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Stabilization of switched linear neutral systems with time-scheduled feedback control strategy

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***Abstract*—This paper is concerned with the stabilization of continuous-time switched linear neutral systems with tighter bounds on mode-dependent average dwell time. By introducing a novel time-scheduled control strategy, a switching-signal-based multiple discontinuous Lyapunov function (SMDLF) is constructed for continuous-time switched linear neutral systems, where the discontinuous points of the Lyapunov function during each mode-running interval are dynamically adjusted according to the actual system switching signal. First, sufficient criteria are established for global uniform exponential stability (GUES) of switched nonlinear systems. Then, based on the constructed SMDLF, we derive less conservative stability conditions for switched linear neutral systems. Furthermore, a set of time-dependent state feedback controllers are designed to guarantee the GUES of the underlying system. In the end, a simulation example with comparison results is provided to verify the effectiveness and advantages of the presented approach.**

***Index Terms*—Time-scheduled control strategy, multiple discontinuous Lyapunov function, switched linear neutral systems, mode-dependent average dwell time**

I. Introduction

In the past few decades, many efforts have been devoted to neutral systems, which not only contain the time delay of the state, but also the time delay of the state derivative. Actually, many physical processes can be modeled by function differential equations of neutral systems such as partial element equivalent circuit, flexible systems, VLSI systems and population ecology [1]–[8]. However, some unavoidable factors, e.g. unexpected faults, random environment variation, and abrupt changes, may occur in man-made or physical systems. In these cases, compared with neutral systems, switched neutral systems have a great advantage in modeling these complex dynamical behaviors [9]–[12]. For instance, in [9], the oilwell drilling system was modeled by switched neutral systems, where the stick slip behavior can be great reduced by the derived results. In [10], a switched neutral system was applied to model lossless transmission lines systems. In recent years, a number of meaningful works have been carried out for the analysis and synthesis of switched neutral systems [13]–[18]. To mention a few, the asynchronous feedback control was investigated for switched linear neutral systems with average dwell time (ADT) switching strategy [13]. In [15], the authors paid attention to the dynamic output feedback control for uncertain switched neutral systems with time-varying delays. In [17], the observer-based output feedback control was considered for switched linear neutral systems under the framework of event-triggered mechanism along with asynchronous switching.

As is well known, stability is one of the most important research topics in control systems, and common Lyapunov function (CLF) and multiple Lyapunov function (MLF) are two commonly used ways to deal with the stability of switched systems. Typically, CLF method is utilized for switched systems under arbitrary switching. To ensure the stability of the switched system under arbitrary switching, it is necessary to ensure that all subsystems have a common Lyapunov function. Howbeit, it is unrealistic to require all subsystems to share a common Lyapunov function in actual situations. Hence, it is of great significance to analyze switched systems with constrained switching. It is worth noting that the MLF approach has greater flexibility in dealing with various problems of switched systems with constrained switching and can reduce the conservatism of theoretical results.

In term of MLF method, many interesting achievements have been reported, such as [19]–[33]. For instance, the stability and stabilization were discussed for switched linear systems under mode-dependent average dwell time (MDADT) [20]. In reference [22], utilizing sampled-data control strategy, the authors made a discussion on the problem of reliable state estimation for switched neutral systems with actuator faults. However, the constructed MLF in these results were continuous during each mode-running interval, which is still conservative in analysis and synthesis of switched systems. Therefore, multiple discontinuous Lyapunov function (MDLF) is raised for switched systems, while each MLF is discontinuous during the operation of any subsystem [34], [35]. For continuous-time and discrete-time switched systems, by exploiting MDLF approach, references [34] and [35] respectively investigated the switching stabilization and l_2-gain analysis problems. While the division of the actual mode-running time is fixed for different subsystems [34], [35]. That is to say, for subsystem i, regardless of the actual running time of the subsystem, the authors uniformly divided the running time into fixed segments, which is obviously somewhat unreasonable since the stability of switched systems is closely related to the switching signal. One problem here is that whether we can

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involve the influence of the switching signal, which is more flexible for the controller design and practical application. This requires the construction of the corresponding function to be able to dynamically adjust the discontinuous points according to the actual running time of the system. To the best of the authors' knowledge, there are no related discussions regarding the stability of switched neutral systems, which promotes our work.

Motivated by the above discussions, the purpose of this paper is to deal with the exponential stability for continuous-time switched linear neutral systems with hybrid switching. Different from the previous MDLF in [34], [35], a switching-signal-based MDLF (SMDLF) is constructed for continuous-time switched linear neutral systems in terms of the proposed time-scheduled control scheme. Then, less conservative stability conditions are developed for the considered system by exploiting the constructed SMDLF. Furthermore, we establish a theorem to cope with the controller design based on the analysis results. Finally, the correctness and merits of the derived results are demonstrated by an illustrate example. The remainder of the article is organized as follows. Section II gives problem formulation. Section III shows the main content of this article. In Section IV, a numerical example is provided to prove the correctness and effectiveness of the proposed method, and Section V concludes this paper.

Notation: $\mathbb{Z}^+$ denotes the set of nonnegative integers, and $\mathbb{Z}_{[a,b]}$ represents the set $\{k \in \mathbb{Z}^+ | a \leqslant k \leqslant b\}$. $\mathbb{R}^n$ stands for the n-dimensional Euclidean space. A function $\rho : [0, \infty) \longrightarrow [0, \infty)$ is said to be of class $\mathcal{K}_\infty$ if it is continuous, strictly increasing and unbounded with $\rho(0) = 0$. Matrix $\mathcal{M} > 0 (\leqslant 0)$ means $\mathcal{M}$ is positive (nonpositive) definite and real symmetric. $\lfloor s \rfloor$ signifies the maximum integer not greater than s, where $s > 0$.$\| \cdot \|$ indicates Euclidean vector norm. The notation $*$ in a symmetry matrix refers to the symmetry part of the matrix. $\lambda_{\min}(\mathcal{M})$ and $\lambda_{\max}(\mathcal{M})$ represent the maximum and minimum eigenvalues of matrix $\mathcal{M}$, respectively.

