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Stabilization for switched stochastic neutral systems under asynchronous switching

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ABSTRACT

In this paper, the mean-square exponential stability of switched stochastic neutral systems with time-varying delay under asynchronous switching is investigated. A new Lyapunov function dependent on the controllers' switching signal is constructed, which can counteract the difficulty of controller design to stabilize the system under asynchronous switching. Moreover, the value of the Lyapunov function is allowed to increase during mismatched periods. Based on the average dwell time approach, a sufficient condition expressed by a set of linear matrix inequalities (LMIs) is derived to guarantee the mean-square exponential stability for the closed-loop system, and the controllers with asynchronous switching are designed. Finally, a numerical example illustrates the effectiveness of the results.

1. Introduction

Switched systems are an important class of hybrid systems which are composed of several dynamical subsystems and a switching rule that orchestrates the switching among them to ensure stability and satisfied performance. Many physical and engineering systems can be modeled as switched systems, such as chemical processes, power electronic, automatic highway systems [3,15,20]. In the past few decades, switched systems have attracted considerable attention. Many remarkable achievements have been made on the issues such as controllability, observability, stability, stabilization, observer design, filtering, and fault detection [1,4,7,14,16–18]. Among them, stability analysis and stabilization control problems are the main concerns, and large numbers of excellent results have been published; see, e.g., [4,12,13,17,21] and the references therein.

As is well known, time delay as a source of instability and poor performance often appears in many dynamical systems, such as neural networks, nuclear reactors, manual control and microwave oscillator systems. Actually, there are many physical processes which can be described by differential equations of neutral type, i.e. neutral systems, where time delay appears both in state and state derivative. In the literature, various analysis techniques have been utilized to study the neutral systems, see for example [2,11,27]. However, due to the complicated behavior of switched neutral systems, only a few results on such systems have appeared [8,26,28]. On the other hand, stochastic phenomena exist widely in engineering applications and social systems. Stochastic systems have received much attention and many results have been reported [2,6,30]. It should be noted that so far the synthesis issue for switched stochastic neutral systems has not been fully investigated.

In the ideal case, the switching of the controllers coincides exactly with that of corresponding subsystems. In engineering application, however, since it inevitably takes some time to identify the active subsystem and apply for the matched controller, the switching time of controllers may lag behind that of practical subsystems, which results in asynchronous switch-

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ing between the controllers and system modes. The necessities of considering asynchronous switching for efficient controller design have been shown in a class of chemical systems [10]. Some primary studies on the asynchronous switching problems for switched systems have been proposed [5,23–25]. Recently, Lyapunov stability theory is used to investigate the problem [9,22,29]. In [9], asynchronous H_∞ filtering is considered for discrete-time switched systems. [22] analyzes the robust reliable control for a class of uncertain switched neutral systems under asynchronous switching. Sufficient conditions for the stability of such systems are given by two inequalities. In [29], stability, L_2-gain and asynchronous H_∞ control are considered for discrete-time switched systems.

Based on the above discussion, switched stochastic neutral time-delay systems under asynchronous switching are worth studying. To the best of our knowledge, few results on the stabilization for such systems have been reported, which motivates this study for us. Applying Itô's differential formula and the Lyapunov stability theory, a sufficient condition with respect to mean-square exponential stability of the given system is obtained and the controllers with asynchronous switching are designed. The presented condition is applicable to the case where the subsystems are unstable during mismatched periods resulted from asynchronous switching between subsystems and the corresponding controllers. Furthermore, the switching signal of the Lyapunov function constructed in the paper is dependent on the controllers' switching signal, which is convenient for the analysis of the proposed issue.

The rest of this paper is organized as follows. In Section 2, the problem to be studied is formulated and some definitions and a lemma are introduced. In Section 3, based on the average dwell time approach, the controllers are developed to ensure the mean-square exponential stability of the switched stochastic neutral system under asynchronous switching. A numerical example is provided to illustrate the method in Section 4 and we conclude this paper in Section 5.

