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New results on stability analysis of neutral-type delay systems

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ABSTRACT
This paper is concerned with stability analysis of linear neutral-type delay systems. The main novelty of this paper comes from proposing a new stability analysis method called a delay-mode-based functional (DMBF) method for nonliear neutral-type delay systems. This is achieved by weakening the condition that when the Lyapunov–Krasovskii functional (LKF) method is used, a functional is required for a neutral-type delay system with a delay varying in a delay interval. With the help of the DMBF method, new stability criteria of linear neutral-type delay systems are obtained. The effectiveness of the obtained criteria is illustrated by two examples from the literature.

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Neutral-type delay systems; time-varying delays; stability analysis; delay-mode-based functionals

1. Introduction

Neutral-type delay systems, which contain delays both in system states and derivatives of states, are often encountered in many engineering systems, such as population ecology, heat exchangers and so on (Fridman, 2001; Han, 2002; Maharajan et al., 2018; Vadivoo et al., 2018; Wu et al., 2004). It is generally recognised that delays may lead to poor performance of delay systems and even make delay systems unstable (Liu & Seuret, 2017; Richard, 2003; Senthilraj et al., 2016; Seuret & Gouaisbaut, 2013; Wang et al., 2021; Zeng et al., 2015). Thus, much attention has been paid to stability analysis of neutral-type delay systems (Mazenc, 2015). When asked about stability analysis of delay systems, most people believe that the Lyapunov–Krasovskii functional (LKF) method is an effective tool to derive stability criteria (Gu et al., 2003; Pratap et al., 2018; Seuret et al., 2018; Sowmiya et al., 2018). The main idea of the method is that if there exists an LKF for a neutral-type delay system with a delay varying in a delay interval, the system is stable. Although the LKF method plays a large role, what calls for special attention is that how to obtain less conservative stability criteria is still an open problem (Fridman, 2014). Therefore, many useful improvements are introduced for the LKF method, such as the descriptor model transformation (Fridman & Shaked, 2003), the free-weighting matrix technique (He et al., 2004), the delay-partitioning technique (Balasubramaniam et al., 2012; Kwon et al., 2012), the augmented LKF approach (Lu et al., 2014), integral inequalities (Sun et al., 2009) and so on. However, in practice, it is a fact that properties of a neutral-type delay system with a delay varying in different delay intervals are different. As a consequence, the condition that an LKF should exist for a neutral-type delay system with a delay varying in a delay interval seems too strong. In this case, it is reasonable to obtain better stability criteria by weakening the strong condition.

Motivated by the above discussion, a stability analysis problem of linear neutral-type delay systems is further investigated and less conservative stability criteria are obtained in this paper. In order to realise the goal of this paper, a new stability analysis method called a delay-mode-based functional (DMBF) method is proposed for nonlinear neutral-type delay systems. This is achieved via two steps. The first step is to introduce a new delay form which contains some functions called delay modes and a function called a delay-mode-varying function. What needs illustration is that the delay modes are used to show a delay varying in different delay subintervals and the delay-mode-varying function is used to show switching between the delay modes. It means that if the new delay form is used, different properties of a neutral-type delay system with a delay varying in different delay subintervals can be taken into full consideration. This is why the new delay form is better than the commonly used one. The other step is to develop a new functional called a DMBF and establish a new stability criterion of nonlinear neutral-type delay systems via the new functional. It is noted that for a nonlinear neutral-type delay system with different delay modes, different LKFs are required in the DMBF. Due to the form of the new functional, the strong condition in the LKF method can be weakened. It implies that the DMBF method is more effective than the LKF method. Based on the DMBF method, new stability criteria of linear neutral-type delay systems are obtained. Two examples are given to show that the new criteria are less conservative than some existing ones which are derived via the LKF method. The main contributions of this paper are as follows.

(1) By weakening the condition in the LKF method, a new method is presented for stability analysis of nonlinear neutral-type delay systems.
(2) The new method is applied to linear neutral-type delay systems and less conservative stability criteria are obtained.

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The rest of this paper is organised as follows. Preliminaries are given in Section 2. In Section 3, a new stability analysis method is proposed and new stability criteria are obtained. Section 4 provides two examples to show the effectiveness of the derived criteria. Conclusions are drawn in Section 5.

Notations: $\mathbb{R}^n$ refers to the n-dimensional Euclidean space. $\mathbb{R}_+$ and $\mathbb{Z}_+$ mean the set of nonnegative real numbers and the set of nonnegative integers, respectively. $\mathbb{R}^{m\times n}$ is the set of $m\times n$ real matrices. $\mathbb{S}^n$ ($\mathbb{S}^n_+$) represents the set of $n\times n$ real symmetric (positive definite) matrices. Denote $x(t+\theta)=x_t(\theta), \forall\theta\in[-h,0]$. $\mathcal{C}([-h,0],\mathbb{R}^n)$ is the space of continuous functions $\psi:[-h,0]\to\mathbb{R}^n$ with the norm $\|\psi\|_{\mathcal{C}}=\sup_{\forall\theta\in[-h,0]}\|\psi(\theta)\|$. $\mathcal{L}_2([-h,0],\mathbb{R}^n)$ is the space of functions $\psi:[-h,0]\to\mathbb{R}^n$ with the norm $\|\psi\|_{\mathcal{L}_2}=[\int_{-h}^0|\phi(s)|^2ds]^{\frac{1}{2}}$. $\mathcal{W}([-h,0],\mathbb{R}^n)$ is the space of continuous functions $\psi:[-h,0]\to\mathbb{R}^n$ with $\dot\psi\in\mathcal{L}_2([-h,0],\mathbb{R}^n)$ and the norm $\|\psi\|_{\mathcal{W}}=\|\psi\|_{\mathcal{C}}+\|\psi\|_{\mathcal{L}_2}$. diag$\{\cdot\}$ stands for the block-diagonal matrix. $I_{m\times n}$ (I_n) is the $m\times n$ ($n\times n$) identity matrix and $0_{m\times n}$ (0_n) is the $m\times n$ ($n\times n$) zero matrix. For a matrix A with appropriate dimensions, Sym$\{A\}$ means $A+A^{\mathrm{T}}$. For matrices A, B and C with appropriate dimensions, $\left[\begin{smallmatrix}A & B\\ * & C\end{smallmatrix}\right]$ stands for $\left[\begin{smallmatrix}A & B\\ B^{\mathrm{T}} & C\end{smallmatrix}\right]$.

2. Preliminaries

Consider a neutral-type delay system

$$\begin{aligned}&\dot x(t)=Ax(t)+Bx(t-\tau(t))+C\dot x(t-d),\quad \forall t\in[t_0,+\infty),\\ &x_{t_0}(\theta)=\phi(\theta),\quad \forall\theta\in[-h,0],\end{aligned}\tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the system state, $A\in\mathbb{R}^{n\times n}, B\in\mathbb{R}^{n\times n}, C\in\mathbb{R}^{n\times n}$, and all the eigenvalues of matrix C are inside the unit circle, $\tau(t)$ called the time-varying delay is a continuous function satisfying $0\le\tau(t)\le h$, d is the constant delay satisfying $0<d\le h$, $h>0$ is a real constant, $t_0\ge0$ is the initial moment, and $\phi(\theta)\in\mathcal{W}([-h,0],\mathbb{R}^n)$ is the initial function.

Definition 2.1: System (1) is said to be globally uniformly asymptotically stable (GUAS), if

$$\|x(t)\|\le\alpha(\|\phi\|_{\mathcal{W}},t-t_0),\quad\forall t\in[t_0,+\infty),\tag{2}$$

holds, where α is a function of class $\mathcal{KL}$.

