# Reliable Stabilization of Delta Operator Switched Linear Systems with Persistent Dwell-time\*

## Hao Hu

College of Information Engineering
Nanjing University Of Finance and Economics
Nanjing, P.R.C.
njandyhu@163.com

Abstract—Here, a persistent dwell-time(PDT) approach is employed for reliable stabilization of Delta operator(DO) switched systems. For DO switched nonlinear systems, the switching stabilization criterion is proposed by using PDT approach. Based on this stability criterion, the reliable controller and the switching law with PDT are designed to make sure that the closed-loop system is globally uniformly asymptotically stable(GUAS). The performance of the proposed approach is illustrated by the simulation results.

Index Terms—switched systems, delta operator, reliable control, persistent dwell-time, actuator fault

#### I. INTRODUCTION

According to the theoretical and practical significance, switched systems have been extensively concerned for a long period. Some new complete and useful results of switched systems(SSs) are proposed in the works [1] and [2]. Some other novel results occurs in this year [3] and [4]. Refs. [3] designs filters for continuous-time(CT) switched linear systems(SLSs), where the novel thing is a new condition of  $H_{\infty}$  approach. There are two common approach of switched systems: state-dependent switching and time-dependent switching. A new time-dependent switching control approach is introduced in Refs. [4] for 2-D SLSs. As the approach employed in this research, time-dependent switching control is stated in the following paragraph. Refs [5] and [6] concern on some control problems for discrete-time switched systems(DTSSs).

Average dwell-time(ADT) approaches has been studied for a long time [7]. But there still is an approach to be developed which is called persistent dwell-time(PDT) approach [8]. Refs. [8] systematically introduces the relation among the different time switching control approaches. Firstly,  $\mathcal{S}$  is the set of switching signals, and it is introduced to conveniently state the results. Secondly, the following relations are proved.

$$\begin{split} \mathcal{S}_{\text{dwell}}[\tau_D] &= \mathcal{S}_{\text{average}}[\tau_D, 1] = \mathcal{S}_{p-\text{dwell}}[\tau_D, 0] \\ &\subseteq \mathcal{S}_{\text{average}}[\tau_D, N_0] \subseteq \mathcal{S}_{p-\text{dwell}}[\gamma \tau_D, T], \\ \forall \tau_D &> 0, N_0 \geqslant 1, \gamma \in (0, 1), \\ T &:= (N_0 - \gamma/1 - \gamma) \ \gamma \tau_D. \end{split}$$

Dwell-time approach, ADT approach, and PDT approach are studied in [8]. Form the above relationships, it can be deduced that PDT approach has the lowest conservatism. Due to this performance, PDT approach enters into the views

This research is founded by the Natural Science Foundation of Jiangsu Province of China (No.BK20171481).

of many scholars in recent years [9]–[12]. A quasi-time-dependent Lyapunov function method is employed to design the  $H_{\infty}$  filter for a class of DTSSs [9]. Refs [10] deals with the non-weighted  $H_{\infty}$  stat estimation problem for discrete-time neural networks. The asynchronous  $H_{\infty}$  control problem is dealt with in [11]. Based on PDT approach, the asynchronous control problem for SLSs is settled [12].

In view of the advantages of DO systems [13] and the PDT approach [8], this paper aims at the reliable control problem of DO switched systems(DOSSs) with PDT. As for DO systems, some control problems can be found in [14] and [15]. The composition of this paper is stated as follows. In Section II, the problem of this study is stated. In Section III, a stability criterion for DO switched non-linear systems(SNLSs) is first proposed by using PDT approach, and then it is used to introduce the result on reliable control for the a class of DOSLSs. In Section IV, a simulation example is employed to illustrate the main result. Section V provides the conclusion of this study.

# II. PROBLEM FORMULATION

Consider the following DOSLS.

$$\delta x(t) = A_{\sigma}x(t) + B_{\sigma}u^{f}(t) \tag{1}$$

where  $\sigma(t): \mathbb{R}^+ \to \bar{N} = \{1,2,\cdots,N\}$  is a switching law(SL);  $x(t) \in \mathbb{R}^n$  is the state vector;  $u^f(t) \in \mathbb{R}^m$  denotes the control input with actuator faults;  $y(t) \in \mathbb{R}^m$  represents the output vector;  $A_i \in \mathbb{R}^{n \times n}, \ B_i \in \mathbb{R}^{n \times m}$  and  $C_i \in \mathbb{R}^{m \times n}$  stand for known constant matrices, where  $i=1,2,\cdots,N$ . The DO  $\delta$  is defined as follow.

