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Asynchronous control for switched systems by using persistent dwell time modeling[☆]

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ABSTRACT

This work is concerned with stability and control for continuous-time switched systems with asynchronous switching. The persistent dwell time (PDT) switching is used to model the asynchronous characteristic in switched systems. Compared with the common dwell time or average dwell time switching widely studied in the existing literature, the PDT switching is known to be more general. The stability and $\mathcal{L}_2$-gain analysis are derived firstly for the switched system with PDT switching, and the Lyapunov-like function is allowed to increase during the mismatched period between the controller and the system mode. Then, the corresponding controller design scheme is proposed. Finally, the effectiveness of the provided method is illustrated with two examples.

1. Introduction

Switched systems, which consist of a family of continuous- or discrete-time subsystems and a switching signal governing the switching among them, have been gaining persistent attention in the last decade. Typical applications of switched systems include networked control systems, vehicle industry, biological systems, flight control systems, power electronics and so on [1–6]. In the context of switched systems, stability analysis, stabilization and some other fundamental issues have been widely studied, and many significant results can be found in the existing literature [7–12].

Meanwhile, several switching signals are frequently adopted to describe the switching characteristics, such as the dwell time (DT) and average dwell time (ADT) switching. As a class of important switching signals, the persistent dwell time (PDT) switching is more general compared with DT and ADT switching since it covers these two switching laws as special cases [13]. For PDT switching, there exists an infinite number of disjoint intervals without switchings and their length is no smaller than a dwell time τ_D (termed as τ-portion), and the length between two consecutive τ-portions is no greater than a period of persistence T (termed as T-portions) [14]. The stability analysis for switched linear systems with PDT switching is primarily investigated in [13]. Furthermore, the control and filtering issues are explored for switched systems based on PDT switching [15–17].

Typically, to solve the stabilization problem of switched system, a set of mode-dependent controllers is adopted instead of mode-independent ones in order to reduce conservatism. For simplicity, the controllers and system modes are generally considered to be synchronous all the time [18]. However, it is impractical in most cases since it takes time to identify the working mode of the system and apply the corresponding controller when switching occurs [19]. As a consequence, there is always a mismatch between the corresponding controller and the system mode, i.e. asynchronous phenomena in switched systems. Recently, these issues have been extensively studied [19–25]. To mention a few, the stabilization scheme is designed for switched linear systems with ADT and asynchronous switching in [25], and based on this result, $\mathcal{H}_\infty$ control is investigated in [20]. It is noteworthy that a common assumption in the aforementioned results is that the maximum mismatched time between the controller and the subsystem is restricted to be less than the minimum dwell time, which means that the subsystem and the controller will be matched before next switching. It is straightforward that such an assumption will lead to a certain conservatism. To get rid of this limitation, we explore the properties of PDT and utilize it to describe the asynchronous phenomena. In this case, the maximum switching delay between controllers and subsystems is allowed to be larger than the minimum dwell time of subsystems.

In this paper, we concentrate on the stability and stabilization for continuous-time switched systems with asynchronous switching. Compared with the existing literature, the main improvements are twofold. First, we convert the arbitrary switched

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system with asynchronous switching to a switched system under the PDT scheme. In this way, the length of mismatched intervals is permissible to be greater than the dwell time, which relaxed the constraint in most of existing results. Second, an $\mathcal{H}_\infty$ control scheme is developed for continuous-time switched system with asynchronous switching, which can guarantee a non-weighted $\mathcal{H}_\infty$ performance. Evidently, the non-weighted $\mathcal{H}_\infty$ index is more general and with a more explicit physical meaning compared with the weighted one in most of related results [20,22]. The remainder of this paper is arranged as follows. In Section 2, some preliminaries are introduced for future development. In Section 3, sufficient conditions are derived to guarantee the stability and $\mathcal{L}_2$-gain performance, upon which a set of controllers for asynchronous switched systems is designed with non-weighted disturbance attenuation in Section 4. Then, two examples are provided to illustrate the effectiveness of the developed control scheme in Section 5. Finally, we conclude the paper in Section 6.

Notation: $\mathbb{R}^n$ represents the n-dimensional Euclidean space, $\mathbb{Z}^+$ represents the set of nonnegative integers, $\mathbb{Z}_{\geq s}$ denotes the set $\{k \in \mathbb{Z}^+ | k \geq s\}$. The notation $\|\cdot\|$ refers to the Euclidean vector norm. The norm of a vector function $\xi(t)$, defined for all $t \geq t_0$, denoted by $\|\xi(t)\|_2$, is equal to $\sqrt{\int_{t_0}^{\infty} \|\xi(t)\|^2\, dt}$. The set of all vector functions such that $\|\xi\|_2^2 < \infty$ is denoted by $\mathcal{L}_2[0, \infty)$. A function $\beta : [0, \infty) \to [0, \infty)$ is said to be of class $\mathcal{K}_\infty$ if it is continuous, strictly increasing, unbounded and $\beta(0) = 0$. The superscripts "T" and "-1" represent matrix transposition and matrix inverse respectively. In addition, in symmetric block matrices or long matrix expressions, we use $*$ as an ellipsis for the terms that are introduced by symmetry. For a matrix X, we denote $X_\perp$ as the orthogonal complement. The notation $Y > 0 (\geq 0)$ means that Y is real symmetric and positive definite (semi-positive definite). The dimensions of matrices which are not explicitly stated, are assumed to be compatible with algebraic operations.

2. Problem formulation

Consider a class of continuous-time switched linear systems:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + E_{\sigma(t)}\omega(t), \tag{1}$$

$$z(t) = G_{\sigma(t)}x(t) + L_{\sigma(t)}\omega(t), \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the system state vector, $u(t) \in \mathbb{R}^m$ is the control input, $z(t) \in \mathbb{R}^q$ is the controlled output, $\omega(t) \in \mathbb{R}^r$ is the exogenous disturbance input which belongs to $\mathcal{L}_2[0, \infty)$. $\sigma(t) : [0, \infty) \to \mathcal{M} = \{1, 2, \ldots, M\}$ is the switching signal with M being the number of subsystems, which is assumed to be a piecewise constant (from the right) function depending on time t.

In this paper, we model the asynchronous switching by using PDT switching signal. The following definition is recalled firstly.

