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Adaptive Neural Output Feedback Controller Design with Reduced-Order Observer for a Class of Uncertain Nonlinear SISO Systems

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***Abstract*— An adaptive output feedback control is studied for uncertain nonlinear single-input–single-output systems with partial unmeasured states. In the scheme, a reduced-order observer (ROO) is designed to estimate those unmeasured states. By employing radial basis function neural networks and incorporating the ROO into a new backstepping design, an adaptive output feedback controller is constructively developed. A prominent advantage is its ability to balance the control action between the state feedback and the output feedback. In addition, the scheme can be still implemented when all the states are not available. The stability of the closed-loop system is guaranteed in the sense that all the signals are semiglobal uniformly ultimately bounded and the system output tracks the reference signal to a bounded compact set. A simulation example is given to validate the effectiveness of the proposed scheme.**

***Index Terms*— Adaptive neural control, nonlinear systems, output feedback control, reduced-order observer.**

I. Introduction

During the past two decades, the adaptive control of uncertain systems has attracted much attention. Several typical results have been designed in [1]–[3]. A main restriction in these results is that the uncertainties are required to satisfy linearly parameterized conditions with known functions. But this assumption is difficult to be ensured in practice.

In the recent years, many researchers have devoted much effort to deal with the problem of adaptive tracking control for nonlinear systems with completely unknown functions. By using the approximation property of the neural network (NN) or the fuzzy logic systems, several elegant adaptive control strategies have been proposed in [4]–[16] for uncertain nonlinear systems. However, a major constraint in these results is that the system state variables are assumed to be measurable. If the system states are unavailable, these results cannot be applied in practice. In the last decade, much progress has been

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made in the field of adaptive output feedback control by using the universal approximators for nonlinear systems with completely unknown functions [17]–[27]. From the results obtained in [17]–[27], we find a common property that the full-order observer (FOO) is designed to estimate the system state variables, i.e., the order of the observer is equal to the order of the nonlinear system observed. In the designed observer, the available *a priori* knowledge with respect to the system output is also observed by using an estimation variable which is used in the design procedure. In contrast to the FOO, the reduced-order observer (ROO) estimates only those states that are not directly measured. An observer in a lower dimension was explored in [28] for linear systems by Luenherger. An approach for the design of ROO for descriptor systems in linear form was presented in [29]. Two global output feedback control approaches with ROO were proposed in [30] and [31] for nonlinear systems with known functions. Based on a high-gain ROO, a global output feedback for nonlinear systems with known functions and satisfying Lipschitz condition was investigated in [32]. However, linear systems [28], [29], nonlinear systems with known function [30], [31], or systems satisfying the Lipschitz condition [32] rarely occur in practice. Therefore, it is necessary to design an adaptive output feedback controller with ROO to stabilize nonlinear systems without the above restriction conditions.

In this paper, we address an adaptive neural output feedback control scheme for a class of uncertain nonlinear systems with both the measured and unmeasured states based on the backstepping design procedure. It is the first time that the adaptive NN output feedback scheme with ROO is investigated to deal with a class of nonlinear systems with both measured and unmeasured states. Then, the order of the designed observer is less than that of the systems observed. It is proven via Lyapunov stability theory that all the signals of the closed-loop system are semiglobal uniformly ultimately bounded (SGUUB) and the tracking error converges to a small compact set.

II. Problem Statement and Preliminaries

Consider the uncertain single-input–single-output system nonlinear systems as follows:

$$\begin{cases} \dot{x}_i = f_i(x_1, \ldots, x_i) + x_{i+1}, i = 1, \ldots, n-1 \\ \dot{x}_n = f_n(x_1, \ldots, x_n) + b_0 u \\ y = x_1 \end{cases} \tag{1}$$

where $x = [x_1, \ldots, x_n]^T \in R^n$, $u \in R$ and $y \in R$ are the state, the input and the output of the systems, respectively; $f_i(x_1, \ldots, x_i), i = 1, \ldots, n$ are unknown smooth continuous functions; b_0 is a known constant; $x_i, i = 1, \ldots, \rho$ are available, and $x_j, j = \rho+1, \ldots, n$ are unavailable. Let $\bar{x}_i = [x_1, \ldots, x_i]^T$, $i = 1, \ldots, n$, and $\bar{x}'_{i-\rho} = \left[x_{\rho+1}, \ldots, x_i\right]^T, i = \rho+1, \ldots, n$.

The control objective is to construct an adaptive neural controller for (1) such that: 1) all the signals in the resulting closed-loop system are SGUUB, and 2) y tracks the reference signal $y_r(t)$ to a bounded compact set where $y_r(t)$ and its kth order derivative $y_r^{(k)}(t)$, $k = 1, \ldots, n$ are assumed to be bounded and continuous.

Based on the approximation property of the radial basis function NN (RBFNN) [33], on the set Ω_i which is a compact set, for any $\bar{x}_i \in \Omega_i$, $h_i(\bar{x}_i), i = 1, \ldots, n$ can be approximated as

$$h_i(\bar{x}_i) = W_i^{*T} S_i(\bar{x}_i) + \varepsilon_i^* \tag{2}$$

where $h_i(\bar{x}_i) = f_i(\bar{x}_i), i = 1, \ldots, \rho$ and $h_i(\bar{x}_i) = f_i(\bar{x}_i) - k_i \times f_\rho(\bar{x}_\rho), i = \rho+1, \ldots, n$ with $K = \left[k_{\rho+1}, \ldots, k_n\right]^T$ being the design parameters to be specified later; and W_i^* and ε_i^* are the optimal weight vector and the minimal approximation error, respectively. One needs to make the following assumption.

Assumption 1: On a compact set Ω_i, there exist the constants ω_i and ε_i such that $\left\|W_i^*\right\| \le \omega_i, \left|\varepsilon_i^*\right| \le \varepsilon_i$.