$$\delta x(t) = \left\{ \begin{array}{l} \dot{x}(t), T = 0\\ \frac{x(t+T) - x(t)}{T}, T \neq 0 \end{array} \right.,$$

in which  $T \geqslant 0$  stands for the sampling period. Consider the state feedback(SF).

$$u(t) = K_{\sigma}x(t) \tag{2}$$

where  $K_i \in \mathbb{R}^{m \times p}$  is the feedback gain matrix for the *i*-th mode of the system (1),  $i = 1, 2, \dots, N$ .

The gain fault model is formulated as follows.

$$u^f(t) = F_\sigma u(t) \tag{3}$$

where the fault gain matrix

$$F_i = \operatorname{diag}(f_{i1}, f_{i2}, \cdots, f_{ip}) \ (i \in \overline{N})$$

can be described as follows.

$$f_{ij} \; (i \in \bar{N}, \; j \in \{1, 2, \cdots, p\})$$
 satisfy

$$0 \leqslant f_{dij} \leqslant f_{ij} \leqslant f_{uij}, f_{uij} \geqslant 1 \geqslant f_{dij},$$

in which  $f_{dij}$  and  $f_{uij}$  are given numbers.  $f_{ij} = 0$  signifies that the j-th sensor of the i-th mode outage;  $f_{ij}=1$  stands for that the j-th sensor of the i-th mode is fault-free;  $0 < f_{dij} \le$  $f_{ij} \leqslant f_{uij}$  and  $f_{ij} \neq 1$  signifies that the j-th sensor of the *i*-th mode is of partial fault.

For the sake of simplicity, we denote

$$F_{ui} := \operatorname{diag}(f_{ui1}, f_{ui2}, \cdots, f_{uip}),$$
  
 $F_{di} := \operatorname{diag}(f_{di1}, f_{di2}, \cdots, f_{dip}).$ 

For convenience, the following symbols are defined

$$F_{0i} := \frac{1}{2}(F_{ui} + F_{di}), \ F_{1i} := \frac{1}{2}(F_{ui} - F_{di}).$$

Thus, the fault matrix  $F_i$  can be formulated as

$$F_i = F_{0i} + F_{1i}\Sigma_i,$$

in which  $\Sigma_i = \operatorname{diag}(\tilde{\sigma}_{i1}, \tilde{\sigma}_{i2}, \cdots, \tilde{\sigma}_{im_i}) \in \mathbb{R}^{m_i \times m_i}, -1 \leqslant \tilde{\sigma}_{ij} \leqslant 1, i \in \bar{N}, j \in \{1, 2, \cdots, p\}.$ 

Then (1), (2) and (3) constitute the following closed-loop switched system(CLSS).

$$\delta x(t) = \tilde{A}_{\sigma} x(t) \tag{4}$$

where  $\tilde{A}_i = A_i + B_i F_{0i} K_i + B_i F_{1i} \Sigma_i K_i$ ,  $i = 1, 2, \dots, N$ . The PDT approach is considered for studying the CLSS(4).

[8] Consider switching instants  $t_0, t_1,$  $\cdots, t_s, \cdots$  with  $t_0 = 0$ . A positive scalar  $\tau_m$  is said to be the persistent dwell time (PDT) if there exists an infinite number of disjoint intervals of length no smaller than  $\tau_m$  on which  $\sigma$  is constant, and consecutive intervals with this property are separated by no more than  $\tau_M$ , where  $\tau_M$  is called the period of persistence.

Based on the definition of globally uniformly asymptotic stability for DTSSs, we propose the following definition for DOSSs.

Now, introduce a DOSNLS as follow.

$$\delta x(t) = f_{\sigma}(x(t)). \tag{5}$$

**Definition 2** For a certain switching signals  $\sigma$ , the DOSNLS (5) is GUAS if there exists a kind of  $\mathcal{K}_{\infty}$  function  $\kappa$  such that the state response signal of the system (5) satisfies the inequality  $||x(t)|| \le \kappa(||x(t_0)||)$  for an initial condition  $x(t_0)$ ,  $\forall t \geqslant t_0 \text{ and } \lim_{t \to \infty} x(t) = 0.$ 

The switching instants  $t_0, t_1, \dots, t_s, \dots$  appear in Definition 1 are also employed through out this paper. To illustrate PDT, suppose the moment  $t_{s_p}$  is the beginning instant of the p-th stage in the switched system, where  $p = 1, 2, \cdots$ . In the p-th stage,  $\tau_m$ -portion denotes the p-th stage consists of the running time of the mode and  $\tau_M$ -portion stands for the period of persistence. Then the following facts can be obtained.

