Brief paper

Sampled-data-based stabilization of switched linear neutral systems☆

Jun Fu [a], Tai-Fang Li [b], Tianyou Chai [a], Chun-Yi Su [c]

[a] State Key Laboratory of Synthetical Automation for Process Industries, Northeastern University, 110189, China
[b] College of Engineering, Bohai University, Jinzhou, 121013, China
[c] Department of Mechanical Engineering, Concordia University, Montreal, Quebec, Canada, H3H 1M8

ABSTRACT

With the development of digital control technology, sampled-data control shows its prominent superiority for most practical industries. In the framework of sampled-data control, this paper studies the stabilization problem for a class of switched linear neutral systems meanwhile taking into account asynchronous switching. By utilizing the relationship between the sampling period and the dwell time of switched neutral systems, a bond between the sampling period and the *average* dwell time is revealed to form a switching condition, under which and certain control gains conditions exponential stability of the closed-loop systems is guaranteed. A simple example is given to demonstrate the effectiveness of the proposed method.

© 2016 Elsevier Ltd. All rights reserved.

1. Introduction

A switched system consists of a family of continuous-time or discrete-time subsystems and a switching law to determine which subsystem is active within certain time interval. Lots of works are devoted to switched systems in the past two decades, see, for example Branicky (1998), Chen and Zheng (2010), Fu, Ma, and Chai (2015), Lian, Ge, and Han (2013), Liberzon (2003), Lin and Antsaklis (2009), Sun, Du, Shi, Wang, and Wang (2014), Sun and Ge (2005), Sun, Liu, David, and Wang (2008), Sun, Zhao, and Hill (2006), Wu and Dong (2006), Xiang, Sun, and Chen (2012), Zhai, Hu, Yasuda, and Michel (2001), Zhang and Gao (2010), Zhang and Yu (2009), Zhang, Zhuang, and Shi (2015), Zhang, Zhuang, Shi, and Zhu (2015), Zhao and Hill (2008) and Zhao, Shi, and Zhang (2012). Among all problems studied for switched systems, asynchronous switching stemming from the delay between the active subsystem and its matched controller is one important issue (Lian et al., 2013; Wang, Zhao, & Jiang, 2013; Xiang et al., 2012; Zhang & Gao, 2010;

Zhao et al., 2012). A majority of existing literature on asynchronous switching focus on either for continuous-time switched systems with continuous controllers or for discrete-time switched systems with discrete-time controllers. The authors of Zhang and Gao (2010) study asynchronously switching control of switched systems with average dwell time technique in both continuous-time and discrete-time cases, where the asynchronous delay is not necessarily less than the dwell time. Refs. Lian et al. (2013) and Zhao et al. (2012) introduce asynchronously switching control methods for deterministic switched linear systems and stochastic ones, respectively. Both methods also deploy the average dwell time technique. Since most practical systems are continuous-time, for which one usually either directly designs continuous-time controllers or first discretizes the continuous systems and then develops the corresponding discrete-time controllers, for example, those methods in Lian et al. (2013), Wang et al. (2013) and Zhang and Gao (2010). However, from a practical implementation point of view, sampled-data controllers are more favorable in practical applications due to the rapid progress of computer and digital technologies and non-approximate treatment of the intersample behavior from sampled-data control's own features, which results in no degradation of the closed-loop performance (Chen & Francis, 1991, 1995; Hara, Yamamoto, & Fujioka, 1996; Hu, Lam, Cao, & Shao, 2003). Given the advantages of sampled-data control, this paper considers a particular class of switched systems called switched neutral system (Krishnasamy & Balasubramaniam, 2015a; Liu, Liu, & Zhong, 2008; Li, Zhao, & Qi, 2014; Wang et al., 2013; Xiang, Sun, & Chen, 2011; Xiang, Sun, & Mahmoud, 2012;

Xiong, Zhong, Ye, & Wu, 2009; Zhang, Liu, Zhu, & Zhong, 2007; Zhang & Yu, 2012) which is first defined in Sun, Fu, Sun, and Zhao (2005), and study its stabilization *in the framework of sampled-data control* meanwhile taking into account asynchronous switching.

To this considered problem, the most relevant works are Feng and Song (2011), Krishnasamy and Balasubramaniam (2015b), Liberzon (2014), Lien, Chen, Yu, and Chung (2012) and Wang, Xing, Zhou, Wang, and Yang (2014). The authors of Wang et al. (2014) propose a finite-time stabilizer for a class of switched linear systems under asynchronous switching. The authors of Liberzon (2014) present an important result on sampled-data quantized state feedback stabilization of switched linear systems by using an encoding and control strategy. In Feng and Song (2011), the authors investigate the stabilization problem of switched linear systems for both the known switching process and the unknown switching setting, for which cases the dwell time technique and the online adaptive estimation method are combined with the sampled-data control, respectively. In Feng and Song (2011), Liberzon (2014) and Wang et al. (2014), only the delay-free switched linear systems are considered. In Lien et al. (2012), the authors study the robust delay-dependent H_∞ control for a class of switched linear delay systems. However, the authors convert the sampled-data control problem into a time-delay one and used time-varying delay approach instead of sampled-data feedback control input. Furthermore, impacts from sampling or from asynchronous switching are not considered on stabilization of switched systems. Although all these references give sampled-data control strategies for switched systems with their own spectacular features, they are obviously not able to cope with stabilization of switched *neutral* systems under either asynchronous or synchronous switching because time delays appear not only in states but also in state derivative of the considered system (see Eq. (1)). In Krishnasamy and Balasubramaniam (2015b), the authors study the sampled-data control for switched neutral systems under synchronous switching which possesses great extent conservativeness. Thus one may wonder whether or not it is possible to propose a new control stabilization method for switched neutral systems, even for switched linear ones, in the framework of sampled-data control under asynchronous switching? This paper provides an affirmative answer.

From the motivation above, this paper focuses on a class of switched linear neutral systems and introduces a sampled-data-based controller for *asynchronously* switching stabilization. By utilizing the relationship between the sampling period and the dwell time of switched systems, a bond between the sampling period and the *average* dwell time is revealed to form a switching condition. Subject to this switching condition and certain control gains related constraints, exponential stability of the closed-loop switched neutral system can be guaranteed. The main features of this paper are as follows. (1) We, for the first time, propose a sample-data-based stabilization method for switched linear neutral systems under asynchronous switching. (2) Delays in the switched neutral systems are time-varying and also appear in the state derivative. (3) A bond between the sampling period and the *average* dwell time is revealed to form a switching condition. (4) Introducing free-weighting matrices obtains the decoupled constraints (30) and (31), and therefore technically reduces the computational complexity compared to the result of Wang et al. (2013).

This paper is organized as follows. Section 2 describes the problem statement and gives some useful definitions and lemmas. Section 3 gives the controller design of stabilizing the switched neutral system under sampled-data input. An example is presented in Section 4, and followed by the conclusion in Section 5.

Notations: $C_r = C([-r, 0], R^n)$ denotes the Banach space of continuous vector functions mapping the interval $[-r, 0]$ into R^n with the topology of uniform convergence. $\|\varphi\|_C = \sup_{-r \leq t \leq 0} \|\varphi(t)\|$ denotes the norm of a function $\varphi \in C_r$. R^n is the n-dimensional Euclidean space. $Q > 0 (Q < 0)$ means that the matrix Q is positive definite (negative definite). Q^T and Q^{-1} present the transpose and the inverse of the matrix Q, respectively. $\|\cdot\|$ is the Euclidean norm. $*$ symbolizes the elements below the main diagonal of a symmetric matrix. $\underline{\lambda}(Q)$ and $\bar{\lambda}(Q)$ are the smallest and the largest eigenvalue of a matrix Q, respectively. $\mathcal{N}$ is the set of nonnegative integers. $\text{diag}\{\cdots\}$ denotes a block-diagonal matrix.