Notations: Throughout this paper, the notations are standard. The superscript "T" denotes matrix transposition, "$*$" the symmetric term in a symmetric matrix, $diag\{\cdots\}$ a block-diagonal matrix and I the identity matrix. $\mathbb{R}^n$ stands for the n-dimensional Euclidean space, $\mathcal{E}\{\cdot\}$ the expectation operator and $\|\cdot\|$ the Euclidean norm. $P > 0$ means that P is real symmetric and positive definite, and $\lambda_{\min}(P)$ ($\lambda_{\max}(P)$) is the minimum (maximum) eigenvalue of P.

2. Problem formulation and preliminaries

Consider the following switched stochastic neutral system described by the Itô's form:

$$\begin{aligned} &d[x(t) - Cx(t-\tau(t))] = [A_{\sigma(t)}x(t) + A_{d\sigma(t)}x(t-\tau(t)) + B_{\sigma(t)}u(t)]dt + [F_{\sigma(t)}x(t) + F_{d\sigma(t)}x(t-\tau(t))]d\varpi(t), \\ &x(s) = \varphi(s), \quad s \in [-h, 0], \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $\varpi(t)$ is a one-dimensional Brownian motion satisfying $\mathcal{E}\{d\varpi(t)\} = 0$ and $\mathcal{E}\{d\varpi^2(t)\} = dt$, the switching signal $\sigma(t):[t_0,\infty) \to M = \{1,2,\ldots,l\}$ is a piecewise continuous (from the right) function, where l is the number of subsystems. Specifically, denote $\sigma(t):\{(t_0,\sigma(t_0)),\ldots,(t_k,\sigma(t_k)),\ldots,k = 0,1,2,\ldots\}$, where t_0 is the initial switching instant, and t_k is the kth switching instant. For any $i \in M$, A_i, A_{di}, B_i, F_i, F_{di}, and C are constant matrices, $\varphi(s) \in \mathbb{R}^n$ is the initial condition, $\tau(t)$ denotes the time-varying delay satisfying

$$0 \leqslant \tau(t) \leqslant h, \quad \dot{\tau}(t) \leqslant d < 1. \tag{2}$$

For system (1), we consider the piecewise constant state feedback given by

$$u(t) = K_{\sigma'(t)}x(t), \tag{3}$$

where $\sigma'(t)$ is the switching signal of the controllers. Ideally, the controllers switch synchronously with the switching of subsystems, that is $\sigma'(t) = \sigma(t)$. In practice, since it inevitably takes some time to identify the system modes and apply the matched controllers, the switching instants of the controllers lag behind those of the subsystems, that is, $\sigma'(t):\{(t_0,\sigma(t_0)), (t_1 + \Delta_1,\sigma(t_1)),\ldots,(t_k + \Delta_k,\sigma(t_k)),\ldots\}$, where the lag time Δ_k satisfies $\Delta_k < \upsilon$ and υ is a known constant. Correspondingly, the running time is divided into two parts: matched periods $[t_{k-1} + \Delta_{k-1}, t_k)$, $k = 1,2,3,\ldots$, with $\Delta_0 = 0$ and mismatched periods $[t_k, t_k + \Delta_k)$, $k = 1,2,3,\ldots$.

Suppose that the ith subsystem is identified at the switching instant t_{k-1},$\sigma(t_{k-1}) = i$, and the jth subsystem at the switching instant t_k, $\sigma(t_k) = j$, then the corresponding switching controllers are activated at the switching instants $t_{k-1} + \Delta_{k-1}$ and $t_k + \Delta_k$, respectively.

For the sake of simplicity, define

$$\begin{aligned} &\widetilde{A}_i = A_i + B_iK_i, \quad f_i(t) = \widetilde{A}_ix(t) + A_{di}x(t-\tau(t)), \quad g_i(t) = F_ix(t) + F_{di}x(t-\tau(t)), \\ &\widetilde{A}_{ij} = A_j + B_jK_i, \quad f_{ij}(t) = \widetilde{A}_{ij}x(t) + A_{dj}x(t-\tau(t)), \quad \mathcal{D}(t) = x(t) - Cx(t-\tau(t)). \end{aligned}$$

Then in the matched periods, $t \in [t_{k-1} + \Delta_{k-1}, t_k)$, the closed-loop system of ith subsystem with the ith controller can be written as

$$d\mathcal{D}(t) = f_i(t)dt + g_i(t)d\varpi(t).$$

In the mismatched periods, $t \in [t_k, t_k + \Delta_k)$, the closed-loop system of jth subsystem with the ith controller can be written as

$$d\mathcal{D}(t) = f_{ij}(t)dt + g_j(t)d\varpi(t).$$

At the end of this section, we introduce some definitions, an assumption, and a lemma for the development of our results.

