**Exercise 4.6** Determine the space frame screw axes $\mathcal{S}_i$ for the WAM robot in Figure 4.8.

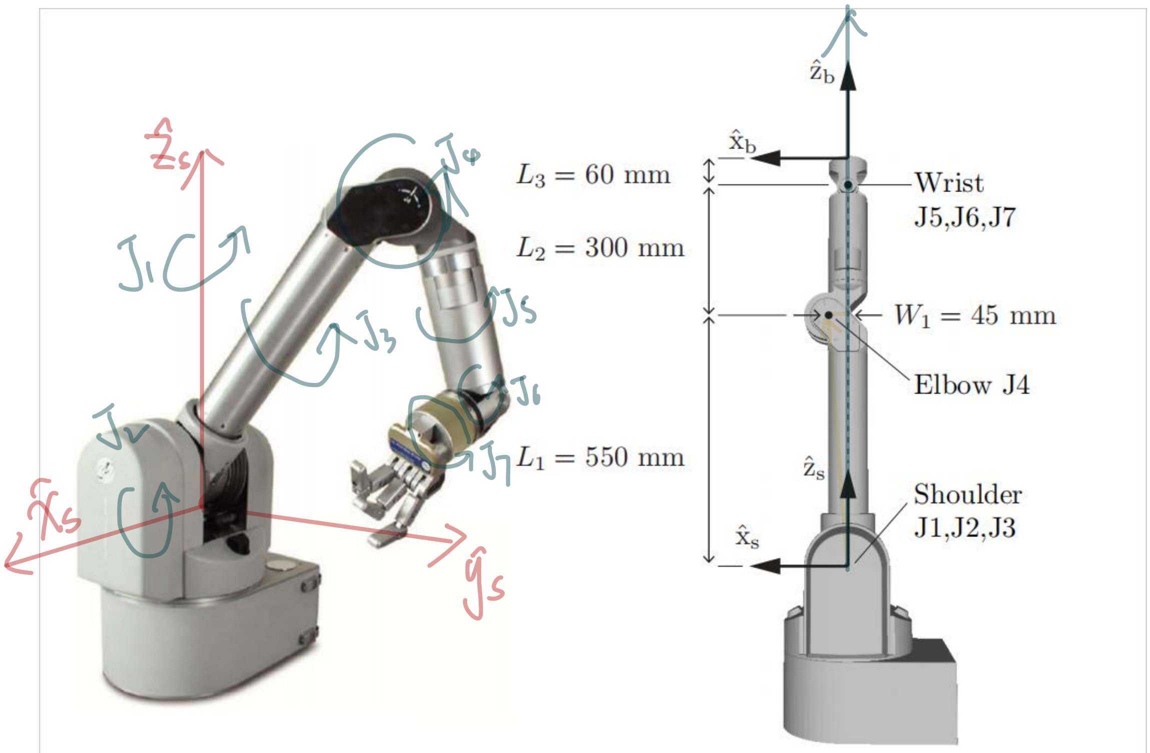



**Figure 4.8:** Barrett Technology's WAM 7R robot arm at its zero configuration (right). At the zero configuration, axes 1, 3, 5, and 7 are along $\hat{z}_s$ and axes 2, 4, and 6 are aligned with $\hat{y}_s$ out of the page. Positive rotations are given by the right-hand rule. Axes 1, 2, and 3 intersect at the origin of {s} and axes 5, 6, and 7 intersect at a point 60mm from {b}. The zero configuration is singular, as discussed in Section 5.3.

Note: in modern robotics, body screw axis $\mathcal{B}_i$ means the screw axis of joint $i$ expressed in frame b, i.e. ${}^b\mathcal{S}_i$ in our notation.

$\hat{\omega}_i \quad {}^s\mathcal{S}_i$

$${}^s\mathcal{S}_7 = \begin{bmatrix} {}^s\omega_7 \\ {}^s v_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad {}^s\mathcal{S}_6 = \begin{bmatrix} {}^s\omega_6 \\ {}^s v_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -(L_1+L_2+L_3) \\ 0 \\ 0 \end{bmatrix} \qquad {}^s\mathcal{S}_5 = \begin{bmatrix} {}^s\omega_5 \\ {}^s v_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$${}^s\mathcal{S}_4 = \begin{bmatrix} {}^s\omega_4 \\ {}^s v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ L_1 \\ 0 \\ -W_1 \end{bmatrix} \qquad {}^s\mathcal{S}_3 = \begin{bmatrix} {}^s\omega_3 \\ {}^s v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad {}^s\mathcal{S}_2 = \begin{bmatrix} {}^s\omega_2 \\ {}^s v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$${}^s\mathcal{S}_1 = \begin{bmatrix} {}^s\omega_1 \\ {}^s v_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

**Exercise 4.9** The spatial RRPPRR open chain of Figure 4.15 is shown in its zero position. Determine the end-effector zero position configuration $M$, the screw axes $\mathcal{S}_i$ in $\{0\}$, and the screw axes $\mathcal{B}_i$ in $\{b\}$.

