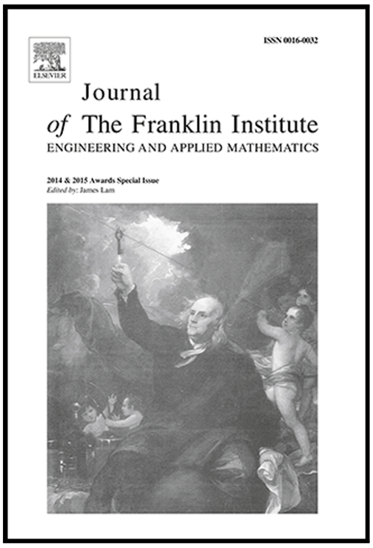

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Stability analysis of micro-grid frequency control system with two additive time-varying delay

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Abstract

Participation of open communication network may cause time delays in micro-grid load frequency control(LFC) system. This paper examines the stability problem of micro-grid LFC system with two additive time delays and load disturbance. A novel LyapunovKrasovskii functional(LKF) is proposed with the consideration of more augmented terms. With the help of the proposed LKF, a new stability condition is developed by using the auxiliary function-based integral inequality. The relationship is built among the admitted maximum values of two delays, the controller parameters and the bounds of the nonlinear load disturbances. The main results can guide the determination of time delay requirements for micro-grid LFC system with communication network. Finally, the validity of the presented approach is illustrated by simulation case studies based on Matlab.

Keywords: stability analysis, micro-grid LFC, time-varying delay

1. Introduction

With the advance of renewable energy techniques, the integration of these techniques and power system has gradually become a focus of research. The micro-grid system encompasses several distributed generations, storage units and loads. The different types of distributed generations observably increase comprehensive performance and flexibility for power systems, which include

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photo-voltaic(PV) generation, micro-gas turbine and fuel cell etc [1, 2, 3]. Therefore, the extension of micro-grid technique can fully promote the large-scale access of distributed generations and renewable energy sources. Micro-grid is a significant direction of future power system[4].

For a power system, frequency stability and controllability should be maintained. Since the renewable energy and distributed generations in micro-grid are irregular and changeable, LFC strategies are utilized to ensure a steady operation with expected frequency[5]. For this purpose, the LFC framework with PI control is employed in this paper. In addition, a micro-grid central controller(MGCC) is designed to coordinate and control all distributed generators in micro-grid system through communication network. In conventional LFC system, a dedicated communication channel is applied which neglects the transmission delay[6]. Drawing a comparison between dedicated communication channel and open communication network, we find that the open communication network has some more advanced features. The open communication network realizes information exchange of large scale and large amount of data with low installation cost, which makes it possible to analyze and control power system from a global perspective. Hence, the micro-grid LFC system demands an open communication network for information transmission[7, 8, 9, 10]. But the use of open communication network for information transmission inevitably leads to some problems such as time delay, network congestion and quantification[11, 12]. Therefore, how to raise the time delay bound and reduce transmission consumption evolve into an important research domain[13, 14, 15, 16].

Owing to the expansive application of open communication network, researchers pay close attention to the stability analysis of time-delay LFC system. For traditional power system perspective, numerous research efforts have been devoted to acquire the stability criteria for LFC system. In [17] the stability conditions for LFC system with multiple time-invariant delay were obtained by utilizing the simple LKF. Zhang et al. [18] further investigated delay-dependent stability of LFC system emphasizing on multi-area and deregulated environment. Snmez et al. [19] proposed an exact method based on frequency domain to provide accurate delay margin for multi-area LFC system. An improved stability condition was presented by employing a novel augmented LKF and the infinite-series-based Inequality [20]. In [21] the truncated Bessel-Legendre inequality was applied to establish the less conservative stability criteria for LFC system. However, in contrast to the plentiful existing research on traditional LFC power system, lesser results about LFC of micro-grid system have been derived. Gndz et al. [22] presented an exact approach that consider both gain and phase margins to determine the delay bound. In [23] the stability problem of the multiple delayed LFC system with participation of electric vehicle aggregators has been investigated. Ramakrishnan et al. [24] studied the problem of stability analysis for micro-grid LFC system with time-varying delay. Therefore, it is necessary to further investigate the influence of time delay on micro-grid LFC system.

Most studies concentrate on system with single time delay which is regarded as a combination of the total delays caused by the communication channel. For power system with open communication network, signal transmission may experience different network segments, resulting in time delays with different characteristics [25]. Then, those time delays can not be considered as a whole. However, the problem of stability of micro-grid LFC system with two additive delays has not been investigated until now.

In this paper, we focus on the stability analysis of micro-grid LFC system with two additive time-varying delays and propose new stability criteria. Firstly, a new model of micro-grid LFC system is proposed, which considers two additive time-varying delays and load uncertainties. Secondly, by constructing novel LKF combined with auxiliary function-based integral inequality, less conservative stability criteria are developed in the LMI form. Thirdly, we explore the interaction among the admitted maximum delay, the control design gain and load disturbance by using the proposed stability criteria. Finally, several numerical examples and simulation case studies are given to show the improvements of the presented stability criteria.

Notations: Throughout this paper, $\mathbb{R}^n$ stands for the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ denotes the set of all $n \times m$ real matrices. X^T and X^{-1} refer to the transpose and the inverse of X, respectively. $\mathbb{N}$ denotes the set of nonnegative integers, and $\mathbb{N}^+$ is the set of positive integers. $X > Y$ $(X \geq Y)$ means that the matrix $X - Y$ is positive definite (positive semi-definite). The notation $diag\{\cdots\}$ means a block-diagonal matrix. For $X \in \mathbb{S}_n^+$, represents X is symmetric and positive definite matrix. $*$ denotes the symmetric term in a symmetric matrix and we define $Sym\{X\} = X + X^T$.

