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Stabilization of Switched Linear Neutral Systems: An Event-Triggered Sampling Control Scheme

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***Abstract*—Event-triggered control is an effective control strategy, capable of reducing the amount of communications and retaining a satisfactory closed-loop performance. In this paper, we aim at proposing an event-triggered sampling mechanism and studying observer-based output feedback control for switched linear neutral systems with mixed time-varying delays. Different from the conventional event-triggered control strategies, our proposed one transmits not only the state but also the switching information to the controller, which is advantageous in applications where the measured outputs and the switching information have to be transmitted over a communication network. Moreover, asynchronous switchings may be caused between the subsystems and their matched sub-controllers under the proposed sampling mechanism, which increases difficulties in analyzing stability. We develop a sufficient condition, under which the proposed control scheme guarantees globally exponential stability of the closed-loop system meanwhile taking into account asynchronous switchings. Finally, an illustrative example is given to show the effectiveness of the proposed method.**

***Index Terms*—Switched neutral systems, event-triggered control, asynchronous switchings.**

I. INTRODUCTION

Switched neutral systems have been paid lots of attention in the last two decades due to their importance from both theoretical and practical points of view. Many practical systems can be modeled as switched neutral systems, such as drilling system [1] and partial element equivalent circuits (PEEC's) [2]. A switched neutral system is a switched system whose subsystems consist of neutral systems (see [3–6] for the details of neutral systems). There have been lots of works concerned with stability analysis and stabilization problem of switched neutral systems, e.g. [7–12]. However, most of the existing works for switched systems focus on stabilization with average dwell time method [13–16] in the framework of continuous-time feedback control. From a practical implementation point of view, sampled-data controllers are more favorable due to the rapid progress of computer and digital technologies and non-approximate treatment of the inter-sample behavior from sampled-data control's own features. The sampled-data control of switched neutral systems therefore possesses important significance on both theoretical research and practical engineering applications.

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Stability analysis and synthesis of sampled-data control systems have been one of the focuses in recent years. In conventional sampled-data control systems, the periodic sampling is popular since it helps simplify the analysis and design of control systems [17–20]. However, the periodicity brought by the periodic sampling would lead to unnecessary waste of communication and computation resources. Therefore, the event-triggered control is developed in the digital implementation of real time control systems, by which the control task is executed after an occurrence of an external event, generated by some well-designed event-triggered mechanisms. The conspicuous advantage of the event-triggering control is that it can significantly reduce the number of control task executions while retaining a satisfactory closed-loop performance. To date, in event-triggered control there have been several different event-triggered mechanisms and control strategies [21–29]. However, it is worth pointing out that these works all focus on *non-switched* systems. There are few theoretical results that study event-triggered control of switched linear neutral systems, even on switched linear systems, see, e.g., [30–32]. Moreover, most event-triggered controllers are based on available state information for feedback. However, in many control applications, full state measurements are not available for feedback.

Motivated by the above analysis, we study the observer-based output feedback stabilization problem of switched linear neutral systems with mixed time-varying delays under an event-triggered sampling mechanism, and for the first time we achieve event-triggered control for switched linear neutral systems. When the triggering condition is violated, asynchronous switchings may be caused during the switching process. Different from [7], which studies asynchronous switching control in continuous feedback form, our method is in the framework of event-triggered sampling control while considering asynchronous switchings. Combining the event-triggered control and the average-dwell-time-based switching policy, a sufficient condition is proposed to guarantee exponential stability of the resulting closed-loop system.

The paper is organized as follows. Section II describes the controlled switched linear neutral system and gives design of a switched observer. Section III develops an observer-based event-triggered sampling mechanism and analyzes stability of the obtained closed-loop system. Zeno behavior is excluded from the implicity defined sampling times. A simulation result is presented in Section IV, and Section V concludes the paper.

Notations: $\mathbb{R}^n$ is the n-dimensional Euclidean space. $\mathbb{N}$ is the set of nonnegative integers. $\underline{\lambda}(P)$ and $\bar{\lambda}(P)$ denote the minimum and maximum eigenvalue of a symmetric matrix P, respectively, and $P > 0$ denotes that P is positive definite.

II. Preliminaries

Consider the continuous-time switched linear neutral system

$$\begin{cases} \dot{x}(t) - C_\sigma \dot{x}(t-h(t)) \\ \qquad = A_\sigma x(t) + B_\sigma x(t-\tau(t)) + D_\sigma u(t), \ t > t_0 \\ y(t) = E_\sigma x(t), \\ x_{t_0} = x(t_0+\theta) = \varphi(\theta), \ \theta \in [-r, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is system state, $u(t) \in \mathbb{R}^m$ is control input, $y(t) \in \mathbb{R}^p$ is measurable output, $\sigma : [0, \infty) \to \mathcal{M} = \{1, 2, \cdots, m\}$ is a switching signal that orchestrates switching between subsystems, A_i, B_i, C_i, D_i and $E_i, i \in \mathcal{M}$ are real matrices of appropriate dimensions, which define the subsystem i, each subsystem is controllable and detectable, and matrix C_i satisfies $\|C_i\| < 1$ and $C_i \neq 0$, $\tau(t)$ and $h(t)$ are both time-varying delays, which satisfy

$$\begin{aligned} &0 < \tau(t) \leq \tau, \ \dot{\tau}(t) \leq \hat{\tau} < 1, \\ &0 < h(t) \leq h, \ \dot{h}(t) \leq \hat{h} < 1 \end{aligned} \tag{2}$$

where $\tau, \hat{\tau}, h$ and $\hat{h}$ are constants. $\varphi(\theta)$ is a continuously differential vector initial function on $[-r, 0]$, $r = \max\{\tau, h\}$, t_0 is the initial time. Corresponding to the switching signal σ, there exists a switching sequence

$$\{x_{t_0} : (l_0, t_0), (l_1, t_1), \cdots, (l_i, t_i), \cdots | l_i \in \mathcal{M}, \forall i \in \mathbb{N}\} \tag{3}$$

which means that the l_ith subsystem is active when $t \in [t_i, t_{i+1})$, where t_i is the switching instant. Without loss of generality, we assume the state trajectory $x(\cdot)$ is continuous everywhere. Moreover, we use $N_\sigma(t, s)$ to denote the number of discontinuities of the switching signal σ on a semi-open interval $(s, t]$, and use τ_d and τ_a to denote a minimum dwell time and an average dwell time, respectively, defined in [34].

For subsystem i, we construct the observer

$$\begin{aligned} \dot{\hat{x}}(t) - C_i \dot{\hat{x}}(t-h(t)) &= A_i \hat{x}(t) + B_i \hat{x}(t-\tau(t)) \\ &\quad + D_i u(t) + L_i(y(t) - E_i \hat{x}(t)) \end{aligned} \tag{4}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is observer state, L_i is observer gain of subsystem i. We define error $\mathbf{e}(t)$ to be $\mathbf{e}(t) = x(t) - \hat{x}(t)$. From (1) and (4), we obtain the dynamic sub-error system

$$\dot{\mathbf{e}}(t) - C_i \dot{\mathbf{e}}(t-h(t)) = (A_i - L_i E_i)\mathbf{e}(t) + B_i \mathbf{e}(t-\tau(t)). \tag{5}$$

Definition 1: If system (5) is exponentially stable, then (4) is an exponential observer of subsystem i.

Definition 2: [7] The equilibrium $x^* = 0$ of system (1) is said to be globally uniformly exponentially stable under σ, if the solution $x(t)$ of system (1) satisfies

$$\|x(t)\| \leq \kappa e^{-\lambda(t-t_0)} \|x(t_0)\|_r, \quad \forall t \geq t_0$$

for positive constants κ and λ, where

$$\|x(t_0)\|_r = \sup_{-r \leq \theta \leq 0} \{\|x(t_0+\theta)\|, \|\dot{x}(t_0+\theta)\|\}.$$

Lemma 1: For any real vectors u, v and matrix $Q > 0$ with compatible dimension, the following inequality holds

$$u^T v + v^T u \leq u^T Q u + v^T Q^{-1} v.$$

Lemma 2: [33] For any constant matrix $M > 0$, scalars r_1, r_2 satisfying $r_1 < r_2$, and a vector function $\omega : [r_1, r_2] \to \mathbb{R}^n$ such that the integrations concerned are well defined, then

$$\begin{aligned} &-(r_2 - r_1) \int_{r_1}^{r_2} \omega^T(s) M \omega(s) ds \\ &\qquad \leq -\int_{r_1}^{r_2} \omega^T(s) ds M \int_{r_1}^{r_2} \omega(s) ds. \end{aligned}$$

Based on Definition 1, we propose a lemma to guarantee that (4) exponentially estimates the state of subsystem i.

