

Monopoles, Domain Walls, Cosmic Strings, and Their Implications for Gravitational Waves

Zhao-Huan Yu (余钊煥)

School of Physics, Sun Yat-Sen University

<https://yzhxxzxy.github.io>



第十四届新物理研讨会

Jinan, July 21, 2025

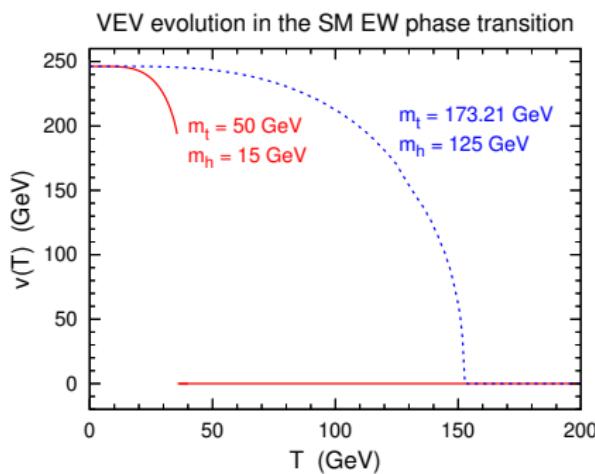
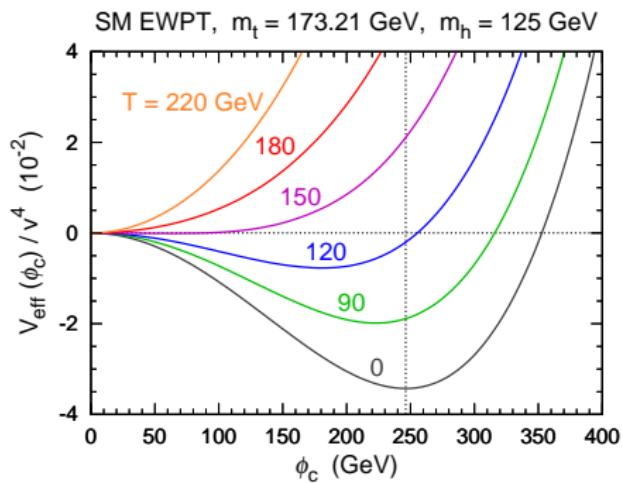


Cosmological Phase Transition

 Spontaneously broken symmetries in field theories can be restored at sufficiently high temperatures due to thermal corrections to the effective potential

 In the history of the Universe, spontaneous symmetry breaking manifests itself as a cosmological phase transition

If the **vacuum manifold** has **nontrivial topological structures**, **topological defects** would be formed after the phase transition



Symmetry Breaking $G \rightarrow H$

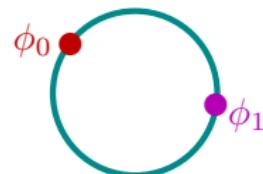
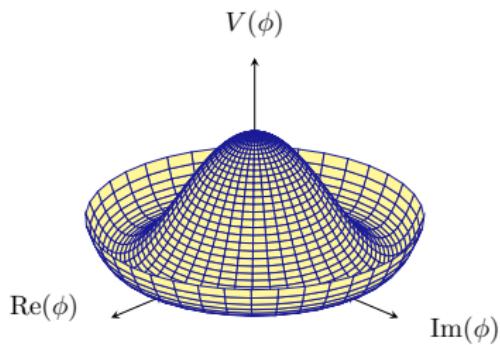
 Consider that **some scalar fields** acquire nonzero **vacuum expectation values** (VEVs), which **break** a **symmetry group** G to a **subgroup** H ($G \rightarrow H$)

C For a **VEV** ϕ_0 , the action by any element $h \in H$ is **trivial**: $h\phi_0 = \phi_0$

A **nontrivial** action on ϕ_0 : $g\phi_0 = \phi_1$, $g \in G$, $g \notin H$  $gh\phi_0 = \phi_1$

All the **nontrivial** transformations are given by the **left cosets** of H (e.g., gH), which constitute the **coset space** G/H

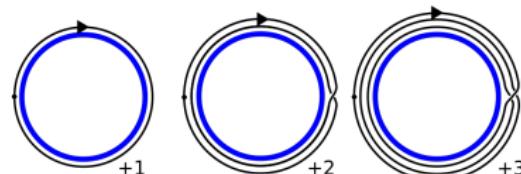
 G/H is **isomorphic** to the **manifold** consisting of all **degenerate vacua**



Vacuum manifold

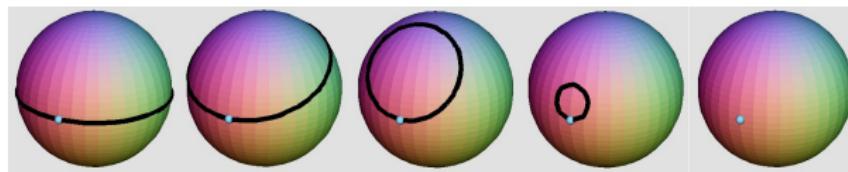
Homotopy Groups $\pi_n(G/H)$

The **topology** of the **vacuum manifold** G/H can be characterized by its **n -th homotopy group** $\pi_n(G/H)$, which is constituted by the **homotopy classes** of the mappings from an **n -dimensional sphere** S^n into G/H



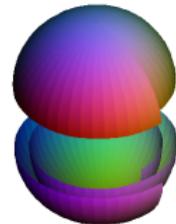
$$\pi_1(S^1) = \mathbb{Z}$$

A homotopy class is identified
with a winding number n



$$\pi_1(S^2) = 1 \text{ (trivial)}$$

Any continuous mapping from
 S^1 to S^2 can be continuously
deformed to a 1-point mapping



$$\pi_2(S^2) = \mathbb{Z}$$

Mappings from S^2 to S^2 can be visualized as wrapping
a twisted plastic bag around a ball n times

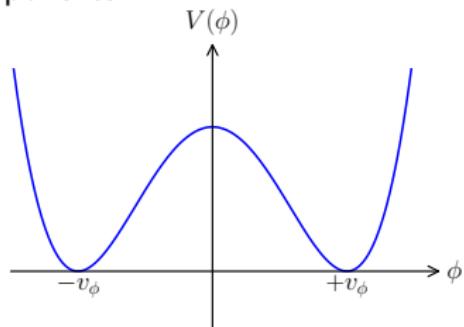
👉 How S^2 can be wrapped twice around another S^2

Topological Defects

.ribbon A **nontrivial** $\pi_n(G/H)$ leads **topological defects** [Kibble, J. Phys. A9 (1976) 1387]

red square **Nontrivial** $\pi_0(G/H)$: two or more disconnected components

blue hand **Domain walls** (2-dim topological defects)



- $G/H \cong Z_2$
- $\pi_0(G/H) = Z_2$

Topological Defects

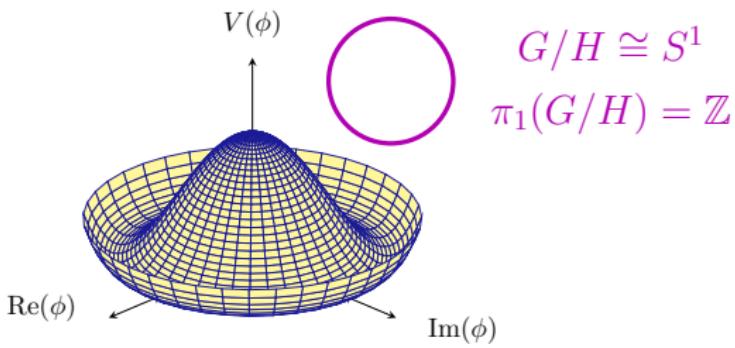
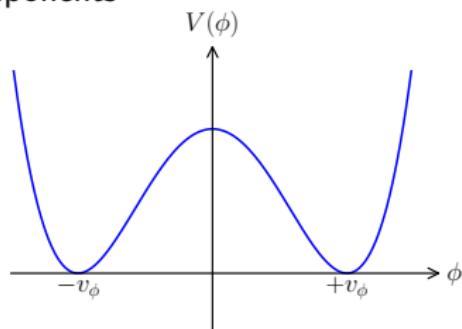
⌚ A nontrivial $\pi_n(G/H)$ leads topological defects [Kibble, J. Phys. A9 (1976) 1387]

 **Nontrivial** $\pi_0(G/H)$: two or more disconnected components

👉 Domain walls (2-dim topological defects)

Nontrivial $\pi_1(G/H)$: incontractable closed paths

👉 Cosmic strings (1-dim topological defects)



- $G/H \cong Z_2$
 - $\pi_0(G/H) = Z_2$

Topological Defects

⌚ A nontrivial $\pi_n(G/H)$ leads topological defects [Kibble, J. Phys. A9 (1976) 1387]

 **Nontrivial** $\pi_0(G/H)$: two or more disconnected components

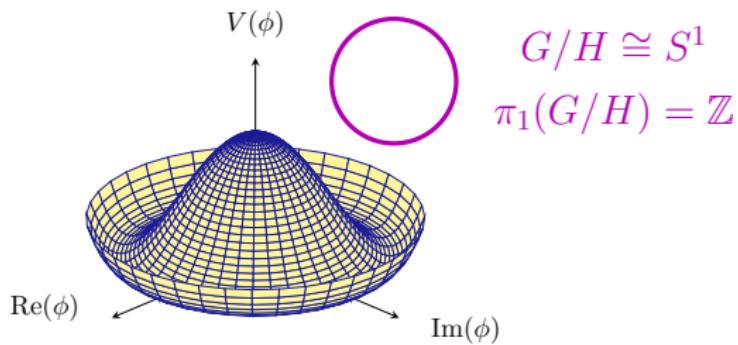
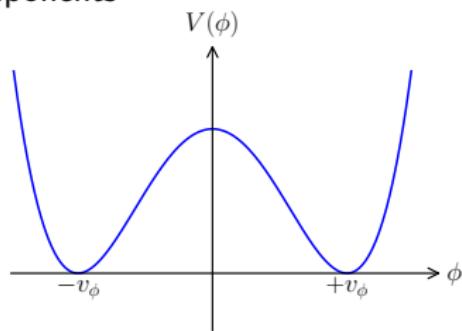
👉 Domain walls (2-dim topological defects)

