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Brief paper

Sampled-data-based stabilization of switched linear neutral systems[☆]

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ABSTRACT

With the development of digital control technology, sampled-data control shows its prominent superiority for most practical industries. In the framework of sampled-data control, this paper studies the stabilization problem for a class of switched linear neutral systems meanwhile taking into account asynchronous switching. By utilizing the relationship between the sampling period and the dwell time of switched neutral systems, a bond between the sampling period and the *average* dwell time is revealed to form a switching condition, under which and certain control gains conditions exponential stability of the closed-loop systems is guaranteed. A simple example is given to demonstrate the effectiveness of the proposed method.

1. Introduction

A switched system consists of a family of continuous-time or discrete-time subsystems and a switching law to determine which subsystem is active within certain time interval. Lots of works are devoted to switched systems in the past two decades, see, for example Branicky (1998), Chen and Zheng (2010), Fu, Ma, and Chai (2015), Lian, Ge, and Han (2013), Liberzon (2003), Lin and Antsaklis (2009), Sun, Du, Shi, Wang, and Wang (2014), Sun and Ge (2005), Sun, Liu, David, and Wang (2008), Sun, Zhao, and Hill (2006), Wu and Dong (2006), Xiang, Sun, and Chen (2012), Zhai, Hu, Yasuda, and Michel (2001), Zhang and Gao (2010), Zhang and Yu (2009), Zhang, Zhuang, and Shi (2015), Zhang, Zhuang, Shi, and Zhu (2015), Zhao and Hill (2008) and Zhao, Shi, and Zhang (2012). Among all problems studied for switched systems, asynchronous switching stemming from the delay between the active subsystem and its matched controller is one important issue (Lian et al., 2013; Wang, Zhao, & Jiang, 2013; Xiang et al., 2012; Zhang & Gao, 2010; Zhao et al., 2012). A majority of existing literature on asynchronous switching focus on either for continuous-time switched systems with continuous controllers or for discrete-time switched systems with discrete-time controllers. The authors of Zhang and Gao (2010) study asynchronously switching control of switched systems with average dwell time technique in both continuous-time and discrete-time cases, where the asynchronous delay is not necessarily less than the dwell time. Refs. Lian et al. (2013) and Zhao et al. (2012) introduce asynchronously switching control methods for deterministic switched linear systems and stochastic ones, respectively. Both methods also deploy the average dwell time technique. Since most practical systems are continuous-time, for which one usually either directly designs continuous-time controllers or first discretizes the continuous systems and then develops the corresponding discrete-time controllers, for example, those methods in Lian et al. (2013), Wang et al. (2013) and Zhang and Gao (2010). However, from a practical implementation point of view, sampled-data controllers are more favorable in practical applications due to the rapid progress of computer and digital technologies and non-approximate treatment of the intersample behavior from sampled-data control's own features, which results in no degradation of the closed-loop performance (Chen & Francis, 1991, 1995; Hara, Yamamoto, & Fujioka, 1996; Hu, Lam, Cao, & Shao, 2003). Given the advantages of sampled-data control, this paper considers a particular class of switched systems called switched neutral system (Krishnasamy & Balasubramaniam, 2015a; Liu, Liu, & Zhong, 2008; Li, Zhao, & Qi, 2014; Wang et al., 2013; Xiang, Sun, & Chen, 2011; Xiang, Sun, & Mahmoud, 2012;

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Xiong, Zhong, Ye, & Wu, 2009; Zhang, Liu, Zhu, & Zhong, 2007; Zhang & Yu, 2012) which is first defined in Sun, Fu, Sun, and Zhao (2005), and study its stabilization *in the framework of sampled-data control* meanwhile taking into account asynchronous switching.

To this considered problem, the most relevant works are Feng and Song (2011), Krishnasamy and Balasubramaniam (2015b), Liberzon (2014), Lien, Chen, Yu, and Chung (2012) and Wang, Xing, Zhou, Wang, and Yang (2014). The authors of Wang et al. (2014) propose a finite-time stabilizer for a class of switched linear systems under asynchronous switching. The authors of Liberzon (2014) present an important result on sampled-data quantized state feedback stabilization of switched linear systems by using an encoding and control strategy. In Feng and Song (2011), the authors investigate the stabilization problem of switched linear systems for both the known switching process and the unknown switching setting, for which cases the dwell time technique and the online adaptive estimation method are combined with the sampled-data control, respectively. In Feng and Song (2011), Liberzon (2014) and Wang et al. (2014), only the delay-free switched linear systems are considered. In Lien et al. (2012), the authors study the robust delay-dependent H_∞ control for a class of switched linear delay systems. However, the authors convert the sampled-data control problem into a time-delay one and used time-varying delay approach instead of sampled-data feedback control input. Furthermore, impacts from sampling or from asynchronous switching are not considered on stabilization of switched systems. Although all these references give sampled-data control strategies for switched systems with their own spectacular features, they are obviously not able to cope with stabilization of switched *neutral* systems under either asynchronous or synchronous switching because time delays appear not only in states but also in state derivative of the considered system (see Eq. (1)). In Krishnasamy and Balasubramaniam (2015b), the authors study the sampled-data control for switched neutral systems under synchronous switching which possesses great extent conservativeness. Thus one may wonder whether or not it is possible to propose a new control stabilization method for switched neutral systems, even for switched linear ones, in the framework of sampled-data control under asynchronous switching? This paper provides an affirmative answer.

From the motivation above, this paper focuses on a class of switched linear neutral systems and introduces a sampled-data-based controller for *asynchronously* switching stabilization. By utilizing the relationship between the sampling period and the dwell time of switched systems, a bond between the sampling period and the *average* dwell time is revealed to form a switching condition. Subject to this switching condition and certain control gains related constraints, exponential stability of the closed-loop switched neutral system can be guaranteed. The main features of this paper are as follows. (1) We, for the first time, propose a sample-data-based stabilization method for switched linear neutral systems under asynchronous switching. (2) Delays in the switched neutral systems are time-varying and also appear in the state derivative. (3) A bond between the sampling period and the *average* dwell time is revealed to form a switching condition. (4) Introducing free-weighting matrices obtains the decoupled constraints (30) and (31), and therefore technically reduces the computational complexity compared to the result of Wang et al. (2013).

This paper is organized as follows. Section 2 describes the problem statement and gives some useful definitions and lemmas. Section 3 gives the controller design of stabilizing the switched neutral system under sampled-data input. An example is presented in Section 4, and followed by the conclusion in Section 5.

Notations: $C_r = C([-r, 0], R^n)$ denotes the Banach space of continuous vector functions mapping the interval $[-r, 0]$ into R^n with the topology of uniform convergence. $\|\varphi\|_C = \sup_{-r\le t\le 0}\|\varphi(t)\|$ denotes the norm of a function $\varphi \in C_r$. R^n is the n-dimensional Euclidean space. $Q > 0 (Q < 0)$ means that the matrix Q is positive definite (negative definite). Q^T and Q^{-1} present the transpose and the inverse of the matrix Q, respectively. $\|\cdot\|$ is the Euclidean norm. $*$ symbolizes the elements below the main diagonal of a symmetric matrix. $\underline{\lambda}(Q)$ and $\bar{\lambda}(Q)$ are the smallest and the largest eigenvalue of a matrix Q, respectively. $\mathcal{N}$ is the set of nonnegative integers. diag$\{\cdots\}$ denotes a block-diagonal matrix.

