Adaptive Output-Feedback Neural Control of Switched Uncertain Nonlinear Systems With Average Dwell Time

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***Abstract*—This paper investigates the problem of adaptive neural tracking control via output-feedback for a class of switched uncertain nonlinear systems without the measurements of the system states. The unknown control signals are approximated directly by neural networks. A novel adaptive neural control technique for the problem studied is set up by exploiting the average dwell time method and backstepping. A switched filter and different update laws are designed to reduce the conservativeness caused by adoption of a common observer and a common update law for all subsystems. The proposed controllers of subsystems guarantee that all closed-loop signals remain bounded under a class of switching signals with average dwell time, while the output tracking error converges to a small neighborhood of the origin. As an application of the proposed design method, adaptive output feedback neural tracking controllers for a mass-spring-damper system are constructed.**

***Index Terms*—Adaptive neural control, average dwell time, output tracking, switched nonlinear systems.**

I. Introduction

In THE last decades, the problem of adaptive tracking control of nonswitched uncertain nonlinear systems has attracted much attention using universal function approximators, such as neural networks (NNs) or fuzzy logic systems to parameterize the unknown nonlinearities [8], [9], [21], [29], [32], [39], [43], [48]. Meanwhile, several control methodologies, such as adaptive neural or fuzzy backstepping methods, have been proposed, which achieve adaptive tracking control results by exploiting certain feedback structures [7], [11], [36], [38]. In particular, the class of nonswitched uncertain nonlinear systems in strict-feedback form has received particular attention, since adaptive neural or fuzzy controllers can be constructed recursively in the framework of the traditional backstepping design [4], [14], [33]. In many real systems, however, state variables are often rarely fully measured, which demands observer-based control schemes [16], [45]. Thus, when the system state is unmeasured, the adaptive neural or fuzzy output-feedback control method is an effective way to control the nonswitched uncertain nonlinear systems in strict-feedback form, which has also attracted a considerable amount of research effort [6], [10], [35], [45].

Switched systems constitute a special class of hybrid systems which consist of a family of subsystems, either continuous-time or discrete-time subsystems, and a switching law, which defines a specific subsystem that is active at each instant of time. Recently, the analysis and synthesis problems of switched systems, especially stability analysis and global stabilization, have been extensively studied in [2], [3], [12], [13], [34], and [47]. An important reason for the wide interest in the study of these problems is the variety of fields where the systems involved can be effectively modeled as switched systems. Such systems include robotic, mechatronic and mechanical systems, gene regulatory networks, switching power converters [19], [20], [28], just name a few. However, a switched system does not necessarily inherit properties of the individual subsystems [1], [27], [30], [41]. For example, asymptotic stability of a switched system is not necessarily established for arbitrary switching signals even if all of the subsystems exhibit this property [20]. Therefore, the study of system properties is very difficult for switched systems. A typical and useful system structure often plays an important role because such a structure of a given system can sometimes be utilized to obtain interesting results, even in the absence of a general theory. Recently, for a special class of switched systems, switched nonlinear systems in strict-feedback form have been studied systematically by the backstepping technique. In [24] and [26], adaptive disturbance rejection and H_∞ control problem for switched nonlinear systems in strict-feedback form and p-normal form are studied on the basis of the multiple Lyapunov functions method, respectively; global stabilization for switched nonlinear systems in strict-feedback form under arbitrary switchings is achieved in [25] and [37]. The abovementioned results are based on the assumption that all system nonlinearities are known. However, this assumption is usually unrealistic in practice. In fact, many complex system dynamics in real world are too difficult to be explicitly formulated. In such case, the results in [24]–[26] and [37] are not directly applicable. Therefore, for switched uncertain nonlinear systems, it is desirable to use the adaptive control based on universal function approximators [32], [39].

Recently, in [15], an adaptive neural tracking control approach has been successfully applied to the analysis and design of adaptive controllers for switched uncertain nonlinear

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systems in strict-feedback form based on the backstepping technique. The result in [15] is based on the assumption that the states of the system are measured directly. As a matter of fact, this assumption does not hold in many practical systems. If the system states are not available, the adaptive neural control scheme above will become ineffective. Naturally, a new adaptive output-feedback neural control approach using an estimated states will be required for switched uncertain nonlinear systems. However, to the best of our knowledge, there are no results available in switched uncertain nonlinear systems on adaptive output-feedback neural control by the backstepping technique. This is mainly due to the complexity arising from interaction between the system structure and switching. Thus, an issue naturally arises: when the system states of a switched uncertain nonlinear system are unmeasured, how to solve the adaptive output-feedback neural control problem by exploiting the switched system theory and the universal approximation property of NNs?

Motivated by the above considerations, this paper studies adaptive output-feedback neural tracking control for switched uncertain nonlinear systems. In the control design, the radial basis function NNs are used to approximate the unknown nonlinear functions. Also, in order to avoid different coordinate transformations for subsystems, a common basis function vector for different subsystems at each step of backstepping is chosen. Compared with the vast existing literature on switched and nonswitched nonlinear systems, the results of this paper have four distinct features.

1) A new adaptive output-feedback neural tracking control technique for switched uncertain nonlinear systems is set up by exploiting the average dwell time method and the backstepping technique for the first time. A sufficient condition for the adaptive output-feedback neural tracking control problem is derived. Also, we extend the adaptive output-feedback neural tracking control problem from its original nonswitched nonlinear version to a switched nonlinear version. To the best of our knowledge, there are no results on adaptive output-feedback neural control available for switched uncertain nonlinear systems up to now.
2) In order to reduce the conservativeness caused by adoption of a common observer in [25] and a common update law in [26] and [34] for all subsystems, a switched filter is designed to estimate the unmeasurable system states, and different update laws for different subsystems are also designed. Meanwhile, we simultaneously construct adaptive output-feedback controllers of subsystems and give a class of switching signals with average dwell time.
3) The classical average dwell time method in [20] cannot be directly applied to handle the adaptive output-feedback neural control problem of switched nonlinear systems since the exponential decline property of Lyapunov functions for individual subsystems is no longer needed in this paper. In order to solve the problem under study, we improve the classical average dwell time method in this paper.
4) Unlike the existing results in [24]–[26] and [37] where all system functions are known, the system functions considered in this paper are unknown. Also, the proposed result extends the stabilizing results of switched nonlinear systems with known nonlinearities in [25] and [37] to switched uncertain nonlinear systems.

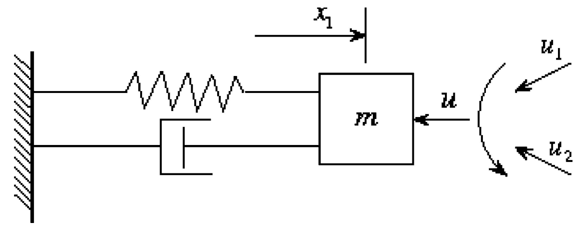

Fig. 1. Mass-spring-damper system with controller switching.

This paper is organized as follows. In Section II, a motivating example is presented. Section III provides the system description and preliminaries. The main result of this paper is presented in Section IV. In Section V, an illustrative example is established. Finally, conclusions are drawn in Section VI.

Notation: The interval $[0, \infty)$ in the space of real numbers $\mathbb{R}$ is denoted by $\mathbb{R}_+$. $\|\cdot\|$ denotes the standard Euclidean norm or the induced matrix 2-norm. I_n denotes a $n \times n$ identity matrix. $\lambda_{\max}(*)$ $(\lambda_{\min}(*))$ denotes the largest (smallest) eigenvalue of the symmetric matrix $*$. $\bar{y}_d^{(i)} = [y_d, y_d^{(1)}, \ldots, y_d^{(i)}]^T$, $i = 1, \ldots, n$, denote the vector of y_d and up to its ith order time derivative. A function $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ is of class $\mathcal{K}$ if α is continuous, strictly increasing, and $\alpha(0) = 0$. If α is also unbounded, it is of class $\mathcal{K}_\infty$ (see [20]).

II. Motivating Example

The output tracking control of mechanical systems has received a lot of attention over the past few decades [12], [17], [42]. One of the major control problems which has been well investigated is the velocity regulation of the motors in a mechanical vibration [17], [31].

In what follows, we present a mass-spring-damper system with controller switching shown in Fig. 1. The mass-spring-damper system with nonlinear stiffness and damping is described by

$$\dot{x}_1 = x_2 \tag{1a}$$
$$\dot{x}_2 = -\frac{1}{m} f(x_1) - \frac{1}{m} g(x_2) + \frac{1}{m} u \tag{1b}$$
$$y = x_1. \tag{1c}$$

The given reference signal is $y_d(t) = 1/100 \sin t$.

This mechanical system has been studied from the point of view of switching control under the assumption that all nonlinearities of the system are known [25], [31], [42]. However, this assumption is often hard to hold because no precise knowledge about the system nonlinearities is known, or the nonlinearities may change with time. In these cases, the approaches in [25], [31], and [42] become infeasible. Here, we consider a realistic circumstance that $f(x_1)$ and $g(x_2)$ are unknown smooth nonlinear functions with $f(0) = 0$ and $g(0) = 0$, and $m > 0$ is an unknown constant. Meanwhile, suppose that we are only allowed to apply two prespecified candidate controllers $u_k = -\Delta f_k(x) + m v_k$, $k = 1, 2$, with $\Delta f_k(0) = 0$

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to the system (1) and switch between them. This results in the following switched uncertain nonlinear system:

$$\dot{x}_1 = x_2 \tag{2a}$$

$$\dot{x}_2 = v_{\sigma(t)} - \frac{1}{m}[f(x_1) + \Delta f_{\sigma(t)}(x)] - \frac{1}{m}g(x_2) \tag{2b}$$

$$y = x_1 \tag{2c}$$

where the function $\sigma(t) : \mathbb{R}_+ \to M = \{1, 2\}$ is a switching signal which is assumed to be a piecewise continuous (from the right) function of time. It is worth pointing out that, no results about output-feedback of the switched uncertain nonlinear system (2) have been reported. There are two main issues to be addressed: 1) when the system states are unavailable, how to solve the adaptive output-feedback tracking problem by exploiting the switched system theory and the universal approximation property and 2) a switched system does not inherit properties of its subsystems. Hence, how to find a sufficient condition for the solvability of the problem above of switched uncertain nonlinear systems without the measurements of the system states by means of suitably constrained switching? The study of adaptive output-feedback neural control of switched uncertain nonlinear systems is of great significance and remains an open area. Therefore, our control objective here is to design output-feedback controllers of subsystems which stabilize the system (2) under some switchings. The objective will be achieved in the sequel.

In the following, we first present a systematic design procedure for the adaptive output-feedback neural tacking control problem of a more general switched uncertain nonlinear system (3).