Nontrivial $\pi_1(G/H)$: incontractable closed paths

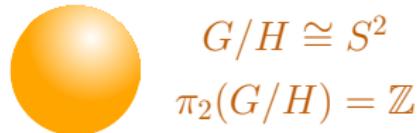
👉 Cosmic strings (1-dim topological defects)

Nontrivial $\pi_2(G/H)$: incontractable spheres

👉 Monopoles (0-dim topological defects)

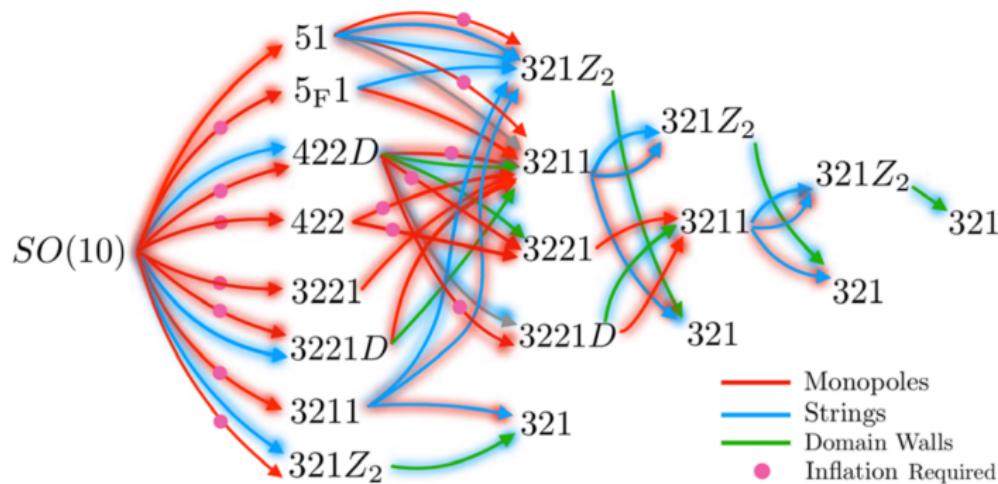


- $G/H \cong Z_2$
 - $\pi_0(G/H) = Z_2$



Topological Defects in GUTs

 **Monopoles**, **cosmic strings**, and **domain walls** are commonly predicted in **grand unified theories (GUTs)**



$$51 = SU(5) \times U(1)_X/Z_5, \quad 5F1 = SU(5)_{\text{flipped}} \times U(1)_{\text{flipped}}/Z_5$$

$$422 = SU(4)_C \times SU(2)_L \times SU(2)_R/Z_2, \quad 3221 = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}/Z_6$$

$$3211 = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X/Z_6, \quad 321 = SU(3)_C \times SU(2)_L \times U(1)_Y/Z_6$$

[Dunsky, Ghoshal, Murayama, Sakakihara, White, 2111.08750, PRD]

't Hooft-Polyakov Monopole

 The 't Hooft-Polyakov monopole is a **static solution** with **finite energy** in a **nonabelian gauge theory** [['t Hooft, NPB 79 \(1974\) 276; Polyakov, JETP Lett. 20 \(1974\) 194](#)]

Consider a $SU(2)$ gauge theory, spontaneously broken to $U(1)_{EM}$ by a $SU(2)$ -triplet Higgs field ϕ^a ($a = 1, 2, 3$; e is elementary electric charge)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{2}(D_\mu\phi)^a(D^\mu\phi)^a - V(\phi), \quad V(\phi) = \frac{\lambda}{4}(|\phi|^2 - v^2)^2$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \textcolor{red}{e} \varepsilon^{abc} A_\mu^b A_\nu^c, \quad D_\mu \phi^a = \partial_\mu \phi^a + \textcolor{red}{e} \varepsilon^{abc} A_\mu^b \phi^c$$

Defining $r \equiv |\mathbf{x}|$, the '**t Hooft-Polyakov monopole**' corresponds to the **ansatz**

$$\phi^a(\mathbf{x}) = \frac{vf(r)x^a}{r}, \quad A^{ai}(\mathbf{x}) = \frac{a(r)\varepsilon^{aij}x^j}{er^2}, \quad f(\infty) = a(\infty) = 1, \quad f(0) = a(0) = 0$$

 Its **mass** (the total energy of the solution) is given by

$$M = \int d^3x \left[\frac{1}{2} B^{ai} B^{ai} + \frac{1}{2} (D^i \phi)^a (D^i \phi)^a + V(\phi) \right], \quad B^{ai} = \frac{1}{2} \varepsilon^{ijk} F^{ajk}$$

Magnetic Charge and Mass

 Rewrite $\frac{1}{2}B^{ai}B^{ai} + \frac{1}{2}(D^i\phi)^a(D^i\phi)^a = \frac{1}{2}[B^{ai} + (D^i\phi)^a]^2 - \nabla \cdot (\mathbf{B}^a\phi^a)$

For vacuum field configuration with **winding number** $n = 1$, Gauss's theorem gives

$$\int d^3x \nabla \cdot (\mathbf{B}^a \phi^a) = \int d\sigma \cdot \mathbf{B}^a \phi^a = Q_M v$$

[Srednicki, *Quantum Field Theory*, Chapter 92]

 $Q_M = -\frac{4\pi}{e}$ is the **magnetic charge** of the '**'t Hooft-Polyakov monopole**' with

$$M = |Q_M|v + \int d^3x \left\{ \frac{1}{2} [B^{ai} + (D^i \phi)^a]^2 + V(\phi) \right\}$$

👉 **Bogomolny bound** $M \geq |Q_M|v$ [Bogomolny, Sov.J.Nucl.Phys. 24 (1976) 449]

In the limit $\lambda \rightarrow 0$ with $B^{ai} = -(D^i\phi)^a$, the **Bogomolny bound** is saturated, leading to **explicit solution** for the '**t Hooft-Polyakov monopole**' with $M = |Q_M|v$:

$$\phi^a = \frac{x^a}{er^2} \left(\frac{evr}{\tanh evr} - 1 \right), \quad A^{ai} = \frac{\varepsilon^{aij} x^j}{er^2} \left(1 - \frac{evr}{\sinh evr} \right)$$

[Prasad & Sommerfield, PRL 35, 760 (1975)]

Generic Monopoles

 The **magnetic charge** of a **generic monopole** with a **winding number n** is

$$Q_M = -\frac{4\pi n}{e}, \quad n \in \mathbb{Z}$$

 Its **mass** is given by $M = |Q_M|v + \int d^3x \left\{ \frac{1}{2} [B^{ai} + \text{sign}(n)(D^i\phi)^a]^2 + V(\phi) \right\}$

 **Bogomolny bound** becomes $M \geq |Q_M|v = \frac{4\pi|n|v}{e}$

 $n = +1$ and $n = -1$ corresponds to the '**t Hooft-Polyakov monopole**' and its **antimonopole**, respectively

 For $\lambda > 0$, a **monopole** with **winding number $n \neq \pm 1$** is **unstable** against breaking up into $|n|$ **monopoles** with **winding number ± 1** , which are **stable**

Generic Monopoles

The **magnetic charge** of a **generic monopole** with a **winding number n** is

$$Q_M = -\frac{4\pi n}{e}, \quad n \in \mathbb{Z}$$

Its **mass** is given by $M = |Q_M|v + \int d^3x \left\{ \frac{1}{2} [B^{ai} + \text{sign}(n)(D^i\phi)^a]^2 + V(\phi) \right\}$

Bogomolny bound becomes $M \geq |Q_M|v = \frac{4\pi|n|v}{e}$

$n = +1$ and $n = -1$ corresponds to the '**t Hooft-Polyakov monopole**' and its **antimonopole**, respectively

For $\lambda > 0$, a **monopole** with **winding number $n \neq \pm 1$** is **unstable** against breaking up into $|n|$ **monopoles** with **winding number ± 1** , which are **stable**

If a **SU(2)-doublet** field is added, then its components have **electric charges** $\pm \frac{e}{2}$, which is the smallest charges; all **possible electric charges** is $Q_E = \frac{je}{2}$, $j \in \mathbb{Z}$

Therefore, the possible **electric** and **magnetic** charges obey $Q_E Q_M = 2\pi k$, $k \in \mathbb{Z}$

This is the **Dirac charge quantization condition** from general considerations

Magnetic Monopoles in GUTs

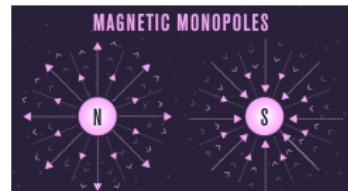
Skateboard icon: The homotopy group relevant to monopoles is $\pi_2(G/H)$

Scooter icon: If $\pi_1(G) = \pi_2(G) = 1$, then $\pi_2(G/H) = \pi_1(H)$

[Vachaspati, hep-ph/0101270]

Bicycle icon: Because of $\pi_1[U(1)] = \mathbb{Z}$, $H = U(1)$ leads to monopoles

Scooter icon: For a GUT with $G \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_{EM}$, generation of stable magnetic monopoles with a typical mass $M \sim 10^{15}$ GeV is inevitable



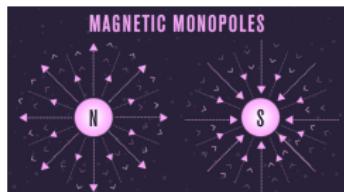
Magnetic Monopoles in GUTs

💡 The **homotopy group** relevant to **monopoles** is $\pi_2(G/H)$

If $\pi_1(G) = \pi_2(G) = 1$, then $\pi_2(G/H) = \pi_1(H)$

[Vachaspati, hep-ph/0101270]

Because of $\pi_1[U(1)] = \mathbb{Z}$, $H = U(1)$ leads to **monopoles**



For a **GUT** with $G \rightarrow \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y \rightarrow \text{SU}(3)_C \times \text{U}(1)_{\text{EM}}$, generation of **stable magnetic monopoles** with a typical mass $M \sim 10^{15} \text{ GeV}$ is **inevitable**

Such magnetic monopoles would be copiously produced in the early universe

[Guth & Tye, PRL 44, 631 (1980)]

