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ORIGINAL RESEARCH PAPER

Stability analysis of networked micro-grid load frequency control system

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Abstract In this paper, the problem of ascertaining delay-dependent stability of networked micro-grid load frequency control systems with time-varying and time-invariant delays has been addressed using Lyapunov approach. In the networked micro-grid control system, it is observed that transfer of incremental frequency variable (feedback variable—an indication of the in-balance between the generation and the demand) through open communication links to effect closed-loop load frequency control introduces time-delays in the feedback path. The feedback delays are generally time-varying in nature, and invariably, they exert a detrimental effect to the overall performance of the closed-loop system paving way to instability. In this paper, using the classical Lyapunov–Krasovskii functional approach combined with appropriate inequalities, less conservative delay-dependent stability criteria are presented in linear matrix inequality (LMI) framework for networked micro-grid control system with time-varying and time-invariant delays. In the sequel, the presented results are validated on a standard benchmark system.

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1 Introduction

In recent times, the power system is evolving in the form of small pockets of independent entities called micro-grids. A typical micro-grid system connected to utility grid encompasses several distributed and interconnected generator units, loads and energy storage units; refer Fig. 1. The different types of these distributed sources include gas-based micro-turbine, wind-turbine, solar PV panel, fuel cell and electrolyzer system. The load essentially consists of domestic and small/medium-scale industrial loads. From the power system grid perspective, a micro-grid system can be considered as a group of controllable generator units and loads; refer [1], and the references cited therein. With increasing penetration of renewable energy sources into the utility grid in recent times, design and control of distributed generation in micro-grid environment have emerged as an active area of research. The advantages of the micro-grid system include increase in reliability and security of the power system, decrease in the cost of operation by effectively utilizing the micro-sources during peak load condition, supplementing base loads and reduction in green house effects.

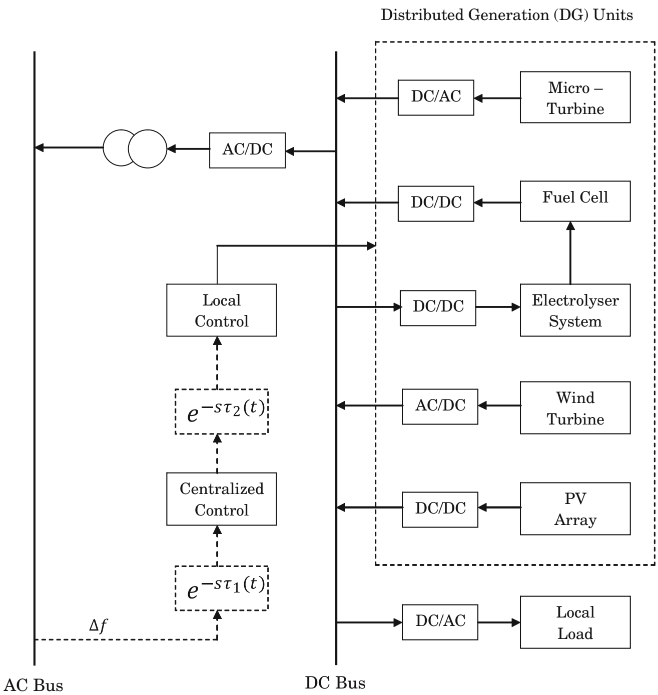

Fig. 1 A typical micro-grid system with communication delays

In this paper, a typical micro-grid closed-loop load frequency control system shown in Fig. 2 is considered wherein wind and PV generators serve as primary sources. As power from these renewable sources are intermittent in nature, a gas based micro-turbine unit is employed cater the base load. In addition, when unexpected real power imbalance occurs in power system, regulation of grid frequency may not be possible with micro-turbine itself; in such a scenario, for frequency compensation, a fuel cell and electrolyzer system are appropriately integrated into the micro-grid system. This system model is taken from [2].

