

标准模型拉氏量和费曼规则

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目 录

1	约定	2
2	标准模型概述	3
3	量子色动力学	3
4	电弱规范理论	6
4.1	Brout–Englert–Higgs 机制	7
4.2	费米子电弱规范相互作用	15
4.3	电弱规范场的自相互作用	19
5	R_ξ 规范下电弱拉氏量和费曼规则	22
6	内外线一般费曼规则	36
7	常用单位和标准模型参数	38

1 约定

本文采用有理化的自然单位制，推导过程参考文献 [1, 2, 3, 4]。

Minkowski 度规张量

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}. \quad (1)$$

Pauli 矩阵

$$\sigma^1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} & -i \\ i & \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}, \quad (2)$$

$$\sigma^\mu \equiv (1, \boldsymbol{\sigma}), \quad \bar{\sigma}^\mu \equiv (1, -\boldsymbol{\sigma}). \quad (3)$$

Weyl 表象中的 Dirac 矩阵

$$\gamma^\mu = \begin{pmatrix} & \sigma^\mu \\ \bar{\sigma}^\mu & \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -\mathbf{1} & \\ & \mathbf{1} \end{pmatrix}. \quad (4)$$

左右手投影算符

$$P_L \equiv \frac{1}{2}(1 - \gamma^5) = \begin{pmatrix} \mathbf{1} & \\ & \mathbf{0} \end{pmatrix}, \quad P_R \equiv \frac{1}{2}(1 + \gamma^5) = \begin{pmatrix} \mathbf{0} & \\ & \mathbf{1} \end{pmatrix} \quad (5)$$

用于定义左手旋量场 $\psi_L \equiv P_L \psi$ 和右手旋量场 $\psi_R \equiv P_R \psi$ 。Levi-Civita 符号的约定取

$$\varepsilon^{0123} = \varepsilon^{123} = +1. \quad (6)$$

费曼规则约定如下。

- 对于指向相互作用顶点的动量 p ，时空导数 ∂_μ 在动量空间费曼规则里贡献一个 $-ip_\mu$ 因子。
- 实线表示费米子，实线上的箭头表示费米子数流动的方向。
- 虚线表示标量玻色子，虚线上的箭头表示玻色子数流动的方向。
- 螺旋线表示胶子；波浪线表示其它规范玻色子，波浪线上的箭头表示玻色子数流动的方向。
- 点线表示鬼粒子，点线上的箭头表示鬼粒子数流动的方向。
- 如果没有额外箭头标记，动量方向与粒子线上的箭头方向一致；否则与额外箭头方向一致。

2 标准模型概述

粒子物理标准模型是一个 $SU(3)_C \times SU(2)_L \times U(1)_Y$ 规范理论。模型中有三代费米子, 包括三代中微子 $\nu_i = \nu_e, \nu_\mu, \nu_\tau$, 三代带电轻子 $\ell_i = e, \mu, \tau$, 三代上型夸克 $u_i = u, c, t$ 和三代下型夸克 $d_i = d, s, b$ ($i = 1, 2, 3$)。规范玻色子传递费米子之间的规范相互作用。

$SU(3)_C$ 部分描述夸克的强相互作用, 称为量子色动力学 (Quantum Chromodynamics, QCD), 相应的规范玻色子是胶子。 $SU(2)_L \times U(1)_Y$ 部分统一描述夸克和轻子的电磁和弱相互作用, 称为电弱规范理论。理论中有一个 Higgs 二重态, 通过 Brout-Englert-Higgs (BEH) 机制引发规范群的自发对称性破缺, 使 $SU(2)_L \times U(1)_Y$ 群破缺为 $U(1)_{EM}$ 群。 $U(1)_{EM}$ 规范理论称为量子电动力学 (Quantum Electrodynamics, QED)。

破缺前, 理论中存在 4 个无质量的规范玻色子和 4 个 Higgs 自由度; 左手费米子和右手费米子都没有质量, 具有不同的量子数。

破缺后, 3 个规范玻色子与 3 个 Higgs 自由度结合, 从而获得质量, 成为 W^\pm 和 Z^0 玻色子, 传递弱相互作用。剩下的 1 个无质量规范玻色子是光子, 即是 $U(1)_{EM}$ 群的规范玻色子, 传递电磁相互作用。剩下的 1 个中性 Higgs 自由度称为 Higgs 玻色子。费米子与 Higgs 二重态的 Yukawa 耦合导致左手费米子和右手费米子获得质量, 组合成 Dirac 费米子。

理论中的中微子没有右手分量, 因而没有获得质量。1998 年实验发现中微子振荡, 证明中微子具有质量, 所以需要扩充标准模型才能正确描述中微子物理。

3 量子色动力学

QCD 的拉氏量表达为

$$\mathcal{L}_{\text{QCD}} = \sum_q \bar{q}(i\gamma^\mu D_\mu - m_q)q - \frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu}, \quad q = u, d, s, c, b, t, \quad a = 1, \dots, 8, \quad (7)$$

其中

$$D_\mu = \partial_\mu - ig_s G_\mu^a t^a, \quad G^{a\mu\nu} \equiv \partial^\mu G^{a\nu} - \partial^\nu G^{a\mu} + g_s f^{abc} G^{b\mu} G^{c\nu}. \quad (8)$$

q 为夸克旋量场 $SU(3)_C$ 三重态, $SU(3)_C$ 规范场 G_μ^a 对应于胶子 g , g_s 是 $SU(3)_C$ 规范耦合常数。 $t^a = \lambda^a/2$ 是 $SU(3)_C$ 群基础表示的生成元, 其中 λ^a 为 Gell-Mann 矩阵。 $SU(3)_C$ 生成元对易关系为 $[t^a, t^b] = if^{abc}t^c$, 结构常数 f^{abc} 是全反对称的, 非零分量为

$$f^{123} = 1, \quad f^{147} = f^{246} = f^{257} = f^{345} = f^{516} = f^{637} = \frac{1}{2}, \quad f^{458} = f^{678} = \frac{\sqrt{3}}{2}. \quad (9)$$

由

$$-\frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu} = -\frac{1}{4}(\partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g_s f^{abc} G_\mu^b G_\nu^c)(\partial^\mu G^{a\nu} - \partial^\nu G^{a\mu} + g_s f^{ade} G^{d\mu} G^{e\nu})$$

$$\begin{aligned}
&= -\frac{1}{2}[(\partial_\mu G_\nu^a)(\partial^\mu G^{a\nu}) - (\partial_\mu G_\nu^a)(\partial^\nu G^{a\mu})] - g_s f^{abc}(\partial_\mu G_\nu^a)G^{b\mu}G^{c\nu} \\
&\quad - \frac{1}{4}g_s^2 f^{abc}f^{ade}G_\mu^b G_\nu^c G^{d\mu}G^{e\nu},
\end{aligned} \tag{10}$$

推出

$$\begin{aligned}
\mathcal{L}_{\text{QCD}} &= \sum_q [\bar{q}(i\gamma^\mu \partial_\mu - m_q)q + g_s G_\mu^a \bar{q}\gamma^\mu t^a q] + \frac{1}{2}[(\partial_\mu G_\nu^a)(\partial^\nu G^{a\mu}) - (\partial_\mu G_\nu^a)(\partial^\mu G^{a\nu})] \\
&\quad - g_s f^{abc}(\partial_\mu G_\nu^a)G^{b\mu}G^{c\nu} - \frac{1}{4}g_s^2 f^{abc}f^{ade}G_\mu^b G_\nu^c G^{d\mu}G^{e\nu}.
\end{aligned} \tag{11}$$

设用于固定胶子场规范的函数 $G^a(x) = \partial^\mu G_\mu^a(x) - \omega^a(x)$, 其中 $\omega^a(x)$ 是某个任意函数, 规范固定条件是 $G^a(x) = 0$ 。这是 Lorenz 规范的推广, $\omega^a(x) = 0$ 对应于 Lorenz 规范。在路径积分量子化中, 以中心为 $\omega^a(x) = 0$ 的 Gauss 权重对 $\omega^a(x)$ 作泛函积分, 有

$$\int \mathcal{D}\omega^a \exp \left[-i \int d^4x \frac{1}{2\xi} (\omega^a)^2 \right] \delta(G^a) = \exp \left[-i \int d^4x \frac{1}{2\xi} (\partial^\mu G_\mu^a)^2 \right]. \tag{12}$$

可见, 拉氏量中的规范固定项为

$$\mathcal{L}_{\text{QCD,GF}} = -\frac{1}{2\xi} (\partial^\mu G_\mu^a)^2. \tag{13}$$

ξ 的任何一个取值对应于一种规范。 $\xi = 1$ 称为 Feynman-'t Hooft 规范, $\xi = 0$ 称为 Landau 规范。于是, 胶子传播子相关拉氏量为

$$\begin{aligned}
\mathcal{L}_{\text{QCD,prop}} &= \frac{1}{2} \left[(\partial_\mu G_\nu^a)(\partial^\nu G^{a\mu}) - (\partial_\mu G_\nu^a)(\partial^\mu G^{a\nu}) - \frac{1}{\xi} (\partial^\mu G_\mu^a)^2 \right] \\
&\rightarrow \frac{1}{2} G_\mu^a \left[g^{\mu\nu} \partial^2 - \left(1 - \frac{1}{\xi} \right) \partial^\mu \partial^\nu \right] G_\nu^a.
\end{aligned} \tag{14}$$

变换到动量空间, 得

$$-g^{\mu\nu} p^2 + \left(1 - \frac{1}{\xi} \right) p^\mu p^\nu, \tag{15}$$

它的逆矩阵是

$$-\frac{1}{p^2} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} (1 - \xi) \right], \tag{16}$$

这是因为

$$\begin{aligned}
&-\frac{1}{p^2} \left[g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2} (1 - \xi) \right] \left[-g^{\mu\nu} p^2 + \left(1 - \frac{1}{\xi} \right) p^\mu p^\nu \right] \\
&= \delta_\rho^\nu - \frac{p_\rho p^\nu}{p^2} \left(1 - \frac{1}{\xi} \right) - \frac{p_\rho p^\nu}{p^2} (1 - \xi) + \frac{p_\rho p^\nu}{p^2} (1 - \xi) \left(1 - \frac{1}{\xi} \right) = \delta_\rho^\nu.
\end{aligned} \tag{17}$$

从而胶子传播子的形式为

$$\frac{-i\delta^{ab}}{p^2 + i\epsilon} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} (1 - \xi) \right]. \quad (18)$$

$SU(3)_C$ 规范变换为

$$q \rightarrow Uq, \quad G_\mu^a t^a \rightarrow U G_\mu^a t^a U^\dagger + \frac{i}{g_s} U \partial_\mu U^\dagger, \quad (19)$$

其中 $U(x) = \exp[i\alpha^a(x)t^a]$ 。胶子场的无穷小规范变换形式是

$$\begin{aligned} G_\mu^a t^a &\rightarrow (1 + i\alpha^a t^a) G_\mu^b t^b (1 - i\alpha^c t^c) + \frac{i}{g_s} (1 + i\alpha^a t^a) \partial_\mu (1 - i\alpha^c t^c) \\ &= G_\mu^b t^b + i\alpha^a G_\mu^b [t^a, t^b] + \frac{1}{g_s} (\partial_\mu \alpha^c) t^c + \mathcal{O}(\alpha^2) = G_\mu^a t^a - f^{abc} \alpha^a G_\mu^b t^c + \frac{1}{g_s} (\partial_\mu \alpha^a) t^a + \mathcal{O}(\alpha^2) \\ &= \left(G_\mu^a + f^{abc} G_\mu^b \alpha^c + \frac{1}{g_s} \partial_\mu \alpha^a \right) t^a + \mathcal{O}(\alpha^2), \end{aligned} \quad (20)$$

即

$$\delta G_\mu^a = \frac{1}{g_s} \partial_\mu \alpha^a + f^{abc} G_\mu^b \alpha^c = \left(\frac{1}{g_s} \delta^{ac} \partial_\mu + f^{abc} G_\mu^b \right) \alpha^c, \quad (21)$$

因而规范固定函数 G^a 的无穷小规范变换为

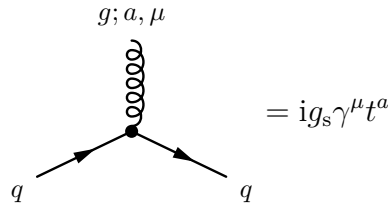
$$\delta G^a = \partial^\mu \delta G_\mu^a = \frac{1}{g_s} \delta^{ac} \partial^2 \alpha^c + f^{abc} \partial^\mu G_\mu^b \alpha^c, \quad g_s \frac{\delta G^a}{\delta \alpha^c} = \delta^{ac} \partial^2 + g_s f^{abc} \partial^\mu G_\mu^b. \quad (22)$$

Faddeev-Popov 鬼场的拉氏量是

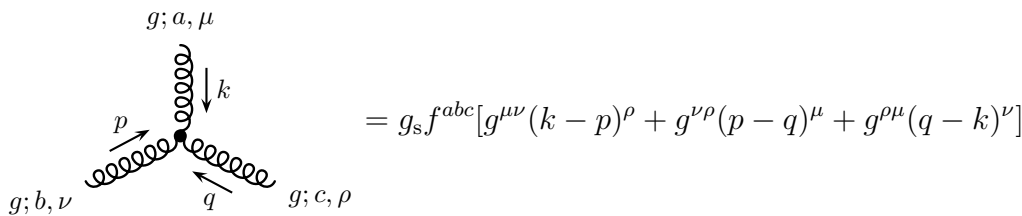
$$\mathcal{L}_{\text{QCD,FP}} = -\bar{\eta}_g^a \left(g_s \frac{\delta G^a}{\delta \alpha^c} \right) \eta_g^c = -\bar{\eta}_g^a (\delta^{ac} \partial^2 + g_s f^{abc} \partial^\mu G_\mu^b) \eta_g^c \rightarrow -\bar{\eta}_g^a \delta^{ab} \partial^2 \eta_g^b + g_s f^{abc} (\partial^\mu \bar{\eta}_g^a) G_\mu^b \eta_g^c. \quad (23)$$

下面列出 QCD 费曼规则。

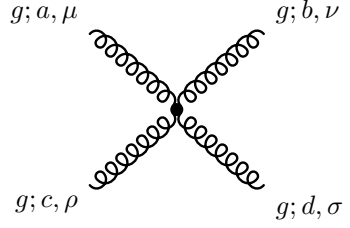
QCD 耦合顶点：



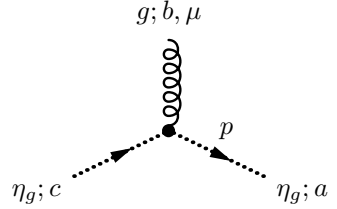
$$= i g_s \gamma^\mu t^a \quad (24)$$



$$= g_s f^{abc} [g^{\mu\nu} (k-p)^\rho + g^{\nu\rho} (p-q)^\mu + g^{\rho\mu} (q-k)^\nu] \quad (25)$$

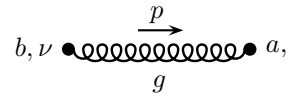


$$= -ig_s^2 [f^{abe} f^{cde} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) + f^{ace} f^{bde} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}) + f^{ade} f^{bce} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma})] \quad (26)$$



$$= -g_s f^{abc} p^\mu \quad (27)$$

胶子传播子:



$$= \frac{-i\delta^{ab}}{p^2 + i\epsilon} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} (1 - \xi) \right] \quad (28)$$

鬼粒子传播子:



$$= \frac{i\delta^{ab}}{p^2 + i\epsilon} \quad (29)$$

4 电弱规范理论

电弱规范理论的规范群是 $SU(2)_L \times U(1)_Y$ ，每一代左手旋量场构成 2 个 $SU(2)_L$ 二重态

$$L_{iL} = \begin{pmatrix} P_L \nu_i \\ P_L \ell_i \end{pmatrix} = \begin{pmatrix} \nu_{iL} \\ \ell_{iL} \end{pmatrix}, \quad Q_{iL} = \begin{pmatrix} P_L u'_i \\ P_L d'_i \end{pmatrix} = \begin{pmatrix} u'_{iL} \\ d'_{iL} \end{pmatrix}, \quad i = 1, 2, 3. \quad (30)$$

它们的协变导数是

$$D_\mu = \partial_\mu - ig W_\mu^a \tau^a - ig' B_\mu Y, \quad (31)$$

其中 $W_\mu^a(x)$ ($a = 1, 2, 3$) 是 $SU(2)_L$ 规范场, $B_\mu(x)$ 是 $U(1)_Y$ 规范场, g 和 g' 分别是 $SU(2)_L$ 和 $U(1)_Y$ 的规范耦合常数。

$$\tau^a = \frac{\sigma^a}{2} \quad (32)$$

是 $SU(2)_L$ 群 2 维表示的生成元, 对应于弱同位旋。生成元 τ^3 的本征值是弱同位旋第 3 分量, 记为 T^3 。 Y 是弱超荷。各代右手旋量场 $\ell_{iR} = P_R \ell_i$ 、 $u'_{iR} = P_R u'_i$ 和 $d'_{iR} = P_R d'_i$ 是 $SU(2)_L$ 单态, 协变导数为

$$D_\mu = \partial_\mu - ig' B_\mu Y. \quad (33)$$

表 1 列出费米子场的电荷 Q 、弱同位旋第 3 分量 T^3 、弱超荷 Y 、重子数 B 和轻子数 $L_e/L_\mu/L_\tau$ ，其中电荷 Q 由 T^3 和 Y 定义，

$$Q \equiv T^3 + Y. \quad (34)$$

表 1: 标准模型费米子场的量子数。

统一记号	第一代	第二代	第三代	Q	T^3	Y	B	$L_e/L_\mu/L_\tau$
$L_{iL} = \begin{pmatrix} \nu_{iL} \\ \ell_{iL} \end{pmatrix}$	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	0 -1	1/2 -1/2	-1/2 -1/2	0 0	1 1
$Q_{iL} = \begin{pmatrix} u'_{iL} \\ d'_{iL} \end{pmatrix}$	$\begin{pmatrix} u'_L \\ d'_L \end{pmatrix}$	$\begin{pmatrix} c'_L \\ s'_L \end{pmatrix}$	$\begin{pmatrix} t'_L \\ b'_L \end{pmatrix}$	2/3 -1/3	1/2 -1/2	1/6 1/6	1/3 1/3	0 0
ℓ_{iR}	e_R	μ_R	τ_R	-1	0	-1	0	1
u'_{iR}	u'_R	c'_R	t'_R	2/3	0	2/3	1/3	0
d'_{iR}	d'_R	s'_R	b'_R	-1/3	0	-1/3	1/3	0