They should remain to the **present day** with a **large number density** against the **null observation** [Zel'dovich & Khlopov, PLB **79** (1978) 239; Preskill, PRL **43**, 1365 (1979)]

 This **magnetic-monopole problem** can be solved by assuming that the **cosmic inflation** occurs below the temperature where magnetic monopoles can be produced

Domain Walls

 **Domain walls (DWs)** are two-dimensional topological defects which could be formed when a **discrete symmetry** of the **scalar potential** is spontaneously broken in the early Universe $V(\phi)$

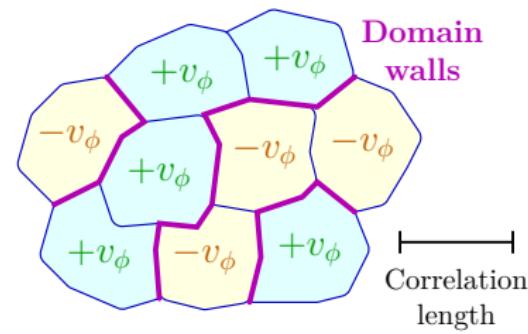
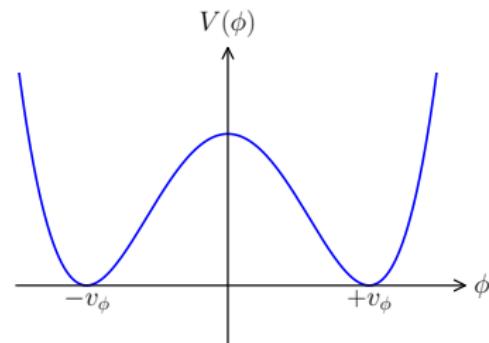
Consider a **real scalar field** $\phi(x)$ with a **spontaneously broken Z_2 symmetry** $\phi \rightarrow -\phi$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi) \partial^\mu \phi - V_0, \quad V_0 = \frac{\lambda}{4}(\phi^2 - v_\phi^2)^2$$

The Z_2 -conserving potential V_0 has two degenerate minima at $\phi = \pm v_\phi$

After the spontaneous symmetry breaking, $\phi(x)$ takes either $+v_\phi$ or $-v_\phi$, and two different domains can appear

 DWs are produced around the **boundary** of the **two domains**



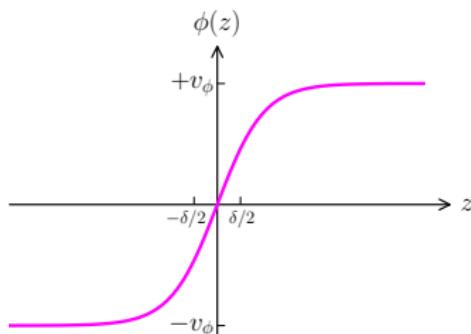
Domain Wall Configuration

Consider a **static planar DW configuration** lying perpendicular to the z -axis

 Solving the **equation of motion** $\frac{d^2\phi}{dz^2} - \frac{dV_0}{d\phi} = 0$ with **boundary conditions**

$\lim_{z \rightarrow +\infty} \phi(z) = \pm v_\phi$, we obtain a **kink solution**

$$\phi(z) = v_\phi \tanh \frac{z}{\delta}, \quad \delta \equiv \left(\sqrt{\frac{\lambda}{2}} v_\phi \right)^{-1}$$



 The DW locates at $z = 0$ with a thickness δ

separating **two domains** with $\phi(z) > 0$ and $\phi(z) < 0$

The **energy-momentum tensor** for this static solution is

$$T^{\mu\nu}(z) = \left[\frac{d^2\phi(z)}{dz^2} \right]^2 \text{diag}(+1, -1, -1, 0)$$

The **DW tension** (surface energy density) is $\sigma = \int_{-\infty}^{+\infty} dz T^{00}(z) = \frac{4}{3} \sqrt{\frac{\lambda}{2}} v_\phi^3$

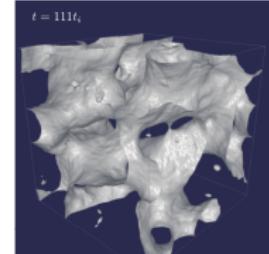
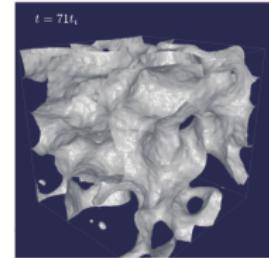
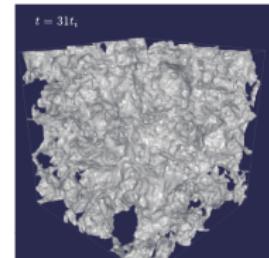
[Saikawa, 1703.02576, Universe]

Evolution of Domain Walls

After DWs are created, the tension σ acts to stretch them up to the horizon size if the friction F_f is small, and they would enter the scaling regime with energy density $\rho_{\text{DW}} = \frac{\mathcal{A}\sigma}{t}$

[Press, Ryden, Spergel, ApJ 347, 590 (1989)]

$\mathcal{A} \approx 0.8 \pm 0.1$ is a numerical factor given by lattice simulation
[Hiramatsu, Kawasaki, Saikawa, 1309.5001, JCAP]



[Hiramatsu *et al.*, 1002.1555, JCAP]

Evolution of Domain Walls

↗ After **DWs** are created, the **tension** σ acts to **stretch** them up to the **horizon size** if the **friction** F_f is **small**, and they would enter the **scaling regime** with **energy density** $\rho_{\text{DW}} = \frac{\mathcal{A}\sigma}{t}$

[Press, Ryden, Spergel, ApJ 347, 590 (1989)]

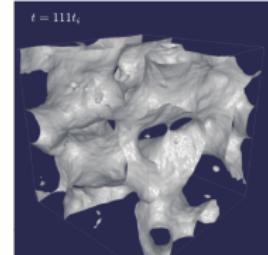
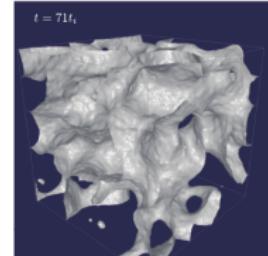
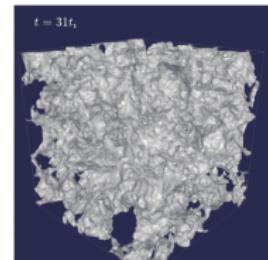
📊 $\mathcal{A} \approx 0.8 \pm 0.1$ is a numerical factor given by lattice simulation
[Hiramatsu, Kawasaki, Saikawa, 1309.5001, JCAP]

⚡ This implies that **DWs** are **diluted more slowly** than **radiation** ($\rho_r \propto t^{-2}$) and **matter** ($\rho_m \propto t^{-3/2}$) as the Universe expands

🏔 If DWs are **stable**, they would soon **dominate** the Universe with a **state parameter** $w = \frac{p_{\text{DW}}}{\rho_{\text{DW}}} = -\frac{2}{3}$

🌋 This implies that the **scale factor** evolves as $a(t) \propto t^2$; such a **rapid expansion** is **incompatible** with **standard cosmology**

🚫 Therefore, **stable DWs** results in a **cosmological problem**
[Zeldovich, Kobzarev, Okun, Zh.Eksp.Teor.Fiz. 67 (1974) 3] [Hiramatsu et al., 1002.1555, JCAP]



Biased Domain Walls



It is **allowed** if **DWs collapse** at a very early epoch [Vilenkin, PRD 23 (1981) 852; Gelmini, Gleiser, Kolb, PRD 39 (1989) 1558; Larsson, Sarkar, White, hep-ph/9608319, PRD]

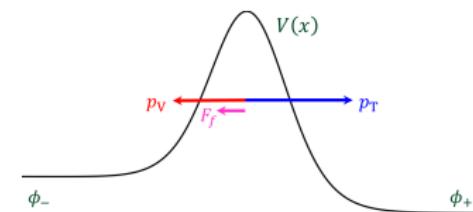
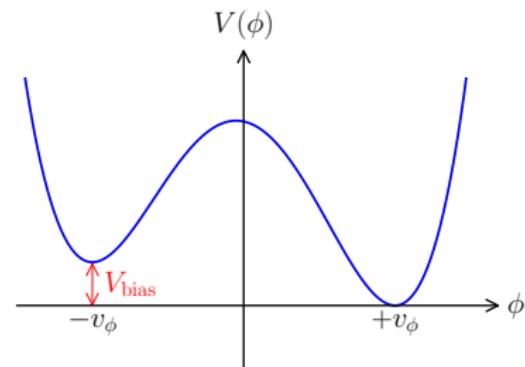
Such **unstable DWs** can be realized if the Z_2 symmetry is **explicitly broken** by a **small potential term**

$$V_1 = \epsilon v_\phi \phi \left(\frac{\phi^2}{3} - v_\phi^2 \right)$$

This gives an **energy bias** among the two minima of the potential:

$$V_{\text{bias}} \equiv V(-v_\phi) - V(+v_\phi) = \frac{4}{3} \epsilon v_\phi^4$$

The potential bias provides a **pressure** $p_V \sim V_{\text{bias}}$ acting on the DWs, against the **tension force per unit area** $p_T \sim \rho_{\text{DW}} \propto T^2$



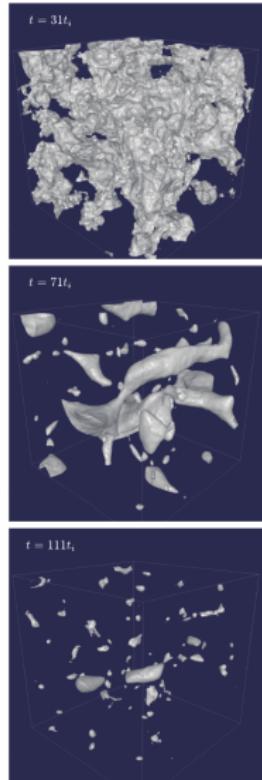
Collapsing Domain Walls and Gravitational Waves

Since $p_T \propto T^2$, the **pressure** p_V would eventually **surpass** the **tension force** at **sufficient low temperatures**

