Automatica 46 (2010) 2100–2104

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Technical communique

A new result on stability analysis for stochastic neutral systems[☆]

Yun Chen[a,b], Wei Xing Zheng[b,*], Anke Xue[a]

[a] Institute of Information and Control, Hangzhou Dianzi University, Hangzhou 310018, China

[b] School of Computing and Mathematics, University of Western Sydney, Penrith South DC NSW 1797, Australia

ARTICLE INFO

Article history:
Received 26 November 2009
Received in revised form
24 June 2010
Accepted 16 July 2010
Available online 31 August 2010

Keywords:
Stochastic neutral systems
Mean-square exponential stability
Delay-dependent criterion
Generalized Finsler lemma

ABSTRACT

This paper discusses the mean-square exponential stability of stochastic linear systems of neutral type. Applying the Lyapunov–Krasovskii theory, a linear matrix inequality-based delay-dependent stability condition is presented. The use of model transformations, cross-term bounding techniques or additional matrix variables is all avoided, thus the method leads to a simple criterion and shows less conservatism. The new result is derived based on the generalized Finsler lemma (GFL). GFL reduces to the standard Finsler lemma in the absence of stochastic perturbations, and it can be used in the analysis and synthesis of stochastic delay systems. Moreover, GFL is also employed to obtain stability criteria for a class of stochastic neutral systems which have different discrete and neutral delays. Numerical examples including a comparison with some recent results in the literature are provided to show the effectiveness of the new results.

1. Introduction

Dynamical systems modeled by neutral functional differential equations are generally called neutral systems in the literature. The study of neutral systems has received considerable attention during the past few decades (see, e.g., Gu, Kharitonov, and Chen (2003), Han (2002), He, Wu, She, and Liu (2004), Li and Liu (2009) and Suplin, Fridman, and Shaked (2006) and the references therein). Recently, increasing efforts have been made to investigate stochastic retarded/neutral systems, since stochastic perturbations exist in many real-world systems (Mao, 1997). It is noticed that the methods in Gao, Lam, and Wang (2006), Wang and Ho (2003) and Xu, Shi, Chu, and Zou (2006) are delay-independent, so they may be restrictive, especially when the delays are small. In order to reduce the conservatism, the delay-dependent stability for stochastic systems was studied in Basin and Rodkina (2008), Chen, Guan, and Lu (2005), Chen, Xue, Zhou, and Lu (2008), Chen, Zheng, and Shen (2009), Huang and Mao (2009), Rodkina and Basin (2006, 2007) and Yue and Han (2005), respectively. Thereinto, the model transformation method together with bounding techniques for cross-terms were extended to consider the stability of stochastic linear delay systems (Chen et al., 2005). Resorting to some slack matrix variables, delay-dependent results were obtained in Chen et al. (2008) and Yue and Han (2005). Based on the free-weighting matrix technique, the mean-square exponential stability for stochastic delay systems of neutral type was studied in Chen et al. (2009) and Huang and Mao (2009). In addition, the almost sure exponential stability of stochastic neutral systems was also discussed in Huang and Mao (2009). Moreover, for nonlinear stochastic neutral systems, various delay-dependent stability results were obtained in Basin and Rodkina (2008) and Rodkina and Basin (2006, 2007).

However, model transformations may lead to additional dynamics of the original systems (see Gu et al., 2003), and cross-term bounding techniques can bring conservatism. Moreover, as pointed out by Xu and Lam (2007), in some cases free matrix variables may not be useful to the reduction of conservatism. Besides, those variables will increase the computational burden and make it rather difficult to synthesize systems. Therefore, there is a need to establish some new stability results with less conservatism and lower computational cost such that the design purposes can be achieved more easily. The above consideration has motivated the present work.

This paper focuses on the stability analysis of linear stochastic neutral-type systems. By Lyapunov–Krasovskii functional theory, a new delay-dependent mean-square exponential stability criterion is formulated in terms of linear matrix inequality (LMI). The presented condition is simple and efficient, because none of the model transformations, bounding techniques for cross-terms

☆ This work was supported partially by the National Basic Research Program of China (973 Program) under grant 2009CB320602 the National Natural Science Foundation of China under grants 60974006, 60974138, the Zhejiang Provincial Natural Science Foundation of China under grant Y1090465, and by a research grant from the Australian Research Council. This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Keqin Gu under the direction of Editor André L. Tits.