III. Problem Formulations and Preliminaries

A. Switched Nonlinear Systems

We consider a class of switched uncertain nonlinear systems described by

$$\dot{x}_1 = x_2 + f_{1\sigma(t)}(x_1) + d_{1\sigma(t)}(t) \tag{3a}$$

$$\cdots$$

$$\dot{x}_{n-1} = x_n + f_{n-1,\sigma(t)}(x_1, \ldots, x_{n-1}) + d_{n-1,\sigma(t)}(t) \tag{3b}$$

$$\dot{x}_n = u_{\sigma(t)} + f_{n\sigma(t)}(x) + d_{n\sigma(t)}(t) \tag{3c}$$

$$y = x_1 \tag{3d}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n$ is the system state, $y \in \mathbb{R}$ is the output, m is the number of subsystems, for each k, $u_k \in \mathbb{R}$ is the control input of the kth subsystem. The function $\sigma(t) : \mathbb{R}_+ \to M = \{1, 2, \ldots, m\}$ is a switching signal which is assumed to be a piecewise continuous (from the right) function of time. Denote $\bar{x}_i = [x_1, \ldots, x_i]^T$, $i = 1, \ldots, n$. $f_{ik}(\cdot)$, $i = 1, \ldots, n$, $k \in M$, are unknown smooth (i.e., C^∞) nonlinear functions, $d_{ik}(\cdot)$, $i = 1, \ldots, n$, $k \in M$, are the unknown external disturbances and satisfy $|d_{ik}(t)| \le \bar{d}_{ik}$ with $\bar{d}_{ik}$ being constants. In addition, we assume that the state of the system (3) does not jump at the switching instants, i.e., the solution is everywhere continuous, which is a standard assumption in the switched system literature [20], [41].

Now, we recall the definition of average dwell time, which plays a key role in recent literature of switched nonlinear control [20]. A switching signal σ has average dwell time τ_a if there exist positive numbers N_0 and τ_a such that

$$N_\sigma(T, t) \le N_0 + \frac{T - t}{\tau_a} \quad \forall\, T \ge t \ge 0 \tag{4}$$

where $N_\sigma(T, t)$ is the number of switches occurring in the interval $[t, T)$.

Let $T > 0$ be an arbitrary time. Denote by $t_1, \ldots, t_{N_\sigma(T,0)}$ the switching times on the interval $(0, T)$ (by convention, $t_0 := 0$). When $t \in [t_j, t_{j+1})$, $\sigma(t) = k_j$, that is, the k_jth subsystem is active. Also, we assume $k_j \ne k_{j+1}$ for all j.

Obviously, the structure of the switched nonlinear system (3) is much more general than the structure of the switched and nonswitched nonlinear systems in [4], [22], [23], [25], [37], and [40]. In addition, the problem of adaptive neural tracking control by output-feedback of the switched uncertain nonlinear system (3) has also not been addressed till now.

For a given reference signal $y_d(t)$, our control objective is to construct adaptive output-feedback controllers of subsystems for the system (3) such that for bounded initial conditions, all the signals in the resulting closed-loop system remain bounded under a class of switching signals with average dwell time, and furthermore, for any given constant $\varsigma > 0$, $\lim_{t\to\infty} |y(t) - y_d(t)|^2 \le \varsigma^2$.

To this end, we make the following assumption.

Assumption 1: The reference signal $y_d(t)$ and its time derivatives up to the nth order are bounded and available.

Assumption 1 is commonly used for adaptive tracking control of nonswitched nonlinear systems [4], [5], [45].

B. Function Approximation Using Radial Basis Function NNs

In control engineering, as a class of linear parameterized networks, the radial basis function (RBF) NNs are usually used as a tool for modeling nonlinear functions because of their good capabilities in function approximation. The RBF NNs can be described as $W^T S(Z)$ with input vector $Z \in \Omega \subset \mathbb{R}^n$, weight vector $W \in \mathbb{R}^l$, node number l, and basis function vector $S(Z) \in \mathbb{R}^l$. Meanwhile, universal approximation results in [18] and [33] indicate that, if l is chosen sufficiently large, then $W^T S(Z)$ can approximate any continuous function to any desired accuracy over a compact set $\Omega \subset \mathbb{R}^n$. Thus, the RBF NNs will be used as an approximator to approximate an unknown continuous function in this paper.

As pointed out in [33], for a given $\varepsilon > 0$ and any continuous function $f(Z)$ defined on $\Omega \subset \mathbb{R}^n$, there exists an RBF NN $W^T S(Z)$ such that

$$f(Z) = W^T S(Z) + \delta(Z), \quad |\delta(Z)| \le \varepsilon \tag{5}$$

where $Z \in \Omega \subset \mathbb{R}^n$ is the input vector, $W = [w_1, w_2, \ldots, w_l]^T$ is the weight vector, $l > 1$ is the number of the NN nodes and $S(Z) = [s_1(Z), \ldots, s_l(Z)]^T$, with $s_i(Z)$ being chosen as the commonly used Gaussian functions, which have the form

$$s_i(Z) = \exp\left[\frac{-(Z - \mu_i)^T (Z - \mu_i)}{\phi_i^2}\right], \quad i = 1, 2, \ldots l$$

where $\mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{\text{in}}]^T$ is the center of the receptive field and ϕ_i is the width of the Gaussian function.

With the aid of RBF NNs, we will obtain the main result of this paper, i.e., adaptive output-feedback neural tracking control is achievable by designing adaptive output-feedback controllers of subsystems and giving a class of switching signals with average dwell time, for the system (3).

IV. Main Result

In this section, a systematic design procedure for the adaptive output-feedback neural tracking control problem of the system (3) is presented. It will be divided into three parts as follows. First of all, we design a proper switched filter for estimating the unmeasured states. Then, the backstepping technique is applied to construct output-feedback controllers of subsystems. Finally, we give stability analysis of the resulting closed-loop system on the basis of the average dwell time method. Since applying the backstepping recursive design scheme results in different coordinate transformations for different subsystems, the key point is to handle the different coordinate transformations for subsystems.

A. Switched Filter Design

This section gives the design procedure of a switched filter to estimate the system state since the state variables of the system (3) are not available.

Design the switched filter as

$$\dot{\hat{x}}_1 = \hat{x}_2 - l_{1\sigma(t)}\hat{x}_1 \tag{6a}$$
$$\cdots$$
$$\dot{\hat{x}}_{n-1} = \hat{x}_n - l_{n-1,\sigma(t)}\hat{x}_1 \tag{6b}$$
$$\dot{\hat{x}}_n = u_{\sigma(t)} - l_{n\sigma(t)}\hat{x}_1 \tag{6c}$$

where $\hat{x}_i$, $i = 1, \ldots, n$, are the estimates of x_i, for each k, $k \in M$, u_k is the control input of the kth subsystem, the function $\sigma(t) : \mathbb{R}_+ \to M = \{1, 2, \ldots, m\}$ is a switching signal which is assumed to be a piecewise continuous (from the right) function of time, and l_{ik}, $i = 1, \ldots, n$, $k \in M$, are the design parameters such that the matrices

$$A_k = \begin{bmatrix} -l_{1k} & \\ \vdots & I_{n-1} \\ -l_{nk} & 0 \cdots 0 \end{bmatrix}, \quad k \in M$$

are Hurwitz, which mean that for any given positive definite symmetric matrices $Q_k > 0$, there exist some matrices $P_k > 0$ satisfying

$$A_k^T P_k + P_k A_k = -Q_k. \tag{7}$$

For each $i = 1, \ldots, n$, define $e_i = x_i - \hat{x}_i$. It then follows from (3) and (6) that:

$$\begin{aligned} \dot{e}_1 &= e_2 - l_{1\sigma(t)}e_1 + f_{1\sigma(t)}(x_1) + l_{1\sigma(t)}y + d_{1\sigma(t)}(t) \\ &\cdots \\ \dot{e}_{n-1} &= e_n - l_{n-1,\sigma(t)}e_1 \\ &\quad + f_{n-1,\sigma(t)}(\bar{x}_{n-1}) + l_{n-1,\sigma(t)}y + d_{n-1,\sigma(t)}(t) \\ \dot{e}_n &= -l_{n\sigma(t)}e_1 + f_{n\sigma(t)}(x) + l_{n\sigma(t)}y + d_{n\sigma(t)}(t) \end{aligned}$$

which can be written in the compact form

$$\dot{e} = A_{\sigma(t)}e + F_{\sigma(t)}(x) + D_{\sigma(t)}(t) \tag{8}$$

where $A_{\sigma(t)} : \mathbb{R}_+ \to \{A_1, \ldots, A_m\}$, $e = [e_1, \ldots, e_n]^T$, $F_k(x) = [f_{1k} + l_{1k}y, \ldots, f_{nk} + l_{nk}y]^T$ and $D_k(t) = [d_{1k}, \ldots, d_{nk}]^T$. Therefore, combining (3), (6), and (8), the switched system can be taken as

$$\dot{e} = A_{\sigma(t)}e + F_{\sigma(t)}(x) + D_{\sigma(t)}(t) \tag{9a}$$
$$\dot{y} = \hat{x}_2 + e_2 + f_{1\sigma(t)}(x_1) + d_{1\sigma(t)}(t) \tag{9b}$$
$$\dot{\hat{x}}_2 = \hat{x}_3 - l_{2\sigma(t)}\hat{x}_1 \tag{9c}$$
$$\cdots$$
$$\dot{\hat{x}}_n = u_{\sigma(t)} - l_{n\sigma(t)}\hat{x}_1 \tag{9d}$$

where y, $\hat{x}_i$, l_{ik}, A_k, P_k, and Q_k are available for control design.

Remark 1: It is worth noting that the switched filter (6) is dependent on the subscript $\sigma(t)$, which implies that each subsystem of the system (3) has its own filter. These filters are different from each other. In contrast with the existing results in switched nonlinear systems, different filters for different subsystems are designed to reduce the conservativeness caused by adoption of a common observer for all subsystems [25], which is more reasonable in practical applications.

B. Adaptive Output-Feedback Control Design

Based on the switched filter designed (6), in this section, we construct output-feedback controllers for the system (3) via the backstepping technique.