 $\begin{array}{l} t_{s_p+1}-t_{s_p}\geqslant \tau_m \text{ and } t_{s_{p+1}}-t_{s_p}\leqslant \tau_M. \\ \text{To the interval } [t_{s_p+1},t_{s_{p+1}}), \text{ let } \mathcal{N}(t_{s_p+1},t_{s_{p+1}}) \text{ be the} \end{array}$ switching times within it and  $au_{\sigma(t_{s_p+r})}$  be the running time of the mode activated at the switching instant  $t_{s_p+r}$ , where r= $1, 2, \cdots, \mathcal{N}(t_{s_p+1}, t_{s_{p+1}})$ . Then it is obvious that  $\tau_{\sigma(t_{s_p+r})} \geqslant \tau_m$  and  $\sum_{r=1}^{\mathcal{N}(t_{s_p+1}, t_{s_{p+1}})} \tau_{\sigma(t_{s_p+r})} \leqslant \tau_M$ .

$$au_m$$
 and  $\sum_{j=1}^{N(t_{s_p+1},t_{s_{p+1}})} au_{\sigma(t_{s_p+r})} \leqslant au_M.$ 

The purpose of this work is to find a switching SF controller (2) and a SL  $\sigma(t)$  by using PDT approach, such that for all admissible fault matrices  $F_i$ , the CLSS (4) is GUAS.

In addition, the following necessary lemmas are introduced.

**Lemma 1** [16] Assume that A and B are matrices of appropriate dimensions. Let  $H = diag(H_1, H_2, \dots, H_s)$ , in which  $H_1, H_2, \dots, H_s$  are uncertain matrices that satisfy  $H_i^{\mathsf{T}} H_i \leqslant I$ ,  $i = 1, 2, \dots, s$ . Then, for arbitrary positive numbers  $\epsilon_1, \epsilon_2, \cdots, \epsilon_s$ , one has

$$AHB + B^{\mathsf{T}}H^{\mathsf{T}}A^{\mathsf{T}} \leqslant A\Xi A^{\mathsf{T}} + B^{\mathsf{T}}\Xi^{-1}B$$

in which  $\Xi = \operatorname{diag}(\epsilon_1 I, \epsilon_2 I, \cdots, \epsilon_s I)$ .

Lemma 2 [13] The property of DO: for two time functions x(t) and y(t)

$$\delta(x(t)y(t)) = \delta(x(t))y(t) + x(t)\delta(y(t)) + T\delta(x(t))\delta(y(t)).$$

## III. MAIN RESULTS

A. Stability of DOSNLSs

For the DOSNLS (5), a stability criterion is firstly proposed by using PDT approach.

**Theorem 1** For two given numbers  $\alpha \in (0, \frac{1}{T})$  and  $\mu > 1$ , if there exist a family Lyapunov functions  $V_{\sigma(t)}$  and two class  $\mathcal{K}_{\infty}$  functions  $\kappa_1$  and  $\kappa_2$  such that  $\forall \sigma(t) = i \in \bar{N}$ ,

$$\kappa_1(\|x(t)\|) \leqslant V_i(x(t)) \leqslant \kappa_2(\|x(t)\|),$$
(6)

$$\delta V_i(t) \leqslant -\alpha V_i(t),$$
 (7)

for any  $i, j \in \bar{N}$ 

$$V_i(t) \leqslant \mu V_i(t),$$
 (8)

then the switched system (5) is GUAS for PDT switching signals satisfying

$$-\frac{\ln \mu}{\ln(1-T\alpha)} < \frac{\tau_m + \tau_M}{\tau_M + T}.$$
 (9)