This makes **DWs collapse** and false vacuum domains shrink

The **annihilation temperature** T_{ann} , at which **DWs collapse**, can be estimated by solving $p_V(T_{\text{ann}}) \simeq p_T(T_{\text{ann}})$:

$$T_{\text{ann}} = \frac{34.1 \text{ MeV}}{\sqrt{\mathcal{A}}} \left[\frac{g_*(T_{\text{ann}})}{10} \right]^{-1/4} \left(\frac{\sigma}{\text{TeV}^3} \right)^{-1/2} \left(\frac{V_{\text{bias}}}{\text{MeV}^4} \right)^{1/2}$$



[Hiramatsu *et al.*, 1002.1555, JCAP]

Collapsing Domain Walls and Gravitational Waves

Since $p_T \propto T^2$, the **pressure** p_V would eventually **surpass** the **tension force** at **sufficient low temperatures**

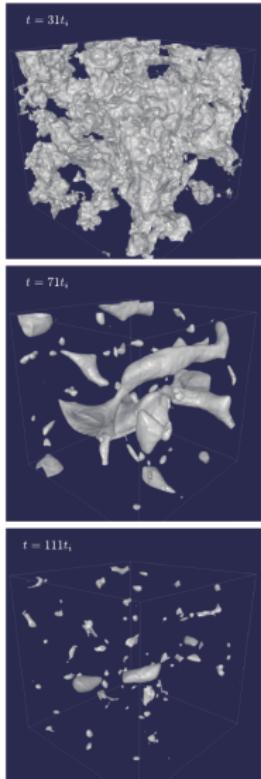
This makes **DWs collapse** and false vacuum domains shrink

The **annihilation temperature** T_{ann} , at which **DWs collapse**, can be estimated by solving $p_V(T_{\text{ann}}) \simeq p_T(T_{\text{ann}})$:

$$T_{\text{ann}} = \frac{34.1 \text{ MeV}}{\sqrt{\mathcal{A}}} \left[\frac{g_*(T_{\text{ann}})}{10} \right]^{-1/4} \left(\frac{\sigma}{\text{TeV}^3} \right)^{-1/2} \left(\frac{V_{\text{bias}}}{\text{MeV}^4} \right)^{1/2}$$

It is expected that such **collapsing domain walls** produce **Gravitational Waves** (GWs) [Preskill *et al.*, NPB 363 (1991) 207; Gleiser, Roberts, astro-ph/9807260, PRL]

A **stochastic gravitational wave background (SGWB)** could be formed and remain to the present time



[Hiramatsu *et al.*, 1002.1555, JCAP]

SGWB Spectrum from Collapsing DWs

The **SGWB spectrum** is commonly characterized by $\Omega_{\text{GW}}(f) = \frac{f}{\rho_c} \frac{d\rho_{\text{GW}}}{df}$

ρ_{GW} is the **GW energy density**, and ρ_c is the critical energy density

The SGWB from **collapsing DWs** can be estimated by **numerical simulations**

[Hiramatsu, Kawasaki, Saikawa, 1002.1555, 1309.5001, JCAP]

The **present SGWB spectrum** induced by collapsing DWs can be evaluated by

$$\Omega_{\text{GW}}(f) h^2 = \Omega_{\text{GW}}^{\text{peak}} h^2 \times \begin{cases} \left(\frac{f}{f_{\text{peak}}}\right)^3, & f < f_{\text{peak}} \\ \frac{f_{\text{peak}}}{f}, & f > f_{\text{peak}} \end{cases}$$

$$\Omega_{\text{GW}}^{\text{peak}} h^2 = 7.2 \times 10^{-18} \tilde{\epsilon}_{\text{GW}} \mathcal{A}^2 \left[\frac{g_{*s}(T_{\text{ann}})}{10} \right]^{-4/3} \left[\frac{\sigma(T_{\text{ann}})}{\text{TeV}^3} \right]^2 \left(\frac{T_{\text{ann}}}{10 \text{ MeV}} \right)^{-4}$$

$$f_{\text{peak}} = 1.1 \times 10^{-9} \text{ Hz} \left[\frac{g_*(T_{\text{ann}})}{10} \right]^{1/2} \left[\frac{g_{*s}(T_{\text{ann}})}{10} \right]^{-1/3} \frac{T_{\text{ann}}}{10 \text{ MeV}}$$

$\tilde{\epsilon}_{\text{GW}} = 0.7 \pm 0.4$ is derived from numerical simulation

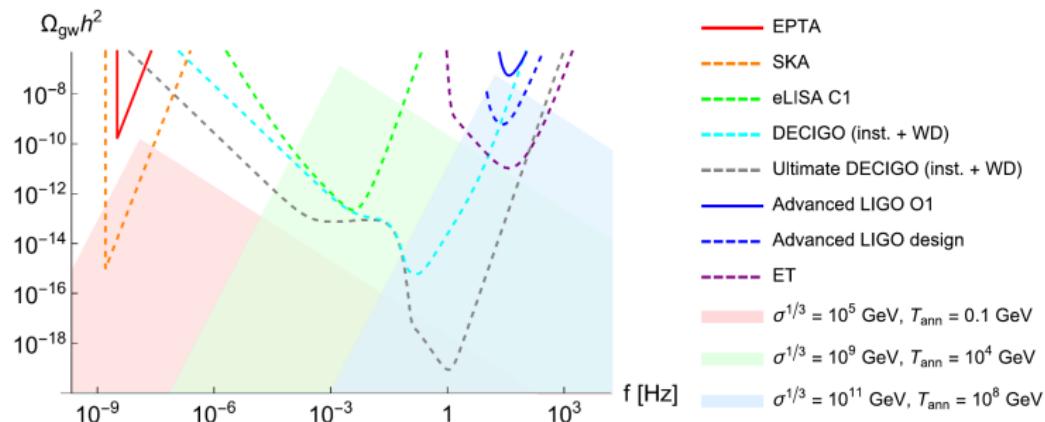
SGWB Spectra Compared to Sensitivity Curves

 By adjusting the **DW tension σ** and the **potential bias V_{bias}** , the **SGWB spectra** from **collapsing DWs** could fall in the sensitivity bands of various GW experiments

 **Pulsar timing arrays (PTAs)** in 10^{-9} – 10^{-7} Hz: **NANOGrav, PPTA, EPTA, CPTA, IPTA, SKA, ...**

 **Space-borne interferometers** in 10^{-4} – 10^0 Hz: **LISA, TianQin, Taiji, BBO, DECIGO, ...**

 **Ground-based interferometers** in 10^0 – 10^4 Hz: **LIGO, Virgo, KAGRA, CE, ET, ...**



[Saikawa, 1703.02576, Universe]

Strong Evidence for a nHz SGWB from PTAs

On June 29, 2023, four **PTA collaborations**

NANOGrav [2306.16213, ApJL; 2306.16219, ApJL],
CPTA [2306.16216, RAA], **PPTA** [2306.16215, ApJL],
 and **EPTA** [2306.16214, 2306.16227, A&A] reported
strong evidence for a **nHz SGWB** with expected
Hellings–Downs correlations

Potential **GW sources** include

Supermassive black hole binaries

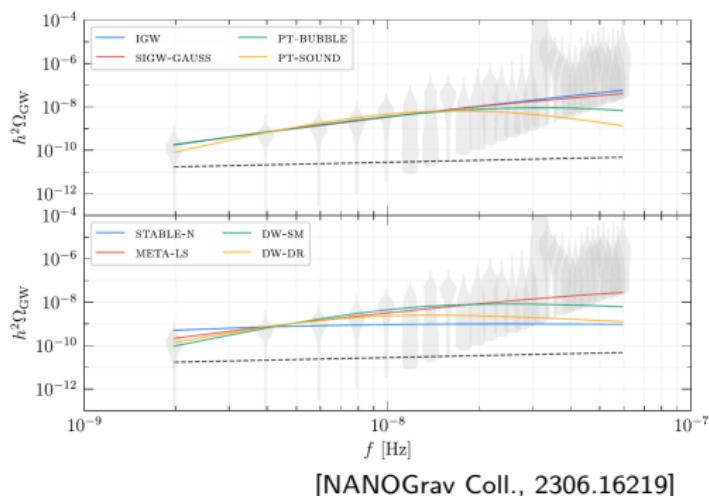
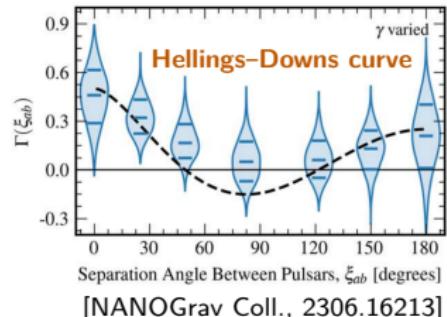
Inflation

Scalar-induced GWs

First-order phase transitions

Cosmic strings

Collapsing domain walls



nHz SGWB from DWs and Freeze-in Dark Matter

For interpreting the **nHz SGWB observation**, we assume that it comes from **collapsing DWs** arising from the **spontaneous breaking** of a Z_2 **symmetry** in a scalar field theory

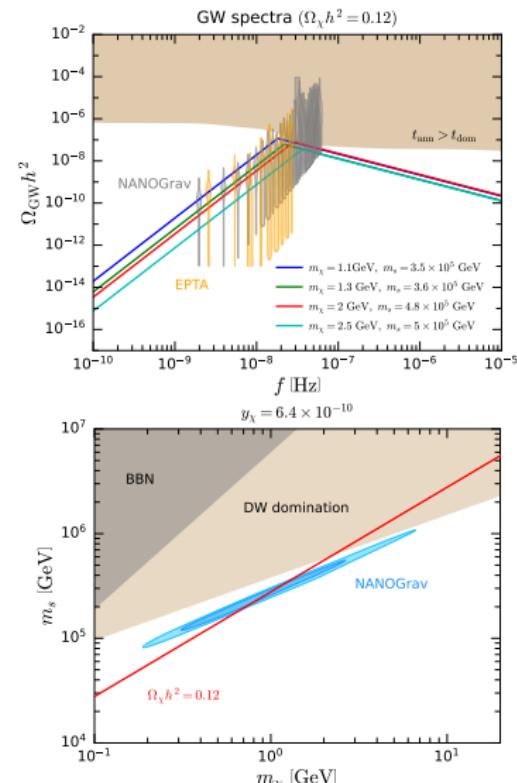
A **tiny Z_2 -violating potential term** with $\epsilon \sim \mathcal{O}(10^{-26})$ is required to reproduce the result