Lemma 3: For given positive scalars $\alpha, \tau, h, \hat{\tau} < 1$ and $\hat{h} < 1$, system (5) is exponentially stable if there exist matrices $P_i > 0, Q_i > 0, R_i > 0, W_i$ and $S_i, i \in \mathcal{M}$ satisfy

$$\begin{bmatrix} \Omega_i^{11} & \Omega_i^{12} & S_i^T B_i & S_i^T C_i \\ * & \Omega_i^{22} & S_i^T B_i & S_i^T C_i \\ * & * & \Omega_i^{33} & 0 \\ * & * & * & \Omega_i^{44} \end{bmatrix} < 0 \tag{6}$$

where

$$\begin{aligned} &\Omega_i^{11} = \alpha P_i + Q_i + S_i^T A_i - W_i E_i + A_i^T S_i - E_i^T W_i^T, \\ &\Omega_i^{12} = P_i - S_i^T + A_i^T S_i - E_i^T W_i^T, \ \Omega_i^{22} = -S_i^T - S_i + R_i, \\ &\Omega_i^{33} = -(1-\hat{\tau}) e^{-\alpha\tau} Q_i, \ \Omega_i^{44} = -(1-\hat{h}) e^{-\alpha h} R_i, \end{aligned}$$

then (4) is an exponential observer of subsystem i and observer gain L_i is given by $L_i = S_i^{-T} W_i$.

Proof: Choose Lyapunov-Krasovskii functional

$$\begin{aligned} V_i(t) = \mathbf{e}^T(t) P_i \mathbf{e}(t) &+ \int_{t-\tau(t)}^{t} \mathbf{e}^T(s) e^{\alpha(s-t)} Q_i \mathbf{e}(s) ds \\ &+ \int_{t-h(t)}^{t} \dot{\mathbf{e}}^T(s) e^{\alpha(s-t)} R_i \dot{\mathbf{e}}(s) ds. \end{aligned} \tag{7}$$

Taking its time derivative of (7) along solutions of system (5), we have

$$\begin{aligned} \dot{V}_i(t) + \alpha V_i(t) &\leq \dot{\mathbf{e}}^T(t) P_i \mathbf{e}(t) + \mathbf{e}^T(t) P_i \dot{\mathbf{e}}(t) \\ &+ \mathbf{e}^T(t)(\alpha P_i + Q_i)\mathbf{e}(t) + \dot{\mathbf{e}}^T(t) R_i \dot{\mathbf{e}}(t) \\ &- (1-\hat{\tau}) \mathbf{e}^T(t-\tau(t)) e^{-\alpha\tau} Q_i \mathbf{e}(t-\tau(t)) \\ &- (1-\hat{h}) \dot{\mathbf{e}}^T(t-h(t)) e^{-\alpha h} R_i \dot{\mathbf{e}}(t-h(t)). \end{aligned} \tag{8}$$

From equation (5), for any invertible matrix S_i with appropriate dimensions, we have the identity

$$\begin{aligned} &-2[\mathbf{e}^T(t) \ \ \dot{\mathbf{e}}^T(t)] S_i^T [\dot{\mathbf{e}}(t) - C_i \dot{\mathbf{e}}(t-h(t)) \\ &\qquad - (A_i - L_i E_i)\mathbf{e}(t) - B_i \mathbf{e}(t-\tau(t))] = 0. \end{aligned} \tag{9}$$

Adding the left-hand side of equality (9) into inequality (8), we have

$$\dot{V}_i(t) + \alpha V_i(t) \leq \zeta^T(t) \Omega_i \zeta(t) \tag{10}$$

where

$$\zeta^T(t) = [\mathbf{e}^T(t) \ \ \dot{\mathbf{e}}^T(t) \ \ \mathbf{e}^T(t-\tau(t)) \ \ \dot{\mathbf{e}}^T(t-h(t))],$$

$$\tilde{\Omega}_i = \begin{bmatrix} \tilde{\Omega}_i^{11} & \tilde{\Omega}_i^{12} & S_i^T B_i & S_i^T C_i \\ * & \Omega_i^{22} & S_i^T B_i & S_i^T C_i \\ * & * & -(1-\hat{\tau}) e^{-\alpha\tau} Q_i & 0 \\ * & * & * & -(1-\hat{h}) e^{-\alpha h} R_i \end{bmatrix},$$

$$\begin{aligned} &\tilde{\Omega}_i^{11} = \alpha P_i + Q_i + S_i^T (A_i - L_i E_i) + (A_i - L_i E_i)^T S_i, \\ &\tilde{\Omega}_i^{12} = P_i - S_i^T + (A_i - L_i E_i)^T S_i. \end{aligned}$$

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Let $W_i = S_i^T L_i$. Then $\dot{V}_i(t) + \alpha V_i(t) < 0$ follows from inequality (6). Integrating inequality $\dot{V}_i(t) + \alpha V_i(t) < 0$ from t_0 to t gives $V_i(t) \leq e^{-\alpha(t-t_0)} V_i(t_0)$, which guarantees that system (5) is exponentially stable. From Definition 1, we know that Lemma 3 guarantees (4) is an exponential observer of subsystem i. ■

From (4) and (5), we can obtain a switching observer

$$\begin{aligned} \dot{\hat{x}}(t) - C_\sigma \dot{\hat{x}}(t-h(t)) &= A_\sigma \hat{x}(t) \\ &+ B_\sigma \hat{x}(t-\tau(t)) + D_\sigma u(t) + L_\sigma E_\sigma \mathbf{e}(t) \end{aligned} \tag{11}$$

and a dynamic error switched system

$$\begin{aligned} &\dot{\mathbf{e}}(t) - C_\sigma \dot{\mathbf{e}}(t-h(t)) \\ &= (A_\sigma - L_\sigma E_\sigma)\mathbf{e}(t) + B_\sigma \mathbf{e}(t-\tau(t)). \end{aligned} \tag{12}$$

III. Event-triggered control

In this section, we aim at constructing an event-triggered detection mechanism and a switching controller to guarantee stability of (1). We first develop a triggering condition as

$$\|\hat{\mathbf{e}}(t)\|^2 \geq \eta \|\xi(t)\|^2 \tag{13}$$

where $\hat{\mathbf{e}}(t) = \hat{x}(t) - \hat{x}(\hat{t}_k), \xi(t) = [\hat{x}^T(t)\ \mathbf{e}^T(t)]^T, \eta > 0$ is a threshold, and $\{\hat{t}_k\}_{k=0}^{\infty}$ with $\hat{t}_k < \hat{t}_{k+1}$ denotes the time instants when an event is triggering. Detection mechanism receives state information from observer and monitors the triggering condition (13) continuously to determine whether an event is generated or not. Control input is determined by both the state and the switching information transmitted by the detector. Controller updates the newest state and switching information when an event happens and holds the information until the next event happens. Asynchronization is thus caused which may lead to instability of the closed-loop system. We introduce an assumption that $\tau_m < \tau_d$, where τ_m demotes the maximal asynchronous period. With the state $\hat{x}(\hat{t}_k)$ sampled at the time instant $\hat{t}_k$, the next sampling instant $\hat{t}_{k+1}$ can be determined by

$$\hat{t}_{k+1} = \inf \left\{ t > \hat{t}_k \middle| \|\hat{\mathbf{e}}(t)\|^2 = \eta \|\xi(t)\|^2 \right\}. \tag{14}$$

