标准模型拉氏量和费曼规则

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目　录

1 约定

本文采用有理化的自然单位制，推导过程参考文献 [1, 2, 3, 4]。

Minkowski 度规张量

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}. \tag{1}$$

Pauli 矩阵

$$\sigma^1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} & -\mathrm{i} \\ \mathrm{i} & \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}, \tag{2}$$

$$\sigma^\mu \equiv (1, \boldsymbol{\sigma}), \quad \bar{\sigma}^\mu \equiv (1, -\boldsymbol{\sigma}). \tag{3}$$

Weyl 表象中的 Dirac 矩阵

$$\gamma^\mu = \begin{pmatrix} & \sigma^\mu \\ \bar{\sigma}^\mu & \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -\mathbf{1} & \\ & \mathbf{1} \end{pmatrix}. \tag{4}$$

左右手投影算符

$$P_{\mathrm{L}} \equiv \frac{1}{2}(1 - \gamma^5) = \begin{pmatrix} \mathbf{1} & \\ & \mathbf{0} \end{pmatrix}, \quad P_{\mathrm{R}} \equiv \frac{1}{2}(1 + \gamma^5) = \begin{pmatrix} \mathbf{0} & \\ & \mathbf{1} \end{pmatrix} \tag{5}$$

用于定义左手旋量场 $\psi_{\mathrm{L}} \equiv P_{\mathrm{L}}\psi$ 和右手旋量场 $\psi_{\mathrm{R}} \equiv P_{\mathrm{R}}\psi$。Levi-Civita 符号的约定取

$$\varepsilon^{0123} = \varepsilon^{123} = +1. \tag{6}$$

费曼规则约定如下。

- 对于指向相互作用顶点的动量 p，时空导数 ∂_μ 在动量空间费曼规则里贡献一个 $-\mathrm{i}p_\mu$ 因子。

- 实线表示费米子，实线上的箭头表示费米子数流动的方向。

- 虚线表示标量玻色子，虚线上的箭头表示玻色子数流动的方向。

- 螺旋线表示胶子；波浪线表示其它规范玻色子，波浪线上的箭头表示玻色子数流动的方向。

- 点线表示鬼粒子，点线上的箭头表示鬼粒子数流动的方向。

- 如果没有额外箭头标记，动量方向与粒子线上的箭头方向一致；否则与额外箭头方向一致。

2 标准模型概述

粒子物理标准模型是一个 $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ 规范理论。模型中有三代费米子，包括三代中微子 $\nu_i = \nu_e, \nu_\mu, \nu_\tau$，三代带电轻子 $\ell_i = e, \mu, \tau$，三代上型夸克 $u_i = u, c, t$ 和三代下型夸克 $d_i = d, s, b$ ($i = 1, 2, 3$)。规范玻色子传递费米子之间的规范相互作用。

$\mathrm{SU}(3)_\mathrm{C}$ 部分描述夸克的强相互作用，称为量子色动力学 (Quantum Chromodynamics, QCD)，相应的规范玻色子是胶子。$\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ 部分统一描述夸克和轻子的电磁和弱相互作用，称为电弱规范理论。理论中有一个 Higgs 二重态，通过 Brout–Englert–Higgs (BEH) 机制引发规范群的自发对称性破缺，使 $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ 群破缺为 $\mathrm{U}(1)_\mathrm{EM}$ 群。$\mathrm{U}(1)_\mathrm{EM}$ 规范理论称为量子电动力学 (Quantum Electrodynamics, QED)。

破缺前，理论中存在 4 个无质量的规范玻色子和 4 个 Higgs 自由度；左手费米子和右手费米子都没有质量，具有不同的量子数。

破缺后，3 个规范玻色子与 3 个 Higgs 自由度结合，从而获得质量，成为 $W^\pm$ 和 Z^0 玻色子，传递弱相互作用。剩下的 1 个无质量规范玻色子是光子，即是 $\mathrm{U}(1)_\mathrm{EM}$ 群的规范玻色子，传递电磁相互作用。剩下的 1 个中性 Higgs 自由度称为 Higgs 玻色子。费米子与 Higgs 二重态的 Yukawa 耦合导致左手费米子和右手费米子获得质量，组合成 Dirac 费米子。

理论中的中微子没有右手分量，因而没有获得质量。1998 年实验发现中微子振荡，证明中微子具有质量，所以需要扩充标准模型才能正确描述中微子物理。

3 量子色动力学

QCD 的拉氏量表达为

$$\mathcal{L}_{\mathrm{QCD}} = \sum_q \bar{q}(\mathrm{i}\gamma^\mu D_\mu - m_q)q - \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu}, \quad q = u, d, s, c, b, t, \quad a = 1, \cdots, 8, \tag{7}$$

其中

$$D_\mu = \partial_\mu - \mathrm{i}g_\mathrm{s}G^a_\mu t^a, \quad G^{a\mu\nu} \equiv \partial^\mu G^{a\nu} - \partial^\nu G^{a\mu} + g_\mathrm{s}f^{abc}G^{b\mu}G^{c\nu}. \tag{8}$$

q 为夸克旋量场 $\mathrm{SU}(3)_\mathrm{C}$ 三重态，$\mathrm{SU}(3)_\mathrm{C}$ 规范场 G^a_μ 对应于胶子 g，g_s 是 $\mathrm{SU}(3)_\mathrm{C}$ 规范耦合常数。$t^a = \lambda^a/2$ 是 $\mathrm{SU}(3)_\mathrm{C}$ 群基础表示的生成元，其中 λ^a 为 Gell-Mann 矩阵。$\mathrm{SU}(3)_\mathrm{C}$ 生成元对易关系为 $[t^a, t^b] = \mathrm{i}f^{abc}t^c$，结构常数 f^{abc} 是全反对称的，非零分量为

$$f^{123} = 1, \quad f^{147} = f^{246} = f^{257} = f^{345} = f^{516} = f^{637} = \frac{1}{2}, \quad f^{458} = f^{678} = \frac{\sqrt{3}}{2}. \tag{9}$$

由

$$-\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} = -\frac{1}{4}(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_\mathrm{s}f^{abc}G^b_\mu G^c_\nu)(\partial^\mu G^{a\nu} - \partial^\nu G^{a\mu} + g_\mathrm{s}f^{ade}G^{d\mu}G^{e\nu})$$

$$= -\frac{1}{2}[(\partial_\mu G^a_\nu)(\partial^\mu G^{a\nu}) - (\partial_\mu G^a_\nu)(\partial^\nu G^{a\mu})] - g_\mathrm{s}f^{abc}(\partial_\mu G^a_\nu)G^{b\mu}G^{c\nu}$$

$$-\frac{1}{4}g_{\mathrm{s}}^2 f^{abc}f^{ade}G_\mu^b G_\nu^c G^{d\mu}G^{e\nu}, \tag{10}$$

推出

$$\mathcal{L}_{\mathrm{QCD}} = \sum_q [\bar{q}(\mathrm{i}\gamma^\mu\partial_\mu - m_q)q + g_{\mathrm{s}}G_\mu^a\bar{q}\gamma^\mu t^a q] + \frac{1}{2}[(\partial_\mu G_\nu^a)(\partial^\nu G^{a\mu}) - (\partial_\mu G_\nu^a)(\partial^\mu G^{a\nu})]$$

$$-g_{\mathrm{s}}f^{abc}(\partial_\mu G_\nu^a)G^{b\mu}G^{c\nu} - \frac{1}{4}g_{\mathrm{s}}^2 f^{abc}f^{ade}G_\mu^b G_\nu^c G^{d\mu}G^{e\nu}. \tag{11}$$

设用于固定胶子场规范的函数 $G^a(x) = \partial^\mu G_\mu^a(x) - \omega^a(x)$，其中 $\omega^a(x)$ 是某个任意函数，规范固定条件是 $G^a(x) = 0$。这是 Lorenz 规范的推广，$\omega^a(x) = 0$ 对应于 Lorenz 规范。在路径积分量子化中，以中心为 $\omega^a(x) = 0$ 的 Gauss 权重对 $\omega^a(x)$ 作泛函积分，有

$$\int \mathcal{D}\omega^a \exp\left[-\mathrm{i}\int \mathrm{d}^4x\,\frac{1}{2\xi}(\omega^a)^2\right]\delta(G^a) = \exp\left[-\mathrm{i}\int \mathrm{d}^4x\,\frac{1}{2\xi}(\partial^\mu G_\mu^a)^2\right]. \tag{12}$$

可见，拉氏量中的规范固定项为

$$\mathcal{L}_{\mathrm{QCD,GF}} = -\frac{1}{2\xi}(\partial^\mu G_\mu^a)^2. \tag{13}$$

ξ 的任何一个取值对应于一种规范。$\xi = 1$ 称为 Feynman-'t Hooft 规范，$\xi = 0$ 称为 Landau 规范。于是，胶子传播子相关拉氏量为

$$\mathcal{L}_{\mathrm{QCD,prop}} = \frac{1}{2}\left[(\partial_\mu G_\nu^a)(\partial^\nu G^{a\mu}) - (\partial_\mu G_\nu^a)(\partial^\mu G^{a\nu}) - \frac{1}{\xi}(\partial^\mu G_\mu^a)^2\right]$$

$$\rightarrow \frac{1}{2}G_\mu^a\left[g^{\mu\nu}\partial^2 - \left(1 - \frac{1}{\xi}\right)\partial^\mu\partial^\nu\right]G_\nu^a. \tag{14}$$

变换到动量空间，得

$$-g^{\mu\nu}p^2 + \left(1 - \frac{1}{\xi}\right)p^\mu p^\nu, \tag{15}$$

它的逆矩阵是

$$-\frac{1}{p^2}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1 - \xi)\right], \tag{16}$$

这是因为

$$-\frac{1}{p^2}\left[g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2}(1 - \xi)\right]\left[-g^{\mu\nu}p^2 + \left(1 - \frac{1}{\xi}\right)p^\mu p^\nu\right]$$

$$= \delta_\rho{}^\nu - \frac{p_\rho p^\nu}{p^2}\left(1 - \frac{1}{\xi}\right) - \frac{p_\rho p^\nu}{p^2}(1 - \xi) + \frac{p_\rho p^\nu}{p^2}(1 - \xi)\left(1 - \frac{1}{\xi}\right) = \delta_\rho{}^\nu. \tag{17}$$

从而胶子传播子的形式为

$$\frac{-\mathrm{i}\delta^{ab}}{p^2 + \mathrm{i}\epsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1 - \xi)\right]. \tag{18}$$

SU(3)$_{\rm C}$ 规范变换为

$$q \to Uq, \quad G_\mu^a t^a \to U G_\mu^a t^a U^\dagger + \frac{\rm i}{g_{\rm s}} U \partial_\mu U^\dagger, \tag{19}$$

其中 $U(x) = \exp[{\rm i}\alpha^a(x)t^a]$。胶子场的无穷小规范变换形式是

$$\begin{aligned}
G_\mu^a t^a &\to (1 + {\rm i}\alpha^a t^a) G_\mu^b t^b (1 - {\rm i}\alpha^c t^c) + \frac{\rm i}{g_{\rm s}} (1 + {\rm i}\alpha^a t^a) \partial_\mu (1 - {\rm i}\alpha^c t^c) \\
&= G_\mu^b t^b + {\rm i}\alpha^a G_\mu^b [t^a, t^b] + \frac{1}{g_{\rm s}} (\partial_\mu \alpha^c) t^c + \mathcal{O}(\alpha^2) = G_\mu^a t^a - f^{abc} \alpha^a G_\mu^b t^c + \frac{1}{g_{\rm s}} (\partial_\mu \alpha^a) t^a + \mathcal{O}(\alpha^2) \\
&= \left(G_\mu^a + f^{abc} G_\mu^b \alpha^c + \frac{1}{g_{\rm s}} \partial_\mu \alpha^a \right) t^a + \mathcal{O}(\alpha^2),
\end{aligned} \tag{20}$$

即

$$\delta G_\mu^a = \frac{1}{g_{\rm s}} \partial_\mu \alpha^a + f^{abc} G_\mu^b \alpha^c = \left(\frac{1}{g_{\rm s}} \delta^{ac} \partial_\mu + f^{abc} G_\mu^b \right) \alpha^c, \tag{21}$$
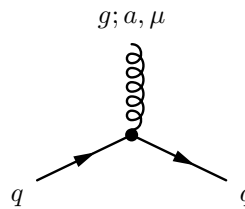
因而规范固定函数 G^a 的无穷小规范变换为

$$\delta G^a = \partial^\mu \delta G_\mu^a = \frac{1}{g_{\rm s}} \delta^{ac} \partial^2 \alpha^c + f^{abc} \partial^\mu G_\mu^b \alpha^c, \quad g_{\rm s} \frac{\delta G^a}{\delta \alpha^c} = \delta^{ac} \partial^2 + g_{\rm s} f^{abc} \partial^\mu G_\mu^b. \tag{22}$$
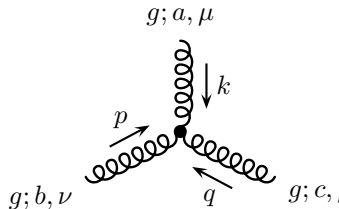
Faddeev-Popov 鬼场的拉氏量是

$$\mathcal{L}_{\rm QCD,FP} = -\bar\eta_g^a \left(g_{\rm s} \frac{\delta G^a}{\delta \alpha^c} \right) \eta_g^c = -\bar\eta_g^a (\delta^{ac} \partial^2 + g_{\rm s} f^{abc} \partial^\mu G_\mu^b) \eta_g^c \to -\bar\eta_g^a \delta^{ab} \partial^2 \eta_g^b + g_{\rm s} f^{abc} (\partial^\mu \bar\eta_g^a) G_\mu^b \eta_g^c. \tag{23}$$
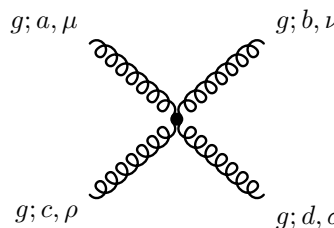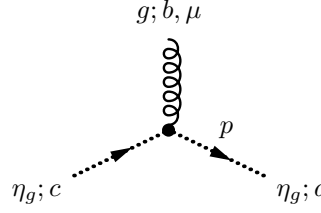
下面列出 QCD 费曼规则。

QCD 耦合顶点:

$$= {\rm i}g_{\rm s} \gamma^\mu t^a \tag{24}$$

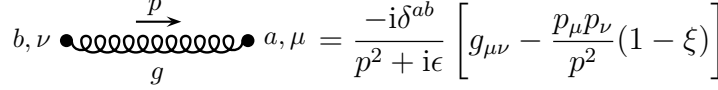$$= g_{\rm s} f^{abc} [g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu] \tag{25}$$

$$\begin{aligned}
= -{\rm i}g_{\rm s}^2 [&f^{abe} f^{cde} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) \\
&+ f^{ace} f^{bde} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}) \\
&+ f^{ade} f^{bce} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma})]
\end{aligned} \tag{26}$$

$$= -g_{\mathrm{s}} f^{abc} p^{\mu} \tag{27}$$

胶子传播子：

$$= \frac{-\mathrm{i}\delta^{ab}}{p^2 + \mathrm{i}\epsilon} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1-\xi) \right] \tag{28}$$

鬼粒子传播子：

$$= \frac{\mathrm{i}\delta^{ab}}{p^2 + \mathrm{i}\epsilon} \tag{29}$$

4 电弱规范理论

电弱规范理论的规范群是 $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$，每一代左手旋量场构成 2 个 $\mathrm{SU(2)_L}$ 二重态

$$L_{i\mathrm{L}} = \begin{pmatrix} P_{\mathrm{L}}\nu_i \\ P_{\mathrm{L}}\ell_i \end{pmatrix} = \begin{pmatrix} \nu_{i\mathrm{L}} \\ \ell_{i\mathrm{L}} \end{pmatrix}, \quad Q_{i\mathrm{L}} = \begin{pmatrix} P_{\mathrm{L}}u_i' \\ P_{\mathrm{L}}d_i' \end{pmatrix} = \begin{pmatrix} u_{i\mathrm{L}}' \\ d_{i\mathrm{L}}' \end{pmatrix}, \quad i = 1, 2, 3. \tag{30}$$

它们的协变导数是

$$D_\mu = \partial_\mu - \mathrm{i}g W_\mu^a \tau^a - \mathrm{i}g' B_\mu Y, \tag{31}$$

其中 $W_\mu^a(x)$ $(a = 1, 2, 3)$ 是 $\mathrm{SU(2)_L}$ 规范场，$B_\mu(x)$ 是 $\mathrm{U(1)_Y}$ 规范场，g 和 g' 分别是 $\mathrm{SU(2)_L}$ 和 $\mathrm{U(1)_Y}$ 的规范耦合常数。

$$\tau^a = \frac{\sigma^a}{2} \tag{32}$$

是 $\mathrm{SU(2)_L}$ 群 2 维表示的生成元，对应于弱同位旋。生成元 τ^3 的本征值是弱同位旋第 3 分量，记为 T^3。Y 是弱超荷。各代右手旋量场 $\ell_{i\mathrm{R}} = P_{\mathrm{R}}\ell_i$、$u_{i\mathrm{R}}' = P_{\mathrm{R}}u_i'$ 和 $d_{i\mathrm{R}}' = P_{\mathrm{R}}d_i'$ 是 $\mathrm{SU(2)_L}$ 单态，协变导数为

$$D_\mu = \partial_\mu - \mathrm{i}g' B_\mu Y. \tag{33}$$

表 1 列出费米子场的电荷 Q、弱同位旋第 3 分量 T^3、弱超荷 Y、重子数 B 和轻子数 $L_e/L_\mu/L_\tau$，其中电荷 Q 由 T^3 和 Y 定义，

$$Q \equiv T^3 + Y. \tag{34}$$

4.1 Brout–Englert–Higgs 机制

由于左手费米子和右手费米子参与不同的 $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ 规范相互作用，耦合左右手费米子场的质量项会破坏规范对称性。另一方面，规范对称性也禁止规范玻色子具有质量。为了让费米子和弱规范玻色子获得质量，需要引入 BEH 机制，使 $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ 规范对称性自发破

表 1: 标准模型费米子场的量子数。

统一记号	第一代	第二代	第三代	Q	T^3	Y	B	$L_e/L_\mu/L_\tau$
$L_{iL} = \begin{pmatrix} \nu_{iL} \\ \ell_{iL} \end{pmatrix}$	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	0 -1	$1/2$ $-1/2$	$-1/2$ $-1/2$	0 0	1 1
$Q_{iL} = \begin{pmatrix} u'_{iL} \\ d'_{iL} \end{pmatrix}$	$\begin{pmatrix} u'_L \\ d'_L \end{pmatrix}$	$\begin{pmatrix} c'_L \\ s'_L \end{pmatrix}$	$\begin{pmatrix} t'_L \\ b'_L \end{pmatrix}$	$2/3$ $-1/3$	$1/2$ $-1/2$	$1/6$ $1/6$	$1/3$ $1/3$	0 0
ℓ_{iR}	e_R	μ_R	τ_R	-1	0	-1	0	1
u'_{iR}	u'_R	c'_R	t'_R	$2/3$	0	$2/3$	$1/3$	0
d'_{iR}	d'_R	s'_R	b'_R	$-1/3$	0	$-1/3$	$1/3$	0

缺。因此，引入 Higgs 标量场

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}, \tag{35}$$

其中 ϕ^+ 和 ϕ^0 都是复标量场。Φ 是 $SU(2)_L$ 二重态，弱超荷是

$$Y_H = \frac{1}{2}. \tag{36}$$

Higgs 场的协变动能项和势能项为

$$\mathcal{L}_H = (D^\mu\Phi)^\dagger (D_\mu\Phi) - V_H(\Phi), \quad V_H(\Phi) = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2, \tag{37}$$

