

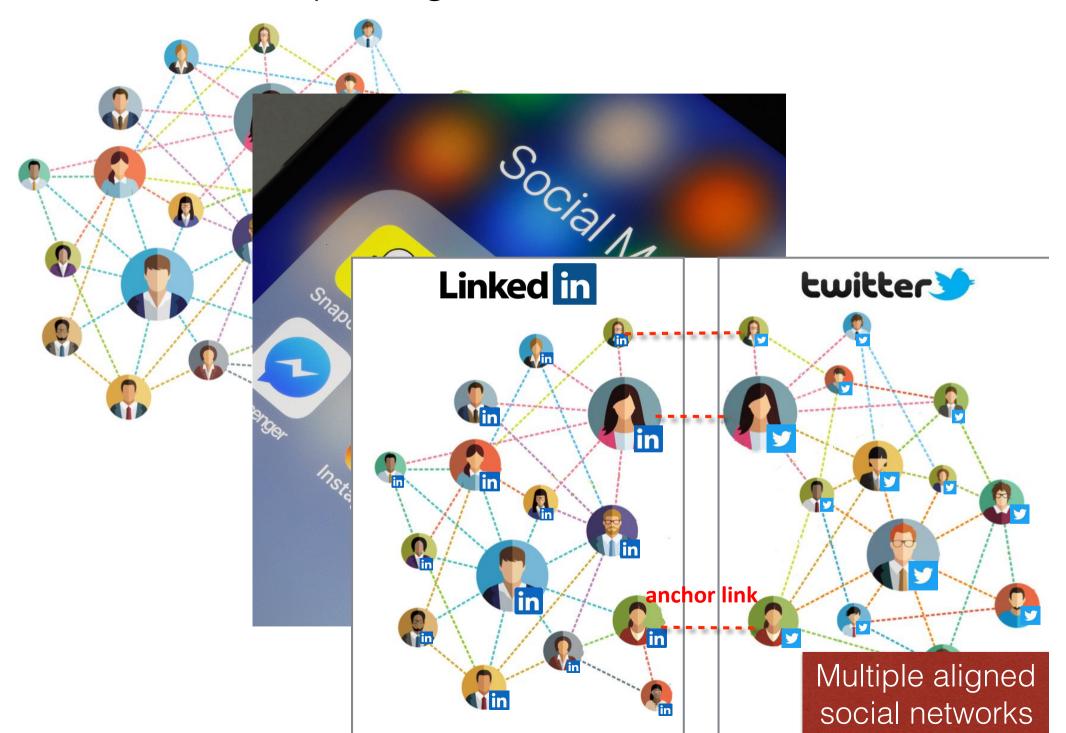
Collective Link Prediction Oriented Network Embedding with Hierarchical Graph Attention

Yizhu Jiao¹, Yun Xiong¹, Jiawei Zhang² and Yangyong Zhu¹

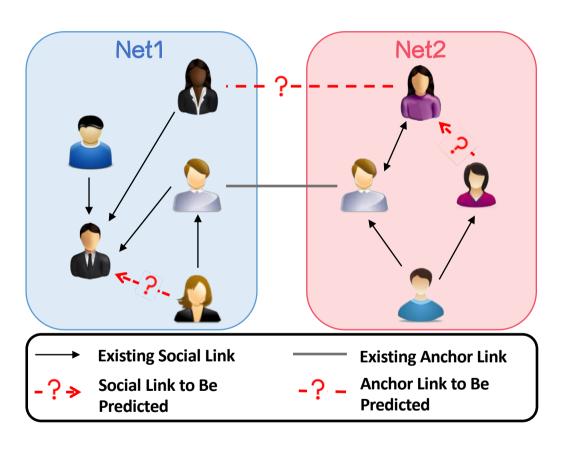
¹ Shanghai Key Laboratory of Data Science, Fudan University, China ² IFM Lab, Florida State University, USA



Multiple aligned social networks



Problem Studied: Collective Link Prediction



Problem Definition:

 It covers several different link formation prediction tasks simultaneously including both the intra-network social link prediction and the inter-network anchor link prediction.

