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Dynamic Surface Control of Nonlinear Systems

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Abstract

A new method is proposed for designing stable controllers with arbitrarily small tracking error for uncertain, mismatched nonlinear systems. This method is another "synthetic input technique", similar to backstepping and multiple surface control methods, but with an important addition, $r-1$ low pass filters are included in the design, where r is the relative degree of the output to be controlled. It is shown that these low pass filters allow a design where the model is not differentiated, thus ending the complexity arising due to the "explosion of terms" that has made other methods difficult to implement in practice. This paper presents the method and proves stability via a Composite Lyapunov method.

1. Introduction

Tremendous strides have been made in the past twenty five years in the area of controller design for nonlinear systems. Variable Structure Control or Sliding mode Control, [5],[23] uses a discontinuous control structure to guarantee perfect tracking for a class of systems satisfying "matching" conditions. Retaining the concept of an "attractive" surface but eliminating the control discontinuities, the method of sliding control [21] is currently being applied in many different applications.

The paper of Hunt, Su and Meyer [11] initiated a surge of interest in feedback linearization and more generally in the application of differential geometry to nonlinear control [12] [18] [19].

Recently the area of robust nonlinear control has received a great deal of attention in the literature. Many methods employ a Lyapunov Synthesis approach where the controlled variable is chosen to make the time derivative of a Lyapunov function candidate negative definite. Corless and Leitmann [3] have applied this approach to open loop stable, mismatched nonlinear systems.

A design methodology that has received a great deal of interest recently is "Integrator Backstepping". The recent book by Kristic, Kanellakopoulous, and Kokotovic [16] develops the backstepping approach to the point of a step by step design procedure. Integrator Backstepping has the problem of an "explosion of terms". The following example illustrates the backstepping approach as well as a difficulty that this paper seeks to solve.

$$\begin{aligned}\dot{x}_1 &= f_1(x_1) + x_2 + \Delta f_1(x_1)\\ \dot{x}_2 &= u\end{aligned}$$

where $f_1, \Delta f_1(x_1)$ are non-Lipschitz nonlinearities. The function $f_1(x_1)$ is assumed completely known, while Δf_1 is uncertain. However it is bounded by a known nonlinearity, $\rho_1(x_1)$. The goal is to regulate the system in the presence of mismatched non-Lipschitz uncertainty. Let

$$\begin{aligned}z_1 &= x_1\\ \dot{z}_1 &= f_1 + x_2 + \Delta f_1(x_1)\\ z_2 &:= x_2 - x_{2d}\\ \Rightarrow \dot{z}_1 &= f_1 + z_2 + x_{2d} + \Delta f_1(x_1)\\ \Rightarrow \dot{z}_2 &= u - \dot{x}_{2d}\end{aligned}$$

Using the idea of nonlinear damping [16] in an Integrator Backstepping controller, consider the following choice of x_{2d}, u, for some $\epsilon > 0$,

$$\begin{aligned}x_{2d} &= -f_1(x_1) - z_1\frac{\rho_1^2(x_1)}{2\epsilon} - K_1 z_1\\ \phi_1 &= \frac{\partial f_1}{\partial x_1} + \frac{z_1\rho_1}{2\epsilon}\frac{\partial \rho_1(x_1)}{\partial x_1} + \frac{\rho_1^2(x_1)}{2\epsilon} + K_1\\ u &= (x_2 + f_1)\phi_1 - \frac{\phi_1^2\rho_1^2 z_2}{2\epsilon} - K_2 z_2 - z_1\end{aligned}$$

Consequently, the closed loop system is governed by

$$\begin{aligned}\dot{z}_1 &= z_2 - K_1 z_1 - \frac{z_1\rho_1^2(x_1)}{2\epsilon} + \Delta f_1(x_1)\\ \dot{z}_2 &= -K_2 z_2 - z_1 + \phi_1\Delta f_1(x_1) - \frac{z_2\rho_1^2\phi_1^2}{2\epsilon}\end{aligned}$$

We define a Lyapunov function candidate,

$$V = \frac{z_1^2}{2} + \frac{z_2^2}{2}$$

$$\Rightarrow \dot{V} = z_1(z_2 - Kz_1 - \frac{z_1\rho_1^2(x_1)}{2\epsilon} + \Delta f_1(x_1)) + z_2(-z_1 - Kz_2 + \phi_1\Delta f_1(x_1) - \frac{z_2\rho_1^2\phi_1^2}{2\epsilon})$$