Similar to the traditional backstepping technique, the recursive design procedure contains n steps. From initial step to step $n-1$, the virtual controllers α_i of subsystems are recursively designed. The controllers u_k of subsystems are constructed at Step n. Meanwhile, at each step, the RBF NN $W_{ik}^T S_i(Z_i)$ is employed to approximate the unknown nonlinear function, i.e., unknown control signal $\hat{\alpha}_{ik}(Z_i)$. Before proceeding with the adaptive output-feedback neural tracking control, define first constants as

$$\Theta_i = \max\left\{\|W_{ik}\|^2 : k \in M\right\}, \quad i = 0, 1, 2, \ldots, n \tag{10}$$

where the function $\hat{\alpha}_{ik}$ and the vector Z_i will be specified in each step. Obviously, Θ_i is an unknown constant since $\|W_{ik}\|$ is an unknown constant. Furthermore, define $\tilde{\Theta}_i = \Theta_i - \hat{\Theta}_i$, $i = 1, 2, \ldots, n$, where $\hat{\Theta}_i$ is the estimate of Θ_i.

1) Initial Step: For the reference signal y_d, define the tracking error $\xi_1 = x_1 - y_d = y - y_d$ and construct the Lyapunov function as

$$V_{1k} = \frac{1}{r}e^T P_k e + \frac{1}{2}\xi_1^2 + \frac{1}{2r}\tilde{\Theta}_1^2$$

where $r > 0$ is a design parameter. Then, it follows from (7) and (9) that:

$$\begin{aligned} \dot{V}_{1k} &= -\frac{1}{r}e^T Q_k e + \frac{2}{r}e^T P_k[F_k + D_k] \\ &\quad + \xi_1[\hat{x}_2 + e_2 + f_{1k} + d_{1k} - \dot{y}_d] - \frac{1}{r}\tilde{\Theta}_1\dot{\hat{\Theta}}_1. \end{aligned} \tag{11}$$

In what follows, the RBF NN $W_{0ik}^T S_0(Z_0)$ is used to approximate the unknown nonlinear function $f_{ik} + l_{ik}y$, $i = 1, \ldots, n$, such that for any given constant $\varepsilon_{0i} > 0$

$$f_{ik} + l_{ik}y = W_{0ik}^T S_0(Z_0) + \delta_{0ik}(Z_0), \ |\delta_{0ik}(Z_0)| \le \varepsilon_{0i}$$

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where $Z_0 = x$, and $\delta_{0ik}(Z_0)$ is the approximation error. Furthermore, using the definition of $F_k(x)$ shows

$$F_k(x) = W_{0k}^T S_0(Z_0) + \delta_{0k}(Z_0),\ \|\delta_{0k}(Z_0)\| \le \varepsilon_0$$

where $W_{0k} = [W_{01k}^T, \ldots, W_{0nk}^T]$, $\delta_{0k}(Z_0) = [\delta_{01k}(Z_0), \ldots, \delta_{0nk}(Z_0)]^T$, and $\varepsilon_0 > 0$ is a constant. Thus, by completion of square and $0 < S_0^T(Z_0)S_0(Z_0) \le l$, we have

$$\begin{aligned}\frac{2}{r}e^T P_k[F_k + D_k] + \xi_1[e_2 + d_{1k}] &\\ \le \frac{4}{r}\|e\|^2 + \frac{1}{r}\|P_k\|^2\big[\Theta_0 l + \varepsilon_0^2 + \|\bar{D}_k\|^2\big] &\\ + \frac{r}{4}\xi_1^2 + \frac{1}{2\rho^2}\xi_1^2 + \frac{1}{2}\rho^2\bar{d}_{1k}^2 &\end{aligned} \tag{12}$$

where l is the number of NN weights, $\rho > 0$ is a design constant and $\bar{D}_k = [\bar{d}_{1k}, \ldots, \bar{d}_{nk}]^T$. Combining (11) and (12) results in

$$\begin{aligned}\dot{V}_{1k} \le & -\frac{1}{r}[\lambda_{\min}(Q_k) - 4]\|e\|^2 + \xi_1[\hat{x}_2 + \bar{f}_{1k}(Z_1)] + \frac{1}{2}\rho^2\bar{d}_{1k}^2\\ & + \frac{1}{r}\|P_k\|^2\big[\Theta_0 l + \varepsilon_0^2 + \|\bar{D}_k\|^2\big] - \frac{1}{2}\xi_1^2 - \frac{1}{r}\tilde{\Theta}_1\dot{\hat{\Theta}}_1\end{aligned} \tag{13}$$

where $Z_1 = [x_1, \hat{\Theta}_1, \bar{y}_d^{(1)}]^T$, $\bar{f}_{1k}(Z_1) = f_{1k} + r/4\xi_1 + 1/2\rho^2\xi_1 + 1/2\xi_1 - \dot{y}_d$. Apparently, if we choose $\hat{\alpha}_{1k}(Z_1) = -[\ell_1\xi_1 + \bar{f}_{1k}]$, where $\ell_1 > 0$ is a design parameter, then (13) can be rewritten in the following form:

$$\begin{aligned}\dot{V}_{1k} \le & -\frac{1}{r}[\lambda_{\min}(Q_k) - 4]\|e\|^2 - \ell_1\xi_1^2 + \xi_1[\hat{x}_2 - \hat{\alpha}_{1k}] + \frac{1}{2}\rho^2\bar{d}_{1k}^2\\ & + \frac{1}{r}\|P_k\|^2\big[\Theta_0 l + \varepsilon_0^2 + \|\bar{D}_k\|^2\big] - \frac{1}{2}\xi_1^2 - \frac{1}{r}\tilde{\Theta}_1\dot{\hat{\Theta}}_1.\end{aligned} \tag{14}$$

However, $\hat{\alpha}_{1k}$ cannot be implemented in practice since it contains the unknown C^∞ function f_{1k}. The RBF NN $W_{1k}^T S_1(Z_1)$ is utilized to approximate $\hat{\alpha}_{1k}$, such that for any given constant $\varepsilon_1 > 0$

$$\hat{\alpha}_{1k} = W_{1k}^T S_1(Z_1) + \delta_{1k}(Z_1), \quad |\delta_{1k}(Z_1)| \le \varepsilon_1 \tag{15}$$

where $\delta_{1k}(Z_1)$ denotes the approximation error. Consequently, one deduces from (10), (15) and completion of square that

$$\begin{aligned}-\xi_1\hat{\alpha}_{1k} &= -\xi_1 W_{1k}^T S_1(Z_1) - \xi_1\delta_{1k}(Z_1)\\ &\le \frac{1}{2a_1^2}\xi_1^2\Theta_1 S_1^T(Z_1)S_1(Z_1) + \frac{1}{2}\xi_1^2 + \frac{1}{2}a_1^2 + \frac{1}{2}\varepsilon_1^2\end{aligned} \tag{16}$$

where $a_1 > 0$ is a design parameter. Let the virtual control signal and the update law as

$$\alpha_1(Z_1) = -\frac{1}{2a_1^2}\xi_1\hat{\Theta}_1 S_1^T(Z_1)S_1(Z_1) \tag{17a}$$

$$\dot{\hat{\Theta}}_1 = \frac{r}{2a_1^2}\xi_1^2 S_1^T(Z_1)S_1(Z_1) - \bar{\ell}_1\hat{\Theta}_1 \tag{17b}$$

where $\bar{\ell}_1 > 0$ is also a design parameter. Then, it follows that $\xi_1\alpha_1 = -(1/2a_1^2)\xi_1^2\hat{\Theta}_1 S_1^T S_1$. Substituting (16) and (17) into (14) yields

$$\dot{V}_{1k} \le \Psi_{1k} + \xi_1(\hat{x}_2 - \alpha_1) \tag{18}$$

where $\Psi_{1k} = -(1/r)[\lambda_{\min}(Q_k) - 4]\|e\|^2 - \ell_1\xi_1^2 + (1/r)\|P_k\|^2 [\Theta_0 l + \varepsilon_0^2 + \|\bar{D}_k\|^2] + (1/2)[a_1^2 + \varepsilon_1^2] + (1/2)\rho^2\bar{d}_{1k}^2 + (\bar{\ell}_1/r)\tilde{\Theta}_1\hat{\Theta}_1$.

Step 2: Define $\xi_2 = \hat{x}_2 - \alpha_1$ and construct the following Lyapunov function:

$$V_{2k} = V_{1k} + \frac{1}{2}\xi_2^2 + \frac{1}{2r}\tilde{\Theta}_2^2.$$

Then, the time derivative of V_{2k} is

$$\dot{V}_{2k} = \dot{V}_{1k} + \xi_2(\hat{x}_3 - l_{2k}\hat{x}_1 - \dot{\alpha}_1) - \frac{1}{r}\tilde{\Theta}_2\dot{\hat{\Theta}}_2 \tag{19}$$

where $\dot{\alpha}_1 = (\partial\alpha_1/\partial x_1)(\hat{x}_2 + e_2 + f_{1k} + d_{1k}) + \sum_{l=0}^{1}(\partial\alpha_1/\partial y_d^{(l)})y_d^{(l+1)} + (\partial\alpha_1/\partial\hat{\Theta}_1)\dot{\hat{\Theta}}_1$. By using the completion of square, one has

$$\begin{aligned}-\xi_2\frac{\partial\alpha_1}{\partial x_1}(e_2 + d_{1k}) \le & \frac{1}{r}\|e\|^2 + \frac{r}{4}\xi_2^2\left[\frac{\partial\alpha_1}{\partial x_1}\right]^2\\ & + \frac{1}{2\rho^2}\xi_2^2\left[\frac{\partial\alpha_1}{\partial x_1}\right]^2 + \frac{1}{2}\rho^2\bar{d}_{1k}^2.\end{aligned} \tag{20}$$

Substituting (17b), (18), and (20) into (19), we arrive at

$$\dot{V}_{2k} \le \Psi_{1k} + \frac{1}{r}\|e\|^2 + \frac{\rho^2}{2}\bar{d}_{1k}^2 + \xi_2[\hat{x}_3 + \bar{f}_{2k}(Z_2)] - \frac{1}{2}\xi_2^2 - \frac{1}{r}\tilde{\Theta}_2\dot{\hat{\Theta}}_2 \tag{21}$$

where $Z_2 = [x_1, \hat{x}_1, \hat{x}_2, \hat{\Theta}_1, \bar{y}_d^{(2)}]^T$, $\bar{f}_{2k}(Z_2) = \xi_1 - l_{2k}\hat{x}_1 - (\partial\alpha_1/\partial x_1)(\hat{x}_2 + f_{1k}) - \sum_{l=0}^{1}(\partial\alpha_1/\partial y_d^{(l)})y_d^{(l+1)} - (\partial\alpha_1/\partial\hat{\Theta}_1)\dot{\hat{\Theta}}_1 + (r/4)\xi_2[\partial\alpha_1/\partial x_1]^2 + (1/2\rho^2)\xi_2[\partial\alpha_1/\partial x_1]^2 + (1/2)\xi_2$. Obviously, if we choose $\hat{\alpha}_{2k}(X_2) = -[\ell_2\xi_2 + \bar{f}_{2k}]$, where $\ell_2 > 0$ is a design parameter, then (21) becomes

$$\begin{aligned}\dot{V}_{2k} \le & \Psi_{1k} + \frac{1}{r}\|e\|^2 + \frac{\rho^2}{2}\bar{d}_{1k}^2 + \xi_2[\hat{x}_3 - \hat{\alpha}_{2k}]\\ & - \ell_2\xi_2^2 - \frac{1}{2}\xi_2^2 - \frac{1}{r}\tilde{\Theta}_2\dot{\hat{\Theta}}_2.\end{aligned} \tag{22}$$