Let $\hat{W}_i$ denote the estimation of W_i^*, and define $\tilde{W}_i = \hat{W}_i - W_i^*$. The parameter adaptation law for $\hat{W}_i$ is given as

$$\dot{\hat{W}}_i = -\Gamma_i \left[\gamma_i \hat{W}_i - z_i S_i(\chi_i)\right] \tag{3}$$

where $\Gamma_i = \Gamma_i^T > 0$ and $\gamma_i > 0$ are the design parameters, and χ_i is defined as

$$\chi_i = \begin{cases} \bar{x}_i, & \text{when } i = 1, \ldots, \rho \\ \left[\bar{x}_\rho^T, \hat{\bar{x}}_{i-\rho}^{\prime T}\right]^T, & \text{when } i = \rho+1, \ldots, n \end{cases} \tag{4}$$

with $\hat{\bar{x}}'_i, i = 1, \ldots, n-\rho$ to be specified in the following. Based on (3) and Assumption 1, it follows that

$$\begin{aligned} \tilde{W}_i^T \left[-z_i S_i(\chi_i) + \Gamma_i^{-1} \dot{\hat{W}}_i\right] &= -\gamma_i \tilde{W}_i^T \hat{W}_i \\ &= -\gamma_i \tilde{W}_i^T \tilde{W}_i - \gamma_i \tilde{W}_i^T W_i^* \\ &\le -\frac{\gamma_i}{2} \tilde{W}_i^T \tilde{W}_i + \frac{\gamma_i}{2} \omega_i^2. \end{aligned} \tag{5}$$

III. Controller Designs and Stability Analysis

The system (1) can be rewritten as

$$\dot{\bar{x}}_\rho = A_\rho \bar{x}_\rho + B_\rho x_{\rho+1} + \sum_{m=1}^{\rho} B_m f_m(\bar{x}_m) \tag{6}$$

$$\dot{\bar{x}}'_{n-\rho} = A_{n-\rho} \bar{x}'_{n-\rho} + \sum_{m=\rho+1}^{n} B_{m-\rho} f_m(\bar{x}_m) + B_{n-\rho} b_0 u \tag{7}$$

where

$$A_\rho = \begin{bmatrix} 0 & & I_{\rho-1} & \\ \vdots & & & \\ 0 & 0 & \cdots & 0 \end{bmatrix}_{\rho\times\rho}, A_{n-\rho} = \begin{bmatrix} 0 & & I_{n-\rho-1} & \\ \vdots & & & \\ 0 & 0 & \cdots & 0 \end{bmatrix}_{(n-\rho)\times(n-\rho)}$$

$B_m = [0, \ldots, 0, 1]_{m\times 1}^T, m = 1, \ldots, \max\{\rho, n-\rho\}$.

Define $\hat{\bar{x}}'_{n-\rho} = \left[\hat{x}_{\rho+1}, \ldots, \hat{x}_n\right]^T = \bar{\phi}_{n-\rho} + K x_\rho = \left[\phi_{\rho+1} + k_{\rho+1} x_\rho, \ \ldots, \phi_n + k_n x_\rho\right]^T$ to denote the estimation of $\bar{x}'_{n-\rho}$. ϕ_i satisfies

$$\begin{cases} \dot{\phi}_i = \phi_{i+1} + \hat{W}_i^T S_i(\chi_i) + k_{i+1} x_\rho \\ \qquad - k_i\left(\phi_{\rho+1} + k_{\rho+1} x_\rho\right), i = \rho+1, \ldots, n-1 \\ \dot{\phi}_n = b_0 u + \hat{W}_n^T S_n(\chi_n) - k_n\left(\phi_{\rho+1} + k_{\rho+1} x_\rho\right) \end{cases} \tag{8}$$

where χ_i is defined in (4) with $\hat{\bar{x}}_{i-\rho}^{\prime T} = \left[\hat{x}_{\rho+1}, \ldots, \hat{x}_i\right]^T, i = \rho+1, \ldots, n$. Equation (8) can be repressed in the equivalent form

$$\dot{\bar{\phi}}_{n-\rho} = A_{n-\rho}\bar{\phi}_{n-\rho} + \sum_{m=\rho+1}^{n} B_{m-\rho}\hat{W}_m^T S_m(\chi_m) + A_{n-\rho}Kx_\rho - K\left(\phi_{\rho+1} + k_{\rho+1}x_\rho\right) + B_{n-\rho}b_0 u.$$

Using (6), (7), and the above equation, it is easy to obtain

$$\dot{\tilde{\bar{x}}}_{n-\rho} = A_{n-\rho}\tilde{\bar{x}}_{n-\rho} + \sum_{m=\rho+1}^{n} B_{m-\rho}\left[f_m(\bar{x}_m) - \hat{W}_m^T S_m(\chi_m)\right] - K\left(x_{\rho+1} - \phi_{\rho+1} - k_{\rho+1}x_\rho\right) - Kf_\rho\left(\bar{x}_\rho\right)$$

where $\tilde{\bar{x}}_{n-\rho} = \bar{x}'_{n-\rho} - \hat{\bar{x}}'_{n-\rho}$.

Noting that $Kf_\rho\left(\bar{x}_\rho\right) = \sum_{m=\rho+1}^{n} B_{m-\rho}k_m f_\rho\left(\bar{x}_\rho\right)$ and $K\left(x_{\rho+1} - \phi_{\rho+1} - k_{\rho+1}x_\rho\right) = K\tilde{x}_{\rho+1} = KC_{n-\rho}^T\tilde{\bar{x}}_{n-\rho}$ with $C_{n-\rho} = [1, 0, \ldots, 0]_{(n-\rho)\times 1}^T$, the above equation becomes

$$\dot{\tilde{\bar{x}}}_{n-\rho} = \bar{A}_{n-\rho}\tilde{\bar{x}}_{n-\rho} + \sum_{m=\rho+1}^{n} B_{m-\rho}\left[h_m(\bar{x}_m) - \hat{W}_m^T S_m(\chi_m)\right] \tag{9}$$

where K is chosen such that

$$\bar{A}_{n-\rho} = A_{n-\rho} - KC_{n-\rho}^T = \begin{bmatrix} -k_{\rho+1} & 1\ 0 \cdots 0 \\ \vdots & \vdots\ \vdots \cdots \vdots \\ -k_{n-\rho-1} & 0\ 0 \cdots 1 \\ -k_{n-\rho} & 0\ 0 \cdots 0 \end{bmatrix}$$

is a Hurwitz matrix. Then, there must exist positive definite matrixes $P = P^T > 0$ and $Q = Q^T > 0$ so that

$$\bar{A}_{n-\rho}^T P + P\bar{A}_{n-\rho} = -Q \tag{10}$$

where Q is required to satisfy $Q^* = Q - (n-\rho+2)I > 0$.

