Stabilization of a Class of Switched Linear Neutral Systems Under Asynchronous Switching

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***Abstract*—This technical note concerns the stabilization problem for a class of switched linear neutral systems in which time delays appear in both the state and the state derivatives. In addition, the switching signal of the switched controller also involves time delays, which makes the switching between the controller and the system asynchronous. Based on a new integral inequality and the piecewise Lyapunov-Krasovskii functional technique, a condition for global uniform exponential stability of the switched neutral system under an average dwell time (ADT) scheme is proposed. Then, the corresponding solvability condition for the controller is established. Finally, a numerical example is given to illustrate the effectiveness of the proposed theory.**

***Index Terms*—Asynchronous switching, average dwell time (ADT), switched neutral systems, time delays.**

I. Introduction

In practice, many system models are described by functional differential equations of neutral type. The models involve not only time delays in the state but also in the state derivatives [1]–[4]. Increasing efforts have been devoted to cope with this type of systems [5]–[8]. On the other hand, as a special class of hybrid systems, switched systems have been attracting considerable attention due to the significance both in theory development and practical applications. For stability analysis and synthesis of switched systems, a large number of results have been obtained, see the papers [9]–[13]. These results have been extended to switched delay systems [14], [15]. If each subsystem of a switched system is a neutral system, then the switched system is called a switched neutral system. In fact, switched neutral systems are ubiquitous in distributed networks containing lossless transmission lines, heat exchangers, population ecology, etc. For instance, a switched neutral system was exploited to model a partial element equivalent circuit in [16]. In [17], the behavior of the oilwell drilling system at the bottom end was described by a switched neutral system. Compared with the switched systems with state delays only, switched neutral systems are much more complicated and very few results have been available. The stability analysis and control synthesis for switched neutral systems were reported in [18] and the references therein.

However, only in ideal cases, the controller's switching is synchronized with the system's switching. In many situations, when the switching signal available to the controller is a delayed version of the system's switching, the closed-loop system can have asynchronous switching. For instance, when the system and the controller communicate via a communication channel, it requires some time to identify the active system and apply the corresponding controller [19]–[25]. Obviously, this is a case that is more realistic and of great significance to study. The necessity of considering asynchronous switching for controller design was shown in digital control loops [22]. In this case, the uncertain nature of the switching signal is due to the inherent inter-sample switching problem. Moreover, for a multi-agent network with switching topology and communication delays, time delays also exist in switching signal. In [26], asynchronous switching of switched linear systems with single input was discussed. Based on the norm estimation of state transmission matrix, the stability condition of the closed-loop system under a dwell time scheme was provided. In [27], the asynchronously switched control problem for a class of switched linear systems with average dwell time (ADT) was concerned. But the switched linear system does not involve state delays and neutral delays both in [26] and [27]. In addition, the constant switching delays are rare in practical systems. Work [28] addressed switched systems with both state delays and uncertain switching delays, but all subsystems are required to be exponentially stable not only with the matched controllers, but also with the mismatched controllers. Time-varying switching delay was considered in [19] for a class of time-delay feedback switched linear systems. In contrast with the case with state delays only, with neutral delays only, and the case with switching delays only, due to the delayed state and the delayed switching signal lie in two different types of sets [19], how to deal with the case where the state delays, neutral delays, and switching delays coexist is a challenging issue [24]. Almost no results on this topic have been reported by now, which motivates our present study.

The purpose of this technical note is to study the stabilization problem for a class of switched neutral systems. The contribution of this technical note lies in three aspects. First, we consider a more general class of switched systems, in which time delays not only appear in the state and the state derivatives, but also appear in the controller's switching signal. This class of systems cover those considered in [17], [19], [20], and [28] as special cases, where only state delays or switched delays, or neutral delays are involved. Second, through choosing a Lyapunov-Krasovskii functional of an integral type, and by further allowing the Lyapunov-Krasovskii functional to increase during the running time of the active subsystem with the mismatched controller, no subsystem is required to be stable during the mismatched periods. Moreover, the Lyapunov-Krasovskii functional is allowed to increase both at the switching instants of the system and the switching instants of the controller. Third, we derive a sufficient condition to guarantee the global uniform exponential stability (GUES) of the closed-loop system under an ADT scheme, and the corresponding existence condition for stabilizing controllers is established. To show the effectiveness of the obtained results, we present a relevant example.

The organization of the technical note is as follows. The problem formulation is stated in Section II, followed by the main results in Section III. A numerical example is given in Section IV. Section V contains conclusion.

Notations: Throughout this technical note, R^n denotes the n-dimensional Euclidean space. For a matrix P, $P > 0$ means that P is positive definite; $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and minimum eigenvalues of P, respectively. I and 0 denote the identity matrix and zero matrix with appropriate dimensions, respectively . P^T and P^{-1} denote the transpose and the inverse of a square matrix P. $\|\cdot\|$ denotes Euclidean vector norm; $*$ denotes the symmetry part of a symmetry matrix; $diag\{\ldots\}$ stands for a block-diagonal matrix. $\mathbb{N}$ denotes the set of nonnegative integer numbers, and define $\mathbb{N}^+ = \mathbb{N}/\{0\}$.

