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Robust stability of uncertain delay-differential systems of neutral type ☆

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Abstract

This paper focuses on the stability problem for neutral delay-differential systems with norm-bounded, and possibly time-varying, uncertainty. The time-delay is assumed constant and known. A stability criterion that is formulated in a linear matrix inequality (LMI) form is derived. Numerical examples show that the results obtained by this new criterion significantly improve the estimate of stability limit over some existing results in the literature. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Stability; Time-delay; Neutral-type; Uncertainty; Linear matrix inequality

1. Introduction

The stability of time-delay systems of neutral type has been widely investigated in the last two decades. The practical examples of neutral delay-differential systems include the distributed networks containing lossless transmission lines (Brayton, 1966; Kolmanovskii & Myshkis, 1992), and population ecology (Kuang, 1993). The existence of delay in a practical system may induce instability, oscillation and poor performance (Malek-Zavarei & Jamshidi, 1987). Current efforts on this topic can be divided into two categories (Mori, 1985), namely *delay-independent* stability criteria (Hu & Hu, 1996; Li, 1988; Slemrod & Infante, 1972) and *delay-dependent* stability criteria (Han, 2001; Khusainov & Yun'kova, 1988; Park & Won, 2000).

Many of delay-independent stability conditions were formulated in terms of matrix measure and matrix norm or existence of a positive-definite solution to an auxiliary algebraic Riccati matrix equation. Although these conditions are easy to check, they required the matrix measure to be negative or the parameters to be tuned. Moreover, abandonment of information on the delay necessarily causes conservativeness of the criteria especially when the delay is comparatively small. Delay-dependent stability results, which take the delay into account, are often less conservative than the delay-independent stability results. Recently, Park and Won (2000) has presented a, delay-dependent, sufficient condition that guaranteed the stability of neutral delay-differential systems. Han (2001) has obtained some stability criteria that showed significant improvements over the results in Khusainov and Yun'kova (1988) and Park and Won (2000).

In this paper, based on some model transformation techniques, we propose a delay-dependent criterion that is formulated in terms of a linear matrix inequality (LMI). Numerical examples show that the results obtained in this paper are less conservative than results in Khusainov and Yun'kova (1988), Park and Won (2000) and Han (2001).

Notation. For a symmetric matrix W, "$W > 0$" denotes that W is positive-definite matrix. Let I be an identity matrix of appropriate dimension. $\mathscr{C}([-h,0],\mathbb{R}^n)$ stands for the set of continuous $\mathbb{R}^n$ valued functions on $[-h,0]$ and let $x_t \in \mathscr{C}([-h,0],\mathbb{R}^n)$ be a segment of system trajectory defined as $x_t(\theta) = x(t+\theta)$, $-h \leqslant \theta \leqslant 0$ and

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denotes $\|\phi\|_c = \sup_{-h\leqslant\theta\leqslant 0}\|\phi(\theta)\|$ as the norm for $\phi\in\mathscr{C}([-h,0],\mathbb{R}^n)$. Use $\|\ \|$ to stand for either the Euclidean vector norm or the induced matrix 2-norm and denote $\sigma_{\max}(W)$ as the maximum singular value of the matrix W.

2. Problem statement

Consider the uncertain linear neutral delay-differential systems described by the following equation

$$\dot{x}(t) - C\dot{x}(t-h) = A(t)x(t) + B(t)x(t-h), \tag{1}$$

$$x(\theta) = \phi(\theta), \quad \forall\theta\in[-h,\ 0], \tag{2}$$

where $x(t)\in\mathbb{R}^n$ is the state, $h>0$ is the constant time-delay, $\phi(\cdot)$ is a continuous vector valued initial function, $C\in\mathbb{R}^{n\times n}$ is a known real constant matrix; $A(t)\in\mathbb{R}^{n\times n}$, $B(t)\in\mathbb{R}^{n\times n}$ are uncertain matrices, not known completely, except that they are within a compact set Ω which we will refer to as the uncertainty set

$$(A(t),B(t))\in\Omega\subset\mathbb{R}^{n\times 2n} \qquad \text{for all } t\in[0,\infty)$$

In this paper, we will attempt to formulate a practically computable criterion to check the stability of the above system.