Introducing the coordinate transformation

$$\begin{cases} z_1 = x_1 - y_r, z_i = x_i - \alpha_{i-1}, i = 2, \ldots, \rho \\ z_i = \phi_i - \alpha_{i-1}, i = \rho+1, \ldots, n \end{cases} \tag{11}$$

where

$$\alpha_1 = -c_1 z_1 - \frac{1}{2\delta_1}z_1 - \hat{W}_1^T S_1(\chi_1) + \dot{y}_r \tag{12}$$

$$\alpha_i = -z_{i-1} - c_i z_i - \frac{1}{2\delta_i}z_i - \hat{W}_i^T S_i(\chi_i) + \Psi_{i-1} - \frac{1}{2\eta_{i-1}}z_i\sum_{m=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_m}\right)^2\left(1 + \|S_m(\chi_m)\|^2\right),\ i = 2, \ldots, \rho \tag{13}$$

$$\begin{aligned}\alpha_i = &-z_{i-1} - c_i z_i - k_{i+1}x_\rho + k_{\rho+1}\left(\phi_{\rho+1} + k_{\rho+1}x_\rho\right) \\ &+\Psi_{i-1} - 2\hat{W}_i^T S_i(\chi_i) - \frac{1}{2\delta_i}z_i\|S_i(\chi_i)\|^2 \\ &-\frac{1}{2}z_i\left(\frac{\partial\alpha_{i-1}}{\partial x_\rho}\right)^2 - \frac{1}{2\eta_{i-1}}z_i\sum_{m=1}^{\rho}\left(\frac{\partial\alpha_{i-1}}{\partial x_m}\right)^2 \\ &\times\left(1 + \|S_m(\chi_m)\|^2\right), i = \rho+1, \ldots, n-1\end{aligned} \tag{14}$$

$$\Psi_i = \sum_{m=1}^{i}\frac{\partial\alpha_i}{\partial\hat{W}_m}\dot{\hat{W}}_m + \sum_{m=1}^{i+1}\frac{\partial\alpha_i}{\partial y_r^{(m-1)}}y_r^{(m)} + \sum_{m=1}^{i}\frac{\partial\alpha_i}{\partial x_m}x_{m+1},\ i = 1, \ldots, \rho-1$$

$$\Psi_\rho = \sum_{m=1}^{\rho}\frac{\partial\alpha_\rho}{\partial\hat{W}_m}\dot{\hat{W}}_m + \sum_{m=1}^{\rho+1}\frac{\partial\alpha_\rho}{\partial y_r^{(m-1)}}y_r^{(m)} + \sum_{m=1}^{\rho-1}\frac{\partial\alpha_\rho}{\partial x_m}x_{m+1} + \frac{\partial\alpha_\rho}{\partial x_\rho}\left(z_{\rho+1} + k_{\rho+1}x_\rho + \alpha_\rho\right) - \frac{1}{2\delta_\rho}z_\rho - \frac{1}{2}z_\rho$$

$$\begin{aligned}\Psi_i = &\sum_{m=1}^{i}\frac{\partial\alpha_i}{\partial\hat{W}_m}\dot{\hat{W}}_m + \sum_{m=1}^{i+1}\frac{\partial\alpha_i}{\partial y_r^{(m-1)}}y_r^{(m)} + \sum_{m=\rho+1}^{i}\frac{\partial\alpha_i}{\partial\phi_m}\dot{\phi}_m \\ &+\sum_{m=1}^{\rho-1}\frac{\partial\alpha_\rho}{\partial x_m}x_{m+1} + \frac{\partial\alpha_\rho}{\partial x_\rho}\left(z_{\rho+1} + k_{\rho+1}x_\rho + \alpha_\rho\right) \\ &i = \rho+1, \ldots, n-1\end{aligned}$$

with c_i, δ_i, and η_i to be the positive design constants.

Step 1: The time derivative of $z_1 = x_1 - y_r$ is $\dot{z}_1 = W_1^{*T}S_1(\chi_1) + \varepsilon_1^* + x_2 - \dot{y}_r$. By using (12), it follows that $\dot{z}_1 = -c_1 z_1 - \tilde{W}_1^T S_1(\chi_1) + \varepsilon_1^* + z_2 - (1/2\delta_1)z_1$.

Consider the Lyapunov function candidate

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}\tilde{W}_1^T\Gamma_1^{-1}\tilde{W}_1. \tag{15}$$

Using Assumption 1 and Young's inequality, one has

$$z_1\varepsilon_1^* \le \frac{1}{2\delta_1}z_1^2 + \frac{\delta}{2_1}\left(\varepsilon_1^*\right)^2 \le \frac{1}{2\delta_1}z_1^2 + \frac{\delta_1}{2}\varepsilon_1^2. \tag{16}$$

By taking $i = 1$ in (5) and using (16), $\dot{V}_1$ is

$$\dot{V}_1 \le -c_1 z_1^2 - \gamma_1\tilde{W}_1^T\tilde{W}_1/2 + \sigma_1 + z_1 z_2 \tag{17}$$

where $\sigma_1 = \gamma_1 w_1^2/2 + \delta_1\varepsilon_1^2/2$.

