2022 IEEE 21st International Conference on Sciences and Techniques of Automatic Control and Computer Engineering (STA) | 978-1-6654-8261-5/22/$31.00 ©2022 IEEE | DOI: 10.1109/STA56120.2022.10019028

Robust stability analysis for uncertain neutral systems with time-delays

Nidhal Khorchani
MACS,
National Engineering School of Gabes (ENIG)
Gabes University, 6029, Tunisia
khorchani.nidhal@gmail.com

Wiem Jebri Jemai
MACS,
National Engineering School of Gabes (ENIG)
Gabes University, 6029, Tunisia
wiem.jeb@gmail.com

Rafika El Harabi
MACS,
National Engineering School of Gabes (ENIG)
Gabes University, 6029, Tunisia
rafikaharabi@yahoo.fr

Hassen Dahman
a- Departement of Automatic and Electrical Engineering,
National Engineering School of Gabes (ENIG),
b- LaPhyMNE Laboratory (LR05ES14), FSG, Gabes
Gabes University, 6029, Tunisia
has.dahman@gmail.com

***Abstract*—The presented work looks into the robust stability study of a class of neutral state time-delay systems according to parameter uncertainties. In fact, the open-loop stability analysis is come to have through means of determining the Linear Matrix Inequalities (LMIs) feasibility condition taking into consideration H_∞ optimization formulation. Afterwards, a new condition is proposed to reveal that the system is able to be asymptotically stable whenever the presence of parameter uncertainties, disturbances and faults; for that, the Lyapunov-Krasovskii theory is used. To highlight the efficiency of the proved theorems, simulation outcomes on a pedagogical example for uncertain cases are shown and discussed.**

***Index Terms*—Class of neutral systems with state time-delays, Stability study, Lyapunov-Krasovskii, LMI, H_∞, Robustness, Parameter uncertainties.**

I. INTRODUCTION

Robust stability analysis for uncertain neutral systems together with time delays in linear case have been a subject of many researches. Besides, time-delays and uncertainties have an impact on the dynamical responses of large-scale complex systems (electrical circuits, chemical reactors, neutral networks...etc).

In fact, numerous physical complex systems not solely rely on the sate values at present and past times, but also, on state derivatives with time delays. For that, this kind of systems is called neutral systems. Moreover, the presence of time-delay phenomena can produce instability of such systems. From this point of view, a large number of works dedicated to stability of investigated systems taking into account time-delays as pointed out in [1] and [2] On the other hand, the systems with a state feedback have been studied for many researches [3]. Thus, some works cited in [4] and [5] are directed towards the stability of considered systems (i.e neutral time-delay systems) with parameter uncertainties. In practice, models considering the characteristics of neutral systems with time-delays may be able to represent dynamic behaviors of different phenomena occurring in many physical complicated processes. Noting that both stability and stabilization studies for neutral time-delay systems are subjects widely discussed by many researchers. The derivative of the past state in the associated model can introduce an significant hardness for their analysis. In this work, we will expand the stability dedicated to time-delay systems to neutral systems with time delays for uncertainties situation. To this end, the work of Routh and Hurwitz [6] remain great references for testing the system stability. Then, Lyapunov method has mathematically reformulated in a rigorous way the notion of the stabilization. After that, Razumikhin has proposed an extension of the Lyapunov function. In the early years 60 Krasovskii studied an extension of quadratic Lyapunov functions within the framework of linear delay systems. A variety of Lyapunov Krasovskii functional construction methods for particular equations have been introduced and have often been linked to the convex optimization [6], [7] and [8]. The LMI tool is utilized in order to facilitate the convexity problem resolution. As can be seen in earlier works, the stability issues related to time-delay systems have received new attention and are broadly treated; but not yet for a class of neutral time-delay systems with uncertainties.

