \quad \backslash \\ W; \mu \quad \gamma; \nu \end{array} &= iem_W g^{\mu\nu} = \frac{ieg v}{2} g^{\mu\nu} \\
 \begin{array}{c} \phi \\ | \\ \bullet \\ / \quad \backslash \\ W; \mu \quad \gamma; \nu \end{array} &= iem_W g^{\mu\nu} = \frac{ieg v}{2} g^{\mu\nu}
 \end{aligned} \quad (238)$$

$$\begin{aligned}
 \begin{array}{c} \phi \\ | \\ \bullet \\ / \quad \backslash \\ W; \mu \quad Z; \nu \end{array} &= -igs_W^2 m_Z g^{\mu\nu} = -\frac{ig^2 s_W^2 v}{2c_W} g^{\mu\nu} \\
 \begin{array}{c} \phi \\ | \\ \bullet \\ / \quad \backslash \\ W; \mu \quad Z; \nu \end{array} &= -igs_W^2 m_Z g^{\mu\nu} = -\frac{ig^2 s_W^2 v}{2c_W} g^{\mu\nu}
 \end{aligned} \quad (239)$$

$$\begin{array}{c} \gamma; \mu \\ | \\ \bullet \\ / \quad \backslash \\ \phi \quad \phi \end{array} \quad p \quad q \quad = -ie(p+q)^\mu \quad (240)$$

$$\begin{array}{c} Z; \mu \\ | \\ \bullet \\ / \quad \backslash \\ \phi \quad \phi \end{array} \quad p \quad q \quad = -\frac{ig(c_W^2 - s_W^2)}{2c_W} (p+q)^\mu \quad \begin{array}{c} Z; \mu \\ | \\ \bullet \\ / \quad \backslash \\ H \quad \chi \end{array} \quad p \quad q \quad = \frac{g}{2c_W} (p+q)^\mu \quad (241)$$

$$\begin{array}{c} W; \mu \\ | \\ \bullet \\ / \quad \backslash \\ H \quad \phi \end{array} \quad p \quad q \quad = -\frac{ig}{2} (p+q)^\mu \quad \begin{array}{c} W; \mu \\ | \\ \bullet \\ / \quad \backslash \\ \phi \quad H \end{array} \quad p \quad q \quad = -\frac{ig}{2} (p+q)^\mu \quad (242)$$

$$\begin{array}{c} W; \mu \\ | \\ \bullet \\ / \quad \backslash \\ \chi \quad \phi \end{array} \quad p \quad q \quad = \frac{g}{2} (p+q)^\mu \quad \begin{array}{c} W; \mu \\ | \\ \bullet \\ / \quad \backslash \\ \phi \quad \chi \end{array} \quad p \quad q \quad = -\frac{g}{2} (p+q)^\mu \quad (243)$$

标量玻色子与电弱规范玻色子的四线性耦合:

$$\begin{array}{ccc}
\begin{array}{c} H \quad H \\ \diagdown \quad \diagup \\ \bullet \\ \diagup \quad \diagdown \\ \text{wavy line} \end{array} & = \frac{ig^2}{2c_W^2} g^{\mu\nu} & \begin{array}{c} H \quad H \\ \diagdown \quad \diagup \\ \bullet \\ \diagup \quad \diagdown \\ \text{gluon line} \end{array} = \frac{ig^2}{2} g^{\mu\nu}
\end{array} \quad (244)$$

$$\begin{array}{c} \diagup \\ \bullet \\ \diagdown \end{array} \begin{array}{c} \text{---} \\ \text{---} \end{array} = \frac{ig^2}{2c_W^2} g^{\mu\nu} \quad \begin{array}{c} \diagup \\ \bullet \\ \diagdown \end{array} \begin{array}{c} \text{---} \\ \text{---} \end{array} = \frac{ig^2}{2} g^{\mu\nu} \quad (245)$$

$$\begin{array}{c} \phi \\ \swarrow \\ \bullet \\ \searrow \\ \phi \end{array} = 2ie^2 g^{\mu\nu} \quad \begin{array}{c} \phi \\ \swarrow \\ \bullet \\ \searrow \\ \phi \end{array} = \frac{ieg(c_W^2 - s_W^2)}{c_W} g^{\mu\nu} \quad (246)$$