4.1 Brout-Englert-Higgs 机制

由于左手费米子和右手费米子参与不同的 $SU(2)_L \times U(1)_Y$ 规范相互作用，耦合左右手费米子场的质量项会破坏规范对称性。另一方面，规范对称性也禁止规范玻色子具有质量。为了让费米子和弱规范玻色子获得质量，需要引入 BEH 机制，使 $SU(2)_L \times U(1)_Y$ 规范对称性自发破缺。因此，引入 Higgs 标量场

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}, \quad (35)$$

其中 ϕ^+ 和 ϕ^0 都是复标量场。 Φ 是 $SU(2)_L$ 二重态，弱超荷是

$$Y_H = \frac{1}{2}. \quad (36)$$

Higgs 场的协变动能项和势能项为

$$\mathcal{L}_H = (D^\mu \Phi)^\dagger (D_\mu \Phi) - V_H(\Phi), \quad V_H(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2, \quad (37)$$

其中协变导数为

$$D_\mu \Phi = (\partial_\mu - ig' B_\mu Y_H - ig W_\mu^a \tau^a) \Phi. \quad (38)$$

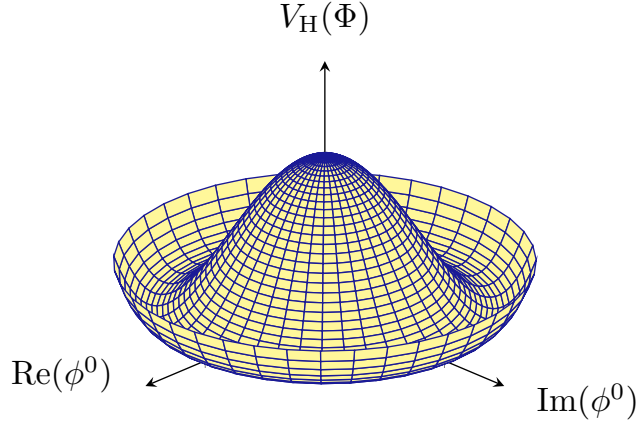


图 1: Higgs 场势能示意图。这里压缩掉 $\text{Re}(\phi^+)$ 和 $\text{Im}(\phi^+)$ 两个维度。

当 $\lambda > 0$ 且 $\mu^2 > 0$ 时, Higgs 场势能 $V_H(\Phi)$ 呈现出图 1 所示墨西哥草帽状的形式, 势能最小值位于方程

$$\Phi^\dagger \Phi = [\text{Re}(\phi^+)]^2 + [\text{Im}(\phi^+)]^2 + [\text{Re}(\phi^0)]^2 + [\text{Im}(\phi^0)]^2 = \frac{v^2}{2} \quad (39)$$

对应的 3 维球面上, 其中 $v \equiv \sqrt{\mu^2/\lambda}$, 满足

$$\mu^2 = \lambda v^2. \quad (40)$$

Higgs 场的真空期待值位于这个 3 维球面上的某一点, 不失一般性, 可将它取为

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. \quad (41)$$

其它真空期待值可通过 $\text{SU}(2)_L \times \text{U}(1)_Y$ 整体变换

$$\langle \Phi \rangle \rightarrow \exp(i\alpha^a \tau^a) \exp(i\alpha^Y Y_H) \langle \Phi \rangle \quad (42)$$

得到, 因为 $\langle \Phi^\dagger \Phi \rangle$ 在这样的变换下保持不变。若 $\alpha^1 = \alpha^2 = 0$ 且 $\alpha^3 = \alpha^Y$, 则

$$\exp(i\alpha^a \tau^a) \exp(i\alpha^Y Y_H) = \exp[i\alpha^3(\sigma^3 + \mathbf{1})/2] = \exp \left[i\alpha^3 \begin{pmatrix} 1 & \\ & 0 \end{pmatrix} \right] = \begin{pmatrix} e^{i\alpha^3} & \\ & 1 \end{pmatrix}, \quad (43)$$

而 $\langle \Phi \rangle$ 在此变换下不变。因此, 有 1 个方向的规范对称性没有受到破坏, 只有 3 个方向的规范对称性发生自发破缺。根据 Goldstone 定理, 破缺后存在 3 个无质量的 Nambu-Goldstone 玻色子。最终, 有 3 个规范玻色子结合 Nambu-Goldstone 玻色子, 通过 BEH 机制获得质量。

以 $\langle \Phi \rangle$ 为基础, 将 Higgs 场参数化为

$$\Phi(x) = \exp \left[-i \frac{\chi^a(x)}{v} \tau^a \right] \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \quad (44)$$

其中 $\chi^a(x)$ 和 $H(x)$ 都是实标量场。 $\exp[-i\chi^a(x)\tau^a/v]$ 因子能够通过 $SU(2)_L$ 规范变换消去, 因而可将 $\Phi(x)$ 直接取为

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \quad \Phi^\dagger \Phi = \frac{1}{2}(v + H)^2. \quad (45)$$

此时 Higgs 场只剩下一个物理自由度 $H(x)$, 对应于 Higgs 玻色子, 这种取法称为么正规范。

在么正规范下, 势能项化为

$$\begin{aligned} -V_H(\Phi) &= \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 = \frac{1}{2} \mu^2 (v + H)^2 - \frac{1}{4} \lambda (v + H)^4 \\ &= \frac{1}{2} \mu^2 (v^2 + H^2 + 2vH) - \frac{1}{4} \lambda (v^4 + 4v^2 H^2 + H^4 + 4v^3 H + 2v^2 H^2 + 4vH^3) \\ &= \frac{1}{4} \mu^2 v^2 + \frac{1}{4} (\mu^2 - \lambda v^2) v^2 + (\mu^2 - \lambda v^2) vH + \frac{1}{2} (\mu^2 - \lambda v^2) H^2 - \lambda v^2 H^2 - \lambda v H^3 - \frac{1}{4} \lambda H^4 \\ &= \frac{1}{8} m_H^2 v^2 - \frac{1}{2} m_H^2 H^2 - \frac{1}{2} \frac{m_H^2}{v} H^3 - \frac{1}{8} \frac{m_H^2}{v^2} H^4, \end{aligned} \quad (46)$$

其中 Higgs 玻色子的质量为

$$m_H \equiv \sqrt{2} \mu, \quad m_H^2 = 2\mu^2 = 2\lambda v^2. \quad (47)$$

由于

$$g' B_\mu Y_H + g W_\mu^a \tau^a = \frac{1}{2} \begin{pmatrix} g' B_\mu + g W_\mu^3 & g(W_\mu^1 - iW_\mu^2) \\ g(W_\mu^1 + iW_\mu^2) & g' B_\mu - g W_\mu^3 \end{pmatrix}, \quad (48)$$

Higgs 场真空期待值 v 对协变导数 $D_\mu \Phi$ 的贡献为

$$\begin{aligned} D_\mu \Phi &\supset -i(g' B_\mu Y_H + g W_\mu^a \tau^a) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \\ &= -\frac{iv}{2\sqrt{2}} \begin{pmatrix} g' B_\mu + g W_\mu^3 & g(W_\mu^1 - iW_\mu^2) \\ g(W_\mu^1 + iW_\mu^2) & g' B_\mu - g W_\mu^3 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \supset -\frac{iv}{2\sqrt{2}} \begin{pmatrix} g(W_\mu^1 - iW_\mu^2) \\ g' B_\mu - g W_\mu^3 \end{pmatrix}, \end{aligned} \quad (49)$$

故协变动能项 $(D^\mu \Phi)^\dagger (D_\mu \Phi)$ 中正比于 v^2 的项是

$$(D^\mu \Phi)^\dagger (D_\mu \Phi) \supset \frac{v^2}{8} [g^2 |W_\mu^1 - iW_\mu^2|^2 + (g' B_\mu - g W_\mu^3)^2] = \frac{v^2}{8} (g^2 W^{a\mu} W_\mu^a + g'^2 B^\mu B_\mu - 2gg' B^\mu W_\mu^3). \quad (50)$$

这些项是规范玻色子的质量项，重新表达为

$$\mathcal{L}_{\text{GBM}} = \frac{1}{2} m_W^2 (W^{1\mu} W_\mu^1 + W^{2\mu} W_\mu^2) + \frac{1}{2} \begin{pmatrix} W^{3\mu} & B^\mu \end{pmatrix} M_{W^3 B}^2 \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}, \quad (51)$$

其中

$$m_W \equiv \frac{1}{2} g v \quad (52)$$

是 W_μ^1 和 W_μ^2 获得的质量，而 $W^{3\mu}$ 和 B^μ 的质量平方矩阵为

$$M_{W^3 B}^2 \equiv \frac{v^2}{4} \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}. \quad (53)$$

为了使 $M_{W^3 B}^2$ 矩阵对角化，定义

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \equiv \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}, \quad (54)$$

其中

$$s_W \equiv \sin \theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}, \quad c_W \equiv \cos \theta_W = \frac{g}{\sqrt{g^2 + g'^2}}, \quad (55)$$

θ_W 称为 Weinberg 角，也称为弱混合角。从后面的讨论可以看出 $A_\mu(x)$ 就是电磁场，对应于光子。 $Z_\mu(x)$ 对应于矢量玻色子 Z 。反过来，有

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}. \quad (56)$$

由

$$\begin{aligned} M_{W^3 B}^2 \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} &= \frac{(g^2 + g'^2)v^2}{4} \begin{pmatrix} c_W^2 & -s_W c_W \\ -s_W c_W & s_W^2 \end{pmatrix} \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \\ &= \frac{(g^2 + g'^2)v^2}{4} \begin{pmatrix} c_W & 0 \\ -s_W & 0 \end{pmatrix} \end{aligned} \quad (57)$$

得

$$\begin{aligned} \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} M_{W^3B}^2 \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} &= \frac{(g^2 + g'^2)v^2}{4} \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} c_W & 0 \\ -s_W & 0 \end{pmatrix} \\ &= \frac{(g^2 + g'^2)v^2}{4} \begin{pmatrix} 1 & \\ & 0 \end{pmatrix}, \end{aligned} \quad (58)$$

因此

$$\begin{aligned} \frac{1}{2} \begin{pmatrix} W^{3\mu} & B^\mu \end{pmatrix} M_{W^3B}^2 \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} &= \frac{1}{2} \begin{pmatrix} Z^\mu & A^\mu \end{pmatrix} \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} M_{W^3B}^2 \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \\ &= \frac{(g^2 + g'^2)v^2}{8} \begin{pmatrix} Z^\mu & A^\mu \end{pmatrix} \begin{pmatrix} 1 & \\ & 0 \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \frac{1}{2} m_Z^2 Z^\mu Z_\mu, \end{aligned} \quad (59)$$

其中

$$m_Z \equiv \frac{1}{2} \sqrt{g^2 + g'^2} v = \frac{gv}{2c_W} = \frac{m_W}{c_W} \quad (60)$$

是 Z 玻色子的质量，而光子没有质量。另一方面，用质量相同的实矢量场 W_μ^1 和 W_μ^2 线性组合出复矢量场

$$W_\mu^+ \equiv \frac{1}{\sqrt{2}}(W_\mu^1 - iW_\mu^2), \quad (61)$$

它的厄米共轭为

$$W_\mu^- \equiv (W_\mu^+)^{\dagger} = \frac{1}{\sqrt{2}}(W_\mu^1 + iW_\mu^2), \quad (62)$$

则

$$W_\mu^1 = \frac{1}{\sqrt{2}}(W_\mu^+ + W_\mu^-), \quad W_\mu^2 = \frac{i}{\sqrt{2}}(W_\mu^+ - W_\mu^-). \quad (63)$$

从而

$$\begin{aligned} \frac{1}{2}(W^{1\mu}W_\mu^1 + W^{2\mu}W_\mu^2) &= \frac{1}{4}[(W^{+\mu} + W^{-\mu})(W_\mu^+ + W_\mu^-) - (W^{+\mu} - W^{-\mu})(W_\mu^+ - W_\mu^-)] \\ &= W^{+\mu}W_\mu^-, \end{aligned} \quad (64)$$

(51) 式化为

$$\mathcal{L}_{\text{GBM}} = m_W^2 W^{+\mu} W_\mu^- + \frac{1}{2} m_Z^2 Z^\mu Z_\mu. \quad (65)$$

复矢量场 W_μ^\pm 描述一对正反矢量玻色子 W^\pm ，质量为 m_W 。可见，BEH 机制使传递弱相互作用的规范玻色子 W^\pm 和 Z 获得了质量。

接下来用质量本征态 W_μ^\pm 、 A_μ 和 Z_μ 表达协变动能项 $(D^\mu\Phi)^\dagger(D_\mu\Phi)$ 。注意到

$$A_\mu = s_W W_\mu^3 + c_W B_\mu, \quad Z_\mu = c_W W_\mu^3 - s_W B_\mu, \quad (66)$$

$$B_\mu = c_W A_\mu - s_W Z_\mu, \quad W_\mu^3 = s_W A_\mu + c_W Z_\mu, \quad (67)$$

有

$$\begin{aligned} g' B_\mu + g W_\mu^3 &= g'(c_W A_\mu - s_W Z_\mu) + g(s_W A_\mu + c_W Z_\mu) = \frac{2gg'}{\sqrt{g^2 + g'^2}} A_\mu + \frac{g^2 - g'^2}{\sqrt{g^2 + g'^2}} Z_\mu \\ &= 2e A_\mu + \frac{g}{c_W} (c_W^2 - s_W^2) Z_\mu, \end{aligned} \quad (68)$$

其中

$$e \equiv \frac{gg'}{\sqrt{g^2 + g'^2}} = g s_W = g' c_W. \quad (69)$$

后面的讨论将表明 e 就是单位电荷量。再利用

$$g' B_\mu - g W_\mu^3 = g'(c_W A_\mu - s_W Z_\mu) - g(s_W A_\mu + c_W Z_\mu) = -\left(\frac{gs_W^2}{c_W} + gc_W\right) Z_\mu = -\frac{g}{c_W} Z_\mu, \quad (70)$$

得

$$\begin{aligned} g' B_\mu Y_H + g W_\mu^a \tau^a &= \frac{1}{2} \begin{pmatrix} g' B_\mu + g W_\mu^3 & g(W_\mu^1 - iW_\mu^2) \\ g(W_\mu^1 + iW_\mu^2) & g' B_\mu - g W_\mu^3 \end{pmatrix} \\ &= \begin{pmatrix} e A_\mu + \frac{g}{2c_W} (c_W^2 - s_W^2) Z_\mu & \frac{g}{\sqrt{2}} W_\mu^+ \\ \frac{g}{\sqrt{2}} W_\mu^- & -\frac{g}{2c_W} Z_\mu \end{pmatrix}. \end{aligned} \quad (71)$$

在么正规范下,

$$\begin{aligned} (D^\mu\Phi)^\dagger(D_\mu\Phi) &= \left| \begin{pmatrix} \partial_\mu - ie A_\mu - \frac{ig}{2c_W} (c_W^2 - s_W^2) Z_\mu & -\frac{ig}{\sqrt{2}} W_\mu^+ \\ -\frac{ig}{\sqrt{2}} W_\mu^- & \partial_\mu + \frac{ig}{2c_W} Z_\mu \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix} \right|^2 \\ &= \frac{1}{2} \begin{pmatrix} \frac{ig}{\sqrt{2}} W^{-\mu}(v + H) & \partial^\mu H - \frac{ig}{2c_W} Z^\mu(v + H) \end{pmatrix} \begin{pmatrix} -\frac{ig}{\sqrt{2}} W_\mu^+(v + H) \\ \partial_\mu H + \frac{ig}{2c_W} Z_\mu(v + H) \end{pmatrix} \\ &= \frac{1}{2} (\partial^\mu H)(\partial_\mu H) + (v + H)^2 \left(\frac{g^2}{4} W_\mu^+ W^{-\mu} + \frac{g^2}{8c_W^2} Z_\mu Z^\mu \right) \\ &= \frac{1}{2} (\partial^\mu H)(\partial_\mu H) + m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu \end{aligned}$$

$$+gm_W HW_\mu^+ W^{-\mu} + \frac{gm_Z}{2c_W} H Z_\mu Z^\mu + \frac{g^2}{4} H^2 W_\mu^+ W^{-\mu} + \frac{g^2}{8c_W^2} H^2 Z_\mu Z^\mu. \quad (72)$$

除了 W^\pm 和 Z 玻色子的质量项之外, 还出现了 Higgs 玻色子 H 与 W^\pm 、 Z 的三线性和四线性耦合项。

Higgs 场 $\Phi(x)$ 的弱超荷为 $+1/2$ 。引入 $\Phi(x)$ 的共轭态

$$\tilde{\Phi}(x) = i\sigma^2 \Phi^*(x) = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix} \begin{pmatrix} \phi^-(x) \\ \phi^{0*}(x) \end{pmatrix} = \begin{pmatrix} \phi^{0*}(x) \\ -\phi^-(x) \end{pmatrix}, \quad (73)$$

其中 $\phi^- \equiv (\phi^+)^*$, 则 $\tilde{\Phi}(x)$ 是弱超荷为 $-1/2$ 的 $SU(2)_L$ 二重态。在么正规范下, $\tilde{\Phi}(x)$ 化为

$$\tilde{\Phi}(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} & 1 \\ -1 & \end{pmatrix} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}. \quad (74)$$

用 $\Phi(x)$ 、 $\tilde{\Phi}(x)$ 和费米子场组成满足 $SU(2)_L \times U(1)_Y$ 规范对称性的 Yukawa 相互作用拉氏量

$$\mathcal{L}_Y = -\tilde{y}_{d,ij} \bar{Q}_{iL} d'_{jR} \Phi - \tilde{y}_{u,ij} \bar{Q}_{iL} u'_{jR} \tilde{\Phi} - y_{\ell_i} \bar{L}_{iL} \ell_{iR} \Phi + \text{H.c.}, \quad (75)$$