Further, the RBF NN $W_{2k}^T S_2(Z_2)$ is used to approximate the unknown nonlinear function $\hat{\alpha}_{2k}$. For any given constant $\varepsilon_2 > 0$

$$\hat{\alpha}_{2k} = W_{2k}^T S_2(Z_2) + \delta_{2k}(Z_2), \quad |\delta_{2k}(Z_2)| \le \varepsilon_2$$

where $\delta_{2k}(Z_2)$ is the approximation error. Then, a simple calculation yields

$$\begin{aligned}-\xi_2\hat{\alpha}_{2k} &= -\xi_2 W_{2k}^T S_2(Z_2) - \xi_2\delta_{2k}(Z_2)\\ &\le \frac{1}{2a_2^2}\xi_2^2\Theta_2 S_2^T(Z_2)S_2(Z_2) + \frac{1}{2}\xi_2^2 + \frac{1}{2}a_2^2 + \frac{1}{2}\varepsilon_2^2\end{aligned} \tag{23}$$

where $a_2 > 0$ is a design parameter. Now, construct the virtual control signal and the update law as

$$\alpha_2(Z_2) = -\frac{1}{2a_2^2}\xi_2\hat{\Theta}_2 S_2^T(Z_2)S_2(Z_2) \tag{24a}$$

$$\dot{\hat{\Theta}}_2 = \frac{r}{2a_2^2}\xi_2^2 S_2^T(Z_2)S_2(Z_2) - \bar{\ell}_2\hat{\Theta}_2 \tag{24b}$$

where $\bar{\ell}_2 > 0$ is also a design parameter. Putting together (22)–(24) gives

$$\dot{V}_{2k} \le \Psi_{2k} + \xi_2(\hat{x}_3 - \alpha_2) \tag{25}$$

where $\Psi_{2k} = -(1/r)[\lambda_{\min}(Q_k) - 5]\|e\|^2 - \sum_{i=1}^{2}\ell_i\xi_i^2 + (1/r)\|P_k\|^2[\Theta_0 l + \varepsilon_0^2 + \|\bar{D}_k\|^2] + (1/2)\sum_{i=1}^{2}[a_i^2 + \varepsilon_i^2] + \rho^2\bar{d}_{1k}^2 + \sum_{i=1}^{2}(\bar{\ell}_i/r)\tilde{\Theta}_i\hat{\Theta}_i$.

2) Inductive Step: Suppose at the jth step, there exist a Lyapunov function V_{jk} and the following change of coordinates:

$$\xi_1 = x_1 - y_d \tag{26a}$$
$$\xi_i = \hat{x}_i - \alpha_{i-1}(Z_{i-1}), \quad i = 2, \ldots, j+1 \tag{26b}$$

such that

$$\dot{V}_{jk} \le \Psi_{jk} + \xi_j \xi_{j+1} \tag{27}$$

where $Z_i = \left[x_1, \hat{x}_1, \ldots, \hat{x}_i, \hat{\Theta}_1, \ldots, \hat{\Theta}_i, \bar{y}_d^{(i)}\right]^T$

$$\begin{aligned}\Psi_{jk} = & -\frac{1}{r}[\lambda_{\min}(Q_k) - 3 - j]\|e\|^2 - \sum_{i=1}^{j} \ell_i \xi_i^2 \\ & + \frac{1}{r}\|P_k\|^2\left[\Theta_0 l + \varepsilon_0^2 + \|\bar{D}_k\|^2\right] \\ & + \frac{1}{2}\sum_{i=1}^{j}\left[a_i^2 + \varepsilon_i^2\right] + \frac{j}{2}\rho^2 \bar{d}_{1k}^2 + \sum_{i=1}^{j}\frac{\bar{\ell}_i}{r}\tilde{\Theta}_i\hat{\Theta}_i.\end{aligned} \tag{28}$$

To complete the induction, at the $(j+1)$th step, construct the following Lyapunov function as:

$$V_{j+1,k} = V_{jk} + \frac{1}{2}\xi_{j+1}^2 + \frac{1}{2r}\tilde{\Theta}_{j+1}^2. \tag{29}$$

Differentiating $V_{j+1,k}$ yields

$$\dot{V}_{j+1,k} = \dot{V}_{jk} + \xi_{j+1}(\hat{x}_{j+2} - l_{j+1,k}\hat{x}_1 - \dot{\alpha}_j) - \frac{1}{r}\tilde{\Theta}_{j+1}\dot{\hat{\Theta}}_{j+1} \tag{30}$$

where $\dot{\alpha}_j = (\partial\alpha_j/\partial x_1)(\hat{x}_2 + e_2 + f_{1k} + d_{1k}) + \sum_{i=1}^{j}(\partial\alpha_j/\partial\hat{x}_i)\dot{\hat{x}}_i + \sum_{i=1}^{j}(\partial\alpha_j/\partial\hat{\Theta}_i)\dot{\hat{\Theta}}_i + \sum_{i=0}^{j}(\partial\alpha_j/\partial y_d^{(i)})y_d^{(i+1)}$. From the completion of square, the following inequality holds:

$$\begin{aligned}-\xi_{j+1}\frac{\partial\alpha_j}{\partial x_1}(e_2 + d_{1k}) \le & \frac{1}{r}\|e\|^2 + \frac{r}{4}\xi_{j+1}^2\left[\frac{\partial\alpha_j}{\partial x_1}\right]^2 \\ & + \frac{1}{2\rho^2}\xi_{j+1}^2\left[\frac{\partial\alpha_j}{\partial x_1}\right]^2 + \frac{1}{2}\rho^2\bar{d}_{1k}^2.\end{aligned} \tag{31}$$

Then, one can deduce from (17b), (27), (40), and (31) that

$$\begin{aligned}\dot{V}_{j+1,k} \le & \Psi_{jk} + \frac{1}{r}\|e\|^2 + \frac{1}{2}\rho^2\bar{d}_{1k}^2 \\ & + \xi_{j+1}[\hat{x}_{j+2} + \bar{f}_{j+1,k}] - \frac{1}{2}\xi_{j+1}^2 - \frac{1}{r}\tilde{\Theta}_{j+1}\dot{\hat{\Theta}}_{j+1}\end{aligned} \tag{32}$$

where $\bar{f}_{j+1,k} = \xi_j - l_{j+1,k}\hat{x}_1 - (\partial\alpha_j/\partial x_1)(\hat{x}_2 + f_{1k}) - \sum_{i=1}^{j}(\partial\alpha_j/\partial\hat{x}_i)\dot{\hat{x}}_i - \sum_{i=1}^{j}(\partial\alpha_j/\partial\hat{\Theta}_i)\dot{\hat{\Theta}}_i - \sum_{i=0}^{j}(\partial\alpha_j/\partial y_d^{(i)})y_d^{(i+1)} + (r/4)\xi_{j+1}[\partial\alpha_j/\partial x_1]^2 + (1/2\rho^2)\xi_{j+1}[\partial\alpha_j/\partial x_1]^2 + (1/2)\xi_{j+1}$. Apparently, if we take $\hat{\alpha}_{j+1,k}(Z_{j+1}) = -[\ell_{j+1}\xi_{j+1} + \bar{f}_{j+1,k}]$, where $\ell_{j+1} > 0$ is a design parameter, then (32) can be written as

$$\begin{aligned}\dot{V}_{j+1,k} \le & \Psi_{jk} + \frac{1}{r}\|e\|^2 + \frac{1}{2}\rho^2\bar{d}_{1k}^2 + \xi_{j+1}[\hat{x}_{j+2} - \hat{\alpha}_{j+1,k}] \\ & - \ell_{j+1}\xi_{j+1}^2 - \frac{1}{2}\xi_{j+1}^2 - \frac{1}{r}\tilde{\Theta}_{j+1}\dot{\hat{\Theta}}_{j+1}.\end{aligned} \tag{33}$$

Since $\hat{\alpha}_{j+1,k}$ is an unknown nonlinear function, the RBF NN $W_{j+1,k}^T S_{j+1}(Z_{j+1})$ is now employed to approximate it. Thus, for any given constant $\varepsilon_{j+1} > 0$, there exists an RBF NN $W_{j+1,k}^T S_{j+1}(Z_{j+1})$ such that

$$\begin{aligned}\hat{\alpha}_{j+1,k} &= W_{j+1,k}^T S_{j+1}(Z_{j+1}) + \delta_{j+1,k}(Z_{j+1}) \\ |\delta_{j+1,k}(Z_{j+1})| &\le \varepsilon_{j+1}\end{aligned}$$

where $\delta_{j+1,k}(Z_{j+1})$ is the approximation error. Further, it is easy to verify that

$$\begin{aligned}-\xi_{j+1}\hat{\alpha}_{j+1,k} = & -\xi_{j+1}W_{j+1,k}^T S_{j+1}(Z_{j+1}) - \xi_{j+1}\delta_{j+1,k}(Z_{j+1}) \\ \le & \frac{1}{2a_{j+1}^2}\xi_{j+1}^2\Theta_{j+1}S_{j+1}^T S_{j+1} \\ & + \frac{1}{2}\xi_{j+1}^2 + \frac{1}{2}a_{j+1}^2 + \frac{1}{2}\varepsilon_{j+1}^2\end{aligned} \tag{34}$$

where $a_{j+1} > 0$ is a design parameter. Now, construct the virtual control signal and the update law as

$$\alpha_{j+1}(Z_{j+1}) = -\frac{1}{2a_{j+1}^2}\xi_{j+1}\hat{\Theta}_{j+1}S_{j+1}^T(Z_{j+1})S_{j+1}(Z_{j+1}) \tag{35a}$$

$$\dot{\hat{\Theta}}_{j+1} = \frac{r}{2a_{j+1}^2}\xi_{j+1}^2 S_{j+1}^T(Z_{j+1})S_{j+1}(Z_{j+1}) - \bar{\ell}_{j+1}\hat{\Theta}_{j+1} \tag{35b}$$

where $\bar{\ell}_{j+1} > 0$ is also a design parameter. It follows from (33)–(35) that:

$$\dot{V}_{j+1,k} \le \Psi_{j+1,k} + \xi_{j+1}(\hat{x}_{j+2} - \alpha_{j+1}). \tag{36}$$

This completes the inductive proof. Using this inductive argument above, we conclude that at Step n, there exist a common output-feedback controller of different subsystems and the update law as

$$u_k = \alpha_n(Z_n) = -\frac{1}{2a_n^2}\xi_n\hat{\Theta}_n S_n^T(Z_n)S_n(Z_n) \tag{37a}$$

$$\dot{\hat{\Theta}}_n = \frac{r}{2a_n^2}\xi_n^2 S_n^T(Z_n)S_n(Z_n) - \bar{\ell}_n\hat{\Theta}_n, \ k \in M \tag{37b}$$

and Lyapunov functions $V_k = V_{nk} = (1/r)e^T P_k e + (1/2)\sum_{i=1}^{n}\xi_i^2 + (1/2r)\sum_{i=1}^{n}\tilde{\Theta}_i^2$, such that

$$\dot{V}_{nk} \le \Psi_{nk} \tag{38}$$

where Ψ_{nk} is defined by (28) with $j = n$.