II. Problem Formulation

In this paper, we consider the continuous-time switched linear neutral systems as

$$\begin{aligned} \dot{x}(t) - & B_{\sigma(t)} \dot{x}(t-h(t)) \\ & = A_{\sigma(t)} x(t) + C_{\sigma(t)} x(t-d(t)) + D_{\sigma(t)} u(t), \\ x(\theta) & = \chi(\theta), \quad \forall \theta \in [-r, 0], \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$ represents the state vector, and $u(t) \in \mathbb{R}^{n_u}$ denotes the control input. $\sigma : [0, \infty) \to \mathcal{S} = \{1, 2, \cdots, S\}$ is a piecewise constant function responding to the switching signal. Define a time sequence $\{t_k, k \in \mathbb{Z}^+\}$ satisfying $0 = t_0 < t_1 < \cdots < t_k < \cdots$, and t_k is the switching instant. For subsystem $i, \forall i \in \mathcal{S}, A_i, B_i, C_i$, and D_i are real matrices with suitable dimensions, and matrix B_i meets $\|B_i\| < 1$. In addition, $h(t)$ and $d(t)$ are time-varying delays, and $0 < h(t) \leqslant \hat{h}, \dot{h}(t) \leqslant \bar{h} < 1, 0 < d(t) \leqslant \hat{d}, \dot{d}(t) \leqslant \bar{d} < 1$, where $\hat{h}, \hat{d}, \bar{h}, \bar{d} \in \mathbb{Z}^+$.$\chi(\theta)$ is a continuously differential vector initial function on $[-r, 0]$ with $r = \max\{\hat{h}, \hat{d}\}$.

Remark 1: For the switched neutral system (1), if χ is a continuously differentiable function on $[-r, 0]$, and the function $x(t) - B_{\sigma(t)} x(t - h(t))$ is continuously differentiable on $(0, \infty)$ with a right-hand derivative at 0, then there exists a unique solution $x(-r, \infty) \to \mathbb{R}^{n_x}$ of (1) that coincides with χ on $[-r, 0]$ [5], [6].

Throughout the paper, we denote $\mathcal{N}_{\sigma i}(t, T)$ as the number that the system switches to the i-th subsystem during a time interval $[t, T)$, and the overall running time of the i-th subsystem is described by $\mathcal{R}_i(t, T), i \in \mathcal{S}$. In this paper, the switching signal $\sigma(t)$ is satisfied the mode-dependent hybrid dwell time (DT) switching, which is an extension of the assumption 1 in [36].

Assumption 1: Suppose that subsystem i is running during the time interval $[t_k, t_{k+1})$, the switching signal $\sigma(t)$ satisfies 1) a mode-dependent DT $\tau_{di} > 0$ such that $t_{k+1} - t_k \geqslant \tau_{di}$ holds.
2) a MDADT $\tau_{ai} > \tau_{di}$ and $\mathcal{N}_{0i} \geqslant 1$ such that

$$\mathcal{N}_{\sigma i}(t, T) \leqslant \mathcal{N}_{0i} + \frac{\mathcal{R}_i(t, T)}{\tau_{ai}}, \quad \forall T \geqslant t \geqslant 0.$$

Definition 1: [13] For the switched linear neutral system (1) and switching signal σ, the equilibrium x^* is said to be globally uniformly exponentially stable (GUES), if there exist numbers $\iota > 0, \kappa > 0$ and the solution $x(t)$ of model (1) meets

$$\|x(t)\| \leqslant \iota e^{-\kappa(t-t_0)} \|x(t_0)\|_r, \quad \forall t \geqslant t_0$$

with

$$\|x(t_0)\|_r = \sup_{\theta \in [-r,0]} \{\|x(t_0 + \theta)\|, \|\dot{x}(t_0 + \theta)\|\}.$$

The main purpose of this paper is to design a set of time-dependent controllers to guarantee the GUES of switched linear neutral systems by constructing a new MDLF. At the same time, the derived switching signal has smaller lower bounds on MDADT, which has greater flexibility in practical applications.

III. Main Results

For the continuous-time switched linear neutral systems (1), we propose a time-scheduled control strategy by introducing a time scheduler $q_t (q_t \in \mathbb{Z}^+)$. Every time q_t is updated, the controller will update accordingly. Meanwhile, we give a maximum number of updates to balance the amount of calculation and system performance. Combining with Assumption 1, the mode-running interval $[t_k, t_{k+1})$ is divided into three portions, which is shown in Fig. 1. The first portion is $[t_k, t_k + n_i m) (n_i \in \mathbb{Z}^+)$, and the second one is $[t_k + n_i m, t_k + N_i m) (n_i \leqslant N_i \in \mathbb{Z}^+)$ with the last portion $[t_k + N_i m, t_{k+1})$. In this case, we can compute this scheduler by

$$q_t = \begin{cases} \lfloor \frac{t - t_k}{m} \rfloor, & t \in [t_k, t_k + n_i m) \\ \left\lfloor \frac{t - (t_k + n_i m)}{m} \right\rfloor + n_i, & t \in [t_k + n_i m, t_k + N_i m) \\ N_i, & t \in [t_k + N_i m, t_{k+1}) \end{cases} \tag{2}$$

where t_k denotes the switching instant of the k-th switching and the i-th mode is activated in $[t_k, t_{k+1})$, $i \in \mathcal{S}$, $k \in \mathbb{Z}^+$. $m > 0$ is a fixed period given in advance and n_i is a mode-dependent maximum integer satisfying $n_i m \leqslant \tau_{di}$. In

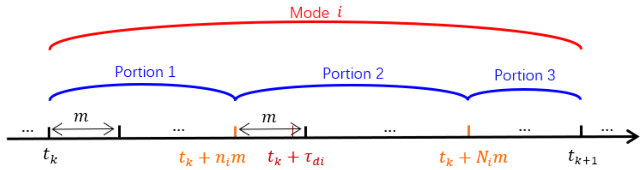

Fig. 1. Mode-running interval $[t_k, t_{k+1})$.

addition, the positive integer N_i is a prechosen maximum step of controller for the i-th mode, $i \in \mathcal{S}$.

Remark 2: According to Assumption 1 and (2), we know the minimum update of controller for the i-th subsystem is n_i during each time interval in which the i-th mode is running. Furthermore, during $[t_k + n_i m, t_{k+1})$, we divide it into two parts to discuss the update of controller by exploiting the prechosen parameter N_i. Meanwhile, it is possible that $N_i m > t_{k+1} - t_k$, $i \in \mathcal{S}$. At such moment, q_t is described by

$$q_t = \begin{cases} \lfloor \frac{t-t_k}{m} \rfloor, & t \in [t_k, t_k + n_i m) \\ \left\lfloor \frac{t-(t_k+n_i m)}{m} \right\rfloor + n_i, & t \in [t_k + n_i m, t_{k+1}) \end{cases}$$

According to the above rules, we have $0 \leqslant q_t \leqslant N_i$, $i \in \mathcal{S}$, and obviously, q_t is dependent on the actual switching signal.

Remark 3: In practice, for any subsystem i, the running time of the mode i may change at any time if the switching signal is not known ahead. From (2) and remark 1, $q_{t_{\max}}$(the maximum value of q_t) belongs to $\{n_i, \cdots, N_i\}$. Namely, the maximum update of the controller for every mode-running interval can dynamically adjust according to the actual switching signal of the system based on the design of q_t.

First, we consider the GUES of the switched nonlinear system (3) with mode-dependent hybrid DT switching. By constructing suitable SMDLF, a stability criterion is provided in the following lemma.