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II. Problem Formulation

Consider the following switched neutral system:

$$\begin{aligned}\dot{x}(t) &= A_{\sigma(t)}x(t) + B_{\sigma(t)}x(t-\tau) + C_{\sigma(t)}\dot{x}(t-h) \\ &\quad + D_{\sigma(t)}u(t), \qquad (1)\\ x(\theta) &= \psi(\theta),\ \forall\theta \in [-H, 0],\ H = \max\{\tau, h\} \qquad (2)\end{aligned}$$

where $x(t) \in R^n$ is the state, $u(t) \in R^p$ is the input; h and τ are time delays; $\psi(\theta)$ is the initial vector function that is continuously differentiable on $[-H, 0]$; $\sigma(t) : [0, \infty) \longrightarrow \mathcal{M} = \{1, 2, \ldots, m\}$ is a piecewise constant function of time t, called switching signal. Corresponding to $\sigma(t)$, we have the switching sequence

$$\{x_{t_0} : (l_0, t_0), (l_1, t_1), \ldots, (l_i, t_i), \ldots, \mid l_i \in \mathcal{M},\ i \in \mathbb{N}\}$$

which means that the l_ith subsystem is active when $t \in [t_i, t_{i+1})$. A_{l_i}, B_{l_i}, C_{l_i}, D_{l_i} are known real constant matrices of appropriate dimensions. We assume that the state of the system does not jump at the switching instants and that only finitely many switches can occur in any finite interval.

In this technical note, the control signal going into the plant is of the form

$$u(t) = K_{\sigma(t-\tau_d(t))}x(t) \qquad (3)$$

where $\tau_d(t)$ is the uncertain switching delay, satisfying $0 \leq \tau_d(t) \leq \tau_d$. Here we assume that the maximal switching delay τ_d is known a priori without loss of generality, and $\tau_d \leq t_{i+1} - t_i, i \in \mathbb{N}$.

Hence the corresponding closed-loop system is given by

$$\begin{aligned}\dot{x}(t) = (A_{\sigma(t)} + D_{\sigma(t)}K_{\sigma(t-\tau_d(t))})x(t) + B_{\sigma(t)}x(t-\tau)\\ +C_{\sigma(t)}\dot{x}(t-h).\end{aligned}$$

We introduce the following definitions, which will be used in the sequel.

Definition 1: [14]: The equilibrium $x^* = 0$ of the system (1) is said to be globally uniformly exponentially stable (GUES) under $\sigma(t)$, if the solution $x(t)$ of system (1) satisfies

$$\|x(t)\| \leq \kappa e^{-\lambda(t-t_0)}\|x(t_0)\|_H,\ \forall t \geq t_0$$

for constants $\kappa > 0$ and $\lambda > 0$, where

$$\|x(t)\|_H = \sup_{-H \leq \theta \leq 0}\{\|x(t+\theta)\|, \|\dot{x}(t+\theta)\|\}.$$

Definition 2: [29]: For any $T_2 > T_1 \geq 0$, let $N_\sigma(T_1, T_2)$ denote the number of switching of $\sigma(t)$ over (T_1, T_2). If

$$N_\sigma(T_1, T_2) \leq N_0 + \frac{T_2 - T_1}{\tau_a}$$

holds for $\tau_a > 0$ and $N_0 \geq 0$, then τ_a is called average dwell time (ADT) and N_0 is called a chatter bound.

Lemma 1: (Schur Complement): For given

$$S = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} < 0$$

where $S_{11} = S_{11}^T, S_{22} = S_{22}^T$, the following are equivalent:

$$\begin{aligned}&(1)\ \ S_{11} < 0, S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0;\\ &(2)\ \ S_{22} < 0, S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0.\end{aligned}$$

Lemma 2: [30]: For any constant matrix $0 < R = R^T \in R^{n\times n}$, scalar $r > 0$, vector function $\omega : [0, r] \longrightarrow R^n$ such that the integrations concerned are well defined, then

$$\left(\int_0^r \omega(s)ds\right)^T R \left(\int_0^r \omega(s)ds\right) \leq r\int_0^r \omega^T(s)R\omega(s)ds.$$

Lemma 3: [31]: Let $x(t) \in R^n$ be a vector-valued function with first-order continuous-derivative entries. Then the following integral inequality holds for any matrices $N_1, N_2 \in R^{n\times n}$ and $X = X^T > 0$, and a scalar $h \geq 0$:

$$\begin{aligned}&-\int_{t-h}^{t} \dot{x}^T(s)X\dot{x}(s)ds\\ &\leq \xi^T(t)\begin{bmatrix} N_1^T + N_1 & -N_1^T + N_2 \\ * & -N_2^T - N_2 \end{bmatrix}\xi(t)\\ &\quad + h\xi^T(t)\begin{bmatrix} N_1^T \\ N_2^T \end{bmatrix} X^{-1}\begin{bmatrix} N_1 & N_2 \end{bmatrix}\xi(t)\end{aligned}$$

where $\xi^T(t) = [\,x^T(t) \quad x^T(t-h)\,]$.

III. Main Results

In this section, we present conditions for the existence of the stabilizing controller for GUES of the switched neutral system (1).