* Corresponding author. Tel.: +61 2 4736 0608; fax: +61 2 4736 0867.
E-mail addresses: cloudscy@hdu.edu.cn (Y. Chen), w.zheng@uws.edu.au (W.X. Zheng), akxue@hdu.edu.cn (A. Xue).

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doi:10.1016/j.automatica.2010.08.007

(including Jensen inequality) and additional free matrix variables are involved. In such a formulation, a generalized version of the standard Finsler lemma (see Qiu, Feng, & Yang, 2009), named the generalized Finsler lemma (GFL) in this paper, is applied. The GFL becomes the standard Finsler lemma if the systems are not driven by stochastic noises. The GFL can be used in the analysis and design of stochastic delay systems, just as the standard Finsler lemma for delay systems in the deterministic context (Coutinho & de Souza, 2008; Du, Lam, Shu, & Wang, 2009; Qiu et al., 2009; Suplin et al., 2006). Furthermore, GFL is applied to obtain a stability condition for a class of stochastic neutral systems whose discrete delay and neutral delay are not equal. Finally, illustrative examples are given to demonstrate the usefulness of the proposed method.

Notations: Throughout this paper, the notations are standard. $\mathbf{E}\{\cdot\}$ is the expectation operator; the image of the corresponding matrix transformation $\mathbf{T}$ is denoted as $\mathrm{im}(\mathbf{T})$; the kernel (or null) space of a given matrix $\mathcal{B} \in \mathbb{R}^{M\times N}$ is represented as $\ker(\mathcal{B})$; moreover, if $\mathrm{rank}(\mathcal{B}) = r < N$, we also let a full rank matrix $\mathcal{B}^{\perp} \in \mathbb{R}^{N\times(N-r)}$ denote the right orthogonal complement of $\mathcal{B}$, i.e., $\mathcal{B}\mathcal{B}^{\perp} = 0$ and $\mathcal{B}^{\perp}\mathcal{B}^{\perp T} > 0$. $(\mathbf{\Omega}, \mathcal{F}, \mathcal{P})$ is a complete probability space, where $\mathbf{\Omega}$ is the sample space, and $\mathcal{F}$ is a σ-algebra of subsets of $\mathbf{\Omega}$.

2. Main results

Consider the following stochastic neutral system

$$\begin{cases} \mathrm{d}[x(t) - Cx(t-h)] = [Ax(t) + Bx(t-h)]\,\mathrm{d}t \\ \quad + [Fx(t) + Gx(t-h)]\,\mathrm{d}w(t) \\ x(s) = \phi(s), \quad s \in [-h,\ 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state; $w(t)$ is a scalar Wiener process (or Brownian motion) defined on the complete probability space $(\mathbf{\Omega}, \mathcal{F}, \mathcal{P})$; $h > 0$ is a scalar indicating the time delay; $\phi(\cdot) \in \mathbb{R}^n$ is the given initial condition and $\mathbf{E}\{\|\phi(\cdot)\|^2\}$ is uniformly bounded; $A, B, C, F, G \in \mathbb{R}^{n\times n}$ are known constant matrices.

This paper is concerned with the mean-square exponential stability analysis for system (1), so we introduce the following definition.

Definition 1. System (1) is said to be mean-square exponentially stable if there exist scalars $\gamma > 0, \alpha > 0$ such that $\mathbf{E}\{\|x(t)\|^2\} \le \gamma \mathrm{e}^{-\alpha t} \sup_{-h\le s\le 0} \mathbf{E}\{\|\phi(s)\|^2\}$.

2.1. New stability criterion

The first result of this paper can be stated as follows.

Theorem 1. *For given a scalar $h > 0$, system (1) is mean-square exponentially stable, if there exist matrices $P > 0$, $Q = \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} > 0$, $R > 0$ satisfying*

$$\Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & -RC & hA^T R \\ * & \Gamma_{22} & -Q_2 + (C+I)^T RC & hB^T R \\ * & * & -Q_3 - C^T RC & 0 \\ * & * & * & -R \end{bmatrix} < 0 \tag{2}$$

where $\Gamma_{11} = P^T A + A^T P + Q_1 - R + F^T PF$, $\Gamma_{12} = PB - A^T PC + Q_2 + R(C+I) + F^T PG$, $\Gamma_{22} = -B^T PC - C^T PB - Q_1 + Q_3 - (C+I)^T R(C+I) + G^T PG$.