Remark 2: According to the design procedure, we can also design the different output-feedback controllers and the update law for different subsystems

$$u_k = \alpha_{nk}(Z_n) = -\frac{1}{2a_{nk}^2}\xi_n\hat{\Theta}_n S_{nk}^T(Z_n)S_{nk}(Z_n) \tag{39a}$$

$$\dot{\hat{\Theta}}_n = \frac{r}{2a_{nk}^2}\xi_n^2 S_{nk}^T(Z_n)S_{nk}(Z_n) - \bar{\ell}_n\hat{\Theta}_n, \ k \in M \tag{39b}$$

where $\ell_{nk} > 0$ and $a_{nk} > 0$ are some design parameters. A simple calculation yields

$$\begin{aligned}\dot{V}_{nk} \le & \Psi_{n-1,k} + \frac{1}{r}\|e\|^2 + \frac{1}{2}\rho^2\bar{d}_{1k}^2 \\ & - \ell_{nk}\xi_n^2 + \frac{1}{2}[a_{nk}^2 + \varepsilon_{nk}^2] + \frac{\bar{\ell}_n}{r}\tilde{\Theta}_n\hat{\Theta}_n \le \Psi_{nk}\end{aligned}$$

where $\ell_n = \min_{k\in M}\{\ell_{nk}\}$ and $a_n^2 + \varepsilon_n^2 = \max_{k\in M}\{a_{nk}^2 + \varepsilon_{nk}^2\}$.

On the other hand, for the system (3), it is worth noting that, in the systematic design procedure above, a common coordinate transformation for different subsystems at each step of backstepping is designed, which effectively avoids individual coordinate transformations for subsystems when applying backstepping. It should be also noticed that, in [26] and [34], a common update law is used to estimate all the vector parameters in different subsystems. However, most switched systems do not possess a common update law for all subsystems from both practical and theoretical points of view. In this paper, we design different update laws (39) for different subsystems, which is more reasonable in practical applications.

C. Stability Analysis

In this section, the boundedness of all the signals in the resulting closed-loop system will be proved. First of all, for notational convenience, let

$$a_0 = \min_{k\in M}\{[\lambda_{\min}(Q_k)-3-n]/\lambda_{\max}(P_k), 2\ell_i, \bar{\ell}_i, \quad i=1,\ldots,n\} \tag{40}$$

$$\mu = \max\left\{\frac{\lambda_{\max}(P_k)}{\lambda_{\min}(P_l)}, \quad k,l\in M\right\}. \tag{41}$$

Obviously, the inequalities $\lambda_{\min}(Q_k) - 3 - n > 0$, $k \in M$, can be obtained by appropriately choosing $Q_k > 0$. It is easy to see that $a_0 > 0$ and $\mu \geq 1$ are known constants. The main result in this paper is summarized in the following theorem.

Theorem 1: For the reference signal $y_d(t)$, consider the system (3) satisfying Assumption 1. Suppose that for $i = 1,\ldots,n$, $k \in M$, the packaged unknown functions $\hat{\alpha}_{ik}$ can be approximated by NNs in the sense that the approximating errors δ_{ik} are bounded. Then, for bounded initial conditions, the adaptive output-feedback neural controllers (17b), (24b), (35b), and (37) render all the signals in the resulting closed-loop system to remain bounded for every switching signal $\sigma(t)$ with average dwell time $\tau_a > (\log\mu/a_0)$. Furthermore, for any given constant $\varsigma > 0$, we can appropriately choose design parameters of subsystems, such that $\lim_{t\to\infty}|y(t) - y_d(t)|^2 \leq \varsigma^2$.

Proof: The proof includes two steps. We will first prove the semiglobal stability of the closed-loop system in part 1), and the convergence of output tracking error in part 2), respectively.

1) For stability analysis of the closed-loop system, construct the Lyapunov functions

$$V_k(X) = \frac{1}{r}e^T P_k e + \frac{1}{2}\sum_{i=1}^{n}\xi_i^2 + \frac{1}{2r}\sum_{i=1}^{n}\tilde{\Theta}_i^2, \quad k\in M$$

where $X = [e^T, \xi_1,\ldots,\xi_n, \tilde{\Theta}_1,\ldots,\tilde{\Theta}_n]^T$. It is easy to deduce that there exist $\underline{\alpha}$, $\bar{\alpha} \in \mathcal{K}_\infty$, such that $\underline{\alpha}(\|X\|) \leq V_k(X) \leq \bar{\alpha}(\|X\|)$. Meanwhile, by (41), one has $V_k(X(t)) \leq \mu V_l(X(t))$, $\forall k,l \in M$. In view of (28) and (38), we have

$$\begin{aligned}\dot{V}_k \leq& -\frac{1}{r}[\lambda_{\min}(Q_k) - 3 - n]\|e\|^2 - \sum_{i=1}^{n}\ell_i\xi_i^2\\ &+\frac{1}{r}\|P_k\|^2\big[\Theta_0 l + \varepsilon_0^2 + \|\bar{D}_k\|^2\big]\\ &+\frac{1}{2}\sum_{i=1}^{n}\big[a_i^2+\varepsilon_i^2\big] + \frac{n}{2}\rho^2\bar{d}_{1k}^2 + \sum_{i=1}^{n}\frac{\bar{\ell}_i}{r}\tilde{\Theta}_i\hat{\Theta}_i.\end{aligned} \tag{42}$$

For the terms $(\bar{\ell}_i/r)\tilde{\Theta}_i\hat{\Theta}_i$, $i = 1,\ldots,n$, the following inequalities hold:

$$\frac{\bar{\ell}_i}{r}\tilde{\Theta}_i\hat{\Theta}_i \leq -\frac{\bar{\ell}_i}{2r}\tilde{\Theta}_i^2 + \frac{\bar{\ell}_i}{2r}\Theta_i^2, \quad i=1,\ldots,n. \tag{43}$$

Then, it follows from (42) and (43) that:

$$\begin{aligned}\dot{V}_k \leq& -\frac{1}{r}[\lambda_{\min}(Q_k)-3-n]\|e\|^2 - \sum_{i=1}^{n}\ell_i\xi_i^2\\ &-\sum_{i=1}^{n}\frac{\bar{\ell}_i}{2r}\tilde{\Theta}_i^2 + b_0 \leq -a_0V_k + b_0\end{aligned} \tag{44}$$

where

$$\begin{aligned}b_0 = \max_{k\in M}\Bigg\{&\frac{1}{r}\|P_k\|^2\big[\Theta_0 l + \varepsilon_0^2 + \|\bar{D}_k\|^2\big]\\ &+\frac{1}{2}\sum_{i=1}^{n}\big[a_i^2+\varepsilon_i^2\big] + \frac{n}{2}\rho^2\bar{d}_{1k}^2 + \sum_{i=1}^{n}\frac{\bar{\ell}_i}{2r}\Theta_i^2\Bigg\} > 0.\end{aligned} \tag{45}$$

It is easy to see that the function $W(t) = e^{a_0t}V_{\sigma(t)}\big(X(t)\big)$ is piecewise differentiable along solutions of the system (3). In view of (44), on each interval $[t_j, t_{j+1})$, one has

$$\begin{aligned}\dot{W}(t) &= a_0e^{a_0t}V_{\sigma(t)}(X(t)) + e^{a_0t}\dot{V}_{\sigma(t)}(X(t))\\ &\leq b_0e^{a_0t}, \quad t\in[t_j, t_{j+1}).\end{aligned}$$

This, together with $V_k(X(t)) \leq \mu V_l(X(t))$, $\forall k,l \in M$, implies that

$$\begin{aligned}W(t_{j+1}) &= e^{a_0t_{j+1}}V_{\sigma(t_{j+1})}(X(t_{j+1}))\\ &\leq \mu e^{a_0t_{j+1}}V_{\sigma(t_j)}(X(t_{j+1})) = \mu W(t_{j+1}^-)\\ &\leq \mu\left[W(t_j) + \int_{t_j}^{t_{j+1}} b_0e^{a_0t}dt\right].\end{aligned} \tag{46}$$

Pick an arbitrary $T > t_0 = 0$. Iterating the inequality (46) from $j = 0$ to $j = N_\sigma(T,0) - 1$, we obtain that

$$\begin{aligned}W(T^-) \leq& W(t_{N_\sigma(T,0)}) + \int_{t_{N_\sigma(T,0)}}^{T} b_0e^{a_0t}dt\\ \leq& \mu\Bigg[W(t_{N_\sigma(T,0)-1}) + \int_{t_{N_\sigma(T,0)-1}}^{t_{N_\sigma(T,0)}} b_0e^{a_0t}dt\\ &+\mu^{-1}\int_{t_{N_\sigma(T,0)}}^{T} b_0e^{a_0t}dt\Bigg]\\ \leq& \cdots\\ \leq& \mu^{N_\sigma(T,0)}\Bigg[W(0) + \sum_{j=0}^{N_\sigma(T,0)-1}\mu^{-j}\int_{t_j}^{t_{j+1}} b_0e^{a_0t}dt\\ &+\mu^{-N_\sigma(T,0)}\int_{t_{N_\sigma(T,0)}}^{T} b_0e^{a_0t}dt\Bigg].\end{aligned} \tag{47}$$

Since $\tau_a > (\log \mu / a_0)$, for any $\delta \in (0, a_0 - (\log \mu / \tau_a))$, one has $\tau_a > (\log \mu / a_0 - \delta)$. By (4), it holds that

$$N_\sigma(T, t) \leq N_0 + \frac{(a_0 - \delta)(T - t)}{\log \mu} \quad \forall\, T \geq t \geq 0.$$