Definition 1 (*[13]*). For a switching signal σ, if there exists an infinite number of disjoint intervals of length no smaller than a positive constant τ_D, and consecutive intervals with this property are separated by no more than T, then τ_D and T are called the persistent dwell time and the period of persistence, respectively.

Remark 1. It has been demonstrated that the DT and ADT switching can be termed as special cases of the PDT switching [16,17]. Therefore, the PDT switching law is more general.

As displayed in Fig. 1, the interval is divided into a number of stages under the PDT scheme, while each stage consists of a τ-portion and a T-portion. In the τ-portion, a certain subsystem is activated with the running time at least τ_D. In the T-portion,

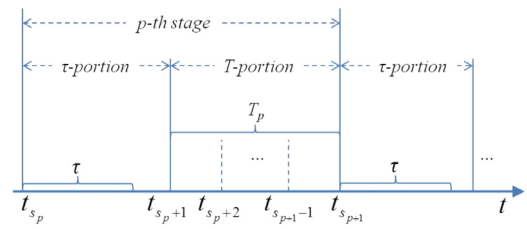

Fig. 1. Illustration of PDT.

multiple switching may occur but the length of a T-portion is no greater than T. Here, let t_{s_p+1} and $t_{s_{p+1}}$ denote the next switching instant after t_{s_p} in the pth stage and the instant switching into the $(p+1)$th stage, respectively. Then, the T-portion of the pth stage can be described as the interval $[t_{s_p+1}, t_{s_{p+1}})$. Define the switching frequency in the T-portion of the pth stage as $f_p = N(t_{s_p+1}, t_{s_{p+1}})/T(t_{s_p+1}, t_{s_{p+1}})$ with $N(t_{s_p+1}, t_{s_{p+1}})$ and $T(t_{s_p+1}, t_{s_{p+1}})$ denoting the total switching number and the length during $[t_{s_p+1}, t_{s_{p+1}})$. f_p, $p \in \mathbb{Z}^+$ is assumed to be no greater than a constant f.

Remark 2. It can be seen that in the T-portion of the pth stage, the total switching number satisfies $N(t_{s_p+1}, t_{s_{p+1}}) = f_p(t_{s_{p+1}} - t_{s_p+1}) \leq fT$. Actually, it is impossible that infinite times of switching occurs in a finite interval. Here, we introduce the switching frequency f_p to restrict the switching numbers in the T-portion.

Here, our purpose is to design the following state-feedback controllers for the system (1)–(2),

$$u(t) = K_{\sigma(t-\varphi(t))}x(t), \tag{3}$$

where $\varphi(t)$ satisfying $0 \leq \varphi(t) \leq \varphi_{\max}$ is the time for detecting the mode of the system and applying the corresponding controller, which can also be treated as the mismatched time after a system switching happens. Without loss of generality, the max switching delay $\varphi_{\max}$ is assumed to be known a priori.

By using the property of PDT, we transform the given switched system (1)–(2) into a switched system based on PDT switching. When the actual dwell time of a subsystem is larger than $\varphi_{\max}$, it is considered to belong to the τ-portion, otherwise, it is considered to belong to the T-portion. By this arrangement, the arbitrary switched system with asynchronous switching is converted to a switched system under the PDT scheme, and apparently $\varphi_{\max} \leq \tau_D$. The corresponding closed-loop system is given by

$$\dot{x}(t) = (A_{\sigma(t)} + B_{\sigma(t)}K_{\sigma(t-\varphi(t))})x(t) + E_{\sigma(t)}\omega(t), \tag{4}$$

$$z(t) = G_{\sigma(t)}x(t) + L_{\sigma(t)}\omega(t). \tag{5}$$

At the end of this section, we introduce the following definitions and lemma which will be used in the sequel.

Definition 2 (*[26]*). The switched system (4) with $\omega(t) \equiv 0$ is globally uniformly asymptotically stable (GUAS) under certain switching signal σ if for initial condition $x(t_0)$, there exists a class of $\mathcal{K}_\infty$ functions κ such that the solution of the system (4) satisfies the inequality $\|x(t)\| \leq \kappa(\|x(t_0)\|)$, $\forall t \geq t_0$, and $\|x(t)\| \to 0$ as $t \to \infty$.

Definition 3 (*[8]*). For $\gamma > 0$, the switched system (4)–(5) is said to be GUAS with an $\mathcal{L}_2$-gain, if under zero initial condition, the system (4)–(5) is GUAS and $\|z(t)\|_2^2 \leq \gamma^2 \|\omega(t)\|_2^2$ holds for any non-zero $\omega(t) \in \mathcal{L}_2[0, \infty)$.

Lemma 1 (*Finsler's Lemma [27]*). *Given $x \in \mathbb{R}^n$, $Q = Q^T \in \mathbb{R}^{n\times n}$, and $H \in \mathbb{R}^{m\times n}$ such that $m < n$, the following statements are equivalent:*

(i) $x^T Qx < 0, \forall x : Hx = 0, x \neq 0$;
(ii) $H_\perp^T QH_\perp < 0$;
(iii) $\exists F \in \mathbb{R}^{n\times m} : Q + FH + H^T F^T < 0$.

3. Stability and $\mathcal{L}_2$-gain analysis

In this section, the stability and $\mathcal{L}_2$-gain analysis of continuous-time asynchronous switched systems modeled by PDT switching will be presented.

Owing to the presence of asynchronism, the mismatched controllers will be applied to the system for a certain time. In the paper, it is assumed that the Lyapunov functions could increase during the mismatched period. Within the interval $[t_u, t_w)$, the separation is given as $[t_u, t_w) = \mathcal{T}_\uparrow(t_u, t_w) \cup \mathcal{T}_\downarrow(t_u, t_w)$, while $\mathcal{T}_\uparrow(t_u, t_w)$ and $\mathcal{T}_\downarrow(t_u, t_w)$ stand for the unions of dispersed intervals during which the subsystem and the controller are mismatched and matched, respectively. Meanwhile, the length of $\mathcal{T}_\uparrow(t_u, t_w)$ and $\mathcal{T}_\downarrow(t_u, t_w)$ is denoted as $T_\uparrow(t_u, t_w)$ and $T_\downarrow(t_u, t_w)$, respectively, and $T(t_u, t_w) = T_\uparrow(t_u, t_w) + T_\downarrow(t_u, t_w)$. It is obvious that $\varphi_{\max} = \max T_\uparrow(t_{s_p}, t_{s_p+1})$.

