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Robust reliable stabilization of uncertain switched neutral systems with delayed switching

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A R T I C L E I N F O

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A B S T R A C T

This paper investigates the problem of robust reliable control for a class of uncertain switched neutral systems under asynchronous switching, where the switching instants of the controller experience delays with respect to those of the system and the parameter uncertainties are assumed to be norm-bounded. A state feedback controller is proposed to guarantee exponential stability and reliability for switched neutral systems, and the dwell time approach is utilized for the stability analysis and controller design. A numerical example is given to illustrate the effectiveness of the proposed method.

1. Introduction

Switched systems are composed of a finite number of continuous-time or discrete-time subsystems and a switching signal specifying the switching between these subsystems. Such system has attracted considerable attention during the past several decades, because various real-world systems, such as chemical processing [1], communication networks, traffic control [2,3], control of manufacturing systems [4,5], automotive engine control and aircraft control [6] can be modeled as switched systems. Many works in the field of stability analysis and control synthesis for switched systems have appeared (see [7–17]).

It is noticed that the time-delay phenomenon exists widely in engineering and social systems, which may cause instability or bad system performance in control systems. Neutral system is an important class of time-delay systems, which depends not only on the delays of state and but also on the delays of state derivative. Some practical examples of neutral systems include distributed networks, heat exchanges, and processes including steam [18]. Recently, there have been increasing research activities in this direction (see [19–21] and the references cited therein).

On the other hand, the actuators may be subjected to failures in real environment. A control system is said to be reliable if it retains certain properties when there exist failures. It should be noted that in normal cases, a controller with fixed gain is easily implemented, and could meet the requirement in practical applications. But when failure occurs, the conventional controller will become conservative and may not satisfy certain control performance indexes. Reliable control is a kind of effective control approach to improve system reliability, whose objective is to design a controller with suitable structure to guarantee stability and satisfactory performance in the case of actuator malfunction. Since the concept of reliable control was given by Siljak in 1970s, the problem of reliable control has drawn amount of scholars' attention. Several approaches to the design of the reliable controllers have been proposed, and some of which have been further extended to investigate the problem of reliable control for switched systems (see [22–25]).

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Recently, some researchers began to pay attention to switched neutral system because of its numerous applications in real systems. The issues of stability analysis and control synthesis for switched neutral system have been studied in [26–30]. However, as pointed out in [31], there inevitably exists asynchronous switching between the controller and the system in actual operation, *i.e*. the switching instants of the controller exceed or lag behind those of the system. Some results on stabilization of switched neutral systems with the delayed controller switching have been proposed in [32–36]. To the best of our knowledge, the issue of robust reliable control for switched neutral systems under asynchronous switching has not been investigated, which motivated our study.

In this paper, we are interested in designing a reliable stabilizing controller for switched neutral system with delayed switching and actuator fault such that the closed-loop system is exponentially stable. The dwell time approach is utilized for the stability analysis and controller design. The remainder of the paper is organized as follows. In Section 2, problem formulation and some necessary lemmas are given. In Section 3, based on the dwell time approach, stability and stabilization for switched neutral systems with delayed switching are addressed, and sufficient conditions for the existence of a reliable stabilizing controller are derived in terms of a set of matrix inequalities. A numerical example is provided to illustrate the effectiveness of the proposed approach in Section 4. Concluding remarks are given in Section 5.

Notations: Throughout this paper, the superscript "T" denotes the transpose, and the notation $X \geqslant Y$ ($X > Y$) means that matrix $X - Y$ is positive semi-definite (positive definite, respectively). $\|x(t)\|$ denotes the Euclidean norm. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and minimum eigenvalues of matrix P, respectively. I represents identity matrix with appropriate dimension. $diag\{a_i\}$ denotes diagonal matrix with the diagonal elements a_i, $i = 1, 2, \ldots, n$. The asterisk $*$ in a matrix is used to denote term that is induced by symmetry. The set of all nonnegative integers is represented by Z^+.

2. Problem formulation and preliminaries

Consider the following uncertain switched neutral system with actuator fault

$$\dot{x}(t) - C_{\sigma(t)}\dot{x}(t-\tau_1) = \widehat{A}_{\sigma(t)}x(t) + \widehat{B}_{\sigma(t)}x(t-\tau_2) + D_{\sigma(t)}u^f(t), \tag{1}$$

$$x(t_0+\theta) = \varphi(\theta), \quad \theta \in [-\tau, 0], \tag{2}$$

where $x(t) \in R^n$ is the state vector, $u^f(t) \in R^l$ is the control input of actuator fault, $\tau = \max\{\tau_1, \tau_2\}$, $\varphi(\theta)$ is a continuous vector-valued initial function. The function $\sigma(t) : [t_0, \infty) \to \underline{N} = \{1, 2, \ldots, N\}$ is a switching signal which is deterministic, piecewise constant and right continuous, corresponding to it, the switching sequence $\Sigma = \{(t_0, \sigma(t_0)), (t_1, \sigma(t_1)), \ldots, (t_k, \sigma(t_k)), \ldots\}$, $k \in Z^+$, where t_0 is the initial time and t_k denotes the kth switching instant, this means that the matrices $\left(\widehat{A}_{\sigma(t)}, \widehat{B}_{\sigma(t)}, C_{\sigma(t)}, D_{\sigma(t)}\right)$ are allowed to take values in the finite set $\left\{\left(\widehat{A}_1, \widehat{B}_1, C_1, D_1\right), \ldots, \left(\widehat{A}_k, \widehat{B}_k, C_k, D_k\right), \ldots\right\}$ at arbitrary time. Moreover, $\sigma(t) = i$ means that the ith subsystem is activated. N denotes the number of subsystems.