Proof. For the sake of simplicity, the following notations are adopted:

$$\begin{aligned} z(t) &= x(t) - Cx(t-h), \\ f(t) &= Ax(t) + Bx(t-h), \\ g(t) &= Fx(t) + Gx(t-h). \end{aligned} \tag{3}$$

System (1) then reads as

$$\mathrm{d}z(t) = f(t)\mathrm{d}t + g(t)\mathrm{d}w(t). \tag{4}$$

Choose the following Lyapunov–Krasovskii functional

$$\begin{aligned} V(t) &= z^T(t)Pz(t) + \int_{t-h}^{t} \zeta^T(s)Q\zeta(s)\mathrm{d}s \\ &\quad + h\int_{-h}^{0}\int_{t+\theta}^{t} f^T(s)Rf(s)\mathrm{d}s\mathrm{d}\theta \end{aligned} \tag{5}$$

where $\zeta(t) = [x^T(t)\ x^T(t-h)]^T$. According to Itô's differential formula (Mao, 1997), the stochastic differential is

$$\mathrm{d}V(t) = \mathcal{L}V(t)\mathrm{d}t + 2z^T(t)Pg(t)\mathrm{d}w(t) \tag{6}$$

with the infinitesimal operator

$$\begin{aligned} \mathcal{L}V(t) &= 2z(t)^T Pf(t) + z(t)^T Pz(t) \\ &\quad + \zeta^T(t)Q\zeta(t) - \zeta^T(t-h)Q\zeta(t-h) \\ &\quad + f^T(t)(h^2R)f(t) - \int_{t-h}^{t} f^T(s)(hR)f(s)\mathrm{d}s \\ &= \frac{1}{h}\int_{t-h}^{t} \xi^T\Theta\xi\mathrm{d}s \end{aligned}$$

where $\xi = [f^T(t)x^T(t)x^T(t-h)x^T(t-2h)f^T(s)]^T$, and

$$\Theta = \begin{bmatrix} h^2R & P & -PC & 0 & 0 \\ * & Q_1 + F^T PF & Q_2 + F^T PG & 0 & 0 \\ * & * & Q_3 - Q_1 + G^T PG & -Q_2 & 0 \\ * & * & * & -Q_3 & 0 \\ * & * & * & * & -h^2R \end{bmatrix}.$$

Taking mathematical expectation of both sides of (6) and by virtue of $\mathbf{E}\{\mathrm{d}w(t)\} = 0$, we have

$$\mathbf{E}\left\{\frac{\mathrm{d}V(t)}{\mathrm{d}t}\right\} = \mathbf{E}\{\mathcal{L}V(t)\} = \frac{1}{h}\int_{t-h}^{t} \mathbf{E}\{\xi^T\Theta\xi\}\mathrm{d}s. \tag{7}$$

It is clear that $\mathbf{E}\{\xi^T\Theta\xi\} < 0$ guarantees the mean-square stability of system (1) (see Mao, 1997). Meanwhile, integrating both sides of (4) from $t-h$ to t and taking mathematical expectation yields

$$\frac{1}{h}\int_{t-h}^{t} \mathbf{E}\{z(t) - z(t-h) - hf(s)\}\mathrm{d}s = 0. \tag{8}$$

Selecting a matrix

$$\mathcal{B}^{\perp} = \begin{bmatrix} A^T & I & 0 & 0 & \dfrac{I}{h} \\ B^T & 0 & I & 0 & -\dfrac{C^T + I}{h} \\ 0 & 0 & 0 & I & \dfrac{C^T}{h} \end{bmatrix}^T \tag{9}$$

then we claim that $\mathbf{E}\{\xi^T\Theta\xi\} < 0$ if and only if the following condition is satisfied:

$$\mathcal{B}^{\perp T}\Theta\mathcal{B}^{\perp} < 0. \tag{10}$$

This claim will be proved in Proposition 1 to be given in the next subsection. So, $\Gamma_0 = \mathcal{B}^{\perp T}\Theta\mathcal{B}^{\perp} < 0$ ensures the mean-square stability of system (1). In view of (9),