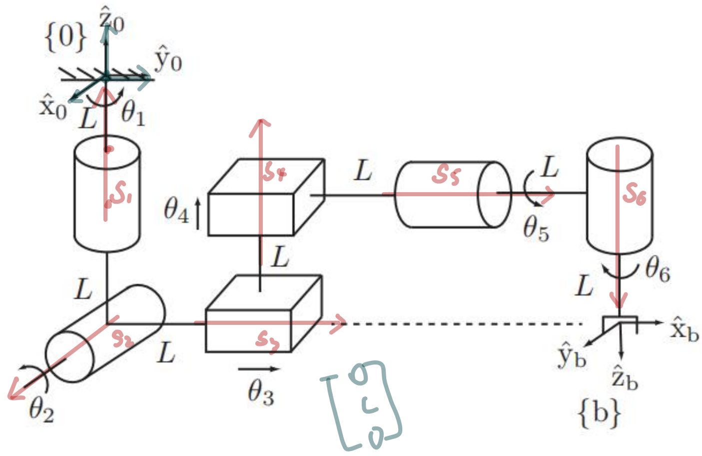



**Figure 4.15:** A spatial RRPPRR open chain with prescribed fixed and end-effector frames.

$$^0S_6 = \begin{bmatrix} ^0W_6 \\ ^0V_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \\ -3L \\ 0 \\ 0 \end{bmatrix} \quad ^0S_5 = \begin{bmatrix} ^0W_5 \\ ^0V_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ L \\ 0 \\ 0 \end{bmatrix} \quad ^0S_4 = \begin{bmatrix} ^0W_4 \\ ^0V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$^0S_3 = \begin{bmatrix} ^0W_3 \\ ^0V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad ^0S_2 = \begin{bmatrix} ^0W_2 \\ ^0V_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ -2L \\ 0 \end{bmatrix} \quad ^0S_1 = \begin{bmatrix} ^0W_1 \\ ^0V_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 3L \\ 0 & 0 & -1 & -2L \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0B_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad ^0B_5 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ -L \\ 0 \end{bmatrix} \quad ^0B_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

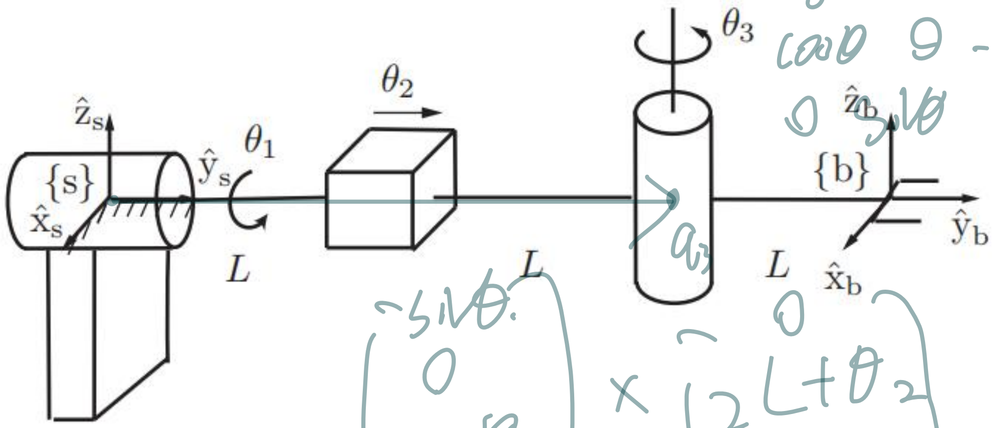
$$^0B_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad ^0B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad ^0B_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ -3L \end{bmatrix} \quad \begin{bmatrix} -1 \\ 0 \\ -3L \\ 0 \end{bmatrix}$$

**Exercise 5.8** The RPR robot of Figure 5.20 is shown in its zero position. The fixed and end-effector frames are respectively denoted {s} and {b}.

(a) Find the space Jacobian $J_s(\theta)$ for arbitrary configurations $\theta \in \mathbb{R}^3$.

$$\begin{matrix} 0 & -\cos\theta & 0 \\ \cos\theta & 0 & -\sin\theta \\ 0 & \sin\theta & 0 \end{matrix}$$




$$\begin{bmatrix} -\sin\theta_1 \\ 0 \\ \cos\theta_1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 2L+\theta_2 \\ 0 \end{bmatrix}$$

**Figure 5.20:** RPR robot.

$$J_s(\theta) = [\, S_1 \mid S_2(\theta_1) \mid S_3(\theta_1, \theta_2) \,]$$

$$S_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad S_2(\theta_1) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad S_3(\theta_1, \theta_2) = \begin{bmatrix} \sin\theta_1 \\ 0 \\ \cos\theta_1 \\ (2L+\theta_2)\cos\theta_1 \\ 0 \\ -(2L+\theta_2)\sin\theta_1 \end{bmatrix}$$

$$J_s(\theta) = \begin{bmatrix} 0 & 0 & \sin\theta_1 \\ 1 & 0 & 0 \\ 0 & 0 & \cos\theta_1 \\ 0 & 0 & (2L+\theta_2)\cos\theta_1 \\ 0 & 1 & 0 \\ 0 & 0 & -(2L+\theta_2)\sin\theta_1 \end{bmatrix}$$