Theorem 1: Consider the switched neutral system (1). Assume $\|C_{l_i}\| < 1,\ \forall l_i \in \mathcal{M}$ and let $h \geq 0, \tau \geq 0, \tau_d \geq 0, \alpha > 0, \beta > 0$, and $\mu \geq 1$ be given constants. Suppose that there exist matrices $\bar{P}_{l_i} > 0, \bar{Q}_{l_i} > 0, \bar{S}_{l_i} > 0, \bar{P}_{l_il_j} > 0, \bar{Q}_{l_il_j} > 0, \bar{S}_{l_il_j} > 0$, and $M_{l_i}, \forall l_i, l_j \in \mathcal{M}, l_i \neq l_j$, such that

$$\begin{aligned}&\Sigma_{l_i}\\ &= \begin{bmatrix} \Sigma_{11} & B_{l_i}\bar{Q}_{l_i} & C_{l_i}\bar{S}_{l_i} & \bar{P}_{l_i}A_{l_i}^T + M_{l_i}^T D_{l_i}^T & \bar{P}_{l_i} \\ * & -e^{-\alpha\tau}\bar{Q}_{l_i} & 0 & \bar{Q}_{l_i}B_{l_i}^T & 0 \\ * & * & -e^{-\alpha h}\bar{S}_{l_i} & \bar{S}_{l_i}C_{l_i}^T & 0 \\ * & * & * & -\bar{S}_{l_i} & 0 \\ * & * & * & * & -\bar{Q}_{l_i} \end{bmatrix}\\ &< 0, \qquad (4)\end{aligned}$$

$$\begin{aligned}&\Pi_{l_il_j}\\ &= \begin{bmatrix} \Pi_{11} & B_{l_i}\bar{Q}_{l_il_j} & C_{l_i}\bar{S}_{l_il_j} & 0 & \Pi_{15} & \Pi_{16} & \bar{P}_{l_il_j} \\ * & -e^{-\alpha\tau}\bar{Q}_{l_il_j} & 0 & 0 & 0 & \bar{Q}_{l_il_j}B_{l_i}^T & 0 \\ * & * & -e^{-\alpha h}\bar{S}_{l_il_j} & 0 & 0 & \bar{S}_{l_il_j}C_{l_i}^T & 0 \\ * & * & * & \Pi_{44} & 0 & 0 & 0 \\ * & * & * & * & \Pi_{55} & 0 & 0 \\ * & * & * & * & * & -\bar{S}_{l_il_j} & 0 \\ * & * & * & * & * & * & -\bar{Q}_{l_il_j} \end{bmatrix}\\ &< 0, \qquad (5)\end{aligned}$$

$$\begin{aligned}&\bar{P}_{l_il_j} \leq \mu\bar{P}_{l_i},\ \bar{Q}_{l_il_j} \leq \mu\bar{Q}_{l_i},\ \bar{S}_{l_il_j} \leq \mu\bar{S}_{l_i},\\ &\bar{P}_{l_j} \leq \mu\bar{P}_{l_il_j},\ \bar{Q}_{l_j} \leq \mu\bar{Q}_{l_il_j},\ \bar{S}_{l_j} \leq \mu\bar{S}_{l_il_j} \qquad (6)\end{aligned}$$

where $\Sigma_{11} = A_{l_i}\bar{P}_{l_i} + \bar{P}_{l_i}A_{l_i}^T + D_{l_i}M_{l_i} + M_{l_i}^T D_{l_i}^T + \alpha\bar{P}_{l_i}, \Pi_{11} = A_{l_i}\bar{P}_{l_il_j} + \bar{P}_{l_il_j}A_{l_i}^T + D_{l_i}M_{l_j}\bar{P}_{l_j}^{-1}\bar{P}_{l_il_j} + \bar{P}_{l_il_j}\bar{P}_{l_j}^{-1}M_{l_j}^T D_{l_i}^T - \beta\bar{P}_{l_il_j}$, $\Pi_{15} = (\alpha+\beta)e^{-\alpha h}\bar{P}_{l_il_j}, \Pi_{16} = \bar{P}_{l_il_j}A_{l_i}^T + \bar{P}_{l_il_j}\bar{P}_{l_j}^{-1}M_{l_j}^T D_{l_i}^T$, $\Pi_{44} = -\alpha + \beta/\tau e^{-\alpha\tau}\bar{Q}_{l_il_j}, \Pi_{55} = -2(\alpha+\beta)e^{-\alpha h}I + h(\alpha + \beta)e^{-\alpha h}\bar{S}_{l_il_j}$.

Then the controller (3) can guarantee that system (1) is GUES for any switching signal with ADT satisfying

$$\tau_a > \tau_a^* = \frac{2\ln\mu + (\alpha+\beta)\tau_d}{\alpha}. \qquad (7)$$

Moreover, the controller gains are given by $K_{l_i} = M_{l_i}\bar{P}_{l_i}^{-1}$.

Proof: Due to the switching delay, the controller K_{l_j}, i.e., $K_{l_{i-1}}$, is still active for the time $\tau_d(t_i)$ after the l_jth subsystem has been switched to the l_ith subsystem. Thus, we have, for all $l_i, l_j \in \mathcal{M}$, $l_i \neq l_j$

$$\dot{x}(t) = \begin{cases} (A_{l_0} + D_{l_0}K_{l_0})x(t) + B_{l_0}x(t-\tau) + C_{l_0}\dot{x}(t-h) \\ t \in [t_0, t_1) \\ (A_{l_i} + D_{l_i}K_{l_j})x(t) + B_{l_i}x(t-\tau) + C_{l_i}\dot{x}(t-h) \\ t \in [t_i, t_i + \tau_d(t_i)), i \in \mathbb{N}^+ \\ (A_{l_i} + D_{l_i}K_{l_i})x(t) + B_{l_i}x(t-\tau) + C_{l_i}\dot{x}(t-h) \\ t \in [t_i + \tau_d(t_i), t_{i+1}), i \in \mathbb{N}^+. \end{cases} \quad (8)$$

