Nonlinear Analysis: Hybrid Systems 33 (2019) 380–392

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

Asynchronous stabilization of switched neutral systems: A cooperative stabilizing approach

Guopin Liu [a], Changchun Hua [a,*], Xinping Guan [b]

[a] Institute of Electrical Engineering, Yanshan University, 066004, Qinhuangdao, China
[b] Department of Automation, Shanghai Jiao Tong University, 200240, Shanghai, China

ARTICLE INFO

Article history:
Received 1 September 2017
Accepted 26 December 2018
Available online 15 February 2019

Keywords:
Switched neutral systems
Asynchronous switching
Unstable subsystems
Cooperative stabilizing

ABSTRACT

This paper considers the stabilization problem for a class of switched neutral systems under asynchronous switching. A cooperative stabilization approach is presented, which means that controllers and switching signals are designed cooperatively to stabilize the switched system. Based on the Lyapunov–Krasovskii functional approach, a sufficient condition is provided to guarantee the global asymptotical stability of the switched neutral systems. Meanwhile, the controller gain is obtained by solving the established linear matrix inequalities (LMIs). The proposed approach allows the Lyapunov-like function to increase not only at the running time in mode-identifying process but also in normal-working period with matched controller. Thus, switched systems, which contain unstable and uncontrollable subsystems, can be stabilized by the proposed scheme. A numerical example further demonstrates the validity of the developed results.

1. Introduction

Time delay is an unavoidable phenomenon in various practical systems, which may lead to unsatisfactory performances and even cause instabilities in dynamic systems. Consequently, considerable attention has been paid to the stability analysis and control for time-delay systems [1–4]. As a special class of delay systems, neutral systems contain delays not simply in the states, but also in the derivatives of the states. The neutral delays usually arise when state-derivative feedback or output-derivative feedback is provided in systems which have delay in the input [5]. Neutral systems exist in numerous practical engineering systems. Applications can be found in the areas of transmission line oscillator [6], partial element equivalent circuits [7] and so on. Therefore, increasing efforts have been paid to address this kind of systems, see, for example [5,8–10] and the references therein.

On the other hand, switched systems are one important class of hybrid systems, which consist of a family of subsystems (also called system modes) and a switching law governing the switching among them. It provided an unified framework for mathematical modeling of numerous physical systems [11]. In the past few decades, switched systems have attracted considerable attention in the last few decades. Some early works and general introduction can be found in [12–20] and the references therein. Switched neutral system is a switched system in which the subsystems are neutral systems (for details one can refer to [21–23]). Since switching signal plays an key role in this kind of systems, considerable methodologies have been proposed with some specified classes of switching signals, especially with (average) dwell time [24]. For instance, via average dwell time and descriptor system approach, [25] addressed the exponential stability analysis problem for switched neutral systems with mixed time-delays. However, average dwell time approach generally requires that the average active

* Corresponding author.
E-mail address: cch@ysu.edu.cn (C. Hua).

https://doi.org/10.1016/j.nahs.2018.12.006
1751-570X/

time between consecutive switchings no less than a constant, which means the number of switches on any time interval can grow at most as an affine function of the interval length. Furthermore, although many works have been reported for switched systems with unstable subsystems [26–28], few of them consider the neutral aspects in their systems.

Generally, in the previous results on switched systems, a very common assumption is that the candidate controllers are switched synchronously with the system modes. However, only in ideal cases, switchings of the controllers and the system modes are coincident. In practice, it takes time to identify the system modes and apply the matched controller, and so there generally exist the phenomena of asynchronous switching between system modes and controller candidates. In this work, the notion "asynchronous" means that there is a time lag or delay between switching of the controller and the system modes. The research for asynchronous switching is also motivated by a wide range of applications, such as in PWM-driven boost converter [29], networked control system [30], etc. Hence, some works have been reported on the investigation of asynchronous switching. [31] concerned the robust control for switched nonlinear systems with both uncertain switching delays and state delays, but all subsystems are required to be exponentially stable with both matched and mismatched controllers. In [23,32] and [33], authors derived a sufficient condition to guarantee the global uniform exponential stability (GUES) for a class of switched systems under an ADT scheme, and the result of [31] was relaxed that no subsystem is required to be stable during the mismatched periods as well. However, exponential stability is still required for all subsystems in the normal-working period with matched controllers.

Motivated by the aforementioned discussions, this paper investigates the asynchronously stabilization problem for a class of switched linear neutral systems. A sufficient condition for global asymptotical stability is presented under a cooperative stabilizing switching signal. Moreover, the controller gain is achieved by solving established LMIs. Compared with existing literatures, the contribution of this paper lies in two aspects. First, we permit the existence of unstable subsystems in the normal-working period, which relax the requirement in [23,32,33] that all the Lyapunov function must be decreasing with matched controllers. Second, we adopt a novel switching scheme which is superior to traditional average dwell time scheme. More specifically, the switching signal can have a switching frequency that grows linearly with time.

The rest of this paper is organized as follows. Systems formulation and preliminaries are introduced in Section 2. The main results are presented in Section 3. Then a numerical simulation is given in Section 4. Section 5 concludes the work of this paper.

2. Problem formulation and preliminaries

Consider the switched neutral system described by the following equations:

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}x(t) + B_{\sigma(t)}x(t-r) + C_{\sigma(t)}\dot{x}(t-h) \\ &\quad + D_{\sigma(t)}u(t) \\ x(\theta) &= \psi(\theta), \forall\theta \in [-H, 0], H = \max\{t, h\} \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^p$ are the state vector and the control input, respectively. $\sigma(t) : [0, +\infty) \to \mathcal{P}$ is a piecewise constant function that specifies the index of the active system with $\mathcal{P} := \{1, 2, \ldots, N\}$ being a finite index set and N as the number of subsystems; h and r are the state delay and neutral delay, respectively. $\psi(\theta)$ is the continuously differentiable initial vector function on $[-H, 0]$. In this paper, both the Zeno and impulsive conditions are assumed to be excluded consideration for the switched system (1). The switching sequence is expressed as

$$\aleph_p = \{\big(\sigma(t_0), t_0\big), \ldots \big(\sigma(t_k), t_k\big), \ldots | \sigma(t_k) \in \mathcal{P}, k = 0, 1, \ldots\}$$

where t_0 is the initial time. The $\sigma(t_k)$th subsystem is active when $t \in [t_k, t_{k+1})$ and t_k is the switching instant. For $i \in \mathcal{P}$, A_i, B_i, C_i, D_i are known real constant matrices of appropriate dimensions.

In the presence of asynchronous switching, the candidate controllers are in the form of $u(t) = K_{\sigma(t-\tau(t))}x(t)$, where $\tau(t)$ represents the switching delay and satisfying $0 < \tau(t) < \tau_d < t_{k+1} - t_k, \forall k \in \mathbb{N}$. Thus, the corresponding closed-loop system turns as

$$\begin{aligned} \dot{x}(t) &= (A_{\sigma(t)} + D_{\sigma(t)}K_{\sigma(t-\tau(t))})x(t) + B_{\sigma(t)}x(t-r) \\ &\quad + C_{\sigma(t)}\dot{x}(t-h) \end{aligned} \tag{2}$$

For the closed-loop system (2), we will verify its asymptotic stability by proving $\lim_{t\to\infty} x(t) = 0$. Now we introduce the following assumptions for later development.

Assumption 1. For all $i, j \in \mathcal{P}$, there exists constant $\mu_{ij} > 0$ and the respective Lyapunov-like functions have the relationship as

$$V_j(x(t)) \leq \mu_{ij}V_i(x(t)) \;\; \forall x(t) \in \mathbb{R}^n \tag{3}$$

whenever the switching from the ith subsystem to the jth subsystem.

In this paper, we adopt a novel class of switching signals to address the stabilization problem of the considered switched system in the presence of asynchronous switching. Now, we present some preliminaries on the switching signals.

Define $h : [0, +\infty) \to [0, +\infty)$ as a continuous monotone strictly increasing function, satisfying $h(0) = 0$ and $\lim_{t\to+\infty} h(t) = +\infty$, i.e., $h(t) \in \mathcal{K}_\infty$. In this paper, $h(t)$ is a crucial function and some of the following functions are related to it, such as $\nu_t(t)$ and $\eta_i^h(j, t), i = 1, 2, j \in \mathcal{P}$. Since $h(t) \in \mathcal{K}_\infty$ and according to the definitions of $\nu_t(t)$ and $\eta_i^h(j, t), i = 1, 2, j \in \mathcal{P}$, the switching number and running time can be more general instead of affine functions on t. The detailed restriction on switching signals can be seen in the main results of this paper.

