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Further results on robust stability of neutral system with mixed time-varying delays and nonlinear perturbations

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ABSTRACT

This paper studies delay-dependent robust stability problem for neutral system with mixed time-varying delays. The uncertainties under consideration are nonlinear time-varying parameter perturbations and norm-bounded uncertainties, respectively. Based on Lyapunov functional approach and linear matrix inequality technology, some improved delay-dependent stability conditions are derived by introducing free-weighting matrices. Numerical examples are given to demonstrate the effectiveness and less conservativeness of the proposed methods.

1. Introduction

A neutral time delay system contains delays both in its state, and in its derivatives of state. Such system can be found in such places as population ecology [1], distributed networks containing lossless transmission lines [2], heat exchangers, robots in contact with rigid environments [3], etc. Because of its wider application, the problem of the stability of delay-differential neutral system has received considerable attention by many scholars in the last two decades [4–11].

It is well known nonlinearities, as time delays, may cause instability and poor performance of practical systems, which have driven many researchers to study the problem of nonlinear perturbed systems with state delays during the recent years [11–22]. In [16], stability criteria are derived by using matrix properties and decomposition technique. While the matrix measure should be negative which leads to its more conservativeness. In [17], a model transformation technique is used to deal with the stability of system with time-varying for delays and nonlinear perturbations. In [13], based on a descriptor model transformation combined with a matrix decomposition approach, the robust stability of uncertain systems with time-varying discrete delay is studied by applying an integral inequality. However, these model transformations often introduce additional dynamics which leads to relatively conservative results. In [15], the neutral-delay-dependent and discrete-delay-dependent stability criterion is obtained without using a fixed model transformation, but this stability criterion is only applicable to the system with a constant neutral delay. In [14], the neutral delay and the discrete delay are all time-varying, while the derivative of the discrete delay is less than 1 which limits its bigger application. In addition, other papers have presented criteria that depend only on the size of the discrete delays, and not on the size of the neutral delays [11,14,17,22]. Therefore their methods have a conservatism which can be improved upon. Recently, He et al. [23,24,4] and Wu et al. [5] propose a new method for dealing with time-delay systems, which employs free weighting matrices to express the relationships between the terms in the Leibinz–Newton formula. The method therein reduces the conservativeness of methods involving a fixed model transformation. To the best of our knowledge, few results have been reported in the

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literature concerning the problem of robust stability of the neutral systems with nonlinear perturbations and mixed time-varying neutral and discrete delays.

In this paper, the problem of robustly delay-dependent stability for uncertain neutral systems with mixed time-varying delays is studied. The uncertainties under consideration are nonlinear time-varying parameter perturbations and norm-bounded uncertainties, respectively. Based on Lyapunov functional approach and linear matrix inequality technology, some new delay-dependent stability conditions are derived without resorting to model transformation and estimates for cross terms. The proposed criterion is both neutral-delay dependent and discrete-delay dependent, and at the same time, is dependent on the derivative of the discrete and neutral delays. Therefore the methods in this paper are less conservative than those produced by previous approaches. Finally, Numerical examples are given to demonstrate the effectiveness of the proposed method.

Notation: Throughout this paper, T stands for matrix transposition. R^n is the n-dimensional Euclidean space. $R^{n\times m}$ is the set of all $n \times m$-dimensional matrice. I denotes the identity matrix of appropriate dimensions. $P > 0$ means that P is positive definite. $*$ represents the elements below the main diagonal of a symmetric matrix.

2. Problem statement

Consider the following neutral system with mixed time-varying delays and nonlinear perturbations

$$\begin{cases}\dot{x}(t) = Ax(t) + Bx(t-h(t)) + C\dot{x}(t-\tau(t)) + f_1(x(t),t) + f_2(x(t-h(t)),t) + f_3(\dot{x}(t-\tau(t)),t),\\ x(\theta) = \phi(\theta), \quad \dot{x}(\theta) = \varphi(\theta), \quad \forall\theta \in [-\max(h,\tau), 0],\end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $A, B, C \in R^{n\times n}$ are constant matrices with appropriate dimensions. $h(t), \tau(t)$ are time-varying discrete delay and neutral delay respectively, and they are assumed to satisfy

$$0 \le h(t) \le h, \quad \dot{h}(t) \le h_d, \tag{2}$$

$$0 \le \tau(t) \le \tau, \quad \dot{\tau}(t) \le \tau_d < 1, \tag{3}$$

where h, τ, h_d, τ_d are constants. $\phi(\theta), \varphi(\theta)$ are the initial condition functions that are continuously differentiable on $[-\max(h,\tau), 0]$. $f_1(x(t),t), f_2(x(t-h(t)),t), f_3(\dot{x}(t-\tau(t)),t)$ are unknown nonlinear perturbations. They satisfy that $f_1(0,t) = 0, f_2(0,t) = 0, f_3(0,t) = 0$, and

$$\begin{aligned} &f_1^{\mathrm{T}}(x(t),t)f_1(x(t),t) \le \alpha^2 x^{\mathrm{T}}(t)x(t),\\ &f_2^{\mathrm{T}}(x(t-h(t)),t)f_2(x(t-h(t)),t) \le \beta^2 x^{\mathrm{T}}(t-h(t))x(t-h(t)),\\ &f_3^{\mathrm{T}}(\dot{x}(t-\tau(t)),t)f_3(\dot{x}(t-\tau(t)),t) \le \gamma^2 \dot{x}^{\mathrm{T}}(t-\tau(t))\dot{x}(t-\tau(t)),\end{aligned} \tag{4}$$

where $\alpha \ge 0, \beta \ge 0$ and $\gamma \ge 0$ are given constants, for simplicity, $f_1 := f_1(x(t),t), f_2 := f_2(x(t-h(t)),t), f_3 = f_3(\dot{x}(t-\tau(t)),t)$.

3. Stability analysis

In this section, we first present a delay-dependent robust criterion for the system (1) with uncertainty (4).

Theorem 1. *For given scalars α, β and γ, system (1) with uncertainty (4) and mixed time-varying delays satisfying (2) and (3) is asymptotically stable if the $\|C\| + \gamma < 1$ and there exists positive definite matrices $P > 0, R_i > 0, Q > 0, M_i > 0, V_i > 0, W_i > 0$, and appropriately dimensioned matrices $X_i = [X_{i1}^{\mathrm{T}}, X_{i2}^{\mathrm{T}}, \dots, X_{i9}^{\mathrm{T}}]^{\mathrm{T}}, Y_i = [Y_{i1}^{\mathrm{T}}, Y_{i2}^{\mathrm{T}}, \dots, Y_{i9}^{\mathrm{T}}]^{\mathrm{T}}, Z_i = [Z_{i1}^{\mathrm{T}}, Z_{i2}^{\mathrm{T}}, \dots, Z_{i9}^{\mathrm{T}}]^{\mathrm{T}}$ and scalars $\varepsilon_j \ge 0$, $i = 1, 2, j = 1, 2, 3$, such that the following symmetric linear matrix inequality holds:*

$$\begin{pmatrix} \Omega & hX_1 & hY_1 & hZ_1 & \tau X_2 & \tau Y_2 & \tau Z_2 & h\Gamma^{\mathrm{T}}(V_1+V_2) & \tau\Gamma^{\mathrm{T}}(W_1+W_2) & \Gamma^{\mathrm{T}}Q\\ * & -hV_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & -hV_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & -hV_2 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & -\tau W_1 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & -\tau W_1 & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & -\tau W_2 & 0 & 0 & 0\\ * & * & * & * & * & * & * & -h(V_1+V_2) & 0 & 0\\ * & * & * & * & * & * & * & * & -\tau(W_1+W_2) & 0\\ * & * & * & * & * & * & * & * & * & -Q \end{pmatrix} < 0, \tag{5}$$

where

$$\Omega = \Omega_1 + \Omega_2 + \Omega_2^{\mathrm{T}},$$

$$\Omega_1 = \begin{pmatrix} \Omega_{11} & PB & 0 & PC & 0 & 0 & P & P & P \\ * & \Omega_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -R_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Omega_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -(1-\tau_d)M_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -M_2 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_3 I \end{pmatrix},$$

$$\Omega_{11} = PA + A^{\mathrm{T}}P + R_1 + R_2 + M_1 + M_2 + \varepsilon_1\alpha^2 I,$$
$$\Omega_{22} = -(1-h_d)R_1 + \varepsilon_2\beta^2 I,$$
$$\Omega_{44} = -(1-\tau_d)Q + \varepsilon_3\gamma^2 I,$$
$$\Omega_2 = (X_1 + Z_1 + X_2 + Z_2, -X_1 + Y_1, -Y_1 - Z_1, 0, -X_2 + Y_2, -Y_2 - Z_2, 0, 0, 0),$$
$$\Gamma = (A, B, 0, C, 0, 0, I, I, I).$$