We propose that this Z_2 -violating potential is **radiatively induced** by a **feeble Yukawa coupling** y_χ between the scalar field and a **fermion field** χ

It is also responsible for **dark matter (DM)** production via the **freeze-in mechanism**

Combining the **PTA data** and the **observed DM relic density**, the model parameters can be narrowed down to small ranges

[Z Zhang, CF Cai, YH Su, SY Wang, **ZHY**, HH Zhang, 2307.11495, PRD]



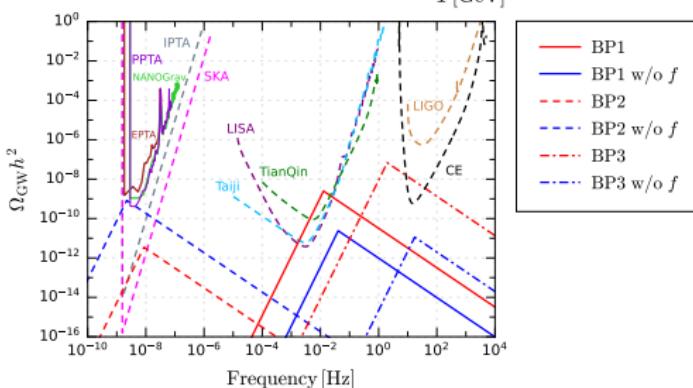
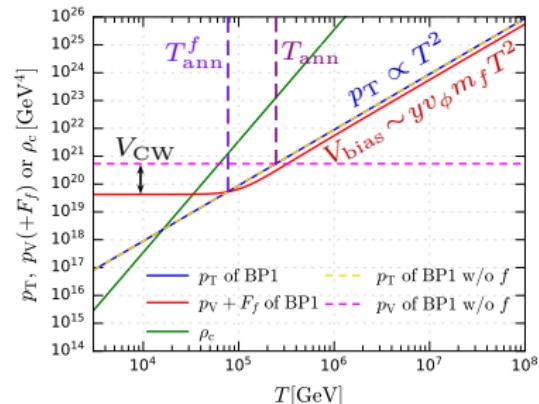
Z₂-violating Coupling to Thermalized Fermions

 We study DWs formed through spontaneous breaking of an approximate Z_2 symmetry

Dynamics of DWs is influenced by **quantum** and **thermal corrections** induced by a Z_2 -**violating coupling** to **thermalized fermions**

 The thermal effects make the **potential bias** V_{bias} dependent on the **temperature** and may lead to notable variations in the **DW annihilation temperature** T_{ann} , in addition to the shift caused by **Coleman-Weinberg corrections**

 This could substantially alter the **SGWB spectrum** produced by DWs, providing observable signatures for future GW detection experiments



[QQ Zeng, X He, ZHY, JM Zheng, 2501.10059, PRD]

Cosmic Strings from U(1) Gauge Symmetry Breaking

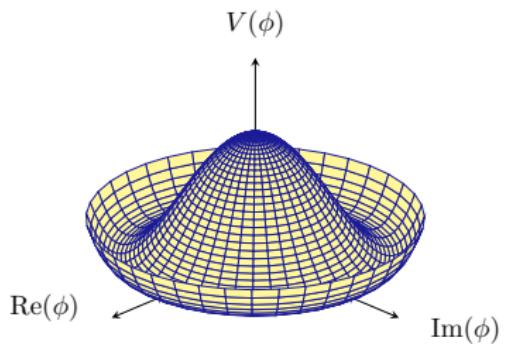
Consider the **Abelian Higgs model** with a **complex scalar field** ϕ

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} (D^\mu \phi)^\dagger (D_\mu \phi) - V(\phi), \quad V(\phi) = \frac{\lambda_\phi}{4} (|\phi|^2 - v_\phi^2)^2$$

The covariant derivative of ϕ is $D_\mu \phi = (\partial_\mu - i g A_\mu) \phi$

The field strength tensor of the **U(1) gauge field** A^μ is $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

The Mexican-hat potential $V(\phi)$ leads to **degenerate vacua** $\langle\phi\rangle = v_\phi e^{i\theta}/\sqrt{2}$



Cosmic Strings from U(1) Gauge Symmetry Breaking

Consider the **Abelian Higgs model** with a **complex scalar field** ϕ

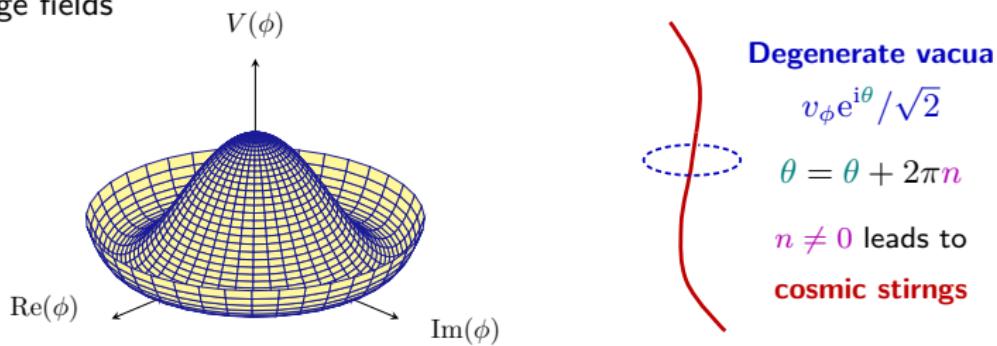
$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} (D^\mu \phi)^\dagger (D_\mu \phi) - V(\phi), \quad V(\phi) = \frac{\lambda_\phi}{4} (|\phi|^2 - v_\phi^2)^2$$

- The covariant derivative of ϕ is $D_\mu \phi = (\partial_\mu - i g A_\mu) \phi$

The field strength tensor of the **$U(1)$ gauge field A^μ** is $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

The **Mexican-hat potential** $V(\phi)$ leads to **degenerate vacua** $\langle\phi\rangle \equiv v_\phi e^{i\theta}/\sqrt{2}$

The spontaneous breaking of the **$U(1)$ gauge symmetry** in the early Universe would induce **cosmic strings (CSs)**, which are concentrated with energies of scalar and gauge fields



Soliton Configuration

The **ansatz** for a **static solution (soliton)** representing a **string** along the z axis is

$$\phi(\rho, \varphi) = v_\phi f(\rho) U(\varphi), \quad \mathbf{A}(\rho, \varphi) = \frac{i}{q} a(\rho) U(\varphi) \nabla U^\dagger(\varphi)$$

$$U(\varphi) \equiv e^{in\varphi}, \quad f(\infty) = a(\infty) = 1, \quad f(0) = a(0) = 0$$

 (ρ, φ) are polar coordinates in the xy plane

 $n \in \mathbb{Z}$ is the **winding number**

 The **soliton** with $n = 1$ is called a **Nielsen-Olesen vortex**

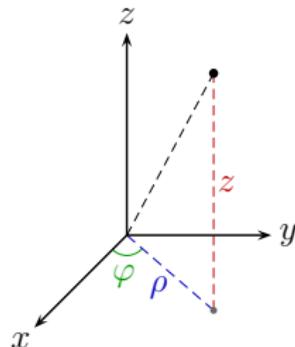
[Nielsen & Olesen, NPB **61** (1973) 45]

 The **energy per unit length** of the **soliton** is [Srednicki, *Quantum Field Theory*]

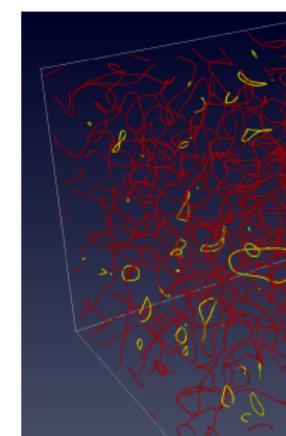
$$\mu = 2\pi v_\phi^2 \int_0^\infty d\rho \rho \left[f'^2 + \frac{n^2}{\rho^2} (a-1)^2 f^2 + \frac{\lambda v_\phi^2}{4} (f^2 - 1)^2 + \frac{n^2 a'^2}{g^2 v_\phi^2 \rho^2} \right]$$

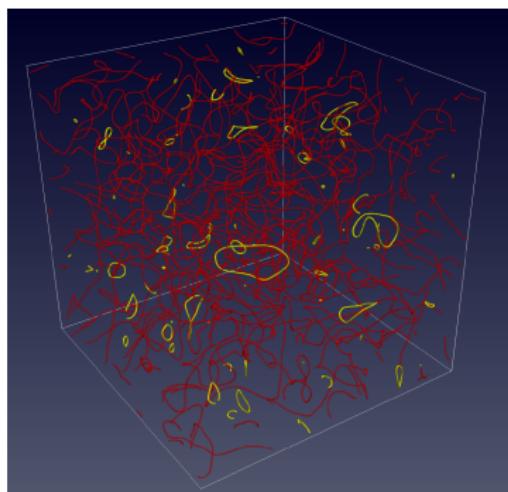
For $\lambda_\phi > g^2$, one can prove a **Bogomolny bound** $\mu > 2\pi v_\phi^2 |n|$

A **soliton** with **winding number** $n \neq \pm 1$ is **unstable** against breaking up into $|n|$ **stable solitons**, each with **winding number** ± 1



Cosmic Strings

- We can **translate** and **boost** a **stable soliton** with $n = \pm 1$
 - It behaves like a **particle** in **two space dimensions**
 - In **three space dimensions**, the soliton becomes a **Nielsen–Olesen string**
 - It is a structure **localized in two directions**, but **extended in the third**
 - Such strings can **bend**, and even form **closed loops**
 - When they are formed in the early universe, we call them **cosmic strings**
 - Therefore, a **network** of **cosmic strings** would be formed after the **spontaneous breaking** of the **$U(1)$ gauge symmetry**