3. Main results

Let us decompose the matrix $B(t)$ as $B(t)=B_1+B_2(t)$, where B_1 is a constant matrix. Rewrite system (1) in the following form:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[x(t) - Cx(t-h) + B_1\int_{t-h}^{t} x(\xi)\,\mathrm{d}\xi\right] = [A(t)+B_1]x(t) + B_2(t)x(t-h). \tag{3}$$

Define the operator $\mathscr{D}:\ \mathscr{C}([-h,0],\mathbb{R}^n)\to\mathbb{R}^n$ as

$$\mathscr{D}x_t = x(t) - Cx(t-h) + B_1\int_{t-h}^{t} x(\xi)\,\mathrm{d}\xi.$$

We have the following lemma.

Lemma 1. *Given a scalar $\bar{h}>0$, system (1)–(2) is asymptotically stable for any constant time-delay h satisfying $0\leqslant h\leqslant\bar{h}$ if the operator $\mathscr{D}$ is stable and there exist $n\times n$ matrices $P>0$, $R>0$ and $W>0$ satisfying the following LMI:*

$$\Xi(t,\bar{h}) = \begin{bmatrix} \Xi_{11}(t) & \Xi_{12}(t) & \Xi_{13}(t) \\ \Xi_{12}^{\mathrm{T}}(t) & \Xi_{22}(t) & \Xi_{23}(t) \\ \Xi_{13}^{\mathrm{T}}(t) & \Xi_{23}^{\mathrm{T}}(t) & \Xi_{33}(t) \end{bmatrix} > 0, \tag{4}$$

where

$$\Xi_{11}(t) = -[A(t)+B_1]^{\mathrm{T}}P - P[A(t)+B_1] - \bar{h}R - W,$$

$$\Xi_{12}(t) = [A(t)+B_1]^{\mathrm{T}}PC - PB_2(t),$$

$$\Xi_{13}(t) = -\bar{h}[A(t)+B_1]^{T}PB_1,$$

$$\Xi_{22}(t) = W + B_2^{\mathrm{T}}(t)PC + C^{\mathrm{T}}PB_2(t),$$

$$\Xi_{23}(t) = -\bar{h}B_2^{\mathrm{T}}(t)PB_1,$$

$$\Xi_{33}(t) = \bar{h}R.$$

Proof. Let us consider the following Lyapunov–Krasovskii functional candidate.

$$V(t,x_t) = (\mathscr{D}x_t)^{\mathrm{T}}P(\mathscr{D}x_t) + \int_{t-h}^{t} x^{\mathrm{T}}(\xi)Wx(\xi)\,\mathrm{d}\xi + \int_{t-h}^{t}(h-t+\xi)x^{\mathrm{T}}(\xi)Rx(\xi)\,\mathrm{d}\xi,$$

where matrices $P>0$, $R>0$ and $W>0$ are solutions of (4).

It is easy to see that the functional $V(t,x_t)$ satisfies the condition

$$\alpha_1\|(\mathscr{D}x_t)\|^2 \leqslant V(t,x_t) \leqslant \alpha_2\|x_t\|_c^2,$$

where $\alpha_1=\lambda_{\min}(P)$ and $\alpha_2=\lambda_{\max}(P)\,(1+\|C\|+h\,\|B_1\|) + \frac{1}{2}h^2\lambda_{\max}(R) + h\lambda_{\max}(W)$.

