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Robust finite-time H_∞ control for a class of uncertain switched neutral systems

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ABSTRACT

This paper investigates the robust finite-time H_∞ control problem for a class of uncertain switched neutral systems with unknown time-varying disturbance. The uncertainties under consideration are norm bounded. By using the average dwell time approach, a sufficient condition for finite-time boundedness of switched neutral systems is derived. Then, finite-time H_∞ performance analysis for switched neutral systems is developed, and a robust finite-time H_∞ state feedback controller is proposed to guarantee that the closed-loop system is finite-time bounded with H_∞ disturbance attenuation level γ. All the results are given in terms of linear matrix inequalities (LMIs). Finally, two numerical examples are provided to show the effectiveness of the proposed method.

1. Introduction

Switched system is an important class of hybrid system, which consists of a family of subsystems and a switching rule specifying which subsystem will be activated along the system trajectory at a time instant. In the last decades, switched systems have received considerable attentions for their significant application in various fields, and a great number of excellent works have been developed (see [1,2], and the references cited therein).

It is well known that the time-delays as inherent features of many dynamic systems, exist widely in practical engineering systems and may cause instability or undesirable system performance. Over the past decades, a great deal of progress has been made on the research of time-delay system (see [3,4]). Neutral system is a special class of time-delay system, which depends not only on the delays of state but also on the delays of state derivative. The primary motivation for studying such neutral system comes partly from the fact that neutral system has numerous applications in the control of engineering and social systems, such as networks, heat exchanges, and processes including steam [5]. In addition, there exist a large number of time-delay systems which can be transformed into neutral systems.

Over the years, many research efforts have been devoted to the study of switched neutral systems (see [6–14]). To name a few, the problem of stability and control synthesis was developed for a class of switched neutral systems in [9,10], the problem of robust non-fragile H_∞ control and reliable H_∞ control for switched neutral systems were investigated in [12,13], respectively, and the issue of robust sliding mode control for uncertain switched neutral systems was discussed in [14]. However, most of the existing literature has focused on Lyapunov asymptotic stability for switched neutral systems, the behavior of which is over an infinite time interval. Lyapunov asymptotic stability depicts steady state performance of a dynamic system, and it could not reflect transient state performance. There exists a system that is asymptotically stable but could be unusable in practical engineering for its bad transient characteristics. For the systems which work in a short time

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interval, such as communication network system, missile system and robot control system, main concern is the behavior over a fixed finite time interval. To tackle the problem, Dorato proposed the concept of finite-time stability (or short-time stability) in 1961 (see [15]). In the last decade, many results have been obtained for this type of stability (see [16–23]). It is worth pointing out that finite-time stability and Lyapunov asymptotic stability are different concepts, and they are independent of each other. A system is finite-time stable if its state retains certain pre-specified bound in the fixed time interval in the case that the initial bound is given.

Recently, it can be found some papers related to finite-time stability for switched systems. For example, based on the average dwell time technique, the problem of finite-time boundedness for switched linear system with time-delay was developed in [24], and sufficient conditions that guarantee the finite-time weighted L_2-gain of the closed-loop system was derived. The problem of finite-time stability analysis and H_∞ stabilization for switched nonlinear discrete-time systems was investigated in [25]. However, to the best of our knowledge, the issue of robust finite-time H_∞ control for uncertain switched neutral systems has not been investigated, which motivated our study. Due to the fact that the existence of switching may have great influence on the finite-time stability of a switched system, *i.e.*. even when all the subsystems are finite-time stable, the switched system may be not finite-time stable, which increases difficulties for us to discuss this topic.

In this paper, we will focus on designing a robust finite-time H_∞ controller for a class of uncertain switched neutral systems such that the corresponding closed-loop system is finite-time stable with H_∞ disturbance attenuation level γ. The remainder of the paper is organized as follows. In Section 2, problem formulation and some necessary lemmas are given. In Section 3, based on the dwell time approach, finite-time boundedness and finite-time H_∞ performance for switched neutral system are addressed, and sufficient conditions for the existence of a robust finite-time H_∞ state feedback controller are proposed in terms of a set of matrix inequalities. Numerical examples are provided to show the effectiveness of the proposed approach in Section 4. Concluding remarks are given in Section 5.

Notation: The notation used in this paper is standard. R^n denotes n dimensional Euclidean, the superscript "T" denotes the transpose, and the notation $X \geqslant Y (X > Y)$ means that matrix $X - Y$ is positive semi-definite (positive definite). $\|x(t)\|$ represents the Euclidean norm. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and minimum eigenvalues of matrix P, respectively. I is an identity matrix with appropriate dimension. $diag\{a_i\}$ denotes the diagonal matrix with the diagonal elements a_i, $i = 1,2,\ldots$ The asterisk $*$ in a matrix is used to denote a term that is induced by symmetry.

2. Problem formulation and preliminaries

Consider the following switched neutral system

$$\dot{x}(t) - \widehat{C}_{\sigma(t)}\dot{x}(t-\tau) = \widehat{A}_{\sigma(t)}x(t) + \widehat{B}_{\sigma(t)}x(t-h) + D_{\sigma(t)}w(t) + E_{\sigma(t)}u(t) \quad (1)$$

$$z(t) = \widehat{F}_{\sigma(t)}x(t) + G_{\sigma(t)}w(t) + H_{\sigma(t)}u(t) \quad (2)$$

$$x(t_0+\theta) = \varphi(\theta), \theta \in [-H, 0] \quad (3)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^l$ is the control input, $w(t) \in R^p$ is the disturbance input which belongs to $L_2[0,\infty), z(t) \in R^q$ is the controlled output, $H = \max\{\tau, h\}$, $\varphi(\theta)$ is a continuous vector-valued initial function. The function $\sigma(t):[0,\infty) \to \underline{N} = \{1,2,\ldots,N\}$ is a switching signal which is deterministic, piecewise constant and right continuous, corresponding to it, the switching sequence $\Sigma = \{(t_0,\sigma(t_0)),(t_1,\sigma(t_1)),\ldots,(t_k,\sigma(t_k)),\ldots\}\,(k = 1,2,\ldots)$, where t_0 is the initial time and t_k denotes the kth switching instant, N denotes the number of subsystems. Moreover, $\sigma(t) = i$ means that the ith subsystem is activated.

For each $i \in \underline{N}, \widehat{A}_i, \widehat{B}_i, \widehat{C}_i, \widehat{F}_i$ are uncertain real-valued matrices with appropriate dimensions. We assume that the uncertainties are norm-bounded and of the form

$$[\widehat{A}_i\widehat{B}_i\widehat{C}_i\widehat{F}_i] = [A_iB_iC_iF_i] + L_i\Xi_i(t)[M_{1i}M_{2i}M_{3i}M_{4i}] \quad (4)$$

where A_i, B_i, C_i, F_i, L_i, M_{1i}, M_{2i}, M_{3i}, M_{4i} are known real-valued constant matrices with appropriate dimensions, $\Xi_i(t)$ are unknown and possibly time-varying matrix with Lebesgue measurable elements satisfying

$$\Xi_i^T(t)\Xi_i(t) \leqslant I$$

Assumption 1. For a given time constant T_f, the external disturbance $w(t)$ satisfies

$$\int_0^{T_f} w^T(t)w(t)dt \leqslant d^2 \quad (6)$$

Definition 1 [26]. For any $T_2 > T_1 \geqslant 0$, let $N_\sigma(T_1, T_2)$ denote the switching number of $\sigma(t)$ on an interval (T_1, T_2). If

$$N_\sigma(T_1, T_2) \leqslant N_0 + \frac{T_2 - T_1}{\tau_a} \quad (7)$$

holds for given $N_0 \geqslant 0$, $\tau_a > 0$, then the constant τ_a is called the average dwell time and N_0 is the chatter bound.

Without loss of generality, we choose $N_0 = 0$ throughout this paper.

