Automatica 45 (2009) 517–524

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

A discrete delay decomposition approach to stability of linear retarded and neutral systems[☆]

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ARTICLE INFO

Article history:
Received 2 April 2007
Received in revised form
8 June 2008
Accepted 25 August 2008
Available online 9 December 2008

Keywords:
Time-delay
Retarded systems
Neutral systems
Stability
Linear matrix inequality (LMI)

ABSTRACT

This paper is concerned with stability of linear time-delay systems of both retarded and neutral types by using some *new* simple quadratic Lyapunov–Krasovskii functionals. These Lyapunov–Krasovskii functionals consist of two parts. One part comes from some existing Lyapunov–Krasovskii functionals employed in [Han, Q.-L. (2005a). Absolute stability of time-delay systems with sector-bounded nonlinearity. Automatica, 41, 2171–2176; Han, Q.-L. (2005b). A new delay-dependent stability criterion for linear neutral systems with norm-bounded uncertainties in all system matrices. International Journal of Systems Science, 36, 469–475]. The other part is constructed by uniformly dividing the discrete delay interval into multiple segments and choosing proper functionals with different weighted matrices corresponding to different segments. Then using these new simple quadratic Lyapunov–Krasovskii functionals, some new discrete delay-dependent stability criteria are derived for both retarded systems and neutral systems. It is shown that these criteria for retarded systems and neutral systems are always less conservative than the ones in Han (2005a) and Han (2005b) cited above, respectively. Numerical examples also show that the results obtained in this paper significantly improve the estimate of the discrete delay limit for stability over some other existing results.

1. Introduction

Time-delays are frequently encountered in many fields of science and engineering, including communication network, manufacturing systems, biology, economy and other areas (Bellman & Cooke, 1963; Brayton, 1966; Gu, Kharitonov, & Chen, 2003; Hale & Verduyn Lunel, 1993; Kolmanovskii & Myshkis, 1999; Niculescu, 2001). In the last three decades, the stability of linear time-delay systems of both retarded and neutral types has been widely studied and many significant results based on frequency domain approaches and time domain approaches have been reported, see, e.g., Chen (1995), Chen, Gu, and Nett (1995), Chiasson (1985), Fridman and Shaked (2002), Gu (2001), Han (2002, 2005a,b), He, Wu, She, and Liu (2004), Han and Yue (2007), Park (1999), Hertz, Jury, and Zeheb (1984), Hale, Infante, and Tsen (1985), Lien and Chen (2003), Moon, Park, Kwon, and Lee (2001) and Wu, He, and She (2004). Current efforts can be divided into two classes: namely, frequency domain approaches and time domain approaches. Frequency domain approach-based stability criteria have been long in existence (Chen, 1995; Chen et al., 1995; Chiasson, 1985; Hale et al., 1985; Hertz et al., 1984). For some recent developments in the frequency domain, we mention frequency-sweeping and matrix-pencils methods (Chen, 1995; Chen et al., 1995). Both frequency-sweeping criteria and matrix-pencils-based criteria are necessary and sufficient conditions for delay-dependent and delay-independent stability for systems with commensurate delays, the reader is referred to Chapter 2 in Gu et al. (2003). Compared with frequency domain approaches, time domain approaches have two advantages: first, one can easily handle nonlinearities and time-varying uncertainties; second, on the basis of time domain stability criteria, one can easily address issues about controller synthesis and filter design for time-delay systems.

In the time domain approach, the direct Lyapunov method is a powerful tool for studying stability of linear time-delay systems of both retarded and neutral types. The existing results are either based on the Lyapunov–Krasovskii Theorem or the Razumikhin Theorem. The results derived by using the Lyapunov–Krasovskii Theorem are usually less conservative than those obtained by using the Razumikhin Theorem (Gu et al., 2003). In this paper, we will focus on the Lyapunov–Krasovskii method and propose some new Lyapunov–Krasovskii functionals to study the stability of linear retarded and neutral systems. We begin with linear retarded systems.

[☆] This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Hitay Ozbay under the direction of Editor Ian R. Petersen.

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doi:10.1016/j.automatica.2008.08.005

Consider the system described by

$$\dot{x}(t) = Ax(t) + Bx(t-h), \tag{1}$$

with

$$x(\theta) = \phi(\theta), \quad \forall \theta \in [-h, 0], \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the state vector of the system; $h \geq 0$ is the constant *discrete delay*; $\phi(\cdot)$ is a continuous vector valued initial function; $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times n}$ are constant matrices.

In the existing literature, there are *complete* quadratic Lyapunov–Krasovskii functionals and *simple* quadratic Lyapunov–Krasovskii functionals for estimating the maximum allowed time-delay bound the system can tolerate and still maintain stability (Gu et al., 2003; Niculescu, 2001). Notice that the existence of a *complete* quadratic Lyapunov–Krasovskii functional is a sufficient and necessary condition for asymptotic stability of the system described by (1) and (2) (Section 5.6, Gu et al. (2003)). Using the *complete* Lyapunov–Krasovskii functionals (Gu, 2001; Gu et al., 2003), one can obtain the maximum allowed time-delay bound which is very close to the analytical delay limit for stability. Throughout this paper, we assume that the system described by (1) and (2) is asymptotically stable at $h = 0$. Using the continuity property of retarded systems (Theorem 1.5, p. 18, Gu et al. (2003)), we define the analytical delay limit for stability of the system as

$$h^{\text{analytical}} := \min\{h \geq 0 | \det(j\omega I - A - Be^{-jh\omega}) = 0 \text{ for some } \omega \in \mathbb{R}\}.$$

In Section 2.1 (p. 33, Gu et al. (2003)), the analytical delay limit for stability is called the *delay margin* of the system. The frequency at $h^{\text{analytical}}$ such that $\det(j\omega I - A - Be^{-jh\omega}) = 0$ represents the first contact or crossing the characteristic roots from the stable region to the unstable one. One can use the *direct method* in Section 2.2.3 (Gu et al., 2003) to calculate the analytical delay limit for stability.

Notice also that the existence of a *simple* quadratic Lyapunov–Krasovskii functional, is only a sufficient condition for asymptotic stability of the system described by (1) and (2) (Gu et al., 2003; Niculescu, 2001). Employing *simple* quadratic Lyapunov–Krasovskii functionals usually yields conservative results. However, the results derived by using the *simple* quadratic Lyapunov–Krasovskii functionals can be easily applied to controller synthesis and filter design. Hence, it is still an attractive topic for finding some simple quadratic Lyapunov–Krasovskii functionals, by which one can derive less conservative results.