Lemma 2. *Consider a continuous-time switched system $\dot{x}(t) = g_\sigma(x(t))$, and let $\alpha > 0$, $\beta > 0$, and $\mu > 1$ be given constants. For a prescribed period of persistence T and frequency f, $\forall \sigma = i \in \mathcal{M}$, suppose that there exist functions $V_i : \mathbb{R}^n \to \mathbb{R}$, and two class $\mathcal{K}_\infty$ functions κ_1 and κ_2 such that*

$$\kappa_1(\|x(t)\|) \leq V_i(x(t)) \leq \kappa_2(\|x(t)\|), \tag{6}$$

$$\dot{V}_i(x(t)) \leq \begin{cases} -\alpha V_i(x(t)), \forall t \in \mathcal{T}_\downarrow(t_l, t_{l+1}), \\ \beta V_i(x(t)), \forall t \in \mathcal{T}_\uparrow(t_l, t_{l+1}), \end{cases} \tag{7}$$

and $\forall(\sigma(t_s) = i, \sigma(t_s^-) = j) \in \mathcal{M} \times \mathcal{M}, i \neq j$,

$$V_i(x(t_s)) \leq \mu V_j(x(t_s)). \tag{8}$$

Then, the switched system is GUAS for any PDT switching signals satisfying

$$\tau_D > \tau_D^* = \frac{(Tf + 1)\ln\mu + \alpha\varphi_{\max} + \beta(T + \varphi_{\max})}{\alpha}. \tag{9}$$

Proof. Consider $\sigma(t_{s_p}) = m$, $\sigma(t_{s_{p+1}}) = n$. Combining (7) with (8), we obtain

$$\begin{aligned} &V_n(x(t_{s_{p+1}})) \\ &\leq \mu \exp\{\beta(t_{s_{p+1}} - t_{s_{p+1}-1})\} V_{\sigma(t_{s_{p+1}-1})}(x(t_{s_{p+1}-1})) \\ &\leq \cdots \\ &\leq \mu^{N(t_{s_p+1}, t_{s_{p+1}})} \exp\{\beta(t_{s_{p+1}} - t_{s_p+1})\} V_{\sigma(t_{s_p+1})}(x(t_{s_p+1})) \\ &\leq \mu^{N(t_{s_p+1}, t_{s_{p+1}})} \exp\{\beta(t_{s_{p+1}} - t_{s_p+1})\}\mu \\ &\quad \times \exp\{-\alpha(t_{s_p+1} - t_{s_p} - \varphi(t_{s_p})) + \beta\varphi(t_{s_p})\} V_m(x(t_{s_p})) \\ &\leq \mu^{Tf+1} \exp\{\beta T - \alpha\tau_D + (\alpha + \beta)\varphi_{\max}\} V_m(x(t_{s_p})). \end{aligned}$$

If the switching signal PDT satisfies (9),

$$\mu^{Tf+1} e^{\beta T - \alpha\tau_D + (\alpha+\beta)\varphi_{\max}} < 1$$

holds. Denote

$$\zeta = \mu^{Tf+1} e^{\beta T - \alpha\tau_D + (\alpha+\beta)\varphi_{\max}},$$

we conclude that

$$V_{\sigma(t_{s_p})}(x(t_{s_p})) \leq \zeta^{p-1} V_{\sigma(t_{s_1})}(x(t_{s_1})).$$

Note that $t_{s_1} = t_0$, from (6), we have

$$\|x(t_{s_p})\| \leq \kappa_1^{-1}(\zeta^{p-1}\kappa_2(\|x(t_0)\|)).$$

From (7) and (8), $\forall t \in [t_{s_p}, t_{s_{p+1}})$,

$$\|x(t)\| \leq \kappa_3 \|x(t_0)\|$$

holds, where

$$\kappa_3(\cdot) = \kappa_1^{-1}(\mu^{Tf} e^{\beta(T+\varphi_{\max})})\kappa_2(\kappa_1^{-1}(\zeta^{p-1}\kappa_2(\cdot))).$$

According to Definition 2, the switched system is GUAS, which completes the proof. ■

Remark 3. Fast switching may happen in the system, which may lead to some complex phenomena, such as the overlapped detection delay and etc. [21]. It would be hard to identify the actual mode of the system in T-portion, which contains the modes with actual dwell time smaller than the maximum switching delay of controllers. Thus in Lemma 2, we suppose that the controllers and the modes are all asynchronous during the T-portion to deal with the worst case that may happen in the system.

Remark 4. When considering the asynchronous phenomena, a common assumption in quantities of existing results is that the minimum dwell time is larger than the $\varphi_{\max}$ [19,25]. In this situation, T-portion in our results is non-existing, (9) will be degenerated to

$$\tau_D > \tau_D^* = \frac{\ln\mu + (\alpha + \beta)\varphi_{\max}}{\alpha}.$$

This result is identical with Lemma 3 in [25], which means it could be treated as a special case considered in this paper.

Based on Lemma 2, we present the criterion for $\mathcal{L}_2$-gain analysis of switched systems with asynchronous switching.

Lemma 3. *Consider a continuous-time switched system*

$$\begin{aligned} &\dot{x}(t) = g_{\sigma(t)}(x(t), \omega(t)), \\ &z(t) = h_{\sigma(t)}(x(t), \omega(t)), \end{aligned}$$

and let $\alpha > 0$, $\beta > 0$, and $\mu > 1$ be given constants. For a prescribed period of persistence T and frequency f, $\forall \sigma = i \in \mathcal{M}$, suppose that there exist functions $V_{\sigma(t)} : \mathbb{R}^n \to \mathbb{R}$, two class $\mathcal{K}_\infty$ functions κ_1 and κ_2, and a scalar γ such that (6) and (8) *are satisfied, and*

$$\dot{V}_i(x(t)) \leq \begin{cases} -\alpha V_i(x(t)) - \Gamma(t), \forall t \in \mathcal{T}_\downarrow(t_l, t_{l+1}), \\ \beta V_i(x(t)) - \Gamma(t), \forall t \in \mathcal{T}_\uparrow(t_l, t_{l+1}), \end{cases} \tag{10}$$

where $\Gamma(t) = \|z(t)\|^2 - \gamma^2 \|\omega(t)\|^2$.