Definition 2 (*Finite-time stability*). For a given time constant T_f, switched neutral system (1)–(3) with $u(t) \equiv 0$ and $w(t) \equiv 0$ is said to be finite-time stable with respect to $(c_1^2, c_2^2, T_f, R, \sigma(t))$ if

$$\sup_{-H \leqslant t_0 \leqslant 0} \{x^T(t_0)Rx(t_0), \dot{x}^T(t_0)R\dot{x}(t_0)\} \leqslant c_1^2 \Rightarrow x^T(t)Rx(t) < c_2^2, \quad t \in (0, T_f] \tag{8}$$

where $c_2 > c_1 > 0$, R is a positive definite matrix and $\sigma(t)$ is a switching signal.

Remark 1. Consider system (1)–(3) with $u(t) \equiv 0$ and $w(t) \equiv 0$, the system is said to be uniformly finite-time stable with respect to (c_1^2, c_2^2, T_f, R) if condition (8) holds for any switching signal.

Definition 3 (*Finite-time boundedness*). For a given time constant T_f, switched neutral system (1)–(3) with $u(t) \equiv 0$ is said to be finite-time bounded with respect to $\left(c_1^2, c_2^2, T_f, d^2, R, \sigma(t)\right)$ if condition (8) holds. Where $c_2 > c_1 > 0$, R is a positive definite matrix, $\sigma(t)$ is a switching signal and $w(t)$ satisfy (6).

Definition 4 (*Finite-time H_∞ performance*). For a given time constant T_f, switched neutral system (1)–(3) with $u(t) \equiv 0$ is said to have finite-time H_∞ performance with respect to $\left(0, c_2^2, T_f, d^2, \gamma, R, \sigma(t)\right)$ if the system is finite-time bounded and the following inequality holds

$$\int_0^{T_f} z^T(s)z(s)ds \leqslant \gamma^2 \int_0^{T_f} w^T(s)w(s)ds \tag{9}$$

where $c_2 > 0$, $\gamma > 0$, R is a positive definite matrix, $\sigma(t)$ is a switching signal and $w(t)$ satisfy (6).

Definition 5 (*Robust finite-time H_∞ control*). For a given time constant T_f, switched neutral system (1)–(3) is said to be robust finite-time stabilizable with H_∞ disturbance attenuation level γ, if there exists a controller $u(t) = K_{\sigma(t)}x(t)$, where $t \in (0, T_f]$, such that

(i) The corresponding closed-loop system is finite-time bounded.
(ii) Under zero initial condition, inequality (9) holds for any $w(t)$ satisfying (6).

The following lemma plays an important role in our later development.

Lemma 1 [27]. *Let U, V, W and X be real matrices of appropriate dimensions with X satisfying $X = X^T$, then for all $V^TV \leqslant I$, $X + UVW + W^TV^TU^T < 0$, if and only if there exists a scalar $\delta > 0$ such that $X + \delta UU^T + \delta^{-1}W^TW < 0$.*

3. Main results

3.1. Finite-time boundedness analysis

In this subsection, we consider the problem of finite-time boundedness for the following unforced switched neutral system

$$\dot{x}(t) - C_{\sigma(t)}\dot{x}(t-\tau) = A_{\sigma(t)}x(t) + B_{\sigma(t)}x(t-h) + D_{\sigma(t)}w(t) \tag{10}$$
$$x(t_0 + \theta) = \varphi(\theta), \quad \theta \in [-H, 0] \tag{11}$$

Theorem 1. *Consider system (10) and (11), let $\widetilde{P}_i = R^{-\frac{1}{2}}P_iR^{-\frac{1}{2}}$, $\widetilde{Q}_i = R^{-\frac{1}{2}}Q_iR^{-\frac{1}{2}}$, $\widetilde{S}_i = R^{-\frac{1}{2}}S_iR^{-\frac{1}{2}}$. If there exist positive scalars α, λ_1, λ_2, λ_3, λ_4 and positive definite symmetric matrices P_i, Q_i, S_i, T_i with appropriate dimensions, such that*

$$\begin{pmatrix} \widetilde{P}_iA_i^T + A_i\widetilde{P}_i - \alpha\widetilde{P}_i & B_i\widetilde{Q}_i & C_i\widetilde{S}_i & D_i & \widetilde{P}_iA_i^T & \widetilde{P}_i \\ * & -e^{\alpha h}\widetilde{Q}_i & 0 & 0 & \widetilde{Q}_iB_i^T & 0 \\ * & * & -e^{\alpha\tau}\widetilde{S}_i & 0 & \widetilde{S}_iC_i^T & 0 \\ * & * & * & -T_i & D_i^T & 0 \\ * & * & * & * & -\widetilde{S}_i & 0 \\ * & * & * & * & * & -\widetilde{Q}_i \end{pmatrix} < 0 \tag{12}$$

$$\lambda_1R^{-1} < \widetilde{P}_i < R^{-1}, \quad \lambda_2R^{-1} < \widetilde{Q}_i, \quad \lambda_3R^{-1} < \widetilde{S}_i, \quad T_i < \lambda_4I \tag{13}$$

$$\begin{pmatrix} -c_2^2 e^{-\alpha T_f} + d^2\lambda_4 & c_1 & c_1 & c_1 \\ * & -\lambda_1 & 0 & 0 \\ * & * & -\frac{1}{h}e^{-\alpha h}\lambda_2 & 0 \\ * & * & * & -\frac{1}{\tau}e^{-\alpha\tau}\lambda_3 \end{pmatrix} < 0 \tag{14}$$

Then, under the following average dwell time scheme

$$\tau_a > \tau_a^* = \frac{T_f \ln\mu}{\ln\left(c_2^2 e^{-\alpha T_f}\right) - \ln\left[\left(\frac{1}{\lambda_1} + \frac{he^{\alpha h}}{\lambda_2} + \frac{\tau e^{\alpha\tau}}{\lambda_3}\right)c_1^2 + d^2\lambda_4\right]} \tag{15}$$

the system is finite-time bounded with respect to $\left(c_1^2, c_2^2, T_f, d^2, R, \sigma(t)\right)$, *where* $\mu > 1$ *satisfying*

$$\widetilde{P}_j < \mu\widetilde{P}_i, \quad \widetilde{Q}_j < \mu\widetilde{Q}_i, \quad \widetilde{S}_j < \mu\widetilde{S}_i \tag{16}$$

Proof. Choose the following piecewise Lyapunov function candidate for system (10) and (11)

$$V(t) = V_{\sigma(t)}(t)$$

the form of each $V_i(x)(i \in \underline{N})$ is given by

$$V_i(t) = V_{1i}(t) + V_{2i}(t) + V_{3i}(t) \tag{17}$$

where $V_{1i}(t) = x^T(t)\widetilde{P}_i^{-1}x(t)$,

$$V_{2i}(t) = \int_{t-h}^{t} x^T(s)e^{\alpha(t-s)}\widetilde{Q}_i^{-1}x(s)ds$$
$$V_{3i}(t) = \int_{t-\tau}^{t} \dot{x}^T(s)e^{\alpha(t-s)}\widetilde{S}_i^{-1}\dot{x}(s)ds$$

Along the trajectory of (10) and (11), we have

$$\dot{V}_{1i}(t) = 2x^T(t)\widetilde{P}_i^{-1}\dot{x}(t)$$
$$\dot{V}_{2i}(t) = x^T(t)\widetilde{Q}_i^{-1}x(t) - x^T(t-h)e^{\alpha h}\widetilde{Q}_i^{-1}x(t-h) + \alpha\int_{t-h}^{t} x^T(s)e^{\alpha(t-s)}\widetilde{Q}_i^{-1}x(s)ds$$
$$\dot{V}_{3i}(t) = \dot{x}^T(t)\widetilde{S}_i^{-1}\dot{x}(t) - \dot{x}^T(t-\tau)e^{\alpha\tau}\widetilde{S}_i^{-1}\dot{x}(t-\tau) + \alpha\int_{t-\tau}^{t} \dot{x}^T(s)e^{\alpha(t-s)}\widetilde{S}_i^{-1}\dot{x}(s)ds$$