2. Problem statement

Consider the switched neutral system

$$\begin{cases} \dot{x}(t) = A_\sigma x(t) + B_\sigma x(t - \tau(t)) + C_\sigma \dot{x}(t - h(t)) + D_\sigma u(t) \\ x(t_0 + \theta) = \varphi(\theta), \quad \theta \in [-r, 0] \end{cases} \quad (1)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control vector, $\sigma : [0, \infty) \to \mathcal{M} = \{1, 2, \ldots, m\}$ is a right-continuous, piecewise constant function called the switching signal, $\{(A_i, B_i, C_i, D_i) : i \in \mathcal{M}\}$ is a collection of matrix pairs defining the individual subsystem of the system (1), and all the eigenvalues of matrix C_i are inside the unit circle, $\tau(t)$ and $h(t)$ denote the discrete time-varying delay and the neutral time-varying delay, respectively, which satisfy

$$0 < \tau(t) \leq \tau, \qquad \dot{\tau}(t) \leq \hat{\tau} < 1,$$
$$0 < h(t) \leq h, \qquad \dot{h}(t) \leq \hat{h} < 1, \quad (2)$$

where $\tau, \hat{\tau}, h$ and $\hat{h}$ are constants. $\varphi(\theta)$ is a continuously differential vector initial function on $[-r, 0]$, $r = \max\{\tau, h\}$. We denote the number of discontinuities of the switching signal σ on the interval $(s, t]$ by $N_\sigma(t, s)$. For all $i \in \mathcal{M}$, we assume that the subsystem i is stabilizable and there is no jump at all switching instants.

For the purpose of this paper, the definitions of average dwell time and exponential stability of the switched linear neutral system are introduced below.

Definition 1 (*Liberzon, 2003*)**.** If there exists a number $\tau_d > 0$ such that any two switches are separated by at least τ_d, then τ_d is called the dwell time. In addition, if there exist two positive numbers $\tau_a > \tau_d$ and $N_0 \geq 1$ such that

$$N_\sigma(t, s) \leq N_0 + \frac{t - s}{\tau_a} \quad \forall t \geq s \geq 0, \quad (3)$$

then τ_a is called the average dwell time.

Definition 2 (*Wang et al., 2013*)**.** The system (1) is said to be exponentially stable under a switching law σ, if the solution $x(t)$ of the system (1) satisfies

$$\|x(t)\| \leq \kappa e^{-\lambda(t-t_0)} \|x_{t_0}\|_c, \quad t \geq t_0 \quad (4)$$

for constants $\kappa \geq 1$ and $\lambda > 0$, where $\|x_{t_0}\|_c = \sup_{-r \leq \theta \leq 0} \{\|x(t_0 + \theta)\|, \|\dot{x}(t_0 + \theta)\|\}$.

State measurements are taken at times $t_k := k\tau_s, k \in \mathcal{N}$, where τ_s is a fixed sampling period. When the sampler gets the state information, it also passes the switching signal into the controller. However, the sampling time and the switching time do not happen synchronously. The delayed time will last to the next sampling time. During this period, the active subsystem would be unstable. Thus the control objective is to stabilize the system (1) by designing a sampled-data controller meanwhile taking into account asynchronous switchings. In order to simplify the analysis process, we assume the sampling period τ_s is no larger than the dwell time τ_d.

Suppose that the subsystem i is active on $[t_k, t_{k+1})$. The control law produced by the sampler with zero-holder on $[t_k, t_{k+1})$ can be represented as

$$u(t) = u(t_k) = u(t - (t - t_k))$$
$$= u(t - \eta(t)) = K_i x(t - \eta(t)), \tag{5}$$

where $\eta(t) = t - t_k$ is piecewise with derivative $\dot{\eta}(t) = 1$ for $t \neq t_k$. Moreover, $\eta(t) < t_{k+1} - t_k = \tau_s$.

Before ending this section, we present an important lemma for our main results.

Lemma 1 (Gu, 2000). *For any constant matrix $M > 0$, scalars r_1, r_2 satisfying $r_1 < r_2$, and a vector function $\omega : [r_1, r_2] \to R^n$ such that the integrations concerned are well defined, then*

$$-(r_2 - r_1) \int_{r_1}^{r_2} \omega^T(s) M \omega(s) ds$$
$$\leq -\left(\int_{r_1}^{r_2} \omega(s) ds \right)^T M \left(\int_{r_1}^{r_2} \omega(s) ds \right). \tag{6}$$

3. Main results

We aim to stabilize the switched neutral system (1) under the assumption $\tau_s \leq \tau_d$. This assumption guarantees that at most one switch occurs within each sampling interval. If no switching occurs within a sampling interval, then the active subsystem is synchronous with its matched controller. If one switching happens within a sampling interval, then the active subsystem is synchronous with its corresponding controller *before* switching, but becomes asynchronous *after* that until the next sampling time begins.

Thus, we now analyze stability of the system (1) in two cases: sampling interval with no switch and sampling interval with one switch.

3.1. Sampling interval with no switch

Consider the case when $\sigma(t_k) = \sigma(t_{k+1}) = i \in \mathcal{M}$. Subsystem i is active on the whole interval $[t_k, t_{k+1})$. The closed-loop dynamic is given by

$$\dot{x}(t) = A_i x(t) + B_i x(t - \tau(t)) + C_i \dot{x}(t - h(t))$$
$$+ D_i K_i x(t - \eta(t)). \tag{7}$$

Construct the Lyapunov–Krasovskii functional candidate for the system (7)

$$V_i(t) = x^T(t) P_i x(t) + \int_{t-\tau_s}^{t} x^T(s) Q_i e^{\lambda_s(s-t)} x(s) ds$$
$$+ \int_{t-\tau(t)}^{t} x^T(s) R_i e^{\lambda_s(s-t)} x(s) ds$$
$$+ \int_{t-h(t)}^{t} \dot{x}^T(s) M_i e^{\lambda_s(s-t)} \dot{x}(s) ds$$
$$+ \int_{t-\alpha\eta(t)}^{t} x^T(s) N_i e^{\lambda_s(s-t)} x(s) ds$$
$$+ \int_{-\tau_s}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) S_i e^{\lambda_s(s-t)} \dot{x}(s) ds d\theta, \tag{8}$$

where $P_i, Q_i, R_i, M_i, N_i, S_i$ are positive definite symmetric matrices, and α is a constant satisfying $0 < \alpha < 1$.

Remark 1. From the construction of the Lyapunov–Krasovskii functional candidate (8), we expect to obtain a sufficient condition

of stabilizing the system (1), which depends on the bounds of the discrete time-varying delay, the neutral time-varying delay, the derivative of the discrete time-varying delay, the derivative of the neutral time-varying delay and the sampling period.

When the subsystem i is active, for $\forall \lambda_s > 0$, taking the time derivative of the Lyapunov–Krasovskii functional (8) along solutions of the system (7) gives

$$\dot{V}_i(t) + \lambda_s V_i(t) \leq 2x^T(t) P_i \dot{x}(t) + \lambda_s x^T(t) P_i x(t) + x^T(t) Q_i x(t)$$
$$- e^{-\lambda_s \tau_s} x^T(t - \tau_s) Q_i x(t - \tau_s) + x^T(t) R_i x(t)$$
$$- (1 - \hat{\tau}) e^{-\lambda_s \tau} x^T(t - \tau(t)) R_i x(t - \tau(t))$$
$$+ \dot{x}^T(t) M_i \dot{x}(t) + x^T(t) N_i x(t) + \tau_s \dot{x}^T(t) S_i \dot{x}(t)$$
$$- (1 - \hat{h}) e^{-\lambda_s h} \dot{x}^T(t - h(t)) M_i \dot{x}(t - h(t))$$
$$- (1 - \alpha) e^{-\alpha\lambda_s \tau_s} x^T(t - \alpha\eta(t)) N_i x(t - \alpha\eta(t))$$
$$- e^{-\lambda_s \tau_s} \int_{t-\alpha\eta(t)}^{t} \dot{x}^T(s) S_i \dot{x}(s) ds$$
$$- e^{-\lambda_s \tau_s} \int_{t-\eta(t)}^{t-\alpha\eta(t)} \dot{x}^T(s) S_i \dot{x}(s) ds$$
$$- e^{-\lambda_s \tau_s} \int_{t-\tau_s}^{t-\eta(t)} \dot{x}^T(s) S_i \dot{x}(s) ds. \tag{9}$$