其中协变导数为

$$D_\mu\Phi = (\partial_\mu - ig'B_\mu Y_H - igW_\mu^a\tau^a)\Phi. \tag{38}$$

当 $\lambda > 0$ 且 $\mu^2 > 0$ 时，Higgs 场势能 $V_H(\Phi)$ 呈现出图 1 所示墨西哥草帽状的形式，势能最小值位于方程

$$\Phi^\dagger\Phi = [\text{Re}(\phi^+)]^2 + [\text{Im}(\phi^+)]^2 + [\text{Re}(\phi^0)]^2 + [\text{Im}(\phi^0)]^2 = \frac{v^2}{2} \tag{39}$$

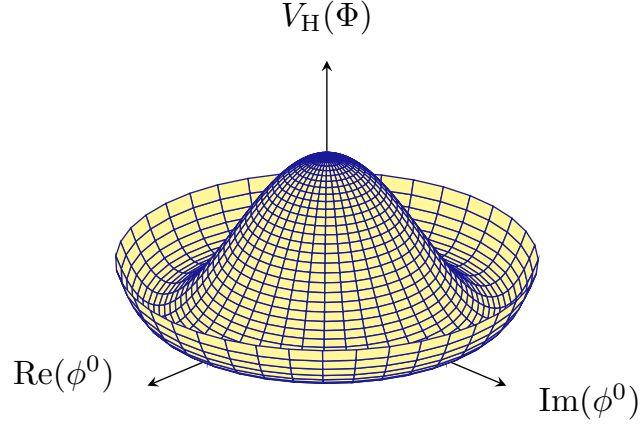对应的 3 维球面上，其中 $v \equiv \sqrt{\mu^2/\lambda}$，满足

$$\mu^2 = \lambda v^2. \tag{40}$$

图 1: Higgs 场势能示意图。这里压缩掉 $\mathrm{Re}(\phi^+)$ 和 $\mathrm{Im}(\phi^+)$ 两个维度。

Higgs 场的真空期待值位于这个 3 维球面上的某一点，不失一般性，可将它取为

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{41}$$

其它真空期待值可通过 $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{Y}}$ 整体变换

$$\langle\Phi\rangle \rightarrow \exp(\mathrm{i}\alpha^a \tau^a) \exp(\mathrm{i}\alpha^Y Y_H) \langle\Phi\rangle \tag{42}$$

得到，因为 $\langle\Phi^\dagger\Phi\rangle$ 在这样的变换下保持不变。若 $\alpha^1 = \alpha^2 = 0$ 且 $\alpha^3 = \alpha^Y$，则

$$\exp(\mathrm{i}\alpha^a \tau^a) \exp(\mathrm{i}\alpha^Y Y_H) = \exp[\mathrm{i}\alpha^3(\sigma^3 + \mathbf{1})/2] = \exp\left[\mathrm{i}\alpha^3 \begin{pmatrix} 1 & \\ & 0 \end{pmatrix}\right] = \begin{pmatrix} \mathrm{e}^{\mathrm{i}\alpha^3} & \\ & 1 \end{pmatrix}, \tag{43}$$

而 $\langle\Phi\rangle$ 在此变换下不变。因此，有 1 个方向的规范对称性没有受到破坏，只有 3 个方向的规范对称性发生自发破缺。根据 Goldstone 定理，破缺后存在 3 个无质量的 Nambu-Goldstone 玻色子。最终，有 3 个规范玻色子结合 Nambu-Goldstone 玻色子，通过 BEH 机制获得质量。

以 $\langle\Phi\rangle$ 为基础，将 Higgs 场参数化为

$$\Phi(x) = \exp\left[-\mathrm{i}\frac{\chi^a(x)}{v} \tau^a\right] \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \tag{44}$$

其中 $\chi^a(x)$ 和 $H(x)$ 都是实标量场。$\exp[-\mathrm{i}\chi^a(x)\tau^a/v]$ 因子能够通过 $\mathrm{SU}(2)_{\mathrm{L}}$ 规范变换消去，因而可将 $\Phi(x)$ 直接取为

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \quad \Phi^\dagger\Phi = \frac{1}{2}(v + H)^2. \tag{45}$$

此时 Higgs 场只剩下一个物理自由度 $H(x)$，对应于 Higgs 玻色子，这种取法称为**幺正规范**。

在幺正规范下，势能项化为

$$
\begin{aligned}
-V_{\mathrm{H}}(\Phi) &= \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2 = \frac{1}{2}\mu^2(v+H)^2 - \frac{1}{4}\lambda(v+H)^4 \\
&= \frac{1}{2}\mu^2(v^2 + H^2 + 2vH) - \frac{1}{4}\lambda(v^4 + 4v^2H^2 + H^4 + 4v^3H + 2v^2H^2 + 4vH^3) \\
&= \frac{1}{4}\mu^2v^2 + \frac{1}{4}(\mu^2 - \lambda v^2)v^2 + (\mu^2 - \lambda v^2)vH + \frac{1}{2}(\mu^2 - \lambda v^2)H^2 - \lambda v^2 H^2 - \lambda v H^3 - \frac{1}{4}\lambda H^4 \\
&= \frac{1}{8}m_H^2 v^2 - \frac{1}{2}m_H^2 H^2 - \frac{1}{2}\frac{m_H^2}{v}H^3 - \frac{1}{8}\frac{m_H^2}{v^2}H^4,
\end{aligned}
\tag{46}
$$

其中 Higgs 玻色子的质量为

$$
m_H \equiv \sqrt{2}\mu, \quad m_H^2 = 2\mu^2 = 2\lambda v^2.
\tag{47}
$$

由于

$$
g'B_\mu Y_H + gW_\mu^a\tau^a = \frac{1}{2}\begin{pmatrix} g'B_\mu + gW_\mu^3 & g(W_\mu^1 - \mathrm{i}W_\mu^2) \\ g(W_\mu^1 + \mathrm{i}W_\mu^2) & g'B_\mu - gW_\mu^3 \end{pmatrix},
\tag{48}
$$

Higgs 场真空期待值 v 对协变导数 $D_\mu\Phi$ 的贡献为

$$
\begin{aligned}
D_\mu\Phi &\supset -\mathrm{i}(g'B_\mu Y_H + gW_\mu^a\tau^a)\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \\
&= -\frac{\mathrm{i}v}{2\sqrt{2}}\begin{pmatrix} g'B_\mu + gW_\mu^3 & g(W_\mu^1 - \mathrm{i}W_\mu^2) \\ g(W_\mu^1 + \mathrm{i}W_\mu^2) & g'B_\mu - gW_\mu^3 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \supset -\frac{\mathrm{i}v}{2\sqrt{2}}\begin{pmatrix} g(W_\mu^1 - \mathrm{i}W_\mu^2) \\ g'B_\mu - gW_\mu^3 \end{pmatrix},
\end{aligned}
\tag{49}
$$

故协变动能项 $(D^\mu\Phi)^\dagger(D_\mu\Phi)$ 中正比于 v^2 的项是

$$
(D^\mu\Phi)^\dagger(D_\mu\Phi) \supset \frac{v^2}{8}[g^2|W_\mu^1 - \mathrm{i}W_\mu^2|^2 + (g'B_\mu - gW_\mu^3)^2] = \frac{v^2}{8}(g^2 W^{a\mu}W_\mu^a + g'^2 B^\mu B_\mu - 2gg'B^\mu W_\mu^3).
\tag{50}
$$

这些项是规范玻色子的质量项，重新表达为

$$
\mathcal{L}_{\mathrm{GBM}} = \frac{1}{2}m_W^2(W^{1\mu}W_\mu^1 + W^{2\mu}W_\mu^2) + \frac{1}{2}\begin{pmatrix} W^{3\mu} & B^\mu \end{pmatrix}M_{W^3B}^2\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix},
\tag{51}
$$

其中

$$
m_W \equiv \frac{1}{2}gv
\tag{52}
$$

是 W_μ^1 和 W_μ^2 获得的质量，而 $W^{3\mu}$ 和 B^μ 的质量平方矩阵为

$$
M_{W^3B}^2 \equiv \frac{v^2}{4}\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}.
\tag{53}
$$

为了使 $M_{W^3B}^2$ 矩阵对角化，定义

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \equiv \begin{pmatrix} c_{\mathrm{W}} & -s_{\mathrm{W}} \\ s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}, \tag{54}$$

其中

$$s_{\mathrm{W}} \equiv \sin\theta_{\mathrm{W}} = \frac{g'}{\sqrt{g^2 + g'^2}}, \quad c_{\mathrm{W}} \equiv \cos\theta_{\mathrm{W}} = \frac{g}{\sqrt{g^2 + g'^2}}, \tag{55}$$

θ_{W} 称为 Weinberg 角，也称为弱混合角。从后面的讨论可以看出 $A_\mu(x)$ 就是电磁场，对应于光子。$Z_\mu(x)$ 对应于矢量玻色子 Z。反过来，有

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} c_{\mathrm{W}} & s_{\mathrm{W}} \\ -s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}. \tag{56}$$

由

$$\begin{aligned} M_{W^3B}^2 \begin{pmatrix} c_{\mathrm{W}} & s_{\mathrm{W}} \\ -s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} &= \frac{(g^2 + g'^2)v^2}{4} \begin{pmatrix} c_{\mathrm{W}}^2 & -s_{\mathrm{W}}c_{\mathrm{W}} \\ -s_{\mathrm{W}}c_{\mathrm{W}} & s_{\mathrm{W}}^2 \end{pmatrix} \begin{pmatrix} c_{\mathrm{W}} & s_{\mathrm{W}} \\ -s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} \\ &= \frac{(g^2 + g'^2)v^2}{4} \begin{pmatrix} c_{\mathrm{W}} & 0 \\ -s_{\mathrm{W}} & 0 \end{pmatrix} \end{aligned} \tag{57}$$

得

$$\begin{aligned} \begin{pmatrix} c_{\mathrm{W}} & -s_{\mathrm{W}} \\ s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} M_{W^3B}^2 \begin{pmatrix} c_{\mathrm{W}} & s_{\mathrm{W}} \\ -s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} &= \frac{(g^2 + g'^2)v^2}{4} \begin{pmatrix} c_{\mathrm{W}} & -s_{\mathrm{W}} \\ s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} \begin{pmatrix} c_{\mathrm{W}} & 0 \\ -s_{\mathrm{W}} & 0 \end{pmatrix} \\ &= \frac{(g^2 + g'^2)v^2}{4} \begin{pmatrix} 1 & \\ & 0 \end{pmatrix}, \end{aligned} \tag{58}$$

因此

$$\begin{aligned} \frac{1}{2} \begin{pmatrix} W^{3\mu} & B^\mu \end{pmatrix} M_{W^3B}^2 \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} &= \frac{1}{2} \begin{pmatrix} Z^\mu & A^\mu \end{pmatrix} \begin{pmatrix} c_{\mathrm{W}} & -s_{\mathrm{W}} \\ s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} M_{W^3B}^2 \begin{pmatrix} c_{\mathrm{W}} & s_{\mathrm{W}} \\ -s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \\ &= \frac{(g^2 + g'^2)v^2}{8} \begin{pmatrix} Z^\mu & A^\mu \end{pmatrix} \begin{pmatrix} 1 & \\ & 0 \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \frac{1}{2} m_Z^2 Z^\mu Z_\mu, \end{aligned} \tag{59}$$

其中

$$m_Z \equiv \frac{1}{2}\sqrt{g^2 + g'^2}\, v = \frac{gv}{2c_{\mathrm{W}}} = \frac{m_W}{c_{\mathrm{W}}} \tag{60}$$

是 Z 玻色子的质量，而光子没有质量。另一方面，用质量相同的实矢量场 W_μ^1 和 W_μ^2 线性组合

出复矢量场

$$W_\mu^+ \equiv \frac{1}{\sqrt{2}}(W_\mu^1 - \mathrm{i}W_\mu^2), \tag{61}$$

它的厄米共轭为

$$W_\mu^- \equiv (W_\mu^+)^\dagger = \frac{1}{\sqrt{2}}(W_\mu^1 + \mathrm{i}W_\mu^2), \tag{62}$$

则

$$W_\mu^1 = \frac{1}{\sqrt{2}}(W_\mu^+ + W_\mu^-), \quad W_\mu^2 = \frac{\mathrm{i}}{\sqrt{2}}(W_\mu^+ - W_\mu^-). \tag{63}$$

从而

$$\begin{aligned}
\frac{1}{2}(W^{1\mu}W_\mu^1 + W^{2\mu}W_\mu^2) &= \frac{1}{4}[(W^{+\mu} + W^{-\mu})(W_\mu^+ + W_\mu^-) - (W^{+\mu} - W^{-\mu})(W_\mu^+ - W_\mu^-)] \\
&= W^{+\mu}W_\mu^-,
\end{aligned} \tag{64}$$

[51] 式化为

$$\mathcal{L}_{\mathrm{GBM}} = m_W^2 W^{+\mu}W_\mu^- + \frac{1}{2}m_Z^2 Z^\mu Z_\mu. \tag{65}$$

复矢量场 $W_\mu^\pm$ 描述一对正反矢量玻色子 $W^\pm$，质量为 m_W。可见，BEH 机制使传递弱相互作用的规范玻色子 $W^\pm$ 和 Z 获得了质量。

接下来用质量本征态 $W_\mu^\pm$ 和 Z_μ 表达协变动能项 $(D^\mu\Phi)^\dagger(D_\mu\Phi)$。注意到

$$A_\mu = s_{\mathrm{W}}W_\mu^3 + c_{\mathrm{W}}B_\mu, \quad Z_\mu = c_{\mathrm{W}}W_\mu^3 - s_{\mathrm{W}}B_\mu, \tag{66}$$

$$B_\mu = c_{\mathrm{W}}A_\mu - s_{\mathrm{W}}Z_\mu, \quad W_\mu^3 = s_{\mathrm{W}}A_\mu + c_{\mathrm{W}}Z_\mu, \tag{67}$$

有

$$\begin{aligned}
g'B_\mu + gW_\mu^3 &= g'(c_{\mathrm{W}}A_\mu - s_{\mathrm{W}}Z_\mu) + g(s_{\mathrm{W}}A_\mu + c_{\mathrm{W}}Z_\mu) = \frac{2gg'}{\sqrt{g^2 + g'^2}}A_\mu + \frac{g^2 - g'^2}{\sqrt{g^2 + g'^2}}Z_\mu \\
&= 2eA_\mu + \frac{g}{c_{\mathrm{W}}}(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)Z_\mu,
\end{aligned} \tag{68}$$

其中

$$e \equiv \frac{gg'}{\sqrt{g^2 + g'^2}} = gs_{\mathrm{W}} = g'c_{\mathrm{W}}. \tag{69}$$

后面的讨论将表明 e 就是单位电荷量。再利用

$$g'B_\mu - gW_\mu^3 = g'(c_{\mathrm{W}}A_\mu - s_{\mathrm{W}}Z_\mu) - g(s_{\mathrm{W}}A_\mu + c_{\mathrm{W}}Z_\mu) = -\left(\frac{gs_{\mathrm{W}}^2}{c_{\mathrm{W}}} + gc_{\mathrm{W}}\right)Z_\mu = -\frac{g}{c_{\mathrm{W}}}Z_\mu, \tag{70}$$

得

$$g'B_\mu Y_H + gW_\mu^a \tau^a = \frac{1}{2}\begin{pmatrix} g'B_\mu + gW_\mu^3 & g(W_\mu^1 - \mathrm{i}W_\mu^2) \\ g(W_\mu^1 + \mathrm{i}W_\mu^2) & g'B_\mu - gW_\mu^3 \end{pmatrix}$$

$$
= \begin{pmatrix} eA_\mu + \dfrac{g}{2c_{\mathrm W}}(c_{\mathrm W}^2 - s_{\mathrm W}^2)Z_\mu & \dfrac{g}{\sqrt 2}\,W_\mu^+ \\[2mm] \dfrac{g}{\sqrt 2}\,W_\mu^- & -\dfrac{g}{2c_{\mathrm W}}\,Z_\mu \end{pmatrix}. \tag{71}
$$

在幺正规范下,

$$
\begin{aligned}
(D^\mu\Phi)^\dagger(D_\mu\Phi) &= \left| \begin{pmatrix} \partial_\mu - \mathrm i eA_\mu - \dfrac{\mathrm i g}{2c_{\mathrm W}}(c_{\mathrm W}^2 - s_{\mathrm W}^2)Z_\mu & -\dfrac{\mathrm i g}{\sqrt 2}\,W_\mu^+ \\[2mm] -\dfrac{\mathrm i g}{\sqrt 2}\,W_\mu^- & \partial_\mu + \dfrac{\mathrm i g}{2c_{\mathrm W}}\,Z_\mu \end{pmatrix} \frac{1}{\sqrt 2}\begin{pmatrix} 0 \\ v+H \end{pmatrix} \right|^2 \\[3mm]
&= \frac{1}{2}\left(\dfrac{\mathrm i g}{\sqrt 2}\,W^{-\mu}(v+H) \quad \partial^\mu H - \dfrac{\mathrm i g}{2c_{\mathrm W}}Z^\mu(v+H) \right) \begin{pmatrix} -\dfrac{\mathrm i g}{\sqrt 2}\,W_\mu^+(v+H) \\[2mm] \partial_\mu H + \dfrac{\mathrm i g}{2c_{\mathrm W}}\,Z_\mu(v+H) \end{pmatrix} \\[3mm]
&= \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + (v+H)^2\left(\frac{g^2}{4}\,W_\mu^+ W^{-\mu} + \frac{g^2}{8c_{\mathrm W}^2}\,Z_\mu Z^\mu \right) \\[3mm]
&= \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2}m_Z^2 Z_\mu Z^\mu \\[2mm]
&\quad + gm_W HW_\mu^+ W^{-\mu} + \frac{gm_Z}{2c_{\mathrm W}}\,HZ_\mu Z^\mu + \frac{g^2}{4}\,H^2 W_\mu^+ W^{-\mu} + \frac{g^2}{8c_{\mathrm W}^2}\,H^2 Z_\mu Z^\mu. \tag{72}
\end{aligned}
$$

除了 $W^\pm$ 和 Z 玻色子的质量项之外, 还出现了 Higgs 玻色子 H 与 $W^\pm$、Z 的三线性和四线性耦合项。

Higgs 场 $\Phi(x)$ 的弱超荷为 $+1/2$。引入 $\Phi(x)$ 的共轭态

$$
\tilde\Phi(x) = \mathrm i\sigma^2\Phi^*(x) = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix}\begin{pmatrix} \phi^-(x) \\ \phi^{0*}(x) \end{pmatrix} = \begin{pmatrix} \phi^{0*}(x) \\ -\phi^-(x) \end{pmatrix}, \tag{73}
$$

其中 $\phi^- \equiv (\phi^+)^*$, 则 $\tilde\Phi(x)$ 是弱超荷为 $-1/2$ 的 $\mathrm{SU}(2)_{\mathrm L}$ 二重态。在幺正规范下, $\tilde\Phi(x)$ 化为

$$
\tilde\Phi(x) = \frac{1}{\sqrt 2}\begin{pmatrix} & 1 \\ -1 & \end{pmatrix}\begin{pmatrix} 0 \\ v+H(x) \end{pmatrix} = \frac{1}{\sqrt 2}\begin{pmatrix} v+H(x) \\ 0 \end{pmatrix}. \tag{74}
$$

用 $\Phi(x)$、$\tilde\Phi(x)$ 和费米子场组成满足 $\mathrm{SU}(2)_{\mathrm L}\times\mathrm U(1)_{\mathrm Y}$ 规范对称性的 Yukawa 相互作用拉氏量

$$
\mathcal L_{\mathrm Y} = -\tilde y_{d,ij}\bar Q_{i\mathrm L}d'_{j\mathrm R}\Phi - \tilde y_{u,ij}\bar Q_{i\mathrm L}u'_{j\mathrm R}\tilde\Phi - y_{\ell_i}\bar L_{i\mathrm L}\ell_{i\mathrm R}\Phi + \mathrm{H.c.}, \tag{75}
$$

