# 标准模型的拉氏量和 Feynman 规则

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## 目　录

# 1　约定

本文采用有理化的自然单位制，推导过程参考文献 [1, 2, 3, 4]，协变导数的约定与 *Review of Particle Physics* [?] 第 9、10、11 章一致。

Minkowski 度规张量

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}. \tag{1}$$

Pauli 矩阵

$$\sigma^1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} & -\mathrm{i} \\ \mathrm{i} & \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}, \tag{2}$$

$$\sigma^\mu \equiv (1, \boldsymbol{\sigma}), \quad \bar{\sigma}^\mu \equiv (1, -\boldsymbol{\sigma}). \tag{3}$$

Weyl 表象中的 Dirac 矩阵

$$\gamma^\mu = \begin{pmatrix} & \sigma^\mu \\ \bar{\sigma}^\mu & \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -\mathbf{1} & \\ & \mathbf{1} \end{pmatrix}. \tag{4}$$

左右手投影算符

$$P_{\mathrm{L}} \equiv \frac{1}{2}(1-\gamma^5) = \begin{pmatrix} \mathbf{1} & \\ & \mathbf{0} \end{pmatrix}, \quad P_{\mathrm{R}} \equiv \frac{1}{2}(1+\gamma^5) = \begin{pmatrix} \mathbf{0} & \\ & \mathbf{1} \end{pmatrix} \tag{5}$$

用于定义左手旋量场 $\psi_{\mathrm{L}} \equiv P_{\mathrm{L}}\psi$ 和右手旋量场 $\psi_{\mathrm{R}} \equiv P_{\mathrm{R}}\psi$。Levi-Civita 符号的约定取

$$\varepsilon^{0123} = \varepsilon^{123} = +1. \tag{6}$$

Feynman 规则约定如下。

- 对于指向相互作用顶点的动量 $p$，时空导数 $\partial_\mu$ 在动量空间 Feynman 规则里贡献一个 $-\mathrm{i}p_\mu$ 因子。
- 实线表示费米子，实线上的箭头表示费米子数流动的方向。
- 虚线表示标量玻色子，虚线上的箭头表示玻色子数流动的方向。
- 螺旋线表示胶子；波浪线表示其它规范玻色子，波浪线上的箭头表示玻色子数流动的方向。
- 点线表示鬼粒子，点线上的箭头表示鬼粒子数流动的方向。
- 如果没有额外箭头标记，动量方向与粒子线上的箭头方向一致；否则与额外箭头方向一致。

# 2 标准模型概述

粒子物理标准模型是一个 $\mathrm{SU}(3)_{\mathrm{C}}\times\mathrm{SU}(2)_{\mathrm{L}}\times\mathrm{U}(1)_{\mathrm{Y}}$ 规范理论。模型中有三代费米子，包括三代中微子 $\nu_i=\nu_e,\nu_\mu,\nu_\tau$，三代带电轻子 $\ell_i=e,\mu,\tau$，三代上型夸克 $u_i=u,c,t$ 和三代下型夸克 $d_i=d,s,b$ $(i=1,2,3)$。规范玻色子传递费米子之间的规范相互作用。

$\mathrm{SU}(3)_{\mathrm{C}}$ 部分描述夸克的强相互作用,称为量子色动力学 (Quantum Chromodynamics, QCD)，相应的规范玻色子是胶子。$\mathrm{SU}(2)_{\mathrm{L}}\times\mathrm{U}(1)_{\mathrm{Y}}$ 部分统一描述夸克和轻子的电磁和弱相互作用，称为电弱规范理论。理论中有一个 Higgs 二重态，通过 Brout–Englert–Higgs (BEH) 机制引发规范群的自发对称性破缺，使 $\mathrm{SU}(2)_{\mathrm{L}}\times\mathrm{U}(1)_{\mathrm{Y}}$ 群破缺为 $\mathrm{U}(1)_{\mathrm{EM}}$ 群。$\mathrm{U}(1)_{\mathrm{EM}}$ 规范理论称为量子电动力学 (Quantum Electrodynamics, QED)。

破缺前，理论中存在 4 个无质量的规范玻色子和 4 个 Higgs 自由度；左手费米子和右手费米子都没有质量，具有不同的量子数。

破缺后，3 个规范玻色子与 3 个 Higgs 自由度结合，从而获得质量，成为 $W^\pm$ 和 $Z^0$ 玻色子，传递弱相互作用。剩下的 1 个无质量规范玻色子是光子，即是 $\mathrm{U}(1)_{\mathrm{EM}}$ 群的规范玻色子，传递电磁相互作用。剩下的 1 个中性 Higgs 自由度称为 Higgs 玻色子。费米子与 Higgs 二重态的 Yukawa 耦合导致左手费米子和右手费米子获得质量，组合成 Dirac 费米子。

理论中的中微子没有右手分量，因而没有获得质量。1998 年实验发现中微子振荡，证明中微子具有质量，所以需要扩充标准模型才能正确描述中微子物理。

# 3 量子色动力学

QCD 的拉氏量表达为

$$\boxed{\mathcal{L}_{\mathrm{QCD}}=\sum_q\bar{q}(\mathrm{i}\gamma^\mu D_\mu-m_q)q-\frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu},\quad q=u,d,s,c,b,t,\quad a=1,\cdots,8,}\tag{7}$$

其中

$$\boxed{D_\mu=\partial_\mu+\mathrm{i}g_{\mathrm{s}}G^a_\mu t^a,\quad G^a_{\mu\nu}\equiv\partial_\mu G^a_\nu-\partial_\nu G^a_\mu-g_{\mathrm{s}}f^{abc}G^b_\mu G^c_\nu.}\tag{8}$$

$q$ 为夸克旋量场 $\mathrm{SU}(3)_{\mathrm{C}}$ 三重态，$\mathrm{SU}(3)_{\mathrm{C}}$ 规范场 $G^a_\mu$ 对应于胶子 $g$，$g_{\mathrm{s}}$ 是 $\mathrm{SU}(3)_{\mathrm{C}}$ 规范耦合常数。$t^a=\lambda^a/2$ 是 $\mathrm{SU}(3)_{\mathrm{C}}$ 群基础表示的生成元，其中 $\lambda^a$ 为 Gell-Mann 矩阵。$\mathrm{SU}(3)_{\mathrm{C}}$ 生成元对易关系为 $[t^a,t^b]=\mathrm{i}f^{abc}t^c$，结构常数 $f^{abc}$ 是全反对称的，非零分量为

$$f^{123}=1,\quad f^{147}=f^{246}=f^{257}=f^{345}=f^{516}=f^{637}=\frac{1}{2},\quad f^{458}=f^{678}=\frac{\sqrt{3}}{2}.\tag{9}$$

由

$$-\frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu}=-\frac{1}{4}(\partial_\mu G^a_\nu-\partial_\nu G^a_\mu-g_{\mathrm{s}}f^{abc}G^b_\mu G^c_\nu)(\partial^\mu G^{a,\nu}-\partial^\nu G^{a,\mu}-g_{\mathrm{s}}f^{ade}G^{d,\mu}G^{e,\nu})$$

$$
\begin{aligned}
&= -\frac{1}{2}[(\partial_\mu G^a_\nu)(\partial^\mu G^{a,\nu}) - (\partial_\mu G^a_\nu)(\partial^\nu G^{a,\mu})] + g_{\mathrm{s}} f^{abc}(\partial_\mu G^a_\nu) G^{b,\mu} G^{c,\nu} \\
&\quad -\frac{g_{\mathrm{s}}^2}{4} f^{abc} f^{ade} G^b_\mu G^c_\nu G^{d,\mu} G^{e,\nu},
\end{aligned}
\tag{10}
$$

推出

$$
\begin{aligned}
\mathcal{L}_{\mathrm{QCD}} &= \sum_q [\bar{q}(\mathrm{i}\gamma^\mu \partial_\mu - m_q) q - g_{\mathrm{s}} G^a_\mu \bar{q} \gamma^\mu t^a q] + \frac{1}{2}[(\partial_\mu G^a_\nu)(\partial^\nu G^{a,\mu}) - (\partial_\mu G^a_\nu)(\partial^\mu G^{a,\nu})] \\
&\quad + g_{\mathrm{s}} f^{abc}(\partial_\mu G^a_\nu) G^{b,\mu} G^{c,\nu} - \frac{g_{\mathrm{s}}^2}{4} f^{abc} f^{ade} G^b_\mu G^c_\nu G^{d,\mu} G^{e,\nu}.
\end{aligned}
\tag{11}
$$

设用于固定胶子场规范的函数 $G^a(x) = \partial^\mu G^a_\mu(x) - \omega^a(x)$，其中 $\omega^a(x)$ 是某个任意函数，规范固定条件是 $G^a(x) = 0$。这是 Lorenz 规范的推广，$\omega^a(x) = 0$ 对应于 Lorenz 规范。在路径积分量子化中，以中心为 $\omega^a(x) = 0$ 的 Gauss 权重对 $\omega^a(x)$ 作泛函积分，有

$$
\int \mathcal{D}\omega^a \exp\left[-\mathrm{i}\int \mathrm{d}^4 x\, \frac{1}{2\xi}(\omega^a)^2\right] \delta(G^a) = \exp\left[-\mathrm{i}\int \mathrm{d}^4 x\, \frac{1}{2\xi}(\partial^\mu G^a_\mu)^2\right].
\tag{12}
$$

可见，拉氏量中的规范固定项为

$$
\mathcal{L}_{\mathrm{QCD,GF}} = -\frac{1}{2\xi}(\partial^\mu G^a_\mu)^2.
\tag{13}
$$

$\xi$ 的任何一个取值对应于一种规范。$\xi = 1$ 称为 Feynman-'t Hooft 规范，$\xi = 0$ 称为 Landau 规范。于是，胶子传播子相关拉氏量为

$$
\begin{aligned}
\mathcal{L}_{\mathrm{QCD,prop}} &= \frac{1}{2}\left[(\partial_\mu G^a_\nu)(\partial^\nu G^{a,\mu}) - (\partial_\mu G^a_\nu)(\partial^\mu G^{a,\nu}) - \frac{1}{\xi}(\partial^\mu G^a_\mu)^2\right] \\
&\to \frac{1}{2} G^a_\mu \left[g^{\mu\nu}\partial^2 - \left(1 - \frac{1}{\xi}\right)\partial^\mu \partial^\nu\right] G^a_\nu.
\end{aligned}
\tag{14}
$$

这里 $\to$ 代表丢弃一些全散度项。变换到动量空间，得

$$
-g^{\mu\nu} p^2 + \left(1 - \frac{1}{\xi}\right) p^\mu p^\nu,
\tag{15}
$$

它的逆矩阵是

$$
-\frac{1}{p^2}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1 - \xi)\right],
\tag{16}
$$

这是因为

$$
\begin{aligned}
&-\frac{1}{p^2}\left[g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2}(1 - \xi)\right]\left[-g^{\mu\nu} p^2 + \left(1 - \frac{1}{\xi}\right) p^\mu p^\nu\right] \\
&= \delta_\rho{}^\nu - \frac{p_\rho p^\nu}{p^2}\left(1 - \frac{1}{\xi}\right) - \frac{p_\rho p^\nu}{p^2}(1 - \xi) + \frac{p_\rho p^\nu}{p^2}(1 - \xi)\left(1 - \frac{1}{\xi}\right) = \delta_\rho{}^\nu.
\end{aligned}
\tag{17}
$$

从而胶子传播子的形式为

$$\frac{-\mathrm{i}\delta^{ab}}{p^2+\mathrm{i}\epsilon}\left[g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}(1-\xi)\right]. \tag{18}$$

$\mathrm{SU(3)_C}$ 规范变换为

$$q'=Uq,\quad (G^a_\mu t^a)'=UG^a_\mu t^a U^\dagger-\frac{\mathrm{i}}{g_\mathrm{s}}U\partial_\mu U^\dagger, \tag{19}$$

其中 $U(x)=\exp[\mathrm{i}\alpha^a(x)t^a]$。胶子场的无穷小规范变换形式是

$$\begin{aligned}(G^a_\mu t^a)' &= (1+\mathrm{i}\alpha^a t^a)G^b_\mu t^b(1-\mathrm{i}\alpha^c t^c)-\frac{\mathrm{i}}{g_\mathrm{s}}(1+\mathrm{i}\alpha^a t^a)\partial_\mu(1-\mathrm{i}\alpha^c t^c)\\ &= G^b_\mu t^b+\mathrm{i}\alpha^c G^b_\mu[t^c,t^b]-\frac{1}{g_\mathrm{s}}(\partial_\mu\alpha^c)t^c+\mathcal{O}(\alpha^a\alpha^b)\\ &= G^a_\mu t^a-f^{cba}\alpha^c G^b_\mu t^a-\frac{1}{g_\mathrm{s}}(\partial_\mu\alpha^a)t^a+\mathcal{O}(\alpha^a\alpha^b)\\ &= \left(G^a_\mu+f^{abc}G^b_\mu\alpha^c-\frac{1}{g_\mathrm{s}}\,\partial_\mu\alpha^a\right)t^a+\mathcal{O}(\alpha^a\alpha^b),\end{aligned} \tag{20}$$

即 $G^a_\mu$ 的无穷小变化为

$$\delta G^a_\mu=(G^a_\mu)'-G^a_\mu=-\frac{1}{g_\mathrm{s}}\,\partial_\mu\alpha^a+f^{abc}G^b_\mu\alpha^c=-\frac{1}{g_\mathrm{s}}(D_\mu\alpha)^a=-\frac{1}{g_\mathrm{s}}\,D^{ac}_\mu\alpha^c, \tag{21}$$

其中 $(D_\mu\alpha)^a=\partial_\mu\alpha^a-g_\mathrm{s}f^{abc}G^b_\mu\alpha^c$ 是 $\mathrm{SU(3)_C}$ 伴随表示中的协变导数，而

$$D^{ac}_\mu=\delta^{ac}\partial_\mu-g_\mathrm{s}f^{abc}G^b_\mu. \tag{22}$$

因此，规范固定函数 $G^a$ 的无穷小变化为

$$\delta G^a=\partial^\mu\delta G^a_\mu=-\frac{1}{g_\mathrm{s}}\,\partial^\mu D^{ac}_\mu\alpha^c, \tag{23}$$

故

$$\frac{\delta G^a}{\delta\alpha^c}=-\frac{1}{g_\mathrm{s}}\,\partial^\mu D^{ac}_\mu=-\frac{1}{g_\mathrm{s}}\,\delta^{ac}\partial^2+f^{abc}\partial^\mu G^b_\mu. \tag{24}$$

根据 Grassmann 数的积分式

$$\left(\prod_i\int\mathrm{d}\theta_i^*\mathrm{d}\theta_i\right)\exp(-\theta_i^*B_{ij}\theta_j)=\det(B), \tag{25}$$

$\delta G^a/\delta\alpha^c$ 的行列式可用 Faddeev-Popov 鬼场 $\eta^a_g$ 和 $\bar{\eta}^a_g$ 表达为

$$\det\left(\frac{\delta G^a}{\delta\alpha^c}\right)=\det\left(-\frac{1}{g_\mathrm{s}}\,\partial^\mu D^{ac}_\mu\right)=\int\mathcal{D}\eta^a_g\,\mathcal{D}\bar{\eta}^c_g\exp\left[\mathrm{i}\int\mathrm{d}^4x\,\bar{\eta}^a_g(-\partial^\mu D^{ac}_\mu)\eta^c_g\right], \tag{26}$$

这里 $-1/g_{\rm s}$ 因子被吸收到鬼场 $\eta_g^a$ 和 $\bar{\eta}_g^a$ 的归一化中。注意到

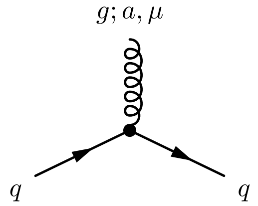
$$-\partial^\mu D_\mu^{ac} = g_{\rm s}\frac{\delta G^a}{\delta\alpha^c} = -\delta^{ac}\partial^2 + g_{\rm s} f^{abc}\partial^\mu G_\mu^b, \tag{27}$$

Faddeev-Popov 鬼场在拉氏量中的贡献是

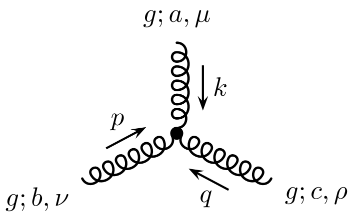
$$\begin{aligned}\mathcal{L}_{\rm QCD,FP} &= \bar{\eta}_g^a(-\partial^\mu D_\mu^{ac})\eta_g^c = \bar{\eta}_g^a\left(g_{\rm s}\frac{\delta G^a}{\delta\alpha^c}\right)\eta_g^c = \bar{\eta}_g^a(-\delta^{ac}\partial^2 + g_{\rm s}f^{abc}\partial^\mu G_\mu^b)\eta_g^c \\ &\to -\bar{\eta}_g^a\delta^{ab}\partial^2\eta_g^b - g_{\rm s}f^{abc}(\partial^\mu\bar{\eta}_g^a)G_\mu^b\eta_g^c.\end{aligned} \tag{28}$$

下面列出 QCDFeynman 规则。

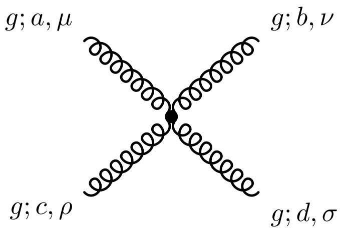
QCD 耦合顶点:



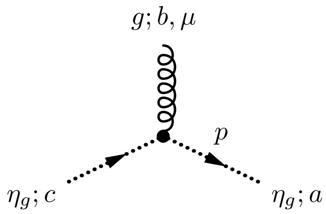

$$= -{\rm i}g_{\rm s}\gamma^\mu t^a \tag{29}$$




$$= -g_{\rm s}f^{abc}[g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu] \tag{30}$$

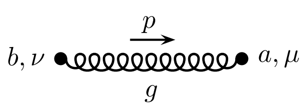



$$\begin{aligned}= -{\rm i}g_{\rm s}^2[&f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\ &+ f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\ &+ f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})]\end{aligned} \tag{31}$$



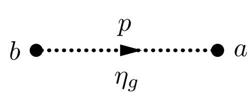

$$= g_{\rm s}f^{abc}p^\mu \tag{32}$$

胶子传播子:




$$= \frac{-{\rm i}\delta^{ab}}{p^2 + {\rm i}\epsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1-\xi)\right] \tag{33}$$

鬼粒子传播子:




$$= \frac{{\rm i}\delta^{ab}}{p^2 + {\rm i}\epsilon} \tag{34}$$

# 4 电弱规范理论

电弱规范理论的规范群是 $\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_\mathrm{Y}$，每一代左手旋量场构成 2 个 $\mathrm{SU}(2)_\mathrm{L}$ 二重态

$$\boxed{L_{i\mathrm{L}}=\begin{pmatrix}P_\mathrm{L}\nu_i\\P_\mathrm{L}\ell_i\end{pmatrix}=\begin{pmatrix}\nu_{i\mathrm{L}}\\\ell_{i\mathrm{L}}\end{pmatrix},\quad Q_{i\mathrm{L}}=\begin{pmatrix}P_\mathrm{L}u'_i\\P_\mathrm{L}d'_i\end{pmatrix}=\begin{pmatrix}u'_{i\mathrm{L}}\\d'_{i\mathrm{L}}\end{pmatrix},\quad i=1,2,3.} \tag{35}$$

它们的协变导数是

$$\boxed{D_\mu=\partial_\mu+\mathrm{i}gW^a_\mu\tau^a+\mathrm{i}g'B_\mu Y,} \tag{36}$$

其中 $W^a_\mu(x)$ $(a=1,2,3)$ 是 $\mathrm{SU}(2)_\mathrm{L}$ 规范场，$B_\mu(x)$ 是 $\mathrm{U}(1)_\mathrm{Y}$ 规范场，$g$ 和 $g'$ 分别是 $\mathrm{SU}(2)_\mathrm{L}$ 和 $\mathrm{U}(1)_\mathrm{Y}$ 的规范耦合常数，取 $g>0$ 且 $g'>0$。

$$\tau^a=\frac{\sigma^a}{2} \tag{37}$$

是 $\mathrm{SU}(2)_\mathrm{L}$ 群 2 维表示的生成元，对应于弱同位旋。生成元 $\tau^3$ 的本征值是弱同位旋第 3 分量，记为 $T^3$。$Y$ 是弱超荷。各代右手旋量场 $\ell_{i\mathrm{R}}=P_\mathrm{R}\ell_i$、$u'_{i\mathrm{R}}=P_\mathrm{R}u'_i$ 和 $d'_{i\mathrm{R}}=P_\mathrm{R}d'_i$ 是 $\mathrm{SU}(2)_\mathrm{L}$ 单态，协变导数为

$$\boxed{D_\mu=\partial_\mu+\mathrm{i}g'B_\mu Y.} \tag{38}$$

表 1 列出费米子场的电荷 $Q$、弱同位旋第 3 分量 $T^3$、弱超荷 $Y$、重子数 $B$ 和轻子数 $L_e/L_\mu/L_\tau$，其中电荷 $Q$ 由 $T^3$ 和 $Y$ 定义，

$$\boxed{Q\equiv T^3+Y.} \tag{39}$$

表 1: 标准模型费米子场的量子数。

| 统一记号 | 第一代 | 第二代 | 第三代 | $Q$ | $T^3$ | $Y$ | $B$ | $L_e/L_\mu/L_\tau$ |
|---|---|---|---|---|---|---|---|---|
| $L_{i\mathrm{L}}=\begin{pmatrix}\nu_{i\mathrm{L}}\\\ell_{i\mathrm{L}}\end{pmatrix}$ | $\begin{pmatrix}\nu_{e\mathrm{L}}\\e_\mathrm{L}\end{pmatrix}$ | $\begin{pmatrix}\nu_{\mu\mathrm{L}}\\\mu_\mathrm{L}\end{pmatrix}$ | $\begin{pmatrix}\nu_{\tau\mathrm{L}}\\\tau_\mathrm{L}\end{pmatrix}$ | $0$<br>$-1$ | $1/2$<br>$-1/2$ | $-1/2$<br>$-1/2$ | $0$<br>$0$ | $1$<br>$1$ |
| $Q_{i\mathrm{L}}=\begin{pmatrix}u'_{i\mathrm{L}}\\d'_{i\mathrm{L}}\end{pmatrix}$ | $\begin{pmatrix}u'_\mathrm{L}\\d'_\mathrm{L}\end{pmatrix}$ | $\begin{pmatrix}c'_\mathrm{L}\\s'_\mathrm{L}\end{pmatrix}$ | $\begin{pmatrix}t'_\mathrm{L}\\b'_\mathrm{L}\end{pmatrix}$ | $2/3$<br>$-1/3$ | $1/2$<br>$-1/2$ | $1/6$<br>$1/6$ | $1/3$<br>$1/3$ | $0$<br>$0$ |
| $\ell_{i\mathrm{R}}$ | $e_\mathrm{R}$ | $\mu_\mathrm{R}$ | $\tau_\mathrm{R}$ | $-1$ | $0$ | $-1$ | $0$ | $1$ |
| $u'_{i\mathrm{R}}$ | $u'_\mathrm{R}$ | $c'_\mathrm{R}$ | $t'_\mathrm{R}$ | $2/3$ | $0$ | $2/3$ | $1/3$ | $0$ |
| $d'_{i\mathrm{R}}$ | $d'_\mathrm{R}$ | $s'_\mathrm{R}$ | $b'_\mathrm{R}$ | $-1/3$ | $0$ | $-1/3$ | $1/3$ | $0$ |