Proof. Choose a Lyapunov functional candidate for the system (1) to be

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t), \tag{6}$$

where

$$V_1(t) = x^{\mathrm{T}}(t)Px(t), \tag{7}$$

$$V_2(t) = \int_{t-h(t)}^{t} x^{\mathrm{T}}(s)R_1x(s)\mathrm{d}s + \int_{t-h}^{t} x^{\mathrm{T}}(s)R_2x(s)\mathrm{d}s, \tag{8}$$

$$V_3(t) = \int_{t-\tau(t)}^{t} x^{\mathrm{T}}(s)M_1x(s)\mathrm{d}s + \int_{t-\tau}^{t} x^{\mathrm{T}}(s)M_2x(s)\mathrm{d}s, \tag{9}$$

$$V_4(t) = \int_{t-\tau(t)}^{t} \dot{x}^{\mathrm{T}}(s)Q\dot{x}(s)\mathrm{d}s, \tag{10}$$

$$V_5(t) = \int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s)(V_1 + V_2)\dot{x}(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s)(W_1 + W_2)\dot{x}(s)\mathrm{d}s\mathrm{d}\theta, \tag{11}$$

where $P = P^{\mathrm{T}} > 0, R_i = R_i^{\mathrm{T}} > 0, M_i = M_i^{\mathrm{T}} > 0, Q = Q^{\mathrm{T}} > 0, V_i = V_i^{\mathrm{T}}$ and $W_i = W_i^{\mathrm{T}} > 0$ are to be determined. Now calculate the derivative of $V(t)$ along the trajectory of the system (1):

$$\dot{V}_1(t) = 2x^{\mathrm{T}}(t)PAx(t) + 2x^{\mathrm{T}}(t)PBx(t-h(t)) + 2x^{\mathrm{T}}(t)PC\dot{x}(t-\tau(t)) + 2x^{\mathrm{T}}(t)Pf_1 + 2x^{\mathrm{T}}(t)Pf_2 + 2x^{\mathrm{T}}(t)Pf_3,$$
$$\dot{V}_2(t) \le x^{\mathrm{T}}(t)(R_1 + R_2)x(t) - (1-h_d)x^{\mathrm{T}}(t-h(t))R_1x(t-h(t)) - x^{\mathrm{T}}(t-h)R_2x(t-h),$$
$$\dot{V}_3(t) \le x^{\mathrm{T}}(t)(M_1 + M_2)x(t) - (1-\tau_d)x^{\mathrm{T}}(t-\tau(t))M_1x(t-\tau(t)) - x^{\mathrm{T}}(t-\tau)M_2x(t-\tau),$$
$$\dot{V}_4(t) \le \dot{x}^{\mathrm{T}}(t)Q\dot{x}(t) - (1-\tau_d)\dot{x}^{\mathrm{T}}(t-\tau(t))Q\dot{x}(t-\tau(t)),$$
$$\begin{aligned} \dot{V}_5(t) \;=\; & \dot{x}^{\mathrm{T}}(t)\big(h(V_1 + V_2) + \tau(W_1 + W_2)\big)\dot{x}(t) - \int_{t-h(t)}^{t} \dot{x}^{\mathrm{T}}(s)V_1\dot{x}(s)\mathrm{d}s \\ & - \int_{t-h}^{t-h(t)} \dot{x}^{\mathrm{T}}(s)V_1\dot{x}(s)\mathrm{d}s - \int_{t-h}^{t} \dot{x}^{\mathrm{T}}(s)V_2\dot{x}(s)\mathrm{d}s - \int_{t-\tau(t)}^{t} \dot{x}^{\mathrm{T}}(s)W_1\dot{x}(s)\mathrm{d}s \\ & - \int_{t-\tau}^{t-\tau(t)} \dot{x}^{\mathrm{T}}(s)W_1\dot{x}(s)\mathrm{d}s - \int_{t-\tau}^{t} \dot{x}^{\mathrm{T}}(s)W_2\dot{x}(s)\mathrm{d}s. \end{aligned} \tag{12}$$

For any real matrices X_i, Y_i, Z_i $(i = 1, 2)$ with appropriate dimensions, by using the Leibinz–Newton formula one has

$$2\zeta^{\mathrm{T}}(t)X_1\left[x(t) - x(t-h(t)) - \int_{t-h(t)}^{t} \dot{x}(s)\mathrm{d}s\right] = 0, \tag{13}$$

$$2\zeta^{\mathrm{T}}(t)Y_1\left[x(t-h(t)) - x(t-h) - \int_{t-h}^{t-h(t)} \dot{x}(s)\mathrm{d}s\right] = 0, \tag{14}$$

$$2\zeta^{\mathrm{T}}(t)Z_1\left[x(t)-x(t-h)-\int_{t-h}^{t}\dot{x}(s)\mathrm{d}s\right]=0, \tag{15}$$

$$2\zeta^{\mathrm{T}}(t)X_2\left[x(t)-x(t-\tau(t))-\int_{t-\tau(t)}^{t}\dot{x}(s)\mathrm{d}s\right]=0, \tag{16}$$

$$2\zeta^{\mathrm{T}}(t)Y_2\left[x(t-\tau(t))-x(t-\tau)-\int_{t-\tau}^{t-\tau(t)}\dot{x}(s)\mathrm{d}s\right]=0, \tag{17}$$

$$2\zeta^{\mathrm{T}}(t)Z_2\left[x(t)-x(t-\tau)-\int_{t-\tau}^{t}\dot{x}(s)\mathrm{d}s\right]=0, \tag{18}$$

where

$$\zeta^{\mathrm{T}}(t)=[x^{\mathrm{T}}(t),x^{\mathrm{T}}(t-h(t)),x^{\mathrm{T}}(t-h),\dot{x}^{\mathrm{T}}(t-\tau(t)),x^{\mathrm{T}}(t-\tau(t)),x^{\mathrm{T}}(t-\tau),f_1^{\mathrm{T}},f_2^{\mathrm{T}},f_3^{\mathrm{T}}].$$

Next, from (4), we can obtain for any scalars $\varepsilon_1>0$, $\varepsilon_2>0$ and $\varepsilon_3>0$,

$$\begin{aligned}
&\varepsilon_1\Big(\alpha^2x^{\mathrm{T}}(t)x(t)-f_1^{\mathrm{T}}(x(t),t)f_1(x(t),t)\Big)\geq 0,\\
&\varepsilon_2\Big(\beta^2x^{\mathrm{T}}(t-h(t))x(t-h(t))-f_2^{\mathrm{T}}(x(t-h(t)),t)f_2(x(t-h(t)),t)\Big)\geq 0,\\
&\varepsilon_3\Big(\gamma^2\dot{x}^{\mathrm{T}}(t-\tau(t))\dot{x}(t-\tau(t))-f_3^{\mathrm{T}}(\dot{x}(t-\tau(t)),t)f_3(\dot{x}(t-\tau(t)),t)\Big)\geq 0.
\end{aligned} \tag{19}$$

