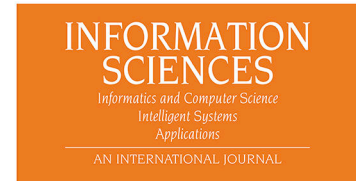




Robust stability analysis and feedback control for networked control systems with additive uncertainties and signal communication delay via matrices transformation information method



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Title page

Robust stability analysis and feedback control for networked control systems with additive uncertainties and signal communication delay via matrices transformation information method

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Abstract The interval type-2 Takagi-Sugeno (T-S) fuzzy dynamic output feedback and H-infinity stability analysis is studied for a class of networked control systems with multiple time-varying additive uncertainties, time-varying signal communication delay and external disturbance. Firstly, the interval type-2 T-S fuzzy is employed to denote the system plant. Secondly, the multiple time-varying additive uncertainties are introduced in the controller design and the state variables depending on additive uncertainties. Thirdly, the delay-dependent Lyapunov-Krasovskii functional with double integral terms is designed to derive the less conservative stability conditions in terms of linear matrix inequalities (LMIs). The characteristic of the state variables are reflected effectively by employing the controller with multiple time-varying additive uncertainties. The closed-loop system is asymptotically stable with prescribed H-infinity performance index γ by employing the matrices transformation information. The less conservative stability conditions are derived and extended into the networked control system without additive uncertainties. Finally, simulations are presented to show the effectiveness of the proposed methods.

Keywords type-2 Takagi-Sugeno; networked control; additive uncertainties; communication delay; stability analysis.

1. Introduction

Networked control systems have developed rapidly because of the advantages of the data network technology in the real world [1, 2]. In the networked control systems, the control signals are transmitted over the communication network. Thus, the networked control systems have applied widely in the real world, such as the autonomous underwater vehicles, unmanned aerial vehicles, smart power grids and so on [3-5]. Recently, the networked control systems have received much attention because the networked control systems have many advantages compared with the conventional point-to-point control systems, such as the signal transmission flexibility, low installation cost, easy diagnosis maintenance and so on [6-8]. Unlike the conventional information technology, the protection of data confidentiality and integrity is not enough in the networked control systems, because the system plant can be affected by the parametric uncertainties, signal communication delays and external disturbances [9, 10]. There are some challenging issues to be solved in the data transmission of the networked control systems based on the imperfect channels and limited bandwidth [11]. In fact, the system stability and control performance of networked control systems are affected by the parametric uncertainties, signal communication delays and external disturbances [12]. Thus, the parametric uncertainties, signal communication delays and external disturbances are three major challenging issues in the communication networks of the networked control systems. Based on the above reasons, this paper investigates a class of networked control systems with multiple time-varying additive uncertainties, time-varying signal communication delay and external disturbance.

In practice, the networked control systems and signal transmission process are nonlinear, thus it may be difficult to achieve the controller synthesis and stability analysis of the networked control systems. However, there are many effective methods have been proposed for the networked control systems, such as the event-triggered control, adaptive sliding mode control, fuzzy model-based control and so on [13, 14, 15]. It is well known that the fuzzy model-based control is an effective method to deal with the complex nonlinear systems and many important research results about the fuzzy model-based control have been obtained [16]. The fuzzy model-based method can divide the nonlinear system into some linear subsystems and connect all the linear subsystems with the membership functions to approximate the original nonlinear system [16]. For example, the Takagi-Sugeno (T-S) fuzzy control with communication constraint was proposed for a class of networked control systems with nonlinearities and network induced delay, and the membership-function-dependent stability conditions were derived to reduce the number of slack variables [17]. Based on the event-triggered mechanism, the T-S fuzzy distributed control was proposed for a class networked control systems with transmission delay, and the asynchronous issue between fuzzy system and fuzzy controller was handled by introducing the triggering conditions [18]. From [17] and [18], it can be seen that the above research results were obtained based on the type-1 T-S fuzzy model. It should be noticed that the system nonlinearities can be captured effectively by the type-1 T-S fuzzy set, but it is difficult to handle the parametric uncertainties in the membership functions [19]. The parameter uncertainties are unavoidable in the practice applications of the networked control systems, such as the limited accuracy of measurement, random variation of parameters, unavoidable noise in the controller design and different definitions of linguistic variables by different experts [20]. Thus, the type-1 T-S fuzzy model has been extended to the type-2 T-S fuzzy model, which improves the ability to handle the

parametric uncertainties. In the type-2 T-S fuzzy model, the upper and lower membership functions can be employed to describe the parametric uncertainties and provide the greater degree of freedom with higher modeling precision and control precision. Thus, the type-2 T-S fuzzy model has the better ability in representing and capturing uncertainties, especially when the nonlinear system plant suffers the parameter uncertainties while the type-1 fuzzy sets do not contain uncertain information. In addition, the type-1 fuzzy sets cannot deal with the linguistic and numerical uncertainties associated with changing unstructured environments completely [21]. The problem of determining the exact membership functions in the fuzzy control system is often relating to the linguistic and numerical uncertainties associated with changing unstructured environments. The structure of type-2 fuzzy sets is similar to the structure of type-1 fuzzy sets, and the difference between the type-2 fuzzy sets and type-1 fuzzy sets is in the output processing [22]. The output processor includes a type reducer and a defuzzifier to generate a type-1 fuzzy system output from the type reducer or generate a crisp number from the defuzzifier [22]. Thus, the type reduction can captures more information about the rule uncertainties than the defuzzified value (a crisp number). A type-2 fuzzy system is characterised by the fuzzy membership function, i.e., the grade of each fuzzy membership element is a fuzzy set in the interval $[0, 1]$. However, the grade of each fuzzy membership element is a crisp number in the interval $[0, 1]$ for the type-1 fuzzy system. The main advantage of using type-2 fuzzy control is that the system plants have a higher stability in the simulations. If the disturbance is considered in the type-2 fuzzy control, the results show that the type-2 fuzzy logic system has the better stability characteristics. Thus, the type-2 T-S fuzzy model has more advantages than type-1 T-S fuzzy model in dealing with the parametric uncertainties. Based on the above reasons, the type-2 T-S fuzzy model is employed to design the novel controller for reducing the effects caused by the multiple time-varying additive uncertainties, time-varying signal communication delay and external disturbance in this paper.

The state variables can reflect the internal characteristics of the control system, and some important research results about the state feedback control have been obtained [23]. For example, the state feedback control with average dwell-time parameter was proposed for a class of stochastic nonlinear switched systems with unknown nonlinearities, such that the switched system was semiglobally uniformly ultimately bounded in probability [24]. The state feedback stabilization control was proposed for a class of stochastic nonlinear high order systems with unknown time-varying powers and stochastic inverse dynamics, and the adaptive algebraic method was designed based on the parameter separation principle to guarantee the global stability of the closed-loop system [25]. However, it may be difficult to obtain all the state variables of the system plant in practice, thus many previous works assume the state variables can be measured. Sometimes this assumption is impossible in practice and it may cause an unobservable system, then the output feedback control is proposed. Compared with state feedback control, the output feedback control is more challenging because of the limited information of the state variables. The output feedback control can be classified into two categories: static output feedback control and dynamic output feedback control [26]. For the static output feedback control, some important research results have been also obtained [26]. For example, the sampled-data static output feedback control was proposed for a class of T-S fuzzy singular systems with parametric uncertainties and L_2/L_∞ disturbances, and some sufficient conditions in terms of linear matrix inequalities (LMIs) were derived [27]. Considering the transmission failures between the controller and actuator, the static output feedback controller with iterative algorithm was designed to against the actuator faults, and the prescribed dissipative performance was guaranteed [28]. From [27] and [28], it can be seen that the static output feedback control is easy for implementation, but some strict constraint conditions should be considered, such as the structural constraint conditions of Lyapunov matrix in [27] and linearization constraint conditions of output matrix in [28]. Compared with the static output feedback control, the dynamic output feedback control is more flexible and the required conditions are less conservative in the system. Based on the above reasons, combining the type-2 T-S fuzzy technique and dynamic output feedback technique, the interval type-2 T-S fuzzy dynamic output feedback control is proposed and the prescribed H-infinity performance is guaranteed in this paper.

The contributions are presented as follows: (1) The interval type-2 T-S fuzzy model is employed to approximate the networked control system. The membership functions are employed to approximate the networked control system. The interval type-2 T-S fuzzy models allow the parametric uncertainties existing in the membership functions. It has been validated that the interval type-2 T-S fuzzy model is less conservative than type-1 fuzzy model on approximating the uncertainties. (2) The interval type-2 T-S fuzzy model is employed to design the controller and the design conditions are relaxed. The state variables of the controller depend on the additive uncertainties and the design flexibility is enhanced. The basic characteristics of the external disturbance are uncertain and random in the practical systems. Thus, the external disturbances are considered in the system plant in this paper. However, the output feedback control is proposed because it may be difficult to obtain the state information of the system. The output feedback control has the better ability to reflect the internal characteristics of the control system. Thus, combining the advantages of the type-2 T-S fuzzy model and output feedback control, some important results have been obtained. (3) Three classes of stability conditions are derived: (i) The delay-dependent Lyapunov-Krasovskii functional is designed, and the prescribed H-infinity performance is guaranteed. (ii) The controller gain matrices are determined via the Schur complement. (iii) The less conservative stability conditions are extended into the networked control system without additive uncertainties.

This paper is organized as follows: The interval type-2 T-S fuzzy model is employed to approximate the system plant in **Section 2**. The controller with multiple time-varying additive uncertainties is designed in **Section 3**. The main results are presented in **Section 4**. Simulations are presented in **Section 5** and conclusions are presented in **Section 6**.

Notations: R^n denotes the n -dimensional Euclidean space, $L_2[0, \infty)$ denotes the space of square-integrable vector function. $A < 0 (\leq 0)$ denotes the negative definite (semi-negative definite) matrix with appropriate dimension, $A > 0 (\geq 0)$ denotes the positive definite (semi-positive definite) matrix with appropriate dimension. “ I ” denotes the identity matrix with appropriate dimension, “ 0 ” denotes the zero matrix with appropriate dimension. The superscript “ T ” denotes the matrix transposition, the superscript “ -1 ” denotes the matrix inverse. $\sum_{i=1}^r h_i(\bullet)$ denotes the summation of $h_i(\bullet)$ with $i=1, 2, \dots, r$, $\prod_{j=1}^g \eta_j(\bullet)$ denotes the product of $\eta_j(\bullet)$ with $j=1, 2, \dots, g$. “ $*$ ” denotes the symmetry vector, $\text{diag}\{r_1 \ r_2 \ \dots \ r_n\}$ denotes the diagonal matrix with r_1, r_2, \dots and r_n .

2. System formulation

Consider a class of networked control systems

$$\begin{cases} \dot{x}(t) = (A + GF_1(t)E_A)x(t) + (A_\tau + GF_1(t)E_{A_\tau})x(t - \tau(t)) + B_u u(t) + B_\omega \omega(t) \\ z(t) = C_z x(t) + D_z u(t) \\ y(t) = C_y x(t) \\ x(t) = \phi(t), \quad -\tau_2 \leq t \leq 0 \end{cases} \quad (1)$$

Plant rule i : if $\theta_1(t)$ is M_{i1} , $\theta_2(t)$ is M_{i2} , ..., and $\theta_g(t)$ is M_{ig} , then

$$\begin{cases} \dot{x}(t) = (A_i + G_i F_{i1}(t)E_{A_i})x(t) + (A_{\tau i} + G_i F_{i1}(t)E_{A_{\tau i}})x(t - \tau(t)) + B_{ui} u(t) + B_{\omega i} \omega(t) \\ z(t) = C_{zi} x(t) + D_{zi} u(t) \\ y(t) = C_{yi} x(t) \\ x(t) = \phi(t), \quad -\tau_2 \leq t \leq 0 \end{cases} \quad (2)$$

where

$$M = \{((x, u), \mu_M(x, u)) | \forall x \in x, \forall u \in j_x \subseteq [0, 1]\}$$

with

$$0 \leq \mu_M(x, u) \leq 1$$

where $j_x \subseteq [0, 1]$ is the primary membership of x , and $\mu_M(x, u)$ is the secondary set. The type-2 membership grade is a subset in $[0, 1]$. For each primary membership, there is a secondary membership and it defines the possibilities for the primary membership. The uncertainty is denoted by a region, and it is called the uncertain footprint. The interval type-2 membership function is obtained if $\mu_M(x, u) = 1$ for $\forall u \in j_x \subseteq [0, 1]$. The uniform shading of the uncertain footprint denotes the entire interval type-2 fuzzy set, and it can be described by upper membership function $\bar{\mu}_M(x)$ and lower membership function $\underline{\mu}_M(x)$ [18]. $\theta_1(t)$, $\theta_2(t)$, ... and $\theta_g(t)$ are the premise variables of the system, M_{ij} ($i=1, 2, \dots, r$, $j=1, 2, \dots, g$) is the fuzzy set of the system, r is the number of the fuzzy rules, and g is the number of the premise variables. $x(t) \in R^n$, $y(t) \in R^p$, $z(t) \in R^q$, $u(t) \in R^m$ and $\phi(t)$ are the state variable, measured output, control output, control input and initial condition of the system, respectively. A_i , $A_{\tau i}$, B_{ui} , $B_{\omega i}$, C_{zi} , C_{yi} and D_{zi} are the system gain matrices with appropriate dimensions.

$G_i F_{i1}(t)E_{A_i}$ and $G_i F_{i1}(t)E_{A_{\tau i}}$ are the multiple time-varying additive uncertainties satisfying

$$F_{i1}^T(t)F_{i1}(t) \leq I, \quad i=1, 2, \dots, r \quad (3)$$

where $F_{i1}(t)$ is the uncertainties. G_i , E_{A_i} and $E_{A_{\tau i}}$ are the additive factors of $F_{i1}(t)$.

$\tau(t)$ is the time-varying signal communication delay and

$$\begin{cases} 0 \leq \tau_1 \leq \tau(t) \leq \tau_2 \\ \dot{\tau}(t) \leq \tau_D \end{cases} \quad (4)$$

where τ_1 is the lower bound of $\tau(t)$, τ_2 is the upper bound of $\tau(t)$, and τ_D is the upper bound of $\dot{\tau}(t)$.

$\omega(t) \in L_2[0, \infty)$ is the external disturbance and

$$\begin{cases} 0 \leq \omega_l \leq \omega(t) \leq \omega_u \\ \dot{\omega}_l(t) \leq \omega_d \end{cases} \quad (5)$$

where ω_l is the lower bound of $\omega(t)$, ω_u is the upper bound of $\omega(t)$, and ω_d is the upper bound of $\dot{\omega}_l(t)$.

Applying fuzzy inference [29], one can obtain

$$\begin{cases} \dot{x}(t) = (\bar{A}_i(h) + \bar{G}_i(h)F_{li}(t)\bar{E}_{Ai}(h))x(t) + (\bar{A}_{ri}(h) + \bar{G}_i(h)F_{li}(t)\bar{E}_{Ari}(h))x(t - \tau(t)) + \bar{B}_{ui}(h)u(t) + \bar{B}_{oi}(h)\omega(t) \\ z(t) = \bar{C}_{zi}(h)x(t) + \bar{D}_{zi}(h)u(t) \\ y(t) = \bar{C}_{yi}(h)x(t) \end{cases} \quad (6)$$

with

$$\begin{cases} \bar{A}_i(h) = \sum_{l=1}^r h_l(\theta(t))A_l, & \bar{A}_{ri}(h) = \sum_{l=1}^r h_l(\theta(t))A_{li} \\ \bar{B}_{ui}(h) = \sum_{l=1}^r h_l(\theta(t))B_{li}, & \bar{B}_{oi}(h) = \sum_{l=1}^r h_l(\theta(t))B_{li} \\ \bar{C}_{zi}(h) = \sum_{l=1}^r h_l(\theta(t))C_{li}, & \bar{C}_{yi}(h) = \sum_{l=1}^r h_l(\theta(t))C_{li} \\ \bar{D}_{zi}(h) = \sum_{l=1}^r h_l(\theta(t))D_{li}, & \bar{G}_i(h) = \sum_{l=1}^r h_l(\theta(t))G_l \\ \bar{E}_{Ai}(h) = \sum_{l=1}^r h_l(\theta(t))E_{li}, & \bar{E}_{Ari}(h) = \sum_{l=1}^r h_l(\theta(t))E_{li} \end{cases} \quad (7)$$

where $\theta(t) = [\theta_1(t) \ \theta_2(t) \ \dots \ \theta_g(t)]^T$, $h_i(\theta(t))$ is the membership function satisfying

$$\begin{cases} h_i(\theta(t)) = h_{iU}(\theta(t))\bar{v}_i(\theta(t)) + h_{iL}(\theta(t))\underline{v}_i(\theta(t)) \\ h_i(\theta(t)) \geq 0, \quad \sum_{i=1}^r h_i(\theta(t)) = 1 \end{cases} \quad (8)$$

$\bar{v}_i(x(t))$ and $\underline{v}_i(x(t))$ are the nonlinear functions satisfying

$$\begin{cases} 0 \leq \bar{v}_i(\theta(t)) \leq 1 \\ 0 \leq \underline{v}_i(\theta(t)) \leq 1 \\ \bar{v}_i(\theta(t)) + \underline{v}_i(\theta(t)) = 1 \end{cases} \quad (9)$$

$h_{iU}(\theta(t))$ is the upper membership function, and $h_{iL}(\theta(t))$ is the lower membership function satisfying

$$\begin{cases} h_{iU}(\theta(t)) = \prod_{j=1}^g \bar{\eta}_{F_{ij}}(\theta_j(t)) \\ h_{iL}(\theta(t)) = \prod_{j=1}^g \underline{\eta}_{F_{ij}}(\theta_j(t)) \\ \tilde{h}_i(\theta(t)) = [h_{iL}(\theta(t)) \ h_{iU}(\theta(t))] \end{cases} \quad (10)$$

where $\tilde{h}_i(\theta(t))$ is the firing strength. $\bar{\eta}_{F_{ij}}(\theta_j(t))$ is the upper grade of the membership function, and $\underline{\eta}_{F_{ij}}(\theta_j(t))$ is the lower grade of the membership function satisfying

$$0 \leq \underline{\eta}_{F_{ij}}(\theta_j(t)) \leq \bar{\eta}_{F_{ij}}(\theta_j(t)) \leq 1 \quad (11)$$

Remark 1. Next, \bar{A}_i , \bar{A}_{ri} , \bar{B}_{ui} , \bar{B}_{oi} , \bar{C}_{zi} , \bar{C}_{yi} , \bar{D}_{zi} , \bar{G}_i , \bar{E}_{Ai} and \bar{E}_{Ari} are used to denote $\bar{A}_i(h)$, $\bar{A}_{ri}(h)$, $\bar{B}_{ui}(h)$, $\bar{B}_{oi}(h)$, $\bar{C}_{zi}(h)$, $\bar{C}_{yi}(h)$, $\bar{D}_{zi}(h)$, $\bar{G}_i(h)$, $\bar{E}_{Ai}(h)$ and $\bar{E}_{Ari}(h)$, respectively.

