Fuzzy $\mathcal{H}_\infty$ Sliding Mode Control of Persistent Dwell-Time Switched Nonlinear Systems

Jing Wang, Haitao Wang, Huaicheng Yan, *Member, IEEE,* Yueying Wang, *Senior Member, IEEE,* Hao Shen, *Member, IEEE*

***Abstract*—In this paper, the fuzzy $\mathcal{H}_\infty$ sliding mode control problem for continuous-time switched nonlinear systems is investigated. The switching signal conforms to the persistent dwell-time switching mechanism. The first objective of this paper is to construct a switched integral sliding surface that not only accommodates the switched nonlinear model, but ensures that the sliding mode dynamics are globally uniformly exponentially stable and have an $\mathcal{H}_\infty$ performance by the equivalent controller derived from the sliding surface. Another aim is to integrate the switched sliding mode control law to force the system trajectories to the sliding surface in a finite amount of time. Then, on the basis of the Lyapunov function technique, the specific form of the sliding mode control law gains is given first, and later the finite-time reachability of the sliding surface is ensured. Finally, the validity of the proposed switched sliding mode control method is validated by a numerical example.**

***Index Terms*—Switched nonlinear systems, persistent dwell-time switching mechanism, external disturbance, sliding mode control.**

I. Introduction

MANY practical engineering applications involve the coupling between continuous dynamics and discrete events, such as the opening and closing of the valves and the thermostats turning heat on or off, etc [1]–[6]. Such systems are called hybrid systems. Being a category of high-level abstract hybrid systems, a switched system is defined by a set of limited dynamic systems and a mapping of a switching mechanism, which typically activates the corresponding subsystem over time [7]–[10]. Moreover, the main motivation for studying switched systems is the consideration that the parameters and structures of systems will inevitably change and exhibit the characteristic of switching due to the influence of many factors. In view of this, meaningful contributions to the switched system have emerged (see, e.g., [11]–[13] and the relative references).

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J. Wang, H. Wang and H. Shen are with the AnHui Province Key Laboratory of Special Heavy Load Robot and the School of Electrical and Information Engineering, Anhui University of Technology, Ma'anshan 243032, China (e-mail: jingwang08@126.com (J. Wang); haitaowang323@gmail.com (H. Wang); haoshen10@gmail.com (H. Shen)). H. Yan is with the School of Information Science and Engineering, East China University of Science and Technology, Shanghai 200237, China (e-mail: hcyan@ecust.edu.cn). Y. Wang is with the School of Mechatronic Engineering and Automation, Shanghai University, Shanghai 201620, China (e-mail: wyy676@126.com).

It should be noted that the above-mentioned literature mainly regards the switching mechanisms of dwell-time (DT) [14] and average dwell-time (ADT) [15] as the focus, but both of them have some properties that greatly limit the freedom of their applications. For DT switching, each switching interval is restricted to be not less than a positive scalar $\tau_{\mathcal{D}}$, which will limit its fast frequency switching in a sense. In comparison, ADT switching relaxes the restriction for DT switching. On the basis of DT switching, ADT switching allows that its ADT is greater than or equal to a positive scalar $\tau_{\mathcal{A}}$, which satisfies the requirement for the existence of fast frequency switching. However, due to the existence of the chatter bound, ADT switching also has the defect of the limited switching frequency. How to relax the limitation of the switching frequency is of extraordinary significance for the research of switched systems. As mentioned in [16], persistent dwell-time (PDT) switching can overcome the barrier of limited switching frequency to a certain extent. In essence, the PDT switching signal is composed of countless alternating τ-portion and T-portion, in which the switching of the τ-portion corresponds to slow frequency switching, while the switching of the T-portion can be considered as arbitrary switching (fast frequency switching). On account of this, the PDT switching mechanism (PDTSM) has great potential in describing the switched systems that contain a mixture of fast and slow switching frequencies (e.g., systems with sudden and intermittent failures). Therefore, compared with the universality of DT and ADT switchings in the applications, PDT switching is more universal. Nonetheless, due to the complexity of PDTSM, relevant research remains somewhat open and challenging.

At another research frontier, as a variable structure control method, sliding mode control (SMC) has attracted extensive research attention for its powerful features (e.g., fast convergence and strong robustness, etc.) [17]–[22]. Essentially, SMC is the process of forcing the state trajectories to a predefined sliding surface for a finite period of time through an effective discontinuous control law, and then maintaining that states and achieving the desired performance. Relevant works on SMC can be found in [23]–[27]. Just to name a few, in [28], the authors researched the stabilization issue for stochastic Takagi-Sugeno (T-S) fuzzy switched systems through the utilization of the integral SMC method, where the time delays and uncertainties were also taken into account. The authors in [25] studied the problem of finite-time bounded for nonlinear systems based on the SMC method. In [26], based on the event-triggered mechanism, the SMC approach for uncertain stochastic systems was developed, in which the situation of limited communication capacity was considered. Although the results on SMC have been quite fruitful, little literature considers the switching mechanism simultaneously, and studying the PDT switched system based on the SMC approach is even more challenging, which is one of the motivations for our research.

In addition, it deserves to be noticed that the T-S fuzzy model is recognized as a highly powerful approach to deal with nonlinearities [4], [29]–[31]. The T-S fuzzy model is stitched together from multiple affine models or local linear subsystems, which can achieve arbitrary accuracy to approximate nonlinear systems [6], [32]. Therefore, the SMC problem for T-S fuzzy systems has attracted tremendous focus. For example, the authors in [33] designed a novel SMC method for T-S fuzzy singularly perturbed systems, and the matched/unmatched uncertainties were also considered. In [34], the authors studied the SMC issue for T-S fuzzy systems based on the weighted try-once-discard protocol. However, it is worth noting that when PDTSM is of consideration, the SMC problem regarding T-S fuzzy systems has not been studied, which deserves further research.

In this paper, the issue of SMC for PDT-switched T-S fuzzy systems is investigated. The main contributions can be summarized as follows:

(I) In contrast to the DT switching mechanism (DTSM) and ADT switching mechanism (ADTSM), a more general PDTSM is introduced as a first attempt to research the SMC issue for continuous-time switched T-S fuzzy systems. And, a newer method [35] of handling the normalized membership function is used, which yields less conservative results to a certain extent.

(II) In order to better accommodate the characteristics of the switched T-S fuzzy model, a new switched fuzzy integral sliding surface (SFISS) is designed. In particular, a state-dependent matrix is introduced into the SFISS with more relaxed existential conditions compared to the strict requirements for the system matrices in literature [18], [36]. Then, based on the Lyapunov function technique, sufficient conditions are derived to guarantee that the sliding mode dynamics (SMDs) are globally uniformly exponentially stable (GUES) and satisfy an $\mathcal{H}_\infty$ performance.

(III) A novel switched fuzzy SMC law, which can ensure that the system states reach SFISS in a finite time in the presence of the external disturbance and matched uncertainty, is constructed by combining SMDs and SFISS. Furthermore, the validity of the proposed method is verified by a numerical example. The effect of PDT switching signal on system performance is explored.

The remainder of this paper is typeset as follows. Section II gives a description of the problem, including an introduction to PDTSM, the construction of the system model and SFISS, and some necessary concepts, followed by the stability and reachability analysis in Section III. Section IV presents the corresponding simulation validation. Section V concludes this paper.

