

Voluntary defense strategy and quantized sample-data control for T-S fuzzy networked control systems with stochastic cyber-attacks and its application[☆]

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ABSTRACT

This work studies mainly the dissipative analysis issue of T-S fuzzy networked control systems (NCSs) with stochastic cyber-attacks (SCAs) and voluntary defense strategy (VDS), which has strong application background in the network information security field. Firstly, a novel time-delay-product relaxed condition (TDPRC) is introduced, which fully excavates time-varying delay (TVD) information in the established condition. Secondly, an improved boundary circulatory function (BLF) is developed, which takes advantage of the characteristics of the sampling moments. Here, when $t \in [t_k, t_{k+1})$, we only require $Q_{d(t)}$ to be an asymmetric matrix in $V_c(x(t))$. Then, a novel criterion and the corresponding control algorithm are developed by using reciprocally convex matrix inequality (RCMI), proper integral inequalities, and the linear convex combination method (LCCM). Furthermore, a new quantized sample-data and VDS controller under SCAs is established, which can prove that T-S fuzzy NCSs under a better dissipative index are asymptotically stable (AS). Lastly, a numerical simulation of the dynamic equation of truck-trailer system to certify the theory's feasibility and validity.

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1. Introduction

With the popularity of the network, the use of the network has become indispensable in people's life. The emergence of the network has brought a lot of convenience to our life [1,2] and also can strengthen the information transmission of

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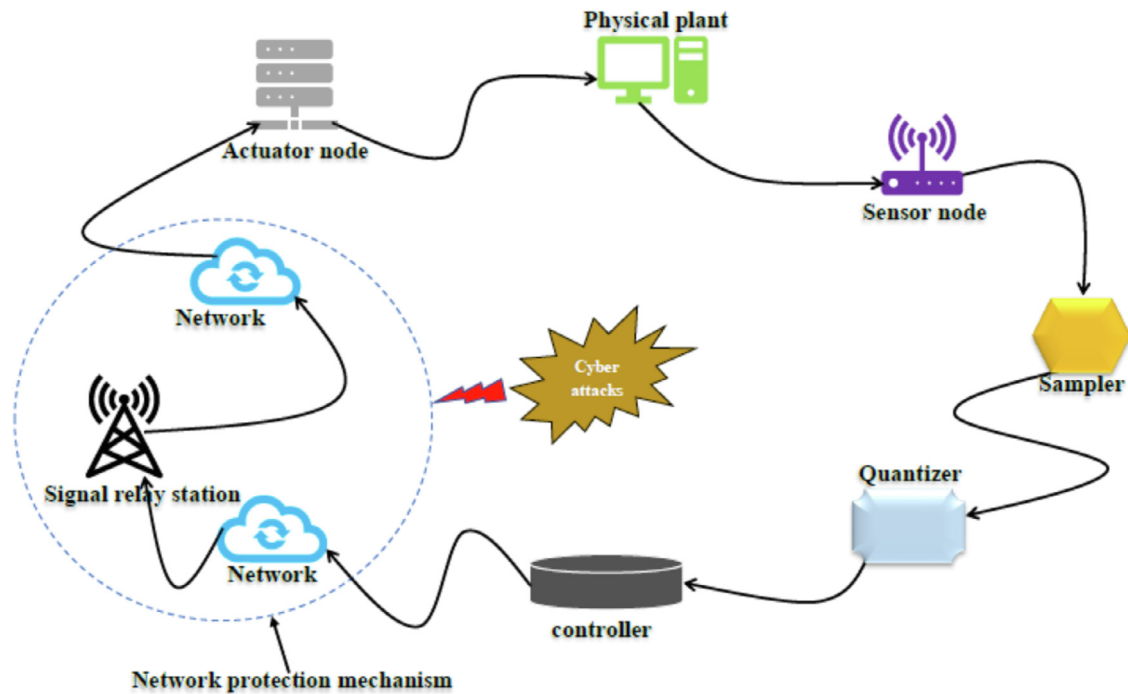


Fig. 1. Cyber attack and protection mechanism diagram.

each country [3,4]. In recent years, networked control systems (NCSs) have attracted more and more attention. NCSs can be expressed as a network system, network transmission, and cloud computer control, while computer control can be expressed as a multi latitude complex system [5–7].

In the NCSs, the signals sent by the controller and received by the receiver are transmitted to each other in the form of data packets through the network [8–10]. However, network transmission is not absolutely stable, and there will always be dynamic changes of data. Therefore, dynamic changes of data have a certain impact on the confidentiality, integrity, and effectiveness of data transmission in the data transmission process. Thus, dynamic changes of data in data transmission are generally accepted by NCSs.

Therefore, the phenomenon of abnormal dynamic changes of data in the network is a popular issue that has been studied in recent years [11–15]. Generally, this phenomenon is called cyber-attacks (CAs) (Fig. 1) in network engineering. At present, CAs are mainly divided into DoS attacks [16,17], spoofing attacks [18,19], stealth integrity attacks [20], SCAs [21,22] and other common attack methods. In [22], the authors studied the wormhole attack on the network control system, where the adversary uses a high-gain antenna in the out-of-band wormhole to establish an in-band wormhole link between two geographically distant areas of the network. In [23], researchers are concerned with the problem of distributed event-triggered controller design for NCSs with SCAs. In [24], this paper involved robust event-based stability of uncertain network control systems under quantification and DoS attacks. In [25], the authors concerned about the event-based security control of a discrete-time random system with multiplicative noise, which is subject to both random DoS attacks and random spoofing attacks. In [26], the H_∞ filtering for networked systems with hybrid-triggered communication mechanisms and SCAs was studied.

SCAs may be a more aggressive network of attacks by randomly adding or blocking packet transmission between measurement and control channels. Unlike traditional delay and packet changes, SCAs may lead to more serious situations, resulting in the degradation or even instability of control performance. SCAs have strong randomness, and the time point and attack duration can not be determined, which is entirely unpredictable. Therefore, based on the characteristics of SCAs, the research motivation of this paper is motivated. At present, many researchers have only studied the attributes of SCAs, but they have not developed appropriate and effective countermeasures. Making effective countermeasures to protect the system through reasonable control is also one of the main problems we are facing, which significantly stimulates our research motivation.

On the other hand, the sampling controllers are widely used in the research on NCSs [27–30]. The sampling controller uses intermittently observed signal values, such as set point signals, drive deviation signals, or signals representing the controlled variable to affect the control effect. The application of sampling control helps improve the control precision and anti-interference ability of the system and also helps to improve the utilization and versatility of the controller. With the popularization of microcomputers, sampling control shows its superiority. In addition, effectively solving the nonlinear prob-

Table 1
Notations and Descriptions.

Notation	$\mathbb{R}^{l \times k}$	\mathcal{I}	0
Description	$l \times k$ matrices	identity matrix	zero matrix
Notation	\mathcal{Q}^{-1}	\mathcal{Q}^T	$\ \cdot\ $
Description	inverse of matrix \mathcal{Q}	transpose of matrix \mathcal{Q}	Euclidean norm in \mathbb{R}^m
Notation	$\mathcal{P} > 0$	$\text{diag}\{\dots\}$	\mathbb{R}^m
Description	\mathcal{P} positive definite matrix	stands for a block diagonal matrix	m-dimensional Euclidean space

lem in the NCSs line is also a hot spot in current research, which dramatically stimulates our research momentum. Therefore, we will conduct an in-depth study of T-S fuzzy NCSs.

In this work, we develop a new dissipative criterion and the responding desired SDQ controller to ensure that the T-S fuzzy NCSs under SCAs are AS. Based on the above discussions, compared with the current works in [21,27,28,36,41,43,48], the major research contributions of this paper are as follows:

(1) This paper proposes a T-S fuzzy NCSs with SCA and VDS, which designs a defense mechanism against CAs carries out by hackers. This has strong application and development potential in the field of network information security.

(2) A novel TDPRC is developed, which fully taps the TVD information under this condition. In addition, an improved BLF is introduced to construct a new LKF, which better optimizes the performance of the algorithm.

(3) A new quantized sample data and VDS controller under SCAs is achieved to ensure self-defense under the premise of CAs by hackers, and to ensure that the T-S fuzzy NCS with a better dissipation index is AS.

The related mathematical symbols used in this article are defined in Table 1.

2. Preliminaries

In this article, we research the following TSFNCS which has TVDs [39]:

◆ Rule i : IF $\vartheta_1(t)$ is \mathcal{J}_{i1} , \dots , and $\vartheta_g(t)$ is \mathcal{J}_{ig} ,

$$\begin{cases} \dot{x}(t) = \mathcal{A}_{1i}x(t) + \mathcal{G}_{1i}x(t-d(t)) + \mathcal{B}_{1i}\tilde{u}(t) + \mathcal{D}_{1i}\omega(t), \\ z(t) = \mathcal{A}_{2i}x(t) + \mathcal{G}_{2i}x(t-d(t)) + \mathcal{B}_{2i}\tilde{u}(t) + \mathcal{D}_{2i}\omega(t), \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $\tilde{u}(t) \in \mathbb{R}^m$ is a control signal to be attacked; $\omega(t) \in \mathcal{L}_2[0, \infty)$ indicates the disturbance from external input; $z(t)$ is the estimated output vector; \mathcal{A}_{1i} , \mathcal{A}_{2i} , \mathcal{B}_{1i} , \mathcal{B}_{2i} , \mathcal{G}_{1i} , \mathcal{G}_{2i} , \mathcal{D}_{1i} , \mathcal{D}_{2i} are constant matrices.

The delay $d(t)$ is a TVD differentiable function satisfying [50]:

$$0 \leq d(t) \leq d, \quad |\dot{d}(t)| \leq \mu, \quad (2)$$

where d and μ are given constants.

Let $h_i(\vartheta(t))$ be the normalized membership function for inferring fuzzy set $\mu_i(\vartheta(t))$, i.e.

$$h_i(\vartheta(t)) = \frac{\mu_i(\vartheta(t))}{\sum_{i=1}^r \mu_i(\vartheta(t))}, \quad \vartheta(t) = [\vartheta_1(t), \dots, \vartheta_g(t)], \quad \mu_i(\vartheta(t)) = \prod_{j=1}^g \mathcal{J}_{ij}(\vartheta_j(t)).$$

$\mathcal{J}_{ij}(\vartheta(t))$ is degree of membership of $\vartheta_j(t)$ in \mathcal{J}_{ij} . It is supposed that

$$\mu_i(\vartheta(t)) \geq 0, i = 1, 2, \dots, r, \quad \sum_{i=1}^r \mu_i(\vartheta(t)) > 0, \forall t \geq 0. \quad (3)$$

Then

$$h_i(\vartheta(t)) \geq 0, i = 1, 2, \dots, r, \quad \sum_{i=1}^r h_i(\vartheta(t)) = 1, \forall t \geq 0. \quad (4)$$

Utilizing central average defuzzer, product reasoning and single-instance defuzzer, fuzzy model (1) is shown below

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^r h_i(\vartheta(t))[\mathcal{A}_{1i}x(t) + \mathcal{G}_{1i}x(t-d(t)) + \mathcal{B}_{1i}\tilde{u}(t) + \mathcal{D}_{1i}\omega(t)], \\ z(t) = \sum_{i=1}^r h_i(\vartheta(t))[\mathcal{A}_{2i}x(t) + \mathcal{G}_{2i}x(t-d(t)) + \mathcal{B}_{2i}\tilde{u}(t) + \mathcal{D}_{2i}\omega(t)]. \end{cases} \quad (5)$$

In this paper, the sampling control laws are used to stabilize (1), as shown below

R^j : IF $\vartheta_j(t)$ is \mathcal{J}_{j1} , $\vartheta_j(t)$ is $\mathcal{J}_{j1}, \dots, \vartheta_g(t)$ is \mathcal{J}_{jg} ,

THEN

$$\tilde{u}(t) = u(t) + \beta(t)\mathcal{K}f(u(t)) + (1 - \beta(t))\mathcal{K}g(u(t)). \quad (6)$$

The defuzzified controller rules are given by the following formula

$$\tilde{u}(t) = \sum_{j=1}^r h_j(\vartheta(t)) [u(t) + \beta(t) \mathcal{K}_j f(u(t)) + (1 - \beta(t)) \mathcal{K}_j g(u(t))], \quad (7)$$

where it can be supposed that the control signal is produced by a ZOH function with a sampling instant t_k satisfying the hold times $0 = t_0 < t_1 < t_2 < \dots < t_k < \dots$, t_k and $\lim_{t \rightarrow +\infty} t_k = \infty$ ($k = 0, 1, 2, \dots$). Suppose the sampling period is $0 < t_{k+1} - t_k \triangleq h_m$.

Remark 1. Malicious CAs have threatened information security in recent years, such as file confidentiality, communication integrity, and data transmission effectiveness. CAs have gradually become the main reason for hindering the effective communication of NCSs. Therefore, network security issues have recently become an important research direction in research and development. In addition, many methods have been proposed in engineering applications to detect and defend CAs in NCSs. However, in actual engineering applications, because controllers, sensors, actuators, and other intelligent system control components communicate through network data to achieve remote control, it is vulnerable to a series of CAs. At present, the types of CAs are mainly divided into: DoS attacks [16,17], spoofing attacks [18,19], stealth integrity attacks [20], SCAs [21,22], etc. However, there are few studies based on TSFNCS under SCAs, which motivates the research motivation of this paper.

Remark 2. Not like the existing research methods [16–22], how to effectively defend and counter SCAs has become one of the main problems that need to be overcome. Therefore, this paper designs a VDS function $g(u(t))$, effectively protecting system stability. When the system suffers from SCAs, the VDS function will automatically activate the protection function. In addition, the activation of the VDS function mainly depends on whether SCAs appear or not. This mechanism better protects the system effectively and consolidates the normal operation of the system.

Moreover, the input signal can be indicated as

$$u(t) = \mathcal{K}x(t_k), \quad t_k \leq t < t_{k+1}. \quad (8)$$

Then, we introduce the quantizer $q(\cdot)$ to quantize the input signal, the quantizer is designed as [32,33,40]

$$u = \mathcal{K}_j [q_1(x_1(t_k)), q_2(x_2(t_k)), \dots, q_n(x_n(t_k))]^T, \quad (9)$$

where $q(\cdot)$ is a quantizer.

$$q_m(x_m(t_k)) = -q_m(-x_m(t_k)). \quad (10)$$

In the design of the quantizer, the quantization level set $q_m(u_m)$ is defined as

$$\mathcal{J}_m = \left\{ \pm \ell_m^{(j)} \mid \ell_m^{(j)} = (\rho)^j \ell_m^{(0)}, \right\} \cup \{ \pm \ell_m^{(0)} \} \cup \{0\}, \quad 0 < \rho_m < 1, \ell_m^{(0)} > 0, j = \pm 1, \pm 2, \pm 3, \dots, \quad (11)$$

where $\ell_m^{(0)}$ is the sub-quantizer of m th, $q_m(u_m)$ original quantized value, and ρ_m with quantization density correlates. The definition of the logarithmic quantizer $q_m(u_m)$ is as follows

$$q_m(x_m(t_k)) = \begin{cases} \ell_m^{(j)}, & \text{if } \frac{1}{1+\epsilon_m} \ell_m^{(j)} < x_m(t_k) \leq \frac{1}{1-\epsilon_m} \ell_m^{(j)}, \\ & x_m(t_k) > 0, \quad j = \pm 1, \pm 2, \dots, \\ 0, & \text{if } x_m(t_k) = 0, \\ -q_m(-u_m), & \text{if } x_m(t_k) < 0, \quad m = 1, 2, 3, \dots, n, \end{cases} \quad (12)$$

where $\epsilon_m = \frac{1-\gamma_m}{1+\gamma_m}$.

