ISA Transactions ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

ISA Transactions

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Robust H_∞ control for a class of 2-D discrete delayed systems

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ARTICLE INFO

Article history:
Received 28 July 2013
Received in revised form
29 October 2013
Accepted 14 December 2013
This paper was recommended for publication by Prof. A.B. Rad

Keywords:
2-D systems
Delayed perturbations
Robust H_∞ control

ABSTRACT

In this paper, we deal with the problem of robust H_∞ control for a class of 2-D discrete uncertain systems with delayed perturbations described by the Roesser state-space model (RM). The problem to be addressed is the design of robust controllers via state feedback such that the stability of the resulting closed-loop system is guaranteed and a prescribed H_∞ performance level is ensured for all delayed perturbations. By utilizing the Lyapunov method and some results, H_∞ controllers are given. The results are delay-dependent and can be expressed in terms of linear matrix inequalities (LMIs). Finally, some numerical examples are given to illustrate the effectiveness of the proposed results.

1. Introduction

Time delays are frequently encountered in various engineering systems [1,38–40]. It is known that time delays are natural components of dynamic processes [2]. In practical industrial fields, such as virtual laboratories, chemical processes, time delayed perturbations may cause systems instability, oscillations or degraded performances [2]. Recently, stability analysis of systems with delayed perturbations has received considerable attention [1–11]. By using the positive definite solution of the Lyapunov equation, Kim [1] investigated the robust stability of linear systems with time-varying perturbations in the time-delayed states. By using the Riccati matrix inequality framework, Ooba and Funahashi [8] also addressed this problem. A stabilizing control law for exponential stability of a class of nonlinear dynamical systems with delayed perturbations was designed using Lyapunov stability theory in [5]. A new sufficient delay dependent exponential stability condition for a class of linear time-varying systems with nonlinear delayed perturbations was derived by using an improved Lyapunov–Krasovskii functional in [6]. By employing an improved Razumikhin-type theorem, robust stability and stabilizability conditions for a class of linear systems subject to delayed time-varying nonlinear perturbations were derived in [7]. A less conservative delay-dependent robust stability condition for linear time-varying delay systems under nonlinear perturbation was derived in [9], using integral inequality approach to express the relationship of Leibniz–Newton formula terms in the within the framework of LMIs. In the literature, the Lyapunov method and some strategies are mainly used.

The study of two-dimensional (2-D) systems has received considerable attention and various approaches to deal with it, for 2-D dynamical systems have many important applications [12]. Some stability results of 2-D systems have been reported in the literature [12–17], etc. Some algebraic algorithms are used and some criteria in terms of LMIs are provided. A great deal of research has been devoted to the stabilization problem [18–28], etc. In [18], the classical definition of the H_2 performance is extended to 2-D systems and an original sufficient condition was presented for evaluation of the H_2 performance, and systematic design methods for the H_2 and mixed H_2/H_∞ control. In [19], the authors presented a state-space solution to the problem of H_∞ control of 2-D systems. The problem of robust H_∞ control for uncertain 2-D discrete systems with a class of generalized Lipschitz nonlinearities has been investigated in [24]. Recently, robust guaranteed cost control [22,28], linear quadratic Gaussian control [21], functional observers [27], finite frequency filtering [25], etc. have been investigated.

Time delays correspond to transportation time or computation time, encountered for instance during the processing of visual image which is intrinsically 2-D [29]. Thus, it becomes appropriate to study 2-D delayed systems. Many important and useful results have been reported in the literature [29–36]. Recently, much attention has been focused on the robust H_∞ control problem. To mention a few, the problem of robust H_∞ control for uncertain 2-D discrete state delayed systems with a class of generalized Lipschitz nonlinearities was studied in [30]. Delay-independent and delay-dependent output feedback H_∞ controllers for 2-D

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0019-0578/$ - see front matter © 2013 ISA. Published by Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.isatra.2013.12.017

systems with delays in the states were developed in [31]. Delay-dependent H_∞ control for uncertain 2-D discrete systems with state delay in the Roesser model was considered in [32].