*Proof:* For the case  $\mu\leqslant \frac{1}{1-T\alpha}$ , the inequality (9) holds for arbitrarily switching law  $\sigma(t)$ , because  $\tau_m\geqslant T$ . So we just consider the other case  $\mu>\frac{1}{1-T\alpha}$ . Suppose that  $\sigma(t_{s_{p+1}})=j$  and  $\sigma(t_{s_p})=i$ . Note that the

inequality (7) can be also expressed as

$$V_i(t+T) \leqslant (1-T\alpha)V_i(t). \tag{10}$$

It follows from (8) and (10) that

$$V_{j}(x(t_{s_{p+1}}))$$

$$\leq \mu V_{\sigma(t_{s_{p+1}-1})}(x(t_{s_{p+1}}))$$

$$\leq \mu \beta^{\frac{t_{s_{p+1}}-t_{s_{p+1}-1}}{T}} V_{\sigma(t_{s_{p+1}-1})}(x(t_{s_{p+1}-1}))$$

$$\leq \mu^{2} \beta^{\frac{t_{s_{p+1}}-t_{s_{p+1}-1}}{T}} V_{\sigma(t_{s_{p+1}-2})}(x(t_{s_{p+1}-1}))$$

$$\leq \cdots$$

$$\leq \mu^{\mathcal{N}(t_{s_{p}},t_{s_{p+1}})} \beta^{\frac{t_{s_{p+1}}-t_{s_{p+1}}}{T}} \beta^{\frac{\tau_{m}}{T}} V_{i}(x(t_{s_{p}})). \tag{11}$$

where  $\beta = 1 - T\alpha$ .

Because  $\mu\beta > 1$ , thus

$$\mu^{\frac{t_{s_{p+1}}-t_{s_{p+1}}}{T}+1}\beta^{\frac{t_{s_{p+1}}-t_{s_{p+1}}}{T}}\leqslant \mu^{\frac{\tau_{M}}{T}+1}\beta^{\frac{\tau_{M}}{T}}.$$

According to (11) and noting that

$$\mathcal{N}(t_{s_p}, t_{s_{p+1}}) = \frac{t_{s_{p+1}} - t_{s_p+1}}{T} + 1,$$

it is obtained that

$$V_j(x(t_{s_{p+1}})) \leqslant \gamma V_i(x(t_{s_p})), \tag{12}$$

where  $\gamma = \mu^{\frac{\tau_M}{T}+1} \beta^{\frac{\tau_M+\tau_m}{T}}$ 

Note that  $t_{s_1} = t_0$ . Based on (12), we can deduce that

$$V_j(x(t_{s_{p+1}})) \leqslant \gamma^{p-1} V_{\sigma(t_0)}(x(t_0)).$$

It follows from (6) that

$$||x(t_{s_p})|| \leq \kappa_1^{-1}(\gamma^{p-1}V_{\sigma(t_0)}(x_0)) \leq \kappa_1^{-1}(\gamma^{p-1}\kappa_2^{-1}(||x_0||)) = \kappa_1^{-1}(\kappa_2^{-1}(\gamma^{p-1}||x_0||)).$$
 (13)

It follows from (9) that  $\gamma < 1$ . According to that  $\kappa_1$  and  $\kappa_2$  are class  $\mathcal{K}_{\infty}$  functions, so that  $\kappa_1^{-1}\kappa_2^{-1}(\cdot)$  is also class  $\mathcal{K}_{\infty}$  function. By using (13), we can obtain that

$$||x(t_{s_p})|| \le \kappa_1^{-1}(\kappa_2^{-1}(||x_0||)).$$
 (14)

For any  $t\geqslant t_0$ , there exists  $p\in\mathbb{N}_+$  such that  $t\in[t_{s_p},t_{s_{p+1}}).$  Then (7) implies

$$||x(t)|| \le ||x(t_{s_n})||$$
 (15)

The following inequality form inequalities (14) and (15)

$$||x(t)|| \le \kappa_1^{-1}(\kappa_2^{-1}(||x_0||)).$$

By employing (12), it can be obtained that  $\lim_{p\to\infty} ||x(t_{s_p})|| = 0$ . Then it is also true that  $\lim_{t\to\infty} x(t) = 0$ . Based on Definition 1, we can conclude that the system (5) is GUAS for PDT switching signals satisfying (9).

#### B. Reliable stabilization

Based on Theorem 1, the reliable stabilization problem for the DOSS (1) is dealt with.