In micro-grid systems, to compensate for continuous change in load demand, real power generation control is required to keep the frequency constant since frequency is an indication of the in-balance between the generation and the demand [3]. For this purpose, load frequency control (LFC) using PI control strategy is employed so that all distributed generation units are operated and controlled cohesively to ensure a stable operation with a desirable frequency and voltage profile in the system [4]. To facilitate this, in addition to a well designed local controller, a micro-grid central

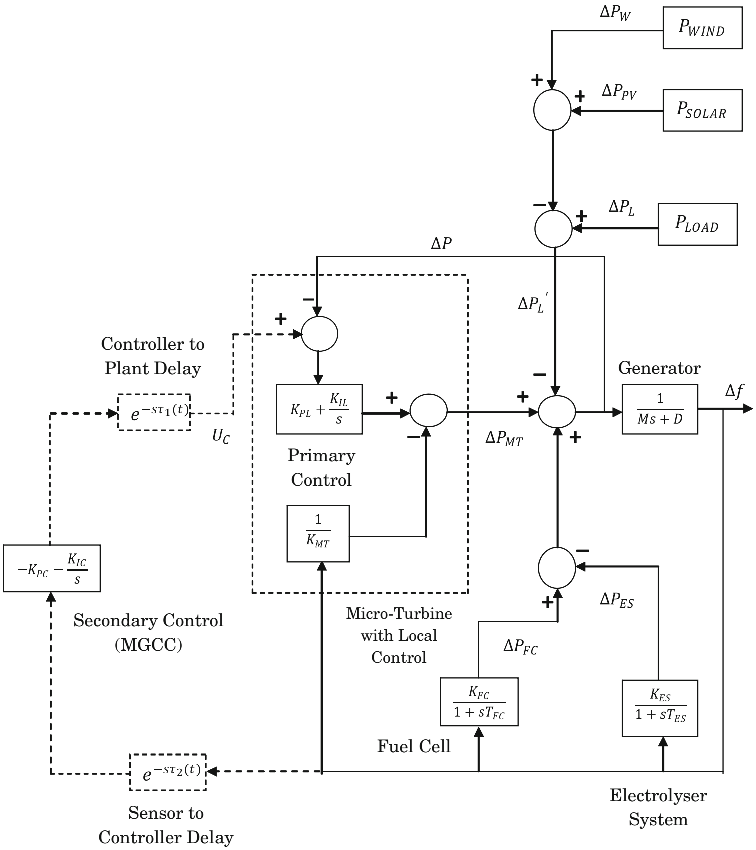

Fig. 2 Micro-grid system with time-varying delays

controller (MGCC) is also employed for load frequency control of the multi-area micro-grid systems. The main functions of MGCC are to obtain measurement from the power system and control information from local controllers, and to decide and implement necessary control actions for regulating frequency and voltage. This centralized control loop is completed through an open communication channel as shown in Fig. 1; this, in turn, introduces time-varying delays in the feedback path.

The feedback loop delays are inevitable in such a distributed system scenario where the system to be controlled and the controller are connected through a communication channel in which the measured/controlled data (that are realized in the form of discrete packets of information) experience buffering, processing and propagation at various internodes. These network-induced feedback loop delays pose serious limitations to achievable performance of the closed-loop system; in dire situations, when the delay margin exceeds a critical value, the micro-grid system will become unstable and trip from the grid. The loss of synchronism may sometimes lead to catastrophic chain of tripping of various generation units connected to the grid leading to islanding of regional power grids, and sometimes, blackouts [5, 6].

Hence, stable delay margin (i.e. maximum delay bound that the networked controlled system can accommodate without losing asymptotic stability) for networked micro-grid systems must be computed for various subsets of controller parameters through a less conservative delay-dependent stability analysis procedure so that they will serve as a practical guideline for fine tuning of controller parameters at the implementation stage even with partial knowledge about network delay size [7]. This, in turn, will enable the operating personnel to achieve optimal performance from the closed-loop LFC system under delayed control inputs. Delay-dependent stability criteria are basically sufficient conditions that are employed to compute the stable margin for the network delays within which the power system controlled through a remote control station remains asymptotically stable in the sense of Lyapunov.

In this paper, using the classical Lyapunov–Krasovskii functional approach [8] combined with appropriate inequalities, less conservative criteria are presented for ascertaining delay-dependent stability of micro-grid system for two cases: time-varying and time-invariant network delays. For time-varying delay case, though the controller to actuator delay $\tau_1(t)$ and sensor to controller delay $\tau_2(t)$ may have dissimilar characteristics [9], they are combined into a single time-varying delay as $\tau(t) = \tau_1(t) + \tau_2(t)$; on the other hand, for time-invariant delay case, the network delays are assumed as $\tau_1(t) = \tau_1$ and $\tau_2(t) = \tau_2$, $\forall t$. In [2], that is recently reported, using transcendental characteristic equation approach, an analytic procedure has been proposed to compute stable delay margin for micro-grid system with time-invariant communication delays; however, for the case of time-varying delay, the only result reported in literature [10] is derived through a conservative procedure. To the best of authors' knowledge, the problem of delay-dependent stability of micro-grid system either with time-varying or time-invariant communication delays has not been addressed using Lyapunov–Krasovskii functional approach so far. In this paper, we have addressed this problem and presented less conservative stability criteria.