2. Problem statement

Consider the switched neutral system

$$\begin{cases}\dot{x}(t) = A_\sigma x(t) + B_\sigma x(t-\tau(t)) + C_\sigma \dot{x}(t-h(t)) + D_\sigma u(t)\\ x(t_0+\theta) = \varphi(\theta), \quad \theta \in [-r, 0]\end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control vector, $\sigma : [0, \infty) \to \mathcal{M} = \{1, 2, \ldots, m\}$ is a right-continuous, piecewise constant function called the switching signal, $\{(A_i, B_i, C_i, D_i) : i \in \mathcal{M}\}$ is a collection of matrix pairs defining the individual subsystem of the system (1), and all the eigenvalues of matrix C_i are inside the unit circle, $\tau(t)$ and $h(t)$ denote the discrete time-varying delay and the neutral time-varying delay, respectively, which satisfy

$$\begin{aligned} &0 < \tau(t) \le \tau, \qquad \dot{\tau}(t) \le \hat{\tau} < 1,\\ &0 < h(t) \le h, \qquad \dot{h}(t) \le \hat{h} < 1, \end{aligned} \tag{2}$$

where $\tau, \hat{\tau}, h$ and $\hat{h}$ are constants. $\varphi(\theta)$ is a continuously differential vector initial function on $[-r, 0]$, $r = \max\{\tau, h\}$. We denote the number of discontinuities of the switching signal σ on the interval $(s, t]$ by $N_\sigma(t, s)$. For all $i \in \mathcal{M}$, we assume that the subsystem i is stabilizable and there is no jump at all switching instants.

For the purpose of this paper, the definitions of average dwell time and exponential stability of the switched linear neutral system are introduced below.

Definition 1 (*Liberzon, 2003*)**.** If there exists a number $\tau_d > 0$ such that any two switches are separated by at least τ_d, then τ_d is called the dwell time. In addition, if there exist two positive numbers $\tau_a > \tau_d$ and $N_0 \ge 1$ such that

$$N_\sigma(t, s) \le N_0 + \frac{t-s}{\tau_a} \quad \forall t \ge s \ge 0, \tag{3}$$

then τ_a is called the average dwell time.

Definition 2 (*Wang et al., 2013*)**.** The system (1) is said to be exponentially stable under a switching law σ, if the solution $x(t)$ of the system (1) satisfies

$$\|x(t)\| \le \kappa e^{-\lambda(t-t_0)}\|x_{t_0}\|_c, \quad t \ge t_0 \tag{4}$$

for constants $\kappa \ge 1$ and $\lambda > 0$, where $\|x_{t_0}\|_c = \sup_{-r\le\theta\le 0}\{\|x(t_0+\theta)\|, \|\dot{x}(t_0+\theta)\|\}$.

State measurements are taken at times $t_k := k\tau_s, k \in \mathcal{N}$, where τ_s is a fixed sampling period. When the sampler gets the state information, it also passes the switching signal into the controller. However, the sampling time and the switching time do not happen synchronously. The delayed time will last to the next sampling time. During this period, the active subsystem would be unstable. Thus the control objective is to stabilize the system (1) by designing a sampled-data controller meanwhile taking into account asynchronous switchings. In order to simplify the analysis process, we assume the sampling period τ_s is no larger than the dwell time τ_d.

Suppose that the subsystem i is active on $[t_k, t_{k+1})$. The control law produced by the sampler with zero-holder on $[t_k, t_{k+1})$ can be represented as

$$\begin{aligned} u(t) &= u(t_k) = u(t-(t-t_k)) \\ &= u(t-\eta(t)) = K_i x(t-\eta(t)), \end{aligned} \tag{5}$$

where $\eta(t) = t - t_k$ is piecewise with derivative $\dot{\eta}(t) = 1$ for $t \neq t_k$. Moreover, $\eta(t) < t_{k+1} - t_k = \tau_s$.

Before ending this section, we present an important lemma for our main results.

Lemma 1 (Gu, 2000). *For any constant matrix $M > 0$, scalars r_1, r_2 satisfying $r_1 < r_2$, and a vector function $\omega : [r_1, r_2] \to R^n$ such that the integrations concerned are well defined, then*

$$\begin{aligned} &-(r_2-r_1)\int_{r_1}^{r_2}\omega^T(s)M\omega(s)ds \\ &\le -\left(\int_{r_1}^{r_2}\omega(s)ds\right)^T M\left(\int_{r_1}^{r_2}\omega(s)ds\right). \end{aligned} \tag{6}$$

3. Main results

We aim to stabilize the switched neutral system (1) under the assumption $\tau_s \le \tau_d$. This assumption guarantees that at most one switch occurs within each sampling interval. If no switching occurs within a sampling interval, then the active subsystem is synchronous with its matched controller. If one switching happens within a sampling interval, then the active subsystem is synchronous with its corresponding controller *before* switching, but becomes asynchronous *after* that until the next sampling time begins.

Thus, we now analyze stability of the system (1) in two cases: sampling interval with no switch and sampling interval with one switch.

3.1. Sampling interval with no switch

Consider the case when $\sigma(t_k) = \sigma(t_{k+1}) = i \in \mathcal{M}$. Subsystem i is active on the whole interval $[t_k, t_{k+1})$. The closed-loop dynamic is given by

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + B_i x(t-\tau(t)) + C_i\dot{x}(t-h(t)) \\ &\quad + D_i K_i x(t-\eta(t)). \end{aligned} \tag{7}$$

Construct the Lyapunov–Krasovskii functional candidate for the system (7)

$$\begin{aligned} V_i(t) &= x^T(t)P_i x(t) + \int_{t-\tau_s}^{t} x^T(s)Q_i e^{\lambda_s(s-t)}x(s)ds \\ &\quad + \int_{t-\tau(t)}^{t} x^T(s)R_i e^{\lambda_s(s-t)}x(s)ds \\ &\quad + \int_{t-h(t)}^{t} \dot{x}^T(s)M_i e^{\lambda_s(s-t)}\dot{x}(s)ds \\ &\quad + \int_{t-\alpha\eta(t)}^{t} x^T(s)N_i e^{\lambda_s(s-t)}x(s)ds \\ &\quad + \int_{-\tau_s}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)S_i e^{\lambda_s(s-t)}\dot{x}(s)dsd\theta, \end{aligned} \tag{8}$$

where $P_i, Q_i, R_i, M_i, N_i, S_i$ are positive definite symmetric matrices, and α is a constant satisfying $0 < \alpha < 1$.

Remark 1. From the construction of the Lyapunov–Krasovskii functional candidate (8), we expect to obtain a sufficient condition of stabilizing the system (1), which depends on the bounds of the discrete time-varying delay, the neutral time-varying delay, the derivative of the discrete time-varying delay, the derivative of the neutral time-varying delay and the sampling period.