**Theorem 2** For two given numbers  $\alpha \in (0, \frac{1}{T})$  and  $\mu > 1$ , if there exist positive definite symmetric matrices  $X_1, X_2, \cdots, X_N \in \mathbb{R}^{n \times n}$ , positive definite diagonal matrices  $U_1, U_2, \cdots, U_N \in \mathbb{R}^{m \times m}$ , and matrices  $Y_1, Y_2, \cdots, Y_N \in \mathbb{R}^{m \times n}$  such that

$$\begin{bmatrix} \operatorname{He}(\Xi_i) + \alpha X_i + \Omega_i & \Xi_i^{\mathrm{T}} + \Omega_i & Y_i^{\mathrm{T}} \\ \Xi_i + \Omega_i & -\frac{1}{T} X_i + \Omega_i & 0 \\ Y_i & 0 & -U_i \end{bmatrix} < 0, \quad (16)$$

and for any  $i, j \in \overline{N}$ , it is holds that

$$\begin{bmatrix} -\mu X_j & X_j \\ X_j & X_i \end{bmatrix} < 0, \tag{17}$$

where  $\text{He}(\Xi_i) = \Xi_i + \Xi_i^T$ ,  $\Xi_i = A_i X_i + B_i F_{0i} Y_i$ ,  $\Omega_i = B_i F_{1i} U_i F_{1i}^T B_i^T$ , then the CLSS(4) is GUAS for PDT switching signals meeting (9) and feedback gain matrices which are designed by

$$K_i = Y_i X_i^{-1}, \tag{18}$$

in which  $i = 1, 2, \dots, N$ .

*Proof:* By using Schur complement lemma(SCL), the LMI (17) is equivalent to

$$-\mu X_j + X_j X_i^{-1} X_j < 0,$$

i.e.

$$X_i^{-1} < \mu X_j^{-1}. (19)$$

For any  $s \in \overline{N}$ , denote  $P_s = X_s^{-1}$  and then  $P_1, P_2, \cdots, P_N$  are positive definite symmetric matrices.

Take switching Lyapunov function  $V(x(t)) = V_{\sigma(t)}(x(t))$  for CLSS (4), where  $V_s(x(t)) = x^{\mathsf{T}}(t)P_sx(t)$ ,  $s = 1, 2, \dots, N$ . Then the matrix inequality (19) elicits that

$$V_i(t) \leqslant \mu V_i(t). \tag{20}$$

By using SCL, we can obtain that the LMI (16) is equivalent to

$$\begin{bmatrix} \operatorname{He}(\Xi_{i}) + \alpha X_{i} & \Xi_{i}^{\mathrm{T}} \\ \Xi_{i} & -\frac{1}{T}X_{i} \end{bmatrix} + \begin{bmatrix} B_{i}F_{1i} \\ B_{i}F_{1i} \end{bmatrix} U_{i} \begin{bmatrix} B_{i}F_{1i} \\ B_{i}F_{1i} \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} Y_{i}^{\mathrm{T}} \\ 0 \end{bmatrix} U_{i}^{-1} [Y_{i}, 0] < 0.$$
(21)

Based on Lemma 1, we can deduce that the matrix inequality(MI) (21) implies the following matrix inequality.

$$\begin{bmatrix} \operatorname{He}(\Xi_{i}) + \alpha X_{i} & \Xi_{i}^{\mathrm{T}} \\ \Xi_{i} & -\frac{1}{T} X_{i} \end{bmatrix} + \operatorname{He}\left(\begin{bmatrix} B_{i} F_{1i} \\ B_{i} F_{1i} \end{bmatrix} \Sigma_{i} [Y_{i}, 0] \right) < 0$$
 (22)

According to (18), the MI (22) can be rewritten as follows.

$$\begin{bmatrix} \tilde{A}_i X_i + X_i \tilde{A}_i^{\mathsf{T}} + \alpha X_i & X_i \tilde{A}_i^{\mathsf{T}} \\ \tilde{A}_i X_i & -\frac{1}{T} X_i \end{bmatrix} < 0.$$
 (23)

From SCL, it can be seen that the MI (23) is equivalent to

$$\tilde{A}_i X_i + X_i \tilde{A}_i^{\mathsf{T}} + T X_i \tilde{A}_i^{\mathsf{T}} X_i^{-1} \tilde{A}_i X_i + \alpha X_i < 0.$$
 (24)

By pre- and post-multiplying  $P_i$  in the double sides of the MI (24), it is true that the MI (24) is equivalent to

$$P_i \tilde{A}_i + \tilde{A}_i^{\mathsf{T}} P_i + T \tilde{A}_i^{\mathsf{T}} P_i \tilde{A}_i + \alpha P_i < 0. \tag{25}$$