However, the results for 2-D discrete delayed systems available in the literature are mainly on delayed state. To the best of the authors′ knowledge, the corresponding problems of stability analysis and control on 2-D uncertain systems with delayed perturbations have been investigated in [36] firstly. Research in this area should be very important, and it need to be studied deeply, which motivate us to do the present work. Stimulated by the Lyapunov method used in [2,33] and H_∞ performance definition in [19], based on [36], we present an approach for robust stability analysis and H_∞ control for a class of 2-D discrete systems with delayed perturbations in this paper. It is different from the previous approaches for 2-D discrete delayed systems, because there are perturbations in the delayed state. Hence, based on the property of perturbation, we utilize the Lyapunov method and some other strategy to establish results for stability of such 2-D systems. Based on results of stability, H_∞ state feedback controllers are proposed such that the corresponding closed-loop systems are asymptotically stable.

In this paper, we discuss the problem of robust H_∞ control of 2-D discrete systems setting with delayed unstructured and structured perturbations. Different from other 2-D delayed systems, there are nonlinear perturbations in delayed state. We utilize the Lyapunov method and some other strategy to establish results for robust H_∞ control of such 2-D systems. The results are delay-dependent and can be rearrange to LMIs.

The paper is organized as follows. In Section 2, the problem to be tackled is stated. In Section 3, the problem of robust H_∞ controllers is designed for 2-D systems with delayed perturbations. Finally, some numerical examples are provided to illustrate the presented technique in Section 4. A brief conclusion is given in Section 5.

Throughout this paper, the following notations are used. $\mathcal{R}^n$, $\mathcal{R}^{n\times m}$ denote the set of real numbers, the n dimensional Euclidean space, the set of all real $n\times m$ matrices, respectively. I is the identity matrix with compatible dimension and $diag\{\cdots\}$ denotes a block diagonal matrix. $X \ge Y$ (respectively, $X > Y$), where X and Y are real symmetric matrices, means that $X-Y$ is positive-semidefinite (respectively, positive definite). The superscript X^T is the transpose of X. $\|X\|$ refers to either the Euclidean vector norm or the induced matrix 2-norm. The $*$ is used as an ellipsis for the terms that are implied by symmetry.

2. Problem formulation

Consider the 2-D discrete uncertain systems with delayed perturbations given by

$$\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = A\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + E(i,j)\begin{bmatrix} x^h(i-d,j) \\ x^v(i,j-d) \end{bmatrix} + B_0\omega(i,j)+B_1u(i,j), \tag{1}$$

$$z(i,j) = C_0\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + C_1\omega(i,j)+C_2u(i,j), \tag{2}$$

where $x^h(i,j)\in\mathcal{R}^{n_1}$, $x^v(i,j)\in\mathcal{R}^{n_2}(n_1+n_2=n)$, $z(i,j)\in\mathcal{R}^q$, $\omega(i,j)\in\mathcal{R}^p$, and $u(i,j)\in\mathcal{R}^m$ represent the horizontal state, the vertical state, the controlled output, the noise input and the control input, respectively. A, B_0, B_1, C_0, C_1, and C_2 are the system real matrices with appropriate dimension. The positive integer d represents delay in the whole dynamic process. The boundary conditions are specified as $X(0)=[X^{hT}(0)X^{vT}(0)]^T$, where

$$\begin{cases} X^h(0)=\{x^h(i,j);\ \forall j\ge 0; i=-d,-d+1,\dots,0\}, \\ X^v(0)=\{x^v(i,j);\ \forall i\ge 0; j=-d,-d+1,\dots,0\}. \end{cases} \tag{3}$$

The boundary conditions are assumed to satisfy

$$\sum_{j=0}^{\infty}\sum_{i=-d}^{0} x^{hT}(i,j)x^h(i,j)<\infty, \qquad \sum_{i=0}^{\infty}\sum_{j=-d}^{0} x^{vT}(i,j)x^v(i,j)<\infty.$$

$E(i,j)\in\mathcal{R}^{(n_1+n_2)\times(n_1+n_2)}$ represents the varying nonlinear perturbation in the delayed state. We consider two cases.