From Young's Inequality,

$$\frac{z_1^2\rho_1^2}{2\epsilon} + \frac{\epsilon}{2} \geq |z_1|\rho_1 \geq z_1 f_1(x_1)$$

$$\frac{z_2^2\phi_1^2\rho_1^2}{2\epsilon} + \frac{\epsilon}{2} \geq z_2\phi_1 f_1(x_1)$$

As a result of the above control law, $\dot{V} \leq -2KV + \epsilon$ which results in ultimately uniformly bounded regulation. Since ϵ is arbitrary, the ultimate error bound in regulation can be made arbitrarily small. In the above example, we see the beginning of the "complexity due to the explosion of terms" arising from the calculation of $\dot{x}_{2d}$ as well as the presence of uncertainty in $\dot{x}_{2d}$. The requirement on the nonlinear functions, f_1 and ρ_1 is very clear - they should be C^1. For a nonlinear system in strict feedback form with a relative degree, n, the requirement on f_1, ρ_1 is more stringent - they should be C^n functions.

A procedure similar to backstepping, called Multiple Surface Sliding control (MSS) [8] [27] was developed to simplify the controller design of systems where model differentiation was difficult. (Earlier work on variable structure controller based on a similar hierarchical and block control principle can also be found in [24], [4], [17] and [25].) Let us apply MSS control to the previous example. Let

$$\begin{aligned}
S_1 &:= z_1 = x_1 \\
\Rightarrow \dot{S}_1 &= f_1 + x_2 + \Delta f_1(S_1) \\
S_2 &:= z_2 = x_2 - x_{2d} \\
\Rightarrow \dot{S}_1 &= f_1 + S_2 + x_{2d} + \Delta f_1(S_1)
\end{aligned}$$

We now choose x_{2d} to make $S_1\dot{S}_1 < 0$ assuming S_2 will be driven to zero. A reasonable choice for x_{2d} is

$$x_{2d} = -f_1 - KS_1 - \rho_1 sgn(S_1)$$

The dynamics of S_1 is given by:

$$\begin{aligned}
\dot{S}_1 &= S_2 - KS_1 + \Delta f_1(S_1) - \rho_1 sgn(S_1) \\
\dot{S}_2 &= u - \dot{x}_{2d} := -KS_2
\end{aligned}$$

Thus, $u = \dot{x}_{2d} - KS_2$. As a result of the control law that is chosen, let

$$\begin{aligned}
V &= \frac{S_1^2 + S_2^2}{2} \\
\dot{V} &= S_1\dot{S}_1 + S_2\dot{S}_2 \\
&\leq -K(S_1^2 + S_2^2) + S_1 S_2
\end{aligned}$$

which can be made negative definite for a choice of $K > \frac{1}{2}$. The difficulty with this scheme is with obtaining $\dot{x}_{2d}$, since $\dot{S}_1$ involves $\Delta f_1(S_1)$, which is uncertain. This problem has been dealt with in an ad hoc way by numerical differentiation, i.e.

$$\dot{x}_{2d}(n) \approx \frac{x_{2d}(n) - x_{2d}(n-1)}{\Delta T}$$

Reference [8] also discusses the uses of a low pass filter to smooth the signal produced by the above equation. This ad hoc approach has worked well in many experimental applications ranging from active suspensions [1] to fuel-injection control [9] to throttle/brake control on automated vehicles [6].