The derivative of $V(t,x_t)$ along the trajectory of system (3) is given by

$$\begin{aligned}
\dot{V}(t,x_t) = {} & -x^{\mathrm{T}}(t)\{-[A(t)+B_1]^{\mathrm{T}}P - P[A(t)+B_1]\}x(t) \\
& -2x^{\mathrm{T}}(t)\{[A(t)+B_1]^{\mathrm{T}}PC - PB_2(t)\}x(t-h) \\
& -2x^{\mathrm{T}}(t)\{-[A(t)+B_1]^{\mathrm{T}}PB_1\}\left(\int_{t-h}^{t} x(\xi)\,\mathrm{d}\xi\right) \\
& -x^{\mathrm{T}}(t-h)\left[B_2^{\mathrm{T}}(t)PC + C^{\mathrm{T}}PB_2(t)\right]x(t-h) \\
& -2x^{\mathrm{T}}(t-h)\left[-B_2^{\mathrm{T}}(t)PB_1\right]\left(\int_{t-h}^{t} x(\xi)\,\mathrm{d}\xi\right) \\
& +hx^{\mathrm{T}}(t)Rx(t) - \int_{t-h}^{t} x^{\mathrm{T}}(\xi)Rx(\xi)\,\mathrm{d}\xi \\
& +x^{\mathrm{T}}(t)Wx(t) - x^{\mathrm{T}}(t-h)Wx(t-h)
\end{aligned}$$

Then

$$\begin{aligned}
\dot{V}(t,x_t) = {} & \\
& -\frac{1}{h}\int_{t-h}^{t} x^{\mathrm{T}}(t)\{-[A(t)+B_1]^{\mathrm{T}}P \\
& -P[A(t)+B_1] - hR - W\}x(t)\,\mathrm{d}\xi \\
& -\frac{1}{h}\int_{t-h}^{t} 2x^{\mathrm{T}}(t)\{[A(t)+B_1]^{\mathrm{T}}PC - PB_2(t)\}x(t-h)\,\mathrm{d}\xi \\
& -\frac{1}{h}\int_{t-h}^{t} 2x^{\mathrm{T}}(t)\{-h[A(t)+B_1]^{\mathrm{T}}PB_1\}x(\xi)\,\mathrm{d}\xi \\
& -\frac{1}{h}\int_{t-h}^{t} x^{\mathrm{T}}(t-h)[W + B_2^{\mathrm{T}}(t)PC + C^{\mathrm{T}}PB_2(t)]x(t-h)\,\mathrm{d}\xi
\end{aligned}$$

$$-\frac{1}{h}\int_{t-h}^{t} 2x^{\mathrm{T}}(t-h)[-hB_2^{\mathrm{T}}(t)PB_1]x(\xi)\,\mathrm{d}\xi$$

$$-\frac{1}{h}\int_{t-h}^{t} x^{\mathrm{T}}(\xi)(hR)x(\xi)\,\mathrm{d}\xi$$

The above can be written as

$$\dot V(t,x_t) = -\frac{1}{h}\int_{t-h}^{t}\left[x^{\mathrm{T}}(t)\ x^{\mathrm{T}}(t-h)\ x^{\mathrm{T}}(\xi)\right]\cdot \Xi(t,h) \times \begin{bmatrix} x(t) \\ x(t-h) \\ x(\xi) \end{bmatrix} \mathrm{d}\xi.$$

Since $\Xi(t,h)$ is monotonic non-increasing with respect to h (in the sense of positive semi-definiteness), (4) implies that $\dot V(t,x_t) \leqslant -\varepsilon\|x(t)\|^2$ for sufficiently small $\varepsilon > 0$. Noting that the operator $\mathscr{D}$ is stable, therefore, system (1)– (2) is asymptotically stable according to Theorem 8.1 (pp. 292–293, Hale & Lunel, 1993). □