Let $\hat{t}_0 = t_0$. Without loss of generality, we suppose that n

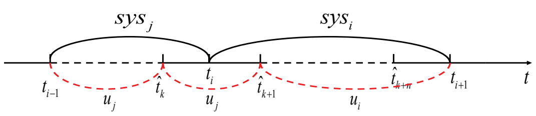

Fig. 1: Asynchronous switching and sampling instants.

samplings occur on the *non-switched* interval $[t_i, t_{i+1})$ and $\hat{t}_{k+1}$ is the first sampling instant on this interval, see Fig. 1. Moreover, from (2), we assume that subsystem j is the l_{i-1}th subsystem activated on the interval $[t_{i-1}, t_i)$ and subsystem i is the l_ith subsystem activated on $[t_i, t_{i+1})$. Then for $\forall t \in [t_i, t_{i+1})$, the observer-based controller is set to

$$u = \begin{cases} K_j \hat{x}(\hat{t}_k), & t \in [t_i, \hat{t}_{k+1}) \\ K_i \hat{x}(\hat{t}_{k+1}), & t \in [\hat{t}_{k+1}, \hat{t}_{k+2}) \\ \cdots & \\ K_i \hat{x}(\hat{t}_{k+n}), & t \in [\hat{t}_{k+n}, t_{i+1}) \end{cases} \tag{15}$$

where K_i and K_j are controller gains of subsystems i and j, respectively. On the consecutive sampling interval, controller only updates the information at sampling instants. A zero-order holder is introduced to keep the control signal continuously. For $\forall t \in \{[t_i, \hat{t}_{k+1}), \cdots, [\hat{t}_{k+n}, t_{i+1})\}$, $\hat{\mathbf{e}}(t) = \hat{x}(t) - \hat{x}(\hat{t}_{k+j})$ holds for all $j = 0, 1, \cdots, n$. Thus when subsystem i is active on interval $[t_i, t_{i+1})$, the closed-loop form of (4) is written as

$$\begin{aligned} &\dot{\hat{x}}(t) - C_i \dot{\hat{x}}(t-h(t)) = \\ &\begin{cases} (A_i + D_i K_j)\hat{x}(t) + B_i \hat{x}(t-\tau(t)) \\ \qquad -D_i K_j \hat{\mathbf{e}}(t) + L_i E_i \mathbf{e}(t), \ t \in [t_i, \hat{t}_{k+1}) \\ (A_i + D_i K_i)\hat{x}(t) + B_i \hat{x}(t-\tau(t)) \\ \qquad -D_i K_i \hat{\mathbf{e}}(t) + L_i E_i \mathbf{e}(t), \ t \in [\hat{t}_{k+1}, t_{i+1}). \end{cases} \end{aligned} \tag{16}$$

Recall $\mathbf{e}(t) = x(t) - \hat{x}(t)$ and $\xi(t) = [\hat{x}^T(t)\ \mathbf{e}^T(t)]^T$, we have

$$\begin{aligned} &\dot{\xi}(t) - \bar{C}_i \dot{\xi}(t-h(t)) = \\ &\begin{cases} \bar{A}_{ij}\xi(t) + \bar{B}_i \xi(t-\tau(t)) + \bar{D}_{ij}\tilde{\mathbf{e}}(t), t \in [t_i, \hat{t}_{k+1}) \\ \bar{A}_i \xi(t) + \bar{B}_i \xi(t-\tau(t)) + \bar{D}_i \tilde{\mathbf{e}}(t), t \in [\hat{t}_{k+1}, t_{i+1}) \end{cases} \end{aligned} \tag{17}$$

where

$$\bar{A}_{ij} = \begin{bmatrix} A_i + D_i K_j & L_i E_i \\ 0 & A_i - L_i E_i \end{bmatrix}, \bar{D}_{ij} = \begin{bmatrix} -D_i K_j & 0 \\ 0 & 0 \end{bmatrix},$$
$$\bar{B}_i = \begin{bmatrix} B_i & 0 \\ 0 & B_i \end{bmatrix}, \bar{C}_i = \begin{bmatrix} C_i & 0 \\ 0 & C_i \end{bmatrix}, \bar{D}_{ii} = \begin{bmatrix} -D_i K_i & 0 \\ 0 & 0 \end{bmatrix},$$
$$\bar{A}_{ii} = \begin{bmatrix} A_i + D_i K_i & L_i E_i \\ 0 & A_i - L_i E_i \end{bmatrix}, \ \tilde{\mathbf{e}}(t) = \begin{bmatrix} \hat{\mathbf{e}}(t) \\ 0 \end{bmatrix}.$$

Now we analyze stability of the generalized system

$$\begin{aligned} &\dot{\xi}(t) - \bar{C}_{\sigma_i} \dot{\xi}(t-h(t)) \\ &= \bar{A}_{\sigma_i \sigma_j} \xi(t) + \bar{B}_{\sigma_i} \xi(t-\tau(t)) + \bar{D}_{\sigma_i \sigma_j} \tilde{\mathbf{e}}(t) \end{aligned} \tag{18}$$

which is equivalent to system (1) under control input (15), where

$$\bar{A}_{\sigma_i \sigma_j} = \begin{bmatrix} A_{\sigma_i} + D_{\sigma_i} K_{\sigma_j} & L_{\sigma_i} E_{\sigma_i} \\ 0 & A_{\sigma_i} - L_{\sigma_i} E_{\sigma_i} \end{bmatrix}, \bar{D}_{\sigma_i \sigma_j} =$$
$$\begin{bmatrix} -D_{\sigma_i} K_{\sigma_j} & 0 \\ 0 & 0 \end{bmatrix}, \bar{B}_{\sigma_i} = \begin{bmatrix} B_{\sigma_i} & 0 \\ 0 & B_{\sigma_i} \end{bmatrix}, \bar{C}_{\sigma_i} = \begin{bmatrix} C_{\sigma_i} & 0 \\ 0 & C_{\sigma_i} \end{bmatrix}.$$

The following theorem presents a sufficient condition for exponential stability of the closed-loop system (18).

Theorem 1: Consider system (18) with sampling instants determined by (14). For given positive scalars $h, \tau, \hat{h} < 1, \hat{\tau} < 1, \mu > 1, \lambda_s, \lambda_u, \eta$ and τ_m if there exist matrices $\hat{P}_{ij} > 0$, $\hat{Q}_{ij} > 0, \hat{R}_{ij} > 0, \hat{N}_{ij} > 0, \hat{P}_i > 0, \hat{Q}_i > 0, \hat{R}_i > 0, \hat{N}_i > 0, \hat{M}_i, \hat{M}_j, G_i$ and G_j for $\forall i, j \in \mathcal{M}$ such that

$$\Pi_{ij} = \begin{bmatrix} \Pi_{ij}^{11} & \Pi_{ij}^{12} \\ * & \Pi_{ij}^{22} \end{bmatrix} < 0, \tag{19}$$

$$\Pi_i = \begin{bmatrix} \Pi_i^{11} & \Pi_i^{12} \\ * & \Pi_i^{22} \end{bmatrix} < 0, \tag{20}$$

$$\begin{aligned} &\hat{P}_{ij} \leq \mu \hat{P}_i, \ \hat{Q}_{ij} \leq \mu \hat{Q}_i, \ \hat{R}_{ij} \leq \mu \hat{R}_i, \\ &\hat{P}_i \leq \mu \hat{P}_{ij}, \ \hat{Q}_i \leq \mu \hat{Q}_{ij}, \ \hat{R}_i \leq \mu \hat{R}_{ij} \end{aligned} \tag{21}$$