For each $i \in \underline{N}$, $\widehat{A}_i, \widehat{B}_i$ are uncertain real-valued matrices with appropriate dimensions and satisfy

$$\begin{bmatrix} \widehat{A}_i & \widehat{B}_i \end{bmatrix} = [A_i \quad B_i] + H_iF_i(t)[E_{1i} \quad E_{2i}], \tag{3}$$

where A_i, B_i, C_i, D_i, H_i,E_{1i}, E_{2i} are known real-valued constant matrices with appropriate dimensions; $F_i(t)$ are unknown and possibly time-varying matrix with Lebesgue measurable elements, and their Euclidean norm satisfy

$$F_i^T(t)F_i(t) \leqslant I. \tag{4}$$

The control input of actuator fault $u^f(t)$ can be described as

$$u^f(t) = \Omega_{\sigma(t)}u(t), \tag{5}$$

where $u(t) = K_{\sigma(t)}x(t)$ is the control input to be designed, $\Omega_i (i \in \underline{N})$ are the actuator fault matrices with the following form

$$\Omega_i = diag\{\omega_{i1}, \omega_{i2}, \ldots, \omega_{in}\}, \tag{6}$$

where $0 \leqslant \omega_{lik} \leqslant \omega_{ik} \leqslant \omega_{uik}$, $\omega_{uik} \geqslant 1$.

For simplicity, we define

$$\Omega_{i0} = diag\{\tilde{\omega}_{i1}, \tilde{\omega}_{i2}, \ldots, \tilde{\omega}_{in}\}, \quad \tilde{\omega}_{ik} = \frac{1}{2}(\omega_{lik} + \omega_{uik}), \tag{7}$$

$$\Xi_i^2 = diag\{\xi_{i1}^2, \xi_{i2}^2, \ldots, \xi_{in}^2\}, \quad \xi_{ik}^2 = \frac{\omega_{uik} - \omega_{lik}}{\omega_{lik} + \omega_{uik}}, \tag{8}$$

$$\Theta_i = diag\{\theta_{i1}, \theta_{i2}, \ldots, \theta_{in}\}, \quad \theta_{ik} = \frac{\omega_{ik} - \tilde{\omega}_{ik}}{\tilde{\omega}_{ik}}. \tag{9}$$

Thus, we have

$$\Omega_i = \Omega_{i0}(I + \Theta_i), \quad |\Theta_i| \leqslant \Xi_i^2 \leqslant I, \tag{10}$$

where $|\Theta_i| = diag\{|\theta_{i1}|, |\theta_{i2}|, \ldots, |\theta_{in}|\}$.

However, in actual operation, there inevitably exists asynchronous switching between the controller and the system. Without loss of generality, we only consider the case that the switching instants of the controller experience delays with respect to the switching instants of the system. Denote $\sigma'(t)$ the switching signal of the controller, the switching instants of the controller can be described as

$$t_1 + \Delta_1, t_2 + \Delta_2, \ldots, t_k + \Delta_k, \ldots, \quad k \in Z^+, \tag{11}$$

where $\Delta_k < \inf_{k>0}(t_{k+1} - t_k)$, Δ_k represents the delayed period, and it is said to be mismatched period.

Remark 1. Mismatched period $\Delta_k < \inf_{k>0}(t_{k+1} - t_k)$ guarantees that there always exists a period that the controller and the system operate synchronously, and this period is said to be matched period in the later section.

Definition 1 [37]. The equilibrium $x^* = 0$ of systems (1) and (2) is said to be exponentially stable under $\sigma(t)$ if the solution $x(t)$ of systems (1) and (2) satisfies

$$\|x(t)\| \leqslant \alpha\|x(t_0)\|_h e^{-\beta(t-t_0)}, \quad t \geqslant t_0 \tag{12}$$

for constants $\alpha \geqslant 1$ and $\beta > 0$, where $\|x(t_0)\|_h = \sup_{-\tau \leqslant \theta \leqslant 0}\{\|x(t_0+\theta)\|, \|\dot{x}(t_0+\theta)\|\}$, and $\|\cdot\|$ denotes the Euclidean norm.

Definition 2 [38]. For any $T_2 > T_1 \geqslant 0$, let $N_\sigma(T_1,T_2)$ denote the switching number of $\sigma(t)$ on an interval (T_1,T_2). If

$$N_\sigma(T_1, T_2) \leqslant N_0 + \frac{T_2 - T_1}{\tau_a}, \tag{13}$$

hold for given $N_0 \geqslant 0$, $\tau_a > 0$, then the constant τ_a is called the average dwell time and N_0 is the chatter bound.

The following lemmas will be essential for the proofs in our later development.

Lemma 1 [39]. *For matrices R_1, R_2 with appropriate dimension, there exists a positive scalar $\varepsilon > 0$, such that*

$$R_1\Sigma(t)R_2 + R_2^T\Sigma^T(t)R_1^T \leqslant \varepsilon R_1 U R_1^T + \varepsilon^{-1} R_2^T U R_2$$

holds, where $\Sigma(t)$ is time-varying diagonal matrix, U is known real-value matrix satisfying $|\Sigma(t)| \leqslant U$.