Then, we have the following inequality:

$$\begin{cases} (1 - \epsilon_m)x_m(t_k) \leq \ell_m^{(j)} \leq (1 + \epsilon_m)x_m(t_k), & x_m(t_k) \geq 0, \\ (1 + \epsilon_m)x_m(t_k) \leq \ell_m^{(j)} \leq (1 - \epsilon_m)x_m(t_k), & x_m(t_k) < 0. \end{cases} \quad (13)$$

The quantizer as follows:

$$q(x(t_k)) = x(t_k) + w(x(t_k)), \quad (14)$$

where

$$w(x(t_k)) = [w_1(x_1(t_k)), w_2(x_2(t_k)), \dots, w_n(x_n(t_k))].$$

Based on the proposed result [34], we have

$$-\epsilon_i [x_m(t_k)]^2 \leq x_m(t_k) w_i(x_i(t_k)) \leq \epsilon_i [x_m(t_k)]^2. \quad (15)$$

Then, we have

$$\tilde{u}(t) = \sum_{i=1}^r h_j(\vartheta(t)) [\mathcal{K}_j [x(t_k) + w(x(t_k)) + \beta(t) f(u(t)) + (1 - \beta(t)) g(u(t))]], \quad t_k \leq t < t_{k+1}. \quad (16)$$

Remark 3. First of all, in order to effectively reduce the congestion of the transmission channel in the network transmission, the research of the quantizer is significant. In addition, because the bandwidth in network transmission is limited, the quantized signal can only be transmitted through a certain network speed, so the optimization effect of the quantizer will have a certain impact on the performance of T-S fuzzy NCSs. In this paper, we introduce the concept of quantization strategy to optimize the controller signal.

(5) with controller (16) can be represented as

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i(\vartheta(t)) h_j(\vartheta(t)) [\mathcal{A}_{1i} x(t) + \mathcal{G}_{1i} x(t-d(t)) + \mathcal{B}_{1i} \mathcal{K}_j x(t_k) + \mathcal{B}_{1i} \mathcal{K}_j \\ \quad \times w(x(t_k)) + \beta(t) \mathcal{B}_{1i} \mathcal{K}_j f(u(t)) + (1-\beta(t)) \mathcal{B}_{1i} \mathcal{K}_j g(u(t)) + \mathcal{D}_{1i} \omega(t)], \\ z(t) = \sum_{i=1}^r \sum_{j=1}^r h_i(\vartheta(t)) h_j(\vartheta(t)) [\mathcal{A}_{2i} x(t) + \mathcal{G}_{2i} x(t-d(t)) + \mathcal{B}_{2i} \mathcal{K}_j x(t_k) + \mathcal{B}_{2i} \mathcal{K}_j \\ \quad \times w(x(t_k)) + \beta(t) \mathcal{B}_{2i} \mathcal{K}_j f(u(t)) + (1-\beta(t)) \mathcal{B}_{2i} \mathcal{K}_j g(u(t)) + \mathcal{D}_{2i} \omega(t)], \end{cases} \quad (17)$$

where $\beta(t)$ is a stochastic variable describing the CAs that occur randomly, which is mutually independent events. It satisfies the Bernoulli distribution and is defined as

$$\beta(t) = \begin{cases} 0, & \text{Network transmission works normally,} \\ 1, & \text{Network transmission under attack,} \end{cases}$$

with

$$\text{Prob}\{\beta(t) = 1\} = \beta, \quad \text{Prob}\{\beta(t) = 0\} = 1 - \beta,$$

where $0 \leq \beta \leq 1$ is a constant and $\mathbb{E}\{\beta(t)\}$ is the expectation of $\beta(t)$, and we can get

$$\mathbb{E}\{\beta(t) - \beta\} = 0, \quad \mathbb{E}\{(\beta(t) - \beta)^2\} = \beta(1 - \beta).$$

In addition, $f(u(t))$ and $g(u(t))$ are nonlinear functions, which are related to $u(t)$ and satisfies the **Assumption 1**.

This paper proposes the energy injection rate function $\psi(\omega, z)$, which satisfies the following

$$\int_{t_0}^{t_1} |\psi(\omega(t), z(t))| dt < +\infty, \quad t_1 \geq t_0 \geq 0. \quad (18)$$

For any disturbance $\omega(t) > 0$, the following inequality (19) holds. Then, we think that the system (17) is the dissipation rate $\psi(\omega, z)$:

$$\int_{t_0}^{t_1} \psi(\omega(s), z(s)) ds \geq 0, \quad \forall t_1 > t_0. \quad (19)$$

According to [27], we can get \mathbb{Q} , \mathbb{S} and \mathbb{R} are known the matrixes of suitable dimensions and the function $\psi(\omega, z)$ as follows:

$$\psi(\omega, z) = z^T \mathbb{Q} z + 2z^T \mathbb{S} \omega + \omega^T \mathbb{R} \omega, \quad (20)$$

Definition 1. [27] The existence of $\gamma > 0$ and $\psi(\omega, z)$ make the system meet the strict $((Q), (S), (R)) - \gamma$ -Dissipative, the following inequality holds:

$$\int_{t_0}^{t_1} \psi(\omega(s), z(s)) ds \geq \gamma \int_{t_0}^{t_1} \omega^T(s) \omega(s) ds, \quad \forall t_1 > t_0.$$

Assumption 1 [21] For any $u_1(t), u_2(t) \in \mathbb{R}^{n_2}$, which meet the Lipschitz condition with the following form

$$\|f(u_1(t)) - f(u_2(t))\|_2 \leq \|F(u_1(t) - u_2(t))\|_2, \quad \|g(u_1(t)) - g(u_2(t))\|_2 \leq \|G(u_1(t) - u_2(t))\|_2,$$

where F is the upper bound of $f(\cdot)$ and G is the upper bound of $g(\cdot)$.

Lemma 1. [7] For given a scalar $\lambda \in (0, 1)$, SMs $\pi_1 > 0, \pi_2 > 0$, any matrices v_1 and v_2 , there are the following inequalities:

$$\begin{bmatrix} \frac{1}{\lambda} \pi_1 & 0 \\ 0 & \frac{1}{1-\lambda} \pi_2 \end{bmatrix} \geq \begin{bmatrix} \pi_1 + (1-\lambda)F_1 & (1-\lambda)v_1 + \lambda v_2 \\ * & \pi_2 + \lambda F_2 \end{bmatrix},$$

where $F_1 = \pi_1 - v_2 \pi_2^{-1} v_2^T$, $F_2 = \pi_2 - v_1^T \pi_1^{-1} v_1$.

Lemma 2. [35] There is a SM $\Upsilon > 0$, scalar η, δ ($\eta < \delta$) and x in $[\eta, \delta] \rightarrow \mathbb{R}^n$. The following inequalities hold:

$$\begin{cases} (\delta - \eta) \int_{\eta}^{\delta} \dot{x}^T(s) \Upsilon \dot{x}(s) ds \geq \Gamma_1^T \Upsilon \Gamma_1, \\ (\delta - \eta) \int_{\eta}^{\delta} x^T(s) \Upsilon x(s) ds \geq \hat{\Gamma}_1^T \Upsilon \hat{\Gamma}_1, \\ (\delta - \eta) \int_{\eta}^{\delta} \dot{x}^T(s) \Upsilon \dot{x}(s) ds \geq \Gamma_1^T \Upsilon \Gamma_1 + 3\Gamma_2^T \Upsilon \Gamma_2, \\ (\delta - \eta) \int_{\eta}^{\delta} x^T(s) \Upsilon x(s) ds \geq \hat{\Gamma}_1^T \Upsilon \hat{\Gamma}_2 + 3\hat{\Gamma}_2^T \Upsilon \hat{\Gamma}_2, \end{cases}$$

$$(\delta - \eta) \int_{\eta}^{\delta} \dot{x}^T(s) \Upsilon \dot{x}(s) ds \geq \Gamma_1^T \Upsilon \Gamma_1 + 3\Gamma_2^T \Upsilon \Gamma_2 + 5\Gamma_3^T \Upsilon \Gamma_3,$$

where $\Gamma_1 = x(\delta) - x(\eta)$, $\chi_2 = x(\delta) + x(\eta) - \frac{2}{\delta - \eta} \int_{\eta}^{\delta} x(s) ds$, $\hat{\Gamma}_1 = \int_{\eta}^{\delta} x(s) ds$, $\hat{\Gamma}_2 = \int_{\eta}^{\delta} x(s) ds - \frac{2}{\delta - \eta} \int_{\eta}^{\delta} \int_{\theta}^{\delta} x(s) ds d\theta$, and $\Gamma_3 = x(\delta) - x(\eta) + \frac{6}{\delta - \eta} \int_{\eta}^{\delta} x(s) ds - \frac{12}{(\delta - \eta)^2} \int_{\eta}^{\delta} \int_{\theta}^{\delta} x(s) ds d\theta$.

3. Main results

In this part, we have established a new QSD controller for (17). For simplicity, Appendix A shows some definitions related to compliance.

Theorem 1. The positive scalars d , μ and h_m are known. The system (17) with TVD $d(t)$ satisfying (2) is AS and strict dissipative, there give SMs $\mathcal{H}_1 > 0$, $\mathcal{H}_3 > 0$, $\mathcal{R}_1 > 0$, $\mathcal{R}_2 > 0$, \mathcal{P}_1 , \mathcal{P}_2 , \mathcal{U}_1 , \mathcal{U}_2 , \mathcal{Q}_1 , \mathcal{Q}_2 , \mathcal{H}_2 , \mathcal{H}_4 and any matrices \mathcal{S}_n , $n = 1, 2, \dots, 4$ have appropriate dimensions so as to satisfy the following LMIs, for $i, j = 1, 2, \dots, r$:

$$\begin{bmatrix} \mathcal{U}_F + \mathcal{P}_{d(t)=0} & \frac{\sqrt{d}}{d} F_3^T \mathcal{S}_1^T & \frac{\sqrt{d-0}}{d} F_2^T \mathcal{S}_2^T \\ * & \mathcal{U}_1 & 0 \\ * & * & \mathcal{U}_2 \end{bmatrix} > 0, \quad \begin{bmatrix} \mathcal{U}_F + \mathcal{P}_{d(t)=d} & \frac{\sqrt{d}}{d} F_3^T \mathcal{S}_1^T & \frac{\sqrt{d-d}}{d} F_2^T \mathcal{S}_2^T \\ * & \mathcal{U}_1 & 0 \\ * & * & \mathcal{U}_2 \end{bmatrix} > 0, \quad (21)$$

$$\begin{bmatrix} \Sigma_a & \frac{\sqrt{d}}{d} \mathcal{E}_1^T \mathcal{S}_2 & \Gamma_1^T & \Pi_1 & \Gamma_1^T & \Pi_2 \\ * & -\Lambda_2 & 0 & 0 & 0 & 0 \\ * & * & -\mathcal{S}_1 \mathcal{I} & 0 & 0 & 0 \\ * & * & * & -\mathcal{S}_1^{-1} \mathcal{I} & 0 & 0 \\ * & * & * & * & -\mathcal{S}_2 \mathcal{I} & 0 \\ * & * & * & * & * & -\mathcal{S}_2^{-1} \mathcal{I} \end{bmatrix} < 0, \quad \begin{bmatrix} \Sigma_b & \frac{\sqrt{d}}{d} \mathcal{E}_2^T \mathcal{S}_3 & \Gamma_1^T & \Pi_1 & \Gamma_1^T & \Pi_2 \\ * & -\Lambda_1 & 0 & 0 & 0 & 0 \\ * & * & -\mathcal{S}_1 \mathcal{I} & 0 & 0 & 0 \\ * & * & * & -\mathcal{S}_1^{-1} \mathcal{I} & 0 & 0 \\ * & * & * & * & -\mathcal{S}_2 \mathcal{I} & 0 \\ * & * & * & * & * & -\mathcal{S}_2^{-1} \mathcal{I} \end{bmatrix} < 0, \quad (22)$$

$$\begin{bmatrix} \Sigma_c & \frac{\sqrt{d}}{d} \mathcal{E}_2^T \mathcal{S}_3 & \Gamma_1^T & \Pi_1 & \Gamma_1^T & \Pi_2 \\ * & -\Lambda_1 & 0 & 0 & 0 & 0 \\ * & * & -\mathcal{S}_1 \mathcal{I} & 0 & 0 & 0 \\ * & * & * & -\mathcal{S}_1^{-1} \mathcal{I} & 0 & 0 \\ * & * & * & * & -\mathcal{S}_2 \mathcal{I} & 0 \\ * & * & * & * & * & -\mathcal{S}_2^{-1} \mathcal{I} \end{bmatrix} < 0, \quad \begin{bmatrix} \Sigma_d & \frac{\sqrt{d}}{d} \mathcal{E}_1^T \mathcal{S}_2 & \Gamma_1^T & \Pi_1 & \Gamma_1^T & \Pi_2 \\ * & -\Lambda_2 & 0 & 0 & 0 & 0 \\ * & * & -\mathcal{S}_1 \mathcal{I} & 0 & 0 & 0 \\ * & * & * & -\mathcal{S}_1^{-1} \mathcal{I} & 0 & 0 \\ * & * & * & * & -\mathcal{S}_2 \mathcal{I} & 0 \\ * & * & * & * & * & -\mathcal{S}_2^{-1} \mathcal{I} \end{bmatrix} < 0, \quad (23)$$

$$\begin{bmatrix} \Sigma_e & \frac{\sqrt{d}}{d} \mathcal{E}_1^T \mathcal{S}_2 & \Gamma_1^T & \Pi_1 & \Gamma_1^T & \Pi_2 \\ * & -\Lambda_2 & 0 & 0 & 0 & 0 \\ * & * & -\mathcal{S}_1 \mathcal{I} & 0 & 0 & 0 \\ * & * & * & -\mathcal{S}_1^{-1} \mathcal{I} & 0 & 0 \\ * & * & * & * & -\mathcal{S}_2 \mathcal{I} & 0 \\ * & * & * & * & * & -\mathcal{S}_2^{-1} \mathcal{I} \end{bmatrix} < 0, \quad \begin{bmatrix} \Sigma_f & \frac{\sqrt{d}}{d} \mathcal{E}_2^T \mathcal{S}_3 & \Gamma_1^T & \Pi_1 & \Gamma_1^T & \Pi_2 \\ * & -\Lambda_1 & 0 & 0 & 0 & 0 \\ * & * & -\mathcal{S}_1 \mathcal{I} & 0 & 0 & 0 \\ * & * & * & -\mathcal{S}_1^{-1} \mathcal{I} & 0 & 0 \\ * & * & * & * & -\mathcal{S}_2 \mathcal{I} & 0 \\ * & * & * & * & * & -\mathcal{S}_2^{-1} \mathcal{I} \end{bmatrix} < 0, \quad (24)$$

$$\begin{bmatrix} \Sigma_g & \frac{\sqrt{d}}{d} \mathcal{E}_2^T \mathcal{S}_3 & \Gamma_1^T & \Pi_1 & \Gamma_1^T & \Pi_2 \\ * & -\Lambda_1 & 0 & 0 & 0 & 0 \\ * & * & -\mathcal{S}_1 \mathcal{I} & 0 & 0 & 0 \\ * & * & * & -\mathcal{S}_1^{-1} \mathcal{I} & 0 & 0 \\ * & * & * & * & -\mathcal{S}_2 \mathcal{I} & 0 \\ * & * & * & * & * & -\mathcal{S}_2^{-1} \mathcal{I} \end{bmatrix} < 0, \quad \begin{bmatrix} \Sigma_h & \frac{\sqrt{d}}{d} \mathcal{E}_1^T \mathcal{S}_2 & \Gamma_1^T & \Pi_1 & \Gamma_1^T & \Pi_2 \\ * & -\Lambda_2 & 0 & 0 & 0 & 0 \\ * & * & -\mathcal{S}_1 \mathcal{I} & 0 & 0 & 0 \\ * & * & * & -\mathcal{S}_1^{-1} \mathcal{I} & 0 & 0 \\ * & * & * & * & -\mathcal{S}_2 \mathcal{I} & 0 \\ * & * & * & * & * & -\mathcal{S}_2^{-1} \mathcal{I} \end{bmatrix} < 0, \quad (25)$$

$$\mathcal{H}_{1,(d(t)=d)} > 0, \quad \mathcal{H}_{2,(d(t)=0)} > 0, \quad (26)$$

where other relevant parameters are shown in Appendix B.