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where

$$\Pi_{ij}^{11} = \begin{bmatrix} \bar{\Pi}_{ij}^{11} & \bar{\Pi}_{ij}^{12} & \bar{\Pi}_{ij}^{13} & 0 & \bar{\Pi}_{ij}^{15} & 0 & \bar{\Pi}_{ij}^{17} & 0 \\ * & \bar{\Pi}_{ij}^{22} & \bar{\Pi}_{ij}^{23} & \bar{\Pi}_{ij}^{24} & 0 & \bar{\Pi}_{ij}^{26} & 0 & \bar{\Pi}_{ij}^{28} \\ * & * & \bar{\Pi}_{ij}^{33} & 0 & \bar{\Pi}_{ij}^{35} & 0 & \bar{\Pi}_{ij}^{37} & 0 \\ * & * & * & \bar{\Pi}_{ij}^{44} & 0 & \bar{\Pi}_{ij}^{46} & 0 & \bar{\Pi}_{ij}^{48} \\ * & * & * & * & \bar{\Pi}_{ij}^{55} & 0 & 0 & 0 \\ * & * & * & * & * & \bar{\Pi}_{ij}^{66} & 0 & 0 \\ * & * & * & * & * & * & \bar{\Pi}_{ij}^{77} & 0 \\ * & * & * & * & * & * & * & \bar{\Pi}_{ij}^{88} \end{bmatrix},$$

$$\Pi_{ij}^{12} = \begin{bmatrix} \bar{\Pi}_{ij}^{19} & 0 & 0 & 0 & D_iG_j & 0 \\ 0 & \bar{\Pi}_{ij}^{210} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & D_iG_j \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Pi_{ij}^{22} = \begin{bmatrix} \bar{\Pi}_{ij}^{99} & 0 & 0 & 0 & 0 & 0 \\ * & \bar{\Pi}_{ij}^{1010} & 0 & 0 & 0 & 0 \\ * & * & \bar{\Pi}_{ij}^{1111} & 0 & 0 & 0 \\ * & * & * & \bar{\Pi}_{ij}^{1212} & 0 & 0 \\ * & * & * & * & -\hat{N}_{ij} & 0 \\ * & * & * & * & * & -\hat{N}_{ij} \end{bmatrix},$$

$$\begin{aligned} \bar{\Pi}_{ij}^{11} &= A_i\hat{M}_j + D_iG_j + \hat{M}_j^TA_i^T + G_j^TD_i^T + \hat{Q}_{ij} \\ &\quad - \lambda_u\hat{P}_{ij} + 2\eta\hat{N}_{ij} - \frac{(\lambda_s+\lambda_u)e^{-\lambda_s h}}{h}\hat{R}_{ij}, \\ \bar{\Pi}_{ij}^{12} &= L_iE_i\hat{M}_j,\ \bar{\Pi}_{ij}^{15} = \bar{\Pi}_{ij}^{26} = \bar{\Pi}_{ij}^{35} = \bar{\Pi}_{ij}^{46} = B_i\hat{M}_j, \\ \bar{\Pi}_{ij}^{17} &= \bar{\Pi}_{ij}^{28} = \bar{\Pi}_{ij}^{37} = \bar{\Pi}_{ij}^{48} = C_i\hat{M}_j, \\ \bar{\Pi}_{ij}^{22} &= A_i\hat{M}_j - L_iE_i\hat{M}_j + \hat{M}_j^TA_i^T - \hat{M}_j^TE_i^TL_i^T + \hat{Q}_{ij} \\ &\quad - \lambda_u\hat{P}_{ij} + 2\eta\hat{N}_{ij} - \frac{(\lambda_s+\lambda_u)e^{-\lambda_s h}}{h}\hat{R}_{ij}, \\ \bar{\Pi}_{ij}^{13} &= \hat{P}_{ij} - \hat{M}_j + \hat{M}_j^TA_i^T + G_j^TD_i^T,\ \bar{\Pi}_{ij}^{23} = \hat{M}_j^TE_i^TL_i^T, \\ \bar{\Pi}_{ij}^{24} &= \hat{P}_{ij} - \hat{M}_j + \hat{M}_j^TA_i^T - \hat{M}_j^TE_i^TL_i^T, \\ \bar{\Pi}_{ij}^{33} &= \bar{\Pi}_{ij}^{44} = \hat{R}_{ij} - \hat{M}_j - \hat{M}_j^T, \\ \bar{\Pi}_{ij}^{55} &= \bar{\Pi}_{ij}^{66} = -(1-\hat{\tau})e^{-\lambda_s\tau}\hat{Q}_{ij}, \\ \bar{\Pi}_{ij}^{77} &= \bar{\Pi}_{ij}^{88} = -(1-\hat{h})e^{-\lambda_s h}\hat{R}_{ij}, \\ \bar{\Pi}_{ij}^{19} &= \bar{\Pi}_{ij}^{210} = -\bar{\Pi}_{ij}^{99} = -\bar{\Pi}_{ij}^{1010} = \frac{(\lambda_s+\lambda_u)e^{-\lambda_s h}}{h}\hat{R}_{ij}, \\ \bar{\Pi}_{ij}^{1111} &= \bar{\Pi}_{ij}^{1212} = -\frac{(\lambda_s+\lambda_u)e^{-\lambda_s\tau}}{\tau}\hat{Q}_{ij}, \end{aligned}$$

$$\Pi_{i}^{11} = \begin{bmatrix} \bar{\Pi}_{i}^{11} & L_iE_i\hat{M}_i & \bar{\Pi}_{i}^{13} & 0 & B_i\hat{M}_i \\ * & \bar{\Pi}_{i}^{22} & \hat{M}_i^TE_i^TL_i^T & \bar{\Pi}_{i}^{24} & 0 \\ * & * & \bar{\Pi}_{i}^{33} & 0 & B_i\hat{M}_i \\ * & * & * & \bar{\Pi}_{i}^{44} & 0 \\ * & * & * & * & \bar{\Pi}_{i}^{55} \end{bmatrix},$$

$$\Pi_{i}^{12} = \begin{bmatrix} 0 & C_i\hat{M}_i & 0 & D_iG_i & 0 \\ B_i\hat{M}_i & 0 & C_i\hat{M}_i & 0 & 0 \\ 0 & C_i\hat{M}_i & 0 & 0 & D_iG_i \\ B_i\hat{M}_i & 0 & C_i\hat{M}_i & 0 & 0 \end{bmatrix},$$

$$\Pi_{i}^{22} = \begin{bmatrix} \bar{\Pi}_{i}^{66} & 0 & 0 & 0 & 0 \\ * & \bar{\Pi}_{i}^{77} & 0 & 0 & 0 \\ * & * & \bar{\Pi}_{i}^{88} & 0 & 0 \\ * & * & * & -\hat{N}_i & 0 \\ * & * & * & * & -\hat{N}_i \end{bmatrix},$$

$$\begin{aligned} \bar{\Pi}_{i}^{11} &= \hat{M}_i^TA_i^T + G_i^TD_i^T + A_i\hat{M}_i + D_iG_i + \hat{Q}_i + \lambda_s\hat{P}_i \\ &\quad + 2\eta\hat{N}_i, \\ \bar{\Pi}_{i}^{22} &= \hat{M}_i^TA_i^T - \hat{M}_i^TE_i^TL_i^T + A_i\hat{M}_i - L_iE_i\hat{M}_i + \hat{Q}_i \\ &\quad + \lambda_s\hat{P}_i + 2\eta\hat{N}_i, \\ \bar{\Pi}_{i}^{13} &= \hat{P}_i - \hat{M}_i + \hat{M}_i^TA_i^T + G_i^TD_i^T, \\ \bar{\Pi}_{i}^{24} &= \hat{P}_i - \hat{M}_i + \hat{M}_i^TA_i^T - \hat{M}_i^TE_i^TL_i^T, \\ \bar{\Pi}_{i}^{33} &= \bar{\Pi}_{i}^{44} = \hat{R}_i - \hat{M}_i - \hat{M}_i^T, \\ \bar{\Pi}_{i}^{55} &= \bar{\Pi}_{i}^{66} = -(1-\hat{\tau})e^{-\lambda_s\tau}\hat{Q}_i, \\ \bar{\Pi}_{i}^{77} &= \bar{\Pi}_{i}^{88} = -(1-\hat{h})e^{-\lambda_s h}\hat{R}_i, \end{aligned}$$

then system (18) is globally exponentially stable for any switching signal with average dwell time τ_a satisfying

$$\tau_a > \frac{2\ln\mu + \tau_m(\lambda_u+\lambda_s)}{\lambda_s} \tag{22}$$

and the controller gain is given by $K_i = G_i\hat{M}_i^{-1}$.