Assumption 1. $\rho(C) < 1$, where $\rho(C)$ denotes the spectral radius of C.

The assumption ensures that the operator $\mathcal{D}(t) = x(t) - Cx(t - \tau(t))$ is stable.

Definition 1. The equilibrium $x^* = 0$ of system (1) is said to be mean-square exponentially stable if

$$\mathcal{E}\{\|\mathcal{D}(t)\|^2\} \leqslant \kappa \sup_{-h\leqslant\theta\leqslant 0} \mathcal{E}\{\|x(t_0+\theta)\|^2\}e^{-\lambda(t-t_0)}, \ \forall t \geqslant t_0$$

is satisfied for constants $\kappa \geqslant 1$ and $\lambda > 0$.

Definition 2 [4]. For any constants $T_2 > T_1 \geqslant 0$, let $N_\sigma(T_1,T_2)$ denote the number of switching of σ over (T_1,T_2). If $N_\sigma(T_1,T_2)$ - $_\sigma(T_1,T_2) \leqslant N_0 + (T_2 - T_1)/T_a$ holds for $T_a > 0$, $N_0 \geqslant 0$, then T_a is called the average dwell time. As commonly used in the literature, we choose $N_0 = 0$.

Lemma 1 [19]. *Let D, S and W > 0 be real matrices with appropriate dimensions. Then for any nonzero vectors x and y with appropriate dimensions, we have*

$$2x^T DSy \leqslant x^T DWD^T x + y^T S^T W^{-1} Sy.$$

3. Main results

The objective of the paper is to design the controllers such that the closed-loop system is stabilizable under asynchronous switching. In the sequel, applying the average dwell time approach, we give sufficient conditions for the mean-square exponential stability of system (1), and the controllers are designed.

3.1. Stability analysis

The following result presents a sufficient condition of the mean-square exponential stability for system (1).

Theorem 1. *Given positive constants α, β and $\mu \geqslant 1$, for $i,j \in M$, $i \neq j$, if there exist symmetric and positive definite matrices P_i and Q_i, such that*

$$P_i \leqslant \mu P_j, \quad Q_i \leqslant \mu Q_j, \tag{4}$$

$$\Phi_i = \begin{bmatrix} \Lambda_{i11} & \Lambda_{i12} \\ * & \Lambda_{i22} \end{bmatrix} < 0, \tag{5}$$

$$\Xi_i = \begin{bmatrix} \Upsilon_{i11} & \Upsilon_{i12} \\ * & \Upsilon_{i22} \end{bmatrix} < 0, \tag{6}$$

where

$$\begin{aligned}
\Lambda_{i11} &= P_i\widetilde{A}_i + \widetilde{A}_i^T P_i + \widetilde{A}_i^T P_i \widetilde{A}_i + F_i^T P_i F_i + Q_i + \alpha P_i,\\
\Lambda_{i12} &= P_i A_{di} + \widetilde{A}_i^T P_i A_{di} + F_i^T P_i F_{di} - \alpha P_i C,\\
\Lambda_{i22} &= (1+\alpha)C^T P_i C + A_{di}^T P_i A_{di} + F_{di}^T P_i F_{di} - (1-d)e^{-\alpha h} Q_i,\\
\Upsilon_{i11} &= P_i\widetilde{A}_{ij} + \widetilde{A}_{ij}^T P_i + \widetilde{A}_{ij}^T P_i \widetilde{A}_{ij} + F_j^T P_i F_j + Q_i - \beta P_i,\\
\Upsilon_{i12} &= P_i A_{dj} + \widetilde{A}_{ij}^T P_i A_{dj} + F_j^T P_i F_{dj} + \beta P_i C,\\
\Upsilon_{i22} &= (1-\beta)C^T P_i C + A_{dj}^T P_i A_{dj} + F_{dj}^T P_i F_{dj} - (1-d)Q_i.
\end{aligned}$$