In this paper we introduce a dynamic extension to MSS control that overcomes the problem of explosion of terms associated with Integrator Backstepping and the problem of finding derivatives of reference (desired) trajectories for the i-th state for the MSS scheme. The first structured approach to the use of dynamic filters can be found in the dissertation of Gerdes [7]. To illustrate how the proposed method overcomes the shortcomings of the previous methods, a controller is designed for the example discussed earlier in this section.

$$\begin{aligned}
S_1 &= x_1 \\
\dot{S}_1 &= f_1(x_1) + x_2 + \Delta f_1(x_1) \\
\bar{x}_2 &:= -f_1(x_1) - K_1 S_1 - S_1\frac{\rho_1^2}{2\epsilon}
\end{aligned}$$

If x_2 were to track $\bar{x}_2$ asymptotically, S_1 would converge to a neighborhood about 0. In order to avoid the problem faced by the MSS scheme, $\bar{x}_2$ is passed through a first order filter, i.e.

$$\tau\dot{x}_{2d} + x_{2d} = \bar{x}_2, \quad x_{2d}(0) := \bar{x}_2(0)$$

S_2 is defined as

$$S_2 := x_2 - x_{2d}$$

Differentiation of x_{2d} is now possible and u is chosen to drive S_2 to zero.

$$u = \dot{x}_2 - K_2 S_2 = \frac{\bar{x}_2 - x_{2d}}{\tau} - K_2 S_2$$

We note that this control law does not involve model differentiation and thus has prevented the explosion of terms. There are two important advantages associated with Dynamic Surface Controller (DSC). It prevents the problem of "explosion of terms" and the requirement on the smoothness of f_1, ρ_1 is relaxed. In order to design a DSC, f_1, ρ_1 are required to be C^1 functions, irrespective of the order of the system. The following sections will provide details of the DSC and the stability of the closed loop system under DSC.

2. Controller Design for Lipschitz nonlinear systems

Consider the following nonlinear system in strict feedback form:

$$\dot{x}_1 = x_2 + f_1(x_1) + \Delta f_1(x_1)$$

$$
\begin{aligned}
\dot{x}_2 &= x_3 + f_2(x_1, x_2) + \Delta f_2(x_1, x_2) \\
&\vdots \\
\dot{x}_{n-1} &= x_n + f_{n-1}(x_1, \dots, x_{n-1}) + \Delta f_n(x_1, \dots, x_{n-1}) \\
\dot{x}_n &= u
\end{aligned}
$$

Assumptions for analysis:

- Δf_i is an unknown Lipschitz nonlinearity, but the Lipschitz constants are known.
- f_i is a known Lipschitz nonlinearity, and is a C^1 function of its arguments.
- $f_i(0, \dots, 0) = 0$ and $\Delta f_i(0, \dots, 0) = 0$.

2.1. DSC Controller Design

Let the error in tracking a desired trajectory, x_{1d} be S_1.

$$
\begin{aligned}
S_1 &:= x_1 - x_{1d} \\
\dot{S}_1 &= x_2 + f_1(x_1) - \dot{x}_{1d} + \Delta f_1(x_1)
\end{aligned}
$$

Define x_{2d} in such a way that

$$
\tau_2 \dot{x}_{2d} + x_{2d} = \dot{x}_{1d} - K_1 S_1 - f_1(x_1)
$$

K_1, τ_2 are positive design parameters that will be chosen later. Define the second surface error, S_2 to be

$$
S_2 := x_2 - x_{2d}
$$

Continuing this design i times, $i \leq n-1$,

$$
\begin{aligned}
S_i &:= x_i - x_{id} \\
\dot{S}_i &= x_{i+1} + f_i(x_1, \dots, x_i) - \dot{x}_{id} + \Delta f_i(x_1, \dots, x_i)
\end{aligned}
$$

Define x_{i+1d} in such a way that

$$
\tau_{i+1} \dot{x}_{i+1d} + x_{i+1d} = \dot{x}_{id} - K_i S_i - f_i(x_1, \dots, x_i)
$$

K_i, τ_{i+1} are positive design constants that we will choose later. Finally,

$$
S_n := x_n - x_{nd}
$$

$$
\dot{S}_n = u - \dot{x}_{nd} \Rightarrow u = \dot{x}_{nd} - K_n S_n, \quad K_n > 0
$$

Theorem 1 (Exponential regulation and BIBO stability of DSC controller for Lipschitz nonlinear systems): Consider the nonlinear system in strict feedback form described at the beginning of the section. Given <u>any</u> unknown Lipschitz nonlinearity with known Lipschitz constants, there exists a set of surface gains, $K_1, \dots, K_n$ and filter time constants, $\tau_2, \dots, \tau_n$ such that the Dynamic Surface Controller guarantees:

- Exponential stable regulation of the state about the origin.
- Arbitrarily small Bounded Error Tracking of bounded trajectories.