$$\Gamma_0 = \begin{bmatrix} \mathcal{A}_1 & \mathcal{A}_2 & -RC \\ * & \mathcal{A}_3 & -Q_2 + (C+I)^T RC \\ * & * & -Q_3 - C^T RC \end{bmatrix} \tag{11}$$

where $\mathcal{A}_1 = P^TA + A^TP + Q_1 - R + F^TPF + h^2A^TRA$, $\mathcal{A}_2 = PB - A^TPC + Q_2 + R(C+I) + F^TPG + h^2A^TRB$, $\mathcal{A}_3 = -B^TPC - C^TPB - Q_1 + Q_3 - (C+I)^TR(C+I) + G^TPG + h^2B^TRB$. Applying Schur complements to $\Gamma_0 < 0$, then $\Gamma < 0$ can be deduced immediately.

We now proceed to prove the mean-square exponential stability of system (1). Denoting $c = \lambda_{\min}\{-\Gamma_0\} > 0$, it follows from $\Gamma_0 < 0$ and (7) that

$$\mathbf{E}\left\{\frac{\mathrm{d}V(t)}{\mathrm{d}t}\right\} = \mathbf{E}\{\mathcal{L}V(t)\} \le -c\mathbf{E}\{\|x(t)\|^2\}. \tag{12}$$

Observing (5), there surely exist finite scalars $\delta_1 > 0$, $\delta_2 > 0$ such that $\delta_1\|x(t)\|^2 \le V(t) \le \delta_2\|x(t)\|^2$ and $-\|x(t)\|^2 \le -\frac{1}{\delta_2}V(t) < 0$. From this and (12), we have $\mathbf{E}\left\{\frac{\mathrm{d}V(t)}{\mathrm{d}t}\right\} \le -c\mathbf{E}\{\|x(t)\|^2\} \le -\frac{c}{\delta_2}\mathbf{E}\{V(t)\}$. Integrating both sides of the above inequality over the interval $[0, T]$ by separation of variables results in

$$\mathbf{E}\{V(T)\} \le \mathbf{E}\{V(0)\}\mathrm{e}^{-\frac{c}{\delta_2}T}. \tag{13}$$

Then, there is a scalar $\gamma_1 > 0$ such that $\mathbf{E}\{V(0)\} \le \gamma_1 \sup_{-h\le s\le 0}\mathbf{E}\{\|\phi(s)\|^2\}$. Consequently, we get

$$\delta_1\mathbf{E}\{\|x(T)\|^2\} \le \gamma_1\mathrm{e}^{-\frac{c}{\delta_2}T}\sup_{-h\le s\le 0}\mathbf{E}\{\|\phi(s)\|^2\} \tag{14}$$

and the inequality in Definition 1, where $\gamma = \frac{\gamma_1}{\delta_1}$, $\alpha = \frac{c}{\delta_2}$. Therefore, system (1) is mean-square exponentially stable. The proof is thus completed. □

Remark 1. In Theorem 1, the mean-square exponential stability of system (1) has been proved concisely. Following similar lines as in Huang and Mao (2009), the almost sure exponential stability of system (1) can also be shown. The detailed process is omitted here for brevity.

Remark 2. In the case of stability analysis for delayed systems, model transformations may introduce additional dynamics to the original systems (Gu et al., 2003), while cross-terms bounding techniques and free-weighting matrices may increase the conservatism and computational burden of the results, respectively. It is clear to see that in deriving Theorem 1, none of model transformations, cross-terms bounding techniques (including Jensen inequality (Gu et al., 2003; Huang & Mao, 2009)) and free-weighting matrices have been employed. However, the free-weighting matrix approach was applied in both Chen et al. (2009) and Huang and Mao (2009). Precisely speaking, six and eleven additional free matrices were introduced in Chen et al. (2009) and Huang and Mao (2009), respectively. Furthermore, Jensen inequality and some other bounding techniques for cross-terms were also used in Chen et al. (2009) and Huang and Mao (2009). Besides, a negative term was neglected in inequality (12) of Chen et al. (2009). Therefore, our method is more appealing since it shows less conservatism and involves fewer computational variables, as will be illustrated by numerical examples in Section 3.