Thus

$$\dot{V}(x(t)) - \alpha V(x(t)) - w^T(t)T_i w(t) = X^T(t)\phi_i X(t)$$

where $X^T(t) = \begin{pmatrix} x^T(t) & x^T(t-h) & \dot{x}^T(t-\tau) & w^T(t) \end{pmatrix}$,

$$\phi_i = \begin{pmatrix} A_i^T\widetilde{P}_i^{-1} + \widetilde{P}_i^{-1}A_i + \widetilde{Q}_i^{-1} - \alpha\widetilde{P}_i^{-1} & \widetilde{P}_i^{-1}B_i & \widetilde{P}_i^{-1}C_i & \widetilde{P}_i^{-1}D_i \\ * & -e^{\alpha h}\widetilde{Q}_i^{-1} & 0 & 0 \\ * & * & -e^{\alpha\tau}\widetilde{S}_i^{-1} & 0 \\ * & * & * & -T_i \end{pmatrix} + \begin{pmatrix} A_i^T \\ B_i^T \\ C_i^T \\ D_i^T \end{pmatrix} \widetilde{S}_i^{-1}\begin{pmatrix} A_i & B_i & C_i & D_i \end{pmatrix} \tag{18}$$

using Schur complement lemma, we can get from (12) that

$$\begin{pmatrix} \widetilde{P}_iA_i^T + A_i\widetilde{P}_i + \widetilde{P}_i\widetilde{Q}_i^{-1}\widetilde{P}_i - \alpha\widetilde{P}_i & B_i\widetilde{Q}_i & C_i\widetilde{S}_i & D_i & \widetilde{P}_iA_i^T \\ * & -e^{\alpha h}\widetilde{Q}_i & 0 & 0 & \widetilde{Q}_iB_i^T \\ * & * & -e^{\alpha\tau}\widetilde{S}_i & 0 & \widetilde{S}_iC_i^T \\ * & * & * & -T_i & D_i^T \\ * & * & * & * & -\widetilde{S}_i \end{pmatrix} < 0 \tag{19}$$

Using $diag\left\{\widetilde{P}_i^{-1}, \widetilde{Q}_i^{-1}, \widetilde{S}_i^{-1}, I, I\right\}$ to pre- and post-multiply the left term of (19), we have

$$\begin{pmatrix} A_i^T\widetilde{P}_i^{-1}+\widetilde{P}_i^{-1}A_i+\widetilde{Q}_i^{-1}-\alpha\widetilde{P}_i^{-1} & \widetilde{P}_i^{-1}B_i & \widetilde{P}_i^{-1}C_i & \widetilde{P}_i^{-1}D_i & A_i^T \\ * & -e^{\alpha h}\widetilde{Q}_i^{-1} & 0 & 0 & B_i^T \\ * & * & -e^{\alpha\tau}\widetilde{S}_i^{-1} & 0 & C_i^T \\ * & * & * & -T_i & D_i^T \\ * & * & * & * & -\widetilde{S}_i \end{pmatrix} < 0 \tag{20}$$

by Schur complement lemma, we have that (20) is equivalent to $\phi_i < 0$.

Thus we have

$$\dot{V}(x(t)) - \alpha V(x(t)) - w^T(t)T_iw(t) < 0 \tag{21}$$

it can be obtained from (16) and (21) that, for $t \in [t_k, t_{k+1})$,

$$\begin{aligned} V(t) &< e^{\alpha(t-t_k)}V(t_k) + \int_{t_k}^{t} e^{\alpha(t-s)}w^T(s)T_iw(s)ds < e^{\alpha(t-t_k)}\mu V(t_k^-) + \int_{t_k}^{t} e^{\alpha(t-s)}w^T(s)T_iw(s)ds \\ &< e^{\alpha(t-t_k)}\mu\left[e^{\alpha(t_k-t_{k-1})}V(t_{k-1}) + \int_{t_{k-1}}^{t_k} e^{\alpha(t_k-s)}w^T(s)T_iw(s)ds\right] + \int_{t_k}^{t} e^{\alpha(t-s)}w^T(s)T_iw(s)ds \\ &= e^{\alpha(t-t_{k-1})}\mu V(t_{k-1}) + \mu\int_{t_{k-1}}^{t_k} e^{\alpha(t-s)}w^T(s)T_iw(s)ds + \int_{t_k}^{t} e^{\alpha(t-s)}w^T(s)T_iw(s)ds < \cdots \\ &< e^{\alpha(t-0)}\mu^{N_\sigma(0,t)}V(0) + \mu^{N_\sigma(0,t)}\int_{0}^{t_1} e^{\alpha(t-s)}w^T(s)T_iw(s)ds + \mu^{N_\sigma(t_1,t)}\int_{t_1}^{t_2} e^{\alpha(t-s)}w^T(s)T_iw(s)ds + \cdots \\ &\quad + \mu\int_{t_{k-1}}^{t_k} e^{\alpha(t-s)}w^T(s)T_iw(s)ds + \int_{t_k}^{t} e^{\alpha(t-s)}w^T(s)T_iw(s)ds \\ &= e^{\alpha(t-0)}\mu^{N_\sigma(0,t)}V(0) + \int_0^t e^{\alpha(t-s)}\mu^{N_\sigma(s,t)}w^T(s)T_iw(s)ds < e^{\alpha t}\mu^{N_\sigma(0,t)}V(0) + \mu^{N_\sigma(0,t)}e^{\alpha t}\int_0^t w^T(s)T_iw(s)ds \\ &< e^{\alpha T_f}\mu^{N_\sigma(0,T_f)}\left[V(0) + \int_0^{T_f} w^T(s)T_iw(s)ds\right] < e^{\alpha T_f}\mu^{N_\sigma(0,T_f)}[V(0) + \lambda_{\max}(T_i)d^2] \end{aligned} \tag{22}$$

From Definition 1, we know $N_\sigma(0, T_f) < \frac{T_f}{\tau_a}$. Noting that $T_i < \lambda_4 I$, we have

$$V(t) < e^{(\alpha+\frac{\ln\mu}{\tau_a})T_f}[V(0) + \lambda_4 d^2] \tag{23}$$

then

$$V(t) = V_i(t) \geqslant x^T(t)\widetilde{P}_i^{-1}x(t) = x^T(t)R^{\frac{1}{2}}P_i^{-1}R^{\frac{1}{2}}x(t) \geqslant \frac{1}{\lambda_{max}(P_i)}x^T(t)Rx(t)$$

Noting that $\lambda_1 R^{-1} < \widetilde{P}_i < R^{-1}$, we have $\lambda_{\max}(P_i) < 1$, then

$$V(t) > V_{1i}(t) > x^T(t)Rx(t) \tag{24}$$

On the other hand

$$\begin{aligned} V(0) &= x^T(0)\widetilde{P}_i^{-1}x(0) + \int_{-h}^{0} x^T(s)e^{-\alpha s}\widetilde{Q}_i^{-1}x(s)ds + \int_{-\tau}^{0}\dot{x}^T(s)e^{-\alpha s}\widetilde{S}_i^{-1}\dot{x}(s)ds \\ &\leqslant \lambda_{\max}\left(P_i^{-1}\right)x^T(0)Rx(0) + he^{\alpha h}\lambda_{\max}\left(Q_i^{-1}\right)\sup_{-h\leqslant\theta\leqslant 0}\{x^T(\theta)Rx(\theta)\} + \tau e^{\alpha\tau}\lambda_{\max}\left(S_i^{-1}\right)\sup_{-\tau\leqslant\theta\leqslant 0}\{\dot{x}^T(\theta)R\dot{x}(\theta)\} \\ &\leqslant \left(\frac{1}{\lambda_{\min}(P_i)} + \frac{he^{\alpha h}}{\lambda_{\min}(Q_i)} + \frac{\tau e^{\alpha\tau}}{\lambda_{\min}(S_i)}\right)\sup_{-H\leqslant\theta\leqslant 0}\{x^T(\theta)Rx(\theta), \dot{x}^T(\theta)R\dot{x}(\theta)\} \end{aligned}$$