Then, for any PDT switching signal satisfying

$$\tau_D > \tau_D^* = \max\left\{\frac{\nu_1 \ln\mu + \alpha\varphi_{\max} + \beta\nu_2}{\alpha}, \frac{1}{f}\right\}, \tag{11}$$

the switched system is GUAS with an $\mathcal{L}_2$-gain no greater than

$$\gamma_l = \sqrt{-\frac{\mu^{\nu_1}\alpha e^{\nu_2\theta}}{\nu_3}}\gamma, \tag{12}$$

with $\nu_1 = Tf + 1$, $\nu_2 = T + \varphi_{\max}$, $\theta = \alpha + \beta$, and $\nu_3 = \frac{\nu_1 \ln\mu + \nu_2\theta}{T + \tau_D} - \alpha$.

Proof. First of all, consider $\omega(t) \equiv 0$, (7) can be ensured by (10). Therefore, one can conclude that the switched system is GUAS from Lemma 2.

Then, consider $\omega(t) \neq 0$, from (8) and (10), we have

$$\begin{aligned} &V_{\sigma(t_{s_{p+1}})}(x(t_{s_{p+1}})) \\ &\leq \mu \exp\{\beta(t_{s_{p+1}} - t_{s_{p+1}-1})\} V_{\sigma(t_{s_{p+1}-1})}(x(t_{s_{p+1}-1})) \\ &\quad - \int_{t_{s_{p+1}-1}}^{t_{s_{p+1}}} \exp\{\beta(t_{s_{p+1}} - l)\}\Gamma(l)dl \end{aligned}$$

$$
\begin{aligned}
&\leq \cdots \\
&\leq \mu^{N(t_{s_p+1},t_{s_{p+1}})} \exp\{\beta(t_{s_{p+1}} - t_{s_p+1}) + \beta\varphi(t_{s_p}) \\
&\quad - \alpha(t_{s_p+1} - t_{s_p} - \varphi(t_{s_p}))\} V_{\sigma(t_{s_p})}(x(t_{s_p})) \\
&\quad - \mu^{N(t_{s_p+1},t_{s_{p+1}})} \exp\{\beta(t_{s_{p+1}} - t_{s_p+1})\} \\
&\quad \times \int_{t_{s_p}+\varphi(t_{s_p})}^{t_{s_p+1}} \exp\{-\alpha(t_{s_p+1} - l)\}\Gamma(l)dl \\
&\quad - \mu^{N(t_{s_p+1},t_{s_{p+1}})} \exp\{\beta(t_{s_{p+1}} - t_{s_p+1})\} \\
&\quad \times \int_{t_{s_p}}^{t_{s_p}+\varphi(t_{s_p})} \exp\{-\alpha(t_{s_p+1} - t_{s_p} - \varphi(t_{s_p})) \\
&\quad + \beta_{\sigma(t_{s_p})}(t_{s_p} + \varphi(t_{s_p}) - l)\}\Gamma(l)dl \\
&\quad - \cdots - \int_{t_{s_{p+1}-1}}^{t_{s_{p+1}}} \exp\{\beta(t_{s_{p+1}} - l)\}\Gamma(l)dl \\
&\leq \cdots \\
&\leq \mu^{N(t_{s_1+1},t_{s_{p+1}})} \exp\{\beta(t_{s_{p+1}} - t_{s_p+1}) + \beta\varphi(t_{s_p}) \\
&\quad - \alpha(t_{s_p+1} - t_{s_p} - \varphi(t_{s_p}))\} \\
&\quad \times \cdots \exp\{\beta(t_{s_2} - t_{s_1+1}) + \beta_{\sigma(t_{s_1})}\varphi(t_{s_1}) \\
&\quad - \alpha(t_{s_1+1} - t_{s_1} - \varphi(t_{s_1}))\} V_{\sigma(t_{s_1})}(x(t_{s_1})) \\
&\quad - \mu^{N(t_{s_1+1},t_{s_{p+1}})} \exp\{\beta(t_{s_{p+1}} - t_{s_p+1}) + \beta\varphi(t_{s_p}) \\
&\quad - \alpha(t_{s_p+1} - t_{s_p} - \varphi(t_{s_p}))\} \cdots \exp\{\beta(t_{s_2} - t_{s_1+1})\} \\
&\quad \times \int_{t_{s_1}+\varphi(t_{s_1})}^{t_{s_1+1}} \exp\{-\alpha(t_{s_1+1} - l)\}\Gamma(l)dl \\
&\quad - \mu^{N(t_{s_1+1},t_{s_{p+1}})} \exp\{\beta(t_{s_{p+1}} - t_{s_p+1}) + \beta\varphi(t_{s_p}) \\
&\quad - \alpha(t_{s_p+1} - t_{s_p} - \varphi(t_{s_p}))\} \cdots \exp\{\beta(t_{s_2} - t_{s_1+1})\} \\
&\quad \times \int_{t_{s_1}}^{t_{s_1}+\varphi(t_{s_1})} \exp\{-\alpha(t_{s_1+1} - t_{s_1} - \varphi(t_{s_1})) \\
&\quad + \beta(t_{s_1} + \varphi(t_{s_1}) - l)\}\Gamma(l)dl \\
&\quad - \cdots - \int_{t_{s_{p+1}-1}}^{t_{s_{p+1}}} \exp\{\beta(t_{s_{p+1}} - l)\}\Gamma(l)dl
\end{aligned}
$$

Note that $t_{s_1} = t_0$, under zero initial condition, we have $V_{\sigma(t_{s_1})}(x(t_{s_1})) = 0$. Thus, for $\forall t \in [t_{s_p}, t_{s_{p+1}})$, we can get

$$
\int_{t_0}^{t} \mu^{N(l,t)} \exp\{-\alpha\mathcal{T}_{\downarrow}(l,t) + \beta\mathcal{T}_{\uparrow}(l,t)\}\Gamma(l)dl \leq 0. \tag{13}
$$

