New Stability Criteria of Delayed Load Frequency Control Systems via Infinite-Series-Based Inequality

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***Abstract*—A new approach is proposed for the stability problem of delayed load frequency control (LFC) scheme with fixed and time-varying delay cases included in the current paper. New stability criteria with delay dependence in terms of linear matrix inequalities for LFC systems are derived by a novel augmented Lyapunov–Krasovski (L–K) functional. Our proof deployment for system stability of power grids employs the further improved integral inequality in the form of infinite series, which turns out to be less conservative than Wirtinger's inequality that encompasses Jensen inequality. Simulation case studies are carried out to show the effectiveness and superiority of the presented delay-dependent PI-type LFC design scheme.**

***Index Terms*—Augmented Lyapunov–Krasovski (L–K) functional, load frequency control (LFC), power system, time-varying delay.**

I. Introduction

FREQUENCY stability is critical in the safe and stable operation of power systems, which reflects the basic position of power systems in supply and demand. The fluctuation of load frequency would bring serious consequences to the safety operation of power systems, plants, and users: it can cause the generator and auxiliary engine to deviate from operation status, thus reduces mechanical efficiency, makes power plant operation deviate from economic efficiency, and also has an adverse effect on the entire economic operation of the power grid. In addition, when the frequency is too low, the frequency fluctuations can even threaten the security of the entire network operation. Therefore, research on the load frequency analysis and control of power systems is considered to be the important elements necessary to ensure the safe operation of power systems. Load frequency control (LFC) strategies have been employed effectively in distributed power systems for the development of smart grid [1], [2]. Maintaining frequency and power interchanges at the scheduled values with neighborhood areas is the significant aim of LFC.

Modern power systems usually require a wide area open communication network to transmit information concerned. The usage of these networks causes inevitable unreliable factors, such as time delays, packet losses, latent faults, and etc. Consequently, as one of the most important unreliable factors, time delay phenomenon in power control system and the impact of network-induced delay on communication-based power grid has attracted vast attention from academic research (see [3]–[7], and references therein). The model for LFC scheme, which is equipped with communication channels, is a class of typical time-delay systems. From the viewpoint of stability analysis, it is of much importance to seek the tolerable upper bound on the communication network delay that preserves the delayed power systems with LFC scheme to remain stable.

The frequency domain method is utilized to judge the stability of the power system by calculating the eigenvalues [8] when computing the delay margin. The time domain method based on Lyapunov stability theory is abroad exploited. Among some inequality-based stability conditions, the linear matrix inequality (LMI) approach becomes a powerful and popular means to tackle the stability issues of power systems with time-varying delays (see [4], [6], and [9]–[12]). In order to diminish the conservatism for the stability conditions of time-varying delay systems, many approaches were developed. The main efforts have been focused on two aspects: one is the techniques of constructing L–K functional, such as delay-division functional, functional with matrices dependent on the time delays [13], functional including the ones with triple-integral terms [14], and quadratic terms which is multiplied by a higher degree scalar function [15]. The other is the analyzing methods for estimating the derivatives of L–K functionals with respect to time, such as improved majorization technique, free weighting matrix method [16], integral inequality including Jensen inequality [11], Wirtinger inequality [17], auxiliary function-based integral inequality [18], and

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convex combination ideas incorporating linear convex analysis [19], reciprocal convex technique [20], and quadratic convex approach [21]. Increasing research for stability of power systems have been done in the facet of the delay-dependent criteria in these years, because delay-dependent ones are apt to have less conservatism than the delay independent ones particularly in case of the small delay [10]–[12].

For stability analysis and performance lifting of the delayed LFC power systems, there exist a few excellent works reported in the literature. Bevrani and Hiyama [22] bestowed focus on the impacts of delay for the stability analysis in LFC systems. But, the time delay margin of the LFC is not provided. The delay-dependent stability of the LFC scheme revealed the relation between the allowable delay upper bound and the PI controller gains has been investigated in [10], however, it affords a high computation cost. A further improved result was given for PID-type multiarea LFC schemes in a deregulated environment by [11] using Jensen integral inequality. Although considered in the modeling of the above works, the load disturbance is not taken into account either in the stability analysis or simulation, thus the application of the criterion and results are subject to limitation. Design of a robust controller is investigated in consideration of delay and disturbance effects during the design stage in [12]. Similarly in [9], an LFC design approach based on an LMI framework is put forward to design a robust H_∞ controller in face of delayed control signals. Further in [23], it is worth noting that the bounded nonlinear perturbation model was built for the disturbed loads. In this paper, the different L–K-based analysis method for the LFC time-varying delay system model is developed, i.e., the augmented L–K functional including single and double integral terms, the error term between the original state and the integrated state contains more information than the routine L–K functional adopted in [23], the truncated infinite-series-based integral inequality is more relaxed than Jensen inequality employed in [23], and besides the reciprocally convex combination, the sufficient quadratic convex combination technique is properly embedded into our proof procedure.

In the present paper, we are motivated to obtain new delay dependent LMI stability criteria of LFC power systems. An enhanced integral inequality, which takes the form of a truncated first infinite series terms tighter than Wirtinger inequality, is resorted to estimate the time derivative of the proposed L–K functional together with a reciprocally convex combination inequality. As far as the authors are concerned, it is for the first time, via this new integral inequality, to do research on the stability problem for delayed LFC systems. A time-varying delay system model is cast to capture the features of the power systems equipped with a PI-type load frequency controller. And a sufficient quadratic convex combination like lemma is adopted in the closed-loop stability analysis. The combined reciprocally convex and quadratically convex approach can be seen as another unique feature of this study. Moreover, case studies are carried out based on one-area or two-area LFC scheme with simulation exemplifications to verify the effectiveness and superiority of the criterion derived.

TABLE I
NOTATIONS

$\Delta f_i(t)$	Deviation of frequency
$\Delta P_{mi}(t)$	Output of generator/turbine
$\Delta P_{vi}(t)$	Valve position of generator
$\Delta P_{di}(t)$	Disturbance of load
M_i	Moment of inertia of generator
D_i	Damping constant of generator
T_{gi}	Time-constant of governor
T_{chi}	Time-constant of turbine
$\hat{R}_i$	Speed droop
β_i	Frequency bias factor
$\mathrm{ACE}_i(t)$	Area control error
K_{Pi}	Proportional gain
K_{Ii}	Integral gain
$\Delta P_{\mathrm{tie}-i}(t)$	Power transfer of tie-line
T_{ij}	Synchronizing coefficient of tie-line

II. LFC Modeling

Fig. 1 depicts a block diagram of a PI-based LFC scheme for power system classically described in [1], [2], [10], and [23]. Table I gives the notations employed for the ith area for the entire LFC system. The index i should be suppressed for the one-area LFC system, such as $\Delta f_1 = \Delta f$, and so on with $\Delta P_{\mathrm{tie}-1} = 0$.