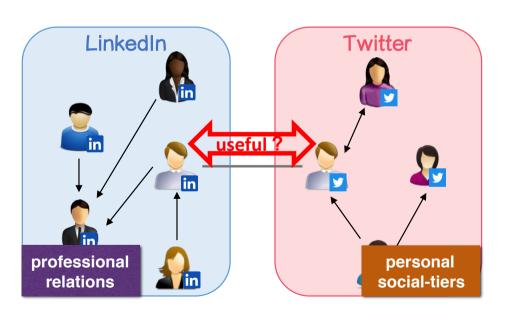
Why Study Together:

 take advantage of their strong correlations to enhance the prediction performance

Main Challenges

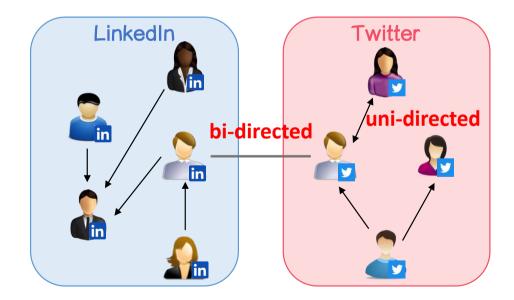
Network characteristic differences

- Each network reveals different aspects of the users.
- Information transfer could also deteriorate the performance of intranetwork link prediction

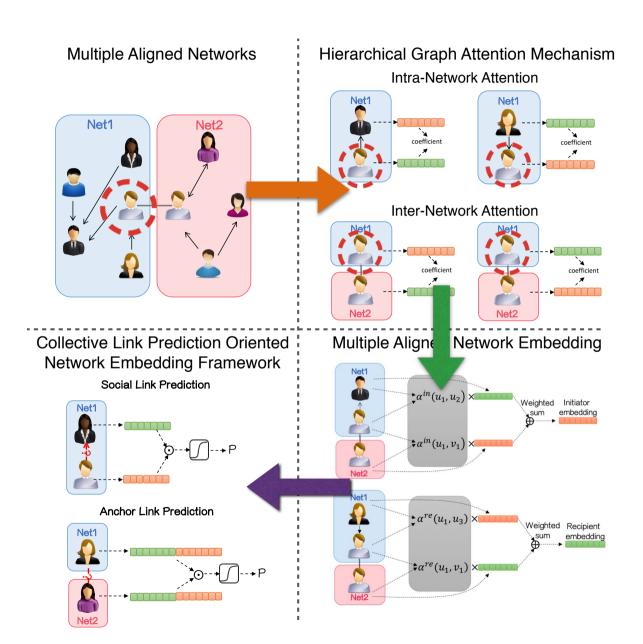


Link directivity differences

- The intra-network social links are usually uni-directed while the internetwork anchor links are bi-directed.
- Such different directivity properties should be carefully considered.



Proposed Framework: **HGANE** (Hierarchical Graph Attention based Network Embedding)



step 1: Hierarchical graph attention mechanism can pay more or less attention to different neighbor nodes and networks to resolve the *network characteristic differences* problems.

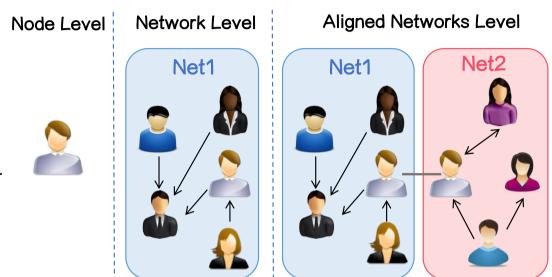
step 2: Multiple aligned network embedding can be learnt by aggregating information from both the *intra-network neighbors* and *inter-network partners*.

step 3: HGANE incorporates the collective link prediction task into a unified framework and balances between multiple prediction tasks.

Hierarchical Graph Attention Mechanism

Multiple aligned networks have a hierarchical structure...

- For the target node, the relevance of different neighbors is different.
- For the target network, other networks are differentially informative since they have different characteristics.



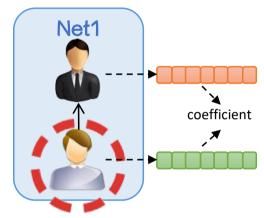
We proposed hierarchical graph attention mechanism including

- Intra-network social attention (at the node level)
- Inter-network anchor attention (at the network level)

to resolve the problems of *network characteristic differences* and *link directivity challenges* in the multiple aligned networks.