The total running time in pth stage is $\tau_p + T_p$, where τ_p and T_p are the running time in τ-portion and T-portion, respectively. Since $T_p \leq T$, and $\tau_p \geq \tau_D$, from (11), one can easily obtain $(f\tau_D - 1)(T_p - T) \leq 0$. Assume that the interval $[l, t)$ is in the pth stage, we have

$$
\left(\frac{t-l}{T_p + \tau_D} + 1\right)(T_p f + 1) \leq \left(\frac{t-l}{T + \tau_D} + 1\right)(Tf + 1)
$$

for any time within the stage. Meanwhile, the relationship also holds for other stages since p is arbitrary. Then by summing up the stages between $[l, t)$, we know

$$
0 \leq N(l,t) \leq \left(\frac{t-l}{T + \tau_D} + 1\right)(Tf + 1). \tag{14}
$$

By similar manipulation, one can also have

$$
0 \leq \mathcal{T}_{\uparrow}(l,t) \leq \left(\frac{t-l}{T + \tau_D} + 1\right)(T + \varphi_{\max}). \tag{15}
$$

Together with (13) and (14)–(15), we conclude that

$$
\begin{aligned}
&\int_{t_0}^{t} \exp\{-\alpha(t-l)\}\|z(l)\|^2 dl \\
&\leq \gamma^2 \int_{t_0}^{t} \mu^{\left(\frac{t-l}{T+\tau_D}+1\right)(Tf+1)} \exp\{-\alpha(t-l) \\
&\quad + (\alpha+\beta)\left(\frac{t-l}{T+\tau_D} + 1\right)(T + \varphi_{\max})\}\|\omega(l)\|^2 dl,
\end{aligned}
$$

which indicates that

$$
\begin{aligned}
&\int_{t_0}^{t} \exp\{-\alpha(t-l)\}\|z(l)\|^2 dl \\
&\leq \gamma^2\mu^{\nu_1}e^{\nu_2\theta} \int_{t_0}^{t} \exp\{\nu_3(t-l)\}\|\omega(l)\|^2 dl,
\end{aligned}
$$

where $\nu_1 = Tf + 1$, $\nu_2 = T + \varphi_{\max}$, $\theta = \alpha + \beta$, and $\nu_3 = \frac{\nu_1 \ln\mu + \nu_2\theta}{T + \tau_D} - \alpha$. It is noted that $\nu_3 < 0$ can be ensured by (11).

Furthermore, we have

$$
\begin{aligned}
&\int_{t_0}^{\infty}\int_{t_0}^{t} \exp\{-\alpha(t-l)\}\|z(l)\|^2 dldt \\
&\leq \gamma^2\mu^{\nu_1}e^{\nu_2\theta} \int_{t_0}^{\infty}\int_{t_0}^{t} \exp\{\nu_3(t-l)\}\|\omega(l)\|^2 dldt,
\end{aligned}
$$

which is equivalent to

$$
\begin{aligned}
&\int_{t_0}^{\infty}\int_{l}^{\infty} \exp\{-\alpha(t-l)\}\|z(l)\|^2 dtdl \\
&\leq \gamma^2\mu^{\nu_1}e^{\nu_2\theta} \int_{t_0}^{\infty}\int_{l}^{\infty} \exp\{\nu_3(t-l)\}\|\omega(l)\|^2 dtdl,
\end{aligned}
$$

Since

$$
\begin{aligned}
&\int_{l}^{\infty} \exp\{-\alpha(t-l)\}\|z(l)\|^2 dt = \frac{1}{\alpha}\|z(l)\|^2, \\
&\int_{l}^{\infty} \exp\{\nu_3(t-l)\}\|\omega(l)\|^2 dt = -\frac{1}{\nu_3}\|\omega(l)\|^2,
\end{aligned}
$$

we can obtain

$$
\int_{t_0}^{\infty}\|z(l)\|^2 dl \leq \gamma_l^2 \int_{t_0}^{\infty}\|\omega(l)\|^2 dl,
$$

where

$$
\gamma_l = \sqrt{-\frac{\mu^{\nu_1}\alpha e^{\nu_2\theta}}{\nu_3}}\gamma.
$$

Therefore, the switched system is GUAS with an $\mathcal{L}_2$-gain no greater than γ_l, which ends the proof. ■

Remark 5. The $\mathcal{L}_2$-gain analysis has been widely investigated for switched systems, and most of the results are based on ADT switching. These results lead to a weighted disturbance attenuation [12,20], which is lacking an explicit physical meaning in reality compared with the non-weighted one as ours [17].

In the following, we present a sufficient criterion which guarantees the stability and $\mathcal{L}_2$-gain analysis of continuous-time linear switched system (4)–(5).

Here, for $\sigma(t) = i \in \mathcal{M}$, we rewrite the continuous-time linear switched system (4)–(5) as

$$
\begin{cases}\dot{x}(t) = \bar{A}_i x(t) + E_i\omega(t) \\ z(t) = G_i x(t) + F_i\omega(t),\end{cases} \forall t \in \mathcal{T}_{\downarrow}(t_l, t_{l+1}), \tag{16}
$$

$$
\begin{cases}\dot{x}(t) = \hat{A}_{ij} x(t) + E_i\omega(t) \\ z(t) = G_i x(t) + F_i\omega(t),\end{cases} \forall t \in \mathcal{T}_{\uparrow}(t_l, t_{l+1}), \tag{17}
$$

where $\bar{A}_i = A_i + B_iK_i$, and $\hat{A}_{ij} = A_i + B_iK_j$, $\forall(i,j) \in \mathcal{M}\times\mathcal{M}$, $i \neq j$. Then we present the following lemma.