$$\begin{array}{c} \phi \\ \swarrow \\ \bullet \\ \searrow \\ \phi \end{array} \begin{array}{c} \text{---} \\ \swarrow \\ \bullet \\ \searrow \\ \text{---} \end{array} \begin{array}{c} \phi \\ \swarrow \\ \bullet \\ \searrow \\ \phi \end{array} = \frac{ig^2(c_W^2 - s_W^2)^2}{2c_W^2} g^{\mu\nu} \begin{array}{c} \phi \\ \swarrow \\ \bullet \\ \searrow \\ \phi \end{array} \begin{array}{c} \text{---} \\ \swarrow \\ \bullet \\ \searrow \\ \text{---} \end{array} \begin{array}{c} \phi \\ \swarrow \\ \bullet \\ \searrow \\ \phi \end{array} \quad (247)$$

$$\begin{aligned}
 & \text{Diagram 1: } H \text{ (dashed line) decaying into } \phi \text{ (dashed line) via a loop of } t \text{ (solid line) and } W \text{ (wavy line).} \\
 & \quad = \frac{ie g}{2} g^{\mu\nu} \\
 & \text{Diagram 2: } H \text{ (dashed line) decaying into } \phi \text{ (dashed line) via a loop of } t \text{ (solid line) and } Z \text{ (wavy line).} \\
 & \quad = \frac{ie g}{2} g^{\mu\nu}
 \end{aligned}
 \tag{248}$$

$$= -\frac{ig^2 s_W^2}{2c_W} g^{\mu\nu} \quad = -\frac{ig^2 s_W^2}{2c_W} g^{\mu\nu} \quad (249)$$

$$\begin{array}{c} \chi \\ \diagup \\ \bullet \\ \diagdown \\ \gamma; \mu \end{array} \begin{array}{c} \phi \\ \diagdown \\ \bullet \\ \diagup \\ W; \nu \end{array} = -\frac{eg}{2} g^{\mu\nu} \begin{array}{c} \chi \\ \diagup \\ \bullet \\ \diagdown \\ \gamma; \mu \end{array} \begin{array}{c} \phi \\ \diagdown \\ \bullet \\ \diagup \\ W; \nu \end{array} = \frac{eg}{2} g^{\mu\nu} \quad (250)$$

$$\begin{aligned}
 \text{Left Diagram} &= \frac{g^2 s_W^2}{2c_W} g^{\mu\nu} \\
 \text{Right Diagram} &= -\frac{g^2 s_W^2}{2c_W} g^{\mu\nu}
 \end{aligned} \quad (251)$$

鬼粒子与标量玻色子的耦合:

$$\begin{aligned}
 &= -\frac{ig\xi_Z m_Z}{2c_W} \\
 &= -\frac{ig^2\xi_Z v}{4c_W^2}
 \end{aligned} \quad (252)$$

$$\begin{aligned}
 \text{Left Diagram} &= -\frac{ig\xi_W m_W}{2} \\
 &= -\frac{ig^2\xi_W v}{4} \\
 \text{Right Diagram} &= -\frac{ig\xi_W m_W}{2} \\
 &= -\frac{ig^2\xi_W v}{4}
 \end{aligned} \quad (253)$$

$$\begin{aligned}
 \text{Left Diagram} &= \frac{g\xi_W m_W}{2} \\
 &= \frac{g^2\xi_W v}{4} \\
 \text{Right Diagram} &= -\frac{g\xi_W m_W}{2} \\
 &= -\frac{g^2\xi_W v}{4}
 \end{aligned} \quad (254)$$

$$\begin{aligned}
 \text{Left Diagram} &= -ie\xi_W m_W \\
 &= -\frac{ieg\xi_W v}{2} \\
 \text{Right Diagram} &= -ie\xi_W m_W \\
 &= -\frac{ieg\xi_W v}{2}
 \end{aligned} \quad (255)$$

$$\begin{aligned}
 \text{Left Diagram} &= -\frac{ig(c_W^2 - s_W^2)\xi_W m_W}{2c_W} \\
 &= -\frac{ig^2(c_W^2 - s_W^2)\xi_W v}{4c_W} \\
 \text{Right Diagram} &= \frac{ig\xi_Z m_Z}{2} \\
 &= \frac{ig^2\xi_Z v}{4c_W}
 \end{aligned} \quad (256)$$

$$\begin{aligned}
\begin{array}{c} \phi \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^- \quad \eta^Z \end{array} &= \frac{ig\xi_Z m_Z}{2} = \frac{ig^2\xi_Z v}{4c_W} \\
\begin{array}{c} \phi \\ \uparrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^Z \quad \eta^- \end{array} &= -\frac{ig(c_W^2 - s_W^2)\xi_W m_W}{2c_W} = -\frac{ig^2(c_W^2 - s_W^2)\xi_W v}{4c_W}
\end{aligned} \tag{257}$$