From Lemma 1, we have

$$-e^{-\lambda_s \tau_s} \int_{t-\alpha\eta(t)}^{t} \dot{x}^T(s) S_i \dot{x}(s) ds$$
$$\leq \frac{-e^{-\lambda_s \tau_s}}{\alpha \tau_s} \left(\int_{t-\alpha\eta(t)}^{t} \dot{x}(s) ds \right)^T S_i \left(\int_{t-\alpha\eta(t)}^{t} \dot{x}(s) ds \right)$$
$$= \frac{-e^{-\lambda_s \tau_s}}{\alpha \tau_s} (x^T(t) - x^T(t - \alpha\eta(t))) S_i (x(t) - x(t - \alpha\eta(t))),$$

$$-e^{-\lambda_s \tau_s} \int_{t-\eta(t)}^{t-\alpha\eta(t)} \dot{x}^T(s) S_i \dot{x}(s) ds$$
$$\leq \frac{-e^{-\lambda_s \tau_s}}{(1-\alpha) \tau_s} \left(\int_{t-\eta(t)}^{t-\alpha\eta(t)} \dot{x}(s) ds \right)^T S_i \left(\int_{t-\eta(t)}^{t-\alpha\eta(t)} \dot{x}(s) ds \right)$$
$$= \frac{-e^{-\lambda_s \tau_s}}{(1-\alpha) \tau_s} (x^T(t - \alpha\eta(t)) - x^T(t - \eta(t)))$$
$$\times S_i (x(t - \alpha\eta(t)) - x(t - \eta(t))),$$

$$-e^{-\lambda_s \tau_s} \int_{t-\tau_s}^{t-\eta(t)} \dot{x}^T(s) S_i \dot{x}(s) ds < 0.$$

Moreover, from the system equation (7), for any matrix W_i with appropriate dimensions, we have

$$-2(\dot{x}^T(t) \ x^T(t)) W_i^T (\dot{x}(t) - A_i x(t) - B_i x(t - \tau(t))$$
$$- C_i \dot{x}(t - h(t)) - D_i K_i x(t - \eta(t))) = 0. \tag{10}$$

Utilizing the above three inequalities and adding (10) into (9), we obtain

$$\dot{V}_i(t) + \lambda_s V_i(t) \leq \xi^T(t) \Omega_i \xi(t), \tag{11}$$

where

$$\xi^T(t) = (x^T(t) \ x^T(t - \tau(t)) \ \dot{x}^T(t - h(t))$$
$$\times x^T(t - \eta(t)) \ x^T(t - \tau_s) \ x^T(t - \alpha\eta(t)) \ \dot{x}^T(t)),$$

$$\Omega_i = \begin{bmatrix} \Omega_{i,11} & W_i^T B_i & W_i^T C_i & \Omega_{i,14} & 0 & \Omega_{i,16} & \Omega_{i,17} \\ * & \Omega_{i,22} & 0 & 0 & 0 & 0 & B_i^T W_i \\ * & * & \Omega_{i,33} & 0 & 0 & 0 & C_i^T W_i \\ * & * & * & \Omega_{i,44} & 0 & \Omega_{i,46} & \Omega_{i,47} \\ * & * & * & * & \Omega_{i,55} & 0 & 0 \\ * & * & * & * & * & \Omega_{i,66} & 0 \\ * & * & * & * & * & * & \Omega_{i,77} \end{bmatrix},$$

$\Omega_{i,11} = R_i + N_i + \lambda_s P_i + Q_i - \dfrac{e^{-\lambda_s \tau_s}}{\alpha \tau_s} S_i + W_i^T A_i + A_i^T W_i,$

$\Omega_{i,14} = W_i^T D_i K_i, \qquad \Omega_{i,16} = \dfrac{e^{-\lambda_s \tau_s}}{\alpha \tau_s} S_i,$

$\Omega_{i,22} = -(1 - \hat{\tau}) e^{-\lambda_s \tau} R_i, \qquad \Omega_{i,33} = -(1 - \hat{h}) e^{-\lambda_s h} M_i,$

$\Omega_{i,44} = -\dfrac{e^{-\lambda_s \tau_s}}{(1 - \alpha) \tau_s} S_i, \qquad \Omega_{i,46} = \dfrac{e^{-\lambda_s \tau_s}}{(1 - \alpha) \tau_s} S_i,$

$\Omega_{i,47} = D_i^T K_i^T W_i, \qquad \Omega_{i,55} = -Q_i e^{-\lambda_s \tau_s},$

$\Omega_{i,66} = -\dfrac{e^{-\lambda_s \tau_s}}{(1 - \alpha) \tau_s} S_i - \dfrac{e^{-\lambda_s \tau_s}}{\alpha \tau_s} S_i - (1 - \alpha) e^{-\alpha \lambda_s \tau_s} N_i,$

$\Omega_{i,17} = P_i + A_i^T W_i - W_i^T, \qquad \Omega_{i,77} = M_i + \tau_s Q_i - W_i - W_i^T.$

From (11), we know that $\Omega_i < 0$ implies

$$\dot{V}_i(t) + \lambda_s V_i(t) < 0. \tag{12}$$

Integrating the inequality (12) gives

$$V_i(t) < e^{-\lambda_s(t - t_k)} V_i(t_k) \leq \nu V_i(t_k), \tag{13}$$

where $\nu := \max \left\{ e^{-\lambda_s(t - t_k)}, 0 \leq t - t_k < \tau_s \right\}.$

3.2. Sampling interval with one switch

Consider the case that one switch has occurred on the interval $[t_k, t_{k+1})$, that is $\sigma(t_k) = i$ and $\sigma(t_{k+1}) = j \neq i$. The controller knows that one switch occurs from subsystem i to j on the interval $[t_k, t_{k+1})$ but does not know when it happens. Assume that the time of the switch from i to j be $t_k + \bar{t}$, where $\bar{t} \in (0, \tau_s]$. When $t \in [t_k, t_k + \bar{t})$, the subsystem i is active. We derive as above and obtain that

$$V_i(t) < e^{-\lambda_s(t - t_k)} V_i(t_k). \tag{14}$$

When $t \in [t_k + \bar{t}, t_{k+1})$, the resulting closed-loop dynamic becomes

$$\begin{aligned} \dot{x}(t) = A_j x(t) &+ B_j x(t - \tau(t)) \\ &+ C_j \dot{x}(t - h(t)) + D_j K_i x(t - \eta(t)). \end{aligned} \tag{15}$$

Construct the Lyapunov–Krasovskii functional candidate for the system (15)

$$\begin{aligned} V_j(t) = x^T(t) P_j x(t) &+ \int_{t - \tau_s}^{t} x^T(s) Q_j e^{\lambda_s(s - t)} x(s) ds \\ &+ \int_{t - h(t)}^{t} \dot{x}^T(s) M_j e^{\lambda_s(s - t)} \dot{x}(s) ds \\ &+ \int_{t - \alpha\eta(t)}^{t} x^T(s) N_j e^{\lambda_s(s - t)} x(s) ds \\ &+ \int_{t - \tau(t)}^{t} x^T(s) R_j e^{\lambda_s(s - t)} x(s) ds \\ &+ \int_{-\tau_s}^{0} \int_{t + \theta}^{t} \dot{x}^T(s) S_j e^{\lambda_s(s - t)} \dot{x}(s) ds d\theta, \end{aligned} \tag{16}$$

where $P_j, Q_j, R_j, M_j, N_j, S_j$ are positive definite symmetric matrices.

When the subsystem j is active, for $\forall \lambda_u > 0$, taking the time derivative of (16) along solutions of the system (15) gives

$$\begin{aligned} \dot{V}_j(t) - \lambda_u V_j(t) \leq\ & 2x^T(t) P_j \dot{x}(t) + \lambda_s x^T(t) P_j x(t) + x^T(t) Q_j x(t) \\ & - e^{-\lambda_s \tau_s} x^T(t - \tau_s) Q_j x(t - \tau_s) + x^T(t) R_j x(t) \\ & - (1 - \hat{\tau}) e^{-\lambda_s \tau} x^T(t - \tau(t)) R_j x(t - \tau(t)) \\ & + \dot{x}^T(t) M_j \dot{x}(t) + x^T(t) N_j x(t) + \tau_s \dot{x}^T(t) S_j \dot{x}(t) \\ & - (1 - \hat{h}) e^{-\lambda_s h} \dot{x}^T(t - h(t)) M_j \dot{x}(t - h(t)) \\ & - (1 - \alpha) e^{-\alpha \lambda_s \tau_s} x^T(t - \alpha\eta(t)) N_j x(t - \alpha\eta(t)) \\ & - e^{-\lambda_s \tau_s} \int_{t - \alpha\eta(t)}^{t} \dot{x}^T(s) S_j \dot{x}(s) ds \\ & - e^{-\lambda_s \tau_s} \int_{t - \eta(t)}^{t - \alpha\eta(t)} \dot{x}^T(s) S_j \dot{x}(t) ds \\ & - e^{-\lambda_s \tau_s} \int_{t - \tau_s}^{t - \eta(t)} \dot{x}^T(s) S_j \dot{x}(t) ds \\ & - (\lambda_s + \lambda_u) e^{-\lambda_s \tau_s} \int_{t - \tau_s}^{t} x^T(s) Q_j x(s) ds \\ & - (\lambda_s + \lambda_u) e^{-\lambda_s \tau} \int_{t - \tau(t)}^{t} x^T(s) R_j x(s) ds \\ & - (\lambda_s + \lambda_u) e^{-\lambda_s h} \int_{t - h(t)}^{t} \dot{x}^T(s) M_j \dot{x}(s) ds \\ & - (\lambda_s + \lambda_u) e^{-\alpha \lambda_s \tau_s} \int_{t - \alpha\eta(t)}^{t} x^T(s) N_j x(s) ds \\ & - (\lambda_s + \lambda_u) \int_{-\tau_s}^{0} \int_{t + \theta}^{t} \dot{x}^T(s) S_j e^{\lambda_s(s - t)} \dot{x}(s) ds d\theta. \end{aligned} \tag{17}$$