## 4.1 Brout–Englert–Higgs 机制

由于左手费米子和右手费米子参与不同的 $\mathrm{SU(2)_L\times U(1)_Y}$ 规范相互作用，耦合左右手费米子场的质量项会破坏规范对称性。另一方面，规范对称性也禁止规范玻色子具有质量。为了让费米子和弱规范玻色子获得质量，需要引入 BEH 机制，使 $\mathrm{SU(2)_L\times U(1)_Y}$ 规范对称性自发破缺。因此，引入 Higgs 标量场

$$\boxed{\Phi(x)=\begin{pmatrix}\phi^+(x)\\ \phi^0(x)\end{pmatrix},} \tag{40}$$

其中 $\phi^+$ 和 $\phi^0$ 都是复标量场。$\Phi$ 是 $\mathrm{SU(2)_L}$ 二重态，弱超荷是

$$Y_H=\frac{1}{2}. \tag{41}$$

Higgs 场的协变动能项和势能项为

$$\boxed{\mathcal{L}_\mathrm{H}=(D^\mu\Phi)^\dagger(D_\mu\Phi)-V_\mathrm{H}(\Phi),\quad V_\mathrm{H}(\Phi)=-\mu^2\Phi^\dagger\Phi+\lambda(\Phi^\dagger\Phi)^2,} \tag{42}$$

其中协变导数为

$$D_\mu\Phi=(\partial_\mu+\mathrm{i}g'B_\mu Y_H+\mathrm{i}gW_\mu^a\tau^a)\Phi. \tag{43}$$

当 $\lambda>0$ 且 $\mu^2>0$ 时，Higgs 场势能 $V_\mathrm{H}(\Phi)$ 呈现出图 1 所示墨西哥草帽状的形式，势能最小值位于方程

$$\Phi^\dagger\Phi=[\mathrm{Re}(\phi^+)]^2+[\mathrm{Im}(\phi^+)]^2+[\mathrm{Re}(\phi^0)]^2+[\mathrm{Im}(\phi^0)]^2=\frac{v^2}{2} \tag{44}$$

对应的 3 维球面上，其中 $v\equiv\sqrt{\mu^2/\lambda}$，满足

$$\boxed{\mu^2=\lambda v^2.} \tag{45}$$

Higgs 场的真空期待值位于这个 3 维球面上的某一点，不失一般性，可将它取为

$$\langle\Phi\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}. \tag{46}$$

其它真空期待值可通过 $\mathrm{SU(2)_L\times U(1)_Y}$ 整体变换

$$\langle\Phi\rangle\to\exp(\mathrm{i}\alpha^a\tau^a)\exp(\mathrm{i}\alpha^Y Y_H)\,\langle\Phi\rangle \tag{47}$$

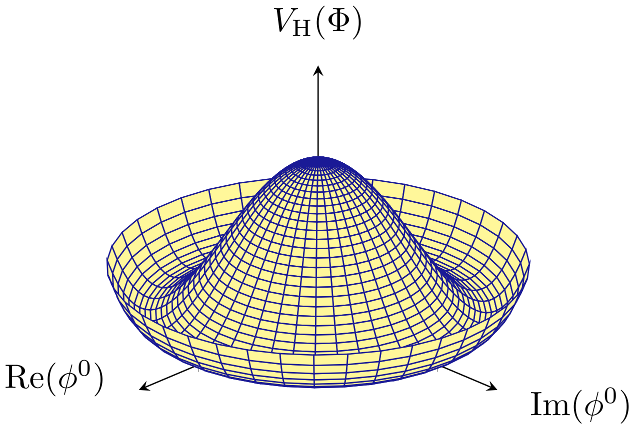



图 1: Higgs 场势能示意图。这里压缩掉 $\mathrm{Re}(\phi^+)$ 和 $\mathrm{Im}(\phi^+)$ 两个维度。

得到，因为 $\langle\Phi^\dagger\Phi\rangle$ 在这样的变换下保持不变。若 $\alpha^1=\alpha^2=0$ 且 $\alpha^3=\alpha^Y$，则

$$\exp(\mathrm{i}\alpha^a\tau^a)\exp(\mathrm{i}\alpha^Y Y_H)=\exp[\mathrm{i}\alpha^3(\sigma^3+\mathbf{1})/2]=\exp\left[\mathrm{i}\alpha^3\begin{pmatrix}1&\\&0\end{pmatrix}\right]=\begin{pmatrix}e^{\mathrm{i}\alpha^3}&\\&1\end{pmatrix},\tag{48}$$

而 $\langle\Phi\rangle$ 在此变换下不变。因此，在 $\mathrm{SU(2)_L}\times\mathrm{U(1)_Y}$ 的 4 维群空间中有 1 个方向的规范对称性没有受到破坏，只有 3 个方向的规范对称性发生自发破缺。根据 Goldstone 定理，破缺后存在 3 个无质量的 Nambu-Goldstone 玻色子。最终，有 3 个规范玻色子结合 Nambu-Goldstone 玻色子，通过 BEH 机制获得质量。

以 $\langle\Phi\rangle$ 为基础，将 Higgs 场参数化为

$$\Phi(x)=\exp\left[-\mathrm{i}\,\frac{\chi^a(x)}{v}\,\tau^a\right]\frac{1}{\sqrt{2}}\begin{pmatrix}0\\v+H(x)\end{pmatrix},\tag{49}$$

其中 $\chi^a(x)$ 和 $H(x)$ 都是实标量场。$\exp[-\mathrm{i}\chi^a(x)\tau^a/v]$ 因子能够通过 $\mathrm{SU(2)_L}$ 规范变换消去，因而可将 $\Phi(x)$ 直接取为

$$\boxed{\Phi(x)=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\v+H(x)\end{pmatrix},\quad\Phi^\dagger\Phi=\frac{1}{2}(v+H)^2.}\tag{50}$$

此时 Higgs 场只剩下一个物理自由度 $H(x)$，对应于 Higgs 玻色子，这种取法称为**幺正规范**。

在幺正规范下，势能项化为

$$\begin{aligned}&-V_{\rm H}(\Phi)=\mu^2\Phi^\dagger\Phi-\lambda(\Phi^\dagger\Phi)^2=\frac{\mu^2}{2}(v+H)^2-\frac{\lambda}{4}(v+H)^4\\&=\frac{\mu^2}{2}(v^2+H^2+2vH)-\frac{\lambda}{4}(v^4+4v^2H^2+H^4+4v^3H+2v^2H^2+4vH^3)\\&=\frac{1}{4}\mu^2v^2+\frac{1}{4}(\mu^2-\lambda v^2)v^2+(\mu^2-\lambda v^2)vH+\frac{1}{2}(\mu^2-\lambda v^2)H^2-\lambda v^2H^2-\lambda vH^3-\frac{\lambda}{4}H^4\end{aligned}$$

$$= \frac{1}{8}\,m_H^2 v^2 - \frac{1}{2}\,m_H^2 H^2 - \frac{1}{2}\frac{m_H^2}{v}\,H^3 - \frac{1}{8}\frac{m_H^2}{v^2}\,H^4, \tag{51}$$

其中 Higgs 玻色子的质量为

$$\boxed{m_H \equiv \sqrt{2}\mu, \quad m_H^2 = 2\mu^2 = 2\lambda v^2.} \tag{52}$$

由于

$$g' B_\mu Y_H + g W_\mu^a \tau^a = \frac{1}{2}\begin{pmatrix} g' B_\mu + g W_\mu^3 & g(W_\mu^1 - \mathrm{i} W_\mu^2) \\ g(W_\mu^1 + \mathrm{i} W_\mu^2) & g' B_\mu - g W_\mu^3 \end{pmatrix}, \tag{53}$$

Higgs 场真空期待值 $v$ 对协变导数 $D_\mu \Phi$ 的贡献为

$$\begin{aligned} D_\mu \Phi &\supset \mathrm{i}(g' B_\mu Y_H + g W_\mu^a \tau^a)\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \\ &= \frac{\mathrm{i}v}{2\sqrt{2}}\begin{pmatrix} g' B_\mu + g W_\mu^3 & g(W_\mu^1 - \mathrm{i} W_\mu^2) \\ g(W_\mu^1 + \mathrm{i} W_\mu^2) & g' B_\mu - g W_\mu^3 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \supset \frac{\mathrm{i}v}{2\sqrt{2}}\begin{pmatrix} g(W_\mu^1 - \mathrm{i} W_\mu^2) \\ g' B_\mu - g W_\mu^3 \end{pmatrix}, \end{aligned} \tag{54}$$

故协变动能项 $(D^\mu \Phi)^\dagger (D_\mu \Phi)$ 中正比于 $v^2$ 的项是

$$(D^\mu \Phi)^\dagger (D_\mu \Phi) \supset \frac{v^2}{8}[g^2 |W_\mu^1 - \mathrm{i} W_\mu^2|^2 + (g' B_\mu - g W_\mu^3)^2] = \frac{v^2}{8}(g^2 W^{a,\mu} W_\mu^a + g'^2 B^\mu B_\mu - 2gg' B^\mu W_\mu^3). \tag{55}$$

这些项是规范玻色子的质量项，重新表达为

$$\mathcal{L}_{\mathrm{GBM}} = \frac{1}{2}\,m_W^2 (W^{1,\mu} W_\mu^1 + W^{2,\mu} W_\mu^2) + \frac{1}{2}\begin{pmatrix} W^{3,\mu} & B^\mu \end{pmatrix} M_{W^3B}^2 \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}, \tag{56}$$

其中

$$\boxed{m_W \equiv \frac{1}{2}\,g v} \tag{57}$$

是 $W_\mu^1$ 和 $W_\mu^2$ 获得的质量，而 $W^{3\mu}$ 和 $B^\mu$ 的质量平方矩阵为

$$M_{W^3B}^2 \equiv \frac{v^2}{4}\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}. \tag{58}$$

为了使 $M_{W^3B}^2$ 矩阵对角化，定义

$$\boxed{\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \equiv \begin{pmatrix} c_{\mathrm{W}} & -s_{\mathrm{W}} \\ s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix}\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix},} \tag{59}$$

其中

$$\boxed{s_{\mathrm{W}} \equiv \sin\theta_{\mathrm{W}} = \frac{g'}{\sqrt{g^2+g'^2}}, \quad c_{\mathrm{W}} \equiv \cos\theta_{\mathrm{W}} = \frac{g}{\sqrt{g^2+g'^2}},} \tag{60}$$

$\theta_{\mathrm{W}}$ 称为 Weinberg 角，也称为弱混合角。从后面的讨论可以看出 $A_\mu(x)$ 就是电磁场，对应于光子。$Z_\mu(x)$ 对应于矢量玻色子 $Z$。反过来，有

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} c_{\mathrm{W}} & s_{\mathrm{W}} \\ -s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}. \tag{61}$$

由

$$\begin{aligned} M_{W^3B}^2 \begin{pmatrix} c_{\mathrm{W}} & s_{\mathrm{W}} \\ -s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} &= \frac{(g^2+g'^2)v^2}{4} \begin{pmatrix} c_{\mathrm{W}}^2 & -s_{\mathrm{W}}c_{\mathrm{W}} \\ -s_{\mathrm{W}}c_{\mathrm{W}} & s_{\mathrm{W}}^2 \end{pmatrix} \begin{pmatrix} c_{\mathrm{W}} & s_{\mathrm{W}} \\ -s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} \\ &= \frac{(g^2+g'^2)v^2}{4} \begin{pmatrix} c_{\mathrm{W}} & 0 \\ -s_{\mathrm{W}} & 0 \end{pmatrix} \end{aligned} \tag{62}$$

得

$$\begin{aligned} \begin{pmatrix} c_{\mathrm{W}} & -s_{\mathrm{W}} \\ s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} M_{W^3B}^2 \begin{pmatrix} c_{\mathrm{W}} & s_{\mathrm{W}} \\ -s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} &= \frac{(g^2+g'^2)v^2}{4} \begin{pmatrix} c_{\mathrm{W}} & -s_{\mathrm{W}} \\ s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} \begin{pmatrix} c_{\mathrm{W}} & 0 \\ -s_{\mathrm{W}} & 0 \end{pmatrix} \\ &= \frac{(g^2+g'^2)v^2}{4} \begin{pmatrix} 1 & \\ & 0 \end{pmatrix}, \end{aligned} \tag{63}$$

因此

$$\begin{aligned} \frac{1}{2} \begin{pmatrix} W^{3,\mu} & B^\mu \end{pmatrix} M_{W^3B}^2 \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} &= \frac{1}{2} \begin{pmatrix} Z^\mu & A^\mu \end{pmatrix} \begin{pmatrix} c_{\mathrm{W}} & -s_{\mathrm{W}} \\ s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} M_{W^3B}^2 \begin{pmatrix} c_{\mathrm{W}} & s_{\mathrm{W}} \\ -s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \\ &= \frac{(g^2+g'^2)v^2}{8} \begin{pmatrix} Z^\mu & A^\mu \end{pmatrix} \begin{pmatrix} 1 & \\ & 0 \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \frac{1}{2} m_Z^2 Z^\mu Z_\mu, \end{aligned} \tag{64}$$

其中

$$\boxed{m_Z \equiv \frac{1}{2}\sqrt{g^2+g'^2}\, v = \frac{gv}{2c_{\mathrm{W}}} = \frac{m_W}{c_{\mathrm{W}}}} \tag{65}$$

是 $Z$ 玻色子的质量，而光子没有质量。另一方面，用质量相同的实矢量场 $W_\mu^1$ 和 $W_\mu^2$ 线性组合出复矢量场

$$\boxed{W_\mu^+ \equiv \frac{1}{\sqrt{2}}(W_\mu^1 - \mathrm{i}W_\mu^2),} \tag{66}$$

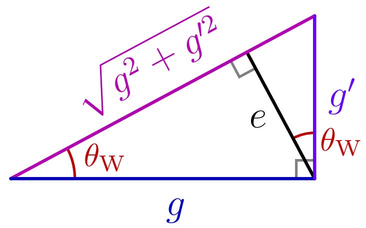



图 2: $g$、$g'$、$e$ 和 $\theta_\mathrm{W}$ 的关系。

它的厄米共轭为

$$W_\mu^- \equiv (W_\mu^+)^\dagger = \frac{1}{\sqrt{2}}(W_\mu^1 + \mathrm{i}W_\mu^2), \tag{67}$$

则

$$W_\mu^1 = \frac{1}{\sqrt{2}}(W_\mu^+ + W_\mu^-), \quad W_\mu^2 = \frac{\mathrm{i}}{\sqrt{2}}(W_\mu^+ - W_\mu^-). \tag{68}$$

从而

$$\begin{aligned} \frac{1}{2}(W^{1,\mu}W_\mu^1 + W^{2,\mu}W_\mu^2) &= \frac{1}{4}[(W^{+,\mu} + W^{-,\mu})(W_\mu^+ + W_\mu^-) - (W^{+,\mu} - W^{-,\mu})(W_\mu^+ - W_\mu^-)] \\ &= W^{+,\mu}W_\mu^-, \end{aligned} \tag{69}$$

(56) 式化为

$$\mathcal{L}_\mathrm{GBM} = m_W^2 W^{+,\mu}W_\mu^- + \frac{1}{2}m_Z^2 Z^\mu Z_\mu. \tag{70}$$

复矢量场 $W_\mu^+$ 描述一对正反矢量玻色子 $W^\pm$，质量为 $m_W$。可见，BEH 机制使传递弱相互作用的规范玻色子 $W^\pm$ 和 $Z$ 获得了质量。

接下来用质量本征态 $W_\mu^\pm$、$A_\mu$ 和 $Z_\mu$ 表达协变动能项 $(D^\mu\Phi)^\dagger(D_\mu\Phi)$。注意到

$$A_\mu = s_\mathrm{W}W_\mu^3 + c_\mathrm{W}B_\mu, \quad Z_\mu = c_\mathrm{W}W_\mu^3 - s_\mathrm{W}B_\mu, \tag{71}$$

$$B_\mu = c_\mathrm{W}A_\mu - s_\mathrm{W}Z_\mu, \quad W_\mu^3 = s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu, \tag{72}$$

有

$$\begin{aligned} g'B_\mu + gW_\mu^3 &= g'(c_\mathrm{W}A_\mu - s_\mathrm{W}Z_\mu) + g(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu) = \frac{2gg'}{\sqrt{g^2+g'^2}}A_\mu + \frac{g^2-g'^2}{\sqrt{g^2+g'^2}}Z_\mu \\ &= 2eA_\mu + \frac{g}{c_\mathrm{W}}(c_\mathrm{W}^2 - s_\mathrm{W}^2)Z_\mu, \end{aligned} \tag{73}$$

其中

$$\boxed{e \equiv \frac{gg'}{\sqrt{g^2+g'^2}} = gs_\mathrm{W} = g'c_\mathrm{W}.} \tag{74}$$

后面的讨论将表明 $e$ 就是单位电荷量。$g$、$g'$、$e$ 和 $\theta_\mathrm{W}$ 的关系如图 2 所示。再利用

$$g'B_\mu - gW_\mu^3 = g'(c_\mathrm{W}A_\mu - s_\mathrm{W}Z_\mu) - g(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu) = -\left(\frac{gs_\mathrm{W}^2}{c_\mathrm{W}} + gc_\mathrm{W}\right)Z_\mu = -\frac{g}{c_\mathrm{W}}Z_\mu, \tag{75}$$

得

$$\begin{aligned} g'B_\mu Y_H + gW_\mu^a\tau^a &= \frac{1}{2}\begin{pmatrix} g'B_\mu + gW_\mu^3 & g(W_\mu^1 - \mathrm{i}W_\mu^2) \\ g(W_\mu^1 + \mathrm{i}W_\mu^2) & g'B_\mu - gW_\mu^3 \end{pmatrix} \\ &= \begin{pmatrix} eA_\mu + \dfrac{g}{2c_\mathrm{W}}(c_\mathrm{W}^2 - s_\mathrm{W}^2)Z_\mu & \dfrac{g}{\sqrt{2}}W_\mu^+ \\ \dfrac{g}{\sqrt{2}}W_\mu^- & -\dfrac{g}{2c_\mathrm{W}}Z_\mu \end{pmatrix}. \end{aligned} \tag{76}$$

在幺正规范下，

$$\begin{aligned} (D^\mu\Phi)^\dagger(D_\mu\Phi) &= \left|\begin{pmatrix} \partial_\mu + \mathrm{i}eA_\mu + \dfrac{\mathrm{i}g}{2c_\mathrm{W}}(c_\mathrm{W}^2 - s_\mathrm{W}^2)Z_\mu & \dfrac{\mathrm{i}g}{\sqrt{2}}W_\mu^+ \\ \dfrac{\mathrm{i}g}{\sqrt{2}}W_\mu^- & \partial_\mu - \dfrac{\mathrm{i}g}{2c_\mathrm{W}}Z_\mu \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+H \end{pmatrix}\right|^2 \\ &= \frac{1}{2}\left(-\frac{\mathrm{i}g}{\sqrt{2}}W^{-,\mu}(v+H) \quad \partial^\mu H + \frac{\mathrm{i}g}{2c_\mathrm{W}}Z^\mu(v+H)\right)\begin{pmatrix} \dfrac{\mathrm{i}g}{\sqrt{2}}W_\mu^+(v+H) \\ \partial_\mu H - \dfrac{\mathrm{i}g}{2c_\mathrm{W}}Z_\mu(v+H) \end{pmatrix} \\ &= \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + (v+H)^2\left(\frac{g^2}{4}W_\mu^+W^{-,\mu} + \frac{g^2}{8c_\mathrm{W}^2}Z_\mu Z^\mu\right) \\ &= \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + m_\mathrm{W}^2 W_\mu^+W^{-,\mu} + \frac{1}{2}m_Z^2 Z_\mu Z^\mu \\ &\quad + gm_\mathrm{W}HW_\mu^+W^{-,\mu} + \frac{gm_Z}{2c_\mathrm{W}}HZ_\mu Z^\mu + \frac{g^2}{4}H^2W_\mu^+W^{-,\mu} + \frac{g^2}{8c_\mathrm{W}^2}H^2Z_\mu Z^\mu. \end{aligned} \tag{77}$$

除了 $W^\pm$ 和 $Z$ 玻色子的质量项之外，还出现了 Higgs 玻色子 $H$ 与 $W^\pm$、$Z$ 的三线性和四线性耦合项。

Higgs 场 $\Phi(x)$ 的弱超荷为 $+1/2$。引入 $\Phi(x)$ 的共轭态

$$\boxed{\tilde{\Phi}(x) = \mathrm{i}\sigma^2\Phi^*(x) = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix}\begin{pmatrix} \phi^-(x) \\ \phi^{0*}(x) \end{pmatrix} = \begin{pmatrix} \phi^{0*}(x) \\ -\phi^-(x) \end{pmatrix},} \tag{78}$$

其中 $\phi^- \equiv (\phi^+)^*$，则 $\tilde{\Phi}(x)$ 是弱超荷为 $-1/2$ 的 $\mathrm{SU(2)_L}$ 二重态。在幺正规范下，$\tilde{\Phi}(x)$ 化为

$$\tilde{\Phi}(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} & 1 \\ -1 & \end{pmatrix}\begin{pmatrix} 0 \\ v+H(x) \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} v+H(x) \\ 0 \end{pmatrix}. \tag{79}$$

用 $\Phi(x)$、$\tilde{\Phi}(x)$ 和费米子场组成满足 $\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_\mathrm{Y}$ 规范对称性的 Yukawa 相互作用拉氏量

$$\boxed{\mathcal{L}_\mathrm{Y} = -\tilde{y}_{d,ij}\bar{Q}_{i\mathrm{L}}d'_{j\mathrm{R}}\Phi - \tilde{y}_{u,ij}\bar{Q}_{i\mathrm{L}}u'_{j\mathrm{R}}\tilde{\Phi} - y_{\ell_i}\bar{L}_{i\mathrm{L}}\ell_{i\mathrm{R}}\Phi + \mathrm{H.c.},} \tag{80}$$

其中 H.c. 表示厄米共轭，Yukawa 耦合常数 $\tilde{y}_{d,ij}$ 和 $\tilde{y}_{u,ij}$ 联系着不同代的夸克场，而 $y_{\ell_i}$ 只联系同一代的轻子场。在幺正规范下，利用

$$\bar{Q}_{i\mathrm{L}}\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\bar{u}'_{i\mathrm{L}} & \bar{d}'_{i\mathrm{L}}\end{pmatrix}\begin{pmatrix}0\\ v+H\end{pmatrix} = \frac{1}{\sqrt{2}}(v+H)\bar{d}'_{i\mathrm{L}}, \tag{81}$$

$$\bar{Q}_{i\mathrm{L}}\tilde{\Phi} = \frac{1}{\sqrt{2}}\begin{pmatrix}\bar{u}'_{i\mathrm{L}} & \bar{d}'_{i\mathrm{L}}\end{pmatrix}\begin{pmatrix}v+H\\ 0\end{pmatrix} = \frac{1}{\sqrt{2}}(v+H)\bar{u}'_{i\mathrm{L}}, \tag{82}$$

$$\bar{L}_{i\mathrm{L}}\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\bar{\nu}_{i\mathrm{L}} & \bar{\ell}_{i\mathrm{L}}\end{pmatrix}\begin{pmatrix}0\\ v+H\end{pmatrix} = \frac{1}{\sqrt{2}}(v+H)\bar{\ell}_{i\mathrm{L}}, \tag{83}$$

推出

$$\mathcal{L}_\mathrm{Y} = -\frac{1}{\sqrt{2}}(v+H)(\tilde{y}_{d,ij}\bar{d}'_{i\mathrm{L}}d'_{j\mathrm{R}} + \tilde{y}_{u,ij}\bar{u}'_{i\mathrm{L}}u'_{j\mathrm{R}} + y_{\ell_i}\bar{\ell}_{i\mathrm{L}}\ell_{i\mathrm{R}} + \mathrm{H.c.}). \tag{84}$$