Then combining Eqs. (12)–(19) yields

$$\begin{aligned}
\dot{V}(t)\;&=\dot{V}_1(t)+\dot{V}_2(t)+\dot{V}_3(t)+\dot{V}_4(t)+\dot{V}_5(t)+\varepsilon_1\Big(\alpha^2x^{\mathrm{T}}(t)x(t)-f_1^{\mathrm{T}}(x(t),t)f_1(x(t),t)\Big)\\
&\quad+\varepsilon_2\Big(\beta^2x^{\mathrm{T}}(t-h(t))x(t-h(t))-f_2^{\mathrm{T}}(x(t-h(t)),t)f_2(x(t-h(t)),t)\Big)\\
&\quad+\varepsilon_3\Big(\gamma^2\dot{x}^{\mathrm{T}}(t-\tau(t))\dot{x}(t-\tau(t))-f_3^{\mathrm{T}}(\dot{x}(t-\tau(t)),t)f_3(\dot{x}(t-\tau(t)),t)\Big)\\
&\leq x^{\mathrm{T}}(t)(PA+A^{\mathrm{T}}P+R_1+R_2+M_1+M_2+\varepsilon_1\alpha^2I)x(t)+x^{\mathrm{T}}(t)(PB+B^{\mathrm{T}}P)x(t-h(t))\\
&\quad+x^{\mathrm{T}}(t)(PC+C^{\mathrm{T}}P)\dot{x}(t-\tau(t))+x^{\mathrm{T}}(t-h(t))\Big(-(1-h_d)R_1+\varepsilon_2\beta^2I\Big)x(t-h(t))\\
&\quad-x^{\mathrm{T}}(t-h)R_2x(t-h)-(1-\tau_d)x^{\mathrm{T}}(t-\tau(t))M_1x(t-\tau(t))-x^{\mathrm{T}}(t-\tau)M_2x(t-\tau)\\
&\quad+\dot{x}^{\mathrm{T}}(t-\tau(t))\Big(-(1-\tau_d)Q+\varepsilon_3\gamma^2I\Big)\dot{x}(t-\tau(t))+x^{\mathrm{T}}(2P)f_1+x^{\mathrm{T}}(2P)f_2+x^{\mathrm{T}}(t)(2P)f_3\\
&\quad-\varepsilon_1f_1^{\mathrm{T}}f_1-\varepsilon_2f_2^{\mathrm{T}}f_2-\varepsilon_3f_3^{\mathrm{T}}f_3+\dot{x}^{\mathrm{T}}(t)\Big(h(V_1+V_2)+\tau(W_1+W_2)+Q\Big)\dot{x}(t)\\
&\quad-\int_{t-h(t)}^{t}\dot{x}^{\mathrm{T}}(s)V_1\dot{x}(s)\mathrm{d}s-\int_{t-h}^{t-h(t)}\dot{x}^{\mathrm{T}}(s)V_1\dot{x}(s)\mathrm{d}s-\int_{t-h}^{t}\dot{x}^{\mathrm{T}}(s)V_2\dot{x}(s)\mathrm{d}s\\
&\quad-\int_{t-\tau(t)}^{t}\dot{x}^{\mathrm{T}}(s)W_1\dot{x}(s)\mathrm{d}s-\int_{t-\tau}^{t-\tau(t)}\dot{x}^{\mathrm{T}}(s)W_1\dot{x}(s)\mathrm{d}s-\int_{t-\tau}^{t}\dot{x}^{\mathrm{T}}(s)W_2\dot{x}(s)\mathrm{d}s\\
&\quad+2\zeta^{\mathrm{T}}(t)X_1\left[x(t)-x(t-h(t))-\int_{t-h(t)}^{t}\dot{x}(s)\mathrm{d}s\right]+2\zeta^{\mathrm{T}}(t)Y_1\left[x(t-h(t))-x(t-h)-\int_{t-h}^{t-h(t)}\dot{x}(s)\mathrm{d}s\right]\\
&\quad+2\zeta^{\mathrm{T}}(t)Z_1\left[x(t)-x(t-h)-\int_{t-h}^{t}\dot{x}(s)\mathrm{d}s\right]+2\zeta^{\mathrm{T}}(t)X_2\left[x(t)-x(t-\tau(t))-\int_{t-\tau(t)}^{t}\dot{x}(s)\mathrm{d}s\right]\\
&\quad+2\zeta^{\mathrm{T}}(t)Y_2\left[x(t-\tau(t))-x(t-\tau)-\int_{t-\tau}^{t-\tau(t)}\dot{x}(s)\mathrm{d}s\right]+2\zeta^{\mathrm{T}}(t)Z_2\left[x(t)-x(t-\tau)-\int_{t-\tau}^{t}\dot{x}(s)\mathrm{d}s\right]\\
&=\zeta^{\mathrm{T}}(t)\Big(\Omega_1+\Omega_2+\Omega_2^{\mathrm{T}}+h\Gamma^{\mathrm{T}}(V_1+V_2)\Gamma+\tau\Gamma^{\mathrm{T}}(W_1+W_2)\Gamma+\Gamma^{\mathrm{T}}Q\Gamma+hX_1V_1^{-1}X_1^{\mathrm{T}}\\
&\quad+hY_1V_1^{-1}Y_1^{\mathrm{T}}+hZ_1V_2^{-1}Z_1^{\mathrm{T}}+\tau X_2W_1^{-1}X_2^{\mathrm{T}}+\tau Y_2W_1^{-1}Y_2^{\mathrm{T}}+\tau Z_2W_2^{-1}Z_2^{\mathrm{T}}\Big)\zeta(t)
\end{aligned}$$

$$
\begin{aligned}
&- \int_{t-h(t)}^{t} (\zeta^{\mathrm{T}}(t)X_1 + \dot{x}^{\mathrm{T}}(s)V_1)V_1^{-1}(X_1^{\mathrm{T}}\zeta(t) + V_1\dot{x}(s))\mathrm{d}s \\
&- \int_{t-h}^{t-h(t)} (\zeta^{\mathrm{T}}(t)Y_1 + \dot{x}^{\mathrm{T}}(s)V_1)V_1^{-1}(Y_1^{\mathrm{T}}\zeta(t) + V_1\dot{x}(s))\mathrm{d}s \\
&- \int_{t-h}^{t} (\zeta^{\mathrm{T}}(t)Z_1 + \dot{x}^{\mathrm{T}}(s)V_2)V_2^{-1}(Z_1^{\mathrm{T}}\zeta(t) + V_2\dot{x}(s))\mathrm{d}s \\
&- \int_{t-\tau(t)}^{t} (\zeta^{\mathrm{T}}(t)X_2 + \dot{x}^{\mathrm{T}}(s)W_1)W_1^{-1}(X_2^{\mathrm{T}}\zeta(t) + W_1\dot{x}(s))\mathrm{d}s \\
&- \int_{t-\tau}^{t-\tau(t)} (\zeta^{\mathrm{T}}(t)Y_2 + \dot{x}^{\mathrm{T}}(s)W_1)W_1^{-1}(Y_2^{\mathrm{T}}\zeta(t) + W_1\dot{x}(s))\mathrm{d}s \\
&- \int_{t-\tau}^{t} (\zeta^{\mathrm{T}}(t)Z_2 + \dot{x}^{\mathrm{T}}(s)W_2)W_2^{-1}(Z_2^{\mathrm{T}}\zeta(t) + W_2\dot{x}(s))\mathrm{d}s. \qquad (20)
\end{aligned}
$$