The aim of this paper is to provide less conservative stability criteria of system (1). Before ending this section, the following lemma is recalled.

Lemma 2.1 (Zeng et al., 2015)**:** *For a differentiable function $x(t):[a,b]\to\mathbb{R}^n$, if there exist matrices $Z_i\in\mathbb{S}^{3n}$, $M_i\in\mathbb{R}^{3n\times n}$, $X\in\mathbb{R}^{3n\times3n}$, $R\in\mathbb{S}^n_+$, $i=1,2$, satisfying*

$$\Delta=\begin{bmatrix}Z_1 & X & M_1\\ * & Z_2 & M_2\\ * & * & R\end{bmatrix}\ge0,\tag{3}$$

then

$$-\int_a^b\dot x^{\mathrm{T}}(s)R\dot x(s)\,\mathrm{d}s\le\zeta^{\mathrm{T}}\Phi\zeta\tag{4}$$

holds, where

$$\zeta=\left[x^{\mathrm{T}}(b),x^{\mathrm{T}}(a),\frac{\int_a^b x^{\mathrm{T}}(s)\,\mathrm{d}s}{b-a}\right]^{\mathrm{T}},$$

$$\Phi=(b-a)\left(Z_1+\frac{Z_2}{3}\right)+\mathrm{Sym}\{M_1\Pi_1+M_2\Pi_2\},$$

$$\Pi_1=[I_n\ \ -I_n\ \ 0_n],\qquad \Pi_2=[-I_n\ \ -I_n\ \ 2I_n].$$

3. Stability analysis

It is generally recognised that the LKF method is effective for stability analysis of neutral-type delay systems. However, the problem that how to obtain less conservative stability criteria is still unsolved. For the aim of this paper, a new stability analysis method for nonlinear neutral-type delay systems is provided in this section, and then, new stability criteria of linear neutral-type delay systems are obtained based on the new method.

3.1 A new stability analysis method

Consider a nonlinear neutral-type delay system

$$\begin{aligned}&\dot x(t)=f(t,x(t-\tau(t)),\dot x(t-d)),\quad\forall t\in[t_0,+\infty),\\ &x_{t_0}(\theta)=\phi(\theta),\quad\forall\theta\in[-h,0],\end{aligned}\tag{5}$$

where $f:[t_0,+\infty)\times\mathcal{W}([-h,0],\mathbb{R}^n)\times\mathcal{L}_2([-h,0],\mathbb{R}^n)\to\mathbb{R}^n$ is continuous in all arguments and is locally Lipschitz in the second and third arguments. Note that a Lipschitz constant for the third argument of f is less than 1. If $f(t,x(t-\tau(t)),\dot x(t-d))=Ax(t)+Bx(t-\tau(t))+C\dot x(t-d)$, system (5) will reduce to system (1). The LKF method is a well-known method for dealing with stability analysis of delay systems. In order to show the effectiveness of the LKF method, the following lemma is recalled.

Lemma 3.1 (Fridman, 2014)**:** *If there exist a continuous functional $\mathcal{V}(t,x_t,\dot x_t):[t_0,+\infty)\times\mathcal{W}([-h,0],\mathbb{R}^n)\times\mathcal{L}_2([-h,0],\mathbb{R}^n)\to\mathbb{R}_+$, and functions α_i, $i=1,2,3$, of class $\mathcal{K}_\infty$ such that*

$$\alpha_1(\|x_t(0)\|)\le\mathcal{V}(t,x_t,\dot x_t)\le\alpha_2(\|x_t\|_{\mathcal{W}}),\tag{6}$$

$$\dot{\mathcal{V}}(t,x_t,\dot x_t)\le-\alpha_3(\|x_t(0)\|),\tag{7}$$

$\forall\,(t,x_t,\dot x_t)\in[t_0,+\infty)\times\mathcal{W}([-h,0],\mathbb{R}^n)\times\mathcal{L}_2([-h,0],\mathbb{R}^n)$, *then system (5) is GUAS.*

Remark 3.1: Lemma 3.1 suggests that if there exists an LKF satisfying (6) and (7), the stability of system (5) can be guaranteed.

It should be noted that the delay interval $[0,h]$ can be divided into $\mathbf{p}\ge2$ delay subintervals: $[h_0,h_1],\ldots,[h_{\mathbf{p}-1},h_{\mathbf{p}}]$, where h_i, $i=0,1,\ldots,\mathbf{p}$, are real constant satisfying $0=h_0<h_1<\cdots<h_{\mathbf{p}}=h$. It is a fact that properties of system (5) with the delay $\tau(t)$ varying in different delay subintervals are different. So, the condition that an LKF is required for system (5) with the delay $\tau(t)$ varying in the interval $[0,h]$ seems too strong. For weakening the condition, a new delay form $\tau_{\chi(t)}(t)$ is put

forward based on $\tau(t)$. Here, $\tau_i(t), i = 1, \ldots, \mathbf{p}$, are called delay modes satisfying

$$\tau_1(t) = \begin{cases} \tau(t), & \forall\, t \in \{s | \tau(s) \in [h_0, h_1]\}, \\ \hat{\tau}_1(t), & \forall\, t \in \{s | \tau(s) \notin [h_0, h_1]\}, \end{cases}$$
$$\tau_2(t) = \begin{cases} \tau(t), & \forall\, t \in \{s | \tau(s) \in [h_1, h_2]\}, \\ \hat{\tau}_2(t), & \forall\, t \in \{s | \tau(s) \notin [h_1, h_2]\}, \end{cases} \tag{8}$$
$$\vdots$$
$$\tau_{\mathbf{p}}(t) = \begin{cases} \tau(t), & \forall\, t \in \{s | \tau(s) \in [h_{\mathbf{p}-1}, h_{\mathbf{p}}]\}, \\ \hat{\tau}_{\mathbf{p}}(t), & \forall\, t \in \{s | \tau(s) \notin [h_{\mathbf{p}-1}, h_{\mathbf{p}}]\}, \end{cases}$$

where $\hat{\tau}_i(t), i = 1, \ldots, \mathbf{p}$, are continuous functions satisfying

$$h_0 < \hat{\tau}_1 < h_1 < \hat{\tau}_2 < \cdots < h_{\mathbf{p}-1} < \hat{\tau}_{\mathbf{p}} < h_{\mathbf{p}}. \tag{9}$$

And $\chi(t) : [t_0, +\infty) \to \mathbb{Z}[1, \mathbf{p}] = \{1, \ldots, \mathbf{p}\}$ called a delay-mode-varying function is used to show switching between the delay modes. $t_1, t_2, \ldots$ called delay-mode-varying instants are real constants satisfying $t_0 < t_1 < t_2 < \cdots$. When $t \in [t_i, t_{i+1}), i \in \mathbb{Z}_+$, it is said that the $\chi(t_i)$th delay mode is activated.

Remark 3.2: Note that $\tau_{\chi(t)}(t)$ is from $\tau(t)$, but it is more detailed than $\tau(t)$. Due to the form of $\tau_{\chi(t)}(t)$, the information that the delay varies in different delay subintervals can be fully considered.

In the following, an example is given to show that $\tau_{\chi(t)}(t)$ is effective. When $\tau(t) = 1 + \sin(t)$, it is assumed that $\mathbf{p} = 2, h_1 = 1, h_2 = 2$. Then, it is clear that

$$\tau_1(t) = \begin{cases} 0.5, & \forall\, t \in (2k\pi, (2k+1)\pi), \\ 1 + \sin(t), & \forall\, t \in [(2k+1)\pi, (2k+2)\pi], \end{cases}$$
$$\tau_2(t) = \begin{cases} 1 + \sin(t), & \forall\, t \in [2k\pi, (2k+1)\pi], \\ 1.5, & \forall\, t \in ((2k+1)\pi, (2k+2)\pi), \end{cases}$$

$\forall\, k \in \mathbb{Z}_+$, are admissible.