$\tilde{y}_{d,ij}$ 和 $\tilde{y}_{u,ij}$ 可看作 $3\times 3$ 矩阵 $\tilde{y}_d$ 和 $\tilde{y}_u$ 的元素。$\tilde{y}_d\tilde{y}_d^\dagger$ 和 $\tilde{y}_u\tilde{y}_u^\dagger$ 是厄米矩阵，必定可以分别通过幺正矩阵 $U_d$ 和 $U_u$ 对角化成 $y_\mathrm{D}^2$ 和 $y_\mathrm{U}^2$ 两个对角元为实数的对角矩阵，满足 $U_d^\dagger\tilde{y}_d\tilde{y}_d^\dagger U_d = y_\mathrm{D}^2$ 和 $U_u^\dagger\tilde{y}_u\tilde{y}_u^\dagger U_u = y_\mathrm{U}^2$，即

$$\tilde{y}_d\tilde{y}_d^\dagger = U_d y_\mathrm{D}^2 U_d^\dagger,\quad \tilde{y}_u\tilde{y}_u^\dagger = U_u y_\mathrm{U}^2 U_u^\dagger. \tag{85}$$

符合这两条式子的 $\tilde{y}_d$ 和 $\tilde{y}_u$ 可以表达为

$$\tilde{y}_d = U_d y_\mathrm{D} K_d^\dagger,\quad \tilde{y}_u = U_u y_\mathrm{U} K_u^\dagger, \tag{86}$$

其中对角矩阵 $y_\mathrm{D}$ 和 $y_\mathrm{U}$ 满足 $y_\mathrm{D}y_\mathrm{D} = y_\mathrm{D}^2$ 和 $y_\mathrm{U}y_\mathrm{U} = y_\mathrm{U}^2$，而 $K_d^\dagger$ 和 $K_u^\dagger$ 是两个幺正矩阵。

将 $y_\mathrm{D}$ 和 $y_\mathrm{U}$ 表示成

$$y_\mathrm{D} = \begin{pmatrix}y_{d_1} & & \\ & y_{d_2} & \\ & & y_{d_3}\end{pmatrix} = \begin{pmatrix}y_d & & \\ & y_s & \\ & & y_b\end{pmatrix},\quad y_\mathrm{U} = \begin{pmatrix}y_{u_1} & & \\ & y_{u_2} & \\ & & y_{u_3}\end{pmatrix} = \begin{pmatrix}y_u & & \\ & y_c & \\ & & y_t\end{pmatrix}. \tag{87}$$

通过幺正变换定义

$$d_{i\mathrm{L}} \equiv (U_d^\dagger)_{ij}d'_{j\mathrm{L}},\quad d_{i\mathrm{R}} \equiv (K_d^\dagger)_{ij}d'_{j\mathrm{R}},\quad u_{i\mathrm{L}} \equiv (U_u^\dagger)_{ij}u'_{j\mathrm{L}},\quad u_{i\mathrm{R}} \equiv (K_u^\dagger)_{ij}u'_{j\mathrm{R}}, \tag{88}$$

则 $\bar{d}_{i\mathrm{L}} = \bar{d}'_{j\mathrm{L}}U_{d,ji}$，$\bar{u}_{i\mathrm{L}} = \bar{u}'_{j\mathrm{L}}U_{u,ji}$，从而

$$\tilde{y}_{d,ij}\bar{d}'_{i\mathrm{L}}d'_{j\mathrm{R}} = \bar{d}'_{i\mathrm{L}}(U_d y_d K_d^\dagger)_{ij}d'_{j\mathrm{R}} = \bar{d}'_{i\mathrm{L}}U_{d,ik}y_{d_k}(K_d^\dagger)_{kj}d'_{j\mathrm{R}} = y_{d_k}\bar{d}_{k\mathrm{L}}d_{k\mathrm{R}} = y_{d_i}\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}}, \tag{89}$$

$$\tilde{y}_{u,ij}\bar{u}'_{i\mathrm{L}}u'_{j\mathrm{R}} = \bar{u}'_{i\mathrm{L}}(U_u y_u K_u^\dagger)_{ij}u'_{j\mathrm{R}} = y_{u_i}\bar{u}_{i\mathrm{L}}u_{i\mathrm{R}}, \tag{90}$$

故

$$\begin{aligned}\mathcal{L}_\mathrm{Y} &= -\frac{1}{\sqrt{2}}(v+H)(y_{d_i}\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}} + y_{u_i}\bar{u}_{i\mathrm{L}}u_{i\mathrm{R}} + y_{\ell_i}\bar{\ell}_{i\mathrm{L}}\ell_{i\mathrm{R}} + \mathrm{H.c.})\\ &= -m_{d_i}\bar{d}_i d_i - m_{u_i}\bar{u}_i u_i - m_{\ell_i}\bar{\ell}_i\ell_i - \frac{m_{d_i}}{v}H\bar{d}_i d_i - \frac{m_{u_i}}{v}H\bar{u}_i u_i - \frac{m_{\ell_i}}{v}H\bar{\ell}_i\ell_i,\end{aligned} \tag{91}$$

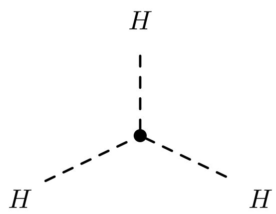
其中前三项是费米子质量项，后三项是 Higgs 玻色子与费米子的 Yukawa 耦合项。于是，三代夸克和带电轻子获得了质量

$$\boxed{m_{d_i} \equiv \frac{1}{\sqrt{2}}y_{d_i}v, \quad m_{u_i} \equiv \frac{1}{\sqrt{2}}y_{u_i}v, \quad m_{\ell_i} \equiv \frac{1}{\sqrt{2}}y_{\ell_i}v.} \tag{92}$$

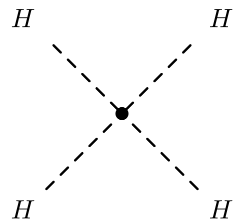
$d'_{i\mathrm{L}}$、$d'_{i\mathrm{R}}$、$u'_{i\mathrm{L}}$ 和 $u'_{i\mathrm{R}}$ 称为规范本征态（也称为味本征态），$d_{i\mathrm{L}}$、$d_{i\mathrm{R}}$、$u_{i\mathrm{L}}$ 和 $u_{i\mathrm{R}}$ 称为质量本征态。

下面给出幺正规范下 Higgs 场的顶点 Feynman 规则。

Higgs 玻色子自耦合：

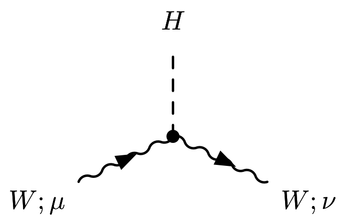



$$= -\frac{3\mathrm{i}m_H^2}{v} = -6\mathrm{i}\lambda v$$

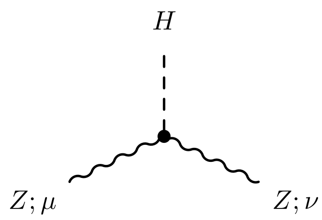



$$= -\frac{3\mathrm{i}m_H^2}{v^2} = -6\mathrm{i}\lambda \tag{93}$$

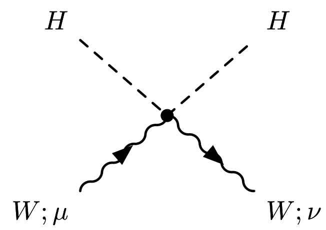
Higgs 玻色子与电弱规范玻色子的耦合：



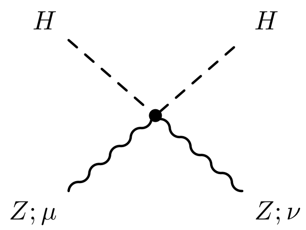

$$= \mathrm{i}gm_W g^{\mu\nu} = \frac{\mathrm{i}g^2 v}{2}g^{\mu\nu}$$




$$= \frac{\mathrm{i}gm_Z}{c_\mathrm{W}}g^{\mu\nu} = \frac{\mathrm{i}g^2 v}{2c_\mathrm{W}^2}g^{\mu\nu} \tag{94}$$

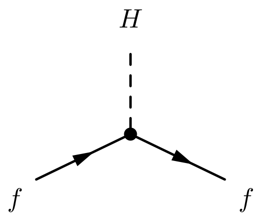



$$= \frac{\mathrm{i}g^2}{2}g^{\mu\nu}$$




$$= \frac{\mathrm{i}g^2}{2c_\mathrm{W}^2}g^{\mu\nu} \tag{95}$$

Higgs 玻色子与费米子 $f = d_i, u_i, \ell_i$ 的耦合:




$$= -\frac{\mathrm{i}m_f}{v} = -\frac{\mathrm{i}y_f}{\sqrt{2}} \tag{96}$$

## 4.2 费米子电弱规范相互作用

(88) 式意味着

$$d'_{i\mathrm{L}} = U_{d,ij} d_{j\mathrm{L}}, \quad d'_{i\mathrm{R}} = K_{d,ij} d_{j\mathrm{R}}, \quad u'_{i\mathrm{L}} = U_{u,ij} u_{j\mathrm{L}}, \quad u'_{i\mathrm{R}} = K_{u,ij} u_{j\mathrm{R}}, \tag{97}$$

从而

$$\bar{d}'_{i\mathrm{L}} \gamma^\mu d'_{i\mathrm{L}} = \bar{d}_{j\mathrm{L}} (U_d^\dagger)_{ji} \gamma^\mu U_{d,ik} d_{k\mathrm{L}} = \bar{d}_{j\mathrm{L}} \delta_{jk} \gamma^\mu d_{k\mathrm{L}} = \bar{d}_{i\mathrm{L}} \gamma^\mu d_{i\mathrm{L}} \tag{98}$$

同理有

$$\bar{u}'_{i\mathrm{L}} \gamma^\mu u'_{i\mathrm{L}} = \bar{u}_{i\mathrm{L}} \gamma^\mu u_{i\mathrm{L}}, \quad \bar{d}'_{i\mathrm{R}} \gamma^\mu d'_{i\mathrm{R}} = \bar{d}_{i\mathrm{R}} \gamma^\mu d_{i\mathrm{R}}, \quad \bar{u}'_{i\mathrm{R}} \gamma^\mu u'_{i\mathrm{R}} = \bar{u}_{i\mathrm{R}} \gamma^\mu u_{i\mathrm{R}}. \tag{99}$$

另一方面,

$$\bar{u}'_{i\mathrm{L}} \gamma^\mu d'_{i\mathrm{L}} = \bar{u}_{j\mathrm{L}} (U_u^\dagger)_{ji} \gamma^\mu U_{d,ik} d_{k\mathrm{L}} = \bar{u}_{i\mathrm{L}} \gamma^\mu V_{ij} d_{j\mathrm{L}} \tag{100}$$

$$\bar{d}'_{i\mathrm{L}} \gamma^\mu u'_{i\mathrm{L}} = \bar{d}_{j\mathrm{L}} (U_d^\dagger)_{ji} \gamma^\mu U_{u,ik} u_{k\mathrm{L}} = \bar{d}_{j\mathrm{L}} V_{ji}^\dagger \gamma^\mu u_{i\mathrm{L}} \tag{101}$$

其中

$$\boxed{V \equiv U_u^\dagger U_d} \tag{102}$$

称为 Cabibbo-Kobayashi-Maskawa (CKM) 矩阵，其厄米共轭矩阵为 $V^\dagger = U_d^\dagger U_u$。注意，幺正矩阵 $U_u$ 和 $U_d$ 的起源不同，因而一般来说它们是不相等的，从而 $V$ 不是恒等矩阵。

$\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ 规范不变的费米子协变动能项为

$$\mathcal{L}_{\mathrm{EWF}} = \bar{Q}_{i\mathrm{L}} \mathrm{i} \not{D} Q_{i\mathrm{L}} + \bar{u}'_{i\mathrm{R}} \mathrm{i} \not{D} u'_{i\mathrm{R}} + \bar{d}'_{i\mathrm{R}} \mathrm{i} \not{D} d'_{i\mathrm{R}} + \bar{L}_{i\mathrm{L}} \mathrm{i} \not{D} L_{i\mathrm{L}} + \bar{\ell}_{i\mathrm{R}} \mathrm{i} \not{D} \ell_{i\mathrm{R}}. \tag{103}$$

根据 $Q$ 的定义 (39)，有

$$\begin{aligned} g'YB_\mu + gT^3 W_\mu^3 &= g'Y(c_\mathrm{W} A_\mu - s_\mathrm{W} Z_\mu) + gT^3(s_\mathrm{W} A_\mu + c_\mathrm{W} Z_\mu) \\ &= e(Y + T^3) A_\mu + \left( g c_\mathrm{W} T^3 - \frac{g s_\mathrm{W}}{c_\mathrm{W}} s_\mathrm{W} Y \right) Z_\mu = QeA_\mu + \frac{g}{c_\mathrm{W}} (T^3 c_\mathrm{W}^2 - Y s_\mathrm{W}^2) Z_\mu \\ &= QeA_\mu + \frac{g}{c_\mathrm{W}} (T^3 - Q s_\mathrm{W}^2) Z_\mu, \end{aligned} \tag{104}$$

故

$$\begin{aligned}D_\mu Q_{i\mathrm{L}} &= (\partial_\mu + \mathrm{i}g' B_\mu Y + \mathrm{i}g W^a_\mu \tau^a) Q_{i\mathrm{L}}\\
&= \partial_\mu Q_{i\mathrm{L}} + \mathrm{i}\begin{pmatrix} g' Y B_\mu + g T^3 W^3_\mu & \dfrac{g}{2}(W^1_\mu - \mathrm{i}W^2_\mu) \\ \dfrac{g}{2}(W^1_\mu + \mathrm{i}W^2_\mu) & g' Y B_\mu + g T^3 W^3_\mu \end{pmatrix} Q_{i\mathrm{L}}\\
&= \partial_\mu Q_{i\mathrm{L}} + \mathrm{i}\begin{pmatrix} QeA_\mu + \dfrac{g}{c_\mathrm{W}}(T^3 - Qs^2_\mathrm{W})Z_\mu & \dfrac{g}{\sqrt{2}} W^+_\mu \\ \dfrac{g}{\sqrt{2}} W^-_\mu & QeA_\mu + \dfrac{g}{c_\mathrm{W}}(T^3 - Qs^2_\mathrm{W})Z_\mu \end{pmatrix}\begin{pmatrix} u'_{i\mathrm{L}} \\ d'_{i\mathrm{L}}\end{pmatrix}\\
&= \partial_\mu Q_{i\mathrm{L}} + \mathrm{i}\begin{pmatrix} \left[QeA_\mu + \dfrac{g}{c_\mathrm{W}}(T^3 - Qs^2_\mathrm{W})Z_\mu\right] u'_{i\mathrm{L}} + \dfrac{g}{\sqrt{2}} W^+_\mu d'_{i\mathrm{L}} \\ \dfrac{g}{\sqrt{2}} W^-_\mu u'_{i\mathrm{L}} + \left[QeA_\mu + \dfrac{g}{c_\mathrm{W}}(T^3 - Qs^2_\mathrm{W})Z_\mu\right] d'_{i\mathrm{L}}\end{pmatrix}. \end{aligned} \tag{105}$$

于是

$$\begin{aligned}\bar{Q}_{i\mathrm{L}}\mathrm{i}\not{D} Q_{i\mathrm{L}} &\supset -\left[QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs^2_\mathrm{W})Z_\mu\right]\bar{u}'_{i\mathrm{L}}\gamma^\mu u'_{i\mathrm{L}} - \left[QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs^2_\mathrm{W})Z_\mu\right]\bar{d}'_{i\mathrm{L}}\gamma^\mu d'_{i\mathrm{L}}\\
&\quad -\frac{g}{\sqrt{2}} W^+_\mu \bar{u}'_{i\mathrm{L}}\gamma^\mu d'_{i\mathrm{L}} - \frac{g}{\sqrt{2}} W^-_\mu \bar{d}'_{i\mathrm{L}}\gamma^\mu u'_{i\mathrm{L}}\\
&= -\left(QeA_\mu + \frac{g}{c_\mathrm{W}} g_\mathrm{L} Z_\mu\right)\bar{u}_i\gamma^\mu \frac{1-\gamma^5}{2} u_i - \left(QeA_\mu + \frac{g}{c_\mathrm{W}} g_\mathrm{L} Z_\mu\right)\bar{d}_i\gamma^\mu \frac{1-\gamma^5}{2} d_i\\
&\quad -\frac{g}{\sqrt{2}} W^+_\mu \bar{u}_i\gamma^\mu P_\mathrm{L} V_{ij} d_j - \frac{g}{\sqrt{2}} W^-_\mu \bar{d}_j V^\dagger_{ji}\gamma^\mu P_\mathrm{L} u_i,\end{aligned} \tag{106}$$

其中左手耦合系数

$$\boxed{g_\mathrm{L} \equiv T^3 - Qs^2_\mathrm{W}.} \tag{107}$$

另一方面,

$$\begin{aligned}D_\mu d'_{i\mathrm{R}} &= (\partial_\mu + \mathrm{i}g' B_\mu Y) d'_{i\mathrm{R}} = \partial_\mu d'_{i\mathrm{R}} + \mathrm{i}g' Q(c_\mathrm{W} A_\mu - s_\mathrm{W} Z_\mu) d'_{i\mathrm{R}}\\
&= \partial_\mu d'_{i\mathrm{R}} + \mathrm{i}QeA_\mu d'_{i\mathrm{R}} - \frac{\mathrm{i}g}{c_\mathrm{W}} Q s^2_\mathrm{W} Z_\mu d'_{i\mathrm{R}},\end{aligned} \tag{108}$$

则

$$\begin{aligned}&\bar{u}'_{i\mathrm{R}}\mathrm{i}\not{D} u'_{i\mathrm{R}} + \bar{d}'_{i\mathrm{R}}\mathrm{i}\not{D} d'_{i\mathrm{R}}\\
&\supset -\left(QeA_\mu - \frac{g}{c_\mathrm{W}} Q s^2_\mathrm{W} Z_\mu\right)\bar{u}'_{i\mathrm{R}}\gamma^\mu u'_{i\mathrm{R}} - \left(QeA_\mu - \frac{g}{c_\mathrm{W}} Q s^2_\mathrm{W} Z_\mu\right)\bar{d}'_{i\mathrm{R}}\gamma^\mu d'_{i\mathrm{R}}\\
&= -\left(QeA_\mu + \frac{g}{c_\mathrm{W}} g_\mathrm{R} Z_\mu\right)\bar{u}_i\gamma^\mu \frac{1+\gamma^5}{2} u_i - \left(QeA_\mu + \frac{g}{c_\mathrm{W}} g_\mathrm{R} Z_\mu\right)\bar{d}_i\gamma^\mu \frac{1+\gamma^5}{2} d_i,\end{aligned} \tag{109}$$

其中右手耦合系数

$$\boxed{g_{\mathrm{R}} \equiv -Qs_{\mathrm{W}}^2.} \tag{110}$$

引入矢量流和轴矢量流耦合系数

$$\boxed{g_{\mathrm{V}} \equiv g_{\mathrm{L}} + g_{\mathrm{R}} = T^3 - 2Qs_{\mathrm{W}}^2, \quad g_{\mathrm{A}} \equiv g_{\mathrm{L}} - g_{\mathrm{R}} = T^3,} \tag{111}$$

得

$$\begin{aligned}
&\bar{Q}_{i\mathrm{L}} \mathrm{i} \not{D} Q_{i\mathrm{L}} + \bar{u}'_{i\mathrm{R}} \mathrm{i} \not{D} u'_{i\mathrm{R}} + \bar{d}'_{i\mathrm{R}} \mathrm{i} \not{D} d'_{i\mathrm{R}} \\
\supset\ & -Qe\bar{u}_i\gamma^\mu u_i A_\mu - Qe\bar{d}_i\gamma^\mu d_i A_\mu - \frac{g}{2c_{\mathrm{W}}} \bar{u}_i\gamma^\mu(g_{\mathrm{V}} - g_{\mathrm{A}}\gamma^5)u_i Z_\mu - \frac{g}{2c_{\mathrm{W}}} \bar{d}_i\gamma^\mu(g_{\mathrm{V}} - g_{\mathrm{A}}\gamma^5)d_i Z_\mu \\
& - \frac{g}{\sqrt{2}} W_\mu^+ \bar{u}_i \gamma^\mu P_{\mathrm{L}} V_{ij} d_j - \frac{g}{\sqrt{2}} W_\mu^- \bar{d}_j V_{ji}^\dagger \gamma^\mu P_{\mathrm{L}} u_i.
\end{aligned} \tag{112}$$

同理，有

$$\begin{aligned}
\bar{L}_{i\mathrm{L}} \mathrm{i} \not{D} L_{i\mathrm{L}} + \bar{\ell}_{i\mathrm{R}} \mathrm{i} \not{D} \ell_{i\mathrm{R}} \supset & -Qe\bar{\ell}_i\gamma^\mu \ell_i A_\mu - \frac{g}{2c_{\mathrm{W}}} \bar{\ell}_i\gamma^\mu(g_{\mathrm{V}} - g_{\mathrm{A}}\gamma^5)\ell_i Z_\mu - \frac{g}{2c_{\mathrm{W}}} \bar{\nu}_i\gamma^\mu(g_{\mathrm{V}} - g_{\mathrm{A}}\gamma^5)\nu_i Z_\mu \\
& - \frac{g}{\sqrt{2}} W_\mu^+ \bar{\nu}_i \gamma^\mu P_{\mathrm{L}} \ell_i - \frac{g}{\sqrt{2}} W_\mu^- \bar{\ell}_i \gamma^\mu P_{\mathrm{L}} \nu_i.
\end{aligned} \tag{113}$$

总结起来，将费米子电弱规范相互作用写成流耦合的形式，

$$\boxed{\mathcal{L}_{\mathrm{EWF}} \supset -A_\mu J_{\mathrm{EM}}^\mu - Z_\mu J_Z^\mu - W_\mu^+ J_W^{+,\mu} - W_\mu^- J_W^{-,\mu},} \tag{114}$$

其中 $f$ 代表任意费米子场，电磁流

$$\boxed{J_{\mathrm{EM}}^\mu \equiv \sum_f Q_f e \bar{f} \gamma^\mu f,} \tag{115}$$

弱中性流

$$\boxed{J_Z^\mu \equiv \frac{g}{2c_{\mathrm{W}}} \sum_f \bar{f}\gamma^\mu(g_{\mathrm{V}}^f - g_{\mathrm{A}}^f\gamma^5)f = \frac{g}{c_{\mathrm{W}}} \sum_f (g_{\mathrm{L}}^f \bar{f}_{\mathrm{L}}\gamma^\mu f_{\mathrm{L}} + g_{\mathrm{R}}^f \bar{f}_{\mathrm{R}}\gamma^\mu f_{\mathrm{R}}),} \tag{116}$$

弱带电流

$$\boxed{J_W^{+,\mu} \equiv \frac{g}{\sqrt{2}}(\bar{u}_i\gamma^\mu V_{ij} P_{\mathrm{L}} d_j + \bar{\nu}_i \gamma^\mu P_{\mathrm{L}} \ell_i), \quad J_W^{-,\mu} \equiv (J_W^{+\mu})^\dagger = \frac{g}{\sqrt{2}}(\bar{d}_j V_{ji}^\dagger \gamma^\mu P_{\mathrm{L}} u_i + \bar{\ell}_i \gamma^\mu P_{\mathrm{L}} \nu_i).} \tag{117}$$