Remark 3. If the matrix $\mathcal{B}$ is chosen as

$$\mathcal{B} = \begin{bmatrix} -I & A & B & 0 & 0 \\ 0 & -I & (C+I) & -C & hI \end{bmatrix} \tag{15}$$

then there holds

$$\frac{1}{h}\int_{t-h}^{t}\mathbf{E}\{\mathcal{B}\xi\}\mathrm{d}s = 0. \tag{16}$$

It is easy to check that $\mathrm{rank}(\mathcal{B}) = 2n < 5n$ and $\mathcal{B}\mathcal{B}^{\perp} = 0$. That is, the columns of $\mathcal{B}^{\perp}$ form a basis of $\ker(\mathcal{B})$, or $\mathcal{B}^{\perp}$ is the right orthogonal complement of matrix $\mathcal{B}$. Thus, Theorem 1, together with (15) and (16), reveals that if there exist a nonzero stochastic vector $\xi \in \mathbb{R}^N$ and a matrix $\mathcal{B} \in \mathbb{R}^{M\times N}$ satisfying $\mathrm{rank}(\mathcal{B}) = r \le N$ and $\int_{t-h}^{t}\mathbf{E}\{\mathcal{B}\xi\}\mathrm{d}s = 0$, $\mathbf{E}\{\xi^T\Theta\xi\} < 0$ holds for a symmetric matrix $\Theta \in \mathbb{R}^{N\times N}$ if and only if $\mathcal{B}^{\perp T}\Theta\mathcal{B}^{\perp} < 0$.

2.2. Generalized Finsler lemma (GFL)

In the proof of Theorem 1 and Remark 3, the generalized version of the standard Finsler lemma (see Qiu et al., 2009 for details), called the generalized Finsler lemma (GFL) in this paper, is applied. In what follows, we give a detailed presentation of the GFL and its proof.

Proposition 1 (*Generalized Finsler Lemma (GFL)*)**.** *Consider a stochastic vector $\theta \in \mathbb{R}^N$, a symmetric matrix $\Theta \in \mathbb{R}^{N\times N}$ and a matrix $\mathcal{B} \in \mathbb{R}^{M\times N}$ with $\mathrm{rank}(\mathcal{B}) = r < N$. Let $\mathcal{B}^{\perp}$ represent the right orthogonal complement of $\mathcal{B}$, i.e., $\mathcal{B}\mathcal{B}^{\perp} = 0$, then the following four statements are equivalent:*

(T_1) $\mathbf{E}\{\theta^T\Theta\theta\} < 0,\ \forall\theta \ne 0,\ t > t_0,\ \int_{t_0}^{t}\mathbf{E}\{\mathcal{B}\theta\}\mathrm{d}s = 0$
(T_2) $\mathcal{B}^{\perp T}\Theta\mathcal{B}^{\perp} < 0$
(T_3) $\exists\varepsilon \in \mathbb{R} : \Theta - \varepsilon\mathcal{B}^T\mathcal{B} < 0$
(T_4) $\exists\Lambda \in \mathbb{R}^{N\times M} : \Theta + \Lambda\mathcal{B} + \mathcal{B}^T\Lambda^T < 0$

Proof. $(T_1) \Rightarrow (T_4)$: Without loss of generality, assume that an $N \times N$-dimensional matrix $S = [\,S_1 \quad S_2\,]$is nonsingular such that the columns of S_2 span $\ker(\mathcal{B})$. Then $\mathcal{B}S$ has the structure $[B_1\ 0]$, where B_1 is of full column rank. Pre- and post-multiplying the inequality in (T_4) with S^T and S, respectively, we have

$$S^T\Theta S + S^T\Lambda(\mathcal{B}S) + (\mathcal{B}S)^T\Lambda^TS < 0 \tag{17}$$

i.e.,

$$S^T\Theta S + \begin{bmatrix} S_1^T \\ S_2^T \end{bmatrix}\Lambda\begin{bmatrix} B_1 & 0 \end{bmatrix} + \begin{bmatrix} B_1^T \\ 0 \end{bmatrix}\Lambda^T\begin{bmatrix} S_1 & S_2 \end{bmatrix} < 0. \tag{18}$$