鬼粒子与电弱规范玻色子的耦合：

$$\begin{array}{c} \gamma; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^+ \quad \eta^+ \end{array} = -iep^\mu \qquad \begin{array}{c} \gamma; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^- \quad \eta^- \end{array} = iep^\mu \tag{258}$$

$$\begin{array}{c} Z; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^+ \quad \eta^+ \end{array} = -igc_W p^\mu \qquad \begin{array}{c} Z; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^- \quad \eta^- \end{array} = igc_W p^\mu \tag{259}$$

$$\begin{array}{c} W; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^\gamma \quad \eta^+ \end{array} = iep^\mu \qquad \begin{array}{c} W; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^+ \quad \eta^\gamma \end{array} = iep^\mu \tag{260}$$

$$\begin{array}{c} W; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^- \quad \eta^\gamma \end{array} = -iep^\mu \qquad \begin{array}{c} W; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^\gamma \quad \eta^- \end{array} = -iep^\mu \tag{261}$$

$$\begin{array}{c} W; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^Z \quad \eta^+ \end{array} = igc_W p^\mu \qquad \begin{array}{c} W; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^+ \quad \eta^Z \end{array} = igc_W p^\mu \tag{262}$$

$$\begin{array}{c} W; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^- \quad \eta^Z \end{array} = -igc_W p^\mu \qquad \begin{array}{c} W; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^Z \quad \eta^- \end{array} = -igc_W p^\mu \tag{263}$$

6 内外线一般 Feynman 规则

本节列出一些通用的内外线 Feynman 规则。

内线 Feynman 规则如下。

- 实标量玻色子传播子: $\bullet \text{---} \xrightarrow{p} \text{---} \bullet = \frac{i}{p^2 - m^2 + i\epsilon}$

- 复标量玻色子传播子: $\bullet \text{---} \xrightarrow{p} \text{---} \bullet = \frac{i}{p^2 - m^2 + i\epsilon}$

- Dirac 费米子传播子: $\bullet \xrightarrow{p} \bullet = \frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon}$

- 无质量实矢量玻色子传播子:

$$\nu \bullet \xrightarrow{p} \bullet \mu = \frac{-ig^{\mu\nu}}{p^2 + i\epsilon} \quad (\text{Feynman 规范})$$

$$\nu \bullet \xrightarrow{p} \bullet \mu = \frac{-i(g^{\mu\nu} - p^\mu p^\nu / p^2)}{p^2 + i\epsilon} \quad (\text{Landau 规范})$$

- 有质量实矢量玻色子传播子:

$$\nu \bullet \xrightarrow{p} \bullet \mu = \frac{-i(g^{\mu\nu} - p^\mu p^\nu / m^2)}{p^2 - m^2 + i\epsilon} \quad (\text{么正规规范})$$

$$\nu \bullet \xrightarrow{p} \bullet \mu = \frac{-ig^{\mu\nu}}{p^2 - m^2 + i\epsilon} \quad (\text{Feynman 规范})$$

- 有质量复矢量玻色子传播子:

$$\nu \bullet \xrightarrow{p} \bullet \mu = \frac{-i(g^{\mu\nu} - p^\mu p^\nu / m^2)}{p^2 - m^2 + i\epsilon} \quad (\text{么正规规范})$$

$$\nu \bullet \xrightarrow{p} \bullet \mu = \frac{-ig^{\mu\nu}}{p^2 - m^2 + i\epsilon} \quad (\text{Feynman 规范})$$

实标量场外线 Feynman 规则如下。

- 实标量玻色子入射外线: $\text{---} \xrightarrow{p} \bullet = 1$

- 实标量玻色子出射外线: $\bullet \xrightarrow{p} \text{---} = 1$

复标量场外线 Feynman 规则如下。

- 正标量玻色子入射外线: $\text{---} \xrightarrow{p} \bullet = 1$

- 反标量玻色子入射外线: $\text{---} \xleftarrow{p} \bullet = 1$

- 正标量玻色子出射外线: $\bullet \dashrightarrow^p \text{---} = 1$

- 反标量玻色子出射外线: $\bullet \dashleftarrow^p \text{---} = 1$

以 λ 代表自旋极化指标 (如螺旋度), **Dirac 旋量场外线** Feynman 规则如下。

- Dirac 正费米子入射外线: $\lambda \xrightarrow{p} \bullet = u(\mathbf{p}, \lambda)$

- Dirac 反费米子入射外线: $\lambda \xleftarrow{p} \bullet = \bar{v}(\mathbf{p}, \lambda)$