Before stating the main results, the mathematical model of the micro-grid system is presented in state-space framework in the next section.

2 Mathematical model

The mathematical models of various components of the micro-grid control system are presented in the coming sub-sections.

2.1 Gas-based micro-turbine

The transfer function model the micro-turbine is developed taking into account the linear speed drop characteristics between the power and frequency. This transfer function is given as follows:

$$G_{MT}(s) = \frac{\Delta P_{MT}}{\Delta f} = -\frac{1}{K_{MT}}, \tag{1}$$

where Δf, ΔP_{MT} and K_{MT} represent frequency deviation, change in output power and drop characteristics of the micro-turbine, respectively.

2.2 Fuel cell and electrolyzer

A fuel cell with an electrolyzer system is utilized to compensate for real power imbalance when the local controller of the micro turbine becomes less effective for substantial variations in load. A part of the wind power is utilized by the aqua electrolyzer to produce hydrogen for fuel cell. The transfer function model of fuel cell and electrolyzer are given as follows:

$$G_{FC}(s) = \frac{\Delta P_{FC}}{\Delta f} = \frac{K_{FC}}{1 + sT_{FC}}, \tag{2}$$

$$G_{ES}(s) = \frac{\Delta P_{ES}}{\Delta f} = \frac{K_{ES}}{1 + sT_{ES}}, \tag{3}$$

where ΔP_{FC}, K_{FC} and T_{FC} represent the change in output power, the gain, time constant of the fuel cell, respectively, while ΔP_{ES}, K_{ES} and T_{ES} denote the same variables of the electrolyzer.

2.3 Extended load

The extended load demand $\Delta P_L^{'}$ consists of housing load ΔP_L, wind power and PV generation and is expressed as follows:

$$\Delta P_L^{'} = \Delta P_L - \Delta P_{PV} - \Delta P_W. \tag{4}$$

The dynamics of fluctuating power generators—PV and wind power—are not considered in the modelling of the test system employed for study. Hence, the characteristic equation and the stability of the time-delayed closed-loop micro-grid system do not depend on the variations in solar and wind power generations; nevertheless, these uncertainties do not substantially affect delay margins results.

2.4 Local and central controller

The local and micro-grid central controller [($G_{LC}(s)$ and $G_{CC}(s)$, respectively] are configured with PI control law. The controller transfer functions are given below:

$$G_{LC}(s) = K_{PL} + \frac{K_{IL}}{s}, \tag{5}$$

$$G_{CC}(s) = K_{PC} + \frac{K_{IC}}{s}, \tag{6}$$

where K_{PX} and K_{IX} represent proportional and integral gains of the controller.

2.5 State-space model

The state-space model of the overall system including central and local controller encompassing network-induced delays is derived in the following autonomous framework:

$$\dot{x}(t) = Ax(t) + A_d x(t - \tau(t)), \tag{7}$$

$$x(t) = \phi(t), t \in [-\bar{\tau}, 0], \tag{8}$$

where the system matrices $A \in \mathcal{R}^{5\times5}$ and $A_d \in \mathcal{R}^{5\times5}$ are as follows:

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & K_{IC} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ 0 & 0 & -\frac{1}{T_{FC}} & 0 & \frac{K_{FC}}{T_{FC}} \\ 0 & 0 & 0 & -\frac{1}{T_{ES}} & \frac{K_{ES}}{T_{ES}} \\ 0 & \frac{1}{M} & \frac{1}{M} & -\frac{1}{M} & \frac{D}{M} \end{bmatrix},$$

$$A_d = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

with state vector $x(t) \in \mathcal{R}^{5\times1}$ being $x(t) = [K_{IC} \int \Delta f dt \Delta P_{MT} \Delta P_{FC}\ \Delta P_{ES} \Delta f]^T$ (refer, Fig. 1). The elements of the matrices (in terms of system parameters) are given below:

$$
\begin{aligned}
a_{21} &= 0,\\
a_{22} &= \frac{1}{1+K_{PL}}\left[-K_{IL}-\frac{1}{MK_{MT}}\right],\\
a_{23} &= \frac{1}{1+K_{PL}}\left[\frac{K_{PL}}{T_{FC}}-K_{IL}-\frac{1}{MK_{MT}}\right],\\
a_{24} &= \frac{1}{1+K_{PL}}\left[-\frac{K_{PL}}{T_{ES}}+K_{IL}+\frac{1}{MK_{MT}}\right],\\
a_{25} &= \frac{1}{1+K_{PL}}\left[-\frac{K_{PL}K_{FC}}{T_{FC}}+\frac{K_{PL}K_{ES}}{T_{ES}}+\frac{D}{MK_{MT}}\right],\\
d_{21} &= -\frac{K_{IL}}{1+K_{PL}},\\
d_{22} &= -\frac{K_{PL}K_{PC}}{M(1+K_{PL})},\\
d_{23} &= -\frac{K_{PL}K_{PC}}{M(1+K_{PL})},\\
d_{24} &= \frac{K_{PL}K_{PC}}{M(1+K_{PL})},\\
d_{25} &= \frac{1}{1+K_{PL}}\left[-K_{IL}K_{PC}+\frac{K_{PL}K_{PC}D}{M}-K_{PL}K_{IC}\right].
\end{aligned}
$$

The time-varying delay is assumed to satisfy the following condition:

$$0 \le \tau(t) \le \bar{\tau}; \quad \dot{\tau}(t) < \mu < \infty, \tag{9}$$

where $\bar{\tau}$ is the upper bound of the time-varying delay and μ is upper bound of the delay-derivative. No restriction is imposed on the upper bound of the delay-derivative. For time-invariant delay case, the following assumptions hold good:

$$\tau_1(t) = \tau_1, \tau_2(t) = \tau_2, \forall t; \quad \tau = \tau_1 + \tau_2. \tag{10}$$

For deducing the main results that are presented in the next section, following lemmas are required to be understood:

Lemma 1 [11] *For any vectors* ζ_1, ζ_2, *matrices* $R > 0$, S *and real numbers* $\alpha_1 > 0$, $\alpha_2 > 0$ *with* $\alpha_1 + \alpha_2 = 1$ *satisfying*

$$\begin{bmatrix} R & S \\ S^T & R \end{bmatrix} \ge 0, \tag{11}$$

following inequality condition holds:

$$-\frac{1}{\alpha_1}\zeta_1^T R \zeta_1 - \frac{1}{\alpha_2}\zeta_2^T R \zeta_2 \le -\begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix}^T \begin{bmatrix} R & S \\ S^T & R \end{bmatrix} \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix}. \tag{12}$$

Lemma 2 [12] *For given symmetric positive definite matrix R, and for any differentiable signal ω in* $[a, b] \to \mathcal{R}^n$, *following inequality holds*:

$$\int_b^a \dot{\omega}^T(u) R \dot{\omega}(u) du \geq \frac{1}{b-a} \begin{bmatrix} \omega(b) \\ \omega(a) \\ \frac{1}{b-a}\int_a^b \omega(u) du \end{bmatrix}^T \begin{bmatrix} 4R & 2R & -6R \\ \star & 4R & -6R \\ \star & \star & 12R \end{bmatrix} \begin{bmatrix} \omega(b) \\ \omega(a) \\ \frac{1}{b-a}\int_a^b \omega(u) du \end{bmatrix}. \tag{13}$$

3 Main results

The delay-dependent stability criteria for the networked micro-grid frequency control system with time-varying delay and time-invariant delays are stated in the following theorems:

Theorem 1 *For a given delay bound* $\bar{\tau}$, *the networked micro-grid system* (7) *with the time-varying delay satisfying* (9) *is asymptotically stable in the sense of Lyapunov, if there exist real symmetric positive definite matrices P, Q and R; free matrix S of appropriate dimension such that the following LMIs hold*:

$$\begin{bmatrix} R & S \\ S^T & R \end{bmatrix} \geq 0; \tag{14}$$

$$\Pi(\bar{\tau}) < 0. \tag{15}$$

where

$$\begin{aligned} \Pi(\bar{\tau}) &= e_1 P e_4^T + e_4 P e_1^T + e_1 Q e_1^T - e_3 Q e_3^T + e_4(\bar{\tau}^2 R) e_4^T \\ &\quad - \begin{bmatrix} e_3^T - e_2^T \\ e_1^T - e_2^T \end{bmatrix}^T \begin{bmatrix} R & S \\ S^T & R \end{bmatrix} \begin{bmatrix} e_3^T - e_2^T \\ e_1^T - e_2^T \end{bmatrix}, \end{aligned} \tag{16}$$

with $e_1 = [I \quad 0 \quad 0]^T$, $e_2 = [0 \quad I \quad 0]^T$, $e_3 = [0 \quad 0 \quad I]^T$ *and* $e_4 = (A e_1^T + A_d e_2^T)^T$.