Based on $\tau_1(t), \tau_2(t)$ and $\tau(t)$, it is clear that $\chi(t)$ satisfies

$$\chi(t) = \begin{cases} 2, & \forall\, t \in [2k\pi, (2k+1)\pi), \\ 1, & \forall\, t \in [(2k+1)\pi, (2k+2)\pi), \end{cases}$$

$\forall\, k \in \mathbb{Z}_+$. As illustrated in Figure 1, replacing $\tau(t)$ with $\tau_{\chi(t)}(t)$ is feasible.

With the help of $\tau_{\chi(t)}(t)$, system (5) can be rewritten as

$$\begin{aligned} &\dot{x}(t) = f(t, x(t - \tau_{\chi(t)}(t)), \dot{x}(t - d)), \quad \forall\, t \in [t_0, +\infty), \\ &x_{t_0}(\theta) = \phi(\theta), \quad \forall\, \theta \in [-h, 0]. \end{aligned} \tag{10}$$

Then, stability analysis of system (5) is converted to stability analysis of system (10).

Definition 3.1 (Zhao et al., 2012)**:** For given constants $b > a$ and any delay-mode-varying function $\chi(t), \mathcal{N}_i[a, b]$ is assumed to be the number that the ith mode is activated in the time interval $[a, b)$ and $\mathcal{T}_i[a, b]$ is assumed to be the activation time of the ith delay mode in the time interval $[a, b)$, where $i \in \mathbb{Z}[1, \mathbf{p}]$. If

$$\mathcal{N}_i[a, b] \le \mathcal{N}_{0i} + \frac{\mathcal{T}_i[a, b]}{\mathfrak{T}_i}, \quad i \in \mathbb{Z}[1, \mathbf{p}], \tag{11}$$

holds, then constants $\mathfrak{T}_i > 0$ and $\mathcal{N}_{0i} \ge 0$ are called a mode-dependent average dwell time and a mode-dependent chatter bound, respectively.

According to the form of system (10), the following functional called a delay-mode-based functional (DMBF) is developed:

$$\mathcal{V}(t, x_t, \dot{x}_t) = \mathcal{V}_{\chi(t)}(t, x_t, \dot{x}_t), \tag{12}$$

where $\mathcal{V}_i(t, x_t, \dot{x}_t), \forall\, i \in \mathbb{Z}[1, \mathbf{p}]$, are LKFs.

Remark 3.3: Compared with the LKF in Lemma 3.1, the DMBF has the advantage that different LKFs are constructed for system (10) with different delay modes. It is clear that the condition in the LKF method can be weakened if the DMBF is adopted. In addition, when the LKFs in the DMBF are the same, the DMBF will reduce to an LKF. It is clear that the DMBF is more general.

By using the DMBF and the introduced definition, the following lemma can be derived.

Lemma 3.2: *For given real constants* $\lambda_i > 0, \mu_i \ge 1, \forall\, i \in \mathbb{Z}[1, \mathbf{p}]$, *if there exist continuously differentiable functionals* $\mathcal{V}_i(t, x_t, \dot{x}_t) : [t_0, +\infty) \times \mathcal{W}([-\tau_2, 0], \mathbb{R}^n) \times \mathcal{L}_2([-\tau_2, 0], \mathbb{R}^n) \to$

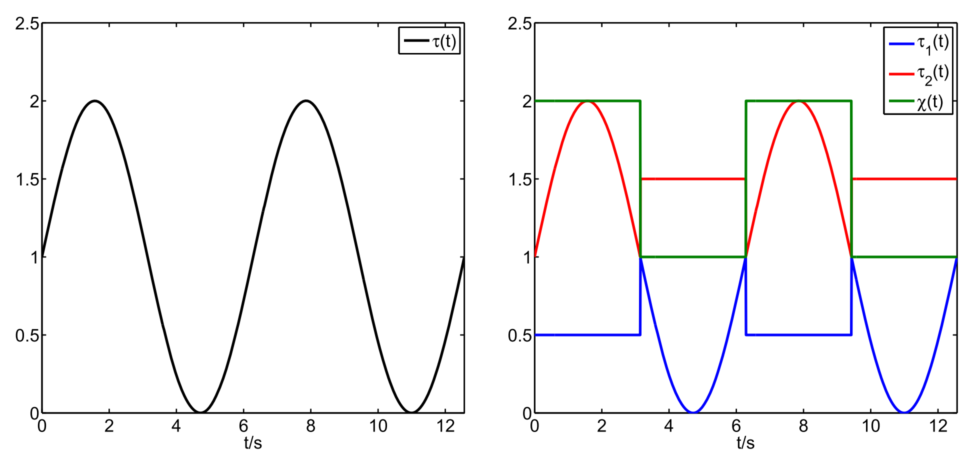

Figure 1. Given $\tau(t)$ and $\tau_{\chi(t)}(t)$.

$\mathbb{R}_+, \forall\, i \in \mathbb{Z}[1,\mathbf{p}]$, *and functions* $\alpha_i, i = 1, 2$, *of class* $\mathcal{K}_\infty$, *such that*

$$\alpha_1(\|x_t(0)\|) \le \mathcal{V}_i(t, x_t, \dot{x}_t) \le \alpha_2(\|x_t\|_{\mathcal{W}}),\quad \forall\, i \in \mathbb{Z}[1,\mathbf{p}], \tag{13}$$

$$\dot{\mathcal{V}}_i(t, x_t, \dot{x}_t) + \lambda_i \mathcal{V}_i(t, x_t, \dot{x}_t) \le 0, \quad \forall\, i \in \mathbb{Z}[1,\mathbf{p}], \tag{14}$$

and

$$\mathcal{V}_i(t, x_t, \dot{x}_t) \le \mu_i \mathcal{V}_j(t, x_t, \dot{x}_t),\quad \forall\,(i,j) \in \mathbb{Z}[1,\mathbf{p}] \times \mathbb{Z}[1,\mathbf{p}],\ \ i \ne j, \tag{15}$$

then system (10) *is GUAS for any delay-mode-varying function satisfying*

$$\mathfrak{T}_i > \frac{\ln(\mu_i)}{\lambda_i}, \quad \forall\, i \in \mathbb{Z}[1,\mathbf{p}]. \tag{16}$$

Proof: When (14) holds, it is true that

$$\mathcal{V}_i(t, x_t, \dot{x}_t) \le e^{-\lambda_i(t-t_0)} \mathcal{V}_i(t_0, x_{t_0}, \dot{x}_{t_0}), \quad \forall\, t \in [t_0, +\infty). \tag{17}$$