Notations: The notations used in this paper are standard. $\mathcal{A}^+$: the left inverse of matrix $\mathcal{A}$; $\mathcal{U}^\perp(x) \in \mathbf{R}^{s\times(s-w)}$: the matrix with independent columns spanning the null space of $\mathcal{U}(x) \in \mathbf{R}^{s\times w}$; $\mathrm{span}\{\mathcal{D}\}$: the set consists of all linear combinations of vector group $\mathcal{D}$; $\mathcal{R}(\mathcal{A})$: the rank of matrix $\mathcal{A}$. For the rest of the notations, please refer to [37].

II. Problem Formulation

A. PDTSM

$\vartheta(t)$ is considered to denote the switching signal with PDT switching property in this paper, which is a right continuous piecewise constant function taking values in a given set $\mathcal{J} = \{1, 2, \ldots, J\}$, and J represents the number of activated subsystem modes. The concept of PDTSM is given as follows.

Definition 1: [38] For the switching signal $\vartheta(t)$ with PDT switching property, there are two scalars $\tau_\mathcal{P} > 0$ (the persistent dwell-time) and $\mathcal{T}_\mathcal{P} > 0$ (the period of persistence) satisfying that:

(i) There exists an infinite number of non-adjacent intervals not smaller than $\tau_\mathcal{P}$ in length with $\vartheta(t)$ being constant in these intervals.

(ii) The above-mentioned intervals are separated by intervals less than or equal to $\mathcal{T}_\mathcal{P}$.

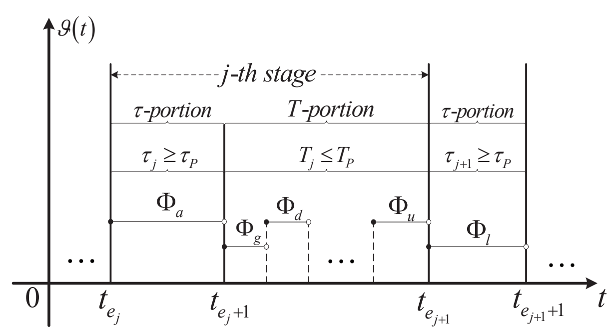

Fig. 1. A possible scenario of PDTSM.

From Fig. 1, it can be observed that the PDT switching signal consists of multiple stages, and each stage is composed of two parts: a τ-portion and a T-portion. During the τ-portion, only one subsystem with a duration no less than $\tau_\mathcal{P}$ is activated accordingly. In the T-portion, the switchings can be regarded as arbitrary switchings. For $\forall j \in \mathbb{Z}_{\geq 1}$, the following inequality can be derived:

$$T_j = \sum\nolimits_{v=1}^{\Delta\left(t_{e_j+1}, t_{e_{j+1}}\right)} \Phi_{\vartheta\left(t_{e_j+v}\right)} \leq \mathcal{T}_\mathcal{P}$$

which implies that the duration of the T-portion in the j-th stage does not exceed $\mathcal{T}_\mathcal{P}$; $\Phi_{\vartheta\left(t_{e_j+v}\right)}$ and $\Delta\left(t_{e_j+1}, t_{e_{j+1}}\right)$ denote the duration of the activated subsystem and the overall switching times in interval $\left[t_{e_j+1}, t_{e_{j+1}}\right)$, respectively. Moreover, $\Phi_{\vartheta\left(t_{e_j+v}\right)} < \tau_\mathcal{P}$.

Remark 1: For $\forall j \in \mathbb{Z}_{\geq 1}$, f_j satisfying $f_j \triangleq \Delta\left(t_{e_j+1}, t_{e_{j+1}}\right)/T_j$ is utilized to represent the switching frequency of the T-portion in the j-th stage, we have $1/f_j \leq \tau_\mathcal{P}$ from $\Phi_{\vartheta\left(t_{e_j+v}\right)} < \tau_\mathcal{P}$. Then, denoting $f \triangleq \max_{j\in\mathbb{Z}_{\geq 1}} f_j$, $1/f \leq \tau_\mathcal{P}$ is clearly held.

Remark 2: Zeno behavior [39] will not occur in the T-portion of each stage as the switching frequency is assumed to be not infinite. Then, for $\forall j \in \mathbb{Z}_{\geq 1}$, supposing that the interval $[n_1, n_2)$ in the T-portion of the j-th stage, it holds that

$$\begin{aligned} 0 \leq \Delta(n_1, n_2) &\leq \left(\frac{n_2 - n_1}{T_j + \tau_j} + 1\right)(T_j f_j + 1) \\ &\leq \left(\frac{n_2 - n_1}{\mathcal{T_P} + \tau_\mathcal{P}} + 1\right)(\mathcal{T_P} f + 1) \end{aligned}$$

where T_j and τ_j represent the duration in the T-portion and τ-portion, respectively; $\mathcal{T_P} \geq T_j$; $\tau_\mathcal{P} \leq \tau_j$. Moreover, as the inequalities hold in any interval of each stage, the above inequalities can be extended to any interval $[t_1, t_2)$ that

$$0 \leq \Delta(t_1, t_2) \leq \left(\frac{t_2 - t_1}{\mathcal{T_P} + \tau_\mathcal{P}} + 1\right)(\mathcal{T_P} f + 1). \tag{1}$$

Remark 3: As mentioned in [37], the switching frequency of DTSM and ADTSM needs to be restricted, which inevitably weakens the application range of DTSM and ADTSM. For PDTSM, the durations of the activated subsystems in the T-portion are limited, namely, the length of each subsystem is less than $\tau_\mathcal{P}$, leading to a certain degree of arbitrary switching in the T-portion, which greatly alleviates the problem of fast switching frequency limitation that DTSM and ADTSM encounter. Moreover, DTSM and ADTSM can be considered as special cases of PDTSM. In summary, PDTSM is the more general switching mechanism.

Remark 4: For continuous-time PDT-switched systems, there are three important parameters: the switching frequency f, the variation rates at the sampling instants and the switching instants ($1 > \zeta > 0$ and $\varrho > 1$). $\varrho > 1$ indicates that the energy function can show an upward trend at the switching instants, which may give more freedom to the applications compared to the traditional requirement [24], [40] of a decreasing energy function value. It should also be noted that these three parameters have an effect on the system performance, and studying the relationship between these parameters and the system performance can facilitate further understanding, hence the corresponding analysis is given in Section IV.