Denote $Y = S^T\Theta S = \begin{bmatrix} Y_1 & Y_2 \\ Y_2^T & Y_3 \end{bmatrix}$, $\begin{bmatrix} S_1^T \\ S_2^T \end{bmatrix}\Lambda\begin{bmatrix} B_1 & 0 \end{bmatrix} = \begin{bmatrix} Z_1 & 0 \\ Z_2^T & 0 \end{bmatrix}$. As a result, (18) becomes

$$\begin{bmatrix} Y_1 + Z_1 + Z_1^T & Y_2 + Z_2 \\ Y_2^T + Z_2^T & Y_3 \end{bmatrix} < 0. \tag{19}$$

Since $\ker(\mathcal{B}S) = \mathrm{im}(\mathbf{T})$, where $\mathbf{T} = \begin{bmatrix} 0 \\ I \end{bmatrix}$, (T_1) just means that $Y_3 < 0$. For any fixed matrix Y_2, we can find a matrix Z_1 such that (19) and (18) hold, which implies the desired result (T_4).
$(T_4) \Leftrightarrow (T_3)$: Selecting $\Lambda = -\frac{\varepsilon}{2}\mathcal{B}^T$ leads to the equivalence of (T_3) and (T_4) directly.
(T_3) or $(T_4) \Rightarrow (T_2)$: Performing congruence transformations to (T_3) or (T_4) by $\mathcal{B}^{\perp T}$ and $\mathcal{B}^{\perp}$, and observing $\mathcal{B}\mathcal{B}^{\perp} = 0$, then (T_2) can be obtained easily.
$(T_2) \Rightarrow (T_1)$: For any given symmetric matrix $\Theta \in \mathbb{R}^{N\times N}$ and matrix $\mathcal{B} \in \mathbb{R}^{M\times N}$, (T_2) is equivalent to $\mathcal{B}^{\perp T}\Theta\mathcal{B}^{\perp} < 0$. And it is also equivalent to $\eta^T\mathcal{B}^{\perp T}\Theta\mathcal{B}^{\perp}\eta < 0$, where η is a nonzero stochastic vector. Choose a stochastic vector $\theta \ne 0$ satisfying $\theta = \mathcal{B}^{\perp}\eta$. Then we obtain $\mathcal{B}\theta = 0$ and $\theta^T\Theta\theta < 0$, which means that (T_1) is satisfied. □

Remark 4. It is worth noting that the vector ξ in Theorem 1 is in the stochastic setting, due to the existence of stochastic perturbation $w(t)$. Just because of this, mathematical expectation and integration have been performed on system (4). Otherwise, (8), (15) and (16) could not be obtained, and it would fail to establish Theorem 1 and Proposition 1 as well. Thus, mathematical expectation and integral manipulations on system (4) play a key role in our method. On the other hand, if there is no stochastic perturbation in system (1), Proposition 1 reduces to the standard Finsler lemma (see Qiu et al. (2009)) by removing the mathematical expectation operator. In Coutinho and de Souza (2008) and Suplin et al. (2006), the first and fourth equivalent conditions of the standard Finsler lemma were applied, so some additional free matrix variables were introduced there. It was proved in Xu and Lam (2007) that the result of Suplin et al. (2006) can be simplified to some equivalent versions without using any redundant matrix variables. Since GFL is an extension of the standard Finsler lemma, it can be applied to dynamical systems in the stochastic case. Actually, the first two equivalent statements of GFL have been used to derive Theorem 1. Because Theorem 1 does not involve any additional matrix variables, it leads to a simpler stability condition and a lower computational cost. For stochastic delay systems, if the last two statements of GFL are employed, then one will also obtain results including some redundant matrix variables like in deterministic delay systems (Suplin et al., 2006). Thus, when studying the stochastic delay systems, from the computational complexity point of view, the first two equivalent statements of GFL are usually preferred.

2.3. Application of GFL

In system (1), the discrete delay is assumed to be equal to the neutral delay. In order to illustrate the application of GFL to stochastic delay systems, the following system will be considered:

$$\mathrm{d}\bar{z}(t) = f(t)\mathrm{d}t + g(t)\mathrm{d}w(t), \qquad x(s) = \phi(s), \quad s \in [-H, 0] \tag{20}$$

where $\bar{z}(t) = x(t) - Cx(t-\tau)$, and $H = \max\{h, \tau\}$, where $\tau > 0$ is the neutral delay. For system (20), we can obtain the following stability condition.