When the subsystem i is active, for $\forall \lambda_s > 0$, taking the time derivative of the Lyapunov–Krasovskii functional (8) along solutions of the system (7) gives

$$\begin{aligned} \dot{V}_i(t) + \lambda_s V_i(t) &\le 2x^T(t)P_i\dot{x}(t) + \lambda_s x^T(t)P_i x(t) + x^T(t)Q_i x(t) \\ &\quad - e^{-\lambda_s\tau_s}x^T(t-\tau_s)Q_i x(t-\tau_s) + x^T(t)R_i x(t) \\ &\quad - (1-\hat{\tau})e^{-\lambda_s\tau}x^T(t-\tau(t))R_i x(t-\tau(t)) \\ &\quad + \dot{x}^T(t)M_i\dot{x}(t) + x^T(t)N_i x(t) + \tau_s\dot{x}^T(t)S_i\dot{x}(t) \\ &\quad - (1-\hat{h})e^{-\lambda_s h}\dot{x}^T(t-h(t))M_i\dot{x}(t-h(t)) \\ &\quad - (1-\alpha)e^{-\alpha\lambda_s\tau_s}x^T(t-\alpha\eta(t))N_i x(t-\alpha\eta(t)) \\ &\quad - e^{-\lambda_s\tau_s}\int_{t-\alpha\eta(t)}^{t}\dot{x}^T(s)S_i\dot{x}(s)ds \\ &\quad - e^{-\lambda_s\tau_s}\int_{t-\eta(t)}^{t-\alpha\eta(t)}\dot{x}^T(s)S_i\dot{x}(s)ds \\ &\quad - e^{-\lambda_s\tau_s}\int_{t-\tau_s}^{t-\eta(t)}\dot{x}^T(s)S_i\dot{x}(s)ds. \end{aligned} \tag{9}$$

From Lemma 1, we have

$$\begin{aligned} &-e^{-\lambda_s\tau_s}\int_{t-\alpha\eta(t)}^{t}\dot{x}^T(s)S_i\dot{x}(s)ds \\ &\le \frac{-e^{-\lambda_s\tau_s}}{\alpha\tau_s}\left(\int_{t-\alpha\eta(t)}^{t}\dot{x}(s)ds\right)^T S_i\left(\int_{t-\alpha\eta(t)}^{t}\dot{x}(s)ds\right) \\ &= \frac{-e^{-\lambda_s\tau_s}}{\alpha\tau_s}(x^T(t)-x^T(t-\alpha\eta(t)))S_i(x(t)-x(t-\alpha\eta(t))), \end{aligned}$$

$$\begin{aligned} &-e^{-\lambda_s\tau_s}\int_{t-\eta(t)}^{t-\alpha\eta(t)}\dot{x}^T(s)S_i\dot{x}(s)ds \\ &\le \frac{-e^{-\lambda_s\tau_s}}{(1-\alpha)\tau_s}\left(\int_{t-\eta(t)}^{t-\alpha\eta(t)}\dot{x}(s)ds\right)^T S_i\left(\int_{t-\eta(t)}^{t-\alpha\eta(t)}\dot{x}(s)ds\right) \\ &= \frac{-e^{-\lambda_s\tau_s}}{(1-\alpha)\tau_s}(x^T(t-\alpha\eta(t))-x^T(t-\eta(t))) \\ &\quad \times S_i(x(t-\alpha\eta(t))-x(t-\eta(t))), \end{aligned}$$

$$-e^{-\lambda_s\tau_s}\int_{t-\tau_s}^{t-\eta(t)}\dot{x}^T(s)S_i\dot{x}(s)ds < 0.$$

Moreover, from the system equation (7), for any matrix W_i with appropriate dimensions, we have

$$\begin{aligned} &-2(\dot{x}^T(t)\;\; x^T(t))W_i^T(\dot{x}(t) - A_i x(t) - B_i x(t-\tau(t)) \\ &\quad - C_i\dot{x}(t-h(t)) - D_i K_i x(t-\eta(t))) = 0. \end{aligned} \tag{10}$$

Utilizing the above three inequalities and adding (10) into (9), we obtain

$$\dot{V}_i(t) + \lambda_s V_i(t) \le \xi^T(t)\Omega_i\xi(t), \tag{11}$$

where

$$\begin{aligned} \xi^T(t) &= (x^T(t)\; x^T(t-\tau(t))\; \dot{x}^T(t-h(t)) \\ &\quad \times x^T(t-\eta(t))\; x^T(t-\tau_s)\; x^T(t-\alpha\eta(t))\; \dot{x}^T(t)), \end{aligned}$$

$$\Omega_i = \begin{bmatrix} \Omega_{i,11} & W_i^T B_i & W_i^T C_i & \Omega_{i,14} & 0 & \Omega_{i,16} & \Omega_{i,17} \\ * & \Omega_{i,22} & 0 & 0 & 0 & 0 & B_i^T W_i \\ * & * & \Omega_{i,33} & 0 & 0 & 0 & C_i^T W_i \\ * & * & * & \Omega_{i,44} & 0 & \Omega_{i,46} & \Omega_{i,47} \\ * & * & * & * & \Omega_{i,55} & 0 & 0 \\ * & * & * & * & * & \Omega_{i,66} & 0 \\ * & * & * & * & * & * & \Omega_{i,77} \end{bmatrix},$$

$$\Omega_{i,11} = R_i + N_i + \lambda_s P_i + Q_i - \frac{e^{-\lambda_s \tau_s}}{\alpha \tau_s} S_i + W_i^T A_i + A_i^T W_i,$$

$$\Omega_{i,14} = W_i^T D_i K_i, \qquad \Omega_{i,16} = \frac{e^{-\lambda_s \tau_s}}{\alpha \tau_s} S_i,$$

$$\Omega_{i,22} = -(1-\hat{\tau}) e^{-\lambda_s \tau} R_i, \qquad \Omega_{i,33} = -(1-\hat{h}) e^{-\lambda_s h} M_i,$$

$$\Omega_{i,44} = -\frac{e^{-\lambda_s \tau_s}}{(1-\alpha)\tau_s} S_i, \qquad \Omega_{i,46} = \frac{e^{-\lambda_s \tau_s}}{(1-\alpha)\tau_s} S_i,$$

$$\Omega_{i,47} = D_i^T K_i^T W_i, \qquad \Omega_{i,55} = -Q_i e^{-\lambda_s \tau_s},$$

$$\Omega_{i,66} = -\frac{e^{-\lambda_s \tau_s}}{(1-\alpha)\tau_s} S_i - \frac{e^{-\lambda_s \tau_s}}{\alpha \tau_s} S_i - (1-\alpha) e^{-\alpha \lambda_s \tau_s} N_i,$$

$$\Omega_{i,17} = P_i + A_i^T W_i - W_i^T, \qquad \Omega_{i,77} = M_i + \tau_s Q_i - W_i - W_i^T.$$

From (11), we know that $\Omega_i < 0$ implies

$$\dot{V}_i(t) + \lambda_s V_i(t) < 0. \tag{12}$$

Integrating the inequality (12) gives

$$V_i(t) < e^{-\lambda_s (t-t_k)} V_i(t_k) \le \nu V_i(t_k), \tag{13}$$

where $\nu := \max\left\{ e^{-\lambda_s (t-t_k), 0 \le t-t_k < \tau_s} \right\}$.