Then system (1) is mean-square exponentially stable under asynchronous switching for any switching signal with the average dwell time satisfying

$$T_a > T_a^* = \frac{\ln\mu + (\alpha+\beta)(h+\upsilon)}{\alpha}. \tag{7}$$

Proof 1. As formulated in the pervious subsection, there exist two periods during the whole running time: matched and mismatched periods. Then, the proof will be presented in two parts.

Firstly, when $t \in [t_{k-1} + \Delta_{k-1}, t_k)$, system (1) runs in matched periods. Choose the Lyapunov functional candidate as follows:

$$V_{1\sigma'}(t) = \mathcal{D}^T(t)P_{\sigma'}\mathcal{D}(t) + \int_{t-\tau(t)}^{t} e^{\alpha(s-t)}x^T(s)Q_{\sigma'}x(s)ds. \tag{8}$$

According to Itô's differential formula, the stochastic differential is

$$dV_{1i}(t) = \mathcal{L}V_{1i}(t)dt + 2\mathcal{D}^T(t)P_i g_i(t)d\varpi(t), \tag{9}$$

with the infinitesimal operator

$$\mathcal{L}V_{1i}(t) = x^T(t)Q_i x(t) - (1-\dot{\tau}(t))e^{-\alpha\tau(t)}x^T(t-\tau(t))Q_i x(t-\tau(t)) - \alpha\int_{t-\tau(t)}^{t} e^{\alpha(s-t)}x^T(s)Q_i x(s)ds + 2\mathcal{D}^T(t)P_i f_i(t) + g_i^T(t)P_i g_i(t). \tag{10}$$

In addition, it is obtained from Lemma 1 that

$$-2[Cx(t-\tau(t))]^T P_i f_i(t) \leqslant x^T(t-\tau(t))C^T P_i C x(t-\tau(t)) + f_i^T(t)P_i f_i(t). \tag{11}$$

Then, combining (2), (10) and (11), it follows that

$$\mathcal{L}V_{1i}(t) + \alpha V_{1i}(t) \leqslant \eta^T(t)\Phi_i\eta(t), \tag{12}$$

where $\eta(t) = \begin{bmatrix} x^T(t) & x^T(t-\tau(t)) \end{bmatrix}^T$.

In conjunction with (5), we can get

$$\mathcal{L}V_{1i}(t) \leqslant -\alpha V_{1i}(t). \tag{13}$$

Then using (9) and (13), we have

$$d[e^{\alpha t}V_{1i}(t)] = \alpha e^{\alpha t}V_{1i}(t)dt + e^{\alpha t}dV_{1i}(t) \leqslant e^{\alpha t}[\alpha V_{1i}(t)dt - \alpha V_{1i}(t)dt + 2\mathcal{D}^T(t)P_i g_i(t)d\varpi(t)] = 2e^{\alpha t}\mathcal{D}^T(t)P_i g_i(t)d\varpi(t). \tag{14}$$

Integrating both sides of (14) from $t_{k-1} + \Delta_{k-1}$ to t and taking expectation, we have

$$\mathcal{E}\{V_{1i}(t)\} \leqslant \mathcal{E}\{V_{1i}(t_{k-1}+\Delta_{k-1})\}e^{-\alpha(t-t_{k-1}-\Delta_{k-1})}, \quad t_{k-1}+\Delta_{k-1} \leqslant t < t_k. \tag{15}$$

Secondly, when $t \in [t_k, t_k + \Delta_k)$, system (1) runs in mismatched periods. Consider the following Lyapunov functional candidate

$$V_{2\sigma'}(t) = \mathcal{D}^T(t)P_{\sigma'}\mathcal{D}(t) + \int_{t-\tau(t)}^{t} e^{\beta(t-s)}x^T(s)Q_{\sigma'}x(s)ds. \tag{16}$$

