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Augmented Lyapunov functional and delay-dependent stability criteria for neutral systems

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SUMMARY

In this paper, an augmented Lyapunov functional is proposed to investigate the asymptotic stability of neutral systems. Two methods with or without decoupling the Lyapunov matrices and system matrices are developed and shown to be equivalent to each other. The resulting delay-dependent stability criteria are less conservative than the existing ones owing to the augmented Lyapunov functional and the introduction of free-weighting matrices. The delay-independent criteria are obtained as an easy corollary. Numerical examples are given to demonstrate the effectiveness and less conservativeness of the proposed methods. Copyright © 2005 John Wiley & Sons, Ltd.

KEY WORDS: neutral systems; delay-dependent stability; linear matrix inequality (LMI); Lyapunov functional

1. INTRODUCTION

Neutral systems are frequently encountered in various engineering systems, including population ecology, heat exchangers and repetitive control [1–3]. The existing stability criteria for neutral systems can be classified into two types: delay-dependent ones which include information on the size of delays [4–17], and delay-independent ones which are applicable to delays of arbitrary size [18–20]. Delay-dependent results are usually less conservative than delay-independent ones, especially when the size of the delay is small.

In the past two decades, the fixed model transformation and parameterized model transformation are usually employed to obtain the delay-dependent conditions for neutral system. Four basic fixed model transformations are addressed in Reference [12]. Among them, the descriptor model transformation method combined with Park's or Moon *et al.*'s inequalities [21, 22] is the most efficient [11, 12, 14]. Recently, in order to reduce the conservatism, a free-weighting matrix method is proposed in References [16, 23–25], in which the bounding techniques on some cross-product terms are not involved. Especially, Wu *et al.* [17] combined

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the parameterized model transformation into the Lyapunov functional and employed the free-weighting matrix method to obtain less conservative delay-dependent stability criteria for neutral systems. We notice that it is hard to further reduce the conservatism by using the same types of Lyapunovs as in the above works.

In this paper, we propose an augmented Lyapunov functional, which takes into account the delay term. Owing to the augmented Lyapunov functional, improved delay-dependent stability criteria for neutral systems are derived by using the free-weighting matrix method. We first develop a delay-dependent condition by introducing some free-weighting matrices with the aid of the Leibniz–Newton formula. Then, we derive an alternative delay-dependent criterion by retaining the derivative of the state and introducing additional free-weighting matrices with the aid of the system equation. The relationship between two methods is clearly established. Finally, numerical examples are given to demonstrate the effectiveness and the merit of the proposed method.

2. PROBLEM FORMULATION

Consider the following neutral system Σ:

$$\begin{cases} \dot{x}(t) - C\dot{x}(t-\tau) = Ax(t) + A_d x(t-\tau), & t > 0 \\ x(t) = \phi(t), & t \in [-\tau, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathscr{R}^n$ is the state vector; $\tau > 0$ is a constant time delay; and A, A_d and C are constant matrices with appropriate dimensions. $\phi(t)$ denotes an initial condition which is continuous vector-valued initial function of $t \in [-\tau, 0]$. Define operator $\mathscr{D}$: $C([-\tau, 0], R^n) \rightarrow R^n$ as

$$\mathscr{D}x_t = x(t) - Cx(t-\tau)$$

Definition 1 (Hale and Verduyn Lund [2])
Operator $\mathscr{D}$ is said to be stable if the zero solution of the homogeneous difference equation $\mathscr{D}x_t = 0, \ t \geqslant 0, \ x_0 = \psi \in \{\phi \in C([-\tau, 0] : \mathscr{D}\phi = 0\}$ is uniformly asymptotically stable.

The stability of operator $\mathscr{D}$ is necessary for the stability of neutral system Σ, which is always satisfied when $\|C\| < 1$.

3. THE RESULTS

By introducing an augmented Lyapunov functional, we obtain a delay-dependent stability criterion for system Σ as follows.