For the switched systems with asynchronous switching, denote $N_\sigma(0, t)$ as the number of switches on the interval $[0, t)$. Define the *h-frequency of switching* at t as

$$\nu_t(t) := \frac{N_{\sigma(t)}}{h(t)}, \quad t > 0 \tag{4}$$

We divide the interval into the mismatched period $\mathcal{M}_1$ and matched period $\mathcal{M}_2$. Define the kth holding time of a switching signal σ in the matched period as

$$S_{k+1} = t_{k+1} - (t_k + \tau(t_k)), \quad k = 0, 1, \ldots \tag{5}$$

Then, for each $j \in \mathcal{P}$, define the *h-frequency of activation* of subsystem j in mismatched period as

$$\eta_1^h(j, t) := \sum_{i:\sigma(t_i)=j} \frac{\tau(t_i)}{h(t)}, \quad t > 0 \tag{6}$$

In matched period, the corresponding *h-frequency of activation* of subsystem j is defined as

$$\eta_2^h(j, t) := \sum_{i:\sigma(t_i)=j} \frac{S_{i+1}}{h(t)}, \quad t > 0 \tag{7}$$

Let $E(\mathcal{P})$ be the set for admissible switch from subsystem m to subsystem n, $\forall m, n \in \mathcal{P}$, which is represented by a sequential pair (m, n). For each pair $(m, n) \in E(\mathcal{P})$, we define the *transition frequency* from the mth subsystem to the nth subsystem as

$$\rho_{mn}(t) := \frac{\sharp\{m \to n\}_t}{N_\sigma}, \quad t > 0 \tag{8}$$

where $\sharp\{m \to n\}_t$ is the transition number from subsystem m to subsystem n in the time interval $[0, t)$.

Now, define the *asymptotic upper density* of ν_h, ρ_{mn} as

$$\hat{\nu}_h := \lim_{t\to+\infty} \sup \nu_h(t) \tag{9}$$

$$\hat{\rho}_{mn} := \lim_{t\to+\infty} \sup \rho_{mn}(t) \tag{10}$$

Similarly, the *asymptotic upper densities* of $\eta_1^h(j, t)$, $\eta_2^h(j, t)$ are defined as

$$\hat{\eta}_1^h(j) : = \lim_{t\to+\infty} \sup \eta_1^h(j, t), \tag{11a}$$

$$\hat{\eta}_2^h(j) : = \lim_{t\to+\infty} \sup \eta_2^h(j, t) \tag{11b}$$

In addition, we also give the definition of *asymptotic lower densities* of $\eta_2^h(j, t)$ as

$$\check{\eta}_2^h(j) := \lim_{t\to+\infty} \inf \eta_2^h(j, t) \tag{12}$$

Let $\mathcal{P}_{m_1}$ denote the index set systems during the mode-identifying period or mismatched period. Meanwhile, denote $\mathcal{P}_{m_2}^s, \mathcal{P}_{m_2}^u \in \mathcal{P}_{m_2}$ as the stable and unstable indices sets during the normal-working period, respectively. Namely,

$$\mathcal{P}_{m_1} = \{i \in \mathcal{P} | \sigma(t) = i, \forall t \in [t_k, t_k + \tau(t_k)), k = 0, 1, \ldots\}$$
$$\mathcal{P}_{m_2} = \{i \in \mathcal{P} | \sigma(t) = i, \forall t \in [t_k + \tau(t_k), t_{k+1}), k = 0, 1, \ldots\}$$

Clearly, it holds that $\mathcal{P} = \mathcal{P}_{m_1} = \mathcal{P}_{m_2} = \mathcal{P}_{m_2}^s \cup \mathcal{P}_{m_2}^u$.

To establish the main result of this paper, we review two useful lemmas that will be used in the sequel.

Lemma 1. *For any symmetric matrix $R > 0$ and a vector function $x : [a, b] \to R^n$ such that the integrations concerned are well defined, then*

$$(a - b) \int_a^b x^T(s)Rx(s)\,\mathrm{d}s \geq \left(\int_a^b x(s)\,\mathrm{d}s\right)^T R \left(\int_a^b x(s)\,\mathrm{d}s\right)$$

Lemma 2 ([34]). *For any symmetric matrix $R > 0$ and a vector function $x : [a, b] \to \mathbf{R}^n$ such that the integrations concerned are well defined, then*

$$\int_a^b \int_\theta^b \dot{x}^T(s)R\dot{x}(s)\,\mathrm{d}s\mathrm{d}u \geq \frac{1}{(b-a)^2}(2\varpi_1^T R\varpi_1 + 4\varpi_2^T R\varpi_2)$$

where

$$\varpi_1^T = (b-a)x^T(b) - \int_a^b x^T(s)\,\mathrm{d}s$$
$$\varpi_2^T = -\frac{(b-a)}{2}x^T(b) - \int_a^b x^T(s)\,\mathrm{d}s + \frac{3}{(b-a)}\int_a^b\int_\theta^b x^T(s)\,\mathrm{d}s\mathrm{d}\theta$$

3. Main results

Now, we are ready to present the existence condition of a set of asynchronous stabilizing controllers for the system (1).

Theorem 1. *Consider the switched neural system (1). Suppose that there exist class $\mathcal{K}_\infty$ functions $\kappa_1(\|x\|)$, $\kappa_2(\|x\|)$, continuously differentiable functions $V_{\sigma(t)} : \mathbf{R}^n \to \mathbf{R}$ satisfying Assumption 1 and $\kappa_1(\|x\|) \leq V_{\sigma(t)}(x) \leq \kappa_2(\|x\|)$. Let α_i, β_i be given constants with $\beta_i > 0$ for $i \in \mathcal{P}_{m1}$, $\alpha_i > 0$ for $i \in \mathcal{P}_{m2}^s$ and $\alpha_i < 0$ for $i \in \mathcal{P}_{m2}^u$, respectively. If there exist matrices $\bar{P}_i > 0, \bar{Q}_i > 0, \bar{R}_i > 0, \bar{S}_i > 0, \bar{M}_i > 0, \bar{W}_i > 0, U_i, \bar{P}_j > 0, \bar{Q}_j > 0, \bar{R}_j > 0, \bar{S}_j > 0, \bar{M}_j > 0, U_j$ for $\forall i, j \in \mathcal{P}, i \neq j$ such that*

$$\bar{\Pi}_i = \begin{bmatrix} \bar{\Pi}_{i,11} & \bar{\Pi}_{i,12} & C_i\bar{W}_i & \frac{1}{h}e^{\frac{-\alpha_i h}{2}}\bar{M}_i & \bar{\Pi}_{i,15} \\ * & \bar{\Pi}_{i,22} & 0 & 0 & \bar{W}_i^T B_i^T \\ * & * & -e^{-\alpha_i h}\bar{S}_i & 0 & \bar{W}_i^T C_i^T \\ * & * & * & -\frac{1}{h}e^{-\alpha_i h}\bar{M}_i & 0 \\ * & * & * & * & \bar{\Pi}_{i,55} \end{bmatrix} < 0 \tag{13}$$

$$\bar{\Psi}_j = \begin{bmatrix} \bar{\Psi}_{j,11} & 0 & \frac{6(\alpha_j+\beta_j)}{r^2}e^{a_{r,j}}\bar{R}_j & 0 & \bar{\Psi}_{j,15} \\ * & \bar{\Psi}_{j,22} & \frac{12(\alpha_j+\beta_j)}{r^3}e^{a_{r,j}}\bar{R}_j & 0 & 0 \\ * & * & \bar{\Psi}_{j,33} & 0 & 0 \\ * & * & * & \bar{\Psi}_{j,44} & \bar{\Psi}_{j,45} \\ * & * & * & * & \bar{\Psi}_{j,55} \end{bmatrix} < 0 \tag{14}$$

where $\bar{\Pi}_{i,11} = \alpha_i\bar{P}_i + \bar{Q}_i - \frac{1}{r}\bar{R}_i - \frac{1}{h}\bar{M}_i + A_i\bar{W}_i + \bar{W}_i^T A_i^T + D_iU_i + U_i^T D_i^T$, $\bar{\Pi}_{i,12} = \frac{1}{r}e^{\frac{-\alpha_i r}{2}}\bar{R}_i + B_i\bar{W}_i$, $\bar{\Pi}_{i,15} = \bar{P}_i + \bar{W}_i^T A_i^T + U_i^T D_i^T - \bar{W}_i$, $\bar{\Pi}_{i,22} = e^{-\alpha_i r}(-\frac{1}{r}\bar{R}_i - \bar{Q}_i)$, $\bar{\Pi}_{i,55} = r\bar{R}_i + \bar{S}_i + h\bar{M}_i - \bar{W}_i^T - \bar{W}_i$

$$\bar{\Psi}_{j,11} = \begin{bmatrix} \bar{\Omega}_{j,11} & \bar{\Omega}_{j,12} & C_j\bar{W}_i & \frac{1}{h}e^{\frac{-\alpha_j h}{2}}\bar{M}_j & \bar{\Omega}_{j,15} \\ * & \bar{\Omega}_{j,22} & 0 & 0 & \bar{W}_i^T B_j^T \\ * & * & -e^{-\alpha_j h}\bar{S}_i & 0 & \bar{W}_i^T C_j^T \\ * & * & * & -\frac{1}{h}e^{-\alpha_j h}\bar{M}_j & 0 \\ * & * & * & * & \bar{\Omega}_{j,55} \end{bmatrix}$$