From the condition $V_i > 0$ and $W_i > 0$, $i = 1, 2$ which it implies

$$
\begin{aligned}
\dot{V}(t) \le \zeta^{\mathrm{T}}(t)\Big(&\Omega_1 + \Omega_2 + \Omega_2^{\mathrm{T}} + h\Gamma^{\mathrm{T}}(V_1 + V_2)\Gamma + \tau\Gamma^{\mathrm{T}}(W_1 + W_2)\Gamma + \Gamma^{\mathrm{T}}Q\Gamma + hX_1V_1^{-1}X_1^{\mathrm{T}} \\
&+ hY_1V_1^{-1}Y_1^{\mathrm{T}} + hZ_1V_2^{-1}Z_1^{\mathrm{T}} + \tau X_2W_1^{-1}X_2^{\mathrm{T}} + \tau Y_2W_1^{-1}Y_2^{\mathrm{T}} + \tau Z_2W_2^{-1}Z_2^{\mathrm{T}}\Big)\zeta(t). \qquad (21)
\end{aligned}
$$

Denote

$$
\begin{aligned}
\Xi = {} & \Omega_1 + \Omega_2 + \Omega_2^{\mathrm{T}} + h\Gamma^{\mathrm{T}}(V_1 + V_2)\Gamma + \tau\Gamma^{\mathrm{T}}(W_1 + W_2)\Gamma + \Gamma^{\mathrm{T}}Q\Gamma + hX_1V_1^{-1}X_1^{\mathrm{T}} \\
& + hY_1V_1^{-1}Y_1^{\mathrm{T}} + hZ_1V_2^{-1}Z_1^{\mathrm{T}} + \tau X_2W_1^{-1}X_2^{\mathrm{T}} + \tau Y_2W_1^{-1}Y_2^{\mathrm{T}} + \tau Z_2W_2^{-1}Z_2^{\mathrm{T}}.
\end{aligned}
$$

If matrix inequality (5) is feasible, then by Schur complement we can get $\Xi < 0$. Next, Let $\lambda = \lambda_{\min}(-\Xi)$, from (21) it follows that $\dot{V}(t) \le -\lambda\|x(t)\|^2$. Therefore according to Hale [25], if $\|C\| + \gamma < 1$ and there exist symmetric positive definite matrices $P > 0$, $R_i > 0$, $Q > 0$, $M_i > 0$, $V_i > 0$, $W_i > 0$, $i = 1, 2$ and scalars $\varepsilon_1 \ge 0$, $\varepsilon_2 \ge 0$ and $\varepsilon_3 \ge 0$ such that the LMI (5) is satisfied, then system (1) with uncertainty (4) is asymptotically stable. This completes the proof. □

Remark 1. Note that Theorem 1 gives a delay-dependent and rate-dependent stability criterion for delays satisfying (2) and (3). However in [11,14,17,22], the proposed criteria depend only on the size of the discrete delays, and not on the size of the neutral delays. At the same time, some papers, such as [14,15], require the delay differential conditions be more strict than ours in which implies our criterion is less conservative than the previous approaches.

Remark 2. Observe that $-\int_{t-h}^{t}\dot{x}^{\mathrm{T}}(s)V_1\dot{x}(s)\mathrm{d}s = -\int_{t-h(t)}^{t}\dot{x}^{\mathrm{T}}(s)V_1\dot{x}(s)\mathrm{d}s - \int_{t-h}^{t-h(t)}\dot{x}^{\mathrm{T}}(s)V_1\dot{x}(s)\mathrm{d}s$, which can be seen that the term $-\int_{t-h}^{t-h(t)}\dot{x}^{\mathrm{T}}(s)V_1\dot{x}(s)\mathrm{d}s$ is retained when estimating the upper bound of $\dot{V}_5(t)$ in Theorem 1. While the term $-\int_{t-h}^{t-h(t)}\dot{x}^{\mathrm{T}}(s)V_1\dot{x}(s)\mathrm{d}s$ was often ignored in previous works [24,5,7]. Thus the obtained criterion shows less conservative than the existing ones [24,5,7].

Next, by choosing $R_1 = M_1 = W_1 = Q = 0$, we can get a delay-dependent and rate-independent stability criterion.

Corollary 1. *For given scalars α, β and γ, system (1) with uncertainty (4) and mixed time-varying delays satisfying $0 \le h(t) \le h, 0 \le \tau(t) \le \tau$ is asymptotically stable if the $\|C\| + \gamma < 1$ and there exist positive definite matrices $P > 0, R_2 > 0, M_2 > 0, V_1 > 0, V_2 > 0, W_2 > 0$, and appropriately dimensioned matrices $\widehat{X}_1 = [X_{11}^{\mathrm{T}}, X_{12}^{\mathrm{T}}, \ldots, X_{18}^{\mathrm{T}}]^{\mathrm{T}}, \widehat{Y}_1 = [Y_{11}^{\mathrm{T}}, Y_{12}^{\mathrm{T}}, \ldots, Y_{18}^{\mathrm{T}}]^{\mathrm{T}}, \widehat{Z}_i = [Z_{i1}^{\mathrm{T}}, Z_{i2}^{\mathrm{T}}, \ldots, Z_{i8}^{\mathrm{T}}]^{\mathrm{T}}$ and scalars $\varepsilon_j \ge 0, j = 1, 2, 3$, such that the following symmetric linear matrix inequality holds:*

$$
\begin{pmatrix}
\widehat{\Omega} & h\widehat{X}_1 & h\widehat{Y}_1 & h\widehat{Z}_1 & \tau\widehat{Z}_2 & h\widehat{\Gamma}^{\mathrm{T}}(V_1 + V_2) & \tau\widehat{\Gamma}^{\mathrm{T}}W_2 \\
* & -hV_1 & 0 & 0 & 0 & 0 & 0 \\
* & * & -hV_1 & 0 & 0 & 0 & 0 \\
* & * & * & -hV_2 & 0 & 0 & 0 \\
* & * & * & * & -\tau W_2 & 0 & 0 \\
* & * & * & * & * & -h(V_1 + V_2) & 0 \\
* & * & * & * & * & * & -\tau W_2
\end{pmatrix} < 0, \qquad (22)
$$

where

$$\widehat{\Omega} = \widehat{\Omega} + \widehat{\Omega}_2 + \widehat{\Omega}_2^{\mathrm{T}},$$
$$\widehat{\Omega} = \begin{pmatrix} \widehat{\Omega}_{11} & PB & 0 & PC & 0 & P & P & P \\ * & \varepsilon_2\beta^2 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -R_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \varepsilon_3\gamma^2 I & 0 & 0 & 0 & 0 \\ * & * & * & * & -M_2 & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & * & * & -\varepsilon_3 I \end{pmatrix},$$
$$\widehat{\Omega}_{11} = PA + A^{\mathrm{T}}P + R_2 + M_2 + \varepsilon_1\alpha^2 I,$$
$$\widehat{\Omega}_2 = (\widehat{X}_1 + \widehat{Z}_1 + \widehat{Z}_2, -\widehat{X}_1 + \widehat{Y}_1, -\widehat{Y}_1 - \widehat{Z}_1, 0, -\widehat{Z}_2, 0, 0, 0),$$
$$\widehat{\Gamma} = (A, B, 0, C, 0, I, I, I).$$

Proof. Choose a suitable Lyapunov functional candidate for the system (1) to be

$$V(t) = x^{\mathrm{T}}(t)Px(t) + \int_{t-h}^{t} x^{\mathrm{T}}(s)R_2x(s)\mathrm{d}s + \int_{t-\tau}^{t} x^{\mathrm{T}}(s)M_2x(s)\mathrm{d}s + \int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s)(V_1+V_2)\dot{x}(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s)W_2\dot{x}(s)\mathrm{d}s\mathrm{d}\theta.$$