Theorem 2. *For given scalars $h > 0, \tau > 0$, system (20) is mean-square exponentially stable, if there exist matrices $P > 0, Q = \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} > 0, T = \begin{bmatrix} T_1 & T_2 & T_4 \\ * & T_3 & T_5 \\ * & * & T_6 \end{bmatrix} > 0, R > 0, S > 0$ satisfying*

$$\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & -RC & -SC & hA^TR & \tau A^TS \\ * & \Psi_{22} & \Psi_{23} & \Psi_{24} & 0 & hB^TR & \tau B^TS \\ * & * & \Psi_{33} & \Psi_{34} & \Psi_{35} & 0 & 0 \\ * & * & * & \Psi_{44} & -T_5 & 0 & 0 \\ * & * & * & * & \Psi_{55} & 0 & 0 \\ * & * & * & * & * & -R & 0 \\ * & * & * & * & * & * & -S \end{bmatrix} < 0 \tag{21}$$

where $\Psi_{11} = P^TA + A^TP + Q_1 + T_1 - R - S + F^TPF$, $\Psi_{12} = PB + T_2 + R + F^TPG$, $\Psi_{22} = -Q_1 + T_3 - R + G^TPG$, $\Psi_{13} = -A^TPC + Q_2 + T_4 + RC + S(C+I)$, $\Psi_{23} = -B^TPC + T_5 - RC$, $\Psi_{33} = Q_3 + T_6 - T_1 - C^TRC - (C+I)^TS(C+I)$, $\Psi_{24} = -Q_2 + RC$, $\Psi_{34} = -T_2 + C^TRC$, $\Psi_{44} = -Q_3 - T_3 - C^TRC$, $\Psi_{35} = (C+I)^TSC - T_4$, $\Psi_{55} = -C^TSC - T_6$.

Proof. Taking mathematical expectation and integrating both sides of system (20) on the intervals $[t-h, t]$ and $[t-\tau, t]$, respectively, we get

$$\begin{aligned} &\frac{1}{h}\int_{t-h}^{t} \mathbf{E}\{-\bar{z}(t) + \bar{z}(t-h) + hf(s)\}\mathrm{d}s = 0 \\ &\frac{1}{\tau}\int_{t-\tau}^{t} \mathbf{E}\{-\bar{z}(t) + \bar{z}(t-\tau) + \tau f(\mu)\}\mathrm{d}\mu = 0. \end{aligned} \tag{22}$$

Eqs. (3) and (22) imply

$$\frac{1}{h\tau}\int_{t-h}^{t}\int_{t-\tau}^{t} \mathbf{E}\{\mathcal{B}\xi\}\mathrm{d}s\mathrm{d}\mu = 0 \tag{23}$$

where $\xi = [f^T(t)x^T(t)x^T(t-h)x^T(t-\tau)x^T(t-h-\tau)x^T(t-2\tau)f^T(s)f^T(\mu)]^T$, and

$$\mathcal{B} = \begin{bmatrix} -I & A & B & 0 & 0 & 0 & 0 & 0 \\ 0 & -I & I & C & -C & 0 & hI & 0 \\ 0 & -I & 0 & (C+I) & 0 & -C & 0 & \tau I \end{bmatrix}.$$

The right orthogonal complement of $\mathcal{B}$ can be written as

$$\mathcal{B}^{\perp} = \begin{bmatrix} A^T & I & 0 & 0 & 0 & 0 & \frac{I}{h} & \frac{I}{\tau} \\ B^T & 0 & I & 0 & 0 & 0 & -\frac{I}{h} & 0 \\ 0 & 0 & 0 & I & 0 & 0 & -\frac{C^T}{h} & -\frac{C^T+I}{\tau} \\ 0 & 0 & 0 & 0 & I & 0 & \frac{C^T}{h} & 0 \\ 0 & 0 & 0 & 0 & 0 & I & 0 & \frac{C^T}{\tau} \end{bmatrix}^T. \tag{24}$$