Remark 2. The efficient model predictive control and exponential mean-square stability analysis were proposed for a class of interval type-2 T-S fuzzy systems [30], without considering additive uncertainties. The optimal finite-horizon control and the asymptotic stability analysis were proposed for a class of networked control systems [31], without considering signal communication delay. The robust adaptive sliding mode control and stochastic stability analysis were proposed for a class of switched networked control systems [32], without considering external disturbance. Compared with [30-32], the additive uncertainties, signal communication delay and external disturbance are considered in this paper. Besides, the interval type-2 T-S fuzzy model has the better ability than type-1 fuzzy model because it offers a distinctive framework to denote the system plant [19].

The schematic diagram of the proposed methods is shown in **Figure 1**.

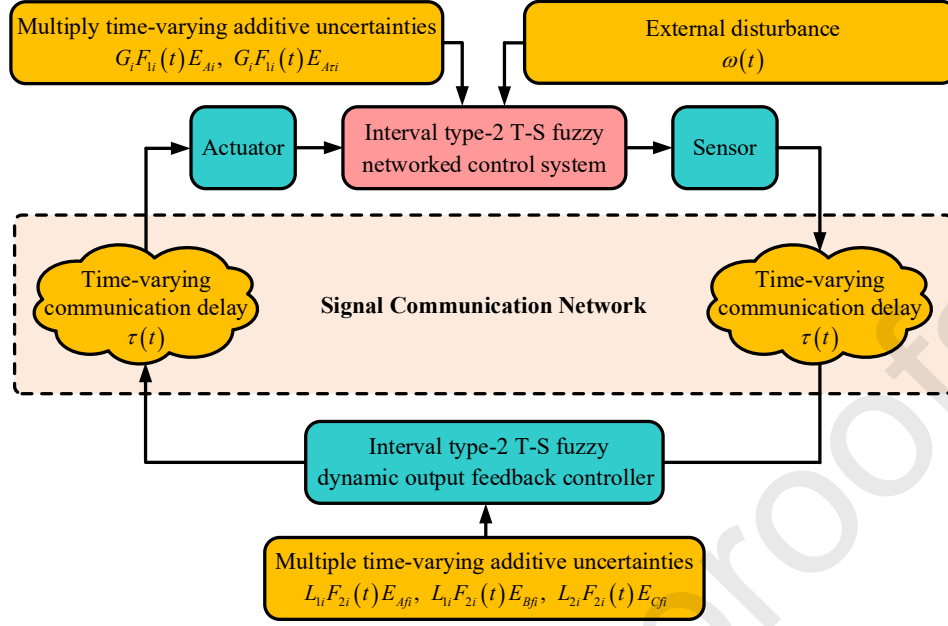


Figure 1. Schematic diagram of the proposed methods.

3. Controller design

The dynamic output feedback controller with multiple time-varying additive uncertainties is designed

$$\begin{cases} \dot{x}_f(t) = (A_f + L_1 F_2(t) E_{Af}) x_f(t) + (B_f + L_1 F_2(t) E_{Bf}) y(t) \\ u(t) = (C_f + L_2 F_2(t) E_{Cf}) x_f(t) \end{cases} \quad (12)$$

Controller rule i : if $\theta_1(t)$ is M_{i1} , $\theta_2(t)$ is M_{i2} , ..., and $\theta_g(t)$ is M_{ig} , then

$$\begin{cases} \dot{x}_f(t) = (A_{fi} + L_{1i} F_{2i}(t) E_{Afi}) x_f(t) + (B_{fi} + L_{1i} F_{2i}(t) E_{Bfi}) y(t) \\ u(t) = (C_{fi} + L_{2i} F_{2i}(t) E_{Cfi}) x_f(t) \end{cases} \quad (13)$$

where $\theta_1(t)$, $\theta_2(t)$, ..., and $\theta_g(t)$ are the premise variables of the controller, M_{ij} ($i=1, 2, \dots, r$, $j=1, 2, \dots, g$) is the fuzzy set of the controller, r is the number of the fuzzy rules, and g is the number of the premise variables. $x_f(t)$ is the state variable of the controller. A_{fi} , B_{fi} and C_{fi} are the controller gain matrices.

$L_{1i} F_{2i}(t) E_{Afi}$, $L_{1i} F_{2i}(t) E_{Bfi}$ and $L_{2i} F_{2i}(t) E_{Cfi}$ are the multiple time-varying additive uncertainties

$$F_{2i}^T(t) F_{2i}(t) \leq I, \quad i=1, 2, \dots, r \quad (14)$$

where $F_{2i}(t)$ is the uncertainties. L_{1i} , L_{2i} , E_{Afi} , E_{Bfi} and E_{Cfi} are the additive factors of $F_{2i}(t)$.

Applying fuzzy inference [29], one can obtain

$$\begin{cases} \dot{x}_f(t) = (\bar{A}_{fi}(h) + \bar{L}_{1i}(h) F_{2i}(t) \bar{E}_{Afi}(h)) x_f(t) + (\bar{B}_{fi}(h) + \bar{L}_{1i}(h) F_{2i}(t) \bar{E}_{Bfi}(h)) y(t) \\ u(t) = (\bar{C}_{fi}(h) + \bar{L}_{2i}(h) F_{2i}(t) \bar{E}_{Cfi}(h)) x_f(t) \end{cases} \quad (15)$$

with

$$\begin{cases} \bar{A}_{fi}(h) = \sum_{i=1}^r h_i(\theta(t)) A_{fi}, & \bar{B}_{fi}(h) = \sum_{i=1}^r h_i(\theta(t)) B_{fi}, & \bar{C}_{fi}(h) = \sum_{i=1}^r h_i(\theta(t)) C_{fi}, & \bar{L}_{1i}(h) = \sum_{i=1}^r h_i(\theta(t)) L_{1i} \\ \bar{L}_{2i}(h) = \sum_{i=1}^r h_i(\theta(t)) L_{2i}, & \bar{E}_{Afi}(h) = \sum_{i=1}^r h_i(\theta(t)) E_{Afi}, & \bar{E}_{Bfi}(h) = \sum_{i=1}^r h_i(\theta(t)) E_{Bfi}, & \bar{E}_{Cfi}(h) = \sum_{i=1}^r h_i(\theta(t)) E_{Cfi} \end{cases} \quad (16)$$

where $\theta(t) = [\theta_1(t) \ \theta_2(t) \ \dots \ \theta_g(t)]^T$, $h_i(\theta(t))$ is the membership function satisfying

$$\begin{cases} h_i(\theta(t)) = \underline{h}_i(\theta(t)) + \bar{h}_i(\theta(t)) \\ h_i(\theta(t)) \geq 0, \quad \sum_{i=1}^r h_i(\theta(t)) = 1 \end{cases} \quad (17)$$

in which

$$\begin{cases} \underline{h}_i(\theta(t)) = \frac{h_{iL}(\theta(t))}{\sum_{k=1}^r (h_{kL}(\theta(t)) + h_{kU}(\theta(t)))} \\ \bar{h}_i(\theta(t)) = \frac{h_{iU}(\theta(t))}{\sum_{k=1}^r (h_{kL}(\theta(t)) + h_{kU}(\theta(t)))} \end{cases} \quad (18)$$

Remark 3. Next, \bar{A}_{fi} , \bar{B}_{fi} , \bar{C}_{fi} , \bar{L}_{1i} , \bar{L}_{2i} , \bar{E}_{Afi} , \bar{E}_{Bfi} and \bar{E}_{Cfi} are used to denote $\bar{A}_{fi}(h)$, $\bar{B}_{fi}(h)$, $\bar{C}_{fi}(h)$, $\bar{L}_{1i}(h)$, $\bar{L}_{2i}(h)$, $\bar{E}_{Afi}(h)$, $\bar{E}_{Bfi}(h)$ and $\bar{E}_{Cfi}(h)$, respectively.

Applying (15) to (16), the closed-loop system is obtained

$$\begin{cases} \dot{\eta}(t) = \mathcal{A}(h)\eta(t) + \mathcal{A}_\tau(h)\eta(t - \tau(t)) + \mathcal{B}_\omega(h)\omega(t) \\ z(t) = \mathcal{C}_z(h)\eta(t) \end{cases} \quad (19)$$

where

$$\begin{aligned} \eta(t) &= \begin{bmatrix} x(t) \\ x_f(t) \end{bmatrix}, \quad \mathcal{A}(h) = \sum_{i=1}^r \sum_{j=1}^r \sum_{k=1}^r h_i(\theta(t)) (\underline{h}_j(\theta(t)) + \bar{h}_j(\theta(t))) h_k(\theta(t)) \begin{bmatrix} A_i + G_i F_{1i}(t) E_{Ai} & B_{ui} C_{fi} + B_{ui} L_{2j} F_{2j}(t) E_{Cfi} \\ B_{fi} C_{yk} + L_{1j} F_{2j}(t) E_{Bfi} & A_{fi} + L_{1j} F_{2j}(t) E_{Afi} \end{bmatrix}, \\ \mathcal{A}_\tau(h) &= \sum_{i=1}^r \sum_{j=1}^r h_i(\theta(t)) \begin{bmatrix} A_{\tau i} + G_i F_{1i}(t) E_{A\tau i} & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathcal{B}_\omega(h) = \sum_{i=1}^r h_i(\theta(t)) \begin{bmatrix} B_{oi} \\ 0 \end{bmatrix}, \\ \mathcal{C}_z(h) &= \sum_{i=1}^r \sum_{j=1}^r \sum_{k=1}^r h_i(\theta(t)) (\underline{h}_j(\theta(t)) + \bar{h}_j(\theta(t))) \begin{bmatrix} C_{zi} & D_{zi} C_{fi} + D_{zi} L_{2j} F_{2j}(t) E_{Cfi} \end{bmatrix}. \end{aligned}$$

Lemma 1 [33]. For a given scalar $\varepsilon > 0$, if there exist the matrices Φ , Ψ and $\Omega = \Omega^T$ satisfying

$$\Omega + \varepsilon^{-1}(\varepsilon\Phi)(\varepsilon\Phi)^{-1} + \varepsilon^{-1}\Psi^T\Psi < 0 \quad (20)$$

then

$$\Omega + \Phi F(t)\Psi + \Psi^T F^T(t)\Phi^T < 0 \quad (21)$$

where

$$F^T(t)F(t) \leq I \quad (22)$$

Lemma 2. For a given scalar $\lambda > 0$, if there exist the matrices Φ , Ψ and $\Omega = \Omega^T$ satisfying

$$\begin{bmatrix} \Omega & \lambda^{-1}\Phi + \lambda\Psi \\ * & -2I \end{bmatrix} < 0 \quad (23)$$

then

$$\Omega + \Phi\Psi^T + \Psi\Phi^T < 0 \quad (24)$$

Proof. Applying Schur complement to (23)

$$\Omega + (\lambda^{-1}\Phi + \lambda\Psi)(0.5I)(\lambda^{-1}\Phi - \lambda\Psi)^T < 0 \quad (25)$$

and (25) is equivalent to

$$(2\Omega + \Phi\Psi^T + \Psi\Phi^T) + \lambda^{-2}\Phi\Phi^T + \lambda^2\Psi\Psi^T < 0 \quad (26)$$

then applying **Lemma 1** to (26), one can obtain (24). **(Q. E. D.)**

Remark 4. In this paper, the objective is to design the controller for the networked control system such that

- (i) the closed-loop system is asymptotically stable for any initial condition and $\omega(t) = 0$;
- (ii) the prescribed H-infinity performance is guaranteed for zero initial condition and $\omega(t) \neq 0$

$$\int_0^\infty z^T(t)z(t)dt \leq \gamma^2 \int_0^\infty \omega^T(t)\omega(t)dt \quad (27)$$

where $\gamma > 0$ is the performance index.

Remark 5. The interval type-2 T-S fuzzy model has the nice ability to facilitate the controller design [29]. Besides, it may be

difficult to obtain the state information in the state feedback control [34]. Moreover, the static output feedback control is easy to implement, but some strict design conditions should be considered [35]. Unlike state feedback and static output feedback, the dynamic characteristic of the state variables can be reflected by the dynamic output feedback [36]. Thus, combining the interval type-2 T-S fuzzy model and dynamic output feedback, the controller is designed in this paper. Unlike other works, the additive uncertainties are introduced to design the controller for relaxing the design conditions.

4. Main results

4.1 Stability conditions

In this section, the stability conditions are derived, such that the closed-loop system is asymptotically stable with prescribed H-infinite performance index γ .

Theorem 1. For the given scalars $\tau_2 > 0$, $\tau_D > 0$, $\delta > 0$ and $\gamma > 0$, if there exist the matrices $P > 0$, $\tilde{Q}_1 > 0$, $\tilde{Q}_2 > 0$, $\tilde{Z} > 0$, $\tilde{M}_{1ij} > 0$, $\tilde{M}_{2ij} > 0$, $\tilde{M}_{3ij} > 0$, \tilde{N}_{1ij} , \tilde{N}_{2ij} , \tilde{N}_{3ij} , \tilde{X}_{11ij} , \tilde{X}_{12ij} , \tilde{X}_{13ij} , \tilde{X}_{22ij} , \tilde{X}_{23ij} and \tilde{X}_{33ij} for $i, j = 1, 2, \dots, r$ satisfying

$$\begin{bmatrix} \tilde{X}_{11ij} & \tilde{X}_{12ij} & \tilde{X}_{13ij} & \tilde{M}_{1ij} \\ * & \tilde{X}_{22ij} & \tilde{X}_{23ij} & \tilde{M}_{2ij} \\ * & * & \tilde{X}_{33ij} & \tilde{M}_{3ij} \\ * & * & * & \tilde{Z} \end{bmatrix} \geq 0 \quad (28)$$

$$\begin{bmatrix} \tilde{X}_{11ij} & \tilde{X}_{12ij} & \tilde{X}_{13ij} & \tilde{N}_{1ij} \\ * & \tilde{X}_{22ij} & \tilde{X}_{23ij} & \tilde{N}_{2ij} \\ * & * & \tilde{X}_{33ij} & \tilde{N}_{3ij} \\ * & * & * & \tilde{Z} \end{bmatrix} \geq 0 \quad (29)$$

$$\tilde{\Omega}(t) = \begin{bmatrix} \bar{\Omega}(t) + \mathcal{N}(t) & \tau_1 \Psi^T(t) \\ * & \tau_2 (\delta^2 \tilde{Z} - 2\delta P) \end{bmatrix} < 0 \quad (30)$$

with

$$\begin{cases} \bar{\Omega}(t) = \begin{bmatrix} \tilde{\mathcal{F}}_{11} & P\mathcal{A}_r(t) & 0 & P\mathcal{B}_w(t) \\ * & (\tau_D - 1)\tilde{Q}_1 & 0 & 0 \\ * & * & -\tilde{Q}_2 & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} \\ \mathcal{N}(t) = \begin{bmatrix} \tau_2 \tilde{X}(t) + \mathcal{M}(t) + \mathcal{M}^T(t) & 0 \\ 0 & 0 \end{bmatrix}, \quad \Psi(t) = [P\mathcal{A}(t) \quad P\mathcal{A}_r(t) \quad 0 \quad P\mathcal{B}_w(t)] \end{cases} \quad (31)$$

where

$$\begin{cases} \tilde{\mathcal{F}}_{11} = P\mathcal{A}(h) + \mathcal{A}^T(h)P + \tilde{Q}_1 + \tilde{Q}_2 + \mathcal{C}_z^T(h)\mathcal{C}_z(h) \\ \tilde{X}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i(\theta(t)) (h_j(\theta(t)) + \bar{h}_j(\theta(t))) \begin{bmatrix} \tilde{X}_{11ij} & \tilde{X}_{12ij} & \tilde{X}_{13ij} \\ * & \tilde{X}_{22ij} & \tilde{X}_{23ij} \\ * & * & \tilde{X}_{33ij} \end{bmatrix} \\ \mathcal{M}(t) = [\tilde{M}(t) \quad (\tilde{N}(t) - \tilde{M}(t)) \quad -\tilde{N}(t)] \end{cases} \quad (32)$$

in which

$$\begin{cases} \tilde{M}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i(\theta(t)) (h_j(\theta(t)) + \bar{h}_j(\theta(t))) \begin{bmatrix} \tilde{M}_{1ij} \\ \tilde{M}_{2ij} \\ \tilde{M}_{3ij} \end{bmatrix} \\ \tilde{N}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i(\theta(t)) (h_j(\theta(t)) + \bar{h}_j(\theta(t))) \begin{bmatrix} \tilde{N}_{1ij} \\ \tilde{N}_{2ij} \\ \tilde{N}_{3ij} \end{bmatrix} \end{cases} \quad (33)$$

then the closed-loop system is asymptotically stable with prescribed H-infinity performance index γ .