B. System description

Consider the continuous-time PDT-switched T-S fuzzy system by the following rules (for convenience, denote $\vartheta(t) = r$):

Model rule l: **IF** $\varphi_1(t)$ is $\mathcal{X}_1^l$ and $\cdots$ and $\varphi_a(t)$ is $\mathcal{X}_a^l$, **THEN**

$$\begin{cases} \dot{x}(t) = A_{lr}x(t) + B_{lr}(u(t) + f_r(x(t))) + C_{lr}\omega(t) \\ y(t) = D_{lr}x(t) + E_{lr}\omega(t), \quad l \in \mathcal{M} = \{1, 2, \ldots, M\} \end{cases} \tag{2}$$

where $x(t) \in \mathbf{R}^{n_x}$, $u(t) \in \mathbf{R}^{n_u}$, $y(t) \in \mathbf{R}^{n_y}$ and $\omega(t) \in \mathbf{R}^{n_w}$ are the state vector, the control input, the controlled output and the disturbance belonging to $\mathcal{L}_2[0, \infty)$, respectively; $f_r(x(t))$ is the nonlinear function, which signifies the matched uncertainty and satisfies $\|f_r(x(t))\| \leq \alpha_r \|x(t)\|$ with α_r being a positive constant. $\varphi(t) = [\varphi_1(t), \varphi_2(t), \ldots, \varphi_a(t)]$ is the premise variable; $\mathcal{X}_e^l(e = 1, 2, \ldots, M)$ is utilized to denote the fuzzy set with M being the number of fuzzy rules. A_{lr}, B_{lr}, C_{lr}, D_{lr} and E_{lr} are the appropriate known constant matrices. Then, by utilizing the fuzzy inference method, system (2) can be re-expressed as

$$\begin{cases} \dot{x}(t) = \sum_{l=1}^{M} h_l(\varphi(t))[A_{lr}x(t) + B_{lr}(u(t) \\ \quad + f_r(x(t))) + C_{lr}\omega(t)] \\ y(t) = \sum_{l=1}^{M} h_l(\varphi(t))[D_{lr}x(t) + E_{lr}\omega(t)] \end{cases} \tag{3}$$

where $h_l(\varphi(t))$ is the normalized membership function satisfying

$$h_l(\varphi(t)) = \frac{\prod_{e=1}^{a} \mathcal{X}_e^l(\varphi_e(t))}{\sum_{l=1}^{M}\prod_{e=1}^{a} \mathcal{X}_e^l(\varphi_e(t))} \geq 0 \tag{4}$$

with $\mathcal{X}_e^l(\varphi_e(t))$ representing the grade of membership of $\varphi_e(t)$ in $\mathcal{X}_e^l$ for $l \in \mathcal{M}$, and $\sum_{l=1}^{M} h_l(\varphi(t)) = 1$.

C. Sliding surface design

In order to better adapt to the characteristics of the PDT-switched fuzzy model, a novel SFISS is constructed as

$$\begin{aligned} s(t) \;=\; & \int_0^t G(x)\,dx - \int_0^t \sum_{l=1}^{M}\sum_{k=1}^{M} G(x(\tau)) \\ & \times h_l(\varphi(\tau))h_k(\varphi(\tau))(A_{lr}x(\tau) \\ & + B_{lr}L_{kr}x(\tau))d\tau \end{aligned} \tag{5}$$

in which $\sum_{k=1}^{M} h_k(\varphi(\tau))L_{kr}$ is the nominal part of SMC law, for $\forall r \in \mathcal{J}$, L_{kr} are parameter matrices to be designed. Especially, $G(x) \triangleq \digamma(x)\mathcal{B}_r^T$ is the projection matrix, where $\mathcal{B}_r \triangleq \sum_{l=1}^{M} h_l(\varphi(\tau))B_{lr}$ with full column rank is the state-dependent input matrix and $\digamma(x)$ is a full rank matrix.

For the continuous-time system and the designed SFISS, $\dot{s}(t) = s(t) = 0$ hold as long as the trajectories of the system reach SFISS. Therefore, in an activated subsystem (i.e., $t \in [t_{e_j}, t_{e_j+1})$), the following equivalent controller can be obtained according to $\dot{s}(t) = 0$, as

$$\begin{aligned} u_e(t) \;=\; & \sum_{k=1}^{M} h_k(\varphi(t))L_{kr}x(t) - (G(x)\mathcal{B}_r)^{-1}G(x) \\ & \times \sum_{l=1}^{M} h_l(\varphi(t))C_{lr}\omega(t) - f_r(x(t)). \end{aligned} \tag{6}$$

By substituting the controller (6) into the fuzzy system (3), SMDs are derived below

$$\begin{cases} \dot{x}(t) = \sum_{l=1}^{M}\sum_{k=1}^{M} h_l(\varphi(t))h_k(\varphi(t)) \\ \quad \times [(A_{lr} + B_{lr}L_{kr})x(t) + \mathcal{T}(x)C_{lr}\omega(t)] \\ y(t) = \sum_{l=1}^{M} h_l(\varphi(t))[D_{lr}x(t) + E_{lr}\omega(t)] \end{cases} \tag{7}$$

where $\mathcal{T}(x) \triangleq I - \mathcal{B}_r(G(x)\mathcal{B}_r)^{-1}G(x)$ is considered to be the transition matrix of disturbance.

Remark 5: Substituting $G(x)$ into the transition matrix $\mathcal{T}(x)$, as mentioned in [40], yields

$$\begin{aligned} \mathcal{T}(x) &= I - \mathcal{B}_r\left(\digamma(x)\mathcal{B}_r^T\mathcal{B}_r\right)^{-1}\digamma(x)\mathcal{B}_r^T \\ &= I - \mathcal{B}_r\left(\mathcal{B}_r^T\mathcal{B}_r\right)^{-1}\mathcal{B}_r^T = I - \mathcal{B}_r\mathcal{B}_r^+ = \mathcal{T}^T(x) \end{aligned}$$

and

$$\begin{aligned} \mathcal{T}^T(x)\mathcal{T}(x) &= \left[I - \mathcal{B}_r\mathcal{B}_r^+\right]\left[I - \mathcal{B}_r\mathcal{B}_r^+\right] \\ &= I - \mathcal{B}_r\mathcal{B}_r^+ - \mathcal{B}_r\mathcal{B}_r^+ + \mathcal{B}_r\mathcal{B}_r^+\mathcal{B}_r\mathcal{B}_r^+ \end{aligned}$$

$$= \quad I - \mathcal{B}_r\mathcal{B}_r^+ = \mathcal{T}(x)$$

which implies that $\mathcal{T}(x)$ is an idempotent matrix, and obviously, its eigenvalues can only be 0 or 1. Note also that $\mathcal{R}(I - \mathcal{B}_r\mathcal{B}_r^+) \neq 0$ because $\mathcal{R}(\mathcal{B}_r\mathcal{B}_r^+) < n$. Therefore, at least one of the eigenvalues of $\mathcal{T}(x)$ is 1, which yields

$$\|\mathcal{T}(x)\| = 1.$$

D. Necessary concepts

Before proceeding, some necessary definitions and lemmas are provided.

Definition 2: [38] SMDs (7) are GUES, if under zero disturbance conditions, for scalars $m \in (0,\infty)$ and $c \in (0,1)$, every switching signal $\vartheta(t)$ and initial condition $x(t_0)$, the inequality $\|x(t)\| \leq mc^{t-t_0}\|x(t_0)\|$, $\forall t \geq t_0$ holds with the solutions of SMDs (7).