Proof. Choose a suitable LKF as shown below

$$V(x(t)) = \sum_{i=a}^e V_i(x(t)), \quad t \in [t_k, t_{k+1}), \quad (27)$$

where

$$V_a(x(t)) = \alpha_1^T(t) \mathcal{P}_{d(t)} \alpha(t)_1,$$

$$V_b(x(t)) = d(t) \alpha_2^T(t) \mathcal{U}_1 \alpha_2(t) + (d - d(t)) \alpha_3^T(t) \mathcal{U}_2 \alpha_3(t),$$

$$V_c(x(t)) = (t_{k+1} - t)(t - t_k) \eta^T(t) \mathcal{Q}_{d(t)} \eta(t),$$

$$V_d(x(t)) = \int_{t-d(t)}^t \dot{x}^T(s) \mathcal{H}_{1,d(t)} \dot{x}(s) ds + \int_{t-d}^{t-d(t)} \dot{x}^T(s) \mathcal{H}_{2,d(t)} \dot{x}(s) ds,$$

$$V_e(x(t)) = \int_{-d(t)}^0 \int_{t+\theta}^t \dot{x}^T(s) \mathcal{R}_1 \dot{x}(s) ds d\theta + \int_{-d}^{-d(t)} \int_{t+\theta}^t \dot{x}^T(s) \mathcal{R}_2 \dot{x}(s) ds d\theta,$$

$\xi(t)$, $\alpha_i(t)$, $\eta(t)$, $\mathcal{P}_{d(t)}$, $\mathcal{H}_{1,d(t)}$, $\mathcal{H}_{2,d(t)}$ have been in [Appendix A](#).

First, in order to relax the restrictions $V(x_t) > 0$, $V_a(x(t)) + V_b(x(t))$ can be combined into

$$\begin{aligned} V_{ab}(x(t)) &= V_a(x(t)) + V_b(x(t)) \\ &= \alpha_1^T(t) \left(\mathcal{P}_{d(t)} + d(t) \begin{bmatrix} \bar{e}_1 \\ \bar{e}_4 \end{bmatrix}^T \mathcal{U}_1 \begin{bmatrix} \bar{e}_1 \\ \bar{e}_4 \end{bmatrix} + (d - d(t)) \begin{bmatrix} \bar{e}_1 \\ \bar{e}_5 \end{bmatrix}^T \mathcal{U}_2 \begin{bmatrix} \bar{e}_1 \\ \bar{e}_5 \end{bmatrix} \right) \\ &= \alpha_1^T(t) \left(\mathcal{P}_{d(t)} + F_1^T [d(t) \mathcal{U}_1 + (d - d(t)) \mathcal{U}_2] \times F_1 + \text{Sym}\{F_1^T \mathcal{U}_1 F_2 + F_1^T \mathcal{U}_2 F_3\} \right. \\ &\quad \left. + \frac{F_2^T \mathcal{U}_1 F_2}{d(t)} + \frac{F_3^T \mathcal{U}_2 F_3}{d - d(t)} \right) \alpha_1(t). \end{aligned} \quad (28)$$

If (21) hold, using the Schur complement lemma (SCL),

$$\mathcal{U}_F + \mathcal{P}_{d(t)} - \frac{d(t)}{d^2} F_3^T S_1^T \mathcal{U}_1^{-1} S_1 F_3 - \frac{d(t)}{d^2} F_2^T S_2 \mathcal{U}_2^{-1} S_2^T F_2 > 0. \quad (29)$$

Then based on [Lemma 1](#), we can get

$$\begin{aligned} \frac{F_2^T \mathcal{U}_1 F_2}{d(t)} + \frac{F_3^T \mathcal{U}_2 F_3}{d - d(t)} &= \begin{bmatrix} F_2 \\ F_3 \end{bmatrix}^T \begin{bmatrix} \frac{1}{d(t)} \mathcal{U}_1 & 0 \\ 0 & \frac{1}{d - d(t)} \mathcal{U}_2 \end{bmatrix} \begin{bmatrix} F_2 \\ F_3 \end{bmatrix} \\ &\geq \frac{2d - d(t)}{d^2} F_2^T \mathcal{U}_1 F_2 + \frac{d + d(t)}{d^2} F_3^T \mathcal{U}_2 F_3 + \text{Sym} \left\{ \frac{d - d(t)}{d^2} F_2^T S_1 F_3 \right. \\ &\quad \left. + \frac{d(t)}{d^2} F_2^T S_2 F_3 \right\} - \frac{d(t)}{d^2} F_3^T S_1^T \mathcal{U}_1^{-1} F_3 - \frac{d - d(t)}{d^2} F_2^T S_2 \mathcal{U}_2^{-1} S_2^T F_2. \end{aligned} \quad (30)$$

Thus, $V_{ab}(x(t))$ can be approximately estimated as

$$\begin{aligned} V_{ab}(x(t)) &= V_a(x(t)) + V_b(x(t)) \\ &\geq \alpha_1^T(t) \left(\mathcal{P}_{d(t)} + F_1^T (d(t) \mathcal{U}_1 + (d - d(t)) \mathcal{U}_2) F_1 + \text{Sym}\{F_1^T \mathcal{U}_1 F_2 + F_1^T \mathcal{U}_2 F_3\} \right. \\ &\quad \left. + \frac{2d - d(t)}{d^2} F_2^T \mathcal{U}_1 F_2 + \frac{d + d(t)}{d^2} F_3^T \mathcal{U}_2 F_3 + \text{Sym} \left\{ \frac{d - d(t)}{d^2} F_2^T S_1 F_3 \right. \right. \\ &\quad \left. \left. + \frac{d(t)}{d^2} F_2^T S_2 F_3 \right\} - \frac{d(t)}{d^2} F_3^T S_1^T \mathcal{U}_1^{-1} F_3 - \frac{d - d(t)}{d^2} F_2^T S_2 \mathcal{U}_2^{-1} S_2^T F_2 \right) \alpha_1(t) \\ &= \alpha_1^T(t) \left(\mathcal{U}_F + \mathcal{P}_{d(t)} - \frac{d(t)}{d^2} F_3^T S_1^T \mathcal{U}_1^{-1} F_3 - \frac{d - d(t)}{d^2} F_2^T S_2 \mathcal{U}_2^{-1} S_2^T F_2 \right) \alpha_1(t). \end{aligned} \quad (31)$$

When (30) holds, a small enough $\kappa > 0$ can be found to make (21) hold.

Calculating the derivative of $V(x(t))$, we can have

$$\dot{V}(x(t)) = \sum_{i=a}^e \dot{V}_i(x(t)), \quad t \in [t_k, t_{k+1}).$$

$$\begin{aligned}
\dot{V}_a(x(t)) &= \dot{d}(t)\alpha_1^T(t)\mathcal{P}_2\alpha_1(t) + 2\alpha_1^T(t)\mathcal{P}_{d(t)}\dot{\alpha}_1(t) \\
&= \dot{d}(t)\alpha_1^T(t)\mathcal{P}_2\alpha_1(t) + 2\alpha_1^T(t)\mathcal{P}_{d(t)} \begin{bmatrix} \dot{x}(t) \\ (1-\dot{d}(t))x(t-d(t)) \\ \dot{x}(t-d) \\ x(t) - (1-\dot{d}(t))x(t-d(t)) \\ (1-\dot{d}(t))x(t-d(t)) - x(t-d) \end{bmatrix} \\
&= \xi^T(t)\Xi_1\xi(t),
\end{aligned} \tag{32}$$

$$\begin{aligned}
\dot{V}_b(x(t)) &= \dot{d}(t)\alpha_2^T(t)\mathcal{U}_1\alpha_2(t) + 2d(t)\alpha_2^T(t)\mathcal{U}_1\dot{\alpha}_2(t) \\
&\quad -\dot{d}(t)\alpha_3^T(t)\mathcal{U}_2\alpha_3(t) + 2(d-d(t))\alpha_3^T(t)\mathcal{U}_2\dot{\alpha}_3(t) \\
&= \dot{d}(t)\alpha_2^T(t)\mathcal{U}_1\alpha_2(t) + 2d(t)\alpha_2^T(t)\mathcal{U}_1 \begin{bmatrix} \dot{x}(t) \\ \frac{x(t)-(1-\dot{d}(t))x(t-d(t))-\dot{d}(t)\theta_1(t)}{d(t)} \end{bmatrix} \\
&\quad -\dot{d}(t)\alpha_3^T(t)\mathcal{U}_2\alpha_3(t) + 2(d-d(t))\alpha_3^T(t)\mathcal{U}_2 \begin{bmatrix} \dot{x}(t) \\ \frac{(1-\dot{d}(t))x(t-d(t))-x(t-d)+d(t)\theta_2(t)}{d-d(t)} \end{bmatrix} \\
&= \xi^T(t)\Xi_2\xi(t),
\end{aligned} \tag{33}$$

$$\begin{aligned}
\dot{V}_c(x(t)) &= (t_{k+1}-t)\eta^T(t)\mathcal{Q}_1\eta(t) - (t-t_k)\eta^T(t)\mathcal{Q}_1\eta(t) + (t-t_k)(d-d(t))\eta^T(t)\mathcal{Q}_2\eta(t) \\
&\quad -(t_{k+1}-t)(d-d(t))\eta^T(t)\mathcal{Q}_2\eta(t) + \dot{d}(t)(t_{k+1}-t)(t-t_k)\eta^T(t)\mathcal{Q}_2\eta(t) \\
&\quad + 2(t_{k+1}-t)(t-t_k)\eta^T(t)\mathcal{Q}_{d(t)}\eta(t) \\
&= \eta^T(t)\mathcal{Q}_{k,t_{k+1}}\eta(t) + \text{Sym}\{(t_{k+1}-t)(t-t_k)\eta^T(t)\mathcal{Q}_{d(t)}\eta(t)\} \\
&= \xi^T(t)\Xi_3\xi(t),
\end{aligned} \tag{34}$$

$$\begin{aligned}
\dot{V}_d(x(t)) &= \dot{x}^T(t)[d(t)\mathcal{H}_1 + \mathcal{H}_2]\dot{x}(t) - (1-\dot{d}(t))\dot{x}^T(t-d(t))[d(t)\mathcal{H}_1 + \mathcal{H}_2]\dot{x}(t-d(t)) \\
&\quad + (1-\dot{d}(t))\dot{x}^T(t-d(t))[(d-d(t))\mathcal{H}_3 + \mathcal{H}_4]\dot{x}(t-d(t)) - \dot{x}^T(t-d)[(d-d(t)) \\
&\quad \times \mathcal{H}_3 + \mathcal{H}_4]\dot{x}(t-d) + \dot{d}(t) \int_t^{t-d(t)} \dot{x}^T(s)\mathcal{H}_1\dot{x}(s)ds - \dot{d}(t) \int_{t-d(t)}^{t-d} \dot{x}^T(s)\mathcal{H}_1\dot{x}(s)ds,
\end{aligned} \tag{35}$$

$$\begin{aligned}
\dot{V}_e(x(t)) &= d(t)\dot{x}^T(t)\mathcal{R}_1\dot{x}(t) + (d-d(t))\dot{x}^T(t)\mathcal{R}_2\dot{x}(t) - (1-\dot{d}(t)) \int_{t-d(t)}^t \dot{x}^T(s)\mathcal{R}_1\dot{x}(s)ds \\
&\quad -\dot{d}(t) \int_{t-d(t)}^t \dot{x}^T(s)\mathcal{R}_2\dot{x}(s)ds - \int_{t-d}^{t-d(t)} \dot{x}^T(s)\mathcal{R}_2\dot{x}(s)ds.
\end{aligned} \tag{36}$$

Combining (35) and (36), we have $\dot{V}_{d,e}(x(t))$ as shown below

$$\begin{aligned}
V_{d,e}(x(t)) &= \dot{x}^T(t)[d(t)\mathcal{R}_1 + (d-d(t))\mathcal{R}_2 + d(t)\mathcal{H}_1 + \mathcal{H}_2]\dot{x}(t) + (1-\dot{d}(t))\dot{x}^T(t-d(t))[(d-d(t)) \\
&\quad \times \mathcal{H}_3 + \mathcal{H}_4 - d(t)\mathcal{H}_1 - \mathcal{H}_2]\dot{x}(t-d(t)) - \dot{x}^T(t-d)[(d-d(t))\mathcal{H}_3 + \mathcal{H}_4]\dot{x}(t-d) \\
&\quad - \int_{t-d(t)}^t \dot{x}^T(s)\Lambda_{1,d(t)}\dot{x}(s)ds - \int_{t-d}^{t-d(t)} \dot{x}^T(s)\Lambda_{2,d(t)}\dot{x}(s)ds.
\end{aligned} \tag{37}$$

According to the Lemma 2, we can get

$$\begin{aligned}
& - \int_{t-d(t)}^t \dot{x}^T(s)\Lambda_{1,d(t)}\dot{x}(s)ds - \int_{t-d}^{t-d(t)} \dot{x}^T(s)\Lambda_{2,d(t)}\dot{x}(s)ds \\
& \leq \xi^T(t) \left(\frac{d-d(t)}{d^2} \mathcal{E}_1^T \mathcal{S}_4^T \overline{\Lambda}_2^{-1} \mathcal{S}_4 \mathcal{E}_1 + \frac{d(t)}{d^2} \mathcal{E}_2^T \mathcal{S}_3^T \overline{\Lambda}_1^{-1} \mathcal{S}_3 \mathcal{E}_2 \right. \\
& \quad \left. - \frac{1}{d} \begin{bmatrix} \mathcal{E}_1 & \mathcal{E}_2 \end{bmatrix}^T \left[\frac{2d-d(t)}{d} \overline{\Lambda}_1 \frac{d-d(t)}{d} \mathcal{S}_3 + \frac{d(t)}{d} \mathcal{S}_4 \right. \right. \\
& \quad \left. \left. * \frac{1}{d-d(t)} \overline{\Lambda}_2 \right] \begin{bmatrix} \mathcal{E}_1 \\ \mathcal{E}_2 \end{bmatrix} \right) \xi(t) \\
& = \xi^T(t) \left\{ \Upsilon_{d(t)} - \frac{1}{d} \left(\frac{2d-d(t)}{d} \mathcal{E}_1^T \overline{\Lambda}_1 \mathcal{E}_1 + \frac{d-d(t)}{d} \mathcal{E}_2^T \mathcal{S}_3^T \mathcal{E}_1 + \frac{d-d(t)}{d} \mathcal{E}_2^T \mathcal{S}_4^T \mathcal{E}_1 + \frac{d-d(t)}{d} \mathcal{E}_1^T \mathcal{S}_3 \mathcal{E}_2 \right. \right.
\end{aligned}$$

$$\left. + \frac{d-d(t)}{d} \mathcal{E}_1^T \mathcal{S}_4 \mathcal{E}_2 + \frac{d-d(t)}{d} \mathcal{E}_2^T \bar{\Lambda}_2 \mathcal{E}_2 \right) \xi(t) \\ = \xi^T(t) (\Upsilon_{d(t)} + \Omega_{d(t)}) \xi(t). \quad (38)$$

Then, $\dot{V}_{d,e}(x(t))$ can be indicated as shown

$$\dot{V}_{d,e}(x(t)) \leq \xi^T(t) (e_4^T [d(t)\mathcal{R}_1 + (d-d(t))\mathcal{R}_2 + d(t)\mathcal{H}_1 + \mathcal{H}_2] e_4 + (1-\dot{d}(t)) e_5^T [(d-d(t))\mathcal{H}_3 + \mathcal{H}_4 \\ - d(t)\mathcal{H}_1 - \mathcal{H}_2] e_5 - e_6^T [(d-d(t))\mathcal{H}_3 + \mathcal{H}_4] e_6 + \Upsilon_{d(t)} + \Omega_{d(t)}) \xi(t), \quad (39)$$

where $\bar{\Lambda}_{1,d(t)}$, $\bar{\Lambda}_{2,d(t)}$, $\Upsilon_{d(t)}$ and $\Omega_{d(t)}$ have been in [Appendix A](#).