$\bar{\Omega}_{j,11} = -\beta_j\bar{P}_j + \bar{Q}_j - \frac{1}{r}\bar{R}_j - \frac{1}{h}\bar{M}_j - 3(\alpha_j+\beta_j)e^{a_{r,j}}\bar{R}_j - 3(\alpha_j+\beta_j)e^{a_{h,j}}\bar{M}_j + A_j\bar{W}_i + \bar{W}_i^T A_j^T + D_jU_i + U_i^T D_j^T$, $\bar{\Omega}_{j,12} = \frac{1}{r}e^{\frac{-\alpha_j r}{2}}\bar{R}_j + B_j\bar{W}_i$, $\bar{\Omega}_{j,15} = \bar{P}_j + \bar{W}_i^T A_j^T + U_i^T D_j^T - \bar{W}_i$, $\bar{\Omega}_{j,22} = e^{-\alpha_j r}(-\frac{1}{r}\bar{R}_j - \bar{Q}_j)$, $\bar{\Omega}_{j,55} = r\bar{R}_j + \bar{S}_j + h\bar{M}_j - \bar{W}_i^T - \bar{W}_i$.

$\bar{\Psi}_{j,15} = \frac{6}{h^2}(\alpha_j+\beta_j)e^{a_{h,j}}\bar{M}_j$, $\bar{\Psi}_{j,22} = (\alpha_j+\beta_j)e^{a_{r,j}}(-\frac{1}{r}\bar{Q}_j - \frac{6}{r^2}\bar{R}_j)$ $\bar{\Psi}_{j,33} = -\frac{36}{r^4}(\alpha_j+\beta_j)e^{a_{r,j}}\bar{R}_j$, $\bar{\Psi}_{j,44} = (\alpha_j+\beta_j)e^{a_{h,j}}(-\frac{1}{h}\bar{S}_j - \frac{6}{h^2}\bar{M}_j)$, $\bar{\Psi}_{j,45} = \frac{12}{h^3}(\alpha_j+\beta_j)e^{a_{h,j}}\bar{M}_j$, $\bar{\Psi}_{j,55} = -\frac{36}{h^4}(\alpha_j+\beta_j)e^{a_{h,j}}\bar{M}_j$, Then, the controller $u(t) = K_{\sigma(t-\tau(t))}x(t)$ can guarantee the global asymptotical stability of the closed-loop system (2) under the cooperative switching signals satisfying

$$\check{\nu}_h := \lim_{t\to+\infty} \inf \nu_h(t) > 0 \tag{15}$$

and

$$\begin{aligned}&\hat{\nu}_h \sum_{(m,n)\in E(\mathcal{P})} \hat{\rho}_{mn} \ln \mu_{mn} + \sum_{i\in\mathcal{P}^u_{m2}} |\alpha_i|\hat{\eta}^h_2(i) - \\ &\sum_{i\in\mathcal{P}^s_{m2}} |\alpha_i|\check{\eta}^h_2(i) + \sum_{i\in\mathcal{P}} |\beta_i|\hat{\eta}^h_1(i) < 0\end{aligned} \tag{16}$$

Moreover, the admissible controller gain can be given by

$$K_i = U_i\bar{W}_i^{-1} \tag{17}$$

Proof. For $(i, j) \in E(\mathcal{P})$, consider the asynchronous switching case. When the subsystem j has been switched, the controller K_i is still active instead of K_j for $\tau(t_{k+1})$, $\sigma(t_{k+1}) = j$. Hence, $\forall i, j \in \mathcal{P}$, $i \neq j$, it holds that

$$\dot{x} = \begin{cases} (A_0 + D_0K_0)x(t) + B_0x(t-r) + C_0\dot{x}(t-h) \\ t \in [t_0, t_1) \\ (A_i + D_iK_i)x(t) + B_ix(t-r) + C_i\dot{x}(t-h) \\ t \in [t_k + \tau(t_k), t_{k+1}), \quad k \in \mathbb{N}^+ \\ (A_j + D_jK_i)x(t) + B_jx(t-r) + C_j\dot{x}(t-h) \\ t \in [t_{k+1}, t_{k+1} + \tau(t_{k+1})), \quad k \in \mathbb{N}^+ \end{cases} \tag{18}$$

Now, consider the piecewise Lyapunov–Krasovskii functional given as follows

$$\begin{aligned} V_{\sigma(t)}(t) =& x^T(t)P_{\sigma(t)}x(t) + \int_{t-r}^{t} e^{\alpha_i(s-t)}x^T(s)Q_{\sigma(t)}x(s)\,\mathrm{d}s \\ &+ \int_{t-r}^{t}\int_{\theta}^{t} e^{\alpha_i(s-t)}\dot{x}^T(s)R_{\sigma(t)}\dot{x}\,\mathrm{d}s\mathrm{d}\theta \\ &+ \int_{t-h}^{t} e^{\alpha_i(s-t)}\dot{x}^T(s)S_{\sigma(t)}\dot{x}(s)\,\mathrm{d}s \\ &+ \int_{t-h}^{t}\int_{\theta}^{t} e^{\alpha_i(s-t)}\dot{x}^T(s)M_{\sigma(t)}\dot{x}(s)\,\mathrm{d}s\mathrm{d}\theta \end{aligned} \tag{19}$$

where $P_{\sigma(t)}, Q_{\sigma(t)}, R_{\sigma(t)}, S_{\sigma(t)}, M_{\sigma(t)}$ are positive matrices, which will be specified later.

Along the trajectories of (18), consider the following two conditions.

(i) For any $\sigma(t) = i \in \mathcal{P}$, $t \in [t_k + \tau(t_k), t_{k+1}]$, subsystem i is active with matched controller. Along the solution of (18), we have

$$\begin{aligned} \dot{V}_i(t) + \alpha_iV_i =& 2x^T(t)P_i\dot{x}(t) + \alpha_ix^T(t)P_ix(t) + x^T(t)Q_ix(t) \\ &- e^{-\alpha_i r}x^T(t-r)Q_ix(t-r) + r\dot{x}^T(t)R_i\dot{x}(t) \\ &- \int_{t-r}^{t} \dot{x}^T(\theta)e^{\alpha_i(\theta-t)}R_i\dot{x}(\theta)\,\mathrm{d}\theta + \dot{x}^T(t)S_i\dot{x}(t) \\ &- e^{-\alpha_i h}\dot{x}^T(t-h)S_i\dot{x}(t-h) + h\dot{x}^T(t)M_i\dot{x}(t) \\ &- \int_{t-h}^{t} \dot{x}^T(\theta)e^{\alpha_i(\theta-t)}M_i\dot{x}(\theta)\,\mathrm{d}\theta \end{aligned} \tag{20}$$

Based on Lemma 1, it holds

$$\begin{aligned} &- \int_{t-r}^{t} \dot{x}^T(\theta)e^{\alpha_i(\theta-t)}R_i\dot{x}(\theta)\,\mathrm{d}\theta \\ &\leq -\frac{1}{r}\left(\int_{t-r}^{t} \dot{x}^T(\theta)e^{\frac{\alpha_i(\theta-t)}{2}}\,\mathrm{d}\theta\right) R_i \left(\int_{t-r}^{t} \dot{x}(\theta)e^{\frac{\alpha_i(\theta-t)}{2}}\,\mathrm{d}\theta\right) \\ &= -\frac{1}{r}(x^T(t) - e^{\frac{-\alpha_i r}{2}}x^T(t-r))R_i(x(t) - e^{\frac{-\alpha_i r}{2}}x(t-r)) \end{aligned} \tag{21}$$