Motivated by the above-mentioned discussion, linear neutral time-delay systems will be addressed in the present paper so as to concentrate on improving the stability analysis in the presence of parameter uncertainties. As a matter of fact, the robust stability well-founded on the optimization technique H_∞ model-matching problem between the fault effects and dynamic performances is less that to give ($\gamma > 0$). The use of Lyapunov-Krasovskii approach can deduce a new stability condition by using Linear Matrix Inequalities for determining this optimization issue and get LMI feasibility conditions.

The rest of the present paper is structured as follows: in

Section II, the problem statement dedicated to neutral time-delay systems is depicted. Afterwards, Section III explains the robust stability analysis for such systems without/with parameter uncertainties and proves later proposed theorems in details. Thereafter, the efficiency of these later is shown in view of a simulation on a numerical example in section IV. At last, Section V encloses this paper with a few concluding remarks.

II. PROBLEM STATEMENT

As stated above, neutral time-delay systems are an extra common class of time-delay systems and described by means of a model taking into account the state derivatives at the present time, the past state values and the past state derivatives in an interval [1] and [8]. Therefore, this type of dynamical systems in certain case is defined as the following:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_h x(t-h) + A_d \dot{x}(t-d) + Bu(t) \\ \qquad\qquad +B_d d(t) + B_f f(t) \\ y(t) = Cx(t) + D_d d(t) + D_f f(t) \\ x(t) = \theta(t) \quad ; \forall t \in [-\xi, 0] \end{cases} \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^m$ and $y(t) \in R^p$ indicate the state vector, the input vector and the output vector, respectively. The constant matrices A, A_h, A_d, B and C are here known and have proper dimensions. While, the disturbance input and the fault vector are denoted by $d(t) \in L_2^P[0, \infty[$ and $f(t) \in R^l$. Besides, the matrices B_f, B_d, D_f and D_d are real known ones. Both h and d represent the constant time-delays. Whereas, $\xi = \max\{h, d\}$ and $\theta(t)$ is a continuous initial function.
Let us consider the parameter uncertainties given through the following forms:

$$\overline{A} = A + \Delta A,\ \overline{A}_h = A_h + \Delta A_h,\ \overline{A}_d = A_d + \Delta A_d, \\ and \quad \overline{B} = B + \Delta B.$$

Thus, the argument Δ of uncertain matrices depends on the time t with ΔA, ΔA_h, ΔA_d and ΔB. As well, the norm bounded parameter uncertainties terms are given as:

$$\begin{bmatrix} \Delta A & \Delta A_h & \Delta A_d & \Delta B \end{bmatrix} = \\ \begin{bmatrix} E_1 \sum_1 F_1 & E_2 \sum_2 F_2 & E_3 \sum_3 F_3 & E_4 \sum_4 F_4 \end{bmatrix} \tag{2}$$

where E_1, E_2, E_3, E_4, F_1, F_2, F_3 and F_4 are known constant matrices. After that, the uncertain character is reduced to the matrices which are assumed that:

$$\textstyle\sum_1^T \sum < I,\ \sum_2^T \sum < I,\ \sum_3^T \sum < I \text{ and } \sum_4^T \sum < I$$

Where $\sum_1$, $\sum_2$, $\sum_3$ and $\sum_4$ denote parameter uncertainties and are added to the nominal matrices cited above (A, A_h, A_d and B). Whilst, the identity matrix, I, possesses a suitable dimension.
Hence, a linear uncertain neutral system with time-delays by adding disturbance and fault signals is rewritten as:

$$\begin{cases} \dot{x}(t) = \bar{A}x(t) + \bar{A}_h x(t-h) + \bar{A}_d \dot{x}(t-d) \\ \qquad\qquad +\bar{B}u(t) + B_d d(t) + B_f f(t) \\ y(t) = Cx(t) + D_d d(t) + D_f f(t) \\ x(t) = \theta(t) \quad ; \forall t \in [-\xi, 0] \end{cases} \tag{3}$$