其中 H.c. 表示厄米共轭, Yukawa 耦合常数 $\tilde y_{d,ij}$ 和 $\tilde y_{u,ij}$ 联系着不同代的夸克场, 而 y_{ℓ_i} 只联系同一代的轻子场。在幺正规范下, 利用

$$
\bar Q_{i\mathrm L}\Phi = \frac{1}{\sqrt 2}\begin{pmatrix} \bar u'_{i\mathrm L} & \bar d'_{i\mathrm L} \end{pmatrix}\begin{pmatrix} 0 \\ v+H \end{pmatrix} = \frac{1}{\sqrt 2}(v+H)\bar d'_{i\mathrm L}, \tag{76}
$$

$$\bar{Q}_{iL}\tilde{\Phi} = \frac{1}{\sqrt{2}}\begin{pmatrix} \bar{u}'_{iL} & \bar{d}'_{iL} \end{pmatrix}\begin{pmatrix} v+H \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}(v+H)\bar{u}'_{iL}, \tag{77}$$

$$\bar{L}_{iL}\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \bar{\nu}_{iL} & \bar{\ell}_{iL} \end{pmatrix}\begin{pmatrix} 0 \\ v+H \end{pmatrix} = \frac{1}{\sqrt{2}}(v+H)\bar{\ell}_{iL}, \tag{78}$$

推出

$$\mathcal{L}_{Y} = -\frac{1}{\sqrt{2}}(v+H)(\tilde{y}_{d,ij}\bar{d}'_{iL}d'_{jR} + \tilde{y}_{u,ij}\bar{u}'_{iL}u'_{jR} + y_{\ell_i}\bar{\ell}_{iL}\ell_{iR} + \text{H.c.}). \tag{79}$$

$\tilde{y}_{d,ij}$ 和 $\tilde{y}_{u,ij}$ 可看作 3×3 矩阵 $\tilde{y}_d$ 和 $\tilde{y}_u$ 的元素。$\tilde{y}_d\tilde{y}_d^\dagger$ 和 $\tilde{y}_u\tilde{y}_u^\dagger$ 是厄米矩阵，必定可以分别通过幺正矩阵 U_d 和 U_u 对角化成 y_D^2 和 y_U^2 两个对角元为实数的对角矩阵，满足 $U_d^\dagger\tilde{y}_d\tilde{y}_d^\dagger U_d = y_D^2$ 和 $U_u^\dagger\tilde{y}_u\tilde{y}_u^\dagger U_u = y_U^2$，即

$$\tilde{y}_d\tilde{y}_d^\dagger = U_d y_D^2 U_d^\dagger, \quad \tilde{y}_u\tilde{y}_u^\dagger = U_u y_U^2 U_u^\dagger. \tag{80}$$

符合这两条式子的 $\tilde{y}_d$ 和 $\tilde{y}_u$ 可以表达为

$$\tilde{y}_d = U_d y_D K_d^\dagger, \quad \tilde{y}_u = U_u y_U K_u^\dagger, \tag{81}$$

其中对角矩阵 y_D 和 y_U 满足 $y_D y_D = y_D^2$ 和 $y_U y_U = y_U^2$，而 $K_d^\dagger$ 和 $K_u^\dagger$ 是两个幺正矩阵。

将 y_D 和 y_U 表示成

$$y_D = \begin{pmatrix} y_{d_1} & & \\ & y_{d_2} & \\ & & y_{d_3} \end{pmatrix} = \begin{pmatrix} y_d & & \\ & y_s & \\ & & y_b \end{pmatrix}, \quad y_U = \begin{pmatrix} y_{u_1} & & \\ & y_{u_2} & \\ & & y_{u_3} \end{pmatrix} = \begin{pmatrix} y_u & & \\ & y_c & \\ & & y_t \end{pmatrix}. \tag{82}$$

通过幺正变换定义

$$d_{iL} \equiv (U_d^\dagger)_{ij}d'_{jL}, \quad d_{iR} \equiv (K_d^\dagger)_{ij}d'_{jR}, \quad u_{iL} \equiv (U_u^\dagger)_{ij}u'_{jL}, \quad u_{iR} \equiv (K_u^\dagger)_{ij}u'_{jR}, \tag{83}$$

则 $\bar{d}_{iL} = \bar{d}'_{jL}U_{d,ji}$，$\bar{u}_{iL} = \bar{u}'_{jL}U_{u,ji}$，从而

$$\tilde{y}_{d,ij}\bar{d}'_{iL}d'_{jR} = \bar{d}'_{iL}(U_d y_d K_d^\dagger)_{ij}d'_{jR} = \bar{d}'_{iL}U_{d,ik}y_{d_k}(K_d^\dagger)_{kj}d'_{jR} = y_{d_k}\bar{d}_{kL}d_{kR} = y_{d_i}\bar{d}_{iL}d_{iR}, \tag{84}$$

$$\tilde{y}_{u,ij}\bar{u}'_{iL}u'_{jR} = \bar{u}'_{iL}(U_u y_u K_u^\dagger)_{ij}u'_{jR} = y_{u_i}\bar{u}_{iL}u_{iR}, \tag{85}$$

故

$$\begin{aligned}
\mathcal{L}_{Y} &= -\frac{1}{\sqrt{2}}(v+H)(y_{d_i}\bar{d}_{iL}d_{iR} + y_{u_i}\bar{u}_{iL}u_{iR} + y_{\ell_i}\bar{\ell}_{iL}\ell_{iR} + \text{H.c.}) \\
&= -m_{d_i}\bar{d}_i d_i - m_{u_i}\bar{u}_i u_i - m_{\ell_i}\bar{\ell}_i\ell_i - \frac{m_{d_i}}{v}H\bar{d}_i d_i - \frac{m_{u_i}}{v}H\bar{u}_i u_i - \frac{m_{\ell_i}}{v}H\bar{\ell}_i\ell_i, \tag{86}
\end{aligned}$$

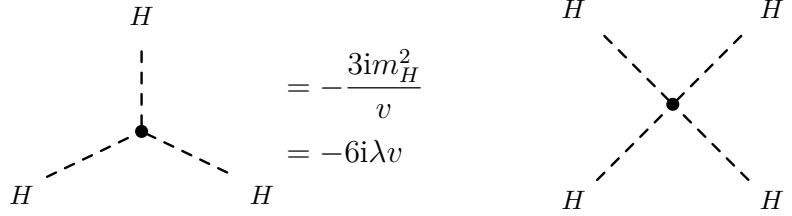其中前三项是费米子质量项，后三项是 Higgs 玻色子与费米子的 Yukawa 耦合项。于是，三代

夸克和带电轻子获得了质量

$$m_{d_i} \equiv \frac{1}{\sqrt{2}}\, y_{d_i} v, \quad m_{u_i} \equiv \frac{1}{\sqrt{2}}\, y_{u_i} v, \quad m_{\ell_i} \equiv \frac{1}{\sqrt{2}}\, y_{\ell_i} v. \tag{87}$$

$d'_{i\mathrm{L}}$、$d'_{i\mathrm{R}}$、$u'_{i\mathrm{L}}$ 和 $u'_{i\mathrm{R}}$ 称为规范本征态，$d_{i\mathrm{L}}$、$d_{i\mathrm{R}}$、$u_{i\mathrm{L}}$ 和 $u_{i\mathrm{R}}$ 称为质量本征态。
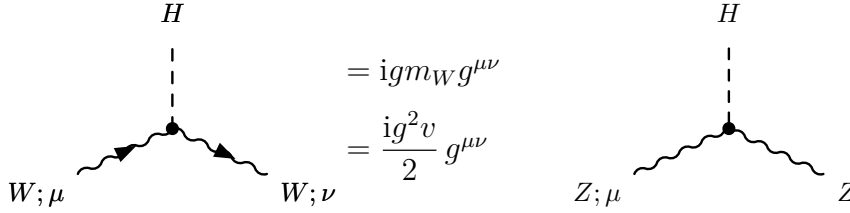
下面给出幺正规范下 Higgs 场的顶点费曼规则。

Higgs 玻色子自耦合：
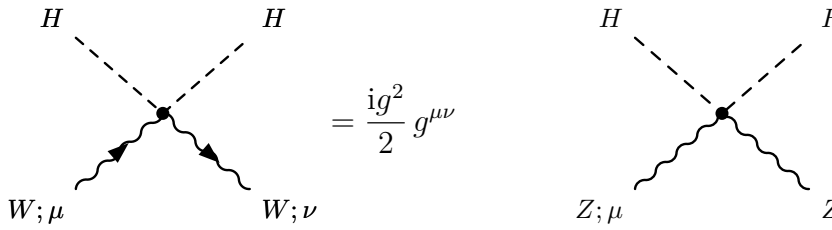
$$\begin{aligned} &= -\frac{3\mathrm{i}m_H^2}{v} \\ &= -6\mathrm{i}\lambda v \end{aligned} \qquad \begin{aligned} &= -\frac{3\mathrm{i}m_H^2}{v^2} \\ &= -6\mathrm{i}\lambda \end{aligned} \tag{88}$$

Higgs 玻色子与电弱规范玻色子的耦合：
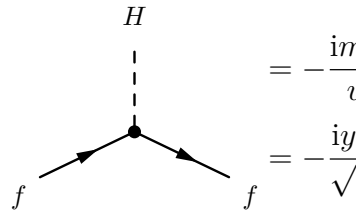
$$\begin{aligned} &= \mathrm{i}g m_W g^{\mu\nu} \\ &= \frac{\mathrm{i}g^2 v}{2}\, g^{\mu\nu} \end{aligned} \qquad \begin{aligned} &= \frac{\mathrm{i}g m_Z}{c_\mathrm{W}}\, g^{\mu\nu} \\ &= \frac{\mathrm{i}g^2 v}{2c_\mathrm{W}^2}\, g^{\mu\nu} \end{aligned} \tag{89}$$

$$= \frac{\mathrm{i}g^2}{2}\, g^{\mu\nu} \qquad\qquad = \frac{\mathrm{i}g^2}{2c_\mathrm{W}^2}\, g^{\mu\nu} \tag{90}$$

Higgs 玻色子与费米子 $f = d_i, u_i, \ell_i$ 的耦合：

$$\begin{aligned} &= -\frac{\mathrm{i}m_f}{v} \\ &= -\frac{\mathrm{i}y_f}{\sqrt{2}} \end{aligned} \tag{91}$$

4.2 费米子电弱规范相互作用

(83) 式意味着

$$d'_{i\mathrm{L}} = U_{d,ij} d_{j\mathrm{L}}, \quad d'_{i\mathrm{R}} = K_{d,ij} d_{j\mathrm{R}}, \quad u'_{i\mathrm{L}} = U_{u,ij} u_{j\mathrm{L}}, \quad u'_{i\mathrm{R}} = K_{u,ij} u_{j\mathrm{R}}, \tag{92}$$

从而

$$\bar{d}'_{i\mathrm{L}}\gamma^\mu d'_{i\mathrm{L}} = \bar{d}_{j\mathrm{L}}(U_d^\dagger)_{ji}\gamma^\mu U_{d,ik}d_{k\mathrm{L}} = \bar{d}_{j\mathrm{L}}\delta_{jk}\gamma^\mu d_{k\mathrm{L}} = \bar{d}_{i\mathrm{L}}\gamma^\mu d_{i\mathrm{L}} \tag{93}$$

同理有

$$\bar{u}'_{i\mathrm{L}}\gamma^\mu u'_{i\mathrm{L}} = \bar{u}_{i\mathrm{L}}\gamma^\mu u_{i\mathrm{L}}, \quad \bar{d}'_{i\mathrm{R}}\gamma^\mu d'_{i\mathrm{R}} = \bar{d}_{i\mathrm{R}}\gamma^\mu d_{i\mathrm{R}}, \quad \bar{u}'_{i\mathrm{R}}\gamma^\mu u'_{i\mathrm{R}} = \bar{u}_{i\mathrm{R}}\gamma^\mu u_{i\mathrm{R}}. \tag{94}$$

另一方面,

$$\bar{u}'_{i\mathrm{L}}\gamma^\mu d'_{i\mathrm{L}} = \bar{u}_{j\mathrm{L}}(U_u^\dagger)_{ji}\gamma^\mu U_{d,ik}d_{k\mathrm{L}} = \bar{u}_{i\mathrm{L}}\gamma^\mu V_{ij}d_{j\mathrm{L}} \tag{95}$$

$$\bar{d}'_{i\mathrm{L}}\gamma^\mu u'_{i\mathrm{L}} = \bar{d}_{j\mathrm{L}}(U_d^\dagger)_{ji}\gamma^\mu U_{u,ik}u_{k\mathrm{L}} = \bar{d}_{j\mathrm{L}}V_{ji}^\dagger\gamma^\mu u_{i\mathrm{L}} \tag{96}$$

其中

$$V \equiv U_u^\dagger U_d \tag{97}$$

称为 Cabibbo-Kobayashi-Maskawa (CKM) 矩阵, 其厄米共轭矩阵为 $V^\dagger = U_d^\dagger U_u$。

$\mathrm{SU(2)_L \times U(1)_Y}$ 规范不变的费米子协变动能项为

$$\mathcal{L}_{\mathrm{EWF}} = \bar{Q}_{i\mathrm{L}}\mathrm{i}\not{D}Q_{i\mathrm{L}} + \bar{u}'_{i\mathrm{R}}\mathrm{i}\not{D}u'_{i\mathrm{R}} + \bar{d}'_{i\mathrm{R}}\mathrm{i}\not{D}d'_{i\mathrm{R}} + \bar{L}_{i\mathrm{L}}\mathrm{i}\not{D}L_{i\mathrm{L}} + \bar{\ell}_{i\mathrm{R}}\mathrm{i}\not{D}\ell_{i\mathrm{R}}. \tag{98}$$

根据 Q 的定义 (34), 有

$$\begin{aligned} g'Y B_\mu + gT^3 W_\mu^3 &= g'Y(c_\mathrm{W}A_\mu - s_\mathrm{W}Z_\mu) + gT^3(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu) \\ &= e(Y + T^3)A_\mu + \left(gc_\mathrm{W}T^3 - \frac{gs_\mathrm{W}}{c_\mathrm{W}}s_\mathrm{W}Y\right)Z_\mu = QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 c_\mathrm{W}^2 - Ys_\mathrm{W}^2)Z_\mu \\ &= QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu, \end{aligned} \tag{99}$$

故

$$\begin{aligned} D_\mu Q_{i\mathrm{L}} &= (\partial_\mu - \mathrm{i}g'B_\mu Y - \mathrm{i}gW_\mu^a\tau^a)Q_{i\mathrm{L}} \\ &= \partial_\mu Q_{i\mathrm{L}} - \mathrm{i}\begin{pmatrix} g'Y B_\mu + gT^3 W_\mu^3 & \frac{g}{2}(W_\mu^1 - \mathrm{i}W_\mu^2) \\ \frac{g}{2}(W_\mu^1 + \mathrm{i}W_\mu^2) & g'Y B_\mu + gT^3 W_\mu^3 \end{pmatrix}Q_{i\mathrm{L}} \\ &= \partial_\mu Q_{i\mathrm{L}} - \mathrm{i}\begin{pmatrix} QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu & \frac{g}{\sqrt{2}}W_\mu^+ \\ \frac{g}{\sqrt{2}}W_\mu^- & QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu \end{pmatrix}\begin{pmatrix} u'_{i\mathrm{L}} \\ d'_{i\mathrm{L}} \end{pmatrix} \\ &= \partial_\mu Q_{i\mathrm{L}} - \mathrm{i}\begin{pmatrix} \left[QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu\right]u'_{i\mathrm{L}} + \frac{g}{\sqrt{2}}W_\mu^+ d'_{i\mathrm{L}} \\ \frac{g}{\sqrt{2}}W_\mu^- u'_{i\mathrm{L}} + \left[QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)\right]d'_{i\mathrm{L}} \end{pmatrix}. \end{aligned} \tag{100}$$

于是

$$
\begin{aligned}
\bar{Q}_{i\mathrm{L}}\mathrm{i}\slashed{D}Q_{i\mathrm{L}} \supset{}& \left[QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu\right]\bar{u}'_{i\mathrm{L}}\gamma^\mu u'_{i\mathrm{L}} + \left[QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu\right]\bar{d}'_{i\mathrm{L}}\gamma^\mu d'_{i\mathrm{L}}\\
&+ \frac{g}{\sqrt{2}}W_\mu^+\bar{u}'_{i\mathrm{L}}\gamma^\mu d'_{i\mathrm{L}} + \frac{g}{\sqrt{2}}W_\mu^-\bar{d}'_{i\mathrm{L}}\gamma^\mu u'_{i\mathrm{L}}\\
={}& \left(QeA_\mu + \frac{g}{c_\mathrm{W}}g_\mathrm{L}Z_\mu\right)\bar{u}_i\gamma^\mu\frac{1-\gamma^5}{2}u_i + \left(QeA_\mu + \frac{g}{c_\mathrm{W}}g_\mathrm{L}Z_\mu\right)\bar{d}_i\gamma^\mu\frac{1-\gamma^5}{2}d_i\\
&+ \frac{g}{\sqrt{2}}W_\mu^+\bar{u}_i\gamma^\mu P_\mathrm{L}V_{ij}d_j + \frac{g}{\sqrt{2}}W_\mu^-\bar{d}_j V_{ji}^\dagger\gamma^\mu P_\mathrm{L}u_i,
\end{aligned}\tag{101}
$$

其中左手耦合系数

$$
g_\mathrm{L} \equiv T^3 - Qs_\mathrm{W}^2.\tag{102}
$$

另一方面,

$$
\begin{aligned}
D_\mu d'_{i\mathrm{R}} &= (\partial_\mu - \mathrm{i}g'B_\mu Y)d'_{i\mathrm{R}} = \partial_\mu d'_{i\mathrm{R}} - \mathrm{i}g'Q(c_\mathrm{W}A_\mu - s_\mathrm{W}Z_\mu)d'_{i\mathrm{R}}\\
&= \partial_\mu d'_{i\mathrm{R}} - \mathrm{i}QeA_\mu d'_{i\mathrm{R}} + \frac{\mathrm{i}g}{c_\mathrm{W}}Qs_\mathrm{W}^2 Z_\mu d'_{i\mathrm{R}},
\end{aligned}\tag{103}
$$

则

$$
\begin{aligned}
&\bar{u}'_{i\mathrm{R}}\mathrm{i}\slashed{D}u'_{i\mathrm{R}} + \bar{d}'_{i\mathrm{R}}\mathrm{i}\slashed{D}d'_{i\mathrm{R}}\\
\supset{}& \left(QeA_\mu - \frac{g}{c_\mathrm{W}}Qs_\mathrm{W}^2 Z_\mu\right)\bar{u}'_{i\mathrm{R}}\gamma^\mu u'_{i\mathrm{R}} + \left(QeA_\mu - \frac{g}{c_\mathrm{W}}Qs_\mathrm{W}^2 Z_\mu\right)\bar{d}'_{i\mathrm{R}}\gamma^\mu d'_{i\mathrm{R}}\\
={}& \left(QeA_\mu + \frac{g}{c_\mathrm{W}}g_\mathrm{R}Z_\mu\right)\bar{u}_i\gamma^\mu\frac{1+\gamma^5}{2}u_i + \left(QeA_\mu + \frac{g}{c_\mathrm{W}}g_\mathrm{R}Z_\mu\right)\bar{d}_i\gamma^\mu\frac{1+\gamma^5}{2}d_i,
\end{aligned}\tag{104}
$$

其中右手耦合系数

$$
g_\mathrm{R} \equiv -Qs_\mathrm{W}^2.\tag{105}
$$

引入矢量流和轴矢量流耦合系数

$$
g_\mathrm{V} \equiv g_\mathrm{L} + g_\mathrm{R} = T^3 - 2Qs_\mathrm{W}^2, \quad g_\mathrm{A} \equiv g_\mathrm{L} - g_\mathrm{R} = T^3,\tag{106}
$$