Proofs in the sequel utilize techniques from Singular Perturbation Theory. Interested readers are referred to [15], [14]. Proof of Theorem 1 is given in [22].

3. DSC for Non Lipschitz systems

Consider a nonlinear system of the following form:

$$
\begin{aligned}
\dot{x}_1 &= x_2 + f_1(x_1) + \Delta f_1(x_1) \\
\dot{x}_2 &= x_3 + f_2(x_1, x_2) + \Delta f_2(x_1, x_2) \\
&\dots \\
\dot{x}_{n-1} &= x_n + f_{n-1}(x_1, \dots, x_{n-1}) + \Delta f_{n-1}(x_1, \dots, x_{n-1}) \\
\dot{x}_n &= u
\end{aligned}
$$

Assumptions for analysis are:

- $|\Delta f_i(x_1, \dots, x_i)| \leq \rho_i(x_1, \dots, x_i)$ where ρ_i is a C^1 function in its arguments. ρ_i is not required to be Lipschitz and Δf_i is not required to be smooth or Lipschitz.
- f_i is a C^1 function in its arguments.

3.1. Controller Design

$$
\begin{aligned}
S_1 &:= x_1 - x_{1d} \\
\tau_2 \dot{x}_{2d} + x_{2d} &= -f_1(x_1) - \frac{S_1 \rho_1^2}{2\epsilon} - K_1 S_1 + \dot{x}_{1d}
\end{aligned}
$$

Here ϵ is an arbitrarily small positive constant which will be chosen later. ϵ is a measure of the regulation (or tracking) accuracy that one desires. Continuing this procedure, for $2 \leq i \leq n-1$

$$
\begin{aligned}
S_i &:= x_i - x_{id} \\
\tau_{i+1} \dot{x}_{i+1d} + x_{i+1d} &= -f_i(x_1, \dots, x_i) - \frac{S_i \rho_i^2}{2\epsilon} - K_i S_i + \dot{x}_{id} \\
S_n &= x_n - x_{nd} \\
u_n &= \dot{x}_{nd} - K_n S_n
\end{aligned}
$$

Theorem 2 (Boundedness of Tracking error using DSC) Consider any non Lipschitz nonlinear system in strict feedback form, described in this section. Given <u>any</u> uncertain non Lipschitz nonlinearity with a known C^1 function as its upper bound, and given any $p > 0, \epsilon > 0$, there exists a set of surface gains, $K_1, \dots, K_n$ and filter time constants, $\tau_2, \dots, \tau_n$ such that the Dynamic Surface Controller guarantees:

- If the desired trajectory is bounded, $x_{1d}^2 + \dot{x}_{1d}^2 + \ddot{x}_{1d}^2 \leq K_0$, then the state of the system is regulated within a ball of radius $R(p, K_0)$, for all initial conditions in a ball of radius p, i.e the closed loop system achieves semiglobally stable tracking. Here, R is a continuous function of p and K_0. $R(0,0) = 0$ if $f_i(0, \dots, 0) = \rho_i(0, \dots, 0) = 0$.
- The tracking (regulation) error eventually resides in a ball of radius ϵ.

Proof:

$$
\begin{aligned}
y_{i+1} &= x_{i+1d} + f_i(x_1, \dots, x_i) + \frac{S_i \rho_i^2(x_1, \dots, x_i)}{2\epsilon} + \frac{y_i}{\tau_i} \\
&\quad + K_i S_i, \quad i \geq 2 \\
y_2 &= x_{2d} + f_1(x_1) + \frac{S_1 \rho_1^2}{2\epsilon} + K_1 S_1 - \dot{x}_{1d}
\end{aligned}
$$

Since $S_i := x_i - x_{id}$, it follows that

$$x_1 = S_1 + x_{1d}$$
$$x_2 = S_2 + y_2 - f_1(x_1) - \frac{S_1\rho_1^2}{2\epsilon} - K_1S_1 + \dot{x}_{1d}$$
$$x_{i+1} = S_{i+1} + y_{i+1} - f_i(x_1,\ldots,x_i) - \frac{S_i\rho_i^2(x_1,\ldots,x_i)}{2\epsilon} - \frac{y_i}{\tau_i} - K_iS_i, \quad i \geq 2$$