Proof: On interval $[t_i, \hat{t}_{k+1})$, subsystem i is activated, but sub-controller u_j is still working. We take Lyapunov-Krasovskii functional candidate as

$$\begin{aligned} V_{ij}(t) &= \xi^T(t)\bar{P}_{ij}\xi(t) + \int_{t-\tau(t)}^{t}\xi^T(s)e^{\lambda_s(s-t)}\bar{Q}_{ij}\xi(s)ds \\ &\quad + \int_{t-h(t)}^{t}\dot{\xi}^T(s)e^{\lambda_s(s-t)}\bar{R}_{ij}\dot{\xi}(s)ds \end{aligned} \tag{23}$$

where $\bar{P}_{ij} = \mathrm{diag}\{\tilde{P}_{ij}, \tilde{P}_{ij}\} > 0$, $\bar{Q}_{ij} = \mathrm{diag}\{\tilde{Q}_{ij}, \tilde{Q}_{ij}\} > 0$, and $\bar{R}_{ij} = \mathrm{diag}\{\tilde{R}_{ij}, \tilde{R}_{ij}\} > 0$. Taking its time derivative of (23) along solutions of (18) gives

$$\begin{aligned} \dot{V}_{ij}(t) - \lambda_uV_{ij}(t) &\le 2\xi^T(t)\bar{P}_{ij}\dot{\xi}(t) + \xi^T(t)\bar{Q}_{ij}\xi(t) \\ &\quad - (1-\hat{\tau})\xi^T(t-\tau(t))e^{-\lambda_s\tau}\bar{Q}_{ij}\xi(t-\tau(t)) \\ &\quad + \dot{\xi}^T(t)\bar{R}_{ij}\dot{\xi}(t) - \lambda_u\xi^T(t)\bar{P}_{ij}\xi(t) \\ &\quad - (1-\hat{h})\dot{\xi}^T(t-h(t))e^{-\lambda_s h}\bar{R}_{ij}\dot{\xi}(t-h(t)) \\ &\quad - (\lambda_s+\lambda_u)e^{-\lambda_s\tau}\int_{t-\tau(t)}^{t}\xi^T(s)\bar{Q}_{ij}\xi(s)ds \\ &\quad - (\lambda_s+\lambda_u)e^{-\lambda_s h}\int_{t-h(t)}^{t}\dot{\xi}^T(s)\bar{R}_{ij}\dot{\xi}(s)ds. \end{aligned} \tag{24}$$

From Lemma 2, we have

$$\begin{aligned} &- (\lambda_s+\lambda_u)e^{-\lambda_s\tau}\int_{t-\tau(t)}^{t}\xi^T(s)\bar{Q}_{ij}\xi(s)ds \\ &\le -\frac{(\lambda_s+\lambda_u)e^{-\lambda_s\tau}}{\tau}\int_{t-\tau(t)}^{t}\xi^T(s)ds\bar{Q}_{ij}\int_{t-\tau(t)}^{t}\xi(s)ds \end{aligned} \tag{25}$$

and

$$\begin{aligned} &- (\lambda_s+\lambda_u)e^{-\lambda_s h}\int_{t-h(t)}^{t}\dot{\xi}^T(s)\bar{R}_{ij}\dot{\xi}(s)ds \\ &\le -\frac{(\lambda_s+\lambda_u)e^{-\lambda_s h}}{h}\int_{t-h(t)}^{t}\dot{\xi}^T(s)ds\bar{R}_{ij}\int_{t-h(t)}^{t}\dot{\xi}(s)ds \\ &= -\frac{(\lambda_s+\lambda_u)e^{-\lambda_s h}}{h} \\ &\quad \times [\xi(t) - \xi(t-h(t))]^T\bar{R}_{ij}[\xi(t) - \xi(t-h(t))]. \end{aligned} \tag{26}$$

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Moreover, from system (18), for any invertible matrix $M_j = \text{diag}\{\tilde{M}_j, \tilde{M}_j\}$ with appropriate dimensions, the following identity holds

$$\begin{aligned} &-2[\xi^T(t) \;\; \dot{\xi}^T(t)]M_j^T[\dot{\xi}(t) - \bar{C}_i\dot{\xi}(t-h(t)) \\ &\quad - \bar{A}_{ij}\xi(t) - \bar{B}_i\xi(t-\tau(t)) - \bar{D}_{ij}\tilde{\mathbf{e}}(t)] = 0. \quad (27) \end{aligned}$$

From Lemma 1, we know that there exist matrices $N_{ij} = \text{diag}\{\tilde{N}_{ij}, \tilde{N}_{ij}\} > 0$ for $\forall i, j \in \mathcal{M}$ satisfying

$$\begin{aligned} &2\xi^T(t)M_j^T\bar{D}_{ij}\tilde{\mathbf{e}}(t) \\ &\quad \le \xi^T(t)M_j^T\bar{D}_{ij}N_{ij}^{-1}\bar{D}_{ij}^TM_j\xi(t) + \tilde{\mathbf{e}}^T(t)N_{ij}\tilde{\mathbf{e}}(t) \quad (28) \end{aligned}$$

and

$$\begin{aligned} &2\dot{\xi}^T(t)M_j^T\bar{D}_{ij}\tilde{\mathbf{e}}(t) \\ &\quad \le \dot{\xi}^T(t)M_j^T\bar{D}_{ij}N_{ij}^{-1}\bar{D}_{ij}^TM_j\dot{\xi}(t) + \tilde{\mathbf{e}}^T(t)N_{ij}\tilde{\mathbf{e}}(t). \quad (29) \end{aligned}$$

Combining (25)-(29) with (24) and taking into account the triggering condition (13), we have

$$\dot{V}_{ij}(t) - \lambda_u V_{ij}(t) \le \varsigma^T(t)\Theta_{ij}\varsigma(t) \quad (30)$$

where

$$\begin{aligned} \varsigma^T(t) = [&\xi^T(t) \;\; \dot{\xi}^T(t) \;\; \xi^T(t-\tau(t)) \;\; \dot{\xi}^T(t-h(t)) \\ &\xi(t-h(t)) \;\; \int_{t-h(t)}^{t}\xi(s)ds], \end{aligned}$$

$$\Theta_{ij} = \begin{bmatrix} \Theta_{ij}^{11} & \Theta_{ij}^{12} & M_j^T\bar{B}_i & M_j^T\bar{C}_i & \Theta_{ij}^{15} & 0 \\ * & \Theta_{ij}^{22} & M_j^T\bar{B}_i & M_j^T\bar{C}_i & 0 & 0 \\ * & * & \Theta_{ij}^{33} & 0 & 0 & 0 \\ * & * & * & \Theta_{ij}^{44} & 0 & 0 \\ * & * & * & * & \Theta_{ij}^{55} & 0 \\ * & * & * & * & * & \Theta_{ij}^{66} \end{bmatrix},$$

$$\begin{aligned} \Theta_{ij}^{11} &= M_j^T\bar{A}_{ij} + \bar{A}_{ij}^TM_j + \bar{Q}_{ij} - \frac{(\lambda_s+\lambda_u)e^{-\lambda_s h}}{h}\bar{R}_{ij} \\ &\quad + 2\eta N_{ij} + M_j^T\bar{D}_{ij}N_{ij}^{-1}\bar{D}_{ij}^TM_j - \lambda_u\bar{P}_{ij}, \\ \Theta_{ij}^{12} &= \bar{P}_{ij} - M_j^T + \bar{A}_{ij}^TM_j, \\ \Theta_{ij}^{22} &= \bar{R}_{ij} - M_j^T - M_j + M_j^T\bar{D}_{ij}N_{ij}^{-1}\bar{D}_{ij}^TM_j, \\ \Theta_{ij}^{33} &= -(1-\hat{\tau})e^{-\lambda_s\tau}\bar{Q}_{ij}, \; \Theta_{ij}^{44} = -(1-\hat{h})e^{-\lambda_s h}\bar{R}_{ij}, \\ \Theta_{ij}^{55} &= -\frac{(\lambda_s+\lambda_u)e^{-\lambda_s h}}{h}\bar{R}_{ij}, \; \Theta_{ij}^{15} = \frac{(\lambda_s+\lambda_u)e^{-\lambda_s h}}{h}\bar{R}_{ij}, \\ \Theta_{ij}^{66} &= -\frac{(\lambda_s+\lambda_u)e^{-\lambda_s\tau}}{\tau}\bar{Q}_{ij}. \end{aligned}$$