得

$$
\begin{aligned}
&\bar{Q}_{i\mathrm{L}}\mathrm{i}\slashed{D}Q_{i\mathrm{L}} + \bar{u}'_{i\mathrm{R}}\mathrm{i}\slashed{D}u'_{i\mathrm{R}} + \bar{d}'_{i\mathrm{R}}\mathrm{i}\slashed{D}d'_{i\mathrm{R}}\\
\supset{}& Qe\bar{u}_i\gamma^\mu u_i A_\mu + Qe\bar{d}_i\gamma^\mu d_i A_\mu + \frac{g}{2c_\mathrm{W}}\bar{u}_i\gamma^\mu(g_\mathrm{V} - g_\mathrm{A}\gamma^5)u_i Z_\mu + \frac{g}{2c_\mathrm{W}}\bar{d}_i\gamma^\mu(g_\mathrm{V} - g_\mathrm{A}\gamma^5)d_i Z_\mu\\
&+ \frac{g}{\sqrt{2}}W_\mu^+\bar{u}_i\gamma^\mu P_\mathrm{L}V_{ij}d_j + \frac{g}{\sqrt{2}}W_\mu^-\bar{d}_j V_{ji}^\dagger\gamma^\mu P_\mathrm{L}u_i.
\end{aligned}\tag{107}
$$

同理，有

$$\bar{L}_{i\mathrm{L}}\mathrm{i}\not{D}L_{i\mathrm{L}} + \bar{\ell}_{i\mathrm{R}}\mathrm{i}\not{D}\ell_{i\mathrm{R}} \supset Qe\bar{\ell}_i\gamma^\mu\ell_i A_\mu + \frac{g}{2c_\mathrm{W}}\bar{\ell}_i\gamma^\mu(g_\mathrm{V}-g_\mathrm{A}\gamma^5)\ell_i Z_\mu + \frac{g}{2c_\mathrm{W}}\bar{\nu}_i\gamma^\mu(g_\mathrm{V}-g_\mathrm{A}\gamma^5)\nu_i Z_\mu$$

$$+ \frac{g}{\sqrt{2}}W_\mu^+\bar{\nu}_i\gamma^\mu P_\mathrm{L}\ell_i + \frac{g}{\sqrt{2}}W_\mu^-\bar{\ell}_i\gamma^\mu P_\mathrm{L}\nu_i. \tag{108}$$

总结起来，将费米子电弱规范相互作用写成流耦合的形式，

$$\mathcal{L}_\mathrm{EWF} \supset A_\mu J_\mathrm{EM}^\mu + Z_\mu J_Z^\mu + W_\mu^+ J_W^{+,\mu} + W_\mu^- J_W^{-,\mu}, \tag{109}$$

其中 f 代表任意费米子场，电磁流

$$J_\mathrm{EM}^\mu \equiv \sum_f Q_f e\bar{f}\gamma^\mu f, \tag{110}$$

弱中性流

$$J_Z^\mu \equiv \frac{g}{2c_\mathrm{W}}\sum_f \bar{f}\gamma^\mu(g_\mathrm{V}^f - g_\mathrm{A}^f\gamma^5)f = \frac{g}{c_\mathrm{W}}\sum_f(g_\mathrm{L}^f\bar{f}_\mathrm{L}\gamma^\mu f_\mathrm{L} + g_\mathrm{R}^f\bar{f}_\mathrm{R}\gamma^\mu f_\mathrm{R}), \tag{111}$$

弱带电流

$$J_W^{+,\mu} \equiv \frac{g}{\sqrt{2}}(\bar{u}_{i\mathrm{L}}\gamma^\mu V_{ij}d_{j\mathrm{L}} + \bar{\nu}_{i\mathrm{L}}\gamma^\mu\ell_{i\mathrm{L}}), \quad J_W^{-,\mu} \equiv (J_W^{+\mu})^\dagger = \frac{g}{\sqrt{2}}(\bar{d}_{j\mathrm{L}}V_{ji}^\dagger\gamma^\mu u_{i\mathrm{L}} + \bar{\ell}_{i\mathrm{L}}\gamma^\mu\nu_{i\mathrm{L}}). \tag{112}$$

对于各种费米子，相关的系数如下，

$$Q_{u_i} = \frac{2}{3}, \qquad\qquad Q_{d_i} = -\frac{1}{3}, \qquad Q_{\nu_i} = 0, \qquad\qquad Q_{\ell_i} = -1; \tag{113}$$

$$g_\mathrm{V}^{u_i} = \frac{1}{2} - \frac{4}{3}s_\mathrm{W}^2, \quad g_\mathrm{A}^{u_i} = \frac{1}{2}; \qquad g_\mathrm{V}^{d_i} = -\frac{1}{2} + \frac{2}{3}s_\mathrm{W}^2, \quad g_\mathrm{A}^{d_i} = -\frac{1}{2}; \tag{114}$$

$$g_\mathrm{V}^{\nu_i} = \frac{1}{2}, \qquad\qquad g_\mathrm{A}^{\nu_i} = \frac{1}{2}; \qquad g_\mathrm{V}^{\ell_i} = -\frac{1}{2} + 2s_\mathrm{W}^2, \quad g_\mathrm{A}^{\ell_i} = -\frac{1}{2}; \tag{115}$$
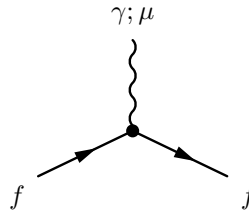
$$g_\mathrm{L}^{u_i} = \frac{1}{2} - \frac{2}{3}s_\mathrm{W}^2, \quad g_\mathrm{R}^{u_i} = -\frac{2}{3}s_\mathrm{W}^2; \quad g_\mathrm{L}^{d_i} = -\frac{1}{2} + \frac{1}{3}s_\mathrm{W}^2, \quad g_\mathrm{R}^{d_i} = \frac{1}{3}s_\mathrm{W}^2; \tag{116}$$

$$g_\mathrm{L}^{\nu_i} = \frac{1}{2}, \qquad\qquad g_\mathrm{R}^{\nu_i} = 0; \qquad g_\mathrm{L}^{\ell_i} = -\frac{1}{2} + s_\mathrm{W}^2, \qquad g_\mathrm{R}^{\ell_i} = s_\mathrm{W}^2. \tag{117}$$

可以看到，电磁流耦合与 QED 耦合完全相同，由此辩认出 A_μ 是电磁场，e 是单位电荷量，由 $Q = T^3 + Y$ 定义的 Q 确实是电荷。为了保持电荷守恒，指定复矢量场 $W_\mu^+(x)$ 携带 $Q = +1$ 的电荷，从而 $W^\pm$ 玻色子的电荷为 ± 1。不同代夸克之间的相互作用只发生在弱带电流耦合中，源自 CKM 矩阵 V 的非对角元。
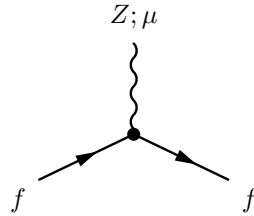
下面给出费米子电弱规范相互作用顶点的费曼规则。

QED 耦合:
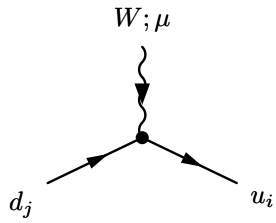
$$= \mathrm{i}Q_f e \gamma^\mu \tag{118}$$
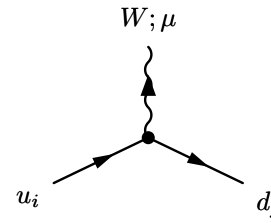
费米子与 Z 玻色子的耦合:
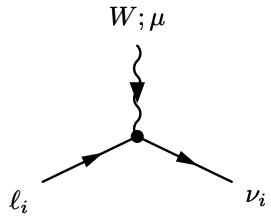
$$= \frac{\mathrm{i}g}{2c_{\mathrm{W}}} \gamma^\mu (g_{\mathrm{V}}^f - g_{\mathrm{A}}^f \gamma^5) \tag{119}$$
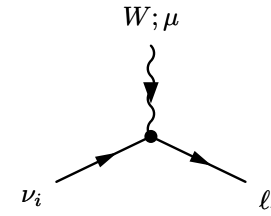
费米子与 $W^\pm$ 玻色子的耦合:

$$= \frac{\mathrm{i}g}{\sqrt{2}} V_{ij} \gamma^\mu P_{\mathrm{L}}$$

$$= \frac{\mathrm{i}g}{\sqrt{2}} V_{ji}^\dagger \gamma^\mu P_{\mathrm{L}} \tag{120}$$

$$= \frac{\mathrm{i}g}{\sqrt{2}} \gamma^\mu P_{\mathrm{L}}$$

$$= \frac{\mathrm{i}g}{\sqrt{2}} \gamma^\mu P_{\mathrm{L}} \tag{121}$$

4.3 电弱规范场的自相互作用

电弱规范场自相互作用的拉氏量是

$$\mathcal{L}_{\mathrm{EWG}} = -\frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}, \tag{122}$$

其中

$$W^{a\mu\nu} \equiv \partial^\mu W^{a\nu} - \partial^\nu W^{a\mu} + g\varepsilon^{abc} W^{b\mu} W^{c\nu}, \quad B^{\mu\nu} \equiv \partial^\mu B^\nu - \partial^\nu B^\mu. \tag{123}$$

利用 (63) 和 (67) 式, 推出

$$
\begin{aligned}
W_\mu^2 W_\nu^3 - W_\mu^3 W_\nu^2 &= \frac{\mathrm{i}}{\sqrt{2}} [(W_\mu^+ - W_\mu^-)(s_{\mathrm{W}} A_\nu + c_{\mathrm{W}} Z_\nu) - (s_{\mathrm{W}} A_\mu + c_{\mathrm{W}} Z_\mu)(W_\nu^+ - W_\nu^-)] \\
&= \frac{\mathrm{i}}{\sqrt{2}} [s_{\mathrm{W}}(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_{\mathrm{W}}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+) \\
&\quad - s_{\mathrm{W}}(W_\mu^- A_\nu - A_\mu W_\nu^-) - c_{\mathrm{W}}(W_\mu^- Z_\nu - Z_\mu W_\nu^-)],
\end{aligned}
\tag{124}
$$

$$W_\mu^3 W_\nu^1 - W_\mu^1 W_\nu^3 = \frac{1}{\sqrt{2}}[(s_{\rm W} A_\mu + c_{\rm W} Z_\mu)(W_\nu^+ + W_\nu^-) - (W_\mu^+ + W_\mu^-)(s_{\rm W} A_\nu + c_{\rm W} Z_\nu)]$$

$$= -\frac{1}{\sqrt{2}}[s_{\rm W}(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_{\rm W}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)$$

$$+ s_{\rm W}(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_{\rm W}(W_\mu^- Z_\nu - Z_\mu W_\nu^-)]. \tag{125}$$

从而

$$W_{\mu\nu}^1 = \partial_\mu W_\nu^1 - \partial_\nu W_\mu^1 + g\varepsilon^{1bc} W_\mu^b W_\nu^c = \partial_\mu W_\nu^1 - \partial_\nu W_\mu^1 + g W_\mu^2 W_\nu^3 - g W_\mu^3 W_\nu^2$$

$$= \frac{1}{\sqrt{2}}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) + \frac{1}{\sqrt{2}}(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) + g(W_\mu^2 W_\nu^3 - g W_\mu^3 W_\nu^2)$$

$$= \frac{1}{\sqrt{2}}\{\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ig[s_{\rm W}(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_{\rm W}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)]\}$$

$$+ \frac{1}{\sqrt{2}}\{\partial_\mu W_\nu^- - \partial_\nu W_\mu^- - ig[s_{\rm W}(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_{\rm W}(W_\mu^- Z_\nu - Z_\mu W_\nu^-)]\}$$

$$= \frac{1}{\sqrt{2}}(F_{\mu\nu}^+ + F_{\mu\nu}^-), \tag{126}$$

其中,

$$F_{\mu\nu}^+ \equiv \partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ie(W_\mu^+ A_\nu - A_\mu W_\nu^+) + igc_{\rm W}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+), \tag{127}$$

$$F_{\mu\nu}^- \equiv \partial_\mu W_\nu^- - \partial_\nu W_\mu^- - ie(W_\mu^- A_\nu - A_\mu W_\nu^-) - igc_{\rm W}(W_\mu^- Z_\nu - Z_\mu W_\nu^-). \tag{128}$$

另一方面,

$$W_{\mu\nu}^2 = \partial_\mu W_\nu^2 - \partial_\nu W_\mu^2 + g\varepsilon^{2bc} W_\mu^b W_\nu^c = \partial_\mu W_\nu^2 - \partial_\nu W_\mu^2 - g W_\mu^1 W_\nu^3 + g W_\mu^3 W_\nu^1$$

$$= \frac{i}{\sqrt{2}}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) - \frac{i}{\sqrt{2}}(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) + g(W_\mu^3 W_\nu^1 - W_\mu^1 W_\nu^3)$$

$$= \frac{i}{\sqrt{2}}\{\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ig[s_{\rm W}(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_{\rm W}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)]\}$$

$$- \frac{i}{\sqrt{2}}\{\partial_\mu W_\nu^- - \partial_\nu W_\mu^- - ig[s_{\rm W}(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_{\rm W}(W_\mu^- Z_\nu - Z_\mu W_\nu^-)]\}$$

$$= \frac{i}{\sqrt{2}}(F_{\mu\nu}^+ - F_{\mu\nu}^-). \tag{129}$$

因此

$$-\frac{1}{4}W_{\mu\nu}^1 W^{1\mu\nu} - \frac{1}{4}W_{\mu\nu}^2 W^{2\mu\nu}$$

$$= -\frac{1}{8}(F_{\mu\nu}^+ + F_{\mu\nu}^-)(F^{+\mu\nu} + F^{-\mu\nu}) + \frac{1}{8}(F_{\mu\nu}^+ - F_{\mu\nu}^-)(F^{+\mu\nu} - F^{-\mu\nu}) = -\frac{1}{2}F_{\mu\nu}^+ F^{-\mu\nu}$$

$$= -\frac{1}{2}[\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ie(W_\mu^+ A_\nu - A_\mu W_\nu^+) + igc_{\rm W}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)]$$

$$\times[\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu} - ie(W^{-\mu} A^\nu - A^\mu W^{-\nu}) - igc_{\rm W}(W^{-\mu} Z^\nu - Z^\mu W^{-\nu})]$$

$$
\begin{aligned}
= &-(\partial_\mu W_\nu^+)(\partial^\mu W^{-\nu}) + (\partial_\mu W_\nu^+)(\partial^\nu W^{-\mu}) \\
&+ie[(\partial_\mu W_\nu^+)W^{-\mu}A^\nu - (\partial_\mu W_\nu^+)W^{-\nu}A^\mu - W_\mu^+(\partial^\mu W^{-\nu})A_\nu + W_\nu^+(\partial^\mu W^{-\nu})A_\mu] \\
&+igc_{\mathrm{W}}[(\partial_\mu W_\nu^+)W^{-\mu}Z^\nu - (\partial_\mu W_\nu^+)W^{-\nu}Z^\mu - W_\mu^+(\partial^\mu W^{-\nu})Z_\nu + W_\nu^+(\partial^\mu W^{-\nu})Z_\mu] \\
&+e^2(W_\mu^+W^{-\nu}A_\nu A^\mu - W_\mu^+W^{-\mu}A_\nu A^\nu) + g^2c_{\mathrm{W}}^2(W_\mu^+W^{-\nu}Z_\nu Z^\mu - W_\mu^+W^{-\mu}Z_\nu Z^\nu) \\
&+egc_{\mathrm{W}}(W_\mu^+W^{-\nu}A_\nu Z^\mu + W_\mu^+W^{-\nu}A^\mu Z_\nu - 2W_\mu^+W^{-\mu}A_\nu Z^\nu).
\end{aligned} \tag{130}
$$

由

$$
\begin{aligned}
W_\mu^1 W_\nu^2 - W_\mu^2 W_\nu^1 &= \frac{\mathrm{i}}{2}(W_\mu^+ + W_\mu^-)(W_\nu^+ - W_\nu^-) - \frac{\mathrm{i}}{2}(W_\mu^+ - W_\mu^-)(W_\nu^+ + W_\nu^-) \\
&= -\mathrm{i}(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+),
\end{aligned} \tag{131}
$$

得到

$$
\begin{aligned}
W_{\mu\nu}^3 &= \partial_\mu W_\nu^3 - \partial_\nu W_\mu^3 + g\varepsilon^{3bc}W_\mu^b W_\nu^c = \partial_\mu W_\nu^3 - \partial_\nu W_\mu^3 + gW_\mu^1 W_\nu^2 - gW_\mu^2 W_\nu^1 \\
&= s_{\mathrm{W}}\partial_\mu A_\nu + c_{\mathrm{W}}\partial_\mu Z_\nu - s_{\mathrm{W}}\partial_\nu A_\mu + c_{\mathrm{W}}\partial_\nu Z_\mu + g(W_\mu^1 W_\nu^2 - W_\mu^2 W_\nu^1) \\
&= s_{\mathrm{W}}(\partial_\mu A_\nu - \partial_\nu A_\mu) + c_{\mathrm{W}}(\partial_\mu Z_\nu - \partial_\nu Z_\mu) - \mathrm{i}g(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+), \\
B_{\mu\nu} &= \partial_\mu(c_{\mathrm{W}}A_\nu - s_{\mathrm{W}}Z_\nu) - \partial_\nu(c_{\mathrm{W}}A_\mu - s_{\mathrm{W}}Z_\mu) \\
&= c_{\mathrm{W}}(\partial_\mu A_\nu - \partial_\nu A_\mu) - s_{\mathrm{W}}(\partial_\mu Z_\nu - \partial_\nu Z_\mu).
\end{aligned} \tag{132}
$$

$$
\tag{133}
$$

于是
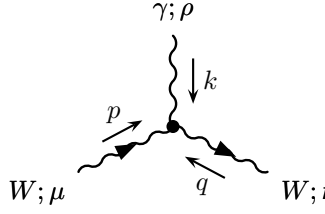
$$
\begin{aligned}
&-\frac{1}{4}W_{\mu\nu}^3 W^{3\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
=&-\frac{1}{2}[(\partial_\mu A_\nu)(\partial^\mu A^\nu) - (\partial_\mu A_\nu)(\partial^\nu A^\mu)] - \frac{1}{2}[(\partial_\mu Z_\nu)(\partial^\mu Z^\nu) - (\partial_\mu Z_\nu)(\partial^\nu Z^\mu)] \\
&+ie[W^{+\mu}W^{-\nu}(\partial_\mu A_\nu) - W^{+\nu}W^{-\mu}(\partial_\mu A_\nu)] + igc_{\mathrm{W}}[W^{+\mu}W^{-\nu}(\partial_\mu Z_\nu) - W^{+\nu}W^{-\mu}(\partial_\mu Z_\nu)] \\
&+\frac{g^2}{2}(W_\mu^+ W^{+\mu}W_\nu^- W^{-\nu} - W_\mu^+ W^{+\nu}W_\nu^- W^{-\mu}).
\end{aligned} \tag{134}
$$