Step i $(i = 2, \ldots, \rho-1)$: In this step, the virtual controller α_i will be constructed. In *Step* $i-1$, we can obtain α_{i-1} which is a function of $x_1, \ldots, x_{i-1}, \hat{W}_1, \ldots, \hat{W}_{i-1}$ and $y_r, \ldots, y_r^{(i-1)}$. Then, $\dot{\alpha}_{i-1}$ can be expressed as

$$\begin{aligned}\dot{\alpha}_{i-1} &= \sum_{m=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{W}_m}\dot{\hat{W}}_m + \sum_{m=1}^{i}\frac{\partial\alpha_{i-1}}{\partial y_r^{(m-1)}}y_r^{(m)} + \sum_{m=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_m}\dot{x}_m \\ &= \Psi_{i-1} + \sum_{m=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_m}W_m^{*T}S_m(\chi_m) + \sum_{m=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_m}\varepsilon_m^*.\end{aligned}$$

The time derivative of $z_i = x_i - \alpha_{i-1}$ is

$$\dot{z}_i = \dot{x}_i - \dot{\alpha}_{i-1} = W_i^{*T}S_i(\chi_m) + \varepsilon_i^* + x_{i+1} - \Psi_{i-1} - \sum_{m=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_m}W_m^{*T}S_m(\chi_m) - \sum_{m=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_m}\varepsilon_m^*. \tag{18}$$

Consider the Lyapunov function candidate

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{2}\tilde{W}_i^T\Gamma_i^{-1}\tilde{W}_i. \tag{19}$$

Using Assumption 1 and Young's inequality, it follows that

$$z_i\varepsilon_i^* \le \frac{1}{2\delta_i}z_i^2 + \frac{\delta_i}{2}\left(\varepsilon_i^*\right)^2 \le \frac{1}{2\delta_i}z_i^2 + \frac{\delta_i}{2}\varepsilon_i^2 \tag{20}$$

$$-z_i\sum_{m=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_m}W_m^{*T}S_m\left(\chi_m\right) - z_i\sum_{m=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_m}\varepsilon_m^*$$
$$-\frac{1}{2\eta_{i-1}}z_i^2\sum_{m=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_m}\right)^2\left(1+\left\|S_m\left(\chi_m\right)\right\|^2\right)$$
$$\le \frac{\eta_{i-1}}{2}\sum_{m=1}^{i-1}\left[\left\|W_m^*\right\|^2 + \left(\varepsilon_m^*\right)^2\right] \le \frac{\eta_{i-1}}{2}\sum_{m=1}^{i-1}\left(w_m^2+\varepsilon_m^2\right). \tag{21}$$

In *Step* $i-1$, it has been obtained that

$$\dot{V}_{i-1} \le -\sum_{m=1}^{i-1}c_m z_m^2 - \frac{1}{2}\sum_{m=1}^{i-1}\gamma_m\tilde{W}_m^T\tilde{W}_m + z_{i-1}z_i + \sigma_{i-1}. \tag{22}$$

Note that when $i=2$, $\dot{V}_1$ is in (17). By introducing $z_{i+1} = x_{i+1}-\alpha_i$, and using (5), (18), and (20)–(22), $\dot{V}_i$ is

$$\dot{V}_i \le -\sum_{m=1}^{i}c_m z_m^2 - \frac{1}{2}\sum_{m=1}^{i}\gamma_m\tilde{W}_m^T\tilde{W}_m + z_i z_{i+1} + \sigma_i \tag{23}$$

where $\sigma_i = \sigma_{i-1} + \frac{\gamma_i}{2}w_i^2 + \frac{\delta_i}{2}\varepsilon_i^2 + \frac{\eta_{i-1}}{2}\sum_{m=1}^{i-1}\left(w_m^2+\varepsilon_m^2\right)$.

Step ρ: $\dot{z}_\rho$ is

$$\dot{z}_\rho = W_\rho^{*T}S_\rho\left(\chi_\rho\right) + \varepsilon_\rho^* + x_{\rho+1} - \Psi_{\rho-1}$$
$$-\sum_{m=1}^{\rho-1}\frac{\partial\alpha_{\rho-1}}{\partial x_m}W_m^{*T}S_m\left(\chi_m\right) - \sum_{m=1}^{\rho-1}\frac{\partial\alpha_{\rho-1}}{\partial x_m}\varepsilon_m^*. \tag{24}$$

Consider the Lyapunov function candidate

$$V_\rho = V_{\rho-1} + \frac{1}{2}z_\rho^2 + \frac{1}{2}\tilde{W}_\rho^T\Gamma_\rho^{-1}\tilde{W}_\rho. \tag{25}$$

Using Young's inequality, it follows that

$$z_\rho\varepsilon_\rho^* \le \frac{1}{2\delta_\rho}z_\rho^2 + \frac{\delta_\rho}{2}\left(\varepsilon_\rho^*\right)^2 \le \frac{1}{2\delta_\rho}z_\rho^2 + \frac{\delta_\rho}{2}\varepsilon_\rho^2 \tag{26}$$

$$z_\rho\tilde{x}_{\rho+1} \le \frac{1}{2}z_\rho^2 + \frac{1}{2}\tilde{x}_{\rho+1}^2 \le \frac{1}{2}z_\rho^2 + \frac{1}{2}\tilde{\bar{x}}_{n-\rho}^T\tilde{\bar{x}}_{n-\rho} \tag{27}$$

$$-\sum_{m=1}^{\rho-1}\frac{\partial\alpha_{\rho-1}}{\partial x_m}W_m^{*T}S_m\left(\chi_m\right) - \sum_{m=1}^{\rho-1}\frac{\partial\alpha_{\rho-1}}{\partial x_m}\varepsilon_m^*$$
$$-\frac{1}{2\eta_{\rho-1}}z_\rho^2\sum_{m=1}^{\rho-1}\left(\frac{\partial\alpha_{\rho-1}}{\partial x_m}\right)^2\left(1+\left\|S_m\left(\chi_m\right)\right\|^2\right)$$
$$\le \frac{\eta_{\rho-1}}{2}\sum_{m=1}^{\rho-1}\left[\left\|W_m^*\right\|^2+\left(\varepsilon_m^*\right)^2\right] \le \frac{\eta_{\rho-1}}{2}\sum_{m=1}^{\rho-1}\left(w_m^2+\varepsilon_m^2\right). \tag{28}$$

By introducing $z_{\rho+1} = \phi_{\rho+1} - \alpha_\rho = \hat{x}_{\rho+1} - k_{\rho+1}x_\rho = x_{\rho+1} - \tilde{x}_{\rho+1} - k_{\rho+1}x_\rho - \alpha_\rho$ and using (5), (13), (23), and (26)–(28), $\dot{V}_\rho$ is

$$\dot{V}_\rho \le -\sum_{m=1}^{\rho}c_m z_m^2 - \frac{1}{2}\sum_{m=1}^{\rho}\gamma_m\tilde{W}_m^T\tilde{W}_m$$
$$+\sigma_\rho + z_\rho z_{\rho+1} + \frac{1}{2}\tilde{\bar{x}}_{n-\rho}^T\tilde{\bar{x}}_{n-\rho} \tag{29}$$

where $\sigma_\rho = \sigma_{\rho-1} + \frac{\gamma_\rho}{2}w_\rho^2 + \frac{\delta_\rho}{2}\varepsilon_\rho^2 + \frac{\eta_{\rho-1}}{2}\sum_{m=1}^{\rho-1}\left(w_m^2+\varepsilon_m^2\right)$.