Following the similar way, in mismatched periods, the following inequality can be obtained:

$$\mathcal{L}V_{2i}(t) - \beta V_{2i}(t) \leqslant \eta^T(t)\Xi_i\eta(t). \tag{17}$$

Furthermore, using (6) yields

$$d[e^{-\beta t}V_{2i}(t)] \leqslant 2e^{-\beta t}\mathcal{D}^T(t)P_i g_j(t)d\varpi(t). \tag{18}$$

Then integrating both sides of (18) from t_k to t and taking expectation, the following inequality can be obtained

$$\mathcal{E}\{V_{2i}(t)\} \leqslant \mathcal{E}\{V_{2i}(t_k)\}e^{\beta(t-t_k)}, \quad t_k \leqslant t < t_k + \Delta_k. \tag{19}$$

In view of (2), we obtain

$$\int_{t-\tau(t)}^{t} e^{\beta(t-s)}x^T(s)Q_i x(s)ds \leqslant e^{\beta h}\int_{t-\tau(t)}^{t} x^T(s)Q_i x(s)ds \leqslant e^{(\alpha+\beta)h}\int_{t-\tau(t)}^{t} e^{\alpha(s-t)}x^T(s)Q_i x(s)ds. \tag{20}$$

Combining (8), (16) and (20), it follows that

$$V_{2i}(t) \leqslant e^{(\alpha+\beta)h}V_{1i}(t). \tag{21}$$

Considering the whole interval $[t_0, t)$, the Lyapunov functional candidate is the combination of (8) and (16):

$$V(t) = \begin{cases} V_{1\sigma'}(t), & t \in [t_{k-1}+\Delta_{k-1}, t_k), \ k = 1,2,3,\ldots, \\ V_{2\sigma'}(t), & t \in [t_k, t_k+\Delta_k), \ k = 1,2,3,\ldots. \end{cases} \tag{22}$$

By means of (4), (15), (19) and (21), $t \in [t_k,\ t_{k+1})$, we can get

$$\begin{aligned} \mathcal{E}\{V(t)\} &\leqslant \theta^{N_\sigma(t_k,t)}\mu^{N_{\sigma'}(t_k,t)}\mathcal{E}\{V(t_k)\}e^{\beta T^+(t_k,t)-\alpha T^-(t_k,t)} \leqslant \theta^{N_\sigma(t_{k-1},t)}\mu^{N_{\sigma'}(t_{k-1},t)}\mathcal{E}\{V(t_{k-1})\}e^{\beta T^+(t_{k-1},t)-\alpha T^-(t_{k-1},t)} \leqslant \cdots \\ &\leqslant \theta^{N_\sigma(t_0,t)}\mu^{N_{\sigma'}(t_0,t)}\mathcal{E}\{V(t_0)\}e^{\beta T^+(t_0,t)-\alpha T^-(t_0,t)} = \theta^{N_\sigma(t_0,t)}\mu^{N_{\sigma'}(t_0,t)}\mathcal{E}\{V(t_0)\}e^{(\alpha+\beta)T^+(t_0,t)-\alpha(t-t_0)} \\ &\leqslant \theta^{N_\sigma(t_0,t)}\mu^{N_{\sigma'}(t_0,t)}\mathcal{E}\{V(t_0)\}e^{\upsilon(\alpha+\beta)N_\sigma(t_0,t)-\alpha(t-t_0)}, \end{aligned} \tag{23}$$

where $\theta = e^{(\alpha+\beta)h}$, $T^-(t_0, t)$ represents the total matched periods and $T^+(t_0, t)$ the total mismatched periods during $[t_0, t]$.