From the Leibniz–Newton formula, the following equations are true for any matrices $\widehat{X}_1, \widehat{Y}_1, \widehat{Z}_1, \widehat{Z}_2$ with appropriate dimensions

$$\begin{aligned} &2\widehat{\zeta}^{\mathrm{T}}(t)\widehat{X}_1\left[x(t) - x(t-h(t)) - \int_{t-h(t)}^{t} \dot{x}(s)\mathrm{d}s\right] = 0, \\ &2\widehat{\zeta}^{\mathrm{T}}(t)\widehat{Y}_1\left[x(t-h(t)) - x(t-h) - \int_{t-h}^{t-h(t)} \dot{x}(s)\mathrm{d}s\right] = 0, \\ &2\widehat{\zeta}^{\mathrm{T}}(t)\widehat{Z}_1\left[x(t) - x(t-h) - \int_{t-h}^{t} \dot{x}(s)\mathrm{d}s\right] = 0, \\ &2\widehat{\zeta}^{\mathrm{T}}(t)\widehat{Z}_2\left[x(t) - x(t-\tau) - \int_{t-\tau}^{t} \dot{x}(s)\mathrm{d}s\right] = 0, \end{aligned} \tag{23}$$

where $\widehat{\zeta}^{\mathrm{T}}(t) = [x^{\mathrm{T}}(t), x^{\mathrm{T}}(t-h(t)), x^{\mathrm{T}}(t-h), \dot{x}^{\mathrm{T}}(t-\tau(t)), x^{\mathrm{T}}(t-\tau), f_1^{\mathrm{T}}, f_2^{\mathrm{T}}, f_3^{\mathrm{T}}]$. Then similar to the proof of Theorem 1 with (19) and (23) being added into the expression of $\dot{V}(t)$, the result follows immediately. This completes the proof. □

If $C = 0$ and $f_3(\dot{x}(t-\tau(t)), t) = 0$, then system (1) reduces to the following system:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t-h(t)) + f_1(x(t), t) + f_2(x(t-h(t), t)), \\ x(\theta) = \phi(\theta), \quad \forall\theta \in [-h, 0]. \end{cases} \tag{24}$$

According to Theorem 1, we have the following corollary for the delay-dependent and rate-dependent stability of system (24).

Corollary 2. *For given scalars α, β, system (24) with uncertainty (4) and time-varying delay satisfying (2) is asymptotically stable if there exists positive definite matrices $P > 0, R_1 > 0, R_2 > 0, V_1 > 0, V_2 > 0$, and appropriately dimensioned matrices $X = [X_1^{\mathrm{T}}, X_2^{\mathrm{T}}, \ldots, X_6^{\mathrm{T}}]^{\mathrm{T}}, Y = [Y_1^{\mathrm{T}}, Y_2^{\mathrm{T}}, \ldots, Y_6^{\mathrm{T}}]^{\mathrm{T}}, Z = [Z_1^{\mathrm{T}}, Z_2^{\mathrm{T}}, \ldots, Z_6^{\mathrm{T}}]^{\mathrm{T}}$ and scalars $\varepsilon_j \geq 0, j = 1, 2$, such that the following symmetric linear matrix inequality holds:*

$$\begin{pmatrix} \widetilde{\Omega} & hX & hY & hZ & h\widetilde{\Gamma}^{\mathrm{T}}(V_1+V_2) \\ * & -hV_1 & 0 & 0 & 0 \\ * & * & -hV_1 & 0 & 0 \\ * & * & * & -hV_2 & 0 \\ * & * & * & * & -h(V_1+V_2) \end{pmatrix} < 0, \tag{25}$$

where

$$\widetilde{\Omega} = \widetilde{\Omega}_1 + \widetilde{\Omega}_2 + \widetilde{\Omega}_2^{\mathrm{T}},$$
$$\widetilde{\Omega}_1 = \begin{pmatrix} PA + A^{\mathrm{T}}P + R_1 + R_2 + \varepsilon_1\alpha^2 I & PB & 0 & P & P \\ * & -(1-h_d)R_1 + \varepsilon_2\beta^2 I & 0 & 0 & 0 \\ * & * & -R_2 & 0 & 0 \\ * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & -\varepsilon_2 I \end{pmatrix},$$
$$\widetilde{\Omega}_2 = (X+Z, -X+Y, -Y-Z, 0, 0),$$
$$\widetilde{\Gamma} = (A, B, 0, I, I).$$

It is seen that Theorem 1 is established by adopting the Leibniz–Newton formula which provides free-weighting matrices X_i, Y_i and Z_i, $i = 1, 2$ as in (5). In the following, we derive an alternative delay-dependent criterion by taking into account the system equation.

Theorem 2. *For given scalars α, β and γ, system (1) with uncertainty (4) and mixed time-varying delays satisfying (2) and (3) is asymptotically stable if the $\|C\| + \gamma < 1$ and there exist positive definite matrices $P > 0$, $R_i > 0$, $Q > 0$, $M_i > 0$, $V_i > 0$, $W_i > 0$, and appropriately dimensioned matrices $\widetilde{X}_i = [X_{i1}^{\mathrm{T}}, X_{i2}^{\mathrm{T}}, \ldots, X_{i,10}^{\mathrm{T}}]^{\mathrm{T}}$, $\widetilde{Y}_i = [Y_{i1}^{\mathrm{T}}, Y_{i2}^{\mathrm{T}}, \ldots, Y_{i,10}^{\mathrm{T}}]^{\mathrm{T}}$, $\widetilde{Z}_i = [Z_{i1}^{\mathrm{T}}, Z_{i2}^{\mathrm{T}}, \ldots, Z_{i,10}^{\mathrm{T}}]^{\mathrm{T}}$, $T = [T_1^{\mathrm{T}}, T_2^{\mathrm{T}}, \ldots, T_{10}^{\mathrm{T}}]^{\mathrm{T}}$ and scalars $\varepsilon_j \geq 0$, $i = 1, 2$, $j = 1, 2, 3$, such that the following symmetric linear matrix inequality holds:*

$$\begin{pmatrix} \Phi & h\widetilde{X}_1 & h\widetilde{Y}_1 & h\widetilde{Z}_1 & \tau\widetilde{X}_2 & \tau\widetilde{Y}_2 & \tau\widetilde{Z}_2 \\ * & -hV_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & -hV_1 & 0 & 0 & 0 & 0 \\ * & * & * & -hV_2 & 0 & 0 & 0 \\ * & * & * & * & -\tau W_1 & 0 & 0 \\ * & * & * & * & * & -\tau W_1 & 0 \\ * & * & * & * & * & * & -\tau W_2 \end{pmatrix} < 0, \tag{26}$$

where

$$\Phi = \Phi_1 + \Phi_2 + \Phi_2^{\mathrm{T}},$$
$$\Phi_1 = \begin{pmatrix} \Phi_{11} & 0 & 0 & 0 & 0 & 0 & P & 0 & 0 & 0 \\ * & \Phi_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -R_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Phi_{44} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -(1-\tau_d)M_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -M_2 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Phi_{77} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & * & * & * & * & -\varepsilon_3 I \end{pmatrix},$$
$$\Phi_{11} = R_1 + R_2 + M_1 + M_2 + \varepsilon_1\alpha^2 I,$$
$$\Phi_{22} = -(1-h_d)R_1 + \varepsilon_2\beta^2 I,$$
$$\Phi_{44} = -(1-\tau_d)Q + \varepsilon_3\gamma^2 I,$$
$$\Phi_{77} = Q + h(V_1 + V_2) + \tau(W_1 + W_2),$$
$$\Phi_2 = (\widetilde{X}_1 + \widetilde{Z}_1 + \widetilde{X}_2 + \widetilde{Z}_2, -\widetilde{X}_1 + \widetilde{Y}_1, -\widetilde{Y}_1 - \widetilde{Z}_1, 0, -\widetilde{X}_2 + \widetilde{Y}_2, -\widetilde{Y}_2 - \widetilde{Z}_2, 0, 0, 0, 0) + T\Lambda + \Lambda^{\mathrm{T}}T^{\mathrm{T}},$$
$$\Lambda = (-A, -B, 0, -C, 0, 0, I, -I, -I, -I).$$