In order to clearly state the motivation and for comparison, we use a well-studied numerical example for system (1) with

$$A = \begin{pmatrix} -2 & 0 \\ 0 & -0.9 \end{pmatrix}, \qquad B = \begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix}.$$

Similar to the case of a scalar delay system (Example 2.4, p. 40, Gu et al. (2003)), using the *direct method* (Gu et al., 2003), we calculate the analytical delay limit for stability for the numerical example as $h^{\text{analytical}} = 6.17258$.

In the existing literature, in order to derive a delay-dependent stability criterion, one transforms system (1) into a system with a distributed delay, i.e.

$$\dot{x}(t) = (A+B)x(t) - B\int_{t-h}^{t} [Ax(\xi) + Bx(\xi - h)]\mathrm{d}\xi. \tag{3}$$

Choose a Lyapunov function

$$V(t, x_t) = x^{\mathrm{T}}(t)Px(t), \quad P = P^{\mathrm{T}} > 0, \tag{4}$$

and apply the Razumikhin Theorem to obtain $h_{\max} = 0.9041$. As pointed out by Gu et al. (2003), for this example, the stability of system (1) is equivalent to that of system (3), and the conservatism of the result is due to the application of the Razumikhin Theorem (Example 5.3, p. 164, Gu et al. (2003)). For the system (1) with different system's matrices, the model transformation (3) may induce additional dynamics (Gu et al., 2003). To reduce the conservatism, instead of transforming system (1) into (3), one transforms it into

$$\dot{x}(t) = (A+B)x(t) - B\int_{t-h}^{t} \dot{x}(\xi)\mathrm{d}\xi. \tag{5}$$

Then choosing a Lyapunov–Krasovskii functional

$$V(t, x_t) = x^{\mathrm{T}}(t)Px(t) + \int_{t-h}^{t} x^{\mathrm{T}}(\xi)Qx(\xi)\mathrm{d}\xi + \int_{-h}^{0}\int_{t+\theta}^{t} x^{\mathrm{T}}(\xi)B^{\mathrm{T}}RBx(\xi)\mathrm{d}\xi\mathrm{d}\theta, \tag{6}$$

where $P = P^{\mathrm{T}} > 0$, $Q = Q^{\mathrm{T}} > 0$, $R = R^{\mathrm{T}} > 0$, and using the bounding technique for some cross term yield $h_{\max} = 4.3588$ (Park, 1999). This result was also derived by decomposing delayed term matrix as $B = B_1 + B_2$ in Han (2002). In Fridman and Shaked (2002), the authors used descriptor system model transformation and bounding technique for cross terms (Moon et al., 2001; Park, 1999) to obtain $h_{\max} = 4.4721$. In He et al. (2004), the authors introduce some slack variables (free-weighting matrices) to derive the same result.

In Han (2005a), the author avoided using model transformation on system (1) and proposed the following Lyapunov–Krasovskii functional

$$V(t, x_t) = x^{\mathrm{T}}(t)Px(t) + \int_{t-h}^{t} x^{\mathrm{T}}(\xi)Qx(\xi)\mathrm{d}\xi + \int_{t-h}^{t} (h - t + \xi)\dot{x}^{\mathrm{T}}(\xi)(hR)\dot{x}(\xi)\mathrm{d}\xi, \tag{7}$$

where $P = P^{\mathrm{T}} > 0$, $Q = Q^{\mathrm{T}} > 0$, and $R = R^{\mathrm{T}} > 0$. Instead of using the bounding technique for some cross term, the author used the following bounding

$$-\int_{t-h}^{t} \dot{x}^{\mathrm{T}}(\xi)(hR)\dot{x}(\xi)\mathrm{d}\xi \leq \begin{pmatrix} x(t) \\ x(t-h) \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} -R & R \\ R & -R \end{pmatrix} \begin{pmatrix} x(t) \\ x(t-h) \end{pmatrix}, \tag{8}$$

to derive the maximum allowed delay bound as $h_{\max} = 4.4721$. Compared with the above mentioned results, the most advantage of the result in Han (2005a) is that the stability condition which was formulated in an LMI form, was very simple and easily applied to controller design, and did only include the Lyapunov–Krasovskii functional matrices variables P, Q and R, which means that no additional matrix variable was involved. From the computation point of view, it is clear to see that testing the result in Han (2005a) is less time-consuming than some existing results in the literature. *However*, the result $h_{\max} = 4.4721$ is not close enough to the analytical delay limit for stability $h^{\text{analytical}} = 6.17258$ and work needs to be done to arrive at a value much closer to the analytical delay limit for stability. Therefore, the natural question is: How can one improve the result by using *simple* quadratic Lyapunov–Krasovskii functionals? The answer to this question will significantly enhance the stability analysis and controller synthesis of time-delay systems. It seems that using the existing *simple* quadratic Lyapunov–Krasovskii functionals can not realize the outcome even if one introduces more *additional* matrices variables apart from Lyapunov–Krasovskii functional matrices variables. One way to solve the problem is to choose a *new* simple quadratic

Lyapunov–Krasovskii functional. For this purpose, we propose the following *new* simple quadratic Lyapunov–Krasovskii functional

$$\begin{aligned} V(t,x_t) &= x^{\mathrm{T}}(t)Px(t) + \int_{t-h}^{t} x^{\mathrm{T}}(\xi)Qx(\xi)\mathrm{d}\xi \\ &\quad + \int_{t-h}^{t}(h-t+\xi)\dot{x}^{\mathrm{T}}(\xi)(hR)\dot{x}(\xi)\mathrm{d}\xi \\ &\quad + \int_{t-\frac{h}{2}}^{t} y^{\mathrm{T}}(\xi)Sy(\xi)\mathrm{d}\xi \\ &\quad + \int_{t-\frac{h}{2}}^{t}\left(\frac{h}{2}-t+\xi\right)\dot{x}^{\mathrm{T}}(\xi)\left(\frac{h}{2}W\right)\dot{x}(\xi)\mathrm{d}\xi \end{aligned} \tag{9}$$

where x_t is defined as $x_t = x(t+\theta)$, $\forall\theta \in [-h, 0]$ and $P = P^{\mathrm{T}} > 0$, $Q = Q^{\mathrm{T}} > 0$, $R = R^{\mathrm{T}} > 0$, $W = W^{\mathrm{T}} \geq 0$, $S = S^{\mathrm{T}} = \begin{pmatrix} S_{11} & S_{12} \\ S_{12}^{\mathrm{T}} & S_{22}\end{pmatrix} \geq 0$, and $y^{\mathrm{T}}(\xi) = \left(x^{\mathrm{T}}(\xi) \quad x^{\mathrm{T}}(\xi - \frac{h}{2})\right)$. In Section 2.1, by using (9), one can derive a delay-dependent stability criterion, Proposition 2 in this paper, which is always less conservative than the one derived in Han (2005a) (see Proposition 4). Applying this new criterion, one obtains the maximum allowed delay bound as $h_{\max} = 5.7175$, which significantly improves the result $h_{\max} = 4.4721$ in the above mentioned references. One can clearly see that we have made a very significant step towards the analytical delay limit for stability of the system.