Intra-Network Social Attention

Each node is represented with two vector representations, the initiator feature and the recipient feature.

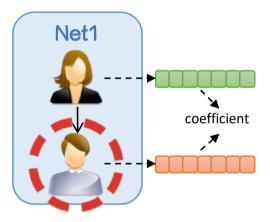


Intra-Network Initiator Attention

 To judge the importance of the intranetwork recipient neighbor to the target initiator.

Definition 2. (Intra-Network Initiator Attention): For the target | **Definition 3.** (Intra-Network Recipient Attention): For the target as

$$e^{in}(u_i, u_j) = \sigma \left(\mathbf{a}_{in}^{(1)^T} \left[\mathbf{W}_{in}^{(1)} \mathbf{u}_i^{in} \middle\| \mathbf{W}_{re}^{(1)} \mathbf{u}_j^{re} \right]\right),$$



Intra-Network Recipient Attention

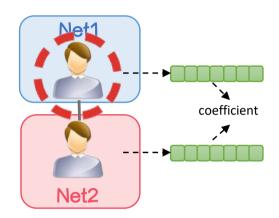
 To judge the importance of the intranetwork initiator neighbor to the target recipient.

node u_i and its intra-network recipient neighbor $u_i \in \mathcal{N}^i(u_i)$, the node u_i and its intra-network initiator neighbor $u_i \in \mathcal{N}^r(u_i)$, the intra-network initiator attention coefficient of u_i to u_j can be given intra-network recipient attention coefficient of u_i to u_j can be given as

$$e^{re}(u_i, u_j) = \sigma\left(\mathbf{a}_{re}^{(1)^T} \left[\mathbf{W}_{re}^{(1)} \mathbf{u}_i^{re} \middle\| \mathbf{W}_{in}^{(1)} \mathbf{u}_j^{in} \right]\right),$$

Inter-Network Anchor Attention

To handle the problem of **network characteristic differences**, the internetwork anchor attention is designed for the network level.

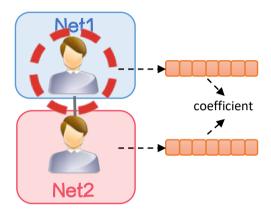


Inter-Network Initiator Attention

 To indicate the importance of information from other networks to the target node as the initiator.

Definition 4. (Inter-Network Initiator Attention): For the target node u_i and its inter-network anchor partner $v_j \in \mathcal{N}^a(u_i)$, the intranetwork initiator attention coefficient of u_i to v_j as the initiator can be given as

$$e^{in}(u_i, v_j) = \sigma\left(\mathbf{a}_{in}^{(1,2)^T} \left[\mathbf{W}_{in}^{(1)} \mathbf{u}_i^{in} \middle\| \mathbf{W}_{in}^{(1,2)} \mathbf{v}_j^{in} \right] \right),$$



Inter-Network Recipient Attention

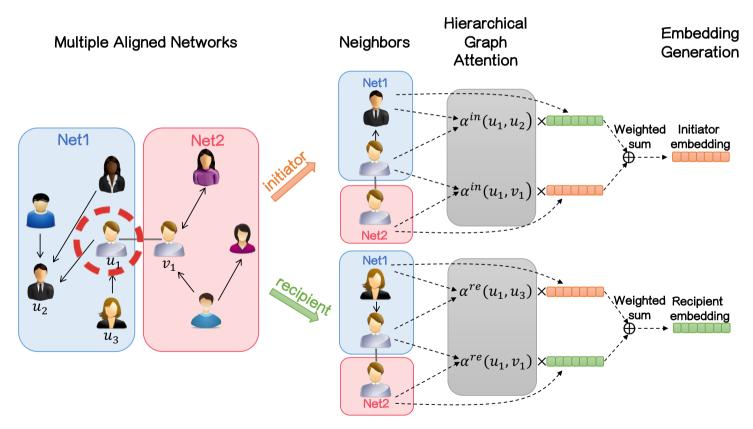
 To indicate the importance of information from other networks to the target node as the recipient.