Proof. The proof of **Theorem 1** is divided into **Step 1** and **Step 2**.

Step 1: The proof of the objective (i) in Remark 4:

Consider the delay-dependent Lyapunov-Krasovskii functional $V(t)$

$$\begin{aligned} V(t) = & \eta^T(t) P \eta(t) + \int_{t-\tau_1}^t \eta^T(s) \tilde{Q} \eta(s) ds + \int_{t-\tau_2}^t \eta^T(s) \tilde{Q}_2 \eta(s) ds + \int_{t-\tau_2}^t \int_{t+\theta}^t \eta^T(s) \tilde{Z} \dot{\eta}(s) ds d\theta \\ & + \int_{\tau_1}^t \int_{\theta-\tau(\theta)}^{\theta} \begin{bmatrix} \varsigma(\theta) \\ \tilde{\eta}(s) \end{bmatrix}^T \begin{bmatrix} \tilde{X}(\theta) & \tilde{M}(\theta) \\ * & \tilde{Z} \end{bmatrix} \begin{bmatrix} \varsigma(\theta) \\ \tilde{\eta}(s) \end{bmatrix} ds d\theta + \int_{\tau_1}^t \int_{\theta-\tau_2}^{\theta-\tau(\theta)} \begin{bmatrix} \varsigma(\theta) \\ \tilde{\eta}(s) \end{bmatrix}^T \begin{bmatrix} \tilde{X}(\theta) & \tilde{N}(\theta) \\ * & \tilde{Z} \end{bmatrix} \begin{bmatrix} \varsigma(\theta) \\ \tilde{\eta}(s) \end{bmatrix} ds d\theta \end{aligned} \quad (34)$$

where

$$\begin{cases} \eta(\theta) = \begin{bmatrix} \eta(\theta) \\ \varsigma(\theta) = \begin{bmatrix} \eta(\theta - \tau(\theta)) \\ \eta(\theta - \tau_2) \end{bmatrix} \\ \eta(s) = \begin{bmatrix} x(s) \\ x_f(s) \end{bmatrix} \end{cases} \quad (35)$$

$$\begin{cases} \tilde{X}(\theta) = \sum_{i=1}^r \sum_{j=1}^r h_i(\theta(\theta)) (\underline{h}_j(\theta(\theta)) + \bar{h}_j(\theta(\theta))) \begin{bmatrix} \tilde{X}_{11ij} & \tilde{X}_{12ij} & \tilde{X}_{13ij} \\ * & \tilde{X}_{21ij} & \tilde{X}_{22ij} \\ * & * & \tilde{X}_{33ij} \end{bmatrix} \\ \tilde{M}(\theta) = \sum_{i=1}^r \sum_{j=1}^r h_i(\theta(\theta)) (\underline{h}_j(\theta(\theta)) + \bar{h}_j(\theta(\theta))) \begin{bmatrix} \tilde{M}_{1ij} \\ \tilde{M}_{2ij} \\ \tilde{M}_{3ij} \end{bmatrix} \\ \tilde{N}(\theta) = \sum_{i=1}^r \sum_{j=1}^r h_i(\theta(\theta)) (\underline{h}_j(\theta(\theta)) + \bar{h}_j(\theta(\theta))) \begin{bmatrix} \tilde{N}_{1ij} \\ \tilde{N}_{2ij} \\ \tilde{N}_{3ij} \end{bmatrix} \end{cases} \quad (36)$$

Taking the time derivative of $V(t)$

$$\begin{aligned} \dot{V}(t) + \|z(t)\|^2 - \gamma^2 \|\omega(t)\|^2 = & 2\eta^T(t) P \left[\mathcal{A}(h) \eta(t) + \mathcal{A}_\tau(h) \eta(t - \tau(t)) + \mathcal{B}_\omega(h) \omega(t) \right] + \tau_2 \varsigma^T(t) \tilde{X}(t) \varsigma(t) \\ & + \eta^T(t) (\tilde{Q}_1 + \tilde{Q}_2) \eta(t) - (1 - \dot{\tau}(t)) \eta^T(t - \tau(t)) \tilde{Q} \eta(t - \tau(t)) \\ & - \eta^T(t - \tau_2) \tilde{Q}_2 \eta(t - \tau_2) + 2\varsigma^T(t) \left[\tilde{M}(t) \eta(t) + (\tilde{N}(t) - \tilde{M}(t)) \eta(t - \tau(t)) - \tilde{N}(t) \eta(t - \tau_2) \right] \\ & - \gamma^2 \omega^T(t) \omega(t) + \eta^T(t) \mathcal{C}_z^T(h) \mathcal{C}_z(h) \eta(t) + \tau_2 \begin{bmatrix} \varsigma(t) \\ \omega(t) \end{bmatrix}^T \\ & \times \begin{bmatrix} P\mathcal{A}(h) & P\mathcal{A}_\tau(h) & 0 & P\mathcal{B}_\omega(h) \end{bmatrix}^T \begin{bmatrix} \varsigma(t) \\ \omega(t) \end{bmatrix} \\ & \times \left[(P - \delta \tilde{Z}) \tilde{Z}^{-1} (P - \delta \tilde{Z}) + 2\delta P - \delta^2 \tilde{Z} \right]^{-1} \begin{bmatrix} P\mathcal{A}(h) & P\mathcal{A}_\tau(h) & 0 & P\mathcal{B}_\omega(h) \end{bmatrix} \begin{bmatrix} \varsigma(t) \\ \omega(t) \end{bmatrix} \\ & \leq \begin{bmatrix} \varsigma(t) \\ \omega(t) \end{bmatrix}^T \left(\bar{\mathcal{Q}}(t) + \mathcal{N}(t) + \tau_2 \Psi^T(t) (2\delta P - \delta^2 \tilde{Z})^{-1} \Psi(t) \right) \begin{bmatrix} \varsigma(t) \\ \omega(t) \end{bmatrix} \end{aligned} \quad (37)$$

where

$$\begin{cases} \eta(t) = \begin{bmatrix} x(t) \\ x_f(t) \end{bmatrix} \\ \varsigma(t) = \begin{bmatrix} \eta(t) \\ \eta(t - \tau(t)) \\ \eta(t - \tau_2) \end{bmatrix} \end{cases} \quad (38)$$

Consider (28)-(30), one can obtain

$$\dot{V}(t) + \|z(t)\|^2 - \gamma^2 \|\omega(t)\|^2 \leq 0 \quad (39)$$

Step 2: The proof of the objective (ii) in Remark 4:

From (2) and (13), one can obtain $\eta(0) = 0$, this means $\dot{\eta}(t) = 0$ for any t . Then

$$V(t) \Big|_{t=0} = \eta^T(0)P\eta(0) + \int_{\tau_1-\tau(t)}^{\tau_1} \eta^T(s)\tilde{Q}_1\eta(s)ds + \int_{\tau_1-\tau_2}^{\tau_1} \eta^T(s)\tilde{Q}_2\eta(s)ds + \int_{\tau_1-\tau_2}^{\tau_1} \int_{\theta}^{\tau_1} \eta^T(s)\tilde{Z}\dot{\eta}(s)dsd\theta = 0 \quad (40)$$

$$V(t) \Big|_{t=\infty} \geq 0 \quad (41)$$

From (40) and (41), one has

$$\int_0^\infty \left(\|z(t)\|^2 - \gamma^2 \|\omega(t)\|^2 \right) dt + \left(V(t) \Big|_{t=\infty} - V(t) \Big|_{t=0} \right) \leq 0 \quad (42)$$

From the proof in **Step 1** and **Step 2**, one can concluded that the proof of **Theorem 1** is completed. **(Q. E. D.)**

Remark 6. From **Theorem 1**, it can be seen that the stability conditions are derived, the closed-loop system is asymptotically stable with prescribed H-infinity performance index γ . The delay-dependent Lyapunov-Krasovskii functional has better ability than conventional Lyapunov-Krasovskii functional to obtain the less conservative stability conditions [37]. Thus, the delay-dependent Lyapunov-Krasovskii functional $V(t)$ is designed in **Theorem 1**. From (34), it can be seen that $V(t)$ depends on $x(t)$, $x_f(t)$, τ_1 and τ_2 at the same time.

Remark 7. Unlike [38, 39], the stability conditions relax the design conditions, and $P^{-1}\tilde{Z}P^{-1}$ was estimated as $(2P - \tilde{Z})^{-1}$ in [38], i.e.,

$$P^{-1}\tilde{Z}P^{-1} \leq (2P - \tilde{Z})^{-1} \quad (43)$$

Similar to [38], $((P - \delta\tilde{Z})\tilde{Z}^{-1}(P - \delta\tilde{Z}) + 2\delta P - \delta^2\tilde{Z})^{-1}$ is estimated as $(2\delta P - \delta^2\tilde{Z})^{-1}$, i.e.,

$$((P - \delta\tilde{Z})\tilde{Z}^{-1}(P - \delta\tilde{Z}) + 2\delta P - \delta^2\tilde{Z})^{-1} \leq (2\delta P - \delta^2\tilde{Z})^{-1} \quad (44)$$

From (43) and (44), it can be seen that $P^{-1}\tilde{Z}P^{-1}$ was replaced by $((P - \delta\tilde{Z})\tilde{Z}^{-1}(P - \delta\tilde{Z}) + 2\delta P - \delta^2\tilde{Z})^{-1}$. Substitute $\delta = 1$ into (44), one can obtain $(2\delta P - \delta^2\tilde{Z})^{-1} = (2P - \tilde{Z})^{-1}$. This means the less conservative stability conditions are derived in **Theorem 1** than [38].

Remark 8. From **Remark 4**, one knows that the objective of this paper is to design the interval type-2 T-S fuzzy dynamic output feedback controller for the networked control system, then the closed-loop system is asymptotically stable with prescribed H-infinity performance index γ . **Theorem 1** is presented for the closed-loop system (19). From **Step 1** in **Theorem 1**, one knows that the closed-loop system is asymptotically stable. From **Step 2** in **Theorem 1**, one knows that the prescribed H-infinity performance is guaranteed. This implies the proposed method is effective in this paper. From **Theorem 1**, one knows that only the asymptotic stability of the closed-loop system is achieved with prescribed H-infinity performance index γ . However, the effectiveness of the controller is not enough for the control objective in this paper, one should present the other theorem to show how to calculate the controller gain matrices A_{fi} , B_{fi} and C_{fi} . The proof of **Theorem 2** implies there exist the feasible solutions of R_i , S_i and T_i to calculate A_{fi} , B_{fi} and C_{fi} .

4.2 Controller gain matrices

In this section, the controller gain matrices A_{fi} , B_{fi} and C_{fi} are determined.

Theorem 2. For the given scalars $\tau_2 > 0$, $\tau_d > 0$, $\delta > 0$, $\gamma > 0$, $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$, if there exist the matrices $F > 0$, $\tilde{F} > 0$, $Q_1 > 0$, $Q_2 > 0$, $Z > 0$, $M_{1ij} > 0$, $M_{2ij} > 0$, $M_{3ij} > 0$, N_{1ij} , N_{2ij} , N_{3ij} , X_{11ij} , X_{12ij} , X_{13ij} , X_{22ij} , X_{23ij} , X_{33ij} , R_i , S_i and T_i for $i, j = 1, 2, \dots, r$ satisfying

$$\begin{bmatrix} X_{11ij} & X_{12ij} & X_{13ij} & M_{1ij} \\ * & X_{22ij} & X_{23ij} & M_{2ij} \\ * & * & X_{33ij} & M_{3ij} \\ * & * & * & Z \end{bmatrix} \geq 0 \quad (45)$$

$$\begin{bmatrix} X_{11ij} & X_{12ij} & X_{13ij} & N_{1ij} \\ * & X_{22ij} & X_{23ij} & N_{2ij} \\ * & * & X_{33ij} & N_{3ij} \\ * & * & * & Z \end{bmatrix} \geq 0 \quad (46)$$

$$\Omega_{ijk} = \begin{bmatrix} \mathcal{F}_{11} & \mathcal{F}_{12} & \mathcal{F}_{13} & \mathcal{A}_{3i} & \tau_2 \mathcal{A}_{1ijk}^T & \tau_1 \mathcal{A}_{4ijk}^T & \mathcal{A}_{5ij}^T & \varepsilon_1 \mathcal{A}_{6ij}^T & \mathcal{A}_{8i} & \varepsilon_2 \mathcal{A}_{9i}^T \\ * & \mathcal{F}_{22} & \mathcal{F}_{23} & 0 & \tau_2 \mathcal{A}_{2i}^T & 0 & 0 & 0 & 0 & \varepsilon_2 \mathcal{A}_{10i}^T \\ * & * & \mathcal{F}_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & \tau_2 \mathcal{A}_{3i}^T & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\tau_D \Delta & 0 & \tau_2 \mathcal{A}_{5ij}^T & 0 & \tau_2 \mathcal{A}_{8i} & 0 \\ * & * & * & * & * & -I & \tau_1 \mathcal{A}_{7ij}^T & 0 & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & * & * & * & * & -\varepsilon_2 I \end{bmatrix} < 0, \quad i, j, k = 1, 2, \dots, r \quad (47)$$

with

$$\begin{cases} \mathcal{A}_{1ijk} = \begin{bmatrix} \bar{A}_i \tilde{F} + \bar{B}_{ui} T_j F & \bar{A}_i F + \bar{B}_{ui} T_j F \\ S_j \bar{C}_{yk} \tilde{F} + R_j F & S_j \bar{C}_{yk} F + R_j F \end{bmatrix}, & \mathcal{A}_{2i} = \begin{bmatrix} \bar{A}_{ri} \tilde{F} & \bar{A}_{ri} F \\ 0 & 0 \end{bmatrix} \\ \mathcal{A}_{3i} = \begin{bmatrix} \bar{B}_{oi} \\ 0 \end{bmatrix}, & \mathcal{A}_{4ijk} = [\bar{C}_{zi} \tilde{F} + \bar{D}_{zi} T_j F \bar{C}_{zk} F + \bar{D}_{zi} T_j F] \\ \mathcal{A}_{5ij} = \begin{bmatrix} \bar{B}_{ui} \bar{L}_{2j} & 0 \\ 0 & \bar{L}_{1j} \end{bmatrix}, & \mathcal{A}_{6ij} = \begin{bmatrix} \bar{E}_{Cfi} F & \bar{E}_{Cfi} F \\ \bar{E}_{Bfi} \bar{C}_{zj} \tilde{F} + \bar{E}_{Afi} F & \bar{E}_{Bfi} \bar{C}_{yj} F + \bar{E}_{Afi} F \end{bmatrix} \\ \mathcal{A}_{7ij} = [\bar{D}_{zi} \bar{L}_{2j} & 0], & \mathcal{A}_{8i} = \begin{bmatrix} \bar{G}_i \\ 0 \end{bmatrix} \\ \mathcal{A}_{9i} = [\bar{E}_{Ai} \tilde{F} & \bar{E}_{Ai} F], & \mathcal{A}_{10i} = [\bar{E}_{Ari} \tilde{F} & \bar{E}_{Ari} F] \end{cases} \quad (48)$$

$$\begin{cases} \mathcal{F}_{11} = \mathcal{A}_{1ijk} + \mathcal{A}_{ijk}^T + Q_1 + Q_2 + \tau_2 X_{1ij} + M_{1ij} + M_{1ij}^T, & \mathcal{F}_{12} = \mathcal{A}_{2i} + \tau_2 X_{12ij} + N_{1ij} - M_{1ij} + M_{2ij}^T \\ \mathcal{F}_{13} = \tau_2 X_{13ij} - N_{1ij} + M_{3ij}^T, & \mathcal{F}_{22} = (\tau_D - 1)Q_1 + \tau_2 X_{22ij} + N_{2ij} + N_{2ij}^T - M_{2ij} - M_{2ij}^T \\ \mathcal{F}_{23} = \tau_2 X_{23ij} - N_{2ij} + N_{3ij}^T - M_{3ij}^T, & \mathcal{F}_{33} = -Q_2 + \tau_2 X_{33ij} - N_{3ij} - N_{3ij}^T \end{cases} \quad (49)$$

$$\Delta = 2\delta\phi_{55} - \delta^2 Z \quad (50)$$

where

$$\phi_{55} = \begin{bmatrix} \tilde{F} & F \\ F & F \end{bmatrix} > 0 \quad (51)$$

then \mathcal{A}_{fi} , \mathcal{B}_{fi} and \mathcal{C}_{fi} can be determined as follows

$$\mathcal{A}_{fi} = R_i F^{-1}, \quad \mathcal{B}_{fi} = S_i, \quad \mathcal{C}_{fi} = T_i F^{-1} \quad (52)$$