其中 H.c. 表示厄米共轭, Yukawa 耦合常数 $\tilde{y}_{d,ij}$ 和 $\tilde{y}_{u,ij}$ 联系着不同代的夸克场, 而 y_{ℓ_i} 只联系同一代的轻子场。在么正规范下, 利用

$$\bar{Q}_{iL} \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \bar{u}'_{iL} & \bar{d}'_{iL} \end{pmatrix} \begin{pmatrix} 0 \\ v + H \end{pmatrix} = \frac{1}{\sqrt{2}} (v + H) \bar{d}'_{iL}, \quad (76)$$

$$\bar{Q}_{iL} \tilde{\Phi} = \frac{1}{\sqrt{2}} \begin{pmatrix} \bar{u}'_{iL} & \bar{d}'_{iL} \end{pmatrix} \begin{pmatrix} v + H \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} (v + H) \bar{u}'_{iL}, \quad (77)$$

$$\bar{L}_{iL} \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \bar{\nu}_{iL} & \bar{\ell}_{iL} \end{pmatrix} \begin{pmatrix} 0 \\ v + H \end{pmatrix} = \frac{1}{\sqrt{2}} (v + H) \bar{\ell}_{iL}, \quad (78)$$

推出

$$\mathcal{L}_Y = -\frac{1}{\sqrt{2}} (v + H) (\tilde{y}_{d,ij} \bar{d}'_{iL} d'_{jR} + \tilde{y}_{u,ij} \bar{u}'_{iL} u'_{jR} + y_{\ell_i} \bar{\ell}_{iL} \ell_{iR} + \text{H.c.}). \quad (79)$$

$\tilde{y}_{d,ij}$ 和 $\tilde{y}_{u,ij}$ 可看作 3×3 矩阵 \tilde{y}_d 和 \tilde{y}_u 的元素。 $\tilde{y}_d \tilde{y}_d^\dagger$ 和 $\tilde{y}_u \tilde{y}_u^\dagger$ 是厄米矩阵, 必定可以分别通过么正矩阵 U_d 和 U_u 对角化成 y_D^2 和 y_U^2 两个对角元为实数的对角矩阵, 满足 $U_d^\dagger \tilde{y}_d \tilde{y}_d^\dagger U_d = y_D^2$ 和 $U_u^\dagger \tilde{y}_u \tilde{y}_u^\dagger U_u = y_U^2$, 即

$$\tilde{y}_d \tilde{y}_d^\dagger = U_d y_D^2 U_d^\dagger, \quad \tilde{y}_u \tilde{y}_u^\dagger = U_u y_U^2 U_u^\dagger. \quad (80)$$

符合这两条式子的 \tilde{y}_d 和 \tilde{y}_u 可以表达为

$$\tilde{y}_d = U_d y_D K_d^\dagger, \quad \tilde{y}_u = U_u y_U K_u^\dagger, \quad (81)$$

其中对角矩阵 y_D 和 y_U 满足 $y_D y_D = y_D^2$ 和 $y_U y_U = y_U^2$, 而 K_d^\dagger 和 K_u^\dagger 是两个么正矩阵。

将 y_D 和 y_U 表示成

$$y_D = \begin{pmatrix} y_{d_1} & & \\ & y_{d_2} & \\ & & y_{d_3} \end{pmatrix} = \begin{pmatrix} y_d & & \\ & y_s & \\ & & y_b \end{pmatrix}, \quad y_U = \begin{pmatrix} y_{u_1} & & \\ & y_{u_2} & \\ & & y_{u_3} \end{pmatrix} = \begin{pmatrix} y_u & & \\ & y_c & \\ & & y_t \end{pmatrix}. \quad (82)$$

通过么正变换定义

$$d_{iL} \equiv (U_d^\dagger)_{ij} d'_{jL}, \quad d_{iR} \equiv (K_d^\dagger)_{ij} d'_{jR}, \quad u_{iL} \equiv (U_u^\dagger)_{ij} u'_{jL}, \quad u_{iR} \equiv (K_u^\dagger)_{ij} u'_{jR}, \quad (83)$$

则 $\bar{d}_{iL} = \bar{d}'_{jL} U_{d,ji}$, $\bar{u}_{iL} = \bar{u}'_{jL} U_{u,ji}$, 从而

$$\tilde{y}_{d,ij} \bar{d}'_{iL} d'_{jR} = \bar{d}'_{iL} (U_d y_d K_d^\dagger)_{ij} d'_{jR} = \bar{d}'_{iL} U_{d,ik} y_{d_k} (K_d^\dagger)_{kj} d'_{jR} = y_{d_k} \bar{d}_{kL} d_{kR} = y_{d_i} \bar{d}_{iL} d_{iR}, \quad (84)$$

$$\tilde{y}_{u,ij} \bar{u}'_{iL} u'_{jR} = \bar{u}'_{iL} (U_u y_u K_u^\dagger)_{ij} u'_{jR} = y_{u_i} \bar{u}_{iL} u_{iR}, \quad (85)$$

故

$$\begin{aligned} \mathcal{L}_Y &= -\frac{1}{\sqrt{2}}(v + H)(y_{d_i} \bar{d}_{iL} d_{iR} + y_{u_i} \bar{u}_{iL} u_{iR} + y_{\ell_i} \bar{\ell}_{iL} \ell_{iR} + \text{H.c.}) \\ &= -m_{d_i} \bar{d}_i d_i - m_{u_i} \bar{u}_i u_i - m_{\ell_i} \bar{\ell}_i \ell_i - \frac{m_{d_i}}{v} H \bar{d}_i d_i - \frac{m_{u_i}}{v} H \bar{u}_i u_i - \frac{m_{\ell_i}}{v} H \bar{\ell}_i \ell_i, \end{aligned} \quad (86)$$

其中前三项是费米子质量项, 后三项是 Higgs 玻色子与费米子的 Yukawa 耦合项。于是, 三代夸克和带电轻子获得了质量

$$m_{d_i} \equiv \frac{1}{\sqrt{2}} y_{d_i} v, \quad m_{u_i} \equiv \frac{1}{\sqrt{2}} y_{u_i} v, \quad m_{\ell_i} \equiv \frac{1}{\sqrt{2}} y_{\ell_i} v. \quad (87)$$

d'_{iL} 、 d'_{iR} 、 u'_{iL} 和 u'_{iR} 称为规范本征态, d_{iL} 、 d_{iR} 、 u_{iL} 和 u_{iR} 称为质量本征态。

下面给出么正规范下 Higgs 场的顶点费曼规则。

Higgs 玻色子自耦合:

$$\begin{aligned} \text{Diagram 1: } & \begin{array}{c} H \\ | \\ \bullet \\ / \quad \backslash \\ H \quad H \end{array} & \begin{aligned} &= -\frac{3im_H^2}{v} \\ &= -6i\lambda v \end{aligned} \\ \text{Diagram 2: } & \begin{array}{c} H \quad H \\ \backslash \quad / \\ \bullet \\ / \quad \backslash \\ H \quad H \end{array} & \begin{aligned} &= -\frac{3im_H^2}{v^2} \\ &= -6i\lambda \end{aligned} \end{aligned} \quad (88)$$

Higgs 玻色子与电弱规范玻色子的耦合:

$$\begin{array}{c}
 \begin{array}{c}
 \text{Diagram 1: } H \text{ (dashed line) connects to a vertex, which then splits into two } W \text{ bosons (wavy lines).} \\
 W; \mu \quad W; \nu
 \end{array}
 \end{array}
 = igm_W g^{\mu\nu}
 = \frac{ig^2 v}{2} g^{\mu\nu}
 \quad (89)$$

$$\begin{array}{c}
 \begin{array}{c}
 \text{Diagram 2: } H \text{ (dashed line) connects to a vertex, which then splits into two } Z \text{ bosons (wavy lines).} \\
 Z; \mu \quad Z; \nu
 \end{array}
 \end{array}
 = \frac{igm_Z}{c_W} g^{\mu\nu}
 = \frac{ig^2 v}{2c_W^2} g^{\mu\nu}
 \quad (90)$$

Higgs 玻色子与费米子 $f = d_i, u_i, \ell_i$ 的耦合:

$$\begin{array}{c}
 \begin{array}{c}
 \text{Diagram 3: } H \text{ (dashed line) connects to a vertex, which then splits into two fermions } f \text{ (solid lines).} \\
 f \quad f
 \end{array}
 \end{array}
 = -\frac{im_f}{v}
 = -\frac{iy_f}{\sqrt{2}}
 \quad (91)$$

4.2 费米子电弱规范相互作用

(83) 式意味着

$$d'_{iL} = U_{d,ij} d_{jL}, \quad d'_{iR} = K_{d,ij} d_{jR}, \quad u'_{iL} = U_{u,ij} u_{jL}, \quad u'_{iR} = K_{u,ij} u_{jR}, \quad (92)$$

从而

$$\bar{d}'_{iL} \gamma^\mu d'_{iL} = \bar{d}_{jL} (U_d^\dagger)_{ji} \gamma^\mu U_{d,ik} d_{kL} = \bar{d}_{jL} \delta_{jk} \gamma^\mu d_{kL} = \bar{d}_{iL} \gamma^\mu d_{iL} \quad (93)$$

同理有

$$\bar{u}'_{iL} \gamma^\mu u'_{iL} = \bar{u}_{iL} \gamma^\mu u_{iL}, \quad \bar{d}'_{iR} \gamma^\mu d'_{iR} = \bar{d}_{iR} \gamma^\mu d_{iR}, \quad \bar{u}'_{iR} \gamma^\mu u'_{iR} = \bar{u}_{iR} \gamma^\mu u_{iR}. \quad (94)$$

另一方面,

$$\bar{u}'_{iL} \gamma^\mu d'_{iL} = \bar{u}_{jL} (U_u^\dagger)_{ji} \gamma^\mu U_{d,ik} d_{kL} = \bar{u}_{iL} \gamma^\mu V_{ij} d_{jL} \quad (95)$$

$$\bar{d}'_{iL} \gamma^\mu u'_{iL} = \bar{d}_{jL} (U_d^\dagger)_{ji} \gamma^\mu U_{u,ik} u_{kL} = \bar{d}_{jL} V_{ji}^\dagger \gamma^\mu u_{iL} \quad (96)$$

其中

$$V \equiv U_u^\dagger U_d \quad (97)$$

称为 Cabibbo-Kobayashi-Maskawa (CKM) 矩阵, 其厄米共轭矩阵为 $V^\dagger = U_d^\dagger U_u$ 。

$SU(2)_L \times U(1)_Y$ 规范不变的费米子协变动能项为

$$\mathcal{L}_{\text{EWF}} = \bar{Q}_{iL} i \not{D} Q_{iL} + \bar{u}'_{iR} i \not{D} u'_{iR} + \bar{d}'_{iR} i \not{D} d'_{iR} + \bar{L}_{iL} i \not{D} L_{iL} + \bar{\ell}_{iR} i \not{D} \ell_{iR}. \quad (98)$$

根据 Q 的定义 (34), 有

$$\begin{aligned} g'Y B_\mu + gT^3 W_\mu^3 &= g'Y(c_W A_\mu - s_W Z_\mu) + gT^3(s_W A_\mu + c_W Z_\mu) \\ &= e(Y + T^3)A_\mu + \left(g c_W T^3 - \frac{g s_W}{c_W} s_W Y \right) Z_\mu = Q e A_\mu + \frac{g}{c_W} (T^3 c_W^2 - Y s_W^2) Z_\mu \\ &= Q e A_\mu + \frac{g}{c_W} (T^3 - Q s_W^2) Z_\mu, \end{aligned} \quad (99)$$

故

$$\begin{aligned} D_\mu Q_{iL} &= (\partial_\mu - i g' B_\mu Y - i g W_\mu^a \tau^a) Q_{iL} \\ &= \partial_\mu Q_{iL} - i \begin{pmatrix} g'Y B_\mu + gT^3 W_\mu^3 & \frac{g}{2} (W_\mu^1 - i W_\mu^2) \\ \frac{g}{2} (W_\mu^1 + i W_\mu^2) & g'Y B_\mu + gT^3 W_\mu^3 \end{pmatrix} Q_{iL} \\ &= \partial_\mu Q_{iL} - i \begin{pmatrix} Q e A_\mu + \frac{g}{c_W} (T^3 - Q s_W^2) Z_\mu & \frac{g}{\sqrt{2}} W_\mu^+ \\ \frac{g}{\sqrt{2}} W_\mu^- & Q e A_\mu + \frac{g}{c_W} (T^3 - Q s_W^2) Z_\mu \end{pmatrix} \begin{pmatrix} u'_{iL} \\ d'_{iL} \end{pmatrix} \\ &= \partial_\mu Q_{iL} - i \begin{pmatrix} \left[Q e A_\mu + \frac{g}{c_W} (T^3 - Q s_W^2) Z_\mu \right] u'_{iL} + \frac{g}{\sqrt{2}} W_\mu^+ d'_{iL} \\ \frac{g}{\sqrt{2}} W_\mu^- u'_{iL} + \left[Q e A_\mu + \frac{g}{c_W} (T^3 - Q s_W^2) Z_\mu \right] d'_{iL} \end{pmatrix}. \end{aligned} \quad (100)$$

于是

$$\begin{aligned} \bar{Q}_{iL} i \not{D} Q_{iL} &\supset \left[Q e A_\mu + \frac{g}{c_W} (T^3 - Q s_W^2) Z_\mu \right] \bar{u}'_{iL} \gamma^\mu u'_{iL} + \left[Q e A_\mu + \frac{g}{c_W} (T^3 - Q s_W^2) Z_\mu \right] \bar{d}'_{iL} \gamma^\mu d'_{iL} \\ &\quad + \frac{g}{\sqrt{2}} W_\mu^+ \bar{u}'_{iL} \gamma^\mu d'_{iL} + \frac{g}{\sqrt{2}} W_\mu^- \bar{d}'_{iL} \gamma^\mu u'_{iL} \\ &= \left(Q e A_\mu + \frac{g}{c_W} g_L Z_\mu \right) \bar{u}_i \gamma^\mu \frac{1 - \gamma^5}{2} u_i + \left(Q e A_\mu + \frac{g}{c_W} g_L Z_\mu \right) \bar{d}_i \gamma^\mu \frac{1 - \gamma^5}{2} d_i \\ &\quad + \frac{g}{\sqrt{2}} W_\mu^+ \bar{u}_i \gamma^\mu P_L V_{ij} d_j + \frac{g}{\sqrt{2}} W_\mu^- \bar{d}_j V_{ji}^\dagger \gamma^\mu P_L u_i, \end{aligned} \quad (101)$$

其中左手耦合系数

$$g_L \equiv T^3 - Q s_W^2. \quad (102)$$

另一方面,

$$D_\mu d'_{iR} = (\partial_\mu - i g' B_\mu Y) d'_{iR} = \partial_\mu d'_{iR} - i g' Q (c_W A_\mu - s_W Z_\mu) d'_{iR}$$

$$= \partial_\mu d'_{iR} - iQeA_\mu d'_{iR} + \frac{ig}{c_W} Qs_W^2 Z_\mu d'_{iR}, \quad (103)$$

则

$$\begin{aligned} & \bar{u}'_{iR} i \not{D} u'_{iR} + \bar{d}'_{iR} i \not{D} d'_{iR} \\ & \supset \left(QeA_\mu - \frac{g}{c_W} Qs_W^2 Z_\mu \right) \bar{u}'_{iR} \gamma^\mu u'_{iR} + \left(QeA_\mu - \frac{g}{c_W} Qs_W^2 Z_\mu \right) \bar{d}'_{iR} \gamma^\mu d'_{iR} \\ & = \left(QeA_\mu + \frac{g}{c_W} g_R Z_\mu \right) \bar{u}_i \gamma^\mu \frac{1+\gamma^5}{2} u_i + \left(QeA_\mu + \frac{g}{c_W} g_R Z_\mu \right) \bar{d}_i \gamma^\mu \frac{1+\gamma^5}{2} d_i, \end{aligned} \quad (104)$$

其中右手耦合系数

$$g_R \equiv -Qs_W^2. \quad (105)$$

引入矢量流和轴矢量流耦合系数

$$g_V \equiv g_L + g_R = T^3 - 2Qs_W^2, \quad g_A \equiv g_L - g_R = T^3, \quad (106)$$

得

$$\begin{aligned} & \bar{Q}_{iL} i \not{D} Q_{iL} + \bar{u}'_{iR} i \not{D} u'_{iR} + \bar{d}'_{iR} i \not{D} d'_{iR} \\ & \supset Qe\bar{u}_i \gamma^\mu u_i A_\mu + Qe\bar{d}_i \gamma^\mu d_i A_\mu + \frac{g}{2c_W} \bar{u}_i \gamma^\mu (g_V - g_A \gamma^5) u_i Z_\mu + \frac{g}{2c_W} \bar{d}_i \gamma^\mu (g_V - g_A \gamma^5) d_i Z_\mu \\ & + \frac{g}{\sqrt{2}} W_\mu^+ \bar{u}_i \gamma^\mu P_L V_{ij} d_j + \frac{g}{\sqrt{2}} W_\mu^- \bar{d}_j \gamma^\mu P_L V_{ji}^\dagger u_i. \end{aligned} \quad (107)$$

同理, 有

$$\begin{aligned} & \bar{L}_{iL} i \not{D} L_{iL} + \bar{\ell}_{iR} i \not{D} \ell_{iR} \supset Qe\bar{\ell}_i \gamma^\mu \ell_i A_\mu + \frac{g}{2c_W} \bar{\ell}_i \gamma^\mu (g_V - g_A \gamma^5) \ell_i Z_\mu + \frac{g}{2c_W} \bar{\nu}_i \gamma^\mu (g_V - g_A \gamma^5) \nu_i Z_\mu \\ & + \frac{g}{\sqrt{2}} W_\mu^+ \bar{\nu}_i \gamma^\mu P_L \ell_i + \frac{g}{\sqrt{2}} W_\mu^- \bar{\ell}_i \gamma^\mu P_L \nu_i. \end{aligned} \quad (108)$$

总结起来, 将费米子电弱规范相互作用写成流耦合的形式,

$$\mathcal{L}_{\text{EWF}} \supset A_\mu J_{\text{EM}}^\mu + Z_\mu J_Z^\mu + W_\mu^+ J_W^{+, \mu} + W_\mu^- J_W^{-, \mu}, \quad (109)$$

其中 f 代表任意费米子场, 电磁流

$$J_{\text{EM}}^\mu \equiv \sum_f Q_f e \bar{f} \gamma^\mu f, \quad (110)$$

弱中性流

$$J_Z^\mu \equiv \frac{g}{2c_W} \sum_f \bar{f} \gamma^\mu (g_V^f - g_A^f \gamma^5) f = \frac{g}{c_W} \sum_f (g_L^f \bar{f}_L \gamma^\mu f_L + g_R^f \bar{f}_R \gamma^\mu f_R), \quad (111)$$