Let real constants $t_1, \ldots, t_{\mathcal{N}}$ be delay-mode-varying instants in the time interval $[t_0, t]$, where $\mathcal{N} = \sum_{\forall\, i\in\mathbb{Z}[1,\mathbf{p}]} \mathcal{N}_i[t_0, t]$. According to (12), (15) and (17), it can be easily obtained that

$$\begin{aligned}
&\mathcal{V}(t, x_t, \dot{x}_t)\\
&\le e^{-\lambda_{\chi(t_{\mathcal{N}})}(t-t_{\mathcal{N}})} \mathcal{V}_{\chi(t_{\mathcal{N}})}(t_{\mathcal{N}}, x_{t_{\mathcal{N}}}, \dot{x}_{t_{\mathcal{N}}})\\
&\le \mu_{\chi(t_{\mathcal{N}})} e^{-\lambda_{\chi(t_{\mathcal{N}})}(t-t_{\mathcal{N}})} \mathcal{V}_{\chi(t_{\mathcal{N}})}(t_{\mathcal{N}}, x_{t_{\mathcal{N}}}, \dot{x}_{t_{\mathcal{N}}})\\
&\le \cdots\\
&\le \prod_{\forall\, i\in\mathbb{Z}[1,\mathbf{p}]} \mu_i^{\mathcal{N}_i[t_0,t]} e^{-\sum_{\forall\, i\in\mathbb{Z}[1,\mathbf{p}]} \lambda_i \mathcal{T}_i[t_0,t]} \mathcal{V}(t_0, x_{t_0}, \dot{x}_{t_0}),
\end{aligned} \tag{18}$$

$\forall\, t \in [t_0, +\infty)$. Then, (11) (16) and (18) can lead to

$$\begin{aligned}
&\mathcal{V}(t, x_t, \dot{x}_t)\\
&\le \prod_{\forall\, i\in\mathbb{Z}[1,\mathbf{p}]} \mu_i^{\mathcal{N}_i[t_0,t]} e^{-\sum_{\forall\, i\in\mathbb{Z}[1,\mathbf{p}]} \lambda_i \mathcal{T}_i[t_0,t]} \mathcal{V}(t_0, x_{t_0}, \dot{x}_{t_0})\\
&\le e^{\sum_{\forall\, i\in\mathbb{Z}[1,\mathbf{p}]} \ln(\mu_i)\mathcal{N}_i[t_0,t]} e^{-\sum_{\forall\, i\in\mathbb{Z}[1,\mathbf{p}]} \lambda_i \mathcal{T}_i[t_0,t]} \mathcal{V}(t_0, x_{t_0}, \dot{x}_{t_0})\\
&\le e^{\sum_{\forall\, i\in\mathbb{Z}[1,\mathbf{p}]} \ln(\mu_i)\mathcal{N}_{0i}[t_0,t]} e^{-\sum_{\forall\, i\in\mathbb{Z}[1,\mathbf{p}]} (\lambda_i - \frac{\ln(\mu_i)}{\mathfrak{T}_i})\mathcal{T}_i[t_0,t]}\\
&\quad \mathcal{V}(t_0, x_{t_0}, \dot{x}_{t_0}).
\end{aligned} \tag{19}$$

Obviously, there must exist a function β_1 of class $\mathcal{KL}$ such that

$$\mathcal{V}(t, x_t, \dot{x}_t) \le \beta_1(\mathcal{V}(t_0, x_{t_0}, \dot{x}_{t_0}), t - t_0), \tag{20}$$

$\forall\, t \in [t_0, +\infty)$. Next, (13) and (20) yield

$$\begin{aligned}
\|x(t)\| &\le \alpha_1^{-1}(\mathcal{V}(t, x_t, \dot{x}_t))\\
&\le \alpha_1^{-1}(\beta_1(\mathcal{V}(t_0, x_{t_0}, \dot{x}_{t_0}), t - t_0))\\
&\le \alpha_1^{-1}(\beta_1(\alpha_2(\|\phi\|_{\mathcal{W}}), t - t_0))\\
&\triangleq \beta_2(\|\phi\|_{\mathcal{W}}, t - t_0), \quad \forall\, t \in [t_0, +\infty),
\end{aligned} \tag{21}$$

where β_2 is a function of class $\mathcal{KL}$. Based on (21), it is clear that system (10) is GUAS. The proof is completed. ■

Remark 3.4: As a result of using the DMBF, a new lemma (i.e. Lemma 3.2) on stability analysis of nonlinear neutral-type delay systems is obtained. Lemma 3.2 shows that if there exists a DMBF satisfying (13), (14) and (15), the stability of system (10) can be ensured. It is clear that the condition in Lemma 3.1 is weakened. This is the main difference between Lemmas 3.1 and 3.2.

When $\mu_i = 1,\ \forall\, i \in \mathbb{Z}[1,\mathbf{p}]$, the following corollary can be obtained from Lemma 3.2.

Corollary 3.1: *For given a real constant* $\lambda > 0$, *if there exist a continuously differentiable functional* $\mathcal{V}(t, x_t, \dot{x}_t) : [t_0, +\infty) \times \mathcal{W}([-\tau_2, 0], \mathbb{R}^n) \times \mathcal{L}_2([-\tau_2, 0], \mathbb{R}^n) \to \mathbb{R}_+$, *and functions* α_i, $i = 1, 2$, *of class* $\mathcal{K}_\infty$, *such that*

$$\alpha_1(\|x_t(0)\|) \le \mathcal{V}(t, x_t, \dot{x}_t) \le \alpha_2(\|x_t\|_{\mathcal{W}}), \tag{22}$$

$$\dot{\mathcal{V}}(t, x_t, \dot{x}_t) + \lambda \mathcal{V}(t, x_t, \dot{x}_t) \le 0, \tag{23}$$

then system (10) *is GUAS.*

Remark 3.5: Note that Corollary 3.1 is a criterion based on the LKF method. It means that when stability analysis of neutral-type delay systems is considered, the DMBF method is more effective than the LKF method.

3.2 New stability criteria

In this subsection, new stability criteria are derived based on the DMBF method. However, since the DMBF method contains $\mathbf{p} \ge 2$ LKFs, stability criteria obtained by using the method may be more complex and involve too many decision variables. So, for convenience, it is assumed that $\mathbf{p} = 2$. Furthermore, the following notations are given for the simplicity of the presentation

$$\tilde{x}(t) = \left[x^{\mathrm{T}}(t), \int_{t-h_1}^{t} x^{\mathrm{T}}(s)\,\mathrm{d}s, \int_{t-h_2}^{t-h_1} x^{\mathrm{T}}(s)\,\mathrm{d}s\right]^{\mathrm{T}},$$

$$\eta_1(t) = [x^{\mathrm{T}}(t), x^{\mathrm{T}}(t-\tau_1), \dot{x}(t-d), \eta_{11}^{\mathrm{T}}(t), \eta_{12}^{\mathrm{T}}(t), \eta_{13}^{\mathrm{T}}(t)]^{\mathrm{T}},$$

$$\eta_2(t) = [x^{\mathrm{T}}(t), x^{\mathrm{T}}(t-\tau_2), \dot{x}(t-d), \eta_{21}^{\mathrm{T}}(t), \eta_{22}^{\mathrm{T}}(t), \eta_{23}^{\mathrm{T}}(t)]^{\mathrm{T}},$$

$$\eta_{11}(t) = [x^{\mathrm{T}}(t-h_1), x^{\mathrm{T}}(t-h_2)]^{\mathrm{T}},$$

$$\eta_{12}(t) = \left[\frac{\int_{t-\tau_1}^{t} x^{\mathrm{T}}(s)\,\mathrm{d}s}{\tau_1}, \frac{\int_{t-h_1}^{t-\tau_1} x^{\mathrm{T}}(s)\,\mathrm{d}s}{h_1-\tau_1}\right]^{\mathrm{T}},$$

$$\eta_{13}(t) = \left[\frac{\int_{t-h_2}^{t-h_1} x^{\mathrm{T}}(s)\,\mathrm{d}s}{h_2-h_1}\right]^{\mathrm{T}},$$

$$\eta_{21}(t) = \eta_{11}(t),$$

$$\eta_{22}(t) = \left[\frac{\int_{t-h_1}^{t} x^{\mathrm{T}}(s)\,\mathrm{d}s}{h_1}\right]^{\mathrm{T}},$$