From (13), we can obtain

$$V(0) < \left(\frac{1}{\lambda_1} + \frac{he^{\alpha h}}{\lambda_2} + \frac{\tau e^{\alpha\tau}}{\lambda_3}\right)\sup_{-H\leqslant\theta\leqslant 0}\{x^T(\theta)Rx(\theta), \dot{x}^T(\theta)R\dot{x}(\theta)\} \tag{25}$$

it holds that

$$x^T(t)Rx(t) < V(t) < e^{(\alpha+\frac{\ln\mu}{\tau_a})T_f}\left[\left(\frac{1}{\lambda_1} + \frac{he^{\alpha h}}{\lambda_2} + \frac{\tau e^{\alpha\tau}}{\lambda_3}\right)c_1^2 + \lambda_4 d^2\right] \tag{26}$$

using Schur complement lemma, we get from (14) that

$$\left(\frac{1}{\lambda_1}+\frac{he^{\alpha h}}{\lambda_2}+\frac{\tau e^{\alpha\tau}}{\lambda_3}\right)c_1^2+\lambda_4 d^2<c_2^2 e^{-\alpha T_f} \tag{27}$$

Substituting (15) into (26), we have

$$x^T(t)Rx(t)<c_2^2 \tag{28}$$

According to Definition 2, we know that system (10) and (11) is finite-time bounded with respect to $\left(c_1^2,c_2^2,T_f,d^2,R,\sigma(t)\right)$. The proof is completed. □

Remark 2. Notice that the LMIs (12)–(14) are related with parameters α, T_f, c_1, c_2, d. In actual applications, the values of T_f, c_1 and $d$are usually prescribed, we can take c_2^2 as optimized variable, *i.e.*, under the condition that the LMIs (12)–(14) have feasible solutions, the value of c_2^2 could be reduced to minimum value with a fixed α. The optimization problem can be formulated as follows:

$$\begin{aligned}&\min\ c_2^2\\&\text{s.t. LMIs }(12)-(14)\end{aligned}$$

Remark 3. The parameter μ in Theorem 1 can be chosen as μ = 1. In this case, we have $\tau_a^*=0$, which implies that switching signals can be arbitrary. For any switching signal, system (10) and (11) is finite-time bounded with respect to $\left(c_1^2,c_2^2,T_f,d^2,R\right)$, *i.e.* the system is uniformly finite-time stable with respect to $\left(c_1^2,c_2^2,T_f,d^2,R\right)$. Then the conditions (12)–(14) can be reduced to

$$\begin{pmatrix}\widetilde{P}A_i^T+A_i\widetilde{P}-\alpha\widetilde{P} & B_i\widetilde{Q} & C_i\widetilde{S} & D_i & \widetilde{P}A_i^T & \widetilde{P}\\ * & -e^{\alpha h}\widetilde{Q} & 0 & 0 & \widetilde{Q}B_i^T & 0\\ * & * & -e^{\alpha\tau}\widetilde{S} & 0 & \widetilde{S}C_i^T & 0\\ * & * & * & -T & D_i^T & 0\\ * & * & * & * & -\widetilde{S} & 0\\ * & * & * & * & * & -\widetilde{Q}\end{pmatrix}<0 \tag{29}$$

$$\widetilde{P}<R^{-1} \tag{30}$$

$$\begin{pmatrix}-c_2^2e^{-\alpha T_f}+d^2\lambda_{\max}(T) & c_1 & c_1 & c_1\\ * & -\lambda_{\min}(P) & 0 & 0\\ * & * & -\frac{1}{h}e^{-\alpha h}\lambda_{\min}(Q) & 0\\ * & * & * & -\frac{1}{\tau}e^{-\alpha\tau}\lambda_{\min}(S)\end{pmatrix}<0 \tag{31}$$

When $w(t)$ = 0 in switched neutral system (10) and (11), we can obtain the following result from Theorem 1.

Corollary 1. *Consider system (10) and (11) with $w(t)$ = 0, let $\widetilde{P}_i=R^{-\frac{1}{2}}P_iR^{-\frac{1}{2}}$, $\widetilde{Q}_i=R^{-\frac{1}{2}}Q_iR^{-\frac{1}{2}}$, $\widetilde{S}_i=R^{-\frac{1}{2}}S_iR^{-\frac{1}{2}}$. If there exist positive scalars α, λ_1, λ_2, λ_3 and positive definite symmetric matrices P_i, Q_i, S_i with appropriate dimensions, such that*

$$\begin{pmatrix}\widetilde{P}_iA_i^T+A_i\widetilde{P}_i-\alpha\widetilde{P}_i & B_i\widetilde{Q}_i & C_i\widetilde{S}_i & \widetilde{P}_iA_i^T & \widetilde{P}_i\\ * & -e^{\alpha h}\widetilde{Q}_i & 0 & \widetilde{Q}_iB_i^T & 0\\ * & * & -e^{\alpha\tau}\widetilde{S}_i & \widetilde{S}_iC_i^T & 0\\ * & * & * & -\widetilde{S}_i & 0\\ * & * & * & * & -\widetilde{Q}_i\end{pmatrix}<0 \tag{32}$$

$$\lambda_1R^{-1}<\widetilde{P}_i<R^{-1},\quad \lambda_2R^{-1}<\widetilde{Q}_i,\quad \lambda_3R^{-1}<\widetilde{S}_i \tag{33}$$

$$\begin{pmatrix}-c_2^2e^{-\alpha T_f} & c_1 & c_1 & c_1\\ * & -\lambda_1 & 0 & 0\\ * & * & -\frac{1}{h}e^{-\alpha h}\lambda_2 & 0\\ * & * & * & -\frac{1}{\tau}e^{-\alpha\tau}\lambda_3\end{pmatrix}<0 \tag{34}$$

Then, under the following average dwell time scheme

$$\tau_a > \tau_a^* = \frac{T_f \ln \mu}{\ln\left(c_2^2 e^{-\alpha T_f}\right) - \ln\left[\left(\frac{1}{\lambda_1} + \frac{he^{\alpha h}}{\lambda_2} + \frac{\tau e^{\alpha\tau}}{\lambda_3}\right)c_1^2\right]} \tag{35}$$

the system is finite-time stable with respect to $(c_1^2,\ c_2^2,\ T_f,\ R,\ \sigma(t))$, *where* $\mu > 1$ *satisfying* (16).