Proof. Applying Schur complement and considering (47)

$$\hat{\Omega}_{ij} + \Phi_{1i} F_{1i}(t) \Psi_{1i} + \Psi_{1i}^T F_{1i}^T(t) \Phi_{1i}^T + \Phi_{2ij} F_{2j}(t) \Psi_{2ij} + \Psi_{2ij}^T F_{2j}^T(t) \Phi_{2ij}^T < 0 \quad (53)$$

where

$$\hat{\Omega}_{ijk} = \begin{bmatrix} \mathcal{F}_{11} & \mathcal{F}_{12} & \mathcal{F}_{13} & \mathcal{A}_{3i} & \tau_2 \mathcal{A}_{1ijk}^T & \mathcal{A}_{4ijk}^T \\ * & \mathcal{F}_{22} & \mathcal{F}_{23} & 0 & \tau_2 \mathcal{A}_{2i}^T & 0 \\ * & * & \mathcal{F}_{33} & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & \tau_2 \mathcal{A}_{3i}^T & 0 \\ * & * & * & * & -\tau_D \Delta & 0 \\ * & * & * & * & * & -I \end{bmatrix} \quad (54)$$

$$\begin{cases} \Phi_{1i} = [\mathcal{A}_{8i} & 0 & 0 & 0 & \tau_2 \mathcal{A}_{8i} & 0]^T \\ \Phi_{2ij} = [\mathcal{A}_{5ij} & 0 & 0 & 0 & \mathcal{A}_{5ij} & \tau_2 \mathcal{A}_{5ij}]^T \end{cases} \quad (55)$$

$$\begin{cases} \Psi_{1i} = [\mathcal{A}_{9i} & \mathcal{A}_{10i} & 0 & 0 & 0 & 0] \\ \Psi_{2ij} = [\mathcal{A}_{6ij} & 0 & 0 & 0 & 0 & 0] \end{cases} \quad (56)$$

$$\sum_{i=1}^r \sum_{j=1}^r \sum_{k=1}^r h_i(\theta(t)) (\bar{h}_j(\theta(t)) + \bar{h}_j(\theta(t))) h_k(\theta(t)) \Xi_{ijk}(t) < 0 \quad (57)$$

with

$$\Xi_{ijk}(t) = \begin{bmatrix} \hat{\theta}_{1ijk}(t) \hat{\lambda}_{4ijk}^T(t) \hat{\lambda}_{4ijk}(t) & \hat{\theta}_{12ij}(t) & \mathcal{F}_{13} & A_{3i} & \tau_2 \hat{\lambda}_{1ijk}^T(t) \\ * & \mathcal{F}_{22} & \mathcal{F}_{23} & 0 & \tau_2 \hat{\lambda}_{2ij}^T(t) \\ * & * & \mathcal{F}_{33} & 0 & 0 \\ * & * & * & -\gamma^2 I & \tau A_{3i}^T \\ * & * & * & * & -\tau_2 \Delta \end{bmatrix} < 0 \quad (58)$$

where

$$\begin{cases} \hat{\lambda}_{1ijk}(t) = \begin{bmatrix} \bar{A}_i + \bar{G}_i F_{1j}(t) \bar{E}_{Ak} & \bar{B}_{ui} T_j + \bar{B}_{ui} \bar{L}_{2j} F_{2j}(t) \bar{E}_{Cjk} \\ S_j \bar{C}_{yk} + \bar{L}_{1j} \bar{F}_{2j}(t) \bar{E}_{Bjk} \bar{C}_{yk} & R_j + \bar{L}_{1j} F_{2j}(t) \bar{E}_{Ajk} \end{bmatrix} \phi_{55} \\ \hat{\lambda}_{2ij}(t) = \begin{bmatrix} \bar{A}_{ri} + \bar{G}_i F_{1i}(t) \bar{E}_{A\tau j} & 0 \\ 0 & 0 \end{bmatrix} \\ \hat{\lambda}_{4ijk}(t) = [\bar{C}_{zi} \quad \bar{D}_{zi} T_j + \bar{D}_{zi} F_{2j}(t) \bar{E}_{Cjk}] \phi_{55} \end{cases} \quad (59)$$

$$\begin{cases} \hat{\theta}_{11ijk}(t) = \hat{\lambda}_{1ijk}(t) + \hat{\lambda}_{1ijk}^T(t) + Q_1 + Q_2 + \tau_2 X_{11ij} + M_{1ij} + M_{1ij}^T \\ \hat{\theta}_{12ij}(t) = \hat{\lambda}_{2i}(t) + \tau_2 X_{12ij} + N_{1ij} - M_{1ij} + M_{2ij}^T \end{cases} \quad (60)$$

Let

$$\mathcal{P} = \text{diag}\{P \quad P \quad P \quad I \quad P\} \quad (61)$$

where

$$P = \phi_{55}^{-1} = \begin{bmatrix} \tilde{F} & F \\ F & F \end{bmatrix}^{-1} \quad (62)$$

Multiplying \mathcal{P}^T and \mathcal{P} into the Pre- and Post- of (58), one can obtain

$$\mathcal{P}^T \sum_{i=1}^r \sum_{j=1}^r \sum_{k=1}^r h_i(\theta(t)) (h_j(\theta(t)) + \bar{h}_j(\theta(t))) h_k(\theta(t)) \Xi_{ijk}(t) \mathcal{P} < 0 \quad (63)$$

Based on the above analysis, it can be seen that (28)-(30) holds with $P > 0$, $\tilde{Q}_1 > 0$, $\tilde{Q}_2 > 0$, $\tilde{Z} > 0$, $\tilde{M}_{1ij} > 0$, $\tilde{M}_{2ij} > 0$, $\tilde{M}_{3ij} > 0$, \tilde{N}_{1ij} , \tilde{N}_{2ij} , \tilde{N}_{3ij} , \tilde{X}_{11ij} , \tilde{X}_{12ij} , \tilde{X}_{13ij} , \tilde{X}_{22ij} , \tilde{X}_{23ij} and \tilde{X}_{33ij} . From **Theorem 1**, it can be seen that $P > 0$, $\tilde{Q}_1 > 0$, $\tilde{Q}_2 > 0$, $\tilde{Z} > 0$, $\tilde{M}_{1ij} > 0$, $\tilde{M}_{2ij} > 0$, $\tilde{M}_{3ij} > 0$, \tilde{N}_{1ij} , \tilde{N}_{2ij} , \tilde{N}_{3ij} , \tilde{X}_{11ij} , \tilde{X}_{12ij} , \tilde{X}_{13ij} , \tilde{X}_{22ij} , \tilde{X}_{23ij} and \tilde{X}_{33ij} are the given matrices, thus $P > 0$, $\tilde{Q}_1 > 0$, $\tilde{Q}_2 > 0$, $\tilde{Z} > 0$, $\tilde{M}_{1ij} > 0$, $\tilde{M}_{2ij} > 0$, $\tilde{M}_{3ij} > 0$, \tilde{N}_{1ij} , \tilde{N}_{2ij} , \tilde{N}_{3ij} , \tilde{X}_{11ij} , \tilde{X}_{12ij} , \tilde{X}_{13ij} , \tilde{X}_{22ij} , \tilde{X}_{23ij} and \tilde{X}_{33ij} can be given as follows

$$\begin{cases} P = \phi_{55}^{-1}, & \tilde{Z} = PZP, & \tilde{Q}_1 = PQ_1P, & \tilde{Q}_2 = PQ_2P \\ \tilde{M}_{1ij} = PM_{1ij}P, & \tilde{M}_{2ij} = PM_{2ij}P, & \tilde{M}_{3ij} = PM_{3ij}P, & \tilde{N}_{1ij} = PN_{1ij}P, & \tilde{N}_{2ij} = PN_{2ij}P, & \tilde{N}_{3ij} = PN_{3ij}P \\ \tilde{X}_{11ij} = PX_{11ij}P, & \tilde{X}_{12ij} = PX_{12ij}P, & \tilde{X}_{13ij} = PX_{13ij}P, & \tilde{X}_{22ij} = PX_{22ij}P, & \tilde{X}_{23ij} = PX_{23ij}P, & \tilde{X}_{33ij} = PX_{33ij}P \end{cases} \quad (64)$$

The proof of **Theorem 2** is completed. (Q. E. D.)

Remark 9. Let $\phi_{55} = \begin{bmatrix} \tilde{F} & F \\ F & F \end{bmatrix} = \begin{bmatrix} \alpha I & \beta I \\ \beta I & \beta I \end{bmatrix}$, where α and β are the scalars and $\alpha > \beta > 0$. The matrix Δ is obtained by substituting ϕ_{55} into (50). Next, (47) is solved by substituting Δ into (47), i.e., F , R_i , S_i and T_i are obtained. Further, A_{fi} , B_{fi} and C_{fi} are determined by substituting F , R_i , S_i and T_i into (52). The similar method was shown in [38, 39]. However, the matrices transformation information in the off-diagonals of (47) is not considered in **Theorem 2**. Thus, the matrices transformation information in the off-diagonals of the LMIs is considered in **Theorem 3**.

4.3 Less conservative stability conditions

In this section, the less conservative stability conditions are derived and extended into the networked control system without additive uncertainties.

Theorem 3. For the given scalars $\tau_2 > 0$, $\tau_d > 0$, $\delta > 0$, $\gamma > 0$, $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ and $\lambda > 0$, if there exist the matrices $F > 0$, $\tilde{F} > 0$, $\tilde{Q}_1 > 0$, $\tilde{Q}_2 > 0$, $\tilde{Z} > 0$, $\tilde{M}_{1ij} > 0$, $\tilde{M}_{2ij} > 0$, $\tilde{M}_{3ij} > 0$, \tilde{N}_{1ij} , \tilde{N}_{2ij} , \tilde{N}_{3ij} , \tilde{X}_{11ij} , \tilde{X}_{12ij} , \tilde{X}_{13ij} , \tilde{X}_{22ij} , \tilde{X}_{23ij} , \tilde{X}_{33ij} , R_i , S_i and T_i for $i, j = 1, 2, \dots, r$ satisfying

$$\begin{bmatrix} \bar{X}_{11ij} & \bar{X}_{12ij} & \bar{X}_{13ij} & \bar{M}_{1ij} \\ * & \bar{X}_{22ij} & \bar{X}_{23ij} & \bar{M}_{2ij} \\ * & * & \bar{X}_{33ij} & \bar{M}_{3ij} \\ * & * & * & \bar{Z} \end{bmatrix} \geq 0 \quad (65)$$

$$\begin{bmatrix} \bar{X}_{11ij} & \bar{X}_{12ij} & \bar{X}_{13ij} & \bar{N}_{1ij} \\ * & \bar{X}_{22ij} & \bar{X}_{23ij} & \bar{N}_{2ij} \\ * & * & \bar{X}_{33ij} & \bar{N}_{3ij} \\ * & * & * & \bar{Z} \end{bmatrix} \geq 0 \quad (66)$$

$$\begin{bmatrix} \Xi_{ijk} & \lambda^{-1}\Phi_i + \lambda\Psi_i \\ * & -2I \end{bmatrix} < 0, \quad i, j, k = 1, 2, \dots, r \quad (67)$$

with

$$\begin{cases} \Psi_i = \begin{bmatrix} [\bar{C}_{yi}\tilde{F} & \bar{C}_{yi}F] & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & S_i \end{bmatrix}^T & 0 & 0 & 0 & [0 & \tau_2 S_i]^T & 0 & 0 & 0 & 0 \\ \Xi_{ijk} = \begin{bmatrix} \bar{\mathcal{F}}_{11} & \mathcal{F}_{12} & \mathcal{F}_{13} & A_{3i} & \tau_2 \tilde{A}_{1ij}^T & \tau_1 A_{4ijk}^T & A_{5ij}^T & \varepsilon_1 A_{6ij}^T & A_{8i} & \varepsilon_2 A_{9i}^T \\ * & \mathcal{F}_{22} & \mathcal{F}_{23} & 0 & \tau_2 \tilde{A}_{2i}^T & 0 & 0 & 0 & 0 & \varepsilon_2 A_{10i}^T \\ * & * & \mathcal{F}_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & \tau_2 A_{3i}^T & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\tau_D \bar{\Delta} & 0 & \tau_2 A_{5ij}^T & 0 & \tau_2 A_{8i} & 0 \\ * & * & * & * & * & -I & \tau_1 A_{7ij}^T & 0 & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & * & * & * & * & -\varepsilon_2 I \end{bmatrix} < 0 \end{cases} \quad (68)$$

where \tilde{A}_{ij} and \tilde{A}_{2i} are the matrices in the off-diagonals of (68) and defined as

$$\begin{cases} \tilde{A}_{ij} = \begin{bmatrix} \bar{A}_i \tilde{F} + \bar{B}_{ui} T_j & \bar{A}_i F + \bar{B}_{ui} T_j \\ R_j & R_j \end{bmatrix} \\ \tilde{A}_{2i} = \begin{bmatrix} \bar{A}_i \tilde{F} + \bar{B}_{ui} T_j & 0 \\ 0 & R_j \end{bmatrix} \\ \bar{\mathcal{F}}_{11} = \tilde{A}_{ij} + \tilde{A}_{ij}^T + Q_1 + Q_2 + \tau_2 X_{1ij} + M_{1ij} + M_{1ij}^T \\ \bar{\Delta} = 2\delta\phi_{35} - \delta^2 \bar{Z} \end{cases} \quad (69)$$

in which

$$\begin{cases} A_{ijk} = \begin{bmatrix} \bar{A}_i \tilde{F} + \bar{B}_{ui} T_j F & \bar{A}_i F + \bar{B}_{ui} T_j F \\ S_j \bar{C}_{yk} \tilde{F} + R_j F & S_j \bar{C}_{yk} F + R_j F \end{bmatrix} \\ A_{ijk} = \tilde{A}_{ij} + \begin{bmatrix} 0 & S_i^T \end{bmatrix}^T \begin{bmatrix} C_{yi} \tilde{F} & C_{yi} F \end{bmatrix} \end{cases} \quad (70)$$

then the closed-loop system is asymptotically stable with prescribed H-infinity performance index γ . A_{fi} , B_{fi} and C_{fi} can be determined

$$\begin{cases} A_{fi} = R_i F^{-1} \\ B_{fi} = S_i \\ C_{fi} = T_i F^{-1} \end{cases} \quad (71)$$

Proof. Applying Schur complement, the following inequality holds

$$\Xi_{ijk} + \Phi_i \Psi_i^T + \Psi_i \Phi_i^T < 0 \quad (72)$$

From (47) and (72), one knows that (47) can be rewritten as

$$\Omega_{ijk} = \Xi_{ijk} + \Phi_i \Psi_i^T + \Psi_i \Phi_i^T < 0 \quad (73)$$

One can obtain (47) if (67) holds. The proof in **Theorem 3** is completed. **(Q. E. D.)**

Remark 10. The objective of **Theorem 3** is to reduce the conservativeness of **Theorem 2**. One knows that A_{ijk} is divided into \tilde{A}_{ij} and $[0 \ S_i^T]^T [C_{yi} \tilde{F} \ C_{yi} F]$. Applying **Lemma 2** to (47), one can obtain (67). The less conservative stability conditions are derived in **Theorem 3** because the matrices transformation information of \tilde{A}_{ij} and \tilde{A}_{2i} of (68) is considered. \tilde{A}_{ij} and \tilde{A}_{2i} are designed in (69), thus the stability conditions can deal with more general networked control system. The **Theorem 3** can be extended into the networked control system without additive uncertainties, and **Corollary 1** is presented.