Proof. Choose the same Lyapunov functional candidate as in (6) for the system (1). It is clear that the following equation holds by system Eq. (1)

$$2\xi^{\mathrm{T}}(t)T\left[\dot{x}(t) - Ax(t) - Bx(t-h(t)) - C\dot{x}(t-\tau(t)) - f_1 - f_2 - f_3\right] = 0, \tag{27}$$

where

$$\xi^{\mathrm{T}}(t) = [x^{\mathrm{T}}(t), x^{\mathrm{T}}(t-h(t)), x^{\mathrm{T}}(t-h), \dot{x}^{\mathrm{T}}(t-\tau(t)), x^{\mathrm{T}}(t-\tau(t)), x^{\mathrm{T}}(t-\tau), \dot{x}(t), f_1^{\mathrm{T}}, f_2^{\mathrm{T}}, f_3^{\mathrm{T}}].$$

Therefore, using the similar equation as (13)–(19) and (27),we can acquire

$$\dot{V}(t) \leq \xi^{\mathrm{T}}(t)\Big(\Phi_1 + \Phi_2 + \Phi_2^{\mathrm{T}} + h\widetilde{X}_1V_1^{-1}\widetilde{X}_1^{\mathrm{T}} + h\widetilde{Y}_1V_1^{-1}\widetilde{Y}_1^{\mathrm{T}} + h\widetilde{Y}_1V_2^{-1}\widetilde{Y}_1^{\mathrm{T}}$$
$$+ \tau\widetilde{X}_2W_1^{-1}\widetilde{X}_2^{\mathrm{T}} + \tau\widetilde{Y}_2W_1^{-1}\widetilde{Y}_2^{\mathrm{T}} + \tau\widetilde{Y}_2W_2^{-1}\widetilde{Y}_2^{\mathrm{T}}\Big)\xi(t). \tag{28}$$

Next, similar to Theorem 1 we can get the result of Theorem 2. This completes the proof. □

Remark 3. It is noted that Theorem 2 is equivalent to Theorem 1 according to the same demonstration in [4] which we omit it. Since the LMI condition of Theorem 2 does not involve the product of system matrices and Lyapunov matrices, Theorem 2 is suitable for dealing with uncertain systems with polytopic-type uncertainties by using parameter-dependent Lyapunov functionals as in [24].

Remark 4. Similar to Theorem 1, Theorem 2 can also result in direct result of rate-independent criterion for system (1) by choosing $R_1 = M_1 = W_1 = Q = 0$.

Remark 5. The method proposed in this paper can be easily extended to the neutral system with multiple mixed neutral and discrete delays and nonlinear perturbations.

4. Norm-bounded uncertainty

In this section, we will present a delay-dependent robust criterion for the system (1) that $f_1(x(t), t), f_2(x(t-h(t)), t)$ and $f_3(\dot{x}(t-\tau(t)), t)$ are norm-bounded uncertainties. That is

$$\begin{aligned} &f_1(x(t), t) = \Delta A(t)x(t),\\ &f_2(x(t-h(t)), t) = \Delta B(t)x(t-h(t)),\\ &f_3(\dot{x}(t-\tau(t)), t) = \Delta C(t)\dot{x}(t-\tau(t)). \end{aligned} \tag{29}$$

The time-varying uncertainties are of the form

$$[\Delta A(t), \Delta B(t), \Delta C(t)] = LF(t)[E_A, E_B, E_C], \tag{30}$$

where E_A, E_B, E_C and L are constant matrices of appropriate dimensions. $F(t)$ is an unknown and possibly time-varying real matrix with Lebesgue measurable elements satisfying

$$F^{\mathrm{T}}(t)F(t) \le I, \quad \forall t > 0. \tag{31}$$

Then system (1) becomes the following system:

$$\dot{x}(t) = (A + LF(t)E_A)x(t) + (B + LF(t)E_B)x(t-h(t)) + (C + LF(t)E_C)\dot{x}(t-\tau(t)). \tag{32}$$

In order to obtain the main results, the following lemmas are first introduced.

Lemma 1 ([26]). *Let A, L, E and $F(t)$ be real matrices of appropriate dimensions with $F^{\mathrm{T}}(t)F(t) \le I$. Then for any symmetric positive definite matrix $P > 0$ and a scalar $\delta > 0$ such that $\delta I - L^{\mathrm{T}}PL > 0$, the following inequality holds:*

$$(A + LF(t)E)^{\mathrm{T}}P(A + LF(t)E) \le A^{\mathrm{T}}PA + A^{\mathrm{T}}PL(\delta I - L^{\mathrm{T}}PL)^{-1}L^{\mathrm{T}}PA + \delta E^{\mathrm{T}}E.$$

Lemma 2 ([27]). *Given matrices $Q = Q^{\mathrm{T}}, H, E$ and $R = R^{\mathrm{T}} > 0$ with appropriate dimensions, then*

$$Q + HFE + E^{\mathrm{T}}F^{\mathrm{T}}H^{\mathrm{T}} < 0$$

for all F satisfying $F^{\mathrm{T}}F \le R$, if and only if there exists an $\varepsilon > 0$ such that

$$Q + \varepsilon HH^{\mathrm{T}} + \varepsilon^{-1}E^{\mathrm{T}}RE < 0.$$

Theorem 3. *The system (32) with mixed time-varying delays satisfying (2) and (3) is robustly stable if there exist a scalar δ satisfying $\delta I - L^{\mathrm{T}}L > 0$, and positive definite matrices $P > 0, R_i > 0, Q > 0, M_i > 0, V_i > 0, W_i > 0$, and appropriately dimensioned matrices $\overline{X}_i = [X_{i1}^{\mathrm{T}}, X_{i2}^{\mathrm{T}}, \ldots, X_{i7}^{\mathrm{T}}]^{\mathrm{T}}, \overline{Y}_i = [Y_{i1}^{\mathrm{T}}, Y_{i2}^{\mathrm{T}}, \ldots, Y_{i7}^{\mathrm{T}}]^{\mathrm{T}}, \overline{Z}_i = [Z_{i1}^{\mathrm{T}}, Z_{i2}^{\mathrm{T}}, \ldots, Z_{i7}^{\mathrm{T}}]^{\mathrm{T}}, \overline{T} = [T_1^{\mathrm{T}}, T_2^{\mathrm{T}}, \ldots, T_7^{\mathrm{T}}]^{\mathrm{T}}, i = 1, 2$ and a scalar $\varepsilon \ge 0$ such that the following symmetric linear matrix inequality holds:*

$$\begin{pmatrix} C^{\mathrm{T}}C - I + \delta E_C^{\mathrm{T}}E_C & C^{\mathrm{T}}I \\ LC & -(\delta I - L^{\mathrm{T}}L) \end{pmatrix} < 0, \tag{33}$$

$$\begin{pmatrix} \Theta & h\overline{X}_1 & h\overline{Y}_1 & h\overline{Z}_1 & \tau\overline{X}_2 & \tau\overline{Y}_2 & \tau\overline{Z}_2 & \Gamma_d \\ * & -hV_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -hV_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -hV_2 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\tau W_1 & 0 & 0 & 0 \\ * & * & * & * & * & -\tau W_1 & 0 & 0 \\ * & * & * & * & * & * & -\tau W_2 & 0 \\ * & * & * & * & * & * & * & -\varepsilon I \end{pmatrix} < 0, \tag{34}$$