We consider the following piecewise Lyapunov-Krasovskii functional $V(t) = V_{\tilde{\sigma}}(t)$ with

$$V_{\tilde{\sigma}}(t) = x^T(t)P_{\tilde{\sigma}}x(t) + \int_{t-\tau}^{t} x^T(s)e^{\alpha(s-t)}Q_{\tilde{\sigma}}x(s)ds + \int_{t-h}^{t} \dot{x}^T(s)e^{\alpha(s-t)}S_{\tilde{\sigma}}\dot{x}(s)ds \quad (9)$$

where

$$P_{\tilde{\sigma}}^{-1} = \bar{P}_{\tilde{\sigma}},\ Q_{\tilde{\sigma}}^{-1} = \bar{Q}_{\tilde{\sigma}},\ S_{\tilde{\sigma}}^{-1} = \bar{S}_{\tilde{\sigma}}$$
$$\tilde{\sigma} = \begin{cases} l_0, & t \in [t_0, t_1) \\ l_i l_j, & t \in [t_i, t_i + \tau_d(t_i)),\ i \in \mathbb{N}^+ \\ l_i, & t \in [t_i + \tau_d(t_i), t_{i+1}),\ i \in \mathbb{N}^+. \end{cases}$$

For any $l_i \in \mathcal{M}$, along the trajectories of (8), we have

$$\dot{V}_{l_i}(t) + \alpha V_{l_i}(t) = \xi^T(t)\bar{\Sigma}_{l_i}\xi(t)$$

where

$$\bar{\Sigma}_{l_i} = \begin{bmatrix} \bar{\Sigma}_{11} & \bar{\Sigma}_{12} & P_{l_i}C_{l_i} + A_{l_i}^T S_{l_i}C_{l_i} + K_{l_i}^T D_{l_i}^T S_{l_i}C_{l_i} \\ * & -e^{-\alpha\tau}Q_{l_i} + B_{l_i}^T S_{l_i}B_{l_i} & B_{l_i}^T S_{l_i}C_{l_i} \\ * & * & -e^{-\alpha h}S_{l_i} + C_{l_i}^T S_{l_i}C_{l_i} \end{bmatrix}$$
$$\xi^T(t) = [\,x^T(t) \quad x^T(t-\tau) \quad \dot{x}^T(t-h)\,]$$

where $\bar{\Sigma}_{11} = P_{l_i}(A_{l_i} + D_{l_i}K_{l_i}) + (A_{l_i} + D_{l_i}K_{l_i})^T P_{l_i} + \alpha P_{l_i} + Q_{l_i} + (A_{l_i}^T + K_{l_i}^T D_{l_i}^T)S_{l_i}(A_{l_i} + D_{l_i}K_{l_i}), \bar{\Sigma}_{12} = P_{l_i}B_{l_i} + A_{l_i}^T S_{l_i}B_{l_i} + K_{l_i}^T D_{l_i}^T S_{l_i}B_{l_i}$.

Multiplying both sides of $\bar{\Sigma}_{l_i} < 0$ by $diag\{\bar{P}_{l_i}, \bar{Q}_{l_i}, \bar{S}_{l_i}\}$, using Lemma 1, and letting $K_{l_i}\bar{P}_{l_i} = M_{l_i}$, we have that $\bar{\Sigma}_{l_i} < 0$ is equivalent to $\Sigma_{l_i} < 0$, which implies

$$\dot{V}_{l_i}(t) + \alpha V_{l_i}(t) \leq 0. \quad (10)$$

For any $l_i,\ l_j \in \mathcal{M}$, $l_i \neq l_j$, along the trajectories of (8), from Lemma 2, we have

$$\begin{aligned} &\dot{V}_{l_i l_j}(t) - \beta V_{l_i l_j}(t) \\ &\leq 2x^T(t)P_{l_i l_j}[(A_{l_i} + D_{l_i}K_{l_j})x(t) \\ &\quad + B_{l_i}x(t-\tau) + C_{l_i}\dot{x}(t-h)] \\ &\quad + \dot{x}^T(t)S_{l_i l_j}\dot{x}(t) - \beta x^T(t)P_{l_i l_j}x(t) \\ &\quad + x^T(t)Q_{l_i l_j}x(t) \\ &\quad - x^T(t-\tau)e^{-\alpha\tau}Q_{l_i l_j}x(t-\tau) \\ &\quad - \dot{x}^T(t-h)e^{-\alpha h}S_{l_i l_j}\dot{x}(t-h) \\ &\quad - (\alpha+\beta)e^{-\alpha h}\int_{t-h}^{t} \dot{x}^T(s)S_{l_i l_j}\dot{x}(s)ds \\ &\quad - (\alpha+\beta)\frac{e^{-\alpha\tau}}{\tau}\int_{t-\tau}^{t} x^T(s)dsQ_{l_i l_j}\int_{t-\tau}^{t} x(s)ds. \end{aligned}$$

It follows from Lemma 3 with $N_1 = 0$, $N_2 = I$:

$$\begin{aligned} &-(\alpha+\beta)e^{-\alpha h}\int_{t-h}^{t} \dot{x}^T(s)S_{l_i l_j}\dot{x}(s)ds \\ &\leq (\alpha+\beta)e^{-\alpha h}\{\eta^T(t)\begin{bmatrix} 0 & I \\ * & -2I \end{bmatrix}\eta(t) \\ &\quad + hx^T(t-h)\bar{S}_{l_i l_j}x(t-h)\} \end{aligned}$$

where $\eta(t) = \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}$.