Similarly,

$$-\int_{t-h}^{t}\dot{x}^T(\theta)e^{\alpha_i(\theta-t)}M_i\dot{x}(\theta)\,d\theta$$
$$\leq -\frac{1}{h}(x^T(t)-e^{\frac{-\alpha_i h}{2}}x^T(t-h))M_i(x(t)-e^{\frac{-\alpha_i h}{2}}x(t-h)) \tag{22}$$

In addition, for any matrix W_i with appropriate dimensions, the following condition holds

$$-2[x^T(t),\dot{x}^T(t)]W_i^T[\dot{x}(t)-(A_i+D_iK_i)x(t)-B_ix(t-r)$$
$$-C_i\dot{x}(t-h)]=0 \tag{23}$$

Substituting (21)–(23) into (20), it is easy to deduce

$$\dot{V}_i(t)+\alpha_iV_i\leq\omega^T(t)\Pi_i\omega(t) \tag{24}$$

where $\omega^T(t)=[x^T(t),x^T(t-r),\dot{x}^T(t-h),x^T(t-h),\dot{x}(t)]$,

$$\Pi_i=\begin{bmatrix}\Pi_{i,11} & \Pi_{i,12} & W_i^TC_i & \frac{1}{h}e^{\frac{-\alpha_i h}{2}}M_i & \Pi_{i,15}\\ * & \Pi_{i,22} & 0 & 0 & B_i^TW_i\\ * & * & -e^{-\alpha_i h}S_i & 0 & C_i^TW_i\\ * & * & * & -\frac{1}{h}e^{-\alpha_i h}M_i & 0\\ * & * & * & * & \Pi_{i,55}\end{bmatrix}$$

$\Pi_{i,11}=\alpha_iP_i+Q_i-\frac{1}{r}R_i-\frac{1}{h}M_i+W_i^T(A_i+D_iK_i)+(A_i+D_iK_i)^TW_i$, $\Pi_{i,12}=\frac{1}{r}e^{\frac{-\alpha_i r}{2}}R_i+W_i^TB_i$, $\Pi_{i,15}=P_i+(A_i+D_iK_i)^TW_i-W_i^T$, $\Pi_{i,22}=e^{-\alpha_i r}(-\frac{1}{r}R_i-Q_i)$, $\Pi_{i,55}=rR_i+S_i+hM_i-W_i^T-W_i$

To further facilitate the solution problem by LMI approach, define $\bar{W}_i=W_i^{-1}$ and multiply the left-hand side and right-hand side of $\Pi_i<0$ by diag$\{\bar{W}_i^T,\ldots,\bar{W}_i^T\}$ and diag$\{\bar{W}_i,\ldots,\bar{W}_i\}$, respectively. Letting $K_i=U_i\bar{W}_i^{-1}$, $\bar{P}_i=\bar{W}_i^TP_i\bar{W}_i$, $\bar{Q}_i=\bar{W}_i^TQ_i\bar{W}_i$, $\bar{R}_i=\bar{W}_i^TR_i\bar{W}_i$, $\bar{S}_i=\bar{W}_i^TS_i\bar{W}_i$, $\bar{M}_i=\bar{W}_i^TP_i\bar{M}_i$, we can verify that $\Pi_i<0$ is equivalent to the inequality (13) in Theorem 1, which indicates

$$\dot{V}_i+\alpha_iV_i\leq 0 \tag{25}$$

(ii) For any $t\in[t_{k+1},t_{k+1}+\tau(t_{k+1})]$, $\sigma(t_{k+1}^-)=i$, $\sigma(t_{k+1}^+)=j$, $i,j\in\mathcal{P}$, $i\neq j$, subsystem j is active. However, the controller for the subsystem i is still working for time $\tau(t_{k+1})$. Along the solution of (18), we have

$$\begin{aligned}\dot{V}_j-\beta_jV_j\leq\ & 2x^T(t)P_j\dot{x}(t)-\beta_jx^T(t)P_jx(t)+x^T(t)Q_jx(t)\\ & -e^{-\alpha_j r}x^T(t-r)Q_jx(t-r)+r\dot{x}^T(t)R_j\dot{x}(t)\\ & -\int_{t-r}^{t}\dot{x}^T(\theta)e^{\alpha_j(\theta-t)}R_j\dot{x}(\theta)\,d\theta+\dot{x}^T(t)S_j\dot{x}(t)\\ & -e^{-\alpha_j h}\dot{x}^T(t-h)S_j\dot{x}(t-h)+h\dot{x}^T(t)M_j\dot{x}(t)\\ & -\int_{t-h}^{t}\dot{x}^T(\theta)e^{\alpha_j(\theta-t)}M_j\dot{x}(\theta)\,d\theta\\ & -(\alpha_j+\beta_j)e^{a_{r,j}}\int_{t-r}^{t}x^T(s)Q_jx(s)\,ds\\ & -(\alpha_j+\beta_j)e^{a_{r,j}}\int_{t-r}^{t}\int_{\theta}^{t}\dot{x}^T(s)R_j\dot{x}(s)\,dsd\theta\\ & -(\alpha_j+\beta_j)e^{a_{h,j}}\int_{t-h}^{t}\dot{x}^T(s)S_j\dot{x}(s)\,ds\\ & -(\alpha_j+\beta_j)e^{a_{h,j}}\int_{t-h}^{t}\int_{\theta}^{t}\dot{x}^T(s)M_j\dot{x}(s)\,dsd\theta\end{aligned} \tag{26}$$

where $a_{r,j}=-\alpha_jr$, $a_{h,j}=-\alpha_jh$ for $j\in\mathcal{P}_{m2}^s$ and $a_{r,j}=a_{h,j}=0$ for $j\in\mathcal{P}_{m2}^u$, respectively. Based on Lemma 1, we have

$$-\int_{t-r}^{t}\dot{x}^T(\theta)e^{\alpha_j(\theta-t)}R_j\dot{x}(\theta)\,d\theta$$
$$\leq -\frac{1}{r}(x^T(t)-e^{\frac{-\alpha_j r}{2}}x^T(t-r))R_j(x(t)-e^{\frac{-\alpha_j r}{2}}x(t-r))$$

$$
\begin{aligned}
&-\int_{t-h}^{t}\dot{x}^T(\theta)e^{\alpha_j(\theta-t)}M_j\dot{x}(\theta)\,d\theta\\
&\le -\frac{1}{h}(x^T(t)-e^{\frac{-\alpha_j h}{2}}x^T(t-h))M_j(x(t)-e^{\frac{-\alpha_j h}{2}}x(t-h))\\
&-\int_{t-r}^{t}x^T(s)Q_jx(s)\,ds \le -\frac{1}{r}\int_{t-r}^{t}x^T(s)\,dsQ_j\int_{t-r}^{t}x(s)\,ds\\
&-\int_{t-h}^{t}x^T(s)S_jx(s)\,ds \le -\frac{1}{h}\int_{t-h}^{t}x^T(s)\,dsS_j\int_{t-h}^{t}x(s)\,ds
\end{aligned}
$$

According to Lemma 2, we can obtain

$$
\begin{aligned}
&-\int_{t-r}^{t}\int_{\theta}^{t}\dot{x}^T(s)R_j\dot{x}(s)\,dsd\theta\\
&\le -\Big\{\frac{2}{r^2}\big(rx^T(t)-\int_{t-r}^{t}x^T(s)\,ds\big)R_j\big(rx(t)-\int_{t-r}^{t}x(s)\,ds\big)\\
&+\frac{4}{r^2}\big(-\frac{r}{2}x^T(t)-\int_{t-r}^{t}x^T(s)ds+\frac{3}{r}\int_{t-r}^{t}\int_{\theta}^{t}x^T(s)dsd\theta\big)\\
&\times R_j\big(-\frac{r}{2}x(t)-\int_{t-r}^{t}x(s)ds+\frac{3}{r}\int_{t-r}^{t}\int_{\theta}^{t}x(s)dsd\theta\big)\Big\}
\end{aligned}
\tag{27}
$$

$$
\begin{aligned}
&-\int_{t-h}^{t}\int_{\theta}^{t}\dot{x}^T(s)M_j\dot{x}(s)\,dsd\theta\\
&\le -\Big\{\frac{2}{h^2}\big(hx^T(t)-\int_{t-h}^{t}x^T(s)\,ds\big)M_j\big(hx(t)-\int_{t-h}^{t}x(s)\,ds\big)\\
&+\frac{4}{h^2}\big(-\frac{h}{2}x^T(t)-\int_{t-h}^{t}x^T(s)ds+\frac{3}{h}\int_{t-h}^{t}\int_{\theta}^{t}x^T(s)dsd\theta\big)\\
&\times M_j\big(-\frac{h}{2}x(t)-\int_{t-h}^{t}x(s)ds+\frac{3}{h}\int_{t-h}^{t}\int_{\theta}^{t}x(s)dsd\theta\big)\Big\}
\end{aligned}
\tag{28}
$$