Proof The proof employs the following LK functional in the stability analysis:

$$V(t) = x^T(t) P x(t) + \int_{t-\bar{\tau}}^{t} x^T(s) Q x(s) ds + \tau \int_{-\bar{\tau}}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) R \dot{x}(s) ds d\theta, \tag{17}$$

where the matrix variables P, Q and R are real, symmetric and positive-definite in nature. The time-derivative of the energy functional is bounded in a less conservative manner using reciprocal convex combination lemma stated in Lemma 1. The stability analysis determines the feasibility of negative-definiteness of the time-derivative of the LK functional paving way to the criterion stated in theorem. For detailed derivation, one may refer to [13]. □

Theorem 2 *For a given (time-invariant) delay bound* τ, *networked micro-grid frequency control system* (7) *with time-invariant delay* (10) *is asymptotically stable in the sense of Lyapunov, if there exist real, symmetric, positive definite matrices P, S and R; symmetric matrix Z and free matrix Q of appropriate dimensions such that following LMIs hold*:

$$\Pi(\tau) > 0, \tag{18}$$

$$\Phi(\tau) < 0, \tag{19}$$

where $\Phi(\tau) = \sum_{i=1}^{3} \Phi_i(\tau)$ *with*

$$\Pi(\tau) = \begin{bmatrix} P & Q \\ \star & Z + \frac{1}{\tau} S \end{bmatrix},$$

$$\Phi_1(\tau) = \begin{bmatrix} A^T P + PA + Q + Q^T + S & -Q + PA_d & \tau A^T Q + \tau Z \\ \star & -S & \tau A_d^T Q - \tau Z \\ \star & \star & 0 \end{bmatrix},$$

$$\Phi_2(\tau) = \begin{bmatrix} A^T \\ A_d^T \\ 0 \end{bmatrix} (\tau R) \begin{bmatrix} A^T \\ A_d^T \\ 0 \end{bmatrix}^T,$$

$$\Phi_3(\tau) = -\frac{1}{\tau} \begin{bmatrix} R & -R & 0 \\ \star & R & 0 \\ \star & \star & 0 \end{bmatrix} - \frac{1}{\tau} \begin{bmatrix} 3R & 3R & -6R \\ \star & 3R & -6R \\ \star & \star & 12R \end{bmatrix},$$

where $\star$ *represents transposed terms in the symmetric matrix.*

Proof The proof of this stability criterion employs the following LK functional:

$$\begin{aligned} V(t) = {} & \begin{bmatrix} x(t) \\ \int_{t-\tau}^{t} x(s)ds \end{bmatrix}^T \begin{bmatrix} P & Q \\ \star & Z \end{bmatrix} \begin{bmatrix} x(t) \\ \int_{t-\tau}^{t} x(s)ds \end{bmatrix} \\ & + \int_{t-\tau}^{t} x^T(s) S x(s) ds + \int_{-\tau}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) R \dot{x}(s) ds d\theta. \end{aligned} \tag{20}$$

By bounding the time-derivative of the LK functional with Wirtinger inequality stated in Lemma 2, the delay-dependent stability criterion is derived. For detailed derivation, [12] may be referred. □

4 Benchmark micro-grid system

The parameters of the standard benchmark system taken from [2] are $M = 10$, $D = 1$, $K_{MT} = 0.04$, $K_{FC} = 1$, $T_{FC} = 4$, $K_{ES} = 1$, $T_{ES} = 1$, $K_{PL} = 1$, $K_{IL} = 1$. For this system, the maximum delay bound $\bar{\tau}$ of the time-varying delay for different subset of the centralized controller parameters (K_{PC}, K_{IC}) obtained by the delay-dependent stability criterion stated in Theorem 2 is given in Table 1.

For a given controller parameter set (K_{PC}, K_{IC}), the maximum allowable delay bound $\bar{\tau}$ obtained by Theorem 1 and existing method [10] is given in Table 2. From the table, it is clear that Theorem 1 is less conservative than that of [10].