2. Problem formulation and preliminary

The model of micro-grid LFC system using open communication network is presented in this section. Fig. 1 depicts a typical block diagram for micro-grid system with PI-based LFC scheme [22, 24, 26]. Table 1 details the notations used in the micro-grid LFC systems. In the system model, wind and photovoltaic generators are employed as primary sources. Since the renewable sources of energy are irregular and changeable, a micro-gas turbine unit is involved to afford the base load. In addition, a fuel cell and electrolyser system, for frequency compensation, are incorporated into the micro-grid system when an accidental real power imbalance happens. A MGCC is implemented together with an open communication network.

Remark 1. *Due to the nonlinearity and complexity of the real micro-grid power system, it is difficult to accurately establish the model. In analysis and synthesis of power system, a simplified linear model is commonly employed to characterize the LFC system. With some simplifications, each part of the system model provided in this paper can represent the frequency-dependent main features respectively. Moreover, the two additive time delays $d_1(t)$ and $d_2(t)$ caused by open communication network are considered in the model.*

Base on the observation, the dynamic state-space model for micro-grid LFC system with two additive time-varying delay components and disturbance is

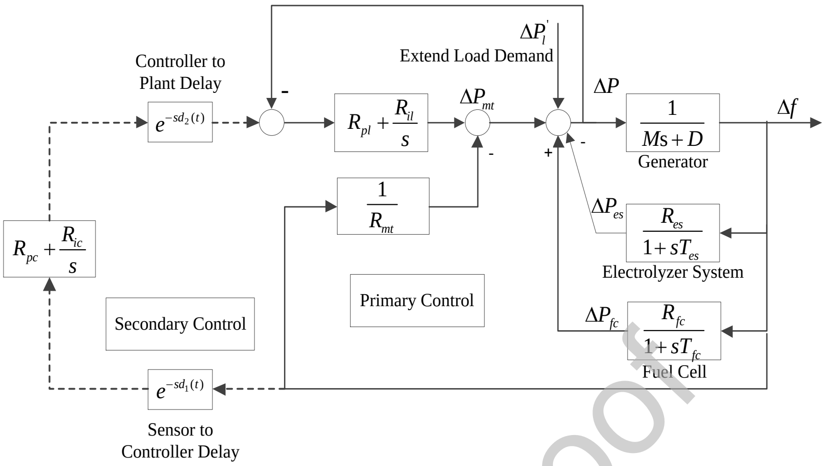

Figure 1: Micro-grid LFC system with two additive time delay

Table 1: Parameters of micro-grid LFC system

Parameter	Description
$\Delta f(t)$	Deviation of frequency
M	Moment of inertia of generator
D	Damping constant of generator
ΔP_{es}	Output power of electrolyser
R_{es}	Gain of electrolyser
T_{es}	Time constant of electrolyser
ΔP_{fc}	Output power of fuel cell
R_{fc}	Gain of fuel cell
T_{fc}	Time constant of fuel cell
$\Delta P_l^{'}$	Disturbance of load
ΔP_{mt}	Change in output power
R_{mt}	Drop characteristics of the micro-turbine
R_{pl}	Proportional gain of local controller
R_{il}	Integral gain of local controller
R_{pc}	Proportional gain of central controller
R_{ic}	Integral gain of central controller

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given as follows:

$$
\begin{aligned}
\dot{x}(t) &= Ax(t) + A_d x(t - d_1(t) - d_2(t)) + K\Delta P_l(t) \\
x(t) &= \varphi(t), t \in [-h, 0]
\end{aligned}
\tag{1}
$$

where $x(t) \in \mathbb{R}^5$ is the state vector, $x(t) = \begin{bmatrix} R_{ic}\int \Delta f dt & \Delta P_{mt} & \Delta P_{fc} & \Delta P_{es} & \Delta f \end{bmatrix}^T$, system matrices $A \in \mathbb{R}^{5\times 5}$ and $A_d \in \mathbb{R}^{5\times 5}$ are given as follows

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & R_{ic} \\ 0 & \epsilon_1 & \epsilon_2 & \epsilon_3 & \epsilon_4 \\ 0 & 0 & -\frac{1}{T_{fc}} & 0 & \frac{R_{fc}}{T_{fc}} \\ 0 & 0 & 0 & -\frac{1}{T_{es}} & \frac{R_{es}}{T_{es}} \\ 0 & \frac{1}{M} & \frac{1}{M} & -\frac{1}{M} & -\frac{D}{M} \end{bmatrix}, \quad A_d = \begin{bmatrix} 0_{5\times 1} & \tau & 0_{5\times 1} & 0_{5\times 1} & 0_{5\times 1} \end{bmatrix}^T \tag{2}$$

where

$$\alpha = \frac{1}{1+R_{pl}}, \quad \epsilon_1 = \alpha\left(-R_{il} - \frac{1}{MR_{mt}}\right), \quad \epsilon_2 = \alpha\left(\frac{R_{pl}}{T_{fc}} - R_{il} - \frac{1}{MR_{mt}}\right)$$
$$\epsilon_3 = \alpha\left(-\frac{R_{pl}}{T_{es}} + R_{il} + \frac{1}{MR_{mt}}\right), \quad \epsilon_4 = \alpha\left(-\frac{R_{pl}R_{fc}}{T_{fc}} + \frac{R_{pl}R_{es}}{T_{es}} + \frac{D}{MR_{mt}}\right)$$
$$\tau = \begin{bmatrix} -\alpha R_{il} & -\alpha\frac{R_{pl}R_{pc}}{M} & -\alpha\frac{R_{pl}R_{pc}}{M} & \alpha\frac{R_{pl}R_{pc}}{M} & \alpha\left(-R_{il}R_{pc} + \frac{R_{pl}R_{pc}D}{M} - R_{pl}R_{ic}\right) \end{bmatrix}^T$$

the two time-varying delays $d_1(t)$ and $d_2(t)$ satisfy the following bounding conditions:

$$0 \le d_1(t) \le h_1, \quad \dot{d}_1(t) \le \mu_1 \tag{3}$$
$$0 \le d_2(t) \le h_2, \quad \dot{d}_2(t) \le \mu_2 \tag{4}$$