Corollary 1. For the given scalars $\tau_2 > 0$, $\tau_D > 0$, $\delta > 0$, $\gamma > 0$, $\varepsilon_1 > 0$ and $\lambda > 0$, if there exist the matrices $F > 0$, $\tilde{F} > 0$, $\hat{Q}_1 > 0$, $\hat{Q}_2 > 0$, $\hat{Z} > 0$, $\hat{M}_{1ij} > 0$, $\hat{M}_{2ij} > 0$, $\hat{M}_{3ij} > 0$, \hat{N}_{1ij} , \hat{N}_{2ij} , \hat{N}_{3ij} , \hat{X}_{1ij} , \hat{X}_{12ij} , \hat{X}_{13ij} , \hat{X}_{22ij} , \hat{X}_{23ij} , \hat{X}_{33ij} , R_i , S_i and T_i for $i, j = 1, 2, \dots, r$ satisfying

$$\begin{bmatrix} \hat{X}_{11ij} & \hat{X}_{12ij} & \hat{X}_{13ij} & \hat{M}_{1ij} \\ * & \hat{X}_{22ij} & \hat{X}_{23ij} & \hat{M}_{2ij} \\ * & * & \hat{X}_{33ij} & \hat{M}_{3ij} \\ * & * & * & \hat{Z} \end{bmatrix} \geq 0 \quad (74)$$

$$\begin{bmatrix} \hat{X}_{11ij} & \hat{X}_{12ij} & \hat{X}_{13ij} & \hat{N}_{1ij} \\ * & \hat{X}_{22ij} & \hat{X}_{23ij} & \hat{N}_{2ij} \\ * & * & \hat{X}_{33ij} & \hat{N}_{3ij} \\ * & * & * & \hat{Z} \end{bmatrix} \geq 0 \quad (75)$$

$$\begin{bmatrix} \hat{\Xi}_{ijk} & \lambda^{-1} \Phi_i + \lambda \Psi_i \\ * & -2I \end{bmatrix} < 0, \quad i, j, k = 1, 2, \dots, r \quad (76)$$

where Ψ_i , Φ_i are defined in (68) and

$$\hat{\Xi}_{ijk} = \begin{bmatrix} \bar{\mathcal{F}}_{11} & \mathcal{F}_{12} & \mathcal{F}_{13} & \mathcal{A}_{3i} & \tau_2 \tilde{\mathcal{A}}_{1ij}^T & \tau_1 \mathcal{A}_{4ijk}^T & \mathcal{A}_{5ij} & \varepsilon_1 \mathcal{A}_{6ij}^T \\ * & \mathcal{F}_{22} & \mathcal{F}_{23} & 0 & \tau_2 \tilde{\mathcal{A}}_{2i}^T & 0 & 0 & 0 \\ * & * & \mathcal{F}_{33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & \tau_2 \mathcal{A}_{3i}^T & 0 & 0 & 0 \\ * & * & * & * & -\tau_D \hat{\Delta} & 0 & \tau_2 \mathcal{A}_{5ij} & 0 \\ * & * & * & * & * & I & \tau_1 \mathcal{A}_{7ij} & 0 \\ * & * & * & * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & * & * & * & -\varepsilon_1 I \end{bmatrix} \quad (77)$$

with

$$\hat{\Delta} = 2\delta\phi_{ss} - \delta^2 \hat{Z} \quad (78)$$

then the closed-loop system is asymptotically stable with prescribed H-infinity performance index γ .

Proof. “Networked control system without additive uncertainties” means $G_i F_{li}(t) E_{Ai} = 0$ and $G_i F_{li}(t) E_{Ari} = 0$ in (2). Via the similar proof in **Theorem 3**, one knows that if the additive uncertainties are not considered, i.e., $G_i F_{li}(t) E_{Ai} = 0$ and $G_i F_{li}(t) E_{Ari} = 0$, the sufficient conditions still hold. Thus, **Theorem 3** can be extended into the networked control system without additive uncertainties. **Corollary 1** enlarges the application scope of the networked control systems in this paper. The proof of **Corollary 1** is completed. (Q. E. D.)

5. Simulation examples

5.1 Example 1

Consider the Chaotic Lorenz system as follows [40]

$$\begin{cases} \dot{x}_1(t) = -\sigma x_1(t) + \sigma x_2(t) \\ \dot{x}_2(t) = r x_1(t) - x_2(t) - x_1(t) x_3(t) \\ \dot{x}_3(t) = x_1(t) x_2(t) - b x_3(t) \end{cases} \quad (79)$$

and it is transformed

$$\begin{cases} \dot{x}(t) = Ax(t) + A_\tau x(t) + B_u u(t) \\ z(t) = C_z x(t) + D_z u(t) \\ y(t) = C_y x(t) \\ x(t) = \phi(t), \quad -\tau_2 \leq t \leq 0 \end{cases} \quad (80)$$

where

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

Consider $GF_1(t)E_A$, $GF_1(t)E_{A_\tau}$, $\tau(t)$ and $\omega(t)$ in (80), one has

$$\begin{cases} \dot{x}(t) = (A + GF_1(t)E_A)x(t) + (A_\tau + GF_1(t)E_{A_\tau})x(t - \tau(t)) + B_u u(t) + B_\omega \omega(t) \\ z(t) = C_z x(t) + D_z u(t) \\ y(t) = C_y x(t) \\ x(t) = \phi(t), \quad -\tau_2 \leq t \leq 0 \end{cases} \quad (81)$$

Plant rule i : if $\theta_1(t)$ is M_{i1} , $\theta_2(t)$ is M_{i2} , ..., and $\theta_g(t)$ is M_{ig} , then

$$\begin{cases} \dot{x}(t) = (A_i + G_i F_{li}(t)E_{A_i})x(t) + (A_{\tau i} + G_i F_{li}(t)E_{A_{\tau i}})x(t - \tau(t)) + B_{ui} u(t) + B_{\omega i} \omega(t) \\ z(t) = C_{zi} x(t) + D_{zi} u(t) \\ y(t) = C_{yi} x(t) \\ x(t) = \phi(t), \quad -\tau_2 \leq t \leq 0, \quad i = 1, 2, 3 \end{cases} \quad (82)$$

A 3-rules fuzzy model is employed, one can obtain

Rule 1: if $\theta_1(t)$ is $(h_{1U}(\theta(t))\bar{v}_1(\theta(t)) + h_{1L}(\theta(t))\underline{v}_1(\theta(t)))$, then

$$\begin{cases} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} -\sigma & \sigma & 0 \\ r & -1 & -M_1 \\ 0 & M_1 & -b \end{bmatrix} x(t) + (A_{\tau 1} + G_1 F_{11}(t)E_{A_{\tau 1}})x(t - \tau(t)) + B_{u1} u(t) + B_{\omega 1} \omega(t) \\ z(t) = C_{z1} x(t) + D_{z1} u(t) \\ y(t) = C_{y1} x(t) \end{cases}$$

Rule 2: if $\theta_2(t)$ is $(h_{2U}(\theta(t))\bar{v}_2(\theta(t)) + h_{2L}(\theta(t))\underline{v}_2(\theta(t)))$, then

$$\begin{cases} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} -\sigma & \sigma & 0 \\ r & -1 & -M_2 \\ 0 & M_2 & -b \end{bmatrix} x(t) + (A_{\tau 2} + G_2 F_{12}(t)E_{A_{\tau 2}})x(t - \tau(t)) + B_{u2} u(t) + B_{\omega 2} \omega(t) \\ z(t) = C_{z2} x(t) + D_{z2} u(t) \\ y(t) = C_{y2} x(t) \end{cases}$$

Rule 3: if $\theta_3(t)$ is $(h_{3U}(\theta(t))\bar{v}_3(\theta(t)) + h_{3L}(\theta(t))\underline{v}_3(\theta(t)))$, then

$$\begin{cases} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0.24 & -2.08 & 0.18 \\ 4.16 & 0.50 & -3.12 \\ 0.64 & 0.45 & 0.40 \end{bmatrix} x(t) + (A_{\tau 3} + G_3 F_{13}(t)E_{A_{\tau 3}})x(t - \tau(t)) + B_{u3} u(t) + B_{\omega 3} \omega(t) \\ z(t) = C_{z3} x(t) + D_{z3} u(t) \\ y(t) = C_{y3} x(t) \end{cases}$$

where $\sigma = 10$, $r = 28$, $b = 8/3$, $M_1 = -20$ and $M_2 = 30$ [40]. A_1 , A_2 , A_3 , $A_{\tau 1}$, $A_{\tau 2}$, $A_{\tau 3}$, B_{u1} , B_{u2} , B_{u3} , $B_{\omega 1}$, $B_{\omega 2}$, $B_{\omega 3}$, C_{z1} , C_{z2} , C_{z3} , C_{y1} , C_{y2} , C_{y3} , D_{z1} , D_{z2} , D_{z3} , G_1 , G_2 , G_3 , E_{A1} , E_{A2} , E_{A3} , $E_{A_{\tau 1}}$, $E_{A_{\tau 2}}$ and $E_{A_{\tau 3}}$ are given as

$$A_1 = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & 20 \\ 0 & -20 & -8/3 \end{bmatrix}, A_2 = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & -30 \\ 0 & 30 & -8/3 \end{bmatrix}, A_3 = \begin{bmatrix} 0.24 & -2.08 & 0.18 \\ 4.16 & 0.50 & -3.12 \\ 0.64 & 0.45 & 0.40 \end{bmatrix}, A_{\tau 1} = \begin{bmatrix} 0.81 & -1.91 & 0.27 \\ 0.90 & 0.63 & -1.54 \\ 0.12 & 1.09 & 0.95 \end{bmatrix}, A_{\tau 2} = \begin{bmatrix} 0.47 & -2.34 & 0.85 \\ 0.58 & 0.24 & -2.69 \\ 0.70 & 2.75 & 0.75 \end{bmatrix},$$

$$A_{\tau 3} = \begin{bmatrix} 0.96 & -3.95 & 0.14 \\ 1.15 & 0.48 & -3.42 \\ 0.97 & 0.80 & 2.91 \end{bmatrix}, B_{u1} = 0.16, B_{u2} = 0.15, B_{u3} = 0.18, B_{\omega 1} = \begin{bmatrix} 1.06 \\ 1.00 \\ 0.82 \end{bmatrix}, B_{\omega 2} = \begin{bmatrix} 0.85 \\ 0.68 \\ 0.60 \end{bmatrix}, B_{\omega 3} = \begin{bmatrix} 1.06 \\ 1.10 \\ 0.22 \end{bmatrix}, C_{z1} = -0.20, C_{z2} = -0.86, \\ C_{z3} = -0.06, C_{y1} = -0.28, C_{y2} = -0.72, C_{y3} = -0.16, D_{z1} = 1.01, D_{z2} = 1.65, D_{z3} = 0.15, G_1 = \begin{bmatrix} 0.19 \\ 1.08 \\ -0.20 \end{bmatrix}, G_2 = \begin{bmatrix} 1.12 \\ 0.28 \\ -0.10 \end{bmatrix}, G_3 = \begin{bmatrix} 0.88 \\ 0.62 \\ -1.05 \end{bmatrix}, \\ E_{A1} = [-1.76 \quad -0.82 \quad 1.09], E_{A2} = [-0.16 \quad -1.50 \quad 0.81], E_{A3} = [-0.71 \quad -0.99 \quad 1.28], E_{A\tau 1} = [-0.88 \quad 1.92 \quad -0.76], \\ E_{A\tau 2} = [-1.56 \quad 0.67 \quad -0.82], E_{A\tau 3} = [-1.82 \quad 0.35 \quad -1.36].$$

The upper membership functions and lower membership functions are shown in **Table 1**.

Table 1. Upper membership functions and lower membership functions.

Upper membership functions	Lower membership functions
$h_{1U}(x_1(t)) = 1 - \frac{1}{1 + e^{-\frac{(x_1(t)+3)}{4}}}$	$h_{1L}(x_1(t)) = 1 - \frac{1}{1 + e^{-\frac{(x_1(t)+6)}{4}}}$
$h_{2U}(x_1(t)) = \frac{1}{1 + e^{-\frac{(x_1(t)-3)}{4}}}$	$h_{2L}(x_1(t)) = \frac{1}{1 + e^{-\frac{(x_1(t)-6)}{4}}}$
$h_{3U}(x_1(t)) = 1 - h_{1L}(x_1(t))$	$h_{3L}(x_1(t)) = 1 - h_{1U}(x_1(t))$

For (82), the controller is designed

$$\begin{cases} \dot{x}_f(t) = (A_f + L_1 F_2(t) E_{Af}) x_f(t) + (B_f + L_1 F_2(t) E_{Bf}) y(t) \\ u(t) = (C_f + L_2 F_2(t) E_{Cf}) x_f(t) \end{cases} \quad (83)$$

Controller rule i : if $\theta_1(t)$ is M_{i1} , $\theta_2(t)$ is M_{i2} , ..., and $\theta_g(t)$ is M_{ig} , then

$$\begin{cases} \dot{x}_f(t) = (A_{fi} + L_{1i} F_{2i}(t) E_{Afi}) x_f(t) + (B_{fi} + L_{1i} F_{2i}(t) E_{Bfi}) y(t) \\ u(t) = (C_{fi} + L_{2i} F_{2i}(t) E_{Cfi}) x_f(t), \quad i = 1, 2, 3 \end{cases} \quad (84)$$

where

$$\begin{cases} L_{11} = \begin{bmatrix} 0.81 \\ 0.90 \\ -0.12 \end{bmatrix}, L_{12} = \begin{bmatrix} 1.47 \\ 0.58 \\ -0.70 \end{bmatrix}, L_{13} = \begin{bmatrix} 0.91 \\ 0.63 \\ -0.09 \end{bmatrix} \\ L_{21} = \begin{bmatrix} 0.34 \\ -1.24 \\ 0.75 \end{bmatrix}, L_{22} = \begin{bmatrix} 0.27 \\ -0.54 \\ 0.95 \end{bmatrix}, L_{23} = \begin{bmatrix} 0.85 \\ -0.69 \\ 1.75 \end{bmatrix} \\ E_{Af1} = [-0.96 \quad 0.15 \quad -0.97], E_{Af2} = [-0.49 \quad 1.76 \quad -0.06], E_{Af3} = [-1.95 \quad 1.48 \quad -0.80] \\ E_{Bf1} = [0.72 \quad 1.54 \quad 1.03], E_{Bf2} = [0.14 \quad 1.42 \quad 1.91], E_{Bf3} = [0.19 \quad 0.18 \quad 1.57] \\ E_{Cf1} = [0.79 \quad -0.95 \quad 0.65], E_{Cf2} = [0.22 \quad -0.95 \quad 1.57], E_{Cf3} = [0.03 \quad -0.84 \quad 0.93] \end{cases} \quad (85)$$

then A_{fi} , B_{fi} and C_{fi} can be determined

$$\begin{cases} A_{fi} = R_i F^{-1} \\ B_{fi} = S_i \\ C_{fi} = T_i F^{-1} \end{cases} \quad (86)$$

Based on **Theorem 3**, τ_2 , τ_D , δ , γ , ε_1 , ε_2 and λ are given as $\tau_2 = 0.5$, $\tau_D = 0.1$, $\delta = 0.12$, $\gamma = 0.65$, $\varepsilon_1 = 0.25$, $\varepsilon_2 = 0.28$ and $\lambda = 1$. Solve the LMIs with $\tau_2 = 0.5$, $\tau_D = 0.1$, $\delta = 0.12$, $\gamma = 0.65$, $\varepsilon_1 = 0.25$, $\varepsilon_2 = 0.28$ and $\lambda = 1$, where τ_2 , τ_D , δ , γ , ε_1 , ε_2 and λ satisfying (65)-(70), then F , R_i , S_i and T_i ($i = 1, 2, 3$) are calculated as

$$\begin{aligned}
F &= \begin{bmatrix} -1.6416 & 0.7725 & 2.4429 \\ -0.4159 & 2.5222 & 0.7306 \\ -0.4479 & 0.7628 & 2.7878 \end{bmatrix} \\
R_1 &= \begin{bmatrix} -1.0500 & -1.8481 & -2.4925 \\ 0.5898 & 1.4199 & 1.7558 \\ 0.7533 & 1.0550 & 1.6492 \end{bmatrix}, \quad S_1 = \begin{bmatrix} -0.9572 & 0.1419 & 0.7922 \\ 0.4854 & -0.4218 & 0.9595 \\ 0.8003 & 0.9157 & -0.6557 \end{bmatrix}, \quad T_1 = \begin{bmatrix} 0.6224 & -1.2040 & 1.3784 \\ -1.0571 & 0.5259 & 1.4963 \\ 0.3313 & -1.3082 & 0.9011 \end{bmatrix} \\
R_2 &= \begin{bmatrix} -1.8344 & -1.5075 & -0.1517 \\ 0.5717 & 0.7609 & 0.1079 \\ 1.5144 & 1.3656 & 1.0616 \end{bmatrix}, \quad S_2 = \begin{bmatrix} -0.0357 & 0.6787 & 0.3922 \\ 0.8491 & -0.7577 & 0.6555 \\ 0.9340 & 0.7431 & -0.1712 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 0.1676 & -0.3048 & 1.9230 \\ -0.4580 & 1.6516 & 0.1564 \\ 1.8267 & -1.0767 & 0.8854 \end{bmatrix} \\
R_3 &= \begin{bmatrix} -1.5583 & -1.1376 & -0.6742 \\ 1.8680 & 0.9388 & 0.3244 \\ 0.2598 & 0.0238 & 1.5886 \end{bmatrix}, \quad S_3 = \begin{bmatrix} -0.0689 & 1.5310 & 0.9757 \\ 0.8775 & -1.5904 & 0.8912 \\ 0.7631 & 0.3737 & -0.2926 \end{bmatrix}, \quad T_3 = \begin{bmatrix} 0.2133 & -1.5489 & 0.1689 \\ -1.9238 & 1.6346 & 0.7996 \\ 0.0093 & -1.7374 & 0.5197 \end{bmatrix}
\end{aligned} \tag{87}$$