Considering above inequality, we can obtain

$$\dot{V}_{l_i l_j}(t) - \beta V_{l_i l_j}(t) \leq \zeta^T(t)\bar{\Pi}_{l_i l_j}\zeta(t)$$

where we get (11), as shown at the bottom of the page, where $\bar{\Pi}_{11} = P_{l_i l_j}A_{l_i} + A_{l_i}^T P_{l_i l_j} + P_{l_i l_j}D_{l_i}K_{l_j} + K_{l_j}^T D_{l_i}^T P_{l_i l_j} + Q_{l_i l_j} - \beta P_{l_i l_j} + (A_{l_i}^T + K_{l_j}^T D_{l_i}^T)S_{l_i l_j}(A_{l_i} + D_{l_i}K_{l_j})$, $\bar{\Pi}_{12} = P_{l_i l_j}B_{l_i} + A_{l_i}^T S_{l_i l_j}B_{l_i} + K_{l_j}^T D_{l_i}^T S_{l_i l_j}B_{l_i}$, $\bar{\Pi}_{13} = P_{l_i l_j}C_{l_i} + A_{l_i}^T S_{l_i l_j}C_{l_i} + K_{l_j}^T D_{l_i}^T S_{l_i l_j}C_{l_i}$, $\bar{\Pi}_{22} = -e^{-\alpha\tau}Q_{l_i l_j} + B_{l_i}^T S_{l_i l_j}B_{l_i}$, $\bar{\Pi}_{33} = -e^{-\alpha h}S_{l_i l_j} + C_{l_i}^T S_{l_i l_j}C_{l_i}$, $\bar{\Pi}_{55} = \Pi_{55}$.

Multiplying both sides of $\bar{\Pi}_{l_i l_j} < 0$ by

$$diag\{\bar{P}_{l_i l_j}, \bar{Q}_{l_i l_j}, \bar{S}_{l_i l_j}, \bar{Q}_{l_i l_j}, I\}$$

and using Lemma 1, and the gains $K_{l_i} = M_{l_i}\bar{P}_{l_i}^{-1}$, we know that $\bar{\Pi}_{l_i l_j} < 0$ is equivalent to $\Pi_{l_i l_j} < 0$, which in turn implies

$$\dot{V}_{l_i l_j}(t) - \beta V_{l_i l_j}(t) \leq 0. \quad (12)$$

Combining (6), (10) with (12), for any $t \in [t_i, t_i + \tau_d(t_i)), i \in \mathbb{N}^+$, we have

$$\begin{aligned} &V_{l_i l_j}(t) \\ &\leq e^{\beta(t-t_i)}V_{l_i l_j}(t_i) \\ &\leq \mu e^{\beta(t-t_i)}V_{l_j}(t_i^-) \\ &\leq \mu e^{\beta(t-t_i)}e^{-\alpha[t_i-(t_{i-1}+\tau_d(t_{i-1}))]} \end{aligned}$$

$$\bar{\Pi}_{l_i l_j} = \begin{bmatrix} \bar{\Pi}_{11} & \bar{\Pi}_{12} & \bar{\Pi}_{13} & 0 & (\alpha+\beta)e^{-\alpha h}I \\ * & \bar{\Pi}_{22} & B_{l_i}^T S_{l_i l_j}C_{l_i} & 0 & 0 \\ * & * & \bar{\Pi}_{33} & 0 & 0 \\ * & * & * & -\frac{\alpha+\beta}{\tau}e^{-\alpha\tau}Q_{l_i l_j} & 0 \\ * & * & * & * & \bar{\Pi}_{55} \end{bmatrix}$$
$$\zeta^T(t) = [\,\xi^T(t) \quad \int_{t-\tau}^{t} x^T(s)ds \quad x^T(t-h)\,] \quad (11)$$

$$
\begin{aligned}
&\times V_{l_j}(t_{i-1}+\tau_d(t_{i-1}))\\
&\leq \mu e^{\beta(t-t_i)} e^{-\alpha[t_i-(t_{i-1}+\tau_d)]} V_{l_j}(t_{i-1}+\tau_d(t_{i-1}))\\
&\leq \cdots\\
&\leq \mu^{2i-1} e^{i\beta\tau_d} e^{-\alpha(t-t_0-i\tau_d)} V_{l_0}(t_0)\\
&\leq \frac{1}{\mu} e^{[2\ln\mu+\alpha\tau_d+\beta\tau_d]N_0}\\
&\quad\times e^{[2\ln\mu/\tau_a+\alpha(\tau_d/\tau_a-1)+\beta\tau_d/\tau_a](t-t_0)} V_{l_0}(t_0). \qquad (13)
\end{aligned}
$$

Similarly, for any $t \in [t_i+\tau_d(t_i), t_{i+1})$, $i \in \mathbb{N}$, we have

$$
\begin{aligned}
&V_{l_i}(t)\\
&\quad\leq e^{-\alpha(t-(t_i+\tau_d(t_i)))} V_{l_i}(t_i+\tau_d(t_i))\\
&\quad\leq \mu e^{-\alpha(t-(t_i+\tau_d))} V_{l_i l_j}((t_i+\tau_d(t_i))^-)\\
&\quad\leq \cdots\\
&\quad\leq \mu^{2i} e^{i\beta\tau_d} e^{-\alpha(t-t_0-i\tau_d)} V_{l_0}(t_0)\\
&\quad\leq e^{[2\ln\mu+\alpha\tau_d+\beta\tau_d]N_0}\\
&\qquad\times e^{[2\ln\mu/\tau_a+\alpha(\tau_d/\tau_a-1)+\beta\tau_d/\tau_a](t-t_0)} V_{l_0}(t_0). \qquad (14)
\end{aligned}
$$