Moreover, $N_\sigma(t_0,t)$ and $N_{\sigma'}(t_0,t)$ have the following relationship

$$\begin{cases} N_\sigma(t_0,t) = N_{\sigma'}(t_0,t), & t \in [t_{k-1} + \Delta_{k-1}, t_k), \ \ k = 1,2,3,\ldots, \\ N_\sigma(t_0,t) = N_{\sigma'}(t_0,t) + 1, & t \in [t_k, t_k + \Delta_k), \ \ k = 1,2,3,\ldots. \end{cases} \tag{24}$$

So we can conclude that

$$\mathcal{E}\{V(t)\} \leqslant (\theta\mu)^{N_\sigma(t_0,t)}\mathcal{E}\{V(t_0)\}e^{\upsilon(\alpha+\beta)N_\sigma(t_0,t)-\alpha(t-t_0)}. \tag{25}$$

By Definition 2, we can get

$$\mathcal{E}\{V(t)\} \leqslant (\mu\theta)^{(\frac{t-t_0}{T_a})}\mathcal{E}\{V(t_0)\}e^{\upsilon(\alpha+\beta)(\frac{t-t_0}{T_a})-\alpha(t-t_0)} = \mathcal{E}\{V(t_0)\}e^{-\left(\alpha-\frac{\ln(\mu\theta)+\upsilon(\alpha+\beta)}{T_a}\right)(t-t_0)}. \tag{26}$$

Notice from (22) that

$$\mathcal{E}\{V(t)\} \geqslant \delta_1\mathcal{E}\{\|\mathcal{D}(t)\|^2\}, \quad \mathcal{E}\{V(t_0)\} \leqslant \delta_2 \sup_{-h\leqslant\theta\leqslant 0} \mathcal{E}\{\|x(t_0+\theta)\|^2\}, \tag{27}$$

where

$$\delta_1 = \min\lambda_{\min}(P_i), \quad \delta_2 = (1+\|C\|)^2 \max\lambda_{\max}(P_i) + h\max\lambda_{\max}(Q_i).$$

Then combining (26) and (27) yields

$$\mathcal{E}\{\|\mathcal{D}(t)\|^2\} \leqslant \frac{1}{\delta_1}\mathcal{E}\{V(t)\} \leqslant \frac{\delta_2}{\delta_1}\sup_{-h\leqslant\theta\leqslant 0}\mathcal{E}\{\|x(t_0+\theta)\|^2\}e^{-(\alpha-\frac{\ln(\mu\theta)+\upsilon(\alpha+\beta)}{T_a})(t-t_0)}. \tag{28}$$

System (1) is mean-square exponentially stable. The proof is completed. □

Remark 1. It is noted that the new Lyapunov function constructed in the form of (22) has two main characteristics. Firstly, the Lyapunov function is dependent on the switching signal of the controllers, which is convenient for the analysis of the controller design under asynchronous switching. Secondly, the subsystems are allowed to be unstable during mismatched periods resulted from asynchronous switching.

Remark 2. In this paper, the lag time Δ_k is unknown with a known upper bound. Although the Lyapunov function is allowed to rise both at switching instants and during the mismatched periods, by limiting the lower bound of the average dwell time T_a^*, the Lyapunov function is decreasing as a whole and the stability of the system is guaranteed.

Remark 3. When $C = 0$, the switched stochastic neutral system (1) reduces to the switched stochastic system:

$$dx(t) = [A_{\sigma(t)}x(t) + A_{d\sigma(t)}x(t-\tau(t)) + B_{\sigma(t)}u(t)]dt + [F_{\sigma(t)}x(t) + F_{d\sigma(t)}x(t-\tau(t))]d\varpi(t),$$
$$x(s) = \varphi(s), \quad s \in [-h, 0].$$

Theorem 1 can cover the result for the above system as a special case.

3.2. Controller design

The following theorem presents the design method of the controllers for system (1).

Theorem 2. *Given positive constants, α, $\beta < 1$ and $\mu \geqslant 1$, for $i,j \in M$, $i \neq j$, if there exist symmetric and positive definite matrices X_i and S_i, and any matrix Y_i satisfying the following LMIs*

$$X_j \leqslant \mu X_i, \quad S_j \leqslant \mu S_i, \tag{29}$$

$$\begin{bmatrix} \Sigma_{i11} & \Sigma_{i12} & \Sigma_{i13} & X_iF_i^T & 0 & X_i \\ * & \Sigma_{i22} & X_iA_{di}^T & X_iF_{di}^T & X_iC^T & 0 \\ * & * & -X_i & 0 & 0 & 0 \\ * & * & * & -X_i & 0 & 0 \\ * & * & * & * & \frac{-X_i}{1+\alpha} & 0 \\ * & * & * & * & * & -S_i \end{bmatrix} < 0, \tag{30}$$