In addition, observe that $N_\sigma(T, 0) - j \leq 1 + N_\sigma(T, t_{j+1})$, $j = 0, 1, \ldots, N_\sigma(T, 0)$, implies

$$\mu^{N_\sigma(T,0)-j} \leq \mu^{1+N_0} e^{(a_0-\delta)(T-t_{j+1})}.$$

In addition, since $\delta < a_0$

$$\int_{t_j}^{t_{j+1}} b_0 e^{a_0 t} dt \leq e^{(a_0-\delta)t_{j+1}} \int_{t_j}^{t_{j+1}} b_0 e^{\delta t} dt. \quad (48)$$

It then follows from (47) and (48) that:

$$W(T^-) \leq \mu^{N_\sigma(T,0)} W(0) + \mu^{1+N_0} e^{(a_0-\delta)T} \int_0^T b_0 e^{\delta t} dt$$

which indicates that

$$\begin{aligned}
\underline{\alpha}(\|X(T)\|) &\leq V_{\sigma(T^-)}(X(T^-)) \\
&\leq e^{N_0 \log \mu} e^{\left(\frac{\log \mu}{\tau_a} - a_0\right)T} \bar{\alpha}(\|X(0)\|) \\
&\quad + \mu^{1+N_0} \frac{b_0}{\delta}(1 - e^{-\delta T}) \\
&\leq e^{N_0 \log \mu} e^{\left(\frac{\log \mu}{\tau_a} - a_0\right)T} \bar{\alpha}(\|X(0)\|) \\
&\quad + \mu^{1+N_0} \frac{b_0}{\delta} \quad \forall\, T > 0. \qquad (49)
\end{aligned}$$

We conclude that, by (49) and $\delta > 0$, if τ_a satisfies $\tau_a > (\log \mu / a_0)$, then for bounded initial conditions, e, ξ_i and $\tilde{\Theta}_i$, $i = 1, \ldots, n$, are bounded. Since Θ_i, $i = 1, \ldots, n$, are constants, $\hat{\Theta}_i$, $i = 1, \ldots, n$, are bounded. Further, by Assumptions 1 and (26b), $\hat{x}_i$, $i = 1, \ldots, n$ are bounded. In addition, using the definition of $e_i = x_i - \hat{x}_i$, it is easy to see that x_i, $i = 1, \ldots, n$ are also bounded. Hence, for bounded initial conditions, all the signals in the closed-loop system (3), (17b), (24b), (35b), and (37) [or (39)] are bounded for every switching signal $\sigma(t)$ with average dwell time $\tau_a > (\log \mu / a_0)$.

2) On the other hand, for any given constant $\varsigma > 0$, the inequality $\mu^{1+N_0}(b_0/\delta) \leq (1/2)\varsigma^2$ can be obtained by appropriately choosing the matrices Q_k, $k \in M$, and the design parameters ℓ_i and $\bar{\ell}_i$, and choosing a_i, ε_i, and ρ sufficiently small and r sufficiently large. In addition, from (49), it follows that:

$$\begin{aligned}
\frac{1}{2}\xi_1^2(T) &\leq e^{N_0 \log \mu} e^{\left(\frac{\log \mu}{\tau_a} - a_0\right)T} \bar{\alpha}(\|X(0)\|) \\
&\quad + \mu^{1+N_0} \frac{b_0}{\delta}(1 - e^{-\delta T}) \quad \forall\, T > 0
\end{aligned}$$

which, together with $\tau_a > (\log \mu / a_0)$, implies that

$$\lim_{t \to \infty} \xi_1^2(t) = \lim_{t \to \infty} |y(t) - y_d(t)|^2 \leq 2\mu^{1+N_0} \frac{b_0}{\delta} \leq \varsigma^2.$$

This completes the proof of Theorem 1. □

Remark 3: In the analysis above, it is easy to see that, since the function approximation property (5) of NN is only guaranteed within a compact set, the stability result proposed in this paper is semiglobal in the sense that, for any compact set, there exist controllers of subsystems with sufficiently large number of NN nodes such that, when the initial states are within the compact set, the boundedness of all the signals are guaranteed for every switching signal $\sigma(t)$ with average dwell time $\tau_a > (\log \mu / a_0)$. However, the NN node number usually cannot be chosen too large due to the possible computation in practical applications. This implies that the NN approximation capability is limited, and some constraints are necessary to guarantee the NN approximation. Hence, an explicit expression of the stability condition is not available since there are no analytical results available in the NN literature (see [44] for details). In fact, Theorem 1 provides a guideline for the designers. From (40), (41), and (45), some suggestions are given for the choice of some key design parameters:

1) increasing ℓ_i and $\bar{\ell}_i$, $i = 1, \ldots, n$, helps to increase a_0 and δ, subsequently reduces $\mu^{1+N_0}(b_0/\delta)$;
2) decreasing a_i, ε_i, and ρ, and increasing r helps to reduce b_0, and reduces $\mu^{1+N_0}(b_0/\delta)$.

However, decreasing a_i will lead to a high gain control scheme. In addition, though $\bar{\ell}_i$ is required to be chosen as a positive constant, a relative small $\bar{\ell}_i$ may not be enough to prevent the NN weight estimates from drifting to very large values in the presence of NN approximation errors, where the large $\hat{\Theta}_i$ might result in a variation of a high gain control [21]. Therefore, in practical applications, the design parameters should be adjusted carefully for improving both stability and performance of the adaptive system.

Remark 4: For switched nonlinear systems with known nonlinearities, Long and Zhao [24], [25] and Wu [37] present some effective approaches of the global stabilization problem and give some conditions to guarantee the solvability of the problem studied. However, the methods in [24], [25], and [37] cannot be directly applied to handle the adaptive neural tracking control problem of (3) since, different from [24], [25], and [37] where all system nonlinearities are known, the system nonlinearities of the system considered in this paper are unknown.

Remark 5: Using the method in [46], we can obtain the initial condition set. By (45) and (49), one has

$$\underline{\alpha}(\|X\|) \leq e^{N_0 \log \mu} \bar{\alpha}(\|X(0)\|) + \mu^{1+N_0} \frac{b_0}{\delta} \quad \forall\, T \geq 0$$

which implies that

$$\|X\| \leq R(\varepsilon_0, \varepsilon_1, \ldots, \varepsilon_n, X(0)) \quad \forall\, T \geq 0$$

where $R(\varepsilon_0, \varepsilon_1, \ldots, \varepsilon_n, X(0)) = \underline{\alpha}^{-1}(e^{N_0 \log \mu} \bar{\alpha}(\|X(0)\|) + \mu^{1+N_0}(b_0/\delta))$. Therefore, we can define the following set as the initial condition set:

$$\Omega_0 = \{X(0) | \{X \big| \|X\| \leq R(0, \ldots, 0, X(0))\} \subset \Omega\}.$$

This set Ω_0 indicates the relation between initial bounding conditions and state trajectories by the functions $\underline{\alpha}$, $\bar{\alpha}$ and the constants $e^{N_0 \log \mu}$, $\mu^{1+N_0}(b_0/\delta)$.

V. Example

In this section, we show the applicability and effectiveness of our approach by an example.

Example 1: Let us revisit the mass-spring-damper system presented in Section II. First of all, following the design procedure in Section IV, we design the switched filter as:

$$\dot{\hat{x}}_1 = \hat{x}_2 - l_{1\sigma(t)}\hat{x}_1 \tag{50a}$$
$$\dot{\hat{x}}_2 = \upsilon_{\sigma(t)} - l_{2\sigma(t)}\hat{x}_1 \tag{50b}$$

and take the design parameters $l_{11} = 2, l_{21} = 1, l_{12} = l_{22} = 4$. Obviously, the matrices A_1 and A_2 are Hurwitz. In addition, we select $Q_1 = 6I_2$ and $Q_2 = 8I_2$. It can be verified that there exist positive-definite symmetric matrices

$$P_1 = \begin{bmatrix} 3 & -3 \\ -3 & 9 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 5 & -4 \\ -4 & 21/4 \end{bmatrix}$$

satisfying $A_k^T P_k + P_k A_k = -Q_k, k \in M$. For each $i = 1, 2$, define $e_i = x_i - \hat{x}_i$. Then, it follows from (2) and (50) that:

$$\dot{e} = A_{\sigma(t)}e + F_{\sigma(t)}(x) \tag{51a}$$
$$\dot{y} = \hat{x}_2 + e_2 \tag{51b}$$
$$\dot{\hat{x}}_2 = \upsilon_{\sigma(t)} - l_{2\sigma(t)}\hat{x}_1 \tag{51c}$$

where $F_k(x) = [l_{1k}y, -1/m[f(x_1) + \Delta f_{\sigma(t)}(x)] - 1/mg(x_2) + l_{2k}y]^T$.

In what follows, we present a systematic design procedure for the adaptive output-feedback neural tracking control problem of the system (2).

Step 1: For the given reference signal $y_d(t) = (1/100)\sin t$, define the tracking error $\xi_1 = y - y_d$ and construct $V_{1k} = (1/r)e^T P_k e + (1/2)\xi_1^2$, where $r > 0$ is a design parameter. Then, the derivative of V_{1k} is

$$\dot{V}_{1k} = -\frac{1}{r}e^T Q_k e + \frac{2}{r}e^T P_k F_k + \xi_1\Big[\hat{x}_2 + e_2 - \frac{1}{100}\cos t\Big]. \tag{52}$$

Using a similar way as in (12), the following inequality can be obtained:

$$\frac{2}{r}e^T P_k F_k + \xi_1 e_2 \le \frac{3}{r}\|e\|^2 + \frac{1}{r}\|P_k\|^2\big[\Theta_0 l + \varepsilon_0^2\big] + \frac{r}{4}\xi_1^2 \tag{53}$$

where $\Theta_0 = \max\{\|W_{0k}\|^2 : k \in M\}$.