A. Single-Area LFC Model

Due to no power exchange of net tie-line in the one-area LFC scheme, the output of the system, that is, the area control error (ACE) is defined as

$$\bar{y}(t) = \mathrm{ACE}(t) = \beta \Delta f(t) \tag{1}$$

where $\beta > 0$ is the frequency bias factor.

A PI-type LFC is designed in consideration of error ACE as the input of the controller as follows

$$u(t) = -K_p \mathrm{ACE}(t) - K_I \int_0^t \mathrm{ACE}(s)ds \tag{2}$$

where K_p and K_I are the proportional gain and, respectively, integral gain. For the sake of simplicity, we aggregate the time delay existing in the transmission of control signal (between the control center and the plant), with that of ACE, and denote the total sum by an exponential block $e^{-sd(t)}$ (here it is worth pointing out that $e^{-sd(t)}$ is not a Laplace operator, but a time-varying delay notation) in Fig. 1. Then the PI-type LFC (2) can be further translated into

$$u(t) = -Ky(t - d(t)) \tag{3}$$

where $y(t) \triangleq [\bar{y}(t) \; \int_0^t \bar{y}(s)ds]^T$, $K = [K_p \; K_I]$, and $d(t)$ stands for the time-varying delay.

The next new virtual state is defined as $x(t) = [\Delta f \; \Delta P_m \; \Delta P_v \; \int_0^t ACE(s)ds]^T$, thus it follows

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t - d(t)) + F\Delta P_d(t) \\ y(t) = Cx(t) \\ x(t) = \phi(t), \; t \in [-h, 0] \end{cases} \tag{4}$$

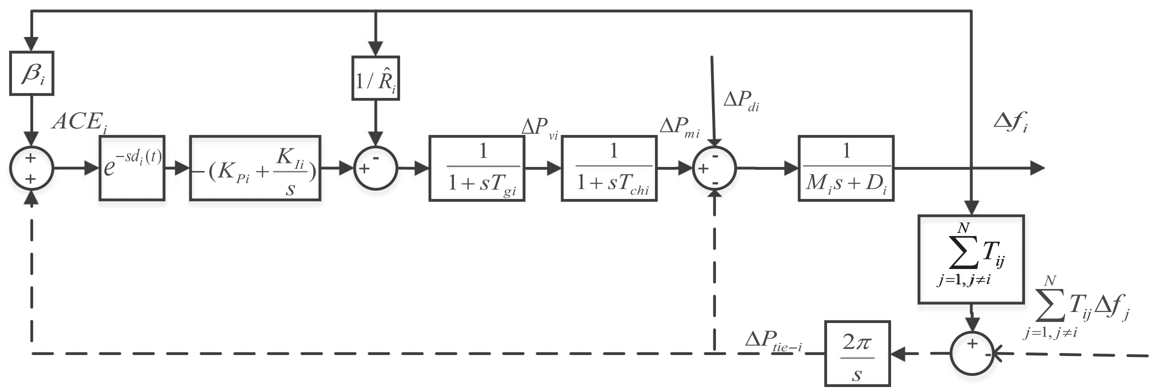

Fig. 1. Multiarea LFC scheme diagram (ith control area); and single-area without dotted lines ($N = 1$).

where

$$A=\begin{bmatrix}-\frac{D}{M} & \frac{1}{M} & 0 & 0\\ 0 & -\frac{1}{T_{ch}} & \frac{1}{T_{ch}} & 0\\ -\frac{1}{\hat{R}T_g} & 0 & -\frac{1}{T_g} & 0\\ \beta & 0 & 0 & 0\end{bmatrix},\ F=\begin{bmatrix}-\frac{1}{M}\\ 0\\ 0\\ 0\end{bmatrix}$$

$$A_d=\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ -\frac{K_P\beta}{T_g} & 0 & 0 & -\frac{K_I}{T_g}\\ 0 & 0 & 0 & 0\end{bmatrix},\ C=\begin{bmatrix}\beta & 0\\ 0 & 0\\ 0 & 0\\ 0 & 1\end{bmatrix}^T.$$

B. Multiarea LFC Model

Fig. 1 shows the classical LFC model of multiarea power systems with time-varying delays (the ith control area (i = 1, 2, ..., N)). It may be noted that we assume all generation units in each control area for equality with a generation unit. The dynamic model of above mentioned LFC system can be described as follows:

$$\begin{cases}\Delta\dot{f}_i=\dfrac{\Delta P_{mi}-\Delta P_{di}-\Delta P_{\text{tie}-i}-D_i\cdot\Delta f_i}{M_i}\\[2mm] \Delta\dot{P}_{mi}=\dfrac{\Delta P_{vi}-\Delta P_{mi}}{T_{chi}}\\[2mm] \Delta\dot{P}_{vi}=\dfrac{u_i(t)-\Delta f_i\cdot\frac{1}{\hat{R}_i}-\Delta P_{vi}}{T_{gi}}\\[2mm] \dot{\int_0^t}\text{ACE}(s)ds=\beta_i\Delta f_i+\Delta P_{\text{tie}-i}\\[2mm] \Delta\dot{P}_{\text{tie}-i}=2\pi\cdot\left(\sum_{j=1,j\neq i}^{N}T_{ij}\Delta f_i-\sum_{j=1,j\neq i}^{N}T_{ij}\Delta f_j\right)\end{cases}$$

where T_{ij} is the tie-line synchronizing coefficient between the ith and jth control area, and $\Delta P_{\text{tie}-i}$ is the net exchange of tie-line power of the ith control area.

The state-space model of the N area LFC systems with time delays is given below:

$$\begin{cases}\dot{x}(t)=Ax(t)+Bu(t)+F\Delta P_d(t)\\ y(t)=Cx(t)\end{cases}\tag{5}$$

where

$$x_i(t)=\begin{bmatrix}\Delta f_i & \Delta P_{mi} & \Delta P_{vi} & \int_0^t \text{ACE}_i(s)ds & \Delta P_{\text{tie}-i}\end{bmatrix}^T$$

$$y_i(t)=\begin{bmatrix}\text{ACE}_i & \int_0^t \text{ACE}_i(s)ds\end{bmatrix}^T$$

$$x(t)=\begin{bmatrix}x_1(t)^T & x_2(t)^T & \cdots & x_N(t)^T\end{bmatrix}^T$$

$$y(t)=\begin{bmatrix}y_1(t)^T & y_2(t)^T & \cdots & y_N(t)^T\end{bmatrix}^T$$

$$u(t)=\begin{bmatrix}u_1(t) & u_2(t) & \cdots & u_N(t)\end{bmatrix}^T$$

$$\Delta P_d(t)=\begin{bmatrix}\Delta P_{d1}(t) & \Delta P_{d2}(t) & \cdots & \Delta P_{dN}(t)\end{bmatrix}^T$$