Proof. The proof is similar to that of Theorem 1, it is omitted here. □

3.2. H_∞ performance analysis

In this subsection, we consider the problem of H_∞ performance analysis for the following unforced switched neutral system

$$\dot{x}(t) - C_{\sigma(t)}\dot{x}(t-\tau) = A_{\sigma(t)}x(t) + B_{\sigma(t)}x(t-h) + D_{\sigma(t)}w(t) \tag{36}$$

$$x(t_0+\theta) = \varphi(\theta), \quad \theta \in [-H, 0] \tag{37}$$

$$z(t) = F_{\sigma(t)}x(t) + G_{\sigma(t)}w(t) \tag{38}$$

Theorem 2. *Consider system* (36)–(38), *let* $\widetilde{P}_i = R^{-\frac{1}{2}}P_iR^{-\frac{1}{2}}, \widetilde{Q}_i = R^{-\frac{1}{2}}Q_iR^{-\frac{1}{2}}, \widetilde{S}_i = R^{-\frac{1}{2}}S_iR^{-\frac{1}{2}}$. *If there exist positive scalars* α, γ, ε *and positive definite symmetric matrices* P_i, Q_i ,S_i *with appropriate dimensions, such that*

$$\begin{pmatrix} \widetilde{P}_iA_i^T + A_i\widetilde{P}_i - \alpha\widetilde{P}_i & B_i\widetilde{Q}_i & C_i\widetilde{S}_i & D_i + \widetilde{P}_iF_i^TG_i & \widetilde{P}_iA_i^T & \widetilde{P}_i & \widetilde{P}_iF_i^T \\ * & -e^{\alpha h}\widetilde{Q}_i & 0 & 0 & \widetilde{Q}_iB_i^T & 0 & 0 \\ * & * & -e^{\alpha\tau}\widetilde{S}_i & 0 & \widetilde{S}_iC_i^T & 0 & 0 \\ * & * & * & -\gamma^2 I + G_i^TG_i & D_i^T & 0 & 0 \\ * & * & * & * & -\widetilde{S}_i & 0 & 0 \\ * & * & * & * & * & -\widetilde{Q}_i & 0 \\ * & * & * & * & * & * & -I \end{pmatrix} < 0 \tag{39}$$

$$\widetilde{P}_i < R^{-1} \tag{40}$$

$$-c_2^2 + e^{\alpha T_f}\gamma^2 d^2 < 0 \tag{41}$$

Then, under the following average dwell time scheme

$$\tau_a > \tau_a^* = \max\left\{\frac{T_f \ln \mu}{\ln\left(c_2^2\right) - \ln(e^{\alpha T_f}\gamma^2 d^2)}, \frac{\ln \mu}{\varepsilon\alpha}\right\} \tag{42}$$

the system is finite-time bounded with H_∞ *performance with respect to* $\left(0, c_2^2, T_f, d^2, \bar{\gamma}, R, \sigma(t)\right)$, *where* $\mu > 1$ *satisfying* (16), H_∞ *performance index is* $\bar{\gamma}$ *which satisfies*

$$\bar{\gamma}^2 = e^{(1+\varepsilon)\alpha T_f}\gamma^2 \tag{43}$$

Proof. Choose the same Lyapunov–Krasovskii functional as in Theorem 1, after some mathematical manipulation, we can get

$$\dot{V}(x(t)) - \alpha V(x(t)) + z^T(t)z(t) - \gamma^2 w^T(t)w(t) = X^T(t)\Psi_i X(t) \tag{44}$$

where $X^T(t) = \begin{pmatrix} x^T(t) & x^T(t-h) & \dot{x}^T(t-\tau) & w^T(t) \end{pmatrix}$,

$$\Psi_i = \begin{pmatrix} A_i^T\widetilde{P}_i^{-1} + \widetilde{P}_i^{-1}A_i + \widetilde{Q}_i^{-1} - \alpha\widetilde{P}_i^{-1} + F_i^TF_i & \widetilde{P}_i^{-1}B_i & \widetilde{P}_i^{-1}C_i & \widetilde{P}_i^{-1}D_i + F_i^TG_i \\ * & -e^{\alpha h}\widetilde{Q}_i^{-1} & 0 & 0 \\ * & * & -e^{\alpha\tau}\widetilde{S}_i^{-1} & 0 \\ * & * & * & -\gamma^2 I + G_i^TG_i \end{pmatrix} + \begin{pmatrix} A_i^T \\ B_i^T \\ C_i^T \\ D_i^T \end{pmatrix}\widetilde{S}_i^{-1}\begin{pmatrix} A_i & B_i & C_i & D_i \end{pmatrix} \tag{45}$$

Using Schur complement lemma, it can be obtained from (39) that

$$\begin{pmatrix} \widetilde{P}_iA_i^T + A_i\widetilde{P}_i + \widetilde{P}_i\widetilde{Q}_i^{-1}\widetilde{P}_i - \alpha\widetilde{P}_i + \widetilde{P}_iF_i^TF_i\widetilde{P}_i & B_i\widetilde{Q}_i & C_i\widetilde{S}_i & D_i + \widetilde{P}_iF_i^TG_i & \widetilde{P}_iA_i^T \\ * & -e^{\alpha h}\widetilde{Q}_i & 0 & 0 & \widetilde{Q}_iB_i^T \\ * & * & -e^{\alpha\tau}\widetilde{S}_i & 0 & \widetilde{S}_iC_i^T \\ * & * & * & -\gamma^2 I + G_i^TG_i & D_i^T \\ * & * & * & * & -\widetilde{S}_i \end{pmatrix} < 0 \tag{46}$$

Using $diag\left\{\widetilde{P}_i^{-1}, \widetilde{Q}_i^{-1}, \widetilde{S}_i^{-1}, I, I\right\}$ to pre- and post-multiply the left term of (46), and by Schur complement lemma, we can get that (46) is equivalent to $\Psi_i < 0$, that is

$$\dot{V}(x(t)) - \alpha V(x(t)) + z^T(t)z(t) - \gamma^2 w^T(t)w(t) < 0 \tag{47}$$

Let $\gamma^2 w^T(s)w(s) - z^T(s)z(s) = \Delta(s)$, following the proof line of (23), we have

$$V(t) < e^{\alpha t}\mu^{N_\sigma(0,t)}V(0) + \int_0^t e^{\alpha(t-s)}\mu^{N_\sigma(s,t)}\Delta(s)ds \tag{48}$$

Under zero initial condition, we have $V(0) = 0$, thus

$$0 < \int_0^t e^{\alpha(t-s)}\mu^{N_\sigma}(s,t)\Delta(s)ds \tag{49}$$

which implies that

$$\int_0^t e^{\alpha(t-s)}\mu^{N_\sigma(s,t)}z^T(s)z(s)ds < \int_0^t e^{\alpha(t-s)}\mu^{N_\sigma(s,t)}\gamma^2 w^T(s)w(s)ds \tag{50}$$

Noting that

$$\int_0^t e^{\alpha(t-s)}\mu^{N_\sigma(s,t)}z^T(s)z(s)ds > \int_0^t z^T(s)z(s)ds \tag{51}$$

$$\int_0^t e^{\alpha(t-s)}\mu^{N_\sigma(s,t)}\gamma^2 w^T(s)w(s)ds < e^{\alpha t}\mu^{N_\sigma(0,t)}\int_0^t \gamma^2 w^T(s)w(s)ds \tag{52}$$

Let $t = T_f$, it can be obtained

$$\int_0^{T_f} z^T(s)z(s)ds < e^{\alpha T_f}\mu^{\frac{T_f}{\tau_a}}\int_0^{T_f} \gamma^2 w^T(s)w(s)ds \tag{53}$$

Since $\tau_a > \frac{\ln\mu}{\varepsilon\alpha}$, then

$$\int_0^{T_f} z^T(s)z(s)ds < e^{(1+\varepsilon)\alpha T_f}\gamma^2\int_0^{T_f} w^T(s)w(s)ds \tag{54}$$

From (43), the following inequality holds

$$\int_0^{T_f} z^T(s)z(s)ds < \bar{\gamma}^2\int_0^{T_f} w^T(s)w(s)ds \tag{55}$$

By Theorem 1, we know that system (36)–(38) is finite-time bounded with H_∞ performance $\bar{\gamma}$. The proof is completed. □