Let the virtual control signal $\alpha_1 = -(\ell_1\xi_1) - (r/4)\xi_1 + (1/100)\cos t$, where $\ell_1 > 0$ is a design parameter. Putting together (52) and (53) gives

$$\begin{aligned}\dot{V}_{1k} \le& -\frac{1}{r}[\lambda_{\min}(Q_k) - 3]\|e\|^2 - \ell_1\xi_1^2 \\ &+ \xi_1[\hat{x}_2 - \alpha_1] + \frac{1}{r}\|P_k\|^2\big[\Theta_0 l + \varepsilon_0^2\big].\end{aligned} \tag{54}$$

Step 2: Define $\xi_2 = \hat{x}_2 - \alpha_1$ and construct the following Lyapunov function:

$$V_k = V_{1k} + \frac{1}{2}\xi_2^2 + \frac{1}{2r}\tilde{\Theta}^2$$

where $\Theta = \max\{\|W_{2k}\|^2 : k \in M\}$. Differentiating V_k yields

$$\dot{V}_k = \dot{V}_{1k} + \xi_2(\upsilon_k - l_{2k}\hat{x}_1 - \dot{\alpha}_1) - \frac{1}{r}\tilde{\Theta}\dot{\hat{\Theta}} \tag{55}$$

where $\dot{\alpha}_1 = (\partial\alpha_1/\partial x_1)(\hat{x}_2 + e_2) + \sum_{l=0}^{1}(\partial\alpha_1/\partial y_d^{(l)})y_d^{(l+1)}$. By using the completion of square, we have

$$-\xi_2\frac{\partial\alpha_1}{\partial x_1}e_2 \le \frac{1}{r}\|e\|^2 + \frac{r}{4}\xi_2^2\left[\frac{\partial\alpha_1}{\partial x_1}\right]^2. \tag{56}$$

Then, one can deduce from (54)–(56) that

$$\begin{aligned}\dot{V}_k \le& -\frac{1}{r}[\lambda_{\min}(Q_k) - 4]\|e\|^2 - \ell_1\xi_1^2 \\ &+ \xi_2[\upsilon_k + \bar{f}_{2k}] + \frac{1}{r}\|P_k\|^2\big[\Theta_0 l + \varepsilon_0^2\big] - \frac{1}{2}\xi_2^2 - \frac{1}{r}\tilde{\Theta}\dot{\hat{\Theta}}\end{aligned} \tag{57}$$

where $Z_2 = [x_1, \hat{x}_1, \hat{x}_2]^T$, $\bar{f}_{2k} = \xi_1 - l_{2k}\hat{x}_1 - (\partial\alpha_1/\partial x_1)\hat{x}_2 - \sum_{l=0}^{1}(\partial\alpha_1/\partial y_d^{(l)})y_d^{(l+1)} + (r/4)\xi_2[\partial\alpha_1/\partial x_1]^2 + (1/2)\xi_2$, $y_d^{(1)} = (1/100)\cos t$, and $y_d^{(2)} = -(1/100)\sin t$. Apparently, if we take $\hat{\alpha}_{2k}(Z_2) = -[\ell_{2k}\xi_2 + \bar{f}_{2k}]$, where $\ell_{2k} > 0$, $k \in M$, are some design parameters, then (57) becomes

$$\begin{aligned}\dot{V}_k \le& -\frac{1}{r}[\lambda_{\min}(Q_k) - 4]\|e\|^2 - \ell_1\xi_1^2 - \ell_{2k}\xi_2^2 \\ &+ \xi_2[\upsilon_k - \hat{\alpha}_{2k}] + \frac{1}{r}\|P_k\|^2[\Theta_0 l + \varepsilon_0^2] - \frac{1}{2}\xi_2^2 - \frac{1}{r}\tilde{\Theta}\dot{\hat{\Theta}}.\end{aligned} \tag{58}$$

Similarly, for any given constant $\varepsilon_2 > 0$, $\hat{\alpha}_{2k}(Z_2)$ can be approximated by the RBF NN $W_{2k}^T S_{2k}(Z_2)$ as

$$\hat{\alpha}_{2k} = W_{2k}^T S_{2k}(Z_2) + \delta_{2k}(Z_2), \ |\delta_{2k}(Z_2)| \le \varepsilon_2.$$

We can also obtain

$$-\xi_2\hat{\alpha}_{2k} \le \frac{1}{2a_{2k}^2}\xi_2^2\Theta S_{2k}^T(Z_2)S_{2k}(Z_2) + \frac{1}{2}a_{2k}^2 + \frac{1}{2}\xi_2^2 + \frac{1}{2}\varepsilon_2^2 \tag{59}$$

where $a_{2k} > 0$, $k \in M$, are some design parameters. Now, for each subsystem, design the output-feedback controller and the update law as

$$\upsilon_k = -\frac{1}{2a_{2k}^2}\xi_2\hat{\Theta}S_{2k}^T(Z_2)S_{2k}(Z_2) \tag{60a}$$
$$\dot{\hat{\Theta}} = \frac{r}{2a_{2k}^2}\xi_2^2 S_{2k}^T(Z_2)S_{2k}(Z_2) - \bar{\ell}_{2k}\hat{\Theta}, \quad k \in M \tag{60b}$$

where $\bar{\ell}_{2k} > 0$, $k \in M$, are also some design parameters. It then follows from (58)–(60) that:

$$\begin{aligned}\dot{V}_k \le& -\frac{1}{r}[\lambda_{\min}(Q_k) - 4]\|e\|^2 - \ell_1\xi_1^2 - \ell_{2k}\xi_2^2 - \frac{\bar{\ell}_{2k}}{2r}\tilde{\Theta}^2 \\ &+ \frac{1}{r}\|P_k\|^2\big[\Theta_0 l + \varepsilon_0^2\big] + \frac{1}{2}\big[a_{2k}^2 + \varepsilon_2^2\big] + \frac{\bar{\ell}_{2k}}{2r}\Theta^2 \\ \le& -a_0 V_k + b_0\end{aligned}$$

where

$$a_0 = \min_{k\in M}\left\{\frac{\lambda_{\min}(Q_k) - 4}{\lambda_{\max}(P_k)}, 2\ell_1, 2\ell_{2k}, \bar{\ell}_{2k}\right\} > 0$$

$$b_0 = \max_{k\in M}\left\{\frac{1}{r}\|P_k\|^2\big[\Theta_0 l + \varepsilon_0^2\big] + \frac{1}{2}\big[a_{2k}^2 + \varepsilon_2^2\big] + \frac{\bar{\ell}_{2k}}{2r}\Theta^2\right\} > 0.$$

On the other hand, it is easy to see that $V_k \le \mu V_l$, $k, l \in M$, where $\mu = \max\{\lambda_{\max}(P_k)/\lambda_{\min}(P_l), k, l \in M\}$. Therefore, the adaptive output-feedback neural tracking control problem of the system (2) is solvable for every switching signal $\sigma(t)$ with average dwell time $\tau_a > (\log\mu/a_0)$ on the basis of Theorem 1.

The simulation is also conducted to verify our design of the example. According to [33], Gaussian RBF NNs arranged on a regular lattice on $\mathbb{R}^n$ can uniformly approximate sufficiently smooth functions on closed, bounded subsets. Furthermore, given only crude estimates of the smoothness of the function being approximated, it is feasible to select the centers and

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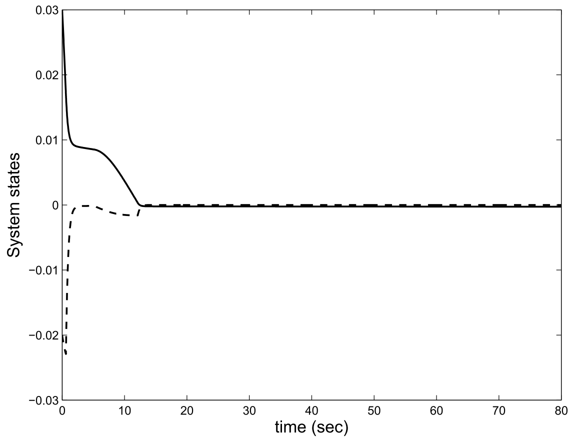

Fig. 2. Responses of x_1 (solid line) and x_2 (dash-dotted) under the switching signal 1.

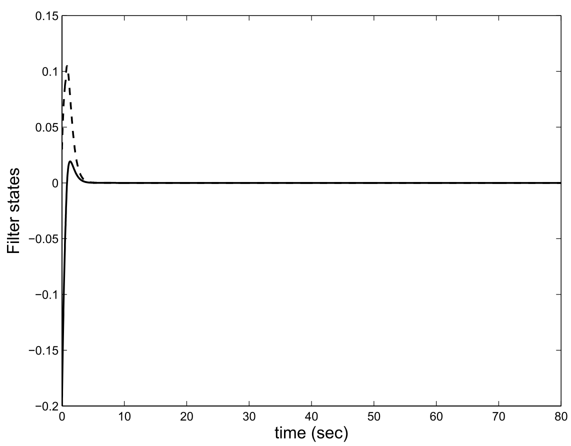

Fig. 3. Responses of $\hat{x}_1$ (solid line) and $\hat{x}_2$ (dash-dotted) under the switching signal 1.

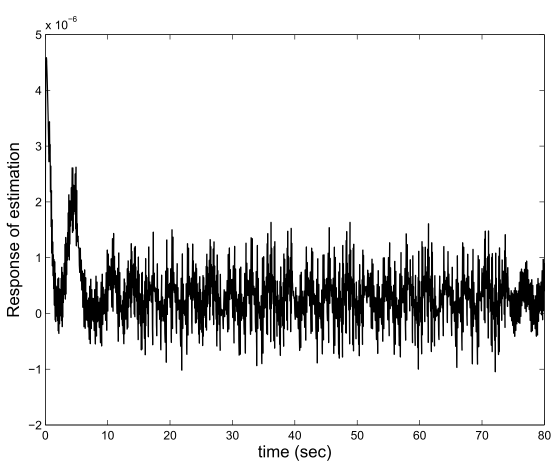

Fig. 4. Response of estimation $\hat{\Theta}$ under the switching signal 1.

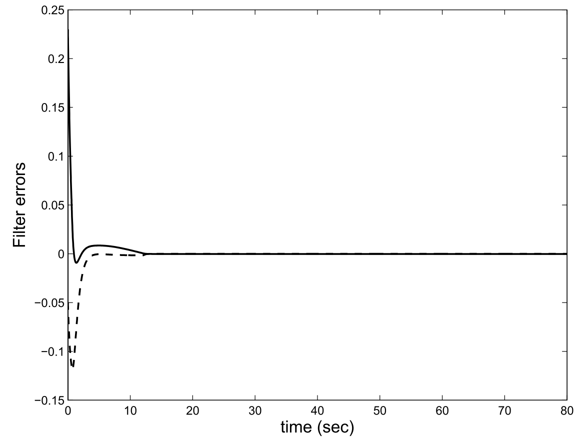

Fig. 5. Responses of $x_1 - \hat{x}_1$ (solid line) and $x_2 - \hat{x}_2$ (dash-dotted) under the switching signal 1.

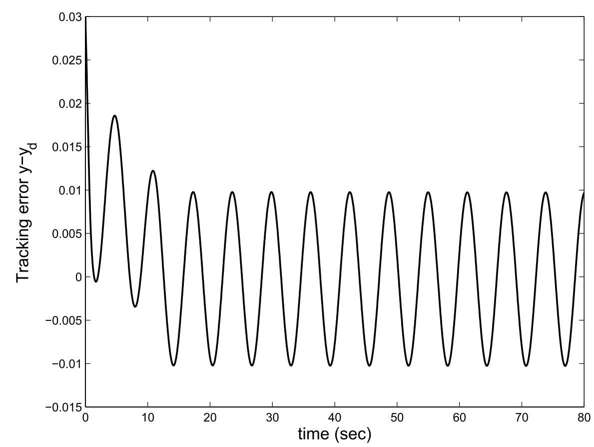

Fig. 6. Tracking error $y - y_d$ under the switching signal 1.