[Kitajima, Nakayama, 2212.13573, JHEP]

Cosmic String Tension



The **tension** of cosmic string (energy per unit length) can be estimated as

$$\mu \simeq \begin{cases} 1.19\pi v_\phi^2 b^{-0.195}, & 0.01 < b < 100, \\ \frac{2.4\pi v_\phi^2}{\ln b}, & b > 100, \end{cases} \quad b \equiv \frac{2g^2}{\lambda_\phi}$$

[Hill, Hodges, Turner, PRD 37, 263 (1988)]



As $\mu \propto v_\phi^2$, a **high symmetry-breaking scale** v_ϕ would lead to cosmic strings with **high tension**



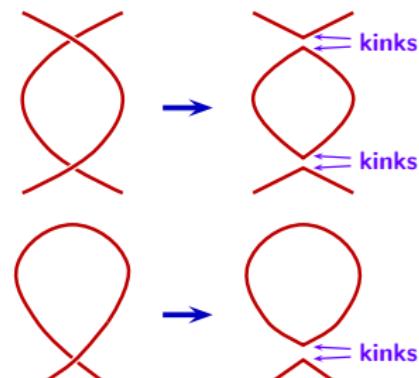
Denoting G as the **Newtonian constant of gravitation**, the **dimensionless quantity** $G\mu$ is commonly used to describe the **CS tension**

Gravitational Waves from Cosmic Strings

According to the analysis of string dynamics, the intersections of long strings could produce closed loops, whose size is smaller than the Hubble radius

Cosmic string loops could further fragment into smaller loops or reconnect to long strings

Loops typically have localized features called “cusps” and “kinks”



Gravitational Waves from Cosmic Strings

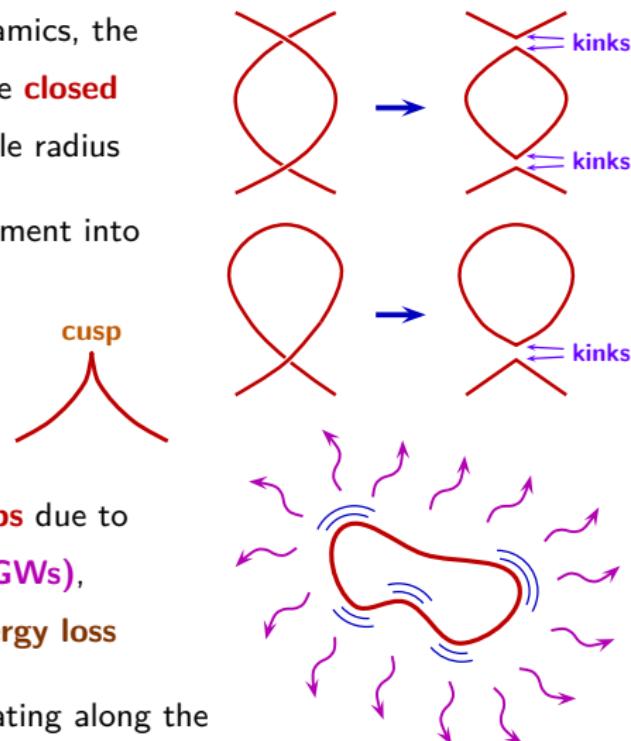
According to the analysis of string dynamics, the intersections of long strings could produce closed loops, whose size is smaller than the Hubble radius

Cosmic string loops could further fragment into smaller loops or reconnect to long strings

Loops typically have localized features called "cusps" and "kinks"

The relativistic oscillations of the loops due to their tension emit Gravitational Waves (GWs), and the loops would shrink because of energy loss

Moreover, the cusps and kinks propagating along the loops could produce GW bursts [Damour & Vilenkin, gr-qc/0004075, PRL]



Power of Gravitational Radiation

🎻 At the **emission time** t_e , a **cosmic string loop** of **length** l emits GWs with **frequencies** $f_e = \frac{2n}{l}$

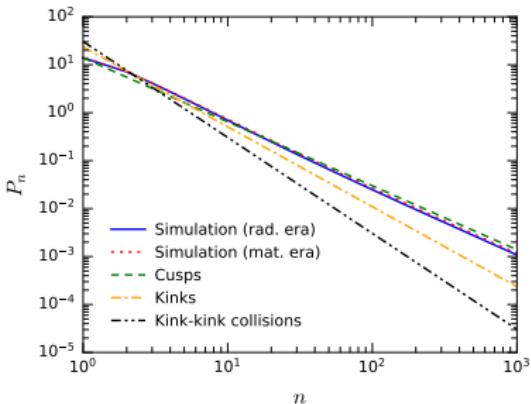
🎵 $n = 1, 2, 3, \dots$ denotes the **harmonic modes** of the loop oscillation

🎺 Denoting P_n as the **power of gravitational radiation** for the harmonic mode n in units of $G\mu^2$, the total power is given by $P = G\mu^2 \sum_n P_n$

🎹 According to the **simulation** of **smoothed cosmic string loops** [Blanco-Pillado & Olum, 1709.02693, PRD], P_n for loops in the **radiation** and **matter** eras are obtained

🥁 The **total dimensionless power** $\Gamma = \sum_n P_n$ is estimated to be ~ 50

🎸 For comparison, analytic studies imply $P_n \simeq \frac{\Gamma}{\zeta(q)n^q}$ with $q = \frac{4}{3}, \frac{5}{3}, 2$ for **cusps**, **kinks**, and **kink-kink collisions**



Stochastic GW Background Induced by Cosmic Strings

 The **energy** of **cosmic strings** is converted into the **energy** of **GWs**, and an **SGWB** is formed due to **incoherent superposition**

 The **SGWB energy density** ρ_{GW} per unit frequency at the present is

$$\frac{d\rho_{\text{GW}}}{df} = G\mu^2 \int_{t_{\text{ini}}}^{t_0} a^5(t) \sum_n \frac{2n P_n}{f^2} n_{\text{CS}} \left(\frac{2na(t)}{f}, t \right) dt$$

 $n_{\text{CS}}(l, t)$ is the **number density per unit length** of **CS loops** with length l at cosmic time t

 $a(t)$ is the **scale factor** satisfying $\frac{da(t)}{dt} = a(t)H(t)$ and $a(t_0) = 1$

 $H(t)$ is the **Hubble rate** and t_{ini} is the cosmic time when the GW emissions start

 The **SGWB spectrum** is commonly represented by

$$\Omega_{\text{GW}}(f) = \frac{f}{\rho_c} \frac{d\rho_{\text{GW}}}{df}, \quad \rho_c \equiv \frac{3H_0^2}{8\pi G}$$

Velocity-dependent One-scale Model

The evolution of the **CS network** can be described using the **velocity-dependent one-scale (VOS) model** [Martins & Shellard, hep-ph/9507335, PRD]

The parameters are the **correlation length L** and the **root-mean-square velocity v** of string segments; the **energy density** of **long strings** is expressed as $\rho = \mu/L^2$

Introducing a **dimensionless quantity** $\xi \equiv L/t$, the evolution equations are

$$t\dot{\xi} = H(1+v^2)t\xi - \xi + \frac{1}{2}\tilde{c}v, \quad t\dot{v} = (1-v^2)\left[\frac{k(v)}{\xi} - 2Htv\right]$$

$$\tilde{c} \simeq 0.23, \quad k(v) = \frac{2\sqrt{2}}{\pi}(1-v^2)(1+2\sqrt{2}v^3)\frac{1-8v^6}{1+8v^6}$$

The solutions converge to **constant values** [Marfatia & YL Zhou, 2312.10455, JHEP]:

$$\xi_r = 0.271, \quad v_r = 0.662, \quad \text{radiation-dominated (RD) era}$$

$$\xi_m = 0.625, \quad v_m = 0.582, \quad \text{matter-dominated (MD) era}$$

This implies that the CS network quickly evolves into a **linear scaling regime** characterized by $L \propto t$

Loop Production Functions

The **CS loop number density** is given by $n_{\text{CS}}(l, t) = \frac{1}{a^3(t)} \int_{t_{\text{ini}}}^t \mathcal{P}(l', t') a^3(t') dt'$

Motivated by **numerical simulations** [Blanco-Pillado, Olum & Shlaer, 1309.6637, PRD], the **loop production functions** can be approximated as

$$\mathcal{P}_r(l, t) = \frac{\mathcal{F}_r \tilde{c} v \delta(\alpha_r \xi - l/t)}{\gamma_v \alpha_r \xi^4 t^5}, \quad \text{RD era}$$

$$\mathcal{P}_m(l, t) = \frac{\mathcal{F}_m \tilde{c} v \Theta(\alpha_m \xi - l/t)}{\gamma_v (l/t)^{1.69} \xi^3 t^5}, \quad \text{MD era}$$

$\gamma_v = (1 - v^2)^{-1/2}$ is the Lorentz factor

At the **loop production time** t_* , we have

$$l_* = l + \Gamma G \mu (t - t_*), \quad \alpha_r \xi_* \simeq 0.1 \text{ and } \alpha_m \xi_* \simeq 0.18$$

Adopting $\mathcal{F}_r = 0.1$ and $\mathcal{F}_m = 0.36$, the obtained **loop number densities** in the **RD** and **MD eras** **agrees** with the **simulation results** in the **scaling regime**

Loop Production Functions

Orange: The **CS loop number density** is given by $n_{\text{CS}}(l, t) = \frac{1}{a^3(t)} \int_{t_{\text{ini}}}^t \mathcal{P}(l', t') a^3(t') dt'$

Green: Motivated by **numerical simulations** [Blanco-Pillado, Olum & Shlaer, 1309.6637, PRD], the **loop production functions** can be approximated as

$$\mathcal{P}_r(l, t) = \frac{\mathcal{F}_r \tilde{c} v \delta(\alpha_r \xi - l/t)}{\gamma_v \alpha_r \xi^4 t^5}, \quad \text{RD era}$$

$$\mathcal{P}_m(l, t) = \frac{\mathcal{F}_m \tilde{c} v \Theta(\alpha_m \xi - l/t)}{\gamma_v (l/t)^{1.69} \xi^3 t^5}, \quad \text{MD era}$$