Definition 5. (Inter-Network Recipient Attention): For the target node u_i and its inter-network anchor partner $v_j \in \mathcal{N}^a(u_i)$, the intranetwork recipient attention coefficient of u_i to v_j as the recipient can be given as

$$e^{re}(u_i, v_j) = \sigma \left(\mathbf{a}_{re}^{(1,2)^T} \left[\mathbf{W}_{re}^{(1)} \mathbf{u}_i^{re} \middle\| \mathbf{W}_{re}^{(1,2)} \mathbf{v}_j^{re} \right]\right),$$

Multiple Aligned Network Embedding

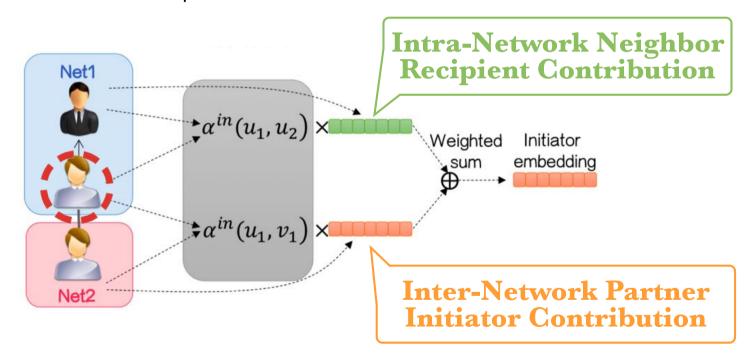
- HGANE extends the traditional graph neural network to the multiple aligned social networks scenario.
- HGANE learns the node representations by aggregating information from both the intra-network neighbors and inter-network partners.



- capture the localized structural features by utilizing information propagated from the intranetwork social neighbors.
- preserve more comprehensive features by leveraging cross-network information transferred by anchor links.

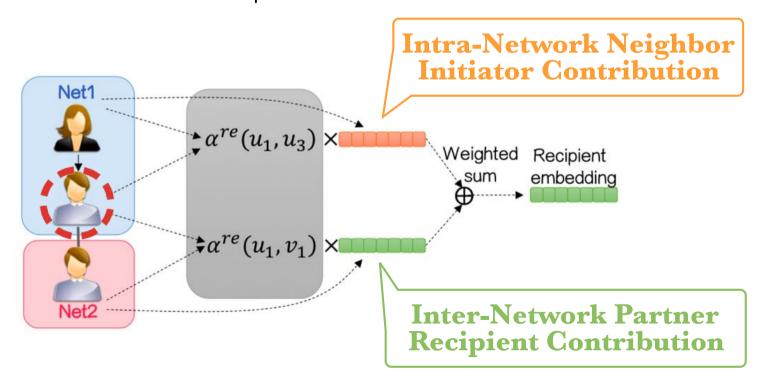
Multiple Aligned Network Embedding

- Formally, each node is represented by two embeddings according to its two roles in social networks considering the problems of link directivity challenges.
- The initiator embedding indicates its features as the initiator in the social network, which depends on its intra-network recipient neighbors and inter-network anchor partners.



Multiple Aligned Network Embedding

- Formally, each node is represented by two embeddings according to its two role in social networks considering the problems of link directivity challenges.
- The recipient embedding indicates its features as the recipient in the social network, which depends on its intra-network initiator neighbors and inter-network anchor partners.

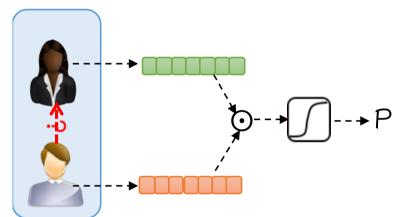


Collective Link Prediction Oriented Network Embedding Optimization Framework

Social Link Prediction

• The probability of the intranetwork social link formation from the initiator u_i pointing to the recipient u_j as

$$p(u_i, u_j) = \sigma(\mathbf{u}_i^{inT} \cdot \mathbf{u}_j^{re})$$



The objective of intra-network social link formation

$$\mathcal{L}_{soc}(u_i, u_j) = \log p(u_i, u_j) + \sum_{\{(u_m, u_n)\}} \log (1 - p(u_m, u_n))$$