Definition 3: [41] SMDs (7) are GUES and have an $\mathcal{H}_\infty$ performance, if SMDs (7) are GUES, and under zero initial conditions and for any nonzero $\omega(t) \in \mathcal{L}_2[0,\infty)$, it holds that

$$\int_{t_0}^{\infty} y^T(t)\, y(t)\, dt \leq \bar{\gamma}^2 \int_{t_0}^{\infty} \omega^T(t)\,\omega(t)\, dt. \qquad (8)$$

Lemma 1: [35] For $\forall l,k \in \mathcal{M}$, consider matrices Λ_{lk} with suitable dimensions, the normalized membership functions $h_l(\varphi(t))$ and $h_k(\varphi(t))$, if there exist matrices Θ_{lk} $(l \neq k)$ satisfying that

$$\Lambda_{lk} + \Lambda_{kl} < \Theta_{lk} + \Theta_{lk}^T, l < k \qquad (9)$$

$$\bar{\Lambda} \triangleq \begin{bmatrix} \Lambda_{11} & \Theta_{12} & \cdots & \Theta_{1M} \\ * & \Lambda_{22} & \cdots & \Theta_{2M} \\ * & * & \ddots & \vdots \\ * & * & * & \Lambda_{MM} \end{bmatrix} < 0. \qquad (10)$$

Then, it holds that:

$$\sum_{l=1}^{M}\sum_{k=1}^{M} h_l(\varphi(t))\, h_k(\varphi(t))\,\Lambda_{lk} < 0. \qquad (11)$$

Proof: According to (4) and (9), it can be derived that:

$$\begin{aligned} &\sum_{l=1}^{M}\sum_{k=1}^{M} h_l(\varphi(t))\, h_k(\varphi(t))\,\Lambda_{lk} \\ < \; &\sum_{l=1}^{M}\Big[h_l^2(\varphi(t))\,\Lambda_{ll} + \sum_{k>l}^{M} h_l(\varphi(t))\, h_k(\varphi(t)) \\ &\times \left(\Theta_{lk} + \Theta_{lk}^T\right)\Big] = \varkappa^T\bar{\Lambda}\varkappa \end{aligned}$$

where $\varkappa \triangleq [h_1(\varphi(t))\ h_2(\varphi(t))\ \cdots\ h_M(\varphi(t))]^T$. It is obvious from (10) that (11) holds. The proof is completed. ■

Lemma 2: [35], [42] For matrices K_1 and K_2 of the appropriate dimensions, any given positive scalar ϵ_r and matrix $\mathcal{S}$ satisfying $\mathcal{S}^T\mathcal{S} \leq I$, then the following inequality is obtained:

$$K_1\mathcal{S}K_2 + (K_1\mathcal{S}K_2)^T \leq \epsilon_r^{-1}K_1K_1^T + \epsilon_r K_2^T K_2.$$

Lemma 3: [34] For any state-dependent full column rank matrix $\mathcal{U}(x) \in \mathbf{R}^{s\times w}$, if the distribution $\mathcal{Z}(x) = \operatorname{span}\{\mathcal{U}_a^\perp(x)\}$ is involutive, as

$$\begin{aligned} \left[\mathcal{U}_a^\perp(x), \mathcal{U}_b^\perp(x)\right] &= \frac{\partial \mathcal{U}_b^\perp(x)}{\partial x}\mathcal{U}_a^\perp(x) - \frac{\partial \mathcal{U}_a^\perp(x)}{\partial x}\mathcal{U}_b^\perp(x) \\ &\in \mathcal{Z}(x), \forall a,b = 1,2,\ldots,s-w \end{aligned}$$

in which $[*,*]$, $\mathcal{U}_a^\perp(x)$ and $\mathcal{U}_b^\perp(x)$ represent the Lie bracket of two vector fields, the a-th and b-th columns of $\mathcal{U}^\perp(x)$, respectively. Then, there exists a vector $\mathcal{V}(x) \in \mathbf{R}^{w\times 1}$ that can ensure that $\frac{\partial \mathcal{V}(x)}{\partial x} = \mathcal{F}(x) = F(x)\,\mathcal{U}^T(x)$ with $F(x) \in \mathbf{R}^{w\times w}$ being a full rank matrix.

Remark 6: In this paper, solving the inequality $\sum_{l=1}^{M}\sum_{k=1}^{M} h_l(\varphi(t))\, h_k(\varphi(t))\,\Lambda_{lk} < 0$ is critical. To the best of the authors' knowledge, the current processing methods are broadly classified into the following three types [34], [41], [43]: (i) $\Lambda_{lk} < 0, \forall l,k \in \mathcal{M}$; (ii) $\Lambda_{ll} < 0, \forall l \in \mathcal{M}$ and $\Lambda_{lk} + \Lambda_{kl} < 0, \forall l,k \in \mathcal{M}, l < k$; (iii) $\Lambda_{ll} < 0, \forall l \in \mathcal{M}$ and $\frac{2}{M-1}\Lambda_{ll} + \Lambda_{lk} + \Lambda_{kl} < 0, \forall l,k \in \mathcal{M}, l \neq k$. The conservativeness of the results obtained from the above processing methods is limited by the number of fuzzy rules. Hence, a comparatively novel approach in [35] is adopted, as shown in Lemma 1, which increases a certain amount of computation but leads to relatively low conservativeness results.

III. Main Results

New results on the globally uniformly exponential stability with an $\mathcal{H}_\infty$ performance index of SMDs (7) are provided based on the Lyapunov function technique in this section. Then, an effective switched fuzzy SMC law is constructed to ensure the finite-time reachability of SFISS.

A. Stability and performance analysis

The GUES analysis for SMDs (7) with an $\mathcal{H}_\infty$ performance index is given subsequently.

Theorem 1: Let scalars $1 > \zeta > 0$, $\varrho > 1$, $f > 0$, $\gamma > 0$, $\mathcal{T}_\mathcal{P} > 0$ and $\tau_\mathcal{P} > 0$ be given constants. Suppose that there exist matrices $\Theta_{lk}^r \triangleq [\Pi_{lk}^r(c,d)]_{c,d\in[1,4]}$ and $\bar{L}_{kr}$, symmetric matrices $P_r > 0$ and $Q_r > 0$, and positive scalars ϵ_r such that the following inequalities hold for any r, a, $b \in \mathcal{J}$, and l, $k \in \mathcal{M}$:

$$\Lambda_{lk}^r + \Lambda_{kl}^r - \Theta_{lk}^r - \Theta_{lk}^{rT} < 0, l < k \qquad (12)$$

$$\begin{bmatrix} \Lambda_{11}^r & \Theta_{12}^r & \cdots & \Theta_{1M}^r \\ * & \Lambda_{22}^r & \cdots & \Theta_{2M}^r \\ * & * & \ddots & \vdots \\ * & * & * & \Lambda_{MM}^r \end{bmatrix} < 0 \qquad (13)$$

$$P_a \leq \varrho P_b,\ a \neq b \qquad (14)$$

$$\frac{\mathcal{T}_\mathcal{P} f + 1}{\zeta}\ln(\varrho) - \frac{1}{f} < \tau_\mathcal{P} < \mathcal{T}_\mathcal{P} \qquad (15)$$

where

$$\Lambda_{lk}^r \triangleq \begin{bmatrix} \hat{Q}_r & 0 & 0 & Q_r D_{lr}^T \\ * & -\gamma^2 I & C_{lr}^T & E_{lr}^T \\ * & * & -\epsilon_r I & 0 \\ * & * & * & -I \end{bmatrix}$$

with

$$\hat{Q}_r \triangleq \operatorname{sym}\{A_{lr}Q_r + B_{lr}\bar{L}_{kr}\} + \zeta Q_r + \epsilon_r I.$$