Note that Lemma 2 leads to

$$\delta V_i(t) = x^{\mathsf{T}}(t)(P_i\tilde{A}_i + \tilde{A}_i^{\mathsf{T}}P_i + T\tilde{A}_i^{\mathsf{T}}P_i\tilde{A}_i)x(t).$$

Hence, from (25), it is obtained that

$$\delta V_i(t) \leqslant -\alpha V_i(t). \tag{26}$$

Note that for any  $i \in \bar{N}$ 

$$\inf_{i \in \bar{N}} \lambda_{\min}(P_i) \cdot ||x(t)|| \leqslant V_i(t)$$

$$\leqslant \sup_{i \in \bar{N}} \lambda_{\max}(P_i) \cdot ||x(t)||. \tag{27}$$

Then, the conditions (20), (26), (7) and (9) indicate that the CLSS(4) satisfies Theorem 1. Therefore the CLSS (4) is GUAS.  $\hfill\Box$ 

#### IV. SIMULATION

Consider a CTSLS with two modes. Take  $T=0.01\mathrm{s}$  and transform it into the system (1) with fault-free, then the parameters are listed as follows.

$$A_{1} = \begin{bmatrix} -3.9307 & 1.9287 & 0.9898 \\ -0.9656 & -2.9700 & 1.9457 \\ 0.0024 & -0.4876 & -1.9850 \end{bmatrix}$$

$$A_{2} = \begin{bmatrix} -1.0073 & -0.9777 & -0.4852 \\ 0.9607 & -3.9307 & -0.9680 \\ 2.9453 & 0.5909 & -2.9676 \end{bmatrix},$$

$$B_{1} = \begin{bmatrix} 1.1666 \\ -1.4737 \\ 0.9937 \end{bmatrix}, B_{2} = \begin{bmatrix} 1.1989 \\ -1.4694 \\ 0.9956 \end{bmatrix}.$$

Consider the fault (3) and the parameters of fault matrixes are given as follows.

$$F_{u1} = F_{u2} = 1$$
,  $F_{d1} = 0.7$ , and  $F_{d2} = 0.8$ .

Take  $\alpha = 5$ ,  $\mu = 1 + \frac{1}{1 - T\alpha}$  and solve the LMIs (16) and (17), we can obtain

$$\begin{split} X_1 &= \begin{bmatrix} 35.5594 & 6.2932 & -1.0600 \\ 6.2932 & 49.4347 & -3.5322 \\ -1.0600 & -3.5322 & 12.6537 \end{bmatrix} \\ X_2 &= \begin{bmatrix} 24.7410 & -0.5233 & -1.9614 \\ -0.5233 & 50.5057 & 0.1720 \\ -1.9614 & 0.1720 & 18.0229 \end{bmatrix}, \\ Y_1 &= \begin{bmatrix} -86.2801, 115.3244, -103.1494 \end{bmatrix}, \\ Y_2 &= \begin{bmatrix} -138.1459, 78.9747, -81.4074 \end{bmatrix}, \\ U_1 &= 833.0830, \text{ and } U_2 = 851.8502. \end{split}$$

Furthermore, it is obtained that

$$K_1 = [-3.0417, 2.1626, -7.8029],$$
  
and  $K_2 = [-5.9622, 1.5195, -5.1802].$ 

Note that the inequality (9) is described as

$$\frac{\tau_M + \tau_m}{\tau_M + T} > 14.0198.$$

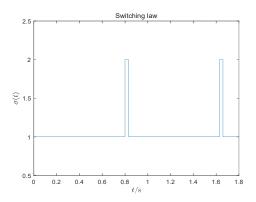


Fig. 1: The switching law with chosen PDT.

Choose a SL  $\sigma(t)$  with PDT satisfying the upper inequality as shown in Fig.1.

Take  $F_1 = 0.8$ ,  $F_2 = 0.9$ ,  $x_0 = (0.03, 0.1, -0.04)^T$  and use the chosen SL above, then the state response curves of the CLSS (4) can be obtained, see Fig.2.