3.2. Sampling interval with one switch

Consider the case that one switch has occurred on the interval $[t_k, t_{k+1})$, that is $\sigma(t_k) = i$ and $\sigma(t_{k+1}) = j \neq i$. The controller knows that one switch occurs from subsystem i to j on the interval $[t_k, t_{k+1})$ but does not know when it happens. Assume that the time of the switch from i to j be $t_k + \bar{t}$, where $\bar{t} \in (0, \tau_s]$. When $t \in [t_k, t_k + \bar{t})$, the subsystem i is active. We derive as above and obtain that

$$V_i(t) < e^{-\lambda_s (t-t_k)} V_i(t_k). \tag{14}$$

When $t \in [t_k + \bar{t}, t_{k+1})$, the resulting closed-loop dynamic becomes

$$\begin{aligned} \dot{x}(t) &= A_j x(t) + B_j x(t-\tau(t)) \\ &\quad + C_j \dot{x}(t-h(t)) + D_j K_i x(t-\eta(t)). \end{aligned} \tag{15}$$

Construct the Lyapunov–Krasovskii functional candidate for the system (15)

$$\begin{aligned} V_j(t) &= x^T(t) P_j x(t) + \int_{t-\tau_s}^{t} x^T(s) Q_j e^{\lambda_s (s-t)} x(s) ds \\ &\quad + \int_{t-h(t)}^{t} \dot{x}^T(s) M_j e^{\lambda_s (s-t)} \dot{x}(s) ds \\ &\quad + \int_{t-\alpha\eta(t)}^{t} x^T(s) N_j e^{\lambda_s (s-t)} x(s) ds \\ &\quad + \int_{t-\tau(t)}^{t} x^T(s) R_j e^{\lambda_s (s-t)} x(s) ds \\ &\quad + \int_{-\tau_s}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) S_j e^{\lambda_s (s-t)} \dot{x}(s) ds d\theta, \end{aligned} \tag{16}$$

where $P_j, Q_j, R_j, M_j, N_j, S_j$ are positive definite symmetric matrices.

When the subsystem j is active, for $\forall \lambda_u > 0$, taking the time derivative of (16) along solutions of the system (15) gives

$$\begin{aligned} \dot{V}_j(t) - \lambda_u V_j(t) &\le 2x^T(t) P_j \dot{x}(t) + \lambda_s x^T(t) P_j x(t) + x^T(t) Q_j x(t) \\ &\quad - e^{-\lambda_s \tau_s} x^T(t-\tau_s) Q_j x(t-\tau_s) + x^T(t) R_j x(t) \\ &\quad - (1-\hat{\tau}) e^{-\lambda_s \tau} x^T(t-\tau(t)) R_j x(t-\tau(t)) \\ &\quad + \dot{x}^T(t) M_j \dot{x}(t) + x^T(t) N_j x(t) + \tau_s \dot{x}^T(t) S_j \dot{x}(t) \\ &\quad - (1-\hat{h}) e^{-\lambda_s h} \dot{x}^T(t-h(t)) M_j \dot{x}(t-h(t)) \\ &\quad - (1-\alpha) e^{-\alpha \lambda_s \tau_s} x^T(t-\alpha\eta(t)) N_j x(t-\alpha\eta(t)) \\ &\quad - e^{-\lambda_s \tau_s} \int_{t-\alpha\eta(t)}^{t} \dot{x}^T(s) S_j \dot{x}(s) ds \\ &\quad - e^{-\lambda_s \tau_s} \int_{t-\eta(t)}^{t-\alpha\eta(t)} \dot{x}^T(s) S_j \dot{x}(t) ds \\ &\quad - e^{-\lambda_s \tau_s} \int_{t-\tau_s}^{t-\eta(t)} \dot{x}^T(s) S_j \dot{x}(t) ds \\ &\quad - (\lambda_s + \lambda_u) e^{-\lambda_s \tau_s} \int_{t-\tau_s}^{t} x^T(s) Q_j x(s) ds \\ &\quad - (\lambda_s + \lambda_u) e^{-\lambda_s \tau} \int_{t-\tau(t)}^{t} x^T(s) R_j x(s) ds \\ &\quad - (\lambda_s + \lambda_u) e^{-\lambda_s h} \int_{t-h(t)}^{t} \dot{x}^T(s) M_j \dot{x}(s) ds \\ &\quad - (\lambda_s + \lambda_u) e^{-\alpha \lambda_s \tau_s} \int_{t-\alpha\eta(t)}^{t} x^T(s) N_j x(s) ds \\ &\quad - (\lambda_s + \lambda_u) \int_{-\tau_s}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) S_j e^{\lambda_s (s-t)} \dot{x}(s) ds d\theta. \end{aligned} \tag{17}$$

From Lemma 1, we have

$$\begin{aligned} &-e^{-\lambda_s \tau_s} \int_{t-\alpha\eta(t)}^{t} \dot{x}^T(s) S_j \dot{x}(s) ds \\ &\quad \le \frac{-e^{-\lambda_s \tau_s}}{\alpha \tau_s} \left(\int_{t-\alpha\eta(t)}^{t} \dot{x}^T(s) ds \right) S_j \left(\int_{t-\alpha\eta(t)}^{t} \dot{x}(s) ds \right) \\ &\quad = \frac{-e^{-\lambda_s \tau_s}}{\alpha \tau_s} (x^T(t) - x^T(t-\alpha\eta(t))) S_j (x(t) - x(t-\alpha\eta(t))), \\ &-e^{-\lambda_s \tau_s} \int_{t-\eta(t)}^{t-\alpha\eta(t)} \dot{x}^T(s) S_j \dot{x}(s) ds \\ &\quad \le \frac{-e^{-\lambda_s \tau_s}}{(1-\alpha)\tau_s} \left(\int_{t-\eta(t)}^{t-\alpha\eta(t)} \dot{x}^T(s) ds \right) S_j \left(\int_{t-\eta(t)}^{t-\alpha\eta(t)} \dot{x}(s) ds \right) \\ &\quad = \frac{-e^{-\lambda_s \tau_s}}{(1-\alpha)\tau_s} (x^T(t-\alpha\eta(t)) - x^T(t-\eta(t))) \\ &\quad\quad \times S_j (x(t-\alpha\eta(t)) - x(t-\eta(t))), \\ &-e^{-\lambda_s \tau_s} \int_{t-\tau_s}^{t-\eta(t)} \dot{x}^T(s) S_j \dot{x}(t) ds < 0, \\ &-(\lambda_s + \lambda_u) e^{-\lambda_s \tau_s} \int_{t-\tau_s}^{t} x^T(s) Q_j x(s) ds \\ &\quad \le \frac{-(\lambda_s + \lambda_u) e^{-\lambda_s \tau_s}}{\tau_s} \int_{t-\tau_s}^{t} x^T(s) ds Q_j \int_{t-\tau_s}^{t} x(s) ds, \\ &-(\lambda_s + \lambda_u) e^{-\lambda_s \tau} \int_{t-\tau(t)}^{t} x^T(s) R_j x(s) ds \\ &\quad \le \frac{-(\lambda_s + \lambda_u) e^{-\lambda_s \tau}}{\tau} \int_{t-\tau(t)}^{t} x^T(s) ds R_j \int_{t-\tau(t)}^{t} x(s) ds, \end{aligned}$$