Then, SMDs (7) are GUES with an $\mathcal{H}_\infty$ performance index $\bar{\gamma} \triangleq \gamma\sqrt{\frac{-\zeta\varrho^{\mathcal{T}_\mathcal{P} f+1}}{\delta}}$, and $\delta \triangleq \frac{\mathcal{T}_\mathcal{P} f+1}{\mathcal{T}_\mathcal{P}+\tau_\mathcal{P}}\ln(\varrho)-\zeta$. Meanwhile, the gain matrices are given by

$$L_{kr} \triangleq \bar{L}_{kr}P_r. \tag{16}$$

Proof: The Lyapunov function is constructed as follows:

$$V_r(x(t)) \triangleq x^T(t)P_r x(t) \tag{17}$$

which yields that

$$V_r(x(t)) \geq \xi_1 \|x(t)\|^2, V_{\alpha(t_0)}(t_0) \leq \xi_2 \|x(t_0)\|^2 \tag{18}$$

with $\xi_1 \triangleq \min_{r\in\mathcal{J}}\{\lambda_{\min}(P_r)\}$ and $\xi_2 \triangleq \max_{r\in\mathcal{J}}\{\lambda_{\max}(P_r)\}$.

For simplicity of expression, define $h_l(\varphi(t)) \triangleq h_l$ and $h_k(\varphi(t)) \triangleq h_k$. Then, the derivative of $V_r(x(t))$ along the trajectory of (7) produces

$$\begin{aligned} &\dot{V}_r(x(t)) \\ =\ &\sum_{l=1}^{M}\sum_{k=1}^{M} h_l h_k \mathrm{sym}\{x^T(t)P_r[A_{lr}x(t) \\ &+B_{lr}L_{kr}x(t)+\mathcal{T}(x)C_{lr}\omega(t)]\}. \end{aligned} \tag{19}$$

On the other hand, according to conditions (12), (13) and Lemma 1, it yields

$$\hat{\Lambda}_{lk}^r \triangleq \sum_{l=1}^{M}\sum_{k=1}^{M} h_l h_k \Lambda_{lk}^r < 0.$$

Denoting $Q_r \triangleq P_r^{-1}$ and $\bar{L}_{kr} \triangleq L_{kr}Q_r$, then pre- and post-multiplying $\hat{\Lambda}_{lk}^r$ by diag$\{P_r, I, I, I\}$ and diag$\{P_r^T, I, I, I\}$, followed by combining Schur complement, one can obtain that

$$\Xi^r \triangleq \begin{bmatrix} \Xi_{11}^r & \hat{D}_r^T\hat{E}_r \\ * & \frac{1}{\epsilon_r}\hat{C}_r^T\hat{C}_r+\hat{E}_r^T\hat{E}_r-\gamma^2 I \end{bmatrix} < 0 \tag{20}$$

where

$$\begin{aligned} \Xi_{11}^r &\triangleq \Xi_{11}^{1r}+\Xi_{11}^{2r}, \hat{C}_r \triangleq \sum_{l=1}^{M} h_l C_{lr}, \hat{E}_r \triangleq \sum_{l=1}^{M} h_l E_{lr} \\ \Xi_{11}^{1r} &\triangleq \zeta Q_r+\epsilon_r P_r P_r^T+\hat{D}_r^T\hat{D}_r, \hat{D}_r \triangleq \sum_{l=1}^{M} h_l D_{lr} \\ \Xi_{11}^{2r} &\triangleq \sum_{l=1}^{M}\sum_{k=1}^{M} h_l h_k \mathrm{sym}\{P_r A_{lr}+P_r B_{lr}L_{kr}\}. \end{aligned}$$

Considering (17), (19), Lemma 2 and Remark 5, it follows that

$$\begin{aligned} \mathcal{J} &\triangleq \dot{V}_r(x(t))+\zeta V_r(x(t))+y^T(t)y(t)-\gamma^2\omega^T(t)\omega(t) \\ &= \eta^T(t)\Xi^r\eta(t) \end{aligned}$$

where $\eta^T(t) \triangleq [x^T(t)\ \ \omega^T(t)]$. It holds that $\mathcal{J} < 0$ from (20). For convenience, denote $V_r(x(t)) \triangleq V_r(t)$.

Letting $\Upsilon(t) \triangleq -y^T(t)y(t)+\gamma^2\omega^T(t)\omega(t)$, one has

$$\dot{V}_r(t) \leq -\zeta V_r(t)+\Upsilon(t). \tag{21}$$

Then, according to conditions (14) and (21), one can deduce

$$\begin{aligned} &V_{\alpha(t_{e_{j+1}-1})}(t_{e_{j+1}}) \\ &\leq \varrho^{\Delta(t_{e_j},t_{e_{j+1}})}\exp(-\zeta(t_{e_{j+1}}-t_{e_j}))V_{\alpha(t_{e_j})}(t_{e_j}) \\ &+\int_{t_{e_j}}^{t_{e_{j+1}}}\varrho^{\Delta(i,t_{e_{j+1}})}\exp(-\zeta(t_{e_{j+1}}-i))\Upsilon(i)\,dt. \end{aligned} \tag{22}$$

For $\forall t \in [t_{e_j}, t_{e_{j+1}})$, considering $\omega(t) \equiv 0$, it can be inferred by virtue of Remark 1 that

$$\begin{aligned} V_{\alpha(t)}(t) &\leq \varrho^{\Delta(t_{e_j},t)}\exp(-\zeta(t-t_{e_j}))V_{\alpha(t_{e_j})}(t_{e_j}) \\ &\quad+\int_{t_{e_j}}^{t}\varrho^{\Delta(i,t)}\exp(-\zeta(t-i))\Upsilon(i)\,dt \\ &\leq \varrho^{\mathcal{T}_\mathcal{P} f+1}V_{\alpha(t_{e_j})}(t_{e_j}). \end{aligned} \tag{23}$$

Letting $t_0 \triangleq t_{e_1}$, and combining inequalities (22) and (23), one can obtain that

$$V_{\alpha(t)}(t) \leq \varrho^{\mathcal{T}_\mathcal{P} f+1}\phi^{j-1}V_{\alpha(t_0)}(t_0)$$

where $\phi \triangleq \exp(\upsilon) \in (0,1)$ and $\upsilon \triangleq (\mathcal{T}_\mathcal{P} f+1)\ln(\varrho)-\zeta((1/f)+\tau_\mathcal{P})$.

By means of condition (18), one can get that

$$\begin{aligned} \|x(t)\|^2 &\leq \frac{\xi_2}{\xi_1}\varrho^{\mathcal{T}_\mathcal{P} f+1}\phi^{j-1}\|x(t_0)\|^2 \\ &= \frac{\xi_2}{\xi_1}\varrho^{\mathcal{T}_\mathcal{P} f+1}\phi^{-1}\bar{\phi}^{t-t_0+1}\|x(t_0)\|^2 \\ &= \frac{\xi_2}{\xi_1}\bar{\phi}\varrho^{\mathcal{T}_\mathcal{P} f+1}\phi^{-1}\bar{\phi}^{t-t_0}\|x(t_0)\|^2 \end{aligned}$$

which leads to

$$\|x(t)\| \leq \sqrt{\frac{\xi_2}{\xi_1}\bar{\phi}\varrho^{\mathcal{T}_\mathcal{P} f+1}\phi^{-1}\bar{\phi}^{t-t_0}}\,\|x(t_0)\|$$

where $\bar{\phi} \triangleq \max_{\forall j\in\mathbb{Z}_{\geq 1}, t\geq t_0}\{\phi^{j/(t-t_0+1)}\}$ and $\bar{\phi} \in (0,1)$. This means that SMDs are GUES according to Definition 2.