$$\eta_{23}(t) = \left[\frac{\int_{t-\tau_2}^{t-h_1} x^{\mathrm{T}}(s)\,\mathrm{d}s}{\tau_2-h_1}, \frac{\int_{t-h_2}^{t-\tau_2} x^{\mathrm{T}}(s)\,\mathrm{d}s}{h_2-\tau_2}\right]^{\mathrm{T}},$$

$$e_i = [0_{n\times(i-1)n}\ I_n\ 0_{n\times(8-i)n}], \quad i = 1, \ldots, 8,$$

$$A_s = [A\ B\ C\ 0_{n\times 5n}],\quad \tau_1=\tau_1(t),\ \ \tau_2=\tau_2(t).$$

Similarly, system (1) can be rewritten as

$$\begin{aligned}\dot{x}(t) &= Ax(t)+Bx(t-\tau_{\chi(t)}(t))+C\dot{x}(t-d),\\ &\quad \forall\, t\in[t_0,+\infty), \qquad (24)\\ x_{t_0}(\theta)&=\phi(\theta),\quad \forall\,\theta\in[-h,0],\end{aligned}$$

where $\tau(t)=\tau_\chi(t)$, $\chi(t)\in\mathbb{Z}[1,2]$. For system (24), the following two DMBFs are chosen according to functional (12)

$$\begin{aligned}\mathcal{V}(t,x_t,\dot{x}_t) &= \mathcal{V}_{\chi(t)}(t,x_t,\dot{x}_t)\\ &= x^{\mathrm T}(t)P_{\chi(t)}x(t)\\ &\quad+\sum_{i=1}^{2}\int_{t-h_i}^{t} x^{\mathrm T}(s)e^{\lambda(s-t)}Q_{\chi(t)i}x(s)\,\mathrm{d}s\\ &\quad+\sum_{i=1}^{2}\int_{-h_i}^{-h_{i-1}}\int_{t+\theta}^{t}\dot{x}^{\mathrm T}(s)e^{\lambda(s-t)}R_{\chi(t)i}\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta\\ &\quad+\int_{t-d}^{t}\dot{x}^{\mathrm T}(s)e^{\lambda(s-t)}S_{\chi(t)}\dot{x}(s)\,\mathrm{d}s, \qquad (25)\end{aligned}$$

$$\begin{aligned}\tilde{\mathcal{V}}(t,x_t,\dot{x}_t) &= \tilde{\mathcal{V}}_{\chi(t)}(t,x_t,\dot{x}_t)\\ &= \tilde{x}^{\mathrm T}(t)\tilde{P}_{\chi(t)}\tilde{x}(t)\\ &\quad+\sum_{i=1}^{2}\int_{t-h_i}^{t} x^{\mathrm T}(s)e^{\lambda(s-t)}Q_{\chi(t)i}x(s)\,\mathrm{d}s\\ &\quad+\sum_{i=1}^{2}\int_{-h_i}^{-h_{i-1}}\int_{t+\theta}^{t}\dot{x}^{\mathrm T}(s)e^{\lambda(s-t)}R_{\chi(t)i}\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta\\ &\quad+\int_{t-d}^{t}\dot{x}^{\mathrm T}(s)e^{\lambda(s-t)}S_{\chi(t)}\dot{x}(s)\,\mathrm{d}s, \qquad (26)\end{aligned}$$

where $\lambda>0$ is a real constant, $P_i\in\mathbb{S}_+^n$, $\tilde{P}_i\in\mathbb{S}_+^{3n}$, $Q_{ij}\in\mathbb{S}_+^n$, $R_{ij}\in\mathbb{S}_+^n$, $S_i\in\mathbb{S}_+^n$, $i=1,2, j=1,2$.

Applying the DMBFs, Lemmas 2.1 and 3.2, the following theorems can be obtained.

Theorem 3.1: *For given real constants $h_2>h_1>0, h_2\ge d>0, \lambda>0, \mu_i\ge 1, i=1,2$, if there exist matrices $P_i\in\mathbb{S}_+^n, Q_{ij}\in\mathbb{S}_+^n, R_{ij}\in\mathbb{S}_+^n, S_i\in\mathbb{S}_+^n, Z_{ip}\in\mathbb{S}^{3n}, M_{ip}\in\mathbb{R}^{3n\times n}, X_{iq}\in\mathbb{R}^{3n\times 3n}$, $i=1,2, j=1,2, p=1,\ldots,6, q=1,2,3$, such that*

$$\Xi_i(\tau_i)<0,\ \Delta_{ij}\ge 0,\quad i=1,2,\ j=1,2,3, \qquad (27)$$

and

$$\begin{aligned}&P_1\le\mu_1P_2, Q_{1i}\le\mu_1Q_{2i}, R_{1i}\le\mu_1R_{2i}, S_1\le\mu_1S_2,\\ &P_2\le\mu_2P_1, Q_{2i}\le\mu_2Q_{1i}, R_{2i}\le\mu_2R_{1i}, S_2\le\mu_2S_1,\quad i=1,2,\end{aligned} \qquad (28)$$

where

$$\begin{aligned}\Xi_1(\tau_1) &= \Theta_1+c_{11}\Upsilon_{11}^{\mathrm T}\Phi_{11}(\tau_1)\Upsilon_{11}+c_{12}\Upsilon_{12}^{\mathrm T}\Phi_{12}(\tau_1)\Upsilon_{12}\\ &\quad+c_{13}\Upsilon_{13}^{\mathrm T}\Phi_{13}\Upsilon_{13},\\ \Xi_2(\tau_2) &= \Theta_2+c_{21}\Upsilon_{21}^{\mathrm T}\Phi_{21}\Upsilon_{21}+c_{22}\Upsilon_{22}^{\mathrm T}\Phi_{22}(\tau_2)\Upsilon_{22}\\ &\quad+c_{23}\Upsilon_{23}^{\mathrm T}\Phi_{23}(\tau_2)\Upsilon_{23},\end{aligned}$$

$$\Theta_i=\begin{bmatrix}\Theta_{11}^i & \Theta_{12}^i & \Theta_{13}^i & 0_n & 0_n & 0_n & 0_n & 0_n\\ * & \Theta_{22}^i & \Theta_{23}^i & 0_n & 0_n & 0_n & 0_n & 0_n\\ * & * & \Theta_{33}^i & 0_n & 0_n & 0_n & 0_n & 0_n\\ * & * & * & \Theta_{44}^i & 0_n & 0_n & 0_n & 0_n\\ * & * & * & * & \Theta_{55}^i & 0_n & 0_n & 0_n\\ * & * & * & * & * & 0_n & 0_n & 0_n\\ * & * & * & * & * & * & 0_n & 0_n\\ * & * & * & * & * & * & * & 0_n\end{bmatrix},$$

$$\begin{aligned}&\Theta_{11}^i=P_iA+A^{\mathrm T}P_i+\lambda P_i+A^{\mathrm T}\Lambda_iA+Q_{i1}+Q_{i2},\\ &\Theta_{12}^i=P_iB+A^{\mathrm T}\Lambda_iA_d,\quad \Theta_{13}^i=P_iC+A^{\mathrm T}\Lambda_iC,\\ &\Theta_{22}^i=B^{\mathrm T}\Lambda_iB,\quad \Theta_{23}^i=B^{\mathrm T}\Lambda_iC,\\ &\Theta_{33}^i=C^{\mathrm T}\Lambda_iC-e^{-\lambda d}S_i,\quad \Theta_{44}^i=-e^{-\lambda h_1}Q_{i1},\\ &\Theta_{55}^i=-e^{-\lambda h_2}Q_{i2},\quad \Lambda_i=h_1R_{i1}+(h_2-h_1)R_{i2}+S_i,\\ &\ \ i=1,2,\\ &c_{11}=e^{-\lambda h_1},\quad c_{12}=c_{11},\quad c_{13}=e^{-\lambda h_2},\\ &c_{21}=e^{-\lambda h_1},\quad c_{22}=e^{-\lambda h_2},\quad c_{23}=c_{22},\end{aligned}$$