弱带电流

$$J_W^{+, \mu} \equiv \frac{g}{\sqrt{2}} (\bar{u}_{iL} \gamma^\mu V_{ij} d_{jL} + \bar{\nu}_{iL} \gamma^\mu \ell_{iL}), \quad J_W^{-, \mu} \equiv (J_W^{+, \mu})^\dagger = \frac{g}{\sqrt{2}} (\bar{d}_{jL} V_{ji}^\dagger \gamma^\mu u_{iL} + \bar{\ell}_{iL} \gamma^\mu \nu_{iL}). \quad (112)$$

对于各种费米子，相关的系数如下，

$$Q_{u_i} = \frac{2}{3}, \quad Q_{d_i} = -\frac{1}{3}, \quad Q_{\nu_i} = 0, \quad Q_{\ell_i} = -1; \quad (113)$$

$$g_V^{u_i} = \frac{1}{2} - \frac{4}{3}s_W^2, \quad g_A^{u_i} = \frac{1}{2}; \quad g_V^{d_i} = -\frac{1}{2} + \frac{2}{3}s_W^2, \quad g_A^{d_i} = -\frac{1}{2}; \quad (114)$$

$$g_V^{\nu_i} = \frac{1}{2}, \quad g_A^{\nu_i} = \frac{1}{2}; \quad g_V^{\ell_i} = -\frac{1}{2} + 2s_W^2, \quad g_A^{\ell_i} = -\frac{1}{2}; \quad (115)$$

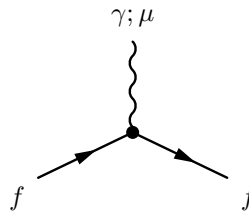
$$g_L^{u_i} = \frac{1}{2} - \frac{2}{3}s_W^2, \quad g_R^{u_i} = -\frac{2}{3}s_W^2; \quad g_L^{d_i} = -\frac{1}{2} + \frac{1}{3}s_W^2, \quad g_R^{d_i} = \frac{1}{3}s_W^2; \quad (116)$$

$$g_L^{\nu_i} = \frac{1}{2}, \quad g_R^{\nu_i} = 0; \quad g_L^{\ell_i} = -\frac{1}{2} + s_W^2, \quad g_R^{\ell_i} = s_W^2. \quad (117)$$

可以看到，电磁流耦合与 QED 耦合完全相同，由此辨认出 A_μ 是电磁场， e 是单位电荷量，由 $Q = T^3 + Y$ 定义的 Q 确实是电荷。为了保持电荷守恒，指定复矢量场 $W_\mu^+(x)$ 携带 $Q = +1$ 的电荷，从而 W^\pm 玻色子的电荷为 ± 1 。不同代夸克之间的相互作用只发生在弱带电流耦合中，源自 CKM 矩阵 V 的非对角元。

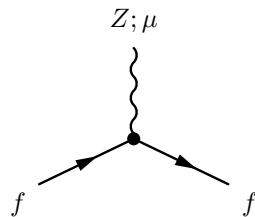
下面给出费米子电弱规范相互作用顶点的费曼规则。

QED 耦合：



$$= iQ_f e \gamma^\mu \quad (118)$$

费米子与 Z 玻色子的耦合：



$$= \frac{ig}{2c_W} \gamma^\mu (g_V^f - g_A^f \gamma^5) \quad (119)$$

费米子与 W^\pm 玻色子的耦合:

$$\begin{aligned}
 \begin{array}{c} W; \mu \\ \downarrow \\ d_j \rightarrow \bullet \rightarrow u_i \end{array} &= \frac{ig}{\sqrt{2}} V_{ij} \gamma^\mu P_L & \begin{array}{c} W; \mu \\ \downarrow \\ u_i \rightarrow \bullet \rightarrow d_j \end{array} &= \frac{ig}{\sqrt{2}} V_{ji}^\dagger \gamma^\mu P_L & (120)
 \end{aligned}$$

$$\begin{aligned}
 \begin{array}{c} W; \mu \\ \downarrow \\ \ell_i \rightarrow \bullet \rightarrow \nu_i \end{array} &= \frac{ig}{\sqrt{2}} \gamma^\mu P_L & \begin{array}{c} W; \mu \\ \downarrow \\ \nu_i \rightarrow \bullet \rightarrow \ell_i \end{array} &= \frac{ig}{\sqrt{2}} \gamma^\mu P_L & (121)
 \end{aligned}$$

4.3 电弱规范场的自相互作用

电弱规范场自相互作用的拉氏量是

$$\mathcal{L}_{\text{EWG}} = -\frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}, \quad (122)$$

其中

$$W^{a\mu\nu} \equiv \partial^\mu W^{a\nu} - \partial^\nu W^{a\mu} + g\epsilon^{abc} W^{b\mu} W^{c\nu}, \quad B^{\mu\nu} \equiv \partial^\mu B^\nu - \partial^\nu B^\mu. \quad (123)$$

利用 (63) 和 (67) 式, 推出

$$\begin{aligned}
 W_\mu^2 W_\nu^3 - W_\mu^3 W_\nu^2 &= \frac{i}{\sqrt{2}} [(W_\mu^+ - W_\mu^-)(s_W A_\nu + c_W Z_\nu) - (s_W A_\mu + c_W Z_\mu)(W_\nu^+ - W_\nu^-)] \\
 &= \frac{i}{\sqrt{2}} [s_W(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+) \\
 &\quad - s_W(W_\mu^- A_\nu - A_\mu W_\nu^-) - c_W(W_\mu^- Z_\nu - Z_\mu W_\nu^-)], \quad (124)
 \end{aligned}$$

$$\begin{aligned}
 W_\mu^3 W_\nu^1 - W_\mu^1 W_\nu^3 &= \frac{1}{\sqrt{2}} [(s_W A_\mu + c_W Z_\mu)(W_\nu^+ + W_\nu^-) - (W_\mu^+ + W_\mu^-)(s_W A_\nu + c_W Z_\nu)] \\
 &= -\frac{1}{\sqrt{2}} [s_W(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+) \\
 &\quad + s_W(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_W(W_\mu^- Z_\nu - Z_\mu W_\nu^-)]. \quad (125)
 \end{aligned}$$

从而

$$\begin{aligned}
 W_{\mu\nu}^1 &= \partial_\mu W_\nu^1 - \partial_\nu W_\mu^1 + g\epsilon^{1bc} W_\mu^b W_\nu^c = \partial_\mu W_\nu^1 - \partial_\nu W_\mu^1 + g(W_\mu^2 W_\nu^3 - gW_\mu^3 W_\nu^2) \\
 &= \frac{1}{\sqrt{2}} (\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) + \frac{1}{\sqrt{2}} (\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) + g(W_\mu^2 W_\nu^3 - gW_\mu^3 W_\nu^2) \\
 &= \frac{1}{\sqrt{2}} \{ \partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ig[s_W(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)] \}
 \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{\sqrt{2}} \{ \partial_\mu W_\nu^- - \partial_\nu W_\mu^- - ig[s_W(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_W(W_\mu^- Z_\nu - Z_\mu W_\nu^-)] \} \\
& = \frac{1}{\sqrt{2}} (F_{\mu\nu}^+ + F_{\mu\nu}^-),
\end{aligned} \tag{126}$$

其中,

$$F_{\mu\nu}^+ \equiv \partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ie(W_\mu^+ A_\nu - A_\mu W_\nu^+) + igc_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+), \tag{127}$$

$$F_{\mu\nu}^- \equiv \partial_\mu W_\nu^- - \partial_\nu W_\mu^- - ie(W_\mu^- A_\nu - A_\mu W_\nu^-) - igc_W(W_\mu^- Z_\nu - Z_\mu W_\nu^-). \tag{128}$$

另一方面,

$$\begin{aligned}
W_{\mu\nu}^2 &= \partial_\mu W_\nu^2 - \partial_\nu W_\mu^2 + g\varepsilon^{2bc}W_\mu^b W_\nu^c = \partial_\mu W_\nu^2 - \partial_\nu W_\mu^2 - gW_\mu^1 W_\nu^3 + gW_\mu^3 W_\nu^1 \\
&= \frac{i}{\sqrt{2}} (\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) - \frac{i}{\sqrt{2}} (\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) + g(W_\mu^3 W_\nu^1 - W_\mu^1 W_\nu^3) \\
&= \frac{i}{\sqrt{2}} \{ \partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ig[s_W(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)] \} \\
&\quad - \frac{i}{\sqrt{2}} \{ \partial_\mu W_\nu^- - \partial_\nu W_\mu^- - ig[s_W(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_W(W_\mu^- Z_\nu - Z_\mu W_\nu^-)] \} \\
&= \frac{i}{\sqrt{2}} (F_{\mu\nu}^+ - F_{\mu\nu}^-).
\end{aligned} \tag{129}$$

因此

$$\begin{aligned}
& -\frac{1}{4}W_{\mu\nu}^1 W^{1\mu\nu} - \frac{1}{4}W_{\mu\nu}^2 W^{2\mu\nu} \\
&= -\frac{1}{8}(F_{\mu\nu}^+ + F_{\mu\nu}^-)(F^{+\mu\nu} + F^{-\mu\nu}) + \frac{1}{8}(F_{\mu\nu}^+ - F_{\mu\nu}^-)(F^{+\mu\nu} - F^{-\mu\nu}) = -\frac{1}{2}F_{\mu\nu}^+ F^{-\mu\nu} \\
&= -\frac{1}{2}[\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ie(W_\mu^+ A_\nu - A_\mu W_\nu^+) + igc_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)] \\
&\quad \times [\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu} - ie(W^{-\mu} A^\nu - A^\mu W^{-\nu}) - igc_W(W^{-\mu} Z^\nu - Z^\mu W^{-\nu})] \\
&= -(\partial_\mu W_\nu^+)(\partial^\mu W^{-\nu}) + (\partial_\mu W_\nu^+)(\partial^\nu W^{-\mu}) \\
&\quad + ie[(\partial_\mu W_\nu^+)W^{-\mu} A^\nu - (\partial_\mu W_\nu^+)W^{-\nu} A^\mu - W_\mu^+(\partial^\mu W^{-\nu})A_\nu + W_\nu^+(\partial^\mu W^{-\nu})A_\mu] \\
&\quad + igc_W[(\partial_\mu W_\nu^+)W^{-\mu} Z^\nu - (\partial_\mu W_\nu^+)W^{-\nu} Z^\mu - W_\mu^+(\partial^\mu W^{-\nu})Z_\nu + W_\nu^+(\partial^\mu W^{-\nu})Z_\mu] \\
&\quad + e^2(W_\mu^+ W^{-\nu} A_\nu A^\mu - W_\mu^+ W^{-\mu} A_\nu A^\nu) + g^2 c_W^2 (W_\mu^+ W^{-\nu} Z_\nu Z^\mu - W_\mu^+ W^{-\mu} Z_\nu Z^\nu) \\
&\quad + egc_W(W_\mu^+ W^{-\nu} A_\nu Z^\mu + W_\mu^+ W^{-\nu} A^\mu Z_\nu - 2W_\mu^+ W^{-\mu} A_\nu Z^\nu).
\end{aligned} \tag{130}$$

由

$$\begin{aligned}
W_\mu^1 W_\nu^2 - W_\mu^2 W_\nu^1 &= \frac{i}{2}(W_\mu^+ + W_\mu^-)(W_\nu^+ - W_\nu^-) - \frac{i}{2}(W_\mu^+ - W_\mu^-)(W_\nu^+ + W_\nu^-) \\
&= -i(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+),
\end{aligned} \tag{131}$$

得到

$$\begin{aligned}
 W_{\mu\nu}^3 &= \partial_\mu W_\nu^3 - \partial_\nu W_\mu^3 + g\varepsilon^{3bc}W_\mu^b W_\nu^c = \partial_\mu W_\nu^3 - \partial_\nu W_\mu^3 + gW_\mu^1 W_\nu^2 - gW_\mu^2 W_\nu^1 \\
 &= s_W \partial_\mu A_\nu + c_W \partial_\mu Z_\nu - s_W \partial_\nu A_\mu - c_W \partial_\nu Z_\mu + g(W_\mu^1 W_\nu^2 - W_\mu^2 W_\nu^1) \\
 &= s_W (\partial_\mu A_\nu - \partial_\nu A_\mu) + c_W (\partial_\mu Z_\nu - \partial_\nu Z_\mu) - ig(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+), \tag{132}
 \end{aligned}$$

$$\begin{aligned}
 B_{\mu\nu} &= \partial_\mu (c_W A_\nu - s_W Z_\nu) - \partial_\nu (c_W A_\mu - s_W Z_\mu) \\
 &= c_W (\partial_\mu A_\nu - \partial_\nu A_\mu) - s_W (\partial_\mu Z_\nu - \partial_\nu Z_\mu). \tag{133}
 \end{aligned}$$

于是

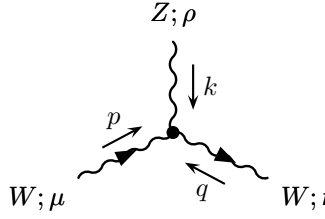
$$\begin{aligned}
 &-\frac{1}{4}W_{\mu\nu}^3 W^{3\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
 &= -\frac{1}{2}[(\partial_\mu A_\nu)(\partial^\mu A^\nu) - (\partial_\mu A_\nu)(\partial^\nu A^\mu)] - \frac{1}{2}[(\partial_\mu Z_\nu)(\partial^\mu Z^\nu) - (\partial_\mu Z_\nu)(\partial^\nu Z^\mu)] \\
 &\quad + ie[W^{+\mu}W^{-\nu}(\partial_\mu A_\nu) - W^{+\nu}W^{-\mu}(\partial_\mu A_\nu)] + igc_W[W^{+\mu}W^{-\nu}(\partial_\mu Z_\nu) - W^{+\nu}W^{-\mu}(\partial_\mu Z_\nu)] \\
 &\quad + \frac{g^2}{2}(W_\mu^+ W^{+\mu} W_\nu^- W^{-\nu} - W_\mu^+ W^{+\nu} W_\nu^- W^{-\mu}). \tag{134}
 \end{aligned}$$

综合起来, 有


$$\begin{aligned}
 \mathcal{L}_{\text{EWG}} &= \frac{1}{2}[(\partial_\mu A_\nu)(\partial^\nu A^\mu) - (\partial_\mu A_\nu)(\partial^\mu A^\nu)] + \frac{1}{2}[(\partial_\mu Z_\nu)(\partial^\nu Z^\mu) - (\partial_\mu Z_\nu)(\partial^\mu Z^\nu)] \\
 &\quad + (\partial_\mu W_\nu^+)(\partial^\nu W^{-\mu}) - (\partial_\mu W_\nu^+)(\partial^\mu W^{-\nu}) + \frac{g^2}{2}(W_\mu^+ W^{+\mu} W_\nu^- W^{-\nu} - W_\mu^+ W^{+\nu} W_\nu^- W^{-\mu}) \\
 &\quad + ie[(\partial_\mu W_\nu^+)W^{-\mu}A^\nu - (\partial_\mu W_\nu^+)W^{-\nu}A^\mu - W^{+\mu}(\partial_\mu W_\nu^-)A^\nu + W^{+\nu}(\partial_\mu W_\nu^-)A^\mu \\
 &\quad + W^{+\mu}W^{-\nu}(\partial_\mu A_\nu) - W^{+\nu}W^{-\mu}(\partial_\mu A_\nu)] \\
 &\quad + igc_W[(\partial_\mu W_\nu^+)W^{-\mu}Z^\nu - (\partial_\mu W_\nu^+)W^{-\nu}Z^\mu - W^{+\mu}(\partial_\mu W_\nu^-)Z^\nu + W^{+\nu}(\partial_\mu W_\nu^-)Z^\mu \\
 &\quad + W^{+\mu}W^{-\nu}(\partial_\mu Z_\nu) - W^{+\nu}W^{-\mu}(\partial_\mu Z_\nu)] \\
 &\quad + e^2(W_\mu^+ W^{-\nu}A_\nu A^\mu - W_\mu^+ W^{-\mu}A_\nu A^\nu) + g^2 c_W^2(W_\mu^+ W^{-\nu}Z_\nu Z^\mu - W_\mu^+ W^{-\mu}Z_\nu Z^\nu) \\
 &\quad + egc_W(W_\mu^+ W^{-\nu}A_\nu Z^\mu + W_\mu^+ W^{-\nu}A^\mu Z_\nu - 2W_\mu^+ W^{-\mu}A_\nu Z^\nu). \tag{135}
 \end{aligned}$$

下面是电弱规范玻色子自耦合的费曼规则:

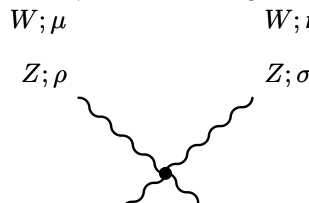
$$= -ie[g^{\mu\nu}(p-q)^\rho + g^{\nu\rho}(q-k)^\mu + g^{\rho\mu}(k-p)^\nu] \tag{136}$$



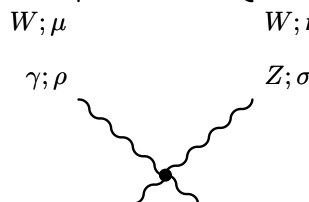
$$= -igc_W[g^{\mu\nu}(p-q)^\rho + g^{\nu\rho}(q-k)^\mu + g^{\rho\mu}(k-p)^\nu] \quad (137)$$



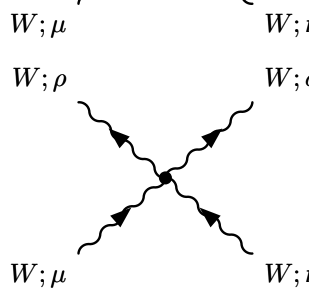
$$= ie^2(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma}) \quad (138)$$



$$= ig^2c_W^2(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma}) \quad (139)$$



$$= iegc_W(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma}) \quad (140)$$



$$= -ig^2(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma}) \quad (141)$$

5 R_ξ 规范下电弱拉氏量和费曼规则

在 R_ξ 规范下, 将 Higgs 场参数化为

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}[v + H(x) + i\chi(x)] \end{pmatrix}, \quad (142)$$