- Dirac 正费米子出射外线: $\bullet \xrightarrow{p} \lambda = \bar{u}(\mathbf{p}, \lambda)$

- Dirac 反费米子出射外线: $\bullet \xleftarrow{p} \lambda = v(\mathbf{p}, \lambda)$

在计算非极化振幅模方时, 可利用自旋求和关系

$$\sum_{\lambda} u(\mathbf{p}, \lambda) \bar{u}(\mathbf{p}, \lambda) = \not{p} + m, \quad \sum_{\lambda} v(\mathbf{p}, \lambda) \bar{v}(\mathbf{p}, \lambda) = \not{p} - m. \quad (264)$$

以 λ 代表自旋极化指标, **实矢量场外线** Feynman 规则如下。

- 实矢量玻色子入射外线: $\lambda; \mu \xrightarrow{p} \bullet = \varepsilon^{\mu}(\mathbf{p}, \lambda)$

- 实矢量玻色子出射外线: $\bullet \xrightarrow{p} \lambda; \mu = \varepsilon^{\mu*}(\mathbf{p}, \lambda)$

复矢量场外线 Feynman 规则如下。

- 正矢量玻色子入射外线: $\lambda; \mu \xrightarrow{p} \bullet = \varepsilon^{\mu}(\mathbf{p}, \lambda)$

- 反矢量玻色子入射外线: $\lambda; \mu \xleftarrow{p} \bullet = \varepsilon^{\mu}(\mathbf{p}, \lambda)$

- 正矢量玻色子出射外线: $\bullet \xrightarrow{p} \lambda; \mu = \varepsilon^{\mu*}(\mathbf{p}, \lambda)$

- 反矢量玻色子出射外线: $\bullet \xleftarrow{p} \lambda; \mu = \varepsilon^{\mu*}(\mathbf{p}, \lambda)$

在计算非极化振幅模方时，若包含无质量矢量玻色子外线，可利用极化求和替换关系

$$\sum_{\lambda} \varepsilon_{\mu}^*(\mathbf{p}, \lambda) \varepsilon_{\nu}(\mathbf{p}, \lambda) \rightarrow -g_{\mu\nu}; \quad (265)$$

若包含有质量矢量玻色子外线，可利用极化求和关系

$$\sum_{\lambda} \varepsilon_{\mu}^*(\mathbf{p}, \lambda) \varepsilon_{\nu}(\mathbf{p}, \lambda) = -g_{\mu\nu} + \frac{p_{\mu} p_{\nu}}{m^2}. \quad (266)$$

7 常用单位和标准模型参数

本节数据来自 Particle Data Group 发布的 2024 版 *Review of Particle Physics* [5]。

在有理化的自然单位制中，光速、约化 Planck 常数和真空介电常数均取为 1，即 $c = \hbar = \varepsilon_0 = 1$ 。从而，速度没有量纲 (dimension)；长度量纲与时间量纲相同，是能量量纲的倒数；能量、质量和动量具有相同的量纲；精细结构常数表达为

$$\alpha = \frac{e^2}{4\pi}, \quad (267)$$

而单位电荷量 $e = \sqrt{4\pi\alpha}$ 是没有量纲的。可以将能量单位电子伏特 (eV) 视作上述有量纲物理量的基本单位。

单位间转换关系为

$$1 = c = 2.99792458 \times 10^{10} \text{ cm} \cdot \text{s}^{-1}, \quad (268)$$

$$1 = \hbar = 6.582119569 \times 10^{-25} \text{ GeV} \cdot \text{s}, \quad (269)$$

$$1 = \hbar c = 1.973269804 \times 10^{-14} \text{ GeV} \cdot \text{cm}, \quad (270)$$

$$1 = (\hbar c)^2 = 3.893793721 \times 10^8 \text{ GeV}^2 \cdot \text{pb}, \quad (271)$$