Step $\rho+1$: $\dot{z}_{\rho+1}$ can be written as

$$\dot{z}_{\rho+1} = \phi_{\rho+2} + \hat{W}_{\rho+1}^T S_{\rho+1}\left(\chi_{\rho+1}\right) + k_{\rho+2}x_\rho$$
$$-k_{\rho+1}\left(\phi_{\rho+1}+k_{\rho+1}x_\rho\right) - \Psi_\rho - \frac{\partial\alpha_\rho}{\partial x_\rho}\tilde{x}_{\rho+1}$$
$$-\sum_{m=1}^{\rho}\frac{\partial\alpha_\rho}{\partial x_m}W_m^{*T}S_m\left(\chi_m\right) - \sum_{m=1}^{\rho}\frac{\partial\alpha_\rho}{\partial x_m}\varepsilon_m^*. \tag{30}$$

Consider the Lyapunov function candidate

$$V_{\rho+1} = V_\rho + \frac{1}{2}\tilde{\bar{x}}_{n-\rho}^T P\tilde{\bar{x}}_{n-\rho} + \frac{1}{2}z_{\rho+1}^2 + \frac{1}{2}\tilde{W}_{\rho+1}^T\Gamma_{\rho+1}^{-1}\tilde{W}_{\rho+1}. \tag{31}$$

Using Assumption 1 and Young's inequality, it follows that

$$-z_{\rho+1}W_{\rho+1}^*S_{\rho+1}\left(\chi_{\rho+1}\right) - \frac{1}{2\delta_{\rho+1}}z_{\rho+1}^2\left\|S_{\rho+1}\left(\chi_{\rho+1}\right)\right\|^2$$
$$\le \frac{\delta_{\rho+1}}{2}\left\|W_{\rho+1}^*\right\|^2 \le \frac{\delta_{\rho+1}}{2}w_{\rho+1}^2 \tag{32}$$

$$-z_{\rho+1}\frac{\partial\alpha_\rho}{\partial x_\rho}\tilde{x}_{\rho+1} - \frac{1}{2}z_{\rho+1}^2\left(\frac{\partial\alpha_\rho}{\partial x_\rho}\right)^2 \le \frac{1}{2}\tilde{x}_{\rho+1}^2$$
$$\le \frac{1}{2}\tilde{\bar{x}}_{n-\rho}^T\tilde{\bar{x}}_{n-\rho} \tag{33}$$

$$-z_{\rho+1}\sum_{m=1}^{\rho}\frac{\partial\alpha_\rho}{\partial x_m}W_m^{*T}S_m\left(\chi_m\right) - z_{\rho+1}\sum_{m=1}^{\rho}\frac{\partial\alpha_\rho}{\partial x_m}\varepsilon_m^*$$
$$-\frac{1}{2\eta_\rho}z_{\rho+1}^2\sum_{m=1}^{\rho}\left(\frac{\partial\alpha_\rho}{\partial x_m}\right)^2\left(1+\left\|S_m\left(\chi_m\right)\right\|^2\right)$$
$$\le \frac{\eta_\rho}{2}\sum_{m=1}^{\rho}\left[\left\|W_m^*\right\|^2+\left(\varepsilon_m^*\right)^2\right] \le \frac{\eta_\rho}{2}\sum_{m=1}^{\rho}\left(w_m^2+\varepsilon_m^2\right) \tag{34}$$

$$\tilde{\bar{x}}_{n-\rho}^T P\sum_{m=\rho+1}^{n}B_m\left[h_m\left(\bar{x}_m\right) - \hat{W}_m^T S_m\left(\chi_m\right)\right] \le \left\|\tilde{\bar{x}}_{n-\rho}^T\right\|$$
$$\times\sum_{m=\rho+1}^{n}\left\|PB_m\bar{\varepsilon}_m\right\| \le \frac{1}{2}\tilde{\bar{x}}_{n-\rho}^T\tilde{\bar{x}}_{n-\rho} + \frac{1}{2}\sum_{m=\rho+1}^{n}\left\|PB_m\bar{\varepsilon}_m\right\|^2 \tag{35}$$

where $\bar{\varepsilon}_m$ is a bounded constant. By using (5), (9), (10), (14), (29), (30), and (32)–(35), $\dot{V}_{\rho+1}$ is

$$\dot{V}_{\rho+1} \le -\sum_{m=1}^{\rho+1}c_m z_m^2 - \frac{1}{2}\sum_{m=1}^{\rho+1}\gamma_m\tilde{W}_m^T\tilde{W}_m$$
$$-\frac{1}{2}\tilde{\bar{x}}_{n-\rho}^T\left(Q-3I\right)\tilde{\bar{x}}_{n-\rho} + \sigma_{\rho+1} + z_{\rho+1}z_{\rho+2} \tag{36}$$

where $\sigma_{\rho+1} = \sigma_\rho + (\gamma_{\rho+1}/2)w_{\rho+1}^2 + (\delta_{\rho+1}/2)w_{\rho+1}^2 + (\eta_\rho/2)\sum_{m=1}^{\rho}\left(w_m^2+\varepsilon_m^2\right) + (1/2)\sum_{m=\rho+1}^{n}\left\|PB_m\bar{\varepsilon}_m\right\|^2$.