where

$$
\begin{aligned}
&\Theta = \Theta_1 + \Theta_2 + \Theta_2^{\mathrm{T}} + +\varepsilon \Gamma_E^{\mathrm{T}} \Gamma_E,\\
&\Theta_1 = \begin{pmatrix}
R_1+R_2+M_1+M_2 & 0 & 0 & 0 & 0 & 0 & P\\
* & -(1-h_d)R_1 & 0 & 0 & 0 & 0 & 0\\
* & * & -R_2 & 0 & 0 & 0 & 0\\
* & * & * & -(1-\tau_d)Q & 0 & 0 & 0\\
* & * & * & * & -(1-\tau_d)M_1 & 0 & 0\\
* & * & * & * & * & -M_2 & 0\\
* & * & * & * & * & * & \Theta_{77}
\end{pmatrix},\\
&\Theta_{77} = Q + h(V_1+V_2) + \tau(W_1+W_2),\\
&\Gamma_E = (E_A, E_B, 0, E_C, 0, 0, 0),\\
&\Gamma_d = -(T_1^{\mathrm{T}}, T_2^{\mathrm{T}}, \cdots, T_7^{\mathrm{T}})^{\mathrm{T}} L\\
&\Theta_2 = (\overline{X}_1 + \overline{Z}_1 + \overline{X}_2 + \overline{Z}_2, -\overline{X}_1 + \overline{Y}_1, -\overline{Y}_1 - \overline{Z}_1, 0, -\overline{X}_2 + \overline{Y}_2, -\overline{Y}_2 - \overline{Z}_2, 0) + \overline{T}\,\overline{\Lambda} + \overline{\Lambda}^{\mathrm{T}}\overline{T}^{\mathrm{T}},\\
&\overline{\Lambda} = (-A, -B, 0, -C, 0, 0, I).
\end{aligned}
$$

Proof. First, by Lemma 1 we can get

$$(C + LF(t)E_C)^{\mathrm{T}}(C + LF(t)E_C) \le C^{\mathrm{T}}C + C^{\mathrm{T}}L(\delta I - L^{\mathrm{T}}L)^{-1}L^{\mathrm{T}}C + \delta E_C^{\mathrm{T}}E_C.$$

If (33) is satisfied, then we have that $(C + LF(t)E_C)^{\mathrm{T}}(C + LF(t)E_C) < I$ that implies $\|C + LF(t)E_C\| < 1$.

Next by removing the 8–10 rows and the 8–10 columns in Φ_1 and replacing A, B and C in (26) with $A+LF(t)E_A$, $B+LF(t)E_B$ and $C + LF(t)E_C$ respectively, we find that (26) for system (32) is equivalent to the following condition

$$\left[\Theta + h\overline{X}_1 V_1^{-1}\overline{X}_1^{\mathrm{T}} + h\overline{Y}_1 V_1^{-1}\overline{Y}_1^{\mathrm{T}} + h\overline{Z}_1 V_2^{-1}\overline{Z}_1^{\mathrm{T}} + \tau\overline{X}_2 W_1^{-1}\overline{X}_2^{\mathrm{T}} + \tau\overline{Y}_2 W_1^{-1}\overline{Y}_2^{\mathrm{T}} + \tau\overline{Z}_2 W_2^{-1}\overline{Z}_2^{\mathrm{T}}\right] + \Gamma_d F(t)\Gamma_E + \Gamma_E^{\mathrm{T}}F^{\mathrm{T}}(t)\Gamma_d^{\mathrm{T}} < 0. \tag{35}$$

By virtue of Lemma 2, a sufficient condition guaranteeing (35) is that there exists a scalar $\varepsilon > 0$ such that

$$\left[\Theta + h\overline{X}_1 V_1^{-1}\overline{X}_1^{\mathrm{T}} + h\overline{Y}_1 V_1^{-1}\overline{Y}_1^{\mathrm{T}} + h\overline{Z}_1 V_2^{-1}\overline{Z}_1^{\mathrm{T}} + \tau\overline{X}_2 W_1^{-1}\overline{X}_2^{\mathrm{T}} + \tau\overline{Y}_2 W_1^{-1}\overline{Y}_2^{\mathrm{T}} + \tau\overline{Z}_2 W_2^{-1}\overline{Z}_2^{\mathrm{T}}\right] + \varepsilon\Gamma_E^{\mathrm{T}}\Gamma_E + \varepsilon^{-1}\Gamma_d\Gamma_d^{\mathrm{T}} < 0. \tag{36}$$

Applying Schur complements, we find that (36) is equivalent to (34). Therefore we can conclude that the system described by (32) with mixed time-varying delays (2) and (3) is robustly stable if the LMI conditions (33) and (34) hold. □

Remark 6. In fact norm-bounded uncertainties can be treated as a special case of nonlinear parameter perturbations, but one can get a less conservative result using Theorem 3 than Theorems 1 and 2.

If $C = 0$ and $E_C = 0$, then system (32) reduces to the following system:

$$\begin{cases}\dot{x}(t) = (A + LF(t)E_A)x(t) + (B + LF(t)E_B)x(t - h(t)),\\ x(\theta) = \phi(\theta), \quad \forall\theta \in [-h, 0].\end{cases} \tag{37}$$

By Theorem 3, we have the following corollary for the delay-dependent stability of system (37).

Corollary 3. *The system (37) with time-varying delay satisfying (2) is robustly stable if there exists positive definite matrices $P > 0$, $R_i > 0$, $V_i > 0$, $i = 1, 2$ and appropriately dimensioned matrices $\widehat{X} = [X_1^{\mathrm{T}}, \ldots, X_4^{\mathrm{T}}]^{\mathrm{T}}$, $\widehat{Y} = [Y_1^{\mathrm{T}}, \ldots, Y_4^{\mathrm{T}}]^{\mathrm{T}}$, $\widehat{Z} = [Z_1^{\mathrm{T}}, \ldots, Z_4^{\mathrm{T}}]^{\mathrm{T}}$, $\widehat{T} = [T_1^{\mathrm{T}}, \ldots, T_4^{\mathrm{T}}]^{\mathrm{T}}$ and a scalar $\varepsilon \ge 0$ such that the following symmetric linear matrix inequality holds:*

$$\begin{pmatrix}
\widehat{\Theta} & h\widehat{X} & h\widehat{Y} & h\widehat{Z} & \widehat{\Gamma}_d\\
* & -hV_1 & 0 & 0 & 0\\
* & * & -hV_1 & 0 & 0\\
* & * & * & -hV_2 & 0\\
* & * & * & * & -\varepsilon I
\end{pmatrix} < 0, \tag{38}$$

where

Table 1
Maximum upper bound of h with $\tau_d = 0$, $h_d = 0.5$ and different values of γ.

$\alpha = 0$					$\alpha = 0.1$				
γ	0	0.1	0.2	0.3	γ	0	0.1	0.2	0.3
Han [14]	0.9328	0.7402	0.5637	0.4042	Han [14]	0.8148	0.6439	0.4864	0.3433
Zhang [15]	0.9488	0.7695	0.6087	0.4667	Zhang [15]	0.8408	0.6841	0.5420	0.4144
Theorem 1	0.9839	0.8024	0.6392	0.4941	Theorem 1	0.8752	0.7166	0.5727	0.4438

Table 2
Maximum upper bound of $h = \tau$ with $\tau_d = 0.5$, $h_d = 0.5$ and different values of γ.