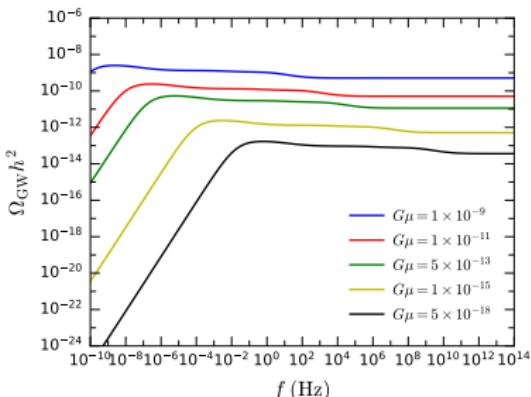
Apple: $\gamma_v = (1 - v^2)^{-1/2}$ is the Lorentz factor

Cherry: At the **loop production time** t_* , we have

$$l_* = l + \Gamma G \mu (t - t_*), \quad \alpha_r \xi_* \simeq 0.1 \text{ and } \alpha_m \xi_* \simeq 0.18$$

Pear: Adopting $\mathcal{F}_r = 0.1$ and $\mathcal{F}_m = 0.36$, the obtained **loop number densities** in the **RD** and **MD eras** **agrees** with the **simulation results** in the **scaling regime**

Avocado: The **SGWB spectra** in the Λ CDM **cosmological model** can be calculated



Scaling Loop Number Density: BOS model

- 🍇 There are other approaches for modeling the $n_{\text{CS}}(l, t)$ in the **scaling regime**
- 🍎 The **BOS model** [Blanco-Pillado, Olum & Shlaer, 1309.6637, PRD] extrapolates the loop production function found in simulations of Nambu-Goto strings
- 🍏 The loop number densities produced in the **radiation** and **matter** era, and that **produced in the radiation era and still surviving in the matter era** are given by

$$n_{\text{CS}}^{\text{r}}(l, t) \simeq \frac{0.18 \theta(0.1t - l)}{t^4(\gamma + \gamma_d)^{5/2}}$$

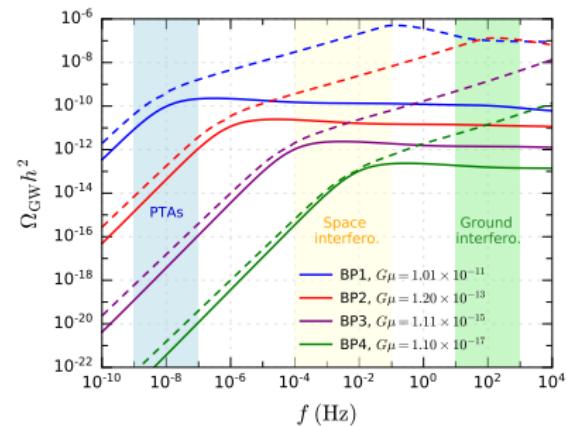
$$n_{\text{CS}}^{\text{m}}(l, t) \simeq \frac{(0.27 - 0.45\gamma^{0.31}) \theta(0.18t - l)}{t^4(\gamma + \gamma_d)^2}$$

$$n_{\text{CS}}^{\text{r} \rightarrow \text{m}}(l, t) \simeq \frac{0.18 t_{\text{eq}}^{1/2} \theta(0.09t_{\text{eq}} - \gamma_d t - l)}{t^{9/2}(\gamma + \gamma_d)^{5/2}}$$

berries $\gamma \equiv \frac{l}{t}$ is a **dimensionless variable**

banana $\gamma_d = -\frac{dl}{dt} \simeq \Gamma G \mu$ is the **loop shrinking rate**

orange $t_{\text{eq}} = 51.1 \pm 0.8 \text{ kyr}$ is the cosmic time at the **matter-radiation equality**



BOS model: solid lines

Scaling Loop Number Density: LRS model

 The **LRS model** [Lorenz, Ringeval & Sakellariadou, 1006.0931, JCAP] takes into account the **gravitational backreaction effect**, which prevents loop production below a certain scale $\gamma_c \simeq 20(G\mu)^{1+2x}$ [Polchinski & Rocha, gr-qc/0702055, PRD]

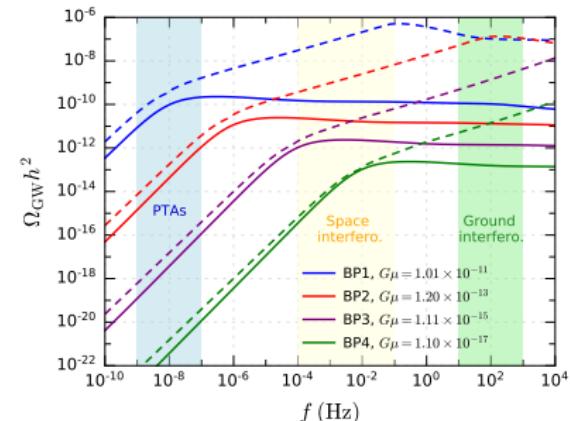
$$n_{CS}(l, t) \simeq \begin{cases} \frac{C}{t^4(\gamma + \gamma_d)^{3-2x}}, & \gamma_d < \gamma \\ \frac{(3\nu - 2\chi - 1)C}{2t^4(1-\chi)\gamma_d\gamma^{2(1-\chi)}}, & \gamma_c < \gamma < \gamma_d \\ \frac{(3\nu - 2\chi - 1)C}{2t^4(1-\chi)\gamma_d\gamma_c^{2(1-\chi)}}, & \gamma < \gamma_c \end{cases}$$

 **RD era:** $\nu = 1/2$, $C \simeq 0.0796$, $\chi \simeq 0.2$

 **MD era:** $\nu = 3/2$, $C \simeq 0.0157$, $\chi \simeq 0.295$

 Smaller $G\mu$ means smaller GW emission power, and loops could survive longer, leading to **more smaller loops** radiating at **higher f**

 The **LRS model** gives a **very high number density** of **small loops** in the $\gamma < \gamma_c$ regime, which significantly contribute to **high frequency GWs**



LRS model: dashed lines

GWs from Cosmic Strings Associated with pNGB Dark Matter

We study the SGWB from cosmic strings

generated in a UV-complete model for pNGB

DM with a spontaneously broken U(1)_X

gauge symmetry [DY Liu, CF Cai, XM Jiang,

[ZH_Y, HH Zhang, 2208.06653, JHEP]

 The DM candidate in this model can naturally evade direct detection bounds

The **bound** on the **DM lifetime** implies a symmetry-breaking scale $v_\Phi > 10^9$ GeV

GWs from Cosmic Strings Associated with pNGB Dark Matter

We study the **SGWB** from **cosmic strings** generated in a UV-complete model for **pNGB DM** with a **spontaneously broken $U(1)_X$** **gauge symmetry** [DY Liu, CF Cai, XM Jiang, ZHY, HH Zhang, 2208.06653, JHEP]

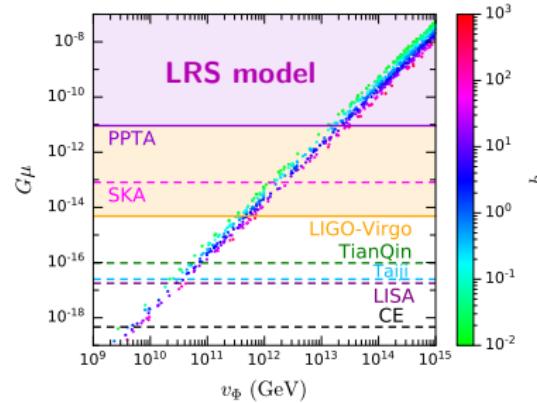
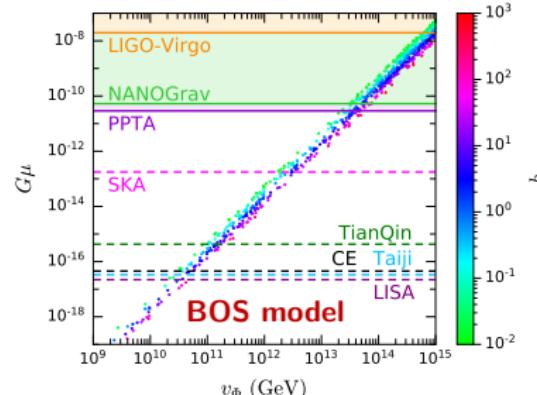
 The DM candidate in this model can naturally evade direct detection bounds

牤 The **bound** on the **DM lifetime** implies a symmetry-breaking scale $v_\Phi > 10^9 \text{ GeV}$

.Constraint from **LIGO-Virgo**, **NANOGrav**,
and **PPTA** have excluded the parameter points
with $v_\Phi \gtrsim 5 \times 10^{13}$ (7×10^{11}) GeV

🦄 The future experiment **LISA** (**CE**) can probe v_Φ down to $\sim 2 \times 10^{10}$ (5×10^9) GeV assuming the **BOS** (**LRS**) model for loop production

[ZY Qiu, ZHY, 2304.02506, CPC]

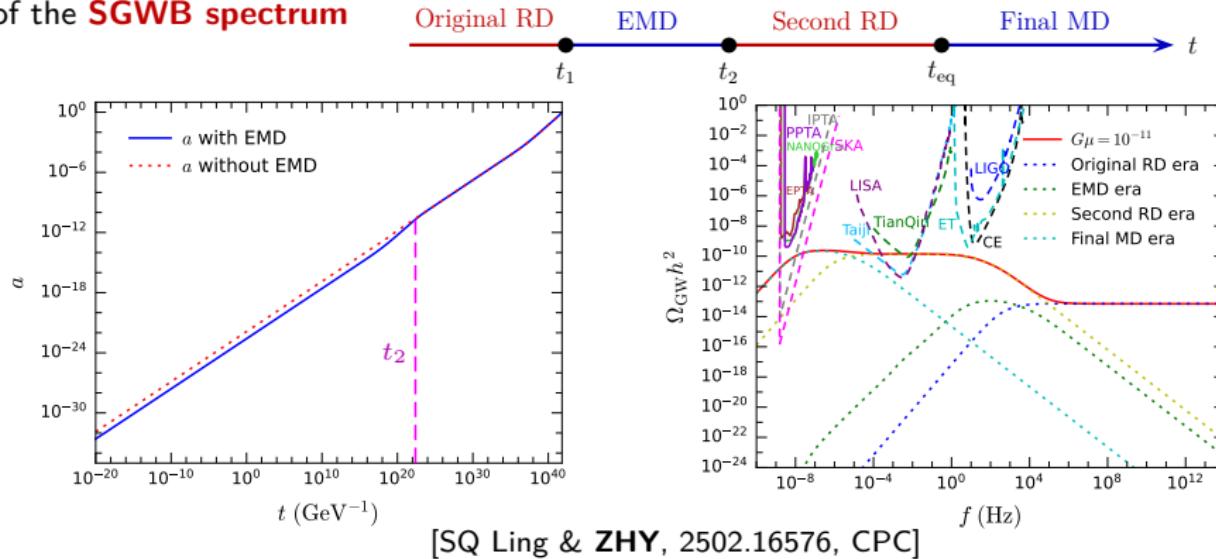


Early Matter-dominated Era and Cosmic String GWs

We investigate the influence of an **early matter-dominated (EMD) era** in cosmic history on the **dynamics** of **cosmic strings** based on the **VOS model**