Step i ($i=\rho+2,\ldots,n-1$): $\dot{z}_i$ is

$$\dot{z}_i = \phi_{i+1} + \hat{W}_i^T S_i(\chi_i) + k_{i+1}x_\rho - k_{\rho+1}(\phi_{\rho+1}+k_{\rho+1}x_\rho) - \Psi_{i-1}$$
$$-\frac{\partial\alpha_\rho}{\partial x_\rho}\tilde{x}_{\rho+1} - \sum_{m=1}^{\rho}\frac{\partial\alpha_\rho}{\partial x_m}W_m^{*T}S_m\left(\chi_m\right) - \sum_{m=1}^{\rho}\frac{\partial\alpha_\rho}{\partial x_m}\varepsilon_m^*. \tag{37}$$

Consider the Lyapunov function candidate

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{2}\tilde{W}_i^T\Gamma_i^{-1}\tilde{W}_i. \tag{38}$$

Using Young's inequality, we obtain

$$-z_i W_i^* S_i(\chi_i) - \frac{1}{2\delta_i} z_i^2 \|S_i(\chi_i)\|^2 \le \frac{\delta_i}{2}\|W_i^*\|^2 \le \frac{\delta_i}{2}\omega_i^2 \quad (39)$$

$$-z_i \frac{\partial\alpha_{i-1}}{\partial x_\rho}\tilde{x}_i - \frac{1}{2}z_i^2\left(\frac{\partial\alpha_{i-1}}{\partial x_\rho}\right)^2 \le \frac{1}{2}\tilde{x}_i^2 \le \frac{1}{2}\tilde{\bar{x}}_{n-\rho}^T\tilde{\bar{x}}_{n-\rho} \quad (40)$$

$$-z_i\sum_{m=1}^{i}\frac{\partial\alpha_{i-1}}{\partial x_m}W_m^{*T}S_m(\chi_m) - z_i\sum_{m=1}^{i}\frac{\partial\alpha_{i-1}}{\partial x_m}\varepsilon_m^* - \frac{1}{2\eta_{i-1}}z_i^2\sum_{m=1}^{i}\left(\frac{\partial\alpha_{i-1}}{\partial x_m}\right)^2\left(1+\|S_m(\chi_m)\|^2\right)$$
$$\le \frac{\eta_{i-1}}{2}\sum_{m=1}^{i}\left[\|W_m^*\|^2+(\varepsilon_m^*)^2\right] \le \frac{\eta_{i-1}}{2}\sum_{m=1}^{i}\left(\omega_m^2+\varepsilon_m^2\right) \quad (41)$$

In *Step* $i-1$, we have obtained

$$\dot{V}_{i-1} \le -\sum_{m=1}^{i-1}c_m z_m^2 - \frac{1}{2}\sum_{m=1}^{i-1}\gamma_m\tilde{W}_m^T\tilde{W}_m$$
$$-\frac{1}{2}\tilde{\bar{x}}_{n-\rho}^T(Q-(i-1-\rho+2)I)\tilde{\bar{x}}_{n-\rho} + \sigma_{i-1} + z_{i-1}z_i. \quad (42)$$

By using (5), (14), (37), and (39)–(42), $\dot{V}_i$ is

$$\dot{V}_i \le -\sum_{m=1}^{i}c_m z_m^2 - \frac{1}{2}\sum_{m=1}^{i}\gamma_m\tilde{W}_m^T\tilde{W}_m$$
$$-\frac{1}{2}\tilde{\bar{x}}_{n-\rho}^T(Q-(i-\rho+2)I)\tilde{\bar{x}}_{n-\rho} + \sigma_i + z_i z_{i+1} \quad (43)$$

where $\sigma_i = \sigma_{i-1} + \frac{\gamma_i}{2}\omega_i^2 + \frac{\delta_i}{2}\omega_i^2 + \frac{\eta_{i-1}}{2}\sum_{m=1}^{i}(\omega_m^2+\varepsilon_m^2)$.

Step n: In a similar way, consider the Lyapunov function

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2}\tilde{W}_n^T\Gamma_n^{-1}\tilde{W}_n. \quad (44)$$

It is easy to obtain

$$u = \frac{1}{b_0}\alpha_n = \frac{1}{b_0}\Bigg[-z_{n-1} - c_n z_n + k_n(\phi_{\rho+1}+k_{\rho+1}x_\rho) + \Psi_{n-1} - 2\hat{W}_n^T S_n(\chi_n) - \frac{1}{2\delta_n}z_n\|S_n(\chi_n)\|^2 - \frac{1}{2}z_n\left(\frac{\partial\alpha_{n-1}}{\partial x_\rho}\right)^2 - \frac{1}{2\eta_{n-1}}z_n\sum_{m=1}^{\rho}\left(\frac{\partial\alpha_{n-1}}{\partial x_m}\right)^2\left(1+\|S_m(\chi_m)\|^2\right)\Bigg] \quad (45)$$

and

$$\dot{V}_n \le -\sum_{m=1}^{n}c_m z_m^2 - \frac{1}{2}\sum_{m=1}^{n}\gamma_m\tilde{W}_m^T\tilde{W}_m - \frac{1}{2}\tilde{\bar{x}}_{n-\rho}^T Q^*\tilde{\bar{x}}_{n-\rho} + \sigma_n \quad (46)$$

where $\sigma_n = \sigma_{n-1} + (\gamma_n+\delta_n)\omega_n^2/2 + \eta_{n-1}\sum_{m=1}^{n}(\omega_m^2+\varepsilon_m^2)/2$.

Equation (46) becomes

$$\dot{V}_n \le -\mu V_n + \sigma_n \quad (47)$$

where $\mu = \min\{2c_i, \gamma_i\lambda_{\min}(\Gamma_i), i=1,\ldots,n, \lambda_{\min}(P^{-1}Q^*)\}$.

Theorem 1: Consider (1). Under Assumption 1 and bounded initial conditions, by constructing $\alpha_i, i=1,\ldots,n-1$ and u, and choosing the adaptation laws $\hat{W}_i, i=1,\ldots,n$, the scheme can guarantee that: 1) all the signals in the closed-loop system are SGUUB, and 2) $y=x_1$ tracks y_r to a compact set defined by $\Omega = \{x_1 \,|\, |x_1-y_r| \le \sqrt{2\sigma_n/\mu}\}$.