From Lemma 1, we have

$$-e^{-\lambda_s \tau_s} \int_{t - \alpha\eta(t)}^{t} \dot{x}^T(s) S_j \dot{x}(s) ds$$
$$\leq \frac{-e^{-\lambda_s \tau_s}}{\alpha \tau_s} \left(\int_{t - \alpha\eta(t)}^{t} \dot{x}^T(s) ds \right) S_j \left(\int_{t - \alpha\eta(t)}^{t} \dot{x}(s) ds \right)$$
$$= \frac{-e^{-\lambda_s \tau_s}}{\alpha \tau_s} (x^T(t) - x^T(t - \alpha\eta(t))) S_j (x(t) - x(t - \alpha\eta(t))),$$

$$-e^{-\lambda_s \tau_s} \int_{t - \eta(t)}^{t - \alpha\eta(t)} \dot{x}^T(s) S_j \dot{x}(s) ds$$
$$\leq \frac{-e^{-\lambda_s \tau_s}}{(1 - \alpha) \tau_s} \left(\int_{t - \eta(t)}^{t - \alpha\eta(t)} \dot{x}^T(s) ds \right) S_j \left(\int_{t - \eta(t)}^{t - \alpha\eta(t)} \dot{x}(s) ds \right)$$
$$= \frac{-e^{-\lambda_s \tau_s}}{(1 - \alpha) \tau_s} (x^T(t - \alpha\eta(t)) - x^T(t - \eta(t)))$$
$$\times S_j (x(t - \alpha\eta(t)) - x(t - \eta(t))),$$

$$-e^{-\lambda_s \tau_s} \int_{t - \tau_s}^{t - \eta(t)} \dot{x}^T(s) S_j \dot{x}(t) ds < 0,$$

$$-(\lambda_s + \lambda_u) e^{-\lambda_s \tau_s} \int_{t - \tau_s}^{t} x^T(s) Q_j x(s) ds$$
$$\leq \frac{-(\lambda_s + \lambda_u) e^{-\lambda_s \tau_s}}{\tau_s} \int_{t - \tau_s}^{t} x^T(s) ds Q_j \int_{t - \tau_s}^{t} x(s) ds,$$

$$-(\lambda_s + \lambda_u) e^{-\lambda_s \tau} \int_{t - \tau(t)}^{t} x^T(s) R_j x(s) ds$$
$$\leq \frac{-(\lambda_s + \lambda_u) e^{-\lambda_s \tau}}{\tau} \int_{t - \tau(t)}^{t} x^T(s) ds R_j \int_{t - \tau(t)}^{t} x(s) ds,$$

$$-(\lambda_s + \lambda_u)e^{-\lambda_s h}\int_{t-h(t)}^{t}\dot{x}^T(s)M_j\dot{x}(s)ds$$

$$\leq \frac{-(\lambda_s + \lambda_u)e^{-\lambda_s h}}{h}\int_{t-h(t)}^{t}\dot{x}^T(s)dsM_j\int_{t-h(t)}^{t}\dot{x}^T(s)ds,$$

$$-(\lambda_s + \lambda_u)e^{-\alpha\lambda_s\tau_s}\int_{t-\alpha\eta(t)}^{t}x^T(s)N_jx(s)ds$$

$$\leq \frac{-(\lambda_s + \lambda_u)e^{-\alpha\lambda_s\tau_s}}{\tau_s}\int_{t-\alpha\eta(t)}^{t}x^T(s)dsN_j\int_{t-\alpha\eta(t)}^{t}x(s)ds,$$

$$-(\lambda_s + \lambda_u)\int_{-\tau_s}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)S_je^{\lambda_s(s-t)}\dot{x}(s)dsd\theta < 0.$$

Moreover, from (15), for any matrices W_i, we have

$$- 2(\dot{x}^T(t)\ x^T(t))W_i^T(\dot{x}(t) - A_jx(t) - B_jx(t - \tau(t))$$
$$- C_j\dot{x}(t - h(t)) - D_jK_ix(t - \eta(t))) = 0. \tag{18}$$

Utilizing the above inequalities and adding (18) into (17), we obtain

$$\dot{V}_j(t) - \lambda_uV_j(t) < \zeta^T(t)\Psi_j\zeta(t), \tag{19}$$

where

$$\zeta^T(t) = \left(\xi^T(t)\ \int_{t-\tau(t)}^{t}x^T(s)ds\ \int_{t-\alpha\eta(t)}^{t}x^T(s)ds\right.$$
$$\left.\times\ \int_{t-h(t)}^{t}x^T(s)ds\ \int_{t-\tau_s}^{t}x^T(s)ds\right),$$

$$\Psi_j = \begin{bmatrix} \Psi_{j,11} & 0 & 0 & 0 & 0 \\ * & \Psi_{j,22} & 0 & 0 & 0 \\ * & * & \Psi_{j,33} & 0 & 0 \\ * & * & * & \Psi_{j,44} & 0 \\ * & * & * & * & \Psi_{j,55} \end{bmatrix},$$

$$\Psi_{j,11} = \begin{bmatrix} \Pi_{j,11} & W_i^TB_j & W_i^TC_j & \Pi_{j,14} & 0 & \Pi_{j,16} & \Pi_{j,17} \\ * & \Pi_{j,22} & 0 & 0 & 0 & 0 & B_j^TW_i \\ * & * & \Pi_{j,33} & 0 & 0 & 0 & C_j^TW_i \\ * & * & * & \Pi_{j,44} & 0 & \Pi_{j,46} & \Pi_{j,47} \\ * & * & * & * & \Pi_{j,55} & 0 & 0 \\ * & * & * & * & * & \Pi_{j,66} & 0 \\ * & * & * & * & * & * & \Pi_{j,77} \end{bmatrix},$$

$\Pi_{j,11} = R_j + N_j - \lambda_uP_j + Q_j - \frac{e^{-\lambda_s\tau_s}}{\alpha\tau_s}S_j + W_i^TA_j + A_j^TW_i,$

$\Pi_{j,14} = W_i^TD_jK_i,\ \Pi_{j,16} = \frac{e^{-\lambda_s\tau_s}}{\alpha\tau_s}S_j,\ \Pi_{j,17} = P_j + A_j^TW_i - W_i^T,$

$\Pi_{j,22} = -(1 - \hat{\tau})e^{-\lambda_s\tau}R_j,\ \Pi_{j,33} = -(1 - \hat{h})e^{-\lambda_s h}M_j,\ \Pi_{j,44} = -\frac{e^{-\lambda_s\tau_s}}{(1-\alpha)\tau_s}S_j,\ \Pi_{j,46} = \frac{e^{-\lambda_s\tau_s}}{(1-\alpha)\tau_s}S_j,\ \Pi_{j,47} = K_i^TD_j^TW_i,\ \Pi_{j,66} = -\frac{e^{-\lambda_s\tau_s}}{(1-\alpha)\tau_s}S_j - \frac{e^{-\lambda_s\tau_s}}{\alpha\tau_s}S_j - (1 - \alpha)e^{-\alpha\lambda_s\tau_s}N_j,\ \Pi_{j,55} = -e^{-\lambda_s\tau_s}Q_j,$