For a particle model related to the **DM dilution mechanism**, we analyze the modifications to the **cosmological scale factor** and the **CS loop number density**

The **EMD era** causes a **characteristic suppression** in the **high-frequency regime** of the **SGWB spectrum**



[SQ Ling & ZHY, 2502.16576, CPC]

Hybrid Topological Defects



For a field theory, such as a **GUT**, where **multiple stages of symmetry breaking** give rise to **topological defects**, **hybrid defects** may form [Vilenkin & Shellard, *Cosmic Strings and Other Topological Defects*; Dunsky, et al., 2111.08750, PRD]



Monopoles attach to **cosmic strings** [Vilenkin, NPB **196** (1982) 240]

$$G \xrightarrow{\text{monopoles}} H \times U(1) \xrightarrow{\text{strings}} H, \quad \pi_1(G/H) = 1$$

- ① **Monopoles** form when G breaks to a subgroup containing a **$U(1)$ symmetry**
- ② **Strings** form and connect to **monopoles** when this **$U(1)$ symmetry** is later broken



Cosmic strings attach to **domain walls** [Kibble, Lazarides, Shafi, PRD **26** (1982) 435]

$$G \xrightarrow{\text{strings}} H \times \mathbb{Z}_2 \xrightarrow{\text{walls}} H, \quad \pi_0(G/H) = 1$$

- ① **Strings** form when G breaks to a subgroup containing a **discrete symmetry** with $\pi_1[G/(H \times \mathbb{Z}_2)] \supset \pi_0(H \times \mathbb{Z}_2) \neq 1$
- ② **Walls** form and connect to **strings** when the same **discrete symmetry** associated with the strings is broken

Strings Eating Monopoles

 Such **hybrid defects** are **unstable**, with **one defect “eating” the other** via the conversion of the **rest mass** of **the latter** into the **kinetic energy** of **the former**, and subsequently, **decaying** via **gravitational waves**

 **Strings eating monopoles** [Lazarides, Shafi, Walsh, NPB **195** (1982) 157]

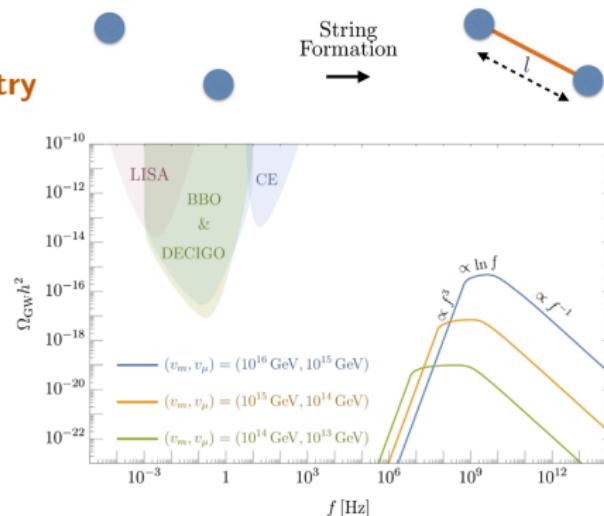
 A monopole network form at a scale v_m

 At temperatures below the **string symmetry**

breaking scale v_μ , the magnetic field of the monopoles squeezes into **flux tubes**

(cosmic strings) connecting each monopole-antimonopole pair

 **GW emission** occurs in a burst, **peaking** at **high frequencies** corresponding to the monopole-antimonopole separation distance at $T \simeq v_\mu$



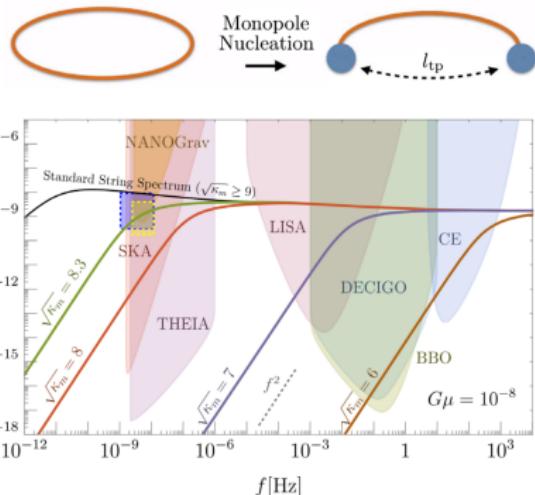
[Dunsky, et al., 2111.08750, PRD]

Monopoles Eating Strings



Monopoles eating strings (metastable cosmic strings)

- ⌚ The symmetry breaking chains are the same as in the previous case
- 🥚 Inflation occurs after monopole formation but before string formation
- 🥞 Because of the absence of monopoles, a normal string network forms
- 🌮 Nevertheless, the strings of tension μ can later become bounded by monopoles of mass m by the Schwinger nucleation of monopole-antimonopole pairs, which cut the strings into pieces bounded by monopoles
- 🚬 Conversion of the string rest mass into the monopole kinetic energy leads to relativistic oscillations of the monopoles before the system decays via gravitational radiation and monopole annihilation ($\kappa_m = m^2/\mu$)



[Dunsky, et al., 2111.08750, PRD]



breaking scale v_σ , walls fill in the space between **strings**



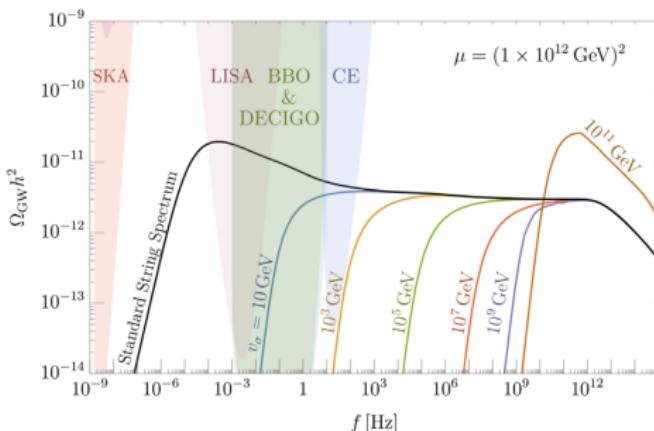
A wall-string network forms, evolves, and finally collapses and decays



 Prior to wall domination at t_* , the **wall-string network** behaves similarly to a **pure string network**, resulting in the **GW spectrum** $\Omega_{\text{GW}} \propto f^0$ at **high frequencies**.



After the network collapses and the largest string-bounded walls decay, Ω_{GW} drops as f^3 at low frequencies



[Dunsky, et al., 2111.08750, PRD]

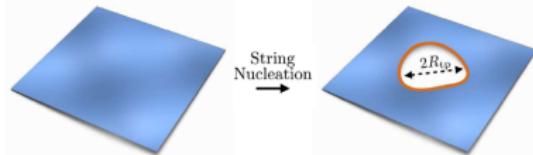
Strings Eating Domain Walls



Strings eating domain walls



 The **symmetry breaking chains** are the **same** as in the previous case



Inflation occurs after **string formation** but before **domain wall formation**



Because of the **absence** of **strings**, a **normal wall network** forms

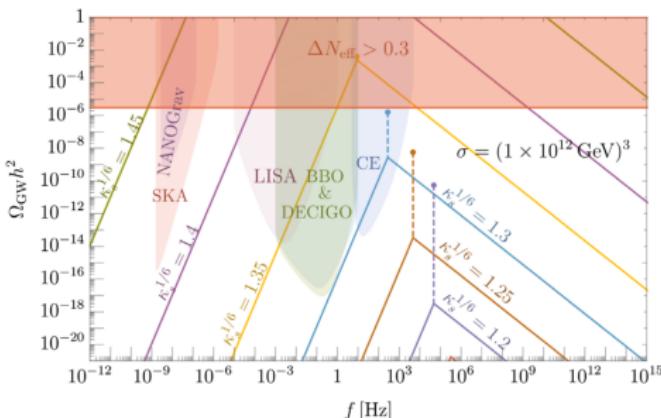


Nevertheless, the **walls** can become bounded by **strings** later by the **Schwinger nucleation of string holes**



 Conversion of wall rest mass into string kinetic energy causes the **string** of **tension** μ to **rapidly expand** and **eat** the **wall** of **tension** σ , causing the **wall network** to decay with **GW**

emissions ($\kappa_s = \mu^3 / \sigma^2$)



[Dunsky, et al., 2111.08750, PRD]

Summary

- In the early Universe, the **spontaneous breaking of symmetries** could lead to **topological defects**, such as **monopoles**, **domain walls** and **cosmic strings**
- **Collapsing domain walls**, **cosmic strings**, or various **hybrid defects** may result in a **stochastic GW background**, which could be probed in future GW experiments

Summary

- In the early Universe, the **spontaneous breaking of symmetries** could lead to **topological defects**, such as **monopoles**, **domain walls** and **cosmic strings**
- **Collapsing domain walls**, **cosmic strings**, or various **hybrid defects** may result in a **stochastic GW background**, which could be probed in future GW experiments

Thanks for your attention!