We have the following zero formula

$$\mathbb{E}\{0 = 2[x^T(t)\mathcal{N}_1 + x^T(t_k)\mathcal{N}_2 + \dot{x}^T(t)\mathcal{N}_3][\sum_{i=1}^r \sum_{j=1}^r h_i(\vartheta(t))h_j(\vartheta(t))\mathcal{A}_{1i}x(t) + \mathcal{G}_{1i}x(t-d(t)) + \mathcal{B}_{1i} \\ \times \mathcal{K}_j x(t_k) + \mathcal{B}_{1i}\mathcal{K}_j w(x(t_k)) + \beta(t)\mathcal{B}_{1i}\mathcal{K}_j f(u(t)) + (1-\beta(t))\mathcal{B}_{1i}\mathcal{K}_j g(u(t)) + \mathcal{D}_{1i}\omega(t) - \dot{x}(t)]\} \\ = \xi^T(t) (\text{Sym}\{\mathcal{L}\varphi\} + 2\beta\mathcal{L}\mathcal{B}_{1i}\mathcal{K}_j f(u(t)) + 2(1-\beta)\mathcal{B}_{1i}\mathcal{K}_j g(u(t))) \xi(t). \quad (40)$$

Next, according to Assumption 1, we have

$$\xi^T(t) (2\beta\mathcal{L}\mathcal{B}_{1i}\mathcal{K}_j f(u(t))) \xi(t) \leq \xi^T(t) (\varsigma_1 \Pi_1 \Pi_1^T + \varsigma_1^{-1} \Gamma_1^T \Gamma_1) \xi(t), \quad (41)$$

$$\xi^T(t) (2(1-\beta)\mathcal{L}\mathcal{B}_{1i}\mathcal{K}_j g(u(t))) \xi(t) \leq \xi^T(t) (\varsigma_2 \Pi_2 \Pi_2^T + \varsigma_2^{-1} \Gamma_1^T \Gamma_1) \xi(t). \quad (42)$$

According to [Definition 1](#), combining with (40)–(42), we can get

$$\mathbb{E}\{\dot{V}(x(t)) - \psi(\omega, z) + \gamma \omega^T(t) \omega(t)\} = \sum_{i=a}^e \dot{V}_i(x(t)) - \psi(\omega, z) + \gamma \omega^T(t) \omega(t) \\ = \xi^T(t) \left\{ \Sigma_{d(t), \dot{d}(t)} + \varsigma_1 \beta^2 \Pi_1 \Pi_1^T + \varsigma_1^{-1} \Gamma_1^T \Gamma_1 \right. \\ \left. + \varsigma_2 (1-\beta)^2 \Pi_2 \Pi_2^T + \varsigma_2^{-1} \Gamma_1^T \Gamma_1 \right\} \xi(t). \quad (43)$$

According to the linear convex combination method (LCCM) [15], $\dot{V}(x(t)) - \psi(\omega, z) + \gamma \omega^T(t) \omega(t) < 0$ holds for all, utilizing the SCL, We believe that (43) is equivalent to the following inequality:

$$\begin{bmatrix} \Sigma_{d(t), \dot{d}(t)} & \frac{\sqrt{d-d(t)}}{d} \mathcal{E}_1^T \mathcal{S}_4 & \frac{\sqrt{d(t)}}{d} \mathcal{E}_2^T \mathcal{S}_3 & \Gamma_1^T & \Pi_1 & \Gamma_1^T & \Pi_2 \\ * & -\bar{\Lambda}_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\bar{\Lambda}_1 & 0 & 0 & 0 & 0 \\ * & * & * & -\varsigma_1 \mathcal{I} & 0 & 0 & 0 \\ * & * & * & * & -\varsigma_1^{-1} \mathcal{I} & 0 & 0 \\ * & * & * & * & * & -\varsigma_2 \mathcal{I} & 0 \\ * & * & * & * & * & * & -\varsigma_2^{-1} \mathcal{I} \end{bmatrix} < 0. \quad (44)$$

Based on (28)–(44), it is easy to draw the following conclusions:

$$\dot{V}(x(t)) - \psi(\omega(t), z(t)) + \gamma \omega^T(t) \omega(t) \leq 0. \quad (45)$$

Under zero initial condition, integrate (45) from t_0 to t_1

$$\gamma \int_{t_0}^{t_1} \omega^T(s) \omega(s) ds - \int_{t_0}^{t_1} \psi(\omega(s), z(s)) ds \leq V(x(t_0)) - V(x(t_1)). \quad (46)$$

According to [Definition 1](#), (17) can be obtained as AS with dissipation performance index γ . \square

As shown below, in order to verify the validity and feasibility of our proposed criteria, we have established a new control Algorithm.

Remark 4. At present, there are many studies on the system's dynamic response when the network is under CAs. Some scholars have only unilaterally studied the effective control of the system by designing a suitable controller when the network is under attack. However, when the network is under the CA, part of the data has been changed, which cannot be ignored for the integrity and confidentiality of the data. Therefore, how to design an effective VDS will be the focus of this paper. We develop a proper VDS function $g(u(t))$ and start the protection mechanism by predicting the appearance of the attack. At the same time, this design method effectively solves the integrity and confidentiality of the data.

Remark 5. In this paper, we consider the $\mathcal{P}_{d(t)} = \mathcal{P}_1 - (d-d(t))\mathcal{P}_2$. (I) When $d-d(t) = 0$, the $\mathcal{P}_{d(t)}$ will become a constant matrix \mathcal{P} . Different from the existing results [27] and [31], this method is first applied in $V_a(x(t))$. (II) Compared with the existing results [28], here we only need to constrain \mathcal{P}_2 and \mathcal{P}_2 to be any SMs. Therefore, this paper only needs to ensure

Algorithm 1 Solve the upper bound of the performance level γ .

Step 1:Input positive scalars $d, \mu, h_m, \varsigma_i, \varepsilon_i, (i=1,2) F, G$ and γ ;

if Solve (21)–(26) and (47)–(52) by using the LMI toolbox in MATLAB and there is a feasible solution;**Then**

Step 2: $\gamma_{n+1} = \gamma_n + 0.0001$ and $n = n + 1$;

if Solve (21)–(26) and (47)–(52), until there is no feasible solution;

Output: Maximum γ_n, κ_j and break;

Otherwise return **Step 1**;

Step 3:Input positive scalars $d, \mu, h_m, \varsigma_i, \varepsilon_i, (i=1,2) F, G$ and γ ;

if Solve (21)–(26) and (47)–(52) by using the LMI toolbox in MATLAB and there is no feasible solution;**Then**

Step 4: $\gamma_{n+1} = \gamma_n - 0.0001$ and $n = n + 1$;

if Solve (21)–(26) and (47)–(52), until there is a feasible solution;

Output: Maximum γ_{n+1}, κ_j and break;

Otherwise return **Step 3**;

that (21) is established, which greatly reduces the conservativeness of the guidelines. (III) $2\alpha_1^T(t)(\mathcal{P}_1 - (d(t))\mathcal{P}_2)\dot{\alpha}_1(t)$ can be derived in $V_a(x(t))$, which can weaken the constraints of the conditions in the LMI and improve the performance of the algorithm. (IV) Moreover, this construction method fully utilizes the information contained in the TVD, which facilitates the construction of a more general LKF and better improves the performance of the control algorithm and reduces the conservativeness of the criterion.

Remark 6. Unlike the existing research method [13], this paper considers the relaxed condition with a TPDF. Here, the positive definiteness of $V_a(x(t))$ and $V_b(x(t))$ are not given separately. On the contrary, it deals with the value of $V_a(x(t)) + V_b(x(t))$ as follow:

$$V_a(x(t)) + V_b(x(t)) \geq \alpha_1^T(t) \begin{bmatrix} \mathcal{U}_F + \mathcal{P}_{d(t)} & \frac{\sqrt{d(t)}}{d} F_3^T \mathcal{S}_1^T & \frac{\sqrt{d-d(t)}}{d} F_2^T \mathcal{S}_2^T \\ * & \mathcal{U}_1 & 0 \\ * & * & \mathcal{U}_2 \end{bmatrix} \alpha_1(t).$$

It can be seen from (31) that $\mathcal{P}_{d(t)} > 0$ are relaxed to (21). Many scholars use this method to weaken the limitation of LKF. Thus, the effectiveness and feasibility of our method can be improved.

Remark 7. In order to better handle the integral terms in (35) and (36)

$$-\int_{t-d(t)}^t \dot{x}^T(s) \Lambda_{1,d(t)} \dot{x}(s) ds - \int_{t-d}^{t-d(t)} \dot{x}^T(s) \Lambda_{2,d(t)} \dot{x}(s) ds.$$

We consider processing integral items through RCMI. Compared with the existing Jensen's inequality, Wirtinger's integral inequality, this method has a tight upper bound, which effectively improves the performance of the algorithm.

Theorem 2. The positive scalars $d, \mu, h_m, \varepsilon_1$ and ε_2 are known. The system (17) with TVD $d(t)$ satisfying (2) is AS and strict dissipative, there give SMs $\tilde{\mathcal{H}}_1 > 0, \tilde{\mathcal{H}}_3 > 0, \tilde{\mathcal{R}}_1 > 0, \tilde{\mathcal{R}}_2 > 0, \tilde{\mathcal{P}}_1, \tilde{\mathcal{P}}_2, \tilde{\mathcal{U}}_1, \tilde{\mathcal{U}}_2, \tilde{\mathcal{Q}}_1, \tilde{\mathcal{Q}}_2, \tilde{\mathcal{H}}_2, \tilde{\mathcal{H}}_4$ and any matrices $\tilde{\mathcal{S}}_n, n = 1, 2, \dots, 4, \mathcal{Y}_j$ and \mathcal{X} have appropriate dimensions so as to satisfy the following LMIs, for $i, j = 1, 2, \dots, r$:

$$\begin{bmatrix} \tilde{\mathcal{U}}_F + \tilde{\mathcal{P}}_{d(t)=0} & \frac{\sqrt{0}}{d} F_3^T \tilde{\mathcal{S}}_1^T & \frac{\sqrt{d-0}}{d} F_2^T \tilde{\mathcal{S}}_2^T \\ * & \tilde{\mathcal{U}}_1 & 0 \\ * & * & \tilde{\mathcal{U}}_2 \end{bmatrix} > 0, \quad \begin{bmatrix} \tilde{\mathcal{U}}_F + \tilde{\mathcal{P}}_{d(t)=d} & \frac{\sqrt{d}}{d} F_3^T \tilde{\mathcal{S}}_1^T & \frac{\sqrt{d-d}}{d} F_2^T \tilde{\mathcal{S}}_2^T \\ * & \tilde{\mathcal{U}}_1 & 0 \\ * & * & \tilde{\mathcal{U}}_2 \end{bmatrix} > 0, \quad (47)$$

$$\begin{bmatrix} \tilde{\Sigma}_a & \frac{\sqrt{d}}{d} \varepsilon_1^T \tilde{\mathcal{S}}_2 & \tilde{\Gamma}_1^T & \tilde{\Pi}_1 & \tilde{\Gamma}_1^T & \tilde{\Pi}_2 \\ * & -\tilde{\Lambda}_2 & 0 & 0 & 0 & 0 \\ * & * & -\mathcal{S}_1 \mathcal{I} & 0 & 0 & 0 \\ * & * & * & -\mathcal{S}_1^{-1} \mathcal{I} & 0 & 0 \\ * & * & * & * & -\mathcal{S}_2 \mathcal{I} & 0 \\ * & * & * & * & * & -\mathcal{S}_2^{-1} \mathcal{I} \end{bmatrix} < 0, \quad \begin{bmatrix} \tilde{\Sigma}_b & \frac{\sqrt{d}}{d} \varepsilon_2^T \tilde{\mathcal{S}}_3 & \tilde{\Gamma}_1^T & \tilde{\Pi}_1 & \tilde{\Gamma}_1^T & \tilde{\Pi}_2 \\ * & -\tilde{\Lambda}_1 & 0 & 0 & 0 & 0 \\ * & * & -\mathcal{S}_1 \mathcal{I} & 0 & 0 & 0 \\ * & * & * & -\mathcal{S}_1^{-1} \mathcal{I} & 0 & 0 \\ * & * & * & * & -\mathcal{S}_2 \mathcal{I} & 0 \\ * & * & * & * & * & -\mathcal{S}_2^{-1} \mathcal{I} \end{bmatrix} < 0, \quad (48)$$

$$\begin{bmatrix} \tilde{\Sigma}_c & \frac{\sqrt{d}}{d} \varepsilon_2^T \tilde{\mathcal{S}}_3 & \tilde{\Gamma}_1^T & \tilde{\Pi}_1 & \tilde{\Gamma}_1^T & \tilde{\Pi}_2 \\ * & -\tilde{\Lambda}_1 & 0 & 0 & 0 & 0 \\ * & * & -\zeta_1 \mathcal{I} & 0 & 0 & 0 \\ * & * & * & -\zeta_1^{-1} \mathcal{I} & 0 & 0 \\ * & * & * & * & -\zeta_2 \mathcal{I} & 0 \\ * & * & * & * & * & -\zeta_2^{-1} \mathcal{I} \end{bmatrix} < 0, \quad \begin{bmatrix} \tilde{\Sigma}_d & \frac{\sqrt{d}}{d} \varepsilon_1^T \tilde{\mathcal{S}}_2 & \tilde{\Gamma}_1^T & \tilde{\Pi}_1 & \tilde{\Gamma}_1^T & \tilde{\Pi}_2 \\ * & -\tilde{\Lambda}_2 & 0 & 0 & 0 & 0 \\ * & * & -\zeta_1 \mathcal{I} & 0 & 0 & 0 \\ * & * & * & -\zeta_1^{-1} \mathcal{I} & 0 & 0 \\ * & * & * & * & -\zeta_2 \mathcal{I} & 0 \\ * & * & * & * & * & -\zeta_2^{-1} \mathcal{I} \end{bmatrix} < 0, \quad (49)$$