$$\begin{aligned}\Phi_{11}(\tau_1)&=(\tau_1-h_1)\left(Z_{11}+\frac{Z_{12}}{3}\right)+\mathrm{Sym}\{M_{11}\Pi_1+M_{12}\Pi_2\},\\ \Phi_{12}(\tau_1)&=(h_2-\tau_1)\left(Z_{13}+\frac{Z_{14}}{3}\right)+\mathrm{Sym}\{M_{13}\Pi_1+M_{14}\Pi_2\},\\ \Phi_{13}&=(h_2-h_1)\left(Z_{15}+\frac{Z_{16}}{3}\right)+\mathrm{Sym}\{M_{15}\Pi_1+M_{16}\Pi_2\},\\ \Phi_{21}&=h_1\left(Z_{21}+\frac{Z_{22}}{3}\right)+\mathrm{Sym}\{M_{21}\Pi_1+M_{22}\Pi_2\},\\ \Phi_{22}(\tau_2)&=(\tau_2-h_1)\left(Z_{23}+\frac{Z_{24}}{3}\right)+\mathrm{Sym}\{M_{23}\Pi_1+M_{24}\Pi_2\},\\ \Phi_{23}(\tau_2)&=(h_2-\tau_2)\left(Z_{25}+\frac{Z_{26}}{3}\right)+\mathrm{Sym}\{M_{25}\Pi_1+M_{26}\Pi_2\},\end{aligned}$$

$$\Upsilon_{11}=\begin{bmatrix}e_1\\ e_2\\ e_6\end{bmatrix},\quad \Upsilon_{12}=\begin{bmatrix}e_2\\ e_4\\ e_7\end{bmatrix},\quad \Upsilon_{13}=\begin{bmatrix}e_4\\ e_5\\ e_8\end{bmatrix},$$

$$\Upsilon_{21}=\begin{bmatrix}e_1\\ e_4\\ e_6\end{bmatrix},\quad \Upsilon_{22}=\begin{bmatrix}e_4\\ e_2\\ e_7\end{bmatrix},\quad \Upsilon_{23}=\begin{bmatrix}e_2\\ e_5\\ e_8\end{bmatrix},$$

$$\Delta_{11}=\begin{bmatrix}Z_{11} & X_{11} & M_{11}\\ * & Z_{12} & M_{12}\\ * & * & R_{11}\end{bmatrix},\quad \Delta_{12}=\begin{bmatrix}Z_{13} & X_{12} & M_{13}\\ * & Z_{14} & M_{14}\\ * & * & R_{12}\end{bmatrix},$$

$$\Delta_{13}=\begin{bmatrix}Z_{15} & X_{13} & M_{15}\\ * & Z_{16} & M_{16}\\ * & * & R_{12}\end{bmatrix},\quad \Delta_{21}=\begin{bmatrix}Z_{21} & X_{21} & M_{21}\\ * & Z_{22} & M_{22}\\ * & * & R_{21}\end{bmatrix},$$

$$\Delta_{22}=\begin{bmatrix}Z_{23} & X_{22} & M_{23}\\ * & Z_{24} & M_{24}\\ * & * & R_{22}\end{bmatrix},\quad \Delta_{23}=\begin{bmatrix}Z_{25} & X_{23} & M_{25}\\ * & Z_{26} & M_{26}\\ * & * & R_{22}\end{bmatrix},$$

and $\Pi_i, i=1,2$, are defined in Lemma 2.1, then system (24) is GUAS for any delay-mode-varying function satisfying

$$\mathfrak{T}_i>\frac{\ln(\mu_i)}{\lambda},\quad i=1,2. \qquad (29)$$

Proof: For functional (25), it is clear that

$$\kappa_1\|x_t(0)\|^2 \le \mathcal{V}_i(t,x_t,\dot{x}_t) \le \kappa_2\|x_t\|^2_{\mathcal{W}}, \quad i=1,2, \tag{30}$$

where $\kappa_i > 0, i=1,2$, are real constant. When $\chi(t)=1$, the following equation can be obtained

$$\begin{aligned}
&\dot{\mathcal{V}}_1(t,x_t,\dot{x}_t)+\lambda\mathcal{V}_1(t,x_t,\dot{x}_t)\\
&\quad= \dot{x}^{\mathrm{T}}(t)P_1x(t)+x^{\mathrm{T}}(t)P_1\dot{x}(t)+\lambda x^{\mathrm{T}}(t)P_1x(t)\\
&\qquad+\sum_{i=1}^{2}x^{\mathrm{T}}(t)Q_{1i}x(t)+\dot{x}^{\mathrm{T}}(t)\Lambda_1\dot{x}(t)\\
&\qquad-\sum_{i=1}^{2}x^{\mathrm{T}}(t-h_i)e^{-\lambda h_i}Q_{1i}x(t-h_i)\\
&\qquad-\dot{x}^{\mathrm{T}}(t-d)e^{-\lambda d}S_1\dot{x}(t-d)\\
&\qquad-\int_{t-\tau_1}^{t}\dot{x}^{\mathrm{T}}(s)e^{\lambda(s-t)}R_{11}\dot{x}(s)\,\mathrm{d}s\\
&\qquad-\int_{t-h_1}^{t-\tau_1}\dot{x}^{\mathrm{T}}(s)e^{\lambda(s-t)}R_{11}\dot{x}(s)\,\mathrm{d}s\\
&\qquad-\int_{t-h_2}^{t-h_1}\dot{x}^{\mathrm{T}}(s)e^{\lambda(s-t)}R_{12}\dot{x}(s)\,\mathrm{d}s.
\end{aligned}\tag{31}$$

The following inequalities are obtained by applying Lemma 2.1

$$\begin{aligned}
-\int_{t-h_1}^{t}\dot{x}^{\mathrm{T}}(s)R_{11}\dot{x}(s)\,\mathrm{d}s &\le \eta_1^{\mathrm{T}}(t)\Upsilon_{11}^{\mathrm{T}}\Phi_{11}(\tau_1)\Upsilon_{11}\eta_1(t)\\
&\quad+\eta_1^{\mathrm{T}}(t)\Upsilon_{12}^{\mathrm{T}}\Phi_{12}(\tau_1)\Upsilon_{12}\eta_1(t),\\
-\int_{t-h_2}^{t-h_1}\dot{x}^{\mathrm{T}}(s)R_{12}\dot{x}(s)\,\mathrm{d}s &\le \eta_1^{\mathrm{T}}(t)\Upsilon_{13}^{\mathrm{T}}\Phi_{13}\Upsilon_{13}\eta_1(t).
\end{aligned}\tag{32}$$

Combining (27), (31) and (32) leads to

$$\begin{aligned}
\dot{\mathcal{V}}_1(t,x_t,\dot{x}_t)+\lambda\mathcal{V}_1(t,x_t,\dot{x}_t) &\le \eta_1^{\mathrm{T}}(t)\Xi_1(\tau_1)\eta_1(t)\\
&\le 0.
\end{aligned}\tag{33}$$

Similarly, when $\eta_2^{\mathrm{T}}(t)\Xi_2(\tau_2)\eta_2(t) < 0, \mathcal{V}_2(t,x_t,\dot{x}_t)$ satisfies

$$\dot{\mathcal{V}}_2(t,x_t,\dot{x}_t)+\lambda\mathcal{V}_2(t,x_t,\dot{x}_t) \le 0. \tag{34}$$

