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A multiple integral approach to stability of neutral time-delay systems

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ABSTRACT

This brief discusses the problem of delay-dependent stability analysis of neutral time-delay systems. First a multiple integral inequality is proposed. Based on the integral inequality, a novel delay-dependent stability criterion is established via a new Lyapunov functional including the multiple integral terms. A numerical example is given to illustrate the less conservatism of the proposed method.

1. Introduction

Time-delay which exists in many real systems is a natural phenomenon and often causes instability and poor performance of the systems.

In the past two decades, considerable attention has been devoted to the stability analysis of systems with time delays owing to the fact that they play an important role in many practical applications [1–17], and a lot of important methods have been proposed including model transformation approach, delay partitioning technique, bounding techniques, discretized Lyapunov functional method, free-weighting matrix approach, descriptor system approach.

Due to the fact that the neutral delay system involves the delays in both its state and the derivative of the state, stability of neutral delay systems proves to be a more complex issue, and the stability problems of neutral time-delay systems have received considerable attention by using above methods during the past decades.

In this brief, a new *multiple integral inequality*, which covers the existing ones as special cases, is introduced to discuss the problem of stability analysis of neutral time-delay systems, and a novel delay-dependent stability criterion is established via a new Lyapunov functional including the multiple integral terms. A numerical example is given to illustrate that the proposed method is effective and lead to less conservative result.

$\mathcal{R}^n$ denotes the n-dimensional Euclidean space, $\mathcal{R}^{m\times n}$ is the set of all $m \times n$ real matrices. $\rho(M)$ denotes the spectral radius of the matrix M The superscript "T" represents the transpose and "$*$" denotes the term that is induced by symmetry.

2. Preliminaries

Consider neutral time-delay system described by:

$$\begin{aligned}&\dot{x}(t) - C\dot{x}(t-d) = Ax(t) + A_d x(t-d),\\ &x(t) = \phi(t), \quad t \in [-d, 0],\end{aligned} \tag{1}$$

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where $x(t) \in \mathcal{R}^n$ is the state vector. $d > 0$ is the constant time delay. $\phi(t)$ is a compatible vector valued initial function. A, A_d and C are known real constant matrices with appropriate dimensions. It is assumed that $\rho(C) < 1$, which guarantees that the differential equation $\mathcal{D}x_t = x(t) - Cx(t-d) = 0$ is asymptotically stable for all d.

We propose the following lemma, which plays a key role in the derivation of the main result.

Lemma 1. *For any symmetric positive-definite constant matrix $Z \in \mathcal{R}^n$ and a scalar $d > 0$, if there exists a vector function $x(s) : [0, d] \to \mathbb{R}^n$ such that the following integrations are well defined, then*

$$-g_l(t) \leqslant -f_l(t)^{\mathrm{T}} Z f_l(t), \tag{2}$$

where l is non-negative integer and

$$g_l(t) = \frac{d^{l+1}}{(l+1)!} \int_{t-d}^{t} \int_{v_l}^{t} \cdots \int_{v_1}^{t} x(s)^{\mathrm{T}} Z x(s) \mathrm{d}s \mathrm{d}v_1 \cdots \mathrm{d}v_l$$
$$f_l(t) = \int_{t-d}^{t} \int_{v_l}^{t} \cdots \int_{v_1}^{t} x(s) \mathrm{d}s \mathrm{d}v_1 \cdots \mathrm{d}v_l.$$

Proof. It can be found that

$$\begin{bmatrix} x(s)^{\mathrm{T}} Z x(s) & x(s)^{\mathrm{T}} \\ * & Z^{-1} \end{bmatrix} = \begin{bmatrix} x(s)^{\mathrm{T}} Z^{1/2} & 0 \\ Z^{-1/2} & 0 \end{bmatrix} \begin{bmatrix} Z^{1/2} x(s) & Z^{-1/2} \\ 0 & 0 \end{bmatrix} \geqslant 0. \tag{3}$$

Hence,

$$\begin{bmatrix} \frac{(l+1)!}{d^{l+1}} g_l(t) & f_l^{\mathrm{T}}(t) \\ * & h_l \end{bmatrix} \geqslant 0, \tag{4}$$

where

$$h_l = \int_{t-d}^{t} \int_{v_l}^{t} \cdots \int_{v_1}^{t} Z^{-1} \mathrm{d}s \mathrm{d}v_1 \cdots \mathrm{d}v_l = \frac{d^{l+1}}{(l+1)!} Z^{-1}.$$

After some simple manipulation including Schur complements, the above inequality yields (2). This completes the proof. □

Remark 1. When $l = 0$, that is, $g_0(t) = d \int_{t-d}^{t} x(s)^{\mathrm{T}} Z x(s) \mathrm{d}s$ and $f_0(t) = \int_{t-d}^{t} x(s) \mathrm{d}s$, inequality (2) reduces to the well-known Jensen inequality [1]. When $l = 1$, inequality (2) reduces to the inequality of [3]. Thus, the existing inequalities can be regarded as the special cases of our inequality.