$$\begin{bmatrix} \tilde{\Sigma}_e & \frac{\sqrt{d}}{d} \varepsilon_1^T \tilde{\mathcal{S}}_2 & \tilde{\Gamma}_1^T & \tilde{\Pi}_1 & \tilde{\Gamma}_1^T & \tilde{\Pi}_2 \\ * & -\tilde{\Lambda}_2 & 0 & 0 & 0 & 0 \\ * & * & -\zeta_1 \mathcal{I} & 0 & 0 & 0 \\ * & * & * & -\zeta_1^{-1} \mathcal{I} & 0 & 0 \\ * & * & * & * & -\zeta_2 \mathcal{I} & 0 \\ * & * & * & * & * & -\zeta_2^{-1} \mathcal{I} \end{bmatrix} < 0, \quad \begin{bmatrix} \tilde{\Sigma}_f & \frac{\sqrt{d}}{d} \varepsilon_2^T \tilde{\mathcal{S}}_3 & \tilde{\Gamma}_1^T & \tilde{\Pi}_1 & \tilde{\Gamma}_1^T & \tilde{\Pi}_2 \\ * & -\tilde{\Lambda}_1 & 0 & 0 & 0 & 0 \\ * & * & -\zeta_1 \mathcal{I} & 0 & 0 & 0 \\ * & * & * & -\zeta_1^{-1} \mathcal{I} & 0 & 0 \\ * & * & * & * & -\zeta_2 \mathcal{I} & 0 \\ * & * & * & * & * & -\zeta_2^{-1} \mathcal{I} \end{bmatrix} < 0, \quad (50)$$

$$\begin{bmatrix} \tilde{\Sigma}_g & \frac{\sqrt{d}}{d} \varepsilon_2^T \tilde{\mathcal{S}}_3 & \tilde{\Gamma}_1^T & \tilde{\Pi}_1 & \tilde{\Gamma}_1^T & \tilde{\Pi}_2 \\ * & -\tilde{\Lambda}_1 & 0 & 0 & 0 & 0 \\ * & * & -\zeta_1 \mathcal{I} & 0 & 0 & 0 \\ * & * & * & -\zeta_1^{-1} \mathcal{I} & 0 & 0 \\ * & * & * & * & -\zeta_2 \mathcal{I} & 0 \\ * & * & * & * & * & -\zeta_2^{-1} \mathcal{I} \end{bmatrix} < 0, \quad \begin{bmatrix} \tilde{\Sigma}_h & \frac{\sqrt{d}}{d} \varepsilon_1^T \tilde{\mathcal{S}}_2 & \tilde{\Gamma}_1^T & \tilde{\Pi}_1 & \tilde{\Gamma}_1^T & \tilde{\Pi}_2 \\ * & -\tilde{\Lambda}_2 & 0 & 0 & 0 & 0 \\ * & * & -\zeta_1 \mathcal{I} & 0 & 0 & 0 \\ * & * & * & -\zeta_1^{-1} \mathcal{I} & 0 & 0 \\ * & * & * & * & -\zeta_2 \mathcal{I} & 0 \\ * & * & * & * & * & -\zeta_2^{-1} \mathcal{I} \end{bmatrix} < 0, \quad (51)$$

$$\tilde{\mathcal{H}}_{1,(d(t)=d)} > 0, \quad \tilde{\mathcal{H}}_{2,(d(t)=0)} > 0, \quad (52)$$

where other relevant parameters are shown in [Appendix C](#).

Proof. Define:

$$\mathcal{N}_1 = \chi^{-1}, \quad \mathcal{N}_2 = \varepsilon_1 \chi^{-1}, \mathcal{N}_3 = \varepsilon_2 \chi^{-1},$$

$$\tilde{\mathcal{P}}_{d(t)} = \text{diag}\{\chi^T, \chi^T, \chi^T, \chi^T, \chi^T\} \mathcal{P}_{d(t)} \text{diag}\{\chi, \chi, \chi, \chi, \chi\}, \quad \tilde{\mathcal{Q}}_k = \text{diag}\{\chi^T, \chi^T, \chi^T\} \mathcal{Q}_k \text{diag}\{\chi, \chi, \chi\},$$

$$\tilde{\mathcal{H}}_i = \text{diag}\{\chi^T\} \mathcal{H}_i \text{diag}\{\chi\}, \quad \tilde{\mathcal{S}}_k = \text{diag}\{\chi^T, \chi^T\} \mathcal{S}_k \text{diag}\{\chi, \chi\}, \quad \tilde{\mathcal{S}}_m = \text{diag}\{\chi^T, \chi^T, \chi^T\} \mathcal{S}_m \text{diag}\{\chi, \chi, \chi\},$$

$$\tilde{\mathcal{U}}_k = \text{diag}\{\chi^T, \chi^T\} \mathcal{U}_k \text{diag}\{\chi, \chi\}, \quad \tilde{\mathcal{R}}_k = \text{diag}\{\chi^T\} \mathcal{R}_k \text{diag}\{\chi\}, \quad i = 1, 2, 3, 4, \quad k = 1, 2, \quad m = 3, 4.$$

We define $\mathcal{K}_j = \mathcal{Y}_j \chi^{-1}$ to solve the controller gain matrixes.

Pre-multiplying and post-multiplying (21)–(26) by $\text{diag}\{\chi^T, \chi^T, \chi^T, \chi^T, \chi^T\}$ and $\text{diag}\{\chi, \chi, \chi, \chi, \chi\}$, we can get LMIs (47)–(52). \square

4. Illustrative example

A physical network control TTS is considered. At the same time, we will verify the actual truck trailer network control system in two situations to verify the effectiveness of the method proposed in this article.

Example 1. Research the following TTS dynamics equation [27,28,44]:

$$\begin{cases} \dot{x}_1(t) = \frac{4}{L} x_1(t) - \frac{4}{l} u(t) + 0.1 \omega_1(t) + 0.1 \omega_2(t), \\ \dot{x}_2(t) = -\frac{4}{L} x_1(t) + 0.1 \omega_1(t) + 0.1 \omega_2(t), \\ \dot{x}_3(t) = -4 \sin(x_2(t) - \frac{1}{L} x_1(t)), \end{cases}$$

where the schematic diagram of the delayed system is shown in [Fig. 2](#) and setting $L = 5.5$ and $l = 2.8$.

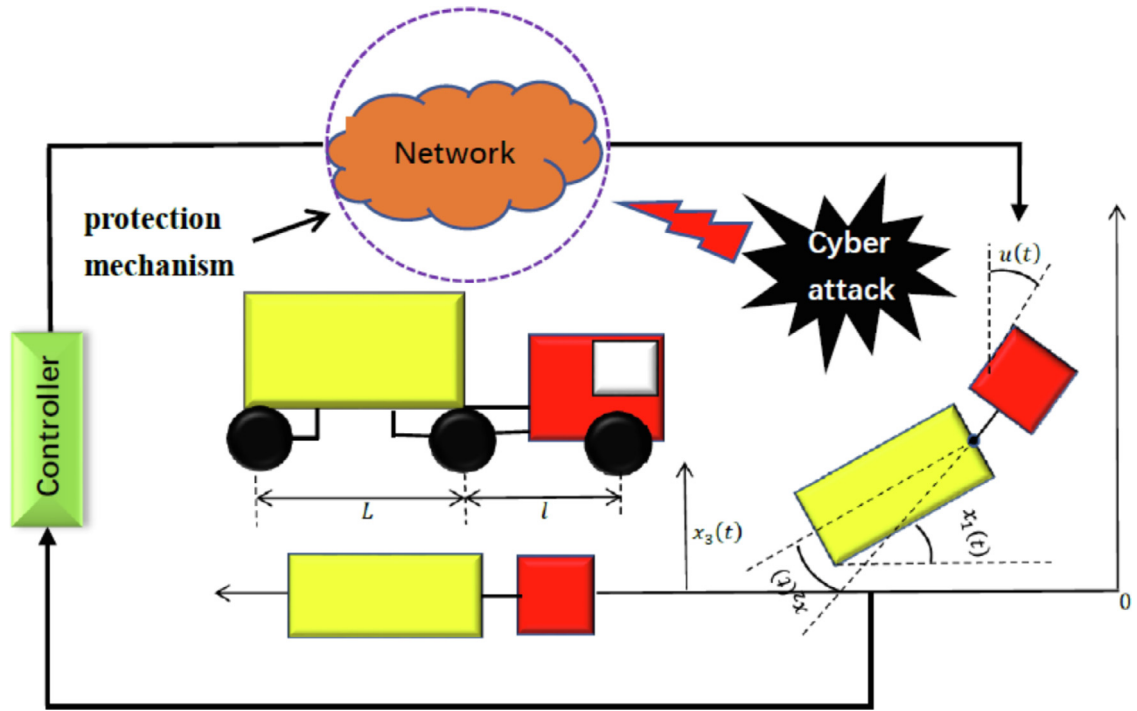


Fig. 2. Truck-trailer model [27].

Set $\vartheta(t) = x_2 - \frac{1}{L}x_1(t)$ and the membership functions as follow

$$h_1(\vartheta(t)) = \begin{cases} \frac{\sin(\vartheta(t)) - \rho\vartheta(t)}{\vartheta(t)(1-\rho)}, & \text{if } \vartheta(t) \neq 0, \\ 1, & \text{if } \vartheta(t) = 0, \end{cases}$$

and $h_2(\vartheta(t)) = 1 - h_1(\vartheta(t))$, where $\rho = \frac{10^{-2}}{\pi}$. Then, apply the following **IF-THEN** rules to define the system.

Plant Rule 1:

IF: $\vartheta(t)$ is about 0,

THEN $\dot{x}(t) = A_1x(t) + B_{11}\tilde{u}(t) + D_{11}\omega(t)$.

Plant Rule 2:

IF: $\vartheta(t)$ is about $\pm\pi$,

THEN $\dot{x}(t) = A_2x(t) + B_{12}\tilde{u}(t) + D_{12}\omega(t)$.

Where the parameters are given as follows:

$$A_{11} = \begin{bmatrix} \frac{4}{L} & 0 & 0 \\ -\frac{4}{L} & 0 & 0 \\ \frac{4}{L} & -4 & 0 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} \frac{4}{L} & 0 & 0 \\ -\frac{4}{L} & 0 & 0 \\ \frac{4\rho}{L} & -4\rho & 0 \end{bmatrix}, \quad B_{11} = B_{12} = \begin{bmatrix} -\frac{10}{7} \\ 0 \\ 0 \end{bmatrix}, \quad D_{21} = D_{22} = \begin{bmatrix} 0.15 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$A_{21} = A_{22} = \begin{bmatrix} 0.1 & 0.2 & 0.2 \\ 0.3 & 0.2 & 0.1 \end{bmatrix}, \quad B_{21} = B_{22} = \begin{bmatrix} 1.2 \\ 1.1 \end{bmatrix}, \quad D_{11} = D_{21} = \begin{bmatrix} 0.15 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$Q = \begin{bmatrix} -0.04 & 0 \\ 0 & -1 \end{bmatrix}, \quad S = \begin{bmatrix} 1.1 & 0.5 \\ 3 & 2 \end{bmatrix}, \quad R = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}.$$

Case A: Safe operation of the system.

When $\mu = 0$, $d = 0$, $F = 0$ and $G = 0$. Table 1 shows the performance level γ under different h_m , which are obtained by Theorem 1. Comparing the results in Table 2, the results obtain in this paper are more conservative than the performance indicators γ obtained by Ge et al. [27] and Wu et al. [28]. Therefore, we can get the conclusion that the Algorithm proposed in this paper has better performance.

Then, when $\varepsilon_1 = \varepsilon_2 = 1$ the controller gains are derived as

$$K_{j1} = \begin{bmatrix} 4.1172 & -0.6913 & 1.2373 \\ 2.3191 & -1.0849 & 3.1948 \end{bmatrix}, \quad K_{j2} = \begin{bmatrix} 4.1172 & -0.6913 & 1.2373 \\ 2.3191 & -1.0849 & 3.1948 \end{bmatrix}.$$

Table 2
Performance level γ for different h_m .

h_m	0.05	0.15	0.25
Wu et al. [28]	0.9971	0.9311	0.8193
Ge et al. [27]	1.3507	1.3211	1.2564
Theorem 1	1.7491	1.6228	1.4081
Improvement	22.7774%	18.5297%	10.7688%

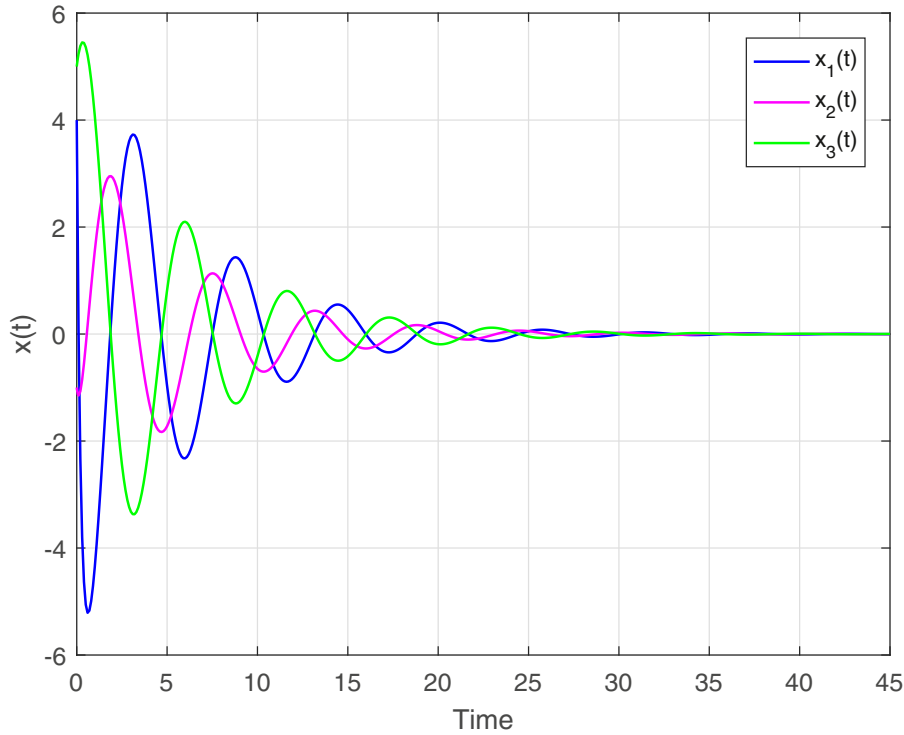


Fig. 3. State of the system via passivity performance.

Table 3
Performance level γ for different h_m .

h_m	0.05	0.15	0.25
Wu et al. [28]	0.9971	0.9311	0.8193
Ge et al. [27]	1.3507	1.3211	1.2564
Case A	1.7491	1.6228	1.4081
Case B	1.5332	1.4670	1.3042

Setting $\gamma = 1.0341$, $h_m = 0.2$, $\omega(t) = [\sin(0.1t)\exp(-0.1t) \sin(0.1t)\exp(-0.1t)]^T$ and $x(0) = [4, -1, 5]^T$. The state running tracks are shown in Fig. 3, and the control input signal is shown in Fig. 4. It can be clearly seen that, with the increase in time, the system tends to be stable without SCAs, which proved the QSDVDS constructed in this paper is reliable and suitable. At the same time, it proves that the system can fully guarantee normal operation without being attacked.

Case B: System under attack.