由此得到

$$1 \text{ s} = 2.997925 \times 10^{10} \text{ cm}, \quad 1 \text{ cm} = 3.335641 \times 10^{-11} \text{ s}, \quad (272)$$

$$1 \text{ s} = 1.519267 \times 10^{24} \text{ GeV}^{-1}, \quad 1 \text{ GeV}^{-1} = 6.582120 \times 10^{-25} \text{ s}, \quad (273)$$

$$1 \text{ cm} = 5.067731 \times 10^{13} \text{ GeV}^{-1}, \quad 1 \text{ GeV}^{-1} = 1.973270 \times 10^{-14} \text{ cm}, \quad (274)$$

$$1 \text{ cm}^2 = 2.568189 \times 10^{27} \text{ GeV}^{-2}, \quad 1 \text{ GeV}^{-2} = 3.893794 \times 10^{-28} \text{ cm}^2, \quad (275)$$

$$1 \text{ cm}^3 \cdot \text{s}^{-1} = 8.566558 \times 10^{16} \text{ GeV}^{-2}, \quad 1 \text{ GeV}^{-2} = 1.167330 \times 10^{-17} \text{ cm}^3 \cdot \text{s}^{-1}. \quad (276)$$

靶 (barn) 是散射截面的常用单位，记作 b，满足

$$1 \text{ b} = 10^{-24} \text{ cm}^2 = 10^9 \text{ nb} = 10^{12} \text{ pb} = 10^{15} \text{ fb} = 10^{18} \text{ ab}, \quad (277)$$

$$1 \text{ pb} = 10^{-36} \text{ cm}^2 = 2.568189 \times 10^{-9} \text{ GeV}^{-2}, \quad 1 \text{ GeV}^{-2} = 3.893794 \times 10^8 \text{ pb}. \quad (278)$$

Fermi 耦合常数是

$$G_F = 1.1663788(6) \times 10^{-5} \text{ GeV}^{-2}, \quad (279)$$

括号内数字代表测量值的 1σ 不确定度, 由树图阶关系式

$$\frac{G_F}{\sqrt{2}} = \frac{1}{2v^2} = \frac{g^2}{8m_W^2}, \quad (280)$$

得到 Higgs 场真空期待值为

$$v = (\sqrt{2}G_F)^{-1/2} = 246.2196 \text{ GeV}. \quad (281)$$

在低能标 (Thomson 极限) 处, 精细结构常数为

$$\alpha = \frac{1}{137.035999178(8)}; \quad (282)$$

在 $\overline{\text{MS}}$ 重整化方案 (以 \wedge 为标志) 中, α^{-1} 跑到到 $\mu = m_Z$ 能标处的数值是

$$\hat{\alpha}^{-1}(m_Z) = 127.930 \pm 0.008. \quad (283)$$

在 $\overline{\text{MS}}$ 方案中, $\mu = m_Z$ 能标处强耦合常数 $\alpha_s = g_s^2/(4\pi)$ 的数值为

$$\hat{\alpha}_s(m_Z) = 0.1180 \pm 0.0009, \quad (284)$$

Weinberg 角 θ_W 的数值对应于

$$\hat{s}_W^2 = \sin^2 \hat{\theta}_W(m_Z) = 0.23129 \pm 0.00004. \quad (285)$$

在标准模型中, 光子、胶子和中微子没有质量, 其它基本粒子的质量为

$$m_W = 80.3692 \pm 0.0133 \text{ GeV}, \quad m_Z = 91.1880 \pm 0.0020 \text{ GeV}, \quad (286)$$

$$m_H = 125.20 \pm 0.11 \text{ GeV}, \quad m_e = 0.51099895000(15) \text{ MeV}, \quad (287)$$

$$m_\mu = 105.6583755(23) \text{ MeV}, \quad m_\tau = 1776.93 \pm 0.09 \text{ MeV}, \quad (288)$$

$$m_u = 2.16 \pm 0.07 \text{ MeV}, \quad m_d = 4.70 \pm 0.07 \text{ MeV}, \quad (289)$$

$$m_s = 93.5 \pm 0.8 \text{ MeV}, \quad m_c = 1.2730 \pm 0.0046 \text{ GeV}, \quad (290)$$

$$m_b = 4.183 \pm 0.007 \text{ GeV}, \quad m_t = 172.57 \pm 0.29 \text{ GeV}. \quad (291)$$

这里, u 、 d 、 s 夸克的质量是 $\mu = 2 \text{ GeV}$ 能标处的 $\overline{\text{MS}}$ 质量, c 和 b 夸克的质量分别是 $\mu = m_c$

和 $\mu = m_b$ 能标处的 $\overline{\text{MS}}$ 质量, 其余粒子的质量均为极点质量 (pole mass)。相应地, 计算出来的 c 、 b 夸克极点质量为

$$m_c^{\text{pole}} = 1.67 \pm 0.07 \text{ GeV}, \quad m_b^{\text{pole}} = 4.78 \pm 0.06 \text{ GeV}. \quad (292)$$