$\Pi_{j,77} = M_j + \tau_sS_j - W_i - W_i^T,\ \Psi_{j,22} = -\frac{(\lambda_s+\lambda_u)e^{-\lambda_s\tau}}{\tau}R_j,\ \Psi_{j,33} = -\frac{(\lambda_s+\lambda_u)e^{-\lambda_s\tau_s}}{\tau_s}N_j,\ \Psi_{j,44} = -\frac{(\lambda_s+\lambda_u)e^{-\lambda_s h}}{h}M_j,\ \Psi_{j,55} = -\frac{(\lambda_s+\lambda_u)e^{-\lambda_s\tau_s}}{\tau_s}Q_j.$

From (19), we know that $\Psi_j < 0$ implies

$$\dot{V}_j(t) - \lambda_uV_j(t) < 0. \tag{20}$$

Integrating the inequality (20) yields

$$V_j(t) < e^{\lambda_u(t-t_k-\bar{t})}V_j(t_k + \bar{t}). \tag{21}$$

From (14) and (21), for $t \in [t_k, t_{k+1})$, we have

$$V_j(t) < e^{\lambda_u(t-t_k-\bar{t})}V_j(t_k + \bar{t})$$
$$\leq \frac{\bar{\lambda}(P_j) + \tau\bar{\lambda}(R_j) + h\bar{\lambda}(M_j) + \alpha\tau_s\bar{\lambda}(N_j) + \tau_s\bar{\lambda}(Q_j) + \tau_s^2\bar{\lambda}(S_j)}{\underline{\lambda}(P_i)}$$

$$\times e^{\lambda_u(t-t_k-\bar{t})}V_i((t_k + \bar{t})^-)$$
$$\leq \frac{\bar{\lambda}(P_j) + \tau\bar{\lambda}(R_j) + h\bar{\lambda}(M_j) + \alpha\tau_s\bar{\lambda}(N_j) + \tau_s\bar{\lambda}(Q_j) + \tau_s^2\bar{\lambda}(S_j)}{\underline{\lambda}(P_i)}$$
$$\times e^{-\lambda_s(t_k+\bar{t}-t_k)}e^{\lambda_u(t-t_k-\bar{t})}V_i(t_k)$$
$$\leq \mu_{ij}V_i(t_k) \leq \mu V_i(t_k), \tag{22}$$

where $V_i((t_k + \bar{t})^-)$ is the function value of $V_i(t)$ right before the switching, and

$$\mu_{ij} := \max\{((\bar{\lambda}(P_j) + \tau\bar{\lambda}(R_j) + h\bar{\lambda}(M_j) + \alpha\tau_s\bar{\lambda}(N_j)$$
$$+ \tau_s\bar{\lambda}(Q_j) + \tau_s^2\bar{\lambda}(S_j))e^{-(\lambda_s+\lambda_u)\bar{t}+\lambda_u(t-t_k)})/\underline{\lambda}(P_i)\}$$

and $\mu := \max_{i,j\in\mathcal{M}}\mu_{ij}$.

3.3. Stability analysis

Definition 1 implies that $N_\sigma(t_{k_0}, t_k) \leq N_0 + \frac{k-k_0}{p}$ for every p such that $\tau_a \geq p\tau_s$. We now derive a lower bound on p to guarantee convergence. From (3), we know that $N_\sigma(t_{k_0}, t_k)$ equals the number $k - k_0$ of intervals of the form (t_l, t_{l+1}), $k_0 \leq l \leq k - 1$, which contain a switch. Combining the conclusions of Sections 3.1 and 3.2, we have the following bound for all $k \geq k_0$

$$V(t_k) = V_\sigma(t_k)$$
$$\leq \mu^{N_0+\frac{k-k_0}{p}}\nu^{k-k_0-N_0-\frac{k-k_0}{p}}V_\sigma(t_0)$$
$$= \left(\frac{\mu}{\nu}\right)^{N_0}\left(\mu^{\frac{1}{p}}\nu^{\frac{p-1}{p}}\right)^{k-k_0}V_\sigma(t_0)$$
$$= \left(\frac{\mu}{\nu}\right)^{N_0}e^{(k-k_0)\ln\left(\mu^{\frac{1}{p}}\nu^{\frac{p-1}{p}}\right)}V_\sigma(t_0). \tag{23}$$

We want to ensure that $\mu^{\frac{1}{p}}\nu^{\frac{p-1}{p}} < 1$, which is equivalent to

$$p > 1 + \frac{\log\mu}{\log\frac{1}{\nu}}. \tag{24}$$

Thus if

$$\tau_a > \left(1 + \frac{\log\mu}{\log\frac{1}{\nu}}\right)\tau_s, \tag{25}$$

then

$$V(t_k) \leq \left(\frac{\mu}{\nu}\right)^{N_0}e^{(k-k_0)\ln\left(\mu^{\frac{1}{p}}\nu^{\frac{p-1}{p}}\right)}V_\sigma(t_0). \tag{26}$$

From the defined piecewise Lyapunov–Krasovskii functional, we have

$$V(t_k) \geq x^T(t)P_ix(t) \geq \underline{\lambda}(P_i)\|x(t)\|^2$$
$$\geq \min_{\forall i\in\mathcal{M}}\underline{\lambda}(P_i)\|x(t)\|^2 = a\|x(t)\|^2, \tag{27}$$

$$V_\sigma(t_0) \leq (\max_{\forall i\in\mathcal{M}}\bar{\lambda}(P_i) + \tau\max_{\forall i\in\mathcal{M}}\bar{\lambda}(R_i)$$
$$+ h\max_{\forall i\in\mathcal{M}}\bar{\lambda}(M_i) + \alpha\tau_s\max_{\forall i\in\mathcal{M}}\bar{\lambda}(N_i)$$
$$+ \tau_s\max_{\forall i\in\mathcal{M}}\bar{\lambda}(Q_i) + \tau_s^2\max_{\forall i\in\mathcal{M}}\bar{\lambda}(S_i))\left\|x_{t_0}\right\|_c^2$$
$$= b\left\|x_{t_0}\right\|_c^2, \tag{28}$$

where

$$a = \min_{\forall i\in\mathcal{M}}\underline{\lambda}(P_i),$$

$$b = \max_{\forall i\in\mathcal{M}}\bar{\lambda}(P_i) + \tau\max_{\forall i\in\mathcal{M}}\bar{\lambda}(R_i) + h\max_{\forall i\in\mathcal{M}}\bar{\lambda}(M_i)$$
$$+ \alpha\tau_s\max_{\forall i\in\mathcal{M}}\bar{\lambda}(N_i) + \tau_s\max_{\forall i\in\mathcal{M}}\bar{\lambda}(Q_i) + \tau_s^2\max_{\forall i\in\mathcal{M}}\bar{\lambda}(S_i).$$

From (26)–(28), we have

$$\|x(t)\|^2 \le \frac{b}{a}\left(\frac{\mu}{\nu}\right)^{N_0} e^{(k-k_0)\ln\left(\mu^{\frac{1}{p}}\nu^{\frac{p-1}{p}}\right)} \|x_{t_0}\|_c^2, \tag{29}$$

which means that the system (1) is exponentially stable.
The following theorem concludes the main result.

Theorem 1. *Consider the switched neutral system* (1). *For some given positive constants* $\alpha < 1$, λ_s, λ_u, τ_s, h, τ, $\hat{\tau} < 1$, $\hat{h} \le 1$, *if there exist matrices* $\bar{P}_i > 0$, $\bar{R}_i > 0$, $\bar{M}_i > 0$, $\bar{N}_i > 0$, $\bar{S}_i > 0$, $\bar{Q}_i > 0$, $\bar{W}_i > 0$, J_i, $\bar{P}_j > 0$, $\bar{R}_j > 0$, $\bar{M}_j > 0$, $\bar{N}_j > 0$, $\bar{S}_j > 0$, $\bar{Q}_j > 0$, J_j *for* $\forall i, j \in \mathcal{M}$, $i \ne j$, *satisfying*