3.3. Robust finite-time H_∞ control

Consider system (1)–(3), under the controller $u(t) = K_{\sigma(t)}x(t)$, $t \in (0, T_f]$, the corresponding closed-loop system is given by

$$\dot{x}(t) - \widehat{C}_{\sigma(t)}\dot{x}(t-\tau) = (\widehat{A}_{\sigma(t)} + E_{\sigma(t)}K_{\sigma(t)})x(t) + \widehat{B}_{\sigma(t)}x(t-h) + D_{\sigma(t)}w(t) \tag{56}$$

$$z(t) = (\widehat{F}_{\sigma(t)} + H_{\sigma(t)}K_{\sigma(t)})x(t) + G_{\sigma(t)}w(t) \tag{57}$$

$$x(t_0+\theta) = \phi(\theta), \quad \theta \in [-H, 0] \tag{58}$$

Theorem 3. *Consider system (1)–(3), let* $\widetilde{P}_i = R^{-\frac{1}{2}}P_iR^{-\frac{1}{2}},\ \widetilde{Q}_i = R^{-\frac{1}{2}}Q_iR^{-\frac{1}{2}},\ \widetilde{S}_i = R^{-\frac{1}{2}}S_iR^{-\frac{1}{2}}$. *If there exist positive scalars* α, ε, δ_{1i}, δ_{2i} *and positive definite symmetric matrices* P_i, Q_i, S_i *with appropriate dimensions, such that the following inequalities hold*

$$\begin{pmatrix} \Theta_{i11} & B_i\widetilde{Q}_i & C_i\widetilde{S}_i & D_i+\widetilde{P}_iF_i^TG_i+Y_i^TH_i^TG_i & \widetilde{P}_iA_i^T+Y_i^TE_i^T+\delta_{1i}L_iL_i^T & \widetilde{P}_i & \widetilde{P}_iF_i^T+Y_i^TH_i^T & \widetilde{P}_iM_{1i}^T & \widetilde{P}_iM_{4i}^T \\ * & -e^{\alpha h}\widetilde{Q}_i & 0 & 0 & \widetilde{Q}_iB_i^T & 0 & 0 & \widetilde{Q}_iM_{2i}^T & 0 \\ * & * & -e^{\alpha\tau}\widetilde{S}_i & 0 & \widetilde{S}_iC_i^T & 0 & 0 & \widetilde{S}_iM_{3i}^T & 0 \\ * & * & * & -\gamma^2I+G_i^TG_i+\delta_{2i}G_i^TL_iL_i^TG_i & D_i^T & 0 & \delta_{2i}G_i^TL_iL_i^T & 0 & 0 \\ * & * & * & * & -\widetilde{S}_i+\delta_{1i}L_iL_i^T & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\widetilde{Q}_i & 0 & 0 & 0 \\ * & * & * & * & * & * & -I+\delta_{2i}L_iL_i^T & 0 & 0 \\ * & * & * & * & * & * & * & -\delta_{1i}I & 0 \\ * & * & * & * & * & * & * & * & -\delta_{2i}I \end{pmatrix} < 0 \quad (59)$$

$$\widetilde{P}_i < R^{-1} \quad (60)$$

$$-c_2^2 + e^{\alpha T_f}\gamma^2 d < 0 \quad (61)$$

Then, under the controller $u(t) = K_{\sigma(t)}x(t),\ K_i = Y_i\widetilde{P}_i^{-1},\ t \in (0, T_f]$, *and the following average dwell time scheme*

$$\tau_a > \tau_a^* = \max\left\{\frac{T_f \ln\mu}{\ln(c_2^2) - \ln(e^{\alpha T_f}\gamma^2 d^2)}, \frac{\ln\mu}{\varepsilon\alpha}\right\} \quad (62)$$

the corresponding closed-loop system is finite-time bounded with H_∞ *performance* $\bar{\gamma}$ *with respect to* $\left(0, c_2^2, T_f, d^2, \bar{\gamma}, R, \sigma(t)\right)$. where $\Theta_{i11} = \widetilde{P}_iA_i^T + Y_i^TE_i^T + A_i\widetilde{P}_i + E_iY_i - \alpha\widetilde{P}_i + \delta_{1i}L_iL_i^T$, $\mu > 1$ satisfying (16), $\bar{\gamma}$ is given by (43).

Proof. Replacing A_i, B_i, C_i, F_i in the left side of (39) with $\widehat{A}_i + E_iK_i, \widehat{B}_i, \widehat{C}_i, \widehat{F}_i + H_iK_i$, and using Schur complement lemma, we can obtain that

$$\prod_i = \begin{pmatrix} \widetilde{P}_i(\widehat{A}_i+E_iK_i)^T+(\widehat{A}_i+E_iK_i)\widetilde{P}_i-\alpha\widetilde{P}_i & \widehat{B}_i\widetilde{Q}_i & \widehat{C}_i\widetilde{S}_i & D_i+\widetilde{P}_i(\widehat{F}_i+H_iK_i)^TG_i & \widetilde{P}_i(\widehat{A}_i+E_iK_i)^T & \widetilde{P}_i & \widetilde{P}_i(\widehat{F}_i+H_iK_i)^T \\ * & -e^{\alpha h}\widetilde{Q}_i & 0 & 0 & \widetilde{Q}_i\widehat{B}_i^T & 0 & 0 \\ * & * & -e^{\alpha\tau}\widetilde{S}_i & 0 & \widetilde{S}_i\widehat{C}_i^T & 0 & 0 \\ * & * & * & -\gamma^2I+G_i^TG_i & D_i^T & 0 & 0 \\ * & * & * & * & -\widetilde{S}_i & 0 & 0 \\ * & * & * & * & * & -\widetilde{Q}_i & 0 \\ * & * & * & * & * & * & -I \end{pmatrix} \quad (63)$$

Let $K_i\widetilde{P}_i = Y_i$, it can be obtained that

$$\prod_i = \begin{pmatrix} \widetilde{P}_i\widehat{A}_i^T+Y_i^TE_i^T+\widehat{A}_i\widetilde{P}_i+E_iY_i-\alpha\widetilde{P}_i & \widehat{B}_i\widetilde{Q}_i & \widehat{C}_i\widetilde{S}_i & D_i+\widetilde{P}_i\widehat{F}_i^TG_i+Y_i^TH_i^TG_i & \widetilde{P}_i\widehat{A}_i^T+Y_i^TE_i^T & \widetilde{P}_i & \widetilde{P}_i\widehat{F}_i^T+Y_i^TH_i^T \\ * & -e^{\alpha h}\widetilde{Q}_i & 0 & 0 & \widetilde{Q}_i\widehat{B}_i^T & 0 & 0 \\ * & * & -e^{\alpha\tau}\widetilde{S}_i & 0 & \widetilde{S}_i\widehat{C}_i^T & 0 & 0 \\ * & * & * & -\gamma^2I+G_i^TG_i & D_i^T & 0 & 0 \\ * & * & * & * & -\widetilde{S}_i & 0 & 0 \\ * & * & * & * & * & -\widetilde{Q}_i & 0 \\ * & * & * & * & * & * & -I \end{pmatrix} \quad (64)$$

Substituting (4) into (64), we have

$$\Pi_i = \Pi_{1i} + \Pi_{2i} + \Pi_{3i} \quad (65)$$

where

$$\Pi_{1i}=\begin{pmatrix} \widetilde{P}_iA_i^T+Y_i^TE_i^T+A_i\widetilde{P}_i+E_iY_i-\alpha\widetilde{P}_i & B_i\widetilde{Q}_i & C_i\widetilde{S}_i & D_i+\widetilde{P}_iF_i^TG_i+Y_i^TH_i^TG_i & \widetilde{P}_iA_i^T+Y_i^TE_i^T & \widetilde{P}_i & \widetilde{P}_iF_i^T+Y_i^TH_i^T \\ * & -e^{\alpha h}\widetilde{Q}_i & 0 & 0 & \widetilde{Q}_iB_i^T & 0 & 0 \\ * & * & -e^{\alpha\tau}\widetilde{S}_i & 0 & \widetilde{S}_iC_i^T & 0 & 0 \\ * & * & * & -\gamma^2I+G_i^TG_i & D_i^T & 0 & 0 \\ * & * & * & * & -\widetilde{S}_i & 0 & 0 \\ * & * & * & * & * & -\widetilde{Q}_i & 0 \\ * & * & * & * & * & * & -I \end{pmatrix} \tag{66}$$