$$A=\begin{bmatrix}A_{11} & A_{12} & \cdots & A_{1N}\\ A_{21} & A_{22} & \cdots & A_{2N}\\ \vdots & \vdots & \ddots & \vdots\\ A_{N1} & A_{N2} & \cdots & A_{NN}\end{bmatrix}$$

$$B=\text{diag}[B_1,\ldots,B_N],\ C=\text{diag}[C_1,\ldots,C_N],$$

$$F=\text{diag}[F_1,\ldots,F_N]$$

$$B_i=\begin{bmatrix}0 & 0 & \frac{1}{T_{gi}} & 0 & 0\end{bmatrix}^T$$

$$C_i=\begin{bmatrix}\beta_i & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 1 & 0\end{bmatrix}$$

$$F_i=\begin{bmatrix}-\frac{1}{M_i} & 0 & 0 & 0 & 0\end{bmatrix}^T$$

$$A_{ii}=\begin{bmatrix}-\frac{D_i}{M_i} & \frac{1}{M_i} & 0 & 0 & -\frac{1}{M_i}\\ 0 & -\frac{1}{T_{chi}} & \frac{1}{T_{chi}} & 0 & 0\\ -\frac{1}{\hat{R}_iT_{gi}} & 0 & -\frac{1}{T_{gi}} & 0 & 0\\ \beta_i & 0 & 0 & 0 & 1\\ 2\pi\sum\limits_{j=1,j\neq i}^{N}T_{ij} & 0 & 0 & 0 & 0\end{bmatrix}$$

$$A_{ij}=\begin{bmatrix}0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ -2\pi T_{ij} & 0 & 0 & 0 & 0\end{bmatrix}.$$

$$T_{ij}=T_{ji}$$

The ACE signal for each control area is the sum of Δf_i multiplied by β_i and the tie-line power exchange $\Delta P_{\text{tie}-i}$:

$$\text{ACE}_i = \beta_i \Delta f_i + \Delta P_{\text{tie}-i}. \tag{6}$$

Each control area error ACE_i sees a PI controller in Fig. 1:

$$\begin{aligned} u_i(t) &= -K_{Pi}\text{ACE}_i(t - d_i(t)) - K_{Ii}\int_0^t \text{ACE}_i(s - d_i(s))ds \\ &= -K_i C_i x_i(t - d_i(t)) \end{aligned} \tag{7}$$

and we can get the closed-loop system dynamic model as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \sum_{i=1}^{N} A_{di}x(t - \bar{d}(t)) + F\Delta P_d(t) \\ &= Ax(t) + \sum_{i=1}^{N} A_{di}x(t - d_i(t)) + F\Delta P_d(t) \end{aligned} \tag{8}$$

where

$$\begin{aligned} x(t-\bar{d}(t)) &\triangleq \text{col}[x_1(t-d_1(t)), x_2(t-d_2(t)), \ldots, x_N(t-d_N(t))] \\ A_{di} &= \text{diag}\begin{bmatrix} 0 & \cdots & -B_i K_i C_i & \cdots & 0 \end{bmatrix} \\ K_i &= \begin{bmatrix} K_{Pi} & K_{Ii} \end{bmatrix}. \end{aligned}$$

Note that the net tie-line power exchange among multiple control areas satisfies the following zero-sum property

$$\sum_{i=1}^{N} \Delta P_{\text{tie}-i} = 0. \tag{9}$$

There are multiple time delays $d_i(t)$, $(i = 1 \ldots N)$ in the multiarea LFC scheme. In order to reduce the computational load of the delay margin for the multiarea LFC scheme, we are working on the assumption that multiple delays are all equal and shown as a single time delay $d(t)$. Then an uncomplicated model is obtained in the same form as (4) below:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t - d(t)) + F\Delta P_d(t) \\ x(t) &= \phi(t), t \in [-h, 0] \end{aligned} \tag{10}$$

where

$$A_d = \sum_{i=1}^{N} A_{di}.$$

And $x(t) \in R^n$ is the system state with $\phi(t)$ denoting its initial condition which is a vector-valued continuous function of $t \in [-h, 0]$. The time-varying delay $d(t)$ fulfills the following condition:

$$0 \le d(t) \le h,\ \dot{d}(t) \le \mu \tag{11}$$

where $h > 0$, and $\mu < 1$ makes systems with time-varying delay well-posed, because the fast time-varying delay case will cause problems with causality, minimality, and inconsistency to arise as indicated by Verriest in [24].

Remark 1: Strictly speaking, the real power system is a system with high nonlinearity and time-varying characteristics, and it is almost impossible to model accurately. Compared with the fast dynamic process, such as grid voltage and power angle, the frequency response of LFC is a slow dynamic process. In the analysis and design of the LFC for the load disturbance, a simplified low-order linear system is often used to characterize the dynamic characteristics of the system in the vicinity of the operating point [2]. Thus, in the modeling of each part of the system, only the simplified model can be used to represent the frequency-dependent main features of each part. As can be seen from Fig. 1, the basic process of LFC implementation for real power systems is: the change of the load of the power system leads to the deviation of the system frequency, the feedback mechanism generates a signal for adjusting the prime mover according to the frequency deviation, then the output power increment of the prime mover is used to compensate the change of the load, so that the system frequency returns to the specified value.

Remark 2: The state-space equation linear model (10) is commonly employed in different sorts of analysis and synthesis on power systems, such as region-wide small-signal stability analysis of delayed power systems, stability analysis of automatic generation control with commensurate delays, power system stabilizer design taking time delays into account, wide-area damping controller design in the presence of time delay, stability and delay margin computation of LFC systems, and so forth.

For the time-varying delay case, the unknown exogenous power system load disturbance is modeled as perturbation in current and lagged state vector similar to [23]

$$F\Delta P_d(t) = f(x(t), x(t - d(t))) \tag{12}$$

satisfying the following norm-bounded condition:

$$\| f(x(t), x(t - d(t))) \| \le \rho \| x(t) \| + \iota \| x(t - d(t)) \| \tag{13}$$

where $\rho \ge 0$ and $\iota \ge 0$ are known scalars. The following version is used in this paper:

$$\begin{aligned} f(x(t), x(t - d(t)))^T f(x(t), x(t - d(t))) &\le \rho^2 x^T(t) G^T G x(t) \\ &+ \iota^2 x^T(t - d(t)) H^T H x(t - d(t)) \end{aligned} \tag{14}$$

where G and H are known constant matrices of appropriate dimensions. The matrices G and H along with the nonnegative scalars ρ and ι quantify the magnitude of the load disturbance to the electrical power system.