Lemma 1: Consider the switched nonlinear system:

$$\dot{x}(t) = f_{\sigma(t)}(x(t)). \tag{3}$$

For given positive integers n_i and N_i satisfying $n_i \leqslant N_i$, and constants $\alpha_i < 0, 0 < \beta_i \leqslant 1, \mu_i > 1, m > 0$ satisfying $\mu_i \beta_i^{n_i} > 1, i \in \mathcal{S}$, if there exists positive definite functions $V_{\sigma(t)} : \mathbb{R}^n \to \mathbb{R}$, and two class of $\mathcal{K}_\infty$ functions κ_1 and κ_2 such that $\forall (i,j) \in \mathcal{S} \times \mathcal{S}$, $i \neq j$, $\phi_i \in \mathbb{Z}_{[0,N_i]}$, $\psi_i \in \mathbb{Z}_{[1,N_i]}$,

$$\kappa_1(\|x(t)\|) \leqslant V_i(x(t), \phi_i) \leqslant \kappa_2(\|x(t)\|), \tag{4}$$

$$\dot{V}_i(x(t), \phi_i) \leqslant \alpha_i V_i(x(t), \phi_i), \tag{5}$$

$$V_i(x(t_k + \psi_i m), \psi_i) \leqslant \beta_i V_i(x(t_k + \psi_i m), \psi_i - 1), \tag{6}$$

$\forall \sigma(t_k) = i \neq j = \sigma(t_{k-1})$, for $t_k - t_{k-1} \geqslant N_j m$

$$V_i(x(t_k), 0) \leqslant \mu_i V_j(x(t_k), N_j), \tag{7}$$

$\forall \sigma(t_k) = i \neq j = \sigma(t_{k-1})$, for $t_k - t_{k-1} < N_j m$

$$V_i(x(t_k), 0) \leqslant \mu_i V_j\left(x(t_k), \left\lfloor \frac{t_k - t_{k-1}}{m} \right\rfloor\right), \tag{8}$$

then, for any mode-dependent hybrid DT switching signal satisfying

$$\tau_{ai} > \tau_{ai}^* = -\frac{\ln \mu_i \beta_i^{n_i}}{\alpha_i}, \tag{9}$$

the system (3) is GUES.

Proof. Choose the following SMDLF for system (3):

$$V_i(x(t), t) = V_i(x(t), q_t), \tag{10}$$

where q_t is defined in (2).

First, when $t \in [t_k, t_k + n_i m)$, q_t is $\lfloor \frac{t-t_k}{m} \rfloor$. In terms of (5), one has

$$\begin{aligned} &\dot{V}_i(x(t), t) - \alpha_i V_i(x(t), t) \\ =&\dot{V}_i\left(x(t), \left\lfloor \frac{t-t_k}{m} \right\rfloor\right) - \alpha_i V_i\left(x(t), \left\lfloor \frac{t-t_k}{m} \right\rfloor\right) \leqslant 0. \end{aligned}$$

In addition, when $t \in [t_k + n_i m, t_k + N_i m)$ and $[t_k + N_i m, t_{k+1})$, q_t is $\left\lfloor \frac{t-(t_k+n_i m)}{m} \right\rfloor + n_i$ and N_i, respectively. Similarly, $\dot{V}_i(x(t), t) - \alpha_i V_i(x(t), t) \leqslant 0$ holds according to (5). Therefore, when $t \in [t_k + \zeta_i m, t_k + (\zeta_i + 1)m)$, $\zeta_i \in \mathbb{Z}_{[0, N_i - 1]}$ and $t \in [t_k + N_i m, t_{k+1})$, we can conclude that

$$\dot{V}_i(x(t), t) \leqslant \alpha_i V_i(x(t), t) \tag{11}$$

always holds for $\forall i \in \mathcal{S}$. Next, when $t = t_k + \psi_i m = t_k^{\psi_i}$, $\psi_i \in \mathbb{Z}_{[1, N_i]}$, condition (6) expresses that

$$V_i(x(t_k^{\psi_i}), t_k^{\psi_i}) \leqslant \beta_i V_i(x(t_k^{\psi_i^-}), t_k^{\psi_i^-}), \tag{12}$$

that is to say, $V_i(x(t), t)$ is discontinuous and decreasing at those time points for $\forall i \in \mathcal{S}$.

On the other hand, consider that the mode j switches to mode i at time instant t_k, and subsystem j is activated in $[t_{k-1}, t_k)$. Denote H_{k-1} as the length of the interval $[t_{k-1}, t_k)$. Then, when $0 < H_{k-1} \leqslant N_j m$, (8) implies that $\forall (i,j) \in \mathcal{S} \times \mathcal{S}$, $i \neq j$,

$$V_i(x(t_k), 0) \leqslant \mu_i V_j\left(x(t_k), \left\lfloor \frac{H_{k-1}}{m} \right\rfloor\right), \tag{13}$$

and (7) indicates that

$$V_i(x(t_k), 0) \leqslant \mu_i V_j(x(t_k), N_j). \tag{14}$$

Hence, combining the definition of q_t, SMDLF (10), (13), and (14), we can derive that for $\sigma(t_k^-) = j \in \mathcal{S}$, $\sigma(t_k) = i \in \mathcal{S}$, $i \neq j$,

$$V_{\sigma(t_k)}(x(t_k), t_k) \leqslant \mu_{\sigma(t_k)} V_{\sigma(t_k^-)}(x(t_k^-), t_k^-). \tag{15}$$

From (11) and (12), we have

$$\begin{aligned} &V_{\sigma(t_k)}(x(t_{k+1}^-), t_{k+1}^-) \\ \leqslant& e^{\alpha_{\sigma(t_k)}(t_{k+1} - t_k^{N_{\sigma(t_k)}})} V_{\sigma(t_k)}(x(t_k^{N_{\sigma(t_k)}}), t_k^{N_{\sigma(t_k)}}) \\ \leqslant& \cdots \\ \leqslant& e^{\alpha_{\sigma(t_k)}(t_{k+1} - t_k)} \beta_{\sigma(t_k)}^{N_{\sigma(t_k)}} V_{\sigma(t_k)}(x(t_k), t_k). \end{aligned} \tag{16}$$

Then, according to Assumption 1, (15) and (16), one can further deduce that

$$\begin{aligned} &V_{\sigma(t_k)}(x(t), t) \\ \leqslant& e^{\alpha_{\sigma(t_k)}(t - t_k)} \beta_{\sigma(t_k)}^{N_{\sigma(t_k)}} \mu_{\sigma(t_k)} V_{\sigma(t_k^-)}(x(t_k), t_k) \\ \leqslant& \cdots \\ \leqslant& e^{\alpha_{\sigma(t_k)}(t - t_k)} \ldots e^{\alpha_{\sigma(t_0)}(t_1 - t_0)} \mu_{\sigma(t_k)} \cdots \mu_{\sigma(t_1)} \end{aligned}$$