Next, the focus is on the $\mathcal{H}_\infty$ performance analysis. Under zero initial conditions, it follows from (23) that

$$V_{\alpha(t)}(t) \leq \int_{t_0}^{t}\varrho^{\Delta(i,t)}\exp(-\zeta(t-i))\Upsilon(i)\,dt.$$

Define $\delta \triangleq \frac{\mathcal{T}_\mathcal{P} f+1}{\mathcal{T}_\mathcal{P}+\tau_\mathcal{P}}\ln(\varrho)-\zeta$. According to $\mathcal{T}_\mathcal{P} > \tau_\mathcal{P} > 1/f$, one has $\delta < 0$, then it holds from (1) that

$$\begin{aligned} &\int_{t_0}^{t}\varrho^{\Delta(i,t)}\exp(-\zeta(t-i))y^T(i)y(i)\,di \\ &\leq \gamma^2\varrho^{\mathcal{T}_\mathcal{P} f+1}\int_{t_0}^{t}\exp(\delta(t-i))\omega^T(i)\omega(i)di \end{aligned}$$

which leads to

$$\begin{aligned} &\int_{t_0}^{\infty}\int_{i}^{\infty}\exp(-\zeta(t-i))y^T(i)y(i)\,dtdi \\ &\leq \gamma^2\varrho^{\mathcal{T}_\mathcal{P} f+1}\int_{t_0}^{\infty}\int_{i}^{\infty}\exp(\delta(t-i))\omega^T(i)\omega(i)dtdi. \end{aligned}$$

It is obvious that the above inequality is equivalent to (8) after the integral computation, which means that SMDs (7) are GUES with an $\mathcal{H}_\infty$ performance index $\bar{\gamma}$. This finishes the proof. ■

Remark 7: In this paper, $G(x)$ is a state-dependent matrix that is introduced into a novel SFISS to accommodate the switched fuzzy model. The conditions for the existence of SFISS are, according to Lemma 3, much less constrained than the strict requirements for system matrices in [18], [36]. Besides, the superiority of SFISS is presented as follows: (a) the matched uncertainty is completely canceled; (b) the disturbance is not magnified during sliding motion.

B. Reachability analysis

In this subsection, an effective switched fuzzy SMC law is designed to ensure that the states of system (3) can be forced to SFISS in a finite amount of time under the influence of disturbance and matched uncertainty. Consider the following SMC law to achieve sliding motion:

Controller rule k: **IF** $\varphi_1(t)$ is $\mathcal{X}_1^k$ and $\cdots$ and $\varphi_a(t)$ is $\mathcal{X}_a^k$, **THEN**

$$u(t) = L_{kr}x(t) - \eta(t)\left(G(x)\mathcal{B}_r\right)^{-1}\operatorname{sgn}(s(t))$$

or equivalently

$$\begin{aligned} u(t) &= \sum\nolimits_{k=1}^{M} h_k(\varphi(t)) L_{kr}x(t) \\ &\quad -\eta(t)\left(G(x)\mathcal{B}_r\right)^{-1}\operatorname{sgn}(s(t)) \end{aligned} \tag{24}$$

where

$$\begin{aligned} \eta(t) &\triangleq \varepsilon + \alpha_r \|G(x)\mathcal{B}_r\| \|x(t)\| \\ &\quad + \sum\nolimits_{l=1}^{M} h_l(\varphi(t)) \|G(x) C_{lr}\| \|\omega(t)\| \end{aligned}$$

in which ε is a positive scalar.

Theorem 2: Consider the continuous-time switched T-S fuzzy system (3) and SFISS (5), the system responses under (24) can be driven to the predefined SFISS $s(t) = 0$ in a finite time.

Proof: Construct the subsequent Lyapunov function:

$$V_r(s(t)) = \frac{1}{2}s^T(t)s(t).$$

It follows from (3), (5) and (24) that:

$$\begin{aligned} \dot{s}(t) &= \sum\nolimits_{l=1}^{M} h_l(\varphi(t)) G(x)\left[B_{lr}f_r(x(t))\right. \\ &\quad \left.+C_{lr}\omega(t) - \eta(t)B_{lr}\left(G(x)\mathcal{B}_r\right)^{-1}\operatorname{sgn}(s(t))\right]. \end{aligned}$$

Then, the derivative of $V_r(s(t))$ can be formulated as

$$\begin{aligned} &\dot{V}_r(s(t)) \\ =\ & s^T(t)\sum\nolimits_{l=1}^{M} h_l(\varphi(t)) G(x)\left[B_{lr}f_r(x(t))\right. \\ & \left.+C_{lr}\omega(t) - \eta(t)B_{lr}\left(G(x)\mathcal{B}_r\right)^{-1}\operatorname{sgn}(s(t))\right] \\ \leq\ & \|s(t)\|\sum\nolimits_{l=1}^{M} h_l(\varphi(t))\left[\alpha_r\|G(x)\mathcal{B}_r\|\|x(t)\|\right. \\ & \left.+\|G(x)C_{lr}\|\|\omega(t)\| - \eta(t)\right] \\ =\ & -\sqrt{2}\varepsilon V_r^{\frac{1}{2}}(s(t)). \end{aligned} \tag{25}$$

It is obvious that $\dot{V}_r(s(t)) < 0$, which means that the system responses under (24) can be driven to the predefined SFISS $s(t) = 0$ in a finite time. This completes the proof. ■

Remark 8: In Theorem 2, the finite-time reachability of SFISS has been demonstrated. In essence, it can be seen from (25) that nonlinearity and disturbance are treated as $\alpha_r\|x(t)\|$ and $\|\omega(t)\|$, respectively, then $\dot{V}_r(s(t)) \leq -\sqrt{2}\varepsilon V_r^{\frac{1}{2}}(s(t))$ can always be obtained by adjusting $\eta(t)$ in the SMC law (24). Thus, despite the presence of nonlinearity and disturbance, the states of the system can always reach SFISS in a finite amount of time, which is the advantage of the SMC method.

