In Catilinam IV ⋆

Marcus Tullius Cicero [a], Julius Caesar [b], Publius Maro Vergilius [c]

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Abstract

Cum M. Cicero consul Nonis Decembribus senatum in aede Iovis Statoris consuleret, quid de iis coniurationis Catilinae sociis fieri placeret, qui in custodiam traditi essent, factum est, ut duae potissimum sententiae proponerentur, una D. Silani consulis designati, qui morte multandos illos censebat, altera C. Caesaris, qui illos publicatis bonis per municipia Italiae distribuendos ac vinculis sempiternis tenendos existimabat.

Key words: Cicero; Catiline; orations.

1 Introduction

Video, patres conscripti, in me omnium vestrum ora atque oculos esse conversos, video vos non solunn de vestro ac rei publicae, verum etiam, si id depulsum sit, de meo periculo esse sollicitos. Est mihi iucunda in malis et grata in dolore vestra erga me voluntas, sed eam, per deos inmortales, deponite atque obliti salutis meae de vobis ac de vestris liberis cogitate. Mihi si haec condicio consulatus data est, ut omnis acerbitates, onunis dolores cruciatusque perferrem, feram non solum fortiter, verum etiam lubenter, dum modo meis laboribus vobis populoque Romano dignitas salusque pariatur.

A novel LOI(linear operator Inequality) approach have been proposed by Peet

A new method converting from DDE to PIE and the method converting from NDS to PIE have been proposed by Representation of Networks and Systems with Delay:DDEs, DDFs, ODE-PDEs and PIEs,comparing with the method used by Delay-Dependent Stability for Load Frequency Control System via Linear Operator Inequality, the new method can convert not only multiple time-delay system but also the nuetral delay system(NDS) even the neutral system with Integral term.

In this article,we consider the stability of the neutral system by using the new method proposed by Peet.

1.1 A subsection

Marcus Tullius Cicero, 106–43 B.C. was a Roman statesman, orator, and philosopher. A major figure in the last years of the Republic, he is best known for his orations against Catiline [1] and for his mastery of Latin prose [**?**]. He was a contemporary of Julius Caesar (Fig. **??**).

2 Problem formulation and preliminaries

Consider a neutral system described by the following equation:

$$\begin{aligned}\dot{x}(t) = A_0 x(t) + B_1 w(t) + B_2 u(t) \\ + \sum_{i=1}^{K} [A_i x(t-\tau_i) + B_{1i} w(t-\tau_i) \\ + B_{2i} u(t-\tau_i) + E_i \dot{x}(t-\tau_i)]\end{aligned} \quad (1)$$

⋆ This paper was not presented at any IFAC meeting. Corresponding author M. T. Cicero. Tel. +XXXIX-VI-mmmxxi. Fax +XXXIX-VI-mmmxxv.

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[1] This footnote should be very brief.

If we do not consider the interfere $w(t)$

$$\dot{x}(t) = A_0 x(t) + B_2 u(t) + \sum_{i=1}^{K} [A_i x(t-\tau_i) + B_{2i} u(t-\tau_i) + E_i \dot{x}(t-\tau_i)] \tag{2}$$

If we also do not consider the output $y(t)$,we can convert Equation (2) to standard NDS(neutral delay system) form Equation (3) by the Equation (4)

$$\begin{bmatrix} \dot{x}(t) \\ z(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A_0 & 0 & B_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \\ u(t) \end{bmatrix} + \sum_{i=1}^{K} \begin{bmatrix} A_i & 0 & B_{2i} & E_i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x(t-\tau_i) \\ w(t-\tau_i) \\ u(t-\tau_i) \\ \dot{x}(t-\tau_i) \end{bmatrix}. \tag{3}$$