综合起来，有
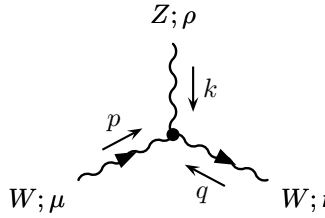
$$
\begin{aligned}
\mathcal{L}_{\mathrm{EWG}} = &\frac{1}{2}[(\partial_\mu A_\nu)(\partial^\nu A^\mu) - (\partial_\mu A_\nu)(\partial^\mu A^\nu)] + \frac{1}{2}[(\partial_\mu Z_\nu)(\partial^\nu Z^\mu) - (\partial_\mu Z_\nu)(\partial^\mu Z^\nu)] \\
&+(\partial_\mu W_\nu^+)(\partial^\nu W^{-\mu}) - (\partial_\mu W_\nu^+)(\partial^\mu W^{-\nu}) + \frac{g^2}{2}(W_\mu^+ W^{+\mu}W_\nu^- W^{-\nu} - W_\mu^+ W^{+\nu}W_\nu^- W^{-\mu}) \\
&+ie[(\partial_\mu W_\nu^+)W^{-\mu}A^\nu - (\partial_\mu W_\nu^+)W^{-\nu}A^\mu - W^{+\mu}(\partial_\mu W_\nu^-)A^\nu + W^{+\nu}(\partial_\mu W_\nu^-)A^\mu \\
&\qquad + W^{+\mu}W^{-\nu}(\partial_\mu A_\nu) - W^{+\nu}W^{-\mu}(\partial_\mu A_\nu)] \\
&+igc_{\mathrm{W}}[(\partial_\mu W_\nu^+)W^{-\mu}Z^\nu - (\partial_\mu W_\nu^+)W^{-\nu}Z^\mu - W^{+\mu}(\partial_\mu W_\nu^-)Z^\nu + W^{+\nu}(\partial_\mu W_\nu^-)Z^\mu \\
&\qquad + W^{+\mu}W^{-\nu}(\partial_\mu Z_\nu) - W^{+\nu}W^{-\mu}(\partial_\mu Z_\nu)] \\
&+e^2(W_\mu^+W^{-\nu}A_\nu A^\mu - W_\mu^+W^{-\mu}A_\nu A^\nu) + g^2c_{\mathrm{W}}^2(W_\mu^+W^{-\nu}Z_\nu Z^\mu - W_\mu^+W^{-\mu}Z_\nu Z^\nu)
\end{aligned}
$$

$$+egc_{\mathrm{W}}(W_\mu^+ W^{-\nu} A_\nu Z^\mu + W_\mu^+ W^{-\nu} A^\mu Z_\nu - 2W_\mu^+ W^{-\mu} A_\nu Z^\nu). \tag{135}$$
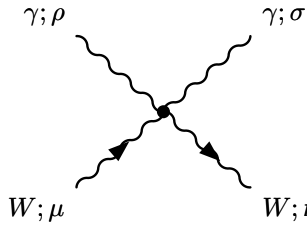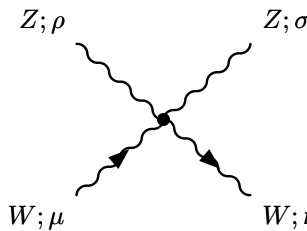
下面是电弱规范玻色子自耦合的费曼规则:

$$= -\mathrm{i}e[g^{\mu\nu}(p-q)^\rho + g^{\nu\rho}(q-k)^\mu + g^{\rho\mu}(k-p)^\nu] \tag{136}$$
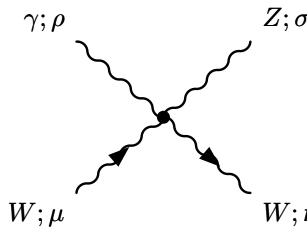
$$= -\mathrm{i}gc_{\mathrm{W}}[g^{\mu\nu}(p-q)^\rho + g^{\nu\rho}(q-k)^\mu + g^{\rho\mu}(k-p)^\nu] \tag{137}$$
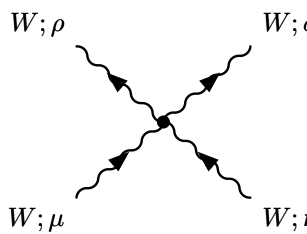
$$= \mathrm{i}e^2(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma}) \tag{138}$$

$$= \mathrm{i}g^2 c_{\mathrm{W}}^2(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma}) \tag{139}$$

$$= \mathrm{i}egc_{\mathrm{W}}(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma}) \tag{140}$$

$$= -\mathrm{i}g^2(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma}) \tag{141}$$

5 R_ξ 规范下电弱拉氏量和费曼规则

在 **R_ξ** 规范下，将 Higgs 场参数化为

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \dfrac{1}{\sqrt{2}}[v + H(x) + \mathrm{i}\chi(x)] \end{pmatrix}, \tag{142}$$

其中 ϕ^+ 和 χ 是 Nambu-Goldstone 标量场。那么，$\tilde{\Phi}(x)$ 的形式是

$$\tilde{\Phi}(x) = \begin{pmatrix} \phi^{0*}(x) \\ -\phi^-(x) \end{pmatrix} = \begin{pmatrix} \dfrac{1}{\sqrt{2}}[v + H(x) - \mathrm{i}\chi(x)] \\ -\phi^-(x) \end{pmatrix}. \tag{143}$$

利用

$$\Phi^\dagger\Phi = \frac{1}{2}(v^2 + H^2 + 2vH + \chi^2) + |\phi^+|^2, \tag{144}$$

$$(\Phi^\dagger\Phi)^2 = \frac{1}{4}(v^2 + H^2 + 2vH + \chi^2)^2 + |\phi^+|^4 + |\phi^+|^2(v^2 + H^2 + 2vH + \chi^2), \tag{145}$$

推出 Higgs 场势能项

$$\begin{aligned}
&-V_{\mathrm{H}}(\Phi) = \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2 \\
&= \frac{\mu^2}{2}(v^2 + H^2 + 2vH + \chi^2) + \mu^2|\phi^+|^2 - \frac{\lambda}{4}(v^2 + H^2 + 2vH + \chi^2)^2 - \lambda|\phi^+|^4 \\
&\quad - \lambda|\phi^+|^2(v^2 + H^2 + 2vH + \chi^2) \\
&= \frac{1}{2}\left(\mu^2 - \frac{\lambda}{2}v^2\right)v^2 + \frac{1}{2}(\mu^2 - 3\lambda v^2)H^2 + (\mu^2 - \lambda v^2)vH + \frac{1}{2}(\mu^2 - \lambda v^2)\chi^2 - \frac{\lambda}{4}H^4 - \frac{\lambda}{4}\chi^4 \\
&\quad - \lambda vH^3 - \frac{\lambda}{2}H^2\chi^2 - \lambda vH\chi^2 + (\mu^2 - \lambda v^2)|\phi^+|^2 - \lambda|\phi^+|^4 - \lambda|\phi^+|^2(H^2 + 2vH + \chi^2) \\
&= \frac{\lambda}{4}v^4 - \lambda v^2 H^2 - \frac{\lambda}{4}H^4 - \frac{\lambda}{4}\chi^4 - \lambda vH^3 - \frac{\lambda}{2}H^2\chi^2 - \lambda vH\chi^2 \\
&\quad - \lambda\phi^+\phi^-(\phi^+\phi^- + H^2 + 2vH + \chi^2) \\
&= \frac{1}{8}m_H^2 v^2 - \frac{1}{2}m_H^2 H^2 - \frac{m_H^2}{2v}H^3 - \frac{m_H^2}{8v^2}H^4 - \frac{m_H^2}{2v}H\chi^2 - \frac{m_H^2}{4v^2}H^2\chi^2 - \frac{m_H^2}{8v^2}\chi^4 \\
&\quad - \frac{m_H^2}{2v^2}\phi^+\phi^-(\phi^+\phi^- + H^2 + 2vH + \chi^2).
\end{aligned} \tag{146}$$

由 $V = U_u^\dagger U_d$ 得到 $V^\dagger = U_d^\dagger U_u$ 和 $U_d = U_u V$，$U_u = U_d V^\dagger$，则

$$\tilde{y}_d = U_d y_{\mathrm{D}} K_d^\dagger = U_u V y_{\mathrm{D}} K_d^\dagger, \quad \tilde{y}_u = U_u y_{\mathrm{U}} K_u^\dagger = U_d V^\dagger y_{\mathrm{U}} K_u^\dagger, \tag{147}$$

故

$$\tilde{y}_{d,ij}\bar{u}'_{i\mathrm{L}}d'_{j\mathrm{R}} = \bar{u}'_{i\mathrm{L}}(U_u V y_{\mathrm{D}} K_d^\dagger)_{ij}d'_{j\mathrm{R}} = \bar{u}'_{i\mathrm{L}}U_{u,ik}V_{kl}y_{d_l}(K_d^\dagger)_{lj}d'_{m\mathrm{R}} = y_{d_j}\bar{u}_{i\mathrm{L}}V_{ij}d_{j\mathrm{R}}, \tag{148}$$

$$\tilde{y}_{u,ij}\bar{d}'_{i\mathrm{L}}u'_{j\mathrm{R}} = \bar{d}'_{i\mathrm{L}}(U_d V^\dagger y_{\mathrm{U}} K_u^\dagger)_{ij}u'_{j\mathrm{R}} = \bar{d}'_{i\mathrm{L}}U_{d,ik}V_{kl}^\dagger y_{u_l}(K_u^\dagger)_{lj}u'_{j\mathrm{R}} = y_{u_i}\bar{d}_{j\mathrm{L}}V_{ji}^\dagger u_{i\mathrm{R}}. \tag{149}$$

结合 (84) 和 (85) 式，得

$$\begin{aligned}
\tilde{y}_{d,ij}\bar{Q}_{i\mathrm{L}}d'_{j\mathrm{R}}\Phi &= \tilde{y}_{d,ij}\left[\bar{u}'_{i\mathrm{L}}d'_{j\mathrm{R}}\phi^+ + \frac{1}{\sqrt{2}}\bar{d}'_{i\mathrm{L}}d'_{j\mathrm{R}}(v+H+\mathrm{i}\chi)\right] \\
&= y_{d_j}\bar{u}_{i\mathrm{L}}V_{ij}d_{j\mathrm{R}}\phi^+ + \frac{y_{d_i}}{\sqrt{2}}\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}}(v+H+\mathrm{i}\chi),
\end{aligned} \tag{150}$$

$$\begin{aligned}
\tilde{y}_{u,ij}\bar{Q}_{i\mathrm{L}}u'_{j\mathrm{R}}\tilde{\Phi} &= \tilde{y}_{u,ij}\left[\frac{1}{\sqrt{2}}\bar{u}'_{i\mathrm{L}}u'_{j\mathrm{R}}(v+H-\mathrm{i}\chi) - \bar{d}'_{i\mathrm{L}}u'_{j\mathrm{R}}\phi^-\right] \\
&= \frac{y_{u_i}}{\sqrt{2}}\bar{u}_{i\mathrm{L}}u_{j\mathrm{R}}(v+H-\mathrm{i}\chi) - y_{u_i}\bar{d}_{j\mathrm{L}}V_{ji}^\dagger u_{i\mathrm{R}}\phi^-.
\end{aligned} \tag{151}$$

从而，Yukawa 相互作用拉氏量化为

$$\begin{aligned}
\mathcal{L}_{\mathrm{Y}} &= -\tilde{y}_{d,ij}\bar{Q}_{i\mathrm{L}}d'_{j\mathrm{R}}\Phi - \tilde{y}_{u,ij}\bar{Q}_{i\mathrm{L}}u'_{j\mathrm{R}}\tilde{\Phi} - y_{\ell_i}\bar{L}_{i\mathrm{L}}\ell_{i\mathrm{R}}\Phi + \mathrm{H.c.} \\
&= -y_{d_j}\bar{u}_{i\mathrm{L}}V_{ij}d_{j\mathrm{R}}\phi^+ - \frac{y_{d_i}}{\sqrt{2}}\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}}(v+H+\mathrm{i}\chi) - \frac{y_{u_i}}{\sqrt{2}}\bar{u}_{i\mathrm{L}}u_{i\mathrm{R}}(v+H-\mathrm{i}\chi) + y_{u_i}\bar{d}_{j\mathrm{L}}V_{ji}^\dagger u_{i\mathrm{R}}\phi^- \\
&\quad -y_{\ell_i}\bar{\nu}_{i\mathrm{L}}\ell_{i\mathrm{R}}\phi^+ - \frac{y_{\ell_i}}{\sqrt{2}}\bar{\ell}_{i\mathrm{L}}\ell_{i\mathrm{R}}(v+H+\mathrm{i}\chi) + \mathrm{H.c.} \\
&= -m_{d_i}\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}} - m_{u_i}\bar{u}_{i\mathrm{L}}u_{i\mathrm{R}} - m_{\ell_i}\bar{\ell}_{i\mathrm{L}}\ell_{i\mathrm{R}} - \frac{m_{d_i}}{v}\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}}(H+\mathrm{i}\chi) - \frac{m_{u_i}}{v}\bar{u}_{i\mathrm{L}}u_{i\mathrm{R}}(H-\mathrm{i}\chi) \\
&\quad -\frac{m_{\ell_i}}{v}\bar{\ell}_{i\mathrm{L}}\ell_{i\mathrm{R}}(H+\mathrm{i}\chi) - \frac{\sqrt{2}m_{d_j}}{v}\bar{u}_{i\mathrm{L}}V_{ij}d_{j\mathrm{R}}\phi^+ + \frac{\sqrt{2}m_{u_i}}{v}\bar{d}_{j\mathrm{L}}V_{ji}^\dagger u_{i\mathrm{R}}\phi^- - \frac{\sqrt{2}m_{\ell_i}}{v}\bar{\nu}_{i\mathrm{L}}\ell_{i\mathrm{R}}\phi^+ + \mathrm{H.c.} \\
&= -m_{d_i}\bar{d}_i d_i - m_{u_i}\bar{u}_i u_i - m_{\ell_i}\bar{\ell}_i\ell_i - \frac{m_{d_i}}{v}H\bar{d}_i d_i - \frac{m_{u_i}}{v}H\bar{u}_i u_i - \frac{m_{\ell_i}}{v}H\bar{\ell}_i\ell_i \\
&\quad -\frac{m_{d_i}}{v}\chi\bar{d}_i\mathrm{i}\gamma^5 d_i + \frac{m_{u_i}}{v}\chi\bar{u}_i\mathrm{i}\gamma^5 u_i - \frac{m_{\ell_i}}{v}\chi\bar{\ell}_i\mathrm{i}\gamma^5\ell_i + \frac{\sqrt{2}V_{ij}}{v}\phi^+\bar{u}_i(m_{u_i}P_{\mathrm{L}} - m_{d_j}P_{\mathrm{R}})d_j \\
&\quad -\frac{\sqrt{2}V_{ji}^\dagger}{v}\phi^-\bar{d}_j(m_{d_j}P_{\mathrm{L}} - m_{u_i}P_{\mathrm{R}})u_i - \frac{\sqrt{2}m_{\ell_i}}{v}(\phi^+\bar{\nu}_i P_{\mathrm{R}}\ell_i + \phi^-\bar{\ell}_i P_{\mathrm{L}}\nu_i).
\end{aligned} \tag{152}$$

利用

$$\begin{aligned}
D_\mu\Phi &= \begin{pmatrix} \partial_\mu - \mathrm{i}eA_\mu - \dfrac{\mathrm{i}g}{2c_{\mathrm{W}}}(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)Z_\mu & -\dfrac{\mathrm{i}g}{\sqrt{2}}W_\mu^+ \\ -\dfrac{\mathrm{i}g}{\sqrt{2}}W_\mu^- & \partial_\mu + \dfrac{\mathrm{i}g}{2c_{\mathrm{W}}}Z_\mu \end{pmatrix}\begin{pmatrix} \phi^+ \\ \dfrac{1}{\sqrt{2}}(v+H+\mathrm{i}\chi) \end{pmatrix} \\
&= \begin{pmatrix} \partial_\mu\phi^+ - \mathrm{i}\left[eA_\mu + \dfrac{g(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)}{2c_{\mathrm{W}}}Z_\mu\right]\phi^+ - \dfrac{\mathrm{i}g}{2}W_\mu^+(H+\mathrm{i}\chi) - \mathrm{i}m_W W_\mu^+ \\ \dfrac{1}{\sqrt{2}}\left[\partial_\mu(H+\mathrm{i}\chi) - \mathrm{i}gW_\mu^-\phi^+ + \dfrac{\mathrm{i}g}{2c_{\mathrm{W}}}Z_\mu(H+\mathrm{i}\chi) + \mathrm{i}m_Z Z_\mu\right] \end{pmatrix},
\end{aligned} \tag{153}$$

将 Higgs 场协变动能项化为

$$(D^\mu\Phi)^\dagger D_\mu\Phi$$

$$= \left|\partial_\mu\phi^+ - \mathrm{i}\left[eA_\mu + \frac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{2c_\mathrm{W}}Z_\mu\right]\phi^+ - \frac{\mathrm{i}g}{2}W_\mu^+(H + \mathrm{i}\chi) - \mathrm{i}m_W W_\mu^+\right|^2$$

$$+ \frac{1}{2}\left|\partial_\mu(H + \mathrm{i}\chi) - \mathrm{i}gW_\mu^-\phi^+ + \frac{\mathrm{i}g}{2c_\mathrm{W}}Z_\mu(H + \mathrm{i}\chi) + \mathrm{i}m_Z Z_\mu\right|^2$$

$$= (\partial^\mu\phi^+)(\partial_\mu\phi^-) + \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + \frac{1}{2}(\partial^\mu\chi)(\partial_\mu\chi)$$

$$+ \left(-\mathrm{i}\partial^\mu\phi^-\left\{\left[eA_\mu + \frac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{2c_\mathrm{W}}Z_\mu\right]\phi^+ + \frac{g}{2}W_\mu^+(H + \mathrm{i}\chi) + m_W W_\mu^+\right\} + \mathrm{H.c.}\right)$$

$$+ \left\{-\frac{\mathrm{i}}{2}\partial^\mu(H - \mathrm{i}\chi)\left[gW_\mu^-\phi^+ - \frac{g}{2c_\mathrm{W}}Z_\mu(H + \mathrm{i}\chi) - m_Z Z_\mu\right] + \mathrm{H.c.}\right\}$$

$$+ \left|\left[eA_\mu + \frac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{2c_\mathrm{W}}Z_\mu\right]\phi^+ + \frac{g}{2}W_\mu^+(H + \mathrm{i}\chi) + m_W W_\mu^+\right|^2$$

$$+ \frac{1}{2}\left|gW_\mu^-\phi^+ - \frac{g}{2c_\mathrm{W}}Z_\mu(H + \mathrm{i}\chi) - m_Z Z_\mu\right|^2$$

$$= (\partial^\mu\phi^+)(\partial_\mu\phi^-) + \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + \frac{1}{2}(\partial^\mu\chi)(\partial_\mu\chi)$$

$$+ m_W^2 W^{-\mu}W_\mu^+ + \frac{1}{2}m_Z^2 Z^\mu Z_\mu + gm_W HW_\mu^+ W^{-\mu} + \frac{gm_Z}{2c_\mathrm{W}}HZ^\mu Z_\mu$$

$$+ \frac{g}{2}[\mathrm{i}W_\mu^+\phi^-\overset{\leftrightarrow}{\partial^\mu}(H + \mathrm{i}\chi) + \mathrm{H.c.}] + \mathrm{i}eA_\mu\phi^-\overset{\leftrightarrow}{\partial^\mu}\phi^+ + \frac{g}{2c_\mathrm{W}}Z_\mu[-\chi\overset{\leftrightarrow}{\partial^\mu}H + \mathrm{i}(c_\mathrm{W}^2 - s_\mathrm{W}^2)\phi^-\overset{\leftrightarrow}{\partial^\mu}\phi^+]$$

$$+ \frac{g^2}{4}W_\mu^+ W^{-\mu}(2\phi^+\phi^- + H^2 + \chi^2) + e^2 A_\mu A^\mu\phi^+\phi^-$$

$$+ \frac{g^2}{4c_\mathrm{W}^2}Z_\mu Z^\mu\left[(c_\mathrm{W}^2 - s_\mathrm{W}^2)^2\phi^+\phi^- + \frac{1}{2}H^2 + \frac{1}{2}\chi^2\right]$$

$$+ \left[\frac{eg}{2}W_\mu^+ A^\mu\phi^-(H + \mathrm{i}\chi) - \frac{g^2 s_\mathrm{W}^2}{2c_\mathrm{W}}W_\mu^+ Z^\mu\phi^-(H + \mathrm{i}\chi) + \mathrm{H.c.}\right] + \frac{eg(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{c_\mathrm{W}}A_\mu Z^\mu\phi^+\phi^-$$

$$+ (em_W A^\mu\phi^+ W_\mu^- - gs_\mathrm{W}^2 m_Z Z^\mu\phi^+ W_\mu^- + \mathrm{H.c.}) + \mathcal{L}_{\mathrm{b1}}, \tag{154}$$