$$-(\lambda_s+\lambda_u)e^{-\lambda_s h}\int_{t-h(t)}^{t}\dot{x}^T(s)M_j\dot{x}(s)ds$$
$$\leq \frac{-(\lambda_s+\lambda_u)e^{-\lambda_s h}}{h}\int_{t-h(t)}^{t}\dot{x}^T(s)dsM_j\int_{t-h(t)}^{t}\dot{x}^T(s)ds,$$
$$-(\lambda_s+\lambda_u)e^{-\alpha\lambda_s\tau_s}\int_{t-\alpha\eta(t)}^{t}x^T(s)N_jx(s)ds$$
$$\leq \frac{-(\lambda_s+\lambda_u)e^{-\alpha\lambda_s\tau_s}}{\tau_s}\int_{t-\alpha\eta(t)}^{t}x^T(s)dsN_j\int_{t-\alpha\eta(t)}^{t}x(s)ds,$$
$$-(\lambda_s+\lambda_u)\int_{-\tau_s}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)S_je^{\lambda_s(s-t)}\dot{x}(s)dsd\theta<0.$$

Moreover, from (15), for any matrices W_i, we have

$$-2(\dot{x}^T(t)\ \ x^T(t))W_i^T(\dot{x}(t)-A_jx(t)-B_jx(t-\tau(t))$$
$$-C_j\dot{x}(t-h(t))-D_jK_ix(t-\eta(t)))=0. \tag{18}$$

Utilizing the above inequalities and adding (18) into (17), we obtain

$$\dot{V}_j(t)-\lambda_uV_j(t)<\zeta^T(t)\Psi_j\zeta(t), \tag{19}$$

where

$$\zeta^T(t)=\left(\xi^T(t)\ \int_{t-\tau(t)}^{t}x^T(s)ds\ \int_{t-\alpha\eta(t)}^{t}x^T(s)ds\right.$$
$$\left.\times\int_{t-h(t)}^{t}x^T(s)ds\ \int_{t-\tau_s}^{t}x^T(s)ds\right),$$

$$\Psi_j=\begin{bmatrix}\Psi_{j,11}&0&0&0&0\\ *&\Psi_{j,22}&0&0&0\\ *&*&\Psi_{j,33}&0&0\\ *&*&*&\Psi_{j,44}&0\\ *&*&*&*&\Psi_{j,55}\end{bmatrix},$$

$$\Psi_{j,11}=\begin{bmatrix}\Pi_{j,11}&W_i^TB_j&W_i^TC_j&\Pi_{j,14}&0&\Pi_{j,16}&\Pi_{j,17}\\ *&\Pi_{j,22}&0&0&0&0&B_j^TW_i\\ *&*&\Pi_{j,33}&0&0&0&C_j^TW_i\\ *&*&*&\Pi_{j,44}&0&\Pi_{j,46}&\Pi_{j,47}\\ *&*&*&*&\Pi_{j,55}&0&0\\ *&*&*&*&*&\Pi_{j,66}&0\\ *&*&*&*&*&*&\Pi_{j,77}\end{bmatrix},$$

$\Pi_{j,11}=R_j+N_j-\lambda_uP_j+Q_j-\frac{e^{-\lambda_s\tau_s}}{\alpha\tau_s}S_j+W_i^TA_j+A_j^TW_i$, $\Pi_{j,14}=W_i^TD_jK_i$, $\Pi_{j,16}=\frac{e^{-\lambda_s\tau_s}}{\alpha\tau_s}S_j$, $\Pi_{j,17}=P_j+A_j^TW_i-W_i^T$, $\Pi_{j,22}=-(1-\hat{\tau})e^{-\lambda_s\tau}R_j$, $\Pi_{j,33}=-(1-\hat{h})e^{-\lambda_s h}M_j$, $\Pi_{j,44}=-\frac{e^{-\lambda_s\tau_s}}{(1-\alpha)\tau_s}S_j$, $\Pi_{j,46}=\frac{e^{-\lambda_s\tau_s}}{(1-\alpha)\tau_s}S_j$, $\Pi_{j,47}=K_i^TD_j^TW_i$, $\Pi_{j,66}=-\frac{e^{-\lambda_s\tau_s}}{(1-\alpha)\tau_s}S_j-\frac{e^{-\lambda_s\tau_s}}{\alpha\tau_s}S_j-(1-\alpha)e^{-\alpha\lambda_s\tau_s}N_j$, $\Pi_{j,55}=-e^{-\lambda_s\tau_s}Q_j$, $\Pi_{j,77}=M_j+\tau_sS_j-W_i-W_i^T$, $\Psi_{j,22}=-\frac{(\lambda_s+\lambda_u)e^{-\lambda_s\tau}}{\tau}R_j$, $\Psi_{j,33}=-\frac{(\lambda_s+\lambda_u)e^{-\lambda_s\tau_s}}{\tau_s}N_j$, $\Psi_{j,44}=-\frac{(\lambda_s+\lambda_u)e^{-\lambda_s h}}{h}M_j$, $\Psi_{j,55}=-\frac{(\lambda_s+\lambda_u)e^{-\lambda_s\tau_s}}{\tau_s}Q_j$.

From (19), we know that $\Psi_j<0$ implies

$$\dot{V}_j(t)-\lambda_uV_j(t)<0. \tag{20}$$

Integrating the inequality (20) yields

$$V_j(t)<e^{\lambda_u(t-t_k-\bar{t})}V_j(t_k+\bar{t}). \tag{21}$$

From (14) and (21), for $t\in[t_k,t_{k+1})$, we have

$$V_j(t)<e^{\lambda_u(t-t_k-\bar{t})}V_j(t_k+\bar{t})$$
$$\leq\frac{\bar{\lambda}(P_j)+\tau\bar{\lambda}(R_j)+h\bar{\lambda}(M_j)+\alpha\tau_s\bar{\lambda}(N_j)+\tau_s\bar{\lambda}(Q_j)+\tau_s^2\bar{\lambda}(S_j)}{\underline{\lambda}(P_i)}$$
$$\times e^{\lambda_u(t-t_k-\bar{t})}V_i((t_k+\bar{t})^-)$$
$$\leq\frac{\bar{\lambda}(P_j)+\tau\bar{\lambda}(R_j)+h\bar{\lambda}(M_j)+\alpha\tau_s\bar{\lambda}(N_j)+\tau_s\bar{\lambda}(Q_j)+\tau_s^2\bar{\lambda}(S_j)}{\underline{\lambda}(P_i)}$$
$$\times e^{-\lambda_s(t_k+\bar{t}-t_k)}e^{\lambda_u(t-t_k-\bar{t})}V_i(t_k)$$
$$\leq\mu_{ij}V_i(t_k)\leq\mu V_i(t_k), \tag{22}$$

where $V_i((t_k+\bar{t})^-)$ is the function value of $V_i(t)$ right before the switching, and

$$\mu_{ij}:=\max\{((\bar{\lambda}(P_j)+\tau\bar{\lambda}(R_j)+h\bar{\lambda}(M_j)+\alpha\tau_s\bar{\lambda}(N_j)$$
$$+\tau_s\bar{\lambda}(Q_j)+\tau_s^2\bar{\lambda}(S_j))e^{-(\lambda_s+\lambda_u)\bar{t}+\lambda_u(t-t_k)})/\underline{\lambda}(P_i)\}$$

and $\mu:=\max_{i,j\in\mathcal{M}}\mu_{ij}$.