Lemma 4. *Consider the continuous-time linear switched system (16)–(17), let $\alpha > 0$, $\beta > 0$, and $\mu > 1$ be given constants. For* *a prescribed period of persistence T and frequency f, suppose that there exist matrices $P_i > 0$, $\forall i \in \mathcal{M}$, and a scalar γ such that $\forall (i,j) \in \mathcal{M} \times \mathcal{M}$, $i \neq j$,*

$$\bar{W}_i^T \bar{\Phi}_i \bar{W}_i < 0, \tag{18}$$

$$\hat{W}_{ij}^T \hat{\Phi}_i \hat{W}_{ij} < 0, \tag{19}$$

$$P_i \leq \mu P_j, \tag{20}$$

where

$$\bar{W}_i = \begin{bmatrix} \bar{A}_i & E_i & 0 \\ I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, \bar{\Phi}_i = \begin{bmatrix} 0 & P_i & 0 & 0 \\ * & \alpha P_i & 0 & G_i^T \\ * & * & -\gamma^2 I & L_i^T \\ * & * & * & -I \end{bmatrix},$$

$$\hat{W}_{ij} = \begin{bmatrix} \hat{A}_{ij} & E_i & 0 \\ I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, \hat{\Phi}_i = \begin{bmatrix} 0 & P_i & 0 & 0 \\ * & -\beta P_i & 0 & G_i^T \\ * & * & -\gamma^2 I & L_i^T \\ * & * & * & -I \end{bmatrix}.$$

Then, for any PDT switching signal satisfying (11), the switched system (16)–(17) is GUAS with an $\mathcal{L}_2$-gain no greater than (12).

Proof. For $\sigma(t) = i \in \mathcal{M}$, choose the following Lyapunov function

$$V_i(x(t)) = x^T(t) P_i x(t). \tag{21}$$

For $\forall t \in \mathcal{T}_{\downarrow}(t_l, t_{l+1})$, combining (16)–(17) with (21), we obtain

$$\dot{V}_i(x(t)) + \alpha V_i(x(t)) + \Gamma(t) = \xi^T(t) \Pi_i \xi(t), \tag{22}$$

where $\xi(t) = \begin{bmatrix} x(t) \\ \omega(t) \end{bmatrix}$, and

$$\Pi_i = \begin{bmatrix} \bar{A}_i^T P_i + P_i \bar{A}_i + \alpha P_i + G_i^T G_i & P_i E_i + G_i^T L_i \\ * & L_i^T L_i - \gamma^2 I \end{bmatrix}.$$

The equality (18) indicates that

$$\begin{bmatrix} \bar{A}_i^T P_i + P_i \bar{A}_i + \alpha P_i & P_i E_i & G_i^T \\ * & -\gamma^2 I & L_i^T \\ * & * & -I \end{bmatrix} < 0. \tag{23}$$

According to Schur complement, (23) is equivalent to $\Pi_i < 0$. From (22), we obtain that

$$\dot{V}_i(x(t)) + \alpha V_i(x(t)) + \Gamma(t) < 0, \forall t \in \mathcal{T}_{\downarrow}(t_l, t_{l+1}).$$

By the similar manipulation, (19) can guarantee

$$\dot{V}_i(x(t)) - \beta V_i(x(t)) + \Gamma(t) < 0, \forall t \in \mathcal{T}_{\uparrow}(t_l, t_{l+1}).$$

Thus, we conclude that (18)–(19) can guarantee (10). On the other hand, (8) can be ensured by (20). According to Lemma 3, we conclude that the switched system (16)–(17) is GUAS with an $\mathcal{L}_2$-gain. The proof is completed. ■

4. Asynchronous control

In this section, our object is to design $\mathcal{H}_\infty$ controllers for the underlying switched system (1)–(2) with asynchronism.

Theorem 1. *Consider the continuous-time linear switched system (4)–(5), let $\alpha > 0$, $\beta > 0$, and $\mu > 1$ be given constants. For a prescribed period of persistence T and frequency f, suppose that there exist matrices $P_i > 0$, K_i, F_i, $\forall i \in \mathcal{M}$, and a scalar γ such that $\forall (i,j) \in \mathcal{M} \times \mathcal{M}$, $i \neq j$, (20) holds, and*

$$\bar{\Psi}_i + F_i T_i + T_i^T F_i^T < 0, \tag{24}$$

$$\hat{\Psi}_i + F_j T_j + T_j^T F_j^T < 0, \tag{25}$$

where

$$\bar{\Psi}_i = \begin{bmatrix} 0 & B_i^T P_i & 0 & 0 \\ * & A_i^T P_i + P_i A_i + \alpha P_i & E_i P_i & G_i^T \\ * & * & -\gamma^2 I & L_i^T \\ * & * & * & -I \end{bmatrix},$$

$$\hat{\Psi}_i = \begin{bmatrix} 0 & B_i^T P_i & 0 & 0 \\ * & A_i^T P_i + P_i A_i - \beta P_i & E_i P_i & G_i^T \\ * & * & -\gamma^2 I & L_i^T \\ * & * & * & -I \end{bmatrix},$$

$$T_i = \begin{bmatrix} -I & K_i & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Then, for any PDT switching signal satisfying (11), the switched system (4)–(5) is GUAS with an $\mathcal{L}_2$-gain no greater than (12). Moreover, the controller gains are given by K_i, $\forall i \in \mathcal{M}$.

Proof. Select (21) as Lyapunov function.

According to Lemma 1, we know that (24) is equivalent to

$$(T_i)_\perp^T \bar{\Psi}_i (T_i)_\perp < 0, \tag{26}$$

We construct S_i for $\forall i \in \mathcal{M}$ such that

$$S_i = (T_i)_\perp = \begin{bmatrix} K_i & 0 & 0 \\ I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}.$$

Then define R_i for $\forall i \in \mathcal{M}$ as

$$R_i = \begin{bmatrix} B_i & A_i & D_i & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}.$$

It can be shown that $\bar{W}_i = R_i S_i$ when noting that $\bar{A}_i = A_i + B_i K_i$, $\forall i \in \mathcal{M}$. Thus we obtain

$$\bar{W}_i^T \bar{\Phi}_i \bar{W}_i = (R_i S_i)^T \bar{\Phi}_i R_i S_i = S_i^T (R_i^T \bar{\Phi}_i R_i) S_i = S_i^T \bar{\Psi}_i S_i.$$

Combining with (26), we conclude that (18) is ensured by (24).