For the case of time-invariant delay, the maximum allowable delay bound for τ obtained by the stability criterion stated in Theorem 1 for different sets of centralized controller parameters is furnished in Table 3.

Remark 1 For the micro-grid system with time invariant delay, the delay bounds presented in Table 3 are more conservative than those obtained by recently reported transcendental characteristic equation approach [2]. This is attributed to the reason that that the presented stability criterion of Theorem 2 is only a sufficient condition. Nevertheless, for time-varying delay, there are no results available in literature using LK functional method, and in this aspect, results presented in this paper are novel.

5 Simulation study

The simulation based study assumes white noise model for intermittent power output from renewable sources viz., wind and solar. Since power electronics based converters are invariably employed for interface with power system grid, this assumption is appropriate. The micro-grid central controller parameters are set at $K_{PC} = 1$ and $K_{IC} = 0.8$. The power system load is kept constant at $P_L = 0.5$ pu through out the study (over which uncertain power from solar and wind are superimposed); the delay value is set at 0 s for $t \in [0, 200]$, and at $t = 200$ s, the time-delay is suddenly introduced to the closed-loop system and the behavior of the system is inferred.

5.1 Time-invariant delay

To ascertain the stability of the closed-loop system with time-invariant delay, at $t = 200$ s, the time-delay τ is suddenly increased from 0 to 2.6 s. The maximum allowable delay bound, in accordance to Table 2, for the chosen MGCC parameters, is $\bar{\tau} = 2.8576$ s. Since $\tau = 2.6 < \bar{\tau}$, the incremental frequency variable $\Delta f(t)$ evolves asymptotically towards equilibrium point $\Delta f(t) = 0$ as shown in Fig. 3. If the delay τ is increased to $\bar{\tau} = 2.8576$, the system is marginally stable as shown in Fig. 4, and

Table 1 Maximum upper delay bound $\bar{\tau}$ for time-varying delay for any μ

$\bar{\tau}$ (s)	K_{PC}					
K_{IC}	0.5	1.0	1.5	2.0	2.5	3.0
0.2	7.7505	9.7176	11.1894	12.1047	12.3069	11.2777
0.4	4.0913	5.1979	6.1624	6.9589	7.5755	8.0058
0.6	2.8639	3.6299	4.3284	4.9438	5.4695	5.9004
0.8	2.2497	**2.8352**	3.3804	3.8761	4.3168	4.6989
1.0	1.8818	2.3552	2.8017	3.2151	3.5913	3.9273

The simulation results are presented for the value shown in bold

Table 2 Maximum upper delay bound $\bar{\tau}$ for time-varying delay

$\bar{\tau}$ (s)	K_{PC}			
K_{IC}	Method	0.1	0.6	1.0
0.1	[10]	10.04	11.20	15.86
	Theorem 1	11.49	15.67	17.98
0.2	[10]	4.81	4.95	6.96
	Theorem 1	5.89	8.17	9.71
0.4	[10]	2.38	2.73	3.32
	Theorem 1	3.13	4.32	5.19
0.6	[10]	1.60	1.76	2.19
	Theorem 1	2.21	3.02	3.62
0.8	[10]	1.21	1.38	1.65
	Theorem 1	1.76	2.36	2.83

Table 3 Maximum upper delay bound for time-invariant delay τ

τ (s)	K_{PC}					
K_{IC}	0.5	1.0	1.5	2.0	2.5	3.0
0.2	7.8710	9.9433	11.5710	12.6685	13.2133	13.2602
0.4	4.1252	5.2644	6.2854	7.1550	7.8493	8.3548
0.6	2.8807	3.6646	4.3943	5.0539	5.6300	6.1120
0.8	2.2600	**2.8576**	3.4239	3.9499	4.4273	4.8489
1.0	1.8896	2.3717	2.8339	3.2701	3.6746	4.0422

The simulation results are presented for the value shown in bold

if the delay is further increased to $\tau = 3 > \bar{\tau}$, the micro-grid system becomes unstable with $\Delta f(t)$ variable evolving unboundedly with time t as shown in Fig. 5. These results clearly substantiate the impact of time-invariant delay on the performance and stability of the networked micro-grid load frequency control system.