And then cite Representation of Networks and Systems with Delay:DDEs, DDFs, ODE-PDEs and PIEs ,we can convert standard NDS form Equation (3) to DDF(Differential Difference Equations) form Eqution (7) by Eqution (4)

then we get the standard DDF form

$$\begin{bmatrix} \dot{x}(t) \\ z(t) \\ y(t) \\ r_i(t) \end{bmatrix} = \begin{bmatrix} A_0 & 0 & B_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ C_{ri} & B_{r1i} & B_{r2i} \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \\ u(t) \end{bmatrix} + \begin{bmatrix} B_v \\ D_{1v} \\ D_{2v} \\ D_{rvi} \end{bmatrix} v(t)$$
$$v(t) = \sum_{i=1}^{K} C_{vi} r(t-\tau_i) \tag{7}$$

And then we try to cite Representation of Networks and Systems with Delay:DDEs, DDFs, ODE-PDEs and PIEs convert standard DDF form Equation (7) to PIE from Equation (8)

$$\mathcal{T}\dot{\mathbf{x}} + \mathcal{B}_{T_2}\dot{u} = \mathcal{A}\mathbf{x} + \mathcal{B}_2 u \tag{8}$$

If we introduce the controller,

$$u(t) = K\mathbf{x}(\mathbf{t}) \tag{9}$$

$$\mathbf{x}(\mathbf{t}) := \begin{bmatrix} x(t) \\ \Phi(t,.) \end{bmatrix}$$

then we got the standard PIE form

$$\mathcal{T}\dot{\mathbf{x}} + \mathcal{B}_{T_2}K\dot{x} = \mathcal{A}\mathbf{x} + \mathcal{B}_2 K x \tag{10}$$

Next we will do some variable substitution

$$\mathcal{T}' = \mathcal{T} + \mathcal{B}_{T_2}K \tag{11}$$

$$\mathcal{A}' = \mathcal{A} + \mathcal{B}_2 K \tag{12}$$

Conversion Formula from NDS to DDF:

$$D_{rvi} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ I & 0 & 0 \end{bmatrix}, \qquad [C_{ri} B_{r1i} B_{r2i}] = \begin{bmatrix} I_n & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_p \\ A_0 & 0 & B_2 \end{bmatrix}, \quad I_{n+q+r} = \begin{bmatrix} B_v \\ D_{1v} \\ D_{2v} \end{bmatrix}$$
$$C_{vi} = \begin{bmatrix} A_i & 0 & B_{2i} & E_i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad C_{vdi}(s) = 0 \tag{4}$$

Conversion Formula from ODE-PDE to DDF to PIE:

$$\mathcal{A} = \mathcal{P}\begin{bmatrix} A_0 & A \\ 0 & \{I_\tau, 0, 0\} \end{bmatrix}, \quad \mathcal{T} = \mathcal{P}\begin{bmatrix} I & 0 \\ T_0 & \{0, T_a, T_b\} \end{bmatrix}, \quad \mathcal{B}_{T_1} = \mathcal{P}\begin{bmatrix} 0 & \varnothing \\ T_1 & \{\varnothing\} \end{bmatrix}, \quad \mathcal{B}_{T_2} = \mathcal{P}\begin{bmatrix} 0 & \varnothing \\ T_2 & \{\varnothing\} \end{bmatrix},$$
$$\mathcal{B}_1 = \mathcal{P}\begin{bmatrix} \mathbf{B_1} & \varnothing \\ 0 & \{\varnothing\} \end{bmatrix}, \quad \mathcal{B}_2 = \mathcal{P}\begin{bmatrix} \mathbf{B_2} & \varnothing \\ 0 & \{\varnothing\} \end{bmatrix}, \quad \mathcal{C}_1 = \mathcal{P}\begin{bmatrix} \mathbf{C_{10}} & \mathbf{C_{11}} \\ \varnothing & \{\varnothing\} \end{bmatrix}, \quad \mathcal{C}_2 = \mathcal{P}\begin{bmatrix} \mathbf{C_{20}} & \mathbf{C_{21}} \\ \varnothing & \{\varnothing\} \end{bmatrix} \tag{5}$$

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then we got a more concise PIE form

$$\mathcal{T}'\dot{\mathbf{x}} = \mathcal{A}'\mathbf{x} \tag{13}$$

remark? It should be noticed that is the difference between $\mathbf{x}$ and x. The reason why $\mathbf{x}$ terms and x terms can be added up is that there is no difference for the operator $\mathcal{B}_{T_2}$ to dual with $\mathbf{x}$ and x.