Noticing (9), and using

$$\int_{t-\tau}^{t} x^T(s)Tx(s)ds \leq \lambda_{\max}(T)\tau\|x(t)\|_\tau^2,\ T>0$$

we have

$$
\begin{aligned}
a\|x(t_0)\|^2 &\leq V_{l_0}(t_0) \leq b\|x(t_0)\|_H^2\\
a\|x(t)\|^2 &\leq V_{l_i}(t)\\
&\leq b\|x(t)\|_H^2,\ t\in[t_i+\tau_d(t_i), t_{i+1})\\
a\|x(t)\|^2 &\leq V_{l_i l_j}(t)\\
&\leq b\|x(t)\|_H^2,\ t\in[t_i, t_i+\tau_d(t_i)) \qquad (15)
\end{aligned}
$$

where

$$
\begin{aligned}
a &= \min_{l_i\neq l_j}\{\lambda_{\min}(P_{l_i}), \lambda_{\min}(P_{l_i l_j})\}\\
b &= \max_{l_i\neq l_j}\{\lambda_{\max}(P_{l_i}), \lambda_{\max}(P_{l_i l_j})\}\\
&\quad+\tau\max_{l_i\neq l_j}\{\lambda_{\max}(Q_{l_i}), \lambda_{\max}(Q_{l_i l_j})\}\\
&\quad+h\max_{l_i\neq l_j}\{\lambda_{\max}(S_{l_i}), \lambda_{\max}(S_{l_i l_j})\}.
\end{aligned}
$$

Thus, from (13)–(15), we have, for any $t\in[t_i, t_{i+1})$, $i\in\mathbb{N}$, (16), as shown at the bottom of the page.

Therefore, if (7) holds, the switched neutral system (1) is GUES. In addition, if the inequalities (4)–(6) have a feasible solution, the stabilizing controller gains are given by $K_{l_i} = M_{l_i}\bar{P}_{l_i}^{-1}$.

Remark 1: The Lyapunov functional is allowed to increase both at the switching instants t_i and $t_i+\tau_d(t_i)$. Besides on the mismatched time interval $[t_i, t_i+\tau_d(t_i))$, the Lyapunov functional can also increase.

Remark 2: If the state delay and neutral delay are removed from the system (1) in our technical note, the system exactly degenerates to the one studied in [20]. Even for such special system, our results are still less conservative by allowing the Lyapunov function to increase at the switching instants $t_i+\tau_d(t_i)$. However, in [20], at the switching instants $t_i+\tau_d(t_i)$, the function is required to be continuous.

Remark 3: It is noticed that the matrix inequalities (4) and (5) are coupled. We can firstly solve (4) to gain $\bar{P}_{l_j}, M_{l_j}$ for $\forall l_j\in\mathcal{M}$, then, we solve (5) by substituting $\bar{P}_{l_j}, M_{l_j}$ into (5). By adjusting the parameter α, β, μ appropriately, we seek the feasible solutions $\bar{P}_{l_j}, \bar{Q}_{l_j}, \bar{S}_{l_j}, M_{l_j}, \bar{P}_{l_i l_j}, \bar{Q}_{l_i l_j}$ and $\bar{S}_{l_i l_j}$ such that (4) and (5) hold. If for the chosen parameters α, β, μ, (4) and (5) have no feasible solution, we can adjust α to be smaller or β, μ to be larger. According to (7), a smaller choice of μ will result in a smaller value of τ_a. Following this guideline, the solution to the matrix inequalities (4) and (5) can be found. This strategy has been commonly used in the literature to design controllers for asynchronous switching, see [19], [24], and [28].

When the switching delay $\tau_d(t)=0$, i.e., the controller's switching and the system's switching is synchronous, we have the following corollary.

Corollary 1: Consider the switched neutral system (1). Assume $\|C_{l_i}\|<1$, $\forall l_i\in\mathcal{M}$ and let $h\geq 0$, $\tau\geq 0$, $\alpha>0$, $\beta>0$, and $\mu\geq 1$. Suppose that there exist matrices $\bar{P}_{l_i}>0$, $\bar{Q}_{l_i}>0$, $\bar{S}_{l_i}>0$, and M_{l_i}, $\forall l_i\in\mathcal{M}$, such that (4) holds with $\bar{P}_{l_j}\leq\mu\bar{P}_{l_i}, \bar{Q}_{l_j}\leq\mu\bar{Q}_{l_i}, \bar{S}_{l_j}\leq\mu\bar{S}_{l_i}$. Then the controller (3) can guarantee that system (1) is GUES for any switching signal with ADT satisfying $\tau_a>\tau_a^*=\ln\mu/\alpha$. The controller gains are given by $K_{l_i}=M_{l_i}\bar{P}_{l_i}^{-1}$.

When $C_{l_i}=0$ and $h=0$ in switched neutral system (1), we have the following corollary.