质子和中子的质量为

$$m_p = 938.27208816(29) \text{ MeV}, \quad m_n = 939.5654205(5) \text{ MeV}. \quad (293)$$

在电弱能标附近作领头阶计算时, 可将单位电荷量取为

$$e = \sqrt{4\pi\hat{\alpha}(m_Z)} = 0.3134142, \quad (294)$$

将强耦合常数取为

$$g_s = \sqrt{4\pi\hat{\alpha}_s(m_Z)} = 1.217716. \quad (295)$$

从树图阶关系计算 Higgs 场四线性耦合常数 λ 和 Yukawa 耦合常数 y_t 、 y_b 、 y_τ 、 y_c , 得

$$\lambda = \frac{m_H^2}{2v^2} = 0.1292806, \quad y_t = \frac{\sqrt{2}m_t}{v} = 0.9911916, \quad y_b = \frac{\sqrt{2}m_b}{v} = 2.402593 \times 10^{-2}, \quad (296)$$

$$y_\tau = \frac{\sqrt{2}m_\tau}{v} = 1.020617 \times 10^{-2}, \quad y_c = \frac{\sqrt{2}m_c}{v} = 7.311739 \times 10^{-3}. \quad (297)$$

耦合常数 g 和 g' 有以下两种取值方式。

1. 根据树图阶关系 $\sin^2 \theta_W = 1 - m_W^2/m_Z^2$ 计算 Weinberg 角, 得

$$s_W^2 = 1 - \frac{m_W^2}{m_Z^2} = 0.2232095, \quad c_W^2 = 1 - s_W^2 = 0.7767905, \quad (298)$$

$$s_W = \sqrt{s_W^2} = 0.4724505, \quad c_W = \sqrt{c_W^2} = 0.8813572, \quad (299)$$

故

$$g = \frac{e}{s_W} = 0.6633800, \quad g' = \frac{e}{c_W} = 0.3556041. \quad (300)$$

2. 根据 $\overline{\text{MS}}$ 方案中 Weinberg 角的数值 (285) 计算 g 和 g' , 得

$$c_W^2 = 1 - \hat{s}_W^2 = 0.76871, \quad s_W = \sqrt{\hat{s}_W^2} = 0.4809262, \quad c_W = \sqrt{c_W^2} = 0.8767611, \quad (301)$$

$$g = \frac{e}{s_W} = 0.6516889, \quad g' = \frac{e}{c_W} = 0.3574682. \quad (302)$$

将 CKM 矩阵参数化为

$$V = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & s_{13}e^{-i\delta} & \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}, \quad (303)$$

其中 $s_{ij} \equiv \sin \theta_{ij}$, $c_{ij} \equiv \cos \theta_{ij}$ 。实验拟合值为

$$s_{12} = 0.22501 \pm 0.00068, \quad s_{23} = 0.04183^{+0.00079}_{-0.00069}, \quad (304)$$

$$s_{13} = 0.003732^{+0.000090}_{-0.000085}, \quad \delta = 1.147 \pm 0.026. \quad (305)$$

如果只讨论第一、二代夸克的混合, 可利用 Cabibbo 转动角 θ_C 将 CKM 矩阵近似地表达为

$$V \simeq \begin{pmatrix} \cos \theta_C & \sin \theta_C & \\ -\sin \theta_C & \cos \theta_C & \\ & & 1 \end{pmatrix}, \quad \sin \theta_C = s_{12} = 0.225. \quad (306)$$

参考文献

- [1] M. E. Peskin and D. V. Schroeder, “An Introduction to Quantum Field Theory,” Reading, USA: Addison-Wesley (1995), 842 pages.
- [2] T. P. Cheng and L. F. Li, “Gauge Theory of Elementary Particle Physics,” Oxford, UK: Clarendon (1984), 536 pages.
- [3] A. Denner, “Techniques for calculation of electroweak radiative corrections at the one loop level and results for W physics at LEP-200,” Fortsch. Phys. **41**, 307 (1993) [arXiv:0709.1075 [hep-ph]].
- [4] 杜东生, 杨茂志, 《粒子物理导论》, 中国北京: 科学出版社 (2014), 402 页。
- [5] S. Navas *et al.* [Particle Data Group], Phys. Rev. D **110**, no.3, 030001 (2024)