3 Stability Analysis

Cite Delay-Dependent Stability for Load FrequencyControl System via Linear Operator Inequality

Theorem 1

$$\mathcal{T}'^*\mathcal{H}\mathcal{A}' + \mathcal{A}'^*\mathcal{H}\mathcal{T}' < 0 \tag{14}$$

Proof:
L-K funtional

$$V(\mathbf{x}) = < \mathcal{T}\mathbf{x}, \mathcal{H}\mathcal{A}\mathbf{x} >_Z \tag{15}$$

$$\begin{aligned} \dot{V}(\mathbf{x}) &= < \mathcal{T}'\mathbf{x}, \mathcal{H}\mathcal{T}'\mathbf{x} >_Z + < \mathcal{A}'\mathbf{x}, \mathcal{H}\mathcal{T}'\mathbf{x} >_Z \\ &= < \mathbf{x}, (\mathcal{T}'^*\mathcal{H}\mathcal{A}' + \mathcal{A}'^*\mathcal{H}\mathcal{T}')\mathbf{x} >_Z \end{aligned} \tag{16}$$

remark? The procedures for calculating the delay margin are provided as follows.?

4 Case Studies Using *PIETOOLS2022a*

$$\begin{aligned} \begin{bmatrix} \dot{x}(t) \\ z(t) \\ y(t) \end{bmatrix} &= \begin{bmatrix} A_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \\ u(t) \end{bmatrix} \\ &+ \begin{bmatrix} A_1 & 0 & 0 & E_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x(t-\tau_1) \\ w(t-\tau_1) \\ u(t-\tau_1) \\ \dot{x}(t-\tau_1) \end{bmatrix} \\ &+ \begin{bmatrix} A_2 & 0 & 0 & E_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x(t-\tau_2) \\ w(t-\tau_2) \\ u(t-\tau_2) \\ \dot{x}(t-\tau_2) \end{bmatrix}. \end{aligned} \tag{17}$$

$$\begin{aligned} &A_0 = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix} \qquad A_1 = \begin{bmatrix} 0 & 0.3 \\ -0.3 & 0 \end{bmatrix} \\ &A_2 = \begin{bmatrix} 0.1 & 0.05 \\ -0.05 & 0.1 \end{bmatrix} \qquad E_1 = \begin{bmatrix} 0 & -0.1 \\ -0.1 & 0 \end{bmatrix} \\ &E_2 = \begin{bmatrix} 0.05 & 0 \\ -0 & 0.05 \end{bmatrix} \\ &\tau_1 = 2.262, \quad \tau_2 = 0.5 \end{aligned} \tag{18}$$

need Tables!!

5 Conclusion

cite Delay-Dependent Stability for Load FrequencyControl System via Linear Operator Inequality

$$\mathcal{T}'^*\mathcal{H}\mathcal{A}' + \mathcal{A}'^*\mathcal{H}\mathcal{T}' < 0 \tag{19}$$

$$
\hat{C}_{vi} = C_{vi}, \qquad D_I = \begin{bmatrix} (I-\sum_{i=1}^K C_{vi}D_{rvi})^{-1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \end{bmatrix},
$$

$$
C_{Ii} = -D_I C_{vi} = \begin{bmatrix} -(I-\sum_{i=1}^K E_i)^{-1}A_i & 0 & -(I-\sum_{i=1}^K E_i)^{-1}B_{2i} & -(I-\sum_{i=1}^K E_i)^{-1}E_i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},
$$

$$
[T_0 \quad T_1 \quad T_2] = \begin{bmatrix} C_{r1}+D_{rv1}C_{vx} & C_{r1}+D_{rv1}D_{vw} & C_{r1}+D_{rv1}D_{vu} \\ C_{r2}+D_{rv2}C_{vx} & C_{r2}+D_{rv2}D_{vw} & C_{r2}+D_{rv2}D_{vu} \\ \vdots & \vdots & \vdots \\ C_{rK}+D_{rvK}C_{vx} & C_{rK}+D_{rvK}D_{vw} & C_{rK}+D_{rvK}D_{vu} \end{bmatrix},
$$