Remark 3: We assume that the unknown load disturbances are modeled as norm-bounded functions of the current and delayed state variables. This assumption is reasonable because the load disturbance will directly influence the state variables of the LFC system. The only limitation is that the norm-bounded condition is imposed on the load disturbance. This limitation can still be considered effective for a restricted bound of the load disturbance in which the PI controller is supposed to exert control and bring the interference power system back to its initial equilibrium condition.

III. Main Results

Before presenting the main theorem, which gives a sufficient stability condition for systems (4) and (10), the following lemmas are introduced, which have an important role in deriving the main results. The connections among these lemmas and the main theorem are depicted in Fig. 2.

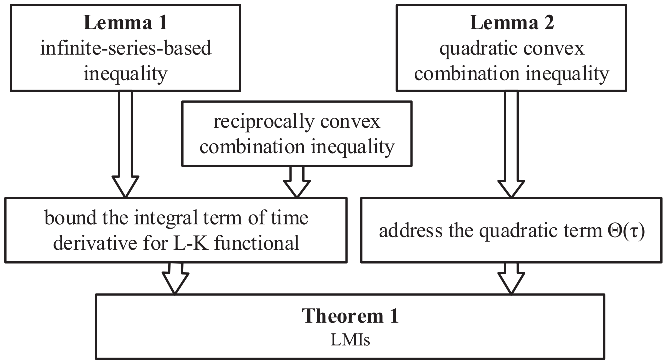

Fig. 2. Connection diagram between lemmas and the main theorem.

Lemma 1: Let $a < b \in \mathbb{R}$, $R = R^T > 0$. Assume that $\forall k = 0, 1, 2, \dots, n-1$,

$$\int_a^b \psi_{2n}(s)\psi_{2k}(s)ds = \int_a^b \psi_{2n+1}(s)\psi_{2k+1}(s)ds = 0$$

for $n = 0, 1, 2, \dots,$ with the following definitions

$$\begin{cases} \psi_{2n}(p) = \left(p - \frac{a+b}{2}\right)^{2n} + \sum_{k=0}^{n-1} a_{nk}\left(p - \frac{a+b}{2}\right)^{2k} \\ \psi_{2n+1}(p) = \left(p - \frac{a+b}{2}\right)^{2n+1} + \sum_{k=0}^{n-1} b_{nk}\left(p - \frac{a+b}{2}\right)^{2k+1} \end{cases}.$$

Then, we have the following integral inequality in the form of infinite series

$$V_{ab}(\omega) = \int_a^b \omega(s)^T R\omega(s)ds \geq \sum_{k=0}^{\infty} \frac{1}{p_k}\Lambda_k^T(\omega)R\Lambda_k(\omega) \quad (15)$$

where $p_k = \int_a^b \psi_k^2(s)ds > 0$, and $\Lambda_k(\omega) = \int_a^b \psi_k(s)\omega(s)ds$.

As a special case,

$$\begin{aligned} -V_{ab}(\dot{x}) \leq & -\frac{1}{b-a}\{\Upsilon_0^T(a,b)R\Upsilon_0(a,b) \\ & + 3\Upsilon_1^T(a,b)R\Upsilon_1(a,b) + 5\Upsilon_2^T(a,b)R\Upsilon_2(a,b)\} \end{aligned} \quad (16)$$

where

$$\Upsilon_0(a,b) = x(b) - x(a),$$

$$\Upsilon_1(a,b) = x(b) + x(a) - \frac{2}{b-a}\int_a^b x(s)ds,$$

$$\Upsilon_2(a,b) = x(b) - x(a) - \frac{12}{(b-a)^2}\int_a^b (s - \frac{a+b}{2})x(s)ds.$$

This useful lemma is adapted from the combination of two facts presented in [25]. It will be employed to bound the integral term of L–K functional time derivative more accurately than ever for stability proof of LFC systems. It is worth noting that the auxiliary-function-based integral inequality (14) of [18] and the second-order B–L inequality in [26] are equivalent to the inequality (16), thus they are included as special cases of infinite-series-based integral inequality (15).

Lemma 2 ([25]): For $a_2, a_1, a_0 \in \mathbb{R}$, $g(x) = a_2x^2 + a_1x + a_0 < 0 \forall x \in [0, h]$, holds true if the following set of inequalities hold

$$(i)g(0) < 0, (ii)g(h) < 0, (iii) - h^2a_2 + g(0) < 0. \quad (17)$$

Up to now, we are in a proper position to set forth the key proposition, in which the necessary knowledge on the LFC system parameters A, A_d included in A_c will be reflected by the stability conditions presented in Theorem 1.

Theorem 1: LFC system (10) with time-varying delay satisfying (11) is globally asymptotically stable if there exist matrices $0 < P \in \mathbb{R}^{3n\times 3n}$, $0 < Q_1 \in \mathbb{R}^{3n\times 3n}$ and $0 < Q_2 \in \mathbb{R}^{n\times n}$, $0 < R \in \mathbb{R}^{n\times n}$ being positive definite matrices. Let $S \in \mathbb{R}^{3n\times 3n}$, $L \in \mathbb{R}^{9n\times n}$, $\sigma \geq 0$, such that the LMIs below hold simultaneously:

$$\text{(i) } \Theta(0) < 0 \quad (18)$$

$$\text{(ii) } -h^2\varpi + \Theta(0) < 0 \quad (19)$$

$$\text{(iii) } \Theta(h) < 0 \quad (20)$$

where

$$\Pi(R,S) = \begin{bmatrix} \mathrm{diag}\{R, 3R, 5R\} & S \\ S^T & \mathrm{diag}\{R, 3R, 5R\} \end{bmatrix} \geq 0. \quad (21)$$

$\Theta(\tau)$ is a quadratic convex function w.r.t. scalar $\tau \in [0, h]$ given by

$$\begin{aligned} \Theta(\tau) = \; & \mathrm{Sym}([e_1^T, \Re(\tau)^T, e_8^T]P \\ & [A_c^T, (e_1 - e_3)^T, (he_1 - \Re(\tau))^T]^T) \\ & + [e_1^T, e_0^T, e_0^T]Q_1[e_1^T, e_0^T, e_0^T]^T \\ & - (1-\mu)[e_2^T, (e_1 - e_2)^T, \tau e_4^T]Q_1[e_2^T, (e_1 - e_2)^T, \tau e_4^T]^T \\ & + \mathrm{Sym}\Bigg(\tau[e_0^T, A_c^T, e_1^T]Q_1\Bigg[e_4^T, (e_1 - e_4)^T, \\ & \tau\left(e_6 + \frac{1}{2}e_4\right)^T\Bigg]^T\Bigg) \\ & + e_1^TQ_2e_1 - e_3^TQ_2e_3 + hA_c^TRA_c \\ & - \frac{1}{h}\begin{bmatrix}\Re_1 \\ \Re_2\end{bmatrix}^T \Pi(R,S)\begin{bmatrix}\Re_1 \\ \Re_2\end{bmatrix} \\ & + \mathrm{Sym}\Bigg(L\Bigg((h-\tau)^2e_7 + \frac{1}{2}(h-\tau)^2e_5 \\ & + \tau^2e_6 + \tau\left(h - \frac{\tau}{2}\right)e_4 - e_8\Bigg)\Bigg) \\ & - e_9^T(\sigma I)e_9 + e_1^T(\sigma\rho^2G^TG)e_1 + e_2^T(\sigma\iota^2H^TH)e_2 \end{aligned}$$