Thus $\Theta_{ij} < 0$ implies $\dot{V}_{ij}(t) \le \lambda_u V_{ij}(t)$. Integrating this inequality from t_i to t, we have

$$V_{ij}(t) \le e^{\lambda_u(t-t_i)}V_{ij}(t_i).$$

On interval $[\hat{t}_{k+1}, t_{i+1})$, since there is no switching occurring, the controller only updates the state information, and subsystem i and sub-controller u_i are active synchronously on $[\hat{t}_{k+1}, t_{i+1})$. We take Lyapunov-Krasovskii functional as

$$\begin{aligned} V_i(t) &= \xi^T(t)\bar{P}_i\xi(t) + \int_{t-\tau(t)}^{t}\xi^T(s)e^{\lambda_s(s-\tau)}\bar{Q}_i\xi(s)ds \\ &\quad + \int_{t-h(t)}^{t}\dot{\xi}^T(s)e^{\lambda_s(s-h)}\bar{R}_i\dot{\xi}(s)ds \quad (31) \end{aligned}$$

where $\bar{P}_i = \text{diag}\{\tilde{P}_i, \tilde{P}_i\} > 0, \bar{Q}_i = \text{diag}\{\tilde{Q}_i, \tilde{Q}_i\} > 0$ and $\bar{R}_i = \text{diag}\{\tilde{R}_i, \tilde{R}_i\} > 0$. Taking the time derivative of (31) along solutions of (18) yields

$$\begin{aligned} \dot{V}_i(t) + \lambda_s V_i(t) &= 2\xi^T(t)\bar{P}_i\dot{\xi}(t) + \lambda_s\xi^T(t)\bar{P}_i\xi(t) \\ &\quad + \xi^T(t)\bar{Q}_i\xi(t) + \dot{\xi}^T(t)\bar{R}_i\dot{\xi}(t) \\ &\quad - (1-\hat{\tau})\xi^T(t-\tau(t))e^{-\lambda_s\tau}\bar{Q}_i\xi(t-\tau(t)) \\ &\quad - (1-\hat{h})\dot{\xi}^T(t-h(t))e^{-\lambda_s h}\bar{R}_i\dot{\xi}(t-h(t)). \quad (32) \end{aligned}$$

Similar to (27), we have from (18) that

$$\begin{aligned} &-2[\xi^T(t) \;\; \dot{\xi}^T(t)]M_i^T[\dot{\xi}(t) - \bar{C}_i\dot{\xi}(t-h(t)) \\ &\quad - \bar{A}_i\xi(t) - \bar{B}_i\xi(t-\tau(t)) - \bar{D}_i\tilde{\mathbf{e}}(t)] = 0. \quad (33) \end{aligned}$$

From Lemma 1, we know that there exist matrices $N_i = \text{diag}\{\tilde{N}_i, \tilde{N}_i\} > 0$ for $\forall i \in \mathcal{M}$ satisfying

$$\begin{aligned} &2\xi^T(t)M_i^T\bar{D}_i\tilde{\mathbf{e}}(t) \\ &\quad \le \xi^T(t)M_i^T\bar{D}_iN_i^{-1}\bar{D}_i^TM_i\xi(t) + \tilde{\mathbf{e}}^T(t)N_i\tilde{\mathbf{e}}(t) \quad (34) \end{aligned}$$

and

$$\begin{aligned} &2\dot{\xi}^T(t)M_i^T\bar{D}_i\tilde{\mathbf{e}}(t) \\ &\quad \le \dot{\xi}^T(t)M_i^T\bar{D}_iN_i^{-1}\bar{D}_i^TM_i\dot{\xi}(t) + \tilde{\mathbf{e}}^T(t)N_i\tilde{\mathbf{e}}(t). \quad (35) \end{aligned}$$

Adding (33) and substituting (34)-(35) into (32) and taking into account the triggering condition (13), we have

$$\dot{V}_i(t) + \lambda_s V_i(t) \le \Gamma^T(t)\Theta_i\Gamma(t) \quad (36)$$

where

$$\Gamma^T(t) = [\xi^T(t) \;\; \dot{\xi}^T(t) \;\; \xi(t-\tau(t)) \;\; \dot{\xi}(t-h(t))],$$

$$\Theta_i = \begin{bmatrix} \Theta_i^{11} & \bar{P}_i - M_i^T + \bar{A}_i^TM_i & M_i^T\bar{B}_i & M_i^T\bar{C}_i \\ * & \Theta_i^{22} & M_i^T\bar{B}_i & M_i^T\bar{C}_i \\ * & * & \Theta_i^{33} & 0 \\ * & * & * & \Theta_i^{44} \end{bmatrix},$$

$$\begin{aligned} \Theta_i^{11} &= M_i^T\bar{A}_i + \bar{A}_i^TM_i + \bar{Q}_i + 2\eta N_i + M_i^T\bar{D}_iN_i^{-1}\bar{D}_i^TM_i \\ &\quad + \lambda_s\bar{P}_i, \; \Theta_i^{22} = \bar{R}_i + M_i^T\bar{D}_iN_i^{-1}\bar{D}_i^TM_i - M_i^T - M_i, \\ \Theta_i^{33} &= -(1-\hat{\tau})e^{-\lambda_s\tau}\bar{Q}_i, \; \Theta_i^{44} = -(1-\hat{h})e^{-\lambda_s h}\bar{R}_i. \end{aligned}$$

From (36), we know that $\Theta_i < 0$ implies $\dot{V}_i(t) \le -\lambda_s V_i(t)$. Integrating this inequality from $\hat{t}_{k+1}$ to t, we have

$$V_i(t) \le e^{-\lambda_s(t-\hat{t}_{k+1})}V_i(\hat{t}_{k+1}).$$

From (21), we have $V_{l_i}(t_i) \le \mu V_{l_{i-1}}(t_i^-)$ for $\forall l_i, l_{i-1} \in \mathcal{M}$ with $\mu > 1$. Note that $i = N_\sigma(t_0, t) \le N_0 + \frac{t-t_0}{\tau_a}$. Then for $t \in [t_i, \hat{t}_{k+1})$, we have

$$\begin{aligned} V_\sigma(t) &= V_{l_il_{i-1}}(t) \le e^{\lambda_u(t-t_i)}V_{l_il_{i-1}}(t_i) \\ &\le \mu e^{\lambda_u(t-t_i)}V_{l_{i-1}}(t_i^-) \\ &\le \mu e^{\lambda_u(t-t_i)}e^{-\lambda_s(t_i-\hat{t}_{k+1-j})}V_{l_{i-1}}(\hat{t}_{k+1-j}) \\ &\le \cdots \\ &\le \mu^{2i-1}e^{\lambda_u(t-t_i+(i-1)\tau_m)}e^{-\lambda_s(t_i-(i-1)\tau_m)}V_{l_0}(t_0) \\ &\le \mu^{2i-1}e^{i\lambda_u\tau_m-\lambda_s(t-i\tau_m)}V_{l_0}(t_0) \\ &= \frac{1}{\mu}e^{i[2\ln\mu+\tau_m(\lambda_u+\lambda_s)]-\lambda_s(t-t_0)}V_{l_0}(t_0) \\ &\le \frac{1}{\mu}e^{[2\ln\mu+\tau_m(\lambda_u+\lambda_s)]N_0} \\ &\quad \times e^{\left[\frac{2\ln\mu}{\tau_a}+\frac{\tau_m(\lambda_u+\lambda_s)}{\tau_a}-\lambda_s\right](t-t_0)}V_{l_0}(t_0) \quad (37) \end{aligned}$$