$$
C_{r1}+D_{rv1}C_{vx} = \begin{bmatrix} I_n \\ 0 \\ A_0 + (I-\sum_{i=1}^K E_i)^{-1}(\sum_{i=1}^K (A_i + E_i A_0) \end{bmatrix},
$$

$$
[C_{vx} \quad D_{vw} \quad D_{vu}] = \begin{bmatrix} (I-\sum_{i=1}^K E_i)^{-1}(\sum_{i=1}^K (A_i+E_iA_0) & (I-\sum_{i=1}^K E_i)^{-1}(E_iB_1) & (I-\sum_{i=1}^K E_i)^{-1}(E_iB_2) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},
$$

$$
T_a(s,\theta) = \begin{bmatrix} D_{rv1}C_{I1} & D_{rv1}C_{I2} & \dots & D_{rv1}C_{IK} \\ D_{rv2}C_{I1} & D_{rv2}C_{I2} & \dots & D_{rv2}C_{IK} \\ \vdots & \vdots & \vdots & \vdots \\ D_{rvK}C_{I1} & D_{rvK}C_{I2} & \dots & D_{rvK}C_{IK} \end{bmatrix},
$$

$$
D_{rv1}C_{I1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -(I-\sum_{i=1}^K E_i)^{-1}A_1 & 0 & -(I-\sum_{i=1}^K E_i)^{-1}B_{21} & -(I-\sum_{i=1}^K E_i)^{-1}E_1 \end{bmatrix},
$$

$$
T_b(s,\theta) = -I_{\sum_i p_i} + T_a(s,\theta) = \begin{bmatrix} -I & 0 & 0 & 0 \\ 0 & -I & 0 & 0 \\ 0 & 0 & -I & 0 \\ -(I-\sum_{i=1}^K E_i)^{-1}A_1 & 0 & -(I-\sum_{i=1}^K E_i)^{-1}B_{21} & -I-(I-\sum_{i=1}^K E_i)^{-1}E_1 \end{bmatrix},
$$

$$
I_\tau = \begin{bmatrix} \frac{1}{\tau_1}I_{p1} & 0 & 0 & 0 \\ 0 & \frac{1}{\tau_2}I_{p2} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \frac{1}{\tau_K}I_{pK} \end{bmatrix}, \quad \begin{bmatrix} A \\ C_{11} \\ C_{21} \end{bmatrix} = \begin{bmatrix} B_v \\ D_{1v} \\ D_{2v} \end{bmatrix} [C_{I1} \dots C_{IK}], \quad \begin{bmatrix} \mathbf{A_0} & \mathbf{B_1} & \mathbf{B_2} \\ \mathbf{C_{10}} & \mathbf{D_{11}} & \mathbf{D_{12}} \\ \mathbf{C_{20}} & \mathbf{D_{21}} & \mathbf{D_{22}} \end{bmatrix} = \begin{bmatrix} A_0 & B_1 & B_2 \\ C_{10} & D_{11} & D_{12} \\ C_{20} & D_{21} & D_{22} \end{bmatrix} [C_{vx} \quad D_{vw} \quad D_{vu}], \tag{6}
$$

PROOF :

$$\begin{aligned}\dot{V}(\mathbf{x}) &=< \mathcal{T}'\mathbf{x}, \mathcal{H}\mathcal{T}'\mathbf{x} >_Z + < \mathcal{A}'\mathbf{x}, \mathcal{H}\mathcal{T}'\mathbf{x} >_Z \\ &=< \mathbf{x}, (\mathcal{T}'^{*}\mathcal{H}\mathcal{A}' + \mathcal{A}'^{*}\mathcal{H}\mathcal{T}')\mathbf{x} >_Z\end{aligned} \tag{20}$$