$$\begin{bmatrix} \bar{\Omega}_{i,11} & B_i\bar{W}_i & C_i\bar{W}_i & D_iJ_i & 0 & \dfrac{e^{-\lambda_s\tau_s}}{\alpha\tau_s}\bar{S}_i & \bar{\Omega}_{i,17} \\ * & \bar{\Omega}_{i,22} & 0 & 0 & 0 & 0 & \bar{W}_i^T B_i^T \\ * & * & \bar{\Omega}_{i,33} & 0 & 0 & 0 & \bar{W}_i^T C_i^T \\ * & * & * & \bar{\Omega}_{i,44} & 0 & \dfrac{e^{-\lambda_s\tau_s}}{(1-\alpha)\tau_s}\bar{S}_i & J_i^T D_i^T \\ * & * & * & * & \bar{\Omega}_{i,55} & 0 & 0 \\ * & * & * & * & * & \bar{\Omega}_{i,66} & 0 \\ * & * & * & * & * & * & \bar{\Omega}_{i,77} \end{bmatrix}$$
$$< 0, \tag{30}$$

$$\begin{bmatrix} \bar{\Psi}_{j,11} & 0 & 0 & 0 & 0 \\ * & \bar{\Psi}_{j,22} & 0 & 0 & 0 \\ * & * & \bar{\Psi}_{j,33} & 0 & 0 \\ * & * & * & \bar{\Psi}_{j,44} & 0 \\ * & * & * & * & \bar{\Psi}_{j,55} \end{bmatrix} < 0, \tag{31}$$

where

$$\bar{\Omega}_{i,11} = \bar{R}_i + \bar{N}_i + \lambda_s\bar{P}_i + \bar{Q}_i - \frac{e^{-\lambda_s\tau_s}}{\alpha\tau_s}\bar{S}_i + A_i\bar{W}_i + \bar{W}_i^T A_i^T,$$

$$\bar{\Omega}_{i,22} = -(1-\hat{\tau})e^{-\lambda_s\tau}\bar{R}_i, \quad \bar{\Omega}_{i,33} = -(1-\hat{h})e^{-\lambda_s h}\bar{M}_i,$$

$$\bar{\Omega}_{i,44} = -\frac{e^{-\lambda_s\tau_s}}{(1-\alpha)\tau_s}\bar{S}_i, \quad \bar{\Omega}_{i,55} = -e^{-\lambda_s\tau_s}\bar{Q}_i,$$

$$\bar{\Omega}_{i,66} = -\frac{e^{-\lambda_s\tau_s}}{(1-\alpha)\tau_s}\bar{S}_i - \frac{e^{-\lambda_s\tau_s}}{\alpha\tau_s}\bar{S}_i - (1-\alpha)e^{-\alpha\lambda_s\tau_s}\bar{N}_i,$$

$$\bar{\Omega}_{i,17} = \bar{P}_i + \bar{W}_i^T A_i^T - \bar{W}_i, \quad \bar{\Omega}_{i,77} = \bar{M}_i + \tau_s\bar{S}_i - \bar{W}_i - \bar{W}_i^T,$$

$$\bar{\Psi}_{j,11} = \begin{bmatrix} \bar{\Pi}_{j,11} & B_j\bar{W}_i & C_j\bar{W}_i & D_iJ_i & 0 & \bar{\Pi}_{j,16} & \bar{\Pi}_{j,17} \\ * & \bar{\Pi}_{j,22} & 0 & 0 & 0 & 0 & \bar{W}_i^T B_j^T \\ * & * & \bar{\Pi}_{j,33} & 0 & 0 & 0 & \bar{W}_i^T C_j^T \\ * & * & * & \bar{\Pi}_{j,44} & 0 & \bar{\Pi}_{j,46} & J_i^T D_j^T \\ * & * & * & * & \bar{\Pi}_{j,55} & 0 & 0 \\ * & * & * & * & * & \bar{\Pi}_{j,66} & 0 \\ * & * & * & * & * & * & \bar{\Pi}_{j,77} \end{bmatrix},$$

$$\bar{\Pi}_{j,11} = \bar{R}_j + \bar{N}_j - \lambda_u\bar{P}_j + \bar{Q}_j - \frac{e^{-\lambda_s\tau_s}}{\alpha\tau_s}\bar{S}_j + A_j\bar{W}_i + \bar{W}_i^T A_j^T, \bar{\Pi}_{j,22} = -(1-\hat{\tau})e^{-\lambda_s\tau}\bar{R}_j, \bar{\Pi}_{j,33} = -(1-\hat{h})e^{-\lambda_s h}\bar{M}_j, \bar{\Pi}_{j,44} = -\frac{e^{-\lambda_s\tau_s}}{(1-\alpha)\tau_s}\bar{S}_j,$$

$$\bar{\Pi}_{j,55} = -e^{-\lambda_s\tau_s}\bar{Q}_j, \bar{\Pi}_{j,66} = -\frac{e^{-\lambda_s\tau_s}}{(1-\alpha)\tau_s}\bar{S}_j - \frac{e^{-\lambda_s\tau_s}}{\alpha\tau_s}\bar{S}_j - (1-\alpha)e^{-\alpha\lambda_s\tau_s}\bar{N}_j,$$

$$\bar{\Pi}_{j,16} = \frac{e^{-\lambda_s\tau_s}}{\alpha\tau_s}\bar{S}_j, \bar{\Pi}_{j,46} = \frac{e^{-\lambda_s\tau_s}}{(1-\alpha)\tau_s}\bar{S}_j, \bar{\Pi}_{j,17} = \bar{P}_j + \bar{W}_i^T A_j^T - \bar{W}_i,$$

$$\bar{\Pi}_{j,77} = \bar{M}_j + \tau_s\bar{S}_j - \bar{W}_i - \bar{W}_i^T, \bar{\Psi}_{j,22} = -\frac{(\lambda_s+\lambda_u)e^{-\lambda_s\tau}}{\tau}\bar{R}_j, \bar{\Psi}_{j,33} = -\frac{(\lambda_s+\lambda_u)e^{-\lambda_s\tau_s}}{\tau_s}\bar{N}_j, \bar{\Psi}_{j,44} = -\frac{(\lambda_s+\lambda_u)e^{-\lambda_s h}}{h}\bar{M}_j, \bar{\Psi}_{j,55} = -\frac{(\lambda_s+\lambda_u)e^{-\lambda_s\tau_s}}{\tau_s}\bar{Q}_j,$$

then the system (1) *is exponentially stable for any switching signals*

with the average dwell time satisfying

$$\tau_a > \left(1 + \frac{\log\mu}{\log\frac{1}{\nu}}\right)\tau_s, \tag{32}$$

where μ *and* ν *are constants, under the sampled-data controller* (5) *with* $K_i = J_i\bar{W}_i^{-1}$.

Proof. It is obvious from the above analysis and thus omitted.

Remark 2. Note that the controller gain K_i cannot be obtained by solving $\Omega_i < 0$ and $\Psi_j < 0$ directly. If, however, we let $\bar{W}_i = W_i^{-1}$, multiply diag$\{\bar{W}_i^T, \ldots, \bar{W}_i^T\}$ and diag$\{\bar{W}_i, \ldots, \bar{W}_i\}$ on the left-hand side and on the right-hand side of the inequalities $\Omega_i < 0$ and $\Psi_j < 0$, respectively, and also let $\bar{R}_i = \bar{W}_i^T R_i\bar{W}_i$, $\bar{N}_i = \bar{W}_i^T N_i\bar{W}_i$, $\bar{P}_i = \bar{W}_i^T P_i\bar{W}_i$, $\bar{Q}_i = \bar{W}_i^T Q_i\bar{W}_i$, $\bar{S}_i = \bar{W}_i^T S_i\bar{W}_i$, $\bar{M}_i = \bar{W}_i^T M_i\bar{W}_i$, $J_i = K_i\bar{W}_i$, $\bar{R}_j = \bar{W}_i^T R_j\bar{W}_i$, $\bar{N}_j = \bar{W}_i^T N_j\bar{W}_i$, $\bar{P}_j = \bar{W}_i^T P_j\bar{W}_i$, $\bar{Q}_j = \bar{W}_i^T Q_j\bar{W}_i$, $\bar{S}_j = \bar{W}_i^T S_j\bar{W}_i$, $\bar{M}_j = \bar{W}_i^T M_j\bar{W}_i$, then we can obtain the inequalities (30) and (31), which are equivalent to $\Omega_i < 0$ and $\Psi_j < 0$. As a result, easily solving (30) and (31) directly gives K_i.

Remark 3. In (32), τ_s can be chosen in advance. μ and ν are computed according to (13) and (22).

Remark 4. In order to obtain the control gain K_i, we introduce the free-weighting matrix W_i when the ith controller is active, (see Eqs. (10) and (18)), so that the control gain K_i can be obtained by solving the conditions (30) and (31) simultaneously as described in Remark 2, which overcomes the technical challenge of solving the *coupled* variables of linear matrix inequalities in Wang et al. (2013).