On the other hand, (25) is equivalent to

$$(T_j)_\perp^T \hat{\Psi}_i (T_j)_\perp < 0, \tag{27}$$

and we have $\hat{W}_{ij} = R_i S_j$ when noting that$\hat{A}_{ij} = A_i + B_i K_j$, $\forall (i,j) \in \mathcal{M} \times \mathcal{M}$, $i \neq j$, which indicates that

$$\hat{W}_{ij}^T \hat{\Phi}_i \hat{W}_j = (R_i S_j)^T \hat{\Phi}_i R_i S_i = S_j^T (R_i^T \hat{\Phi}_i R_i) S_j = S_j^T \hat{\Psi}_i S_j.$$

From (27) we know that (25) can guarantee (19).

Therefore, according to Lemma 4, we conclude that the switched system (1)–(2) is GUAS with an $\mathcal{L}_2$-gain γ_l, and the controller gains are given by K_i, $\forall i \in \mathcal{M}$. The proof is completed. ■

Remark 6. There exist some control schemes for continuous-time switched systems with the asynchronous switching [17,19,20]. However, in [17], the results cannot be solved by the linear matrix inequality (LMI) toolbox. Though the controller design scheme proposed in [19] could be solved by LMI toolbox by using multiple Lyapunov functions, complicated iteration and optimization process are requisites. In this paper, the Lyapunov matrices and design controllers are successfully decoupled by using Finsler's lemma. Moreover, all of the above mentioned results are with the assumption that the actual dwell time of a subsystem is larger than the maximum controller switching delay $\varphi_{\max}$, while the results in Theorem 1 are without such constraints, which means the criterion in this paper is more general and simple to use in practice.

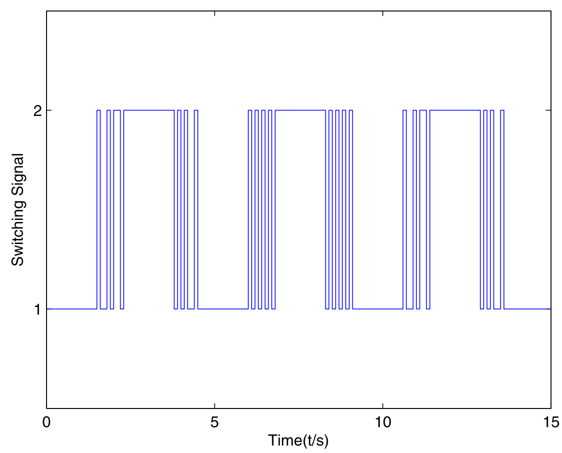

Fig. 2. The switching signal $\sigma(t)$.

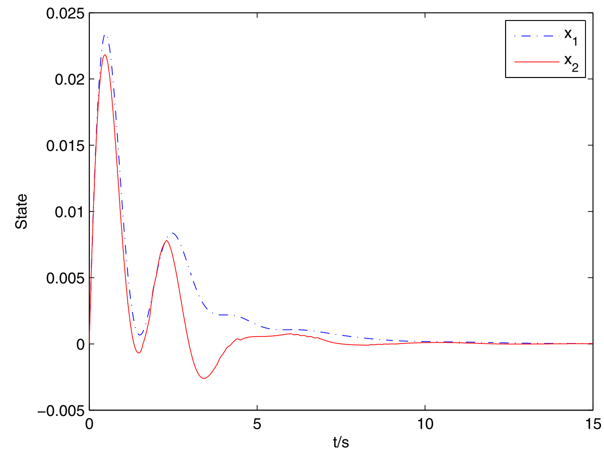

Fig. 3. The state responses of the thermal system.

5. Illustrative examples

In this section, we present two examples to demonstrate the validity of the proposed control scheme. We firstly use a thermal system with linear switched dynamics borrowed from [28].

Example 1. Consider a metal cube immersed in a liquid bath, let $T_1(t)$ and $T_2(t)$ denote the cube temperature and liquid bath temperature, respectively, and $T_o(t)$ stands for the temperature outside the bath. Then we can describe this model as:

$$C_1\frac{dT_1(t)}{dt} = -\frac{1}{R_1}(T_1(t)-T_2(t)),$$
$$C_2\frac{dT_2(t)}{dt} = \frac{1}{R_1}(T_1(t)-T_2(t)) - \frac{1}{R_2}(T_1(t)-T_o(t)),$$

where C_1, C_2 are the capacitances of the cube and the liquid bath, respectively, and R_1 represents the convective resistance between the cube and the bath and R_2 stands for the combined convective/conductive resistance of the container wall and the liquid surface.

Meanwhile, let the output be

$$z(t) = \frac{1}{R_3C_3}(T_1(t)-T_2(t)).$$

Assume that the bath properties C_2, R_2 can be varied, i.e. the system has features of switching. Suppose there exists additional disturbance $\omega(t)$, the system above can be rewritten in terms of (1)–(2), where

$$A_{\sigma(t)} = \begin{bmatrix} -\frac{1}{R_1C_1} & \frac{1}{R_1C_1} \\ \frac{1}{R_1C_{2\sigma(t)}} - \frac{1}{R_{2\sigma(t)}C_{2\sigma(t)}} & -\frac{1}{R_1C_{2\sigma(t)}} \end{bmatrix},$$
$$B_{\sigma(t)} = \begin{bmatrix} 0 \\ \frac{1}{R_{2\sigma(t)}C_{2\sigma(t)}} \end{bmatrix}, G_{\sigma(t)} = \frac{1}{R_{2\sigma(t)}C_{2\sigma(t)}},$$
$$L_{\sigma(t)} = \begin{bmatrix} \frac{1}{R_{3\sigma(t)}C_{3\sigma(t)}} & -\frac{1}{R_{3\sigma(t)}C_{3\sigma(t)}} \end{bmatrix}.$$

For systems with two modes, given $C_1 = 100$, $R_1 = 0.018$, $C_{21} = 50$, $C_{22} = 100$, $R_{21} = 0.2$, $R_{22} = 0.0198$, $C_3 = 1200$, $R_3 = 0.0694$, and

$$E_1 = \begin{bmatrix}0.1 & 0.1\end{bmatrix}^T, E_2 = \begin{bmatrix}0.1 & 0.2\end{bmatrix}^T$$
$$L_1 = 0.2, L_2 = 0.1.$$

Here, select $\alpha = 0.2$, $\beta = 0.1$, $\mu = 1.01$, and assume that $T = 0.8$s, $f = 11.25\text{s}^{-1}$, $\varphi_{\max} = 0.1$s. According to Theorem 1, by given

$$F_1 = F_2 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

we can find admissible solution with $\tau_D^* = 1.0475$ s, and a set of state-feedback controllers can be obtained with

$$K_1 = \begin{bmatrix}-0.9828 & -1.0237\end{bmatrix}, K_2 = \begin{bmatrix}-0.9827 & -1.0250\end{bmatrix}.$$

Under the zero initial condition, consider the disturbance $\omega(t) = 0.9\cos(\pi t)e^{-t}$. The switching signal $\sigma(t)$ is depicted in Fig. 2, and the state responses of the closed-loop system are shown in Fig. 3. We can calculate the maximum noise attenuation $\max\gamma_c(t) = 0.1982$ with

$$\gamma_c(t) = \sqrt{\int_0^t \|z(l)\|^2\,dl \Big/ \int_0^t \|\omega(l)\|^2\,dl}.$$

Then, we provide a more general numerical example.