By induction,

$$\begin{aligned} x_{i+1} &= \psi_i(S_1,S_2,\ldots,S_{i+1},y_2,\ldots,y_{i+1}, \\ &\quad K_1,\ldots,K_i,\tau_2,\ldots,\tau_i,x_{1d},\dot{x}_{1d}) \\ \dot{S}_i &= x_{i+1} + f_i(x_1,\ldots,x_i) + \Delta f_i(x_1,\ldots,x_i) - \dot{x}_{id} \\ &= S_{i+1} + y_{i+1} - K_iS_i - \frac{S_i\rho_i^2}{2\epsilon} + \Delta f_i \\ |\dot{S}_i| &\leq \Theta_i(S_1,\ldots,S_{i+1},y_2,\ldots,y_{i+1},K_1,\ldots,K_i, \\ &\quad \tau_2,\ldots,\tau_i,x_{1d},\dot{x}_{1d}) \end{aligned}$$

Similarly, one can show that

$$\begin{aligned} \frac{d\rho_i}{dt} &= \sum_1^i \frac{\partial\rho_i}{\partial x_j}\dot{x}_j \\ |\frac{d\rho_i}{dt}| &\leq \phi_i(S_1,\ldots,S_{i+1},y_2,\ldots,y_{i+1},K_1,\ldots,K_i, \\ &\quad \tau_2,\ldots,\tau_i,x_{1d},\dot{x}_{1d}) \\ \dot{y}_2 &= -\frac{y_2}{\tau_2} - \frac{\partial f_1}{\partial x_1}\dot{x}_1 - \dot{S}_1\frac{\rho_1^2}{2\epsilon} - \frac{S_1\rho_1}{\epsilon}\frac{\partial\rho_1}{\partial x_1}\dot{x}_1 + K_1\dot{S}_1 \\ &\quad -\ddot{x}_{1d} \end{aligned}$$

Clearly, $|\dot{y}_2 + \frac{y_2}{\tau_2}| \leq \eta_2(S_1,S_2,y_2,K_1,x_{1d},\dot{x}_{1d},\ddot{x}_{1d})$ for some continuous function η_2. This follows from the fact that f_1 and ρ_1 are C^1 functions.

$$\begin{aligned} \dot{y}_{i+1} &= -\frac{y_{i+1}}{\tau_{i+1}} - \sum_1^i \frac{\partial f_i}{\partial x_j}\dot{x}_j - \frac{\rho_i^2}{2\epsilon}\dot{S}_i \\ &\quad -\frac{\rho_iS_i}{\epsilon}\sum_1^i \frac{\partial\rho_i}{\partial x_j}\dot{x}_j + \frac{\dot{y}_i}{\tau_i} + K_i\dot{S}_i \end{aligned}$$

for some continuous function, η_{i+1}. By induction,

$$|\dot{y}_{i+1} + \frac{y_{i+1}}{\tau_{i+1}}| \leq \eta_{i+1}(S_1,\ldots,S_{i+1},y_2,\ldots,y_{i+1}, K_1,\ldots,K_i,\tau_2,\ldots,\tau_i,x_{1d},\dot{x}_{1d},\ddot{x}_{1d})$$

$$\dot{S}_i = S_{i+1} + y_{i+1} - K_iS_i - \frac{S_i\rho_i^2}{2\epsilon} + \Delta f_i$$

$$V_{is} := \frac{S_i^2}{2}, \quad i = 1,\ldots,n$$

$$V_{iy} := \frac{y_{i+1}^2}{2}, \quad i = 1,\ldots,n-1$$

$$\begin{aligned} \dot{V}_{is} &= S_i(S_{i+1} + y_{i+1} - K_iS_i - \frac{S_i\rho_i^2}{2\epsilon} + \Delta f_i) \\ &\leq |S_i|(|S_{i+1}| + |y_{i+1}|) - K_iS_i^2 + \frac{\epsilon}{2} \end{aligned}$$

The last inequality follows from Young's inequality, $\frac{S_i^2\rho_i^2}{2\epsilon} + \frac{\epsilon}{2} \geq |S_i|\rho_i \geq |S_i||\Delta f_i(x_1,\ldots,x_i)|$.