3.3. Stability analysis

Definition 1 implies that $N_\sigma(t_{k_0},t_k)\leq N_0+\frac{k-k_0}{p}$ for every p such that $\tau_a\geq p\tau_s$. We now derive a lower bound on p to guarantee convergence. From (3), we know that $N_\sigma(t_{k_0},t_k)$ equals the number $k-k_0$ of intervals of the form (t_l,t_{l+1}), $k_0\leq l\leq k-1$, which contain a switch. Combining the conclusions of Sections 3.1 and 3.2, we have the following bound for all $k\geq k_0$

$$V(t_k)=V_\sigma(t_k)$$
$$\leq\mu^{N_0+\frac{k-k_0}{p}}\nu^{k-k_0-N_0-\frac{k-k_0}{p}}V_\sigma(t_0)$$
$$=\left(\frac{\mu}{\nu}\right)^{N_0}\left(\mu^{\frac{1}{p}}\nu^{\frac{p-1}{p}}\right)^{k-k_0}V_\sigma(t_0)$$
$$=\left(\frac{\mu}{\nu}\right)^{N_0}e^{(k-k_0)\ln\left(\mu^{\frac{1}{p}}\nu^{\frac{p-1}{p}}\right)}V_\sigma(t_0). \tag{23}$$

We want to ensure that $\mu^{\frac{1}{p}}\nu^{\frac{p-1}{p}}<1$, which is equivalent to

$$p>1+\frac{\log\mu}{\log\frac{1}{\nu}}. \tag{24}$$

Thus if

$$\tau_a>\left(1+\frac{\log\mu}{\log\frac{1}{\nu}}\right)\tau_s, \tag{25}$$

then

$$V(t_k)\leq\left(\frac{\mu}{\nu}\right)^{N_0}e^{(k-k_0)\ln\left(\mu^{\frac{1}{p}}\nu^{\frac{p-1}{p}}\right)}V_\sigma(t_0). \tag{26}$$

From the defined piecewise Lyapunov–Krasovskii functional, we have

$$V(t_k)\geq x^T(t)P_ix(t)\geq\underline{\lambda}(P_i)\|x(t)\|^2$$
$$\geq\min_{\forall i\in\mathcal{M}}\underline{\lambda}(P_i)\|x(t)\|^2=a\|x(t)\|^2, \tag{27}$$

$$V_\sigma(t_0)\leq(\max_{\forall i\in\mathcal{M}}\bar{\lambda}(P_i)+\tau\max_{\forall i\in\mathcal{M}}\bar{\lambda}(R_i)$$
$$+h\max_{\forall i\in\mathcal{M}}\bar{\lambda}(M_i)+\alpha\tau_s\max_{\forall i\in\mathcal{M}}\bar{\lambda}(N_i)$$
$$+\tau_s\max_{\forall i\in\mathcal{M}}\bar{\lambda}(Q_i)+\tau_s^2\max_{\forall i\in\mathcal{M}}\bar{\lambda}(S_i))\left\|x_{t_0}\right\|_c^2$$
$$=b\left\|x_{t_0}\right\|_c^2, \tag{28}$$

where

$$a=\min_{\forall i\in\mathcal{M}}\underline{\lambda}(P_i),$$
$$b=\max_{\forall i\in\mathcal{M}}\bar{\lambda}(P_i)+\tau\max_{\forall i\in\mathcal{M}}\bar{\lambda}(R_i)+h\max_{\forall i\in\mathcal{M}}\bar{\lambda}(M_i)$$
$$+\alpha\tau_s\max_{\forall i\in\mathcal{M}}\bar{\lambda}(N_i)+\tau_s\max_{\forall i\in\mathcal{M}}\bar{\lambda}(Q_i)+\tau_s^2\max_{\forall i\in\mathcal{M}}\bar{\lambda}(S_i).$$

From (26)–(28), we have

$$\|x(t)\|^2 \le \frac{b}{a}\left(\frac{\mu}{\nu}\right)^{N_0} e^{(k-k_0)\ln\left(\mu^{\frac{1}{p}}\nu^{\frac{p-1}{p}}\right)}\left\|x_{t_0}\right\|_c^2, \tag{29}$$

which means that the system (1) is exponentially stable.

The following theorem concludes the main result.

Theorem 1. *Consider the switched neutral system (1). For some given positive constants $\alpha < 1$, λ_s, λ_u, τ_s, h, τ, $\hat{\tau} < 1$, $\hat{h} < 1$, if there exist matrices $\bar{P}_i > 0$, $\bar{R}_i > 0$, $\bar{M}_i > 0$, $\bar{N}_i > 0$, $\bar{S}_i > 0$, $\bar{Q}_i > 0$, $\bar{W}_i > 0$, J_i, $\bar{P}_j > 0$, $\bar{R}_j > 0$, $\bar{M}_j > 0$, $\bar{N}_j > 0$, $\bar{S}_j > 0$, $\bar{Q}_j > 0$, J_j for $\forall i, j \in \mathcal{M}$, $i \neq j$, satisfying*

$$\begin{bmatrix} \bar{\Omega}_{i,11} & B_i\bar{W}_i & C_i\bar{W}_i & D_iJ_i & 0 & \dfrac{e^{-\lambda_s\tau_s}}{\alpha\tau_s}\bar{S}_i & \bar{\Omega}_{i,17} \\ * & \bar{\Omega}_{i,22} & 0 & 0 & 0 & 0 & \bar{W}_i^TB_i^T \\ * & * & \bar{\Omega}_{i,33} & 0 & 0 & 0 & \bar{W}_i^TC_i^T \\ * & * & * & \bar{\Omega}_{i,44} & 0 & \dfrac{e^{-\lambda_s\tau_s}}{(1-\alpha)\tau_s}\bar{S}_i & J_i^TD_i^T \\ * & * & * & * & \bar{\Omega}_{i,55} & 0 & 0 \\ * & * & * & * & * & \bar{\Omega}_{i,66} & 0 \\ * & * & * & * & * & * & \bar{\Omega}_{i,77} \end{bmatrix} < 0, \tag{30}$$