对于各种费米子，相关的系数如下，

$$Q_{u_i}=\frac{2}{3},\qquad Q_{d_i}=-\frac{1}{3},\qquad Q_{\nu_i}=0,\qquad Q_{\ell_i}=-1;\tag{118}$$

$$g_{\mathrm{V}}^{u_i}=\frac{1}{2}-\frac{4}{3}s_{\mathrm{W}}^2,\quad g_{\mathrm{A}}^{u_i}=\frac{1}{2};\quad g_{\mathrm{V}}^{d_i}=-\frac{1}{2}+\frac{2}{3}s_{\mathrm{W}}^2,\quad g_{\mathrm{A}}^{d_i}=-\frac{1}{2};\tag{119}$$

$$g_{\mathrm{V}}^{\nu_i}=\frac{1}{2},\qquad g_{\mathrm{A}}^{\nu_i}=\frac{1}{2};\qquad g_{\mathrm{V}}^{\ell_i}=-\frac{1}{2}+2s_{\mathrm{W}}^2,\quad g_{\mathrm{A}}^{\ell_i}=-\frac{1}{2};\tag{120}$$

$$g_{\mathrm{L}}^{u_i}=\frac{1}{2}-\frac{2}{3}s_{\mathrm{W}}^2,\quad g_{\mathrm{R}}^{u_i}=-\frac{2}{3}s_{\mathrm{W}}^2;\quad g_{\mathrm{L}}^{d_i}=-\frac{1}{2}+\frac{1}{3}s_{\mathrm{W}}^2,\quad g_{\mathrm{R}}^{d_i}=\frac{1}{3}s_{\mathrm{W}}^2;\tag{121}$$

$$g_{\mathrm{L}}^{\nu_i}=\frac{1}{2},\qquad g_{\mathrm{R}}^{\nu_i}=0;\qquad g_{\mathrm{L}}^{\ell_i}=-\frac{1}{2}+s_{\mathrm{W}}^2,\quad g_{\mathrm{R}}^{\ell_i}=s_{\mathrm{W}}^2.\tag{122}$$

可以看到，电磁流耦合与 QED 耦合完全相同，由此辨认出 $A_\mu$ 是电磁场，$e$ 是单位电荷量，由 $Q=T^3+Y$ 定义的 $Q$ 确实是电荷。为了保持电荷守恒，指定复矢量场 $W_\mu^+(x)$ 携带 $Q=+1$ 的电荷，从而 $W^\pm$ 玻色子的电荷为 $\pm1$。不同代夸克之间的相互作用只发生在弱带电流耦合中，源自 CKM 矩阵 $V$ 的非对角元，这是夸克味混合现象。

由于标准模型中没有引入右手中微子场 $\nu_{i\mathrm{R}}$，不能在 (80) 式中写出 $\bar{L}_{i\mathrm{L}}\nu_{j\mathrm{R}}\tilde{\Phi}$ 形式的相互作用项，因此中微子没有通过 BEH 机制获得质量，从而标准模型里不存在轻子味混合。具体来说，如果将左手中微子场、左手带电轻子场和右手带电轻子场的规范本征态分别记为 $\nu'_{i\mathrm{L}}$、$\ell'_{i\mathrm{L}}$ 和 $\ell'_{i\mathrm{R}}$，将 $L_{iL}$ 改写成

$$L_{iL}=\begin{pmatrix}\nu'_{i\mathrm{L}}\\ \ell'_{i\mathrm{L}}\end{pmatrix},\tag{123}$$

而 Yukawa 相互作用项改写为

$$\mathcal{L}_{\mathrm{Y},L\ell\Phi}=-\tilde{y}_{\ell,ij}\bar{L}_{i\mathrm{L}}\ell'_{j\mathrm{R}}\Phi+\mathrm{H.c.},\tag{124}$$

那么幺正规范给出

$$\mathcal{L}_{\mathrm{Y},L\ell\Phi}\supset-\frac{1}{\sqrt{2}}(v+H)\tilde{y}_{\ell,ij}\bar{\ell}'_{i\mathrm{L}}\ell'_{j\mathrm{R}}+\mathrm{H.c.}\tag{125}$$

利用幺正矩阵 $U_\ell$ 和 $K_\ell^\dagger$ 将 Yukawa 耦合矩阵 $\tilde{y}_\ell$ 表达为 $\tilde{y}_{\ell,ij}=U_\ell y_{\mathrm{L}}K_\ell^\dagger$，其中

$$y_{\mathrm{L}}=\begin{pmatrix}y_{\ell_1}&&\\&y_{\ell_2}&\\&&y_{\ell_3}\end{pmatrix}=\begin{pmatrix}y_e&&\\&y_\mu&\\&&y_\tau\end{pmatrix}\tag{126}$$

是对角矩阵。引入左右手带电轻子场的质量本征态

$$\ell_{i\mathrm{L}}\equiv(U_\ell^\dagger)_{ij}\ell'_{j\mathrm{L}},\quad \ell_{i\mathrm{R}}\equiv(K_\ell^\dagger)_{ij}\ell'_{j\mathrm{R}},\tag{127}$$

则

$$\tilde{y}_{\ell,ij}\bar{\ell}'_{i\mathrm{L}}\ell'_{j\mathrm{R}}=\bar{\ell}'_{i\mathrm{L}}(U_\ell y_{\mathrm{L}}K_\ell^\dagger)_{ij}\ell'_{j\mathrm{R}}=y_{\ell_i}\bar{\ell}_{i\mathrm{L}}\ell_{i\mathrm{R}}\tag{128}$$

给出对角化的带电轻子质量项和 Yukawa 相互作用项，与 (91) 式一致。此时，引入左手中微子场的“质量本征态”

$$\nu_{i\mathrm{L}} \equiv (U_\ell^\dagger)_{ij}\nu'_{j\mathrm{L}}\,, \tag{129}$$

那么 $\bar{L}_{i\mathrm{L}}\mathrm{i}\not{D}L_{i\mathrm{L}}$ 中的带电流相互作用算符

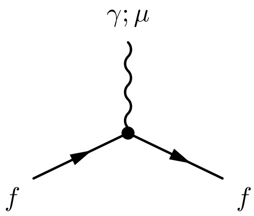
$$W_\mu^+\bar{\nu}'_{i\mathrm{L}}\gamma^\mu\ell'_{i\mathrm{L}} = W_\mu^+\bar{\nu}'_{i\mathrm{L}}\gamma^\mu U_{\ell,ij}\ell_{j\mathrm{L}} = W_\mu^+\bar{\nu}_{j\mathrm{L}}\gamma^\mu\ell_{j\mathrm{L}} = W_\mu^+\bar{\nu}_i\gamma^\mu P_\mathrm{L}\ell_i \tag{130}$$

与 (113) 式中的相应算符是一样的，没有出现轻子味混合，在物理上没有任何不同。简单起见，我们可以让左右手带电轻子场和左手中微子场的质量本征态同时等于它们的规范本征态，从而之前的讨论都是合理的，(80) 式中只需要让 Yukawa 耦合 $y_{\ell_i}$ 联系同一代轻子场。

不过，中微子振荡实验表明中微子具有微小质量，而且存在味混合，这是超出标准模型的新物理。仿照夸克味混合的讨论，需要引入类似于 CKM 矩阵的 Pontecorvo–Maki–Nakagawa–Sakata (PMNS) 矩阵来描述轻子味混合，但这不在本文的讨论范围之内。

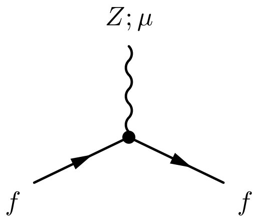
下面给出费米子电弱规范相互作用顶点的 Feynman 规则。

QED 耦合:



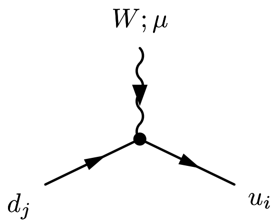

$$= -\mathrm{i}Q_f e\gamma^\mu \tag{131}$$

费米子与 $Z$ 玻色子的耦合:

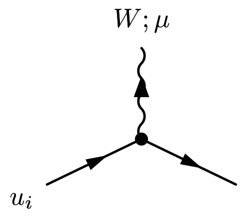



$$= -\frac{\mathrm{i}g}{2c_\mathrm{W}}\gamma^\mu(g_\mathrm{V}^f - g_\mathrm{A}^f\gamma^5) \tag{132}$$

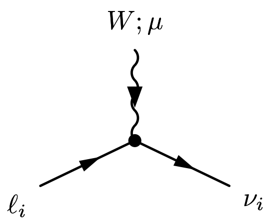
费米子与 $W^\pm$ 玻色子的耦合:



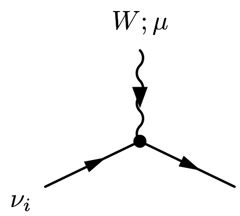

$$= -\frac{\mathrm{i}g}{\sqrt{2}}V_{ij}\gamma^\mu P_\mathrm{L}$$




$$= -\frac{\mathrm{i}g}{\sqrt{2}}V_{ji}^\dagger\gamma^\mu P_\mathrm{L} \tag{133}$$



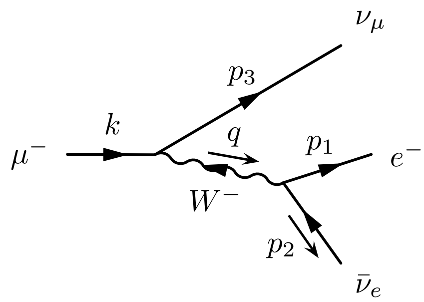

$$= -\frac{\mathrm{i}g}{\sqrt{2}}\gamma^\mu P_\mathrm{L}$$




$$= -\frac{\mathrm{i}g}{\sqrt{2}}\gamma^\mu P_\mathrm{L} \tag{134}$$

考虑 $\mu$ 子衰变的最主要过程 $\mu^- \to e^- \bar{\nu}_e \nu_\mu$，相应的领头阶不变振幅为

$$\mathrm{i}\mathcal{M} =$$




$$\begin{aligned} &= \left(\frac{-\mathrm{i}g}{\sqrt{2}}\right)^2 \bar{u}(p_3)\gamma^\mu P_\mathrm{L} u(k) \, \frac{-\mathrm{i}(g_{\mu\nu} - q_\mu q_\nu/m_W^2)}{q^2 - m_W^2} \, \bar{u}(p_1)\gamma^\nu P_\mathrm{L} v(p_2) \\ &= \frac{\mathrm{i}g^2(g_{\mu\nu} - q_\mu q_\nu/m_W^2)}{8(q^2 - m_W^2)} \, \bar{u}(p_3)\gamma^\mu(1-\gamma^5)u(k)\bar{u}(p_1)\gamma^\nu(1-\gamma^5)v(p_2) \end{aligned} \tag{135}$$

由于 $m_\mu \ll m_W$，$W$ 传播子的四维动量 $q^\mu$ 满足 $q^2 \ll m_W^2$。因此，可在低能近似下忽略 $q^\mu$ 和 $q^2$，将不变振幅化为

$$\mathrm{i}\mathcal{M} \simeq -\frac{\mathrm{i}g^2}{8m_W^2} \, \bar{u}(p_3)\gamma^\mu(1-\gamma^5)u(k)\bar{u}(p_1)\gamma_\mu(1-\gamma^5)v(p_2). \tag{136}$$

可以将这样的振幅看作有效拉氏量

$$\mathcal{L}_\mathrm{eff} = -\frac{G_\mathrm{F}}{\sqrt{2}} \, \bar{\nu}_\mu \gamma^\mu (1-\gamma^5)\mu \, \bar{e}\gamma_\mu(1-\gamma^5)\nu_e + \mathrm{H.c.} \tag{137}$$

的结果，其中 Fermi 常数 $G_\mathrm{F}$ 定义为

$$\boxed{\frac{G_\mathrm{F}}{\sqrt{2}} \equiv \frac{g^2}{8m_W^2}.} \tag{138}$$

可以进一步将 $\mathcal{L}_\mathrm{eff}$ 推广到其它参与弱相互作用的标准模型费米子，而耦合常数 $G_\mathrm{F}$ 是普适的，对所有费米子都适用，这样得到的理论称为**四费米子相互作用理论**。为了解释 $\beta$ 衰变，Enrico Fermi 于 1933 年首次提出这个理论。现在认为它是标准模型弱相互作用的低能有效理论。

忽略电子质量，由以上振幅推出 $\mu$ 子寿命为

$$\tau_\mu = \frac{1}{\Gamma_\mu} \simeq \frac{192\pi^3}{G_\mathrm{F}^2 m_\mu^5}. \tag{139}$$

于是，通过测量 $\mu$ 子寿命可以得到 Fermi 常数的观测值。根据 (57) 式，Fermi 常数 $G_\mathrm{F}$ 与 Higgs 场真空期待值 $v$ 的关系为

$$G_\mathrm{F} = \frac{\sqrt{2}g^2}{8m_W^2} = \frac{\sqrt{2}g^2}{2g^2v^2} = \frac{1}{\sqrt{2}v^2}, \qquad \boxed{v = (\sqrt{2}G_\mathrm{F})^{-1/2}.} \tag{140}$$

## 4.3 电弱规范场的自相互作用

电弱规范场自相互作用的拉氏量是

$$\mathcal{L}_{\mathrm{EWG}} = -\frac{1}{4}W^a_{\mu\nu}W^{a,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \tag{141}$$

其中

$$W^a_{\mu\nu} \equiv \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g\varepsilon^{abc}W^b_\mu W^c_\nu, \quad B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu. \tag{142}$$

利用 (68) 和 (72) 式，推出

$$\begin{aligned}
W^2_\mu W^3_\nu - W^3_\mu W^2_\nu &= \frac{\mathrm{i}}{\sqrt{2}}[(W^+_\mu - W^-_\mu)(s_\mathrm{W}A_\nu + c_\mathrm{W}Z_\nu) - (s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu)(W^+_\nu - W^-_\nu)] \\
&= \frac{\mathrm{i}}{\sqrt{2}}[s_\mathrm{W}(W^+_\mu A_\nu - A_\mu W^+_\nu) + c_\mathrm{W}(W^+_\mu Z_\nu - Z_\mu W^+_\nu) \\
&\quad - s_\mathrm{W}(W^-_\mu A_\nu - A_\mu W^-_\nu) - c_\mathrm{W}(W^-_\mu Z_\nu - Z_\mu W^-_\nu)],
\end{aligned} \tag{143}$$

$$\begin{aligned}
W^3_\mu W^1_\nu - W^1_\mu W^3_\nu &= \frac{1}{\sqrt{2}}[(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu)(W^+_\nu + W^-_\nu) - (W^+_\mu + W^-_\mu)(s_\mathrm{W}A_\nu + c_\mathrm{W}Z_\nu)] \\
&= -\frac{1}{\sqrt{2}}[s_\mathrm{W}(W^+_\mu A_\nu - A_\mu W^+_\nu) + c_\mathrm{W}(W^+_\mu Z_\nu - Z_\mu W^+_\nu) \\
&\quad + s_\mathrm{W}(W^-_\mu A_\nu - A_\mu W^-_\nu) + c_\mathrm{W}(W^-_\mu Z_\nu - Z_\mu W^-_\nu)].
\end{aligned} \tag{144}$$

从而

$$\begin{aligned}
W^1_{\mu\nu} &= \partial_\mu W^1_\nu - \partial_\nu W^1_\mu - g\varepsilon^{1bc}W^b_\mu W^c_\nu = \partial_\mu W^1_\nu - \partial_\nu W^1_\mu - g(W^2_\mu W^3_\nu - W^3_\mu W^2_\nu) \\
&= \frac{1}{\sqrt{2}}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu) + \frac{1}{\sqrt{2}}(\partial_\mu W^-_\nu - \partial_\nu W^-_\mu) - g(W^2_\mu W^3_\nu - gW^3_\mu W^2_\nu) \\
&= \frac{1}{\sqrt{2}}\{\partial_\mu W^+_\nu - \partial_\nu W^+_\mu - \mathrm{i}g[s_\mathrm{W}(W^+_\mu A_\nu - A_\mu W^+_\nu) + c_\mathrm{W}(W^+_\mu Z_\nu - Z_\mu W^+_\nu)]\} \\
&\quad + \frac{1}{\sqrt{2}}\{\partial_\mu W^-_\nu - \partial_\nu W^-_\mu + \mathrm{i}g[s_\mathrm{W}(W^-_\mu A_\nu - A_\mu W^-_\nu) + c_\mathrm{W}(W^-_\mu Z_\nu - Z_\mu W^-_\nu)]\} \\
&= \frac{1}{\sqrt{2}}(F^+_{\mu\nu} + F^-_{\mu\nu}),
\end{aligned} \tag{145}$$

其中，

$$F^+_{\mu\nu} \equiv \partial_\mu W^+_\nu - \partial_\nu W^+_\mu - \mathrm{i}e(W^+_\mu A_\nu - A_\mu W^+_\nu) - \mathrm{i}gc_\mathrm{W}(W^+_\mu Z_\nu - Z_\mu W^+_\nu), \tag{146}$$

$$F^-_{\mu\nu} \equiv \partial_\mu W^-_\nu - \partial_\nu W^-_\mu + \mathrm{i}e(W^-_\mu A_\nu - A_\mu W^-_\nu) + \mathrm{i}gc_\mathrm{W}(W^-_\mu Z_\nu - Z_\mu W^-_\nu). \tag{147}$$

另一方面，

$$\begin{aligned}
W^2_{\mu\nu} &= \partial_\mu W^2_\nu - \partial_\nu W^2_\mu - g\varepsilon^{2bc}W^b_\mu W^c_\nu = \partial_\mu W^2_\nu - \partial_\nu W^2_\mu - g(W^3_\mu W^1_\nu - W^1_\mu W^3_\nu) \\
&= \frac{\mathrm{i}}{\sqrt{2}}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu) - \frac{\mathrm{i}}{\sqrt{2}}(\partial_\mu W^-_\nu - \partial_\nu W^-_\mu) - g(W^3_\mu W^1_\nu - W^1_\mu W^3_\nu)
\end{aligned}$$

$$\begin{aligned}
&= \frac{\mathrm{i}}{\sqrt{2}}\{\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ - \mathrm{i}g[s_\mathrm{W}(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_\mathrm{W}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)]\} \\
&\quad - \frac{\mathrm{i}}{\sqrt{2}}\{\partial_\mu W_\nu^- - \partial_\nu W_\mu^- + \mathrm{i}g[s_\mathrm{W}(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_\mathrm{W}(W_\mu^- Z_\nu - Z_\mu W_\nu^-)]\} \\
&= \frac{\mathrm{i}}{\sqrt{2}}(F_{\mu\nu}^+ - F_{\mu\nu}^-). \qquad (148)
\end{aligned}$$

因此

$$\begin{aligned}
&-\frac{1}{4}W_{\mu\nu}^1 W^{1,\mu\nu} - \frac{1}{4}W_{\mu\nu}^2 W^{2,\mu\nu} \\
&= -\frac{1}{8}(F_{\mu\nu}^+ + F_{\mu\nu}^-)(F^{+,\mu\nu} + F^{-,\mu\nu}) + \frac{1}{8}(F_{\mu\nu}^+ - F_{\mu\nu}^-)(F^{+,\mu\nu} - F^{-,\mu\nu}) = -\frac{1}{2}F_{\mu\nu}^+ F^{-,\mu\nu} \\
&= -\frac{1}{2}[\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ - \mathrm{i}e(W_\mu^+ A_\nu - A_\mu W_\nu^+) - \mathrm{i}gc_\mathrm{W}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)] \\
&\quad \times[\partial^\mu W^{-,\nu} - \partial^\nu W^{-,\mu} + \mathrm{i}e(W^{-,\mu}A^\nu - A^\mu W^{-,\nu}) + \mathrm{i}gc_\mathrm{W}(W^{-,\mu}Z^\nu - Z^\mu W^{-,\nu})] \\
&= -(\partial_\mu W_\nu^+)(\partial^\mu W^{-,\nu}) + (\partial_\mu W_\nu^+)(\partial^\nu W^{-,\mu}) \\
&\quad -\mathrm{i}e[(\partial_\mu W_\nu^+)W^{-,\mu}A^\nu - (\partial_\mu W_\nu^+)W^{-,\nu}A^\mu - W_\mu^+(\partial^\mu W^{-,\nu})A_\nu + W_\nu^+(\partial^\mu W^{-,\nu})A_\mu] \\
&\quad -\mathrm{i}gc_\mathrm{W}[(\partial_\mu W_\nu^+)W^{-,\mu}Z^\nu - (\partial_\mu W_\nu^+)W^{-,\nu}Z^\mu - W_\mu^+(\partial^\mu W^{-,\nu})Z_\nu + W_\nu^+(\partial^\mu W^{-,\nu})Z_\mu] \\
&\quad +e^2(W_\mu^+ W^{-,\nu}A_\nu A^\mu - W_\mu^+ W^{-,\mu}A_\nu A^\nu) + g^2 c_\mathrm{W}^2(W_\mu^+ W^{-,\nu}Z_\nu Z^\mu - W_\mu^+ W^{-,\mu}Z_\nu Z^\nu) \\
&\quad +egc_\mathrm{W}(W_\mu^+ W^{-,\nu}A_\nu Z^\mu + W_\mu^+ W^{-,\nu}A^\mu Z_\nu - 2W_\mu^+ W^{-,\mu}A_\nu Z^\nu). \qquad (149)
\end{aligned}$$

由

$$\begin{aligned}
W_\mu^1 W_\nu^2 - W_\mu^2 W_\nu^1 &= \frac{\mathrm{i}}{2}(W_\mu^+ + W_\mu^-)(W_\nu^+ - W_\nu^-) - \frac{\mathrm{i}}{2}(W_\mu^+ - W_\mu^-)(W_\nu^+ + W_\nu^-) \\
&= -\mathrm{i}(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+), \qquad (150)
\end{aligned}$$

得到

$$\begin{aligned}
W_{\mu\nu}^3 &= \partial_\mu W_\nu^3 - \partial_\nu W_\mu^3 - g\varepsilon^{3bc}W_\mu^b W_\nu^c = \partial_\mu W_\nu^3 - \partial_\nu W_\mu^3 - g(W_\mu^1 W_\nu^2 - W_\mu^2 W_\nu^1) \\
&= s_\mathrm{W}\partial_\mu A_\nu + c_\mathrm{W}\partial_\mu Z_\nu - s_\mathrm{W}\partial_\nu A_\mu - c_\mathrm{W}\partial_\nu Z_\mu - g(W_\mu^1 W_\nu^2 - W_\mu^2 W_\nu^1) \\
&= s_\mathrm{W}(\partial_\mu A_\nu - \partial_\nu A_\mu) + c_\mathrm{W}(\partial_\mu Z_\nu - \partial_\nu Z_\mu) + \mathrm{i}g(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+), \qquad (151) \\
B_{\mu\nu} &= \partial_\mu(c_\mathrm{W}A_\nu - s_\mathrm{W}Z_\nu) - \partial_\nu(c_\mathrm{W}A_\mu - s_\mathrm{W}Z_\mu) \\
&= c_\mathrm{W}(\partial_\mu A_\nu - \partial_\nu A_\mu) - s_\mathrm{W}(\partial_\mu Z_\nu - \partial_\nu Z_\mu). \qquad (152)
\end{aligned}$$

于是

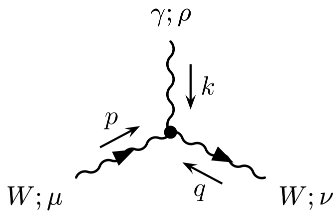
$$\begin{aligned}
&-\frac{1}{4}W_{\mu\nu}^3 W^{3,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
&= -\frac{1}{2}[(\partial_\mu A_\nu)(\partial^\mu A^\nu) - (\partial_\mu A_\nu)(\partial^\nu A^\mu)] - \frac{1}{2}[(\partial_\mu Z_\nu)(\partial^\mu Z^\nu) - (\partial_\mu Z_\nu)(\partial^\nu Z^\mu)] \\
&\quad -\mathrm{i}e[W^{+,\mu}W^{-,\nu}(\partial_\mu A_\nu) - W^{+,\nu}W^{-,\mu}(\partial_\mu A_\nu)] - \mathrm{i}gc_\mathrm{W}[W^{+,\mu}W^{-,\nu}(\partial_\mu Z_\nu) - W^{+,\nu}W^{-,\mu}(\partial_\mu Z_\nu)]
\end{aligned}$$

$$
+\frac{g^2}{2}(W^+_\mu W^{+,\mu}W^-_\nu W^{-,\nu} - W^+_\mu W^{+,\nu}W^-_\nu W^{-,\mu}). \tag{153}
$$