Theorem 1
Given a scalar $\tau \geqslant 0$, the neutral system Σ is asymptotically stable if the operator $\mathscr{D}$ is stable and there exist matrices

$$L = \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{12}^{\mathrm{T}} & L_{22} & L_{23} \\ L_{13}^{\mathrm{T}} & L_{23}^{\mathrm{T}} & L_{33} \end{bmatrix} \geqslant 0 \text{ with } L_{11} > 0$$

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^{\mathrm{T}} & Q_{22} \end{bmatrix} \geqslant 0$$

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12}^{\mathrm{T}} & Z_{22} \end{bmatrix} \geqslant 0 \text{ and } M_i, \quad i = 1, 2, 3$$

such that the following LMI holds:

$$\Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} & -\tau M_1 & A^{\mathrm{T}} S \\ \Gamma_{12}^{\mathrm{T}} & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} & -\tau M_2 & A_d^{\mathrm{T}} S \\ \Gamma_{13}^{\mathrm{T}} & \Gamma_{23}^{\mathrm{T}} & \Gamma_{33} & \Gamma_{34} & -\tau M_3 & C^{\mathrm{T}} S \\ \Gamma_{14}^{\mathrm{T}} & \Gamma_{24}^{\mathrm{T}} & \Gamma_{34}^{\mathrm{T}} & -\tau Z_{11} & -\tau Z_{12} & 0 \\ -\tau M_1^{\mathrm{T}} & -\tau M_2^{\mathrm{T}} & -\tau M_3^{\mathrm{T}} & -\tau Z_{12}^{\mathrm{T}} & -\tau Z_{22} & 0 \\ SA & SA_d & SC & 0 & 0 & -S \end{bmatrix} < 0 \tag{2}$$

where

$$\Gamma_{11} = GA + A^{\mathrm{T}} G^{\mathrm{T}} + L_{13} + L_{13}^{\mathrm{T}} + Q_{11} + \tau Z_{11} + M_1 + M_1^{\mathrm{T}}$$

$$\Gamma_{12} = GB + A^{\mathrm{T}} L_{12} + L_{23}^{\mathrm{T}} - L_{13} + M_2^{\mathrm{T}} - M_1$$

$$\Gamma_{13} = GC + L_{12} + M_3^{\mathrm{T}}$$

$$\Gamma_{14} = \tau(L_{33} + A^{\mathrm{T}} L_{13})$$

$$\Gamma_{22} = A_d^{\mathrm{T}} L_{12} + L_{12}^{\mathrm{T}} A_d - L_{23} - L_{23}^{\mathrm{T}} - Q_{11} - M_2 - M_2^{\mathrm{T}}$$

$$\Gamma_{23} = L_{12}^{\mathrm{T}} C + L_{22} - Q_{12} - M_3^{\mathrm{T}}$$

$$\Gamma_{24} = \tau(-L_{33} + A_d^{\mathrm{T}} L_{13})$$

$$\Gamma_{33} = -Q_{22}$$

$$\Gamma_{34} = \tau(L_{23} + C^{\mathrm{T}} L_{13})$$

$$S = Q_{22} + \tau Z_{22}$$

$$G = L_{11} + Q_{12} + \tau Z_{12}$$

Proof

Choose a Lyapunov functional candidate to be

$$V(x_t) := \zeta_1^{\mathrm{T}}(t)L\zeta_1(t) + \int_{t-\tau}^{t} \zeta_2^{\mathrm{T}}(s)Q\zeta_2(s)\,\mathrm{d}s + \int_{-\tau}^{0}\int_{t+\theta}^{t} \zeta_2^{\mathrm{T}}(s)Z\zeta_2(s)\,\mathrm{d}s\,\mathrm{d}\theta \tag{3}$$

where

$$L = \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{12}^{\mathrm{T}} & L_{22} & L_{23} \\ L_{13}^{\mathrm{T}} & L_{23}^{\mathrm{T}} & L_{33} \end{bmatrix} \geqslant 0 \quad \text{with } L_{11} > 0$$

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^{\mathrm{T}} & Q_{22} \end{bmatrix} \geqslant 0 \quad \text{and} \quad Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12}^{\mathrm{T}} & Z_{22} \end{bmatrix} \geqslant 0$$

are to be determined and

$$\zeta_1(t) = \begin{bmatrix} x(t) \\ x(t-\tau) \\ \int_{t-\tau}^{t} x(s)\,\mathrm{d}s \end{bmatrix}, \quad \zeta_2(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}$$

From the Leibniz–Newton formula, the following equation is true for any matrices M_i with appropriate dimensions, $i = 1, 2, 3$:

$$2[x^{\mathrm{T}}(t)M_1 + x^{\mathrm{T}}(t-\tau)M_2 + \dot{x}^{\mathrm{T}}(t-\tau)M_3]\left[x(t) - \int_{t-\tau}^{t} \dot{x}(s)\,\mathrm{d}s - x(t-\tau)\right] = 0 \tag{4}$$