$$\begin{aligned} \varpi = \; & \frac{1}{2}\frac{d^2\Theta(\tau)}{d\tau^2} \\ = \; & \mathrm{Sym}([e_0^T, 1/2(e_4 - e_5)^T, e_0^T]P[e_0^T, e_0^T, (-e_4 + e_5)^T]^T) \\ & - (1-\mu)[e_0^T, e_0^T, e_4^T]Q_1[e_0^T, e_0^T, e_4^T]^T \\ & + \mathrm{Sym}\Bigg([e_0^T, A_c^T, e_1^T]Q_1\Bigg[e_0^T, e_0^T, \left(e_6 + \frac{1}{2}e_4\right)^T\Bigg]^T\Bigg) \\ & + \mathrm{Sym}\left(L\left(e_6 + e_7 + \frac{1}{2}(e_5 - e_4)\right)\right) \end{aligned}$$

$$\begin{cases} e_i = [0_{n\times(i-1)n}, I_n, 0_{n\times(9-i)n}], i = 1, 2, \ldots, 9;\ e_0 = 0_{n\times 9n} \\ A_c = Ae_1 + A_d e_2 + e_9, \\ \Re(\tau) = \tau e_4 + (h-\tau)e_5, \\ \Re_1 = \mathrm{col}\{e_2 - e_3, e_2 + e_3 - 2e_5, e_2 - e_3 - 12e_7\}, \\ \Re_2 = \mathrm{col}\{e_1 - e_2, e_1 + e_2 - 2e_4, e_1 - e_2 - 12e_6\}. \end{cases} \tag{22}$$

Proof: It is given in Appendix.

Remark 4: The inequality (16) in Lemma 1 is fully taken advantage of bounding the integral term $v_a(x_t) = -\int_{t-h}^{t} \dot{x}^T(s) R\dot{x}(s)ds$ of the L–K functional time derivative. It is easy to see the partial-order relations

$$\begin{aligned} v_a(x_t) \leq & -\frac{1}{h}\{\Upsilon_0^T(t-h,t)R\Upsilon_0(t-h,t)\} \\ & -\frac{1}{h}\{3\Upsilon_1^T(t-h,t)R\Upsilon_1(t-h,t) \\ & +5\Upsilon_2^T(t-h,t)R\Upsilon_2(t-h,t)\} \\ \leq & -\frac{1}{h}\{\Upsilon_0^T(t-h,t)R\Upsilon_0(t-h,t) \\ & +3\Upsilon_1^T(t-h,t)R\Upsilon_1(t-h,t)\} \\ \leq & -\frac{1}{h}\Upsilon_0^T(t-h,t)R\Upsilon_0(t-h,t). \end{aligned} \tag{23}$$

By simple calculation, one can obtain the estimations of the Wirtinger inequality and Jensen inequality to bound $v_a(x_t)$, respectively, the same with the right-hand sides of the second and third inequalities of (23). Compared to the criterion used in [10], an improved less conservative stability criterion was derived by using Jensen inequality in [11]. So we can expect the much less conservative results by Theorem 1 than the results in [10] and [11].

Remark 5: In this paper, splitting the single integral aforementioned into two integrals, taken over the two intervals $[t-h(t), t]$ and $[t-h, t-h(t)]$, and applying the reciprocal convex inequality, yields further less conservative estimation results for time derivative of L–K functional, and contributes to the resulting enhanced stability criteria of time-varying delayed LFC power systems.

Remark 6: In the calculation process for the derivative of the L–K energy functional with respect to time, we make use of the quadratic convex combination like conclusion implied by Lemma 2 for addressing the general quadratic terms with respect to time-varying delay value, contributing to the resultant less conservative stability criteria for delayed LFC systems in Theorem 1.

Remark 7: Notice $\int_{-h}^{0}\int_{t+\eta}^{t} \dot{x}^T(s)R\dot{x}(s)dsd\eta = \int_{t-h}^{t}(h-t+s)\dot{x}^T(s)R\dot{x}(s)ds$ by exchanging the integration order, therefore this delay-dependent functional leads to delay-dependent stability criteria for LFC power systems.

Remark 8: When the LFC system is subjected to a small disturbance or there is no disturbed load, the delay-dependent stability condition can be deduced from Theorem 1 by letting $\rho = \iota = 0$.

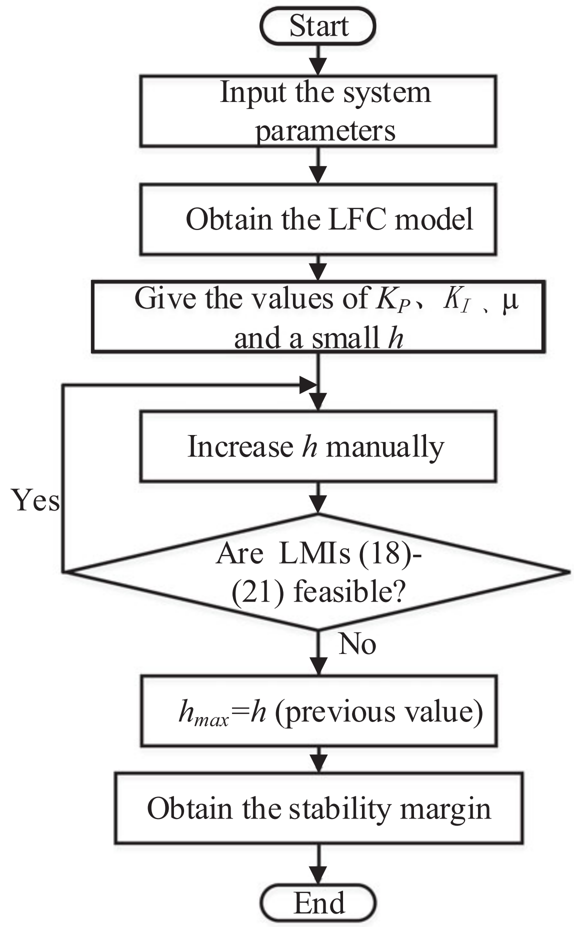

Fig. 3. Flow chart of delay margin computation steps for LFC system.