Employing (65)-(70) with $\tau_2 = 0.5$, $\tau_D = 0.1$, $\delta = 0.12$, $\varepsilon_1 = 0.25$, $\varepsilon_2 = 0.28$, $\lambda = 1$, $\gamma = 0.65$ and substituting (87) into (86), A_{fi} , B_{fi} and C_{fi} ($i = 1, 2, 3$) are calculated as

$$\begin{aligned}
A_{f1} &= \begin{bmatrix} 1.2951 & -0.5602 & -1.8821 \\ -0.8033 & 0.4405 & 1.2183 \\ -0.8868 & 0.2997 & 1.2901 \end{bmatrix}, \quad B_{f1} = \begin{bmatrix} -0.9572 & 0.1419 & 0.7922 \\ 0.4854 & -0.4218 & 0.9595 \\ 0.8003 & 0.9157 & -0.6557 \end{bmatrix}, \quad C_{f1} = \begin{bmatrix} -0.5184 & -0.6576 & 1.1211 \\ 0.6488 & 0.0211 & -0.0373 \\ -0.2235 & -0.6595 & 0.6919 \end{bmatrix} \\
A_{f2} &= \begin{bmatrix} 1.6567 & -0.7055 & -1.3212 \\ -0.5527 & 0.3397 & 0.4340 \\ -1.4754 & 0.5291 & 1.5350 \end{bmatrix}, \quad B_{f2} = \begin{bmatrix} -0.0357 & 0.6787 & 0.3922 \\ 0.8491 & -0.7577 & 0.6555 \\ 0.9340 & 0.7431 & -0.1712 \end{bmatrix}, \quad C_{f2} = \begin{bmatrix} -0.2992 & -0.3444 & 1.0422 \\ 0.1829 & 0.6846 & -0.2836 \\ -1.4561 & -0.5027 & 1.7253 \end{bmatrix} \\
A_{f3} &= \begin{bmatrix} 1.4478 & -0.4753 & -1.3860 \\ -1.6423 & 0.4396 & 1.4402 \\ -0.3740 & -0.1602 & 0.9396 \end{bmatrix}, \quad B_{f3} = \begin{bmatrix} -0.0689 & 1.5310 & 0.9757 \\ 0.8775 & -1.5904 & 0.8912 \\ 0.7631 & 0.3737 & -0.2926 \end{bmatrix}, \quad C_{f3} = \begin{bmatrix} -0.0286 & 0.6856 & 0.2654 \\ 1.3056 & 0.5512 & -1.0017 \\ 0.1204 & -0.8148 & 0.2944 \end{bmatrix}
\end{aligned} \tag{88}$$

The initial values of $x(0)$ is $x(0) = [6.50 \ 6.50 \ 13.70]^T$. $F_{li}(t)$ are given as $F_{11}(t) = F_{12}(t) = F_{13}(t) = \sin t$, thus the additive

uncertainties $G_i F_{li}(t) E_{Ai}$ and $G_i F_{li}(t) E_{Afi}$ ($i = 1, 2, 3$) are given as $G_1 F_{11}(t) E_{A1} = \begin{bmatrix} 0.3344 & -0.1558 & 0.2071 \\ 1.9008 & -0.8856 & 1.1772 \\ -0.3520 & 0.1640 & -0.2180 \end{bmatrix} \sin t$,

$$G_2 F_{12}(t) E_{A2} = \begin{bmatrix} -0.1792 & -1.6800 & 0.9072 \\ -0.0448 & -0.4200 & 0.2268 \\ 0.0160 & 0.1500 & -0.0810 \end{bmatrix} \sin t, \quad G_3 F_{13}(t) E_{A3} = \begin{bmatrix} -0.6248 & -0.8712 & 1.1264 \\ -0.4402 & -0.6138 & 0.7936 \\ 0.7455 & 1.0395 & -1.3440 \end{bmatrix} \sin t,$$

$$G_1 F_{11}(t) E_{A\tau 1} = \begin{bmatrix} -0.1672 & 0.3648 & -0.1444 \\ -0.9504 & 2.0736 & -0.8208 \\ 0.1760 & -0.3840 & 0.1520 \end{bmatrix} \sin t, \quad G_2 F_{12}(t) E_{A\tau 2} = \begin{bmatrix} -1.7472 & 0.7504 & -0.9184 \\ -0.4368 & 0.1876 & -0.2296 \\ 0.1560 & -0.0670 & 0.0820 \end{bmatrix} \sin t \quad \text{and}$$

$$G_3 F_{13}(t) E_{A\tau 3} = \begin{bmatrix} -1.6016 & 0.3080 & -1.1968 \\ -1.1284 & 0.2170 & -0.8432 \\ 1.9110 & -0.3675 & 1.4280 \end{bmatrix} \sin t. \text{ The time-varying delay is given as } \tau(t) = 0.25(1 + \sin t). \text{ The external disturbance is}$$

given as $\omega(t) = |\sin(0.2t)|$. $F_{2i}(t)$ are given as $F_{21}(t) = F_{22}(t) = F_{23}(t) = 0.06 \sin t$, thus the additive uncertainties $L_{1i} F_{2i}(t) E_{Afi}$,

$$L_{1i} F_{2i}(t) E_{Bfi} \quad \text{and} \quad L_{2i} F_{2i}(t) E_{Cfi} \quad (i = 1, 2, 3) \quad \text{are given as} \quad L_{11} F_{21}(t) E_{A\tau 1} = \begin{bmatrix} -0.7776 & 0.1215 & -0.7857 \\ -0.8640 & 0.1350 & -0.8730 \\ 0.1152 & -0.0180 & 0.1164 \end{bmatrix} 0.06 \sin t,$$

$$L_{12} F_{22}(t) E_{A\tau 2} = \begin{bmatrix} -0.7203 & 2.5872 & -0.0882 \\ -0.2842 & 1.0208 & -0.0348 \\ 0.3430 & -1.2320 & 0.0420 \end{bmatrix} 0.06 \sin t, \quad L_{13} F_{23}(t) E_{A\tau 3} = \begin{bmatrix} -1.7745 & 1.3468 & -0.7280 \\ -1.2285 & 0.9324 & -0.5040 \\ 0.1755 & -0.1332 & 0.0720 \end{bmatrix} 0.06 \sin t,$$

$$L_{11}F_{21}(t)E_{Bf1} = \begin{bmatrix} 0.5832 & 1.2474 & 0.8343 \\ 0.6480 & 1.3860 & 0.9270 \\ -0.0864 & -0.1848 & -0.1236 \end{bmatrix} 0.06 \sin t ,$$

$$L_{13}F_{23}(t)E_{Bf3} = \begin{bmatrix} 0.1729 & 0.1638 & 1.4287 \\ 0.1197 & 0.1134 & 0.9891 \\ -0.0171 & -0.0162 & -0.1413 \end{bmatrix} 0.06 \sin t ,$$

$$L_{22}F_{22}(t)E_{Cf2} = \begin{bmatrix} 0.0594 & -0.2565 & 0.4239 \\ -0.1188 & 0.5130 & -0.8478 \\ 0.2090 & -0.9025 & 1.4915 \end{bmatrix} 0.06 \sin t ,$$

$$L_{12}F_{22}(t)E_{Bf2} = \begin{bmatrix} 0.2058 & 2.0874 & 2.8077 \\ 0.0812 & 0.8236 & 1.1078 \\ -0.0980 & -0.9940 & -1.3370 \end{bmatrix} 0.06 \sin t ,$$

$$L_{21}F_{21}(t)E_{Cf1} = \begin{bmatrix} 0.2686 & -0.3230 & 0.2210 \\ -0.9796 & 1.1780 & -0.8060 \\ 0.5925 & -0.7125 & 0.4825 \end{bmatrix} 0.06 \sin t ,$$

$$L_{23}F_{23}(t)E_{Cf3} = \begin{bmatrix} 0.0255 & -0.7140 & 0.7905 \\ -0.0207 & 0.5796 & -0.6417 \\ 0.0525 & -1.4700 & 1.6275 \end{bmatrix} 0.06 \sin t .$$

The responses of $x_1(t)$, $x_2(t)$ and $x_3(t)$ with $x(0) = [6.50 \ 6.50 \ 13.70]^T$ are shown in **Figures 4-5**, respectively. The responses of $x_1(t)$, $x_2(t)$ and $x_3(t)$ with $x(0) = [-1.60 \ -1.80 \ 9.20]^T$ are shown in **Figures 6-7**, respectively. The responses of $u_1(t)$, $u_2(t)$ and $u_3(t)$ are shown in **Figures 8-9**, respectively. The response of $\sqrt{\int_0^\infty z^T(t)z(t)/\int_0^\infty \omega^T(t)\omega(t)}$ with $\omega(t) = |\sin(0.2t)|$ is shown in **Figure 10**. From **Figures 2-3**, it can be seen that the open-loop system is not stable. From **Figure 4-5**, it can be seen that $x_1(t)$, $x_2(t)$ and $x_3(t)$ are convergent. From **Figures 8-9**, it can be seen that the control inputs are bounded.

Remark 11. The response of $\sqrt{\int_0^\infty z^T(t)z(t)/\int_0^\infty \omega^T(t)\omega(t)}$ with $\omega(t) = 1 + \sin(0.2t)$ is shown in **Figure 11**. The response of $\sqrt{\int_0^\infty z^T(t)z(t)/\int_0^\infty \omega^T(t)\omega(t)}$ with $\omega(t) = 0.2(1 + \sin(0.2t))$ is shown in **Figure 12**. From **Figures 10-12**, it can be seen that the responses of $\sqrt{\int_0^\infty z^T(t)z(t)/\int_0^\infty \omega^T(t)\omega(t)}$ are smaller than $\gamma = 0.65$ whether $\omega(t) = |\sin(0.2t)|$, $\omega(t) = 1 + \sin(0.2t)$ and $\omega(t) = 0.2(1 + \sin(0.2t))$.

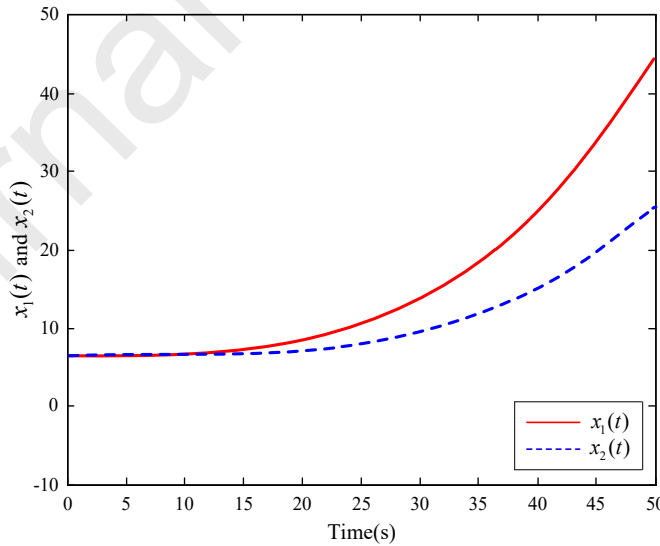


Figure 2. Responses of $x_1(t)$ and $x_2(t)$ for the open-loop system.

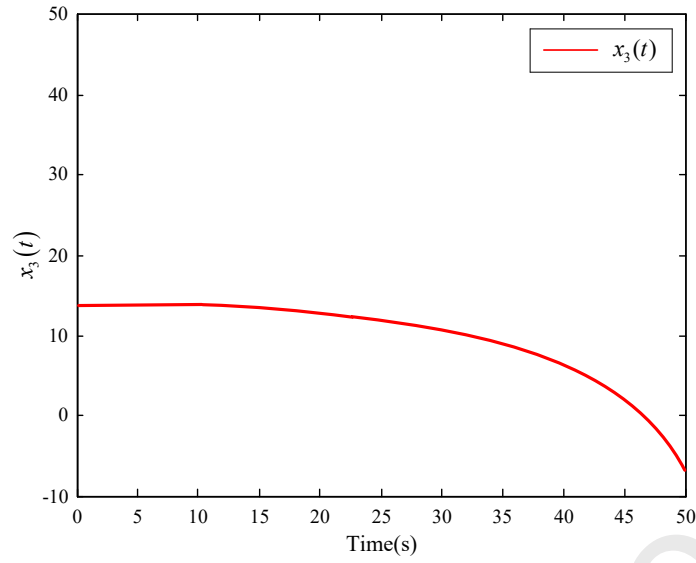


Figure 3. Response of $x_3(t)$ for the open-loop system.

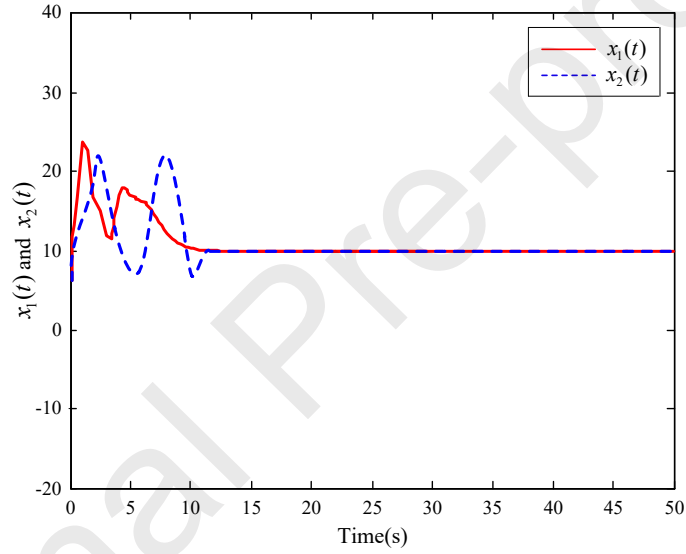


Figure 4. Responses of $x_1(t)$ and $x_2(t)$ with $x(0) = [6.50 \ 6.50 \ 13.70]^T$.

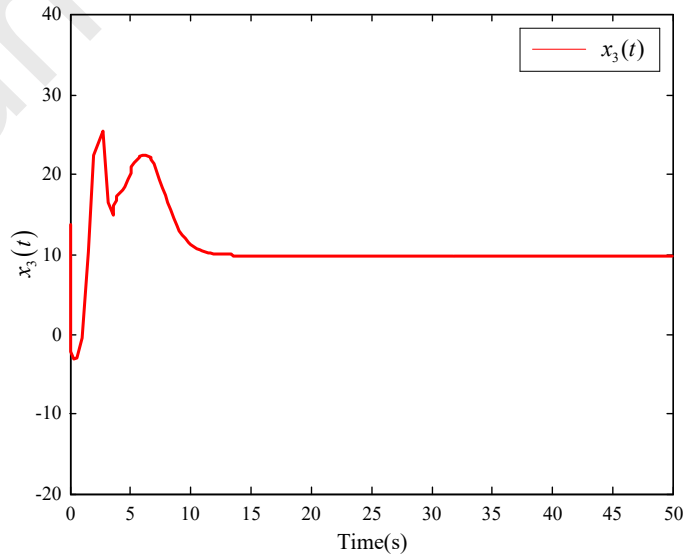


Figure 5. Response of $x_3(t)$ with $x(0) = [6.50 \ 6.50 \ 13.70]^T$.

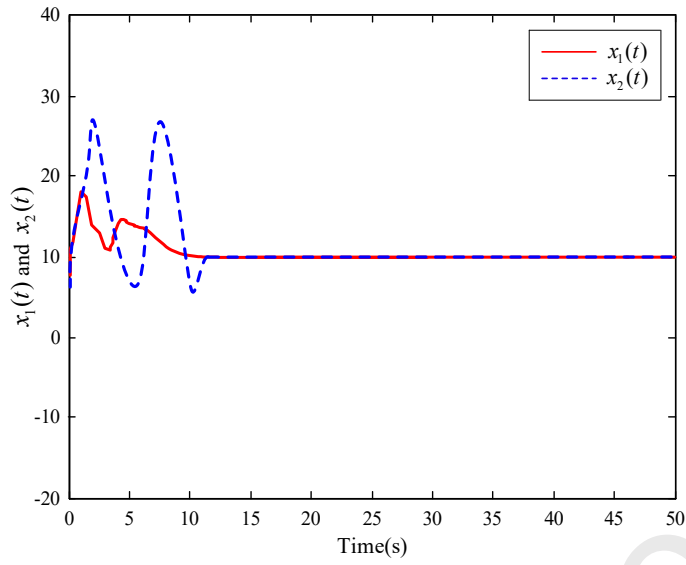


Figure 6. Responses of $x_1(t)$ and $x_2(t)$ with $x(0) = [-1.60 \ -1.80 \ 9.20]^T$.