$$\begin{bmatrix} \bar{\Psi}_{j,11} & 0 & 0 & 0 & 0 \\ * & \bar{\Psi}_{j,22} & 0 & 0 & 0 \\ * & * & \bar{\Psi}_{j,33} & 0 & 0 \\ * & * & * & \bar{\Psi}_{j,44} & 0 \\ * & * & * & * & \bar{\Psi}_{j,55} \end{bmatrix} < 0, \tag{31}$$

where

$$\bar{\Omega}_{i,11} = \bar{R}_i + \bar{N}_i + \lambda_s\bar{P}_i + \bar{Q}_i - \frac{e^{-\lambda_s\tau_s}}{\alpha\tau_s}\bar{S}_i + A_i\bar{W}_i + \bar{W}_i^TA_i^T,$$

$$\bar{\Omega}_{i,22} = -(1-\hat{\tau})e^{-\lambda_s\tau}\bar{R}_i,\ \bar{\Omega}_{i,33} = -(1-\hat{h})e^{-\lambda_s h}\bar{M}_i,$$

$$\bar{\Omega}_{i,44} = -\frac{e^{-\lambda_s\tau_s}}{(1-\alpha)\tau_s}\bar{S}_i,\ \bar{\Omega}_{i,55} = -e^{-\lambda_s\tau_s}\bar{Q}_i,$$

$$\bar{\Omega}_{i,66} = -\frac{e^{-\lambda_s\tau_s}}{(1-\alpha)\tau_s}\bar{S}_i - \frac{e^{-\lambda_s\tau_s}}{\alpha\tau_s}\bar{S}_i - (1-\alpha)e^{-\alpha\lambda_s\tau_s}\bar{N}_i,$$

$$\bar{\Omega}_{i,17} = \bar{P}_i + \bar{W}_i^TA_i^T - \bar{W}_i,\qquad \bar{\Omega}_{i,77} = \bar{M}_i + \tau_s\bar{S}_i - \bar{W}_i - \bar{W}_i^T,$$

$$\bar{\Psi}_{j,11} = \begin{bmatrix} \bar{\Pi}_{j,11} & B_j\bar{W}_i & C_j\bar{W}_i & D_jJ_i & 0 & \bar{\Pi}_{j,16} & \bar{\Pi}_{j,17} \\ * & \bar{\Pi}_{j,22} & 0 & 0 & 0 & 0 & \bar{W}_i^TB_j^T \\ * & * & \bar{\Pi}_{j,33} & 0 & 0 & 0 & \bar{W}_i^TC_j^T \\ * & * & * & \bar{\Pi}_{j,44} & 0 & \bar{\Pi}_{j,46} & J_i^TD_j^T \\ * & * & * & * & \bar{\Pi}_{j,55} & 0 & 0 \\ * & * & * & * & * & \bar{\Pi}_{j,66} & 0 \\ * & * & * & * & * & * & \bar{\Pi}_{j,77} \end{bmatrix},$$

$\bar{\Pi}_{j,11} = \bar{R}_j + \bar{N}_j - \lambda_u\bar{P}_j + \bar{Q}_j - \frac{e^{-\lambda_s\tau_s}}{\alpha\tau_s}\bar{S}_j + A_j\bar{W}_i + \bar{W}_i^TA_j^T$, $\bar{\Pi}_{j,22} = -(1-\hat{\tau})e^{-\lambda_s\tau}\bar{R}_j$, $\bar{\Pi}_{j,33} = -(1-\hat{h})e^{-\lambda_s h}\bar{M}_j$, $\bar{\Pi}_{j,44} = -\frac{e^{-\lambda_s\tau_s}}{(1-\alpha)\tau_s}\bar{S}_j$, $\bar{\Pi}_{j,55} = -e^{-\lambda_s\tau_s}\bar{Q}_j$, $\bar{\Pi}_{j,66} = -\frac{e^{-\lambda_s\tau_s}}{(1-\alpha)\tau_s}\bar{S}_j - \frac{e^{-\lambda_s\tau_s}}{\alpha\tau_s}\bar{S}_j - (1-\alpha)e^{-\alpha\lambda_s\tau_s}\bar{N}_j$, $\bar{\Pi}_{j,16} = \frac{e^{-\lambda_s\tau_s}}{\alpha\tau_s}\bar{S}_j$, $\bar{\Pi}_{j,46} = \frac{e^{-\lambda_s\tau_s}}{(1-\alpha)\tau_s}\bar{S}_j$, $\bar{\Pi}_{j,17} = \bar{P}_j + \bar{W}_i^TA_j^T - \bar{W}_i$, $\bar{\Pi}_{j,77} = \bar{M}_j + \tau_s\bar{S}_j - \bar{W}_i - \bar{W}_i^T$, $\bar{\Psi}_{j,22} = -\frac{(\lambda_s+\lambda_u)e^{-\lambda_s\tau}}{\tau}\bar{R}_j$, $\bar{\Psi}_{j,33} = -\frac{(\lambda_s+\lambda_u)e^{-\lambda_s\tau_s}}{\tau_s}\bar{N}_j$, $\bar{\Psi}_{j,44} = -\frac{(\lambda_s+\lambda_u)e^{-\lambda_s h}}{h}\bar{M}_j$, $\bar{\Psi}_{j,55} = -\frac{(\lambda_s+\lambda_u)e^{-\lambda_s\tau_s}}{\tau_s}\bar{Q}_j$, *then the system (1) is exponentially stable for any switching signals with the average dwell time satisfying*

$$\tau_a > \left(1 + \frac{\log\mu}{\log\frac{1}{\nu}}\right)\tau_s, \tag{32}$$

where μ and ν are constants, under the sampled-data controller (5) with $K_i = J_i\bar{W}_i^{-1}$.

Proof. It is obvious from the above analysis and thus omitted.

Remark 2. Note that the controller gain K_i cannot be obtained by solving $\Omega_i < 0$ and $\Psi_j < 0$ directly. If, however, we let $\bar{W}_i = W_i^{-1}$, multiply diag$\{\bar{W}_i^T, \ldots, \bar{W}_i^T\}$ and diag$\{\bar{W}_i, \ldots, \bar{W}_i\}$ on the left-hand side and on the right-hand side of the inequalities $\Omega_i < 0$ and $\Psi_j < 0$, respectively, and also let $\bar{R}_i = \bar{W}_i^TR_i\bar{W}_i$, $\bar{N}_i = \bar{W}_i^TN_i\bar{W}_i$, $\bar{P}_i = \bar{W}_i^TP_i\bar{W}_i$, $\bar{Q}_i = \bar{W}_i^TQ_i\bar{W}_i$, $\bar{S}_i = \bar{W}_i^TS_i\bar{W}_i$, $\bar{M}_i = \bar{W}_i^TM_i\bar{W}_i$, $J_i = K_i\bar{W}_i$, $\bar{R}_j = \bar{W}_i^TR_j\bar{W}_i$, $\bar{N}_j = \bar{W}_i^TN_j\bar{W}_i$, $\bar{P}_j = \bar{W}_i^TP_j\bar{W}_i$, $\bar{Q}_j = \bar{W}_i^TQ_j\bar{W}_i$, $\bar{S}_j = \bar{W}_i^TS_j\bar{W}_i$, $\bar{M}_j = \bar{W}_i^TM_j\bar{W}_i$, then we can obtain the inequalities (30) and (31), which are equivalent to $\Omega_i < 0$ and $\Psi_j < 0$. As a result, easily solving (30) and (31) directly gives K_i.