The idea of constructing the Lyapunov–Krasovskii functional (9) is that first, we keep all the three terms in (7); second, we uniformly divide the discrete delay interval $[-h, 0]$ into two subintervals $[-h, -\frac{h}{2}]$ and $[-\frac{h}{2}, 0]$, and then on the subintervals we choose different functionals. For the second aspect, more specifically, notice that

$$\begin{aligned} &\int_{t-\frac{h}{2}}^{t} y^{\mathrm{T}}(\xi)Sy(\xi)\mathrm{d}\xi \\ &= \int_{t-\frac{h}{2}}^{t}\begin{pmatrix} x^{\mathrm{T}}(\xi) & x^{\mathrm{T}}\left(\xi-\frac{h}{2}\right)\end{pmatrix}\begin{pmatrix} S_{11} & S_{12} \\ S_{12}^{\mathrm{T}} & S_{22}\end{pmatrix}\begin{pmatrix} x(\xi) \\ x\left(\xi-\frac{h}{2}\right)\end{pmatrix}\mathrm{d}\xi \\ &= \int_{t-\frac{h}{2}}^{t} x^{\mathrm{T}}(\xi)S_{11}x(\xi)\mathrm{d}\xi + \int_{t-h}^{t-\frac{h}{2}} x^{\mathrm{T}}(\xi)S_{22}x(\xi)\mathrm{d}\xi \\ &\quad + 2\int_{t-\frac{h}{2}}^{t} x^{\mathrm{T}}(\xi)S_{12}x\left(\xi-\frac{h}{2}\right)\mathrm{d}\xi \end{aligned}$$

from which one can see that we choose a functional $\int_{t-\frac{h}{2}}^{t} x^{\mathrm{T}}(\xi) S_{11}x(\xi)\mathrm{d}\xi$ on the subinterval $[-\frac{h}{2}, 0]$, a functional $\int_{t-h}^{t-\frac{h}{2}} x^{\mathrm{T}}(\xi)S_{22} x(\xi)\mathrm{d}\xi$ on the subinterval $[-h, -\frac{h}{2}]$, and a "cross term" functional $2\int_{t-\frac{h}{2}}^{t} x^{\mathrm{T}}(\xi)S_{12}x(\xi - \frac{h}{2})\mathrm{d}\xi$ corresponding to both subintervals $[-\frac{h}{2}, 0]$ and $[-h, -\frac{h}{2}]$. Notice also that parallel to the term $\int_{t-h}^{t}(h-t+\xi)\dot{x}^{\mathrm{T}}(\xi)(hR)\dot{x}(\xi)\mathrm{d}\xi$, whose role leads to a sufficient stability condition depending on h, we introduce the term $\int_{t-\frac{h}{2}}^{t}(\frac{h}{2}-t+\xi)\dot{x}^{\mathrm{T}}(\xi)(\frac{h}{2}W)\dot{x}(\xi)\mathrm{d}\xi$ to derive the sufficient stability condition, which depends on $\frac{h}{2}$.

If we set $S_{11} = S_{22} = S_e \geq 0$ and $S_{12} = 0$, then

$$\begin{aligned} &\int_{t-\frac{h}{2}}^{t}\begin{pmatrix} x^{\mathrm{T}}(\xi) & x^{\mathrm{T}}\left(\xi-\frac{h}{2}\right)\end{pmatrix}\begin{pmatrix} S_{11} & S_{12} \\ S_{12}^{\mathrm{T}} & S_{22}\end{pmatrix}\begin{pmatrix} x(\xi) \\ x\left(\xi-\frac{h}{2}\right)\end{pmatrix}\mathrm{d}\xi \\ &= \int_{t-h}^{t} x^{\mathrm{T}}(\xi)S_e x(\xi)\mathrm{d}\xi \end{aligned}$$

which can be absorbed by the term $\int_{t-h}^{t} x^{\mathrm{T}}(\xi)Q_{new}x(\xi)\mathrm{d}\xi$ by introducing a new matrix variable $Q_{new} = Q + S_e$. Clearly, compared with $S_{11} = S_{22} = S_e \geq 0$ and $S_{12} = 0$, the requirement $S = S^{\mathrm{T}} = \begin{pmatrix} S_{11} & S_{12} \\ S_{12}^{\mathrm{T}} & S_{22}\end{pmatrix} \geq 0$ gives a more choice of S_{11}, S_{12}, and S_{22}. Hence, it is expected that the resulting stability criteria will be less conservative.

Since the discrete delay interval $[-h, 0]$ is uniformly divided into two subintervals $[-h, -\frac{h}{2}]$ and $[-\frac{h}{2}, 0]$, for the purpose of distinguishing from the existing methods, in what follows, we refer to the approach based on the Lyapunov–Krasovskii functional (9) as *a discrete delay bi-decomposition approach*.

After understanding the *discrete delay bi-decomposition approach*, one can easily extend the approach to a *discrete delay N-decomposition approach* by uniformly dividing the discrete delay interval $[-h, 0]$ into N-subintervals and choosing different functional on these subintervals, which will be discussed in detail in Section 2.2.

Parallel to the system described by (1) and (2), we now consider stability of the neutral system

$$\frac{\mathrm{d}}{\mathrm{d}t}[x(t) - Cx(t-\tau)] = Ax(t) + Bx(t-h), \tag{10}$$

with

$$x(\theta) = \phi(\theta), \qquad \forall\theta \in [-\max\{h, \tau\}, 0], \tag{11}$$

where $h \geq 0$ is the constant *discrete delay* and $\tau \geq 0$ is the constant *neutral delay*; $\phi(\cdot)$ is a continuous vector valued initial function; $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times n}$ and $C \in \mathbb{R}^{n\times n}$ are constant matrices.