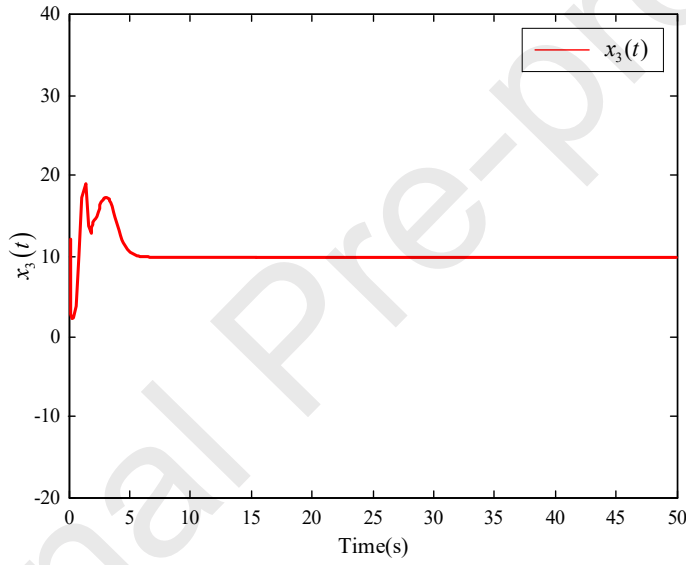


Figure 7. Response of $x_3(t)$ with $x(0) = [-1.60 \ -1.80 \ 9.20]^T$.

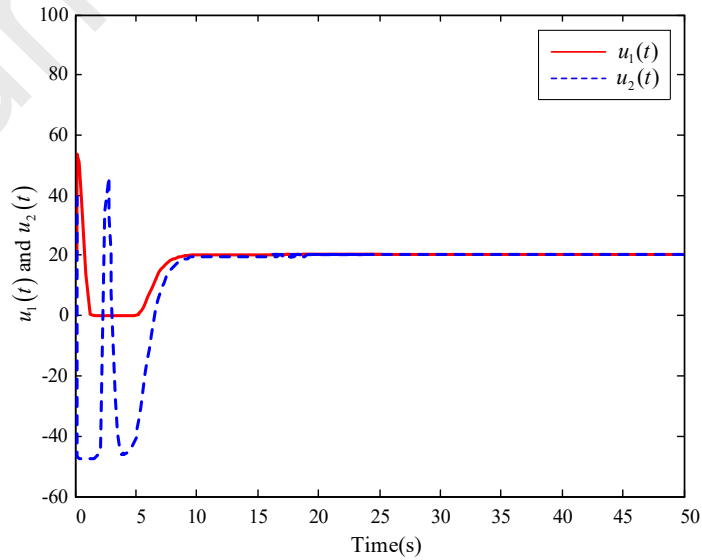


Figure 8 Responses of $u_1(t)$ and $u_2(t)$.

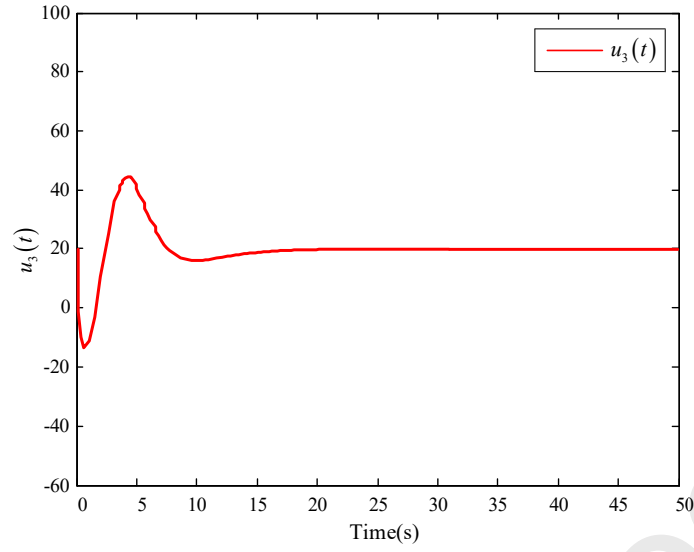


Figure 9. Response of $u_3(t)$.

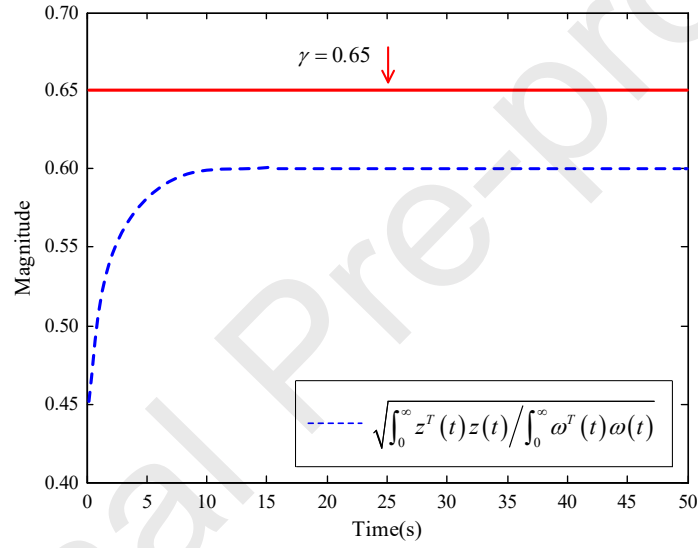


Figure 10. Response of $\sqrt{\int_0^\infty z^T(t)z(t)/\int_0^\infty \omega^T(t)\omega(t)}$ with $\omega(t) = |\sin(0.2t)|$.

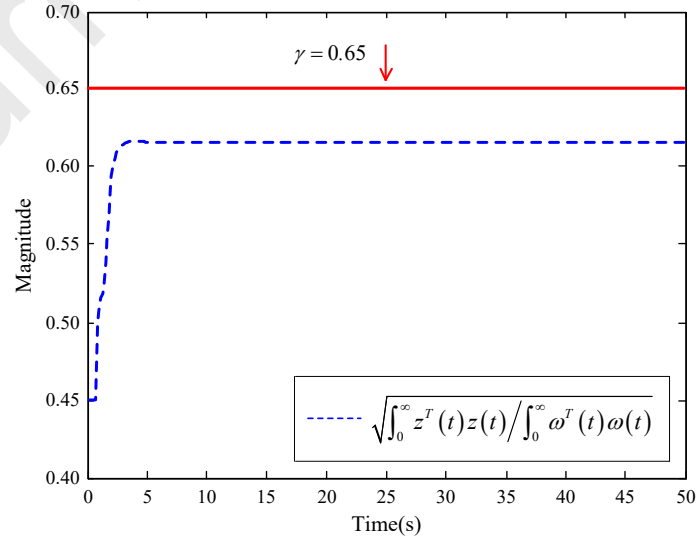


Figure 11. Response of $\sqrt{\int_0^\infty z^T(t)z(t)/\int_0^\infty \omega^T(t)\omega(t)}$ with $\omega(t) = 1 + \sin(0.2t)$.

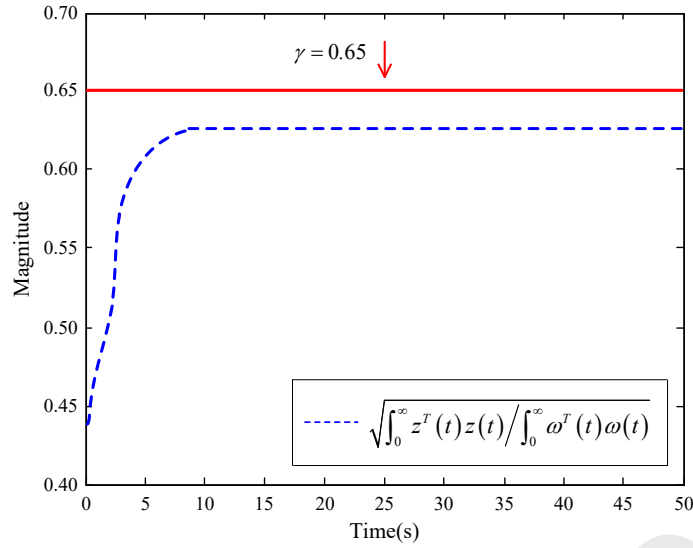


Figure 12. Response of $\sqrt{\int_0^\infty z^T(t)z(t)dt / \int_0^\infty \omega^T(t)\omega(t)dt}$ with $\omega(t) = 0.2(1 + \sin(0.2t))$.

5.2 Example 2

Consider the inverted pendulum system as follows [41-43]

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = \frac{g \sin(x_1(t)) - am_p L (x_2(t))^2 \sin(2x_1(t)) / 2}{4L/3 - am_p L \cos^2(x_1(t))} - \frac{a \cos(x_1(t)) u(t)}{4L/3 - am_p L \cos^2(x_1(t))} + 0.1d(t) \end{cases} \quad (89)$$

where

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} \theta(t) \\ \dot{\theta}(t) \end{bmatrix} \quad (90)$$

$GF_1(t)E_A$, $GF_1(t)E_{A\tau}$, $\tau(t)$ and $\omega(t)$ are introduced in (89), one has

$$\begin{cases} \dot{x}(t) = (A + GF_1(t)E_A)x(t) + (A_\tau + GF_1(t)E_{A\tau})x(t - \tau(t)) + B_u u(t) + B_\omega \omega(t) \\ z(t) = C_z x(t) + D_z u(t) \\ y(t) = C_y x(t) \\ x(t) = \phi(t), \quad -\tau_2 \leq t \leq 0 \end{cases} \quad (91)$$

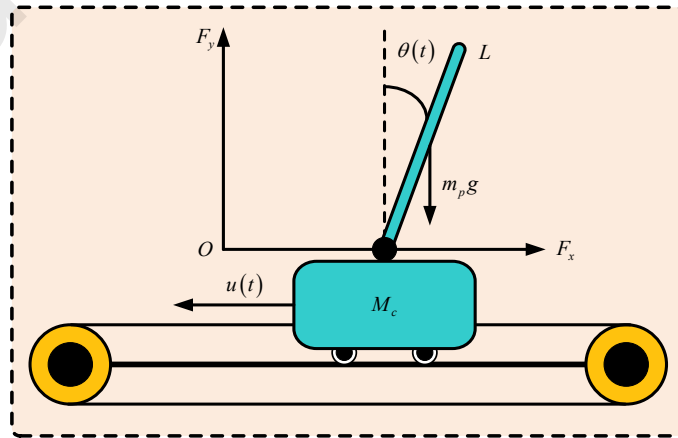


Figure 13. Schematic diagram of the inverted pendulum.

Table 2. Physical parameters of the inverted pendulum [43].

Physical parameters	Symbols	Values	Units
mass of cart	$M_c \in [M_{cmin}, M_{cmax}]$	$[2, 3]$	kg
mass of pendulum	$m_p \in [m_{pmin}, m_{pmax}]$	$[8, 16]$	kg
mass coefficient of pendulum	a	$\frac{1}{m_p + M_c}$	kg^{-1}
length of pendulum	L	0.5	m
angular displacement of pendulum	$\theta(t)$	-----	rad
angular velocity of pendulum	$\dot{\theta}(t)$	-----	rad/s
gravity acceleration	g	9.8	m/s^2

Plant rule i : if $\theta_1(t)$ is M_{i1} , $\theta_2(t)$ is M_{i2} , ..., and $\theta_g(t)$ is M_{ig} , then

$$\begin{cases} \dot{x}(t) = (A_i + G_i F_{li}(t) E_{Ai})x(t) + (A_{ri} + G_i F_{li}(t) E_{Ari})x(t - \tau(t)) + B_{ui}u(t) + B_{\omega i}\omega(t) \\ z(t) = C_{zi}x(t) + D_{zi}u(t) \\ y(t) = C_{yi}x(t), \\ x(t) = \phi(t), \quad -\tau_2 \leq t \leq 0, \quad i = 1, 2 \end{cases} \quad (92)$$

A 2-rules fuzzy model is employed in (92), one has

Rule 1: if $\theta_1(t)$ is $(h_{1U}(\theta(t))\bar{\nu}_1(\theta(t)) + h_{1L}(\theta(t))\underline{\nu}_1(\theta(t)))$, then

$$\begin{cases} \begin{bmatrix} \dot{\theta}(t) \\ \ddot{\theta}(t) \end{bmatrix} = \left(\begin{bmatrix} 0 & 1.12 \\ 1.90 & 0 \end{bmatrix} + G_1 F_{11}(t) E_{A1} \right) x(t) + (A_{r1} + G_1 F_{11}(t) E_{Ar1}) x(t - \tau(t)) + B_{u1}u(t) + B_{\omega 1}\omega(t) \\ z(t) = C_{z1}x(t) + D_{z1}u(t) \\ y(t) = C_{y1}x(t), \end{cases}$$

Rule 2: if $\theta_2(t)$ is $(h_{2U}(\theta(t))\bar{\nu}_2(\theta(t)) + h_{2L}(\theta(t))\underline{\nu}_2(\theta(t)))$, then

$$\begin{cases} \begin{bmatrix} \dot{\theta}(t) \\ \ddot{\theta}(t) \end{bmatrix} = \left(\begin{bmatrix} -a & -0.81 \\ 6.91 & 0 \end{bmatrix} + G_2 F_{12}(t) E_{A2} \right) x(t) + (A_{r2} + G_2 F_{12}(t) E_{Ar2}) x(t - \tau(t)) + B_{u2}u(t) + B_{\omega 2}\omega(t) \\ z(t) = C_{z2}x(t) + D_{z2}u(t) \\ y(t) = C_{y2}x(t), \end{cases}$$

$A_1, A_2, A_{r1}, A_{r2}, B_{u1}, B_{u2}, B_{\omega 1}, B_{\omega 2}, C_{z1}, C_{z2}, C_{y1}, C_{y2}, D_{z1}, D_{z2}, G_1, G_2, E_{A1}, E_{A2}, E_{Ar1}$ and E_{Ar2} are given as follows

$$\begin{cases} A_1 = \begin{bmatrix} 0 & 1.12 \\ 1.90 & 0 \end{bmatrix}, & A_2 = \begin{bmatrix} -a & -0.81 \\ 6.91 & 0 \end{bmatrix}, & A_{r1} = \begin{bmatrix} 0.63 & 1.27 \\ 2.09 & 0.54 \end{bmatrix}, & A_{r2} = \begin{bmatrix} 1.24 & 0.85 \\ 2.75 & 1.69 \end{bmatrix} \\ B_{u1} = \begin{bmatrix} 0.75 \\ 0.60 \end{bmatrix}, & B_{u2} = \begin{bmatrix} -b + 0.76 \\ 0.16 \end{bmatrix}, & B_{\omega 1} = \begin{bmatrix} 0.95 \\ 1.56 \end{bmatrix}, & B_{\omega 2} = \begin{bmatrix} 0.72 \\ 1.36 \end{bmatrix} \\ C_{z1} = -0.6, & C_{z2} = -0.80, & C_{y1} = -0.16, & C_{y2} = -0.22 \\ D_{z1} = \begin{bmatrix} 2.13 \\ 1.68 \end{bmatrix}, & D_{z2} = \begin{bmatrix} 1.28 \\ 1.08 \end{bmatrix}, & G_1 = \begin{bmatrix} 0.80 \\ 0.14 \end{bmatrix}, & G_2 = \begin{bmatrix} 1.03 \\ 1.19 \end{bmatrix} \\ E_{A1} = [1.42 & 2.91], & E_{A2} = [1.18 & 3.57], & E_{Ar1} = [2.37 & 1.87], & E_{Ar2} = [1.66 & 2.85] \end{cases}$$

The upper membership functions and lower membership functions are given in **Table 3**.

Table 3. Upper membership functions and lower membership functions.