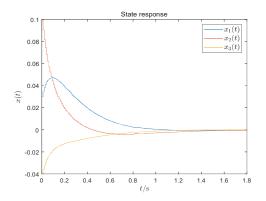


Fig. 2: The closed-loop state response curves

## V. CONCLUSIONS

We considered PDT approach to design a reliable controller and the switching law for a class of DOSLSs subject to actuator continuous fault. The stability criterion was proposed to design the SL by using PDT method. And it was also applied to design the reliable controller for the class of systems under study. In terms of LMIs, the sufficient condition, as the final result, was proposed. The feasibility and availability of the proposed approach was verified through the simulation example.

#### ACKNOWLEDGMENT

I would like to express my gratitude to the editors and the reviewers of this paper.

## REFERENCES

 K. Ali; Y. M. Javad; Moshiri, "Robust switching signal estimation for a class of uncertain nonlinear switched systems," International Journal of Control, vol. 92, no. 5, pp. 1094-1102, 2019.

- [2] J. Lu; Z. She, "Average dwell time based stability analysis for nonautonomous continuous-time switched systems," International Journal of Robust and Nonlinear Control, vol. 29, no. 8, pp. 2333-2350, 2019.
- [3] G. Zong, D. Duan, D. Yang, "Exponential  $H_{\infty}$  filtering of networked linear switched systems with mode-dependent average dwell time: an event-triggered scheme," International Journal of Systems Science, vol. 50, no. 7, pp. 1450-1464, 2019.
- [4] Y. Fan et al., "Quasi-time-dependent stabilisation for 2-D switched systems with persistent dwell-time," International Journal of Systems Science, vol. 50, no. 16, pp. 2885-2897, 2019.
- [5] M. Regaieg, M. Kchaou, J. Bosche, "Robust dissipative observer-based control design for discrete-time switched systems with time-varying delay," IET Control Theory and Applications, vol. 13, no.18, pp. 3026-3039, 2019.
- [6] Z. Gao, Z. Wang, D. Wu, "Input and output strictly passive  $H_{\infty}$  control of discrete-time switched systems," International Journal of Systems Science, vol. 50, no. 15, pp. 2776-2784, 2019.
- [7] Y. Ren, M. J. Er, and G. Sun, "Switched systems with average dwell time: computation of the robust positive invariant set," Automatica, vol. 85, pp. 306-313, 2017.
- [8] J. P. Hespanha, "Uniform stability of switched linear systems extensions of Lasalles invariance principle," IEEE Transactions on Automatic Control, vol.49, no.4, pp. 470-482, 2004.
  [9] L. Zhang, S. Zhuang and P. Shi, "Non-weighted quasi-time-dependent
- [9] L. Zhang, S. Zhuang and P. Shi, "Non-weighted quasi-time-dependent  $H_{\infty}$  filtering for switched linear systems with persistent dwell-time," Automatica, vol. 54, pp. 201-209, 2015.
- [10] R. Rakkiyappan, K. Maheswari, and K. Sivaranjani, "Non-weighted  $H_{\infty}$  state estimation for discrete-time switched neural networks with persistent dwell time switching regularities based on Finsler's lemma," Neurocomputing, vol. 260, pp. 131-141, 2017.
- [11] S. Shi, Z. Fei, Z. Shi, and S. Ren, "Stability and stabilization for discrete-time switched systems with asynchronism," Applied Mathmatics and Computation, vol. 338, pp. 520-536, 2018.
  [12] S. Shi, Z. Shi, Z. Fei, "Asynchronous control for switched systems by
- [12] S. Shi, Z. Shi, Z. Fei, "Asynchronous control for switched systems by using persistent dwell time modeling," Systems and Control Letters, vol. 133, 2019. Accepted
- [13] R. H. Middleton, G. C. Goodwin, Digital Control and Estimation. A Unified Approach. New Jersey: Prentice-Hall, 1990.
- [14] H. Hu, B. Jiang, and H. Yang, "Reliable guaranteed-cost control of delta operator switched systems with actuator faults: mode-dependent average dwell-time approach," IET Control Theory & Applications, vol. 10, no. 1, pp. 17-23, 2016.
- [15] H. Hu, B. Jiang, H. Yang, and A. Shumsky. "Non-fragile reliable D-Stabilization for delta operator switched linear systems," Journal of Franklin Institute, vol. 353, no. 9, pp.1931-1956, 2016.
- [16] Y. S. Lee, Y. S. Moon, W. H. Kwon, and P. G. Park, "Delay-dependent robust  $H_{\infty}$  control for uncertain systems with a state-delay," Automatica, vol.40, pp. 65-72, 2004.