Similarly, for any matrix W_i with appropriate dimensions, the following condition holds

$$
\begin{aligned}
&-2[x^T(t),\dot{x}^T(t)]W_i[\dot{x}(t)-(A_j+D_jK_i)x(t)-B_jx(t-r)\\
&-C_j\dot{x}(t-h)]=0
\end{aligned}
\tag{29}
$$

Considering the above inequalities and (29), the following result can be obtained

$$
\dot{V}_j-\beta_jV_j\le\xi^T(t)\Psi_j\xi(t)
\tag{30}
$$

where $\xi^T(t)=[\omega^T(t),\int_{t-r}^{t}x^T(s)\,ds,\int_{t-r}^{t}\int_{\theta}^{t}x^T(s)\,dsd\theta,\int_{t-h}^{t}x^T(s)\,ds,$
$\int_{t-r}^{t}\int_{\theta}^{t}x^T(s)\,dsd\theta]$,

$$
\Psi_j=\begin{bmatrix}
\Psi_{j,11} & 0 & \frac{6(\alpha_j+\beta_j)}{r^2}e^{a_{r,j}}R_j & 0 & \Psi_{j,15}\\
* & \Psi_{j,22} & \frac{12(\alpha_j+\beta_j)}{r^3}e^{a_{r,j}}R_j & 0 & 0\\
* & * & \Psi_{j,33} & 0 & 0\\
* & * & * & \Psi_{j,44} & \Psi_{j,45}\\
* & * & * & * & \Psi_{j,55}
\end{bmatrix}
$$

$$
\Psi_{j,11}=\begin{bmatrix}
\Omega_{j,11} & \Omega_{j,12} & W_i^TC_j & \frac{1}{h}e^{\frac{-\alpha_j h}{2}}M_j & \Omega_{j,15}\\
* & \Omega_{j,22} & 0 & 0 & B_j^TW_i\\
* & * & -e^{-\alpha_j h}S_i & 0 & C_j^TW_i\\
* & * & * & -\frac{1}{h}e^{-\alpha_j h}M_j & 0\\
* & * & * & * & \Omega_{j,55}
\end{bmatrix}
$$

$\Omega_{j,11}=-\beta_jP_j+Q_j-\frac{1}{r}R_j-\frac{1}{h}M_j-3(\alpha_j+\beta_j)e^{a_{r,j}}R_j-3(\alpha_j+\beta_j)e^{a_{h,j}}M_j+W_i^T(A_j+D_jK_i)+(A_j+D_jK_i)^TW_i$, $\Omega_{j,12}=\frac{1}{r}e^{\frac{-\alpha_j r}{2}}R_j+W_i^TB_j$, $\Omega_{j,15}=P_j+A_j^TW_i+K_i^TD_j^TW_i-W_i^T$, $\Omega_{j,22}=e^{-\alpha_j r}(-\frac{1}{r}R_j-Q_j)$, $\Omega_{j,55}=rR_j+S_j+hM_j-W_i^T-W_i$

$\Psi_{j,15} = \frac{6}{h^2}(\alpha_j+\beta_j)e^{a_{h,j}}M_j$, $\Psi_{j,22} = (-\frac{1}{r}Q_j - \frac{6}{r^2}R_j)(\alpha_j+\beta_j)e^{a_{r,j}}$ $\Psi_{j,33} = -\frac{36}{r^4}(\alpha_j+\beta_j)e^{a_{r,j}}R_j$, $\Psi_{j,44} = (-\frac{1}{h}S_j - \frac{6}{h^2}M_j)(\alpha_j+\beta_j)e^{a_{h,j}}$ $\Psi_{j,45} = \frac{12}{h^3}(\alpha_j+\beta_j)e^{a_{h,j}}M_j$, $\Psi_{j,55} = -\frac{36}{h^4}(\alpha_j+\beta_j)e^{a_{h,j}}M_j$

Multiply the left-hand side and right-hand side of $\Psi_j < 0$ by $\mathrm{diag}\{\bar{W}_i^T, \ldots, \bar{W}_i^T\}$ and $\mathrm{diag}\{\bar{W}_i, \ldots, \bar{W}_i\}$, respectively. Letting $\bar{P}_j = \bar{W}_i^T P_j \bar{W}_i$, $\bar{Q}_j = \bar{W}_i^T Q_j \bar{W}_i$, $\bar{R}_j = \bar{W}_i^T R_j \bar{W}_i$, $\bar{S}_j = \bar{W}_i^T S_j \bar{W}_i$, $\bar{M}_j = \bar{W}_i^T M_j \bar{W}_i$, we can verify that $\Psi_j < 0$ is equivalent to the inequality (14) in Theorem 1, which indicates

$$\dot{V}_j - \beta_j V_j \leq 0 \tag{31}$$

It analyzes two conditions on time interval $[t_k + \tau(t_k), t_{k+1} + \tau(t_{k+1})]$, in which two consecutive modes are involved. Through a simple transformation and combining the above results, we can obtain the following consequence for any $t \in [t_k, t_{k+1}]$, $k \in \mathbb{N}$

$$\dot{V}_{\sigma(t)}(x(t)) \leq \begin{cases} -\alpha_{\sigma(t)}V_{\sigma(t)}(x(t)) & t \in [t_k + \tau(t_k), t_{k+1}] \\ \beta_{\sigma(t)}V_{\sigma(t)}(x(t)) & t \in [t_k, t_k + \tau(t_k)] \end{cases} \tag{32}$$

In the following, we express $t_k + \tau(t_k)$ by T_k for simplification. Then, from (32), we can obtain

$$\begin{aligned} V_{\sigma(t)}(x(t)) &\leq \exp\{-\alpha_{\sigma(T_{N_\sigma})}(t - T_{N_\sigma})\}V_{\sigma(T_{N_\sigma})}(x(T_{N_\sigma})) \\ &\leq \exp\{-\alpha_{\sigma(T_{N_\sigma})}(t - T_{N_\sigma}) + \beta_{\sigma(t_{N_\sigma})}\tau(t_{N_\sigma})\}V_{\sigma(t_{N_\sigma})}(x(t_{N_\sigma})) \end{aligned}$$

Then, in view of (3), iterating the above results in

$$V_{\sigma(t)}(x(t)) \leq \exp(\phi(t))V_{\sigma(0)}(x(0)) \tag{33}$$

where

$$\begin{aligned} \phi(t) = &\sum_{k=0}^{N_\sigma-1} \ln \mu_{\sigma(t_k)\sigma(t_{k+1})} - \alpha_{\sigma(T_{N_\sigma})}(t - T_{N_\sigma}) \\ &- \sum_{k=0}^{N_\sigma-1} \alpha_{\sigma(T_k)}(t_{k+1} - T_k) + \sum_{k=0}^{N_\sigma-1} \beta_{\sigma(t_k)}\tau(t_k) \end{aligned} \tag{34}$$

It is easy to get

$$\begin{aligned} \sum_{k=0}^{N_\sigma-1} \ln \mu_{\sigma(t_k)\sigma(t_{k+1})} &= \sum_{m\in\mathcal{P}} \sum_{k=0}^{N_\sigma-1} \sum_{\substack{m\to n \\ n\in\mathcal{P} \\ m\neq n \\ \sigma(t_k)=m \\ \sigma(t_{k+1})=n}} \ln \mu_{mn} \\ &= N_\sigma \sum_{(m,n)\in E(\mathcal{P})} \ln \mu_{mn} \frac{\sharp\{m\to n\}_t}{N_\sigma} \end{aligned} \tag{35}$$

According to (5), we have

$$\begin{aligned} &\sum_{k=0}^{N_\sigma-1} \alpha_{\sigma(T_k)}(t_{k+1} - T_k) = \sum_{k=0}^{N_\sigma-1} \alpha_{\sigma(T_k)}S_{k+1} \\ &= \sum_{k=0}^{N_\sigma-1} \left(\sum_{i\in\mathcal{P}} 1_{(i)}(\sigma(T_k))\alpha_i S_{k+1} \right) \\ &= -\sum_{i\in\mathcal{P}_{m2}^u} |\alpha_i| \sum_{k:\sigma(T_k)=i} S_{k+1} + \sum_{i\in\mathcal{P}_{m2}^s} |\alpha_i| \sum_{k:\sigma(T_k)=i} S_{k+1} \end{aligned} \tag{36}$$