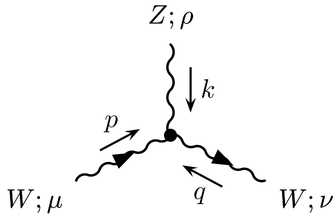
综合起来，有

$$
\begin{aligned}
\mathcal{L}_{\mathrm{EWG}} = & \frac{1}{2}[(\partial_\mu A_\nu)(\partial^\nu A^\mu) - (\partial_\mu A_\nu)(\partial^\mu A^\nu)] + \frac{1}{2}[(\partial_\mu Z_\nu)(\partial^\nu Z^\mu) - (\partial_\mu Z_\nu)(\partial^\mu Z^\nu)] \\
& +(\partial_\mu W^+_\nu)(\partial^\nu W^{-,\mu}) - (\partial_\mu W^+_\nu)(\partial^\mu W^{-,\nu}) + \frac{g^2}{2}(W^+_\mu W^{+,\mu}W^-_\nu W^{-,\nu} - W^+_\mu W^{+,\nu}W^-_\nu W^{-,\mu}) \\
& -\mathrm{i}e[(\partial_\mu W^+_\nu)W^{-,\mu}A^\nu - (\partial_\mu W^+_\nu)W^{-,\nu}A^\mu - W^{+,\mu}(\partial_\mu W^-_\nu)A^\nu + W^{+,\nu}(\partial_\mu W^-_\nu)A^\mu \\
& \qquad +W^{+,\mu}W^{-,\nu}(\partial_\mu A_\nu) - W^{+,\nu}W^{-,\mu}(\partial_\mu A_\nu)] \\
& -\mathrm{i}gc_{\mathrm{W}}[(\partial_\mu W^+_\nu)W^{-,\mu}Z^\nu - (\partial_\mu W^+_\nu)W^{-,\nu}Z^\mu - W^{+,\mu}(\partial_\mu W^-_\nu)Z^\nu + W^{+,\nu}(\partial_\mu W^-_\nu)Z^\mu \\
& \qquad + W^{+,\mu}W^{-,\nu}(\partial_\mu Z_\nu) - W^{+,\nu}W^{-,\mu}(\partial_\mu Z_\nu)] \\
& +e^2(W^+_\mu W^{-,\nu}A_\nu A^\mu - W^+_\mu W^{-,\mu}A_\nu A^\nu) + g^2c^2_{\mathrm{W}}(W^+_\mu W^{-,\nu}Z_\nu Z^\mu - W^+_\mu W^{-,\mu}Z_\nu Z^\nu) \\
& +egc_{\mathrm{W}}(W^+_\mu W^{-,\nu}A_\nu Z^\mu + W^+_\mu W^{-,\nu}A^\mu Z_\nu - 2W^+_\mu W^{-,\mu}A_\nu Z^\nu).
\end{aligned} \tag{154}
$$

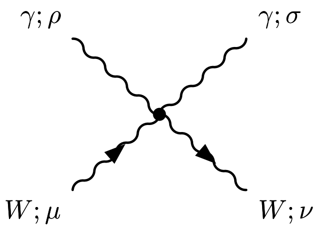
下面是电弱规范玻色子自耦合的 Feynman 规则：



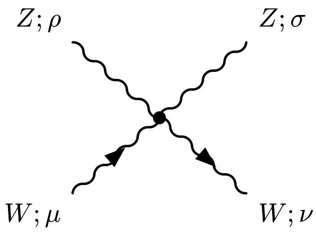

$$
= \mathrm{i}e[g^{\mu\nu}(p-q)^\rho + g^{\nu\rho}(q-k)^\mu + g^{\rho\mu}(k-p)^\nu] \tag{155}
$$



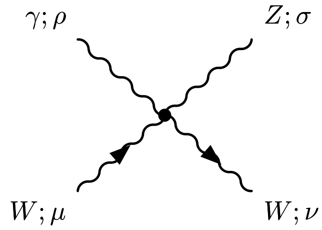

$$
= \mathrm{i}gc_{\mathrm{W}}[g^{\mu\nu}(p-q)^\rho + g^{\nu\rho}(q-k)^\mu + g^{\rho\mu}(k-p)^\nu] \tag{156}
$$



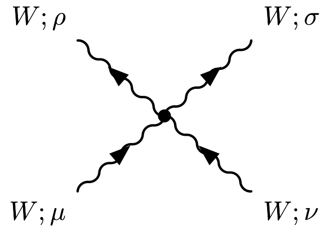

$$
= \mathrm{i}e^2(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma}) \tag{157}
$$




$$
= \mathrm{i}g^2c^2_{\mathrm{W}}(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma}) \tag{158}
$$




$$= \mathrm{i}egc_{\mathrm{W}}(g^{\mu\rho}g^{\nu\sigma}+g^{\mu\sigma}g^{\nu\rho}-2g^{\mu\nu}g^{\rho\sigma}) \tag{159}$$




$$= -\mathrm{i}g^2(g^{\mu\rho}g^{\nu\sigma}+g^{\mu\sigma}g^{\nu\rho}-2g^{\mu\nu}g^{\rho\sigma}) \tag{160}$$

# 5 $\mathrm{R}_\xi$ 规范下电弱拉氏量和 Feynman 规则

在 $\mathbf{R}_\xi$ 规范下，将 Higgs 场参数化为

$$\Phi(x)=\begin{pmatrix}\phi^+(x)\\ \frac{1}{\sqrt{2}}[v+H(x)+\mathrm{i}\chi(x)]\end{pmatrix}, \tag{161}$$

其中 $\phi^+$ 和 $\chi$ 是 Nambu-Goldstone 标量场。那么，$\tilde{\Phi}(x)$ 的形式是

$$\tilde{\Phi}(x)=\begin{pmatrix}\phi^{0*}(x)\\ -\phi^-(x)\end{pmatrix}=\begin{pmatrix}\frac{1}{\sqrt{2}}[v+H(x)-\mathrm{i}\chi(x)]\\ -\phi^-(x)\end{pmatrix}. \tag{162}$$

利用

$$\Phi^\dagger\Phi = \frac{1}{2}(v^2+H^2+2vH+\chi^2)+|\phi^+|^2, \tag{163}$$

$$(\Phi^\dagger\Phi)^2 = \frac{1}{4}(v^2+H^2+2vH+\chi^2)^2+|\phi^+|^4+|\phi^+|^2(v^2+H^2+2vH+\chi^2), \tag{164}$$

推出 Higgs 场势能项

$$\begin{aligned}
&-V_{\mathrm{H}}(\Phi)=\mu^2\Phi^\dagger\Phi-\lambda(\Phi^\dagger\Phi)^2\\
=&\frac{\mu^2}{2}(v^2+H^2+2vH+\chi^2)+\mu^2|\phi^+|^2-\frac{\lambda}{4}(v^2+H^2+2vH+\chi^2)^2-\lambda|\phi^+|^4\\
&-\lambda|\phi^+|^2(v^2+H^2+2vH+\chi^2)\\
=&\frac{1}{2}\left(\mu^2-\frac{\lambda}{2}v^2\right)v^2+\frac{1}{2}(\mu^2-3\lambda v^2)H^2+(\mu^2-\lambda v^2)vH+\frac{1}{2}(\mu^2-\lambda v^2)\chi^2-\frac{\lambda}{4}H^4-\frac{\lambda}{4}\chi^4\\
&-\lambda vH^3-\frac{\lambda}{2}H^2\chi^2-\lambda vH\chi^2+(\mu^2-\lambda v^2)|\phi^+|^2-\lambda|\phi^+|^4-\lambda|\phi^+|^2(H^2+2vH+\chi^2)\\
=&\frac{\lambda}{4}v^4-\lambda v^2H^2-\frac{\lambda}{4}H^4-\frac{\lambda}{4}\chi^4-\lambda vH^3-\frac{\lambda}{2}H^2\chi^2-\lambda vH\chi^2
\end{aligned}$$

$$
\begin{aligned}
&-\lambda\phi^+\phi^-(\phi^+\phi^- + H^2 + 2vH + \chi^2)\\
=\ &\frac{1}{8}\,m_H^2 v^2 - \frac{1}{2}\,m_H^2 H^2 - \frac{m_H^2}{2v}\,H^3 - \frac{m_H^2}{8v^2}\,H^4 - \frac{m_H^2}{2v}\,H\chi^2 - \frac{m_H^2}{4v^2}\,H^2\chi^2 - \frac{m_H^2}{8v^2}\,\chi^4\\
&-\frac{m_H^2}{2v^2}\,\phi^+\phi^-(\phi^+\phi^- + H^2 + 2vH + \chi^2). \qquad (165)
\end{aligned}
$$

由 $V = U_u^\dagger U_d$ 得到 $V^\dagger = U_d^\dagger U_u$ 和 $U_d = U_u V$，$U_u = U_d V^\dagger$，则

$$
\tilde{y}_d = U_d y_{\mathrm{D}} K_d^\dagger = U_u V y_{\mathrm{D}} K_d^\dagger, \quad \tilde{y}_u = U_u y_{\mathrm{U}} K_u^\dagger = U_d V^\dagger y_{\mathrm{U}} K_u^\dagger, \qquad (166)
$$

故

$$
\begin{aligned}
\tilde{y}_{d,ij}\bar{u}'_{i\mathrm{L}}d'_{j\mathrm{R}} &= \bar{u}'_{i\mathrm{L}}(U_u V y_{\mathrm{D}} K_d^\dagger)_{ij} d'_{j\mathrm{R}} = \bar{u}'_{i\mathrm{L}} U_{u,ik} V_{kl} y_{d_l} (K_d^\dagger)_{lj} d'_{j\mathrm{R}} = y_{d_j}\bar{u}_{i\mathrm{L}} V_{ij} d_{j\mathrm{R}}, \qquad (167)\\
\tilde{y}_{u,ij}\bar{d}'_{i\mathrm{L}}u'_{j\mathrm{R}} &= \bar{d}'_{i\mathrm{L}}(U_d V^\dagger y_{\mathrm{U}} K_u^\dagger)_{ij} u'_{j\mathrm{R}} = \bar{d}'_{i\mathrm{L}} U_{d,ik} V^\dagger_{kl} y_{u_l} (K_u^\dagger)_{lj} u'_{j\mathrm{R}} = y_{u_i}\bar{d}_{j\mathrm{L}} V^\dagger_{ji} u_{i\mathrm{R}}. \qquad (168)
\end{aligned}
$$

结合 (89) 和 (90) 式，得

$$
\begin{aligned}
\tilde{y}_{d,ij}\bar{Q}_{i\mathrm{L}}d'_{j\mathrm{R}}\Phi &= \tilde{y}_{d,ij}\left[\bar{u}'_{i\mathrm{L}}d'_{j\mathrm{R}}\phi^+ + \frac{1}{\sqrt{2}}\,\bar{d}'_{i\mathrm{L}}d'_{j\mathrm{R}}(v + H + \mathrm{i}\chi)\right]\\
&= y_{d_j}\bar{u}_{i\mathrm{L}}V_{ij}d_{j\mathrm{R}}\phi^+ + \frac{y_{d_i}}{\sqrt{2}}\,\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}}(v + H + \mathrm{i}\chi), \qquad (169)\\
\tilde{y}_{u,ij}\bar{Q}_{i\mathrm{L}}u'_{j\mathrm{R}}\tilde{\Phi} &= \tilde{y}_{u,ij}\left[\frac{1}{\sqrt{2}}\,\bar{u}'_{i\mathrm{L}}u'_{j\mathrm{R}}(v + H - \mathrm{i}\chi) - \bar{d}'_{i\mathrm{L}}u'_{j\mathrm{R}}\phi^-\right]\\
&= \frac{y_{u_i}}{\sqrt{2}}\,\bar{u}_{i\mathrm{L}}u_{i\mathrm{R}}(v + H - \mathrm{i}\chi) - y_{u_i}\bar{d}_{j\mathrm{L}}V^\dagger_{ji}u_{i\mathrm{R}}\phi^-. \qquad (170)
\end{aligned}
$$

从而，Yukawa 相互作用拉氏量化为

$$
\begin{aligned}
&\mathcal{L}_{\mathrm{Y}} = -\tilde{y}_{d,ij}\bar{Q}_{i\mathrm{L}}d'_{j\mathrm{R}}\Phi - \tilde{y}_{u,ij}\bar{Q}_{i\mathrm{L}}u'_{j\mathrm{R}}\tilde{\Phi} - y_{\ell_i}\bar{L}_{i\mathrm{L}}\ell_{i\mathrm{R}}\Phi + \mathrm{H.c.}\\
=\ &-y_{d_j}\bar{u}_{i\mathrm{L}}V_{ij}d_{j\mathrm{R}}\phi^+ - \frac{y_{d_i}}{\sqrt{2}}\,\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}}(v + H + \mathrm{i}\chi) - \frac{y_{u_i}}{\sqrt{2}}\,\bar{u}_{i\mathrm{L}}u_{i\mathrm{R}}(v + H - \mathrm{i}\chi) + y_{u_i}\bar{d}_{j\mathrm{L}}V^\dagger_{ji}u_{i\mathrm{R}}\phi^-\\
&-y_{\ell_i}\bar{\nu}_{i\mathrm{L}}\ell_{i\mathrm{R}}\phi^+ - \frac{y_{\ell_i}}{\sqrt{2}}\,\bar{\ell}_{i\mathrm{L}}\ell_{i\mathrm{R}}(v + H + \mathrm{i}\chi) + \mathrm{H.c.}\\
=\ &-m_{d_i}\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}} - m_{u_i}\bar{u}_{i\mathrm{L}}u_{i\mathrm{R}} - m_{\ell_i}\bar{\ell}_{i\mathrm{L}}\ell_{i\mathrm{R}} - \frac{m_{d_i}}{v}\,\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}}(H + \mathrm{i}\chi) - \frac{m_{u_i}}{v}\,\bar{u}_{i\mathrm{L}}u_{i\mathrm{R}}(H - \mathrm{i}\chi)\\
&-\frac{m_{\ell_i}}{v}\,\bar{\ell}_{i\mathrm{L}}\ell_{i\mathrm{R}}(H + \mathrm{i}\chi) - \frac{\sqrt{2}m_{d_j}}{v}\,\bar{u}_{i\mathrm{L}}V_{ij}d_{j\mathrm{R}}\phi^+ + \frac{\sqrt{2}m_{u_i}}{v}\,\bar{d}_{j\mathrm{L}}V^\dagger_{ji}u_{i\mathrm{R}}\phi^- - \frac{\sqrt{2}m_{\ell_i}}{v}\,\bar{\nu}_{i\mathrm{L}}\ell_{i\mathrm{R}}\phi^+ + \mathrm{H.c.}\\
=\ &-m_{d_i}\bar{d}_i d_i - m_{u_i}\bar{u}_i u_i - m_{\ell_i}\bar{\ell}_i\ell_i - \frac{m_{d_i}}{v}\,H\bar{d}_i d_i - \frac{m_{u_i}}{v}\,H\bar{u}_i u_i - \frac{m_{\ell_i}}{v}\,H\bar{\ell}_i\ell_i\\
&-\frac{m_{d_i}}{v}\,\chi\bar{d}_i\mathrm{i}\gamma^5 d_i + \frac{m_{u_i}}{v}\,\chi\bar{u}_i\mathrm{i}\gamma^5 u_i - \frac{m_{\ell_i}}{v}\,\chi\bar{\ell}_i\mathrm{i}\gamma^5\ell_i + \frac{\sqrt{2}V_{ij}}{v}\,\phi^+\bar{u}_i(m_{u_i}P_{\mathrm{L}} - m_{d_j}P_{\mathrm{R}})d_j\\
&-\frac{\sqrt{2}V^\dagger_{ji}}{v}\,\phi^-\bar{d}_j(m_{d_j}P_{\mathrm{L}} - m_{u_i}P_{\mathrm{R}})u_i - \frac{\sqrt{2}m_{\ell_i}}{v}\,(\phi^+\bar{\nu}_i P_{\mathrm{R}}\ell_i + \phi^-\bar{\ell}_i P_{\mathrm{L}}\nu_i). \qquad (171)
\end{aligned}
$$

利用

$$
\begin{aligned}
D_\mu\Phi &= \begin{pmatrix} \partial_\mu + \mathrm{i}eA_\mu + \dfrac{\mathrm{i}g}{2c_\mathrm{W}}(c_\mathrm{W}^2 - s_\mathrm{W}^2)Z_\mu & \dfrac{\mathrm{i}g}{\sqrt{2}}\,W_\mu^+ \\ \dfrac{\mathrm{i}g}{\sqrt{2}}\,W_\mu^- & \partial_\mu - \dfrac{\mathrm{i}g}{2c_\mathrm{W}}\,Z_\mu \end{pmatrix} \begin{pmatrix} \phi^+ \\ \dfrac{1}{\sqrt{2}}(v + H + \mathrm{i}\chi) \end{pmatrix} \\
&= \begin{pmatrix} \partial_\mu\phi^+ + \mathrm{i}\left[eA_\mu + \dfrac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{2c_\mathrm{W}}\,Z_\mu\right]\phi^+ + \dfrac{\mathrm{i}g}{2}\,W_\mu^+(H + \mathrm{i}\chi) + \mathrm{i}m_W W_\mu^+ \\ \dfrac{1}{\sqrt{2}}\left[\partial_\mu(H + \mathrm{i}\chi) + \mathrm{i}gW_\mu^-\phi^+ - \dfrac{\mathrm{i}g}{2c_\mathrm{W}}\,Z_\mu(H + \mathrm{i}\chi) - \mathrm{i}m_Z Z_\mu\right] \end{pmatrix}, \qquad (172)
\end{aligned}
$$

将 Higgs 场协变动能项化为

$$
\begin{aligned}
&(D^\mu\Phi)^\dagger D_\mu\Phi \\
={}& \left|\partial_\mu\phi^+ + \mathrm{i}\left[eA_\mu + \frac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{2c_\mathrm{W}}Z_\mu\right]\phi^+ + \frac{\mathrm{i}g}{2}\,W_\mu^+(H + \mathrm{i}\chi) + \mathrm{i}m_W W_\mu^+\right|^2 \\
&+\frac{1}{2}\left|\partial_\mu(H + \mathrm{i}\chi) + \mathrm{i}gW_\mu^-\phi^+ - \frac{\mathrm{i}g}{2c_\mathrm{W}}\,Z_\mu(H + \mathrm{i}\chi) - \mathrm{i}m_Z Z_\mu\right|^2 \\
={}& (\partial^\mu\phi^+)(\partial_\mu\phi^-) + \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + \frac{1}{2}(\partial^\mu\chi)(\partial_\mu\chi) \\
&+\left(\mathrm{i}\partial^\mu\phi^-\left\{\left[eA_\mu + \frac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{2c_\mathrm{W}}Z_\mu\right]\phi^+ + \frac{g}{2}\,W_\mu^+(H + \mathrm{i}\chi) + m_W W_\mu^+\right\} + \mathrm{H.c.}\right) \\
&+\left\{\frac{\mathrm{i}}{2}\,\partial^\mu(H - \mathrm{i}\chi)\left[gW_\mu^-\phi^+ - \frac{g}{2c_\mathrm{W}}\,Z_\mu(H + \mathrm{i}\chi) - m_Z Z_\mu\right] + \mathrm{H.c.}\right\} \\
&+\left|\left[eA_\mu + \frac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{2c_\mathrm{W}}Z_\mu\right]\phi^+ + \frac{g}{2}\,W_\mu^+(H + \mathrm{i}\chi) + m_W W_\mu^+\right|^2 \\
&+\frac{1}{2}\left|gW_\mu^-\phi^+ - \frac{g}{2c_\mathrm{W}}\,Z_\mu(H + \mathrm{i}\chi) - m_Z Z_\mu\right|^2 \\
={}& (\partial^\mu\phi^+)(\partial_\mu\phi^-) + \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + \frac{1}{2}(\partial^\mu\chi)(\partial_\mu\chi) \\
&+m_W^2 W_\mu^+ W^{-,\mu} + \frac{1}{2}\,m_Z^2 Z_\mu Z^\mu + gm_W HW_\mu^+W^{-,\mu} + \frac{gm_Z}{2c_\mathrm{W}}\,HZ_\mu Z^\mu \\
&-\frac{g}{2}[\mathrm{i}W_\mu^+\phi^-\overleftrightarrow{\partial^\mu}(H + \mathrm{i}\chi) + \mathrm{H.c.}] - \mathrm{i}eA_\mu\phi^-\overleftrightarrow{\partial^\mu}\phi^+ - \frac{g}{2c_\mathrm{W}}\,Z_\mu[-\chi\overleftrightarrow{\partial^\mu}H + \mathrm{i}(c_\mathrm{W}^2 - s_\mathrm{W}^2)\phi^-\overleftrightarrow{\partial^\mu}\phi^+] \\
&+\frac{g^2}{4}\,W_\mu^+W^{-,\mu}(2\phi^+\phi^- + H^2 + \chi^2) + e^2A_\mu A^\mu\phi^+\phi^- \\
&+\frac{g^2}{4c_\mathrm{W}^2}\,Z_\mu Z^\mu\left[(c_\mathrm{W}^2 - s_\mathrm{W}^2)^2\phi^+\phi^- + \frac{1}{2}\,H^2 + \frac{1}{2}\,\chi^2\right] \\
&+\left[\frac{eg}{2}\,W_\mu^+A^\mu\phi^-(H + \mathrm{i}\chi) - \frac{g^2s_\mathrm{W}^2}{2c_\mathrm{W}}\,W_\mu^+Z^\mu\phi^-(H + \mathrm{i}\chi) + \mathrm{H.c.}\right] + \frac{eg(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{c_\mathrm{W}}\,A_\mu Z^\mu\phi^+\phi^- \\
&+(em_W A^\mu\phi^+W_\mu^- - gs_\mathrm{W}^2 m_Z Z^\mu\phi^+W_\mu^- + \mathrm{H.c.}) + \mathcal{L}_{\mathrm{b1}}, \qquad (173)
\end{aligned}
$$

其中

$$\mathcal{L}_{\mathrm{b1}}=\mathrm{i}m_W(\partial^\mu\phi^-)W^+_\mu-\mathrm{i}m_W(\partial^\mu\phi^+)W^-_\mu-m_Z(\partial^\mu\chi)Z_\mu. \tag{174}$$

在 R$_\xi$ 规范下，将规范固定函数设为

$$G^\pm=\frac{1}{\sqrt{\xi_W}}(\partial^\mu W^\pm_\mu\pm\mathrm{i}\xi_W m_W\phi^\pm),\quad G^Z=\frac{1}{\sqrt{\xi_Z}}(\partial^\mu Z_\mu+\xi_Z m_Z\chi),\quad G^\gamma=\frac{1}{\sqrt{\xi_\gamma}}\,\partial^\mu A_\mu, \tag{175}$$

它们在路径积分量子化中的泛函积分形式为

$$\begin{aligned}&\int\mathcal{D}\omega^+\int\mathcal{D}\omega^-\int\mathcal{D}\omega^Z\int\mathcal{D}\omega^\gamma\exp\left[-\mathrm{i}\int d^4x\left(\omega^+\omega^-+\frac{1}{2}\,\omega^Z\omega^Z+\frac{1}{2}\,\omega^\gamma\omega^\gamma\right)\right]\\&\qquad\times\delta(G^+-\omega^+)\delta(G^--\omega^-)\delta(G^Z-\omega^Z)\delta(G^\gamma-\omega^\gamma)\\&=\exp\left[-\mathrm{i}\int\mathrm{d}^4x\left(G^+G^-+\frac{1}{2}\,G^ZG^Z+\frac{1}{2}\,G^\gamma G^\gamma\right)\right].\end{aligned} \tag{176}$$