其中

$$\mathcal{L}_{\mathrm{b1}} = -\mathrm{i}m_W(\partial^\mu\phi^-)W_\mu^+ + \mathrm{i}m_W(\partial^\mu\phi^+)W_\mu^- + m_Z(\partial^\mu\chi)Z_\mu. \tag{155}$$

在 R$_\xi$ 规范下，将规范固定函数设为

$$G^\pm = \frac{1}{\sqrt{\xi_W}}(\partial^\mu W_\mu^\pm \mp \mathrm{i}\xi_W m_W\phi^\pm), \quad G^Z = \frac{1}{\sqrt{\xi_Z}}(\partial^\mu Z_\mu - \xi_Z m_Z\chi), \quad G^\gamma = \frac{1}{\sqrt{\xi_\gamma}}\partial^\mu A_\mu, \tag{156}$$

它们在路径积分量子化中的泛函积分形式为

$$\int \mathcal{D}\omega^+ \int \mathcal{D}\omega^- \int \mathcal{D}\omega^Z \int \mathcal{D}\omega^\gamma \exp\left[-\mathrm{i}\int d^4x\left(\omega^+\omega^- + \frac{1}{2}\omega^Z\omega^Z + \frac{1}{2}\omega^\gamma\omega^\gamma\right)\right]$$

$$\times \delta(G^+ - \omega^+)\delta(G^- - \omega^-)\delta(G^Z - \omega^Z)\delta(G^\gamma - \omega^\gamma)$$

$$= \exp\left[-i \int d^4x \left(G^+G^- + \frac{1}{2}G^ZG^Z + \frac{1}{2}G^\gamma G^\gamma\right)\right]. \tag{157}$$

由此得到拉氏量中的规范固定项

$$\mathcal{L}_{\mathrm{EW,GF}} = -G^+G^- - \frac{1}{2}(G^Z)^2 - \frac{1}{2}(G^\gamma)^2$$

$$= -\frac{1}{\xi_W}(\partial^\mu W_\mu^+ - i\xi_W m_W \phi^+)(\partial^\nu W_\nu^- + i\xi_W m_W \phi^-) - \frac{1}{2\xi_Z}(\partial^\mu Z_\mu - \xi_Z m_Z \chi)^2 - \frac{1}{2\xi_\gamma}(\partial^\mu A_\mu)^2$$

$$= -\frac{1}{\xi_W}(\partial^\mu W_\mu^+)(\partial^\nu W_\nu^-) - \frac{1}{2\xi_Z}(\partial^\mu Z_\mu)^2 - \frac{1}{2\xi_\gamma}(\partial^\mu A_\mu)^2 - \xi_W m_W^2 \phi^+ \phi^- - \frac{1}{2}\xi_Z m_Z^2 \chi^2 + \mathcal{L}_{\mathrm{b2}}. \tag{158}$$

可见,Nambu-Goldstone 玻色子 $\phi^\pm$ 和 χ 在 R_ξ 规范下具有依赖于 ξ_W 和 ξ_Z 的非物理质量,

$$m_\phi = \sqrt{\xi_W}\, m_W, \quad m_\chi = \sqrt{\xi_Z}\, m_Z. \tag{159}$$

这里

$$\mathcal{L}_{\mathrm{b2}} = -im_W \phi^-(\partial^\mu W_\mu^+) + im_W \phi^+ \partial^\mu W_\mu^- + m_Z \chi \partial^\mu Z_\mu. \tag{160}$$

由于

$$\mathcal{L}_{\mathrm{b1}} + \mathcal{L}_{\mathrm{b2}} = -im_W \partial^\mu(\phi^- W_\mu^+) + im_W \partial^\mu(\phi^+ W_\mu^-) + m_Z \partial^\mu(\chi Z_\mu), \tag{161}$$

这两项体现为全散度,不会有物理效应。可见,协变动能项中规范场与 Nambu-Goldstone 标量场之间的双线性耦合项 $\mathcal{L}_{\mathrm{b1}}$ 被规范固定项中的 $\mathcal{L}_{\mathrm{b2}}$ 抵消掉,这就是如此选取规范固定函数的目的。

这样一来,电弱规范场传播子相关拉氏量变成

$$\begin{aligned}
\mathcal{L}_{\mathrm{EW,prop}} &= (\partial_\mu W_\nu^+)(\partial^\nu W^{-\mu}) - (\partial_\mu W_\nu^+)(\partial^\mu W^{-\nu}) - \frac{1}{\xi_W}(\partial^\mu W_\mu^+)(\partial^\nu W_\nu^-) + m_W^2 W^{-\mu} W_\mu^+ \\
&\quad + \frac{1}{2}\left[(\partial_\mu Z_\nu)(\partial^\nu Z^\mu) - (\partial_\mu Z_\nu)(\partial^\mu Z^\nu) - \frac{1}{\xi_Z}(\partial^\mu Z_\mu)^2 + m_Z^2 Z^\mu Z_\mu\right] \\
&\quad + \frac{1}{2}\left[(\partial_\mu A_\nu)(\partial^\nu A^\mu) - (\partial_\mu A_\nu)(\partial^\mu A^\nu) - \frac{1}{\xi_\gamma}(\partial^\mu A_\mu)^2\right] \\
&\rightarrow W_\mu^+\left[g^{\mu\nu}(\partial^2 + m_W^2) - \left(1 - \frac{1}{\xi_W}\right)\partial^\mu \partial^\nu\right]W_\nu^- \\
&\quad + \frac{1}{2}Z_\mu\left[g^{\mu\nu}(\partial^2 + m_Z^2) - \left(1 - \frac{1}{\xi_Z}\right)\partial^\mu \partial^\nu\right]Z_\nu \\
&\quad + \frac{1}{2}A_\mu\left[g^{\mu\nu}\partial^2 - \left(1 - \frac{1}{\xi_\gamma}\right)\partial^\mu \partial^\nu\right]A_\nu.
\end{aligned} \tag{162}$$

于是,光子的传播子与胶子形式类似,为

$$\frac{-i}{p^2 + i\epsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1 - \xi_\gamma)\right]. \tag{163}$$

将 $W^\pm$ 传播子相关拉氏量变换到动量空间，得

$$- g^{\mu\nu}(p^2 - m_W^2) + \left(1 - \frac{1}{\xi_W}\right) p^\mu p^\nu = - \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)(p^2 - m_W^2) - \frac{p^\mu p^\nu}{p^2}\frac{p^2 - \xi_W m_W^2}{\xi_W}, \quad (164)$$

它的逆矩阵是

$$- \frac{1}{p^2 - m_W^2}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) - \frac{\xi_W}{p^2 - \xi_W m_W^2}\frac{p_\mu p_\nu}{p^2} = - \frac{1}{p^2 - m_W^2}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_W m_W^2}(1 - \xi_W)\right],$$
$$(165)$$

这是因为由

$$\left(g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2}\right)\frac{p^\mu p^\nu}{p^2} = \frac{p_\rho p^\nu}{p^2} - \frac{p_\rho p^\nu}{p^2} = 0, \quad \left(g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2}\right)\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) = \delta_\rho{}^\nu - \frac{p_\rho p^\nu}{p^2} \quad (166)$$

得

$$\left[-\frac{1}{p^2 - m_W^2}\left(g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2}\right) - \frac{\xi_W}{p^2 - \xi_W m_W^2}\frac{p_\rho p_\mu}{p^2}\right]$$
$$\times \left[-\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)(p^2 - m_W^2) - \frac{p^\mu p^\nu}{p^2}\frac{p^2 - \xi_W m_W^2}{\xi}{}_W\right]$$
$$= \delta_\rho{}^\nu - \frac{p_\rho p^\nu}{p^2} + \frac{p_\rho p^\nu}{p^2} = \delta_\rho{}^\nu. \quad (167)$$

从而，$W^\pm$ 传播子的形式为

$$\frac{-\mathrm{i}}{p^2 - m_W^2 + \mathrm{i}\epsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_W m_W^2}(1 - \xi_W)\right]. \quad (168)$$

同理，Z 传播子的形式为

$$\frac{-\mathrm{i}}{p^2 - m_Z^2 + \mathrm{i}\epsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_Z m_Z^2}(1 - \xi_Z)\right]. \quad (169)$$

电弱规范场的无穷小规范变换形式是

$$\delta W_\mu^a = \frac{1}{g}\,\partial_\mu \alpha^a + \varepsilon^{abc}W_\mu^b \alpha^c, \quad \delta B_\mu = \frac{1}{g'}\,\partial_\mu \alpha^Y. \quad (170)$$

定义

$$\alpha^\pm \equiv \frac{1}{\sqrt{2}}(\alpha^1 \mp \mathrm{i}\alpha^2), \quad \alpha^Z \equiv \alpha^3 - \alpha^Y, \quad \alpha^\gamma \equiv s_W^2 \alpha^3 + c_W^2 \alpha^Y, \quad (171)$$

利用

$$\varepsilon^{1bc}W_\mu^b \alpha^c = W_\mu^2 \alpha^3 - W_\mu^3 \alpha^2, \quad \varepsilon^{2bc}W_\mu^b \alpha^c = -W_\mu^1 \alpha^3 + W_\mu^3 \alpha^1, \quad (172)$$

$$\pm\mathrm{i}\sqrt{2}\alpha^\pm = \pm\mathrm{i}\alpha^1 + \alpha^2, \qquad \pm\mathrm{i}\sqrt{2}W_\mu^\pm = \pm\mathrm{i}W_\mu^1 + W_\mu^2, \quad (173)$$

有

$$\varepsilon^{1bc}W_\mu^b\alpha^c \mp i\varepsilon^{2bc}W_\mu^b\alpha^c = (W_\mu^2\alpha^3 - W_\mu^3\alpha^2) \mp i(-W_\mu^1\alpha^3 + W_\mu^3\alpha^1)$$
$$= (W_\mu^2 \pm iW_\mu^1)\alpha^3 - W_\mu^3(\alpha^2 \pm i\alpha^1)$$
$$= \pm i\sqrt{2}W_\mu^\pm(c_W^2\alpha^Z + \alpha^\gamma) \mp i\sqrt{2}(s_W A_\mu + c_W Z_\mu)\alpha^\pm, \tag{174}$$

$$\varepsilon^{3bc}W_\mu^b\alpha^c = W_\mu^1\alpha^2 - W_\mu^2\alpha^1 = \frac{1}{\sqrt{2}}(W_\mu^+ + W_\mu^-)\frac{i}{\sqrt{2}}(\alpha^+ - \alpha^-) - \frac{i}{\sqrt{2}}(W_\mu^+ - W_\mu^-)\frac{1}{\sqrt{2}}(\alpha^+ + \alpha^-)$$
$$= -i(W_\mu^+\alpha^- - W_\mu^-\alpha^+). \tag{175}$$

因此,

$$\delta W_\mu^+ = \frac{1}{\sqrt{2}}(\delta W_\mu^1 - i\delta W_\mu^2) = \frac{1}{\sqrt{2}g}\partial_\mu(\alpha^1 - i\alpha^2) + \frac{1}{\sqrt{2}}(\varepsilon^{1bc}W_\mu^b\alpha^c - i\varepsilon^{2bc}W_\mu^b\alpha^c)$$
$$= \frac{1}{g}\partial_\mu\alpha^+ - i(s_W A_\mu + c_W Z_\mu)\alpha^+ + iW_\mu^+(c_W^2\alpha^Z + \alpha^\gamma), \tag{176}$$

$$\delta W_\mu^- = (\delta W_\mu^+)^\dagger = \frac{1}{g}\partial_\mu\alpha^- + i(s_W A_\mu + c_W Z_\mu)\alpha^- - iW_\mu^-(c_W^2\alpha^Z + \alpha^\gamma), \tag{177}$$

$$\delta Z_\mu^a = c_W\delta W_\mu^3 - s_W\delta B_\mu = \frac{c_W}{g}\partial_\mu\alpha^3 + c_W\varepsilon^{3bc}W_\mu^b\alpha^c - \frac{s_W}{g'}\partial_\mu\alpha^Y$$
$$= \frac{c_W}{g}\partial_\mu\alpha^Z - ic_W(W_\mu^+\alpha^- - W_\mu^-\alpha^+), \tag{178}$$

$$\delta A_\mu = s_W\delta W_\mu^3 + c_W\delta B_\mu = \frac{s_W}{g}\partial_\mu\alpha^3 + s_W\varepsilon^{3bc}W_\mu^b\alpha^c + \frac{c_W}{g'}\partial_\mu\alpha^Y$$
$$= \frac{1}{e}\partial_\mu\alpha^\gamma - is_W(W_\mu^+\alpha^- - W_\mu^-\alpha^+). \tag{179}$$

另一方面, 根据

$$\alpha^a T^a + \alpha^Y Y_H = \frac{1}{2}(\alpha^a\sigma^a + \alpha^Y) = \frac{1}{2}\begin{pmatrix} \alpha^3 + \alpha^Y & \alpha^1 - i\alpha^2 \\ \alpha^1 + i\alpha^2 & -\alpha^3 + \alpha^Y \end{pmatrix}$$
$$= \frac{1}{2}\begin{pmatrix} 2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z & \sqrt{2}\alpha^+ \\ \sqrt{2}\alpha^- & -\alpha^Z \end{pmatrix}, \tag{180}$$

可知 Higgs 场的无穷小规范变换形式为

$$\delta\Phi = i(\alpha^a T^a + \alpha^Y Y_H)\Phi = \frac{i}{2}\begin{pmatrix} 2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z & \sqrt{2}\alpha^+ \\ \sqrt{2}\alpha^- & -\alpha^Z \end{pmatrix}\begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\chi) \end{pmatrix}$$
$$= \begin{pmatrix} \frac{i}{2}\{\phi^+[2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z] + (v + H + i\chi)\alpha^+\} \\ \frac{1}{\sqrt{2}}\left[i\phi^+\alpha^- - \frac{1}{2}(iv + iH - \chi)\alpha^Z\right] \end{pmatrix} = \begin{pmatrix} \delta\phi^+ \\ \frac{1}{\sqrt{2}}(\delta H + i\delta\chi) \end{pmatrix}. \tag{181}$$

利用

$$\text{Re}(\phi^+\alpha^-) = \frac{1}{2}(\phi^+\alpha^- + \phi^-\alpha^+), \quad \text{Im}(\phi^+\alpha^-) = -\frac{\mathrm{i}}{2}(\phi^+\alpha^- - \phi^-\alpha^+), \tag{182}$$

推出

$$\delta\phi^+ = \frac{\mathrm{i}}{2}\{\phi^+[2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z] + (v + H + \mathrm{i}\chi)\alpha^+\}, \tag{183}$$

$$\delta\phi^- = -\frac{\mathrm{i}}{2}\{\phi^-[2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z] + (v + H - \mathrm{i}\chi)\alpha^-\}, \tag{184}$$

$$\delta H = \frac{1}{2}[\mathrm{i}(\phi^+\alpha^- - \phi^-\alpha^+) + \chi\alpha^Z], \quad \delta\chi = \frac{1}{2}[\phi^+\alpha^- + \phi^-\alpha^+ - (v + H)\alpha^Z]. \tag{185}$$

于是，规范固定函数的无穷小规范变换为

$$\begin{aligned}
\sqrt{\xi_W}\,\delta G^+ &= \partial^\mu \delta W_\mu^+ - \mathrm{i}\xi_W m_W \delta\phi^+ \\
&= \partial^\mu \left[\frac{1}{g}\partial_\mu\alpha^+ - \mathrm{i}(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu)\alpha^+ + \mathrm{i}W_\mu^+(c_\mathrm{W}^2\alpha^Z + \alpha^\gamma)\right] \\
&\quad + \frac{1}{2}\xi_W m_W \left\{\phi^+[2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z] + (v + H + \mathrm{i}\chi)\alpha^+\right\},
\end{aligned} \tag{186}$$

$$\begin{aligned}
\sqrt{\xi_W}\,\delta G^- &= \partial^\mu \delta W_\mu^- + \mathrm{i}\xi_W m_W \delta\phi^- \\
&= \partial^\mu \left[\frac{1}{g}\partial_\mu\alpha^- + \mathrm{i}(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu)\alpha^- - \mathrm{i}W_\mu^-(c_\mathrm{W}^2\alpha^Z + \alpha^\gamma)\right] \\
&\quad + \frac{1}{2}\xi_W m_W \left\{\phi^-[2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z] + (v + H - \mathrm{i}\chi)\alpha^-\right\},
\end{aligned} \tag{187}$$

$$\begin{aligned}
\sqrt{\xi_Z}\,\delta G^Z &= \partial^\mu \delta Z_\mu - \xi_Z m_Z \delta\chi \\
&= \partial^\mu \left[\frac{c_\mathrm{W}}{g}\partial_\mu\alpha^Z - \mathrm{i}c_\mathrm{W}(W_\mu^+\alpha^- - W_\mu^-\alpha^+)\right] \\
&\quad - \frac{1}{2}\xi_Z m_Z \left[\phi^+\alpha^- + \phi^-\alpha^+ - (v + H)\alpha^Z\right],
\end{aligned} \tag{188}$$

$$\sqrt{\xi_\gamma}\,\delta G^\gamma = \partial^\mu \delta A_\mu = \partial^\mu \left[\frac{1}{e}\partial_\mu\alpha^\gamma - \mathrm{i}s_\mathrm{W}(W_\mu^+\alpha^- - W_\mu^-\alpha^+)\right]. \tag{189}$$

因此，

$$\sqrt{\xi_W}g\,\frac{\delta G^+}{\delta\alpha^+} = \partial^2 + \xi_W m_W^2 - \mathrm{i}e\partial^\mu A_\mu - \mathrm{i}gc_\mathrm{W}\partial^\mu Z_\mu + \frac{1}{2}g\xi_W m_W(H + \mathrm{i}\chi), \tag{190}$$

$$\frac{\sqrt{\xi_W}g}{c_\mathrm{W}}\,\frac{\delta G^+}{\delta\alpha^Z} = \mathrm{i}gc_\mathrm{W}\partial^\mu W_\mu^+ + \frac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)\xi_W m_W}{2c_\mathrm{W}}\phi^+, \tag{191}$$

$$\sqrt{\xi_W}e\,\frac{\delta G^+}{\delta\alpha^\gamma} = \mathrm{i}e\partial^\mu W_\mu^+ + e\xi_W m_W\phi^+, \tag{192}$$

$$\sqrt{\xi_W}g\,\frac{\delta G^-}{\delta\alpha^-} = \partial^2 + \xi_W m_W^2 + \mathrm{i}e\partial^\mu A_\mu + \mathrm{i}gc_\mathrm{W}\partial^\mu Z_\mu + \frac{1}{2}g\xi_W m_W(H - \mathrm{i}\chi), \tag{193}$$

$$\frac{\sqrt{\xi_W}g}{c_\mathrm{W}}\,\frac{\delta G^-}{\delta\alpha^Z} = -\mathrm{i}gc_\mathrm{W}\partial^\mu W_\mu^- + \frac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)\xi_W m_W}{2c_\mathrm{W}}\phi^-, \tag{194}$$

$$\sqrt{\xi_W}e\,\frac{\delta G^-}{\delta\alpha^\gamma} = -\mathrm{i}e\partial^\mu W_\mu^- + e\xi_W m_W\phi^-, \tag{195}$$

$$\sqrt{\xi_Z}g\frac{\delta G^Z}{\delta\alpha^+} = \mathrm{i}gc_\mathrm{W}\partial^\mu W_\mu^- - \frac{1}{2}g\xi_Z m_Z\phi^-, \tag{196}$$

$$\sqrt{\xi_Z}g\frac{\delta G^Z}{\delta\alpha^-} = -\mathrm{i}gc_\mathrm{W}\partial^\mu W_\mu^+ - \frac{1}{2}g\xi_Z m_Z\phi^+, \tag{197}$$

$$\frac{\sqrt{\xi_Z}g}{c_\mathrm{W}}\frac{\delta G^Z}{\delta\alpha^Z} = \partial^2 + \xi_Z m_Z^2 + \frac{g\xi_Z m_Z}{2c_\mathrm{W}}H, \tag{198}$$

$$\sqrt{\xi_\gamma}g\frac{\delta G^\gamma}{\delta\alpha^+} = \mathrm{i}e\partial^\mu W_\mu^-, \quad \sqrt{\xi_\gamma}g\frac{\delta G^\gamma}{\delta\alpha^-} = -\mathrm{i}e\partial^\mu W_\mu^+, \quad \sqrt{\xi_\gamma}e\frac{\delta G^\gamma}{\delta\alpha^\gamma} = \partial^2. \tag{199}$$