Corollary 2: Consider the switched system (1) with $C_{l_i}=0$ and $h=0$. Let $\tau\geq 0$, $\tau_d\geq 0$, $\alpha>0$, $\beta>0$, and $\mu\geq 1$ be given constants. Suppose that there exist matrices $\bar{P}_{l_i}>0, \bar{Q}_{l_i}>0, \bar{P}_{l_i l_j}>0, \bar{Q}_{l_i l_j}>0$ and M_{l_i}, $\forall l_i, l_j\in\mathcal{M}, l_i\neq l_j$, such that

$$
\begin{bmatrix}
\Sigma_{11} & B_{l_i}\bar{Q}_{l_i} & \bar{P}_{l_i}\\
* & -e^{-\alpha\tau}\bar{Q}_{l_i} & 0\\
* & * & -\bar{Q}_{l_i}
\end{bmatrix}<0 \qquad (17)
$$

$$
\begin{bmatrix}
\Pi_{11} & B_{l_i}\bar{Q}_{l_i l_j} & 0 & \bar{P}_{l_i l_j}\\
* & -e^{-\alpha\tau}\bar{Q}_{l_i l_j} & 0 & 0\\
* & * & -\frac{\alpha+\beta}{\tau}e^{-\alpha\tau}\bar{Q}_{l_i l_j} & 0\\
* & * & * & -\bar{Q}_{l_i l_j}
\end{bmatrix}<0, \qquad (18)
$$

$$
\begin{aligned}
&\bar{P}_{l_i l_j}\leq\mu\bar{P}_{l_i},\ \bar{Q}_{l_i l_j}\leq\mu\bar{Q}_{l_i},\\
&\bar{P}_{l_j}\leq\mu\bar{P}_{l_i l_j},\ \bar{Q}_{l_j}\leq\mu\bar{Q}_{l_i l_j}. \qquad (19)
\end{aligned}
$$

Then, the controller (3) can guarantee that system (1) with $C_{l_i}=0$ and $h=0$ is GUES for any switching signal with ADT satisfying (7), where Σ_{11} and Π_{11} are defined in Theorem 1. Moreover, the controller gains are given by $K_{l_i}=M_{l_i}\bar{P}_{l_i}^{-1}$.

Remark 4: In [28], the stabilization problem of switched systems with state delays under asynchronous switching was addressed, but all subsystems are required to be exponentially stable with both matched and mismatched controllers. In this technical note, by choosing integral

$$
\|x(t)\|\leq
\begin{cases}
\sqrt{\frac{b}{\mu a}}e^{[\ln\mu+\alpha\tau_d/2+\beta\tau_d/2]N_0}e^{[\ln\mu/\tau_a+\alpha/2(\tau_d/\tau_a-1)+\beta\tau_d/2\tau_a](t-t_0)}\\
\|x(t_0)\|_H,\ t\in[t_i, t_i+\tau_d(t_i)),\\
\sqrt{\frac{b}{a}}e^{[\ln\mu+\alpha\tau_d/2+\beta\tau_d/2]N_0}e^{[\ln\mu/\tau_a+\alpha/2(\tau_d/\tau_a-1)+\beta/2\tau_d/\tau_a](t-t_0)}\\
\|x(t_0)\|_H,\ t\in[t_i+\tau_d(t_i), t_{i+1}).
\end{cases} \qquad (16)
$$

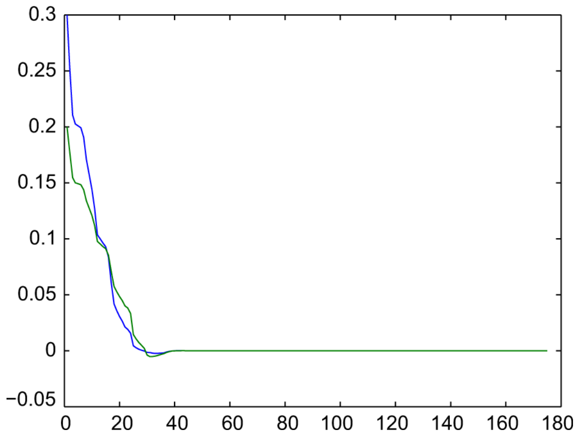

Fig. 1. State response of the closed-loop system with $\tau_d = 0$, $\tau_a^* = 0.5108$ and controller gains (22).

types of Lyapunov-Krasovskii functional, no subsystem is required to be stable with the mismatched controllers.

IV. Numerical Example

In this section, we present an example to illustrate the effectiveness of the proposed method.

Consider the switched neutral system (1) with two subsystems, $Subsystem 1$, as folows:

$$A_1 = \begin{bmatrix} 9 & -0.2 \\ 0 & 2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -0.5 & 0 \\ 0.1 & -0.2 \end{bmatrix}$$
$$C_1 = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0.3 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{20}$$

$Subsystem 2$:

$$A_2 = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -0.4 & 0.1 \\ 0 & -0.2 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}. \tag{21}$$

For $\tau = 0.3$, $h = 0.3$, $\alpha = 1$, $\mu = 1.6667$, assuming $\tau_d = 0$, and solving the conditions in Corollary 1, we can obtain the minimum ADT $\tau_a^* = 0.5108$ and the controller gains

$$K_1 = \begin{bmatrix} -10.6329 & 0.0000 \\ 0.2000 & -3.6329 \end{bmatrix}$$
$$K_2 = \begin{bmatrix} -0.6062 & 0.4209 \\ 1.0271 & -0.4123 \end{bmatrix}. \tag{22}$$

Constructing a possible switching sequence satisfying $\tau_a = 0.5109 > \tau_a^*$, one gets the steady-state response of the closed-loop system with $x_{t_0} = [0.3 \quad 0.2]^T$ as shown in Fig. 1. Then, if there exists switching delay $\tau_d(t) \equiv 0.23$ in the switching signal of the controller, the state response of the closed-loop system is given, see Fig. 2. From Fig. 2, we can see the state response is diverging.