$$\begin{aligned} \dot{V}_{iy} &= y_{i+1}\dot{y}_{i+1} \leq -\frac{y_{i+1}^2}{\tau_{i+1}} + |y_{i+1}|\eta_{i+1}(S_1,\ldots,S_{i+1}, \\ &\quad y_2,\ldots,y_{i+1},K_1,\ldots,K_i,\tau_2,\ldots,\tau_i,x_{1d},\dot{x}_{1d},\ddot{x}_{1d}) \end{aligned}$$

Let $V := V_{1s} + \ldots + V_{ns} + V_{1y} + \ldots + V_{n-1y}$.

Claim 4: Given any $p > 0, K_0 > 0$, such that for all $V(0) \leq p$ and $|x_{1d}|^2 + |\dot{x}_{1d}|^2 + |\ddot{x}_{1d}|^2 \leq K_0$, there exists a set of gains, $K_1,\ldots,K_n$, and filter time constants, $\tau_2,\ldots,\tau_n$, such that $V(t) \leq p \;\; \forall t > 0$ and $\dot{V} \leq -2\alpha_0V + n\epsilon$.

Proof of Claim 4: Consider the set $Q := \{(x_{1d},\dot{x}_{1d},\ddot{x}_{1d}) : |x_{1d}|^2 + |\dot{x}_{1d}|^2 + |\ddot{x}_{1d}|^2 \leq K_0\}$. Clearly Q is compact in R^3. Consider the sets $A_i := \{S_1^2 + y_2^2 + S_2^2 + \ldots + y_i^2 + S_i^2 \leq 2p$. A_i is compact in R^{2i-1}. Also, $A_i \times Q$ is compact in R^{2i+2}. Therefore, η_{i+1} has a maximum, say M_{i+1} on $A_i \times Q$. Choose the following set of gains inductively,

$$K_i = 2 + \alpha_0$$
$$\frac{1}{\tau_{i+1}} = 1 + \frac{M_{i+1}^2}{2\epsilon} + \alpha_0$$

Here, α_0 is a positive constant and is greater than $\frac{n\epsilon}{2p}$. Using Young's inequality

$$\begin{aligned} \dot{V} &= \sum_1^n -(2+\alpha_0)S_i^2 + \frac{2S_i^2 + S_{i+1}^2 + y_{i+1}^2}{2} + \frac{\epsilon}{2} \\ &\quad + \sum_1^{n-1} -(1 + \frac{M_{i+1}^2}{2\epsilon} + \alpha_0)y_{i+1}^2 + \frac{M_{i+1}^2y_{i+1}^2}{2\epsilon}\frac{\eta_{i+1}^2}{M_{i+1}^2} + \frac{\epsilon}{2} \\ &\leq -2\alpha_0V + n\epsilon - (1 - \frac{\eta_{i+1}^2}{M_{i+1}^2})\frac{M_{i+1}^2y_{i+1}^2}{2\epsilon} \end{aligned}$$

On $V(S_1,\ldots,S_n,y_2,\ldots,y_n) = p$, $\eta_{i+1} \leq M_{i+1}$. Therefore,

$$\dot{V} \leq -2\alpha_0p + n\epsilon$$

Since $\alpha_0 > \frac{n\epsilon}{2p}$, it follows that $\dot{V} \leq 0$ on $V = p$. Therefore, $V \leq p$ is an invariant set, i.e if $V(0) \leq p$, then $V(t) \leq p$ for all $t > 0$. It can be inductively shown that x_{i+1d} is bounded by showing that the input to the filter is bounded uniformly. The bound, $M(p,K_0)$, on x_{i+1d} is a continuous function of p and K_0. One can show that $M(0,0) = 0$ if $f_i(0,\ldots,0) = \rho_i(0,\ldots,0) = 0, i = 1,\ldots,n$. Eventually tracking error resides in a ball of radius, $\frac{n\epsilon}{2\alpha_0}$. Choose $\alpha_0 > max\{\frac{n\epsilon}{p}, \frac{n}{2}\}$. This completes the proof for semiglobal regulation and tracking of DSC for non Lipschitz systems.