$$\Pi_{2i}=\begin{pmatrix} L_i \\ 0 \\ 0 \\ 0 \\ L_i \\ 0 \\ 0 \end{pmatrix}\Xi_i(t)\begin{pmatrix} M_{1i}\widetilde{P}_i & M_{2i}\widetilde{Q}_i & M_{3i}\widetilde{S}_i & 0 & 0 & 0 & 0 \end{pmatrix}+\begin{pmatrix} \widetilde{P}_iM_{1i}^T \\ \widetilde{Q}_iM_{2i}^T \\ \widetilde{S}_iM_{3i}^T \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}\Xi_i^T(t)\begin{pmatrix} L_i^T & 0 & 0 & 0 & L_i^T & 0 & 0 \end{pmatrix} \tag{67}$$

$$\Pi_{3i}=\begin{pmatrix} 0 \\ 0 \\ 0 \\ G_i^TL_i \\ 0 \\ 0 \\ L_i \end{pmatrix}\Xi_i(t)\begin{pmatrix} M_{4i}\widetilde{P}_i & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}+\begin{pmatrix} \widetilde{P}_iM_{4i}^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}\Xi_i^T(t)\begin{pmatrix} 0 & 0 & 0 & L_i^TG_i & 0 & 0 & L_i^T \end{pmatrix} \tag{68}$$

By Lemma 1, we can get

$$\Pi_{2i}\leqslant\delta_{1i}\begin{pmatrix} L_i \\ 0 \\ 0 \\ 0 \\ L_i \\ 0 \\ 0 \end{pmatrix}\begin{pmatrix} L_i^T & 0 & 0 & 0 & L_i^T & 0 & 0 \end{pmatrix}+\delta_{1i}^{-1}\begin{pmatrix} \widetilde{P}_iM_{1i}^T \\ \widetilde{Q}_iM_{2i}^T \\ \widetilde{S}_iM_{3i}^T \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}\begin{pmatrix} M_{1i}\widetilde{P}_i & M_{2i}\widetilde{Q}_i & M_{3i}\widetilde{S}_i & 0 & 0 & 0 & 0 \end{pmatrix} \tag{69}$$

$$\Pi_{3i}\leqslant\delta_{2i}\begin{pmatrix} 0 \\ 0 \\ 0 \\ G_i^TL_i \\ 0 \\ 0 \\ L_i \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 & L_i^TG_i & 0 & 0 & 0 \end{pmatrix}+\delta_{2i}^{-1}\begin{pmatrix} \widetilde{P}_iM_{4i}^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}\begin{pmatrix} M_{4i}\widetilde{P}_i & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{70}$$

Using Schur complement lemma, we can obtain from (59) that $\Pi_i < 0$. The proof is completed. □

Remark 4. Note that the LMIs (59)–(61) are related with parameters $\alpha,\ T_f,\ d,\ h,\ \tau,\ R,\ \bar{\gamma}$ and c_2. When parameters α, T_f, d^2, h, τ, R and c_2^2 are prescribed, we could take $\bar{\gamma}^2$ as an optimized variable for non-switched system. However, it can be seen from (62) and (43) that we should reduce the value of ε for obtaining a smaller value of $\bar{\gamma}$, which will enlarge the average dwell time τ_a. By solving optimization problem and adjusting the value of α, the minimum value of $\bar{\gamma}$ can be obtained. In actual applications, we should choose the parameters according to specific requirements, such as considering H_∞ disturbance attenuation level and the average dwell time.

4. Numerical example

In this section we present examples to illustrate the effectiveness of the proposed approach.

Example 1. Consider system (10) and (11) with parameters as follows

$$A_1=\begin{bmatrix}0.2 & 0\\ -1.5 & 0.1\end{bmatrix},\quad B_1=\begin{bmatrix}-0.5 & 0\\ 0 & -0.1\end{bmatrix},\quad C_1=\begin{bmatrix}-0.4 & 0\\ 0.3 & -0.01\end{bmatrix},\quad D_1=\begin{bmatrix}-1 & 0.5\\ -2 & -0.1\end{bmatrix}$$

$$A_2=\begin{bmatrix}0.3 & 0\\ -1 & 0.5\end{bmatrix},\quad B_2=\begin{bmatrix}0.1 & 0\\ -3 & -0.5\end{bmatrix},\quad C_2=\begin{bmatrix}-0.05 & 0\\ 0 & -0.1\end{bmatrix},\quad D_2=\begin{bmatrix}-1 & -1.5\\ 1 & -0.5\end{bmatrix}$$

Choosing $T_f = 5$, $h = 0.2$, $\tau = 0.3$, $c_1 = 0.01$, $d^2 = 0.001$, $R = 0.2I$, we know from Theorem 1 that the optimal value of c_2^2 depends on parameter α. By solving the matrix inequalities (12)–(14), we can get the optimal bound of c_2^2 with different value of α in each subsystems, as shown in Figs. 1 and 2.The smallest bound can be obtained as $c_{2\,\min}^2 = 612.371075$ when $\alpha = 2.19$.

Example 2. Consider system (1)–(3) with parameters as follows

$$A_1=\begin{bmatrix}2 & 0\\ 3 & 3\end{bmatrix},\quad B_1=\begin{bmatrix}-2 & 0\\ -1 & -2\end{bmatrix},\quad C_1=\begin{bmatrix}0.3 & 0\\ 0 & -0.2\end{bmatrix},\quad D_1=\begin{bmatrix}1 & -1\\ 2 & 1\end{bmatrix},\quad E_1=\begin{bmatrix}3 & -3\\ 0 & 4\end{bmatrix},\quad F_1=\begin{bmatrix}1 & 1\\ 0 & 2\end{bmatrix},$$
$$G_1=\begin{bmatrix}0.7 & 0\\ 1 & 0.5\end{bmatrix},\quad H_1=\begin{bmatrix}0.8 & 0\\ 0 & 0.2\end{bmatrix},\quad L_1=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\quad M_1=M_2=M_4=\begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix},$$

$$M_3=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},\quad \Xi_1(t)=\begin{bmatrix}0.8 & 0\\ 0 & 0.8\end{bmatrix},$$

$$A_2=\begin{bmatrix}2 & -1\\ -2 & 3\end{bmatrix},\quad B_2=\begin{bmatrix}-1 & 0\\ -2 & 1\end{bmatrix},\quad C_2=\begin{bmatrix}0.2 & 0\\ 0 & -0.3\end{bmatrix},\quad D_2=\begin{bmatrix}-1 & 0\\ 2 & 0.8\end{bmatrix},\quad E_2=\begin{bmatrix}4 & -1\\ 1 & 6\end{bmatrix},$$

$$F_2=\begin{bmatrix}2 & 1\\ 0 & 3\end{bmatrix},\quad G_2=\begin{bmatrix}1 & 0.5\\ 0.5 & 0.6\end{bmatrix},\quad H_2=\begin{bmatrix}0.8 & 0\\ 1 & -1\end{bmatrix},\quad L_2=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},$$

$$M_1=M_2=M_4=\begin{bmatrix}0.15 & 0\\ 0 & 0.15\end{bmatrix},\quad M_3=\begin{bmatrix}0.08 & 0\\ 0 & 0.08\end{bmatrix},\quad \Xi_2(t)=\begin{bmatrix}0.9 & 0\\ 0 & 0.9\end{bmatrix}.$$

Choosing $T_f = 12,\ h = 0.2,\ \tau = 0.3,\ d^2 = 0.01,\ c_2^2 = 4,\ R = I$, and taking γ^2 as optimized variable with a fixed α, by solving the optimal value problem for each subsystem according to Theorem 3, we get the following conclusion.