Define x_t as $x_t = x(t+\theta)$, $\forall\theta \in [-\max\{h,\tau\}, 0]$, and the operator $\mathscr{D}x_t = x(t) - Cx(t-\tau)$. Throughout this paper, we assume that

Assumption 1. $\rho(C) < 1$, where $\rho(C)$ denotes the spectral radius of the matrix C.

Notice that Assumption 1 guarantees that the operator $\mathscr{D}$ is strongly stable, i.e. the different equation

$$x(t) - Cx(t-\tau) = 0, \quad \tau \geq 0 \tag{12}$$

is stable *independent of delay* τ (Lemma 3.18, p. 108, Gu et al. (2003)).

In the existing literature (Fridman & Shaked, 2002; Han, 2005b; He et al., 2004; Lien & Chen, 2003), under Assumption 1, there are some results available for the system described by (10) and (11) by choosing simple quadratic Lyapunov–Krasovskii functionals which are the similar as the ones mentioned for the system described by (1) and (2). These results are conservative due to the similar facts as the ones mentioned above for the system described by (1) and (2).

In this paper, we will also extend the *discrete delay decomposition approach* to the system described by (10) *and* (11) and derive some new *discrete delay-dependent* stability criteria. These new stability criteria will be include some existing results as their special cases and be less conservative than some existing results.

Notation: The notation used in this paper are the same as the ones in Han (2005a,b).

2. Stability of linear retarded systems

In this section, we are concerned with stability of linear retarded systems. We employ both the *discrete delay bi-decomposition approach* and the *discrete delay N-decomposition approach* to derive some new and less conservative stability criteria. We begin with the *discrete delay bi-decomposition approach*.

2.1. A discrete delay bi-decomposition approach

By using the new Lyapunov–Krasovskii functional (9), we state and establish the following stability criterion.

Proposition 2. *For a given scalar $h > 0$, the system described by (1) and (2) is asymptotically stable if there exist real $n \times n$ matrices $P > 0$, $Q > 0$, $R > 0$, $W \geq 0$, and $S_{11} = S_{11}^{\mathrm{T}}$, S_{12}, $S_{22} = S_{22}^{\mathrm{T}}$ such that*

$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{12}^{\mathrm{T}} & S_{22} \end{pmatrix} \geq 0, \tag{13}$$

and

$$\Phi = \begin{pmatrix} \Phi_{11} & \Phi_{12} & PB+R & \frac{h}{2}A^{\mathrm{T}}W & hA^{\mathrm{T}}R \\ * & \Phi_{22} & -S_{12} & 0 & 0 \\ * & * & \Phi_{33} & \frac{h}{2}B^{\mathrm{T}}W & hB^{\mathrm{T}}R \\ * & * & * & -W & 0 \\ * & * & * & * & -R \end{pmatrix} < 0, \tag{14}$$

where

$$\Phi_{11} = A^{\mathrm{T}}P + PA + Q + S_{11} - W - R; \qquad \Phi_{12} = W + S_{12};$$
$$\Phi_{22} = S_{22} - S_{11} - W; \qquad \Phi_{33} = -S_{22} - Q - R.$$

Proof. It is omitted due to the fact that the proof is much similar to that of Proposition 1 in Han (2005a).

In Han (2005a), the author employed the Lyapunov–Krasovskii functional (7) and derived the following theorem.

Theorem 3 (*Han, 2005a*). *For a given scalar $h > 0$, the system described by (1) and (2) is asymptotically stable if there exist real $n \times n$ matrices $P > 0$, $Q > 0$, and $R > 0$ such that*

$$\Psi = \begin{pmatrix} \Psi_{11} & PB+R & hA^{\mathrm{T}}R \\ * & -Q-R & hB^{\mathrm{T}}R \\ * & * & -R \end{pmatrix} < 0, \tag{15}$$

where

$$\Psi_{11} = A^{\mathrm{T}}P + PA + Q - R.$$

To illustrate the relation between Proposition 2 and Theorem 3, we present the following result.

Proposition 4. *Consider the system described by (1) and (2). For a given scalar $h > 0$, if there exist real $n \times n$ matrices $P > 0$, $Q > 0$, and $R > 0$ such that $\Psi < 0$, then there exist real $n \times n$ matrices $P > 0$, $Q > 0$, $R > 0$, $W \geq 0$, and $S_{11} = S_{11}^{\mathrm{T}}$, S_{12}, $S_{22} = S_{22}^{\mathrm{T}}$ such that $S \geq 0$ and $\Phi < 0$.*

Proof. If there exist real $n \times n$ matrices $P > 0$, $Q > 0$, and $R > 0$ such that $\Psi < 0$, then there exists a sufficiently small scalar $\varepsilon_1 > 0$ such that

$$\Psi + \mathrm{diag}(\varepsilon_1 I, 0, 0) < 0.$$

Choosing ε_2 such that $0 < \varepsilon_2 < \varepsilon_1$, one can see that the following is true

$$\Psi + \mathrm{diag}(\varepsilon_1 I, 0, 0) + \mathrm{diag}(0, -\varepsilon_2 I, 0) < 0,$$

from which we have

$$\begin{pmatrix} \Psi_{11}+\varepsilon_1 I & 0 & PB+R & hA^{\mathrm{T}}R \\ * & \varepsilon_2 I - \varepsilon_1 I & 0 & 0 \\ * & * & -\varepsilon_2 I - Q - R & hB^{\mathrm{T}}R \\ * & * & * & -R \end{pmatrix} < 0.$$

Similarly, we can also choose a sufficiently small scalar $\varepsilon_3 > 0$ such that

$$\begin{pmatrix} \Psi_{11}+\varepsilon_1 I & 0 & PB+R & hA^{\mathrm{T}}R \\ * & \varepsilon_2 I - \varepsilon_1 I & 0 & 0 \\ * & * & -\varepsilon_2 I - Q - R & hB^{\mathrm{T}}R \\ * & * & * & -R \end{pmatrix} + \begin{pmatrix} \frac{h^2}{4}\varepsilon_3 A^{\mathrm{T}}A & 0 & \frac{h^2}{4}\varepsilon_3 A^{\mathrm{T}}B & 0 \\ * & 0 & 0 & 0 \\ * & * & \frac{h^2}{4}\varepsilon_3 B^{\mathrm{T}}B & 0 \\ * & * & * & 0 \end{pmatrix} < 0.$$