IV. A NUMERICAL EXAMPLE

In order to verify the effectiveness of the switched fuzzy SMC scheme used in the paper, a numerical example is shown in this section. Consider the PDT-switched system model with $J = 2$, which is described by the T-S fuzzy model in (3) with $M = 2$ and the following parameters:

$$\begin{aligned} A_{11} &= \begin{bmatrix} -9 & 1 & -2.5 \\ 2 & -0.4 & -0.7 \\ 0.2 & 2.4 & 0.6 \end{bmatrix}, B_{11} = \begin{bmatrix} 0 \\ 0.199 \\ -0.3 \end{bmatrix} \\ A_{12} &= \begin{bmatrix} -10 & 1 & -1 \\ 0 & -0.5 & -6 \\ 0.3 & 0.4 & 0.5 \end{bmatrix}, B_{12} = \begin{bmatrix} 0 \\ 0.2 \\ -0.3 \end{bmatrix} \\ A_{21} &= \begin{bmatrix} -9.9 & 1.1 & -2.75 \\ 2.2 & -0.44 & -0.77 \\ 0.22 & 2.64 & 0.66 \end{bmatrix}, B_{21} = \begin{bmatrix} 0 \\ 0.219 \\ -0.33 \end{bmatrix} \\ A_{22} &= \begin{bmatrix} -11 & 1.1 & -1.1 \\ 0 & -0.55 & -6.6 \\ 0.33 & 0.44 & 0.55 \end{bmatrix}, B_{22} = \begin{bmatrix} 0 \\ 0.22 \\ -0.33 \end{bmatrix} \\ D_{11} &= \begin{bmatrix} 1.4 & 0.9 & 0.8 \end{bmatrix}, D_{21} = \begin{bmatrix} 1.64 & 0.99 & 0.88 \end{bmatrix} \\ D_{12} &= \begin{bmatrix} 0.8 & 1.4 & 0.9 \end{bmatrix}, D_{22} = \begin{bmatrix} 0.88 & 1.64 & 0.99 \end{bmatrix} \\ C_{11} &= \begin{bmatrix} 0.3 \\ 0.2 \\ 0.3 \end{bmatrix}, C_{12} = \begin{bmatrix} 0.4 \\ 0.3 \\ 0.3 \end{bmatrix}, C_{21} = \begin{bmatrix} 0.33 \\ 0.22 \\ 0.33 \end{bmatrix} \\ C_{22} &= \begin{bmatrix} 0.44 \\ 0.33 \\ 0.33 \end{bmatrix}, \begin{matrix} E_{11} = 0.4, E_{12} = 0.2 \\ E_{21} = 0.6, E_{22} = 0.3 \end{matrix} \end{aligned}$$

The normalized membership functions are designed as

$$\begin{aligned} h_1(x_1(t)) &= \begin{cases} 0.5(1 + \frac{x_1(t)}{2}), & |x_1(t)| \leq 2 \\ 0, & |x_1(t)| > 2 \end{cases} \\ h_2(x_1(t)) &= 1 - h_1(x_1(t)). \end{aligned}$$

Then, referring to [24], $F(x) = I$, we have $G(x) = \mathcal{B}_r^T$, and SFISS in (5) can be provided as follows

$$\begin{aligned} s(t) &= \mathcal{B}_r^T\left[x(t) - x(0)\right] - \int_0^t \sum\nolimits_{l=1}^{M}\sum\nolimits_{k=1}^{M} \\ &\quad \times \mathcal{B}_r^T h_l(\varphi(\tau)) h_k(\varphi(\tau))\left(A_{lr}x(\tau) + B_{lr}L_{kr}x(\tau)\right)d\tau. \end{aligned}$$

Moreover, the relevant parameters of PDTSM and $\mathcal{H}_\infty$ performance are given below

$$\begin{aligned} \mathcal{T_P} &= 3\,\text{sec}, \tau_{\mathcal{P}} = 1.54\,\text{sec}, f = 10\text{Hz} \\ \zeta &= 0.75, \varrho = 1.01, \gamma = 1. \end{aligned}$$

Subsequently, Algorithm 1 is given to introduce the design procedure of the gain matrices.

Then, according to Theorem 1, we can get gain matrices as follows

$$\begin{aligned} L_{11} &= \begin{bmatrix} -95.9647 & -264.9103 & -137.4604 \end{bmatrix} \\ L_{12} &= \begin{bmatrix} -280.6531 & -966.4523 & -491.2655 \end{bmatrix} \\ L_{21} &= \begin{bmatrix} -106.7004 & -277.3540 & -145.5029 \end{bmatrix} \\ L_{22} &= \begin{bmatrix} -282.3720 & -970.7920 & -493.4352 \end{bmatrix}. \end{aligned}$$

Based on $\mathcal{T_P}$, $\tau_{\mathcal{P}}$ and f, the switching signal is generated in Fig. 2, which corresponds to PDTSM. Furthermore, the

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Algorithm 1: The Design of Gain Matrices.

Input: The number of subsystems and fuzzy rules, i.e., J and M; the given parameters A_{lr}, B_{lr},C_{lr},D_{lr},E_{lr},$\mathcal{T}_{\mathcal{P}}$,$\tau_{\mathcal{P}}$,$f$,$\zeta$,$\varrho$,$\gamma$; $r \in \{1, 2, \ldots, J\}$, $l \in \{1, 2, \ldots, M\}$;

Output: Gain Matrices L_{kr}.

1 Given the constraints on the unknown parameters, i.e., $P_r > 0$ and $\epsilon_r > 0$;

2 Verify the unknown parameters and conditions (12)-(15). If all constraints are satisfied, go to 3; otherwise, adjust input parameters;

3 Calculate the gain matrices L_{kr} satisfying (16);

4 Check the feasibility of the gain matrices from simulation results. If the simulation results are correct, go to 5; otherwise $L_{kr} = null$;

5 **return** L_{kr}.

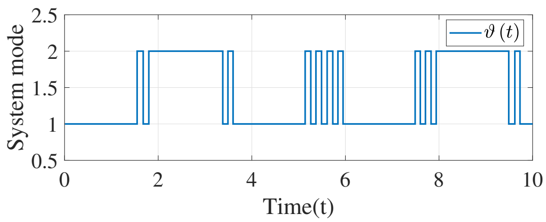

Fig. 2. PDT switching signal with $\mathcal{T}_{\mathcal{P}} = 3\,\mathrm{sec}$ and $\tau_{\mathcal{P}} = 1.54\,\mathrm{sec}$.

nonlinear functions are given as $f_1(x(t)) = f_2(x(t)) = 0.5 \sin \sqrt{x_1^2 + x_2^2 + x_3^2}$ (thus $\alpha_1 = \alpha_2 = 0.5$), and the disturbance is chosen as $\omega(t) = \exp(-0.3t) \sin t$. Then, $\mathrm{sgn}(s(t))$ in the switched fuzzy SMC law $u(t)$ is replaced by $s(t) / (\|s(t)\| + 0.01)$ for the sake of reducing the chattering phenomenon, and set the initial value $x^T(0) = [1\ 0.5\ -0.6]$, Figs. 3-5 depict the corresponding simulation results. Thereinto, Fig. 3 (a) represents the responses of system (3) without control input, and it can be seen that system (3) is unstable under the influence of the matched uncertainty and the disturbance. Under the switched fuzzy SMC law, for the closed-loop system, as illustrated in Fig. 3 (b), its states gradually converge to zero, which verifies the effectiveness of the switched fuzzy SMC law in eliminating the effects of matched uncertainty and the disturbance. Fig. 4 displays the SFISS function, and we can observe that the curve can tend to 0 very rapidly, which validates the finite-time reachability of SFISS. Fig. 5 depicts the switched fuzzy SMC law.

According to Definition 3, the $\mathcal{H}_\infty$ performance index can be calculated under zero initial conditions as

$$\frac{\int_0^{10} y^T(t)\, y(t)\, dt}{\int_0^{10} \omega^T(t)\, \omega(t)\, dt} = 0.3417 < \bar{\gamma} = 1.2175$$

where $\bar{\gamma} = 1.2175$ is derived from the given parameters. Obviously, the resulting $\mathcal{H}_\infty$ performance index satisfies the requirements of Definition 3, and further verifies the robustness of the proposed control scheme.