Calculating the derivative of $V(x_t)$ along the solution of Σ yields

$$\begin{aligned}
\dot{V}(x_t) &= 2\zeta_1^{\mathrm{T}}(t)L\dot{\zeta}_1(t) + \zeta_2^{\mathrm{T}}(t)Q\zeta_2(t) - \zeta_2^{\mathrm{T}}(t-\tau)Q\zeta_2(t-\tau) \\
&\quad + \tau\zeta_2^{\mathrm{T}}(t)Z\zeta_2(t) - \int_{t-\tau}^{t} \zeta_2^{\mathrm{T}}(s)Z\zeta_2(s)\,\mathrm{d}s \\
&= 2\zeta_1^{\mathrm{T}}(t)L\begin{bmatrix} \dot{x}(t) \\ \dot{x}(t-\tau) \\ x(t) - x(t-\tau) \end{bmatrix} + \zeta_2^{\mathrm{T}}(t)Q\zeta_2(t) - \zeta_2^{\mathrm{T}}(t-\tau)Q\zeta_2(t-\tau) \\
&\quad + \tau\zeta_2^{\mathrm{T}}(t)Z\zeta_2(t) - \int_{t-\tau}^{t} \zeta_2^{\mathrm{T}}(s)Z\zeta_2(s)\,\mathrm{d}s \\
&\quad + 2[x^{\mathrm{T}}(t)M_1 + x^{\mathrm{T}}(t-\tau)M_2 + \dot{x}^{\mathrm{T}}(t-\tau)M_3] \\
&\quad \times \left[x(t) - \int_{t-\tau}^{t} \dot{x}(s)\,\mathrm{d}s - x(t-\tau)\right] \\
&:= \frac{1}{\tau}\int_{t-\tau}^{t} \eta_1^{\mathrm{T}}(t,s)\hat{\Gamma}\eta_1(t,s)\,\mathrm{d}s
\end{aligned} \tag{5}$$

where

$$\eta_1(t,s) = [x^{\mathrm{T}}(t)\ x^{\mathrm{T}}(t-\tau)\ \dot{x}^{\mathrm{T}}(t-\tau)\ x^{\mathrm{T}}(s)\ \dot{x}^{\mathrm{T}}(s)]^{\mathrm{T}}$$

$$\hat{\Gamma} = \begin{bmatrix} \Gamma_{11} + A^{\mathrm{T}}SA & \Gamma_{12} + A^{\mathrm{T}}SA_d & \Gamma_{13} + A^{\mathrm{T}}SC & \Gamma_{14} & -\tau M_1 \\ \Gamma_{12}^{\mathrm{T}} + A_d^{\mathrm{T}}SA & \Gamma_{22} + A_d^{\mathrm{T}}SA_d & \Gamma_{23} + A_d^{\mathrm{T}}SC & \Gamma_{24} & -\tau M_2 \\ \Gamma_{13}^{\mathrm{T}} + C^{\mathrm{T}}SA & \Gamma_{23}^{\mathrm{T}} + C^{\mathrm{T}}SA_d & \Gamma_{33} + C^{\mathrm{T}}SC & \Gamma_{34} & -\tau M_3 \\ \Gamma_{14}^{\mathrm{T}} & \Gamma_{24}^{\mathrm{T}} & \Gamma_{34}^{\mathrm{T}} & -\tau Z_{11} & -\tau Z_{12} \\ -\tau M_1^{\mathrm{T}} & -\tau M_2^{\mathrm{T}} & -\tau M_3^{\mathrm{T}} & -\tau Z_{12}^{\mathrm{T}} & -\tau Z_{22} \end{bmatrix}$$

$$S = Q_{22} + \tau Z_{22}$$

and $\Gamma_{ij},\ i = 1,2,3,\ i \leqslant j \leqslant 3$ are defined as in Theorem 1. By Schur complement, the inequality $\Gamma < 0$ is equivalent to $\hat{\Gamma} < 0$, which gives $\dot{V}(x_t) < -\varepsilon\|x(t)\|^2$ for a sufficiently small $\varepsilon > 0$. This proves the asymptotic stability of system Σ. □

It is seen that Theorem 1 is established by adopting the Leibniz–Newton formula which provides free-weighting matrices $M_i,\ i = 1,2,3$ as in (4). In the following, we derive an alternative delay-dependent criterion by taking into account the system equation Σ, which provides another set of free-weighting matrices.