Setting $\mu = 0$, $d = 0$, $m = n = 1$, $\beta = 0.4$, $F = \text{diag}\{0.3 \ 0.4 \ 0.5\}$, $G = \text{diag}\{0.7 \ 0.6 \ 0.5\}$, when the system is under SCAs, we solve the LMIs through Matlab and we get γ , which are shown in Table 3.

Then, when $\varepsilon_1 = \varepsilon_2 = 1$ the controller gains are derived as

$$\mathcal{K}_{j2} = \begin{bmatrix} 3.9710 & -1.0062 & 2.1542 \\ 3.0141 & -2.0717 & 3.6219 \end{bmatrix}, \quad \mathcal{K}_{j2} = \begin{bmatrix} 3.9710 & -1.0062 & 2.1542 \\ 3.0141 & -2.0717 & 3.6219 \end{bmatrix}.$$

Let the initial condition is $x(0) = [1, -3, -5]^T$, $\omega(t) = [\sin(0.1t)\exp(-0.1t), \sin(0.1t)\exp(-0.1t)]^T$, $f(u(t)) = [0.3u_1(t)\sin(0.3\pi t) \ 0.4u_2(t)\sin(0.4\pi t) \ 0.5u_3(t)\sin(0.5\pi t)]^T$, $g(u(t)) = [0.7u_1(t)\sin(0.7\pi t) \ 0.4u_2(t)\sin(0.6\pi t) \ 0.5u_3(t)\sin(0.5\pi t)]^T$, $\gamma = 0.9118$ and $h_m = 0.2$. When the system is under SCA, in order to more intuitively prove the effective

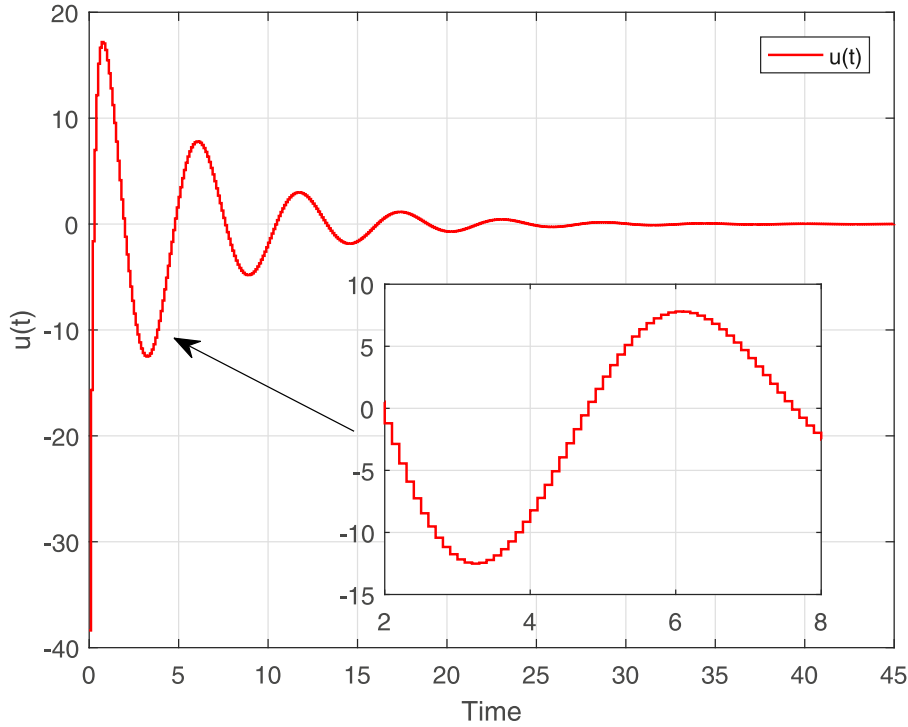


Fig. 4. Control input of the system via passivity performance.

performance of the controller, the system running trajectory is shown in Fig. 5, and the system running trajectory without a controller is shown in Fig. 6. In addition, in Fig. 5, we also provide the system that suffers from the network frequency distribution diagram. It is evident from Fig. 5 that when the system is SCA, the trajectory of the dynamic response will fluctuate slightly. Therefore, the system can effectively reach a stable state under the controller.

In order to better study the dynamic performance of the attack function $f(u(t))$ and its impact on the system when the system is under attack, we give the dynamic response of the attack function $f(u(t))$ as shown in Fig. 7. In addition, when an attack occurs, the VDS we have established will automatically start, and the attack will happen at any time, and the designed VDS will also happen at any time. From Fig. 8, we can know that the dynamic response of the defense function $g(u(t))$ tends to be stable, while the state of the attack function $f(u(t))$ also tends to be stable. The VDS we designed is practical and feasible.

Finally, we show the curve of the control input $u(t)$ as shown in Fig. 9.

5. Conclusion

This paper has mainly focused on the dissipative analysis issue of T-S fuzzy NCSs with SCAs and VDS. Firstly, to obtain more general constraints, a novel TDPRC has been introduced, which can fully excavate TVD information in the received condition. Secondly, an improved BLF has been developed, which can take advantage of the characteristics of the sampling moments. Then, a new criterion and the corresponding control algorithm have been developed by using RCMI, proper integral inequalities, and the LCCM. Furthermore, a desired quantized sample-data and VDS controller under SCAs has been designed to ensure that T-S fuzzy NCSs have been AS and dissipative. Finally, a numerical simulation based on the truck-trailer dynamics equation to certify the theory's feasibility and validity. In the future, we will conduct other research on NCSs, such as: application research of NCSs [37,46]; extending NCSs to switching systems [38,42,45,47] and the others problem of NCSs [43,49,51–53].

Appendix A

$$\begin{aligned}\theta_1(t) &= \int_{t-d(t)}^t \frac{x^T(s)}{d(t)} ds, \theta_2(t) = \int_{t-d}^{t-d(t)} \frac{x^T(s)}{d-d(t)} ds, \\ \theta_3(t) &= \int_{t-d(t)}^t \int_{\theta} \frac{x^T(s)}{d^2(t)} ds d\theta, \theta_4(t) = \int_{t-d}^t \int_{\theta} \frac{x^T(s)}{(d-d(t))^2} ds d\theta,\end{aligned}$$

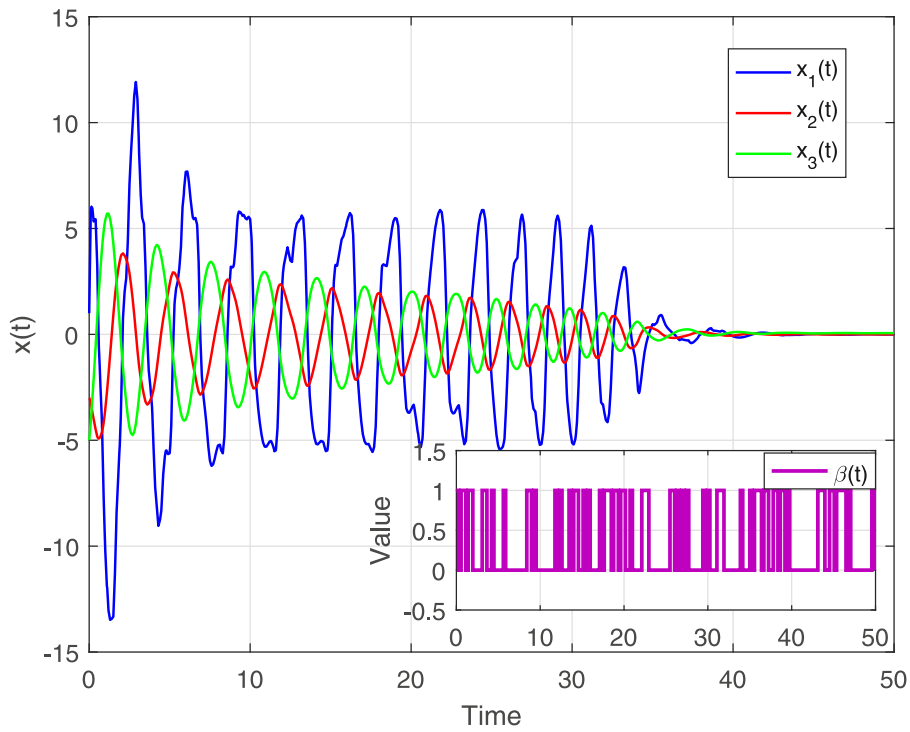


Fig. 5. State of the system via passivity performance.

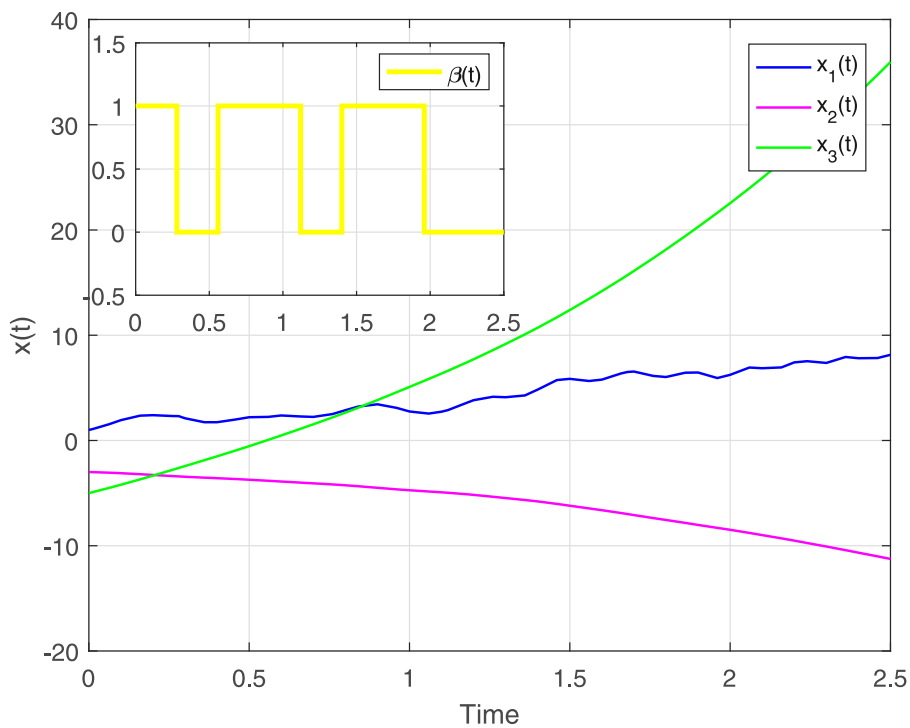


Fig. 6. State of the system without controller via passivity performance.

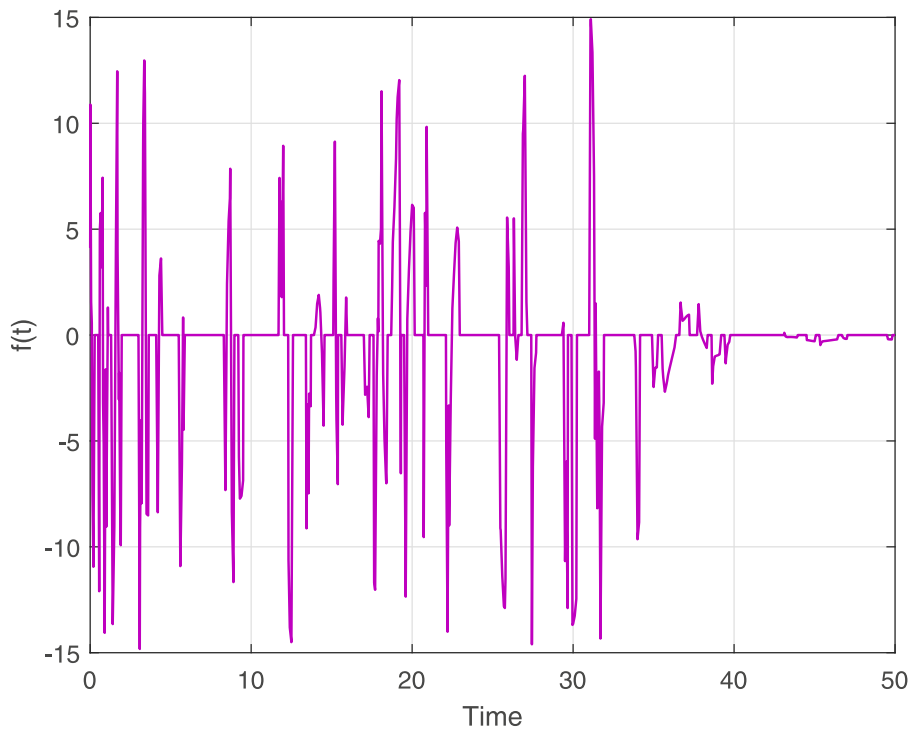


Fig. 7. State of the attack function $f(u(t))$.

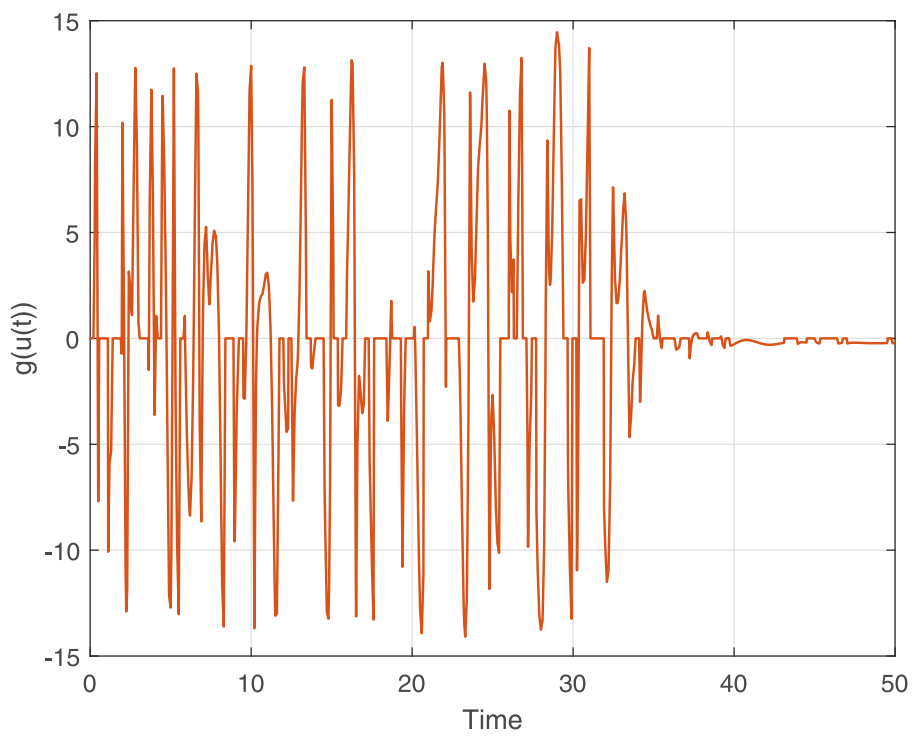


Fig. 8. State of the defense function $g(u(t))$.