For subsystem 1, when $\alpha \in [0, 1.255]$, the LMI in Theorem 3 has feasible solution, and the value of $\gamma_{1\,\min}^2$ is between 1.73 and 1.78 with differentα. For subsystem 2, when $\alpha \in [0, 0.446]$, the LMI has feasible solution, and the value of $\gamma_{2\,\min}^2$ is between 1.81 and 1.88 with different α. The variety of α has little effect on the value of $\gamma_{\min}^2$.

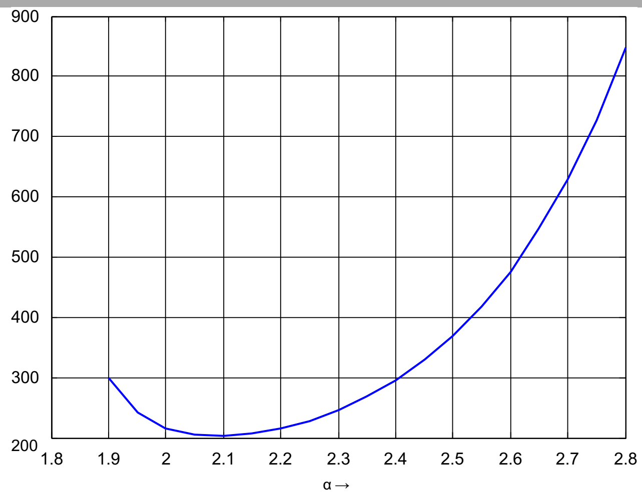

Fig. 1. The optimal bound of c_2^2 with different value of α in subsystem 1.

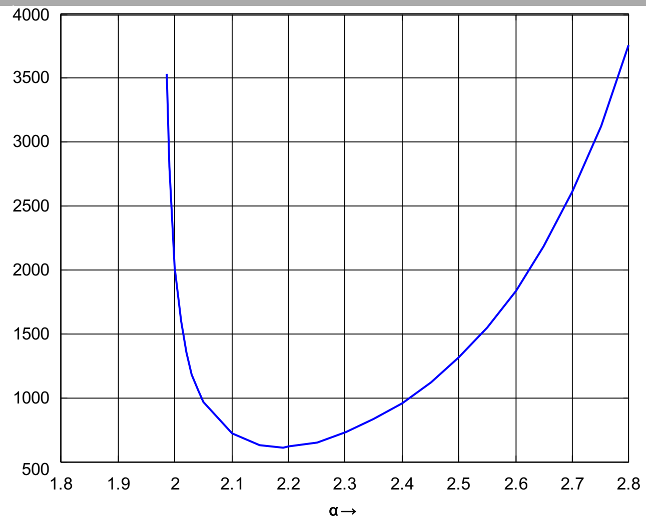

Fig. 2. The optimal bound of c_2^2 with different value of α in subsystem 2.

We choose $\alpha = 0.01$, then $\gamma_{1\,\min}^2 = 1.7733$ and $\gamma_{2\,\min}^2 = 1.8227$. So the value of γ^2 should be chosen as 1.8227. By solving the matrix equalities in Theorem 3, we get

$$P_1 = \begin{bmatrix} 1.1435 & -1.2047 \\ -1.2047 & 1.5085 \end{bmatrix},\quad Q_1 = \begin{bmatrix} 1.4200 & -1.2547 \\ -1.2547 & 1.4149 \end{bmatrix},\quad S_1 = \begin{bmatrix} 40.7618 & 14.2440 \\ 14.2440 & 67.9887 \end{bmatrix},\quad Y_1 = \begin{bmatrix} -3.8074 & -0.2800 \\ 1.7497 & -2.3440 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 2.4146 & 2.2435 \\ 2.2435 & 2.1232 \end{bmatrix},\quad Q_2 = \begin{bmatrix} 6.4753 & 3.1996 \\ 3.1996 & 10.1339 \end{bmatrix},\quad S_2 = \begin{bmatrix} 147.0733 & -118.1740 \\ -118.1740 & 374.7383 \end{bmatrix},\quad Y_2 = \begin{bmatrix} -5.0981 & -11.3077 \\ -1.8704 & -12.4175 \end{bmatrix}.$$

and the controller gain is

$$K_1 = \begin{bmatrix} -22.6735 & -18.3478 \\ -0.9148 & -2.3664 \end{bmatrix},\quad K_2 = \begin{bmatrix} 309.4060 & -340.9595 \\ 503.8082 & -550.3875 \end{bmatrix}.$$

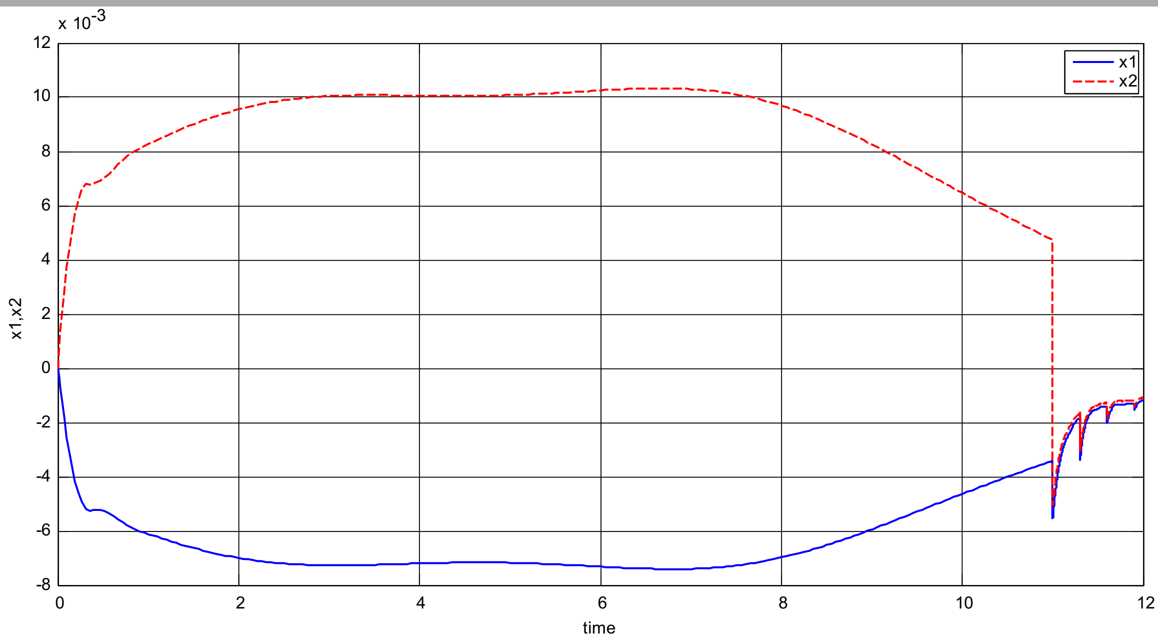

Fig. 3. State trajectories of the closed-loop system.

from (38), we have μ = 123.0086, thus $\tau_a > \tau_a^* = \frac{T_f \ln \mu}{\ln\left(c_2^2\right) - \ln(e^{\alpha T_f}\gamma^2 d^2)} = 10.9553$.

We choose τ_a = 11, in this case, ε can be chosen as 44, it can be obtained that $\bar{\gamma} = 20.0887$. The state trajectories of the closed-loop system are shown in Fig. 3.

5. Conclusion

In this paper, we have investigated the problem of robust finite-time H_∞ control for a class of uncertain switched neutral systems with unknown time-varying disturbance. The dwell time approach is utilized for finite-time boundedness analysis and robust finite-time H_∞ controller design, and the proposed state feedback controller can guarantee that the closed-loop system is finite-time bounded with H_∞ disturbance attenuation level γ. All the results are given in terms of linear matrix inequalities (LMIs), and two numerical examples are provided to show the effectiveness of the proposed method. Future work will focus on extending the proposed method to uncertain switched neutral systems with distributed delays.

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