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$$
\begin{aligned}
&\times \prod_{\substack{i=1,\sigma(t_s)=i,\\ H_s\geqslant N_i m}}^{S} \beta_i^{N_i} \prod_{\substack{i=1,\sigma(t_s)=i,\\ (N_i-1)m\leqslant H_s<N_i m}}^{S} \beta_i^{N_i-1} \cdots \\
&\times \prod_{\substack{i=1,\sigma(t_s)=i,\\ n_i m\leqslant H_s<(n_i+1)m}}^{S} \beta_i^{n_i} V_{\sigma(t_0)}(x(t_0),t_0) \\
\leqslant &\exp\left\{\sum_{i=1}^{S}\alpha_i\mathcal{R}_i(t_0,t)\right\}\prod_{i=1}^{S}(\mu_i\beta_i^{n_i})^{\mathcal{N}_{\sigma i}(t_0,t)} \\
&\times V_{\sigma(t_0)}(x(t_0),t_0). \qquad (17)
\end{aligned}
$$

On the basis of Assumption 1, $\mu_i\beta_i^{n_i} > 1$, and (17), the following inequality holds

$$
\begin{aligned}
&V_{\sigma(t)}(x(t),t) \\
\leqslant &\exp\left\{\sum_{i=1}^{S}\mathcal{N}_{0i}\ln(\mu_i\beta_i^{n_i}) + \sum_{i=1}^{S}\left(\alpha_i + \frac{\ln(\mu_i\beta_i^{n_i})}{\tau_{ai}}\right)\right. \\
&\left.\times \mathcal{R}_i(t_0,t)\right\} V_{\sigma(t_0)}(x(t_0),t_0). \qquad (18)
\end{aligned}
$$

Thus, if the switching signal satisfies (9) for $i \in \mathcal{S}, V_{\sigma(t)}(x(t),t)$ converges to 0 as $t\to\infty$. Then, according to (4), we can conclude that the system (3) is GUES.

Remark 4: The methods proposed in [34], [35] require the actual dwell time of each subsystem in advance, i.e. the switching signal is known ahead, and then divide the mode-running interval into G_i segments, so as to design the controller. Different from the approaches in [34], [35], our method can be applied to the case of unknown switching signal. In addition, if the running time of a subsystem is very short, it is still divided into G_i parts in [34], [35], which leads to frequent controller updates. It also means that the update time is varying for different subsections if each mode-running interval is different. The strategy proposed in this paper updates the controller every m, making it easier to implement. It also enables dynamically adjustment within mode-running intervals according to actual switching signal if the switching signal is not known ahead. It is clear that our method is more flexible and applicable. Note if $N_i = n_i$ and the switching signal is pre-known, the SMDLF method in this paper is degenerated to the MDLF approach in [34], [35].

A. Stability Analysis

In this subsection, the stability analysis of the switched linear neutral system (1) is addressed by exploiting time-scheduled control scheme and constructing novel mode-dependent SMDLF.

For system (1), the time-dependent state feedback controller is expressed as:

$$u(t) = K_{\sigma(t)}(q_t)x(t),$$

where $K_{\sigma(t)}(q_t)$ is the controller gain to be designed, and q_t is defined in (2). Therefore, the closed-loop system is characterized by

$$
\begin{aligned}
\dot{x}(t) - &B_{\sigma(t)}\dot{x}(t-h(t)) \\
&= \bar{A}_{\sigma(t)}(q_t)x(t) + C_{\sigma(t)}x(t-d(t)), \qquad (19)
\end{aligned}
$$

where $\bar{A}_{\sigma(t)}(q_t) = A_{\sigma(t)} + D_{\sigma(t)}K_{\sigma(t)}(q_t)$.

Next, some sufficient conditions, which guarantee switched linear neural system (1) is GUES, are derived in Theorem 1.

Theorem 1: Consider the switched linear neutral systems (1). For given positive integers n_i and N_i satisfying $n_i \leqslant N_i$, and constants $\alpha_i < 0, 0 < \beta_i \leqslant 1, \mu_i > 1, m > 0$ satisfying $\mu_i\beta_i^{n_i} > 1, i \in \mathcal{S}$, if there exists positive definite matrices $P_i(\phi_i), E_i(\phi_i), F_i(\phi_i)$, matrix G_i, such that $\forall (i,j) \in \mathcal{S}\times\mathcal{S}$, $i \neq j$, $\phi_i \in \mathbb{Z}_{[0,N_i]}, \psi_i \in \mathbb{Z}_{[1,N_i]}, \varphi_i \in \mathbb{Z}_{[0,N_i-n_i]}$,

$$\Phi_i(\phi_i) \leqslant 0, \qquad (20)$$

$$
\begin{cases}
P_i(\psi_i) \leqslant \beta_i P_i(\psi_i - 1),\\
E_i(\psi_i) \leqslant \beta_i E_i(\psi_i - 1),\\
F_i(\psi_i) \leqslant \beta_i F_i(\psi_i - 1),
\end{cases} \qquad (21)
$$

$$
\begin{cases}
P_i(0) \leqslant \mu_i P_j(\varphi_j + n_j),\\
E_i(0) \leqslant \mu_i e^{\alpha_j \hat{d}} E_j(\varphi_j + n_j),\\
F_i(0) \leqslant \mu_i e^{\alpha_j \hat{h}} F_j(\varphi_j + n_j),
\end{cases} \qquad (22)
$$

where

$$
\Phi_i(\phi_i) = \begin{bmatrix}
\Phi_{i11}(\phi_i) & \Phi_{i12}(\phi_i) & G_i^T B_i & G_i^T C_i\\
* & \Phi_{i22}(\phi_i) & G_i^T B_i & G_i^T C_i\\
* & * & -\tilde{h}_i F_i(\phi_i) & 0\\
* & * & * & -\tilde{d}_i E_i(\phi_i)
\end{bmatrix},
$$

$$
\begin{aligned}
&\Phi_{i11}(\phi_i) = E_i(\phi_i) - \alpha_i P_i(\phi_i) + G_i^T\bar{A}_i(\phi_i) + \bar{A}_i^T(\phi_i)G_i,\\
&\Phi_{i12}(\phi_i) = P_i(\phi_i) - G_i^T + \bar{A}_i^T(\phi_i)G_i,\\
&\Phi_{i22}(\phi_i) = F_i(\phi_i) - G_i^T - G_i,
\end{aligned}
$$

then, for any mode-dependent hybrid DT switching signal satisfying (9), the system (1) is GUES.