Let $d(t) = d_1(t) + d_2(t)$ and $h = h_1 + h_2$. The load disturbance of micro-grid LFC system is modeled as nonlinear perturbation associated with current and delayed state variables

$$K\Delta P_l(t) = f(t, x(t), x(t-d(t))) \tag{5}$$

satisfy the following norm-bounded condition

$$\|f(\cdot)\| \le \alpha\|x(t)\| + \beta\|x(t-d(t))\| \tag{6}$$

where $\alpha \geq 0$ and $\beta \geq 0$ are known scalars. Then, the more generalized version of the condition (6) is as follow:

$$\begin{aligned} f(\cdot)^T f(\cdot) \leq & \alpha^2 x^T(t) Z^T Z x(t) \\ & + \beta^2 x^T(t-d(t)) X^T X x(t-d(t)) \end{aligned} \tag{7}$$

where Z and X are known constant matrices of appropriate dimensions. The matrices Z and X together with the scalars α and β quantify the magnitude of the load disturbance to the micro-grid system. When the delays $d_1(t)$ and $d_2(t)$ are combined into a single time-varying delay $d(t)$, the micro-grid LFC system (1) is rewrited as (8), which was investigated in [24].

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-d(t)) + K\Delta P_l(t) \\ x(t) &= \varphi(t), t \in [-h, 0] \end{aligned} \tag{8}$$

Remark 2. *We consider the external load disturbance in the micro-grid system and model it as a nonlinear disturbance associated with delay and current system state. Since the load disturbance can affect the state variable of the micro-grid LFC system, the mathematical model of load disturbance are reasonable and feasible. And the norm bounded restriction for load disturbance is also effective in which the PI controller is assumed to restore the perturbed micro-grid LFC system back to its initial equilibrium state.*

The objective of this paper is to derive new stability criteria for micro-grid system with two additive time delay. To prove the main results, the following lemmas are introduced.

Lemma 1. *[27]Let $\nu(s)$ be a differentiable function ν:$[a, b] \to \mathbb{R}^n$, for a given positive definite matrix R, the following inequality holds*

$$\int_a^b \dot{\nu}^T(s) R \dot{\nu}(s) ds \geq \frac{1}{b-a} \sum_{i=1}^{3} (2i-1) \Omega_i^T R \Omega_i \tag{9}$$

where

$$\Omega_1 = \nu(b) - \nu(a), \quad \nu_2 = \nu(b) + \nu(a) - \frac{2}{b-a}\int_a^b \nu(s)ds$$
$$\Omega_3 = \nu(b) - \nu(a) + \frac{6}{b-a}\int_a^b \nu(s)ds - \frac{12}{b-a}\int_a^b\int_s^b \nu(u)duds$$

Lemma 2. *[28]For function $g(x) = \alpha_2 x^2 + \alpha_1 x + \alpha_0 (\alpha_0, \alpha_1, \alpha_2 \in \mathbb{N})$ where $x \in [0, h]$, the inequality $g(x) < 0$ is holds, if*

$$(1) g(0) < 0, \quad (2) g(h) < 0, \quad (3) - h^2\alpha_2 + g(0) < 0$$

110 ## 3. Main Results

Theorem 1. *For given scalars h, μ, system (8) is asymptotically stable with time-varying delay $d(t)$ satisfying $0 \le d(t) \le h$, $\dot{d}(t) \le \mu$, if there exist matrices $P \in \mathbb{S}^+_{3n}$, $U_1 \in \mathbb{S}^+_{3n}$, $U_2 \in \mathbb{S}^+_n$, $R \in \mathbb{S}^+_n$ and any matrices $L \in \mathbb{R}^{9n\times n}$, $S \in \mathbb{R}^{3n\times 3n}$, a scalar $\sigma_1 \ge 0$, satisfying the following LMIs:*

$$\begin{bmatrix} \psi_1 & S \\ * & \psi_1 \end{bmatrix} > 0 \tag{10}$$

$$\Upsilon_{[d(t)=0]} < 0, \quad \Upsilon_{[d(t)=h]} < 0, \quad \Upsilon_{[d(t)=0]} - h^2\varsigma < 0 \tag{11}$$

Proof. It is provided in Appendix section.

Remark 3. *When the disturbance of the LFC system is negligible and information of the upper bound $\dot{d}(t)$ is unknown, the stability condition of the system can be provided by choosing $\alpha = 0$, $\beta = 0$ and $U_1 = 0$ in Theorem 1. This result*
115 *is shown as follows.*

Corollary 1. *For given scalars h, system (8) is asymptotically stable with time-varying delay $d(t)$ satisfying $0 \le d(t) \le h$, $K\Delta P_l(t) = 0$, if there exist matrices $P \in \mathbb{S}^+_{3n}$, $U_2 \in \mathbb{S}^+_n$, $R \in \mathbb{S}^+_n$, any matrices $L \in \mathbb{R}^{9n\times n}$, $S \in \mathbb{R}^{3n\times 3n}$, satisfying the following LMIs:*

$$\begin{bmatrix} \psi_1 & S \\ * & \psi_1 \end{bmatrix} > 0 \tag{12}$$

$$\tilde{\Upsilon}_{[d(t)=0]} < 0, \quad \tilde{\Upsilon}_{[d(t)=h]} < 0, \quad \tilde{\Upsilon}_{[d(t)=0]} - h^2\tilde{\varsigma} < 0 \tag{13}$$

Proof. The proof of Corollary 1 is similar to the proof of Theorem 1. It is omitted here.