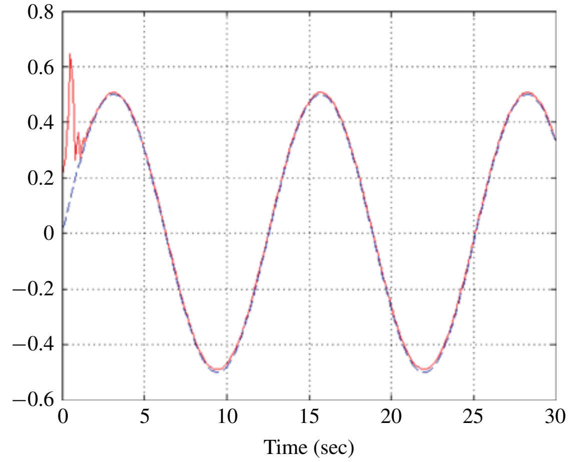

Fig. 1. $y = x_1$ (solid line) and $y_r = 0.5\sin(0.5t)$ (dashed line).

Proof: Multiplying both sides in (47) by $e^{\mu t}$ yields $d(V_n e^{\mu t})/dt \le \sigma_n e^{\mu t}$, and integrating this inequality over $[0,t]$ leads to $0 \le V_n(t) \le [V_n(0) - \sigma_n/\mu]e^{-\mu t} + \sigma_n/\mu$. Since σ_n and μ are positive constants, this implies that

$$0 \le V_n(t) \le V_n(0)e^{-\mu t} + \sigma_n/\mu. \quad (48)$$

Using (15), (19), (25), (31), (38), and (44), it has

$$V_n = \frac{1}{2}\tilde{\bar{x}}_{n-\rho}^T P\tilde{\bar{x}}_{n-\rho} + \frac{1}{2}\sum_{m=1}^{n}z_m^2 + \frac{1}{2}\sum_{m=1}^{n}\tilde{W}_m^T\Gamma_m^{-1}\tilde{W}_m. \quad (49)$$

From (48) and (49), it can be seen that $\tilde{\bar{x}}_{n-\rho}$, z_m, and $\tilde{W}_m, m=1,\ldots,n$ are bounded and one obtains $\|\tilde{\bar{x}}_{n-\rho}\| = \|\bar{x}'_{n-\rho} - \hat{\bar{x}}'_{n-\rho}\| \le \sqrt{2\left[V(0)e^{-\mu t}+\sigma_n/\mu\right]/\lambda_{\min}(P)}$. Since $\tilde{W}_m = \hat{W}_m - W_m^*$ and W_m^* is bounded from Assumption 1, $\hat{W}_m$ is bounded. From $z_1 = x_1 - y_r$, it is easy to know that x_1 is bounded. Because $z_1, \hat{W}_1, x_1, \dot{y}_r$ are bounded, from (12), it can be obtained that α_1 is bounded. Thus, $x_2 = z_2 + \alpha_1$ must be bounded. In a similar way, we can prove in turn that α_i and $x_i, i=1,\ldots,\rho$ are bounded. Then, from $z_{\rho+1} = \phi_{\rho+1} - \alpha_\rho$, it is obvious that $\phi_{\rho+1}$ is bounded. $\alpha_{\rho+1}$ in (14) is also bounded. Similar to the above procedure, it can be in turn proven that $\alpha_{\rho+2},\ldots,\alpha_{n-1}$ and $\phi_{\rho+2},\ldots,\phi_n$ are bounded. According to the definition of the controller u in (45), we can know that u is bounded. It is easy to obtain $|x_1-y_r| = |z_1| \le \sqrt{2[V_n(0)-\sigma_n/\mu]e^{-\mu t}+2\sigma_n/\mu}$. If $V_n(0) = \sigma_n/\mu$, then, it has $|x_1-y_r| \le \sqrt{2\sigma_n/\mu}$. If $V_n(0) \ne \sigma_n/\mu$, we obtain that, given any $\Delta_z > \sqrt{2\sigma_n/\mu}$, there must exist T_z such that for any $t > T_z$, it has $|x_1-y_r| \le \Delta_z$. This completes the proof. ■

IV. Simulation Example

Consider a single-link manipulator with the inclusion of actuator dynamics. The system model is given by

$$\begin{cases} M\ddot{q} + B\dot{q} + N\sin(q) = I \\ D\dot{I} + HI = V - K_m\dot{q} \end{cases} \quad (50)$$

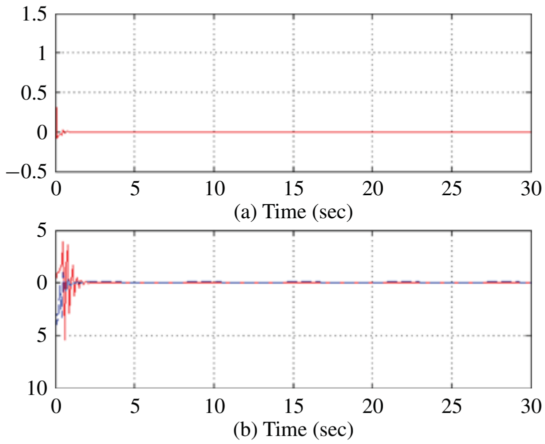

Fig. 2. (a) $\tilde{x}_3$. (b) z_2 (solid line) and z_3 (dashed line).

where $q, \dot{q}$, and $\ddot{q}$ denote the link angular position, velocity, and acceleration, respectively; I is the motor current; V is the voltage to be used as the control input. The parameter values are given as $M = 1, D = 0.05, B = 1, K_m = 10, H = 0.5$, and $N = 10$. Letting $x_1 = q, x_2 = \dot{q}$, $x_3 = I$, and $u = V$, (50) is expressed as the form of (1) where $f_1(x_1) = 0$, $f_2(x_1, x_2) = -10\sin x_1 - x_2$, $f_3(x_1, x_2, x_3) = (-10x_2 - 0.5x_3)/0.05$, $b_0 = 20$ and $y = x_1$ is the system output. The initial conditions are $x_1(0) = x_2(0) = 0.2, x_3(0) = 0.1$.