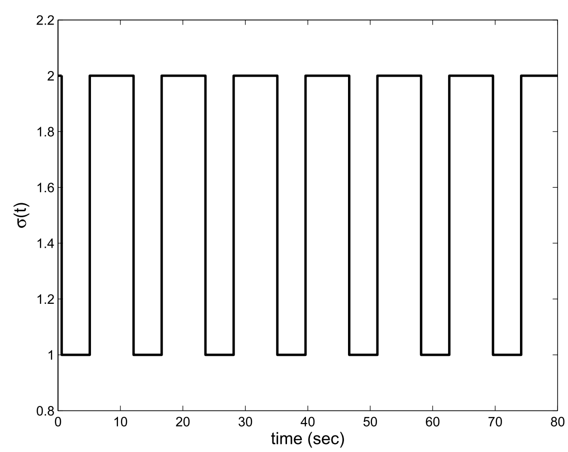

Fig. 7. Switching signal 1.

variances of a finite number of Gaussian nodes, so that the resulting NNs are capable of uniformly approximating the required function to a chosen tolerance everywhere on a prespecified subset. Accordingly, in the following simulation studies, we select the centers and widths on a regular lattice in the respective compact sets. Specifically, we construct the basis function vectors $S_{21}(Z)$ and $S_{22}(Z)$ using 15 and 27 nodes, with the centers μ_{21} and μ_{22} evenly spaced on $[-0.5, 3.5] \times [-4, 4] \times [-8, 8]$ and $[-4, 4] \times [-30, 10] \times [-0.5, 3.5]$, and the widths $\phi_{21} = 2.5$ and $\phi_{22} = 2$, respectively. The results are shown in Figs. 2–7 when $m = 1/3$, $f = 2x_1^2$, $g = x_2^2 \cos x_2$, $\Delta f_1 = x_1^3 \sin(x_1 x_2)$, $\Delta f_2 = x_2 \cos(x_1^2)$, $a_{21} = a_{22} = 1$, $r = 20$, $\ell_1 = 2$, $\ell_{2k} = 3$, $\bar{\ell}_{21} = 20$, $\bar{\ell}_{22} = 32$, $[x_1(0), x_2(0)]^T = [0.03, -0.02]^T$, $[\hat{x}_1(0), \hat{x}_2(0)]^T = [-0.2, 0.03]^T$, and $\hat{\Theta}(0) = 0$. It can be seen from Figs. 2–6 and 8–12 that the adaptive output-feedback neural tracking control problem of

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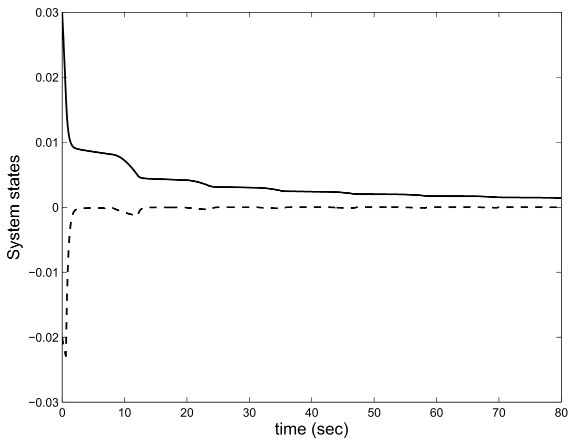

Fig. 8. Responses of x_1 (solid line) and x_2 (dash-dotted) under the switching signal 2.

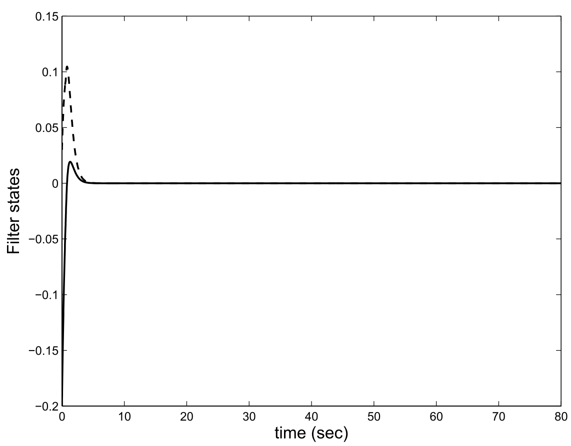

Fig. 9. Responses of $\hat{x}_1$ (solid line) and $\hat{x}_2$ (dash-dotted) under the switching signal 2.

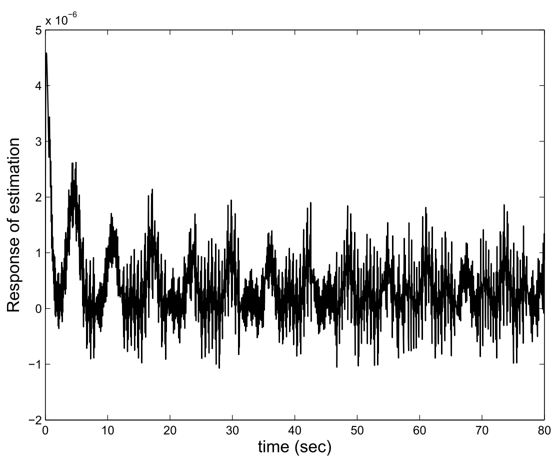

Fig. 10. Response of estimation $\hat{\Theta}$ under the switching signal 2.

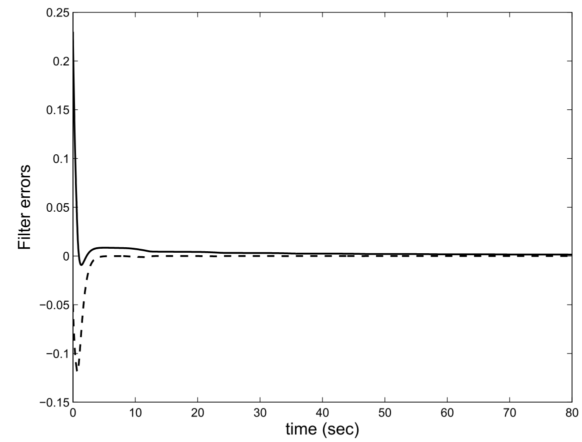

Fig. 11. Responses of $x_1 - \hat{x}_1$ (solid line) and $x_2 - \hat{x}_2$ (dash-dotted) under the switching signal 2.

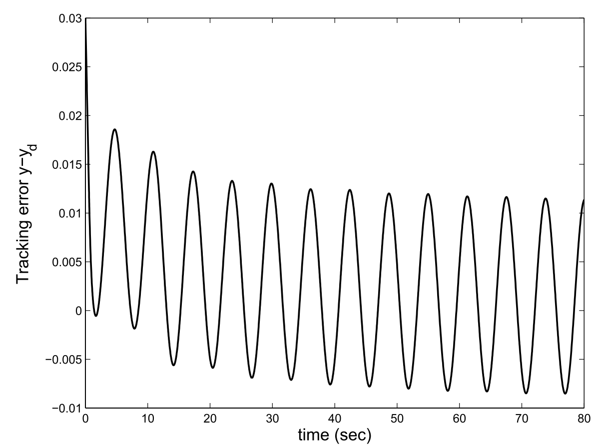

Fig. 12. Tracking error $y - y_d$ under the switching signal 2.

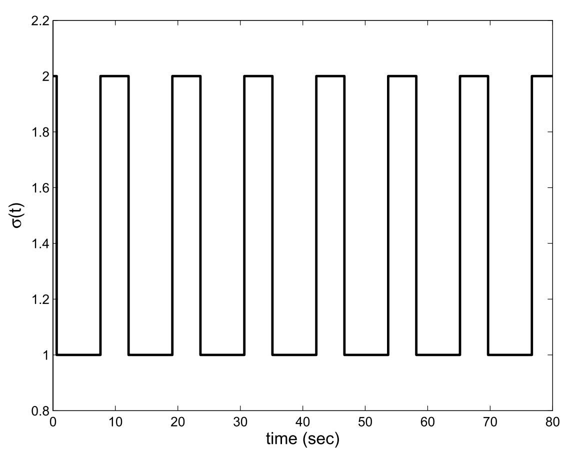

Fig. 13. Switching signal 2.

the resulting closed-loop system (2) and (60) is solvable under every switching signal σ with average dwell time $\tau_a = 11.5 > (\log 9.1199/0.1952)$. The switching signals are given in Figs. 7 and 13. Thus, the simulation results well illustrate the theory presented.

In addition, it can be also seen from Figs. 2–6 and 8–12 that, on the basis of the different switching signals, the stability and performance of the system (2) are different. For example, generating two possible switching signals (Figs. 7 and 13) with the average dwell time property, one can obtain the corresponding state responses of the closed-loop system as shown in Figs. 2 and 8, respectively, for the same initial state condition. It can be seen from the curves that the state response under the switched signal 1 is faster than the state response under the switching law 2. Meanwhile, the tracking errors of the corresponding closed-loop system are also obviously different based on the different switching signals. Therefore, in practical applications, the switching law should be adjusted carefully for improving both stability and performance of the system (2).

VI. Conclusion

This paper has studied the problem of adaptive output-feedback neural tracking control for a class of switched uncertain nonlinear systems. The nonlinear functions in systems are completely unknown. A new adaptive neural tracking control technique based on the average dwell time method and the backstepping technique was established to guarantee that all the signals in the resulting closed-loop system remain bounded and the system output follows the reference signal. It is worth noting that a switched filter and different update laws have designed to reduce the conservativeness caused by adoption of a common observer and a common update law for all subsystems. In addition, the result obtained can be regarded as an extension of the stabilizing results for switched nonlinear systems with known nonlinearities to switched uncertain nonlinear systems.

There are many open problems that deserve further study. For example, when the solvability of the adaptive neural tracking control problem for individual subsystems is not assumed, how to find a sufficient condition for the adaptive neural tracking control of a switched uncertain nonlinear system is a challenging problem.

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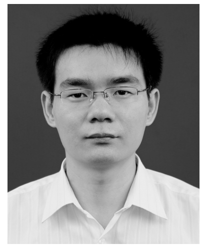

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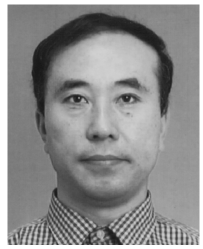

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