Since the characteristics of the delays $d_1(t)$ and $d_2(t)$ may not be identical owing to the network transmission conditions, it is unreasonable to combine them into a single delay as $d(t)$. The stability criteria for micro-grid LFC system with two additive time-varying delays are given as follows:

Theorem 2. *For given scalars h_1, h_2, μ_1 and μ_2, system (1) is asymptotically stable with time-varying delays $d_1(t)$ and $d_2(t)$ satisfying (3) and (4), if there exist matrices $P \in \mathbb{S}_{2n}^{+}$, $U_1 \in \mathbb{S}_n^+$, $U_2 \in \mathbb{S}_n^+$, $U_3 \in \mathbb{S}_n^+$,$U_4 \in \mathbb{S}_n^+$ and $R \in \mathbb{S}_n^+$, any matrices $X_1 \in \mathbb{R}^{2n\times 2n}$, $X_2 \in \mathbb{R}^{2n\times 2n}$, a scalar $\sigma_2 \geq 0$, satisfying the following LMIs:*

$$\begin{bmatrix} \psi_2 & X_2 \\ * & \psi_2 \end{bmatrix} > 0, \quad \begin{bmatrix} \psi_2 & X_3 \\ * & \psi_2 \end{bmatrix} > 0 \tag{14}$$

$$\Psi_{[d(t)=0]} < 0, \quad \Psi_{[d(t)=h]} < 0 \tag{15}$$

Proof. It is provided in Appendix section.

Remark 4. *Enlighten by the [29], the $V_2(t)$ is proposed to exploit more information on the relation of delay components. The three-order auxiliary function-based integral inequality is used in Theorem 1 and Corollary 1, which provides tighter bound than Wirtinger-based inequality. By considering the tradeoff between conservatism and calculation complexity, the two-order auxiliary function-based integral inequality is used in Theorem 2. On the other hand, in order to cooperate with the integral inequality, $\delta_1(t)^T P \delta_1(t)$ is built in $V_2(t)$.*

4. Case study and discussion

In this section, stability results for micro-grid LFC system with single and two additive time-varying delays are presented, respectively. The improvement and validity of the method are illustrated by a comparison with existing literature. The system parameters taken from [22] are $R_{il} = 1$, $R_{mt} = 0.04$, $M = 10$,

$R_{fc} = 1$, $T_{fc} = 4$, $D = 1$, $R_{es} = 1$, $R_{pl} = 1$, and the matrices Z, X correlated with the disturbance are set to $0.1I_n$.

4.1. Time-invariant delay

When $\alpha = 0$, $\beta = 0$ and $d(t)$ as a constant delay ($\mu = 0$), the micro-grid LFC system is deduced to nominal system with time-invariant delay. For different R_{pc} and R_{ic}, the admitted maximum delay for nominal micro-grid LFC system are presented in Table 2 based on Theorem 1. From [22], the accurate delay margin results for nominal micro-grid LFC system with time-invariant delay are provided in Table 3. Comparing the above two tables, under the same parameters, the errors between Table 2 and Table 3 are very small. The results certify that the stability criteria according to Theorem 1 are less conservative. It is worth noting that the Theorem 1 using the Lyapunov theory can deal with the stability problem of micro-gird LFC system with time-varying delay and disturbance, while frequency domain method cannot.

Table 2: Admitted maximum delay for nominal micro-grid LFC system

h/s	R_{pc}					
R_{ic}	0.5	1	1.5	2	2.5	3
0.2	7.8713	9.9443	11.5736	12.6737	13.2219	13.2725
0.4	4.1252	5.2651	6.2859	7.1559	7.8511	8.3577
0.6	2.8807	3.6647	4.3944	5.0543	5.6307	6.1132
0.8	2.2606	2.8577	3.4240	3.9501	4.4277	4.8495
1	1.8896	2.3717	2.8340	3.2702	3.6748	4.0424

Table 3: Accurate delay margin results

h/s	R_{pc}					
R_{ic}	0.5	1	1.5	2	2.5	3
0.2	7.8713	9.9443	11.5736	12.6737	13.2220	13.2727
0.4	4.1252	5.2651	6.2859	7.1560	7.8512	8.3578
0.6	2.8808	3.6647	4.3945	5.0543	5.6308	6.1132
0.8	2.2606	2.8577	3.4240	3.9501	4.4277	4.8496
1	1.8897	2.3718	2.8340	3.2703	3.6748	4.0426

4.2. Time varying delay

For different controller parameters, disturbance conditions and time-varying delay constraints, the admitted maximum delay bound according to Theorem 1 are shown in Table 4. As shown in the Table 4, with the increase of parameters α and β, admitted maximum delay bound decrease. It reflects that the disturbance on micro-grid LFC system is one of the most factors affecting the admitted maximum delay bound.

The admitted maximum delay bound, without restriction for u, are present in Table 5 according to Corollary 1. When micro-grid LFC system have same controller gains, the admitted maximum delay bound calculated by Corollary 1 are larger than ones obtained in [24]. This is owing to the employment of the novel LKF and suitable integral inequality. For nominal micro-grid LKF system, the relationship between the controller gain and the admitted maximum delay bound is illuminated in Table 5. When the controller parameter R_{pc} constant, the admitted maximum delay bound declines as R_{ic} increase, and the tendency is more obvious as the R_{pc} is small. When the controller parameter R_{ic} constant, the admitted maximum delay bound increases initially and decreases afterwards as the R_{pc} increases. The bold fonts in Table 5 represent the maximal delay bound when R_{ic} is fixed. The simulation results are provided to show the validity of the stability method. By setting $R_{pc} = 0.5$, $R_{ic} = 0.4$, $d(t) = \frac{3.044}{2} sin(\frac{1}{3.044}t) + \frac{3.044}{2}$, the load disturbance is considered as $f = 0.0025tanh(x(t - d(t)))$, the frequency deviation response of the micro-grid LFC system are obtained in Fig 2. It is clearly to see that the micro-grid LFC system with time-varying delay satisfying $h < 3.044$ and $\mu < 0.5$ is asymptotically stable.