In addition, the effects of different switching frequency f, energy function variation rates ζ and ϱ on the optimal $\mathcal{H}_\infty$ performance index $\bar{\gamma}_{\min}$ (smaller $\bar{\gamma}_{\min}$ equals strong

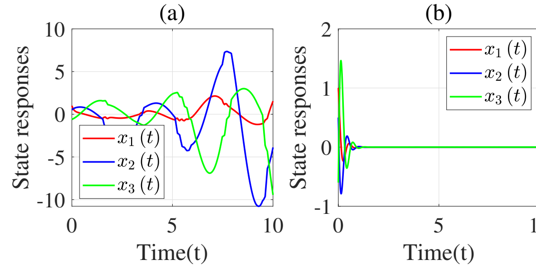

Fig. 3. State responses. (a) open-loop system; (b) closed-loop system.

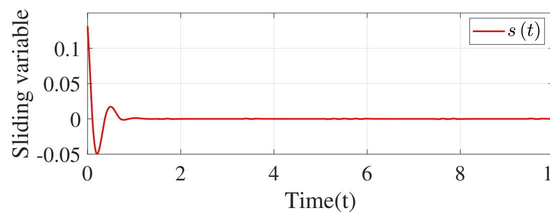

Fig. 4. SFISS function.

system immunity) are investigated. As shown in Tables I-II, by varying the values of the different parameters, the corresponding $\bar{\gamma}_{\min}$ can be derived. For the switching frequency, direct observation of Table I reveals that the reduction in switching frequency has an enhanced effect on improving system performance. From Table II, it can be seen that an increase in ϱ and a decrease in ζ both lead to deterioration in system performance. Consequently, proper adjustment of the switching frequency and the energy function variation rates is essential to improve the immunity of SMDs (7).

V. Conclusion

In this paper, the SMC issue for continuous-time switched T-S fuzzy systems has been investigated, and the PDT switching mechanism has been introduced to describe the variation characteristic of system parameters. In view of the switched T-S fuzzy model, a novel switched fuzzy sliding surface has been constructed, and sufficient conditions have been obtained, which can ensure that SMDs are GUES with an $\mathcal{H}_\infty$ performance index based on the Lyapunov function technique, followed by a switched fuzzy SMC law has been synthesized to guarantee the reachability of the system trajectories in a finite time. Finally, the validity of the switched fuzzy SMC scheme has been verified. The limitations of this paper are summarized in the following three points: (i) the τ-portion of the PDT switching signal is mode-independent, which is somewhat conservative; (ii) the SMC method used in this paper requires the assumption that the upper bound of the nonlinear function is known, which is not very realistic in practical applications; (iii) the fuzzy weighting function processing approach employed in this paper brings a lot of computational complexity. Therefore, in future work, we will work on solving the above problems, such as studying mode-dependent PDT switching mechanisms, adaptive SMC methods, and more superior fuzzy weighting function processing methods.

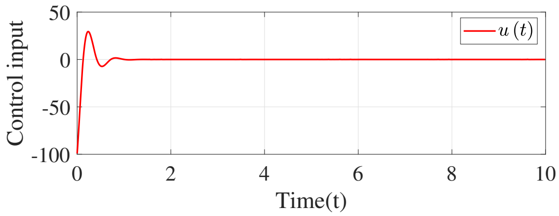

Fig. 5. SMC law.

TABLE I
THE OPTIMAL $\mathcal{H}_\infty$ PERFORMANCE INDEX $\bar{\gamma}_{\min}$ UNDER DIFFERENT f WHEN $\varrho = 1.03, \zeta = 0.95$.

f	8	10	12	14	16
$\bar{\gamma}_{\min}$	1.0130	1.1355	1.2741	1.4327	1.6133

TABLE II
THE OPTIMAL $\mathcal{H}_\infty$ PERFORMANCE INDEX $\bar{\gamma}_{\min}$ UNDER DIFFERENT ϱ AND ζ WHEN $f = 10$Hz.

$\bar{\gamma}_{\min}$	$\zeta = 0.75$	$\zeta = 0.80$	$\zeta = 0.85$	$\zeta = 0.90$	$\zeta = 0.95$
$\varrho = 1.01$	0.7797	0.7773	0.7752	0.7733	0.7717
$\varrho = 1.02$	0.9568	0.9503	0.9446	0.9379	0.9354
$\varrho = 1.03$	1.1789	1.1653	1.1539	1.1441	1.1355
$\varrho = 1.04$	1.4598	1.4351	1.4143	1.3966	1.3808

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Jing Wang received received the Ph.D. degree in power system and automation from Hohai University, Nanjing, China, in 2019. She is currently an Associate Professor with Anhui University of Technology, China.

Her current research interests include Markov jump nonlinear systems, singularly perturbed systems, power systems, nonlinear control.

Haitao Wang received B.Sc. degree in automation from Tongling University, Tongling, China, in 2019. He is currently pursuing the M.S. degree with the School of Electrical and Information Engineering, Anhui University of Technology, Maanshan, China.

His current research interests include switched systems, fuzzy systems, sliding mode control.

Huaicheng Yan (M'07) received Ph.D. degree in control theory and control engineering from Huazhong University of Science and Technology, China, in 2007. From 2007 to 2009, he was a Postdoctoral Fellow with the Chinese University of Hong Kong. From June 2011 to August 2011, he was a Research Fellow with the University of Hong Kong. From February 2012 to August 2012, he was a Research Fellow with the City University of Hong Kong. Currently, he is a Professor with the School of Information Science and Engineering, East China University of Science and Technology, Shanghai, China. His research interests include networked control systems, multi-agent systems, fuzzy systems and smart grids.

Yueying Wang (M'16-SM'18) received the B.Sc. degree in mechanical engineering and automation form the Beijing Institute of Technology, Beijing, China, in 2006, the M. Sc. degree in navigation, guidance, and control, and Ph.D. degree in control science and engineering from Shanghai Jiao Tong University, Shanghai, China, in 2010 and 2015, respectively.

He is currently a Full Professor with the School of Mechatronic Engineering and Automation, Shanghai University, Shanghai. His current research interests include intelligent and hybrid control systems, control of unmanned aerial/surface vehicles. He has served on the editorial board of a number of journals, including IET-Electronics Letters, International Journal of Electronics, International Journal of Fuzzy Systems, International Journal of Control, Automation and Systems, Journal of Electrical Engineering & Technology, and Cyber-Physical Systems.

Hao Shen (M'17) received the Ph.D. degree in control theory and control engineering from Nanjing University of Science and Technology, Nanjing, China, in 2011. Since 2011, he has been with Anhui University of Technology, China, where he is currently a Professor. His current research interests include stochastic hybrid systems, complex networks, fuzzy systems and control, nonlinear control.

Dr. Shen has served on the technical program committee for several international conferences. He is an Associate Editor/Guest Editor for several international journals, including *Journal of The Franklin Institute*, *Applied Mathematics and Computation*, *Neural Processing Letters* and *Transactions of the Institute Measurement and Control*. Prof. Shen was a recipient of the Highly Cited Researcher Award by Clarivate Analytics (formerly, Thomson Reuters) in 2019-2021.