Notice that

$$\begin{pmatrix} -\varepsilon_3 I & \varepsilon_3 I & 0 & 0 \\ * & -\varepsilon_3 I & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{pmatrix} \leq 0.$$

Then, we have

$$\begin{pmatrix} \Psi_{11}+\varepsilon_1 I & 0 & PB+R & hA^{\mathrm{T}}R \\ * & \varepsilon_2 I - \varepsilon_1 I & 0 & 0 \\ * & * & -\varepsilon_2 I - Q - R & hB^{\mathrm{T}}R \\ * & * & * & -R \end{pmatrix} + \begin{pmatrix} -\varepsilon_3 I + \frac{h^2}{4}\varepsilon_3 A^{\mathrm{T}}A & \varepsilon_3 I & \frac{h^2}{4}\varepsilon_3 A^{\mathrm{T}}B & 0 \\ * & -\varepsilon_3 I & 0 & 0 \\ * & * & \frac{h^2}{4}\varepsilon_3 B^{\mathrm{T}}B & 0 \\ * & * & * & 0 \end{pmatrix} < 0,$$

from which, in view of Schur complement, we deduce that $S \geq 0$ and $\Phi < 0$ by setting $S_{11} = \varepsilon_1 I$, $S_{12} = 0$, $S_{22} = \varepsilon_2 I$, and $W = \varepsilon_3 I$. The proof is completed. □

Remark 5. Proposition 4 reveals the fact that Proposition 2 is less conservative than Theorem 3 (Han, 2005a) when using them to judge asymptotic stability of the system described by (1) and (2).

2.2. A discrete delay N-decomposition approach

In this subsection, we extend the approach from discrete delay bi-decomposition to discrete delay N-decomposition with N a given positive integer to derive more general results. In doing so, we choose the Lyapunov–Krasovskii functional

$$\begin{aligned} V(t, x_t) &= x^{\mathrm{T}}(t)Px(t) + \int_{t-h}^{t} x^{\mathrm{T}}(\xi)Qx(\xi)\mathrm{d}\xi \\ &\quad + \int_{t-h}^{t} (h - t + \xi)\dot{x}^{\mathrm{T}}(\xi)(hR)\dot{x}(\xi)\mathrm{d}\xi \\ &\quad + \int_{t-\frac{h}{N}}^{t} z^{\mathrm{T}}(\xi)Sz(\xi)\mathrm{d}\xi \\ &\quad + \int_{t-\frac{h}{N}}^{t} \left(\frac{h}{N} - t + \xi\right)\dot{x}^{\mathrm{T}}(\xi)\left(\frac{h}{N}W\right)\dot{x}(\xi)\mathrm{d}\xi \end{aligned} \tag{16}$$

where

$$z^{\mathrm{T}}(t) = \left(x^{\mathrm{T}}(t) \quad x^{\mathrm{T}}\left(t - \frac{h}{N}\right) \quad \cdots \quad x^{\mathrm{T}}\left(t - \frac{(N-1)h}{N}\right) \right).$$

By using (16), we obtain the following stability criterion.

Proposition 6. *For a given scalar $h > 0$ and a positive integer $N \geq 2$, the system described by (1) and (2) is asymptotically stable if there exist real $n \times n$ matrices $P > 0$, $Q > 0$, $R > 0$, $W \geq 0$, and $S_{ii} = S_{ii}^{\mathrm{T}}$ $(i = 1, 2, \ldots, N)$, S_{ij} $(i < j; i = 1, 2, \ldots, N-1; j = 2, \ldots, N)$ such that*

$$S = S^{\mathrm{T}} = \begin{pmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ * & S_{22} & \cdots & S_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & S_{NN} \end{pmatrix} \geq 0, \tag{17}$$

and

$$\Xi = \begin{pmatrix} \Xi^{(1)} & \Xi^{(2)} & \Xi^{(3)} \\ * & -W & 0 \\ * & * & -R \end{pmatrix} < 0, \tag{18}$$

where

$$\Xi^{(1)} = \begin{pmatrix} \Xi_{11}^{(1)} & \Xi_{12}^{(1)} & S_{13} & \cdots & S_{1N} & PB+R \\ * & \Xi_{22}^{(1)} & \Xi_{23}^{(1)} & \cdots & S_{2N} - S_{1\,N-1} & -S_{1N} \\ * & * & \Xi_{33}^{(1)} & \cdots & S_{3N} - S_{2\,N-1} & -S_{2N} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & \cdots & \Xi_{NN}^{(1)} & -S_{N-1\,N} \\ * & * & * & \cdots & * & \Xi_{N+1\,N+1}^{(1)} \end{pmatrix}$$

with

$$\Xi_{11}^{(1)} = A^{\mathrm{T}}P + PA + Q - W - R + S_{11},$$
$$\Xi_{22}^{(1)} = S_{22} - S_{11} - W,$$
$$\Xi_{33}^{(1)} = S_{33} - S_{22},$$
$$\vdots$$
$$\Xi_{NN}^{(1)} = S_{NN} - S_{N-1\,N-1},$$
$$\Xi_{N+1\,N+1}^{(1)} = -S_{NN} - Q - R,$$
$$\Xi_{12}^{(1)} = W + S_{12}, \qquad \Xi_{23}^{(1)} = S_{23} - S_{12}$$

and

$$\Xi^{(2)} = \begin{pmatrix} \frac{h}{N}A^{\mathrm{T}}W \\ 0 \\ 0 \\ \vdots \\ 0 \\ \frac{h}{N}B^{\mathrm{T}}W \end{pmatrix}, \qquad \Xi^{(3)} = \begin{pmatrix} hA^{\mathrm{T}}R \\ 0 \\ 0 \\ \vdots \\ 0 \\ hB^{\mathrm{T}}R \end{pmatrix}.$$

Similar to Proposition 4, we establish the following result.