其中 ϕ^+ 和 χ 是 Nambu-Goldstone 标量场。那么, $\tilde{\Phi}(x)$ 的形式是

$$\tilde{\Phi}(x) = \begin{pmatrix} \phi^{0*}(x) \\ -\phi^-(x) \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}[v + H(x) - i\chi(x)] \\ -\phi^-(x) \end{pmatrix}. \quad (143)$$

利用

$$\Phi^\dagger \Phi = \frac{1}{2}(v^2 + H^2 + 2vH + \chi^2) + |\phi^+|^2, \quad (144)$$

$$(\Phi^\dagger \Phi)^2 = \frac{1}{4}(v^2 + H^2 + 2vH + \chi^2)^2 + |\phi^+|^4 + |\phi^+|^2(v^2 + H^2 + 2vH + \chi^2), \quad (145)$$

推出 Higgs 场势能项

$$\begin{aligned} -V_H(\Phi) &= \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 \\ &= \frac{\mu^2}{2}(v^2 + H^2 + 2vH + \chi^2) + \mu^2 |\phi^+|^2 - \frac{\lambda}{4}(v^2 + H^2 + 2vH + \chi^2)^2 - \lambda |\phi^+|^4 \\ &\quad - \lambda |\phi^+|^2(v^2 + H^2 + 2vH + \chi^2) \\ &= \frac{1}{2} \left(\mu^2 - \frac{\lambda}{2} v^2 \right) v^2 + \frac{1}{2} (\mu^2 - 3\lambda v^2) H^2 + (\mu^2 - \lambda v^2) vH + \frac{1}{2} (\mu^2 - \lambda v^2) \chi^2 - \frac{\lambda}{4} H^4 - \frac{\lambda}{4} \chi^4 \\ &\quad - \lambda v H^3 - \frac{\lambda}{2} H^2 \chi^2 - \lambda v H \chi^2 + (\mu^2 - \lambda v^2) |\phi^+|^2 - \lambda |\phi^+|^4 - \lambda |\phi^+|^2 (H^2 + 2vH + \chi^2) \\ &= \frac{\lambda}{4} v^4 - \lambda v^2 H^2 - \frac{\lambda}{4} H^4 - \frac{\lambda}{4} \chi^4 - \lambda v H^3 - \frac{\lambda}{2} H^2 \chi^2 - \lambda v H \chi^2 \\ &\quad - \lambda \phi^+ \phi^- (\phi^+ \phi^- + H^2 + 2vH + \chi^2) \\ &= \frac{1}{8} m_H^2 v^2 - \frac{1}{2} m_H^2 H^2 - \frac{m_H^2}{2v} H^3 - \frac{m_H^2}{8v^2} H^4 - \frac{m_H^2}{2v} H \chi^2 - \frac{m_H^2}{4v^2} H^2 \chi^2 - \frac{m_H^2}{8v^2} \chi^4 \\ &\quad - \frac{m_H^2}{2v^2} \phi^+ \phi^- (\phi^+ \phi^- + H^2 + 2vH + \chi^2). \end{aligned} \quad (146)$$

由 $V = U_u^\dagger U_d$ 得到 $V^\dagger = U_d^\dagger U_u$ 和 $U_d = U_u V$, $U_u = U_d V^\dagger$, 则

$$\tilde{y}_d = U_d y_D K_d^\dagger = U_u V y_D K_d^\dagger, \quad \tilde{y}_u = U_u y_U K_u^\dagger = U_d V^\dagger y_U K_u^\dagger, \quad (147)$$

故

$$\tilde{y}_{d,ij} \bar{u}'_{iL} d'_{jR} = \bar{u}'_{iL} (U_u V y_D K_d^\dagger)_{ij} d'_{jR} = \bar{u}'_{iL} U_{u,ik} V_{kl} y_{d_l} (K_d^\dagger)_{lj} d'_{mR} = y_{d_j} \bar{u}_{iL} V_{ij} d_{jR}, \quad (148)$$

$$\tilde{y}_{u,ij} \bar{d}'_{iL} u'_{jR} = \bar{d}'_{iL} (U_d V^\dagger y_U K_u^\dagger)_{ij} u'_{jR} = \bar{d}'_{iL} U_{d,ik} V_{kl}^\dagger y_{u_l} (K_u^\dagger)_{lj} u'_{jR} = y_{u_i} \bar{d}_{jL} V_{ji}^\dagger u_{iR}. \quad (149)$$

结合 (84) 和 (85) 式, 得

$$\begin{aligned} \tilde{y}_{d,ij} \bar{Q}_{iL} d'_{jR} \Phi &= \tilde{y}_{d,ij} \left[\bar{u}'_{iL} d'_{jR} \phi^+ + \frac{1}{\sqrt{2}} \bar{d}'_{iL} d'_{jR} (v + H + i\chi) \right] \\ &= y_{d_j} \bar{u}_{iL} V_{ij} d_{jR} \phi^+ + \frac{y_{d_i}}{\sqrt{2}} \bar{d}_{iL} d_{iR} (v + H + i\chi), \end{aligned} \quad (150)$$

$$\begin{aligned} \tilde{y}_{u,ij} \bar{Q}_{iL} u'_{jR} \tilde{\Phi} &= \tilde{y}_{u,ij} \left[\frac{1}{\sqrt{2}} \bar{u}'_{iL} u'_{jR} (v + H - i\chi) - \bar{d}'_{iL} u'_{jR} \phi^- \right] \\ &= \frac{y_{u_i}}{\sqrt{2}} \bar{u}_{iL} u_{jR} (v + H - i\chi) - y_{u_i} \bar{d}_{jL} V_{ji}^\dagger u_{iR} \phi^-. \end{aligned} \quad (151)$$

从而, Yukawa 相互作用拉氏量化为

$$\begin{aligned}
\mathcal{L}_Y &= -\tilde{y}_{d,ij} \bar{Q}_{iL} d'_{jR} \Phi - \tilde{y}_{u,ij} \bar{Q}_{iL} u'_{jR} \tilde{\Phi} - y_{\ell_i} \bar{L}_{iL} \ell_{iR} \Phi + \text{H.c.} \\
&= -y_{d_j} \bar{u}_{iL} V_{ij} d_{jR} \phi^+ - \frac{y_{d_i}}{\sqrt{2}} \bar{d}_{iL} d_{iR} (v + H + i\chi) - \frac{y_{u_i}}{\sqrt{2}} \bar{u}_{iL} u_{iR} (v + H - i\chi) + y_{u_i} \bar{d}_{jL} V_{ji}^\dagger u_{iR} \phi^- \\
&\quad - y_{\ell_i} \bar{\nu}_{iL} \ell_{iR} \phi^+ - \frac{y_{\ell_i}}{\sqrt{2}} \bar{\ell}_{iL} \ell_{iR} (v + H + i\chi) + \text{H.c.} \\
&= -m_{d_i} \bar{d}_{iL} d_{iR} - m_{u_i} \bar{u}_{iL} u_{iR} - m_{\ell_i} \bar{\ell}_{iL} \ell_{iR} - \frac{m_{d_i}}{v} \bar{d}_{iL} d_{iR} (H + i\chi) - \frac{m_{u_i}}{v} \bar{u}_{iL} u_{iR} (H - i\chi) \\
&\quad - \frac{m_{\ell_i}}{v} \bar{\ell}_{iL} \ell_{iR} (H + i\chi) - \frac{\sqrt{2}m_{d_j}}{v} \bar{u}_{iL} V_{ij} d_{jR} \phi^+ + \frac{\sqrt{2}m_{u_i}}{v} \bar{d}_{jL} V_{ji}^\dagger u_{iR} \phi^- - \frac{\sqrt{2}m_{\ell_i}}{v} \bar{\nu}_{iL} \ell_{iR} \phi^+ + \text{H.c.} \\
&= -m_{d_i} \bar{d}_i d_i - m_{u_i} \bar{u}_i u_i - m_{\ell_i} \bar{\ell}_i \ell_i - \frac{m_{d_i}}{v} H \bar{d}_i d_i - \frac{m_{u_i}}{v} H \bar{u}_i u_i - \frac{m_{\ell_i}}{v} H \bar{\ell}_i \ell_i \\
&\quad - \frac{m_{d_i}}{v} \chi \bar{d}_i i\gamma^5 d_i + \frac{m_{u_i}}{v} \chi \bar{u}_i i\gamma^5 u_i - \frac{m_{\ell_i}}{v} \chi \bar{\ell}_i i\gamma^5 \ell_i + \frac{\sqrt{2}V_{ij}}{v} \phi^+ \bar{u}_i (m_{u_i} P_L - m_{d_j} P_R) d_j \\
&\quad - \frac{\sqrt{2}V_{ji}^\dagger}{v} \phi^- \bar{d}_j (m_{d_j} P_L - m_{u_i} P_R) u_i - \frac{\sqrt{2}m_{\ell_i}}{v} (\phi^+ \bar{\nu}_i P_R \ell_i + \phi^- \bar{\ell}_i P_L \nu_i). \tag{152}
\end{aligned}$$

利用

$$\begin{aligned}
D_\mu \Phi &= \begin{pmatrix} \partial_\mu - ieA_\mu - \frac{ig}{2c_W}(c_W^2 - s_W^2)Z_\mu & -\frac{ig}{\sqrt{2}}W_\mu^+ \\ -\frac{ig}{\sqrt{2}}W_\mu^- & \partial_\mu + \frac{ig}{2c_W}Z_\mu \end{pmatrix} \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\chi) \end{pmatrix} \\
&= \begin{pmatrix} \partial_\mu \phi^+ - i \left[eA_\mu + \frac{g(c_W^2 - s_W^2)}{2c_W}Z_\mu \right] \phi^+ - \frac{ig}{2}W_\mu^+(H + i\chi) - im_W W_\mu^+ \\ \frac{1}{\sqrt{2}} \left[\partial_\mu (H + i\chi) - igW_\mu^- \phi^+ + \frac{ig}{2c_W}Z_\mu (H + i\chi) + im_Z Z_\mu \right] \end{pmatrix}, \tag{153}
\end{aligned}$$

将 Higgs 场协变动能项化为

$$\begin{aligned}
&(D^\mu \Phi)^\dagger D_\mu \Phi \\
&= \left| \partial_\mu \phi^+ - i \left[eA_\mu + \frac{g(c_W^2 - s_W^2)}{2c_W}Z_\mu \right] \phi^+ - \frac{ig}{2}W_\mu^+(H + i\chi) - im_W W_\mu^+ \right|^2 \\
&\quad + \frac{1}{2} \left| \partial_\mu (H + i\chi) - igW_\mu^- \phi^+ + \frac{ig}{2c_W}Z_\mu (H + i\chi) + im_Z Z_\mu \right|^2 \\
&= (\partial^\mu \phi^+) (\partial_\mu \phi^-) + \frac{1}{2} (\partial^\mu H) (\partial_\mu H) + \frac{1}{2} (\partial^\mu \chi) (\partial_\mu \chi) \\
&\quad + \left(-i\partial^\mu \phi^- \left\{ \left[eA_\mu + \frac{g(c_W^2 - s_W^2)}{2c_W}Z_\mu \right] \phi^+ + \frac{g}{2}W_\mu^+(H + i\chi) + m_W W_\mu^+ \right\} + \text{H.c.} \right) \\
&\quad + \left\{ -\frac{i}{2} \partial^\mu (H - i\chi) \left[gW_\mu^- \phi^+ - \frac{g}{2c_W}Z_\mu (H + i\chi) - m_Z Z_\mu \right] + \text{H.c.} \right\} \\
&\quad + \left| \left[eA_\mu + \frac{g(c_W^2 - s_W^2)}{2c_W}Z_\mu \right] \phi^+ + \frac{g}{2}W_\mu^+(H + i\chi) + m_W W_\mu^+ \right|^2
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2} \left| gW_\mu^- \phi^+ - \frac{g}{2c_W} Z_\mu (H + i\chi) - m_Z Z_\mu \right|^2 \\
& = (\partial^\mu \phi^+) (\partial_\mu \phi^-) + \frac{1}{2} (\partial^\mu H) (\partial_\mu H) + \frac{1}{2} (\partial^\mu \chi) (\partial_\mu \chi) \\
& + m_W^2 W_\mu^- W_\mu^+ + \frac{1}{2} m_Z^2 Z_\mu Z_\mu + g m_W H W_\mu^+ W_\mu^- + \frac{g m_Z}{2c_W} H Z_\mu Z_\mu \\
& + \frac{g}{2} [iW_\mu^+ \phi^- \overleftrightarrow{\partial}^\mu (H + i\chi) + \text{H.c.}] + ie A_\mu \phi^- \overleftrightarrow{\partial}^\mu \phi^+ + \frac{g}{2c_W} Z_\mu [-\chi \overleftrightarrow{\partial}^\mu H + i(c_W^2 - s_W^2) \phi^- \overleftrightarrow{\partial}^\mu \phi^+] \\
& + \frac{g^2}{4} W_\mu^+ W_\mu^- (2\phi^+ \phi^- + H^2 + \chi^2) + e^2 A_\mu A^\mu \phi^+ \phi^- \\
& + \frac{g^2}{4c_W^2} Z_\mu Z^\mu \left[(c_W^2 - s_W^2)^2 \phi^+ \phi^- + \frac{1}{2} H^2 + \frac{1}{2} \chi^2 \right] \\
& + \left[\frac{eg}{2} W_\mu^+ A^\mu \phi^- (H + i\chi) - \frac{g^2 s_W^2}{2c_W} W_\mu^+ Z^\mu \phi^- (H + i\chi) + \text{H.c.} \right] + \frac{eg(c_W^2 - s_W^2)}{c_W} A_\mu Z^\mu \phi^+ \phi^- \\
& + (em_W A^\mu \phi^+ W_\mu^- - gs_W^2 m_Z Z^\mu \phi^+ W_\mu^- + \text{H.c.}) + \mathcal{L}_{b1}, \tag{154}
\end{aligned}$$

其中

$$\mathcal{L}_{b1} = -im_W (\partial^\mu \phi^-) W_\mu^+ + im_W (\partial^\mu \phi^+) W_\mu^- + m_Z (\partial^\mu \chi) Z_\mu. \tag{155}$$

在 R_ξ 规范下, 将规范固定函数设为

$$G^\pm = \frac{1}{\sqrt{\xi_W}} (\partial^\mu W_\mu^\pm \mp i\xi_W m_W \phi^\pm), \quad G^Z = \frac{1}{\sqrt{\xi_Z}} (\partial^\mu Z_\mu - \xi_Z m_Z \chi), \quad G^\gamma = \frac{1}{\sqrt{\xi_\gamma}} \partial^\mu A_\mu, \tag{156}$$

它们在路径积分量子化中的泛函积分形式为

$$\begin{aligned}
& \int \mathcal{D}\omega^+ \int \mathcal{D}\omega^- \int \mathcal{D}\omega^Z \int \mathcal{D}\omega^\gamma \exp \left[-i \int d^4x \left(\omega^+ \omega^- + \frac{1}{2} \omega^Z \omega^Z + \frac{1}{2} \omega^\gamma \omega^\gamma \right) \right] \\
& \quad \times \delta(G^+ - \omega^+) \delta(G^- - \omega^-) \delta(G^Z - \omega^Z) \delta(G^\gamma - \omega^\gamma) \\
& = \exp \left[-i \int d^4x \left(G^+ G^- + \frac{1}{2} G^Z G^Z + \frac{1}{2} G^\gamma G^\gamma \right) \right]. \tag{157}
\end{aligned}$$

由此得到拉氏量中的规范固定项

$$\begin{aligned}
\mathcal{L}_{\text{EW,GF}} & = -G^+ G^- - \frac{1}{2} (G^Z)^2 - \frac{1}{2} (G^\gamma)^2 \\
& = -\frac{1}{\xi_W} (\partial^\mu W_\mu^+ - i\xi_W m_W \phi^+) (\partial^\nu W_\nu^- + i\xi_W m_W \phi^-) - \frac{1}{2\xi_Z} (\partial^\mu Z_\mu - \xi_Z m_Z \chi)^2 - \frac{1}{2\xi_\gamma} (\partial^\mu A_\mu)^2 \\
& = -\frac{1}{\xi_W} (\partial^\mu W_\mu^+) (\partial^\nu W_\nu^-) - \frac{1}{2\xi_Z} (\partial^\mu Z_\mu)^2 - \frac{1}{2\xi_\gamma} (\partial^\mu A_\mu)^2 - \xi_W m_W^2 \phi^+ \phi^- - \frac{1}{2} \xi_Z m_Z^2 \chi^2 + \mathcal{L}_{b2}. \tag{158}
\end{aligned}$$

可见, Nambu-Goldstone 玻色子 ϕ^\pm 和 χ 在 R_ξ 规范下具有依赖于 ξ_W 和 ξ_Z 的非物理质量,

$$m_\phi = \sqrt{\xi_W} m_W, \quad m_\chi = \sqrt{\xi_Z} m_Z. \tag{159}$$

这里

$$\mathcal{L}_{b2} = -im_W \phi^- (\partial^\mu W_\mu^+) + im_W \phi^+ \partial^\mu W_\mu^- + m_Z \chi \partial^\mu Z_\mu. \quad (160)$$

由于

$$\mathcal{L}_{b1} + \mathcal{L}_{b2} = -im_W \partial^\mu (\phi^- W_\mu^+) + im_W \partial^\mu (\phi^+ W_\mu^-) + m_Z \partial^\mu (\chi Z_\mu), \quad (161)$$

这两项体现为全散度，不会有物理效应。可见，协变动能项中规范场与 Nambu-Goldstone 标量场之间的双线性耦合项 \mathcal{L}_{b1} 被规范固定项中的 \mathcal{L}_{b2} 抵消掉，这就是如此选取规范固定函数的目的。