(25) and (28) yield

$$\begin{aligned}
\mathcal{V}_1(t,x_t,\dot{x}_t) &\le \mu_1\mathcal{V}_2(t,x_t,\dot{x}_t),\\
\mathcal{V}_2(t,x_t,\dot{x}_t) &\le \mu_2\mathcal{V}_1(t,x_t,\dot{x}_t).
\end{aligned}\tag{35}$$

From Lemma 3.2, it can be seen that system (24) is GUAS. The proof is completed. ■

Theorem 3.2: *For given real constants $h_2 > h_1 > 0, h_2 \ge d > 0, \lambda > 0, \mu_i \ge 1, i=1,2$, if there exist matrices $\tilde{P}_i \in \mathbb{S}_+^{3n}, Q_{ij} \in \mathbb{S}_+^n, R_{ij} \in \mathbb{S}_+^n, S_i \in \mathbb{S}_+^n, Z_{il} \in \mathbb{S}^{3n}, M_{ip} \in \mathbb{R}^{3n\times n}, X_{iq} \in \mathbb{R}^{3n\times 3n}$, $i=1,2, j=1,2, p=1,\ldots,6, q=1,2,3$, such that*

$$\tilde{\Xi}_i(\tau_i) < 0, \quad \Delta_{ij} \ge 0, \quad i=1,2,\ j=1,2,3, \tag{36}$$

and

$$\begin{aligned}
&\tilde{P}_1 \le \mu_1\tilde{P}_2, Q_{1i} \le \mu_1 Q_{2i}, R_{1i} \le \mu_1 R_{2i}, S_1 \le \mu_1 S_2,\\
&\tilde{P}_2 \le \mu_2\tilde{P}_1, Q_{2i} \le \mu_2 Q_{1i}, R_{2i} \le \mu_2 R_{1i}, S_2 \le \mu_2 S_1, \quad i=1,2,
\end{aligned}\tag{37}$$

where

$$\begin{aligned}
\tilde{\Xi}_1(\tau_1) &= \mathrm{Sym}\{\Gamma_{11}^{\mathrm{T}}(\tau_1)\tilde{P}_1\Gamma_{12}\}+\lambda\Gamma_{11}^{\mathrm{T}}(\tau_1)\tilde{P}_1\Gamma_{11}(\tau_1)\\
&\quad+\tilde{\Theta}_1+c_{11}\Upsilon_{11}^{\mathrm{T}}\Phi_{11}(\tau_1)\Upsilon_{11}+c_{12}\Upsilon_{12}^{\mathrm{T}}\Phi_{12}(\tau_1)\Upsilon_{12}\\
&\quad+c_{13}\Upsilon_{13}^{\mathrm{T}}\Phi_{13}\Upsilon_{13},\\
\tilde{\Xi}_2(\tau_2) &= \mathrm{Sym}\{\Gamma_{21}^{\mathrm{T}}(\tau_2)\tilde{P}_2\Gamma_{22}\}+\lambda\Gamma_{21}^{\mathrm{T}}(\tau_2)\tilde{P}_2\Gamma_{21}(d_2)\\
&\quad+\tilde{\Theta}_2+c_{21}\Upsilon_{21}^{\mathrm{T}}\Phi_{21}\Upsilon_{21}+c_{22}\Upsilon_{22}^{\mathrm{T}}\Phi_{22}(\tau_2)\Upsilon_{22}\\
&\quad+c_{23}\Upsilon_{23}^{\mathrm{T}}\Phi_{23}(\tau_2)\Upsilon_{23},
\end{aligned}$$

$$\Gamma_{11}(\tau_1)=\begin{bmatrix} e_1\\ \tau_1e_5+(h_1-\tau_1)e_6\\ (h_2-h_1)e_7\end{bmatrix}, \quad \Gamma_{12}=\begin{bmatrix} A_s\\ e_1-e_3\\ e_3-e_4\end{bmatrix},$$

$$\Gamma_{21}(\tau_2)=\begin{bmatrix} e_1\\ h_1e_5\\ (\tau_2-h_1)e_6+(h_2-\tau_2)e_7\end{bmatrix}, \quad \Gamma_{22}=\Gamma_{12},$$

$$\tilde{\Theta}_i=\begin{bmatrix}
\tilde{\Theta}_{11}^i & \tilde{\Theta}_{12}^i & \tilde{\Theta}_{13}^i & 0_n & 0_n & 0_n & 0_n & 0_n\\
* & \Theta_{22}^i & \Theta_{23}^i & 0_n & 0_n & 0_n & 0_n & 0_n\\
* & * & \Theta_{33}^i & 0_n & 0_n & 0_n & 0_n & 0_n\\
* & * & * & \Theta_{44}^i & 0_n & 0_n & 0_n & 0_n\\
* & * & * & * & \Theta_{55}^i & 0_n & 0_n & 0_n\\
* & * & * & * & * & 0_n & 0_n & 0_n\\
* & * & * & * & * & * & 0_n & 0_n\\
* & * & * & * & * & * & * & 0_n
\end{bmatrix},$$

$$\begin{aligned}
&\tilde{\Theta}_{11}^i = A^{\mathrm{T}}\Lambda_iA+Q_{21}+Q_{22},\\
&\tilde{\Theta}_{12}^i = A^{\mathrm{T}}\Lambda_iB, \quad \tilde{\Theta}_{13}^i = A^{\mathrm{T}}\Lambda_iC, \quad i=1,2,
\end{aligned}$$

and other notations are defined in Theorem 3.1, *then system* (24) *is GUAS for any delay-mode-varying function satisfying* (29).

Proof: The proof of Theorem 3.2 is similar to the proof of Theorem 3.1. Thus, it is omitted here. ■

Remark 3.6: Based on the DMBF method, new stability criteria of linear neutral-type delay systems are derived. The main novelty of the criteria mainly comes from using the DMBF method. When $\mu_1=\mu_2=1$, Theorems 3.1 and 3.2 will reduce to two stability criteria obtained by using the LKF method. Although $\mathbf{p}=2$ is chosen, the obtained stability criteria still seem complicated. It is known to all that when a neutral-type delay system with a high dimension is considered, a complicated criterion may lead to a computational burden. According to the proof of Theorems 3.1 and 3.2, it is clear that for a given DMBF, the integral inequalities, which are more effective and contains fewer decision variables, are needed. Thus, how to propose the kind of inequalities will be further studied in the future.

Table 1. Admissible upper bounds of h based on different criteria (Example 4.1).

Criterion	Admissible upper bound of h
Theorem 3.1 ($\mu_1 = \mu_2 = 1$)	1.28
Theorem 3.2 ($\mu_1 = \mu_2 = 1$)	1.31
Theorem 3.1 ($\mu_1 = \mu_2 = 1.5$)	1.35
Theorem 3.2 ($\mu_1 = \mu_2 = 1.5$)	1.47

Table 2. Admissible upper bounds of h based on different criteria (Example 4.2).

Criterion	Admissible upper bound of h
Zeng et al. (2015)	2.18
Liu and Seuret (2017)	2.24
Seuret et al. (2018)	2.24
Theorem 3.1	2.28
Theorem 3.2	2.56

4. Numerical examples

In this section, two examples are provided to show the effectiveness of the obtained criteria.