Using the Newton-Lebuniz formula,

$$x(t-h) = x(t) - \int_{-h}^{0} \dot{x}(t+\alpha)\, d\alpha$$

Then, the last formula is substituted into system (1), we get the following form:

$$\begin{cases} \dot{x}(t) = (A + A_h)x(t) + A_h \int_{-h}^{0} \dot{x}(t+\alpha) d\alpha + A_d \dot{x}(t-d) \\ \qquad\qquad +Bu(t) + B_d d(t) + B_f f(t) \\ y(t) = Cx(t) + D_f f(t) + D_d d(t) \\ x(t) = \theta(t) \quad ; \forall t \in [-\xi, 0] \end{cases} \tag{4}$$

III. ROBUST DELAY-DEPENDENT STABILITY ISSUE

A. Certain neutral time-delay system case

The sequel investigates stability study task dedicated to the linear certain neutral systems together with time-delays. The purpose is to created a new stability conditions in convex form solved by using the LMI technique. The theorem proposed herein is an extension of the results already found in the research works of [4], [5]. An open-loop system stability criterion is developed for (4). The condition is determined by the below theorem.
Theorem 1:
Given the considered system in the certain case (1). This latter is asymptotically stable, if there exists a scalar γ and positive symmetric definite matrices X, T and Y satisfying this LMI:

$$\begin{bmatrix} M(X,A,A_h) & 0 & A_d Y & B & B_f + C^T D_f & B_d + C^T D_d & XA^T & XC^T & X & A_h Y \\ * & -T & 0 & 0 & 0 & 0 & TA_h^T & 0 & 0 & 0 \\ * & * & -Y & 0 & 0 & 0 & YA_d^T & 0 & 0 & 0 \\ * & * & * & I & 0 & 0 & B^T & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I + D_f^T D_d & 0 & B_f^T & 0 & 0 & 0 \\ * & * & * & * & * & D_d^T D_d & B_d^T & 0 & 0 & 0 \\ * & * & * & * & * & * & -\frac{1}{\Psi} Y & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & * & -T & 0 \\ * & * & * & * & * & * & * & * & * & -\frac{1}{h} Y \end{bmatrix} < 0 \tag{5}$$

$$M(X, A, A_h) = (A + A_h)X + X(A + A_h)^T$$

where $\Psi = (1 + h)$ and the identity matrix is indicated via I. Noting that the transposed element at the symmetric position is designated through the symbol $(*)$ in a symmetric matrix.

The verification condition of the LMI (4) results if the studied system (1) is asymptotically stable and fulfills the H_∞ performance.

$$\|H_{yf}\|_\infty < \gamma \tag{6}$$

Proof:
During this subsection, the focus is about ensuring the optimisation problem (that to say the minimization) between the influence of the fault effect and the output signal(denoting $f(t)$ and $y(t)$, respectively). This performance (5) with initial condition is described by this equation:

$$J = \sup_{f\in\ L_2-0} \frac{\|y\|_2}{\|f\|_2} < \gamma \tag{7}$$

So that study the stability related to the considered system (1), the criterion J will be chosen to minimize the energy as specified below:

$$J = \int_0^\infty y^T(t)y(t) - \gamma^2 f^T(t)f(t)dt < 0 \tag{8}$$

Owing to the unknown inputs $d(t)$, the robust stability condition for the neutral system with time-delays (1) can be determined. Thus, optimization problem is based on the norm H_∞ by giving a scalar γ as shown though the equation (6).We introduce the quadratic Lyapunov-Krasovskii functional $V(x,t)$ with a simple integral chosen according to the states and the states derivatives represented in this form:

$$\begin{aligned} V(x,t) &= x^T(s)Px(s) + \int_{t-h}^t x^T(s)Qx(s)ds \\ &+ \int_{t-d}^t \dot{x}^T(s)S\dot{x}(s)ds + \int_{-h}^0 \left(\int_{t+\alpha}^t \dot{x}^T(s)S\dot{x}(s)ds\right)d\alpha \end{aligned} \tag{9}$$

where the matrices Q, S and P are symmetric positive definite ones. The Lyapunov-Krasovskii function comes by:

$$\begin{aligned} &\dot{V}(x,t) = 2\dot{x}^T(t)Px(t) + x^T(t)Qx(t) - x^T(t-h)Qx(t-h) \\ &+\dot{x}^T(t)S\dot{x}(t) - \dot{x}^T(t-d)S\dot{x}(t-d) + \int_{-h}^0 \left[\dot{x}^T(t)S\dot{x}(t)\right. \\ &\qquad \left. -\dot{x}^T(t+\alpha)S\dot{x}(t+\alpha)\right] d\alpha \\ &= 2\dot{x}^T(t)Px(t) + x^T(t)Qx(t) - x^T(t-h)Qx^T(t-h) \\ &\qquad +(1+h)\dot{x}^T(t)S\dot{x}(t) - \dot{x}^T(t-d)S\dot{x}(t-d) \\ &\qquad\qquad - \int_{-h}^0 \dot{x}^T(t+\alpha)S\dot{x}(t+\alpha)d\alpha \\ &= x^T(t)\left(P(A+A_h) + (A+A_h)^T P + Q\right)x(t) \\ &-2x^T(t)P\int_{-h}^0 A_h\dot{x}(t+\alpha)d\alpha + 2x^T(t)PA_d\dot{x}(t-d) \\ &+2x^T(t)PBu(t) + 2x^T(t)PB_f f(t) \\ &+2x^T(t)PB_d d(t)(t)S\dot{x}(t) - x^T(t-h)Qx^T(t-h) + \Psi\dot{x}^T S\dot{x} \\ &-\dot{x}^T(t-d)S\dot{x}(t-d) - \int_{-h}^0 \dot{x}^T(t+\alpha)S\dot{x}(t+\alpha)d\alpha \\ &-2x^T(t)P\int_{-h}^0 A_h\dot{x}(t+\alpha)d\alpha = -\int_{-h}^0 2x^T(t)PA_h\dot{x}(t+\alpha)d\alpha \\ &\le \int_{-h}^0 x^T(t)PA_hS^{-1}A_h^TPx(t) + \dot{x}^T(t+\alpha)S\dot{x}(t+\alpha)d\alpha \\ &\le hx^T(t)PA_hS^{-1}A_h^TPx(t) + \int_{-h}^0 \dot{x}^T(t+\alpha)S\dot{x}(t+\alpha)d\alpha \end{aligned} \tag{10}$$

Substituting also,

$$\begin{aligned} &\dot{x}^T(S+hS)\dot{x} = \\ &x^T(t)A^T\Psi SAx(t) + 2x^T(t)A^T\Psi SA_hx(t-h) \\ &+2x^T(t)A^T\Psi SA_d\dot{x}(t-d) + 2x^T(t)A^T\Psi SBu(t) \\ &+2x^T(t)A^T\Psi SB_f f(t) + 2x^T(t)A^T\Psi SB_d d(t) \\ &+2x^T(t-h)A_h^T\Psi SA_d\dot{x}(t-d) + 2x^T(t-h)A_h^T\Psi SBu(t) \\ &+2x^T(t-h)A_h^T\Psi SB_f f(t) + 2x^T(t-h)A_h^T\Psi SB_d d(t) \\ &+x^T(t-h)A_h^T\Psi SA_hx(t-h) + 2\dot{x}^T(t-d)A_d^T\Psi SBu(t) \\ &+2\dot{x}^T(t-d)A_d^T\Psi SB_f f(t) + 2\dot{x}^T(t-d)A_d^T\Psi SB_d d(t) \\ &+\dot{x}^T(t-d)A_d^T\Psi SA_d\dot{x}(t-d) + u^T(t)B^T\Psi SBu(t) \\ &+u^T(t)B^T\Psi SB_f f(t) + u^T(t)B^T(1+h)SB_d d(t) \\ &+d^T(t)B_d\Psi SBu(t) + d^T(t)B_d\Psi SB_f f(t) \\ &+d^T B_d\Psi SB_d d(t) + f^T(t)B_f^T SBu(t) \\ &+f^T(t)B_f^T SB_f f(t) + f^T(t)B_f^T SB_d d(t) \end{aligned} \tag{11}$$