Substituting (35) and (36) into (34), one has

$$\begin{aligned} \phi(t) =& N_\sigma \sum_{(m,n)\in E(\mathcal{P})} \ln \mu_{mn} \frac{\sharp\{m\to n\}_t}{N_\sigma} \\ &+ \sum_{i\in\mathcal{P}_{m2}^u} |\alpha_i| \sum_{k:\sigma(T_k)=i} S_{k+1} - \sum_{i\in\mathcal{P}_{m2}^s} |\alpha_i| \sum_{k:\sigma(T_k)=i} S_{k+1} \\ &- \alpha_{\sigma(T_{N_\sigma})}(t - T_{N_\sigma}) + \sum_{k=0}^{N_\sigma-1} \beta_{\sigma(t_k)}\tau(t_k) \\ =& h(t)\left(\frac{N_\sigma}{h(t)} \sum_{(m,n)\in E(\mathcal{P})} \ln \mu_{mn} \frac{\sharp\{m\to n\}_t}{N_\sigma} \right. \end{aligned}$$

$$
\begin{aligned}
& - \alpha_{\sigma(T_{N_\sigma})} \frac{(t - T_{N_\sigma})}{h(t)} + \sum_{i \in \mathcal{P}^u_{m2}} |\alpha_i| \sum_{k:\sigma(T_k)=i} \frac{S_{k+1}}{h(t)} \\
& - \sum_{i \in \mathcal{P}^s_{m2}} |\alpha_i| \sum_{k:\sigma(T_k)=i} \frac{S_{k+1}}{h(t)} + \sum_{k=0}^{N_\sigma - 1} \beta_{\sigma(t_k)} \frac{\tau(t_k)}{h(t)} \Bigg)
\end{aligned} \tag{37}
$$

where $h(t)$ is the mentioned function in Section 2. Define

$$
f(t) = \sum_{(m,n)\in E(\mathcal{P})} \ln \mu_{mn} \frac{\sharp\{m \to n\}_t}{N_\sigma} \tag{38}
$$

and

$$
\begin{aligned}
g(t) = \Bigg(& - \alpha_{\sigma(T_{N_\sigma})} \frac{(t - T_{N_\sigma})}{h(t)} + \sum_{i \in \mathcal{P}^u_{m2}} |\alpha_i| \sum_{k:\sigma(T_k)=i} \frac{S_{k+1}}{h(t)} \\
& - \sum_{i \in \mathcal{P}^s_{m2}} |\alpha_i| \sum_{k:\sigma(T_k)=i} \frac{S_{k+1}}{h(t)} + \sum_{k=0}^{N_\sigma - 1} \beta_{\sigma(t_k)} \frac{\tau(t_k)}{h(t)} \Bigg)
\end{aligned} \tag{39}
$$

In order to guarantee the global asymptotic convergence of the switched system (1), it requires that

$$
\lim_{t\to+\infty} \exp\{h(t)\big(\nu_h(t)f(t) + g(t)\big)\} = 0 \tag{40}
$$

Since $\lim_{t\to+\infty} h(t) = +\infty$, (40) can be satisfied if

$$
\lim_{t\to+\infty} \sup\big(\nu_h(t)f(t) + g(t)\big) < 0 \tag{41}
$$

It is easy to verify that $\limsup_{t\to+\infty}(\nu_h(t)f(t) + g(t)) \le \limsup_{t\to+\infty} \nu_h(t) \limsup_{t\to+\infty} f(t) + \limsup_{t\to+\infty} g(t)$. By (8) and (38), it holds that

$$
\limsup_{t\to+\infty} f(t) \le \sum_{(m,n)\in E(\mathcal{P})} \ln \mu_{mn} \limsup_{t\to+\infty} \rho_{mn}(t) \tag{42}
$$

The hypothesis (15) implies that the term $-\alpha_{\sigma(T_{N_\sigma})}(t - T_{N_\sigma})$ in $g(t)$ is $o(h(t))$ as $t \to +\infty$. Otherwise, it will results $\check{\nu}_h = 0$, which is a contradiction with (15). Thus, $(t - T_{N_\sigma})/h(t) \to 0$ as $t \to +\infty$. By the definition of $g(t)$, we obtain

$$
\begin{aligned}
\limsup_{t\to+\infty}\ g(t) &= \limsup_{t\to+\infty}\Bigg(\sum_{i\in\mathcal{P}^u_{m2}} |\alpha_i| \eta_2^h(i,t) + \sum_{i\in\mathcal{P}} \beta_i \eta_1^h(i,t) \\
& - \sum_{i\in\mathcal{P}^s_{m2}} |\alpha_i| \eta_2^h(i,t) \Bigg) \\
& \le \sum_{i\in\mathcal{P}^u_{m2}} |\alpha_i| \limsup_{t\to+\infty} \eta_2^h(i,t) + \sum_{i\in\mathcal{P}} |\beta_i| \limsup_{t\to+\infty} \eta_1^h(i,t) \\
& - \sum_{i\in\mathcal{P}^s_{m2}} |\alpha_i| \liminf_{t\to+\infty} \eta_2^h(i,t)
\end{aligned} \tag{43}
$$

It is obvious that a sufficient condition for (41) is

$$
\begin{aligned}
& \limsup_{t\to+\infty} \nu_h(t) \sum_{(m,n)\in E(\mathcal{P})} \ln \mu_{mn} \limsup_{t\to+\infty} \rho_{mn}(t) \\
& + \sum_{i\in\mathcal{P}^u_{m2}} |\alpha_i| \limsup_{t\to+\infty} \eta_2^h(i,t) + \sum_{i\in\mathcal{P}} |\beta_i| \limsup_{t\to+\infty} \eta_1^h(i,t) \\
& - \sum_{i\in\mathcal{P}^s_{m2}} |\alpha_i| \liminf_{t\to+\infty} \eta_2^h(i,t) < 0
\end{aligned} \tag{44}
$$

According to the definition of (9), (10), (11), (12), it can be known (44) holds in view of (16). As a result, (41) holds. According to $V_{\sigma(t)}(x(t)) \le \exp(\phi(t)) V_{\sigma(0)}(x(0))$ and $\kappa_1(\|x\|) \le V_{\sigma(t)}(x) \le \kappa_2(\|x\|)$, we have

$$
\|x(t)\| \le \kappa_1^{-1}\big(\kappa_2(\|x(0)\|)\exp(\phi(t))\big)
$$

Obviously, the initial condition x_0 is independent on $\phi(t)$ and $\sigma(t)$. Then, we conclude that for all admissible switching signal $\sigma(t)$ and initial x_0, the global asymptotic convergence can be guaranteed with $\lim_{t\to+\infty} x(t) = 0$. Thus the proof is completed. □

Remark 3. In [23], exponential stability is required for every mode under corresponding matched controller in the normal-working period, namely, $\dot{V}_{\sigma(t)}(\cdot) \le -\alpha V_{\sigma(t)}(\cdot)$ with $\alpha > 0$ for all $t \in [t_i + \tau(t_i), t_{i+1}]$. Superiorly, in (13) and (32) of this paper, unstable systems are permitted in the matched period. Specifically, it allows $\alpha_{\sigma(t)} < 0$ for $\sigma(t) \in \mathcal{P}^u_{m_2}$. Compared with $\alpha_n > 0$ for all $n \in \mathcal{P}$, the existence of $\alpha_n < 0$ for $n \in \mathcal{P}^u_{m_2}$ leads to a more tractable LMI condition in (13). Thus, directly expressed in terms of LMIs, a less conservative stability criteria is proposed for the switched neutral systems.

Remark 4. In this paper, the cooperative stabilization means stabilizing the switched systems by controller and switching signal cooperatively. Unstable and uncontrollable subsystems, which cannot be stabilized simply by the controller, are permitted under the provided switching signal. Meanwhile, the designed controller is also essential for the scheme. It can make the conditions (32) easily be satisfied, which are basis of stability analysis in this paper. In addition, the synergistic controller also can regulate the value of α_n, $n \in \mathcal{P}$, which are key parameters in the constraint (16). Thus, the designed controller can regulate the switching frequency character.