Lemma 2 [40]. *Let U, V, W and X be real matrices of appropriate dimensions with X satisfying $X = X^T$, then for all $V^TV \leqslant I$, $X + UVW + W^TV^TU^T < 0$, if and only if there exists a scalar $\delta > 0$ such that $X + \delta UU^T + \delta^{-1}W^TW < 0$.*

The aim of this paper is to design a reliable switching controller for uncertain switched neutral systems (1) and (2) with asynchronous switching such that the closed-loop system is exponentially stable.

3. Main results

3.1. Stability analysis

In this subsection, we focus on the problem of stability analysis for the non-switched neutral systems.

Lemma 3. *Consider the following neutral system*

$$\dot{x}(t) - C\dot{x}(t-\tau_1) = Ax(t) + Bx(t-\tau_2), \tag{14}$$

$$x(t_0+\theta) = \varphi(\theta), \quad \theta \in [-\tau, 0], \tag{15}$$

where A, B, C are constant matrices with appropriate dimensions. For given positive constant α, if there exist positive definite symmetric matrices P, R_1, R_2 with appropriate dimensions, such that

$$\begin{bmatrix} A^TP^{-1} + P^{-1}A + \alpha P^{-1} + R_1^{-1} & P^{-1}B & P^{-1}C & A^T \\ * & -e^{-\alpha\tau_2}R_1^{-1} & 0 & B^T \\ * & * & -e^{-\alpha\tau_1}R_2^{-1} & C^T \\ * & * & * & -R_2 \end{bmatrix} < 0, \tag{16}$$

then, along the trajectory of systems (14) and (15), there holds the following inequality

$$V(x(t)) < e^{-\alpha(t-t_0)}V(x(t_0)). \tag{17}$$

Proof. Consider Lyapunov–Krasovskii functional candidate

$$V(x(t)) = \sum_{i=1}^{3} V_i(x(t)), \tag{18}$$

where $V_1(x(t)) = x^T(t)P^{-1}x(t),\ V_2(x(t)) = \int_{t-\tau_2}^{t} x^T(s)e^{-\alpha(t-s)}R_1^{-1}x(s)ds,\ V_3(x(t)) = \int_{t-\tau_1}^{t} \dot{x}^T(s)e^{-\alpha(t-s)}R_2^{-1}\dot{x}(s)ds.$

Along the trajectory of (14) and (15), we have

$$\dot{V}(x(t)) = \sum_{i=1}^{3} \dot{V}_i(x(t)), \tag{19}$$

where

$$\dot{V}_1(x(t)) = 2x^T(t)P^{-1}\dot{x}(t),$$

$$\dot{V}_2(x(t)) = x^T(t)R_1^{-1}x(t) - x^T(t-\tau_2)e^{-\alpha\tau_2}R_1^{-1}x(t-\tau_2) - \alpha\int_{t-\tau_2}^{t} x^T(s)e^{-\alpha(t-s)}R_1^{-1}x(s)ds,$$

$$\dot{V}_3(x(t)) = \dot{x}^T(t)R_2^{-1}\dot{x}(t) - \dot{x}^T(t-\tau_1)e^{-\alpha\tau_1}R_2^{-1}\dot{x}(t-\tau_1) - \alpha\int_{t-\tau_1}^{t} \dot{x}^T(s)e^{-\alpha(t-s)}R_2^{-1}\dot{x}(s)ds.$$

We can obtain the following relation

$$\dot{V}(x(t)) + \alpha V(x(t)) = X^T(t)\phi X(t),$$

where $X^T(t) = \begin{bmatrix} x^T(t) & x^T(t-\tau_2) & \dot{x}^T(t-\tau_1) \end{bmatrix}$,

$$\phi = \begin{bmatrix} A^TP^{-1} + P^{-1}A + \alpha P^{-1} + R_1^{-1} & P^{-1}B & P^{-1}C \\ * & -e^{-\alpha\tau_2}R_1^{-1} & 0 \\ * & * & -e^{-\alpha\tau_1}R_2^{-1} \end{bmatrix} + \begin{bmatrix} A^T \\ B^T \\ C^T \end{bmatrix} R_2^{-1}[A \quad B \quad C].$$

According to Schur complement lemma, condition (16) is equivalent to $\phi < 0$, that is to say, $\dot{V}(x(t)) + \alpha V(x(t)) < 0$. The proof is completed. □

Lemma 4. *Consider systems (14) and (15), for a given positive constant β, if there exist positive definite symmetric matrices P, R_1, R_2 with appropriate dimensions, such that*

$$\begin{bmatrix} A^TP^{-1} + P^{-1}A - \beta P^{-1} + R_1^{-1} & P^{-1}B & P^{-1}C & A^T \\ * & -e^{\beta\tau_2}R_1^{-1} & 0 & B^T \\ * & * & -e^{\beta\tau_1}R_2^{-1} & C^T \\ * & * & * & -R_2 \end{bmatrix} < 0. \tag{20}$$

Then, along the trajectory of systems (14) and (15), there holds the following inequality

$$V(x(t)) < e^{\beta(t-t_0)}V(x(t_0)). \tag{21}$$

Proof. Following the similar proof of Lemma 3, Lemma 4 can be derived, it is omitted here. □

Remark 2. Lemmas 3 and 4 provide the methods for the estimation of Lyapunov functional candidate which will be used to design the controller for the switched neutral system under asynchronous switching.