Ultimately, Lyapunov functional is deduced as:

$$\begin{aligned} &= x^T(t)\left(P(A+A_h) + (A+A_h)^T P + Q + h\,PA_h\,S^{-1}A_h^T P\right)x(t) \\ &+2x^T(t)PA_d\dot{x}(t-d) + 2x^T(t)PBu(t) + 2x^T(t)PB_f f(t) \\ &+2x^T(t)PB_d d(t) - x^T(t-h)SQx(t-h) + x^T(t)A^T\Psi SAx(t) \\ &+2x^T(t)A^T\Psi SA_hx(t-h) + 2x^T(t)A^T\Psi SA_d\dot{x}(t-d) \\ &+2x^T(t)A^T\Psi SBu(t) + 2x^T(t)A^T\Psi SB_f f(t) \\ &+2x^T(t)A^T\Psi SB_d d(t) + 2x^T(t-h)A_h^T\Psi SA_d\dot{x}(t-d) \\ &+2x^T(t-h)A_h^T\Psi SBu(t) + 2x^T(t-h)A_h^T\Psi SB_f f(t) \\ &+2x^T(t-h)A_h^T\Psi SB_d d(t) + x^T(t-h)A_h^T\Psi SA_hx(t-h) \\ &+2\dot{x}^T(t-d)A_d^T\Psi SBu(t) + 2\dot{x}^T(t-d)A_d^T\Psi SB_f f(t) \\ &+2\dot{x}^T(t-d)A_d^T\Psi SB_d d(t) + \dot{x}^T(t-d)A_d^T\Psi SA_d\dot{x}(t-d) \\ &+u^T(t)B^T\Psi SBu(t) + u^T(t)B^T\Psi SB_f f(t) \\ &+u^T(t)B^T\Psi SB_d d(t) + d^T(t)B_d\Psi SBu(t) \\ &+d^T B_d\Psi SB_f f(t) + d^T(t)B_d\Psi SB_d d(t) \\ &+f^T(t)B_f^T SBu(t) + f^T(t)B_f^T SB_f f(t) + f^T(t)B_f^T SB_d d(t) \\ &-\dot{x}^T(t-d)S\,\dot{x}(t-d) \end{aligned} \tag{12}$$

In addition, the criterion function to be minimized as follows:

$$\begin{aligned} J = \int_0^\infty y^T(t)y(t) - \gamma^2 f^T(t)f(t) + \dot{V}(x,t)dt + V(x,t)|_{t=0} \\ -V(x,t)|_{t=\infty} \end{aligned} \tag{13}$$

with

$$\begin{aligned} &y^T(t)y(t) = \left[Cx(t) + D_f f(t) + D_d d(t)\right]^T Cx(t) \\ &+D_f f(t) + D_d d(t) = x^T(t)C^TCx(t) + x^T(t)C^TD_f f(t) \\ &+x^T(t)C^TD_d d(t) + f^T(t)D_f^TCx(t) + f^T(t)D_f^TD_f f(t) \\ &+f^T(t)D_f^TD_d d(t) + d^T(t)D_d^TCx(t) + d^T(t)D_d^TD_f f(t) \\ &+d^T(t)D_d^TD_d d(t) \end{aligned} \tag{14}$$

The equality (13) can be simplified as follows:

$$J = \int_0^\infty \left\{\phi^T(t)\,\eta\,\phi(t)\right\}dt \tag{15}$$

wherein:
$\phi(t) =$
$\left[\begin{array}{cccccc} x^T(t) & x^T(t-h) & \dot{x}^T(t-d) & u^T(t) & f^T(t) & d^T(t) \end{array}\right]^T$