Example 4.1: Consider system (1) with

$$A = \begin{bmatrix} -0.9 & 0.2 \\ 0.1 & -0.9 \end{bmatrix}, \quad B = \begin{bmatrix} -1.1 & -0.2 \\ -0.1 & -1.1 \end{bmatrix},$$
$$C = \begin{bmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{bmatrix}.$$

Here, it is assumed that $\lambda = 0.05, h_1 = \frac{h_2}{\theta}, 1 < \theta < 4, d = h_2$. For different $\mu_i, i = 1, 2$, admissible upper bounds of h based on Theorems 3.1 and 3.2 are given in Table 1. From the table, the following can be obtained.

(1) Although the DMBF method is more complex than the LKF method, the obtained criteria based on the DMBF method are less conservative.
(2) Compared with functional (25) and functional (27), one can know that an augmented DMBF may be more effective than a simple DMBF.

Example 4.2: Consider system (1) with

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

It is assumed that $\lambda = 0.01, \mu_1 = 2, \mu_2 = 2, h_1 = \frac{2h_2}{3}, d = h_2$. When $\mathfrak{T}_i > 100 \ln(2), i = 1, 2$, admissible upper bounds of h obtained via Theorems 3.1 and 3.2 are provided in Table 2. It should be noted that the other results are based on the LKF method. Table 2 shows that the obtained criteria based on the DMBF method are less conservative.

5. Conclusions

Stability analysis of neutral-type delay systems has been further investigated in this paper. A new stability analysis method called a DMBF method has been proposed by weakening the strong condition in the LKF method. Then, new stability criteria of neutral-type delay systems have been acquired via the new method. Finally, two examples have been given to show that the obtained criteria are less conservative than some existing criteria based on the LKF method.

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References

Balasubramaniam, P., Krishnasamy, R., & Rakkiyappan, R. (2012). Delay-dependent stability of neutral systems with time-varying delays using delay-decomposition approach. *Applied Mathematical Modelling*, *36*(5), 2253–2261. https://doi.org/10.1016/j.apm.2011.08.024

Fridman, E. (2001). New Lyapunov–Krasovskii functionals for stability of linear retarded and neutral type systems. *Systems & Control Letters*, *43*(4), 309–319. https://doi.org/10.1016/S0167-6911(01)00114-1

Fridman, E. (2014). *Introduction to time-delay systems: Analysis and control*. Birkhüser.

Fridman, E., & Shaked, U. (2003). Delay-dependent stability and H_∞ control : Constant and time-varying delays. *International Journal of Control*, *76*(1), 48–60. https://doi.org/10.1080/0020717021000049151

Gu, K., Kharitonov, V. L., & Chen, J. (2003). *Stability of time delay systems*. Birkhüser.

Han, Q. (2002). Robust stability of uncertain delay-differential systems of neutral type. *Automatica*, *38*(4), 719–723. https://doi.org/10.1016/S0005-1098(01)00250-3

He, Y., Wu, M., She, J., & Liu, G. (2004). Delay-dependent robust stability criteria for uncertain neutral systems with mixed delays. *Systems & Control Letters*, *51*(1), 57–65. https://doi.org/10.1016/S0167-6911(03)00207-X

Kwon, O., Park, M., Park, J., Lee, S., & Cha, E. (2012). New delay-partitioning approaches to stability criteria for uncertain neutral systems with time-varying delays. *Journal of the Franklin Institute*, *349*(9), 2799–2823. https://doi.org/10.1016/j.jfranklin.2012.08.013

Liu, K., & Seuret, A. (2017). Comparison of bounding methods for stability analysis of systems with time-varying delays. *Journal of the Franklin Institute*, *354*(7), 2979–2993. https://doi.org/10.1016/j.jfranklin.2017.02.007

Lu, R., Wu, H., & Bai, J. (2014). New delay-dependent robust stability criteria for uncertain neutral systems with mixed delays. *Journal of the Franklin Institute*, *351*(3), 1386–1399. https://doi.org/10.1016/j.jfranklin.2013.11.001

Maharajan, C., Raja, R., Cao, J., & Rajchakit, G. (2018). Novel global robust exponential stability criterion for uncertain inertial-type BAM neural networks with discrete and distributed time-varying delays via Lagrange sense. *Journal of the Franklin Institute*, *355*(11), 4727–4754. https://doi.org/10.1016/j.jfranklin.2018.04.034

Mazenc, F. (2015). Stability analysis of time-varying neutral time-delay systems. *IEEE Transactions on Automatic Control*, *60*(2), 540–546. https://doi.org/10.1109/TAC.2014.2342095

Pratap, A., Raja, R., Cao, J., Rajchakit, G., & Alsaadi, F. E. (2018). Further synchronization in finite time analysis for time-varying delayed fractional order memristive competitive neural networks with leakage delay. *Neurocomputing*, *317*(2), 110–126. https://doi.org/10.1016/j.neucom.2018.08.016

Richard, J.-P. (2003). Time-delay systems: An overview of some recent advances and open problems. *Automatica*, *39*(10), 1667–1694. https://doi.org/10.1016/S0005-1098(03)00167-5

Senthilraj, S., Raja, R., Zhu, Q., Samidurai, R., & Yao, Z. (2016). New delay-interval-dependent stability criteria for static neural networks with time-varying delays. *Neurocomputing*, *186*(19), 1–7. https://doi.org/10.1016/j.neucom.2015.12.063

Seuret, A., & Gouaisbaut, F. (2013). Wirtinger-based integral inequality: Application to time-delay systems. *Automatica*, *49*(9), 2860–2866. https://doi.org/10.1016/j.automatica.2013.05.030

Seuret, A., Liu, K., & Gouaisbaut, F. (2018). Generalized reciprocally convex combination lemmas and its application to time-delay systems. *Automatica*, *95*(11), 488–493. https://doi.org/10.1016/j.automatica.2018.06.017

Sowmiya, C., Raja, R., Cao, J., & Rajchakit, G. (2018). Enhanced result on stability analysis of randomly occurring uncertain parameters, leakage, and impulsive BAM neural networks with time-varying delays: Discrete–time case. *International Journal of Adaptive Control and Signal Processing*, *32*(7), 1010–1039. https://doi.org/10.1002/acs.2883

Sun, J., Liu, G., & Chen, J. (2009). Delay-dependent stability and stabilization of neutral time-delay systems. *International Journal of Robust and Nonlinear Control*, *19*(12), 1364–1375. https://doi.org/10.1002/rnc.1384

Vadivoo, B. S.., Ramachandran, R., Cao, J., Zhang, H., & Li, X. (2018). Controllability analysis of nonlinear neutral-type fractional-order differential systems with state delay and impulsive effects. *International Journal of Control, Automation and Systems*, *16*(2), 659–669. https://doi.org/10.1007/s12555-017-0281-1

Wang, Y., Liu, H., & Li, X. (2021). A novel method for stability analysis of time-varying delay systems. *IEEE Transactions on Automatic Control*, *66*(3), 1422–1428. https://doi.org/10.1109/TAC.2020.3001422

Wu, M., He, Y., & She, J. (2004). New delay-dependent stability criteria and stabilizing method for neutral systems. *IEEE Transactions on Automatic Control*, *49*(12), 2266–2271. https://doi.org/10.1109/TAC.2004.838484

Zeng, H., He, Y., Wu, M., & She, J. (2015). Free-matrix-based integral inequality for stability analysis of systems with time-varying delay. *IEEE Transactions on Automatic Control*, *60*(10), 2768–2772. https://doi.org/10.1109/TAC.2015.2404271

Zhao, X., Zhang, L., Shi, P., & Liu, M. (2012). Stability and stabilization of switched linear systems with mode-dependent average dwell time. *IEEE Transactions on Automatic Control*, *57*(7), 1809–1815. https://doi.org/10.1109/TAC.2011.2178629