Proof. For switched linear neural system (1), we construct the following SMDLF

$$
\begin{aligned}
&V_i(x(t),t)\\
=&x^T(t)P_i(q_t)x(t) + \int_{t-d(t)}^{t} x^T(s)e^{\alpha_i(t-s)}E_i(q_t)x(s)ds\\
&+\int_{t-h(t)}^{t} \dot{x}^T(s)e^{\alpha_i(t-s)}F_i(q_t)\dot{x}(s)ds. \qquad (23)
\end{aligned}
$$

First, we will proof that condition (20) guarantees (5). For $t\in[t_k, t_k+n_i m)$, we can easily know that q_t is $\lfloor\frac{t-t_k}{m}\rfloor$ from (2), then

$$
\begin{aligned}
&\dot{V}_i(x(t),q_t) - \alpha_i V_i(x(t),q_t)\\
\leqslant &x^T(t)P_i(q_t)\dot{x}(t) + \dot{x}^T(t)P_i(q_t)x(t)\\
&+ x^T(t)[E_i(q_t) - \alpha_i P_i(q_t)]x(t) + \dot{x}^T F_i(q_t)\dot{x}(t)\\
&- \tilde{d}_i x^T(t-d(t))E_i(q_t)x(t-d(t))\\
&- \tilde{h}_i\dot{x}^T(t-h(t))F_i(q_t)\dot{x}(t-h(t)), \qquad (24)
\end{aligned}
$$

where $\tilde{d}_i = e^{\alpha_i\hat{d}}(1-\bar{d}), \tilde{h}_i = e^{\alpha_i\hat{h}}(1-\bar{h})$.

Furthermore, for any slack variables G_i, one has

$$
\begin{aligned}
&-2\big(x^T(t)G_i^T + \dot{x}^T(t)G_i^T\big)\Big(\dot{x}(t) - B_i\dot{x}(t-h(t))\\
&-\bar{A}_i(q_t)x(t) - C_i x(t-d(t))\Big) = 0. \qquad (25)
\end{aligned}
$$

Combining (24) and (25), we can obtain

$$\dot{V}_i(x(t),q_t) - \alpha_i V_i(x(t),q_t) \leqslant \delta^T(t)\Phi_i(q_t)\delta(t), \qquad (26)$$

where $\delta(t)=[x^T(t),\dot{x}^T(t),\dot{x}^T(t-h(t)),x^T(t-d(t))]^T$,

$$\Phi_i(q_t)=\begin{bmatrix}\Phi_{i11}(q_t) & \Phi_{i12}(q_t) & G_i^TB_i & G_i^TC_i\\ * & \Phi_{i22}(q_t) & G_i^TB_i & G_i^TC_i\\ * & * & -\tilde{h}_iF_i(q_t) & q_t\\ * & * & * & -\tilde{d}_iE_i(q_t)\end{bmatrix},$$

$$\begin{aligned}\Phi_{i11}(q_t)=&E_i(q_t)-\alpha_iP_i(q_t)+G_i^T\bar{A}_i(q_t)+\bar{A}_i^T(q_t)G_i,\\ \Phi_{i12}(q_t)=&P_i(q_t)-G_i^T+\bar{A}_i^T(q_t)G_i,\\ \Phi_{i22}(q_t)=&F_i(q_t)-G_i^T-G_i.\end{aligned}$$

From the condition (20), one obtains $\dot{V}_i(x(t),q_t)\leqslant\alpha_iV_i(x(t),q_t)$ for $t\in[t_k,t_k+n_im)$. Moreover, for any mode-running interval $[t_k,t_{k+1})$, if $N_im>t_{k+1}-t_k$, q_t is $\left\lfloor\frac{t-(t_k+n_im)}{m}\right\rfloor+n_i$ when $t\in[t_k+n_im,t_{k+1})$. Obviously, (5) can also be ensured by (20). In addition, if $N_im<t_{k+1}-t_k$, q_t is N_i when $t\in[t_k+N_im,t_{k+1})$, then (5) is also achieved if (20) holds.

Next, at the discontinuous points in interval $[t_k,t_{k+1})$ (excluding switching instant t_k), that is to say, when $t=t_k+\psi_im=t_k^{\psi_i},\psi_i\in\mathbb{Z}_{[1,N_i]}$, we can derive that

$$\begin{aligned}&V_i(x(t_k+\psi_im),\psi_i)-\beta_iV_i(x(t_k+\psi_im),\psi_i-1)\\ =&x^T(t_k^{\psi_i})(P_i(\psi_i)-\beta_iP_i(\psi_i-1))x(t_k^{\psi_i})\\ &+\int_{t_k^{\psi_i}-d(t_k^{\psi_i})}^{t_k^{\psi_i}}x^T(s)e^{\alpha_i(t_k^{\psi_i}-s)}(E_i(\psi_i)\\ &-\beta_iE_i(\psi_i-1))x(s)ds\\ &+\int_{t_k^{\psi_i}-h(t_k^{\psi_i})}^{t_k^{\psi_i}}\dot{x}^T(s)e^{\alpha_i(t_k^{\psi_i}-s)}(F_i(\psi_i)\\ &-\beta_iF_i(\psi_i-1))\dot{x}(s)ds.\end{aligned}$$

Obviously, (6) is satisfied if the condition (21) holds. Meanwhile, at the switching point, conditions (7) and (8) are established in terms of (22). Then, it is clear that the inequality (18) is guaranteed. From (23), it yields that

$$V_{\sigma(t)}(x(t),t)\geqslant a\|x(t)\|^2, \tag{27}$$

and

$$\begin{aligned}V_{\sigma(t_0)}(x(t_0),t_0)\leqslant&\max_{\substack{\phi_i\in\mathbb{Z}_{[0,N_i]},\\ i\in\mathcal{S}}}\lambda_{\max}(P_i(\phi_i))\|\chi\|^2\\ &+\hat{d}\max_{\substack{\phi_i\in\mathbb{Z}_{[0,N_i]},\\ i\in\mathcal{S}}}\lambda_{\max}(E_i(\phi_i))\|\chi\|^2\\ &+\hat{h}\max_{\substack{\phi_i\in\mathbb{Z}_{[0,N_i]},\\ i\in\mathcal{S}}}\lambda_{\max}(F_i(\phi_i))\|\dot{\chi}\|^2\\ \leqslant& b\max\{\|\chi\|,\|\dot{\chi}\|\}^2,\end{aligned} \tag{28}$$

where

$$\begin{aligned}a=&\min_{\substack{\phi_i\in\mathbb{Z}_{[0,N_i]},\\ i\in\mathcal{S}}}\lambda_{\min}(P_i(\phi_i)),\\ b=&\max_{\substack{\phi_i\in\mathbb{Z}_{[0,N_i]},\\ i\in\mathcal{S}}}\lambda_{\max}(P_i(\phi_i))\\ &+\hat{d}\max_{\substack{\phi_i\in\mathbb{Z}_{[0,N_i]},\\ i\in\mathcal{S}}}\lambda_{\max}(E_i(\phi_i))\\ &+\hat{h}\max_{\substack{\phi_i\in\mathbb{Z}_{[0,N_i]},\\ i\in\mathcal{S}}}\lambda_{\max}(F_i(\phi_i)).\end{aligned}$$

Then, combining with (18), (27), (28) and Definition 1, we deduce that the switched linear neutral system (1) is GUES under the switching signal (9).