Remark 9: In order to resolve the stability conditions and get the feasible LMI solution stated in Theorem 1, they are recast into the following constraint optimization problem

$$\max_{P, Q_1, Q_2, R, S, F} h \tag{24}$$

s.t.

$$\Theta(0) < 0$$
$$-h^2\varpi + \Theta(0) < 0$$
$$\Theta(h) < 0.$$

The maximum allowable delay bound that the delayed LFC power system can endure with no loss of stability is solved readily by using the convex optimization software of the MATLAB LMI toolbox. The flow chart of delay margin determination steps for LFC systems are given in Fig. 3 .

IV. Case Studies

A. Theoretical Delay Margin Results

Choose $d(t)$ as a constant delay ($\mu = 0$) and a time-varying delay ($\mu = 0.5$), respectively. Table II shows the parameters of the real power system under study gained from [10]. Table III

TABLE II
PARAMETERS OF TWO AREA LFC SYSTEM

Parameter	T_{ch}/s	T_g/s	$\hat{R}$	D	β	M/s
Area 1	0.3	0.1	0.05	1.0	21	10
Area 2	0.4	0.17	0.05	1.5	21.5	12
$T_{12} = 0.1986$						

TABLE III
MAXIMUM DELAY UPPER BOUND h BY THEOREM 1 FOR ONE AREA LFC SYSTEM

		$\rho = 0, \iota = 0$		$\rho = 0, \iota = 0.025$		$\rho = 0.025, \iota = 0.025$	
K_p	K_I	$\mu = 0$	$\mu = 0.5$	$\mu = 0$	$\mu = 0.5$	$\mu = 0$	$\mu = 0.5$
0	0.2	7.33	6.45	5.94	5.48	5.70	5.29
0	0.4	3.38	2.94	2.98	2.66	2.92	2.61
0	0.6	2.04	1.75	1.84	1.59	1.81	1.57
0	1.0	0.92	0.75	0.82	0.66	0.79	0.64
0.1	0.1	16.07	13.75	11.40	9.83	10.31	9.18
0.1	0.2	7.79	6.80	6.34	5.76	6.16	5.59
0.1	0.4	3.61	3.13	3.18	2.84	3.13	2.80
0.1	0.6	2.19	1.88	2.00	1.71	1.96	1.68
0.1	1.0	1.01	0.83	0.90	0.72	0.87	0.70

TABLE IV
MAXIMUM DELAY UPPER BOUND h FOR NOMINAL ONE AREA LFC SYSTEM

		$\mu = 0$			$\mu = 0.9$		
K_p	K_I	Theorem 1	[10]	[23]	Theorem 1	[10]	[23]
0	0.05	**30.91**	27.92	27.92	**27.26**	20.45	26.37
0	0.1	**15.20**	13.77	13.77	**13.39**	9.93	12.96
0	0.2	**7.33**	6.69	6.69	**6.43**	4.59	6.25
0	0.4	**3.38**	3.12	3.12	**2.91**	1.81	2.85
0	0.6	**2.04**	1.91	1.91	**1.71**	1.01	1.68
0	1.0	**0.92**	0.88	0.88	**0.75**	0.48	0.74
0.1	0.05	**31.61**	27.03	27.05	**22.00**	17.39	20.25
0.1	0.10	**16.02**	13.68	13.69	**12.32**	9.16	11.07
0.1	0.2	**7.79**	6.94	6.94	**6.59**	4.67	5.93
0.1	0.4	**3.61**	3.29	3.29	**3.11**	1.85	2.87
0.1	0.6	**2.19**	2.02	2.02	**1.84**	1.05	1.75
0.1	1.0	**1.01**	0.96	0.96	**0.75**	0.48	0.74

TABLE V
MAXIMUM DELAY UPPER BOUND h FOR NOMINAL TWO AREA LFC SYSTEM

		$\mu = 0$			$\mu = 0.5$		
K_p	K_I	Theorem 1	[10]	[23]	Theorem 1	[10]	[23]
0	0.2	**7.23**	6.60	6.60	**6.41**	5.55	6.14
0	0.4	**3.24**	3.00	3.00	**2.81**	2.36	2.68
0	0.6	**1.86**	1.74	1.74	**1.54**	1.18	1.40
0	1.0	**0.58**	0.57	0.57	**0.41**	0.22	0.35
0.1	0.1	**15.97**	13.65	13.65	**13.73**	11.63	12.58
0.1	0.2	**7.67**	6.88	6.88	**6.75**	5.35	6.34
0.1	0.4	**3.47**	3.17	3.17	**2.84**	2.55	2.83
0.1	0.6	**2.03**	1.86	1.86	**1.53**	1.30	1.51

TABLE VI
DELAY MARGIN $h \propto (K_P, K_I)$ (ONE AREA LFC, $\mu = 0$)

h	K_I						
K_P	0.05	0.1	0.15	0.2	0.4	0.6	1.0
0	30.91	15.20	9.95	7.33	3.38	2.04	0.92
0.05	31.71	15.68	10.27	7.57	3.51	2.12	0.97
0.10	31.61	16.02	10.56	7.79	3.61	2.19	1.01
0.20	30.39	16.43	11.00	8.15	3.79	2.31	1.07
0.40	26.38	15.09	10.71	8.22	3.97	2.42	1.11
0.60	20.69	12.30	8.94	7.05	3.66	2.27	0.94
1.0	0.59	0.58	0.57	0.56	0.51	0.46	0.36

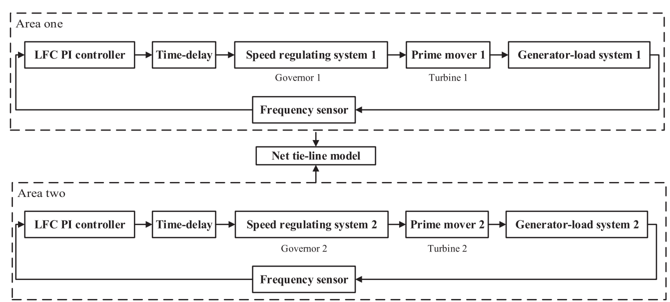

Fig. 4. Scheme diagram of two-area power system model.

lists the maximum delay upper bounds of one area (Area 1) LFC scheme using Theorem 1 for various values of ρ and ι, and under diversified values of LFC gains K_p and K_I. It is obvious that the delay margins calculated by our method are larger than those in [23].

As for nominal system with constant and time-varying delay, from more clear and concise comparisons with [10] and [23], the maximum delay bounds are given in Tables IV and V. As can be seen, the presented results are less conservative than those of [10] and [23]. This is due to the application of infinite-series-based integral inequality and the quadratic convex combination lemma. Results in Table VI reflects that the gain of PI controller is one of the major factors affecting the delay margins.