Proposition 7. *Consider the system described by (1) and (2). For a given scalar $h > 0$ and a positive integer $N \geq 2$, if there exist real $n \times n$ matrices $P > 0$, $Q > 0$, and $R > 0$ such that $\Psi < 0$, then there exist real $n \times n$ matrices $P > 0$, $Q > 0$, $R > 0$, $W \geq 0$, and $S_{ii} = S_{ii}^{\mathrm{T}}$ $(i = 1, 2, \ldots, N)$, S_{ij} $(i < j; i = 1, 2, \ldots, N-1; j = 2, \ldots, N)$ such that $S \geq 0$ and $\Xi < 0$.*

From Proposition 7, one can clearly see that Proposition 6 always provides less conservative results than Theorem 3 (Han, 2005a).

Notice that for different $N \geq 2$, from Proposition 6, we have different sufficient conditions $S \geq 0$ and $\Xi < 0$. One may wonder whether there exists some relation between these sufficient conditions. For the purpose of simplicity, we take S as a block-diagonal matrix and compare the results for the case of the delay N-decomposition and the case of the delay $(N+1)$-decomposition. For the case of the delay $(N+1)$-decomposition, denote the corresponding matrices as $\tilde{S}$ and $\tilde{\Xi}$ in Proposition 6. Then we have

Proposition 8. *Consider the system described by (1) and (2). For a given scalar $h > 0$ and a positive integer $N \geq 2$, if there exist real $n \times n$ matrices $P > 0$, $Q > 0$, $R > 0$, $W \geq 0$, and $S_{ii} = S_{ii}^{\mathrm{T}}$ $(i = 1, 2, \ldots, N)$, $S_{ij} = 0$ $(i < j; i = 1, 2, \ldots, N-1; j = 2, \ldots, N)$ such that $S \geq 0$ and $\Xi < 0$, then there exist real $n \times n$ matrices $P > 0$, $Q > 0$, $R > 0$, $W \geq 0$, and $S_{ii} = S_{ii}^{\mathrm{T}}$ $(i = 1, 2, \ldots, N+1)$, $S_{ij} = 0$ $(i < j; i = 1, 2, \ldots, N; j = 2, \ldots, N+1)$ such that $\tilde{S} \geq 0$ and $\tilde{\Xi} < 0$.*

Proof. If there exist real $n \times n$ matrices $P > 0$, $Q > 0$, $R > 0$, $W \geq 0$, and $S_{ii} = S_{ii}^{\mathrm{T}}$ $(i = 1, 2, \ldots, N)$, $S_{ij} = 0$ $(i < j; i = 1, 2, \ldots, N-1; j = 2, \ldots, N)$ such that $S \geq 0$ and $\Xi < 0$, then there exists a sufficiently small scalar $\varepsilon > 0$ such that

$$\begin{pmatrix} \hat{\Xi}^{(1)} & \Xi^{(2)} & \Xi^{(3)} \\ * & -W & 0 \\ * & * & -R \end{pmatrix} < 0,$$

where $\hat{\Xi}^{(1)}$ is derived from $\Xi^{(1)}$ by replacing $\Xi_{N+1\,N+1}^{(1)}$ with $\hat{\Xi}_{N+1\,N+1}^{(1)} = -S_{NN} - Q - R + \varepsilon I$.

Notice that from (18) it follows that

$$\begin{pmatrix} \hat{\Xi}^{(1)} & \hat{\Xi}^{(2)} & \Xi^{(3)} \\ * & -W & 0 \\ * & * & -R \end{pmatrix} < 0,$$

where $\hat{\Xi}^{(2)}$ is obtained from $\Xi^{(2)}$ by replacing $\frac{h}{N}A^{\mathrm{T}}W$ and $\frac{h}{N}B^{\mathrm{T}}W$ with $\frac{h}{N+1}A^{\mathrm{T}}W$ and $\frac{h}{N+1}B^{\mathrm{T}}W$, respectively.

Define $\tilde{\Xi}^{(1)}$, $\tilde{\Xi}^{(2)}$, and $\tilde{\Xi}^{(3)}$ as

$$\tilde{\Xi}^{(1)} = \begin{pmatrix} \Xi_{11}^{(1)} & W & 0 & \cdots & 0 & 0 & PB+R \\ * & \Xi_{22}^{(1)} & 0 & \cdots & 0 & 0 & 0 \\ * & * & \Xi_{33}^{(1)} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \\ * & * & * & \cdots & \Xi_{NN}^{(1)} & 0 & 0 \\ * & * & * & \cdots & * & -\varepsilon I & 0 \\ * & * & * & \cdots & * & * & \hat{\Xi}_{N+1\,N+1}^{(1)} \end{pmatrix},$$

$$\tilde{\Xi}^{(2)} = \begin{pmatrix} \frac{h}{N+1}A^{\mathrm{T}}W \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ \frac{h}{N+1}B^{\mathrm{T}}W \end{pmatrix}, \qquad \tilde{\Xi}^{(3)} = \begin{pmatrix} hA^{\mathrm{T}}R \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ hB^{\mathrm{T}}R \end{pmatrix}.$$

Then, we have

$$\begin{pmatrix} \tilde{\Xi}^{(1)} & \tilde{\Xi}^{(2)} & \tilde{\Xi}^{(3)} \\ * & -W & 0 \\ * & * & -R \end{pmatrix} < 0.$$

By setting $S_{N+1\,N+1} = S_{NN} - \varepsilon I$ and $S_{ij} = 0$ $(i = 1, 2, \ldots, N; j = N+1)$, we have that $\tilde{S} \geq 0$ and $\tilde{\Xi} < 0$. This completes the proof. □

Remark 9. Proposition 8 shows that the conservatism of the derived results can be reduced by increasing N.

Table 1
$h_{\max}$ using Proposition 6 for different $N \geq 2$.

N	2	3	4	5
$h_{\max}$	5.7175	5.9677	6.0568	6.0983
N	6	7	8	9
$h_{\max}$	6.1209	6.1346	6.1435	6.1496
N	10	15	20	30
$h_{\max}$	6.1539	6.1643	6.1679	6.1705

Remark 10. If the second and third terms in (16) are not taken into account, then we have

$$V(t,x_t) = x^{\mathrm{T}}(t)Px(t) + \int_{t-\frac{h}{N}}^{t} z^{\mathrm{T}}(\xi)Sz(\xi)\mathrm{d}\xi + \int_{t-\frac{h}{N}}^{t} \left(\frac{h}{N} - t + \xi\right)\dot{x}^{\mathrm{T}}(\xi)\left(\frac{h}{N}W\right)\dot{x}(\xi)\mathrm{d}\xi. \quad (19)$$

In Gouaisbaut and Peaucelle (2006), the authors used the quadratic separation framework to study stability of the system described by (1) and (2), and formulated sufficient conditions as a feasibility problem of LMIs. Similar to Gouaisbaut and Peaucelle (2006), one can prove that (19) is a Lyapunov–Krasovskii functional if P, S, and W satisfy some LMIs.