Theorem 2

Given a scalar $\tau \geqslant 0$, the neutral system Σ is asymptotically stable if operator $\mathscr{D}$ is stable and there exist matrices

$$L = \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{12}^{\mathrm{T}} & L_{22} & L_{23} \\ L_{13}^{\mathrm{T}} & L_{23}^{\mathrm{T}} & L_{33} \end{bmatrix} \geqslant 0 \quad \text{with } L_{11} > 0$$

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^{\mathrm{T}} & Q_{22} \end{bmatrix} \geqslant 0$$

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12}^{\mathrm{T}} & Z_{22} \end{bmatrix} \geqslant 0, \quad U,\ M_i \text{ and } T_j, \quad i = 1,2,3,\ j = 1,\ldots,6$$

such that the following LMI holds:

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & \Phi_{16} \\ \Phi_{12}^{\mathrm{T}} & \Phi_{22} & \Phi_{23} & \Phi_{24} & \Phi_{25} & \Phi_{26} \\ \Phi_{13}^{\mathrm{T}} & \Phi_{23}^{\mathrm{T}} & \Phi_{33} & \Phi_{34} & \Phi_{35} & \Phi_{36} \\ \Phi_{14}^{\mathrm{T}} & \Phi_{24}^{\mathrm{T}} & \Phi_{34}^{\mathrm{T}} & \Phi_{44} & \Phi_{45} & \Phi_{46} \\ \Phi_{15}^{\mathrm{T}} & \Phi_{25}^{\mathrm{T}} & \Phi_{35}^{\mathrm{T}} & \Phi_{45}^{\mathrm{T}} & -\tau Z_{11} & -\tau Z_{12} \\ \Phi_{16}^{\mathrm{T}} & \Phi_{26}^{\mathrm{T}} & \Phi_{36}^{\mathrm{T}} & \Phi_{46}^{\mathrm{T}} & -\tau Z_{12}^{\mathrm{T}} & -\tau Z_{22} \end{bmatrix} < 0 \tag{6}$$

where

$$\Phi_{11} = L_{13} + L_{13}^{\mathrm{T}} + Q_{11} + \tau Z_{11} + M_1 + M_1^{\mathrm{T}} - T_1 A - A^{\mathrm{T}} T_1^{\mathrm{T}}$$

$$\Phi_{12} = L_{11} + Q_{12} + \tau Z_{12} + U^{\mathrm{T}} + T_1 - A^{\mathrm{T}} T_2^{\mathrm{T}}$$

$$\Phi_{13} = L_{23}^{\mathrm{T}} - L_{13} + M_2^{\mathrm{T}} - M_1 - T_1 A_d - A^{\mathrm{T}} T_3^{\mathrm{T}}$$

$$\Phi_{14} = L_{12} + M_3^{\mathrm{T}} - T_1 C - A^{\mathrm{T}} T_4^{\mathrm{T}}$$

$$\Phi_{15} = \tau L_{33} - A^{\mathrm{T}} T_5^{\mathrm{T}}$$

$$\Phi_{16} = -\tau M_1 - A^{\mathrm{T}} T_6^{\mathrm{T}}$$

$$\Phi_{22} = Q_{22} + \tau Z_{22} + T_2 + T_2^{\mathrm{T}}$$

$$\Phi_{23} = L_{12} - U + T_3^{\mathrm{T}} - T_2 A_d$$

$$\Phi_{24} = -T_2 C + T_4^{\mathrm{T}}$$

$$\Phi_{25} = \tau L_{13} + T_5^{\mathrm{T}}$$

$$\Phi_{26} = -\tau U + T_6^{\mathrm{T}}$$

$$\Phi_{33} = -L_{23} - L_{23}^{\mathrm{T}} - Q_{11} - M_2 - M_2^{\mathrm{T}} - T_3 A_d - A_d^{\mathrm{T}} T_3^{\mathrm{T}}$$

$$\Phi_{34} = L_{22} - Q_{12} - M_3^{\mathrm{T}} - T_3 C - A_d^{\mathrm{T}} T_4^{\mathrm{T}}$$