B. Simulation Verifications

1) Case for Constant Communication Delay: In order to validate the effectiveness of theoretical results for delay margins, we use MATLAB/Simulink for time domain simulation based on two-area power system model scheme in Fig. 4. Suppose that the load disturbance $F\Delta P_d(t)$ adopts the form in (14) where the matrices H is taken as $0.1I_4$ and $\iota = 0.025$. For the following PI-based controller gains: ($K_p = 0.1$, $K_I = 0.4$), the simulation studies are conducted to observe the variation of the deviation variables Δf (the state variable).

Fig. 5 shows the frequency response of one-area LFC system in cases of containing time-delay and no delay. For $K_p = 0.1$, $K_I = 0.4$, the delay margin obtained by our proposed method is $h_{\max} = 3.18$ s in Table III, and it is $h_{\max} = 3.05$ s by [23] as shown in Table IV. It shows that the system is asymptotically stable at $h_{\max} = 3.18$ s in Fig. 5.

2) Case for Time-Varying Delay: The simulation is realized on the nominal two area LFC system with time-varying delay which is specified with bounds on the maximum value of h and the delay-derivative μ to view the evolution of the derivation variables Δf_1 and Δf_2. In the simulation study, the controller parameters is $K_p = 0$, $K_I = 0.2$. The time-varying delay $0 \le d(t) \le 7.23$ s with $\mu = 0.5$, as shown in Fig. 6, is considered.

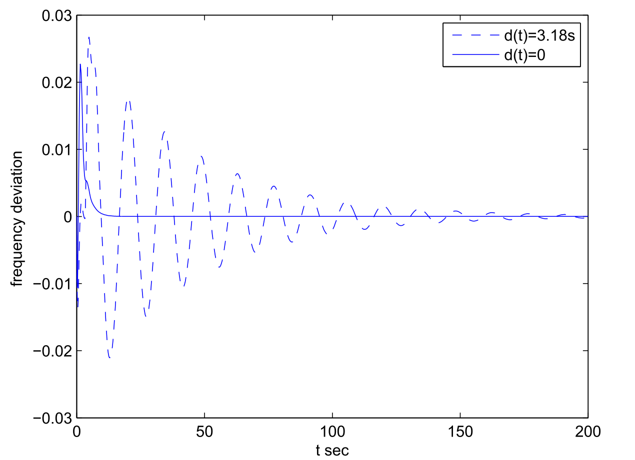

Fig. 5. Frequency responses with $h_{\max} = 3.18\,(\mu = 0)$ and without delay.

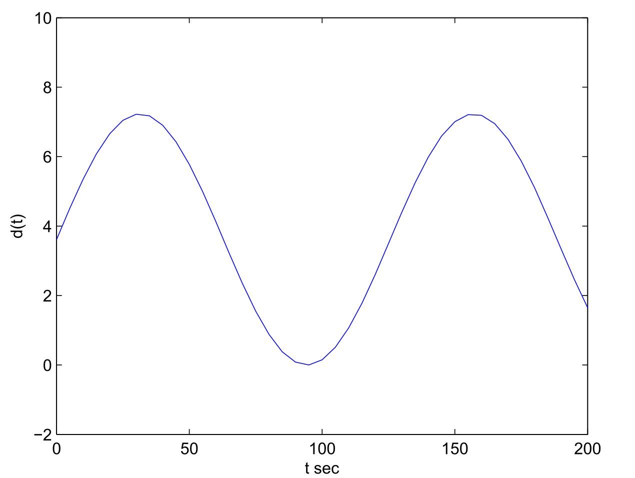

Fig. 6. Time-varying delay $d(t) = 3.615\ \sin(0.05t) + 3.615$.

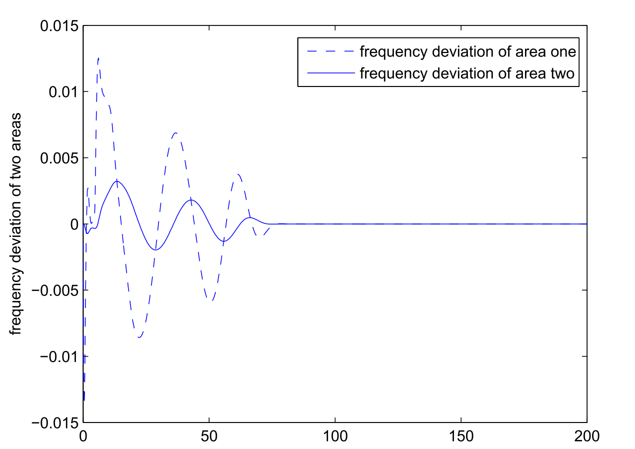

Fig. 7. Δf_1 and Δf_2 with $h_{\max} = 6.41\,(\mu = 0.5)$.

For two-area LFC system having time-varying delay, frequency response is presented in Fig. 7. For $K_p = 0,\ K_I = 0.2$, the delay margin is discovered to be $h_{\max} = 6.41$ s by our proposed method as listed in Table V while $h_{\max} = 6.14$ s through the approach of [23]. It can be seen by simple inspection that the system is asymptotically stable at $h_{\max} = 6.41$ s in Fig. 7.

Remark 10: In previous studies, the typical one-area and two-area LFC schemes, which are commonly used in the existing literature [1], [2], [10], and [23], have been tested to verify the effectiveness of proposed method. A real power system may have more complex construction and more detailed model, for example, the number of areas may be more than two, thus the dimension of state-space model will increase. But our approach is still suitable for studying large systems.

V. Conclusion

In the present paper, the stability problem for LFC systems with time delay has been investigated by using a further improved (infinite series based) single integral inequality possessing less conservatism than Wirtinger inequality, which encompasses the Jensen inequality. It makes contributions to overbounding the time derivative of the constructed augmented L–K functional along with the state equation of the delayed LFC systems, and generates less conservative stability test conditions in the form of LMI. How to obtain the double integral inequality from the constructive infinite series method, and design optimal feedback controllers like in [27] are retained for our later study.

Appendix
Proof of Theorem 1

First, select an augmented L–K functional candidate similar to the one used in [28]:

$$\begin{aligned}
V(x_t) &= \lambda^T(t)P\lambda(t)\\
&\quad + \int_{t-d(t)}^{t} \xi^T(t,s)Q_1\xi(t,s)ds + \int_{t-h}^{t} x^T(s)Q_2x(s)ds\\
&\quad + \int_{t-h}^{t} (s+h-t)\dot{x}^T(s)R\dot{x}(s)ds\\
\lambda(t) &= \mathrm{col}\left[x(t),\ \int_{t-h}^{t} x(s)ds,\ \int_{t-h}^{t} (s+h-t)x(s)ds\right]\\
\xi(t,s) &= \mathrm{col}\left[x(s),\, x(t)-x(s),\ \int_{s}^{t} x(r)dr\right].
\end{aligned} \tag{25}$$