Remark 1. In light of the Definition 3.1 (p. 275) in Hale and Lunel (1993), the operator $\mathscr{D}$ is stable if the difference-integral system $x(t) - Cx(t-h) + B_1\int_{t-h}^{t} x(\xi)\,\mathrm{d}\xi = 0$ is asymptotically stable which is equivalent to the fact that there exists a $\delta > 0$ such that all solutions μ of the characteristic equation

$$\det\left[I - C\mathrm{e}^{-h\mu} + B_1\int_{-h}^{0}\mathrm{e}^{\mu\theta}\,\mathrm{d}\theta\right] = 0$$

satisfy $\mathrm{Re}(\mu) \leqslant -\delta < 0$. A sufficient condition is that the inequality $\|C\| + \bar h\,\|B_1\| < 1$ holds.

Remark 2. If there is no uncertainty in the system matrices, and $B_2(t) = 0$, Lemma 1 reduces to Theorem 1 in Han (2001).

Remark 3. If $B_1 = 0$, through some simple variable changes we can easily obtain that system (1)–(2) is asymptotically stable if that all eigenvalues μ of the matrix C are such that $|\mu(C)| < 1$ and there exist $n \times n$ matrices $\tilde P > 0$, $\tilde W > 0$ satisfying the following LMI:

$$\begin{bmatrix} -A^{\mathrm{T}}(t)\tilde P - \tilde P A(t) - \tilde W & A^{\mathrm{T}}(t)\tilde P C - \tilde P B(t) \\ C^{\mathrm{T}}\tilde P A(t) - B^{\mathrm{T}}(t)\tilde P & \tilde W \end{bmatrix} > 0,$$

which does not include any information on the time-delay. Therefore, the corresponding criterion is independent of delay. The efficiency of Lemma 1 depends on the decomposition of matrix B. The matrix B_1 was chosen such that the operator $\mathscr{D}$ is stable and $A(t) + B_1$ is "more stable" than matrix $A(t)$. This means that one can separate the stabilization part and destabilization part from the delayed term. As pointed out in Goubet-Batholomeus, Dambrine, and Richard, (1997), a partial optimization of decomposition is possible, but not necessary. "Natural" decomposition is easily found and can still obtain good results.

For polytopic uncertainty, it is clearly sufficient that (4) only needs to be satisfied at all the vertices. Now we consider the norm bounded uncertainty described by

$$A(t) = A + \Delta A(t),\ B(t) = B + \Delta B(t), \tag{5}$$

where

$$[\Delta A(t)\ \Delta B(t)] = LF(t)[E_a\ E_b], \tag{6}$$

where $F(t) \in \mathbb{R}^{p\times q}$ is an unknown real and possibly time-varying matrix with Lebesgue measurable elements satisfying

$$\sigma_{\max}(F(t)) \leqslant 1 \tag{7}$$

and L, E_a and E_b are known real constant matrices which characterize how the uncertainty enters the nominal matrices A and B.

Let $B = B_1 + B_2$, then $B_2(t) = B_2 + \Delta B(t)$. Now we state the following result.

Theorem 1. *Given a scalar $\bar h > 0$, the system described by* (1)–(2), *with uncertainty described by* (5)–(7) *is asymptotically stable for any constant time-delay h satisfying $0 \leqslant h \leqslant \bar h$ if the operator $\mathscr{D}$ is stable and there exist $n\times n$ matrices $X > 0$, $Y > 0$, and $Z > 0$ such that*

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} \\ \Gamma_{12}^{\mathrm{T}} & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} \\ \Gamma_{13}^{\mathrm{T}} & \Gamma_{23}^{\mathrm{T}} & \Gamma_{33} & \Gamma_{34} \\ \Gamma_{14}^{\mathrm{T}} & \Gamma_{24}^{\mathrm{T}} & \Gamma_{34}^{\mathrm{T}} & \Gamma_{44} \end{bmatrix} > 0, \tag{8}$$

where

$$\Gamma_{11} = -(A+B_1)^{\mathrm{T}}X - X(A+B_1) - \bar hY - Z - E_a^{\mathrm{T}}E_a,$$