3. Main result

In this section, a new and improved delay-dependent stability criterion is presented for the system (1). For presentation convenience, in the following, we denote

$$\varphi(t) = \begin{bmatrix} x(t)^{\mathrm{T}} & \dot{x}(t-d)^{\mathrm{T}} & x(t-d)^{\mathrm{T}} & f_0(t)^{\mathrm{T}} & f_1(t)^{\mathrm{T}} & \ldots & f_{m-1}(t)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}},$$
$$e_i = \begin{bmatrix} 0_{n \times (i-1)n} & I_n & 0_{n \times (m+3-i)n} \end{bmatrix},$$
$$\delta_i = \frac{d^{i-1}}{(i-1)!} e_1 - e_{i+2},$$
$$\rho(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix},$$

where f_l follows the same definition as that in Lemma 1. This way, system (1) can be rewritten as

$$\dot{x}(t) = \Xi \varphi(t), \tag{5}$$

where

$$\Xi = Ae_1 + Ce_2 + A_d e_3.$$

Now, we have the following main result.

Theorem 1. *For a given positive integer m, the neutral time-delay systems (1) is stable, if there exist symmetric positive-definite matrices $P, Z_l, (l = 0, 1, \ldots m)$ such that*

$$\Omega = \Sigma_1^{\mathrm{T}} P \Sigma_2 + \Sigma_2^{\mathrm{T}} P \Sigma_1 + \sum_{l=0}^{m} \left[\frac{d^l}{l!}\right]^2 \begin{bmatrix} e_1 \\ \Xi \end{bmatrix} Z_l \begin{bmatrix} e_1 \\ \Xi \end{bmatrix} - \begin{bmatrix} e_3 \\ e_2 \end{bmatrix}^{\mathrm{T}} Z_0 \begin{bmatrix} e_3 \\ e_2 \end{bmatrix} - \sum_{l=1}^{m} \begin{bmatrix} e_{l+3} \\ \delta_l \end{bmatrix}^{\mathrm{T}} Z_l \begin{bmatrix} e_{l+3} \\ \delta_l \end{bmatrix} < 0, \tag{6}$$

where

$$\Sigma_1 = \begin{bmatrix} e_1^{\mathrm{T}} & e_3^{\mathrm{T}} & e_4^{\mathrm{T}} & \dots & e_{m+3}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \Sigma_2 = \begin{bmatrix} \Xi^{\mathrm{T}} & e_2^{\mathrm{T}} & \delta_1^{\mathrm{T}} & \delta_2^{\mathrm{T}} & \dots & \delta_m^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}.$$

Proof. Choose a Lyapunov functional candidate to be

$$V(x_t) = V_1(x_t) + \sum_{l=0}^{m} W_l(x_t), \tag{7}$$

where

$$V_1(x_t) = \eta(t)^T P \eta(t),$$

$$W_0(x_t) = \int_{t-d}^{t} \rho(s)^T Z_0 \rho(s) ds,$$

$$W_l(x_t) = \frac{d^l}{l!} \int_{t-d}^{t} \int_{v_l}^{t} \cdots \int_{v_1}^{t} \rho(s)^T Z_l \rho(s) ds dv_1 \cdots dv_l, \; l \geqslant 1,$$

where $\eta(t)$ is given by $\varphi(t)$ without $\dot{x}(t-d)$, and $x_t = x(t+\theta), -d \leqslant \theta \leqslant 0$. Then, the time-derivative of $V(x_t)$ along the solution of the system (1) gives