这样一来，电弱规范场传播子相关拉氏量变成

$$\begin{aligned} \mathcal{L}_{EW,prop} &= (\partial_\mu W_\nu^+) (\partial^\nu W^{-\mu}) - (\partial_\mu W_\nu^+) (\partial^\mu W^{-\nu}) - \frac{1}{\xi_W} (\partial^\mu W_\mu^+) (\partial^\nu W_\nu^-) + m_W^2 W^{-\mu} W_\mu^+ \\ &\quad + \frac{1}{2} \left[(\partial_\mu Z_\nu) (\partial^\nu Z^\mu) - (\partial_\mu Z_\nu) (\partial^\mu Z^\nu) - \frac{1}{\xi_Z} (\partial^\mu Z_\mu)^2 + m_Z^2 Z^\mu Z_\mu \right] \\ &\quad + \frac{1}{2} \left[(\partial_\mu A_\nu) (\partial^\nu A^\mu) - (\partial_\mu A_\nu) (\partial^\mu A^\nu) - \frac{1}{\xi_\gamma} (\partial^\mu A_\mu)^2 \right] \\ &\rightarrow W_\mu^+ \left[g^{\mu\nu} (\partial^2 + m_W^2) - \left(1 - \frac{1}{\xi_W} \right) \partial^\mu \partial^\nu \right] W_\nu^- \\ &\quad + \frac{1}{2} Z_\mu \left[g^{\mu\nu} (\partial^2 + m_Z^2) - \left(1 - \frac{1}{\xi_Z} \right) \partial^\mu \partial^\nu \right] Z_\nu \\ &\quad + \frac{1}{2} A_\mu \left[g^{\mu\nu} \partial^2 - \left(1 - \frac{1}{\xi_\gamma} \right) \partial^\mu \partial^\nu \right] A_\nu. \end{aligned} \quad (162)$$

于是，光子的传播子与胶子形式类似，为

$$\frac{-i}{p^2 + i\epsilon} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} (1 - \xi_\gamma) \right]. \quad (163)$$

将 W^\pm 传播子相关拉氏量变换到动量空间，得

$$-g^{\mu\nu} (p^2 - m_W^2) + \left(1 - \frac{1}{\xi_W} \right) p^\mu p^\nu = - \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) (p^2 - m_W^2) - \frac{p^\mu p^\nu}{p^2} \frac{p^2 - \xi_W m_W^2}{\xi_W}, \quad (164)$$

它的逆矩阵是

$$-\frac{1}{p^2 - m_W^2} \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) - \frac{\xi_W}{p^2 - \xi_W m_W^2} \frac{p_\mu p_\nu}{p^2} = -\frac{1}{p^2 - m_W^2} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_W m_W^2} (1 - \xi_W) \right], \quad (165)$$

这是因为由

$$\left(g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2} \right) \frac{p^\mu p^\nu}{p^2} = \frac{p_\rho p^\nu}{p^2} - \frac{p_\rho p^\nu}{p^2} = 0, \quad \left(g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2} \right) \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) = \delta_\rho^\nu - \frac{p_\rho p^\nu}{p^2} \quad (166)$$

得

$$\begin{aligned} & \left[-\frac{1}{p^2 - m_W^2} \left(g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2} \right) - \frac{\xi_W}{p^2 - \xi_W m_W^2} \frac{p_\rho p_\mu}{p^2} \right] \\ & \times \left[-\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) (p^2 - m_W^2) - \frac{p^\mu p^\nu}{p^2} \frac{p^2 - \xi_W m_W^2}{\xi_W} \right] \\ & = \delta_\rho^\nu - \frac{p_\rho p^\nu}{p^2} + \frac{p_\rho p^\nu}{p^2} = \delta_\rho^\nu. \end{aligned} \quad (167)$$

从而, W^\pm 传播子的形式为

$$\frac{-i}{p^2 - m_W^2 + i\epsilon} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_W m_W^2} (1 - \xi_W) \right]. \quad (168)$$

同理, Z 传播子的形式为

$$\frac{-i}{p^2 - m_Z^2 + i\epsilon} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_Z m_Z^2} (1 - \xi_Z) \right]. \quad (169)$$

电弱规范场的无穷小规范变换形式是

$$\delta W_\mu^a = \frac{1}{g} \partial_\mu \alpha^a + \varepsilon^{abc} W_\mu^b \alpha^c, \quad \delta B_\mu = \frac{1}{g'} \partial_\mu \alpha^Y. \quad (170)$$

定义

$$\alpha^\pm \equiv \frac{1}{\sqrt{2}}(\alpha^1 \mp i\alpha^2), \quad \alpha^Z \equiv \alpha^3 - \alpha^Y, \quad \alpha^\gamma \equiv s_W^2 \alpha^3 + c_W^2 \alpha^Y, \quad (171)$$

利用

$$\varepsilon^{1bc} W_\mu^b \alpha^c = W_\mu^2 \alpha^3 - W_\mu^3 \alpha^2, \quad \varepsilon^{2bc} W_\mu^b \alpha^c = -W_\mu^1 \alpha^3 + W_\mu^3 \alpha^1, \quad (172)$$

$$\pm i\sqrt{2}\alpha^\pm = \pm i\alpha^1 + \alpha^2, \quad \pm i\sqrt{2}W_\mu^\pm = \pm iW_\mu^1 + W_\mu^2, \quad (173)$$

有

$$\begin{aligned} \varepsilon^{1bc} W_\mu^b \alpha^c \mp i\varepsilon^{2bc} W_\mu^b \alpha^c &= (W_\mu^2 \alpha^3 - W_\mu^3 \alpha^2) \mp i(-W_\mu^1 \alpha^3 + W_\mu^3 \alpha^1) \\ &= (W_\mu^2 \pm iW_\mu^1) \alpha^3 - W_\mu^3 (\alpha^2 \pm i\alpha^1) \\ &= \pm i\sqrt{2}W_\mu^\pm (c_W^2 \alpha^Z + \alpha^\gamma) \mp i\sqrt{2}(s_W A_\mu + c_W Z_\mu) \alpha^\pm, \end{aligned} \quad (174)$$

$$\begin{aligned} \varepsilon^{3bc} W_\mu^b \alpha^c &= W_\mu^1 \alpha^2 - W_\mu^2 \alpha^1 = \frac{1}{\sqrt{2}}(W_\mu^+ + W_\mu^-) \frac{i}{\sqrt{2}}(\alpha^+ - \alpha^-) - \frac{i}{\sqrt{2}}(W_\mu^+ - W_\mu^-) \frac{1}{\sqrt{2}}(\alpha^+ + \alpha^-) \\ &= -i(W_\mu^+ \alpha^- - W_\mu^- \alpha^+). \end{aligned} \quad (175)$$

因此,

$$\begin{aligned}\delta W_\mu^+ &= \frac{1}{\sqrt{2}}(\delta W_\mu^1 - i\delta W_\mu^2) = \frac{1}{\sqrt{2}g} \partial_\mu(\alpha^1 - i\alpha^2) + \frac{1}{\sqrt{2}}(\varepsilon^{1bc}W_\mu^b\alpha^c - i\varepsilon^{2bc}W_\mu^b\alpha^c) \\ &= \frac{1}{g} \partial_\mu\alpha^+ - i(s_W A_\mu + c_W Z_\mu)\alpha^+ + iW_\mu^+(c_W^2\alpha^Z + \alpha^\gamma),\end{aligned}\quad (176)$$

$$\delta W_\mu^- = (\delta W_\mu^+)^{\dagger} = \frac{1}{g} \partial_\mu\alpha^- + i(s_W A_\mu + c_W Z_\mu)\alpha^- - iW_\mu^-(c_W^2\alpha^Z + \alpha^\gamma),\quad (177)$$

$$\begin{aligned}\delta Z_\mu^a &= c_W\delta W_\mu^3 - s_W\delta B_\mu = \frac{c_W}{g} \partial_\mu\alpha^3 + c_W\varepsilon^{3bc}W_\mu^b\alpha^c - \frac{s_W}{g'} \partial_\mu\alpha^Y \\ &= \frac{c_W}{g} \partial_\mu\alpha^Z - ic_W(W_\mu^+\alpha^- - W_\mu^-\alpha^+),\end{aligned}\quad (178)$$

$$\begin{aligned}\delta A_\mu &= s_W\delta W_\mu^3 + c_W\delta B_\mu = \frac{s_W}{g} \partial_\mu\alpha^3 + s_W\varepsilon^{3bc}W_\mu^b\alpha^c + \frac{c_W}{g'} \partial_\mu\alpha^Y \\ &= \frac{1}{e} \partial_\mu\alpha^\gamma - is_W(W_\mu^+\alpha^- - W_\mu^-\alpha^+).\end{aligned}\quad (179)$$

另一方面, 根据

$$\begin{aligned}\alpha^a T^a + \alpha^Y Y_H &= \frac{1}{2}(\alpha^a \sigma^a + \alpha^Y) = \frac{1}{2} \begin{pmatrix} \alpha^3 + \alpha^Y & \alpha^1 - i\alpha^2 \\ \alpha^1 + i\alpha^2 & -\alpha^3 + \alpha^Y \end{pmatrix} \\ &= \frac{1}{2} \begin{pmatrix} 2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z & \sqrt{2}\alpha^+ \\ \sqrt{2}\alpha^- & -\alpha^Z \end{pmatrix},\end{aligned}\quad (180)$$

可知 Higgs 场的无穷小规范变换形式为

$$\begin{aligned}\delta\Phi &= i(\alpha^a T^a + \alpha^Y Y_H)\Phi = \frac{i}{2} \begin{pmatrix} 2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z & \sqrt{2}\alpha^+ \\ \sqrt{2}\alpha^- & -\alpha^Z \end{pmatrix} \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\chi) \end{pmatrix} \\ &= \begin{pmatrix} \frac{i}{2}\{\phi^+[2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z] + (v + H + i\chi)\alpha^+\} \\ \frac{1}{\sqrt{2}}\left[i\phi^+\alpha^- - \frac{1}{2}(iv + iH - \chi)\alpha^Z\right] \end{pmatrix} = \begin{pmatrix} \delta\phi^+ \\ \frac{1}{\sqrt{2}}(\delta H + i\delta\chi) \end{pmatrix}.\end{aligned}\quad (181)$$

利用

$$\text{Re}(\phi^+\alpha^-) = \frac{1}{2}(\phi^+\alpha^- + \phi^-\alpha^+), \quad \text{Im}(\phi^+\alpha^-) = -\frac{i}{2}(\phi^+\alpha^- - \phi^-\alpha^+),\quad (182)$$

推出

$$\delta\phi^+ = \frac{i}{2}\{\phi^+[2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z] + (v + H + i\chi)\alpha^+\},\quad (183)$$

$$\delta\phi^- = -\frac{i}{2}\{\phi^-[2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z] + (v + H - i\chi)\alpha^-\},\quad (184)$$

$$\delta H = \frac{1}{2}[i(\phi^+\alpha^- - \phi^-\alpha^+) + \chi\alpha^Z], \quad \delta\chi = \frac{1}{2}[\phi^+\alpha^- + \phi^-\alpha^+ - (v + H)\alpha^Z].\quad (185)$$

于是, 规范固定函数的无穷小规范变换为

$$\begin{aligned}\sqrt{\xi_W} \delta G^+ &= \partial^\mu \delta W_\mu^+ - i\xi_W m_W \delta \phi^+ \\ &= \partial^\mu \left[\frac{1}{g} \partial_\mu \alpha^+ - i(s_W A_\mu + c_W Z_\mu) \alpha^+ + iW_\mu^+ (c_W^2 \alpha^Z + \alpha^\gamma) \right] \\ &\quad + \frac{1}{2} \xi_W m_W \{ \phi^+ [2\alpha^\gamma + (c_W^2 - s_W^2) \alpha^Z] + (v + H + i\chi) \alpha^+ \},\end{aligned}\quad (186)$$

$$\begin{aligned}\sqrt{\xi_W} \delta G^- &= \partial^\mu \delta W_\mu^- + i\xi_W m_W \delta \phi^- \\ &= \partial^\mu \left[\frac{1}{g} \partial_\mu \alpha^- + i(s_W A_\mu + c_W Z_\mu) \alpha^- - iW_\mu^- (c_W^2 \alpha^Z + \alpha^\gamma) \right] \\ &\quad + \frac{1}{2} \xi_W m_W \{ \phi^- [2\alpha^\gamma + (c_W^2 - s_W^2) \alpha^Z] + (v + H - i\chi) \alpha^- \},\end{aligned}\quad (187)$$

$$\begin{aligned}\sqrt{\xi_Z} \delta G^Z &= \partial^\mu \delta Z_\mu - \xi_Z m_Z \delta \chi \\ &= \partial^\mu \left[\frac{c_W}{g} \partial_\mu \alpha^Z - i c_W (W_\mu^+ \alpha^- - W_\mu^- \alpha^+) \right] \\ &\quad - \frac{1}{2} \xi_Z m_Z [\phi^+ \alpha^- + \phi^- \alpha^+ - (v + H) \alpha^Z],\end{aligned}\quad (188)$$

$$\sqrt{\xi_\gamma} \delta G^\gamma = \partial^\mu \delta A_\mu = \partial^\mu \left[\frac{1}{e} \partial_\mu \alpha^\gamma - i s_W (W_\mu^+ \alpha^- - W_\mu^- \alpha^+) \right]. \quad (189)$$

因此,

$$\sqrt{\xi_W} g \frac{\delta G^+}{\delta \alpha^+} = \partial^2 + \xi_W m_W^2 - i e \partial^\mu A_\mu - i g c_W \partial^\mu Z_\mu + \frac{1}{2} g \xi_W m_W (H + i\chi), \quad (190)$$

$$\frac{\sqrt{\xi_W} g}{c_W} \frac{\delta G^+}{\delta \alpha^Z} = i g c_W \partial^\mu W_\mu^+ + \frac{g(c_W^2 - s_W^2) \xi_W m_W}{2 c_W} \phi^+, \quad (191)$$

$$\sqrt{\xi_W} e \frac{\delta G^+}{\delta \alpha^\gamma} = i e \partial^\mu W_\mu^+ + e \xi_W m_W \phi^+, \quad (192)$$

$$\sqrt{\xi_W} g \frac{\delta G^-}{\delta \alpha^-} = \partial^2 + \xi_W m_W^2 + i e \partial^\mu A_\mu + i g c_W \partial^\mu Z_\mu + \frac{1}{2} g \xi_W m_W (H - i\chi), \quad (193)$$

$$\frac{\sqrt{\xi_W} g}{c_W} \frac{\delta G^-}{\delta \alpha^Z} = -i g c_W \partial^\mu W_\mu^- + \frac{g(c_W^2 - s_W^2) \xi_W m_W}{2 c_W} \phi^-, \quad (194)$$

$$\sqrt{\xi_W} e \frac{\delta G^-}{\delta \alpha^\gamma} = -i e \partial^\mu W_\mu^- + e \xi_W m_W \phi^-, \quad (195)$$

$$\sqrt{\xi_Z} g \frac{\delta G^Z}{\delta \alpha^+} = i g c_W \partial^\mu W_\mu^- - \frac{1}{2} g \xi_Z m_Z \phi^-, \quad (196)$$

$$\sqrt{\xi_Z} g \frac{\delta G^Z}{\delta \alpha^-} = -i g c_W \partial^\mu W_\mu^+ - \frac{1}{2} g \xi_Z m_Z \phi^+, \quad (197)$$

$$\frac{\sqrt{\xi_Z} g}{c_W} \frac{\delta G^Z}{\delta \alpha^Z} = \partial^2 + \xi_Z m_Z^2 + \frac{g \xi_Z m_Z}{2 c_W} H, \quad (198)$$

$$\sqrt{\xi_\gamma} g \frac{\delta G^\gamma}{\delta \alpha^+} = i e \partial^\mu W_\mu^-, \quad \sqrt{\xi_\gamma} g \frac{\delta G^\gamma}{\delta \alpha^-} = -i e \partial^\mu W_\mu^+, \quad \sqrt{\xi_\gamma} e \frac{\delta G^\gamma}{\delta \alpha^\gamma} = \partial^2. \quad (199)$$

最后, 得到以下 Faddeev-Popov 鬼场拉氏量,

$$\begin{aligned}
\mathcal{L}_{\text{EWG,FP}} = & -\bar{\eta}^+ \left(\sqrt{\xi_W} g \frac{\delta G^+}{\delta \alpha^+} \right) \eta^+ - \bar{\eta}^Z \left(\sqrt{\xi_Z} g \frac{\delta G^Z}{\delta \alpha^+} \right) \eta^+ - \bar{\eta}^\gamma \left(\sqrt{\xi_\gamma} g \frac{\delta G^\gamma}{\delta \alpha^+} \right) \eta^+ \\
& -\bar{\eta}^- \left(\sqrt{\xi_W} g \frac{\delta G^-}{\delta \alpha^-} \right) \eta^- - \bar{\eta}^Z \left(\sqrt{\xi_Z} g \frac{\delta G^Z}{\delta \alpha^-} \right) \eta^- - \bar{\eta}^\gamma \left(\sqrt{\xi_\gamma} g \frac{\delta G^\gamma}{\delta \alpha^-} \right) \eta^- \\
& -\bar{\eta}^Z \left(\frac{\sqrt{\xi_Z} g}{c_W} \frac{\delta G^Z}{\delta \alpha^Z} \right) \eta^Z - \bar{\eta}^+ \left(\frac{\sqrt{\xi_W} g}{c_W} \frac{\delta G^+}{\delta \alpha^Z} \right) \eta^Z - \bar{\eta}^- \left(\frac{\sqrt{\xi_W} g}{c_W} \frac{\delta G^-}{\delta \alpha^Z} \right) \eta^Z \\
& -\bar{\eta}^\gamma \left(\sqrt{\xi_\gamma} e \frac{\delta G^\gamma}{\delta \alpha^\gamma} \right) \eta^\gamma - \bar{\eta}^+ \left(\sqrt{\xi_W} e \frac{\delta G^+}{\delta \alpha^\gamma} \right) \eta^\gamma - \bar{\eta}^- \left(\sqrt{\xi_W} e \frac{\delta G^-}{\delta \alpha^\gamma} \right) \eta^\gamma \\
\rightarrow & \bar{\eta}^+ \left[-\partial^2 - \xi_W m_W^2 - ie \overleftarrow{\partial}^\mu A_\mu - ig c_W \overleftarrow{\partial}^\mu Z_\mu - \frac{1}{2} g \xi_W m_W (H + i\chi) \right] \eta^+ \\
& + \bar{\eta}^Z \left(ig c_W \overleftarrow{\partial}^\mu W_\mu^- + \frac{1}{2} g \xi_Z m_Z \phi^- \right) \eta^+ + ie (\partial^\mu \bar{\eta}^\gamma) W_\mu^- \eta^+ \\
& + \bar{\eta}^- \left[-\partial^2 - \xi_W m_W^2 + ie \overleftarrow{\partial}^\mu A_\mu + ig c_W \overleftarrow{\partial}^\mu Z_\mu - \frac{1}{2} g \xi_W m_W (H - i\chi) \right] \eta^- \\
& + \bar{\eta}^Z \left(-ig c_W \overleftarrow{\partial}^\mu W_\mu^+ + \frac{1}{2} g \xi_Z m_Z \phi^+ \right) \eta^- - ie (\partial^\mu \bar{\eta}^\gamma) W_\mu^+ \eta^- \\
& + \bar{\eta}^Z \left(-\partial^2 - \xi_Z m_Z^2 - \frac{g \xi_Z m_Z}{2 c_W} H \right) \eta^Z \\
& + \bar{\eta}^+ \left(ig c_W \overleftarrow{\partial}^\mu W_\mu^+ - \frac{g(c_W^2 - s_W^2) \xi_W m_W}{2 c_W} \phi^+ \right) \eta^Z \\
& + \bar{\eta}^- \left(-ig c_W \overleftarrow{\partial}^\mu W_\mu^- - \frac{g(c_W^2 - s_W^2) \xi_W m_W}{2 c_W} \phi^- \right) \eta^Z \\
& - \bar{\eta}^\gamma \partial^2 \eta^\gamma + \bar{\eta}^+ (ie \overleftarrow{\partial}^\mu W_\mu^+ - e \xi_W m_W \phi^+) \eta^\gamma + \bar{\eta}^- (-ie \overleftarrow{\partial}^\mu W_\mu^- - e \xi_W m_W \phi^-) \eta^\gamma. \quad (200)
\end{aligned}$$