Remark 5: According to the constructed SMDLF (23), the derived switching signal (9) is not only dependent on the increase coefficient μ_i and the decay rate α_i in traditional time-independent case, but also the decay coefficient β_i at the discontinuous points in (t_k,t_{k+1}) and the mode-dependent minimum update n_i. It is worth noting that tighter bounds on MDADT can be obtained by decreasing β_i or increasing n_i, which is well illustrated in the simulation section.

B. Time-dependent Feedback Controller Design

Based on Theorem 1, a set of state feedback controllers are designed for the continuous-time switched linear neutral system (1) in the following theorem.

Theorem 2: Consider the switched linear neutral systems (1). For given positive integers n_i and N_i satisfying $n_i\leqslant N_i$, and constants $\alpha_i<0, 0<\beta_i\leqslant1,\mu_i>1,m>0$ satisfying $\mu_i\beta_i^{n_i}>1,i\in\mathcal{S}$, if there exists positive definite matrices $\tilde{P}_i(\phi_i),\tilde{E}_i(\phi_i),\tilde{F}_i(\phi_i)$, matrices $\tilde{G}_i,L_i(\phi_i)$, such that $\forall(i,j)\in\mathcal{S}\times\mathcal{S}$, $i\neq j$, $\phi_i\in\mathbb{Z}_{[0,N_i]},\psi_i\in\mathbb{Z}_{[1,N_i]},\varphi_i\in\mathbb{Z}_{[0,N_i-n_i]}$,

$$\tilde{\Phi}_i(\phi_i)\leqslant0, \tag{29}$$

$$\begin{cases}\tilde{P}_i(\psi_i)\leqslant\beta_i\tilde{P}_i(\psi_i-1),\\ \tilde{E}_i(\psi_i)\leqslant\beta_i\tilde{E}_i(\psi_i-1),\\ \tilde{F}_i(\psi_i)\leqslant\beta_i\tilde{F}_i(\psi_i-1),\end{cases} \tag{30}$$

$$\begin{cases}\tilde{P}_i(0)\leqslant\mu_i\tilde{P}_j(\varphi_j+n_j),\\ \tilde{E}_i(0)\leqslant\mu_ie^{\alpha_j\hat{d}}\tilde{E}_j(\varphi_j+n_j),\\ \tilde{F}_i(0)\leqslant\mu_ie^{\alpha_j\hat{h}}\tilde{F}_j(\varphi_j+n_j),\end{cases} \tag{31}$$

where

$$\tilde{\Phi}_i(\phi_i)\begin{bmatrix}\tilde{\Phi}_{i11}(\phi_i) & \tilde{\Phi}_{i12}(\phi_i) & B_i\tilde{G}_i & C_i\tilde{G}_i\\ * & \tilde{\Phi}_{i22}(\phi_i) & B_i\tilde{G}_i & C_i\tilde{G}_i\\ * & * & -\tilde{h}_i\tilde{F}_i(\phi_i) & 0\\ * & * & * & -\tilde{d}_i\tilde{E}_i(\phi_i)\end{bmatrix},$$

$$\begin{aligned}&\tilde{\Phi}_{i11}(\phi_i)=\tilde{E}_i(\phi_i)-\alpha_i\tilde{P}_i(\phi_i)+A_i\tilde{G}_i+D_iL_i(\phi_i)\\ &\quad+\tilde{G}_i^TA_i^T+L_i^T(\phi_i)D_i^T,\\ &\tilde{\Phi}_{i12}(\phi_i)=\tilde{P}_i(\phi_i)-\tilde{G}_i+\tilde{G}_i^TA_i^T+L_i^T(\phi_i)D_i^T,\\ &\tilde{\Phi}_{i22}(\phi_i)=\tilde{F}_i(\phi_i)-\tilde{G}_i^T-\tilde{G}_i,\end{aligned}$$

then, for any mode-dependent hybrid DT switching signal satisfying (9), the system (1) is GUES with the controller gain $K_i(\phi_i)=L_i(\phi_i)\tilde{G}_j^{-1}$.

Proof. Denote $\tilde{G}_i=G_i^{-1},K_i(\phi_i)\tilde{G}_i=L_i(\phi_i),\tilde{P}_i(\phi_i)=\tilde{G}_i^TP_i(\phi_i)\tilde{G}_i,\tilde{E}_i(\phi_i)=\tilde{G}_i^TE_i(\phi_i)\tilde{G}_i,\tilde{F}_i(\phi_i)=\tilde{G}_i^TF_i(\phi_i)\tilde{G}_i,\Upsilon=\mathrm{diag}\{\tilde{G}_i,\tilde{G}_i,\tilde{G}_i,\tilde{G}_i\}$. Pre- and post-multiplying inequality (20) with Υ^T and Υ, which implies the condition (29). By the similarly process, conditions (30) and (31) can guarantee (21) and (22), respectively. Based on Theorem 1, system (1) is GUES, and the controller gain can be derived by $K_i(\phi_i)=L_i(\phi_i)\tilde{G}_i^{-1}$.

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It is clear that n_i and N_i are both mode-dependent, however, they will be mode-independent if we consider hybrid DT switching in [36]. Then, the following corollary can be derived in a similar way, which considers the GUES of switched linear neutral systems (1) with mode-independent time-scheduled control scheme and SMDLF.

Corollary 1: Consider the switched linear neutral systems (1). For given positive integers n and N satisfying $n \leqslant N$, and constants $\alpha < 0, 0 < \beta \leqslant 1, \mu > 1, m > 0$ satisfying $\mu\beta^n > 1$, if there exists positive definite matrices $\tilde{P}_i(\phi), \tilde{E}_i(\phi), \tilde{F}_i(\phi)$, matrices $\tilde{G}_i, L_i(\phi)$, such that $\forall (i,j) \in \mathcal{S} \times \mathcal{S},\ i \neq j,\ \phi \in \mathbb{Z}_{[0,N]}, \psi \in \mathbb{Z}_{[1,N]}, \varphi \in \mathbb{Z}_{[0,N-n]}$,

$$\tilde{\Phi}_i(\phi) \leqslant 0, \tag{32}$$

$$\begin{cases} \tilde{P}_i(\psi) \leqslant \beta \tilde{P}_i(\psi - 1), \\ \tilde{E}_i(\psi) \leqslant \beta \tilde{E}_i(\psi - 1), \\ \tilde{F}_i(\psi) \leqslant \beta \tilde{F}_i(\psi - 1), \end{cases} \tag{33}$$

$$\begin{cases} \tilde{P}_i(0) \leqslant \mu \tilde{P}_j(\varphi + n), \\ \tilde{E}_i(0) \leqslant \mu e^{\alpha \hat{d}} \tilde{E}_j(\varphi + n), \\ \tilde{F}_i(0) \leqslant \mu e^{\alpha \hat{h}} \tilde{F}_j(\varphi + n), \end{cases} \tag{34}$$