To end this session, consider the numerical example mentioned in the introduction. Applying Proposition 6, we calculate the maximum allowed time-delay $h_{\max}$ for different N and list the results in Table 1. In the case of $N = 2$, the same result as that using Proposition 2 is derived. In the case of $N \geq 3$, one can obtain better results and as N increases, the results approach the analytical delay limit for stability (6.17258).

Remark 11. Notice that for this example, one can see that the result in the case of $N = 2$ significantly improves the existing result while the result in the case of $N = 3$ slightly improves the result in the case of $N = 2$. Moreover, as N increases, testing the result in Proposition 6 is much time-consuming. Notice also that the result in the case of $N = 2$ (Proposition 2) is sufficient for most practical applications. Due to these facts, one can employ the result in the case of $N = 2$ (Proposition 2) for the tradeoff between better results and time-consuming.

3. Stability of linear neutral systems

In this section, by using the similar argument as retarded systems we will establish some new discrete delay-dependent stability criteria for the system described by (10) and (11). In doing so, following the same arrangement as the one in Section 2: first, we employ a discrete delay bi-decomposition approach to present some results; second, we use a discrete delay N-decomposition approach to obtain more general results.

3.1. Stability criteria based on the discrete delay bi-decomposition approach

Choose the Lyapunov–Krasovskii functional

$$\begin{aligned}\tilde{V}_1(t,x_t) &= (\mathcal{D}x_t)^{\mathrm{T}}P(\mathcal{D}x_t) + \int_{t-h}^{t} x^{\mathrm{T}}(\xi)Qx(\xi)\mathrm{d}\xi \\ &\quad + \int_{t-h}^{t}(h-t+\xi)\dot{x}^{\mathrm{T}}(\xi)(hR)\dot{x}(\xi)\mathrm{d}\xi \\ &\quad + \int_{t-\frac{h}{2}}^{t} y^{\mathrm{T}}(\xi)Sy(\xi)\mathrm{d}\xi + \int_{t-\tau}^{t} x^{\mathrm{T}}(\xi)G_1x(\xi)\mathrm{d}\xi \\ &\quad + \int_{t-\frac{h}{2}}^{t}\left(\frac{h}{2}-t+\xi\right)\dot{x}^{\mathrm{T}}(\xi)\left(\frac{h}{2}W\right)\dot{x}(\xi)\mathrm{d}\xi \\ &\quad + \int_{t-\tau}^{t}\dot{x}^{\mathrm{T}}(\xi)G_2\dot{x}(\xi)\mathrm{d}\xi. \end{aligned} \quad (20)$$

Similar to the proof of Proposition 1 in Han (2005b), we conclude that

Proposition 12. *Under Assumption 1, for given scalars $h > 0$ and $\tau > 0$, the system described by (10) and (11) is asymptotically stable if there exist real $n \times n$ matrices $P > 0$, $Q > 0$, $R > 0$, $W \geq 0$, $G_1 > 0$, $G_2 > 0$, and $S_{11} = S_{11}^{\mathrm{T}}$, S_{12}, $S_{22} = S_{22}^{\mathrm{T}}$ such that (13) and*

$$\Upsilon = \begin{pmatrix} \Upsilon^{(1)} & \Upsilon^{(2)} \\ * & \Upsilon^{(3)} \end{pmatrix} < 0, \quad (21)$$

where

$$\Upsilon^{(1)} = \begin{pmatrix} (1,1) & W+S_{12} & PB+R & -A^{\mathrm{T}}PC & 0 \\ * & (2,2) & -S_{12} & 0 & 0 \\ * & * & (3,3) & -B^{\mathrm{T}}PC & 0 \\ * & * & * & -G_1 & 0 \\ * & * & * & * & -G_2 \end{pmatrix},$$

$$\Upsilon^{(2)} = \begin{pmatrix} \frac{h}{2}A^{\mathrm{T}}W & hA^{\mathrm{T}}R & A^{\mathrm{T}}G_2 \\ 0 & 0 & 0 \\ \frac{h}{2}B^{\mathrm{T}}W & hB^{\mathrm{T}}R & B^{\mathrm{T}}G_2 \\ 0 & 0 & 0 \\ \frac{h}{2}C^{\mathrm{T}}W & hC^{\mathrm{T}}R & C^{\mathrm{T}}G_2 \end{pmatrix},$$

$$\Upsilon^{(3)} = \mathrm{diag}(-W, -R, -G_2),$$

with

$$(1,1) \triangleq A^{\mathrm{T}}P + PA + Q + S_{11} + G_1 - W - R,$$

$$(2,2) \triangleq S_{22} - S_{11} - W,\ (3,3) \triangleq -S_{22} - Q - R.$$

Notice that if $\int_{t-\frac{h}{2}}^{t}(\frac{h}{2}-t+\xi)\dot{x}^{\mathrm{T}}(\xi)(\frac{h}{2}W)\dot{x}(\xi)\mathrm{d}\xi$ and $\int_{t-\frac{h}{2}}^{t} y^{\mathrm{T}}(\xi) Sy(\xi)\mathrm{d}\xi$ are not taken into account, then the Lyapunov–Krasovskii functional (20) reduce to the one employed in Han (2005b). The following result is immediately followed.