$$\Gamma_{12} = (A+B_1)^{\mathrm{T}}XC - XB_2 - E_a^{\mathrm{T}}E_b,$$

$$\Gamma_{13} = -\bar h(A+B_1)^{\mathrm{T}}XB_1,$$

$$\Gamma_{14} = -XL,$$

$$\Gamma_{22} = Z + B_2^{\mathrm{T}}XC + C^{\mathrm{T}}XB_2 - E_b^{\mathrm{T}}E_{\mathrm{b}},$$

$$\Gamma_{23} = -\bar hB_2^{\mathrm{T}}XB_1,$$

$$\Gamma_{24} = C^{\mathrm{T}}XL,$$

$$\Gamma_{33} = \bar hY,$$

$$\Gamma_{34} = -\bar hB_1^{\mathrm{T}}XL,$$

$$\Gamma_{44} = I.$$

Proof. Concerning the uncertainties $\Delta A(t) = LF(t)E_a$ and $\Delta B(t) = LF(t)E_b$, $\Xi(t,\bar h) > 0$ in (4) can be written as

$$\Xi_0(\bar{h}) + \begin{bmatrix} -PL \\ C^{\mathrm{T}}PL \\ -\bar{h}B_1^{\mathrm{T}}PL \end{bmatrix} F(t)[E_a\ E_b\ 0]$$

$$+ \begin{bmatrix} E_a^{\mathrm{T}} \\ E_b^{\mathrm{T}} \\ 0 \end{bmatrix} F^{\mathrm{T}}(t)[-L^{\mathrm{T}}P\ L^{\mathrm{T}}PC\ \ -\bar{h}L^{\mathrm{T}}PB_1] > 0,$$

where

$$\Xi_0(\bar{h}) = \begin{bmatrix} \Xi_{11}^0 & \Xi_{12}^0 & \Xi_{13}^0 \\ \Xi_{12}^{0T} & \Xi_{22}^0 & \Xi_{23}^0 \\ \Xi_{13}^{0T} & \Xi_{23}^{0T} & \Xi_{33}^0 \end{bmatrix}$$

with $\Xi_{ij}^0 = \Xi_{ij}(t)\big|_{F(t)=0}$.

By Lemma 2.4 in Xie (1996), a sufficient condition for the above is

$$\lambda\Xi_0(\bar{h}) - \lambda^2 \begin{bmatrix} -PL \\ C^{\mathrm{T}}PL \\ -\bar{h}B_1^{\mathrm{T}}PL \end{bmatrix} [-L^{\mathrm{T}}P \quad L^{\mathrm{T}}PC \quad -\bar{h}L^{\mathrm{T}}PB_1]$$

$$- \begin{bmatrix} E_a^{\mathrm{T}} \\ E_b^{\mathrm{T}} \\ 0 \end{bmatrix} [E_a \quad E_b \quad 0] > 0$$

for some $\lambda > 0$. Introducing the new variables $X = \lambda P$, $Y = \lambda R$, $Z = \lambda W$ and using Schur complement (Boyd, Ghaoui, Feron, and Balakrishnan 1994) yields (8). □

Remark 4. Theorem 1 provides a *delay-dependent* stability criterion for neutral systems with norm-bounded uncertainty in terms of the solvability of a LMI. It is also interesting to note that $\bar{h}$ appears linearly. Therefore, a generalized eigenvalue problem as defined in Boyd et al. (1994) can be formulated to solve the minimum acceptable $1/\bar{h}$, and therefore the maximum $\bar{h}_{\max}$ to maintain asymptotic stability as judged by the criterion.

Remark 5. The results in this paper can be easily extended to uncertain neutral systems with *time-varying* delays.