Remark 5. In this paper, our work is based on the assumption $\tau_s \le \tau_d$ which simplifies the analysis process into two cases, that is no switching or one switching may occur on arbitrary a sampling interval. In fact, our method can be extended to deal with the case of several switchings happen on a sampling interval. We just need analyze the monotony property of Lyapunov–Krasovskii functional on each individual non-switching interval on one sampling interval, and then analyze the overall descend property of Lyapunov–Krasovskii functional to guarantee stability.

Remark 6. The author of Zhang et al. (2007) analyzes stability of switched linear systems without delays from the view of analytical solutions of switched linear systems. However, the proposed method cannot be applied for switched neutral systems since the analytical solution of a switched neutral system is infinite-dimensional and is difficult to show out. Moreover, the feedback controller in Zhang et al. (2007) is given in advance. Our proposed method is by means of the Lyapunov–Krasovskii functional analysis and especially we give the design of the feedback controller.

4. An illustration example

Consider the switched neutral system (1) with two subsystems, where

$$A_1 = \begin{bmatrix} -2.5 & 0.3 \\ 0 & -1.5 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -0.2 & 0.2 \\ 0.1 & -0.1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -0.1 & 0.1 \\ 0 & 0.2 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1.2 & 0 \\ 0.1 & -1.6 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0.3 & 0 \\ 0.1 & 0.1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

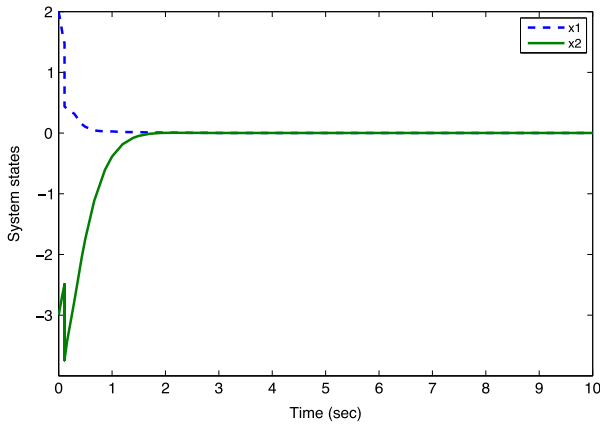$$\tau(t) = 0.1\sin t + 0.3, \quad h(t) = 0.1\sin t + 0.1.$$

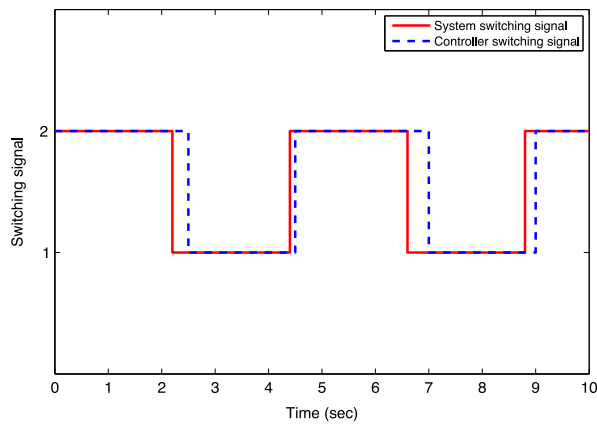**Fig. 1.** State responses of the closed-loop system.

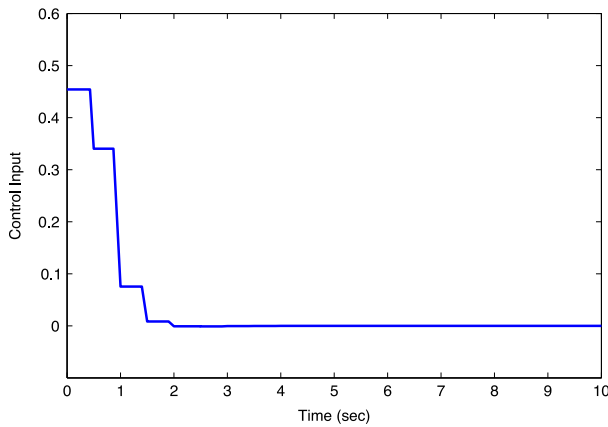**Fig. 2.** Switching signals of the system and the controller.

Fig. 3. The control input.

Let parameters be $\lambda_s = 1, \lambda_u = 1, \alpha = 0.4$ and $\tau_s = 0.5$. Solving the conditions in Theorem 1, we obtain $K_1 = [-0.0723 \quad -0.1996], K_2 = [-0.2809 \quad -0.1149]$. From (32), we obtain the minimum average dwell time $\tau_a^* = 1.9090$. Constructing a certain switching sequence satisfying $\tau_a = 2.2 > \tau_a^*$, we obtain the state responses of the closed-loop system with the initial state $x_0 = (2 \quad -3)^T$ shown in Fig. 1, and the system switching signal and the controller switching signal shown in Fig. 2. The control input is shown in Fig. 3. From Fig. 2, the asynchronous switchings are clearly shown. Thus, from the figures the closed-loop system is stabilized under the sampled-data control input, and the simulation results verified the proposed method.

5. Conclusions

We have presented a result on sampled-data-based state feedback stabilization of a class of switched linear neutral systems under asynchronous switching. A relationship between the average dwell time and the sampling period has been revealed to form a switching condition to guarantee exponential stability. Furthermore, by introducing free-weighting matrices, we have obtained the controller gains by solving the resulting matrix inequalities. Future work will focus on the research of the class of switched linear neutral systems with noise, which is very promising but needs the knowledge of stochastic dynamics, especially the theory of Itô stochastic differential equations. Initial attempt will consider the system with the simple wide-band noise as studied in Ref. Blankenship and Papanicolaou (1978).

Acknowledgments

References

Blankenship, G., & Papanicolaou, G. C. (1978). Stability and control of stochastic systems with wideband noise disturbances. *SIAM Journal on Applied Mathematics*, *42*(3), 437–476.

Branicky, M. (1998). Multiple Lyapunov functions and other analysis tools for switched and hybrid systems. *IEEE Transactions on Automatic Control*, *43*(4), 475–482.

Chen, T., & Francis, B. A. (1991). Input–output stability of sampled-data systems. *IEEE Transactions on Automatic Control*, *36*, 50–58.

Chen, T., & Francis, B. A. (1995). *Optimal sampled-data control systems*. Springer.

Chen, W., & Zheng, W. (2010). Delay-independent minimum dwell time for exponential stability of uncertain switched delay systems. *IEEE Transactions on Automatic Control*, *55*(10), 2406–2413.

Feng, L., & Song, Y. (2011). Stability condition for sampled data based control of linear continuous switched systems. *Systems & Control Letters*, *60*(10), 787–797.

Fu, J., Ma, R., & Chai, T. (2015). Global finite-time stabilization of a class of switched nonlinear systems with the powers of positive odd rational numbers. *Automatica*, *54*, 360–373.

Gu, K. (2000). An integral inequality in the stability problem of time-delay systems. In *Proceedings of 39th IEEE conference on decision control* (pp. 2805–2810).

Hara, S., Yamamoto, Y., & Fujioka, H. (1996). Modern and classical analysis/synthesis methods in sampled-data control. In *Proceedings of the 35th conference on decision and control* (pp. 1251–1255).

Hu, L., Lam, J., Cao, Y., & Shao, H. (2003). A linear matrix inequality approach to robust H_2 sampled-data control for linear uncertain systems. *IEEE Transactions on Systems, Man and Cybernetics*, *33*, 149–155.

Krishnasamy, R., & Balasubramaniam, P. (2015a). A descriptor system approach to the delay-dependent exponential stability analysis for switched neutral systems with nonlinear perturbations. *Nonlinear Analysis. Hybrid Systems*, *15*, 23–26.

Krishnasamy, R., & Balasubramaniam, P. (2015b). Stabilisation analysis for switched neutral systems based on sampled-data control. *International Journal of Systems Science*, *46*(14), 2531–2546.

Lian, J., Ge, Y., & Han, M. (2013). Stabilization for switched stochastic neutral systems under asynchronous switching. *Information Sciences*, *222*, 501–508.

Liberzon, D. (2003). *Switched in systems and control*. Boston: Birkhauser.