Case 1: System (1)–(2) with unstructured perturbations where $E(i,j)$ is assumed to be bounded, i.e.,

$$\|E(i,j)\| \le \eta, \tag{4}$$

and η is a positive constant number.

Case 2: System (1)–(2) with structured perturbations where $E(i,j)$ is assumed to take the form

$$E(i,j)=\sum_{\alpha=1}^{m} q_\alpha(i,j)E_\alpha, \tag{5}$$

where $E_\alpha\in\mathcal{R}^{(n_1+n_2)\times(n_1+n_2)}(\alpha=1,2,\dots,m)$ are real constant matrices and $q_\alpha(i,j)(\alpha=1,2,\dots,m)$ are the varying uncertain parameters.

Definition 1. Given a positive scalar $\gamma>0$, 2-D system (1)–(2) with zero initial boundary condition is said to have an H_∞ performance γ if it is asymptotically stable with the following property:

$$\|z(i,j)\|_2 \le \gamma\|\omega(i,j)\|_2. \tag{6}$$

To simplify the notation, define the vector

$$x_{(\tau,0)}=\begin{bmatrix} x^h(i+\tau,j) \\ x^v(i,j+\tau) \end{bmatrix},$$

where τ is the given integer. Then, system (1)–(2) can be rewritten as

$$x_{(1,0)}(i,j)=Ax_{(0,0)}(i,j)+E(i,j)x_{(-d,0)}(i,j)+B_0\omega(i,j)+B_1u(i,j), \tag{7}$$

$$z(i,j)=C_0x_{(0,0)}(i,j)+C_1\omega(i,j)+C_2u(i,j). \tag{8}$$

In this paper, we will study the stability of unforced system (9) firstly.

$$x_{(1,0)}(i,j)=Ax_{(0,0)}(i,j)+E(i,j)x_{(-d,0)}(i,j). \tag{9}$$

Then, when $\omega(i,j)=0$, we propose a state feedback controller

$$u(i,j)=Kx_{(0,0)}(i,j) \tag{10}$$

for the following system:

$$x_{(1,0)}(i,j)=Ax_{(0,0)}(i,j)+E(i,j)x_{(-d,0)}(i,j)+B_1u(i,j). \tag{11}$$

Then, the closed-loop system with controller (10) can be expressed as

$$x_{(1,0)}(i,j)=(A+B_1K)x_{(0,0)}(i,j)+E(i,j)x_{(-d,0)}(i,j). \tag{12}$$

Next, based on the results, we will design state feedback controller (10) for 2-D discrete system (7)–(8) such that closed-loop system

$$x_{(1,0)}(i,j)=(A+B_1K)x_{(0,0)}(i,j)+E(i,j)x_{(-d,0)}(i,j)+B_0\omega(i,j), \tag{13}$$

$$z(i,j)=(C_0+C_2K)x_{(0,0)}(i,j)+C_1\omega(i,j), \tag{14}$$

has an H_∞ performance and derive the corresponding LMI-based algorithm.

Next, we present an inequality that will be essential in the proof of our main results.

Lemma 1 (*Wang et al.* [37]). *For any $x, y \in R^n$ and any positive definite matrix $P \in R^{n\times n}$, we have*

$$2x^T y \le x^T P x + y^T P^{-1} y.$$

3. Main results

In this section, firstly, we give some results on the problem of robust stability analysis and robust control for system (9) with unstructured delayed perturbations and structured delayed perturbations in [36]. Based on the results, next, we consider H_∞ performance analysis for such systems. Finally, robust H_∞ controllers are given for 2-D discrete system (7)–(8).