Remark 5. Inspired by [33], the free-weighting matrix W_i is introduced as described in (23), (29). By this way, the term $\dot{x}(t)$ can be treated as a variable in vector $\omega(t)$, $\xi(t)$ and thus simplify the computational complexity significantly. In addition, during the running time of the ith controller, the control gain K_i can be obtained directly by solving LMIs (13) and (14) at the same time, which are equivalent to $\Pi_i < 0$ and $\Psi_j < 0$.

Remark 6. For time-delay systems, especially the neutral systems, many results have been reported by Lyapunov–Krasovskii functional method. For instance, the single integral form of quadratic terms was constructed in [23]. Although double integral forms are constructed in the Lyapunov–Krasovskii functional in [33], the results are obtained by the inequality $\int_a^b \int_\theta^b \dot{x}^T(s)R\dot{x}(s)\,\mathrm{d}s\mathrm{d}u \ge 0$, which is more conservative than Lemma 2. In this paper, a double integral inequality is employed (see Lemma 2). Thus, compared with [23] and [33], Theorem 1 involves the information on the double integral of the system state to yields less conservative stability condition.

Remark 7. Based on the proposed cooperative stabilization method in this paper, parameters α_i are allowed to be negative for some $i \in \mathcal{P}$. In order to finish the derivation of the paper, however, $(\alpha_i + \beta_i)$ are required to be positive in inequality (26), which is a conservation for the proposed approach and needs to be further investigated.

Remark 8. The switching signal restriction are specified by inequality (16). It requires that the switching number and working period for unstable or mismatched period cannot be too much, but the working period for stable period should last for enough time. It also offers the guideline for selecting crucial parameters. For example, α_i reflects to the convergence rate of systems state. For $\alpha_i > 0$, the larger it is, the more rapid it converges, and for $\alpha_i < 0$, the larger it is, the slower it diverges. Hence, larger α_i permits better performance. However, larger value of α_i will burden the restriction in (16), even leading to failure to stabilize the system. Hence, parameters should be selected without contravention of this restriction.

4. Examples: numerical and simulation results

In this section, a numerical example is provided to show the effectiveness of the proposed approach.

Consider the switched linear neutral systems consisting of two subsystems described by

Subsystem 1:

$$A_1 = \begin{bmatrix} 1.8 & -0.3 \\ 0 & 2.5 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -0.8 & 0 \\ 0.5 & -0.2 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} -0.2 & 0.5 \\ 0.2 & 0.7 \end{bmatrix} \quad D_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Subsystem 2:

$$A_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 1.5 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.3 & 0 \\ -0.2 & -0.6 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 0.15 & 0 \\ -0.15 & 0.8 \end{bmatrix} \quad D_2 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$$

Observing the above switched system, we have $\mathcal{P}^s_{m2} = \{1\}$, $\mathcal{P}^u_{m2} = \{2\}$. Namely, for the second subsystem, it is impossible to design controller to satisfy Assumption 1 with $\alpha_2 > 0$. Hence, the schemes proposed in [23,33] cannot be applied for the above switched system directly.

Given $\tau(t) = 0.308$, $r = 0.4$, $h = 0.3$, $\alpha_1 = 1$, $\alpha_2 = -2$, $\mu_{ij} = 2$ with $i, j \in \mathcal{P}$, $i \ne j$, $\beta_1 = \beta_2 = 2$. Solving the LMIs in Theorem 1, it obtains the feasible controllers with the following gains

$$K_1 = \begin{bmatrix} -4.3178 & 0.8389 \\ -1.3983 & -8.5772 \end{bmatrix}$$

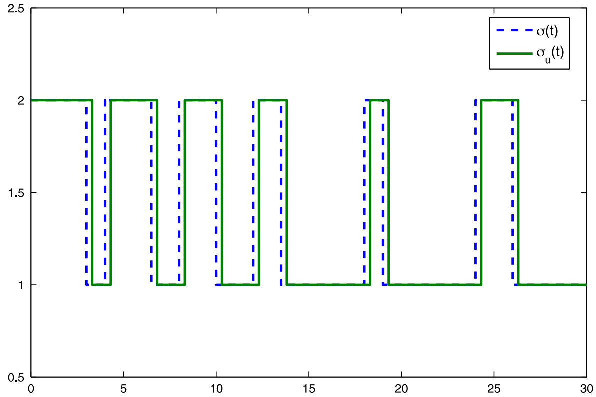

Fig. 1. An eligible switching signal $\sigma(t)$.

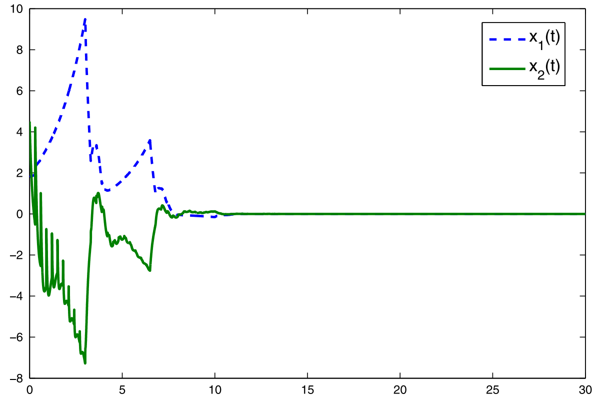

Fig. 2. State response of the closed-loop system.

$$K_2 = \begin{bmatrix} -3.8051 & 0.8461 \\ -1.0301 & -8.2361 \end{bmatrix}$$

Now, we will specify the switching signal. Let $h(t)$ be the identity function, and $N_t^{\sigma} = 0.3t + t^{\frac{1}{3}}$, $\eta_2^h(1,t) = 0.855 - t^{-\frac{1}{3}}$, $\eta_2^h(2,t) = 0.125 + t^{-\frac{1}{3}} - t^{-\frac{2}{3}}$, $\eta_1^h(1,t) = \eta_1^h(2,t) = \frac{1}{2}(0.02 + t^{-\frac{2}{3}})$. $\rho_{mn} = \frac{1}{2}$ for each pair $(m,n) \in E(\mathcal{P})$.

Give the time horizon as $t = 30$. From above specification, it yields $N_t^{\sigma} = 12$, $\eta_2^h(1,t) = 0.533$, $\eta_2^h(2,t) = 0.343$, $\eta_1^h(1,t) = \eta_1^h(2,t) = 0.062$, $\rho_{mn} = \frac{1}{2}$.

Now we verify the condition (16). The specification of $\sigma(t)$ reveals the asymptotics: $\check{\eta}_2^h(1) = 0.855$, $\hat{\eta}_2^h(2) = 0.125$, $\hat{\eta}_1^h(1) = \hat{\eta}_1^h(2) = 0.01$. $\hat{\nu}_h = 0.3$, and $\hat{\rho}_{mn} = \frac{1}{2}$ for each pair $(m,n) \in E(\mathcal{P})$. Thus,

$$\begin{aligned}
&\hat{\nu}_h \sum_{(m,n)\in E(\mathcal{P})} \hat{\rho}_{mn} \ln \mu_{mn} + \Big(\sum_{i\in\mathcal{P}_{m2}^u} |\alpha_i| \hat{\eta}_2^h(i) \\
&- \sum_{i\in\mathcal{P}_{m2}^s} |\alpha_i| \check{\eta}_2^h(i) + \sum_{i\in\mathcal{P}} |\beta_i| \hat{\eta}_1^h(i) \Big) \\
&= 0.416 - 0.425 < 0
\end{aligned}$$

Therefore, (16) holds. Based on the restriction, Fig. 1 demonstrates an eligible switching signal $\sigma(t)$.

Under the above conditions, the state response of the closed loop systems is shown in Fig. 2 with initial value $x_0 = [1.8; 4.5]$. Fig. 3 shows the control input signal. From the simulation figures, it is obvious that the proposed cooperative stabilization scheme can guarantee the stability of the closed-loop system. Thus, the numerical example illustrates the validity of the proposed method.

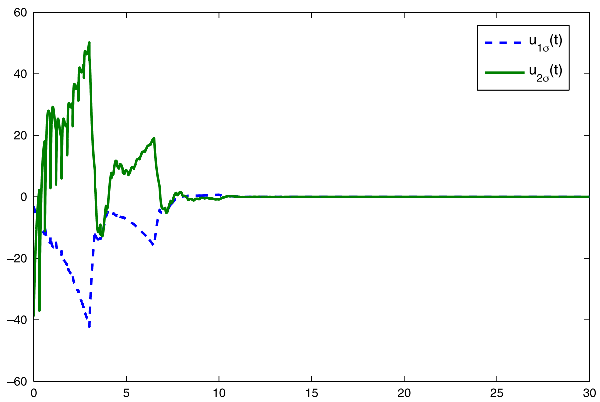

Fig. 3. The trajectory of control input $u(t)$.