4. Examples

Example 1. Consider the neutral delay-differential system

$$\dot{x}(t) - C\dot{x}(t-h) = Ax(t) + Bx(t-h) \tag{11}$$

where

$$A = \begin{bmatrix} -3 & -2.5 \\ 1 & 0.5 \end{bmatrix},\ B = \begin{bmatrix} 1.5 & 2.5 \\ -0.5 & -1.5 \end{bmatrix},$$

$$C = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix},\ |c| < 1.$$

For $c = 0$, system (11) reduces to the system studied in Chen, Gu, and Nett (1994) and the analytical solution for stability, $\bar{h}_{\max}$, was obtained as 2.4184.

Let us decompose the matrix B as $B = B_1 + B_2$, where

$$B_1 = \begin{bmatrix} -0.40 & 0 \\ 0 & -0.40 \end{bmatrix},\ B_2 = \begin{bmatrix} 1.9 & 2.5 \\ -0.5 & -1.1 \end{bmatrix}.$$

Using the approaches in Khusainov and Yun'kova (1988), Park and Won (2000), no conclusion can be made since the corresponding conditions are not satisfied. The maximum time-delay for asymptotic stability $\bar{h}_{\max}$ as judged by the criteria in Han (2001) and in this paper is listed in Table 1. It's clear that as $|c|$ increases, $\bar{h}_{\max}$ decreases. Hence, for this example, the criterion in this paper gives a less conservative result that those in Khusainov and Yun'kova (1988), Park and Won (2000) and Han (2001).

Table 1

c	−0.30	−0.10	0.00	0.10	0.30
Han (2001)	0.700	0.900	1.000	0.900	0.700
This paper	1.664	1.842	1.863	1.842	1.664

Example 2. Consider the following uncertain neutral delay-differential system

$$\begin{aligned}\dot{x}(t) - C\dot{x}(t-h) &= [A + \Delta A(t)]x(t) \\ &\quad + [B + \Delta B(t)]x(t-h),\end{aligned} \tag{12}$$

where

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix},\ B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$

$$C = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix},\ 0 \leqslant c < 1$$

and $\Delta A(t)$ and $\Delta B(t)$ are unknown matrices satisfying $\|\Delta A(t)\| \leqslant \alpha$ and $\|\Delta B(t)\| \leqslant \alpha$, $\forall t$. The above system is of the form of (5) to (7) with $L = \alpha I$ and $E_a = E_b = I$. When $c = 0$, system (12) reduces to the system discussed in de Souza and Li (1999).

The matrix B is naturally decomposed as $B = B_1 + B_2$, where

$$B_1 = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix},\ B_2 = \begin{bmatrix} -0.9 & 0 \\ -1 & -0.9 \end{bmatrix}.$$

Table 2

c	0.00	0.10	0.30
Han (2001)	0.99	0.89	0.69
This paper	4.35	4.33	4.10
c	0.50	0.70	0.90
Han (2001)	0.49	0.29	0.09
This paper	3.62	2.73	0.99

Table 3

c	0.05	0.10	0.15	0.20
$\bar{h}_{\max}$	1.63	1.48	1.33	1.16
c	0.25	0.30	0.35	0.40
$\bar{h}_{\max}$	0.98	0.79	0.59	0.37

Table 4

α	0.00	0.05	0.10
$\bar{h}_{\max}$	4.33	3.61	2.90
α	0.15	0.20	0.25
$\bar{h}_{\max}$	2.19	1.480	0.77

For $c=0$, $\Delta A(t)=0$ and $\Delta B(t)=0$ (i.e. nominal system), the analytical solution for stability, $\bar{h}_{\max}$, was obtained as 6.17258. For $c=0$ and $\alpha=0.2$, by the criterion in de Souza and Li (1999), system (12) is robustly stable for any h satisfying $0 \leqslant h \leqslant 0.4437$. Applying the approach in this paper, the maximum value of $\bar{h}_{\max}$ for the system to have guaranteed robust stability is $\bar{h}_{\max} = 1.7747$ that gives $\bar{h}_{\max}$ almost four times that of the result in de Souza and Li (1999). Hence, for this example, the robust stability criterion here gives a less conservative result than that in de Souza and Li (1999).