鬼粒子的质量为

$$m_{\eta^+} = m_{\eta^-} = \sqrt{\xi_W} m_W, \quad m_{\eta^Z} = \sqrt{\xi_Z} m_Z, \quad m_{\eta^\gamma} = 0. \quad (201)$$

下面给出 R_ξ 规范下的费曼规则。 $\xi_i = 1$ 对应 Feynman-'t Hooft 规范, $\xi_i = 0$ 对应 Landau 规范, $\xi_W, \xi_Z \rightarrow \infty$ 对应幺正规范。在树图计算中, 常取 $\xi_\gamma = 1$ 和 $\xi_W, \xi_Z \rightarrow \infty$ 。在圈图计算中, 常取 $\xi_\gamma = \xi_W = \xi_Z = 1$ 。

传播子:

$$\bullet \text{---} \frac{\overrightarrow{p}}{H} \text{---} \bullet = \frac{i}{p^2 - m_H^2 + i\epsilon} \quad (202)$$

$$\bullet \text{---} \frac{\overrightarrow{p}}{\chi} \text{---} \bullet = \frac{i}{p^2 - \xi_Z m_Z^2 + i\epsilon} \quad (203)$$

$$\bullet \text{---} \frac{\overrightarrow{p}}{\phi} \text{---} \bullet = \frac{i}{p^2 - \xi_W m_W^2 + i\epsilon} \quad (204)$$

$$\nu \text{---}\gamma\text{---}\mu = \frac{-i}{p^2 + i\epsilon} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} (1 - \xi_\gamma) \right] \quad (205)$$

$$\nu \text{---}Z\text{---}\mu = \frac{-i}{p^2 - m_Z^2 + i\epsilon} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_Z m_Z^2} (1 - \xi_Z) \right] \quad (206)$$

$$\nu \text{---}W\text{---}\mu = \frac{-i}{p^2 - m_W^2 + i\epsilon} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_W m_W^2} (1 - \xi_W) \right] \quad (207)$$

$$\text{---}\eta^\gamma\text{---} = \frac{i}{p^2 + i\epsilon} \quad (208)$$

$$\text{---}\eta^Z\text{---} = \frac{i}{p^2 - \xi_Z m_Z^2 + i\epsilon} \quad (209)$$

$$\text{---}\eta^\pm\text{---} = \frac{i}{p^2 - \xi_W m_W^2 + i\epsilon} \quad (210)$$

标量玻色子三线性耦合:

$$\begin{aligned} \begin{array}{c} H \\ | \\ \bullet \\ / \backslash \\ H \quad H \end{array} &= -\frac{3im_H^2}{v} \\ &= -6i\lambda v \end{aligned} \quad \begin{array}{c} H \\ | \\ \bullet \\ / \backslash \\ \chi \quad \chi \end{array} = -\frac{im_H^2}{v} = -2i\lambda v \quad \begin{array}{c} H \\ | \\ \bullet \\ / \backslash \\ \phi \quad \phi \end{array} = -\frac{im_H^2}{v} = -2i\lambda v \quad (211)$$

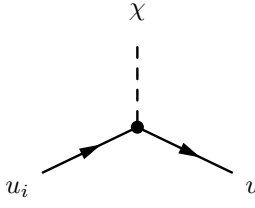
标量玻色子四线性耦合:

$$\begin{aligned} \begin{array}{c} H \quad H \\ \backslash \quad / \\ \bullet \\ / \quad \backslash \\ H \quad H \end{array} &= -\frac{3im_H^2}{v^2} \\ &= -6i\lambda \end{aligned} \quad \begin{array}{c} H \quad H \\ \backslash \quad / \\ \bullet \\ / \quad \backslash \\ \chi \quad \chi \end{array} = -\frac{im_H^2}{v^2} = -2i\lambda \quad \begin{array}{c} \chi \quad \chi \\ \backslash \quad / \\ \bullet \\ / \quad \backslash \\ \chi \quad \chi \end{array} = -\frac{3im_H^2}{v^2} = -6i\lambda \quad (212)$$

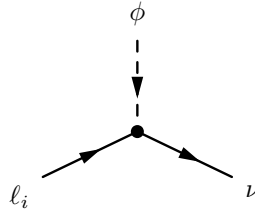
$$\begin{aligned} \begin{array}{c} H \quad H \\ \backslash \quad / \\ \bullet \\ / \quad \backslash \\ \phi \quad \phi \end{array} &= -\frac{im_H^2}{v^2} \\ &= -2i\lambda \end{aligned} \quad \begin{array}{c} \chi \quad \chi \\ \backslash \quad / \\ \bullet \\ / \quad \backslash \\ \phi \quad \phi \end{array} = -\frac{im_H^2}{v^2} = -2i\lambda \quad \begin{array}{c} \phi \quad \phi \\ \backslash \quad / \\ \bullet \\ / \quad \backslash \\ \phi \quad \phi \end{array} = -\frac{2im_H^2}{v^2} = -4i\lambda \quad (213)$$

Yukawa 耦合:

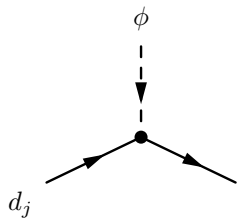
$$\begin{aligned} \begin{array}{c} H \\ | \\ \bullet \\ / \backslash \\ f \quad f \end{array} &= -\frac{im_f}{v} \\ &= -\frac{iy_f}{\sqrt{2}} \end{aligned} \quad \begin{array}{c} \chi \\ | \\ \bullet \\ / \backslash \\ \ell_i \quad \ell_i \end{array} = \frac{m_{\ell_i}}{v} \gamma^5 = \frac{y_{\ell_i}}{\sqrt{2}} \gamma^5 \quad (214)$$



$$\begin{aligned}
 &= -\frac{m_{u_i}}{v} \gamma^5 \\
 &= -\frac{y_{u_i}}{\sqrt{2}} \gamma^5
 \end{aligned}
 \quad
 \begin{aligned}
 &= \frac{m_{d_i}}{v} \gamma^5 \\
 &= \frac{y_{d_i}}{\sqrt{2}} \gamma^5
 \end{aligned}
 \quad (215)$$

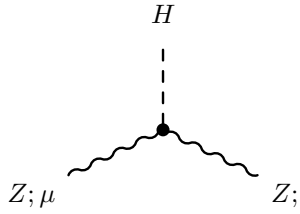


$$\begin{aligned}
 &= -\frac{i\sqrt{2}m_{\ell_i}}{v} P_R \\
 &= -iy_{\ell_i} P_R
 \end{aligned}
 \quad
 \begin{aligned}
 &= -\frac{i\sqrt{2}m_{\ell_i}}{v} P_L \\
 &= -iy_{\ell_i} P_L
 \end{aligned}
 \quad (216)$$

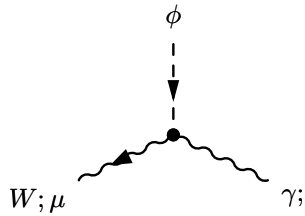


$$\begin{aligned}
 &= \frac{i\sqrt{2}V_{ij}}{v} (m_{u_i} P_L - m_{d_j} P_R) \\
 &= iV_{ij} (y_{u_i} P_L - y_{d_j} P_R)
 \end{aligned}
 \quad
 \begin{aligned}
 &= -\frac{i\sqrt{2}V_{ji}^\dagger}{v} (m_{d_j} P_L - m_{u_i} P_R) \\
 &= -iV_{ji}^\dagger (y_{d_j} P_L - y_{u_i} P_R)
 \end{aligned}
 \quad (217)$$

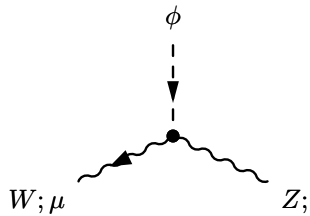
标量玻色子与电弱规范玻色子的三线耦合:



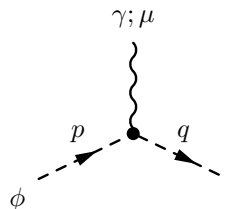
$$\begin{aligned}
 &= \frac{igm_Z}{c_W} g^{\mu\nu} \\
 &= \frac{ig^2 v}{2c_W^2} g^{\mu\nu}
 \end{aligned}
 \quad
 \begin{aligned}
 &= igm_W g^{\mu\nu} \\
 &= \frac{ig^2 v}{2} g^{\mu\nu}
 \end{aligned}
 \quad (218)$$



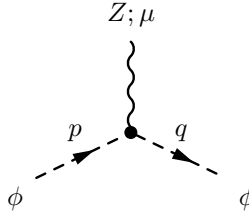
$$\begin{aligned}
 &= iem_W g^{\mu\nu} \\
 &= \frac{ieg v}{2} g^{\mu\nu}
 \end{aligned}
 \quad
 \begin{aligned}
 &= iem_W g^{\mu\nu} \\
 &= \frac{ieg v}{2} g^{\mu\nu}
 \end{aligned}
 \quad (219)$$

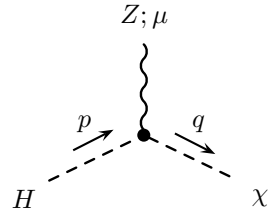


$$\begin{aligned}
 &= -igs_W^2 m_Z g^{\mu\nu} \\
 &= -\frac{ig^2 s_W^2 v}{2c_W} g^{\mu\nu}
 \end{aligned}
 \quad
 \begin{aligned}
 &= -igs_W^2 m_Z g^{\mu\nu} \\
 &= -\frac{ig^2 s_W^2 v}{2c_W} g^{\mu\nu}
 \end{aligned}
 \quad (220)$$

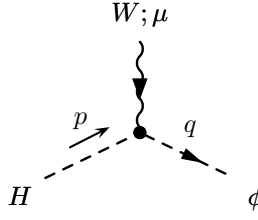


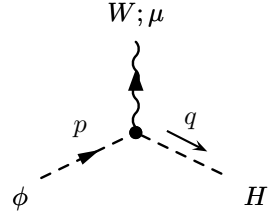
$$= ie(p + q)^\mu \quad (221)$$



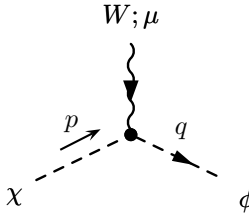
$$= \frac{ig(c_W^2 - s_W^2)}{2c_W} (p+q)^\mu$$


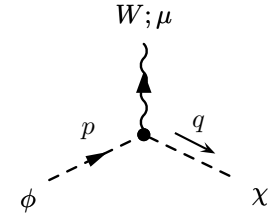
$$= -\frac{g}{2c_W} (p+q)^\mu \quad (222)$$



$$= \frac{ig}{2} (p+q)^\mu$$


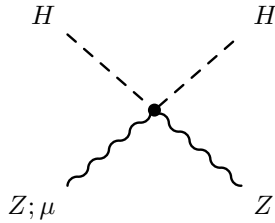
$$= \frac{ig}{2} (p+q)^\mu \quad (223)$$

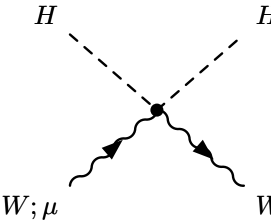


$$= -\frac{g}{2} (p+q)^\mu$$


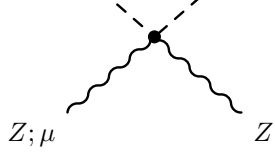
$$= \frac{g}{2} (p+q)^\mu \quad (224)$$

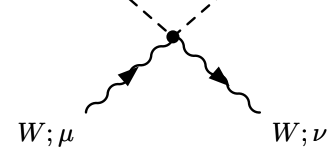
标量玻色子与电弱规范玻色子的四线性耦合:



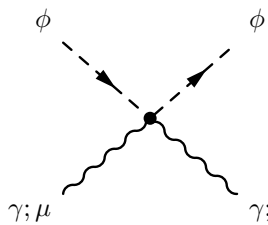
$$= \frac{ig^2}{2c_W^2} g^{\mu\nu}$$


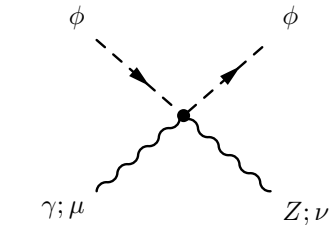
$$= \frac{ig^2}{2} g^{\mu\nu} \quad (225)$$



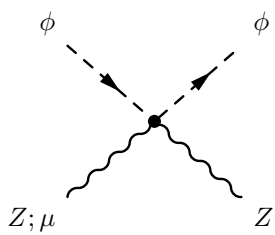
$$= \frac{ig^2}{2c_W^2} g^{\mu\nu}$$


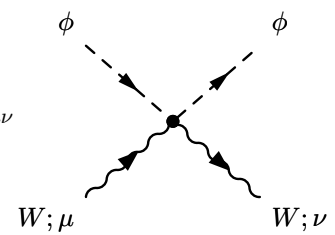
$$= \frac{ig^2}{2} g^{\mu\nu} \quad (226)$$



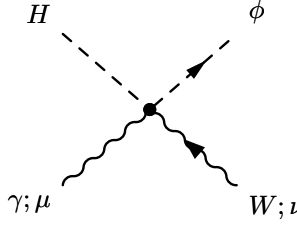
$$= 2ie^2 g^{\mu\nu}$$


$$= \frac{ieg(c_W^2 - s_W^2)}{c_W} g^{\mu\nu} \quad (227)$$

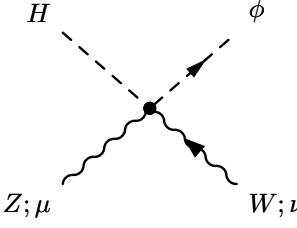


$$= \frac{ig^2(c_W^2 - s_W^2)^2}{2c_W^2} g^{\mu\nu}$$


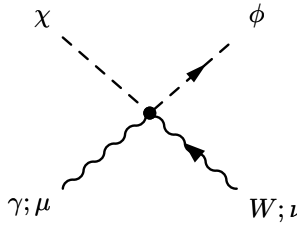
$$= \frac{ig^2}{2} g^{\mu\nu} \quad (228)$$



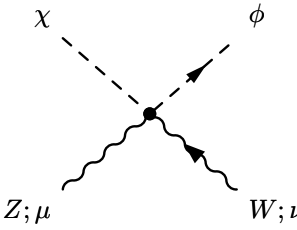
$$= \frac{ieg}{2} g^{\mu\nu} \quad (229)$$



$$= -\frac{ig^2 s_W^2}{2c_W} g^{\mu\nu} \quad (230)$$

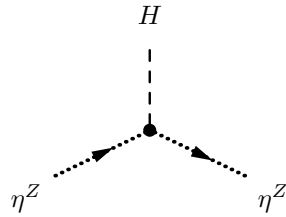


$$= -\frac{eg}{2} g^{\mu\nu} \quad (231)$$



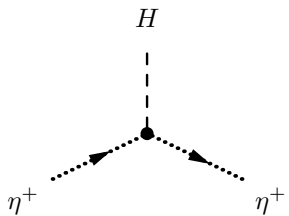
$$= \frac{g^2 s_W^2}{2c_W} g^{\mu\nu} \quad (232)$$

鬼粒子与标量玻色子的耦合:



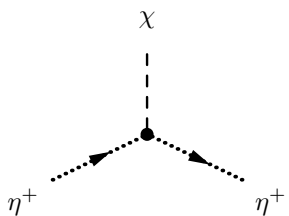
$$= -\frac{ig\xi_Z m_Z}{2c_W} \quad (233)$$

$$= -\frac{ig^2 \xi_Z v}{4c_W^2}$$



$$= -\frac{ig\xi_W m_W}{2} \quad (234)$$

$$= -\frac{ig^2 \xi_W v}{4}$$



$$= \frac{g\xi_W m_W}{2} \quad (235)$$

$$= \frac{g^2 \xi_W v}{4}$$

$$\begin{aligned}
\begin{array}{c} \phi \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^\gamma \quad \eta^+ \end{array} &= -ie\xi_W m_W \\
&= -\frac{ieg\xi_W v}{2}
\end{aligned}
\qquad
\begin{aligned}
\begin{array}{c} \phi \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^\gamma \quad \eta^- \end{array} &= -ie\xi_W m_W \\
&= -\frac{ieg\xi_W v}{2}
\end{aligned} \quad (236)$$

$$\begin{aligned}
\begin{array}{c} \phi \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^Z \quad \eta^+ \end{array} &= -\frac{ig(c_W^2 - s_W^2)\xi_W m_W}{2c_W} \\
&= -\frac{ig^2(c_W^2 - s_W^2)\xi_W v}{4c_W}
\end{aligned}
\qquad
\begin{aligned}
\begin{array}{c} \phi \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^+ \quad \eta^Z \end{array} &= \frac{ig\xi_Z m_Z}{2} \\
&= \frac{ig^2\xi_Z v}{4c_W}
\end{aligned} \quad (237)$$

$$\begin{aligned}
\begin{array}{c} \phi \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^- \quad \eta^Z \end{array} &= \frac{ig\xi_Z m_Z}{2} \\
&= \frac{ig^2\xi_Z v}{4c_W}
\end{aligned}
\qquad
\begin{aligned}
\begin{array}{c} \phi \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^Z \quad \eta^- \end{array} &= -\frac{ig(c_W^2 - s_W^2)\xi_W m_W}{2c_W} \\
&= -\frac{ig^2(c_W^2 - s_W^2)\xi_W v}{4c_W}
\end{aligned} \quad (238)$$