$$\Phi_{35} = -\tau L_{33} - A_d^{\mathrm{T}} T_5^{\mathrm{T}}$$

$$\Phi_{36} = -\tau M_2 - A_d^{\mathrm{T}} T_6^{\mathrm{T}}$$

$$\Phi_{44} = -Q_{22} - T_4 C - C^{\mathrm{T}} T_4^{\mathrm{T}}$$

$$\Phi_{45} = \tau L_{23} - C^{\mathrm{T}} T_5^{\mathrm{T}}$$

$$\Phi_{46} = -\tau M_3 - C^{\mathrm{T}} T_6^{\mathrm{T}}$$

Proof

Choose the same augmented Lyapunov functional as in (3). It is clear that the following equation holds by system equation Σ:

$$2\int_{t-\tau}^{t} \eta_2^{\mathrm{T}}(t,s) T[\dot{x}(t) - C\dot{x}(t-\tau) - Ax(t) - A_d x(t-\tau)]\,\mathrm{d}s = 0 \tag{7}$$

where

$$T = [T_1^{\mathrm{T}}\ T_2^{\mathrm{T}}\ T_3^{\mathrm{T}}\ T_4^{\mathrm{T}}\ T_5^{\mathrm{T}}\ T_6^{\mathrm{T}}]^{\mathrm{T}}$$

$$\eta_2(t,s) = [x^{\mathrm{T}}(t)\ \dot{x}^{\mathrm{T}}(t)\ x^{\mathrm{T}}(t-\tau)\ \dot{x}^{\mathrm{T}}(t-\tau)\ x^{\mathrm{T}}(s)\ \dot{x}^{\mathrm{T}}(s)]^{\mathrm{T}}$$

On the other hand, $\dot{x}(t)$ in $\dot{V}(x_t)$ is retained (contrasting with the proof of Theorem 1, in which $\dot{x}(t)$ is replaced with the system equation Σ) and (4) is slightly modified to

$$\begin{aligned} &2[x^{\mathrm{T}}(t)M_1 + \dot{x}^{\mathrm{T}}(t)U + x^{\mathrm{T}}(t-\tau)M_2 + \dot{x}^{\mathrm{T}}(t-\tau)M_3] \\ &\times \left[x(t) - \int_{t-\tau}^{t} \dot{x}(s)\,\mathrm{d}s - x(t-\tau)\right] = 0 \end{aligned} \tag{8}$$

Similar to the proof of Theorem 1 with (7) and (8) being added into the expression of $\dot{V}(x_t)$, the result follows immediately. □

Remark 1

Theorems 1 and 2 are based on a newly proposed augmented Lyapunov functional of form (3), which contains a structure more general than the traditional ones as those in References [8, 16, 17]. For instance, the first term of (3) involves the delay term $x(t-\tau)$, and thus is more general than the simply case of operator $\mathscr{D}$. This new type of Lyapunov functional enables us to establish less conservative results. For theoretical comparison, let us take Reference [17] for example. It can be verified that the result in Reference [17] is recovered by setting $L_{11} = P_{11}$, $L_{12} = -P_{11}C$, $L_{13} = P_{12}$, $L_{22} = C^{\mathrm{T}}P_{11}C$, $L_{23} = -C^{\mathrm{T}}P_{12}$, $L_{33} = P_{22}$, $Q_{12} = Z_{12} = 0$, $Z_{11} = W$, $Z_{22} = Z$, $Q_{11} = Q$, $Q_{22} = R$, $M_1 = N_1$, $U = N_2$, $M_2 = N_3$, $M_3 = N_4$, $T_5 = 0$ and $T_6 = 0$.

In fact, Theorems 1 and 2 are equivalent to each other, which is shown as follows. Setting

$$J = \begin{bmatrix} I & A^{\mathrm{T}} & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 \\ 0 & A_d^{\mathrm{T}} & I & 0 & 0 & 0 \\ 0 & C^{\mathrm{T}} & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{bmatrix} \tag{9}$$