In the simulation studies, the centers and widths are selected on a regular lattice in the respective compact sets. Note that $f_1(x_1) = 0$, it is not necessary to use RBFNN to approximate it. $\hat{W}_2^T S_2(\chi_2)$ contains eight nodes with centers π_{l_2} $(l_2 = 1, \ldots, 8)$ evenly spaced in $[-2, 2] \times [-2, 2]$, and widths $\upsilon_{l_2} = 0.2$ where χ_2 is defined in (4). $\hat{W}_3^T S_3(\chi_3)$ contains 15 nodes with centers π_{l_3} $(l_3 = 1, \ldots, 15)$ evenly spaced in $[-1.5, 1.5] \times [-1.5, 1.5] \times [-1.5, 1.5]$, and widths $\upsilon_{l_3} = 0.3$ where χ_3 is defined in (4).

The initial values are chosen as $\hat{W}_i(0) = 0$, $i = 2, 3$. We consider the case that x_1, x_2 are available and x_3 is unmeasured. It is easy to know that $\rho = 2$. According to the above design procedure, the observer is designed as $\dot{\phi}_3 = b_0 u + \hat{W}_3^T S_3(\chi_3) - k_3(\phi_3 + k_3 x_2)$ where the initial value for ϕ_3 is shown as $\phi_3(0) = 0.5$. The controller is given in (45). The design parameters are selected as $\delta_1 = \delta_2 = \delta_3 = 2$, $\eta_1 = \eta_2 = 1$, $k_3 = 2$, $c_1 = c_2 = c_3 = 30$, $\gamma_2 = \gamma_3 = 2$, $\Gamma_2 = \mathrm{diag}[3, \ldots, 3]_{8\times 8}$ and $\Gamma_3 = \mathrm{diag}[2, \ldots, 2]_{15\times 15}$.

Figs. 1 and 2 show the simulation results for tracking $y_r = 0.5\sin(0.5t)$. Fig. 1 shows the output tracking trajectory, which shows that a good tracking performance is obtained. Fig. 2(a) illustrates the trajectories of the observer error $\tilde{x}_3$. This figure is sufficient to show that $\hat{x}_3$ is very good to estimate x_3. The boundedness of z_2 and z_3 is given in Fig. 2(b).

V. Conclusion

In this paper, an adaptive output feedback control scheme was presented to solve the tracking problem for the nonlinear systems in the strict feedback form. RBFNNs were used to approximate the unknown functions. A novel adaptive NN observer was designed to estimate the unmeasured states. The order of the observer was reduced compared with the previous works on the adaptive neural observer design. Based on Lyapunov theory, it is proven that all the signals of the resulting closed-loop system are guaranteed to be SGUUB and the tracking error can be reduced to a small compact set.

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Minimising Added Classification Error Using Walsh Coefficients

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***Abstract*— Two-class supervised learning in the context of a classifier ensemble may be formulated as learning an incompletely specified Boolean function, and the associated Walsh coefficients can be estimated without the knowledge of the unspecified patterns. Using an extended version of the Tumer–Ghosh model, the relationship between added classification error and second-order Walsh coefficients is established. In this brief, the ensemble is composed of multilayer perceptron base classifiers, with the number of hidden nodes and epochs systematically varied. Experiments demonstrate that the mean second-order coefficients peak at the same number of training epochs as ensemble test error reaches a minimum.**

***Index Terms*— Classification algorithm, multilayer perceptrons, pattern analysis, pattern recognition.**

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I. Introduction

Walsh coefficients, particularly the Rademacher-Walsh ordering, have previously been used for logic design [1]. In this brief, the second-order Walsh coefficients are used for pattern classification, where the goal is to minimize ensemble test error. The motivation will be explained in terms of the meaning of the spectral coefficients, and since the meaning is not dependent on the ordering, we will refer only to the Walsh coefficients. To understand the significance of the coefficients, the Tumer–Ghosh model [2] for ensemble classifiers will be described. This model defines added classification error as the difference between classifier error and Bayes error. The model provides a framework for understanding relationship between classifier correlation and reduction in error due to combining.

An important design issue for multiple classifier systems is a choice of individual (base) classifier complexity, which is usually set with the help of a validation set or cross-validation techniques [3], [4]. The maximum number of patterns should be reserved for training, which implies that base classifier parameters should ideally be determined from the training set. However, there has been no convincing theory or experimental study to suggest that any measure, computed on the training set, can reliably facilitate optimal ensemble design [5]. It is possible to bootstrap training patterns and use the ensemble out-of-bootstrap error estimate [6], in place of validation, but since each bootstrap replicate uses approximately two-thirds of the patterns, lack of training data can cause degradation of performance. In this brief, the proposed measure based on Walsh coefficients is computed on the training set.

The main contribution is to demonstrate the relationship between second-order Walsh coefficients of a Boolean function and added classification error of an ensemble, an issue that has not been addressed in any previous conference or journal publication. First-order Walsh coefficients were shown to provide a measure of class separability for selecting optimal base classifiers in [7], in which it is also shown that this does not imply optimality of the ensemble. In contrast, in this brief it is shown that second-order Walsh coefficients can be used to determine base classifier complexity for optimal ensemble performance. The motivation for using Walsh coefficients in ensemble design is fully explored in [5] and [7]. The interested reader is further referred to [1], [8] for an understanding of the meaning and applications of Walsh coefficients.

Section II explains the computation of the second-order coefficients, and Section III discusses their relationship with the model of added classification error. In Section IV, mean second-order Walsh coefficients are computed as the number of nodes and training epochs of multilayered perceptron (MLP) base classifiers are systematically varied.

II. Walsh Coefficients

Consider a two-class supervised learning problem of μ training patterns, with the label given to each pattern X_m denoted by $\Omega_m = \Phi(X_m)$, where $m = 1, \ldots, \mu$. It is assumed that there are N parallel base classifiers and that X_m is an N-dimension vector formed from the decisions of the N classifiers, applied to the original patterns which in general