where

$$\tilde{\Phi}_i(\phi) = \begin{bmatrix} \tilde{\Phi}_{i11}(\phi) & \tilde{\Phi}_{i12}(\phi) & B_i\tilde{G}_i & C_i\tilde{G}_i \\ * & \tilde{\Phi}_{i22}(\phi) & B_i\tilde{G}_i & C_i\tilde{G}_i \\ * & * & -\tilde{h}\tilde{F}_i(\phi) & 0 \\ * & * & * & -\tilde{d}\tilde{E}_i(\phi) \end{bmatrix},$$

$$\begin{aligned} \tilde{\Phi}_{i11}(\phi) &= \tilde{E}_i(\phi) - \alpha \tilde{P}_i(\phi) + A_i\tilde{G}_i + D_iL_i(\phi) \\ &\quad + \tilde{G}_i^T A_i^T + L_i^T(\phi) D_i^T, \\ \tilde{\Phi}_{i12}(\phi) &= \tilde{P}_i(\phi) - \tilde{G}_i + \tilde{G}_i^T A_i^T + L_i^T(\phi) D_i^T, \\ \tilde{\Phi}_{i22}(\phi) &= \tilde{F}_i(\phi) - \tilde{G}_i^T - \tilde{G}_i, \end{aligned}$$

then, for any switching signal satisfying

$$\tau_a > \tau_a^* = -\frac{\ln \mu\beta^n}{\alpha}, \tag{35}$$

the system (3) is GUES with the controller gain $K_i(\phi) = L_i(\phi)\tilde{G}_i^{-1}$.

When consider the time-independent case, the controller will not be updated over time during each mode operation, and we have to utilize the traditional MLF method to discuss the state feedback control for switched linear neutral systems.

Corollary 2: Consider the switched linear neutral systems (1). For given constants $\alpha_i < 0, \mu_i > 1$, if there exists positive definite matrices $\tilde{P}_i, \tilde{E}_i, \tilde{F}_i$, matrix $\tilde{G}_i, L_i$, such that $\forall (i,j) \in \mathcal{S} \times \mathcal{S},\ i \neq j$,

$$\tilde{\Phi}_i \leqslant 0, \tag{36}$$

$$\begin{cases} \tilde{P}_i \leqslant \mu_i \tilde{P}_j, \\ \tilde{E}_i \leqslant \mu_i e^{\alpha_j \hat{d}} \tilde{E}_j, \\ \tilde{F}_i \leqslant \mu_i e^{\alpha_j \hat{h}} \tilde{F}_j, \end{cases} \tag{37}$$

where

$$\tilde{\Phi}_i = \begin{bmatrix} \tilde{\Phi}_{i11} & \tilde{\Phi}_{i12} & B_i\tilde{G}_i & C_i\tilde{G}_i \\ * & \tilde{\Phi}_{i22} & B_i\tilde{G}_i & C_i\tilde{G}_i \\ * & * & -\tilde{h}_i\tilde{F}_i & 0 \\ * & * & * & -\tilde{d}_i\tilde{E}_i \end{bmatrix},$$

$$\begin{aligned} \tilde{\Phi}_{i11} &= \tilde{E}_i - \alpha_i \tilde{P}_i + A_i\tilde{G}_i + D_iL_i \\ &\quad + \tilde{G}_i^T A_i^T + L_i^T D_i^T, \\ \tilde{\Phi}_{i12} &= \tilde{P}_i - \tilde{G}_i + \tilde{G}_i^T A_i^T + L_i^T D_i^T, \\ \tilde{\Phi}_{i22} &= \tilde{F}_i - \tilde{G}_i^T - \tilde{G}_i, \end{aligned}$$

then, for any MDADT switching signal satisfying

$$\tau_{ai} > \tau_{ai}^* = -\frac{\ln \mu_i}{\alpha_i}, \tag{38}$$

the system (1) is GUES with the controller gain $K_i = L_i\tilde{G}_i^{-1}$.

IV. An Illustrate Example

In this section, we use an example to illustrate the effectiveness and advantages of the proposed approach in the paper. Here, we consider a water-quality dynamic model for the Nile River [11], [37]. As mentioned in [11], x_1, x_2, and x_3 represent the relative concentration of Algae, Nitrogen group and Phosphate-BOD-DO group, respectively. The system parameters are given as follows:

$$A_1 = \begin{bmatrix} -0.893 & 0.003 & 0 \\ -0.003 & -1.146 & -0.0026 \\ -0.006 & -0.561 & -1.257 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -1.341 & 0.007 & 0 \\ -0.003 & -1.1477 & -0.005 \\ -0.007 & -0.888 & -1.745 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.1 & -2.05 & 0 \\ 0 & -0.021 & 0.01 \\ 0.01 & -0.11 & -0.02 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0.05 & -0.41 & 0.12 \\ 0.1 & 0.02 & 0 \\ -0.15 & 0 & 0.32 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -0.225 & 0 & 0.001 \\ -0.001 & -0.136 & -0.001 \\ -0.002 & -0.041 & -0.233 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} -0.235 & 0 & 0.001 \\ -0.001 & -0.156 & -0.001 \\ -0.002 & -0.041 & -0.363 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.1 \\ 0 & 0.5 \end{bmatrix}, D_2 = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.2 \\ 0 & 0.8 \end{bmatrix},$$

with $h(t) = 0.05\sin t + 0.1, d(t) = 0.1\sin t + 0.2$. Obviously, $\hat{h} = 0.15, \hat{d} = 0.3, \bar{h} = 0.05, \bar{d} = 0.1$. First, we illustrate that the proposed time-scheduled control strategy can provide tighter bounds of MDADT compared with conventional time-independent one. Given $\alpha_1 = \alpha_2 = -1, \mu_1 = 1.8, \mu_2 = 2, m = 0.05, \tau_{d1} = \tau_{d2} = nm, N = 4$, according to (9) and (38), the comparison between Corollary 1 in [13] and Theorem 1 are provided in Table I by choosing different β_i and $n_i, i = 1, 2$. From Table I, we can see that a smaller lower bound on switching signal could be obtained by decreasing β or increasing n. In other words, SMDLF (10) should decrease more at discontinuous points (excluding switching instants) or increase the number of segments that can be divided in the interval $[t_k, t_k + \tau_{di}]$.