4.3. Additive time varying delay

We consider the micro-grid LFC system (1) with two additive time delays when the parameters and time delays satisfy $R_{pc} = 3$, $R_{ic} = 0.2$ and $\mu_1 \leq 0.1$, $\mu_2 \leq 0.8$, respectively. Our object is to obtain the admitted maximum delay

Table 4: Admitted maximum delay bound for different u and load disturbance

		$\alpha=0,\beta=0$		$\alpha=0,\beta=0.025$		$\alpha=\beta=0.025$	
R_{pc}	R_{ic}	$\mu=0$	$\mu=0.5$	$\mu=0$	$\mu=0.5$	$\mu=0$	$\mu=0.5$
0.5	0.2	7.871	7.814	6.154	6.091	5.468	5.432
0.5	0.4	4.125	4.111	3.078	3.044	2.653	2.634
0.5	0.6	2.880	2.874	2.071	2.047	1.741	1.727
1.0	0.2	9.944	9.869	8.312	8.217	7.661	7.598
1.0	0.4	5.265	5.247	4.240	4.185	3.825	3.788
1.0	0.6	3.664	3.657	2.864	2.824	2.538	2.511

Table 5: Admitted maximum delay bound for any u based Corollary 1

h/s	R_{pc}							
R_{ic}	1	2	3	4	5	6	7	8
0.2	9.864	12.469	**12.653**	11.087	9.359	7.959	6.876	6.031
0.4	5.246	7.105	8.249	**8.575**	8.159	7.385	6.578	5.861
0.6	3.656	5.028	6.057	6.678	**6.859**	6.636	6.173	5.637
0.8	2.852	3.933	4.811	5.442	5.802	**5.876**	5.697	5.358
1	2.368	3.257	4.013	4.602	5.007	5.211	**5.213**	5.043

bound h_1 of $d_1(t)$, or h_2 of $d_2(t)$, when one of them is known. By employing Theorem 2 with different disturbance conditions, we obtain the admitted maximum delay bound h_2 of $d_2(t)$ tabulated in Table 6 when $h_1 = 6$, 7 and 8. Similarly, when $h_2 = 3$, 4 and 5, we also have the admitted maximum delay bound h_1 of $d_1(t)$ tabulated in Table 6. The deduced results afford more useful guidelines for micro-grid LFC system considering the two delays difference. The simulation results are carries out for this case: two time-varying delays are $d_1(t) = 2sin(0.05t)+4$, $d_2(t) = \frac{5.494}{2}sin(\frac{1.6}{5.494}t)+\frac{5.494}{2}$(satisfying $0 \le d_1(t) \le 6$, $0 \le d_1(t) \le 5.494$, $\mu_1 \le 0.1$, $\mu_2 \le 0.8$) and the load disturbance is considered as $f = 0.0025tanh(x(t))$. Then, the frequency deviation response of the micro-grid LFC system are displayed in Figure 3, which shows the validity of the method.

Table 6: Admitted maximum delay bound by Theorem 2

Disturbance	Delay bound h_2 for given h_1			Delay bound h_2 for given h_1		
	$h_1=6$	$h_1=7$	$h_1=8$	$h_2=3$	$h_2=4$	$h_2=5$
$\alpha=\beta=0$	6.383	5.391	4.401	9.419	8.405	7.395
$\alpha=0.025,\beta=0$	5.494	4.501	3.513	8.519	7.509	6.497
$\alpha=\beta=0.025$	5.173	4.181	3.192	8.192	7.185	6.177

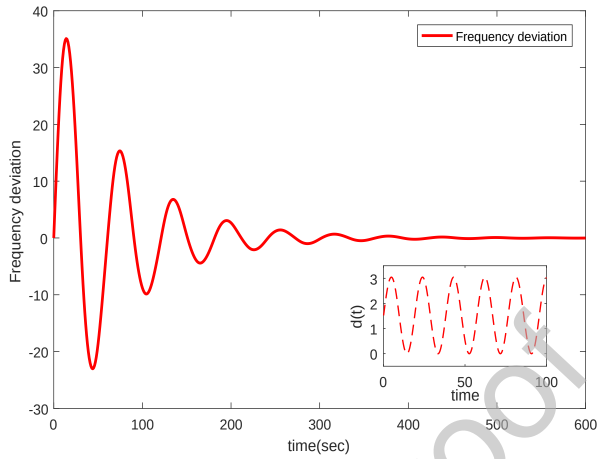

Figure 2: Frequency response under single time-varying delay

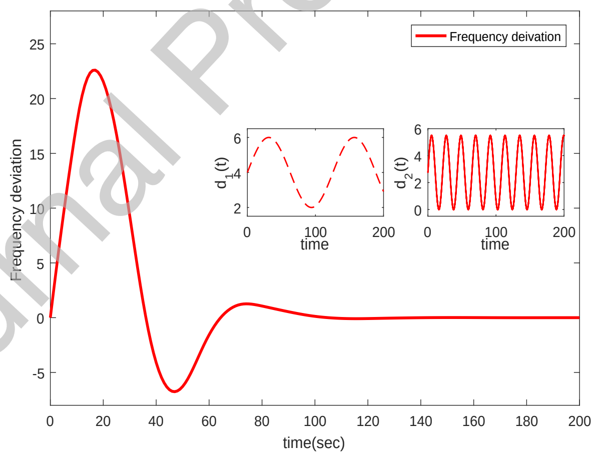

Figure 3: Frequency response under two additive time delays

5. Conclusions

This paper investigates the stability problem for micro-grid LFC system with
190 two additive time-varying delays and load disturbance. Some less conservative

stability conditions are built by employing novel LKF combined with auxiliary function-based integral inequality. Then the impact of the delays on micro-grid system stability can be determined by utilizing the proposed stability criteria. The methods in this paper is helpful to the determination of delay requirement and the design of controller for micro-grid LFC system. How to handle stability problems of the random communication delays and data loss in micro-grid LFC system are later direction of study.