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where $\hat{t}_{k+1-j}$ denotes the $(k+1-j)$th sampling instant, which is also the first sampling instant on $[t_{i-1}, t_i)$. For $t \in [\hat{t}_{k+1}, t_{i+1})$, we have

$$\begin{aligned}
V_\sigma(t) &= V_{l_i}(t) \le e^{-\lambda_s(t-\hat{t}_{k+1})} V_{l_i}(\hat{t}_{k+1}) \\
&\le \mu e^{-\lambda_s(t-\hat{t}_{k+1})} V_{l_i l_{i-1}}(\hat{t}_{k+1}^-) \\
&\le \mu e^{-\lambda_s(t-\hat{t}_{k+1})} e^{\lambda_u(\hat{t}_{k+1}-t_i)} V_{l_i l_{i-1}}(t_i) \\
&\le \cdots \\
&\le \mu^{2i} e^{-\lambda_s[t-(\hat{t}_{k+1}-t_i)-(\hat{t}_{k+1-j}-t_{i-1})-\cdots-(\hat{t}_{k+1-m}-t_1)-\hat{t}_0]} \\
&\quad \times e^{\lambda_u(\hat{t}_{k+1}-t_i+\hat{t}_{k+1-j}-t_{i-1}+\cdots+\hat{t}_{k+1-m}-t_1)} V_{l_0}(t_0) \\
&\le \mu^{2i} e^{i\lambda_u\tau_m - \lambda_s(t-i\tau_m-t_0)} V_{l_0}(t_0) \\
&\le e^{[2\ln\mu+\tau_m(\lambda_u+\lambda_s)]N_0} \\
&\quad \times e^{[\frac{2\ln\mu}{\tau_a}+\frac{\tau_m(\lambda_u+\lambda_s)}{\tau_a}-\lambda_s](t-t_0)} V_{l_0}(t_0). \qquad (38)
\end{aligned}$$

From (23) and (31), we have

$$V_\sigma(t) \ge \min_{l_i,l_j\in\mathcal{M}} (\underline{\lambda}\{\bar{P}_{l_i}, \bar{P}_{l_i l_j}\})\|\xi(t)\|^2 = \alpha\|\xi(t)\|^2, \qquad (39)$$

and

$$\begin{aligned}
&V_\sigma(t_0) \\
&\le (\max_{l_i,l_j\in\mathcal{M}} \bar{\lambda}\{\bar{P}_{l_i}, \bar{P}_{l_i l_j}\} + \tau \max_{l_i,l_j\in\mathcal{M}} \bar{\lambda}\{\bar{Q}_{l_i}, \bar{Q}_{l_i l_j}\})\|\varphi\|^2 \\
&\quad + h \max_{l_i,l_j\in\mathcal{M}} (\bar{\lambda}\{\bar{R}_{l_i}, \bar{R}_{l_i l_j}\})\|\dot{\varphi}\|^2 \le \beta \max\{\|\varphi\|, \|\dot{\varphi}\|\}^2 \qquad (40)
\end{aligned}$$

where $\alpha = \min_{l_i,l_j\in\mathcal{M}} \underline{\lambda}\{\bar{P}_{l_i}, \bar{P}_{l_i l_j}\}, \beta = \max_{l_i,l_j\in\mathcal{M}} \bar{\lambda}\{\bar{P}_{l_i}, \bar{P}_{l_i l_j}\} + \tau \max_{l_i,l_j\in\mathcal{M}} \bar{\lambda}\{\bar{Q}_{l_i}, \bar{Q}_{l_i l_j}\} + h \max_{l_i,l_j\in\mathcal{M}} \bar{\lambda}\{\bar{R}_{l_i}, \bar{R}_{l_i l_j}\}$. Combining (37)-(40), we have

$$\begin{aligned}
\|\xi(t)\|^2 &\le \frac{1}{\alpha}\{V_{l_i}(t), V_{l_i l_j}(t)\} \\
&\le \left\{1, \frac{1}{\mu}\right\} \frac{\beta}{\alpha} e^{[2\ln\mu+\tau_m(\lambda_u+\lambda_s)]N_0} \\
&\quad \times e^{[\frac{2\ln\mu}{\tau_a}+\frac{\tau_m(\lambda_u+\lambda_s)}{\tau_a}-\lambda_s](t-t_0)} \max\{\|\varphi\|, \|\dot{\varphi}\|\}^2. \qquad (41)
\end{aligned}$$

According to Definition 2, exponential stability of system (18) is guaranteed from (41) when τ_a satisfies (22).

It is obvious that inequalities $\Theta_{ij} < 0$ and $\Theta_i < 0$ are both nonlinear. Applying Schur Complement Lemma and substituting matrix parameters of (18) and multiplying $\mathrm{diag}\{\underbrace{\tilde{M}_j^{-T}, \cdots, \tilde{M}_j^{-T}}_{14}\}$ and $\mathrm{diag}\{\underbrace{\tilde{M}_j^{-1}, \cdots, \tilde{M}_j^{-1}}_{14}\}$ on both sides of Θ_{ij} and multiplying $\mathrm{diag}\{\underbrace{\tilde{M}_i^{-T}, \cdots, \tilde{M}_i^{-T}}_{10}\}$ and $\mathrm{diag}\{\underbrace{\tilde{M}_i^{-1}, \cdots, \tilde{M}_i^{-1}}_{10}\}$ on both sides of Θ_i, respectively. Let $\hat{M}_i = \tilde{M}_i^{-1}$, $\hat{M}_j = \tilde{M}_j^{-1}$, $\hat{P}_{ij} = \hat{M}_j^T \tilde{P}_{ij} \hat{M}_j$, $\hat{Q}_{ij} = \hat{M}_j^T \tilde{Q}_{ij} \hat{M}_j$, $\hat{R}_{ij} = \hat{M}_j^T \tilde{R}_{ij} \hat{M}_j$, $\hat{P}_i = \hat{M}_i^T \tilde{P}_i \hat{M}_i$, $\hat{Q}_i = \hat{M}_i^T \tilde{Q}_i \hat{M}_i$, $\hat{R}_i = \hat{M}_i^T \tilde{R}_i \hat{M}_i$, $\hat{N}_i = \hat{M}_i^T \tilde{N}_i \hat{M}_i$, $\hat{N}_{ij} = \hat{M}_j^T \tilde{N}_{ij} \hat{M}_j$, $G_i = K_i \hat{M}_i$ and $G_j = K_j \hat{M}_j$, we thus obtain the equivalent inequalities (19) and (20) of $\Theta_{ij} < 0$ and $\Theta_i < 0$, which can be solved by Matlab LMI toolbox directly. ■

Remark 1: Different from [7], our proposed method is in the framework of event-triggered control, which can reduce the number of control task executions meanwhile retaining stability performance. Moreover, in our analysis process, by introducing the free-weighting matrix $\tilde{M}$, matrix inequalities (19) and (20) obtained in Theorem 1 are uncoupled, which can be solved directly.

At last, we briefly prove that there always exists a lower bound on the inter-event interval to exclude Zeno behavior.

For system (1), the switching instants are denoted by $t_0, t_1, t_2, \cdots$. Implementation of the feedback controller is done by sampling the state at time instants $\hat{t}_0, \hat{t}_1, \hat{t}_2, \cdots$. Suppose $t_0 = \hat{t}_0$. Since $\tau_m < \tau_d$, there must be more than one sampling on the *non-switching* interval $[t_i, t_{i+1})$. Let $T = \hat{t}_{k+1} - \hat{t}_k$ denote the consecutive sampling interval. We prove from two aspects: (i) *One sampling occurs on* $[t_i, t_{i+1})$. We assume that $\hat{t}_k$ is the sampling instant on $[t_i, t_{i+1})$. If $\hat{t}_k = t_i$, then it is obvious that $T \ge \tau_d > 0$. If $\hat{t}_k \in (t_i, t_{i+1})$, then $T > \tau_d - \tau_m > 0$. (ii) *Multiple samplings occur on* $[t_i, t_{i+1})$. Suppose that $\hat{t}_k$ and $\hat{t}_{k+1}$ are any two consecutive sampling instants on $[t_i, t_{i+1})$. For $t_i \le \hat{t}_k < t \le \hat{t}_{k+1} < t_{i+1}$, recall $\hat{\mathbf{e}}(t) = \hat{x}(t) - \hat{x}(\hat{t}_k)$, we thus obtain from (16) that

$$\begin{aligned}
\dot{\hat{\mathbf{e}}}(t) &= \dot{\hat{x}}(t) = A_i\hat{\mathbf{e}}(t) + B_i\hat{x}(t-\tau(t)) \\
&\quad + C_i\dot{\hat{x}}(t-h(t)) + (A_i + D_iK_i)\hat{x}(\hat{t}_k) + L_iE_i\mathbf{e}(t). \quad (42)
\end{aligned}$$