3.1. Robust stability analysis

Here, we state robust stability analysis of system (9) with delayed perturbation (5). The following theorem provides the results.

Theorem 1 (*Ye and Li* [36]). *Consider system (9) with delayed perturbation (4), if there exists a positive symmetric matrix $P = diag\{P_h, P_v\} > 0$, and scalars $\beta_1 > 0, \beta_2 > 0, \lambda > 0$ satisfying*

$$A^T P A - P + A^T P(\lambda I - P)^{-1} P A + \beta_1 I + d\beta_2 I < 0, \tag{15}$$

$$0 < P < \lambda I, \tag{16}$$

then, system (9) is asymptotically stable if the following inequality holds:

$$\eta \le \sqrt{(\beta_1 + \beta_2)/\lambda}. \tag{17}$$

Remark 1. Since there exists $P(\lambda I - P)^{-1} P$ in (15), (15) is not a linear matrix inequality. Next, a sufficient condition of stability in terms of LMIs is presented in Corollary 1. Obviously, the following results are in a strict linear matrix inequality form, we can solve by the Matlab LMI Toolbox easily.

Corollary 1 (*Ye and Li* [36]). *Consider system (9) with delayed perturbation (4), if there exists a positive symmetric matrix $P = diag\{P_h, P_v\} > 0$, and scalars $\beta_1 > 0, \beta_2 > 0, \lambda > 0$ satisfying*

$$\begin{bmatrix} \beta_1 I + d\beta_2 I - P & A^T P & A^T P \\ PA & -P & 0 \\ PA & 0 & P - \lambda I \end{bmatrix} < 0, \tag{18}$$

$$0 < P < \lambda I, \tag{19}$$

then, system (9) is asymptotically stable if

$$\eta \le \sqrt{(\beta_1 + \beta_2)/\lambda} \tag{20}$$

holds.

Here, the following results provide robust stability analysis of system (9) with delayed perturbation (5).

Theorem 2 (*Ye and Li* [36]). *Consider system (9) with delayed perturbation (5), if there exists a positive symmetric matrix $P = diag\{P_h, P_v\} > 0$, and scalars $\beta_1 > 0, \beta_2 > 0, \lambda > 0$ satisfying*

$$A^T P A - P + A^T P(\lambda I - P)^{-1} P A + \beta_1 I + d\beta_2 I < 0, \tag{21}$$

$$0 < P < \lambda I, \tag{22}$$

then, system (9) is asymptotically stable if the following inequality holds:

$$\sum_{\alpha=1}^{m} q_\alpha^2(i,j) \le \frac{\beta_1 + \beta_2}{\lambda \sigma_{\max}^2(E_c)}, \tag{23}$$

where $E_c = [E_1, E_2, \ldots E_m]$.

Corollary 2 (*Ye and Li* [36]). *Consider system (9) with delayed perturbation (5), if there exists a positive symmetric matrix $P = diag\{P_h, P_v\} > 0$, and scalars $\beta_1 > 0, \beta_2 > 0, \lambda > 0$ satisfying*

$$\begin{bmatrix} \beta_1 I + d\beta_2 I - P & A^T P & A^T P \\ PA & -P & 0 \\ PA & 0 & P - \lambda I \end{bmatrix} < 0, \tag{24}$$

and

$$0 < P < \lambda I, \tag{25}$$

then, system (9) is asymptotically stable if the following inequality holds:

$$\sum_{\alpha=1}^{m} q_\alpha^2(i,j) \le \frac{\beta_1 + \beta_2}{\lambda \sigma_{\max}^2(E_c)}, \tag{26}$$

where $E_c = [E_1, E_2, \ldots E_m]$.

Next, the methods of corresponding designing controllers $u(i,j) = Kx_{(0,0)}(i,j)$ for system (11) with delayed perturbation (4) and (5) have been given in the following theorems.