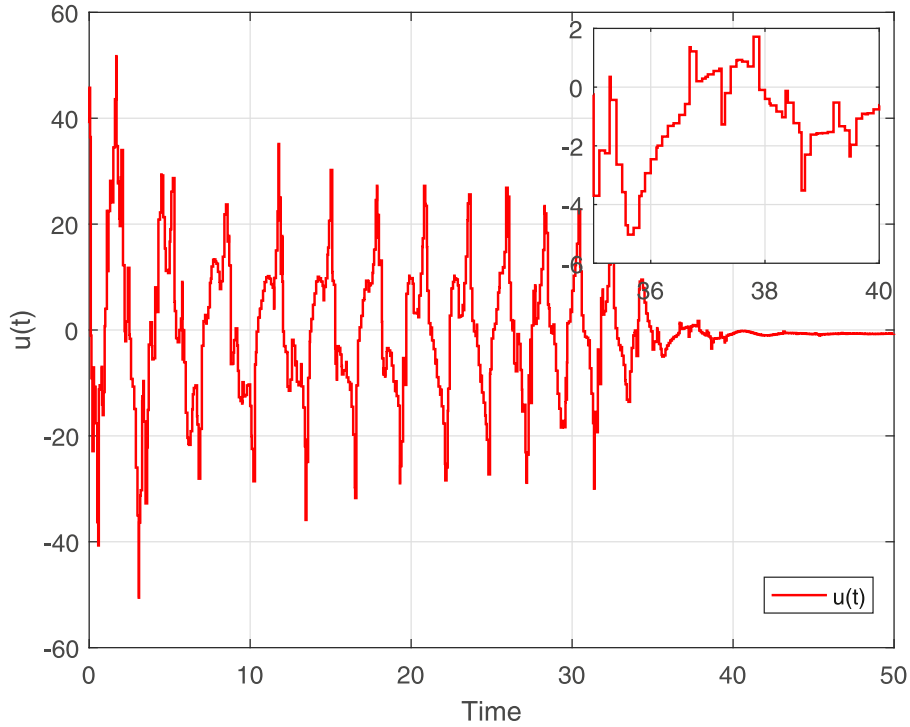


Fig. 9. Control input of the system via passivity performance.

$$\begin{aligned}
 \alpha_1(t) &= \text{col}\{x(t), x(t-d(t)), x(t-d), d(t)\theta_1(t), (d-d(t))\theta_2(t)\}, \\
 \alpha_2(t) &= \text{col}\{x(t), \theta_1(t)\}, \alpha_3(t) = \text{col}\{x(t), \theta_2(t)\}, \\
 \eta(t) &= \text{col}\{x(t), x(t-d(t)), x(t_k)\}, \\
 \xi(t) &= \text{col}\{x(t), x(t-d(t)), x(t-d), \dot{x}(t), \dot{x}(t-d(t)), \dot{x}(t-d), x(t_k), \\
 &\quad w(x(t_k)), f(u(t)), g(u(t)), \theta_1(t), \theta_2(t), \theta_3(t), \theta_4(t), \omega(t)\}, \\
 \mathcal{P}_{d(t)} &= \mathcal{P}_1 - (d-d(t))\mathcal{P}_2, \mathcal{Q}_{d(t)} = \mathcal{Q}_1 - (d-d(t))\mathcal{Q}_2, \\
 \mathcal{H}_{1,d(t)} &= d(t)\mathcal{H}_1 + \mathcal{H}_2, \mathcal{H}_{2,d(t)} = (d-d(t))\mathcal{H}_3 + \mathcal{H}_4.
 \end{aligned}$$

Appendix B

$$\Sigma_{d(t), \dot{d}(t)} = \Xi_1 + \Xi_2 + \Xi_3 + \Xi_4 + \text{Sym}\{L\varphi\} + \gamma e_{12}^T e_{12} - \Psi,$$

$$\Sigma_a = \Sigma_{d(t)=0, \dot{d}(t)=0}^{t=t_k}, \quad \Sigma_b = \Sigma_{d(t)=d, \dot{d}(t)=0}^{t=t_k}, \quad \Sigma_c = \Sigma_{d(t)=d, \dot{d}(t)=\mu}^{t=t_k}, \quad \Sigma_d = \Sigma_{d(t)=0, \dot{d}(t)=\mu}^{t=t_k},$$

$$\Sigma_e = \Sigma_{d(t)=0, \dot{d}(t)=0}^{t=t_{k+1}}, \quad \Sigma_f = \Sigma_{d(t)=d, \dot{d}(t)=0}^{t=t_{k+1}}, \quad \Sigma_g = \Sigma_{d(t)=d, \dot{d}(t)=\mu}^{t=t_{k+1}}, \quad \Sigma_h = \Sigma_{d(t)=0, \dot{d}(t)=\mu}^{t=t_{k+1}},$$

$$\Xi_1 = \text{Sym}\{\Phi_1^T \mathcal{P}_{d(t)} \Phi_2\} + \dot{d}(t) \Phi_1^T \mathcal{P}_2 \Phi_1,$$

$$\Xi_2 = \dot{d}(t) \Phi_3^T \mathcal{U}_1 \Phi_3 - \dot{d}(t) \Phi_4^T \mathcal{U}_2 \Phi_4 + \text{Sym}\{\Phi_3^T \mathcal{U}_1 \Phi_5 + \Phi_4^T \mathcal{U}_2 \Phi_6\},$$

$$\Xi_3 = \Phi_7^T \mathcal{Q}_{t_k, t_{k+1}} \Phi_7 + \text{Sym}\{(t_{k+1} - t)(t - t_k) \Phi_7^T \mathcal{Q}_{d(t)} \Phi_8\},$$

$$\begin{aligned}
 \Xi_4 &= e_4^T [d(t)\mathcal{R}_1 + (d-d(t))\mathcal{R}_2 + d(t)\mathcal{H}_1 + \mathcal{H}_2] e_4 + (1 - \dot{d}(t)) e_5^T \\
 &\quad \times [(d-d(t))\mathcal{H}_3 + \mathcal{H}_4 - d(t)\mathcal{H}_1 - \mathcal{H}_2] e_5 + \Omega_{d(t)},
 \end{aligned}$$

$$\Upsilon_{d(t)} = \frac{d-d(t)}{d^2} \mathcal{E}_1^T \mathcal{S}_4^T \overline{\Lambda}_2^{-1} \mathcal{S}_4 \mathcal{E}_1 + \frac{d(t)}{d^2} \mathcal{E}_2^T \mathcal{S}_3^T \overline{\Lambda}_1^{-1} \mathcal{S}_3 \mathcal{E}_2,$$

$$\Omega_{d(t)} = -\frac{1}{d} \left(\frac{2d-d(t)}{d} \mathcal{E}_1^T \overline{\Lambda}_1 \mathcal{E}_1 + \frac{d-d(t)}{d} \mathcal{E}_2^T \mathcal{S}_3^T \mathcal{E}_1 + \frac{d-d(t)}{d} \mathcal{E}_2^T \mathcal{S}_4^T \mathcal{E}_1 \right. \\ \left. + \frac{d-d(t)}{d} \mathcal{E}_1^T \mathcal{S}_3 \mathcal{E}_2 + \frac{d-d(t)}{d} \mathcal{E}_1^T \mathcal{S}_4 \mathcal{E}_2 + \frac{d-d(t)}{d} \mathcal{E}_2^T \overline{\Lambda}_2 \mathcal{E}_2 \right),$$

$$\mathcal{Q}_{t_k, t_{k+1}} = (t_{k+1} - t) \mathcal{Q}_1 - (t - t_k) \mathcal{Q}_1 + (t - t_k)(d - d(t)) \mathcal{Q}_2 - (t_{k+1} - t)(d - d(t)) \mathcal{Q}_2 + \dot{d}(t)(t_{k+1} - t)(t - t_k) \mathcal{Q}_2,$$

$$\mathcal{L} = e_1^T \mathcal{N}_1 + e_7^T \mathcal{N}_2 + e_4^T \mathcal{N}_3, \varphi = \mathcal{A}_{1i} e_1 + \mathcal{G}_{1i} e_2 + \mathcal{B}_{1i} \mathcal{K}_j e_7 + \mathcal{B}_{1i} \mathcal{K}_j e_8 + \mathcal{D}_{1i} e_{15} - e_4,$$

$$\sigma = \mathcal{A}_{2i} e_1 + \mathcal{G}_{2i} e_2 + \mathcal{B}_{2i} \mathcal{K}_j e_7 + \mathcal{B}_{2i} \mathcal{K}_j e_8 + \beta \mathcal{B}_{2i} \mathcal{K}_j e_7 + (1 - \beta) \mathcal{B}_{2i} \mathcal{K}_j e_8 + \mathcal{D}_{2i} e_{15},$$

$$\Psi = \sigma^T \mathcal{Q} \sigma + 2\sigma^T \mathbb{S} e_{15} + e_{15}^T \mathbb{R} e_{15}, \Gamma_1 = \mathcal{L} \mathcal{B}_{1i} \mathcal{K}_j, \quad \Pi_1 = \beta \mathcal{K}_j^T F^T e_1, \Pi_2 = (1 - \beta) \mathcal{K}_j^T G^T e_1,$$

$$\Phi_1 = \text{col}\{e_1, e_2, e_3, d(t)e_{11}, (d - d(t))e_{12}\}, \Phi_2 = \text{col}\{e_4, (1 - \dot{d}(t))e_2, e_6, e_1 - (1 - \dot{d}(t))e_2, (1 - \dot{d}(t))e_2 - e_3\},$$

$$\Phi_3 = \text{col}\{e_1, e_{11}\}, \quad \Phi_4 = \text{col}\{e_1, e_{12}\}, \Phi_5 = \text{col}\{d(t)e_4, e_1 - (1 - \dot{d}(t))e_2 - \dot{d}(t)e_{11}\},$$

$$\Phi_6 = \text{col}\{(d - d(t))e_4, (1 - \dot{d}(t))e_2 - e_3 + d(t)e_{12}\}, \Phi_7 = \text{col}\{e_1, e_2, e_7\}, \quad \Phi_8 = \text{col}\{e_4, (1 - \dot{d}(t))e_5, \mathbf{0}\},$$

$$F_1 = \text{col}\{\bar{e}_1, \mathbf{0}\}, \quad F_2 = \text{col}\{\mathbf{0}, \bar{e}_4\}, \quad F_3 = \text{col}\{\mathbf{0}, \bar{e}_5\}, \mathcal{E}_1 = \text{col}\{e_1 - e_2, e_1 + e_2 - 2e_{11}, e_1 - e_2 + 6e_{11} - 12e_{13}\},$$

$$\mathcal{E}_2 = \text{col}\{e_2 - e_3, e_2 + e_3 - 2e_{12}, e_2 - e_3 + 6e_{12} - 12e_{14}\}, \Lambda_{1,d(t)} = (1 - \dot{d}(t))\mathcal{R}_1 + \dot{d}(t)\mathcal{R}_2 - \dot{d}(t)\mathcal{H}_1,$$

$$\Lambda_{2,d(t)} = \mathcal{R}_2 + \dot{d}(t)\mathcal{H}_3, \overline{\Lambda}_1 = \begin{bmatrix} \Lambda_{1,d(t)} & 0 & 0 \\ 0 & 3\Lambda_{1,d(t)} & 0 \\ 0 & 0 & 12\Lambda_{1,d(t)} \end{bmatrix}, \overline{\Lambda}_2 = \begin{bmatrix} \Lambda_{2,d(t)} & 0 & 0 \\ 0 & 3\Lambda_{2,d(t)} & 0 \\ 0 & 0 & 12\Lambda_{2,d(t)} \end{bmatrix},$$

$$\bar{e}_i = [0_{n \times (i-1)n} \quad I_{n \times n} \quad 0_{n \times (5-i)}], \quad i = 1, 2, \dots, 5, \quad e_i = [0_{n \times (i-1)n} \quad I_{n \times n} \quad 0_{n \times (15-i)}], \quad i = 1, 2, \dots, 15.$$

Appendix C

$$\Sigma_{d(t), \dot{d}(t)} = \tilde{\Xi}_1 + \tilde{\Xi}_2 + \tilde{\Xi}_3 + \Xi_4 + \text{Sym}\{\tilde{L}\tilde{\varphi}\} + \gamma e_{12}^T e_{12} - \tilde{\Psi},$$

$$\tilde{\Sigma}_a = \tilde{\Sigma}_{d(t)=0, \dot{d}(t)=0}^{t=t_k}, \quad \tilde{\Sigma}_b = \tilde{\Sigma}_{d(t)=d, \dot{d}(t)=0}^{t=t_k}, \quad \tilde{\Sigma}_c = \tilde{\Sigma}_{d(t)=d, \dot{d}(t)=\mu}^{t=t_k}, \quad \tilde{\Sigma}_d = \Sigma_{d(t)=0, \dot{d}(t)=\mu}^{t=t_k},$$

$$\tilde{\Sigma}_e = \tilde{\Sigma}_{d(t)=0, \dot{d}(t)=0}^{t=t_{k+1}}, \quad \tilde{\Sigma}_f = \tilde{\Sigma}_{d(t)=d, \dot{d}(t)=0}^{t=t_{k+1}}, \quad \tilde{\Sigma}_g = \tilde{\Sigma}_{d(t)=d, \dot{d}(t)=\mu}^{t=t_{k+1}}, \quad \tilde{\Sigma}_h = \tilde{\Sigma}_{d(t)=0, \dot{d}(t)=\mu}^{t=t_{k+1}},$$

$$\tilde{\Xi}_1 = \text{Sym}\{\Phi_1^T \tilde{\mathcal{P}}_{d(t)} \Phi_2\} + \dot{d}(t) \Phi_1^T \tilde{\mathcal{P}}_2 \Phi_1,$$

$$\tilde{\Xi}_2 = \dot{d}(t) \Phi_3^T \tilde{\mathcal{U}}_1 \Phi_3 - \dot{d}(t) \Phi_4^T \tilde{\mathcal{U}}_2 \Phi_4 + \text{Sym}\{\Phi_3^T \tilde{\mathcal{U}}_1 \Phi_5 + \Phi_4^T \tilde{\mathcal{U}}_2 \Phi_6\},$$

$$\tilde{\Xi}_3 = \Phi_7^T \tilde{\mathcal{Q}}_{t_k, t_{k+1}} \Phi_7 + \text{Sym}\{(t_{k+1} - t)(t - t_k) \Phi_7^T \tilde{\mathcal{Q}}_{d(t)} \Phi_8\},$$

$$\tilde{\Xi}_4 = e_4^T [d(t)\tilde{\mathcal{R}}_1 + (d - d(t))\tilde{\mathcal{R}}_2 + d(t)\tilde{\mathcal{H}}_1 + \tilde{\mathcal{H}}_2] e_4 + (1 - \dot{d}(t)) e_5^T \\ \times [(d - d(t))\tilde{\mathcal{H}}_3 + \tilde{\mathcal{H}}_4 - d(t)\tilde{\mathcal{H}}_1 - \tilde{\mathcal{H}}_2] e_5 + \tilde{\Omega}_{d(t)},$$

$$\tilde{\Upsilon}_{d(t)} = \frac{d-d(t)}{d^2} \varepsilon_1^T \tilde{S}_4^T \tilde{\Lambda}_2^{-1} \tilde{S}_4 \varepsilon_1 + \frac{d(t)}{d^2} \varepsilon_2^T \tilde{S}_3^T \tilde{\Lambda}_1^{-1} \tilde{S}_3 \varepsilon_2,$$

$$\tilde{\Omega}_{d(t)} = -\frac{1}{d} \left(\frac{2d-d(t)}{d} \varepsilon_1^T \tilde{\Lambda}_1 \varepsilon_1 + \frac{d-d(t)}{d} \varepsilon_2^T \tilde{S}_3^T \varepsilon_1 + \frac{d-d(t)}{d} \varepsilon_2^T \tilde{S}_4^T \varepsilon_1 \right. \\ \left. + \frac{d-d(t)}{d} \varepsilon_1^T \tilde{S}_3 \varepsilon_2 + \frac{d-d(t)}{d} \varepsilon_1^T \tilde{S}_4 \varepsilon_2 + \frac{d-d(t)}{d} \varepsilon_2^T \tilde{\Lambda}_2 \varepsilon_2 \right),$$