Since $\hat{\mathbf{e}}(\hat{t}_k) = 0$, the response of (42) is

$$\begin{aligned}
\hat{\mathbf{e}}(t) = &\int_{\hat{t}_k}^t e^{A_i(t-s)}(B_i\hat{x}(s-\tau(s)) + C_i\dot{\hat{x}}(s-h(s)) \\
&+ (A_i + D_iK_i)\hat{x}(\hat{t}_k) + L_iE_i\mathbf{e}(s))ds. \qquad (43)
\end{aligned}$$

Therefore, we have

$$\begin{aligned}
\|\hat{\mathbf{e}}(t)\| \le &\int_{\hat{t}_k}^t e^{\|A_i\|(t-s)}(\|B_i\hat{x}(s-\tau(s)) + C_i\dot{\hat{x}}(s-h(s))\| \\
&+ \|(A_i + D_iK_i)\hat{x}(\hat{t}_k)\| + \|L_iE_i\mathbf{e}(s)\|)ds. \qquad (44)
\end{aligned}$$

From Theorem 1, we know $\mathbf{e}(t)$ and $\hat{x}(t)$ are convergent on $[\hat{t}_k, \hat{t}_{k+1})$, which implies that there exist positive constants κ_1, κ_2 and λ_1 such that $\|\mathbf{e}(t)\| \le \kappa_1 e^{-\lambda_1(t-\hat{t}_k)}\|\mathbf{e}(\hat{t}_k)\|_r$. Thus

$$\begin{aligned}
\|\hat{\mathbf{e}}(t)\| \le &\int_{\hat{t}_k}^t e^{\|A_i\|(t-s)}(\varphi_1\|\hat{x}(\hat{t}_k)\|_r + \varphi_2\|\hat{x}(\hat{t}_k)\| \\
&+ \kappa_1 e^{-\lambda_1(s-\hat{t}_k)}\|L_iE_i\|\|\mathbf{e}(\hat{t}_k)\|_r)ds \\
\le &\,\phi(\hat{t}_k)\int_{\hat{t}_k}^t e^{\|A_i\|(t-s)}ds - \triangle(\hat{t}_k) \qquad (45)
\end{aligned}$$

where $\phi(\hat{t}_k) = \varphi_1\|\hat{x}(\hat{t}_k)\|_r + \varphi_2\|\hat{x}(\hat{t}_k)\| + \kappa_1\|L_iE_i\|\|\mathbf{e}(\hat{t}_k)\|_r$, $\triangle(\hat{t}_k) > 0, \varphi_1 = \kappa_2(\|B_i\| + \|C_i\|)$ and $\varphi_2 = \|A_i + D_iK_i\|$.

If $\|A_i\| \ne 0$, then we have

$$\|\hat{\mathbf{e}}(t)\| \le \frac{\phi(\hat{t}_k)}{\|A_i\|}\left(e^{\|A_i\|(t-\hat{t}_k)} - 1\right) - \triangle(\hat{t}_k). \qquad (46)$$

Recall the triggering condition (13), the next event will not be generated before $\|\hat{\mathbf{e}}(t)\| = \sqrt{\eta}\|\xi(t)\|$. Thus, the inter-event interval can be lower bounded by

$$T \ge t - \hat{t}_k = \frac{1}{\|A_i\|}\ln\left(\frac{(\sqrt{\eta}\|\xi(t)\| + \triangle(\hat{t}_k))\|A_i\|}{\phi(\hat{t}_k)} + 1\right) > 0.$$

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If $\|A_i\| = 0$, then we have

$$T \geq t - \hat{t}_k = \frac{\sqrt{\eta}\|\xi(t)\| + \triangle(\hat{t}_k)}{\phi(\hat{t}_k)} > 0.$$

From the above reasoning, it can be concluded that there exists a positive lower bound of the minimum inter-event interval, which means that Zeno behavior is theoretically excluded.

IV. An Illustrative Example

In this section, we illustrate the effectiveness of the proposed control strategy by a numerical example. Consider system (1) with two subsystems, where

$$\begin{aligned} A_1 &= \begin{bmatrix} -5 & 0 \\ 0 & -3 \end{bmatrix}, B_1 = \begin{bmatrix} -0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \\ C_1 &= \begin{bmatrix} 0.1 & 0.1 \\ 0 & -0.1 \end{bmatrix}, D_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, E_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -4.5 & 0 \\ 0 & -2.5 \end{bmatrix}, B_2 = \begin{bmatrix} -0.2 & 0 \\ 0 & 0.3 \end{bmatrix}, \\ C_2 &= \begin{bmatrix} 0.2 & 0.1 \\ 0 & -0.1 \end{bmatrix}, D_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, E_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}, \\ h(t) &= 0.2\sin t + 0.1, \ \tau(t) = 0.1\sin t + 0.2. \end{aligned}$$

Let $\alpha = 1$. We solve inequality (6) and obtain

$$L_1 = \begin{bmatrix} -1.1439 \\ -0.0755 \end{bmatrix}, \ L_2 = \begin{bmatrix} -0.9377 \\ 0.2919 \end{bmatrix}.$$

Let parameters be $\eta = 1, \lambda_s = 1, \lambda_u = 0.1, \mu = 1.2$ and $\tau_m = 1$. Then we obtain $\tau_a > 1.4647$ from (22) in Theorem 1. Memorizing values of L_1 and L_2, we solve inequalities (19)-(21) in Theorem 1 and obtain

$$K_1 = \begin{bmatrix} 0.0275 & 0.0057 \\ 0.0210 & -0.0310 \end{bmatrix}, K_2 = \begin{bmatrix} 0.2801 & 0.4761 \\ -0.0068 & -0.2058 \end{bmatrix}.$$

Choose a certain switching signal and the initial state $\hat{x}_0 = \mathbf{e}_0 = [-2 \quad 3]^T$. We obtain the system state responses and the sampled state responses in Fig. 2. Triggered instants are presented in Fig. 3. Switching signals of controlled system and controller are illustrated in Fig. 4. From the simulation results, we can see that system (1) is stabilized under the control input (15) determined by the triggering mechanism (13).

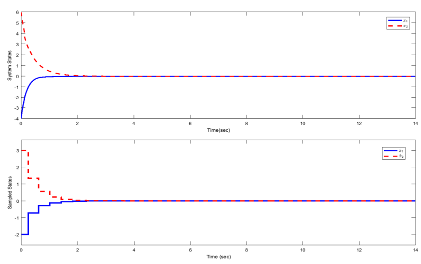

Fig. 2: System state responses and sampled state responses.

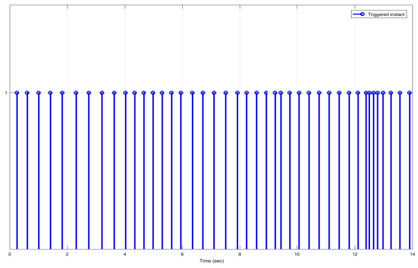

Fig. 3: Event-triggered instants: logical value is true when an event is triggering.

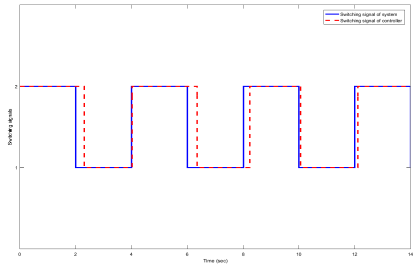

Fig. 4: Switching signals.

V. Conclusions

We have presented a new result on event-triggered control of switched linear neutral systems with mixed time-varying delays. The event is triggered whenever the defined error becomes larger when compared with the state norm. A sufficient condition is obtained to guarantee exponential stability of the closed-loop system subject to an average dwell time constraint. The future work will extend the proposed method to switched sensor networks with time delays [35].

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