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TABLE I
COMPARISON BETWEEN COROLLARY 1 IN [13] AND THEOREM 2

Corollary 1 in [13]	Theorem 2							
	β_1	β_2	$n_1 = n_2 = 1$		$n_1 = n_2 = 2$		$n_1 = n_2 = 3$	
			τ_{a1}^*	τ_{a2}^*	τ_{a1}^*	τ_{a2}^*	τ_{a1}^*	τ_{a2}^*
$\tau_{a1}^* = 0.5878$ $\tau_{a2}^* = 0.6931$	0.96	0.96	0.5470	0.6523	0.5061	0.6115	0.4653	0.5707
	0.92	0.92	0.5044	0.6098	0.4210	0.5264	0.3376	0.4430
	0.88	0.88	0.4600	0.5653	0.3321	0.4375	0.2043	0.3096

TABLE II
COMPARISON BETWEEN COROLLARY 2 AND THEOREM 2

feasible or not feasible (F or NF)		$\alpha_1 = -1.1, \alpha_2 = -1.2, n = 2, m = 0.05$			
μ		$[1.174, 1.224)$	$[1.224, 1.277)$	$[1.277, 1.412)$	$\geqslant 1.412$
Corollary 2		NF	NF	NF	F
	$\beta = 0.94$	NF	NF	F	F
Theorem 2	$\beta = 0.96$	NF	F	F	F
	$\beta = 0.98$	F	F	F	F

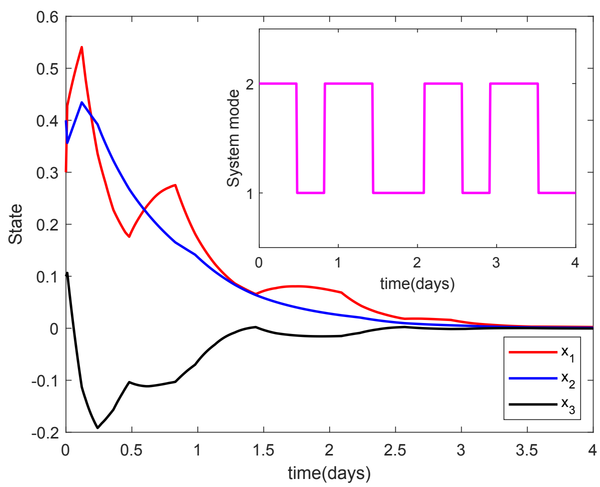

Fig. 2. State responses of the closed-loop system.

Then, set $m = 0.15$, $\tau_{di} = 0.3, N_i = 3, \beta_i = 0.92, i \in \{1, 2\}$, then we have $n_i = 2$ for $i \in \{1, 2\}$ according to the definition of q_t in (2). By solving the inequalities (29)-(31) in Theorem 2, we can obtain the controller gains $K_i(q_t)$ as follows

$$K_1(0) = \begin{bmatrix} -2.1850 & 0.2820 & 0.0607 \\ 0.1190 & -2.3559 & -2.8789 \end{bmatrix},$$

$$K_1(1) = \begin{bmatrix} -1.9403 & 0.2777 & 0.0764 \\ 0.1199 & -1.9711 & -2.4934 \end{bmatrix},$$

$$K_1(2) = \begin{bmatrix} -1.7100 & 0.3156 & 0.0683 \\ 0.1239 & -1.9432 & -2.1979 \end{bmatrix},$$

$$K_1(3) = \begin{bmatrix} -1.7100 & 0.3156 & 0.0683 \\ 0.1239 & -1.9432 & -2.1979 \end{bmatrix},$$

$$K_2(0) = \begin{bmatrix} -1.8740 & 0.0152 & 0.2625 \\ 0.1999 & 0.0922 & -1.6489 \end{bmatrix},$$

$$K_2(1) = \begin{bmatrix} -1.3283 & 0.0321 & 0.2020 \\ 0.1654 & 0.1426 & -1.1269 \end{bmatrix},$$

$$K_2(2) = \begin{bmatrix} -1.6964 & 0.0362 & 0.2601 \\ 0.2068 & -0.0272 & -1.5044 \end{bmatrix},$$

$$K_2(3) = \begin{bmatrix} -0.9992 & 0.0164 & 0.1559 \\ 0.1225 & 0.3821 & -0.7002 \end{bmatrix},$$

and MDADT $\tau_{a1}^* = 0.4210, \tau_{a2}^* = 0.5264$. Here, we will explain in detail how the controller gains are applied in the switched system according to the switching signal. Initially, subsystem 2 runs within the time interval $[0, 0.48)$. According to the control scheduler q_t (2), controller gains $K_2(0), K_2(1), K_2(2), K_2(3)$ are applied sequentially during the running interval $[0, 0.48)$. Immediately afterwards, subsystem 1 runs for 0.34, and controller gains $K_1(0), K_1(1), K_1(2)$ are applied successively. It can be seen that the application of feedback control gain is related to the actual system switching and can be dynamically adjusted according to the running time of each subsystem. Set the initial condition of system (1) as $x(0) = [0.3 \quad 0.4 \quad 0.1]^T$. Fig. 2 shows the state trajectories of the closed-loop system. Clearly, the underlying system is stable with the derived time-dependent state feedback controller gains.

Next, consider $\alpha_1 = -1.1, \alpha_2 = -1.2, \mu_1 = \mu_2 = \mu, m = 0.05$. By solving conditions (36)-(37) in Corollary 2 and (29)-(31) in Theorem 2 respectively, the values of μ to guarantee the feasibility of Corollary 2 and Theorem 2 are shown in Table II. Obviously, a larger feasibility region can be derived by the proposed time-scheduled control strategy comparing with the time-independent case, which reveals the developed method is less conservative than traditional time-independent approach.

Furthermore, we will provide a table to show that our results are less conservative than [34]. First, the number of segments divided within each mode-running interval between our proposed method and [34] remains the same, i.e. $G_i = N_i + 1$(G_i represents the number of segments divided in any mode-running interval of mode i in [34]), $i \in \{1, 2\}$. Choosing $\alpha_1 = -0.2, \alpha_2 = -0.1, \mu_i = \mu, m = 0.05, G_i = 3, i \in \{1, 2\}$. Then, minimum allowable values of μ are recorded in Table III by the approach in [34] and our method. Clearly, our results provide a larger feasible domain than that in [34], which shows the merits and validity of our obtained theory.

V. CONCLUSIONS

This paper aims to develop a new control approach to investigate the GUES of continuous-time switched linear neu-

TABLE III
COMPARISON BETWEEN THE METHOD PROPOSED IN [34] AND THEOREM 2

$\beta_1 = \beta_2 = \beta$	The minimum allowable value of $\mu(\mu_{\min})$	
	The method proposed in [34]	Theorem 2
0.70	2.09	1.45
0.80	1.60	1.27
0.90	1.26	1.13

tral systems. Based on mode-dependent hybrid dwell time switching, we present a time-scheduled control scheme and further construct an improved MDLF, which is suitable for unknown switching signals. By exploiting SMDLF method and some inequality techniques, we deal with the stability analysis of switched nonlinear system and switched linear neutral systems, respectively. Furthermore, new criteria are derived to design a set of state feedback controllers for the GUES of the underlying system. In the end, we give some comparative simulation results to show the merits and validity of the obtained theoretical results.

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