Now, turning to using Theorem 1, for the same τ, h, α, choosing $\mu = 312$, we can obtain a different minimum ADT and the corresponding controller gains for different β and τ_d. It can be tested that a smaller ADT may be impracticable for asynchronous switching. If choosing $\beta = 21$, we obtain the minimum ADT $\tau_a^* = 16.5460$ and the corresponding controller gains

$$K_1 = \begin{bmatrix} -10.8111 & -0.0460 \\ 0.3271 & -3.6725 \end{bmatrix}$$

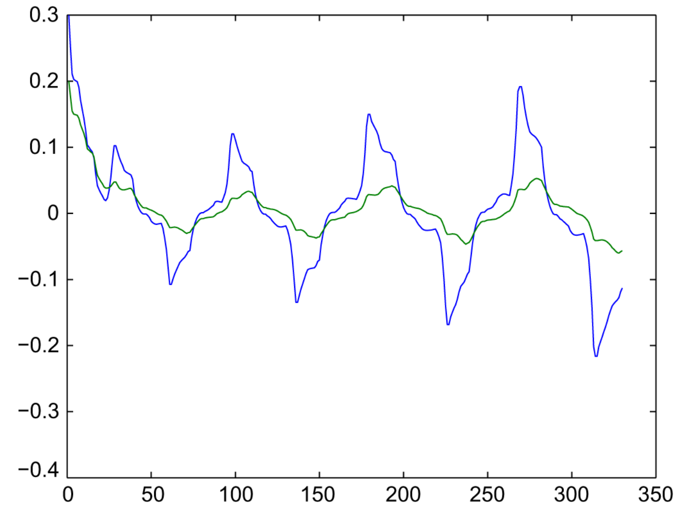

Fig. 2. State response of the closed-loop system with $\tau_d(t) \equiv 0.23$, $\tau_a^* = 0.5108$ and controller gains (22).

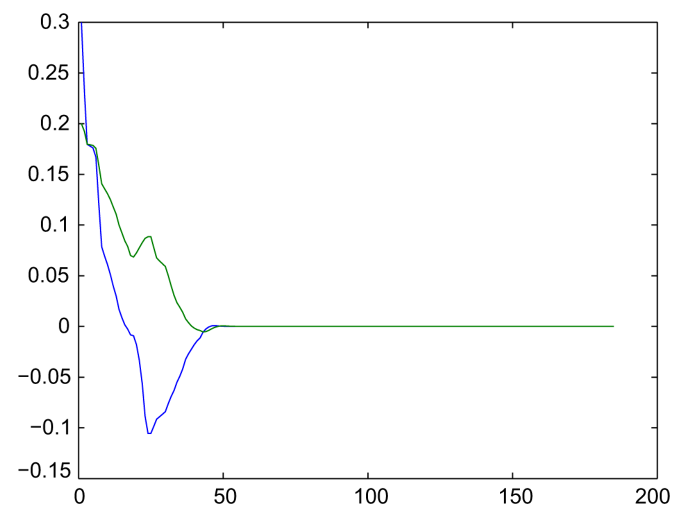

Fig. 3. State response of the closed-loop system with $\tau_d(t) = 0.23$, $\tau_a^* = 16.5460$ and controller gains (23).

$$K_2 = \begin{bmatrix} -0.5725 & -0.1725 \\ 0.8098 & 0.6545 \end{bmatrix}. \tag{23}$$

The state response of the closed-loop system is shown in Fig. 3.

V. Conclusion

We have investigated the stabilization problem for a class of switched neutral systems. The switching signal of the switched controller involves time delays, which leads to the asynchronous switching between the systems and the candidate controllers. By further allowing the Lyapunov-Krasovskii functional to increase during the running time of the active subsystem with the mismatched controllers, we derive a sufficient condition to guarantee the GUES of the closed-loop system under an ADT scheme. Then, the corresponding solvability condition for desired controllers is established. The asynchronous stabilization problem for switched neutral systems is a difficult issue that is far from being well explored. Since multiple Lyapunov function approach is commonly considered less conservative, the design of controllers for switching neutral system with an appropriate switching law using multiple Lyapunov functions is of great significance which deserves further study.

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Boundary Feedback Stabilization of Periodic Fluid Flows in a Magnetohydrodynamic Channel

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***Abstract*—In this technical note, an electrically conducting 2-D channel fluid flow, in the presence of a transverse magnetic field, is investigated. The governing equations are the magnetohydrodynamics equations, which are a coupling between the Navier-Stokes and Maxwell equations. The stability of the Hartmann-Poiseuille profile is achieved by finite-dimensional feedback controllers acting on both normal components of the velocity field and of the magnetic field, on the upper wall only.**

***Index Terms*—Control design, magnetohydrodynamics, stability.**

I. INTRODUCTION

In this note, we consider a 2-D channel flow of an incompressible electrically conducting fluid driven by a pressure gradient and affected by a constant transverse magnetic field B_0. This kind of flow was first investigated both experimentally and theoretically by Hartmann [4].

The governing equations are the magnetohydrodynamic equations (MHD, for short), which are a combination between the Navier-Stokes

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