5. Conclusion

The cooperative stabilization issue is investigated for a class of asynchronously switched linear neutral systems in this paper. Based on one novel kind of switching signal scheme, a sufficient condition is derived to ensure the global asymptotical stability of the considered system. The established condition allows the Lyapunov-like function to increase not only in the period of mode-identifying process but also in normal-working period with matched controller. Thus, the proposed scheme can handle the case with unstable and/or uncontrollable modes. Moreover, in the presence of asynchronous switching, the solvability condition for the cooperative controller is derived by generating admissible gains. Finally, a simulation example is provided to demonstrate the validity of the developed method. There are many other open problems on switched neutral systems, such as reducing the conversation of switching restriction and enlarge the time-delay arrange. Future work may focus on stabilizing neutral switched systems without stable or controllable modes.

Acknowledgments

This work was partially supported by National Natural Science Foundation of China (61751309, 61673335, 61603329, 61603330, 61503323), Basic Research Program of Hebei Province (F2016203467), Hebei Province Applied Basis Research Project (15967629D), Top Talents Project of Hebei Province.

References

[1] K. Gu, L. Kharitonov, J. Chen, Stability of Time-delay Systems, Berlin, Germany: Birkhauser, 2003.
[2] C. Hua, X. Guan, Output feedback stabilization for time-delay nonlinear interconnected systems using neural networks, IEEE Trans. Neural Netw. 19 (4) (2008) 673–688.
[3] X. Li, H.K. Lam, F. Liu, X. Zhao, Stability and stabilization analysis of positive polynomial fuzzy systems with time delay considering piecewise membership functions, IEEE Trans. Fuzzy Syst. 25 (4) (2017) 958–971.
[4] X. Zhang, L. Liu, G. Feng, C. Zhang, Output feedback control of large-scale nonlinear time-delay systems in lower triangular form, Automatica 49 (2013) 3476–3483.
[5] H. Ghadiri, M.R. Jahed-Motlagh, M.B. Yazdi, Robust stabilization for uncertain switched neutral systems with interval time-varying mixed delays, Nonlinear Anal. Hybrid Syst 13 (2014) 2–21.
[6] J.N. Blakely, N.J. Corron, Experimental observation of delay-induced radio frequency chaos in a transmission line oscillator, Chaos 14 (2004) 1035–1041.
[7] A. Bellen, N. Guglielmi, A.E. Ruehli, Methods for linear systems of circuit delay differential equations of neutral type, IEEE Trans. Circ. Syst. I: Fund. Theor. Appl. 46 (1) (1999) 212–215.
[8] H.R. Karimi, Robust delay-dependent H_∞ control of uncertain time-delay systems with mixed neutral, discrete, and distributed time-delays and markovian switching parameters, IEEE Trans. Circuits Syst. I, Reg. Papers 58 (8) (2011) 1910–1923.
[9] J. Lian, Y. Ge, M. Han, Stabilization for switched stochastic neutral systems under asynchronous switching, Inform. Sci. 222 (2013) 501–508.
[10] S.M. Magdi, Robust H_∞ control of linear neutral systems, Automatica 36 (5) (2000) 757–764.
[11] L. Long, J. Zhao, Decentralized adaptive fuzzy output-feedback control of switched large-scale nonlinear systems, IEEE Trans. Fuzzy Syst. 23 (5) (2015) 1844–1860.
[12] Y. Li, S. Sui, S. Tong, Adaptive fuzzy control design for stochastic nonlinear switched systems with arbitrary switchings and unmodeled dynamics, IEEE Trans. Cybern. 47 (2) (2017) 403–414.
[13] D. Liberzon, Switching in Systems and Control, Berlin: Birkhäuser, 2003.
[14] H. Lin, P.J. Antsaklis, Stabilit and stabilizability of switched linear systems: A survey of recent results, IEEE Trans. Automat. Control 54 (2) (2009) 308–322.
[15] B. Niu, H.R. Karimi, H.Q. Wang, Y.L. Liu, Adaptive outpout-feedback controller design for switched nonlinear stochastic systems with a modified average dwell-time method, IEEE Trans. Syst. Man, Cybern. Syst. 47 (7) (2017) 1371–1382.
[16] D. Zhai, A. Lu, D. Ye, Q. Zhang, Adaptive tracking control for a class of switched uncertain nonlinear systems under a new state-dependent switching law, Nonlinear Anal. Hybrid Syst 24 (2017) 227–243.
[17] X. Zhao, S. Yin, H. Li, B. Niu, Switching stabilization for a class of slowly switched systems, IEEE Trans. Automat. Control 60 (1) (2015) 221–226.

[18] T. Li, J. Fu, F. Deng, T. Chai, Stabilization of switched linear neutral systems: an event-triggered sampling scheme, IEEE Trans. Autom. Control (2018) http://dx.doi.org/10.1109/tac.2018.2797160.
[19] T. Li, J. Zhao, G.M. Dimirovski, Hysteresis switching design for stabilization of a class of switched neutral systems, Asian J. Control 15 (4) (2012) 1149–1157.
[20] T. Li, J. Zhao, G.M. Dimirovski, Stability and L_2-gain analysis for switched neutral systems with mixed time-varying delays, J. Franklin Inst. 348 (9) (2011) 2237–2256.
[21] T. Li, J. Zhao, G.M. Dimirovski, Stability and L_2-gain analysis for switched neutral systems with mixed time-varying delays, J. Franklin Inst. 348 (9) (2011) 2237–2256.
[22] S. Liu, Z. Xiang, Exponential H_∞ output tracking control for switched neutral system with time-varying delay and nonlinear perturbations, Circuits Systems Signal Process. 32 (1) (2013) 103–121.
[23] Y. Wang, J. Zhao, B. Jiang, Stabilization of a class of switched linear neutral systems under asynchronous switching, IEEE Trans. Automat. Control 58 (8) (2013) 2114–2119.
[24] X. Zhao, Y. Yin, B. Niu, X. Zheng, Stabilization for a class of switched nonlinear systems with novel average dwell time switching by T-S modeling, IEEE Trans. Cybern. 46 (8) (2016) 1952–1957.
[25] R. Krishnasamy, P. Balasubramaniam, A descriptor system approach to the delay-dependent exponential stability analysis for switched neutral systems with nonlinear perturbations, Nonlinear Anal. Hybrid Syst 15 (2015) 23–36.
[26] G. Zhai, B. Hu, K. Yasuda, A.N. Michel, Stability analysis of switched systems with stable and unstable subsystems: An average dwell time approach, Int. J. Syst. Sci. 32 (8) (2001) 1055–1061.
[27] L.X. Zhang, S. Wang, H.R. Karimi, A. Jasra, Robust finite-time control of switched linear systems and application to a class of servomechanism systems, IEEE/ASME Trans. Mechatronics 20 (5) (2015) 2476–2485.
[28] C. Hua, G. Liu, Y. Li, X. Guan, Adaptive neural tracking control for interconnected switched systems with non-ISS unmodeled dynamics, IEEE Trans. Cybern. (2018) http://dx.doi.org/10.1109/TCYB.2018.2809576.
[29] L.X. Zhang, P. Shi, Stability, L_2-gain and asychronous H_∞ control of discrete-time switched systems with average dwell time, IEEE Trans. Automat. Control 54 (9) (2009) 2193–2200.
[30] D. Antunes, J. Hespanha, C. Silvestre, Stability of networked control systems with asynchronous renewal links: An impulsive systems approach, Automatica 49 (2) (2013) 402–413.
[31] Z. Xiang, R. Wang, Robust control for uncertain switched nonlinear systems with time delay under asynchronous switching, IET Control Theory Appl. 3 (8) (2008) 1041–1050.
[32] L. Zhang, H. Gao, Asynchronously switched control of switched linear systems with average dwell time, Automatica 46 (2010) 953–958.
[33] J. Fu, T.F. Li, T. Chai, C.Y. Su, Sampled-data-based stabilization of switched linear neutral systems, Automatica 72 (2016) 92–99.
[34] C. Hua, S. Wu, X. Yang, X. Guan, Stability analysis of time-delay systems via free-matrix-based double integral inequality, Internat. J. Systems Sci. 48 (2) (2017) 257–263.