then it follows that

$$\Pi = J\Phi J^{\mathrm{T}} = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} & \Gamma_{14} & \Pi_{16} \\ \Pi_{12}^{\mathrm{T}} & \Phi_{22} & \Pi_{23} & \Pi_{24} & \Phi_{25} & \Phi_{26} \\ \Pi_{13}^{\mathrm{T}} & \Pi_{23}^{\mathrm{T}} & \Pi_{33} & \Pi_{34} & \Gamma_{24} & \Pi_{36} \\ \Pi_{14}^{\mathrm{T}} & \Pi_{24}^{\mathrm{T}} & \Pi_{34}^{\mathrm{T}} & \Pi_{44} & \Gamma_{34} & \Pi_{46} \\ \Gamma_{14}^{\mathrm{T}} & \Phi_{25}^{\mathrm{T}} & \Gamma_{24}^{\mathrm{T}} & \Gamma_{34}^{\mathrm{T}} & -\tau Z_{11} & -\tau Z_{12} \\ \Pi_{16}^{\mathrm{T}} & \Phi_{26}^{\mathrm{T}} & \Pi_{36}^{\mathrm{T}} & \Pi_{46}^{\mathrm{T}} & -\tau Z_{12}^{\mathrm{T}} & -\tau Z_{22} \end{bmatrix} \tag{10}$$

where

$$\Pi_{11} = \Gamma_{11} + U^{\mathrm{T}}A + A^{\mathrm{T}}U + A^{\mathrm{T}}SA$$

$$\Pi_{12} = G + T_1 + U^{\mathrm{T}} + A^{\mathrm{T}}S + A^{\mathrm{T}}T_2$$

$$\Pi_{13} = \Gamma_{12} - A^{\mathrm{T}}U + U^{\mathrm{T}}A_d + A^{\mathrm{T}}SA_d$$

$$\Pi_{14} = \Gamma_{13} + U^{\mathrm{T}}C + A^{\mathrm{T}}SC$$

$$\Pi_{16} = -\tau(M_1 + A^{\mathrm{T}}U)$$

$$\Pi_{23} = L_{12} - U + T_2^{\mathrm{T}}A_d + SA_d + T_3^{\mathrm{T}}$$

$$\Pi_{24} = SC + T_2^{\mathrm{T}}C + T_4^{\mathrm{T}}$$

$$\Pi_{33} = \Gamma_{22} - U^{\mathrm{T}}A_d - A_d^{\mathrm{T}}U + A_d^{\mathrm{T}}SA_d$$

$$\Pi_{34} = \Gamma_{23} - U^{\mathrm{T}}C + A_d^{\mathrm{T}}SC$$

$$\Pi_{36} = -\tau(M_2 + A_d^{\mathrm{T}}U)$$

$$\Pi_{44} = \Gamma_{33} + C^{\mathrm{T}}SC$$

$$\Pi_{46} = -\tau(M_3 + C^{\mathrm{T}}U)$$

and S, G and $\Gamma_{ij},\ i = 1,2,3,\ i \leqslant j \leqslant 3$ are defined as in Theorem 1. From (10), it remains to show that $\Pi < 0$ is equivalent to $\hat{\Gamma} < 0$. To this end, if $\Pi < 0$ holds, then $\hat{\Gamma} < 0$ holds since Π reduces to $\hat{\Gamma}$ by removing the second row and the second column in Π and replacing variables M_1, M_2 and M_3 in $\hat{\Gamma}$ by $M_1 + A^{\mathrm{T}}U$, $M_2 + A_d^{\mathrm{T}}U$ and $M_3 + C^{\mathrm{T}}U$, respectively. On the other hand, if $\hat{\Gamma} < 0$, $\Pi < 0$ is feasible by setting $T_1 = -G = -(L_{11} + Q_{12} + \tau Z_{12})$, $T_2 = -S = -(Q_{22} + \tau Z_{22})$, $T_3 = -L_{12}$, $T_4 = 0$, $T_5 = -\tau L_{13}$, $T_6 = 0$ and $U = 0$. Therefore, this shows the equivalence of Theorems 1 and 2.

Remark 2

Theorem 2 is suitable for dealing with uncertain systems with polytopic-type uncertainties by using parameter-dependent Lyapunov functionals as in Reference [23]. This is because the LMI condition of Theorem 2 does not involve the product of system matrices and Lyapunov matrices. As for norm-bounded uncertain case, Theorems 1 and 2 are applicable for the robust stability analysis following a similar line to References [17, 24].

Before concluding this section, we would like to point out that our method can be used to establish delay-independent criteria. To see this, setting $Z = \varepsilon I$, where $\varepsilon > 0$ is a sufficiently small positive scalar, $L_{13} = L_{23} = L_{33} = 0$, $T_5 = T_6 = 0$ and $U = M_1 = M_2 = M_3 = 0$ in conditions of Theorem 2, the following delay-independent result is straightforward.