4. Illustrative Example:

In this section, the following example is considered:

$$\dot{x}_1 = x_2 + \Delta f_1(x_1)$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = u$$
$$y = x_1$$

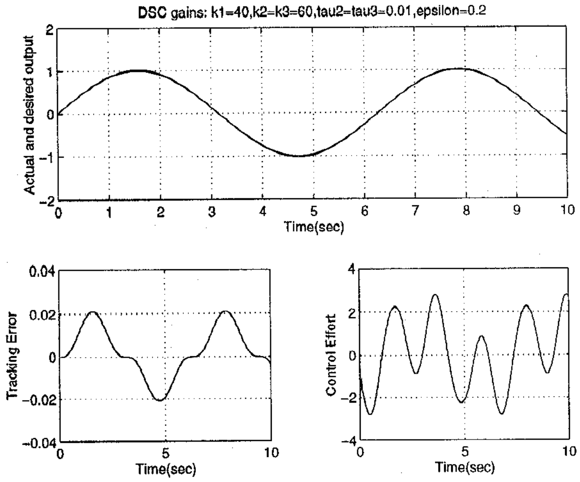

Figure 1: Performance of DSC with filter time constants = 0.01 sec

The control objective is to synthesize a state feedback law for u such that the output, $y(t)$, tracks a reference signal, $r(t)$. The uncertainty $\Delta f_1(x_1)$ is assumed to be bounded by x_1^2. For the simulation study, $\Delta f_1(x_1) = x_1^2 \sin(x_1)$. Details of the backstepping controller design for this system are omitted due to space considerations. Dynamic Surface Controller design for this system is outlined below:

$$\begin{aligned}
S_1 &:= x_1 - r(t) \\
\dot{S}_1 &= x_2 + \Delta f_1(x_1) - \dot{r}(t) \\
\Rightarrow \bar{x}_2 &= \dot{r}(t) - K_1 S_1 - S_1 \frac{x_1^4}{2\epsilon} \\
\tau_2 \dot{x}_{2d} + x_{2d} &= \bar{x}_2, \qquad x_{2d}(0) = \bar{x}_2(0) \\
S_2 &:= x_2 - x_{2d} \\
\dot{S}_2 &= x_3 - \dot{x}_{2d} \\
\Rightarrow \bar{x}_3 &= \dot{x}_{2d} - K_2 S_2 \\
\tau_3 \dot{x}_{3d} + x_{3d} &= \bar{x}_3, \qquad x_{3d}(0) = \bar{x}_3(0) \\
S_3 &:= x_3 - x_{3d} \\
\dot{S}_3 &= u - \dot{x}_{3d} \\
\Rightarrow u &= \dot{x}_{3d} - K_3 S_3
\end{aligned}$$

This illustrative example sheds a light on the explosion of terms that one faces in designing a backstepping controller.

The reference signal, $r(t)$, used in the simulation is $\sin(t)$. The results of the simulation are shown in figures 1 through 3. Figures 1 through 3 illustrate how the filter time constants affect the performance of the system. Three sets of time constants chosen for simulation are: $\{\tau_2 = \tau_3 = 0.01\}$, $\{\tau_2 = \tau_3 = 0.024\}$, and $\{\tau_2 = \tau_3 = 0.028\}$. The surface gains used for all those simulations are $\{K_1 = 40.0, K_2 = 60.0, K_3 = 60.0\}$. $\tau_2 K_1$ equals 0.4 for the first set, 0.96 for the second set and 1.12 for the third set. With the first set of gains and filter time