最后，得到以下 Faddeev-Popov 鬼场拉氏量，

$$
\begin{aligned}
\mathcal{L}_{\mathrm{EWG,FP}} =& -\bar{\eta}^+\left(\sqrt{\xi_W}g\frac{\delta G^+}{\delta\alpha^+}\right)\eta^+ - \bar{\eta}^Z\left(\sqrt{\xi_Z}g\frac{\delta G^Z}{\delta\alpha^+}\right)\eta^+ - \bar{\eta}^\gamma\left(\sqrt{\xi_\gamma}g\frac{\delta G^\gamma}{\delta\alpha^+}\right)\eta^+ \\
& -\bar{\eta}^-\left(\sqrt{\xi_W}g\frac{\delta G^-}{\delta\alpha^-}\right)\eta^- - \bar{\eta}^Z\left(\sqrt{\xi_Z}g\frac{\delta G^Z}{\delta\alpha^-}\right)\eta^- - \bar{\eta}^\gamma\left(\sqrt{\xi_\gamma}g\frac{\delta G^\gamma}{\delta\alpha^-}\right)\eta^- \\
& -\bar{\eta}^Z\left(\frac{\sqrt{\xi_Z}g}{c_\mathrm{W}}\frac{\delta G^Z}{\delta\alpha^Z}\right)\eta^Z - \bar{\eta}^+\left(\frac{\sqrt{\xi_W}g}{c_\mathrm{W}}\frac{\delta G^+}{\delta\alpha^Z}\right)\eta^Z - \bar{\eta}^-\left(\frac{\sqrt{\xi_W}g}{c_\mathrm{W}}\frac{\delta G^-}{\delta\alpha^Z}\right)\eta^Z \\
& -\bar{\eta}^\gamma\left(\sqrt{\xi_\gamma}e\frac{\delta G^\gamma}{\delta\alpha^\gamma}\right)\eta^\gamma - \bar{\eta}^+\left(\sqrt{\xi_W}e\frac{\delta G^+}{\delta\alpha^\gamma}\right)\eta^\gamma - \bar{\eta}^-\left(\sqrt{\xi_W}e\frac{\delta G^-}{\delta\alpha^\gamma}\right)\eta^\gamma \\
\rightarrow& \ \bar{\eta}^+\left[-\partial^2 - \xi_W m_W^2 - \mathrm{i}e\overleftarrow{\partial^\mu}A_\mu - \mathrm{i}gc_\mathrm{W}\overleftarrow{\partial^\mu}Z_\mu - \frac{1}{2}g\xi_W m_W(H + \mathrm{i}\chi)\right]\eta^+ \\
& +\bar{\eta}^Z\left(\mathrm{i}gc_\mathrm{W}\overleftarrow{\partial^\mu}W_\mu^- + \frac{1}{2}g\xi_Z m_Z\phi^-\right)\eta^+ + \mathrm{i}e(\partial^\mu\bar{\eta}^\gamma)W_\mu^-\eta^+ \\
& +\bar{\eta}^-\left[-\partial^2 - \xi_W m_W^2 + \mathrm{i}e\overleftarrow{\partial^\mu}A_\mu + \mathrm{i}gc_\mathrm{W}\overleftarrow{\partial^\mu}Z_\mu - \frac{1}{2}g\xi_W m_W(H - \mathrm{i}\chi)\right]\eta^- \\
& +\bar{\eta}^Z\left(-\mathrm{i}gc_\mathrm{W}\overleftarrow{\partial^\mu}W_\mu^+ + \frac{1}{2}g\xi_Z m_Z\phi^+\right)\eta^- - \mathrm{i}e(\partial^\mu\bar{\eta}^\gamma)W_\mu^+\eta^- \\
& +\bar{\eta}^Z\left(-\partial^2 - \xi_Z m_Z^2 - \frac{g\xi_Z m_Z}{2c_\mathrm{W}}H\right)\eta^Z \\
& +\bar{\eta}^+\left(\mathrm{i}gc_\mathrm{W}\overleftarrow{\partial^\mu}W_\mu^+ - \frac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)\xi_W m_W}{2c_\mathrm{W}}\phi^+\right)\eta^Z \\
& +\bar{\eta}^-\left(-\mathrm{i}gc_\mathrm{W}\overleftarrow{\partial^\mu}W_\mu^- - \frac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)\xi_W m_W}{2c_\mathrm{W}}\phi^-\right)\eta^Z \\
& -\bar{\eta}^\gamma\partial^2\eta^\gamma + \bar{\eta}^+(\mathrm{i}e\overleftarrow{\partial^\mu}W_\mu^+ - e\xi_W m_W\phi^+)\eta^\gamma + \bar{\eta}^-(-\mathrm{i}e\overleftarrow{\partial^\mu}W_\mu^- - e\xi_W m_W\phi^-)\eta^\gamma. \quad (200)
\end{aligned}
$$

鬼粒子的质量为

$$m_{\eta^+} = m_{\eta^-} = \sqrt{\xi_W}\,m_W, \quad m_{\eta^Z} = \sqrt{\xi_Z}\,m_Z, \quad m_{\eta^\gamma} = 0. \tag{201}$$

下面给出 R_ξ 规范下的费曼规则。$\xi_i = 1$ 对应 Feynman-'t Hooft 规范，$\xi_i = 0$ 对应 Landau 规范，$\xi_W, \xi_Z \to \infty$ 对应幺正规范。在树图计算中，常取 $\xi_\gamma = 1$ 和 $\xi_W, \xi_Z \to \infty$。在圈图计算中，常取 $\xi_\gamma = \xi_W = \xi_Z = 1$。

传播子:

$$\begin{aligned}\xrightarrow{\ p\ }_{H} = \frac{\mathrm{i}}{p^2 - m_H^2 + \mathrm{i}\epsilon}\end{aligned} \tag{202}$$

$$\begin{aligned}\xrightarrow{\ p\ }_{\chi} = \frac{\mathrm{i}}{p^2 - \xi_Z m_Z^2 + \mathrm{i}\epsilon}\end{aligned} \tag{203}$$

$$\begin{aligned}\xrightarrow{\ p\ }_{\phi} = \frac{\mathrm{i}}{p^2 - \xi_W m_W^2 + \mathrm{i}\epsilon}\end{aligned} \tag{204}$$

$$\nu \xrightarrow{\ p\ }_{\gamma} \mu = \frac{-\mathrm{i}}{p^2 + \mathrm{i}\epsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1 - \xi_\gamma)\right] \tag{205}$$

$$\nu \xrightarrow{\ p\ }_{Z} \mu = \frac{-\mathrm{i}}{p^2 - m_Z^2 + \mathrm{i}\epsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_Z m_Z^2}(1 - \xi_Z)\right] \tag{206}$$

$$\nu \xrightarrow{\ p\ }_{W} \mu = \frac{-\mathrm{i}}{p^2 - m_W^2 + \mathrm{i}\epsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_W m_W^2}(1 - \xi_W)\right] \tag{207}$$

$$\xrightarrow{\ p\ }_{\eta^\gamma} = \frac{\mathrm{i}}{p^2 + \mathrm{i}\epsilon} \tag{208}$$

$$\xrightarrow{\ p\ }_{\eta^Z} = \frac{\mathrm{i}}{p^2 - \xi_Z m_Z^2 + \mathrm{i}\epsilon} \tag{209}$$

$$\xrightarrow{\ p\ }_{\eta^\pm} = \frac{\mathrm{i}}{p^2 - \xi_W m_W^2 + \mathrm{i}\epsilon} \tag{210}$$

标量玻色子三线性耦合:

$$\begin{aligned} &= -\frac{3\mathrm{i}m_H^2}{v} = -6\mathrm{i}\lambda v \qquad\qquad &= -\frac{\mathrm{i}m_H^2}{v} = -2\mathrm{i}\lambda v \qquad\qquad &= -\frac{\mathrm{i}m_H^2}{v} = -2\mathrm{i}\lambda v \end{aligned} \tag{211}$$

标量玻色子四线性耦合:

$$\begin{aligned} &= -\frac{3\mathrm{i}m_H^2}{v^2} = -6\mathrm{i}\lambda \qquad &= -\frac{\mathrm{i}m_H^2}{v^2} = -2\mathrm{i}\lambda \qquad &= -\frac{3\mathrm{i}m_H^2}{v^2} = -6\mathrm{i}\lambda \end{aligned} \tag{212}$$

$$\begin{aligned} &= -\frac{\mathrm{i}m_H^2}{v^2} = -2\mathrm{i}\lambda \qquad &= -\frac{\mathrm{i}m_H^2}{v^2} = -2\mathrm{i}\lambda \qquad &= -\frac{2\mathrm{i}m_H^2}{v^2} = -4\mathrm{i}\lambda \end{aligned} \tag{213}$$

Yukawa 耦合：

$$
\begin{aligned}
&= -\frac{\mathrm{i}m_f}{v} \\
&= -\frac{\mathrm{i}y_f}{\sqrt{2}}
\end{aligned}
\qquad
\begin{aligned}
&= \frac{m_{\ell_i}}{v}\,\gamma^5 \\
&= \frac{y_{\ell_i}}{\sqrt{2}}\,\gamma^5
\end{aligned}
\tag{214}
$$

$$
\begin{aligned}
&= -\frac{m_{u_i}}{v}\,\gamma^5 \\
&= -\frac{y_{u_i}}{\sqrt{2}}\,\gamma^5
\end{aligned}
\qquad
\begin{aligned}
&= \frac{m_{d_i}}{v}\,\gamma^5 \\
&= \frac{y_{d_i}}{\sqrt{2}}\,\gamma^5
\end{aligned}
\tag{215}
$$

$$
\begin{aligned}
&= -\frac{\mathrm{i}\sqrt{2}m_{\ell_i}}{v}\,P_{\mathrm{R}} \\
&= -\mathrm{i}y_{\ell_i}P_{\mathrm{R}}
\end{aligned}
\qquad
\begin{aligned}
&= -\frac{\mathrm{i}\sqrt{2}m_{\ell_i}}{v}\,P_{\mathrm{L}} \\
&= -\mathrm{i}y_{\ell_i}P_{\mathrm{L}}
\end{aligned}
\tag{216}
$$

$$
\begin{aligned}
&= \frac{\mathrm{i}\sqrt{2}V_{ij}}{v}(m_{u_i}P_{\mathrm{L}} - m_{d_j}P_{\mathrm{R}}) \\
&= \mathrm{i}V_{ij}(y_{u_i}P_{\mathrm{L}} - y_{d_j}P_{\mathrm{R}})
\end{aligned}
\qquad
\begin{aligned}
&= -\frac{\mathrm{i}\sqrt{2}V_{ji}^{\dagger}}{v}(m_{d_j}P_{\mathrm{L}} - m_{u_i}P_{\mathrm{R}}) \\
&= -\mathrm{i}V_{ji}^{\dagger}(y_{d_j}P_{\mathrm{L}} - y_{u_i}P_{\mathrm{R}})
\end{aligned}
\tag{217}
$$

标量玻色子与电弱规范玻色子的三线性耦合：

$$
\begin{aligned}
&= \frac{\mathrm{i}gm_Z}{c_{\mathrm{W}}}\,g^{\mu\nu} \\
&= \frac{\mathrm{i}g^2 v}{2c_{\mathrm{W}}^2}\,g^{\mu\nu}
\end{aligned}
\qquad
\begin{aligned}
&= \mathrm{i}gm_W g^{\mu\nu} \\
&= \frac{\mathrm{i}g^2 v}{2}\,g^{\mu\nu}
\end{aligned}
\tag{218}
$$

$$
\begin{aligned}
&= \mathrm{i}em_W g^{\mu\nu} \\
&= \frac{\mathrm{i}egv}{2}\,g^{\mu\nu}
\end{aligned}
\qquad
\begin{aligned}
&= \mathrm{i}em_W g^{\mu\nu} \\
&= \frac{\mathrm{i}egv}{2}\,g^{\mu\nu}
\end{aligned}
\tag{219}
$$

$$
\begin{aligned}
&= -\mathrm{i}gs_{\mathrm{W}}^2 m_Z g^{\mu\nu} \\
&= -\frac{\mathrm{i}g^2 s_{\mathrm{W}}^2 v}{2c_{\mathrm{W}}}\,g^{\mu\nu}
\end{aligned}
\qquad
\begin{aligned}
&= -\mathrm{i}gs_{\mathrm{W}}^2 m_Z g^{\mu\nu} \\
&= -\frac{\mathrm{i}g^2 s_{\mathrm{W}}^2 v}{2c_{\mathrm{W}}}\,g^{\mu\nu}
\end{aligned}
\tag{220}
$$

$$= \mathrm{i}e(p+q)^\mu \tag{221}$$
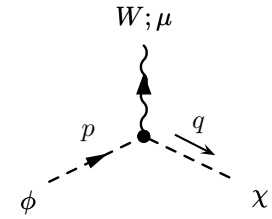
$$= \frac{\mathrm{i}g(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{2c_\mathrm{W}}(p+q)^\mu \qquad = -\frac{g}{2c_\mathrm{W}}(p+q)^\mu \tag{222}$$
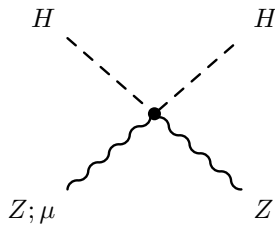
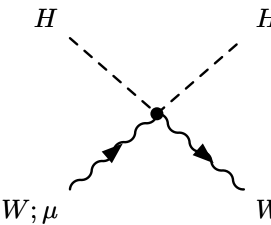$$= \frac{\mathrm{i}g}{2}(p+q)^\mu \qquad = \frac{\mathrm{i}g}{2}(p+q)^\mu \tag{223}$$
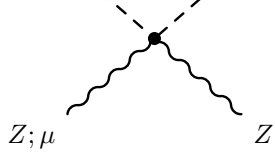
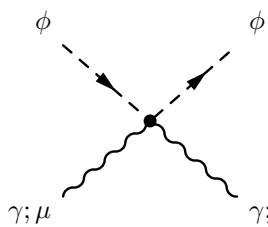$$= -\frac{g}{2}(p+q)^\mu \qquad = \frac{g}{2}(p+q)^\mu \tag{224}$$
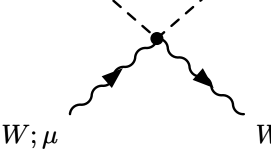
标量玻色子与电弱规范玻色子的四线性耦合:
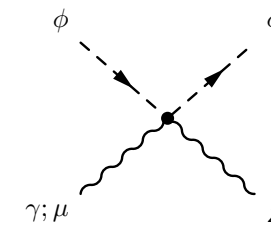
$$= \frac{\mathrm{i}g^2}{2c_\mathrm{W}^2}g^{\mu\nu} \qquad = \frac{\mathrm{i}g^2}{2}g^{\mu\nu} \tag{225}$$
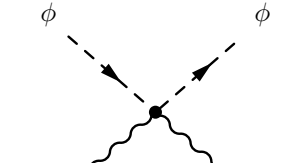
$$= \frac{\mathrm{i}g^2}{2c_\mathrm{W}^2}g^{\mu\nu} \qquad = \frac{\mathrm{i}g^2}{2}g^{\mu\nu} \tag{226}$$
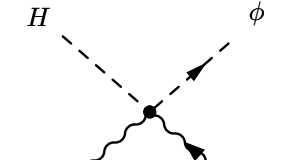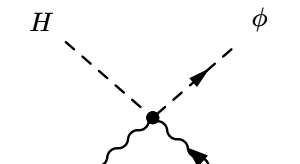
$$= 2\mathrm{i}e^2 g^{\mu\nu} \qquad = \frac{\mathrm{i}eg(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{c_\mathrm{W}}g^{\mu\nu} \tag{227}$$

$$\phi \qquad \phi \qquad = \frac{\mathrm{i}g^2(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)^2}{2c_{\mathrm{W}}^2}\, g^{\mu\nu}$$

$$Z;\mu \qquad Z;\nu$$

$$\phi \qquad \phi \qquad = \frac{\mathrm{i}g^2}{2}\, g^{\mu\nu} \tag{228}$$

$$W;\mu \qquad W;\nu$$

$$H \qquad \phi \qquad = \frac{\mathrm{i}eg}{2}\, g^{\mu\nu}$$

$$\gamma;\mu \qquad W;\nu$$

$$H \qquad \phi \qquad = \frac{\mathrm{i}eg}{2}\, g^{\mu\nu} \tag{229}$$

$$\gamma;\mu \qquad W;\nu$$

$$H \qquad \phi \qquad = -\frac{\mathrm{i}g^2 s_{\mathrm{W}}^2}{2c_{\mathrm{W}}}\, g^{\mu\nu}$$

$$Z;\mu \qquad W;\nu$$

$$H \qquad \phi \qquad = -\frac{\mathrm{i}g^2 s_{\mathrm{W}}^2}{2c_{\mathrm{W}}}\, g^{\mu\nu} \tag{230}$$

$$Z;\mu \qquad W;\nu$$

$$\chi \qquad \phi \qquad = -\frac{eg}{2}\, g^{\mu\nu}$$

$$\gamma;\mu \qquad W;\nu$$

$$\chi \qquad \phi \qquad = \frac{eg}{2}\, g^{\mu\nu} \tag{231}$$

$$\gamma;\mu \qquad W;\nu$$

$$\chi \qquad \phi \qquad = \frac{g^2 s_{\mathrm{W}}^2}{2c_{\mathrm{W}}}\, g^{\mu\nu}$$

$$Z;\mu \qquad W;\nu$$

$$\chi \qquad \phi \qquad = -\frac{g^2 s_{\mathrm{W}}^2}{2c_{\mathrm{W}}}\, g^{\mu\nu} \tag{232}$$

$$Z;\mu \qquad W;\nu$$

鬼粒子与标量玻色子的耦合:

$$H$$

$$\begin{aligned} &= -\frac{\mathrm{i}g\xi_Z m_Z}{2c_{\mathrm{W}}} \\ &= -\frac{\mathrm{i}g^2 \xi_Z v}{4c_{\mathrm{W}}^2} \end{aligned} \tag{233}$$

$$\eta^Z \qquad \eta^Z$$

$$H$$

$$\begin{aligned} &= -\frac{\mathrm{i}g\xi_W m_W}{2} \\ &= -\frac{\mathrm{i}g^2 \xi_W v}{4} \end{aligned}$$

$$\eta^+ \qquad \eta^+$$

$$H$$

$$\begin{aligned} &= -\frac{\mathrm{i}g\xi_W m_W}{2} \\ &= -\frac{\mathrm{i}g^2 \xi_W v}{4} \end{aligned} \tag{234}$$

$$\eta^- \qquad \eta^-$$

$$\frac{\chi}{\eta^+ \qquad \eta^+} \quad = \frac{g\xi_W m_W}{2}$$
$$= \frac{g^2 \xi_W v}{4}$$

$$\frac{\chi}{\eta^- \qquad \eta^-} \quad = -\frac{g\xi_W m_W}{2}$$
$$= -\frac{g^2 \xi_W v}{4} \tag{235}$$

$$\frac{\phi}{\eta^\gamma \qquad \eta^+} \quad = -\mathrm{i}e\xi_W m_W$$
$$= -\frac{\mathrm{i}eg\xi_W v}{2}$$

$$\frac{\phi}{\eta^\gamma \qquad \eta^-} \quad = -\mathrm{i}e\xi_W m_W$$
$$= -\frac{\mathrm{i}eg\xi_W v}{2} \tag{236}$$

$$\frac{\phi}{\eta^Z \qquad \eta^+} \quad = -\frac{\mathrm{i}g(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\xi_W m_W}{2c_{\mathrm{W}}}$$
$$= -\frac{\mathrm{i}g^2(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\xi_W v}{4c_{\mathrm{W}}}$$

$$\frac{\phi}{\eta^+ \qquad \eta^Z} \quad = \frac{\mathrm{i}g\xi_Z m_Z}{2}$$
$$= \frac{\mathrm{i}g^2 \xi_Z v}{4c_{\mathrm{W}}} \tag{237}$$

$$\frac{\phi}{\eta^- \qquad \eta^Z} \quad = \frac{\mathrm{i}g\xi_Z m_Z}{2}$$
$$= \frac{\mathrm{i}g^2 \xi_Z v}{4c_{\mathrm{W}}}$$

$$\frac{\phi}{\eta^Z \qquad \eta^-} \quad = -\frac{\mathrm{i}g(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\xi_W m_W}{2c_{\mathrm{W}}}$$
$$= -\frac{\mathrm{i}g^2(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\xi_W v}{4c_{\mathrm{W}}} \tag{238}$$