3.2. Robust reliable controller design

Consider systems (1) and (2), under asynchronous reliable switching controller $u^f(t) = \Omega_{\sigma'(t)}K_{\sigma'(t)}x(t)$, the corresponding closed-loop system is given by

$$\dot{x}(t) - C_{\sigma(t)}\dot{x}(t-\tau_1) = \left(\widehat{A}_{\sigma(t)} + D_{\sigma(t)}\Omega_{\sigma'(t)}K_{\sigma'(t)}\right)x(t) + \widehat{B}_{\sigma(t)}x(t-\tau_2), \tag{22}$$

$$x(t_0+\theta) = \varphi(\theta), \quad \theta \in [-\tau, 0]. \tag{23}$$

Suppose that the ith subsystem is activated at the switching instant t_k, the jth subsystem is activated at the switching instant t_{k+1}, the corresponding switching controller is activated at the switching instant $t_k + \Delta_k$, $t_{k+1} + \Delta_{k+1}$, respectively.

Let $T^{+}(t_0,t)$ denote the total mismatched period during $[t_0,t)$, $T^{-}(t_0,t)$ denote the total matched period during $[t_0,t)$, then we can get the following result.

Theorem 1. *Consider systems (1) and (2), for given positive constants* α, β, *if there exists positive definite symmetric matrices* P_i, R_{1i}, R_{2i}, P_{ij}, R_{1ij}, R_{2ij}, *matrices* W_i *with appropriate dimensions, and positive scalars* ε_i, δ_i, ε_{ij}, δ_{ij} *such that, for* $i, j \in \underline{N}$, $i \neq j$, *the following inequalities (24) and (25) hold.*

$$\begin{bmatrix} \Pi_{11i} & B_iR_{1i} & C_iR_{2i} & \Pi_{14i} & P_i & W_i^T\Xi_i & P_iE_{1i}^T \\ * & -e^{-\alpha\tau_2}R_{1i} & 0 & R_{1i}B_i^T & 0 & 0 & R_{1i}E_{2i}^T \\ * & * & -e^{-\alpha\tau_1}R_{2i} & R_{2i}C_i^T & 0 & 0 & 0 \\ * & * & * & \Pi_{44i} & 0 & 0 & 0 \\ * & * & * & * & -R_{1i} & 0 & 0 \\ * & * & * & * & * & -\varepsilon_iI & 0 \\ * & * & * & * & * & * & -\delta_iI \end{bmatrix} < 0, \tag{24}$$

$$\begin{bmatrix} \Pi_{11ij} & B_jR_{1ij} & C_jR_{2ij} & \Pi_{14ij} & P_{ij} & P_{ij}K_i^T\Xi_i & P_{ij}E_{1j}^T \\ * & -e^{\beta\tau_2}R_{1ij} & 0 & R_{1ij}B_j^T & 0 & 0 & R_{1ij}E_{2j}^T \\ * & * & -e^{\beta\tau_1}R_{2ij} & R_{2ij}C_j^T & 0 & 0 & 0 \\ * & * & * & \Pi_{44ij} & 0 & 0 & 0 \\ * & * & * & * & -R_{1ij} & 0 & 0 \\ * & * & * & * & * & -\varepsilon_{ij}I & 0 \\ * & * & * & * & * & * & -\delta_{ij}I \end{bmatrix} < 0. \tag{25}$$

Then, under the reliable switching controller $u^f(t) = \Omega_{\sigma'(t)}K_{\sigma'(t)}x(t),\ K_i = W_iP_i^{-1}$ and the following average dwell time scheme

$$\inf_{t>t_0} \frac{T^{-}(t_0,t)}{T^{+}(t_0,t)} \geqslant \frac{\lambda^{+}+\lambda^{*}}{\lambda^{-}-\lambda^{*}}, \quad \tau_a > \tau_a^{*} = \frac{\ln(\mu_1\mu_2)}{\lambda^{*}}, \tag{26}$$

the corresponding closed-loop system is exponentially stable, where

$$\Pi_{11i} = A_iP_i + P_iA_i^T + \alpha P_i + D_i\Omega_{i0}W_i + W_i^T\Omega_{i0}^TD_i^T + \varepsilon_iD_i\Omega_{i0}\Xi_i^2\Omega_{i0}^TD_i^T + \delta_iH_iH_i^T,$$

$$\Pi_{14i} = P_iA_i^T + W_i^T\Omega_{i0}^TD_i^T + \varepsilon_iD_i\Omega_{i0}\Xi_i^2\Omega_{i0}^TD_i^T + \delta_iH_iH_i^T,$$

$$\Pi_{44i} = -R_{2i} + \varepsilon_iD_i\Omega_{i0}\Xi_i^2\Omega_{i0}^TD_i^T + \delta_iH_iH_i^T,$$

$$\Pi_{11ij} = A_jP_{ij} + P_{ij}A_j^T - \beta P_{ij} + D_j\Omega_{i0}K_iP_{ij} + P_{ij}K_i^T\Omega_{i0}^TD_j^T + \varepsilon_{ij}D_j\Omega_{i0}\Xi_i^2\Omega_{i0}^TD_j^T + \delta_{ij}H_jH_j^T,$$

$$\Pi_{14ij} = P_{ij}A_j^T + P_{ij}K_i^T\Omega_{i0}^TD_j^T + \varepsilon_{ij}D_j\Omega_{i0}\Xi_i^2\Omega_{i0}^TD_j^T + \delta_{ij}H_jH_j^T,$$

$$\Pi_{44ij} = -R_{2ij} + \varepsilon_{ij}D_j\Omega_{i0}\Xi_i^2\Omega_{i0}^TD_j^T + \delta_{ij}H_jH_j^T,$$