Corollary 1

For any scalar $\tau \geqslant 0$, the neutral system Σ is asymptotically stable if operator $\mathscr{D}$ is stable and there exist matrices

$$L = \begin{bmatrix} L_{11} & L_{12} \\ L_{12}^{\mathrm{T}} & L_{22} \end{bmatrix} \geqslant 0 \quad \text{with } L_{11} > 0, \quad Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^{\mathrm{T}} & Q_{22} \end{bmatrix} \geqslant 0 \text{ and } T_j, \quad j = 1, \dots, 4$$

such that the following LMI holds:

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} \\ \Psi_{12}^{\mathrm{T}} & \Psi_{22} & \Psi_{23} & \Psi_{24} \\ \Psi_{13}^{\mathrm{T}} & \Psi_{23}^{\mathrm{T}} & \Psi_{33} & \Psi_{34} \\ \Psi_{14}^{\mathrm{T}} & \Psi_{24}^{\mathrm{T}} & \Psi_{34}^{\mathrm{T}} & \Psi_{44} \end{bmatrix} < 0 \tag{11}$$

where

$$\Psi_{11} = Q_{11} - T_1 A - A^{\mathrm{T}} T_1^{\mathrm{T}}$$

$$\Psi_{12} = L_{11} + Q_{12} + T_1 - A^{\mathrm{T}} T_2^{\mathrm{T}}$$

$$\Psi_{13} = -T_1 A_d - A^{\mathrm{T}} T_3^{\mathrm{T}}$$

$$\Psi_{14} = L_{12} - T_1 C - A^{\mathrm{T}} T_4^{\mathrm{T}}$$

$$\Psi_{22} = Q_{22} + T_2 + T_2^{\mathrm{T}}$$

$$\Psi_{23} = L_{12} - T_2 A_d$$

$$\Psi_{24} = -T_2 C + T_4^{\mathrm{T}}$$

$$\Psi_{33} = -Q_{11} - T_3 A_d - A_d^{\mathrm{T}} T_3^{\mathrm{T}}$$

$$\Psi_{34} = L_{22} - Q_{12} - T_3 C - A_d^{\mathrm{T}} T_4^{\mathrm{T}}$$

$$\Psi_{44} = -Q_{22} - T_4 C - C^{\mathrm{T}} T_4^{\mathrm{T}}$$

Table I. Maximum upper bound of τ.

c	0	0.1	0.3	0.5	0.7	0.9
Fridman *et al.* [12]	4.47	3.49	2.06	1.14	0.54	0.13
Han [8]	4.35	4.33	4.10	3.62	2.73	0.99
Wu *et al.* [17]	4.47	4.35	4.13	3.67	2.87	1.41
Theorems 1 and 2	4.47	4.42	4.17	3.69	2.87	1.41

With a similar treatment, an alternative delay-independent result, which is equivalent to Corollary 1, can be obtained as a direct corollary of Theorem 1. For brevity, we omit the details.

4. NUMERICAL EXAMPLES

In this section, we use two numerical examples to show the less conservativeness of our results.

Example 1
Consider the neutral system Σ with

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad A_d = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}, \quad 0 \leqslant c < 1$$

Table I lists the maximum upper bound of τ for this system in case of different c's. It is clear that the method in this paper produces better results than those in References [8, 12, 17].

Example 2
Consider the neutral system Σ with

$$A = \begin{bmatrix} -1.7073 & 0.6856 \\ 0.2279 & -0.6368 \end{bmatrix}, \quad A_d = \begin{bmatrix} -2.5026 & -1.0540 \\ -0.1856 & -1.5715 \end{bmatrix}, \quad C = \begin{bmatrix} 0.0558 & 0.0360 \\ 0.2747 & -0.1084 \end{bmatrix}$$

The upper bound of delay obtained in References [16, 17, 25] are 0.5735, 0.6054 and 0.5937, respectively. By Theorems 1 and 2, the computed value is 0.6189, which is larger than those in References [16, 25]. In addition, setting $C = 0$, this system reduces to a linear nominal system with time-invariant delay. The upper bound of delay obtained in References [11, 12, 14, 23, 24] are all 0.6903, and 0.7163 in Reference [17], while the computed value is 0.7918 by Theorems 1 and 2, which is larger than those in References [11, 12, 14, 17, 23, 24].

5. CONCLUSIONS

In this paper, an augmented Lyapunov functional is constructed to study the delay-dependent stability problems for neutral systems. The resulting criteria are shown less conservative than some existing ones. Numerical examples given have convinced the less conservativeness.

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