5.2 Time-varying delay

The time-varying delay is assumed to have a sinusoidal evolution for simulation study and no restriction is imposed on μ, the upper bound of the delay-derivative. For the considered set of PI controller parameters, according to Table 1, the maximum allowable bound for time-varying delay is $\bar{\tau} = 2.8352$. By retaining the load at 0.5 pu, time-varying delay is applied to the system at $t = 200$ s. If $\max(\tau(t)) < \bar{\tau}$, the system is asymptotically stable; if $\max(\tau(t)) = \bar{\tau}$, the system incremental frequency variable will exhibit marginally stable response and if $\max(\tau(t)) > \bar{\tau}$, the system will become unstable. The evolution of incremental frequency variable $\Delta f(t)$ with t for $\bar{\tau} = 2.5$, $\bar{\tau} = 2.8352$ and $\bar{\tau} = 3.2$ shown in Fig. 6. Figures 7 and 8 exhibiting asymptotically stable, marginally stable and unstable response validates the theoretical results.

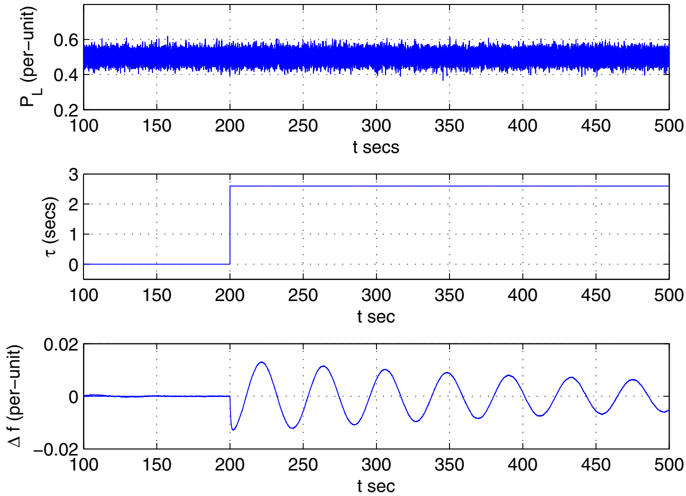

Fig. 3 Time-domain simulation for stable response for $\tau = 2.6$

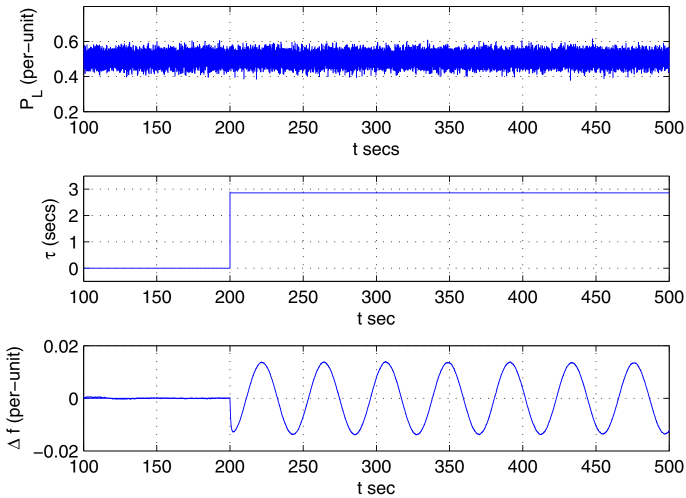

Fig. 4 Time-domain simulation for marginally stable response for $\tau = 2.8576$

6 Conclusion

In this paper, the problem of delay-dependent stability of micro-grid system with time-varying and time-invariant communication delay has been addressed using Lyapunov–Krasovskii functional approach. Various components of a typical micro-grid system are modelled as simple first order transfer functions neglecting higher