• The *final objective* for $G^{(1)}$

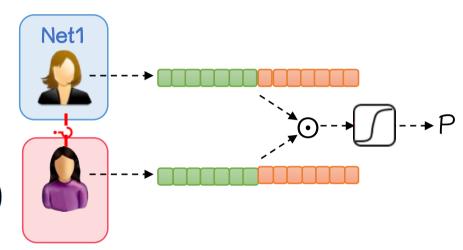
$$\mathcal{L}^{(1)} = \sum_{(u_i, u_i) \in E^{(1)}} \mathcal{L}_{soc}(u_i, u_j)$$

Collective Link Prediction Oriented Network Embedding Optimization Framework

 The probability of the internetwork anchor link formation between the target node u_i and the anchor node v_i

$$p(u_i, v_j) = \sigma((\mathbf{u}_i^{in} || \mathbf{u}_i^{re})^T \cdot (\mathbf{v}_j^{in} || \mathbf{v}_j^{re}))$$

Anchor Link Prediction



The objective of inter-network anchor link formation

$$\mathcal{L}_{ach}(u_i, v_j) = \log p(u_i, v_j) + \sum_{\{(u_m, v_n)\}} \log (1 - p(u_m, v_n))$$

The final objective of network alignment

$$\mathcal{L}^{(1,2)} = \sum_{(u_i,v_j)\in A^{(1,2)}} \left(\mathcal{L}_{ach}(u_i,v_j)\right)$$

Collective Link Prediction Oriented Network Embedding Optimization Framework

 To incorporate the collective link prediction task into a unified framework, we learn the node representations with rich information by jointly training the objective function

$$\mathcal{L}\left(G^{(1)}, G^{(2)}\right) = \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \alpha \cdot \mathcal{L}^{(1,2)} + \beta \cdot \mathcal{L}_{reg}$$

- the objective for the two social networks
- the objective of network alignment
- the regularization term
- Hyperparameter

Experiments

- Datasets
 - Twitter-Foursquare
 - Facebook-Twitter

Table 2: Statistics of datasets

Dataset	#Nodes	#Social Links	#Anchor Links
Twitter	5,223	164,920	2 200
Foursquare	5,392	76,972	3,388
Facebook	4,137	57,528	4,137
Twitter	4,137	147,726	4,137

- Evaluation Metrics
 - AUC

Experiment Settings

Comparison Methods

- Deepwalk, Node2vec, GAT learn node embeddings in the single network
- IONE, DIME, MNN study the single kind of link prediction task.
- CLF proposes and studies the collective link prediction task.

	Multiple Networks	Links Directi. Differe.	Network Charact. Differe.	Predict Social Link	Predict Anchor Link
DeepWalk				$\sqrt{}$	
Node2Vec				V	
GAT				$\sqrt{}$	
IONE	V	V			$\sqrt{}$
DIME	√	$\sqrt{}$		$\sqrt{}$	
MNN	V	$\sqrt{}$		V	
CLF	√			V	$\sqrt{}$
HGANE		$\sqrt{}$	√	V	V

Table 4: Performance comparison with different methods. Soc1, Soc2 and Ach indicate social link prediction in the first and second network and anchor link prediction respectively.