$$\begin{bmatrix} \Omega_{i11} & \Omega_{i12} & \Omega_{i13} & X_iF_j^T & 0 & X_i \\ * & \Omega_{i22} & X_iA_{dj}^T & X_iF_{dj}^T & X_iC^T & 0 \\ * & * & -X_i & 0 & 0 & 0 \\ * & * & * & -X_i & 0 & 0 \\ * & * & * & * & \frac{-X_i}{1-\beta} & 0 \\ * & * & * & * & * & -S_i \end{bmatrix} < 0, \tag{31}$$

where

$$\Sigma_{i11} = A_iX_i + B_iY_i + (A_iX_i + B_iY_i)^T + \alpha X_i,$$
$$\Sigma_{i12} = A_{di}X_i - \alpha CX_i,$$
$$\Sigma_{i13} = (A_iX_i + B_iY_i)^T,$$
$$\Sigma_{i22} = (1-d)e^{-\alpha h}(S_i - 2X_i),$$
$$\Omega_{i11} = A_jX_i + B_jY_i + (A_jX_i + B_jY_i)^T - \beta X_i,$$
$$\Omega_{i12} = A_{dj}X_i + \beta CX_i,$$
$$\Omega_{i13} = (A_jX_i + B_jY_i)^T,$$
$$\Omega_{i22} = (1-d)(S_i - 2X_i).$$

Then system (1) is mean-square exponentially stable under asynchronous switching for any switching signal with the average dwell time satisfying (7). Moreover, the controller gains are constructed by

$$K_i = Y_iX_i^{-1}, \quad i \in M. \tag{32}$$

Proof 2. From $S_i > 0$, we can get $(S_i - X_i)^T S_i^{-1}(S_i - X_i) \geqslant 0$. Then the following inequality can be obtained:

$$S_i - 2X_i \geqslant -X_iS_i^{-1}X_i. \tag{33}$$

Substituting (33) into (30) and performing a congruence transformation by $diag\left\{X_i^{-1}, X_i^{-1}, I, I, I, I\right\}$, we have

$$\begin{bmatrix} \Pi_{i11} & \Pi_{i12} & \Pi_{i13} & F_i^T & 0 & I \\ * & \Pi_{i22} & A_{di}^T & F_{di}^T & C^T & 0 \\ * & * & -X_i & 0 & 0 & 0 \\ * & * & * & -X_i & 0 & 0 \\ * & * & * & * & \frac{-X_i}{1+\alpha} & 0 \\ * & * & * & * & * & -S_i \end{bmatrix} < 0, \tag{34}$$

where

$$\Pi_{i11} = X_i^{-1}A_i + X_i^{-1}B_iY_iX_i^{-1} + (X_i^{-1}A_i + X_i^{-1}B_iY_iX_i^{-1})^T + \alpha X_i^{-1},$$
$$\Pi_{i12} = X_i^{-1}A_{di} - \alpha X_i^{-1}C,$$
$$\Pi_{i13} = \left(A_i + B_iY_iX_i^{-1}\right)^T,$$
$$\Pi_{i22} = -(1-d)e^{-\alpha h}S_i^{-1}.$$

Then setting

$$Y_i = K_iX_i, \quad X_i^{-1} = P_i, \quad S_i^{-1} = Q_i, \tag{35}$$

and using the Schur complement in (34), it can be concluded that (5) holds. This means that (30) implies (5). Following the similar way, (31) implies (6). From (35), the controller gains are given by (32). The proof is completed. □