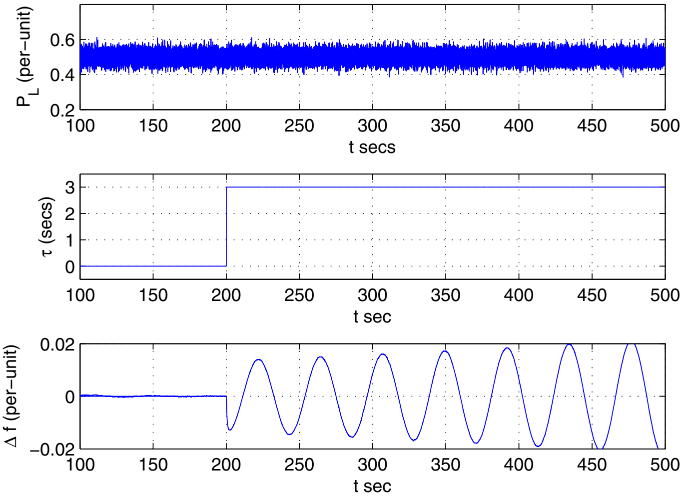

Fig. 5 Time-domain simulation for unstable response for $\tau = 3.0$

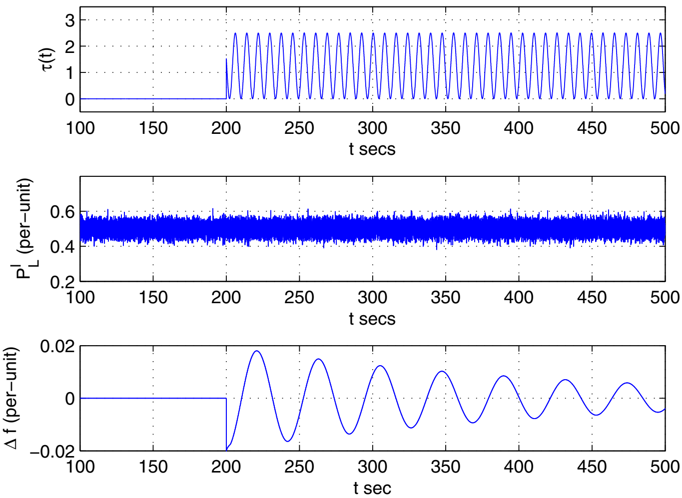

Fig. 6 Time-domain simulation for stable response (time-varying delay)

order dynamics. With PI control technique being employed for local and networked control, time-delayed state-space model is extracted for the overall system using appropriate incremental variables as state variables. Subsequently, less conservative stability criteria are presented in LMI framework for ascertaining delay-dependent stability of networked micro-grid system. The possibility of alleviating the demerits/limitations of the presented results will be explored as a future work.

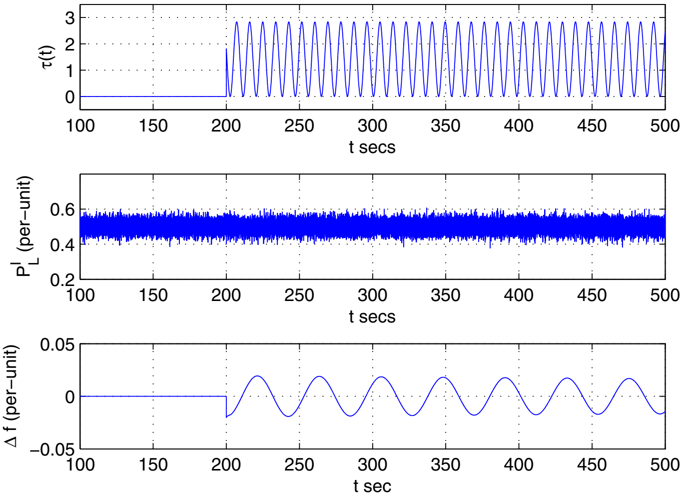

Fig. 7 Time-domain simulation for marginally stable response (time-varying delay)

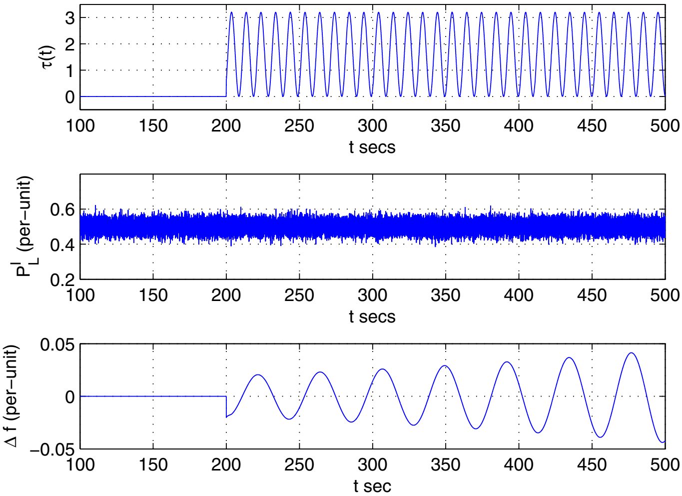

Fig. 8 Time-domain simulation for unstable response (time-varying delay)

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Ethical statement The authors declare that they have understood and fulfilled all the ethical norms imposed by the journal.

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