Choose the Lyapunov–Krasovskii functional

$$\begin{aligned} V(t) &= \bar{z}^T(t)P\bar{z}(t) + h\int_{-h}^{0}\int_{t+\theta}^{t} f^T(s)Rf(s)\mathrm{d}s\mathrm{d}\theta \\ &\quad + \int_{t-\tau}^{t} \varsigma^T(s)T\varsigma(s)\mathrm{d}s + \int_{t-h}^{t} \eta^T(s)Q\eta(s)\mathrm{d}s \\ &\quad + \tau\int_{-\tau}^{0}\int_{t+\theta}^{t} f^T(\mu)Sf(\mu)\mathrm{d}\mu\mathrm{d}\theta \end{aligned} \tag{25}$$

where $\eta(t) = [x^T(t)x^T(t-\tau)]^T$, $\varsigma(t) = [x^T(t)x^T(t-h)x^T(t-\tau)]^T$. Following similar lines as in the proof of Theorem 1, the mean-square exponential stability of (20) can be established directly and the details are omitted for brevity. □

3. Numerical examples

In this section, we provide numerical examples to show the effectiveness of our approach.

Example 1. Consider system (1) with

$$\begin{aligned} &C = \begin{bmatrix} -0.2 & 0 \\ 1 & 0.2 \end{bmatrix}, \qquad A = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.3 \end{bmatrix}, \\ &B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \qquad F = 0.2I, \qquad G = 0.3I. \end{aligned} \tag{26}$$

This system was studied in Huang and Mao (2009). The delay-independent method given in Xu et al. (2006) is not applicable to system (26). Employing the most recent results of Chen et al. (2009) and Huang and Mao (2009), the maximal admissible delay h_M of this example is $h_{\mathrm{M}} = 0.35$ and $h_{\mathrm{M}} = 0.5731$, respectively. If Theorem 1 in this paper is applied, however, then we can conclude that system (26) is mean-square exponentially stable for all delays smaller than $h_{\mathrm{M}} = 0.9421$. It is clear that the reduction of the conservatism by our method is quite significant when compared with Chen et al. (2009) and Huang and Mao (2009).

It should be noted that the numbers of decision variables to be determined in Chen et al. (2009), Huang and Mao (2009) and Theorem 1 of this paper are $\frac{17n^2+5n}{2}$, $\frac{37n^2+7n}{2}$ and $3n^2 + 2n$, respectively. Thus, our result is more computationally efficient than Chen et al. (2009) and Huang and Mao (2009), and it can be used more easily to implement design objectives for stochastic delay systems.

Table 1
τ_M of Example 2.

h	1.5	2.0	2.5	3.0	3.5
τ_M	1.6009	1.4937	1.3731	1.2455	1.1173

Example 2. Consider system (20) with

$$C = \begin{bmatrix} -0.6 & 0 \\ 0 & 0.5 \end{bmatrix}, \qquad A = \begin{bmatrix} -0.4 & 0.2 \\ 0 & -0.5 \end{bmatrix},$$
$$B = \begin{bmatrix} -0.5 & -0.2 \\ 0 & 0.3 \end{bmatrix}, \qquad F = G = 0.2I. \tag{27}$$

Given the value of the discrete delay h, the maximal allowable upper bounds of the neutral delay τ_M of this example obtained by Theorem 2 of this paper are listed in Table 1. However, even if $h = 0.5$, Theorem 1 of Huang and Mao (2009) is still infeasible for system (27).

4. Conclusions

The mean-square exponential stability for stochastic neutral-type systems has been investigated in this paper. A new stability condition has been established based on the Lyapunov–Krasovskii functional method. Different from most of the existing results, the proposed result has been deduced without involving any model transformations, cross-terms bounding techniques and free-weighting matrices. Instead, the generalized Finsler lemma (GFL) has been used in the derivation. The GFL covers the standard Finsler lemma as its special case, and it is applicable to the analysis and design of stochastic systems with delays. Further, the GFL has been used in stability analysis for a class of stochastic neutral systems whose neutral delay is not equal to the discrete delay, and the obtained result is dependent not only upon the discrete delay but also upon the neutral delay. The two numerical examples have illustrated the usefulness of the proposed method.

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