鬼粒子与电弱规范玻色子的耦合:

$$\frac{\gamma;\mu}{\eta^+ \qquad p \qquad \eta^+} \quad = \mathrm{i}ep^\mu$$

$$\frac{\gamma;\mu}{\eta^- \qquad p \qquad \eta^-} \quad = -\mathrm{i}ep^\mu \tag{239}$$

$$\frac{Z;\mu}{\eta^+ \qquad p \qquad \eta^+} \quad = \mathrm{i}gc_{\mathrm{W}}p^\mu$$

$$\frac{Z;\mu}{\eta^- \qquad p \qquad \eta^-} \quad = -\mathrm{i}gc_{\mathrm{W}}p^\mu \tag{240}$$

$$\frac{W;\mu}{\eta^\gamma \qquad p \qquad \eta^+} \quad = -\mathrm{i}ep^\mu$$

$$\frac{W;\mu}{\eta^+ \qquad p \qquad \eta^\gamma} \quad = -\mathrm{i}ep^\mu \tag{241}$$

$$= \mathrm{i}ep^\mu \qquad\qquad = \mathrm{i}ep^\mu \qquad (242)$$

$$= -\mathrm{i}gc_\mathrm{W}p^\mu \qquad\qquad = -\mathrm{i}gc_\mathrm{W}p^\mu \qquad (243)$$

$$= \mathrm{i}gc_\mathrm{W}p^\mu \qquad\qquad = \mathrm{i}gc_\mathrm{W}p^\mu \qquad (244)$$

6 内外线一般费曼规则

本节列出一些通用的内外线费曼规则。

内线费曼规则如下。

- 实标量玻色子传播子: $= \dfrac{\mathrm{i}}{p^2 - m^2 + \mathrm{i}\epsilon}$

- 复标量玻色子传播子: $= \dfrac{\mathrm{i}}{p^2 - m^2 + \mathrm{i}\epsilon}$

- Dirac 费米子传播子: $= \dfrac{\mathrm{i}(\not{p} + m)}{p^2 - m^2 + \mathrm{i}\epsilon}$

- 无质量实矢量玻色子传播子:

$$\nu \qquad \mu = \frac{-\mathrm{i}g^{\mu\nu}}{p^2 + \mathrm{i}\epsilon} \quad \text{（Feynman 规范）}$$

$$\nu \qquad \mu = \frac{-\mathrm{i}(g^{\mu\nu} - p^\mu p^\nu/p^2)}{p^2 + \mathrm{i}\epsilon} \quad \text{（Landau 规范）}$$

- 有质量实矢量玻色子传播子:

$$\nu \qquad \mu = \frac{-\mathrm{i}(g^{\mu\nu} - p^\mu p^\nu/m^2)}{p^2 - m^2 + \mathrm{i}\epsilon} \quad \text{（幺正规范）}$$

$$\nu \qquad \mu = \frac{-\mathrm{i}g^{\mu\nu}}{p^2 - m^2 + \mathrm{i}\epsilon} \quad \text{（Feynman 规范）}$$

- 有质量复矢量玻色子传播子:

$$\nu \;\bullet\!\!\sim\!\!\!\sim\!\!\!\sim\!\!\!\succ\!\!\!\sim\!\!\bullet\; \mu \;=\; \frac{-\mathrm{i}(g^{\mu\nu} - p^\mu p^\nu / m^2)}{p^2 - m^2 + \mathrm{i}\epsilon} \quad (\text{幺正规范})$$

$$\nu \;\bullet\!\!\sim\!\!\!\sim\!\!\!\succ\!\!\!\sim\!\!\bullet\; \mu \;=\; \frac{-\mathrm{i}g^{\mu\nu}}{p^2 - m^2 + \mathrm{i}\epsilon} \quad (\text{Feynman 规范})$$

实标量场外线费曼规则如下。

- 实标量玻色子入射外线： $-\!-\!-\!\!\stackrel{p}{\succ}\!\!-\!-\!-\bullet = 1$

- 实标量玻色子出射外线： $\bullet-\!-\!\stackrel{p}{\succ}\!-\!-\!- = 1$

复标量场外线费曼规则如下。

- 正标量玻色子入射外线： $-\!-\!-\!\stackrel{p}{\blacktriangleright}\!-\!-\bullet = 1$

- 反标量玻色子入射外线： $-\!-\!\stackrel{p}{\blacktriangleleft}\!-\!-\bullet = 1$

- 正标量玻色子出射外线： $\bullet-\!-\!\stackrel{p}{\blacktriangleright}\!-\!-\!- = 1$

- 反标量玻色子出射外线： $\bullet-\!-\!\stackrel{p}{\blacktriangleleft}\!-\!-\!- = 1$

以 λ 代表自旋极化指标（如螺旋度），**Dirac 旋量场**外线费曼规则如下。

- Dirac 正费米子入射外线： $\lambda \xrightarrow{\;p\;}\bullet = u(\mathbf{p}, \lambda)$

- Dirac 反费米子入射外线： $\lambda \stackrel{p}{\underset{\blacktriangleleft}{\longrightarrow}}\bullet = \bar{v}(\mathbf{p}, \lambda)$

- Dirac 正费米子出射外线： $\bullet \xrightarrow{\;p\;} \lambda = \bar{u}(\mathbf{p}, \lambda)$

- Dirac 反费米子出射外线： $\bullet \stackrel{p}{\underset{\blacktriangleleft}{\longrightarrow}} \lambda = v(\mathbf{p}, \lambda)$

在计算非极化振幅模方时，可利用自旋求和关系

$$\sum_\lambda u(\mathbf{p}, \lambda)\bar{u}(\mathbf{p}, \lambda) = \not{p} + m, \quad \sum_\lambda v(\mathbf{p}, \lambda)\bar{v}(\mathbf{p}, \lambda) = \not{p} - m. \tag{245}$$

以 λ 代表自旋极化指标，**实矢量场**外线费曼规则如下。

- 实矢量玻色子入射外线：$\lambda; \mu$ $\overset{p}{\wwavebullet}$ $= \varepsilon^\mu(\mathbf{p}, \lambda)$

- 实矢量玻色子出射外线：$\bullet\overset{p}{\wwave}$ $\lambda; \mu$ $= \varepsilon^{\mu*}(\mathbf{p}, \lambda)$

复矢量场外线费曼规则如下。

- 正矢量玻色子入射外线：$\lambda; \mu$ $\overset{p}{\wave}$ $= \varepsilon^\mu(\mathbf{p}, \lambda)$

- 反矢量玻色子入射外线：$\lambda; \mu$ $\overset{p}{\wave}$ $= \varepsilon^\mu(\mathbf{p}, \lambda)$

- 正矢量玻色子出射外线：$\bullet\overset{p}{\wave}$ $\lambda; \mu$ $= \varepsilon^{\mu*}(\mathbf{p}, \lambda)$

- 反矢量玻色子出射外线：$\bullet\overset{p}{\wave}$ $\lambda; \mu$ $= \varepsilon^{\mu*}(\mathbf{p}, \lambda)$

在计算非极化振幅模方时，若包含无质量矢量玻色子外线，可利用极化求和替换关系

$$\sum_\lambda \varepsilon_\mu^*(\mathbf{p}, \lambda)\varepsilon_\nu(\mathbf{p}, \lambda) \to -g_{\mu\nu} \, ; \tag{246}$$

若包含有质量矢量玻色子外线，可利用极化求和关系

$$\sum_\lambda \varepsilon_\mu^*(\mathbf{p}, \lambda)\varepsilon_\nu(\mathbf{p}, \lambda) = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}. \tag{247}$$

7 常用单位和标准模型参数

本节数据来自 Particle Data Group 发布的 2022 版 *Review of Particle Physics* [5]。

在有理化的自然单位制中，光速、约化 Planck 常数和真空介电常数均取为 1，即 $c = \hbar = \varepsilon_0 = 1$。从而，速度没有量纲 (dimension)；长度量纲与时间量纲相同，是能量量纲的倒数；能量、质量和动量具有相同的量纲；精细结构常数表达为

$$\alpha = \frac{e^2}{4\pi}, \tag{248}$$

而单位电荷量 $e = \sqrt{4\pi\alpha}$ 是没有量纲的。可以将能量单位电子伏特 (eV) 视作上述有量纲物理量的基本单位。

单位间转换关系为

$$1 = c = 2.99792458 \times 10^{10} \text{ cm} \cdot \text{s}^{-1}, \tag{249}$$

$$1 = \hbar = 6.582119569 \times 10^{-25} \text{ GeV} \cdot \text{s}, \tag{250}$$

$$1 = \hbar c = 1.973269804 \times 10^{-14} \text{ GeV} \cdot \text{cm}, \tag{251}$$

$$1 = (\hbar c)^2 = 3.893793721 \times 10^8 \text{ GeV}^2 \cdot \text{pb}, \tag{252}$$

由此得到

$$1 \text{ s} = 2.997925 \times 10^{10} \text{ cm}, \qquad 1 \text{ cm} = 3.335641 \times 10^{-11} \text{ s}, \tag{253}$$

$$1 \text{ s} = 1.519267 \times 10^{24} \text{ GeV}^{-1}, \quad 1 \text{ GeV}^{-1} = 6.582120 \times 10^{-25} \text{ s}, \tag{254}$$

$$1 \text{ cm} = 5.067731 \times 10^{13} \text{ GeV}^{-1}, \quad 1 \text{ GeV}^{-1} = 1.973270 \times 10^{-14} \text{ cm}, \tag{255}$$

$$1 \text{ cm}^2 = 2.568189 \times 10^{27} \text{ GeV}^{-2}, \quad 1 \text{ GeV}^{-2} = 3.893794 \times 10^{-28} \text{ cm}^2, \tag{256}$$

$$1 \text{ cm}^3 \cdot \text{s}^{-1} = 8.566558 \times 10^{16} \text{ GeV}^{-2}, \quad 1 \text{ GeV}^{-2} = 1.167330 \times 10^{-17} \text{ cm}^3 \cdot \text{s}^{-1}. \tag{257}$$

靶 (barn) 是散射截面的常用单位, 记作 b, 满足

$$1 \text{ b} = 10^{-24} \text{ cm}^2 = 10^9 \text{ nb} = 10^{12} \text{ pb} = 10^{15} \text{ fb} = 10^{18} \text{ ab}, \tag{258}$$

$$1 \text{ pb} = 10^{-36} \text{ cm}^2 = 2.568189 \times 10^{-9} \text{ GeV}^{-2}, \quad 1 \text{ GeV}^{-2} = 3.893794 \times 10^8 \text{ pb}. \tag{259}$$

Fermi 耦合常数是

$$G_{\text{F}} = 1.1663788(6) \times 10^{-5} \text{ GeV}^{-2}, \tag{260}$$

括号内数字代表测量值的 1σ 不确定度, 由树图阶关系式

$$\frac{G_{\text{F}}}{\sqrt{2}} = \frac{1}{2v^2} = \frac{g^2}{8m_{\text{W}}^2}, \tag{261}$$

得到 Higgs 场真空期待值为

$$v = (\sqrt{2}G_{\text{F}})^{-1/2} = 246.2196 \text{ GeV}. \tag{262}$$

在低能标 (Thomson 极限) 处, 精细结构常数为

$$\alpha = \frac{1}{137.035999180(10)}; \tag{263}$$

在 $\overline{\text{MS}}$ 重整化方案 (以 ^ 为标志) 中, α^{-1} 跑动到 $\mu = m_Z$ 能标处的数值是

$$\widehat{\alpha}^{-1}(m_Z) = 127.951 \pm 0.009. \tag{264}$$

在 $\overline{\text{MS}}$ 方案中, $\mu = m_Z$ 能标处强耦合常数 $\alpha_{\text{s}} = g_{\text{s}}^2/(4\pi)$ 的数值为

$$\widehat{\alpha}_{\text{s}}(m_Z) = 0.1179 \pm 0.0009, \tag{265}$$

Weinberg 角 θ_{W} 的数值对应于

$$\widehat{s}_{\mathrm{W}}^2 = \sin^2\widehat{\theta}_{\mathrm{W}}(m_Z) = 0.23122 \pm 0.00004\,. \tag{266}$$

在标准模型中，光子、胶子和中微子没有质量，其它基本粒子的质量为

$$m_W = 80.377 \pm 0.012 \text{ GeV}, \qquad m_Z = 91.1876 \pm 0.0021 \text{ GeV}, \tag{267}$$

$$m_H = 125.25 \pm 0.17 \text{ GeV}, \qquad m_e = 0.51099895000(15) \text{ MeV}, \tag{268}$$

$$m_\mu = 105.6583755(23) \text{ MeV}, \quad m_\tau = 1776.86 \pm 0.12 \text{ MeV}, \tag{269}$$

$$m_u = 2.16^{+0.49}_{-0.26} \text{ MeV}, \qquad m_d = 4.67^{+0.48}_{-0.17} \text{ MeV}, \tag{270}$$

$$m_s = 93.4^{+8.6}_{-3.4} \text{ MeV}, \qquad m_c = 1.27 \pm 0.02 \text{ GeV}, \tag{271}$$

$$m_b = 4.18^{+0.03}_{-0.02} \text{ GeV}, \qquad m_t = 172.69 \pm 0.30 \text{ GeV}. \tag{272}$$

这里，u、d、s 夸克的质量是 $\mu = 2$ GeV 能标处的 $\overline{\text{MS}}$ 质量，c 和 b 夸克的质量分别是 $\mu = m_c$ 和 $\mu = m_b$ 能标处的 $\overline{\text{MS}}$ 质量，其余粒子的质量均为极点质量 (pole mass)。相应地，计算出来的 c、b 夸克极点质量为

$$m_c^{\text{pole}} = 1.67 \pm 0.07 \text{ GeV}, \quad m_b^{\text{pole}} = 4.78 \pm 0.06 \text{ GeV}. \tag{273}$$

质子和中子的质量为

$$m_p = 938.27208816(29) \text{ MeV}, \quad m_n = 939.56542052(54) \text{ MeV}. \tag{274}$$

在电弱能标附近作领头阶计算时，可将单位电荷量取为

$$e = \sqrt{4\pi\widehat{\alpha}(m_Z)} = 0.3133885, \tag{275}$$

将强耦合常数取为

$$g_{\mathrm{s}} = \sqrt{4\pi\widehat{\alpha}_{\mathrm{s}}(m_Z)} = 1.217200\,. \tag{276}$$

从树图阶关系计算 Higgs 场四线性耦合常数 λ 和 Yukawa 耦合常数 y_t、y_b、y_τ、y_c，得

$$\lambda = \frac{m_H^2}{2v^2} = 0.1293839, \quad y_t = \frac{\sqrt{2}m_t}{v} = 0.9918808, \quad y_b = \frac{\sqrt{2}m_b}{v} = 2.400870 \times 10^{-2}, \tag{277}$$

$$y_\tau = \frac{\sqrt{2}m_\tau}{v} = 1.020576 \times 10^{-2}, \quad y_c = \frac{\sqrt{2}m_c}{v} = 7.294508 \times 10^{-3}. \tag{278}$$

耦合常数 g 和 g' 有以下两种取值方式。

1. 根据树图阶关系 $\sin^2\theta_{\mathrm{W}} = 1 - m_W^2/m_Z^2$ 计算 Weinberg 角，得

$$s_{\mathrm{W}}^2 = 1 - \frac{m_W^2}{m_Z^2} = 0.2230519, \quad c_{\mathrm{W}}^2 = 1 - s_{\mathrm{W}}^2 = 0.7769481, \tag{279}$$

$$s_{\mathrm{W}} = \sqrt{s_{\mathrm{W}}^2} = 0.4722837, \qquad c_{\mathrm{W}} = \sqrt{c_{\mathrm{W}}^2} = 0.8814466, \tag{280}$$

故

$$g = \frac{e}{s_{\mathrm{W}}} = 0.6635599, \quad g' = \frac{e}{c_{\mathrm{W}}} = 0.3555389\,. \tag{281}$$

2. 根据 $\overline{\mathrm{MS}}$ 方案中 Weinberg 角的数值 (266) 计算 g 和 g'，得

$$c_{\mathrm{W}}^2 = 1 - \widehat{s}_{\mathrm{W}}^2 = 0.76878, \quad s_{\mathrm{W}} = \sqrt{\widehat{s}_{\mathrm{W}}^2} = 0.4808534, \quad c_{\mathrm{W}} = \sqrt{c_{\mathrm{W}}^2} = 0.8768010, \tag{282}$$

$$g = \frac{e}{s_{\mathrm{W}}} = 0.6517340, \quad g' = \frac{e}{c_{\mathrm{W}}} = 0.3574226\,. \tag{283}$$

将 CKM 矩阵参数化为

$$V = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}\mathrm{e}^{-\mathrm{i}\delta} \\ & 1 & \\ -s_{13}\mathrm{e}^{\mathrm{i}\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}, \tag{284}$$

其中 $s_{ij} \equiv \sin\theta_{ij}$，$c_{ij} \equiv \cos\theta_{ij}$。实验拟合值为

$$s_{12} = 0.22500 \pm 0.00067, \qquad s_{13} = 0.00369 \pm 0.00011, \tag{285}$$

$$s_{23} = 0.04182^{+0.00085}_{-0.00074}, \qquad \delta = 1.144 \pm 0.027\,. \tag{286}$$

如果只讨论第一、二代夸克的混合，可利用 Cabibbo 转动角 θ_{C} 将 CKM 矩阵近似地表达为

$$V \simeq \begin{pmatrix} \cos\theta_{\mathrm{C}} & \sin\theta_{\mathrm{C}} & \\ -\sin\theta_{\mathrm{C}} & \cos\theta_{\mathrm{C}} & \\ & & 1 \end{pmatrix}, \quad \sin\theta_{\mathrm{C}} = s_{12} = 0.225\,. \tag{287}$$

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