Next, denote an augmented vector

$$\begin{aligned}
\eta_t = \ \mathrm{col}\Big[&x(t),\, x(t-d(t)),\, x(t-h)\\
&\frac{1}{d(t)}\int_{t-d(t)}^{t} x(s)ds\\
&\frac{1}{h-d(t)}\int_{t-h}^{t-d(t)} x(s)ds\\
&\frac{1}{d(t)^2}\int_{t-d(t)}^{t}\left(s+\frac{d(t)}{2}-t\right)x(s)ds\\
&\frac{1}{(h-d(t))^2}\int_{t-h}^{t-d(t)}\left(s+\frac{d(t)+h}{2}-t\right)x(s)ds\\
&\int_{t-h}^{t}(s+h-t)x(s)ds,\ f(x(t),x(t-d(t)))\Big].
\end{aligned}$$

And computing the derivative of $V(x_t)$ to time along the trajectories of system (10) yields,

$$\begin{aligned}\dot V(x_t) &= 2\lambda^T(t)P\dot\lambda(t) + \xi^T(t,t)Q_1\xi(t,t)\\ &\quad -(1-\dot d(t))\xi^T(t,t_d)Q_1\xi(t,t-d(t))\\ &\quad + \int_{t-d(t)}^{t}\frac{\partial}{\partial t}(\xi^T(t,s)Q_1\xi(t,s))ds\\ &\quad + x^T(t)Q_2x(t) - x^T(t-h)Q_2x(t-h)\\ &\quad + h\dot x^T(t)R\dot x(t) + v_a(x_t)\\ &\le \eta_t^T\{2[e_1,(h-d(t)e_5+d(t)e_4),e_8]^TP\\ &\quad [A_c, e_1-e_3, he_1-(h-d(t)e_5+d(t)e_4)]\\ &\quad + [e_1,0,0]^TQ_1[e_1,0,0]\\ &\quad -(1-\mu)[e_2,e_1-e_2,d(t)e_4]^TQ_1\\ &\quad [e_2,e_1-e_2,d(t)e_4] + 2d(t)[0,A_c,e_1]^TQ_1\\ &\quad \left[e_4, e_1-e_4, d(t)e_6+\frac{d(t)}{2}e_4\right] + e_1^TQ_2e_1\\ &\quad - e_3^TQ_2e_3 + hA_c^TRA_c\}\eta_t + v_a(x_t). \qquad (26)\end{aligned}$$

Using Lemma 1 and reciprocally convex combination inequality (see [20]), sequentially gives

$$\begin{aligned}v_a(x_t) &= -\int_{t-d(t)}^{t}\dot x(s)R\dot x(s)ds - \int_{t-h}^{t-d(t)}\dot x(s)R\dot x(s)ds\\ &\le -\frac{1}{d(t)}\{\Upsilon_0^T(t-d(t),t)R\Upsilon_0(t-d(t),t)\\ &\quad + 3\Upsilon_1^T(t-d(t),t)R\Upsilon_1(t-d(t),t)\\ &\quad + 5\Upsilon_2^T(t-d(t),t)R\Upsilon_2(t-d(t),t)\}\\ &\quad - \frac{1}{h-d(t)}\{\Upsilon_0^T(t-h,t-d(t))R\Upsilon_0(t-h,t-d(t))\\ &\quad + 3\Upsilon_1^T(t-h,t-d(t))R\Upsilon_1(t-h,t-d(t))\\ &\quad + 5\Upsilon_2^T(t-h,t-d(t))R\Upsilon_2(t-h,t-d(t))\}\\ &\le -\frac{1}{h}\begin{bmatrix}\Upsilon(t-h,t-d(t))\\ \Upsilon(t-d(t),t)\end{bmatrix}^T\Pi(R,S)\\ &\quad \begin{bmatrix}\Upsilon(t-h,t-d(t))\\ \Upsilon(t-d(t),t)\end{bmatrix}\\ &= -\frac{1}{h}\eta_t^T\begin{bmatrix}\Re_1\\ \Re_2\end{bmatrix}^T\Pi(R,S)\begin{bmatrix}\Re_1\\ \Re_2\end{bmatrix}\eta_t \qquad (27)\end{aligned}$$

where

$$\Upsilon^T(a,b) = \left[\Upsilon_0^T(a,b), \Upsilon_1^T(a,b), \Upsilon_2^T(a,b)\right]$$

with $\Upsilon_i(a,b), i = 0,1,2$ are defined in (16) of Lemma 1.

Also for any $\sigma \ge 0$, from (14), the following holds:

$$\begin{aligned}&-\sigma f(\cdot)^T f(\cdot) + \sigma\rho^2 x^T G^T Gx(t)\\ &\quad + \sigma\iota^2 x^T(t-d(t))H^T Hx(t-d(t)) \ge 0 \qquad (28)\end{aligned}$$

which can be expressed using the augmented state vector η_t:

$$\begin{aligned}&\eta_t^T((e_9^T(-\sigma I)e_9) + e_1^T(\sigma\rho^2G^TG)e_1\\ &\quad + e_2^T(\sigma\iota^2H^TH)e_2)\eta_t \ge 0. \qquad (29)\end{aligned}$$

In addition, from

$$\begin{aligned}e_8\eta_t &= \int_{t-h}^{t}(s+h-t)x(s)ds\\ &= (h-d(t))^2e_7\eta_t + \frac{(h-d(t))^2}{2}e_5\eta_t\\ &\quad + d(t)^2e_6\eta_t + \left(h-\frac{d(t)}{2}\right)d(t)e_4\eta_t.\end{aligned}$$

Thus, we have

$$\begin{aligned}0 &= 2\eta_t^TL\left((h-d(t))^2\left(e_7+\frac{1}{2}e_5\right)+d^2(t)e_6\right.\\ &\quad \left. + d(t)\left(h-\frac{1}{2}d(t)\right)e_4 - e_8\right)\eta_t. \qquad (30)\end{aligned}$$

By substituting (27) into (26) and by adding the terms in (29) and (30), we have

$$\begin{aligned}\dot V(x_t) &\le \eta_t^T\{\Theta(d(t))\}\eta_t \quad \forall d(t)\in[0,h]\\ &= \eta_t^T\{\Theta(\tau)\}\eta_t \quad \forall \tau\in[0,h]. \qquad (31)\end{aligned}$$

Finally, note that $\eta_t^T\Theta(\tau)\eta_t$ is a convex quadratic function with respect to the scalar τ, from (18)–(20) and Lemma 2,

$$\eta_t^T\Theta(\tau)\eta_t < 0 \quad \forall\tau\in[0,h] \quad \forall\eta_t \ne 0$$

which indicates, by inserting the fact above into (31), one can obtain $\dot V(x_t) < 0, \forall\eta_t \ne 0$. The proof is completely finished.

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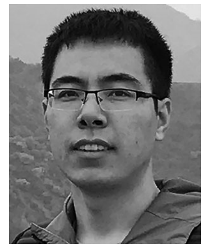

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