$\lambda^{+} = \beta$, $\lambda^{-} = \alpha$, $0 < \lambda^{*} < \lambda^{-}$, μ_1, $\mu_2 > 1$ satisfying

$$P_i^{-1} < \mu_1P_{ij}^{-1}, \quad P_{ij}^{-1} < \mu_2P_i^{-1}, R_{1i}^{-1} < \mu_1R_{1ij}^{-1}, \quad R_{1ij}^{-1} < \mu_2R_{1i}^{-1}, \quad R_{2i}^{-1} < \mu_1R_{2ij}^{-1}, \quad R_{2ij}^{-1} < \mu_2R_{2i}^{-1}. \tag{27}$$

Proof. When $t \in [t_k + \Delta_k, t_{k+1})$, the closed-loop system can be written as

$$\dot{x}(t) - C_i\dot{x}(t-\tau_1) = \left(\widehat{A}_i + D_i\Omega_iK_i\right)x(t) + \widehat{B}_ix(t-\tau_2). \tag{28}$$

For system (28), we consider Lyapunov functional candidate as follows

$$V_i(x(t)) = x^T(t)P_i^{-1}x(t) + \int_{t-\tau_2}^{t} x^T(s)e^{-\alpha(t-s)}R_{1i}^{-1}x(s)ds + \int_{t-\tau_1}^{t} \dot{x}^T(s)e^{-\alpha(t-s)}R_{2i}^{-1}\dot{x}(s)ds.$$

By Lemma 3, we know that if there exist positive definite symmetric matrices P_i, R_{1i}, R_{2i} with appropriate dimensions, such that

$$\begin{bmatrix} \left(\widehat{A}_i + D_i\Omega_iK_i\right)^T P_i^{-1} + P_i^{-1}\left(\widehat{A}_i + D_i\Omega_iK_i\right) + \alpha P_i^{-1} + R_{1i}^{-1} & P_i^{-1}\widehat{B}_i & P_i^{-1}C_i & \widehat{A}_i^T \\ * & -e^{-\alpha\tau_2}R_{1i}^{-1} & 0 & \widehat{B}_i^T \\ * & * & -e^{-\alpha\tau_1}R_{2i}^{-1} & C_i^T \\ * & * & * & -R_{2i} \end{bmatrix} < 0, \tag{29}$$

then the following matrix inequalities hold

$$V_i(x(t)) < e^{-\alpha(t-t_0^i)}V_i(x(t_0^i)), \tag{30}$$

where t_0^i represents the initial value of the ith controller.

From (30), we can obtain that

$$\|x(t)\| < \sqrt{\frac{b_1}{a_1}} \cdot e^{-\alpha(t-t_0^i)/2}\|x(t_0^i)\|_h, \quad t \in [t_k + \Delta_k, t_{k+1}) \tag{31}$$

where

$$a_1 = \frac{1}{\lambda_{\max}(P_i)}, \quad b_1 = \frac{1}{\lambda_{\min}(P_i)} + \frac{\tau_2}{\lambda_{\min}(R_{1i})} + \frac{\tau_1}{\lambda_{\min}(R_{2i})}.$$

When $t \in [t_{k+1}, t_{k+1} + \Delta_{k+1})$, closed-loop system can be written as:

$$\dot{x}(t) - C_j\dot{x}(t-\tau_1) = \left(\widehat{A}_j + D_j\Omega_iK_i\right)x(t) + \widehat{B}_jx(t-\tau_2). \tag{32}$$

For system (32), we consider Lyapunov functional candidate as follows

$$V_{ij}(x(t)) = x^T(t)P_{ij}^{-1}x(t) + \int_{t-\tau_2}^t x^T(s)e^{\beta(t-s)}R_{1ij}^{-1}x(s)ds + \int_{t-\tau_1}^t \dot{x}^T(s)e^{\beta(t-s)}R_{2ij}^{-1}\dot{x}(s)ds.$$

According to Lemma 4, we know that if there exists positive definite symmetric matrices P_{ij}, R_{1ij}, R_{2ij} with appropriate dimensions, such that

$$\begin{bmatrix} \left(\widehat{A}_j + D_j\Omega_iK_i\right)^T P_{ij}^{-1} + P_{ij}^{-1}\left(\widehat{A}_j + D_j\Omega_iK_i\right) - \beta P_{ij}^{-1} + R_{1ij}^{-1} & P_{ij}^{-1}\widehat{B}_j & P_{ij}^{-1}C_{ij} & \left(\widehat{A}_j + D_j\Omega_iK_i\right)^T \\ * & -e^{\beta\tau_2}R_{1ij}^{-1} & 0 & \widehat{B}_j^T \\ * & * & -e^{\beta\tau_1}R_{2ij}^{-1} & C_j^T \\ * & * & * & -R_{2ij} \end{bmatrix} < 0, \tag{33}$$

then the following inequality holds

$$V_{ij}(x(t)) < e^{\beta(t-t_0^{ij})}V_{ij}\left(x\left(t_0^{ij}\right)\right), \tag{34}$$

where t_0^{ij} represents the initial value of the jth subsystem.