$$\tilde{Q}_{t_k, t_{k+1}} = (t_{k+1} - t) \tilde{Q}_1 - (t - t_k) \tilde{Q}_1 + (t - t_k)(d - d(t)) \tilde{Q}_2 \\ - (t_{k+1} - t)(d - d(t)) \tilde{Q}_2 + \dot{d}(t)(t_{k+1} - t)(t - t_k) \tilde{Q}_2,$$

$$\tilde{L} = e_1^T + \varepsilon_1 e_7^T + \varepsilon_2 e_4^T, \tilde{\varphi} = \mathcal{A}_{1i} \chi e_1 + \mathcal{G}_{1i} \chi e_2 + \mathcal{B}_{1i} \mathcal{V}_j e_7 + \mathcal{B}_{1i} \mathcal{V}_j e_8 + \mathcal{D}_{1i} \chi e_{15} - \chi e_4,$$

$$\tilde{\sigma} = \mathcal{A}_{2i} \chi e_1 + \mathcal{G}_{2i} \chi e_2 + \mathcal{B}_{2i} \mathcal{V}_j e_7 + \mathcal{B}_{2i} \mathcal{V}_j e_8 + \beta \mathcal{B}_{2i} \mathcal{V}_j e_7 + (1 - \beta) \mathcal{B}_{2i} \mathcal{V}_j e_8 + \mathcal{D}_{2i} \chi e_{15},$$

$$\tilde{\Psi} = \tilde{\sigma}^T \tilde{Q} \tilde{\sigma} + 2 \tilde{\sigma}^T \mathbb{S} e_{15} + e_{15}^T \mathbb{R} e_{15}, \tilde{\Gamma}_1 = \mathcal{L} \mathcal{B}_{1i} \mathcal{V}_j, \tilde{\Pi}_1 = \beta \mathcal{V}_j^T F^T e_1, \tilde{\Pi}_2 = (1 - \beta) \mathcal{V}_j^T G^T e_1,$$

$$\tilde{\Lambda}_{1,d(t)} = (1 - \dot{d}(t)) \tilde{\mathcal{R}}_1 + \dot{d}(t) \tilde{\mathcal{R}}_2 - \dot{d}(t) \tilde{\mathcal{H}}_1, \tilde{\Lambda}_{2,d(t)} = \tilde{\mathcal{R}}_2 + \dot{d}(t) \tilde{\mathcal{H}}_3,$$

$$\tilde{\tilde{\Lambda}}_1 = \begin{bmatrix} \tilde{\Lambda}_{1,d(t)} & 0 & 0 \\ 0 & 3\tilde{\Lambda}_{1,d(t)} & 0 \\ 0 & 0 & 12\tilde{\Lambda}_{1,d(t)} \end{bmatrix}, \tilde{\tilde{\Lambda}}_2 = \begin{bmatrix} \tilde{\Lambda}_{2,d(t)} & 0 & 0 \\ 0 & 3\tilde{\Lambda}_{2,d(t)} & 0 \\ 0 & 0 & 12\tilde{\Lambda}_{2,d(t)} \end{bmatrix}.$$

References

- [1] N. Shaukat, B. Khan, S.M. Ali, C.A. Mehmood, J. Khan, U. Farid, M. Majid, S.M. Anwar, M. Jawad, Z. Ullah, Survey on electric vehicle transportation within smart grid system, *Renew. Sustain. Energy Rev.* 81 (2018) 1329–1349.
- [2] A. Bemporad, M. Heemels, M. Johansson, *Networked Control Systems*, Springer, London, 2010.
- [3] R.M. Murray, Future directions in control, dynamics, and systems: overview, grand challenges and new courses, *RM Murray Eur. J. Control* 9 (2) (2003) 144–158.
- [4] L. Zhang, H. Gao, O. Kaynak, Network-induced constraints in networked control systems survey, *IEEE Trans. Ind. Inf.* 9 (1) (2013) 403–416.
- [5] W. Zhang, M.S. Branicky, S.M. Phillips, Stability of networked control systems, *IEEE Control Syst.* 21 (1) (2001) 84–99.
- [6] R.A. Gupta, M.Y. Chow, Networked control system: overview and research trends, *IEEE Trans. Ind. Electron.* 57 (7) (2010) 2527–2535.
- [7] X. Ge, F. Yang, Q.L. Han, Distributed networked control systems: a brief overview, *Inf. Sci.* (2015).
- [8] S. Amin, G.A. Schwartz, S.S. Sastry, Security of interdependent and identical networked control systems, *Automatica* 49 (1) (2013) 186–192.
- [9] D. Freirich, E. Fridman, Decentralized networked control of systems with local networks: a time-delay approach, *Automatica* 69 (2016) 201–209.
- [10] C. Peng, J. Zhang, Event-triggered output-feedback h_∞ control for networked control systems with time-varying sampling, *IET Control Theory Appl.* 9 (9) (2015) 1384–1391.
- [11] J.P. Farwell, R. Rohozinski, Stuxnet and the future of cyber war, *Survival* 53 (1) (2011) 23–40.
- [12] E.G. Tian, C. Peng, Memory-based event-triggering h_∞ load frequency control for power systems under deception attacks, *IEEE Trans. Cybern.* (2020), doi:10.1109/TCYB.2020.2972384.
- [13] X. Cai, S.M. Zhong, J. Wang, K.B. Shi, Robust h_∞ control for uncertain delayed T-S fuzzy systems with stochastic packet dropouts, *Appl. Math. Comput.* 385 (2020) 125432.
- [14] H. Yang, M. Shi, Y. Xia, P. Zhang, Security research on wireless networked control systems subject to jamming attacks, *IEEE Trans. Cybern.* 49 (6) (2019) 2022–2031.
- [15] H. Zhang, P. Cheng, L. Shi, J. Chen, Optimal dos attack scheduling in wireless networked control system, *IEEE Trans. Control Syst. Technol.* 24 (3) (2016) 843–852.
- [16] A.Y. Lu, G.H. Yang, Input-to-state stabilizing control for cyberphysical systems with multiple transmission channels under denial-of-service, *IEEE Trans. Automat. Control* 63 (6) (2018) 1813–1820.
- [17] Y. Yuan, F. Sun, Q. Zhu, Resilient control in the presence of dos attack: switched system approach, *Int. J. Control Autom. Syst.* 13 (6) (2015) 1423–1435.
- [18] Z. Pang, G. Liu, Design and implementation of secure networked predictive control systems under deception attacks, *IEEE Trans. Control Syst. Technol.* 20 (5) (2012) 1334–1342.
- [19] D. Ding, Z. Wang, Q. Han, G. Wei, Security control for discrete-time stochastic nonlinear systems subject to deception attacks, *IEEE Trans. Syst. Man Cybern. Syst.* 48(5) 779–789 381 (15) (2018) 1–25.
- [20] Y. Mo, B. Sinopoli, On the performance degradation of cyber-physical systems under stealthy integrity attacks, *IEEE Trans. Automat. Control* 61 (9) (2016) 2618–2624.
- [21] M.H. Xiong, Y.S. Tan, D.S. Du, B.Y. Zhang, S.M. Fei, Observer-based event-triggered output feedback control for fractional-order cyber-physical systems subject to stochastic network attacks, *ISA Trans.* 104 (2020) 15–25.
- [22] P. Lee, A. Clark, L. Bushnell, R. Poovendran, A passivity framework for modeling and mitigating wormhole attacks on networked control systems, *IEEE Trans. Automat. Control* 59 (12) (2018) 3224–3237.
- [23] J.L. Liu, E.G. Tian, X.P. Xie, H. Lin, Distributed event-triggered control for networked control systems with stochastic cyber-attacks, *J. Franklin Inst.* 356 (2019) 10260–10276.
- [24] X.L. Chen, Y.G. Wang, S.L. Hu, Event-based robust stabilization of uncertain networked control systems under quantization and denial-of-service attacks, *Inf. Sci.* 459 (2018) 369–386.

- [25] D. Ding, Z.D. Wang, G.L. Wei, F.E. Alsaadi, Event-based security control for discrete-time stochastic systems, *IET Control Theory Appl.* 10 (15) (2016) 1808–1815.
- [26] J.L. Liu, L.L. Wei, E.G. Tian, S.M. Fei, J. Cao, H_∞ Filtering for networked systems with hybrid-triggered communication mechanism and stochastic cyber attacks, *J. Franklin Inst.* 354 (2017) 8490–8512.
- [27] C. Ge, J.H. Park, C.C. Hua, X.P. Guan, Dissipativity analysis for T-S fuzzy system under memory sampled-data control, *IEEE Trans. Cybern.* (2019) 1–9.
- [28] Z.G. Wu, P. Shi, H. Su, R. Lu, Dissipativity-based sampled-data fuzzy control design and its application to truck-trailer system, *IEEE Trans. Fuzzy Syst.* 23 (5) (2015) 1669–1679.
- [29] X. Cai, J. Wang, S.M. Zhong, K.B. Shi, Y.Q. Tang, Fuzzy quantized sampled-data control for extended dissipative analysis of T-S fuzzy system and its application to WPGSSs, *J. Franklin Inst.* 358 (2) (2021) 1350–1375.
- [30] T.H. Lee, J.H. Park, Stability analysis of sampled-data systems via free-matrix-based time-dependent discontinuous Lyapunov approach, *IEEE Trans. Automat. Control* 62 (7) (2017) 3653–3657.
- [31] C.K. Zhang, Y. He, L. Jiang, M. Wu, Q.G. Wang, An extended reciprocally convex matrix inequality for stability analysis of systems with time-varying delay, *Automatica* 85 (2017) 481–485.
- [32] Z.D. Wang, B. Shen, H.S. Shu, G.L. Wei, Quantized h_∞ control for nonlinear stochastic time-delay systems with missing measurements, *IEEE Trans. Autom. Control* 57 (6) (2012) 1431–1444.
- [33] Z.P. Wang, H.N. Wu, J.L. Wang, H.X. Li, Quantized sampled-data synchronization of delayed reaction-diffusion neural networks under spatially point measurements, *IEEE Trans. Cybern.* (2020), doi:10.1109/TCYB.2019.2960094.
- [34] H.Y. Song, L. Yu, D. Zhang, W.A. Zhang, Finite-time h_∞ control for a class of discrete-time switched time-delay systems with quantized feedback, *Commun. Nonlinear Sci. Numer. Simul.* 17 (12) (2012) 4802–4814.
- [35] F. Long, C.K. Zhang, L. Jiang, Y. He, M. Wu, Stability analysis of systems with time-varying delay via improved Lyapunov-Krasovskii functional, *IEEE Trans. Syst. Man Cybern. Syst.* (2019) 1–10.
- [36] Z.J. Chen, Y.G. Zhang, Q.K. Kong, T. Fang, J. Wang, Observer-based h_∞ control for persistent dwell-time switched networked nonlinear systems under packet dropout, *Appl. Math. Comput.* 415 (15) (2022) 126679.
- [37] X. Cai, K.B. Shi, S.M. Zhong, X.R. Pang, Dissipative sampled-data control for high-speed train systems with quantized measurements, *IEEE Trans. Intell. Transp. Syst.* (2021) 1–12, doi:10.1109/ITITS.2021.3052940.
- [38] J.W. Xia, G.L. Chen, J.H. Park, H. Shen, G.M. Zhuang, Dissipativity-based sampled-data control for fuzzy switched Markovian jump systems, *IEEE Trans. Fuzzy Syst.* 29 (6) (2021) 1325–1339.
- [39] H. Asai, S. Tanaka, K. Uegima, Linear regression analysis with fuzzy model, *IEEE Trans. Syst. Man Cybern. Syst.* (1982) 903–907.
- [40] X. Cai, J. Wang, K.B. Shi, S.M. Zhong, T.T. Jiang, Quantized dissipative control based on T-S fuzzy model for wind generation systems, *ISA Trans.* 96 (2021) 570–583.
- [41] H.T. Wang, X.Y. Chen, J. Wang, H_∞ sliding mode control for PDT-switched nonlinear systems under the dynamic event-triggered mechanism, *Appl. Math. Comput.* 412 (1) (2022) 126474.
- [42] H. Shen, X.H. Hu, J. Wang, J.D. Cao, W.H. Qian, Non-fragile h_∞ synchronization for Markov jump singularly perturbed coupled neural networks subject to double-layer switching regulation, *IEEE Trans. Neural Netw. Learn. Syst.* (2021), doi:10.1109/TNNLS.2021.3107607.
- [43] J.W. Xia, B.M. Li, S.F. Su, W. Sun, H. Shen, Finite-time command filtered event-triggered adaptive fuzzy tracking control for stochastic nonlinear systems, *IEEE Trans. Fuzzy Syst.* 29 (7) (2020) 1815–1825.
- [44] X. Cai, K.B. Shi, K. She, S.M. Zhong, J. Wang, H.C. Yan, New results for T-S fuzzy systems with hybrid communication delays, *Fuzzy Sets Syst.* (2021), doi:10.1016/j.fss.2021.08.018.
- [45] J. Wang, Z.G. Huang, Z.G. Wu, J.D. Cao, H. Shen, Extended dissipative control for singularly perturbed PDT switched systems and its application, *IEEE Trans. Circuits Syst. I Regul. Pap.* 67 (12) (2020) 5281–5289.
- [46] J.C.L. Chan, T.H. Lee, C.P. Tan, A sliding mode observer for robust fault reconstruction in a class of nonlinear non-infinitely observable descriptor systems, *Nonlinear Dyn.* 101 (2) (2020) 1023–1036.
- [47] H.T. Wang, X.Y. Chen, J. Wang, H_∞ sliding mode control for PDT-switched nonlinear systems under the dynamic event-triggered mechanism, *Appl. Math. Comput.* 412 (1) (2021) 126474.
- [48] T.H. Lee, J.H. Park, New methods of fuzzy sampled-data control for stabilization of chaotic systems, *IEEE Trans. Syst. Man Cybern. Syst.* 4 (12) (2018) 2026–2034.
- [49] J. Wang, J.W. Xia, H. Shen, M.P. Xing, J.H. Park, H_∞ synchronization for fuzzy Markov jump chaotic systems with piecewise-constant transition probabilities subject to PDT switching rule, *IEEE Trans. Fuzzy Syst.* 29 (10) (2021) 3082–3092.
- [50] T.H. Lee, M.J. Park, J.H. Park, An improved stability criterion of neural networks with time-varying delays in the form of quadratic function using novel geometry-based conditions, *Appl. Math. Comput.* 404 (2021) 126226.
- [51] K.B. Shi, J. Wang, S.M. Zhong, Y.Y. Tang, J. Cheng, Non-fragile memory filtering of T-S fuzzy delayed neural networks based on switched fuzzy sampled-data control, *Fuzzy Sets Syst.* 394 (2020) 40–644.
- [52] B. Wu, X.H. Chang, X.D. Zhao, Fuzzy h_∞ output feedback control for nonlinear NCSs with quantization and stochastic communication protocol, *IEEE Trans. Fuzzy Syst.* 29 (9) (2021) 2623–2634.
- [53] J. Wang, C.Y. Yang, J.W. Xia, Z.G. Wu, H. Shen, Observer-based sliding mode control for networked fuzzy singularly perturbed systems under weighted try-once-discard protocol, *IEEE Trans. Fuzzy Syst.* (2021), doi:10.1109/TFUZZ.2021.3070125.