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Appendix

For simplicity, some notations for matrices and vectors are introduced as follows:

$$
\begin{aligned}
e_i &= \begin{bmatrix} 0_{n\times(i-1)n} & I_{n\times n} & 0_{n\times(9-i)n} \end{bmatrix}^T (i=1,\dots,9) \\
e_s &= Ae_1^T + A_d e_2^T + e_9^T, \quad e_0 = 0_{9n\times n} \\
\tilde{e}_i &= \begin{bmatrix} 0_{n\times(i-1)n} & I_{n\times n} & 0_{n\times(8-i)n} \end{bmatrix}^T (i=1,\dots,8) \\
\tilde{e}_s &= A\tilde{e}_1^T + A_d \tilde{e}_2^T, \quad \tilde{e}_0 = 0_{8n\times n} \\
\bar{e}_i &= \begin{bmatrix} 0_{n\times(i-1)n} & I_{n\times n} & 0_{n\times(10-i)n} \end{bmatrix}^T (i=1,\dots,10) \\
\bar{e}_s &= A\bar{e}_1^T + A_d \bar{e}_2^T + \bar{e}_{10}^T, \quad \bar{e}_0 = 0_{10n\times n} \\
\psi_1 &= diag\{R, 3R, 5R\}, \quad \psi_2 = diag\{R, 3R\} \\
\upsilon &= \begin{bmatrix} x^T(t) & x^T(t-d(t)) & x^T(t-h) \end{bmatrix} \\
\chi_1(t) &= \Big[\ \upsilon \quad \tfrac{1}{d(t)}\int_{t-d(t)}^{t} x^T(s)ds \quad \tfrac{1}{h-d(t)}\int_{t-h}^{t-d(t)} x^T(s)ds \\
&\quad \tfrac{1}{d^2(t)}\int_{t-d(t)}^{t}\int_s^t x^T(u)duds \quad \tfrac{1}{(h-d(t))^2}\int_{t-h}^{t-d(t)}\int_s^{t-d(t)} x^T(u)duds \\
&\quad \int_{t-h}^{t}\int_s^t x^T(u)duds \quad f^T(t,x(t),x(t-d(t))) \ \Big] \\
\chi_2(t) &= \Big[\ \upsilon \quad x^T(t-d_1(t)) \quad x^T(t-d_1(t)-h_2) \quad \tfrac{1}{d_1(t)}\int_{t-d_1(t)}^{t} x^T(s)ds \\
&\quad \tfrac{1}{d_2(t)}\int_{t-d(t)}^{t-d_1(t)} x^T(s)ds \quad \tfrac{1}{h_2-d_2(t)}\int_{t-d_1(t)-h_2}^{t-d(t)} x^T(s)ds \\
&\quad \tfrac{1}{h_1-d_1(t)}\int_{t-h_1-h_2}^{t-d_1(t)-h_2} x^T(s)ds \quad f^T(t,x(t),x(t-d(t))) \ \Big] \\
\gamma_1 &= \begin{bmatrix} e_1-e_2 & e_1+e_2-2e_4 & e_1-e_2+6e_4-12e_6 \end{bmatrix} \\
\gamma_2 &= \begin{bmatrix} e_2-e_3 & e_2+e_3-2e_5 & e_2-e_3+6e_5-12e_7 \end{bmatrix} \\
\zeta_1 &= \begin{bmatrix} \bar{e}_1-\bar{e}_4 & \bar{e}_1+\bar{e}_4-2\bar{e}_6 & \bar{e}_5-\bar{e}_3 & \bar{e}_5+\bar{e}_3-2\bar{e}_9 \end{bmatrix} \\
\zeta_2 &= \begin{bmatrix} \bar{e}_4-\bar{e}_2 & \bar{e}_4+\bar{e}_2-2\bar{e}_7 & \bar{e}_2-\bar{e}_5 & \bar{e}_2+\bar{e}_5-2\bar{e}_8 \end{bmatrix} \\
\Upsilon_{[d(t)]} &= Sym\Big\{ \begin{bmatrix} e_1 & (h-d(t))e_5+d(t)e_4 & e_8 \end{bmatrix} \\
&\quad \times P \begin{bmatrix} e_s & e_1-e_3 & he_1-(h-d(t))e_5-d(t)e_4 \end{bmatrix}^T \Big\}
\end{aligned}
$$