Liberzon, D. (2014). Finite data-rate feedback stabilization of switched and hybrid linear systems. *Automatica*, *50*(2), 409–420.

Lien, C., Chen, J., Yu, K., & Chung, L. (2012). Robust delay-dependent H_∞ control for uncertain switched time-delay systems via sampled-data state feedback input. *Computers and Mathematics with Applications*, *64*(5), 1187–1196.

Lin, H., & Antsaklis, P. (2009). Stability and stabilizability of switched linear systems: a survey of recent results. *IEEE Transactions on Automatic Control*, *54*, 308–322.

Liu, D., Liu, X., & Zhong, S. (2008). Delay-dependent robust stability and control synthesis for uncertain switched neutral systems with mixed delays. *Applied Mathematics and Computation*, *202*(2), 828–839.

Li, T., Zhao, J., & Qi, Y. (2014). Switching design of stabilizing switched neutral systems with application to lossless transmission lines. *IET Control Theory & Applications*, *17*, 2082–2091.

Sun, X., Du, S., Shi, P., Wang, W., & Wang, L. (2014). Input-to-state stability for nonlinear systems with large delay period based on switching techniques. *IEEE Transactions on Circuits and Systems. I. Regular Papers*, *61*(6), 1789–1800.

Sun, X., Fu, J., Sun, H., & Zhao, J. (2005). Stability of linear switched neutral delay systems. In *Proceedings of the CSEE, 25* (pp. 42–46).

Sun, Z., & Ge, S. (2005). Analysis and synthesis of switched linear control systems. *Automatica, 41*(2), 181–195.

Sun, X., Liu, G., David, R., & Wang, W. (2008). Stability of systems with controller failure and time-varying delay. *IEEE Transactions on Automatic Control, 53*, 941–953.

Sun, X., Zhao, J., & Hill, D. (2006). Stability and L_2-gain analysis for switched delay systems: A delay-dependent method. *Automatica, 42*(10), 1769–1774.

Wang, R., Xing, J., Zhou, C., Wang, P., & Yang, Q. (2014). Finite-time asynchronously switched control of switched systems with sampled data feedback. *Circuits, Systems, and Signal Processing, 33*, 3713–3738.

Wang, Y., Zhao, J., & Jiang, B. (2013). Stabilization of a class of switched linear neutral systems under asynchronous switching. *IEEE Transactions on Automatic Control, 58*, 2114–2119.

Wu, F., & Dong, K. (2006). Gain-scheduling control of LFT systems using parameter-dependent Lyapunov function. *Automatica, 42*, 39–50.

Xiang, Z., Sun, Y., & Chen, Q. (2011). Robust reliable stabilization of uncertain switched neutral systems with delayed switching. *Applied Mathematics and Computation, 217*(23), 9835–9844.

Xiang, Z., Sun, Y., & Chen, Q. (2012). Stabilization for a class of switched neutral systems under asynchronous switching. *Transactions of the Institute of Measurement and Control, 34*, 739–801.

Xiang, Z., Sun, Y., & Mahmoud, M. (2012). Robust finite-time H_∞ control for a class of uncertain switched neutral systems. *Communications in Nonlinear Science and Numerical Simulation, 17*(4), 1766–1778.

Xiong, L., Zhong, S., Ye, M., & Wu, S. (2009). New stability and stabilization for switched neutral control systems. *Chaos, Solitons & Fractals, 42*(3), 1800–1811.

Zhai, G., Hu, B., Yasuda, K., & Michel, A. (2001). Disturbance attenuation properties of time-controlled switched systems. *Journal of the Franklin Institute. Engineering and Applied Mathematics, 338*, 765–779.

Zhang, L., & Gao, H. (2010). Asynchonously switched control of switched linear systems with average dwell time. *Automatica, 46*(5), 953–958.

Zhang, Y., Liu, X., Zhu, H., & Zhong, S. (2007). Stability analysis and control synthesis for a class of switched neutral systems. *Applied Mathematics and Computation, 190*(2), 1258–1266.

Zhang, W., & Yu, L. (2009). Stability analysis for discrete-time switched time-delay systems. *Automatica, 45*(10), 2265–2271.

Zhang, D., & Yu, L. (2012). Exponential stability analysis for neutral switched systems with interval time-varying mixed delays and nonlinear perturbations. *Nonlinear Analysis. Hybrid Systems, 6*(2), 775–786.

Zhang, L., Zhuang, S., & Shi, P. (2015). Non-weighted quasi-time-dependent H_∞ filtering for switched linear systems with persistent dwell-time. *Automatica, 54*, 201–209.

Zhang, L., Zhuang, S., Shi, P., & Zhu, Y. (2015). Uniform tube based stabilization of switched linear systems with mode-dependent persistent dwell-time. *IEEE Transactions on Automatic Control, 60*, 2994–2999.

Zhao, J., & Hill, D. (2008). On stability, L_2-gain and H_∞ control for switched systems. *Automatica, 44*, 1220–1232.

Zhao, X., Shi, P., & Zhang, L. (2012). Asynchronously switched control of a class of slowly switched linear systems. *Systems & Control Letters, 61*(12), 1151–1156.
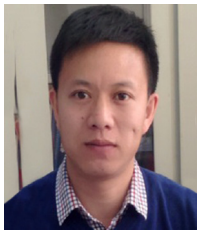
Jun Fu received the Ph.D. degree in mechanical engineering from Concordia University, Montreal, Quebec, Canada, in 2009. He is a postdoctoral fellow/associate in the department of mechanical engineering, Massachusetts Institute of Technology (MIT), USA from 2010 to 2014. He is a full professor at Northeastern University, China. His current research is on mathematical programming, dynamic optimization, switched systems, control of hysteretic systems, and robust control of nonlinear systems. He has authored/co-authored over 60 publications which appeared in journals, conference proceedings and book chapters.

He received the Chinese Government Award for Outstanding Self-financed Ph.D. Student Abroad. He has actively participated in considerable number of conference organizations as session chair including 2011 American Control Conference. He was a founding member of MIT Postdoctoral Association (PDA), and was Chair of Information Flow & IT Committee of MIT PDA. He is a senior member of IEEE.

Tai-Fang Li received her M.S. degree in operational research and cybernetics from Shenyang Normal University, Shenyang, China in 2009, and her Ph.D. degree in control theory and control engineering from Northeastern University, Shenyang, China in 2015. One academic year of her Ph.D. study was spent, for a joint Ph.D. program, in College of Engineering, University of Notre Dame, Notre Dame, IN, USA. She is an assistant professor in College of Engineering, Bohai University, Jinzhou, China. Her current research interests include switched systems, hybrid control, and distributed optimization.

Tianyou Chai received the Ph.D. degree in control theory and engineering in 1985 from Northeastern University, Shenyang, China, where he became a Professor in 1988. He is the founder and Director of the Center of Automation, which became a National Engineering and Technology Research Center and a State Key Laboratory. He is a member of Chinese Academy of Engineering, IFAC Fellow and IEEE Fellow, director of Department of Information Science of National Natural Science Foundation of China.

His current research interests include modeling, control, optimization and integrated automation of complex industrial processes.

He has published 170 peer reviewed international journal papers. His paper titled *Hybrid intelligent control for optimal operation of shaft furnace roasting process* was selected as one of three best papers for the Control Engineering Practice Paper Prize for 2011–2013. He has developed control technologies with applications to various industrial processes. For his contributions, he has won 4 prestigious awards of National Science and Technology Progress and National Technological Innovation, the 2007 Industry Award for Excellence in Transitional Control Research from IEEE Multiple-conference on Systems and Control.

Chun-Yi Su received the Ph.D. degree in control engineering from the South China University of Technology, Guangzhou, China, in 1990. After a seven-year stint with the University of Victoria, he joined Concordia University, Montreal, QC, Canada, in 1998, where he is a Professor and Concordia University Research Chair. His research interests include the application of automatic control theory to mechanical systems. He is particularly interested in control of systems involving hysteresis nonlinearities. He has authored or co-authored over 300 publications in journals, book chapters, and conference proceedings. He has received several best conference paper awards.

Dr. Su has served as an Associate Editor of the IEEE Transactions on Automatic Control, the IEEE Transactions on Control Systems Technology, and the Journal of Control Theory and Applications. He has been on the Editorial Board of 18 journals, including the IFAC Journal of Control Engineering Practice and Mechatronics. He served for many conferences as an Organizing Committee Member.