			Training Ratio										
Dataset	Method		0.2			0.4		0.6			0.8		
		Soc1	Soc2	Ach	Soc1	Soc2	Ach	Soc1	Soc2	Ach	Soc1	Soc2	Ach
	DeepWalk	75.8%	72.5%	57.9%	80.3%	76.9%	63.1%	82.2%	79.7%	67.3%	85.7%	82.5%	75.4%
	Node2Vec	82.5%	77.4%	64.3%	84.6%	80.9%	66.1%	86.4%	84.3%	72.1%	89.3%	88.3%	78.9%
	GAT	85.5%	78.2%	65.5%	91.5%	86.9%	68.9%	92.5%	90.3%	75.8%	92.6%	92.3%	80.9%
Twitter	IONE	83.2%	75.7%	72.1%	86.2%	81.7%	78.0%	88.2%	84.7%	85.6%	88.7%	84.7%	87.4%
_ &	DIME	85.1%	76.2%	74.8%	88.4%	80.3%	76.3%	89.8%	83.0%	82.6%	92.0%	85.2%	84.9%
Foursquare	MNN	89.2%	72.4%	-	92.9%	81.1%	-	94.8%	86.1%	-	96.3%	87.6%	-
	CLF	84.5%	78.7%	70.9%	86.7%	80.5%	75.2%	90.9%	84.2%	83.1%	92.3%	86.5%	87.1%
	HGANE	94.4%	90.3%	76.7%	96.4%	95.1%	85.8%	97.1%	96.8%	90.0%	97.5%	97.5%	93.0%
	DeepWalk	76.3%	70.3%	55.8%	81.5%	75.2%	70.9%	84.0%	81.6%	77.7%	90.9%	86.5%	78.9%
	Node2Vec	83.0%	81.5%	58.6%	86.6%	85.7%	76.2%	88.8%	87.5%	81.0%	91.3%	88.2%	83.2%
	GAT	87.3%	86.1%	60.2%	92.0%	90.0%	78.5%	94.7%	92.8%	83.5%	95.7%	93.4%	85.5%
Facebook	IONE	82.8%	79.1%	77.9%	85.9%	82.6%	85.4%	87.4%	85.1%	89.4%	90.9%	89.1%	92.1%
&	DIME	87.1%	86.2%	74.3%	88.4%	87.3%	81.9%	89.8%	90.0%	85.1%	94.0%	92.2%	87.5%
Twitter	MNN	88.6%	87.1%	-	92.4%	91.3%	-	94.4%	93.1%	-	95.7%	94.8%	-
	CLF	84.9%	81.1%	80.5%	88.7%	85.9%	84.2%	91.4%	88.9%	87.6%	93.1%	90.2%	90.4%
	HGANE	91.8%	90.9%	84.8%	95.2%	94.8%	93.4%	97.1%	96.9%	95.8%	98.1%	97.5%	97.1%

Table 4: Performance comparison with different methods. Soc1, Soc2 and Ach indicate social link prediction in the first and second network and anchor link prediction respectively.

			Training Ratio										
Dataset	Method		0.2			0.4			0.6			0.8	
		Soc1	Soc2	Ach									
	DeepWalk Node2Vec	75.8% 82.5%	72.5% 77.4%	57.9% 64.3%	80.3% 84.6%	76.9% 80.9%	63.1% 66.1%	82.2% 86.4%	79.7% 84.3%	67.3% 72.1%	85.7% 89.3%	82.5% 88.3%	75.4% 78.9%
Twitter & Foursquare	IONE DIME MNN	85.5% 83.2% 85.1%	78.2% 75.7% 76.2%	65.5% 72.1% 74.8%	91.5% 86.2% 88.4%	86.9% 81.7% 80.3%	78.0% 76.3%	92.5% 88.2% 89.8%	90.3% 84.7% 83.0%	75.8% 85.6% 82.6%	92.6% 88.7% 92.0%	92.3% 84.7% 85.2%	80.9% 87.4% 84.9%
	CLF HGANE	89.2% 84.5% 94.4%	72.4% 78.7% 90.3%	- 70.9% 76.7%	92.9% 86.7% 96.4%	81.1% 80.5% 95.1%	- 75.2% 85.8%	94.8% 90.9% 97.1 %	86.1% 84.2% 96.8%	83.1% 90.0 %	96.3% 92.3% 97.5 %	87.6% 86.5% 97.5 %	87.1% 93.0 %
	DeepWalk Node2Vec GAT	76.3% 83.0% 87.3%	70.3% 81.5% 86.1%	55.8% 58.6% 60.2%	81.5% 86.6% 92.0%	75.2% 85.7% 90.0%	70.9% 76.2% 78.5%	84.0% 88.8% 94.7%	81.6% 87.5% 92.8%	77.7% 81.0% 83.5%	90.9% 91.3% 95.7%	86.5% 88.2% 93.4%	78.9% 83.2% 85.5%
Facebook & Twitter	DIME MNN CLF	82.8% 87.1% 88.6% 84.9%	79.1% 86.2% 87.1% 81.1%	77.9% 74.3% - 80.5%	85.9% 88.4% 92.4% 88.7%	82.6% 87.3% 91.3% 85.9%	85.4% 81.9% - 84.2%	87.4% 89.8% 94.4% 91.4%	85.1% 90.0% 93.1% 88.9%	89.4% 85.1% - 87.6%	90.9% 94.0% 95.7% 93.1%	89.1% 92.2% 94.8% 90.2%	92.1% 87.5% - 90.4%
	HGANE	91.8%	90.9%	84.8%	95.2%	94.8%	93.4%	97.1%	96.9%	95.8%	98.1%	97.5%	97.1%

 The methods studying multiple aligned networks alleviate the data insufficiency problem.