By this way, the index J is afforded as:
$\eta(h) =$

$$\begin{bmatrix} M(h) & \Psi A^T S A_h & PA_d + \Psi A^T S A_d & PB + \Psi A^T SB & C^T D_f + PB_f + \Psi A^T SB_f & C^T D_d + PB_d + \Psi A^T SB_d \\ * & \Psi A_h^T S A_h - Q & \Psi A_h^T S A_d & \Psi A_h^T SB & \Psi A_h^T SB_f & \Psi A_h^T SB_d \\ * & * & \Psi A_d^T S A_d - S & \Psi A_d^T S A_d B & \Psi A_d^T SB_f & \Psi A_d^T SB_d \\ * & * & * & \Psi B^T SB - I & \Psi B^T SB_f & \Psi B^T SB_d \\ * & * & * & * & -\gamma^2 I + D_f^T D_d + \Psi B_f^T SB_f & D_f^T D_d + \Psi B_f^T SB_d \\ * & * & * & * & * & D_d^T D_d + \Psi B_d^T SB_d \end{bmatrix} \quad (16)$$

Where
$M(h) = P(A+A_h) + (A+A_h)^T P + Q + hPA_h S^{-1} A_h^T P + \Psi A^T S A + C^T C$
The matrix $\eta(h) < 0$ may hereafter be reformulated with reference to the Schur complement:

$$\begin{bmatrix} M(P,Q) & 0 & PA_d & PB & C^T D_f + PB_f & C^T D_d + PB_d & A^T & PA_h \\ * & -Q & 0 & 0 & 0 & 0 & A_h^T & 0 \\ * & * & -S & 0 & 0 & 0 & A_d^T & 0 \\ * & * & * & -I & 0 & 0 & B^T & 0 \\ * & * & * & * & -\gamma^2 I + D_f^T D_d & D_f^T D_d & B_f^T & 0 \\ * & * & * & * & * & D_d^T D_d & B_d^T & 0 \\ * & * & * & * & * & * & -\frac{1}{\Psi} S^{-1} & 0 \\ * & * & * & * & * & * & * & -\frac{1}{h} Y \end{bmatrix} < 0 \quad (17)$$

where
$M(P,Q) = P(A+A_h) + (A+A_h)^T P + Q + C^T C$

Let us consider $diag(X,T,Y,I,Y)$ and multiply equation (17) on both sides through the last matrix. After that, we apply Schur complement in order to get these LMIs:

$$\begin{bmatrix} M(X,A,A_h) & 0 & A_d Y & B & B_f + C^T D_f & B_d + C^T D_d & XA^T & XC^T & X & A_h Y \\ * & -T & 0 & 0 & 0 & 0 & TA_h^T & 0 & 0 & 0 \\ * & * & -Y & 0 & 0 & 0 & YA_d^T & 0 & 0 & 0 \\ * & * & * & I & 0 & 0 & B^T & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I + D_f^T D_d & 0 & B_f^T & 0 & 0 & 0 \\ * & * & * & * & * & D_d^T D_d & B_d^T & 0 & 0 & 0 \\ * & * & * & * & * & * & -\frac{1}{\Psi} Y & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & * & -T & 0 \\ * & * & * & * & * & * & * & * & * & -\frac{1}{h} Y \end{bmatrix} < 0 \quad (18)$$

where
$M(X,A,A_h) = (A+A_h)X + X(A+A_h)^T$.

Thus, under the condition of Theorem 1, if the inequalities (7) hold then $\dot{V} < 0$.

End proof.

B. *Uncertain neutral time-delays system case*

In this part, we address on the robust stability issue dedicated to the uncertain linear neutral system with time delays (3).