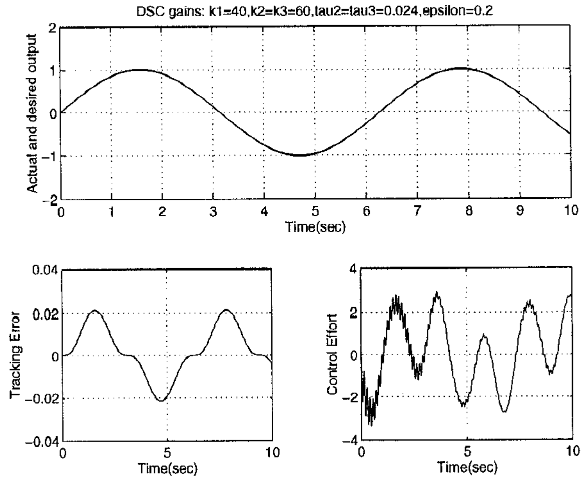

Figure 2: Performance of a DSC with filter time constants = 0.024 sec

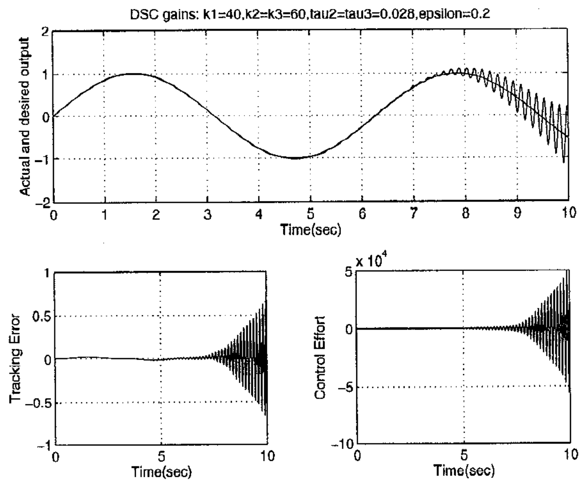

Figure 3: Performance of a DSC with filter time constants = 0.028 sec

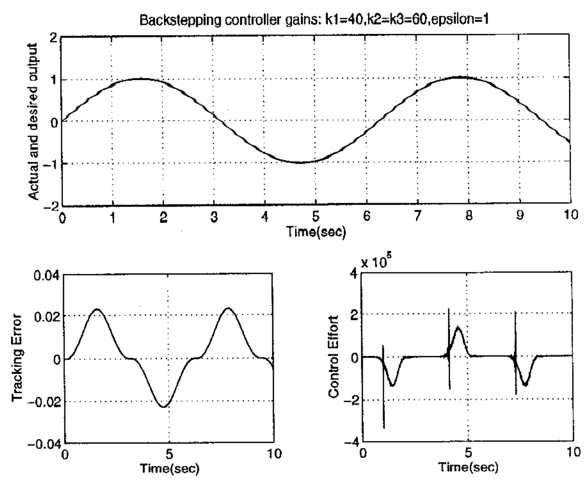

Figure 4: Performance of Backstepping controller

constants, the amplitude of the tracking error is about 2% of the reference signal. With the second set of gains, the control efforts oscillates rapidly initially before converging to a smooth signal similar to the one obtained with the first set of time constants. With the third set of time constants, the controller goes unstable. Performance of the corresponding backstepping controller design, which guarantees exponential stable regulation and arbitrarily small tracking error, is shown in Figure 4.

5. Conclusions

In this paper, Dynamic Surface Controller design is proposed. This controller design is intuitively appealing, and it has "$r-1$" lowpass filters, where r is the relative degree of the output to be controlled. These low pass filters allow a design where the model is not differentiated, at the same time, avoiding the complexity that arises due to the explosion of terms. In this paper, Dynamic Surface Controller is shown to guarantee exponential regulation and bounded tracking error in the presence of Lipschitz mismatched uncertainties in strict feedback form. We have also designed Dynamic Surface Controller for non Lipschitz nonlinear systems. We have shown that Dynamic Surface Controller guarantees arbitrarily tight semi-global regulation. Backstepping algorithm guarantees arbitrarily tight regulation globally. This is the trade-off in performance. In particular, the key feature of the algorithm, which removes the need for differentiations in the controller design and reduces the explosion of terms, is that due to the presence of the auxiliary first-order filters none of the nonlinearities are ever differentiated more than once. This is a crucial point, because it implies that any C^1 nonlinearity can be used. In some of the previous control design schemes, the assumption of globally continuity was needed not only on the original nonlinearities, but also on all the derivatives that were generated during the design process, thus limiting the allowable nonlinearities so much that sometimes only linear functions could be dealt with. Real-time implementation of the software filters poses a hard performance limitation. The maximum bandwidth of the software filters is bounded by the control sampling frequency. In other words, the filter time constants cannot be made arbitrarily small in real-time implementation. Since the DSC tracking requirement is semi-global to start with, we conjecture that the DSC control effort will be smaller in comparison to the globally stabilizing backstepping controller.

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