由此得到拉氏量中的规范固定项

$$\begin{aligned}&\mathcal{L}_{\mathrm{EW,GF}}=-G^+G^--\frac{1}{2}(G^Z)^2-\frac{1}{2}(G^\gamma)^2\\&=-\frac{1}{\xi_W}(\partial^\mu W^+_\mu+\mathrm{i}\xi_W m_W\phi^+)(\partial^\nu W^-_\nu-\mathrm{i}\xi_W m_W\phi^-)-\frac{1}{2\xi_Z}(\partial^\mu Z_\mu+\xi_Z m_Z\chi)^2-\frac{1}{2\xi_\gamma}(\partial^\mu A_\mu)^2\\&=-\frac{1}{\xi_W}(\partial^\mu W^+_\mu)(\partial^\nu W^-_\nu)-\frac{1}{2\xi_Z}(\partial^\mu Z_\mu)^2-\frac{1}{2\xi_\gamma}(\partial^\mu A_\mu)^2-\xi_W m_W^2\phi^+\phi^--\frac{1}{2}\,\xi_Z m_Z^2\chi^2+\mathcal{L}_{\mathrm{b2}}.\end{aligned} \tag{177}$$

可见，Nambu-Goldstone 玻色子 $\phi^\pm$ 和 $\chi$ 在 R$_\xi$ 规范下具有依赖于 $\xi_W$ 和 $\xi_Z$ 的非物理质量，

$$m_\phi=\sqrt{\xi_W}\,m_W,\quad m_\chi=\sqrt{\xi_Z}\,m_Z. \tag{178}$$

这里

$$\mathcal{L}_{\mathrm{b2}}=\mathrm{i}m_W\phi^-(\partial^\mu W^+_\mu)-\mathrm{i}m_W\phi^+\partial^\mu W^-_\mu-m_Z\chi\partial^\mu Z_\mu. \tag{179}$$

$\mathcal{L}_{\mathrm{b1}}$ 与 $\mathcal{L}_{\mathrm{b2}}$ 之和

$$\mathcal{L}_{\mathrm{b1}}+\mathcal{L}_{\mathrm{b2}}=\mathrm{i}m_W\partial^\mu(\phi^-W^+_\mu)-\mathrm{i}m_W\partial^\mu(\phi^+W^-_\mu)-m_Z\partial^\mu(\chi Z_\mu) \tag{180}$$

是全散度，不会有物理效应。可见，协变动能项中规范场与 Nambu-Goldstone 标量场之间的双线性耦合项 $\mathcal{L}_{\mathrm{b1}}$ 被规范固定项中的 $\mathcal{L}_{\mathrm{b2}}$ 抵消掉，这就是如此选取规范固定函数的目的。

这样一来，电弱规范场传播子相关拉氏量变成

$$\begin{aligned}\mathcal{L}_{\mathrm{EW,prop}}\;=\;&(\partial_\mu W^+_\nu)(\partial^\nu W^{-,\mu})-(\partial_\mu W^+_\nu)(\partial^\mu W^{-,\nu})-\frac{1}{\xi_W}(\partial^\mu W^+_\mu)(\partial^\nu W^-_\nu)+m_W^2W^+_\mu W^{-,\mu}\\&+\frac{1}{2}\left[(\partial_\mu Z_\nu)(\partial^\nu Z^\mu)-(\partial_\mu Z_\nu)(\partial^\mu Z^\nu)-\frac{1}{\xi_Z}(\partial^\mu Z_\mu)^2+m_Z^2Z_\mu Z^\mu\right]\end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{2}\left[(\partial_\mu A_\nu)(\partial^\nu A^\mu)-(\partial_\mu A_\nu)(\partial^\mu A^\nu)-\frac{1}{\xi_\gamma}(\partial^\mu A_\mu)^2\right]\\
\to\ & W_\mu^+\left[g^{\mu\nu}(\partial^2+m_W^2)-\left(1-\frac{1}{\xi_W}\right)\partial^\mu\partial^\nu\right]W_\nu^-\\
&+\frac{1}{2}Z_\mu\left[g^{\mu\nu}(\partial^2+m_Z^2)-\left(1-\frac{1}{\xi_Z}\right)\partial^\mu\partial^\nu\right]Z_\nu\\
&+\frac{1}{2}A_\mu\left[g^{\mu\nu}\partial^2-\left(1-\frac{1}{\xi_\gamma}\right)\partial^\mu\partial^\nu\right]A_\nu.
\end{aligned}
\tag{181}
$$

于是，光子的传播子与胶子形式类似，为

$$
\frac{-\mathrm{i}}{p^2+\mathrm{i}\epsilon}\left[g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}(1-\xi_\gamma)\right].
\tag{182}
$$

将 $W^\pm$ 传播子相关拉氏量变换到动量空间，得

$$
-g^{\mu\nu}(p^2-m_W^2)+\left(1-\frac{1}{\xi_W}\right)p^\mu p^\nu=-\left(g^{\mu\nu}-\frac{p^\mu p^\nu}{p^2}\right)(p^2-m_W^2)-\frac{p^\mu p^\nu}{p^2}\frac{p^2-\xi_W m_W^2}{\xi_W},
\tag{183}
$$

它的逆矩阵是

$$
-\frac{1}{p^2-m_W^2}\left(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)-\frac{\xi_W}{p^2-\xi_W m_W^2}\frac{p_\mu p_\nu}{p^2}=-\frac{1}{p^2-m_W^2}\left[g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2-\xi_W m_W^2}(1-\xi_W)\right],
\tag{184}
$$

这是因为由

$$
\left(g_{\rho\mu}-\frac{p_\rho p_\mu}{p^2}\right)\frac{p^\mu p^\nu}{p^2}=\frac{p_\rho p^\nu}{p^2}-\frac{p_\rho p^\nu}{p^2}=0,\quad \left(g_{\rho\mu}-\frac{p_\rho p_\mu}{p^2}\right)\left(g^{\mu\nu}-\frac{p^\mu p^\nu}{p^2}\right)=\delta_\rho{}^\nu-\frac{p_\rho p^\nu}{p^2}
\tag{185}
$$

得

$$
\begin{aligned}
&\left[-\frac{1}{p^2-m_W^2}\left(g_{\rho\mu}-\frac{p_\rho p_\mu}{p^2}\right)-\frac{\xi_W}{p^2-\xi_W m_W^2}\frac{p_\rho p_\mu}{p^2}\right]\\
&\quad\times\left[-\left(g^{\mu\nu}-\frac{p^\mu p^\nu}{p^2}\right)(p^2-m_W^2)-\frac{p^\mu p^\nu}{p^2}\frac{p^2-\xi_W m_W^2}{\xi_W}\right]\\
&=\delta_\rho{}^\nu-\frac{p_\rho p^\nu}{p^2}+\frac{p_\rho p^\nu}{p^2}=\delta_\rho{}^\nu.
\end{aligned}
\tag{186}
$$

从而，$W^\pm$ 传播子的形式为

$$
\frac{-\mathrm{i}}{p^2-m_W^2+\mathrm{i}\epsilon}\left[g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2-\xi_W m_W^2}(1-\xi_W)\right].
\tag{187}
$$

同理，$Z$ 传播子的形式为

$$
\frac{-\mathrm{i}}{p^2-m_Z^2+\mathrm{i}\epsilon}\left[g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2-\xi_Z m_Z^2}(1-\xi_Z)\right].
\tag{188}
$$

电弱规范场的无穷小规范变换形式是

$$\delta W_\mu^a = -\frac{1}{g}\,\partial_\mu\alpha^a + \varepsilon^{abc}W_\mu^b\alpha^c, \quad \delta B_\mu = -\frac{1}{g'}\,\partial_\mu\alpha^Y. \tag{189}$$

定义

$$\alpha^\pm \equiv \frac{1}{\sqrt{2}}(\alpha^1 \mp \mathrm{i}\alpha^2), \quad \alpha^Z \equiv \alpha^3 - \alpha^Y, \quad \alpha^\gamma \equiv s_\mathrm{W}^2\alpha^3 + c_\mathrm{W}^2\alpha^Y, \tag{190}$$

利用

$$\varepsilon^{1bc}W_\mu^b\alpha^c = W_\mu^2\alpha^3 - W_\mu^3\alpha^2, \quad \varepsilon^{2bc}W_\mu^b\alpha^c = -W_\mu^1\alpha^3 + W_\mu^3\alpha^1, \tag{191}$$

$$\pm\mathrm{i}\sqrt{2}\alpha^\pm = \pm\mathrm{i}\alpha^1 + \alpha^2, \qquad \pm\mathrm{i}\sqrt{2}W_\mu^\pm = \pm\mathrm{i}W_\mu^1 + W_\mu^2, \tag{192}$$

有

$$\begin{aligned}
\varepsilon^{1bc}W_\mu^b\alpha^c \mp \mathrm{i}\varepsilon^{2bc}W_\mu^b\alpha^c &= (W_\mu^2\alpha^3 - W_\mu^3\alpha^2) \mp \mathrm{i}(-W_\mu^1\alpha^3 + W_\mu^3\alpha^1)\\
&= (W_\mu^2 \pm \mathrm{i}W_\mu^1)\alpha^3 - W_\mu^3(\alpha^2 \pm \mathrm{i}\alpha^1)\\
&= \pm\mathrm{i}\sqrt{2}W_\mu^\pm(c_\mathrm{W}^2\alpha^Z + \alpha^\gamma) \mp \mathrm{i}\sqrt{2}(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu)\alpha^\pm,
\end{aligned} \tag{193}$$

$$\begin{aligned}
\varepsilon^{3bc}W_\mu^b\alpha^c &= W_\mu^1\alpha^2 - W_\mu^2\alpha^1 = \frac{1}{\sqrt{2}}(W_\mu^+ + W_\mu^-)\frac{\mathrm{i}}{\sqrt{2}}(\alpha^+ - \alpha^-) - \frac{\mathrm{i}}{\sqrt{2}}(W_\mu^+ - W_\mu^-)\frac{1}{\sqrt{2}}(\alpha^+ + \alpha^-)\\
&= -\mathrm{i}(W_\mu^+\alpha^- - W_\mu^-\alpha^+).
\end{aligned} \tag{194}$$

因此,

$$\begin{aligned}
\delta W_\mu^+ &= \frac{1}{\sqrt{2}}(\delta W_\mu^1 - \mathrm{i}\delta W_\mu^2) = -\frac{1}{\sqrt{2}g}\,\partial_\mu(\alpha^1 - \mathrm{i}\alpha^2) + \frac{1}{\sqrt{2}}(\varepsilon^{1bc}W_\mu^b\alpha^c - \mathrm{i}\varepsilon^{2bc}W_\mu^b\alpha^c)\\
&= -\frac{1}{g}\,\partial_\mu\alpha^+ - \mathrm{i}(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu)\alpha^+ + \mathrm{i}W_\mu^+(c_\mathrm{W}^2\alpha^Z + \alpha^\gamma),
\end{aligned} \tag{195}$$

$$\delta W_\mu^- = (\delta W_\mu^+)^\dagger = -\frac{1}{g}\,\partial_\mu\alpha^- + \mathrm{i}(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu)\alpha^- - \mathrm{i}W_\mu^-(c_\mathrm{W}^2\alpha^Z + \alpha^\gamma), \tag{196}$$

$$\begin{aligned}
\delta Z_\mu^a &= c_\mathrm{W}\delta W_\mu^3 - s_\mathrm{W}\delta B_\mu = -\frac{c_\mathrm{W}}{g}\,\partial_\mu\alpha^3 + c_\mathrm{W}\varepsilon^{3bc}W_\mu^b\alpha^c + \frac{s_\mathrm{W}}{g'}\,\partial_\mu\alpha^Y\\
&= -\frac{c_\mathrm{W}}{g}\,\partial_\mu\alpha^Z - \mathrm{i}c_\mathrm{W}(W_\mu^+\alpha^- - W_\mu^-\alpha^+),
\end{aligned} \tag{197}$$

$$\begin{aligned}
\delta A_\mu &= s_\mathrm{W}\delta W_\mu^3 + c_\mathrm{W}\delta B_\mu = -\frac{s_\mathrm{W}}{g}\,\partial_\mu\alpha^3 + s_\mathrm{W}\varepsilon^{3bc}W_\mu^b\alpha^c - \frac{c_\mathrm{W}}{g'}\,\partial_\mu\alpha^Y\\
&= -\frac{1}{e}\,\partial_\mu\alpha^\gamma - \mathrm{i}s_\mathrm{W}(W_\mu^+\alpha^- - W_\mu^-\alpha^+).
\end{aligned} \tag{198}$$

另一方面，根据

$$\alpha^aT^a + \alpha^YY_H = \frac{1}{2}(\alpha^a\sigma^a + \alpha^Y) = \frac{1}{2}\begin{pmatrix}\alpha^3 + \alpha^Y & \alpha^1 - \mathrm{i}\alpha^2\\ \alpha^1 + \mathrm{i}\alpha^2 & -\alpha^3 + \alpha^Y\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z & \sqrt{2}\alpha^+ \\ \sqrt{2}\alpha^- & -\alpha^Z \end{pmatrix}, \tag{199}$$

可知 Higgs 场的无穷小规范变换形式为

$$\begin{aligned} \delta\Phi &= \mathrm{i}(\alpha^a T^a + \alpha^Y Y_H)\Phi = \frac{\mathrm{i}}{2}\begin{pmatrix} 2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z & \sqrt{2}\alpha^+ \\ \sqrt{2}\alpha^- & -\alpha^Z \end{pmatrix}\begin{pmatrix} \phi^+ \\ \dfrac{1}{\sqrt{2}}(v + H + \mathrm{i}\chi) \end{pmatrix} \\ &= \begin{pmatrix} \dfrac{\mathrm{i}}{2}\{\phi^+[2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z] + (v + H + \mathrm{i}\chi)\alpha^+\} \\ \dfrac{1}{\sqrt{2}}\left[\mathrm{i}\phi^+\alpha^- - \dfrac{1}{2}(\mathrm{i}v + \mathrm{i}H - \chi)\alpha^Z\right] \end{pmatrix} = \begin{pmatrix} \delta\phi^+ \\ \dfrac{1}{\sqrt{2}}(\delta H + \mathrm{i}\delta\chi) \end{pmatrix}. \end{aligned} \tag{200}$$

利用

$$\mathrm{Re}(\phi^+\alpha^-) = \frac{1}{2}(\phi^+\alpha^- + \phi^-\alpha^+), \quad \mathrm{Im}(\phi^+\alpha^-) = -\frac{\mathrm{i}}{2}(\phi^+\alpha^- - \phi^-\alpha^+), \tag{201}$$

推出

$$\delta\phi^+ = \frac{\mathrm{i}}{2}\{\phi^+[2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z] + (v + H + \mathrm{i}\chi)\alpha^+\}, \tag{202}$$

$$\delta\phi^- = -\frac{\mathrm{i}}{2}\{\phi^-[2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z] + (v + H - \mathrm{i}\chi)\alpha^-\}, \tag{203}$$

$$\delta H = \frac{1}{2}[\mathrm{i}(\phi^+\alpha^- - \phi^-\alpha^+) + \chi\alpha^Z], \quad \delta\chi = \frac{1}{2}[\phi^+\alpha^- + \phi^-\alpha^+ - (v + H)\alpha^Z]. \tag{204}$$

于是，规范固定函数的无穷小规范变换为

$$\begin{aligned} \sqrt{\xi_W}\,\delta G^+ &= \partial^\mu \delta W_\mu^+ + \mathrm{i}\xi_W m_W \delta\phi^+ \\ &= \partial^\mu\left[-\frac{1}{g}\,\partial_\mu\alpha^+ - \mathrm{i}(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu)\alpha^+ + \mathrm{i}W_\mu^+(c_\mathrm{W}^2\alpha^Z + \alpha^\gamma)\right] \\ &\quad -\frac{1}{2}\,\xi_W m_W\left\{\phi^+[2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z] + (v + H + \mathrm{i}\chi)\alpha^+\right\}, \end{aligned} \tag{205}$$

$$\begin{aligned} \sqrt{\xi_W}\,\delta G^- &= \partial^\mu \delta W_\mu^- - \mathrm{i}\xi_W m_W \delta\phi^- \\ &= \partial^\mu\left[-\frac{1}{g}\,\partial_\mu\alpha^- + \mathrm{i}(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu)\alpha^- - \mathrm{i}W_\mu^-(c_\mathrm{W}^2\alpha^Z + \alpha^\gamma)\right] \\ &\quad -\frac{1}{2}\,\xi_W m_W\left\{\phi^-[2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z] + (v + H - \mathrm{i}\chi)\alpha^-\right\}, \end{aligned} \tag{206}$$

$$\begin{aligned} \sqrt{\xi_Z}\,\delta G^Z &= \partial^\mu \delta Z_\mu + \xi_Z m_Z \delta\chi \\ &= \partial^\mu\left[-\frac{c_\mathrm{W}}{g}\,\partial_\mu\alpha^Z - \mathrm{i}c_\mathrm{W}(W_\mu^+\alpha^- - W_\mu^-\alpha^+)\right] \\ &\quad +\frac{1}{2}\,\xi_Z m_Z\left[\phi^+\alpha^- + \phi^-\alpha^+ - (v + H)\alpha^Z\right], \end{aligned} \tag{207}$$

$$\sqrt{\xi_\gamma}\,\delta G^\gamma = \partial^\mu \delta A_\mu = \partial^\mu\left[-\frac{1}{e}\,\partial_\mu\alpha^\gamma - \mathrm{i}s_\mathrm{W}(W_\mu^+\alpha^- - W_\mu^-\alpha^+)\right]. \tag{208}$$

因此,

$$\sqrt{\xi_W}g\,\frac{\delta G^+}{\delta\alpha^+} = -\partial^2 - \xi_W m_W^2 - \mathrm{i}e\partial^\mu A_\mu - \mathrm{i}gc_\mathrm{W}\partial^\mu Z_\mu - \frac{1}{2}\,g\xi_W m_W(H+\mathrm{i}\chi), \tag{209}$$

$$\frac{\sqrt{\xi_W}g}{c_\mathrm{W}}\,\frac{\delta G^+}{\delta\alpha^Z} = \mathrm{i}gc_\mathrm{W}\partial^\mu W_\mu^+ - \frac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)\xi_W m_W}{2c_\mathrm{W}}\,\phi^+, \tag{210}$$

$$\sqrt{\xi_W}e\,\frac{\delta G^+}{\delta\alpha^\gamma} = \mathrm{i}e\partial^\mu W_\mu^+ - e\xi_W m_W\phi^+, \tag{211}$$

$$\sqrt{\xi_W}g\,\frac{\delta G^-}{\delta\alpha^-} = -\partial^2 - \xi_W m_W^2 + \mathrm{i}e\partial^\mu A_\mu + \mathrm{i}gc_\mathrm{W}\partial^\mu Z_\mu - \frac{1}{2}\,g\xi_W m_W(H-\mathrm{i}\chi), \tag{212}$$

$$\frac{\sqrt{\xi_W}g}{c_\mathrm{W}}\frac{\delta G^-}{\delta\alpha^Z} = -\mathrm{i}gc_\mathrm{W}\partial^\mu W_\mu^- - \frac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)\xi_W m_W}{2c_\mathrm{W}}\,\phi^-, \tag{213}$$

$$\sqrt{\xi_W}e\,\frac{\delta G^-}{\delta\alpha^\gamma} = -\mathrm{i}e\partial^\mu W_\mu^- - e\xi_W m_W\phi^-, \tag{214}$$

$$\sqrt{\xi_Z}g\,\frac{\delta G^Z}{\delta\alpha^+} = \mathrm{i}gc_\mathrm{W}\partial^\mu W_\mu^- + \frac{1}{2}\,g\xi_Z m_Z\phi^-, \tag{215}$$

$$\sqrt{\xi_Z}g\,\frac{\delta G^Z}{\delta\alpha^-} = -\mathrm{i}gc_\mathrm{W}\partial^\mu W_\mu^+ + \frac{1}{2}\,g\xi_Z m_Z\phi^+, \tag{216}$$

$$\frac{\sqrt{\xi_Z}g}{c_\mathrm{W}}\frac{\delta G^Z}{\delta\alpha^Z} = -\partial^2 - \xi_Z m_Z^2 - \frac{g\xi_Z m_Z}{2c_\mathrm{W}}H, \tag{217}$$

$$\sqrt{\xi_\gamma}g\,\frac{\delta G^\gamma}{\delta\alpha^+} = \mathrm{i}e\partial^\mu W_\mu^-, \quad \sqrt{\xi_\gamma}g\,\frac{\delta G^\gamma}{\delta\alpha^-} = -\mathrm{i}e\partial^\mu W_\mu^+, \quad \sqrt{\xi_\gamma}e\,\frac{\delta G^\gamma}{\delta\alpha^\gamma} = -\partial^2. \tag{218}$$

最后，得到以下 Faddeev-Popov 鬼场拉氏量,

$$\begin{aligned}
\mathcal{L}_{\mathrm{EWG,FP}} =\ & \bar{\eta}^+\left(\sqrt{\xi_W}g\,\frac{\delta G^+}{\delta\alpha^+}\right)\eta^+ + \bar{\eta}^Z\left(\sqrt{\xi_Z}g\,\frac{\delta G^Z}{\delta\alpha^+}\right)\eta^+ + \bar{\eta}^\gamma\left(\sqrt{\xi_\gamma}g\,\frac{\delta G^\gamma}{\delta\alpha^+}\right)\eta^+ \\
& +\bar{\eta}^-\left(\sqrt{\xi_W}g\,\frac{\delta G^-}{\delta\alpha^-}\right)\eta^- + \bar{\eta}^Z\left(\sqrt{\xi_Z}g\,\frac{\delta G^Z}{\delta\alpha^-}\right)\eta^- + \bar{\eta}^\gamma\left(\sqrt{\xi_\gamma}g\,\frac{\delta G^\gamma}{\delta\alpha^-}\right)\eta^- \\
& +\bar{\eta}^Z\left(\frac{\sqrt{\xi_Z}g}{c_\mathrm{W}}\frac{\delta G^Z}{\delta\alpha^Z}\right)\eta^Z + \bar{\eta}^+\left(\frac{\sqrt{\xi_W}g}{c_\mathrm{W}}\frac{\delta G^+}{\delta\alpha^Z}\right)\eta^Z + \bar{\eta}^-\left(\frac{\sqrt{\xi_W}g}{c_\mathrm{W}}\frac{\delta G^-}{\delta\alpha^Z}\right)\eta^Z \\
& +\bar{\eta}^\gamma\left(\sqrt{\xi_\gamma}e\,\frac{\delta G^\gamma}{\delta\alpha^\gamma}\right)\eta^\gamma + \bar{\eta}^+\left(\sqrt{\xi_W}e\,\frac{\delta G^+}{\delta\alpha^\gamma}\right)\eta^\gamma + \bar{\eta}^-\left(\sqrt{\xi_W}e\,\frac{\delta G^-}{\delta\alpha^\gamma}\right)\eta^\gamma \\
\to\ & \bar{\eta}^+\left[-\partial^2 - \xi_W m_W^2 + \mathrm{i}e\overleftarrow{\partial^\mu}A_\mu + \mathrm{i}gc_\mathrm{W}\overleftarrow{\partial^\mu}Z_\mu - \frac{1}{2}\,g\xi_W m_W(H+\mathrm{i}\chi)\right]\eta^+ \\
& +\bar{\eta}^Z\left(-\mathrm{i}gc_\mathrm{W}\overleftarrow{\partial^\mu}W_\mu^- + \frac{1}{2}\,g\xi_Z m_Z\phi^-\right)\eta^+ - \mathrm{i}e(\partial^\mu\bar{\eta}^\gamma)W_\mu^-\eta^+ \\
& +\bar{\eta}^-\left[-\partial^2 - \xi_W m_W^2 - \mathrm{i}e\overleftarrow{\partial^\mu}A_\mu - \mathrm{i}gc_\mathrm{W}\overleftarrow{\partial^\mu}Z_\mu - \frac{1}{2}\,g\xi_W m_W(H-\mathrm{i}\chi)\right]\eta^- \\
& +\bar{\eta}^Z\left(\mathrm{i}gc_\mathrm{W}\overleftarrow{\partial^\mu}W_\mu^+ + \frac{1}{2}\,g\xi_Z m_Z\phi^+\right)\eta^- + \mathrm{i}e(\partial^\mu\bar{\eta}^\gamma)W_\mu^+\eta^- \\
& +\bar{\eta}^Z\left(-\partial^2 - \xi_Z m_Z^2 - \frac{g\xi_Z m_Z}{2c_\mathrm{W}}H\right)\eta^Z \\
& +\bar{\eta}^+\left(-\mathrm{i}gc_\mathrm{W}\overleftarrow{\partial^\mu}W_\mu^+ - \frac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)\xi_W m_W}{2c_\mathrm{W}}\,\phi^+\right)\eta^Z
\end{aligned}$$