6 References

cite Delay-Dependent Stability for Load FrequencyControl System via Linear Operator Inequality

$$\mathcal{T}'^{*}\mathcal{H}\mathcal{A}' + \mathcal{A}'^{*}\mathcal{H}\mathcal{T}'<0 \tag{21}$$

$$\begin{aligned}\dot{V}(\mathbf{x}) &=< \mathcal{T}'\mathbf{x}, \mathcal{H}\mathcal{T}'\mathbf{x} >_Z + < \mathcal{A}'\mathbf{x}, \mathcal{H}\mathcal{T}'\mathbf{x} >_Z \\ &=< \mathbf{x}, (\mathcal{T}'^{*}\mathcal{H}\mathcal{A}' + \mathcal{A}'^{*}\mathcal{H}\mathcal{T}')\mathbf{x} >_Z\end{aligned} \tag{22}$$

7 Appendixes

PROOF : lemma2
Given NDS form

$$\begin{aligned}\begin{bmatrix}\dot{x}(t)\\ z(t)\\ y(t)\end{bmatrix} &= \begin{bmatrix}A_0 & 0 & B_2\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x(t)\\ w(t)\\ u(t)\end{bmatrix} \\ &+ \sum_{i=1}^{K}\begin{bmatrix}A_i & 0 & B_{2i} & E_i\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x(t-\tau_i)\\ w(t-\tau_i)\\ u(t-\tau_i)\\ \dot{x}(t-\tau_i)\end{bmatrix}\end{aligned} \tag{23}$$

assume that

$$v(t) = \sum_{i=1}^{K} C_{vi} r_i(t-\tau_i) \tag{24}$$

then we get

$$\begin{bmatrix}\dot{x}(t)\\ z(t)\\ y(t)\end{bmatrix} = \begin{bmatrix}A_0 & 0 & B_2\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x(t)\\ w(t)\\ u(t)\end{bmatrix} + Iv(t) \tag{25}$$

I is a identity matrix,we define

$$I = \begin{bmatrix}B_v\\ D_{1v}\\ D_{2v}\end{bmatrix} \tag{26}$$

so that

$$\begin{bmatrix} \dot{x}(t) \\ z(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A_0 & 0 & B_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \\ u(t) \end{bmatrix} + \begin{bmatrix} B_v \\ D_{1v} \\ D_{2v} \end{bmatrix} v(t) \tag{27}$$

from the above eqution,we get

$$\dot{x}(t) = \begin{bmatrix} A_0 & 0 & B_2 \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \\ u(t) \end{bmatrix} + B_v v(t) \tag{28}$$

we can see $B_v v(t)$ Representing the first part of $v(t)$,so we get

$$B_v v(t) = \begin{bmatrix} I & 0 & 0 \end{bmatrix} v(t) \tag{29}$$

define

$$r_i(t) = \begin{bmatrix} x(t) \\ z(t) \\ y(t) \\ \dot{x}(t) \end{bmatrix} \tag{30}$$

$$\begin{aligned} r_i(t) = \begin{bmatrix} x(t) \\ z(t) \\ y(t) \\ \dot{x}(t) \end{bmatrix} &= \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \\ u(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dot{x}(t) \end{bmatrix} \\ &= \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ A_0 & 0 & B_2 \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \\ u(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ I & 0 & 0 \end{bmatrix} v(t) \end{aligned} \tag{31}$$

Cite [NDS to DDF] we get

$$r_i(t) = \begin{bmatrix} C_{ri} & B_{r1i} & B_{r2i} \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \\ u(t) \end{bmatrix} + D_{rvi} v(t) \tag{32}$$

merge eqution (27) and eqution (30),we get the standard DDF form

$$\begin{bmatrix} \dot{x}(t) \\ z(t) \\ y(t) \\ r_i(t) \end{bmatrix} = \begin{bmatrix} A_0 & 0 & B_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ C_{ri} & B_{r1i} & B_{r2i} \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \\ u(t) \end{bmatrix} + \begin{bmatrix} B_v \\ D_{1v} \\ D_{2v} \\ D_{rvi} \end{bmatrix} v(t) \tag{33}$$

PROOF : lemma4