4. Numerical example

In this section, an example is given to illustrate the effectiveness of the proposed approach. Consider system (1) composed of two subsystems with the following parameters:

$$C = \begin{bmatrix} -0.16 & 0 \\ 0 & -0.18 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -0.75 & -0.10 \\ 0.46 & -0.80 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -0.10 \\ 0.20 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} -0.10 & 0 \\ 0 & -0.10 \end{bmatrix},$$

$$F_1 = \begin{bmatrix} -0.15 & 0 \\ 0 & -0.15 \end{bmatrix}, \quad F_{d1} = \begin{bmatrix} -0.05 & -0.01 \\ 0.01 & -0.03 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1.20 & 0 \\ 0 & -1.20 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.21 \\ -0.60 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} -0.05 & 0.03 \\ 0.04 & -0.03 \end{bmatrix}, \quad F_2 = \begin{bmatrix} -0.20 & 0 \\ 0 & -0.20 \end{bmatrix}, \quad F_{d2} = \begin{bmatrix} -0.03 & 0.01 \\ 0.01 & -0.03 \end{bmatrix}, \quad \tau(t) = 0.3.$$

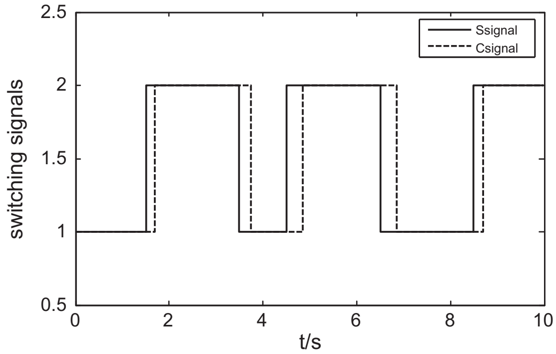

Fig. 1. Switching signals.

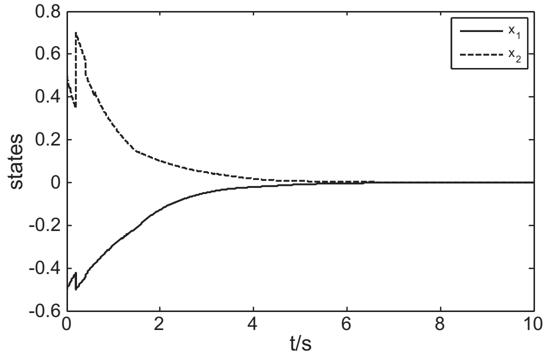

Fig. 2. State responses.

Choose $\alpha = 0.3$, $\beta = 0.3$, $\upsilon = 0.35$, $\mu = 1.1$, $h = 0.3$, then the average dwell time is $T_a > T_a^* = \frac{\ln\mu + (\alpha+\beta)(h+\upsilon)}{\alpha} = 1.6177$. By solving (29)–(31) in Theorem 2, we have

$$X_1 = \begin{bmatrix} 3.0839 & 0.8255 \\ 0.8255 & 5.6156 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 3.1966 & 0.8092 \\ 0.8092 & 5.5194 \end{bmatrix}, \quad S_1 = \begin{bmatrix} 5.8119 & 1.5909 \\ 1.5909 & 9.4085 \end{bmatrix},$$

$$S_2 = \begin{bmatrix} 5.6660 & 1.5219 \\ 1.5219 & 9.4025 \end{bmatrix}, \quad Y_1 = [-2.7554 \quad -4.2049], \quad Y_2 = [0.5359 \quad -1.5880].$$

Then the controller gains constructed by (32) are

$$K_1 = [-0.7220 \quad -0.6430], \quad K_2 = [0.2497 \quad -0.3243].$$

In order to illustrate the effectiveness of the proposed method, simulation curves are shown in Figs. 1 and 2. Fig. 1 shows the switching signals, where solid line (Ssignal) and dashed line (Csignal) represent switching signals of subsystems and controllers, respectively. The state responses are given in Fig. 2 with the initial condition $x(0) = [-0.5 \quad 0.5]^T$. From the simulation curves, we can draw the conclusion that system (1) is mean square exponentially stable by the controller design.

5. Conclusions

The stabilization problem for switched stochastic neutral systems under asynchronous switching has been studied. By using the average dwell time approach, delay-dependent sufficient conditions have been given to guarantee the mean-square exponential stability of such system in terms of LMIs. The controllers have been designed. The Lyapunov function constructed in the paper is dependent on the controllers' switching signal and has been allowed to increase during mismatched periods. Finally, numerical example has illustrated the effectiveness of the results.

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