Theorem 13. *Under Assumption 1, for given scalars $h > 0$ and $\tau > 0$, the system described by (10) and (11) is asymptotically stable if there exist real $n \times n$ matrices $P > 0$, $Q > 0$, $R > 0$, $G_1 > 0$, and $G_2 > 0$ such that*

$$\begin{pmatrix} (1,1) & (1,2) & -A^{\mathrm{T}}PC & 0 & hA^{\mathrm{T}}R & A^{\mathrm{T}}G_2 \\ * & (2,2) & -B^{\mathrm{T}}PC & 0 & hB^{\mathrm{T}}R & B^{\mathrm{T}}G_2 \\ * & * & -G_1 & 0 & 0 & 0 \\ * & * & * & -G_2 & hC^{\mathrm{T}}R & C^{\mathrm{T}}G_2 \\ * & * & * & * & -R & 0 \\ * & * & * & * & * & -G_2 \end{pmatrix} < 0 \quad (22)$$

where

$$(1,1) \triangleq A^{\mathrm{T}}P + PA + Q + G_1 - R,$$

$$(1,2) \triangleq PB + R,\ (2,2) \triangleq -Q - R.$$

Remark 14. Theorem 13 is the corresponding result of Proposition 2 in Han (2005b) without considering uncertainties in system' parameter matrices. Similar to the proof of Proposition 4, one can show that if there exist real $n \times n$ matrices $P > 0$, $Q > 0$, $R > 0$, $G_1 > 0$, and $G_2 > 0$ such that (22) holds, then there exist real $n \times n$

matrices $P > 0$, $Q > 0$, $R > 0$, $W \geq 0$, $G_1 > 0$, $G_2 > 0$, and $S_{11} = S_{11}^{\mathrm{T}}$, S_{12}, $S_{22} = S_{22}^{\mathrm{T}}$ such that (13) and (21) hold, which means that Proposition 12 is always less conservative than Theorem 13 when employing them to judge asymptotic stability of the system described by (10) and (11).

Remark 15. Notice that if we set $S = 0$ and $W = 0$, then the Lyapunov–Krasovskii functional (20) reduces to the one employed in Han (2005b). Moreover, if there exist norm-bounded uncertainties in system' parameter matrices, one can easily derive the corresponding result, which recovers the result in Han (2005b).

3.2. Stability criteria based on the discrete delay N-decomposition approach

In this subsection, we employ a discrete delay N-decomposition approach to derive a more general result and concentrate on presenting the result without the proof due to page limit.

Choose the Lyapunov–Krasovskii functional

$$\begin{aligned}\hat{V}_1(t, x_t) = {} & (\mathcal{D}x_t)^{\mathrm{T}}P(\mathcal{D}x_t) + \int_{t-h}^{t} x^{\mathrm{T}}(\xi)Qx(\xi)\mathrm{d}\xi \\ & + \int_{t-h}^{t} (h - t + \xi)\dot{x}^{\mathrm{T}}(\xi)(hR)\dot{x}(\xi)\mathrm{d}\xi \\ & + \int_{t-\frac{h}{N}}^{t} z^{\mathrm{T}}(\xi)Sz(\xi)\mathrm{d}\xi + \int_{t-\tau}^{t} x^{\mathrm{T}}(\xi)G_1x(\xi)\mathrm{d}\xi \\ & + \int_{t-\frac{h}{N}}^{t} \left(\frac{h}{N} - t + \xi\right)\dot{x}^{\mathrm{T}}(\xi)\left(\frac{h}{N}W\right)\dot{x}(\xi)\mathrm{d}\xi \\ & + \int_{t-\tau}^{t} \dot{x}^{\mathrm{T}}(\xi)G_2\dot{x}(\xi)\mathrm{d}\xi.\end{aligned}$$

We now state and establish the following result.

Proposition 16. *Under Assumption 1, for given scalars $h > 0$, $\tau > 0$, and a positive integer $N \geq 2$, the system described by (10) and (11) is asymptotically stable if there exist real $n \times n$ matrices $P > 0$, $Q > 0$, $R > 0$, $W \geq 0$, $G_1 > 0$, $G_2 > 0$, and $S_{ii} = S_{ii}^{\mathrm{T}}$ $(i = 1, 2, \ldots, N)$, S_{ij} $(i < j; i = 1, 2, \ldots, N = 1; j = 2, \ldots, N)$ such that (17) and*

$$\Omega = \begin{pmatrix} \Omega^{(1)} & \Omega^{(2)} & 0 & \Omega^{(3)} & \Omega^{(4)} & \Omega^{(5)} \\ * & -G_1 & 0 & 0 & 0 & 0 \\ * & * & -G_2 & \frac{h}{N}C^{\mathrm{T}}W & hC^{\mathrm{T}}R & C^{\mathrm{T}}G_2 \\ * & * & * & -W & 0 & 0 \\ * & * & * & * & -R & 0 \\ * & * & * & * & * & -G_2 \end{pmatrix} < 0 \quad (23)$$

where $\Omega^{(1)}$ is derived from $\Xi^{(1)}$ by replacing $\Xi_{11}^{(1)}$ with $\Xi_{11}^{(1)} + G_1$, and $\Omega^{(3)} = \Xi^{(2)}$, $\Omega^{(4)} = \Xi^{(3)}$ with $\Xi^{(i)}$ $(i = 1, 2, 3)$ are defined in Proposition 6, and

$$\Omega^{(2)} = \begin{pmatrix} -A^{\mathrm{T}}PC \\ 0 \\ 0 \\ \vdots \\ 0 \\ -B^{\mathrm{T}}PC \end{pmatrix}, \qquad \Omega^{(5)} = \begin{pmatrix} A^{\mathrm{T}}G_2 \\ 0 \\ 0 \\ \vdots \\ 0 \\ B^{\mathrm{T}}G_2 \end{pmatrix}.$$

Remark 17. In the case of $N = 2$, Proposition 16 reduces to Proposition 12. Similar to Proposition 8, one can establish the similar result, which is omitted due to page limit.

4. Conclusion

The problem of the stability of linear time-delay systems of both retarded and neutral types has been investigated. A *discrete delay decomposition approach* has been proposed for studying the stability problem. For linear retarded systems, first, we have employed the *discrete delay bi-decomposition approach* to derive a stability criterion and have shown that the stability criterion is always less than the result in Han (2005a); second, we have used the *discrete delay N-decomposition approach* to obtain some more general results. For linear neutral systems, when considering the stability, we have required the operator $\mathcal{D}$ is stable, i.e. the different equation $x(t) - Cx(t - \tau) = 0$, $\tau \geq 0$ is stable *independent of neutral delay* τ. Parallel to retarded systems, we have established some *neutral delay-independent* and *discrete delay-dependent* stability criteria by using both the *discrete delay bi-decomposition approach* and the *discrete delay N-decomposition approach*. These criteria include some existing results as their special cases and are much less conservative than some existing results.

Acknowledgements

The research work was partially supported by Central Queensland University for the Research Advancement Awards Scheme Project "Robust Fault Detection, Filtering and Control for Uncertain Systems with Time-Varying Delay" (Jan 2006–Dec 2008) and the Australian Research Council Discovery Project entitled "Variable Structure Control Systems in Networked Environments" (DP0986376).

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