$\alpha = 0$					$\alpha = 0.1$				
γ	0	0.1	0.2	0.3	γ	0	0.1	0.2	0.3
Han [14]	0.8524	0.5936	0.3686	0.1795	Han [14]	0.7434	0.5131	0.3112	0.1398
Theorem 1	0.9075	0.6665	0.4619	0.2930	Theorem 1	0.8084	0.5969	0.4151	0.2618

$$
\begin{aligned}
&\widehat{\Theta} = \widehat{\Theta}_1 + \widehat{\Theta}_2 + \widehat{\Theta}_2^{\mathrm{T}},\\
&\widehat{\Theta}_1 = \begin{pmatrix} R_1+R_2 & 0 & 0 & P \\ * & -(1-h_d)R_1 & 0 & 0 \\ * & * & -R_2 & 0 \\ * & * & * & h(V_1+V_2) \end{pmatrix},\\
&\widehat{\Theta}_2 = (\widehat{X}+\widehat{Z}, -\widehat{X}+\widehat{Y}, -\widehat{Y}-\widehat{Z}, 0) + \widehat{T}\widehat{\Lambda} + \widehat{\Lambda}^{\mathrm{T}}\widehat{T}^{\mathrm{T}},\\
&\widehat{\Gamma}_E = (E_A, E_B, 0, I),\\
&\widehat{\Gamma}_d = -(T_1^{\mathrm{T}}, T_2^{\mathrm{T}}, T_3^{\mathrm{T}}, T_4^{\mathrm{T}})^{\mathrm{T}} L,\\
&\widehat{\Lambda} = (-A, -B, 0, I).
\end{aligned}
$$

5. Illustrative examples

In this section, we use two examples and compare our results with the previous ones to show the effectiveness of ours.

Example 1. Consider the following system as in Han [14] and Zhang [15] with:

$$
\begin{aligned}
&A = \begin{pmatrix} -1.2 & 0.1 \\ -0.1 & -1 \end{pmatrix}, \qquad B = \begin{pmatrix} -0.6 & 0.7 \\ -1 & -0.8 \end{pmatrix}, \qquad C = \begin{pmatrix} c & 0 \\ 0 & c \end{pmatrix},\\
&f_1^{\mathrm{T}}(x(t), t) f_1(x(t), t) \le \alpha^2 x^{\mathrm{T}}(t) x(t),\\
&f_2^{\mathrm{T}}(x(t-h(t)), t) f_2(x(t-h(t)), t) \le \beta^2 x^{\mathrm{T}}(t-h(t)) x(t-h(t)),\\
&f_3^{\mathrm{T}}(\dot{x}(t-\tau(t)), t) f_3(\dot{x}(t-\tau(t)), t) \le \gamma^2 \dot{x}^{\mathrm{T}}(t-\tau(t)) \dot{x}(t-\tau(t)),
\end{aligned}
$$

where $0 \le |c| < 1$, $\alpha \ge 0$, $\beta \ge 0$, $\gamma \ge 0$.

Case I. For $c = 0.1$, $\beta = 0.1$, $\tau = 1$, $\tau_d = 0$, $h_d = 0.5$, and different values of γ, we apply Theorem 1 to calculate the maximal allowable value h that guarantees the asymptotical stability of the system. Table 1 illustrates the numerical results for different γ, $\alpha = 0$ and $\alpha = 0.1$, respectively. It can be seen from Table 1 that the maximum allowable delay h decreases as γ increases. In addition, it is easy to see that our proposed stability criterion gives a much less conservative result than those in [14] and [15].

Case II. Next we will consider this case that the discrete delay and the neutral delay are all time-varying. For $c = 0.1$, $\beta = 0.1$, $\tau_d = h_d = 0.5$, and different values of γ, the maximum upper bounds on the allowable delay of $\tau = h$ obtained from Theorem 1 are listed in Table 2. Note that the proposed criterion in [14] is only neutral-delay dependent, but is discrete-delay-independent which shows more conservativeness than ours.

Case III. For $c = 0$ and $f(\dot{x}(t-\tau(t)), t) = 0$, the maximum value h obtained from Corollary 2 is listed in Table 3. It is clear that the obtained results in our paper are significantly better than those in [12–14,17,18,20].

Example 2. Consider the following uncertain neutral system as in Han [14]

$$
\dot{x}(t) = \begin{pmatrix} -2+\delta_1 & 0 \\ 0 & -1+\delta_2 \end{pmatrix} x(t) + \begin{pmatrix} -1+\delta_3 & 0 \\ -1 & -1+\delta_4 \end{pmatrix} x(t-h(t)) + \begin{pmatrix} c & 0 \\ 0 & c \end{pmatrix} \dot{x}(t-\tau(t)),
$$

where $0 \le |c| < 1$ and δ_1, δ_2, δ_3 and δ_4 are unknown parameters satisfying:

$$
|\delta_1| \le 1.6, \quad |\delta_2| \le 0.05, \quad |\delta_3| \le 0.1, \quad \delta_4 \le 0.3.
$$

Table 3
Maximum upper bound of h with $\beta = 0.1$.

	$\alpha = 0$			$\alpha = 0.1$		
	$h_d = 0$	$h_d = 0.5$	$h_d \geq 1$	$h_d = 0$	$h_d = 0.5$	$h_d \geq 1$
Cao and Lam [17]	0.6811	0.5467	–	0.6129	0.4950	–
Han [13]	1.3279	0.6743	–	1.2503	0.5715	–
Han [14]	2.7424	1.1365	–	1.8753	0.9952	–
Zhang [20]	2.742	1.142	–	1.875	1.009	–
Zou [12]	2.7422	1.1424	–	1.8753	1.0097	–
Chen [18]	2.7423	1.1425	0.7355	1.8753	1.0097	0.7147
Corollary 2	2.7757	1.1849	0.9284	1.8959	1.0512	0.8865

Table 4
Maximum upper bound of h with different c.

$\lvert c \rvert$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
Zhao [9]	0.92	0.73	0.55	0.41	0.29	0.19	0.11	0.04
Han [14]	0.97	0.78	0.60	0.45	0.31	0.19	0.10	0.02
Theorem 3	1.1072	0.9208	0.7516	0.5991	0.4625	0.3402	0.2310	0.1277

Table 5
Maximum upper bound of h with $\tau_d = 0.1$ and different h_d.

h_d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$h_d \geq 1$
Han [14]	0.81	0.77	0.73	0.68	0.62	0.57	0.50	0.42	0.32	0.17	–
Theorem 3	0.9404	0.9109	0.8825	0.8537	0.8253	0.7980	0.7711	0.7472	0.7287	0.7199	0.7216

Case I. For $h_d = 0.1$, $\tau_d = 0$, the maximum values of h is listed in Table 4 for various values of c by applying criteria in [9,14] and in this work. Furthermore, It can be seen from Table 4 that the maximum allowable delay d decreases as c increases.

Case II. For $c = 0.1$, $\tau_d = 0.1$, the effect of h_d on the maximum value h obtained from Theorem 3 is listed in Table 5. In [10,21] the neutral delay is constant, then its stability criterion cannot be applied to systems with time-varying neutral delay. Moveover, the stability criterion in [14,15] cannot be applied to this case that the derivative of the discrete delay is more than 1. It is obvious that the obtained results are significantly better than those in [10,14,15,21].

Case III. When $C = 0$ and $E_C = 0$, by Corollary 3 we can obtain the maximum upper bound on the allowable size to be $h = 1.4011$. However, applying criteria in [8,19,14,22], the maximum value of h for the above system is 0.2412, 1.0, 1.0345 and 1.2093, respectively.

6. Conclusion

This paper has discussed stability criteria for neutral systems with mixed time-varying delays and nonlinear perturbations. Based on Lyapunov functional approach and linear matrix inequality technology, the less conservative delay-dependent stability conditions are derived. The proposed criteria are both neutral-delay dependent and discrete dependent, and at the same time, are dependent on the derivative of the discrete and neutral delays. Numerical examples have shown the effectiveness of the proposed method.

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