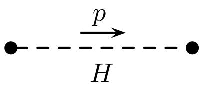
$$
\begin{aligned}
&+\bar{\eta}^-\left(\mathrm{i}gc_\mathrm{W}\overleftarrow{\partial^\mu}W_\mu^- - \frac{g(c_\mathrm{W}^2-s_\mathrm{W}^2)\xi_W m_W}{2c_\mathrm{W}}\,\phi^-\right)\eta^Z\\
&-\bar{\eta}^\gamma\partial^2\eta^\gamma+\bar{\eta}^+(-\mathrm{i}e\overleftarrow{\partial^\mu}W_\mu^+ - e\xi_W m_W\phi^+)\eta^\gamma+\bar{\eta}^-(\mathrm{i}e\overleftarrow{\partial^\mu}W_\mu^- - e\xi_W m_W\phi^-)\eta^\gamma. \qquad (219)
\end{aligned}
$$

可以认为这里通过鬼场 $\eta^\pm$、$\eta^Z$、$\eta^\gamma$ 的归一化吸收了 $-1/g$、$-c_\mathrm{W}/g$、$-1/e$ 因子，通过鬼场 $\bar{\eta}^\pm$、$\bar{\eta}^Z$、$\bar{\eta}^\gamma$ 的归一化吸收了 $1/\sqrt{\xi_W}$、$1/\sqrt{\xi_Z}$、$1/\sqrt{\xi_\gamma}$ 因子。鬼粒子的质量为

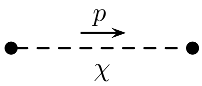
$$
m_{\eta^+}=m_{\eta^-}=\sqrt{\xi_W}\,m_W,\quad m_{\eta^Z}=\sqrt{\xi_Z}\,m_Z,\quad m_{\eta^\gamma}=0. \qquad (220)
$$

下面给出 R$_\xi$ 规范下的 Feynman 规则。$\xi_i=1$ 对应 Feynman-'t Hooft 规范，$\xi_i=0$ 对应 Landau 规范，$\xi_W,\xi_Z\to\infty$ 对应幺正规范。在树图计算中，常取 $\xi_\gamma=1$ 和 $\xi_W,\xi_Z\to\infty$。在圈图计算中，常取 $\xi_\gamma=\xi_W=\xi_Z=1$。

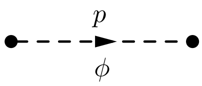
传播子:



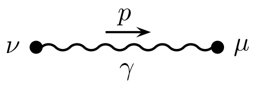

$$= \frac{\mathrm{i}}{p^2-m_H^2+\mathrm{i}\epsilon} \qquad (221)$$



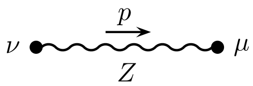

$$= \frac{\mathrm{i}}{p^2-\xi_Z m_Z^2+\mathrm{i}\epsilon} \qquad (222)$$



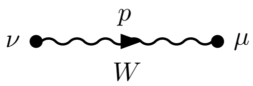

$$= \frac{\mathrm{i}}{p^2-\xi_W m_W^2+\mathrm{i}\epsilon} \qquad (223)$$



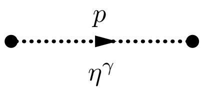

$$= \frac{-\mathrm{i}}{p^2+\mathrm{i}\epsilon}\left[g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}(1-\xi_\gamma)\right] \qquad (224)$$



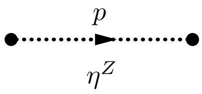

$$= \frac{-\mathrm{i}}{p^2-m_Z^2+\mathrm{i}\epsilon}\left[g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2-\xi_Z m_Z^2}(1-\xi_Z)\right] \qquad (225)$$



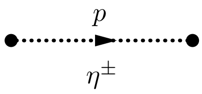

$$= \frac{-\mathrm{i}}{p^2-m_W^2+\mathrm{i}\epsilon}\left[g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2-\xi_W m_W^2}(1-\xi_W)\right] \qquad (226)$$




$$= \frac{\mathrm{i}}{p^2+\mathrm{i}\epsilon} \qquad (227)$$

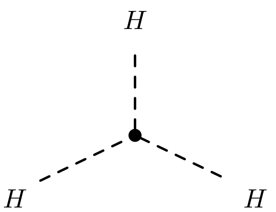



$$= \frac{\mathrm{i}}{p^2-\xi_Z m_Z^2+\mathrm{i}\epsilon} \qquad (228)$$

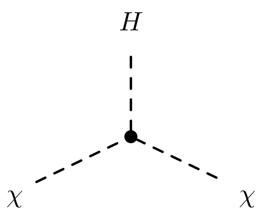



$$= \frac{\mathrm{i}}{p^2-\xi_W m_W^2+\mathrm{i}\epsilon} \qquad (229)$$

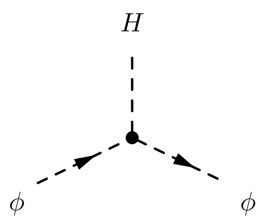
标量玻色子三线性耦合:




$$= -\frac{3\mathrm{i}m_H^2}{v} = -6\mathrm{i}\lambda v$$

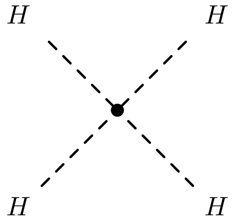



$$= -\frac{\mathrm{i}m_H^2}{v} = -2\mathrm{i}\lambda v$$

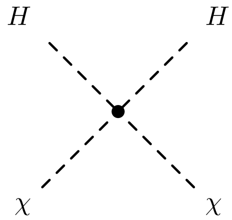



$$= -\frac{\mathrm{i}m_H^2}{v} = -2\mathrm{i}\lambda v \qquad (230)$$

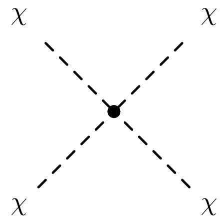
标量玻色子四线性耦合:



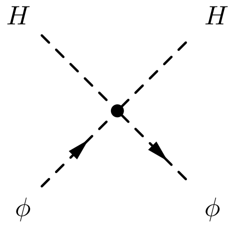

$$= -\frac{3\mathrm{i}m_H^2}{v^2} = -6\mathrm{i}\lambda$$



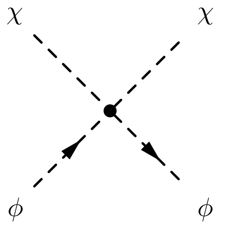

$$= -\frac{\mathrm{i}m_H^2}{v^2} = -2\mathrm{i}\lambda$$



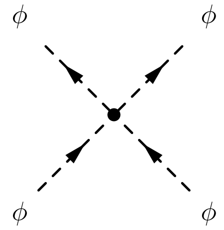

$$= -\frac{3\mathrm{i}m_H^2}{v^2} = -6\mathrm{i}\lambda \tag{231}$$




$$= -\frac{\mathrm{i}m_H^2}{v^2} = -2\mathrm{i}\lambda$$

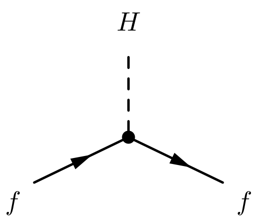



$$= -\frac{\mathrm{i}m_H^2}{v^2} = -2\mathrm{i}\lambda$$

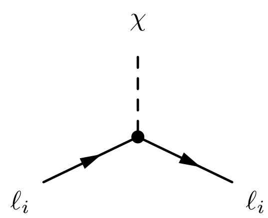



$$= -\frac{2\mathrm{i}m_H^2}{v^2} = -4\mathrm{i}\lambda \tag{232}$$

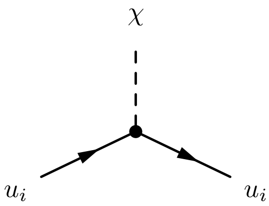
Yukawa 耦合:



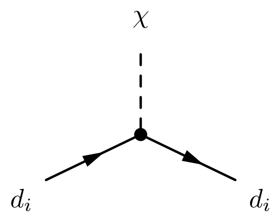

$$= -\frac{\mathrm{i}m_f}{v} = -\frac{\mathrm{i}y_f}{\sqrt{2}}$$



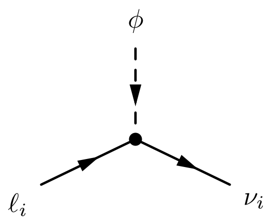

$$= \frac{m_{\ell_i}}{v}\gamma^5 = \frac{y_{\ell_i}}{\sqrt{2}}\gamma^5 \tag{233}$$



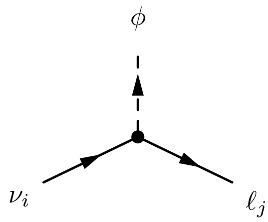

$$= -\frac{m_{u_i}}{v}\gamma^5 = -\frac{y_{u_i}}{\sqrt{2}}\gamma^5$$



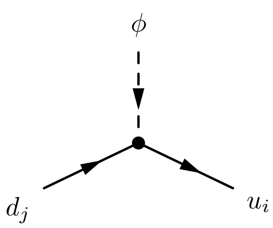

$$= \frac{m_{d_i}}{v}\gamma^5 = \frac{y_{d_i}}{\sqrt{2}}\gamma^5 \tag{234}$$



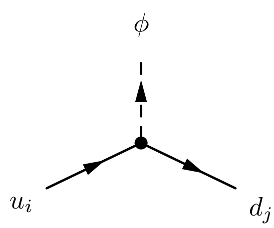

$$= -\frac{\mathrm{i}\sqrt{2}m_{\ell_i}}{v}P_\mathrm{R} = -\mathrm{i}y_{\ell_i}P_\mathrm{R}$$




$$= -\frac{\mathrm{i}\sqrt{2}m_{\ell_i}}{v}P_\mathrm{L} = -\mathrm{i}y_{\ell_i}P_\mathrm{L} \tag{235}$$

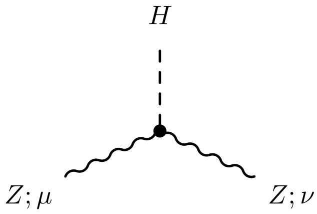



$$= \frac{\mathrm{i}\sqrt{2}V_{ij}}{v}(m_{u_i}P_\mathrm{L} - m_{d_j}P_\mathrm{R}) = \mathrm{i}V_{ij}(y_{u_i}P_\mathrm{L} - y_{d_j}P_\mathrm{R})$$

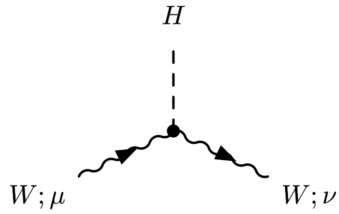



$$= -\frac{\mathrm{i}\sqrt{2}V_{ji}^\dagger}{v}(m_{d_j}P_\mathrm{L} - m_{u_i}P_\mathrm{R}) = -\mathrm{i}V_{ji}^\dagger(y_{d_j}P_\mathrm{L} - y_{u_i}P_\mathrm{R}) \tag{236}$$

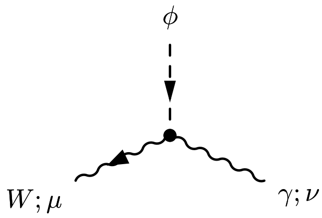
标量玻色子与电弱规范玻色子的三线性耦合:



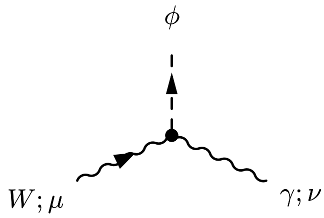

$$= \frac{\mathrm{i}gm_Z}{c_\mathrm{W}}\, g^{\mu\nu} = \frac{\mathrm{i}g^2 v}{2c_\mathrm{W}^2}\, g^{\mu\nu}$$



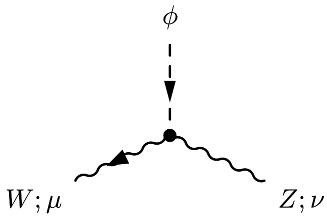

$$= \mathrm{i}gm_W g^{\mu\nu} = \frac{\mathrm{i}g^2 v}{2}\, g^{\mu\nu} \tag{237}$$



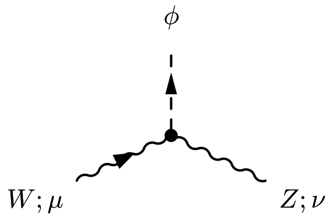

$$= \mathrm{i}em_W g^{\mu\nu} = \frac{\mathrm{i}egv}{2}\, g^{\mu\nu}$$



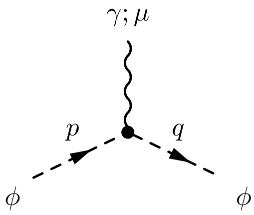

$$= \mathrm{i}em_W g^{\mu\nu} = \frac{\mathrm{i}egv}{2}\, g^{\mu\nu} \tag{238}$$



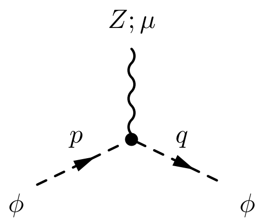

$$= -\mathrm{i}gs_\mathrm{W}^2 m_Z g^{\mu\nu} = -\frac{\mathrm{i}g^2 s_\mathrm{W}^2 v}{2c_\mathrm{W}}\, g^{\mu\nu}$$



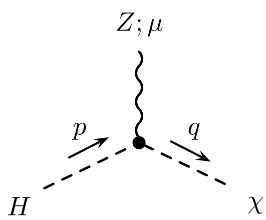

$$= -\mathrm{i}gs_\mathrm{W}^2 m_Z g^{\mu\nu} = -\frac{\mathrm{i}g^2 s_\mathrm{W}^2 v}{2c_\mathrm{W}}\, g^{\mu\nu} \tag{239}$$



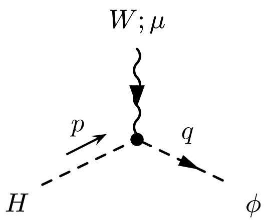

$$= -\mathrm{i}e(p+q)^\mu \tag{240}$$



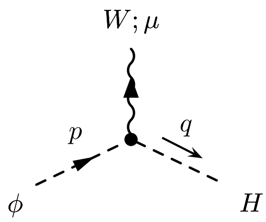

$$= -\frac{\mathrm{i}g(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{2c_\mathrm{W}}\,(p+q)^\mu$$



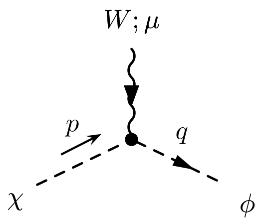

$$= \frac{g}{2c_\mathrm{W}}\,(p+q)^\mu \tag{241}$$



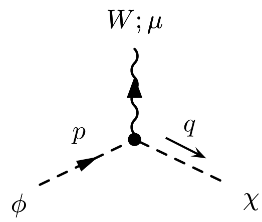

$$= -\frac{\mathrm{i}g}{2}\,(p+q)^\mu$$




$$= -\frac{\mathrm{i}g}{2}\,(p+q)^\mu \tag{242}$$

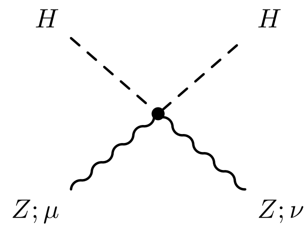



$$= \frac{g}{2}\,(p+q)^\mu$$

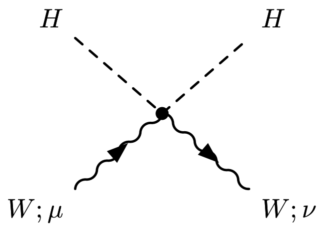



$$= -\frac{g}{2}\,(p+q)^\mu \tag{243}$$

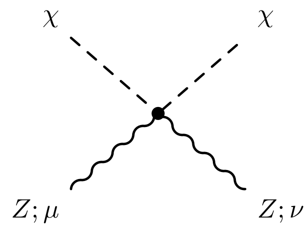
标量玻色子与电弱规范玻色子的四线性耦合:



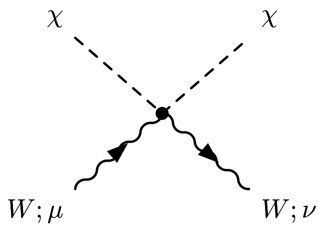

$$= \frac{\mathrm{i}g^2}{2c_\mathrm{W}^2}\, g^{\mu\nu}$$



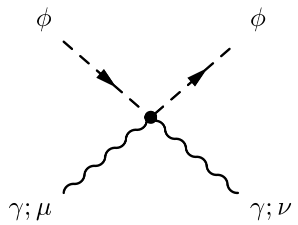

$$= \frac{\mathrm{i}g^2}{2}\, g^{\mu\nu} \tag{244}$$



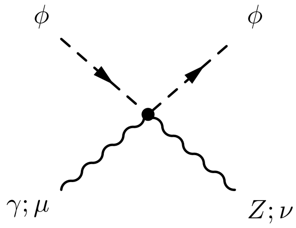

$$= \frac{\mathrm{i}g^2}{2c_\mathrm{W}^2}\, g^{\mu\nu}$$



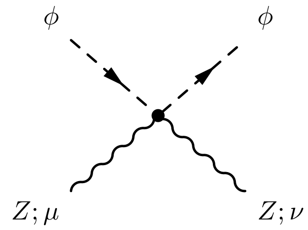

$$= \frac{\mathrm{i}g^2}{2}\, g^{\mu\nu} \tag{245}$$



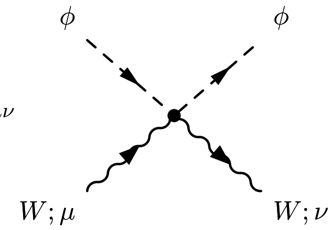

$$= 2\mathrm{i}e^2 g^{\mu\nu}$$



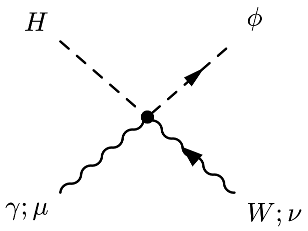

$$= \frac{\mathrm{i}eg(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{c_\mathrm{W}}\, g^{\mu\nu} \tag{246}$$



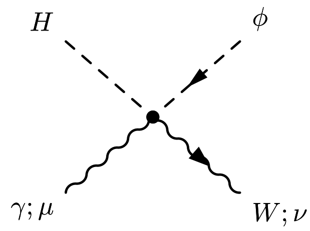

$$= \frac{\mathrm{i}g^2(c_\mathrm{W}^2 - s_\mathrm{W}^2)^2}{2c_\mathrm{W}^2}\, g^{\mu\nu}$$



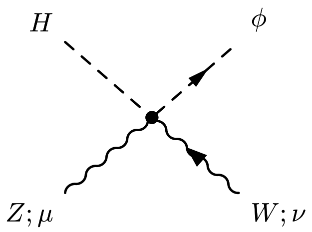

$$= \frac{\mathrm{i}g^2}{2}\, g^{\mu\nu} \tag{247}$$



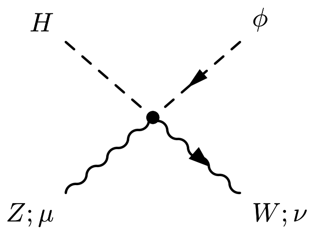

$$= \frac{\mathrm{i}eg}{2}\, g^{\mu\nu}$$



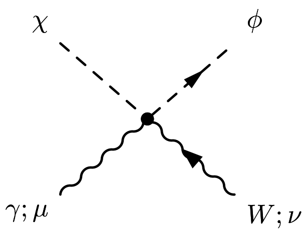

$$= \frac{\mathrm{i}eg}{2}\, g^{\mu\nu} \tag{248}$$



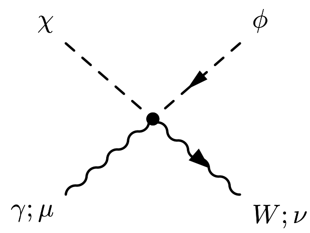

$$= -\frac{\mathrm{i}g^2 s_\mathrm{W}^2}{2c_\mathrm{W}}\, g^{\mu\nu}$$



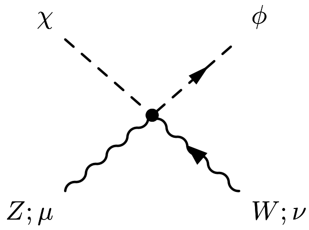

$$= -\frac{\mathrm{i}g^2 s_\mathrm{W}^2}{2c_\mathrm{W}}\, g^{\mu\nu} \tag{249}$$



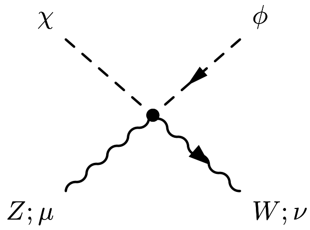

$$= -\frac{eg}{2}\, g^{\mu\nu}$$




$$= \frac{eg}{2}\, g^{\mu\nu} \tag{250}$$

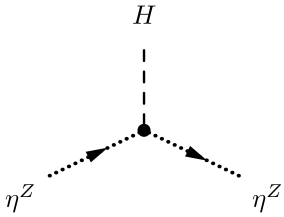



$$= \frac{g^2 s_{\mathrm{W}}^2}{2c_{\mathrm{W}}}\, g^{\mu\nu}$$

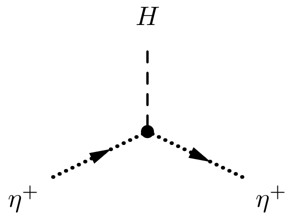



$$= -\frac{g^2 s_{\mathrm{W}}^2}{2c_{\mathrm{W}}}\, g^{\mu\nu} \tag{251}$$

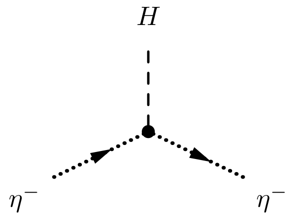
鬼粒子与标量玻色子的耦合:



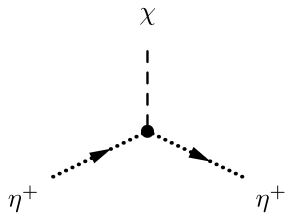

$$= -\frac{\mathrm{i} g \xi_Z m_Z}{2c_{\mathrm{W}}} = -\frac{\mathrm{i} g^2 \xi_Z v}{4c_{\mathrm{W}}^2} \tag{252}$$