Remark 3. In (32), τ_s can be chosen in advance. μ and ν are computed according to (13) and (22).

Remark 4. In order to obtain the control gain K_i, we introduce the free-weighting matrix W_i when the ith controller is active, (see Eqs. (10) and (18)), so that the control gain K_i can be obtained by solving the conditions (30) and (31) simultaneously as described in Remark 2, which overcomes the technical challenge of solving the *coupled* variables of linear matrix inequalities in Wang et al. (2013).

Remark 5. In this paper, our work is based on the assumption $\tau_s \le \tau_d$ which simplifies the analysis process into two cases, that is no switching or one switching may occur on arbitrary a sampling interval. In fact, our method can be extended to deal with the case of several switchings happen on a sampling interval. We just need analyze the monotony property of Lyapunov–Krasovskii functional on each individual non-switching interval on one sampling interval, and then analyze the overall descend property of Lyapunov–Krasovskii functional to guarantee stability.

Remark 6. The author of Zhang et al. (2007) analyzes stability of switched linear systems without delays from the view of analytical solutions of switched linear systems. However, the proposed method cannot be applied for switched neutral systems since the analytical solution of a switched neutral system is infinite-dimensional and is difficult to show out. Moreover, the feedback controller in Zhang et al. (2007) is given in advance. Our proposed method is by means of the Lyapunov–Krasovskii functional analysis and especially we give the design of the feedback controller.

4. An illustration example

Consider the switched neutral system (1) with two subsystems, where

$$A_1 = \begin{bmatrix} -2.5 & 0.3 \\ 0 & -1.5 \end{bmatrix},\qquad B_1 = \begin{bmatrix} -0.2 & 0.2 \\ 0.1 & -0.1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -0.1 & 0.1 \\ 0 & 0.2 \end{bmatrix},\qquad D_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\qquad A_2 = \begin{bmatrix} -1.2 & 0 \\ 0.1 & -1.6 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0.3 & 0 \\ 0.1 & 0.1 \end{bmatrix},\qquad C_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & -0.1 \end{bmatrix},\qquad D_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$\tau(t) = 0.1\sin t + 0.3, \qquad h(t) = 0.1\sin t + 0.1.$

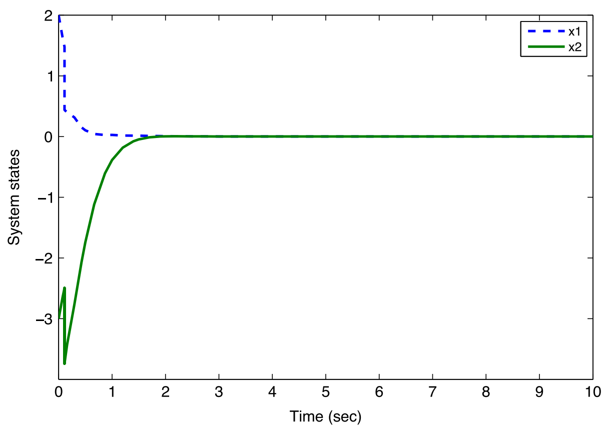

Fig. 1. State responses of the closed-loop system.

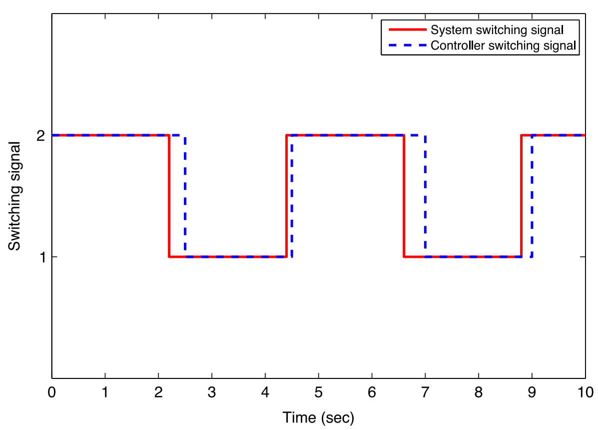

Fig. 2. Switching signals of the system and the controller.

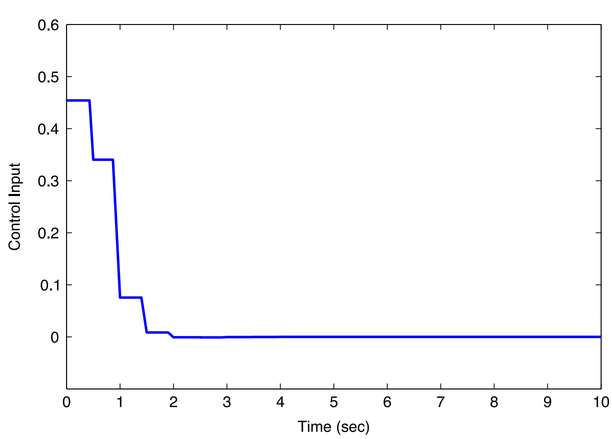

Fig. 3. The control input.

Let parameters be $\lambda_s = 1, \lambda_u = 1, \alpha = 0.4$ and $\tau_s = 0.5$. Solving the conditions in Theorem 1, we obtain $K_1 = [-0.0723 \quad -0.1996], K_2 = [-0.2809 \quad -0.1149]$. From (32), we obtain the minimum average dwell time $\tau_a^* = 1.9090$. Constructing a certain switching sequence satisfying $\tau_a = 2.2 > \tau_a^*$, we obtain the state responses of the closed-loop system with the initial state $x_0 = (2 \quad -3)^T$shown in Fig. 1, and the system switching signal and the controller switching signal shown in Fig. 2. The control input is shown in Fig. 3. From Fig. 2, the asynchronous switchings are clearly shown. Thus, from the figures the closed-loop system is stabilized under the sampled-data control input, and the simulation results verified the proposed method.

5. Conclusions

We have presented a result on sampled-data-based state feedback stabilization of a class of switched linear neutral systems under asynchronous switching. A relationship between the average dwell time and the sampling period has been revealed to form a switching condition to guarantee exponential stability. Furthermore, by introducing free-weighting matrices, we have obtained the controller gains by solving the resulting matrix inequalities. Future work will focus on the research of the class of switched linear neutral systems with noise, which is very promising but needs the knowledge of stochastic dynamics, especially the theory of Itô stochastic differential equations. Initial attempt will consider the system with the simple wide-band noise as studied in Ref. Blankenship and Papanicolaou (1978).

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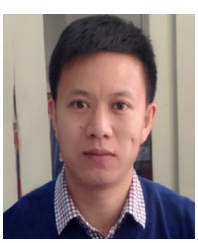

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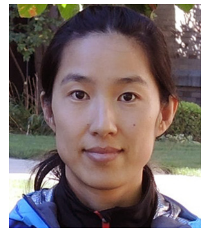

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