Theorem 2:
Given the system (3) with parameter uncertainties. This latter is asymptotically stable, if there exist three matrices K, H and R which are symmetric, positive and definite. The LMI

is feasible such that

$$\left[\begin{array}{cccccccccc} M(H,\overline{A},\overline{A}_h) & 0 & \overline{A}_d R & \overline{B} & B_f + C^T D_f & B_d + C^T D_d & H\overline{A}^T & HC^T & H & \overline{A}_h R \\ * & -K & 0 & 0 & 0 & 0 & K\overline{A}_h^T & 0 & 0 & 0 \\ * & * & -R & 0 & 0 & 0 & RA_d^T & 0 & 0 & 0 \\ * & * & * & I & 0 & 0 & \overline{B}^T & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I + D_f^T D_d & 0 & B_f^T & 0 & 0 & 0 \\ * & * & * & * & * & D_d^T D_d & B_d^T & 0 & 0 & 0 \\ * & * & * & * & * & * & -\frac{1}{\Psi}R & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & * & -K & 0 \\ * & * & * & * & * & * & * & * & * & -\frac{1}{h}R \end{array}\right] < 0 \tag{19}$$

$$M(X,\overline{A},\overline{A}_h) = (\overline{A}+\overline{A}_h)H + H(\overline{A}+\overline{A}_h)^T$$

Remark: The proof details for Theorem 2 are identical to the demonstration of Theorem 1; however, the difference between the LMI (5) and (19) is firstly adding the parameter uncertainties in the system model, secondly, taking the matrix $diag(H,K,R,I,R)$ and multiplying the equation (17) on both sides using this diagonal matrix cited herein.

IV. NUMERICAL EXAMPLE

The proposed theorems will be illustrated in this sequel through a pedagogical example so that to reveal the efficiency of the alternative robust stability analysis associated to linear neutral time-delay systems with/without parameter uncertainties. The considered model is cited in the literature [9].

A. Certain case

Let take into account the system (1) together with defined matrices such as:

$$A = \begin{bmatrix} -8 & 1 \\ 0 & -10 \end{bmatrix}, \; A_h = \begin{bmatrix} -1 & 0 \\ 0.8 & -1 \end{bmatrix},$$
$$A_d = \begin{bmatrix} 0.5 & -1 \\ 0.3 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$C = \begin{bmatrix} 0.7 & 0 \\ 0 & 0.7 \end{bmatrix}$$
$$B_d = \begin{bmatrix} 0 & 0.01 \\ 0.01 & 0 \end{bmatrix}, \; B_f = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$D_d = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad D_f = \begin{bmatrix} 1 & 0.1 \\ 0.1 & 1 \end{bmatrix} \text{ and } h = 1.$$

These matrices are reconstructed and taken so as to satisfy the simulation require. Thus, the studied system (1) is stable after solving the LMI (5) involvoling Theorem 1. The corresponding matrices are deduced as follow:

$$X = \begin{bmatrix} 7.8210 & 2.2378 \\ 2.2378 & 8.3019 \end{bmatrix} > 0$$

$$Y = \begin{bmatrix} 37.9770 & 15.3241 \\ 15.3241 & 57.1763 \end{bmatrix} > 0$$

$$T = \begin{bmatrix} 14.0278 & 23.6799 \\ 23.6799 & 55.9986 \end{bmatrix} > 0$$

It is clear that the matrices Y, T and X are all positive. Therefore, the corresponding system is asymptotically stable.

B. Uncertain case

Along this sequel, let consider the parameter uncertainties linked to the investigated system (3) which is defined using these matrices:

$$E_1 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \; E_2 = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix},$$
$$E_3 = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix} \; and, \quad E_4 = \begin{bmatrix} -0.1 \\ 0 \end{bmatrix}$$
$$F_1 = \begin{bmatrix} 0.001 & 0.3 \end{bmatrix}, \; F_2 = \begin{bmatrix} 0.01 & 0.2 \end{bmatrix},$$
$$F_3 = \begin{bmatrix} 0.01 & 0.2 \end{bmatrix} \; and, \quad F_4 = 0.02$$