$$
\begin{aligned}
&+ \begin{bmatrix} e_1 & e_0 & e_0 \end{bmatrix} U_1 \begin{bmatrix} e_1 & e_0 & e_0 \end{bmatrix}^T \\
&- (1-\mu) \begin{bmatrix} e_2 & e_1 - e_2 & d(t)e_4 \end{bmatrix} U_1 \begin{bmatrix} e_2 & e_1 - e_2 & d(t)e_4 \end{bmatrix}^T \\
&+ d(t)Sym \left\{ \begin{bmatrix} e_4 & e_1 - e_4 & d(t)e_6 \end{bmatrix} U_1 \begin{bmatrix} e_0 & e_s & e_1 \end{bmatrix}^T \right\} + e_1 U_2 e_1^T - e_3 U_2 e_3 \\
&+ h e_s R e_s^T - \frac{1}{h} \begin{bmatrix} \gamma_1 & \gamma_2 \end{bmatrix} \begin{bmatrix} \psi_1 & S \\ * & \psi_1 \end{bmatrix} \begin{bmatrix} \gamma_1 & \gamma_2 \end{bmatrix}^T \\
&+ Sym \left\{ L \left(d(t)^2 e_6^T + (h - d(t))^2 e_7^T + d(t)(h-d(t)) e_4^T - e_8^T \right) \right\} \\
&- e_9 (\sigma_1 I) e_9^T + e_1 (\sigma_1 \alpha^2 Z^T Z) e_1^T + e_2 (\sigma_1 \beta^2 X^T X) e_2^T \\
\tilde{\Upsilon}_{[d(t)]} =& Sym \left\{ \begin{bmatrix} \tilde{e}_1 & (h-d(t))\tilde{e}_5 + d(t)\tilde{e}_4 & \tilde{e}_8 \end{bmatrix} \right. \\
&\left. \times P \begin{bmatrix} \tilde{e}_s & \tilde{e}_1 - \tilde{e}_3 & h\tilde{e}_1 - (h-d(t))\tilde{e}_5 - d(t)\tilde{e}_4 \end{bmatrix}^T \right\} \\
&+ \tilde{e}_1 U_2 \tilde{e}_1^T - \tilde{e}_3 U_2 \tilde{e}_3 + h \tilde{e}_s R \tilde{e}_s^T - \frac{1}{h} \begin{bmatrix} \gamma_1 & \gamma_2 \end{bmatrix} \begin{bmatrix} \psi_1 & S \\ * & \psi_1 \end{bmatrix} \begin{bmatrix} \gamma_1 & \gamma_2 \end{bmatrix}^T \\
&+ Sym \left\{ L \left(d(t)^2 \tilde{e}_6^T + (h-d(t))^2 \tilde{e}_7^T + d(t)(h-d(t)) \tilde{e}_4^T - \tilde{e}_8^T \right) \right\} \\
\tilde{\varsigma} =& Sym \left\{ \begin{bmatrix} \tilde{e}_0 & \tilde{e}_4 - \tilde{e}_5 & \tilde{e}_0 \end{bmatrix} P \begin{bmatrix} \tilde{e}_0 & \tilde{e}_0 & \tilde{e}_5 - \tilde{e}_4 \end{bmatrix}^T + L \left(\tilde{e}_6^T + \tilde{e}_7^T - \tilde{e}_4^T \right) \right\} \\
\Psi_{[d(t)]} =& Sym \left\{ \left[\ \bar{e}_1 \quad d_1(t)\bar{e}_6 + d_2(t)\bar{e}_7 + (h - d_2(t))\bar{e}_8 \right. \right. \\
&\left. \left. + (h_1 - d_1(t))\bar{e}_9 \right] P \begin{bmatrix} \bar{e}_s & \bar{e}_1 - \bar{e}_3 \end{bmatrix}^T \right\} \\
&+ \bar{e}_1 (U_1 + U_2 + U_3 + U_4) \bar{e}_1^T - (1-\mu)\bar{e}_2 U_1 \bar{e}_2^T - \bar{e}_3 U_2 \bar{e}_3^T \\
&- (1-\mu_1)\bar{e}_4 U_3 \bar{e}_4^T - (1-\mu_1)\bar{e}_5 U_4 \bar{e}_5^T + h\bar{e}_s(t) R \bar{e}_s(t) + \zeta_1 \begin{bmatrix} \psi_2 & X_2 \\ * & \psi_2 \end{bmatrix} \zeta_1^T \\
&+ \zeta_2 \begin{bmatrix} \psi_2 & X_3 \\ * & \psi_2 \end{bmatrix} \zeta_2^T - \bar{e}_{10}(\sigma_2 I)\bar{e}_{10}^T + \bar{e}_1(\sigma_2 \alpha^2 Z^T Z)\bar{e}_1^T + \bar{e}_2(\sigma_2 \beta^2 X^T X)\bar{e}_2^T
\end{aligned}
$$

Proof of Theorem 1

Let us construct the following LKF candidate:

$$
\begin{aligned}
V_1(t) =& \delta_1(t)^T P \delta_1(t) + \int_{t-d(t)}^{t} \delta_2(t,s)^T U_1 \delta_2(t,s) ds \\
&+ \int_{t-h}^{t} x^T(s) U_2 x(s) ds + \int_{t-h}^{t} \int_{s}^{t} \dot{x}^T(u) R \dot{x}(u) du ds \qquad (16)
\end{aligned}
$$

where

$$\delta_1(t) = \begin{bmatrix} x^T(t) & \int_{t-h}^{t} x^T(s)ds & \int_{t-h}^{t}\int_{s}^{t} x^T(u)duds \end{bmatrix}^T$$
$$\delta_2(t,s) = \begin{bmatrix} x^T(s) & \int_{s}^{t} \dot{x}^T(u)du & \int_{s}^{t} x^T(u)du \end{bmatrix}^T$$

The time derivative of $V_1(t)$ along the trajectories of (8) can be provided as follows:

$$\begin{aligned}
\dot{V}_1(t) =& \chi_1(t) \Big\{ 2 \begin{bmatrix} e_1 & (h-d(t))e_5 + d(t)e_4 & e_8 \end{bmatrix} \\
& \times P \begin{bmatrix} e_s & e_1 - e_3 & he_1 - (h-d(t))e_5 - d(t)e_4 \end{bmatrix}^T \\
& + \begin{bmatrix} e_1 & e_0 & e_0 \end{bmatrix} U_1 \begin{bmatrix} e_1 & e_0 & e_0 \end{bmatrix}^T \\
& - (1-\dot{d}(t)) \begin{bmatrix} e_2 & e_1 - e_2 & d(t)e_4 \end{bmatrix} U_1 \begin{bmatrix} e_2 & e_1 - e_2 & d(t)e_4 \end{bmatrix}^T \\
& + 2d(t) \begin{bmatrix} e_4 & e_1 - e_4 & d(t)e_6 \end{bmatrix} U_1 \begin{bmatrix} e_0 & e_s & e_1 \end{bmatrix}^T \\
& + e_1 U_2 e_1^T - e_3 U_2 e_3^T + h e_s R e_s^T \Big\} \chi_1^T(t) + V_a(t) \qquad (17)
\end{aligned}$$

Applying the Lemma 1 and reciprocally convex combination[30] together, $V_a(t)$ is expressed as follows

$$\begin{aligned}
V_a(t) =& -\int_{t-d(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds - \int_{t-h}^{t-d(t)} \dot{x}^T(s)R\dot{x}(s)ds \\
\leq & -\frac{1}{h}\chi_1(t) \begin{bmatrix} \gamma_1 & \gamma_2 \end{bmatrix} \begin{bmatrix} \psi_1 & S \\ * & \psi_1 \end{bmatrix} \begin{bmatrix} \gamma_1 & \gamma_2 \end{bmatrix}^T \chi_1^T(t) \qquad (18)
\end{aligned}$$