From (34), we can obtain that

$$\|x(t)\| < \sqrt{\frac{b_2}{a_2}} \cdot e^{\beta(t-t_0^{ij})/2}\|x(t_0^{ij})\|_h, \quad t \in [t_{k+1}, t_{k+1} + \Delta_{k+1}), \tag{35}$$

where $a_2 = \frac{1}{\lambda_{\max}(P_{ij})}$, $b_2 = \frac{1}{\lambda_{\min}(P_{ij})} + \frac{\tau_2}{\lambda_{\min}(R_{1ij})} + \frac{\tau_1}{\lambda_{\min}(R_{2ij})}$.

Denote $t_0, t_1, \ldots, t_k$ the switching instants in $[t_0, t)$, we consider the following piece-wise Lyapunov functional candidate for the closed-loop system

$$V(t) = \begin{cases} x^T(t)P_i^{-1}x(t) + \int_{t-\tau_2}^t x^T(s)e^{-\alpha(t-s)}R_{1i}^{-1}x(s)ds + \int_{t-\tau_1}^t \dot{x}^T(s)e^{-\alpha(t-s)}R_{2i}^{-1}\dot{x}(s)ds \\ t \in [t_{n-1} + \Delta_{n-1}, t_n), \quad n = 1, 2, \ldots, k, \\ x^T(t)P_{ij}^{-1}x(t) + \int_{t-\tau_2}^t x^T(s)e^{\beta(t-s)}R_{1ij}^{-1}x(s)ds + \int_{t-\tau_1}^t \dot{x}^T(s)e^{\beta(t-s)}R_{2ij}^{-1}\dot{x}(s)ds \\ t \in [t_n, t_n + \Delta_n), \quad n = 0, 1, \ldots, k. \end{cases} \tag{36}$$

From conditions (27), (30) and (34), for $t \geqslant t_k + \Delta_k$, we have

$$\begin{aligned} V(t) &< e^{-\alpha(t-t_k-\Delta_k)}V(t_k+\Delta_k) < \mu_1 e^{-\alpha(t-t_k-\Delta_k)}V((t_k+\Delta_k)^-) < \mu_1 e^{-\alpha(t-t_k-\Delta_k)}e^{\beta\Delta_k}V(t_k) < \mu_1\mu_2 e^{-\alpha(t-t_k-\Delta_k)}e^{\beta\Delta_k}V(t_k^-) \\ &< \cdots < (\mu_1\mu_2)^k e^{-\alpha[(t-t_k-\Delta_k)+(t_k-t_{k-1}-\Delta_{k-1})+\cdots+(t_1-t_0-\Delta_0)]+\beta(\Delta_k+\Delta_{k-1}+\cdots+\Delta_0)}V(t_0) = (\mu_1\mu_2)^k e^{-\alpha T^-(t_0,t)+\beta T^+(t_0,t)}V(t_0). \end{aligned} \tag{37}$$

According to Definition 2, we know $k = N_\sigma$, then

$$k \leqslant N_0 + \frac{t - t_0}{\tau_a}. \tag{38}$$

From (26), we can obtain

$$-T^-(t_0, t)\lambda^- + T^+(t_0, t)\lambda^+ \leqslant -\lambda^*(t - t_0). \tag{39}$$

Substituting (38) and (39) into (37), we have

$$V(t) < (\mu_1\mu_2)^{N_0 + (t-t_0)/\tau_a} e^{-\lambda^*(t-t_0)} V(t_0) = (\mu_1\mu_2)^{N_0} e^{[\ln(\mu_1\mu_2)/\tau_a - \lambda^*](t-t_0)} V(t_0). \tag{40}$$

Then the following inequality can be obtained

$$\|x(t)\| < \sqrt{\frac{b}{a}} \cdot (\mu_1\mu_2)^{N_0/2} e^{[\ln(\mu_1\mu_2)/\tau_a - \lambda^*](t-t_0)/2} \|x(t_0)\|_h, \tag{41}$$

where $a = \min_{i,j \in \underline{N}, i \neq j} \left\{ \frac{1}{\lambda_{\max}(P_i)}, \frac{1}{\lambda_{\max}(P_{ij})} \right\}$,

$$b = \max_{i,j \in \underline{N}, i \neq j} \left\{ \frac{1}{\lambda_{\min}(P_i)} + \frac{\tau_2}{\lambda_{\min}(R_{1i})} + \frac{\tau_1}{\lambda_{\min}(R_{2i})}, \frac{1}{\lambda_{\min}(P_{ij})} + \frac{\tau_2}{\lambda_{\min}(R_{1ij})} + \frac{\tau_1}{\lambda_{\min}(R_{2ij})} \right\}.$$

Using $diag\{P_i, R_{1i}, R_{2i}, I\}$ to pre- and post-multiply the left term of (29), and denoting $K_iP_i = W_i$, we get

$$\begin{bmatrix} \widehat{A}_iP_i + P_i\widehat{A}_i^T + D_i\Omega_iW_i + W_i^T\Omega_i^TD_i^T + \alpha P_i + P_iR_{1i}^{-1}P_i & \widehat{B}_iR_{1i} & C_iR_{2i} & P_i\widehat{A}_i^T + W_i^T\Omega_i^TD_i^T \\ * & -e^{-\alpha\tau_2}R_{1i} & 0 & R_{1i}\widehat{B}_i^T \\ * & * & -e^{-\alpha\tau_1}R_{2i} & R_{2i}C_i^T \\ * & * & * & -R_{2i} \end{bmatrix} < 0. \tag{42}$$