While the uncertain parameter characteristics are defined by

$$\textstyle\sum_1 = \sum_2 = \sum_3 = \sum_4 = 0.2.$$

Thereby, the uncertainties matrices are obtained as:

$$\Delta A = \begin{bmatrix} 0.02 & 0.06 \\ 0.02 & 0.06 \end{bmatrix}, \; \Delta B = \begin{bmatrix} -0.4 & 0 \end{bmatrix}$$
$$\Delta A_h = \begin{bmatrix} 0.02 & 0.04 \\ -0.02 & -0.04 \end{bmatrix}, \Delta A_d = \begin{bmatrix} 0.02 & 0.04 \\ -0.02 & -0.04 \end{bmatrix}$$

Referring to Theorem 2, the Linear Matrix Inequality (19) is feasible. Thus, it is clear that the investigated system (3) is stable and the corresponding matrices can be got as:

$$H = \begin{bmatrix} 7.8775 & 2.7021 \\ 2.7021 & 8.7533 \end{bmatrix} > 0$$

$$R = \begin{bmatrix} 40.5155 & 20.3529 \\ 20.3529 & 63.5264 \end{bmatrix} > 0$$

$$K = \begin{bmatrix} 19.7204 & 31.4308 \\ 31.4308 & 63.9750 \end{bmatrix} > 0$$

Therefore, the corresponding open-loop system with parameter uncertainties is asymptotically stable.

V. CONCLUSIONS

The robust delay-dependent stability issue for uncertain neutral systems with state time-delays has been studied throughout this paper. In this regard, two theorems have been developed and proven based on the LMI formulation and the Lyapunov-Krasovskii theory so as to ensure robust asymptotically stability for this class of investigated systems. The numerical example has been used to illustrate the relevance of the stability conditions against disturbances and in faulty situations.

REFERENCES

[1] Li, Zhichen and Yan, Huaicheng and Zhang, Hao and Peng, Yan and Park, Ju H and He, Yong. Stability analysis of linear systems with time-varying delay via intermediate polynomial-based functions, Elsevier, Journal of Automatica, vol. 113, pp. 108756, 2020.

[2] Wu, Qian and Song, Qiankun and Hu, Binxin and Zhao, Zhenjiang and Liu, Yurong and Alsaadi, Fuad E, Robust stability of uncertain fractional order singular systems with neutral and time-varying delays, Neurocomputing ,Elsevier, vol. 401, no. 1, pp. 145-152, (2020).

[3] Soltani, H. and Bel Hadj Ali Naoui, S. and El Harabi, R. and Aitouche, A. and Abdelkrim, M. N, Fault Tolerant Control for Uncertain State Time-delay Systems, Journal of Advances in Systems, Signals and Devices, 2016.

[4] Medvedeva, I.V. and Zhabko, A.P.,A Novel Approach to Robust Stability Analysis of Linear Time-Delay System, Elsevier, Journal of IFAC-PapersOnLine, vol. 48, no. 12, pp. 233–238, 2015

[5] Medvedeva, I.V. and Zhabko, A.P. Synthesis of Razumikhin and Lyapunov–Krasovskii approaches to stability analysis of time-delay systems, Elsevier, Journal of Automatica, vol. 51, pp. 372–377, 2015.

[6] Liu, Muyang and Dassios, Ioannis and Milano, Federico. On the stability analysis of systems of neutral delay differential equations, Springer, Journal of Circuits, Systems, and Signal Processing, vol.38 no.4, pp. 1639–1653, 2019.

[7] S.A. Rodriguez, V. Kharitonov, J-M. Dion, and L. Dugard. Robust stability of neutral systems: a lyapunov-krasovskii constructive approach.International Journal of Robust and Nonlinear Control, vol.14, No.16, 1345-1358, 2004.

[8] Sheng, Zhaoliang and Lin, Chong and Chen, Bing and Wang, Qing-Guo. Asymmetric Lyapunov–Krasovskii functional method on stability of time-delay systems, Wiley Online Library, International Journal of Robust and Nonlinear Control, vol. 31, no. 7, pp. 2847–2854, 2021.

[9] Bai L, Tian Z and Shi S, Design of H_∞ robust fault detection filter for linear uncertain time-delay systems. ISA Transactions, vol.45, no. 4, pp. 491-502, 2006