In addition, for any $\sigma_1 \geq 0$ and $L \in \mathbb{R}^{9n \times n}$, the following holds

$$\begin{aligned}
0 \leq & \chi_1(t) \left(e_9(-\sigma_1 I)e_9^T + e_1(\sigma_1 \alpha^2 Z^T Z)e_1^T + e_2(\sigma_1 \beta^2 X^T X)e_2^T \right) \chi_1^T(t) \\
0 =& 2\chi_1(t) L \left(d(t)^2 e_6^T + (h-d(t))^2 e_7^T + d(t)(h-d(t))e_4^T - e_8^T \right) \chi_1^T(t) \qquad (19)
\end{aligned}$$

Therefore, based on (17)-(19) , we have

$$\dot{V}_1(t) \leq \chi_1(t) \Upsilon_{[d(t)]} \chi_1^T(t) \qquad (20)$$

Obviously the above condition is quadratic, By applying Lemma 2, if

$$\Upsilon_{[d(t)=0]} < 0, \quad \Upsilon_{[d(t)=h]} < 0, \quad \Upsilon_{[d(t)=0]} - h^2 \varsigma < 0 \qquad (21)$$

where

$$\begin{aligned}
\varsigma =& \frac{1}{2}\frac{d^2\Upsilon(d(t))}{d(d(t))^2} = Sym\left\{\begin{bmatrix} e_0 & e_4 - e_5 & e_0 \end{bmatrix} P \begin{bmatrix} e_0 & e_0 & e_5 - e_4 \end{bmatrix}^T\right\} \\
& - (1-u)\begin{bmatrix} e_0 & e_0 & e_4 \end{bmatrix} U_1 \begin{bmatrix} e_0 & e_0 & e_4 \end{bmatrix}^T \\
& + Sym\left\{\begin{bmatrix} e_0 & e_0 & e_6 \end{bmatrix} U_1 \begin{bmatrix} e_0 & e_s & e_1 \end{bmatrix}^T + L\left(e_6^T + e_7^T - e_4^T\right)\right\}
\end{aligned}$$

then the condition (20) holds. Therefore LMIs (10)-(11) are equivalent to (20). The proof is completely finished.

Proof of Theorem 2

Let us construct the following LKF candidate:

$$\begin{aligned}
V_2(x) =& \begin{bmatrix} x(t) \\ \int_{t-h}^{t} x(s)ds \end{bmatrix}^T P \begin{bmatrix} x(t) \\ \int_{t-h}^{t} x(s)ds \end{bmatrix} + \int_{t-d(t)}^{t} x^T(s)U_1x(s)ds \\
& + \int_{t-h}^{t} x^T(s)U_2x(s)ds + \int_{t-d_1(t)}^{t} x^T(s)U_3x(s)ds \\
& + \int_{t-d_1(t)-h_2}^{t} x^T(s)U_4x(s)ds + \int_{t-h}^{t}\int_{t+\theta}^{t} \dot{x}^T(s)R\dot{x}(s)dsd\theta \qquad (22)
\end{aligned}$$

The time derivative of $V_2(t)$ along the trajectories of (1) can be provided as follows:

$$\begin{aligned}
\dot{V}_2(t) =& \chi_2(t)\Big(2\begin{bmatrix} \bar{e}_1 & d_1(t)\bar{e}_6 + d_2(t)\bar{e}_7 + (h-d_2(t))\bar{e}_8 + (h_1 - d_1(t))\bar{e}_9 \end{bmatrix} \\
& \times P\begin{bmatrix} \bar{e}_s & \bar{e}_1 - \bar{e}_3 \end{bmatrix}^T + \bar{e}_1(U_1+U_2+U_3+U_4)\bar{e}_1^T - (1-\dot{d}(t))\bar{e}_2U_1\bar{e}_2^T \\
& - \bar{e}_3U_2\bar{e}_3^T - (1-\dot{d}_1(t))\bar{e}_4U_3\bar{e}_4^T - (1-\dot{d}_1(t))\bar{e}_5U_4\bar{e}_5^T \\
& + h\bar{e}_sR\bar{e}_s^T\Big)\chi_2^T(t) - V_b(t) \qquad (23)
\end{aligned}$$

The upper bound of the $V_b(t)$ on the right side of equation (23) can be obtained using Lemma 1 and reciprocally convex combination as

$$\begin{aligned}
V_b(t) =& \int_{t-d_1(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds + \int_{t-d(t)}^{t-d_1(t)} \dot{x}^T(s)R\dot{x}(s)ds \\
& + \int_{t-d_1(t)-h_2}^{t-d(t)} \dot{x}^T(s)R\dot{x}(s)ds + \int_{t-h}^{t-d_1(t)-h_2} \dot{x}^T(s)R\dot{x}(s)ds \\
\geq& \chi_2(t)\left(\zeta_1\begin{bmatrix} \psi_2 & X_2 \\ * & \psi_2 \end{bmatrix}\zeta_1^T + \zeta_2\begin{bmatrix} \psi_2 & X_3 \\ * & \psi_2 \end{bmatrix}\zeta_2^T\right)\chi_2^T(t) \qquad (24)
\end{aligned}$$

From(7), for $\sigma_2 \geq 0$, the following holds

$$\begin{aligned} 0 \leq & \chi_2(t) \left(\bar{e}_{10}(-\sigma_2 I)\bar{e}_{10}^T + \bar{e}_1(\sigma_2 \alpha^2 Z^T Z)\bar{e}_1^T \right. \\ & \left. + \bar{e}_2(\sigma_2 \beta^2 X^T X)\bar{e}_2^T \right) \chi_2^T(t) \end{aligned} \tag{25}$$

By combining (23)-(25), we have

$$\dot{V}_2 \leq \chi_2(t)\Psi_{[d(t)]}\chi_2^T(t) \tag{26}$$

Therefore if the LMIs (14)-(15) are satisfied, we can deduce that the system(1) is asymptotically stable. This ends the proof.