By Schur complement lemma and Lemma 1, we can obtain that condition (42) can be guaranteed by the following inequality (43).

$$\begin{bmatrix} \widehat{A}_iP_i + P_i\widehat{A}_i^T + D_i\Omega_{i0}W_i + W_i^T\Omega_{i0}^TD_i^T + \alpha P_i + \varepsilon_iD_i\Omega_{i0}\Xi_i^2\Omega_{i0}^TD_i^T & \widehat{B}_iR_{1i} & C_iR_{2i} & P_i\widehat{A}_i^T + W_i^T\Omega_{i0}^TD_i^T + \varepsilon_iD_i\Omega_{i0}\Xi_i^2\Omega_{i0}^TD_i^T & P_i & W_i^T\Xi_i \\ * & -e^{-\alpha\tau_2}R_{1i} & 0 & R_{1i}\widehat{B}_i^T & 0 & 0 \\ * & * & -e^{-\alpha\tau_1}R_{2i} & R_{2i}C_i^T & 0 & 0 \\ * & * & * & -R_{2i} + \varepsilon_iD_i\Omega_{i0}\Xi_i^2\Omega_{i0}^TD_i^T & 0 & 0 \\ * & * & * & * & -R_{1i} & 0 \\ * & * & * & * & * & -\varepsilon_iI \end{bmatrix} < 0. \tag{43}$$

Substituting (3)–(43), and applying lemma 2 and Schur complement lemma, it follows that (43) is equivalent to (24).

Similarly, by Lemmas 1, 2 and Schur complement lemma, it is easy to get that (33) is equivalent to (25). The proof is completed. □

Remark 3. It is noticed that (24) and (25) are mutually dependent. Therefore, we can firstly solve the LMI (24) to obtain the solutions of matrices P_i, W_i. Then (25) can be transformed into the LMI by substituting $K_i = W_iP_i^{-1}$ into (25). By adjusting the parameter α, β, we can find the feasible solutions of P_i, R_{1i}, R_{2i}, W_i, P_{ij}, R_{1ij}, R_{2ij} such that (24) and (25) hold.

4. Numerical example

In this section we present an example to illustrate the effectiveness of the proposed approach. Consider systems (1) and (2) with parameters as follows

$$A_1 = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} -0.2 & 0 \\ 0.5 & -0.1 \end{bmatrix}, \quad D_1 = \begin{bmatrix} -4 & -1 \\ 2 & 1 \end{bmatrix},$$

$$E_{11} = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0 \end{bmatrix}, \quad E_{21} = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0.2 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 0.8 & 0.6 \\ 0.4 & 1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 2 & 3 \\ -4 & -5 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -0.1 & 0 \\ -0.5 & -0.2 \end{bmatrix}, \quad D_2 = \begin{bmatrix} -2 & 1 \\ -1 & 1 \end{bmatrix},$$

$$E_{12} = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.1 \end{bmatrix}, \quad E_{22} = \begin{bmatrix} 0.2 & 0 \\ 0.3 & 0.2 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 0.6 & 0.3 \\ 0.4 & 0.4 \end{bmatrix},$$

$$F_1 = \begin{bmatrix} \sin t & 0 \\ 0 & \sin t \end{bmatrix}, \quad F_2 = \begin{bmatrix} \cos t & 0 \\ 0 & \cos t \end{bmatrix}.$$

The fault matrices $\Omega_i = diag\{\omega_{i1}, \omega_{i2}\}$, $i = 1,2$

$$0.6 \leqslant \omega_{11} \leqslant 0.7, \quad 0.2 \leqslant \omega_{12} \leqslant 0.7, \quad 0.3 \leqslant \omega_{21} \leqslant 0.6, \quad 0.2 \leqslant \omega_{22} \leqslant 0.8.$$

Take $\tau_1 = 0.5$, $\tau_2 = 0.3$, $\alpha = 2$, $\beta = 6$, solving the matrix inequalities in Theorem 1 gives rise to

$$K_1 = \begin{bmatrix} 18.8234 & 7.5050 \\ -11.6482 & -5.7466 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 32.3175 & 1.1808 \\ -5.0367 & -2.3803 \end{bmatrix}.$$

$$P_1 = \begin{bmatrix} 2.1232 & -4.0345 \\ -4.0345 & 8.5620 \end{bmatrix}, \quad R_{11} = \begin{bmatrix} 2.4296 & -4.0505 \\ -4.0505 & 8.0769 \end{bmatrix}, \quad R_{12} = \begin{bmatrix} 14.9833 & -7.9728 \\ -7.9728 & 40.6105 \end{bmatrix},$$