鬼粒子与电弱规范玻色子的耦合：

$$\begin{aligned}
\begin{array}{c} \gamma; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^+ \quad \eta^+ \end{array} &= iep^\mu \\
\begin{array}{c} \gamma; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^- \quad \eta^- \end{array} &= -iep^\mu
\end{aligned} \quad (239)$$

$$\begin{aligned}
\begin{array}{c} Z; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^+ \quad \eta^+ \end{array} &= igc_W p^\mu \\
\begin{array}{c} Z; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^- \quad \eta^- \end{array} &= -igc_W p^\mu
\end{aligned} \quad (240)$$

$$\begin{aligned}
\begin{array}{c} W; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^\gamma \quad \eta^+ \end{array} &= -iep^\mu \\
\begin{array}{c} W; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^+ \quad \eta^\gamma \end{array} &= -iep^\mu
\end{aligned} \quad (241)$$

$$\begin{aligned}
\begin{array}{c} W; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^- \quad \eta^\gamma \end{array} &= iep^\mu \\
\begin{array}{c} W; \mu \\ \downarrow \\ \bullet \\ \swarrow \quad \searrow \\ \eta^\gamma \quad \eta^- \end{array} &= iep^\mu
\end{aligned} \quad (242)$$

$$\begin{array}{ccc}
 \begin{array}{c} W; \mu \\ \downarrow \\ \eta^Z \quad \nearrow \quad \nwarrow \quad \eta^+ \\ p \end{array} & = -igc_W p^\mu & \begin{array}{c} W; \mu \\ \downarrow \\ \eta^+ \quad \nearrow \quad \nwarrow \quad \eta^Z \\ p \end{array} = -igc_W p^\mu
 \end{array} \quad (243)$$

$$\begin{array}{ccc}
 \begin{array}{c} W; \mu \\ \downarrow \\ \eta^- \quad \nearrow \quad \nwarrow \quad \eta^Z \\ p \end{array} & = igc_W p^\mu & \begin{array}{c} W; \mu \\ \downarrow \\ \eta^Z \quad \nearrow \quad \nwarrow \quad \eta^- \\ p \end{array} = igc_W p^\mu
 \end{array} \quad (244)$$

6 内外线一般费曼规则

本节列出一些通用的内外线费曼规则。

内线费曼规则如下。

- 实标量玻色子传播子: $\bullet \dashrightarrow^p \bullet = \frac{i}{p^2 - m^2 + i\epsilon}$

- 复标量玻色子传播子: $\bullet \dashrightarrow^p \bullet = \frac{i}{p^2 - m^2 + i\epsilon}$

- Dirac 费米子传播子: $\bullet \xrightarrow{p} \bullet = \frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon}$

- 无质量实矢量玻色子传播子:

$$\nu \xrightarrow{p} \mu = \frac{-ig^{\mu\nu}}{p^2 + i\epsilon} \quad (\text{Feynman 规范})$$

$$\nu \xrightarrow{p} \mu = \frac{-i(g^{\mu\nu} - p^\mu p^\nu / p^2)}{p^2 + i\epsilon} \quad (\text{Landau 规范})$$

- 有质量实矢量玻色子传播子:

$$\nu \xrightarrow{p} \mu = \frac{-i(g^{\mu\nu} - p^\mu p^\nu / m^2)}{p^2 - m^2 + i\epsilon} \quad (\text{么正规规范})$$

$$\nu \xrightarrow{p} \mu = \frac{-ig^{\mu\nu}}{p^2 - m^2 + i\epsilon} \quad (\text{Feynman 规范})$$

- 有质量复矢量玻色子传播子:

$$\nu \xrightarrow{p} \mu = \frac{-i(g^{\mu\nu} - p^\mu p^\nu / m^2)}{p^2 - m^2 + i\epsilon} \quad (\text{么正规规范})$$

$$\nu \xrightarrow{p} \mu = \frac{-ig^{\mu\nu}}{p^2 - m^2 + i\epsilon} \quad (\text{Feynman 规范})$$

实标量场外线费曼规则如下。

- 实标量玻色子入射外线: $---\xrightarrow{p}\bullet = 1$
- 实标量玻色子出射外线: $\bullet---\xrightarrow{p} = 1$

复标量场外线费曼规则如下。

- 正标量玻色子入射外线: $---\blacktriangleright\xrightarrow{p}\bullet = 1$
- 反标量玻色子入射外线: $---\blacktriangleleft\xrightarrow{p}\bullet = 1$
- 正标量玻色子出射外线: $\bullet---\blacktriangleright\xrightarrow{p} = 1$
- 反标量玻色子出射外线: $\bullet---\blacktriangleleft\xrightarrow{p} = 1$

以 λ 代表自旋极化指标 (如螺旋度), Dirac 旋量场外线费曼规则如下。

- Dirac 正费米子入射外线: $\lambda \longrightarrow\xrightarrow{p}\bullet = u(\mathbf{p}, \lambda)$
- Dirac 反费米子入射外线: $\lambda \longleftarrow\xrightarrow{p}\bullet = \bar{v}(\mathbf{p}, \lambda)$
- Dirac 正费米子出射外线: $\bullet\longrightarrow\xrightarrow{p}\lambda = \bar{u}(\mathbf{p}, \lambda)$
- Dirac 反费米子出射外线: $\bullet\longleftarrow\xrightarrow{p}\lambda = v(\mathbf{p}, \lambda)$

在计算非极化振幅模方时, 可利用自旋求和关系

$$\sum_{\lambda} u(\mathbf{p}, \lambda) \bar{u}(\mathbf{p}, \lambda) = \not{p} + m, \quad \sum_{\lambda} v(\mathbf{p}, \lambda) \bar{v}(\mathbf{p}, \lambda) = \not{p} - m. \quad (245)$$

以 λ 代表自旋极化指标, 实矢量场外线费曼规则如下。

- 实矢量玻色子入射外线: $\lambda; \mu \sim\!\!\!\sim\xrightarrow{p}\bullet = \varepsilon^{\mu}(\mathbf{p}, \lambda)$
- 实矢量玻色子出射外线: $\bullet\sim\!\!\!\sim\xrightarrow{p}\lambda; \mu = \varepsilon^{\mu*}(\mathbf{p}, \lambda)$

复矢量场外线费曼规则如下。

- 正矢量玻色子入射外线: $\lambda; \mu \sim \xrightarrow{p} \bullet = \varepsilon^\mu(\mathbf{p}, \lambda)$
- 反矢量玻色子入射外线: $\lambda; \mu \sim \xleftarrow{p} \bullet = \varepsilon^\mu(\mathbf{p}, \lambda)$
- 正矢量玻色子出射外线: $\bullet \sim \xrightarrow{p} \lambda; \mu = \varepsilon^{\mu*}(\mathbf{p}, \lambda)$
- 反矢量玻色子出射外线: $\bullet \sim \xleftarrow{p} \lambda; \mu = \varepsilon^{\mu*}(\mathbf{p}, \lambda)$

在计算非极化振幅模方时, 若包含无质量矢量玻色子外线, 可利用极化求和替换关系

$$\sum_{\lambda} \varepsilon_{\mu}^*(\mathbf{p}, \lambda) \varepsilon_{\nu}(\mathbf{p}, \lambda) \rightarrow -g_{\mu\nu}; \quad (246)$$

若包含有质量矢量玻色子外线, 可利用极化求和关系

$$\sum_{\lambda} \varepsilon_{\mu}^*(\mathbf{p}, \lambda) \varepsilon_{\nu}(\mathbf{p}, \lambda) = -g_{\mu\nu} + \frac{p_{\mu} p_{\nu}}{m^2}. \quad (247)$$

7 常用单位和标准模型参数

本节数据来自 Particle Data Group 发布的 2024 版 *Review of Particle Physics* [5]。

在有理化的自然单位制中, 光速、约化 Planck 常数和真空介电常数均取为 1, 即 $c = \hbar = \varepsilon_0 = 1$ 。从而, 速度没有量纲 (dimension); 长度量纲与时间量纲相同, 是能量量纲的倒数; 能量、质量和动量具有相同的量纲; 精细结构常数表达为

$$\alpha = \frac{e^2}{4\pi}, \quad (248)$$

而单位电荷量 $e = \sqrt{4\pi\alpha}$ 是没有量纲的。可以将能量单位电子伏特 (eV) 视作上述有量纲物理量的基本单位。

单位间转换关系为

$$1 = c = 2.99792458 \times 10^{10} \text{ cm} \cdot \text{s}^{-1}, \quad (249)$$

$$1 = \hbar = 6.582119569 \times 10^{-25} \text{ GeV} \cdot \text{s}, \quad (250)$$

$$1 = \hbar c = 1.973269804 \times 10^{-14} \text{ GeV} \cdot \text{cm}, \quad (251)$$

$$1 = (\hbar c)^2 = 3.893793721 \times 10^8 \text{ GeV}^2 \cdot \text{pb}, \quad (252)$$

由此得到

$$1 \text{ s} = 2.997925 \times 10^{10} \text{ cm}, \quad 1 \text{ cm} = 3.335641 \times 10^{-11} \text{ s}, \quad (253)$$

$$1 \text{ s} = 1.519267 \times 10^{24} \text{ GeV}^{-1}, \quad 1 \text{ GeV}^{-1} = 6.582120 \times 10^{-25} \text{ s}, \quad (254)$$

$$1 \text{ cm} = 5.067731 \times 10^{13} \text{ GeV}^{-1}, \quad 1 \text{ GeV}^{-1} = 1.973270 \times 10^{-14} \text{ cm}, \quad (255)$$

$$1 \text{ cm}^2 = 2.568189 \times 10^{27} \text{ GeV}^{-2}, \quad 1 \text{ GeV}^{-2} = 3.893794 \times 10^{-28} \text{ cm}^2, \quad (256)$$

$$1 \text{ cm}^3 \cdot \text{s}^{-1} = 8.566558 \times 10^{16} \text{ GeV}^{-2}, \quad 1 \text{ GeV}^{-2} = 1.167330 \times 10^{-17} \text{ cm}^3 \cdot \text{s}^{-1}. \quad (257)$$

靶 (barn) 是散射截面的常用单位, 记作 b, 满足

$$1 \text{ b} = 10^{-24} \text{ cm}^2 = 10^9 \text{ nb} = 10^{12} \text{ pb} = 10^{15} \text{ fb} = 10^{18} \text{ ab}, \quad (258)$$

$$1 \text{ pb} = 10^{-36} \text{ cm}^2 = 2.568189 \times 10^{-9} \text{ GeV}^{-2}, \quad 1 \text{ GeV}^{-2} = 3.893794 \times 10^8 \text{ pb}. \quad (259)$$

Fermi 耦合常数是

$$G_F = 1.1663788(6) \times 10^{-5} \text{ GeV}^{-2}, \quad (260)$$

括号内数字代表测量值的 1σ 不确定度, 由树图阶关系式

$$\frac{G_F}{\sqrt{2}} = \frac{1}{2v^2} = \frac{g^2}{8m_W^2}, \quad (261)$$

得到 Higgs 场真空期待值为

$$v = (\sqrt{2}G_F)^{-1/2} = 246.2196 \text{ GeV}. \quad (262)$$

在低能标 (Thomson 极限) 处, 精细结构常数为

$$\alpha = \frac{1}{137.035999178(8)}; \quad (263)$$

在 $\overline{\text{MS}}$ 重整化方案 (以 \wedge 为标志) 中, α^{-1} 跑动到 $\mu = m_Z$ 能标处的数值是

$$\hat{\alpha}^{-1}(m_Z) = 127.930 \pm 0.008. \quad (264)$$

在 $\overline{\text{MS}}$ 方案中, $\mu = m_Z$ 能标处强耦合常数 $\alpha_s = g_s^2/(4\pi)$ 的数值为

$$\hat{\alpha}_s(m_Z) = 0.1180 \pm 0.0009, \quad (265)$$

Weinberg 角 θ_W 的数值对应于

$$\hat{s}_W^2 = \sin^2 \hat{\theta}_W(m_Z) = 0.23129 \pm 0.00004. \quad (266)$$

在标准模型中, 光子、胶子和中微子没有质量, 其它基本粒子的质量为

$$m_W = 80.3692 \pm 0.0133 \text{ GeV}, \quad m_Z = 91.1880 \pm 0.0020 \text{ GeV}, \quad (267)$$

$$m_H = 125.20 \pm 0.11 \text{ GeV}, \quad m_e = 0.51099895000(15) \text{ MeV}, \quad (268)$$

$$m_\mu = 105.6583755(23) \text{ MeV}, \quad m_\tau = 1776.93 \pm 0.09 \text{ MeV}, \quad (269)$$

$$m_u = 2.16 \pm 0.07 \text{ MeV}, \quad m_d = 4.70 \pm 0.07 \text{ MeV}, \quad (270)$$

$$m_s = 93.5 \pm 0.8 \text{ MeV}, \quad m_c = 1.2730 \pm 0.0046 \text{ GeV}, \quad (271)$$

$$m_b = 4.183 \pm 0.007 \text{ GeV}, \quad m_t = 172.57 \pm 0.29 \text{ GeV}. \quad (272)$$

这里, u 、 d 、 s 夸克的质量是 $\mu = 2 \text{ GeV}$ 能标处的 $\overline{\text{MS}}$ 质量, c 和 b 夸克的质量分别是 $\mu = m_c$ 和 $\mu = m_b$ 能标处的 $\overline{\text{MS}}$ 质量, 其余粒子的质量均为极点质量 (pole mass)。相应地, 计算出来的 c 、 b 夸克极点质量为

$$m_c^{\text{pole}} = 1.67 \pm 0.07 \text{ GeV}, \quad m_b^{\text{pole}} = 4.78 \pm 0.06 \text{ GeV}. \quad (273)$$

质子和中子的质量为

$$m_p = 938.27208816(29) \text{ MeV}, \quad m_n = 939.5654205(5) \text{ MeV}. \quad (274)$$

在电弱能标附近作领头阶计算时, 可将单位电荷量取为

$$e = \sqrt{4\pi\hat{\alpha}(m_Z)} = 0.3134142, \quad (275)$$

将强耦合常数取为

$$g_s = \sqrt{4\pi\hat{\alpha}_s(m_Z)} = 1.217716. \quad (276)$$

从树图阶关系计算 Higgs 场四线性耦合常数 λ 和 Yukawa 耦合常数 y_t 、 y_b 、 y_τ 、 y_c , 得

$$\lambda = \frac{m_H^2}{2v^2} = 0.1292806, \quad y_t = \frac{\sqrt{2}m_t}{v} = 0.9911916, \quad y_b = \frac{\sqrt{2}m_b}{v} = 2.402593 \times 10^{-2}, \quad (277)$$

$$y_\tau = \frac{\sqrt{2}m_\tau}{v} = 1.020617 \times 10^{-2}, \quad y_c = \frac{\sqrt{2}m_c}{v} = 7.311739 \times 10^{-3}. \quad (278)$$

耦合常数 g 和 g' 有以下两种取值方式。

1. 根据树图阶关系 $\sin^2 \theta_W = 1 - m_W^2/m_Z^2$ 计算 Weinberg 角, 得

$$s_W^2 = 1 - \frac{m_W^2}{m_Z^2} = 0.2232095, \quad c_W^2 = 1 - s_W^2 = 0.7767905, \quad (279)$$

$$s_W = \sqrt{s_W^2} = 0.4724505, \quad c_W = \sqrt{c_W^2} = 0.8813572, \quad (280)$$

故

$$g = \frac{e}{s_W} = 0.6633800, \quad g' = \frac{e}{c_W} = 0.3556041. \quad (281)$$

2. 根据 $\overline{\text{MS}}$ 方案中 Weinberg 角的数值 (266) 计算 g 和 g' , 得

$$c_W^2 = 1 - \hat{s}_W^2 = 0.76871, \quad s_W = \sqrt{\hat{s}_W^2} = 0.4809262, \quad c_W = \sqrt{c_W^2} = 0.8767611, \quad (282)$$

$$g = \frac{e}{s_W} = 0.6516889, \quad g' = \frac{e}{c_W} = 0.3574682. \quad (283)$$

将 CKM 矩阵参数化为

$$V = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & s_{13}e^{-i\delta} & \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}, \quad (284)$$

其中 $s_{ij} \equiv \sin \theta_{ij}$, $c_{ij} \equiv \cos \theta_{ij}$ 。实验拟合值为

$$s_{12} = 0.22501 \pm 0.00068, \quad s_{23} = 0.04183^{+0.00079}_{-0.00069}, \quad (285)$$

$$s_{13} = 0.003732^{+0.000090}_{-0.000085}, \quad \delta = 1.147 \pm 0.026. \quad (286)$$

如果只讨论第一、二代夸克的混合, 可利用 Cabibbo 转动角 θ_C 将 CKM 矩阵近似地表达为

$$V \simeq \begin{pmatrix} \cos \theta_C & \sin \theta_C & \\ -\sin \theta_C & \cos \theta_C & \\ & & 1 \end{pmatrix}, \quad \sin \theta_C = s_{12} = 0.225. \quad (287)$$

参考文献

- [1] M. E. Peskin and D. V. Schroeder, “An Introduction to Quantum Field Theory,” Reading, USA: Addison-Wesley (1995), 842 pages.
- [2] T. P. Cheng and L. F. Li, “Gauge Theory of Elementary Particle Physics,” Oxford, UK: Clarendon (1984), 536 pages.
- [3] A. Denner, “Techniques for calculation of electroweak radiative corrections at the one loop level and results for W physics at LEP-200,” Fortsch. Phys. **41**, 307 (1993) [arXiv:0709.1075 [hep-ph]].
- [4] 杜东生, 杨茂志, 《粒子物理导论》, 中国北京: 科学出版社 (2014), 402 页。
- [5] S. Navas *et al.* [Particle Data Group], Phys. Rev. D **110**, no.3, 030001 (2024)