Example 2. Consider a continuous-time switched linear system (1)–(2) with three subsystems, and the corresponding parameters are given by

$$A_1 = \begin{bmatrix} -0.5 & 0.6 \\ 0.83 & -0.55 \end{bmatrix}, B_1 = \begin{bmatrix} 0.1 \\ 0.28 \end{bmatrix}, D_1 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -0.6 & 0.55 \\ 0.5 & -0.3 \end{bmatrix}, B_2 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, D_2 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} -0.5 & 0.56 \\ 0.4 & -0.2 \end{bmatrix}, B_3 = \begin{bmatrix} 0.2 \\ 0.204 \end{bmatrix}, D_3 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},$$
$$G_1 = G_2 = G_3 = \begin{bmatrix}0.1 & -0.1\end{bmatrix}, L_3 = L_2 = L_3 = 0.1.$$

Choose $\alpha = 0.4$, $\beta = 0.2$, $\mu = 1.01$, and $T = 2$s, $f = 5\text{s}^{-1}$, $\varphi_{\max} = 0.1$s. From Theorem 1 we can find $\tau_D^* = 1.4236$s and a set of state-feedback controllers is designed,

$$K_1 = \begin{bmatrix}4.9712 & -14.2508\end{bmatrix},$$
$$K_2 = \begin{bmatrix}9.6190 & -22.4843\end{bmatrix},$$
$$K_3 = \begin{bmatrix}7.1621 & -21.7861\end{bmatrix}.$$

Assume that the disturbance $\omega(t) = \cos(t)e^{-t}$, and the switching signal $\sigma(t)$ is shown in Fig. 4. Without controllers, the state responses of the open-loop system are shown in Fig. 5, which are divergent apparently. The state responses of the closed-loop system are shown in Fig. 6, and $\max\gamma_c(t) = 0.1005$, which indicates the effectiveness of the above designed controllers.

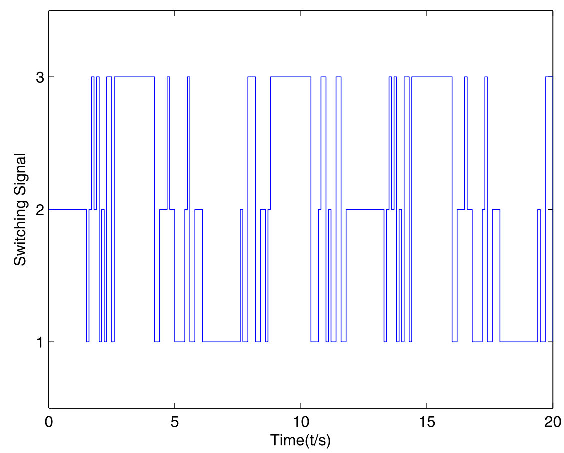

Fig. 4. The switching signal $\sigma(t)$.

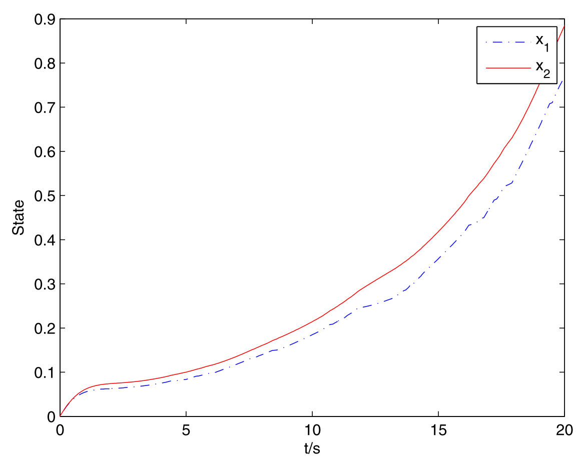

Fig. 5. The state responses of the open-loop system.

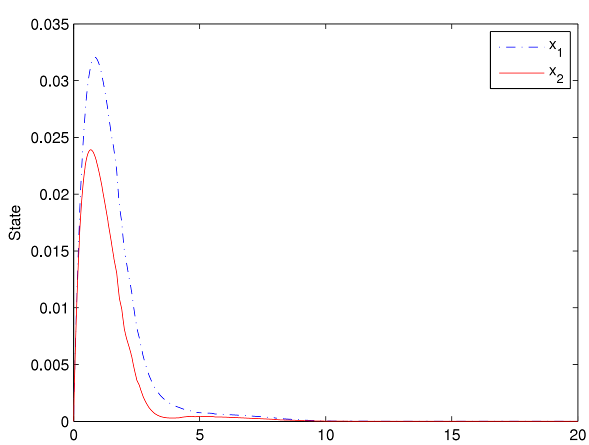

Fig. 6. The state response of the closed-loop system.

6. Conclusions

The $\mathcal{H}_\infty$ control problem for a class of continuous-time switched linear systems is investigated in this paper. The switching between controllers and modes involves time delays, which lead to the asynchronous phenomenon. The PDT switching is used to model this phenomenon, and an extended criterion is addressed to solve the issue of stability and $\mathcal{L}_2$-gain analysis that guaranteed disturbance attenuation. Then, the corresponding controllers are designed based on Finsler's Lemma. Finally, two examples are provided to verify the effectiveness of the acquired control scheme.

Declaration of competing interest

The authors declare that there is no conflict of interest in this paper.

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