Upper membership functions	Lower membership functions
$h_{1U}(x_1(t)) = h_{1L}(x_1(t))$	$h_{1L}(x_1(t)) = 0.6e^{\left(-\frac{x_1^2(t)}{0.16}\right)}$
$h_{2U}(x_1(t)) = h_{2L}(x_1(t))$	$h_{2L}(x_1(t)) = 1 + h_{1L}(x_1(t))$

For (92), the controller is designed

$$\begin{cases} \dot{x}_f(t) = (A_f + L_1 F_2(t) E_{Af}) x_f(t) + (B_f + L_1 F_2(t) E_{Bf}) y(t) \\ u(t) = (C_f + L_2 F_2(t) E_{Cf}) x_f(t) \end{cases} \quad (93)$$

Controller rule i : if $\theta_1(t)$ is M_{i1} , $\theta_2(t)$ is M_{i2} , ..., and $\theta_g(t)$ is M_{ig} , then

$$\begin{cases} \dot{x}_f(t) = (A_{fi} + L_{1i} F_{2i}(t) E_{Afi}) x_f(t) + (B_{fi} + L_{1i} F_{2i}(t) E_{Bfi}) y(t) \\ u(t) = (C_{fi} + L_{2i} F_{2i}(t) E_{Cfi}) x_f(t), \quad i = 1, 2 \end{cases} \quad (94)$$

where

$$\begin{cases} L_{11} = \begin{bmatrix} 1.65 \\ -1.03 \end{bmatrix}, \quad L_{12} = \begin{bmatrix} 0.57 \\ -0.57 \end{bmatrix}, \quad L_{21} = \begin{bmatrix} 1.69 \\ 1.86 \end{bmatrix}, \quad L_{22} = \begin{bmatrix} 0.83 \\ 0.80 \end{bmatrix} \\ E_{Af1} = \begin{bmatrix} 1.35 & -1.51 \end{bmatrix}, \quad E_{Af2} = \begin{bmatrix} 0.75 & -0.55 \end{bmatrix}, \quad E_{Bf1} = \begin{bmatrix} 0.48 & -1.63 \end{bmatrix} \\ E_{Bf2} = \begin{bmatrix} 1.78 & -0.45 \end{bmatrix}, \quad E_{Cf1} = \begin{bmatrix} 1.31 & -0.10 \end{bmatrix}, \quad E_{Cf2} = \begin{bmatrix} 0.34 & -0.24 \end{bmatrix} \end{cases} \quad (95)$$

A_{fi} , B_{fi} and C_{fi} can be determined as

$$A_{fi} = R_i F^{-1}, \quad B_{fi} = S_i, \quad C_{fi} = T_i F^{-1} \quad (96)$$

Based on **Theorem 3**, τ_2 , τ_D , δ , γ , ε_1 , ε_2 and λ are given as $\tau_2 = 0.3$, $\tau_D = 0.1$, $\delta = 0.15$, $\gamma = 0.30$, $\varepsilon_1 = 0.16$, $\varepsilon_2 = 0.36$ and $\lambda = 1$. Solve the LMIs with $\tau_2 = 0.3$, $\tau_D = 0.1$, $\delta = 0.15$, $\gamma = 0.30$, $\varepsilon_1 = 0.16$, $\varepsilon_2 = 0.36$ and $\lambda = 1$, where τ_2 , τ_D , δ , γ , ε_1 , ε_2 and λ satisfying (65)-(70), then F , R_i , S_i and T_i ($i = 1, 2$) are calculated as follows

$$\begin{cases} F = \begin{bmatrix} 0.7060 & 1.2769 \\ 1.0318 & 0.0462 \end{bmatrix}, \quad R_1 = \begin{bmatrix} -0.1943 & 1.3897 \\ 1.6469 & 0.6342 \end{bmatrix}, \quad S_1 = \begin{bmatrix} -1.9004 & 0.8775 \\ 0.0689 & 0.7631 \end{bmatrix}, \quad T_1 = \begin{bmatrix} -1.5310 & 0.3737 \\ 1.5904 & 0.9795 \end{bmatrix} \\ R_2 = \begin{bmatrix} 0.8912 & -1.4187 \\ 1.2926 & 1.5094 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 0.5521 & -1.3102 \\ 1.3594 & 0.3252 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 0.2380 & -1.9195 \\ 0.9967 & 0.6808 \end{bmatrix} \end{cases} \quad (97)$$

Employing (65)-(70) with $\tau_2 = 0.3$, $\tau_D = 0.1$, $\delta = 0.15$, $\gamma = 0.30$, $\varepsilon_1 = 0.16$, $\varepsilon_2 = 0.36$, $\lambda = 1$ and substituting (97) into (96), A_{fi} , B_{fi} and C_{fi} ($i = 1, 2$) are calculated as

$$\begin{cases} A_{f1} = \begin{bmatrix} 1.1230 & -0.9567 \\ 0.4501 & 1.2882 \end{bmatrix}, \quad B_{f1} = \begin{bmatrix} -1.9004 & 0.8775 \\ 0.0689 & 0.7631 \end{bmatrix}, \quad C_{f1} = \begin{bmatrix} 0.3551 & -1.7268 \\ 0.7294 & 1.0423 \end{bmatrix} \\ A_{f2} = \begin{bmatrix} -1.1713 & 1.6652 \\ 1.1656 & 0.4552 \end{bmatrix}, \quad B_{f2} = \begin{bmatrix} 0.5521 & -1.3102 \\ 1.3594 & 0.3252 \end{bmatrix}, \quad C_{f2} = \begin{bmatrix} -1.5500 & 1.2912 \\ 0.5109 & 0.6164 \end{bmatrix} \end{cases} \quad (98)$$

The stability regions by employing **Theorem 2** ('•'), [44] ('•'), [45] ('•') and **Theorem 3** ('•') are shown in **Figures 14-16** with $0 \leq a \leq 100$ and $10 \leq b \leq 130$. The stability regions by employing [44] ('•'), [45] ('•') and **Theorem 3** ('•') are shown in **Figures 17-18** with $0 \leq a \leq 100$ and $-130 \leq b \leq -10$.

From **Figures 14-18**, it can be seen that the larger stability regions are obtained by employing **Theorem 3**. The upper bounds τ_2 with $\tau_D = 0.15$ and different γ by employing **Theorems 2-3** are shown in **Table 4**. The upper bounds τ_2 with $\tau_D = 0.30$ and different γ by employing **Theorems 2-3** are shown in **Table 5**. The lower bounds γ with $\tau_D = 0.15$ and different τ_2 by employing **Theorems 2-3** are shown in **Table 6**. The lower bounds γ with $\tau_D = 0.30$ and different τ_2 by employing **Theorems 2-3** are shown in **Table 7**. The upper bounds τ_2 with $\tau_D = 0.15$ and different γ by employing [44, 45] and **Theorem 3** are shown in **Table 8**. The upper bounds τ_2 with $\tau_D = 0.30$ and different γ by employing [44, 45] and **Theorem 3** are shown in **Table 9**. The lower bounds γ with $\tau_D = 0.15$ and different τ_2 by employing [44, 45] and **Theorem 3** are shown in **Table 10**. The lower bounds γ with $\tau_D = 0.30$

and different τ_2 by employing [44, 45] and **Theorem 3** are shown in **Table 11**. From **Tables 4-5** and **Tables 8-9**, it can be seen that the larger τ_2 are obtained by employing **Theorem 3**. From **Tables 6-7** and **10-11**, it can be seen that the smaller γ are obtained by employing **Theorem 3**.

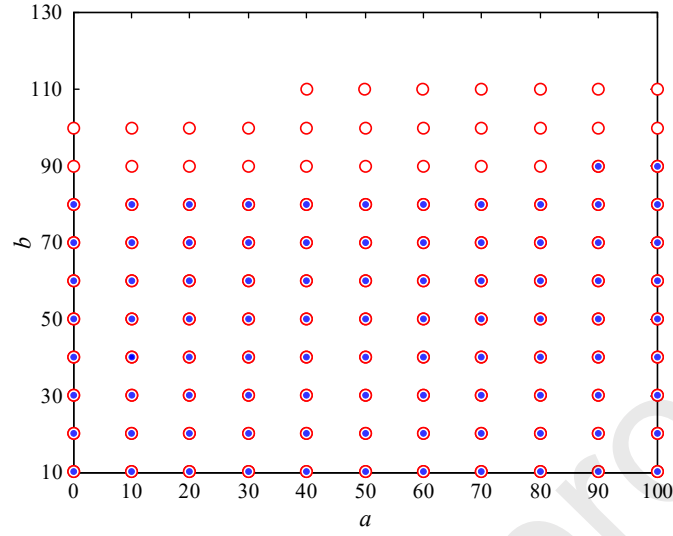


Figure 14. Stability regions by employing **Theorem 2** (‘•’) and **Theorem 3** (‘○’).

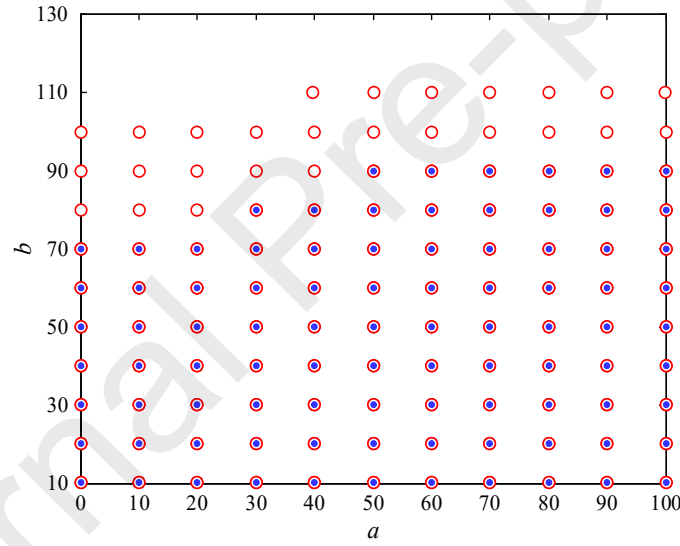


Figure 15. Stability regions by employing [44] (‘•’) and **Theorem 3** (‘○’) for $0 \leq a \leq 100$ and $10 \leq b \leq 130$.

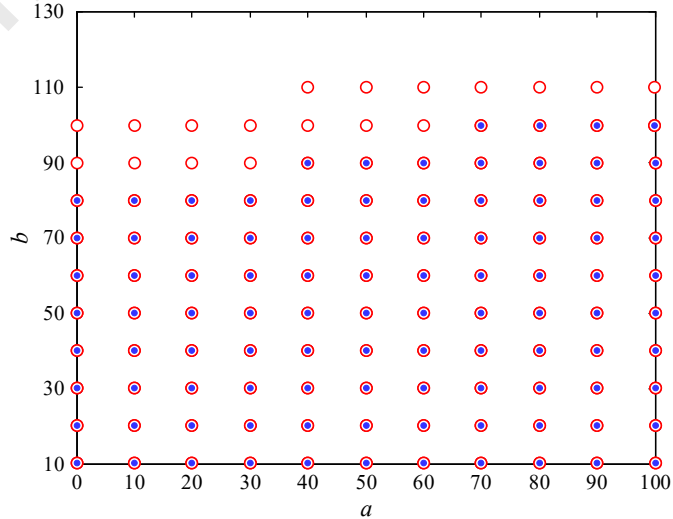
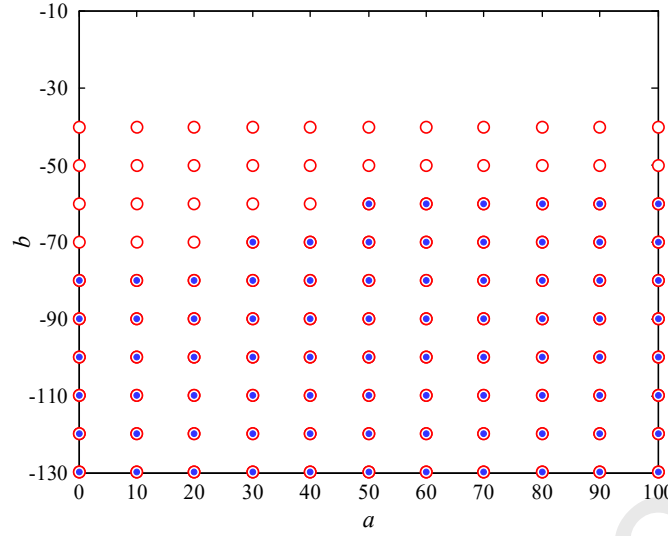
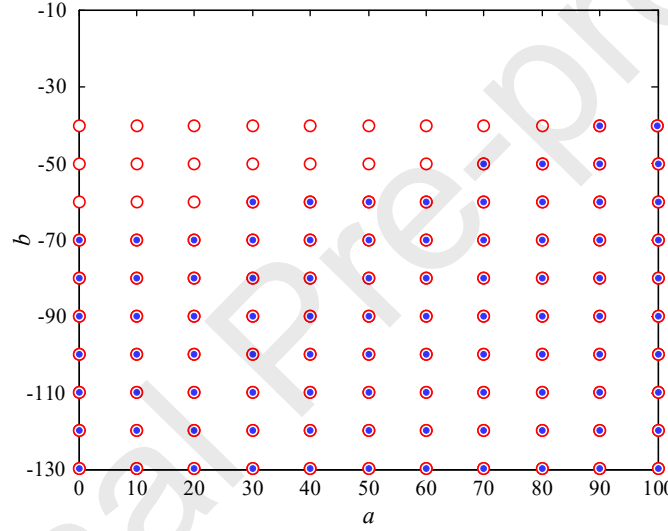


Figure 16. Stability regions by employing [45] (‘•’) and **Theorem 3** (‘○’) for $0 \leq a \leq 100$ and $10 \leq b \leq 130$.**Figure 17.** Stability regions by employing [44] (‘•’) and **Theorem 3** (‘○’) for $0 \leq a \leq 100$ and $-130 \leq b \leq -10$.**Figure 18.** Stability regions by employing [45] (‘•’) and **Theorem 3** (‘○’) for $0 \leq a \leq 100$ and $-130 \leq b \leq -10$.**Table 4.** Upper bounds τ_2 with $\tau_D = 0.15$ and different γ by employing **Theorems 2-3**.

Methods	γ				
	1.6	1.7	1.8	1.9	2.0
Theorem 2	1.9130	2.1852	2.3963	2.5645	2.7626
Theorem 3	2.1176	2.3741	2.5987	2.7341	2.9363

Table 5. Upper bounds τ_2 with $\tau_D = 0.30$ and different γ by employing **Theorems 2-3**.

Methods	γ				
	1.6	1.7	1.8	1.9	2.0
Theorem 2	1.8153	1.9191	2.0376	2.1176	2.2666
Theorem 3	2.0284	2.1060	2.2761	2.3228	2.4005

Table 6. Lower bounds γ with $\tau_D = 0.15$ and different τ_2 by employing **Theorems 2-3**.

Methods	τ_2				
	1.1	1.2	1.3	1.4	1.5
Theorem 2	1.2252	1.2453	1.2997	1.3689	1.4107

Theorem 3	1.2040	1.2217	1.2781	1.3415	1.3932
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Table 7. Lower bounds γ with $\tau_D = 0.30$ and different τ_2 by employing **Theorems 2-3**.

Methods	τ_2				
	1.1	1.2	1.3	1.4	1.5
Theorem 2	1.3474	1.3975	1.4418	1.4901	1.5329
Theorem 3	1.3262	1.3739	1.4203	1.4737	1.5154

Table 8. Upper bounds τ_2 with $\tau_D = 0.15$ and different γ by employing [44, 45] and **Theorem 3**.

Methods	γ				
	1.6	1.7	1.8	1.9	2.0
[44]	1.7181	1.9213	2.1721	2.3972	2.5121
[45]	1.9790	2.1977	2.3648	2.5974	2.7199
Theorem 3	2.1176	2.3741	2.5987	2.7341	2.9363

Table 9. Upper bounds τ_2 with $\tau_D = 0.30$ and different γ by employing [44, 45] and **Theorem 3**.

Methods	γ				
	1.6	1.7	1.8	1.9	2.0
[44]	1.6592	1.7324	1.8483	1.9996	2.0232
[45]	1.8883	1.9988	2.0758	2.1985	2.2299
Theorem 3	2.0284	2.1060	2.2761	2.3228	2.4005

Table 10. Lower bounds γ with $\tau_D = 0.15$ and different τ_2 by employing [44, 45] and **Theorem 3**.

Methods	τ_2				
	1.1	1.2	1.3	1.4	1.5
[44]	1.6738	1.7079	1.7338	1.7605	1.7979
[45]	1.4374	1.4775	1.5018	1.5306	1.5829
Theorem 3	1.2040	1.2217	1.2781	1.3415	1.3932

Table 11. Lower bounds γ with $\tau_D = 0.30$ and different τ_2 by employing [44, 45] and **Theorem 3**.

Methods	τ_2				
	1.1	1.2	1.3	1.4	1.5
[44]	1.7516	1.7857	1.8816	2.0806	2.1957
[45]	1.5241	1.5442	1.6115	1.7679	1.8296
Theorem 3	1.3262	1.3739	1.4203	1.4737	1.5154

Remark 12. “additive uncertainties” denotes the uncertainties are in the form of addition, such as $A_i + G_i F_{li}(t) E_{Ai}$ and $A_{ri} + G_i F_{li}(t) E_{Ari}$ in this paper. “multiplicative uncertainties” denotes the uncertainties are in the form of multiplication, such as $A_i \times G_i F_{li}(t) E_{Ai}$ and $A_{ri} \times G_i F_{li}(t) E_{Ari}$. The additive uncertainties may induce the loss of important data [46], thus the additive uncertainties are investigated in this paper.

6. Conclusion

In this paper, the interval type-2 T-S fuzzy dynamic output feedback control is presented for class of networked control systems. The interval type-2 T-S fuzzy model is employed and the networked control system is approximated. The interval type-2 T-S fuzzy dynamic output feedback controller is designed and the design conditions are relaxed. The additive uncertainties are introduced in

the controller and the design flexibility is enhanced. The closed-loop system is asymptotically stable with prescribed H-infinity performance index γ . The obtained LMIs are solved and controller gain matrices are determined. However, the networked control system has higher computational complexity if the time-varying multiplicative uncertainties are considered. It is significant to design the effective controller for the networked control system with time-varying multiplicative uncertainties. Based on the above analysis, the multiplicative uncertainties $A_i \times G_i F_{li}(t) E_{Ai}$ and $A_{ti} \times G_i F_{li}(t) E_{A_{ti}}$ will be investigated in future. We will consider that how to design the stochastic scheme and the corresponding control algorithm for the system.

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Competing interests

All the authors declare that there is no conflict of interest for the publication of this paper.

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