Table 4: Performance comparison with different methods. Soc1, Soc2 and Ach indicate social link prediction in the first and second network and anchor link prediction respectively.

							Trainin	ng Ratio					
Dataset	Method		0.2			0.4			0.6			0.8	
		Soc1	Soc2	Ach	Soc1	Soc2	Ach	Soc1	Soc2	Ach	Soc1	Soc2	Ach
	DeepWalk	75.8%	72.5%	57.9%	80.3%	76.9%	63.1%	82.2%	79.7%	67.3%	85.7%	82.5%	75.4%
	Node2Vec	82.5%	77.4%	64.3%	84.6%	80.9%	66.1%	86.4%	84.3%	72.1%	89.3%	88.3%	78.9%
	GAT	85.5%	78.2%	65.5%	91.5%	86.9%	68.9%	92.5%	90.3%	75.8%	92.6%	92.3%	80.9%
Twitter	IONE	83.2%	75.7%	72.1%	86.2%	81.7%	78.0%	88.2%	84.7%	85.6%	88.7%	84.7%	87.4%
_ &	DIME	85.1%	76.2%	74.8%	88.4%	80.3%	76.3%	89.8%	83.0%	82.6%	92.0%	85.2%	84.9%
Foursquare	MNN	89.2%	72.4%	-	92.9%	81.1%	-	94.8%	86.1%	-	96.3%	87.6%	-
	CLF	84.5%	78.7%	70.9%	86.7%	80.5%	75.2%	90.9%	84.2%	83.1%	92.3%	86.5%	87.1%
	HGANE	94.4%	90.3%	76.7%	96.4%	95.1%	85.8%	97.1%	96.8%	90.0%	97.5%	97.5%	93.0%
	DeepWalk	76.3%	70.3%	55.8%	81.5%	75.2%	70.9%	84.0%	81.6%	77.7%	90.9%	86.5%	78.9%
	Node2Vec	83.0%	81.5%	58.6%	86.6%	85.7%	76.2%	88.8%	87.5%	81.0%	91.3%	88.2%	83.2%
	GAT	87.3%	86.1%	60.2%	92.0%	90.0%	78.5%	94.7%	92.8%	83.5%	95.7%	93.4%	85.5%
Facebook	IONE	82.8%	79.1%	77.9%	85.9%	82.6%	85.4%	87.4%	85.1%	89.4%	90.9%	89.1%	92.1%
&	DIME	87.1%	86.2%	74.3%	88.4%	87.3%	81.9%	89.8%	90.0%	85.1%	94.0%	92.2%	87.5%
Twitter	MNN	88.6%	87.1%	-	92.4%	91.3%	-	94.4%	93.1%	-	95.7%	94.8%	-
	CLF	84.9%	81.1%	80.5%	88.7%	85.9%	84.2%	91.4%	88.9%	87.6%	93.1%	90.2%	90.4%
	HGANE	91.8%	90.9%	84.8%	95.2%	94.8%	93.4%	97.1%	96.9%	95.8%	98.1%	97.5%	97.1%

 As the training rate λ drops, the performance degradation of HGANE is rather moderate since we handle the problem of network characteristic differences.

 Validation of the design of represent each node with two embeddings to resolve the link directivity differences problem.

Feature	·			Facebook&Twitter			
	Soc1	Soc2	Ach	Soc1	Soc2	Ach	
initiator recipient both	93.4%	93.2%	85.2%	97.0%	94.9%	95.8%	
recipient	93.0%	93.7%	85.6%	97.1%	95.1%	96.2%	
both	97.2%	96.8%	93.0%	98.1%	97.5%	97.1%	

- Our full model outperforms two variants with either the initiator or recipient features.
- The performance of social link prediction within the more dense network can be improved more by distinguishing nodes' initiator and recipient roles.

Hypothesis Verification

 Four hypotheses about how the the target node aggregates information from social neighbors and

anchor partners.

 The hypotheses SC+AD is adopted in our framework.