For $\alpha=0$, Table 2 gives the $\bar{h}_{\max}$ by the criteria in Han (2001) and this paper. It is clear that the new criterion here significantly improve the estimate of stability limit over the results in Han (2001).

For $\alpha = 0.2$, the maximum value $\bar{h}_{max}$ is listed in Table 3 for various parameter c. As c increases, $\bar{h}_{\max}$ decreases.

For $c=0.10$, we now consider the effect of uncertainty bound α on the maximum time-delay for stability $\bar{h}_{\max}$. Table 4 illustrates the numerical results for different α. We can see that $\bar{h}_{\max}$ decreases as α increases.

5. Conclusion

The stability problem for uncertain neutral delay-differential systems has been investigated. A delay-dependent stability criterion has been obtained. Numerical examples have shown significant improvements over some existing results.

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References

Boyd, S, Ghaoui, L.El., Feron, E., & Balakrishnan, V. (1994). *Linear matrix inequalities in systems and control theory*. Philadelphia: SIAM.

Brayton, R. K. (1966). Bifurcation of periodic solutions in a nonlinear difference-differential equation of neutral type. *Quarterly of Applied Mathematics*, *24*, 215–224.

Chen, J., Gu, G., & Nett, C.N. (1994). A new method for computing delay margins for stability of linear delay systems. *In Proceedings of the 33rd IEEE Conference on Decision and Control.* Lake Buena Vista, FL, (pp. 433-437).

Goubet-Batholomeus, A., Dambrine, M., & Richard, J. P. (1997). Stability of perturbed systems with time-varying delays. *Systems & Control Letters*, *31*, 155–163.

Hale, J. K., & Lunel, S. M. V. (1993). *Introduction to functional differential equation*. New York: Springer.

Han, Q.L. (2001). On delay-dependent stability criteria for neutral delay-differential systems. In: *International Journal of Applied Mathematics and Computer Science*, accepted for publication.

Hu, G. D., & Hu, G. D. (1996). Some simple stability criteria of neutral delay-differential systems. *Applied Mathematics and Computation*, *80*, 257–271.

Khusainov, D. Y., & Yun'kova, E. V. (1988). Investigation of the stability of linear systems of neutral type by the Lyapunov functional method. *Diff. Uravn*, *24*, 613–621.

Kolmanovskii, V. B., & Myshkis, A. (1992). *Applied theory of functional differential equations*. Boston: Kluwer Academic Publishers.

Kuang, Y. (1993). *Delay differential equations with applications in population dynamics*. Math. in Sci. Eng. 191, San Diego: Academic Press.

Li, L. M. (1988). Stability of linear neutral delay-differential systems. *Bulletin Australian Mathematical Society*, *38*, 339–344.

Malek-Zavarei, M., & Jamshidi, M. (1987). *Time delay systems: analysis, optimization and applications*. Amsterdam: North-Holland.

Mori, T. (1985). Criteria for asymptotic stability of linear time-delay systems. *IEEE Transactions on Automatic Control*, *AC-30*, 158–161.

Park, J.-H., & Won, S. (2000). Stability analysis for neutral delay-differential systems. *Journal of Franklin Institute*, *337*, 1–9.

Slemrod, M., & Infante, E. F. (1972). Asymptotic stability criteria for linear systems of differential equations of neutral type and their discrete analogues. *Journal of Mathematical Analysis Applications*, *38*, 399–415.

de Souza, C. E., & Li, X. (1999). Delay-dependent robust H_∞ control of uncertain linear state-delayed systems. *Automatica*, *35*, 1313–1321.

Xie, L. (1996). Output feedback H_∞ control of systems with parameter uncertainty. *International Journal of Control*, *63*, 741–750.