$$\dot{V}_1(x_t) = 2\varphi(t)^{\mathrm{T}} \Sigma_1^{\mathrm{T}} P \Sigma_2 \varphi(t), \dot{W}_0(x_t) = \varphi(t)^{\mathrm{T}} \begin{bmatrix} e_1 \\ \Xi \end{bmatrix} Z_0 \begin{bmatrix} e_1 \\ \Xi \end{bmatrix} \varphi(t) - \varphi(t)^{\mathrm{T}} \begin{bmatrix} e_3 \\ e_2 \end{bmatrix}^{\mathrm{T}} Z_0 \begin{bmatrix} e_3 \\ e_2 \end{bmatrix} \varphi(t),$$
$$\dot{W}_l(x_t) = \left[\frac{d^l}{l!}\right]^2 \rho(t) Z_l \rho(t) - \frac{d}{l} W_{l-1}(x_t) \leqslant \left[\frac{d^l}{l!}\right]^2 \varphi(t)^{\mathrm{T}} \begin{bmatrix} e_1 \\ \Xi \end{bmatrix} Z_l \begin{bmatrix} e_1 \\ \Xi \end{bmatrix} \varphi(t) - \varphi(t)^{\mathrm{T}} \begin{bmatrix} e_{l+3} \\ \delta_l \end{bmatrix}^{\mathrm{T}} Z_l \begin{bmatrix} e_{l+3} \\ \delta_l \end{bmatrix} \varphi(t), \tag{8}$$

where Lemma 1 is applied.

We can get from (6) and (8) that

$$\dot{V}(x_t) \leqslant \varphi(t)^{\mathrm{T}} \Omega \varphi(t) < 0. \tag{9}$$

Thus, system (1) is stable. This completes the proof. □

Remark 2. Theorem 1 gives a delay-dependent stability criterion for the neutral time-delay systems (1) by a new Lyapunov functional including the multiple integral terms. It can be seen that the parameter m has relation with the upper bound of the time-delay d and the conservatism of the results. More specifically, the larger m corresponds to the larger d and the less conservatism of Theorem 1.

Remark 3. It is noted that when $m = 2$, the Lyapunov functional (7) reduce to the following Lyapunov functional

$$V(x_t) = \eta(t)^T P \eta(t) + \int_{t-d}^{t} \rho(s)^T Z_0 \rho(s) ds + \sum_{l=1}^{2} W_l(x_t), \tag{10}$$

where

$$W_1(x_t) = d \int_{t-d}^{t} \int_{v_1}^{t} \rho(s)^T Z_1 \rho(s) ds dv_1,$$

$$W_2(x_t) = \frac{d^2}{2} \int_{t-d}^{t} \int_{v_2}^{t} \int_{v_1}^{t} \rho(s)^{\mathrm{T}} Z_2 \rho(s) ds dv_1 dv_2.$$

Letting $Z_0 = Q, dZ_1 = Z, \frac{d^2}{2} Z_2 = diag\{\varepsilon I, R\}$ $(\varepsilon \to 0)$, and

Table 1
Comparisons of maximum allowed time-delay d for Example 1.

c	0	0.1	0.3	0.5
[3]	5.30	5.21	4.85	4.20
Theorem 1 $m = 6$	5.8613	5.7781	5.3946	4.6687
Theorem 1 $m = 7$	5.8779	5.7910	5.4007	4.6703
Theorem 1 $m = 8$	5.9287	5.8427	5.4417	4.6917
Theorem 1 $m = 9$	6.1028	5.9816	5.5173	4.7247

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & 0 \\ * & P_{22} & P_{23} & 0 \\ * & * & P_{33} & 0 \\ * & * & * & \varepsilon I \end{bmatrix},$$

we have that Lyapunov functional (10) reduces to the Lyapunov functional of [3]. Therefore, our result theoretically has less conservatism than results of [3].

4. Numerical example

In this section, an example is applied to show the effectiveness of the result and its advantage over existing methods.

Example 1. Consider system (1) with

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad A_d = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}, 0 \leqslant c < 1$$

The maximum upper bounds on the delay can be found in Table 1 for different c, from which we can find that the stability criterion proposed in this paper gives much better results than that in [3]. Moreover, when $m = 12$, the maximum upper bound on the delay of system with $c = 0$ is 6.17, which is the *theoretical bound* to ensure the stability of system.

5. Conclusion

In this brief, the problem of delay-dependent stability analysis of neutral time-delay systems has been discussed. A new multiple integral inequality has been proposed, based on which, a new delay-dependent stability criterion has been proposed by a novel Lyapunov functional including the multiple integral terms. A numerical example has been given to show the less conservatism of the proposed method. It should be pointed out that the proposed multiple integral inequality can be widely applied to all kinds of systems with time delay and leads to less conservative results.

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