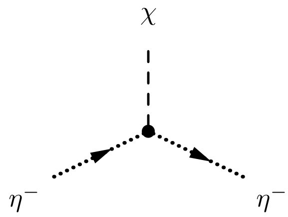

$$= -\frac{\mathrm{i} g \xi_W m_W}{2} = -\frac{\mathrm{i} g^2 \xi_W v}{4}$$



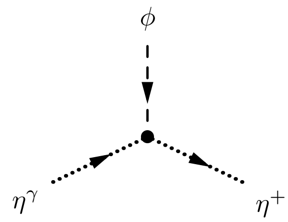

$$= -\frac{\mathrm{i} g \xi_W m_W}{2} = -\frac{\mathrm{i} g^2 \xi_W v}{4} \tag{253}$$



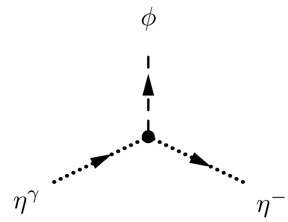

$$= \frac{g \xi_W m_W}{2} = \frac{g^2 \xi_W v}{4}$$



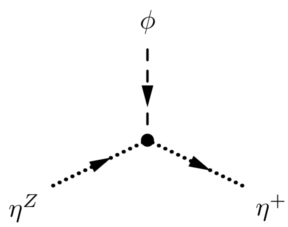

$$= -\frac{g \xi_W m_W}{2} = -\frac{g^2 \xi_W v}{4} \tag{254}$$



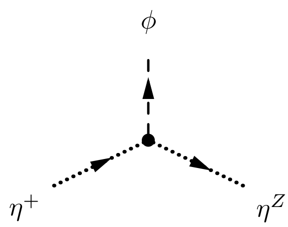

$$= -\mathrm{i} e \xi_W m_W = -\frac{\mathrm{i} e g \xi_W v}{2}$$



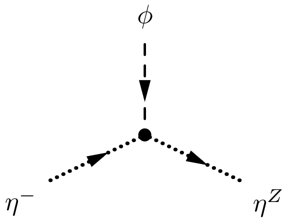

$$= -\mathrm{i} e \xi_W m_W = -\frac{\mathrm{i} e g \xi_W v}{2} \tag{255}$$



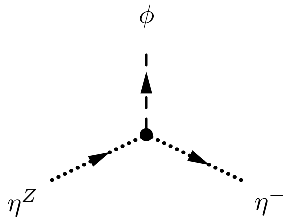

$$= -\frac{\mathrm{i} g (c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2) \xi_W m_W}{2c_{\mathrm{W}}} = -\frac{\mathrm{i} g^2 (c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2) \xi_W v}{4c_{\mathrm{W}}}$$




$$= \frac{\mathrm{i} g \xi_Z m_Z}{2} = \frac{\mathrm{i} g^2 \xi_Z v}{4c_{\mathrm{W}}} \tag{256}$$



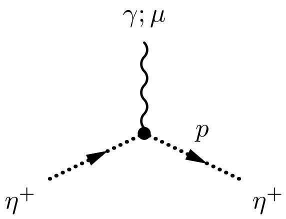

$$= \frac{\mathrm{i}g\xi_Z m_Z}{2}$$
$$= \frac{\mathrm{i}g^2\xi_Z v}{4c_\mathrm{W}}$$



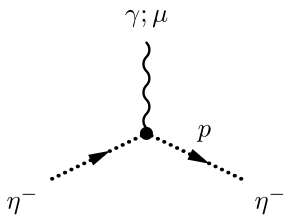

$$= -\frac{\mathrm{i}g(c_\mathrm{W}^2 - s_\mathrm{W}^2)\xi_W m_W}{2c_\mathrm{W}}$$
$$= -\frac{\mathrm{i}g^2(c_\mathrm{W}^2 - s_\mathrm{W}^2)\xi_W v}{4c_\mathrm{W}}$$

(257)

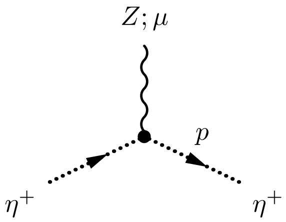
鬼粒子与电弱规范玻色子的耦合:



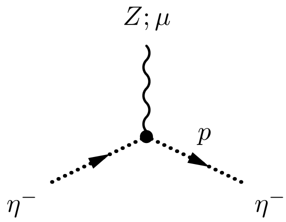

$$= -\mathrm{i}ep^\mu$$



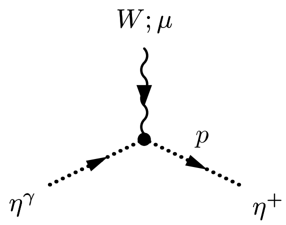

$$= \mathrm{i}ep^\mu \tag{258}$$



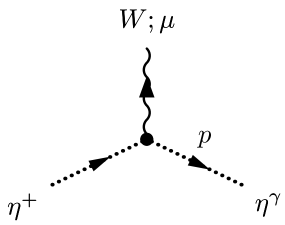

$$= -\mathrm{i}gc_\mathrm{W}p^\mu$$



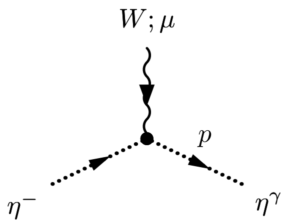

$$= \mathrm{i}gc_\mathrm{W}p^\mu \tag{259}$$



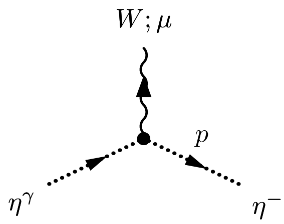

$$= \mathrm{i}ep^\mu$$



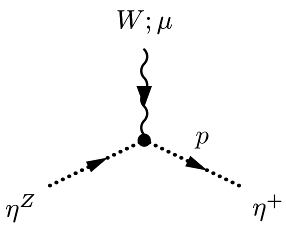

$$= \mathrm{i}ep^\mu \tag{260}$$



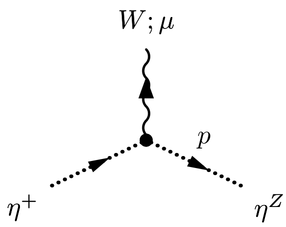

$$= -\mathrm{i}ep^\mu$$



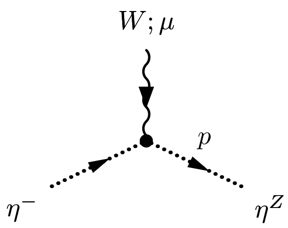

$$= -\mathrm{i}ep^\mu \tag{261}$$



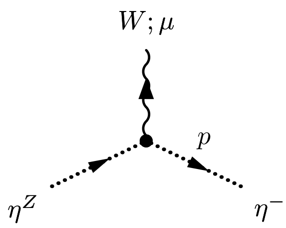

$$= \mathrm{i}gc_\mathrm{W}p^\mu$$




$$= \mathrm{i}gc_\mathrm{W}p^\mu \tag{262}$$




$$= -\mathrm{i}gc_\mathrm{W}p^\mu$$

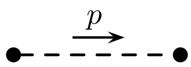
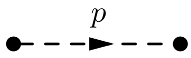
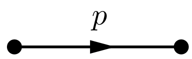


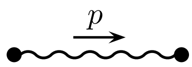

$$= -\mathrm{i}gc_\mathrm{W}p^\mu \tag{263}$$

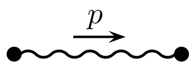
# 6 内外线一般 Feynman 规则

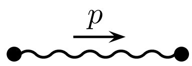
本节列出一些通用的内外线 Feynman 规则。

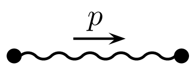
内线 Feynman 规则如下。

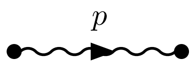
- 实标量玻色子传播子： $= \dfrac{\mathrm{i}}{p^2 - m^2 + \mathrm{i}\epsilon}$
- 复标量玻色子传播子： $= \dfrac{\mathrm{i}}{p^2 - m^2 + \mathrm{i}\epsilon}$
- Dirac 费米子传播子： $= \dfrac{\mathrm{i}(\not{p} + m)}{p^2 - m^2 + \mathrm{i}\epsilon}$
- 无质量实矢量玻色子传播子：

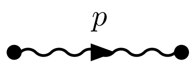
  $\nu$  $\mu \ = \dfrac{-\mathrm{i}g^{\mu\nu}}{p^2 + \mathrm{i}\epsilon}$ （Feynman 规范）

  $\nu$  $\mu \ = \dfrac{-\mathrm{i}(g^{\mu\nu} - p^\mu p^\nu / p^2)}{p^2 + \mathrm{i}\epsilon}$ （Landau 规范）
- 有质量实矢量玻色子传播子：

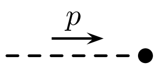
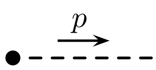
  $\nu$  $\mu \ = \dfrac{-\mathrm{i}(g^{\mu\nu} - p^\mu p^\nu / m^2)}{p^2 - m^2 + \mathrm{i}\epsilon}$ （幺正规范）

  $\nu$  $\mu \ = \dfrac{-\mathrm{i}g^{\mu\nu}}{p^2 - m^2 + \mathrm{i}\epsilon}$ （Feynman 规范）
- 有质量复矢量玻色子传播子：

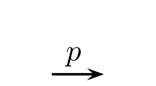
  $\nu$  $\mu \ = \dfrac{-\mathrm{i}(g^{\mu\nu} - p^\mu p^\nu / m^2)}{p^2 - m^2 + \mathrm{i}\epsilon}$ （幺正规范）

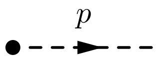
  $\nu$  $\mu \ = \dfrac{-\mathrm{i}g^{\mu\nu}}{p^2 - m^2 + \mathrm{i}\epsilon}$ （Feynman 规范）

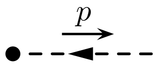
**实标量场**外线 Feynman 规则如下。

- 实标量玻色子入射外线： $= 1$
- 实标量玻色子出射外线： $= 1$

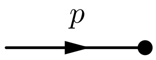
**复标量场**外线 Feynman 规则如下。

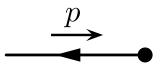
- 正标量玻色子入射外线： $= 1$
- 反标量玻色子入射外线： $= 1$

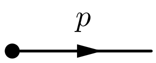
- 正标量玻色子出射外线： $= 1$

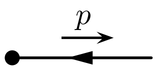
- 反标量玻色子出射外线： $= 1$

以 $\lambda$ 代表自旋极化指标（如螺旋度），**Dirac 旋量场**外线 Feynman 规则如下。

- Dirac 正费米子入射外线：$\lambda$  $= u(\mathbf{p}, \lambda)$

- Dirac 反费米子入射外线：$\lambda$  $= \bar{v}(\mathbf{p}, \lambda)$

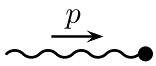
- Dirac 正费米子出射外线： $\lambda$ $= \bar{u}(\mathbf{p}, \lambda)$

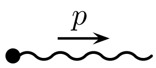
- Dirac 反费米子出射外线： $\lambda$ $= v(\mathbf{p}, \lambda)$

在计算非极化振幅模方时，可利用自旋求和关系

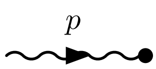
$$\sum_{\lambda} u(\mathbf{p}, \lambda)\bar{u}(\mathbf{p}, \lambda) = \not{p} + m, \quad \sum_{\lambda} v(\mathbf{p}, \lambda)\bar{v}(\mathbf{p}, \lambda) = \not{p} - m. \tag{264}$$

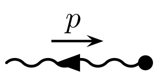
以 $\lambda$ 代表自旋极化指标，**实矢量场**外线 Feynman 规则如下。

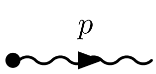
- 实矢量玻色子入射外线：$\lambda; \mu$  $= \varepsilon^{\mu}(\mathbf{p}, \lambda)$

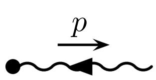
- 实矢量玻色子出射外线： $\lambda; \mu$ $= \varepsilon^{\mu *}(\mathbf{p}, \lambda)$

**复矢量场**外线 Feynman 规则如下。

- 正矢量玻色子入射外线：$\lambda; \mu$  $= \varepsilon^{\mu}(\mathbf{p}, \lambda)$

- 反矢量玻色子入射外线：$\lambda; \mu$  $= \varepsilon^{\mu}(\mathbf{p}, \lambda)$

- 正矢量玻色子出射外线： $\lambda; \mu$ $= \varepsilon^{\mu *}(\mathbf{p}, \lambda)$

- 反矢量玻色子出射外线： $\lambda; \mu$ $= \varepsilon^{\mu *}(\mathbf{p}, \lambda)$

在计算非极化振幅模方时，若包含无质量矢量玻色子外线，可利用极化求和替换关系

$$\sum_{\lambda}\varepsilon_{\mu}^{*}(\mathbf{p},\lambda)\varepsilon_{\nu}(\mathbf{p},\lambda)\to -g_{\mu\nu}\,; \tag{265}$$

若包含有质量矢量玻色子外线，可利用极化求和关系

$$\sum_{\lambda}\varepsilon_{\mu}^{*}(\mathbf{p},\lambda)\varepsilon_{\nu}(\mathbf{p},\lambda) = -g_{\mu\nu}+\frac{p_\mu p_\nu}{m^2}. \tag{266}$$

# 7 常用单位和标准模型参数

本节数据来自 Particle Data Group 发布的 2024 版 *Review of Particle Physics* [5]。

在有理化的自然单位制中，光速、约化 Planck 常数和真空介电常数均取为 1，即 $c=\hbar=\varepsilon_0=1$。从而，速度没有量纲 (dimension)；长度量纲与时间量纲相同，是能量量纲的倒数；能量、质量和动量具有相同的量纲；精细结构常数表达为

$$\alpha=\frac{e^2}{4\pi}, \tag{267}$$

而单位电荷量 $e=\sqrt{4\pi\alpha}$ 是没有量纲的。可以将能量单位电子伏特 (eV) 视作上述有量纲物理量的基本单位。

单位间转换关系为

$$\begin{aligned}
1 &= c = 2.99792458\times 10^{10}\ \mathrm{cm\cdot s^{-1}}, &(268)\\
1 &= \hbar = 6.582119569\times 10^{-25}\ \mathrm{GeV\cdot s}, &(269)\\
1 &= \hbar c = 1.973269804\times 10^{-14}\ \mathrm{GeV\cdot cm}, &(270)\\
1 &= (\hbar c)^2 = 3.893793721\times 10^{8}\ \mathrm{GeV^2\cdot pb}, &(271)
\end{aligned}$$

由此得到

$$\begin{aligned}
1\ \mathrm{s} &= 2.997925\times 10^{10}\ \mathrm{cm}, & 1\ \mathrm{cm} &= 3.335641\times 10^{-11}\ \mathrm{s}, &(272)\\
1\ \mathrm{s} &= 1.519267\times 10^{24}\ \mathrm{GeV^{-1}}, & 1\ \mathrm{GeV^{-1}} &= 6.582120\times 10^{-25}\ \mathrm{s}, &(273)\\
1\ \mathrm{cm} &= 5.067731\times 10^{13}\ \mathrm{GeV^{-1}}, & 1\ \mathrm{GeV^{-1}} &= 1.973270\times 10^{-14}\ \mathrm{cm}, &(274)\\
1\ \mathrm{cm^2} &= 2.568189\times 10^{27}\ \mathrm{GeV^{-2}}, & 1\ \mathrm{GeV^{-2}} &= 3.893794\times 10^{-28}\ \mathrm{cm^2}, &(275)\\
1\ \mathrm{cm^3\cdot s^{-1}} &= 8.566558\times 10^{16}\ \mathrm{GeV^{-2}}, & 1\ \mathrm{GeV^{-2}} &= 1.167330\times 10^{-17}\ \mathrm{cm^3\cdot s^{-1}}. &(276)
\end{aligned}$$

靶 (barn) 是散射截面的常用单位，记作 b，满足

$$1\ \mathrm{b} = 10^{-24}\ \mathrm{cm^2} = 10^{9}\ \mathrm{nb} = 10^{12}\ \mathrm{pb} = 10^{15}\ \mathrm{fb} = 10^{18}\ \mathrm{ab}, \tag{277}$$

$$1\ \mathrm{pb} = 10^{-36}\ \mathrm{cm}^2 = 2.568189 \times 10^{-9}\ \mathrm{GeV}^{-2}, \quad 1\ \mathrm{GeV}^{-2} = 3.893794 \times 10^{8}\ \mathrm{pb}. \tag{278}$$

Fermi 耦合常数是

$$G_{\mathrm{F}} = 1.1663788(6) \times 10^{-5}\ \mathrm{GeV}^{-2}, \tag{279}$$

括号内数字代表测量值的 $1\sigma$ 不确定度，由树图阶关系式

$$\frac{G_{\mathrm{F}}}{\sqrt{2}} = \frac{1}{2v^2} = \frac{g^2}{8m_{\mathrm{W}}^2}, \tag{280}$$

得到 Higgs 场真空期待值为

$$v = (\sqrt{2}G_{\mathrm{F}})^{-1/2} = 246.2196\ \mathrm{GeV}. \tag{281}$$

在低能标（Thomson 极限）处，精细结构常数为

$$\alpha = \frac{1}{137.035999178(8)}; \tag{282}$$

在 $\overline{\mathrm{MS}}$ 重整化方案（以 $\widehat{\ }$ 为标志）中，$\alpha^{-1}$ 跑动到 $\mu = m_Z$ 能标处的数值是

$$\widehat{\alpha}^{-1}(m_Z) = 127.930 \pm 0.008\,. \tag{283}$$

在 $\overline{\mathrm{MS}}$ 方案中，$\mu = m_Z$ 能标处强耦合常数 $\alpha_{\mathrm{s}} = g_{\mathrm{s}}^2/(4\pi)$ 的数值为

$$\widehat{\alpha}_{\mathrm{s}}(m_Z) = 0.1180 \pm 0.0009, \tag{284}$$

Weinberg 角 $\theta_{\mathrm{W}}$ 的数值对应于

$$\widehat{s}_{\mathrm{W}}^2 = \sin^2\widehat{\theta}_{\mathrm{W}}(m_Z) = 0.23129 \pm 0.00004\,. \tag{285}$$

在标准模型中，光子、胶子和中微子没有质量，其它基本粒子的质量为

$$m_W = 80.3692 \pm 0.0133\ \mathrm{GeV}, \quad m_Z = 91.1880 \pm 0.0020\ \mathrm{GeV}, \tag{286}$$

$$m_H = 125.20 \pm 0.11\ \mathrm{GeV}, \quad m_e = 0.51099895000(15)\ \mathrm{MeV}, \tag{287}$$

$$m_\mu = 105.6583755(23)\ \mathrm{MeV}, \quad m_\tau = 1776.93 \pm 0.09\ \mathrm{MeV}, \tag{288}$$

$$m_u = 2.16 \pm 0.07\ \mathrm{MeV}, \quad m_d = 4.70 \pm 0.07\ \mathrm{MeV}, \tag{289}$$

$$m_s = 93.5 \pm 0.8\ \mathrm{MeV}, \quad m_c = 1.2730 \pm 0.0046\ \mathrm{GeV}, \tag{290}$$

$$m_b = 4.183 \pm 0.007\ \mathrm{GeV}, \quad m_t = 172.57 \pm 0.29\ \mathrm{GeV}. \tag{291}$$

这里，$u$、$d$、$s$ 夸克的质量是 $\mu = 2$ GeV 能标处的 $\overline{\mathrm{MS}}$ 质量，$c$ 和 $b$ 夸克的质量分别是 $\mu = m_c$

和 $\mu = m_b$ 能标处的 $\overline{\text{MS}}$ 质量，其余粒子的质量均为极点质量 (pole mass)。相应地，计算出来的 $c$、$b$ 夸克极点质量为

$$m_c^{\text{pole}} = 1.67 \pm 0.07 \text{ GeV}, \quad m_b^{\text{pole}} = 4.78 \pm 0.06 \text{ GeV}. \tag{292}$$

质子和中子的质量为

$$m_p = 938.27208816(29) \text{ MeV}, \quad m_n = 939.5654205(5) \text{ MeV}. \tag{293}$$

在电弱能标附近作领头阶计算时，可将单位电荷量取为

$$e = \sqrt{4\pi\widehat{\alpha}(m_Z)} = 0.3134142, \tag{294}$$

将强耦合常数取为

$$g_{\text{s}} = \sqrt{4\pi\widehat{\alpha}_{\text{s}}(m_Z)} = 1.217716\,. \tag{295}$$

从树图阶关系计算 Higgs 场四线性耦合常数 $\lambda$ 和 Yukawa 耦合常数 $y_t$、$y_b$、$y_\tau$、$y_c$，得

$$\lambda = \frac{m_H^2}{2v^2} = 0.1292806, \quad y_t = \frac{\sqrt{2}m_t}{v} = 0.9911916, \quad y_b = \frac{\sqrt{2}m_b}{v} = 2.402593 \times 10^{-2}, \tag{296}$$

$$y_\tau = \frac{\sqrt{2}m_\tau}{v} = 1.020617 \times 10^{-2}, \quad y_c = \frac{\sqrt{2}m_c}{v} = 7.311739 \times 10^{-3}. \tag{297}$$

耦合常数 $g$ 和 $g'$ 有以下两种取值方式。

1. 根据树图阶关系 $\sin^2\theta_{\text{W}} = 1 - m_W^2/m_Z^2$ 计算 Weinberg 角，得

$$s_{\text{W}}^2 = 1 - \frac{m_W^2}{m_Z^2} = 0.2232095, \quad c_{\text{W}}^2 = 1 - s_{\text{W}}^2 = 0.7767905, \tag{298}$$

$$s_{\text{W}} = \sqrt{s_{\text{W}}^2} = 0.4724505, \quad c_{\text{W}} = \sqrt{c_{\text{W}}^2} = 0.8813572, \tag{299}$$

故

$$g = \frac{e}{s_{\text{W}}} = 0.6633800, \quad g' = \frac{e}{c_{\text{W}}} = 0.3556041\,. \tag{300}$$

2. 根据 $\overline{\text{MS}}$ 方案中 Weinberg 角的数值 (285) 计算 $g$ 和 $g'$，得

$$c_{\text{W}}^2 = 1 - \widehat{s}_{\text{W}}^2 = 0.76871, \quad s_{\text{W}} = \sqrt{\widehat{s}_{\text{W}}^2} = 0.4809262, \quad c_{\text{W}} = \sqrt{c_{\text{W}}^2} = 0.8767611, \tag{301}$$

$$g = \frac{e}{s_{\text{W}}} = 0.6516889, \quad g' = \frac{e}{c_{\text{W}}} = 0.3574682\,. \tag{302}$$

将 CKM 矩阵参数化为

$$V = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}\mathrm{e}^{-\mathrm{i}\delta} \\ & 1 & \\ -s_{13}\mathrm{e}^{\mathrm{i}\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}, \tag{303}$$

其中 $s_{ij} \equiv \sin\theta_{ij}$，$c_{ij} \equiv \cos\theta_{ij}$。实验拟合值为

$$s_{12} = 0.22501 \pm 0.00068, \qquad s_{23} = 0.04183^{+0.00079}_{-0.00069}, \tag{304}$$

$$s_{13} = 0.003732^{+0.000090}_{-0.000085}, \qquad \delta = 1.147 \pm 0.026\,. \tag{305}$$

如果只讨论第一、二代夸克的混合，可利用 Cabibbo 转动角 $\theta_{\mathrm{C}}$ 将 CKM 矩阵近似地表达为

$$V \simeq \begin{pmatrix} \cos\theta_{\mathrm{C}} & \sin\theta_{\mathrm{C}} & \\ -\sin\theta_{\mathrm{C}} & \cos\theta_{\mathrm{C}} & \\ & & 1 \end{pmatrix}, \quad \sin\theta_{\mathrm{C}} = s_{12} = 0.225\,. \tag{306}$$

# 参考文献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[3] A. Denner, “Techniques for calculation of electroweak radiative corrections at the one loop level and results for $W$ physics at LEP-200,” Fortsch. Phys. **41**, 307 (1993) [arXiv:0709.1075 [hep-ph]].

[4] 杜东生，杨茂志，《粒子物理导论》，中国北京：科学出版社 (2014)，402 页。

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