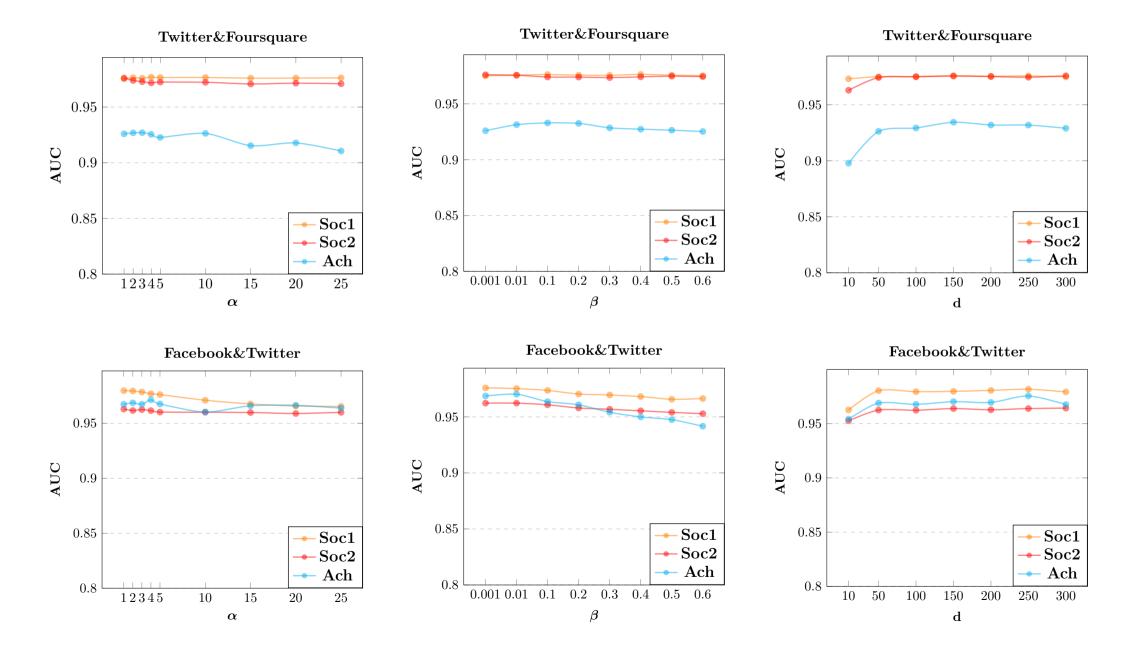
 If DC is replaced with CC, AUC of the anchor link prediction decreases a lot.

 AUC of the social link prediction are affected if we adopt DC for social links.

	Two hypotheses about social neighbors(S)	Two hypotheses about anchor partners(A)
Cross	Net1	Net2
contribution (C)	SC	Net1 AC
Direct	Net1	Net2
contribution (D)	SD	Net1 AD

Hypothesis	Twitte	er&Fours	square	Facebook&Twitter				
	Soc1	Soc2	Ach	Soc1	Soc2	Ach		
SC+AC	97.2%	96.0%	89.4%	98.1%	97.0%	96.0%		
SD+AD	91.8%	91.3%	82.8%	97.0%	95.2%	95.3%		
SD+AC	91.8%	91.3%	82.1%	96.9%	95.3%	94.3%		
SC+AD	97.2%	96.7%	92.4%	98.1%	97.1%	96.7%		

Hyperparameter analysis



Summary

- In this paper, we study the Collective Link Prediction problem over multiple aligned social networks
- Proposed a task oriented network embedding framework with hierarchical graph attention
 - 1. learn node representation by aggregating information from both the intra-network neighbors and inter-network partners
 - 2. hierarchical graph attention mechanism to resolve the network characteristic differences and link directivity challenges
 - 3. incorporate the collective link prediction task objectives into consideration and balances between different prediction tasks
- Conduct extensive experiments on two real-world datasets to test the effectiveness of HGANE



Collective Link Prediction Oriented Network Embedding with Hierarchical Graph Attention

Q&A

Yizhu Jiao¹, Yun Xiong¹, Jiawei Zhang² and Yangyong Zhu¹ {yzjiao18, yunx, yyzhu}@fudan.edu.cn, jiawei@ifmlab.org

Code: https://github.com/yzjiao/HierarchicalGraphAttention