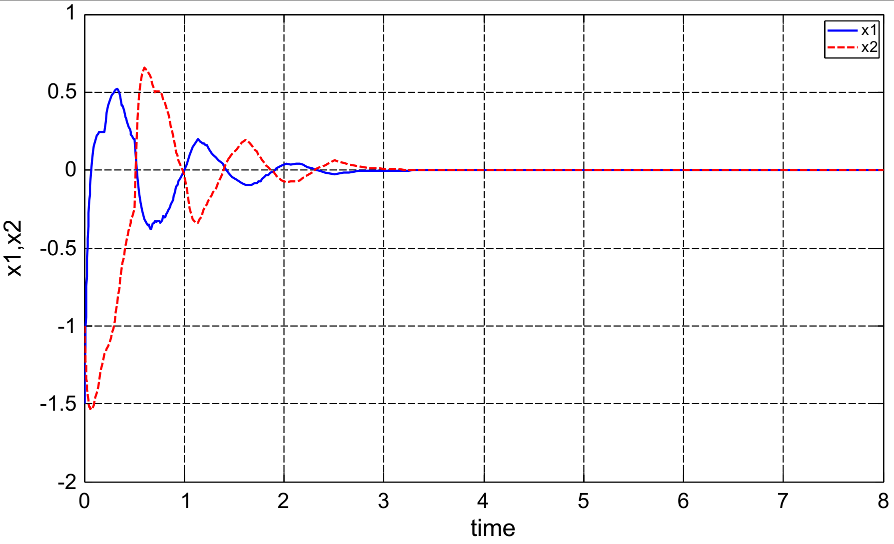

Fig. 1. State trajectories of the closed-loop system.

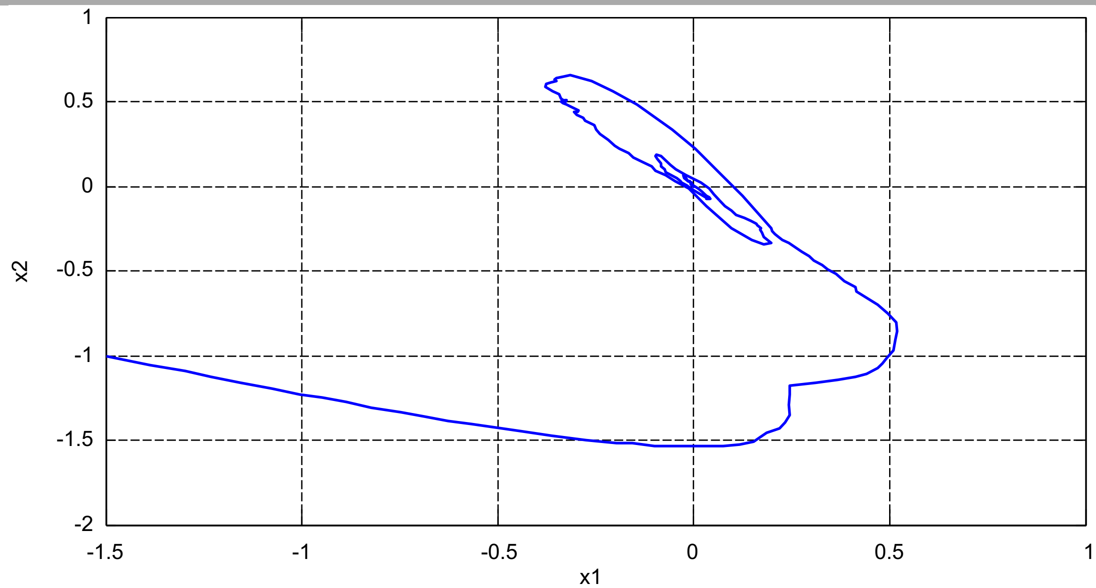

Fig. 2. Phase plane portrait of the closed-loop system.

$$P_{12} = \begin{bmatrix} 0.0539 & -0.0504 \\ -0.0504 & 0.1719 \end{bmatrix}, \quad R_{112} = \begin{bmatrix} 0.7954 & -0.2026 \\ -0.2026 & 0.3864 \end{bmatrix}, \quad R_{212} = \begin{bmatrix} 1.1159 & 0.4358 \\ 0.4358 & 0.8462 \end{bmatrix},$$

$$P_{2} = \begin{bmatrix} 0.3947 & 0.3143 \\ 0.3143 & 2.3980 \end{bmatrix}, \quad R_{21} = \begin{bmatrix} 3.3264 & -1.2587 \\ -1.2587 & 1.2138 \end{bmatrix}, \quad R_{22} = \begin{bmatrix} 14.0713 & 5.9282 \\ 5.9282 & 28.6057 \end{bmatrix},$$

$$P_{21} = \begin{bmatrix} 0.0134 & -0.0148 \\ -0.0148 & 0.2576 \end{bmatrix}, \quad R_{121} = \begin{bmatrix} 0.4821 & -0.1346 \\ -0.1346 & 0.6508 \end{bmatrix}, \quad R_{221} = \begin{bmatrix} 0.9277 & -0.0454 \\ -0.0454 & 0.7785 \end{bmatrix}.$$

Then, according to condition (27), we can get $\mu_1 = 1.7373$ and $\mu_2 = 76.7691$. From (26), it can be obtained that $\tau_a^* = 2.4466$. Choosing $\tau_a = 2.5$, the state trajectories and the phase plane portrait of closed-loop system under asynchronous switching ($\Delta_k = 0.2$, $k = 1,2$) are shown in Figs. 1 and 2, where the initial value $x(0) = [-1.5 \quad -1]^T$.

From the Figs. 1 and 2, it can be observed that reliable controller can guarantee the asymptotic stability and the reliability of the closed-loop system. This demonstrates the effectiveness of the proposed method.

5. Conclusions

This paper focuses on designing the robust reliable controller for a class of uncertain switched neutral systems with delayed switching and actuator failures. A kind of reliable controller design methodology is proposed, and the dwell time approach is utilized for the stability analysis. Sufficient conditions for the existence of such controller are formulated in terms of a set of LMIs. An illustrative example is also given to illustrate the applicability of the proposed approach.

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