Theorem 3 (*Ye and Li* [36]). *Consider system (11) with delayed perturbation (4), if there exists a matrix K, a positive symmetric matrix $P = diag\{P_h, P_v\} > 0$, and scalars $\beta_1 > 0, \beta_2 > 0, \lambda > 0$ satisfying*

$$\begin{aligned}(A + B_1 K)^T P (A + B_1 K) - P + (A + B_1 K)^T P(\lambda I - P)^{-1} P(A + B_1 K) \\ + \beta_1 I + d\beta_2 I < 0,\end{aligned} \tag{27}$$

$$0 < P < \lambda I, \tag{28}$$

and

$$\eta \le \sqrt{(\beta_1 + \beta_2)/\lambda}, \tag{29}$$

then there exists a state feedback controller $u(i,j) = Kx_{(0,0)}(i,j)$ such that system (11) is asymptotically stable.

Corollary 3 (*Ye and Li* [36]). *Consider system (11) with delayed perturbation (4), if there exist positive symmetric matrices $S = diag\{S_h, S_v\} > 0$, $S_1 = diag\{S_{h1}, S_{v1}\} > 0$, $S_2 = diag\{S_{h2}, S_{v2}\} > 0$, $S_3 = diag\{S_{h3}, S_{v3}\} > 0$, and a matrix U satisfying*

$$\begin{bmatrix} S_1 + dS_2 - S & SA^T + U^T B_1^T & SA^T + U^T B_1^T \\ AS + B_1 U & -S & 0 \\ AS + B_1 U & 0 & S - S_3 \end{bmatrix} < 0, \tag{30}$$

$$0 < S < S_3, \tag{31}$$

and

$$S_3 \eta^2 \le S_1 + S_2, \tag{32}$$

then there exists a state feedback controller $u(i,j) = Kx_{(0,0)}(i,j)$ with $K = US^{-1}$ such that system (11) is asymptotically stable.

Theorem 4 (*Ye and Li* [36]). *Consider system (11) with delayed perturbation (5), if there exists a positive symmetric matrix $P = diag\{P_h, P_v\} > 0$, and scalars $\beta_1 > 0, \beta_2 > 0, \lambda > 0$ satisfying*

$$\begin{aligned}(A + B_1 K)^T P (A + B_1 K) - P + (A + B_1 K)^T P(\lambda I - P)^{-1} P(A + B_1 K) \\ + \beta_1 I + d\beta_2 I < 0,\end{aligned} \tag{33}$$

$$0 < P < \lambda I, \tag{34}$$

$$\sum_{\alpha=1}^{m} q_\alpha^2(i,j) \le \frac{\beta_1 + \beta_2}{\lambda \sigma_{\max}^2(E_c)}, \tag{35}$$

where $E_c = [E_1, E_2, \ldots E_m]$. Then, there exists a state feedback controller $u(i,j) = Kx_{(0,0)}(i,j)$ such that system (11) is asymptotically stable.

Corollary 4 (*Ye and Li* [36]). *Consider system (11) with delayed perturbation (5), if there exist positive symmetric matrices $S = diag\{S_h, S_v\} > 0$, $S_1 = diag\{S_{h1}, S_{v1}\} > 0$, $S_2 = diag\{S_{h2}, S_{v2}\} > 0$,*

$S_3 = diag\{S_{h3}, S_{v3}\} > 0$, and a matrix U satisfying

$$\begin{bmatrix} S_1+dS_2-S & SA^T+U^TB_1^T & SA^T+U^TB_1^T \\ AS+B_1U & -S & 0 \\ AS+B_1U & 0 & S-S_3 \end{bmatrix} < 0, \tag{36}$$

$$0 < S < S_3, \tag{37}$$

$$S_3 \sum_{\alpha=1}^{m} q_\alpha^2(i,j)\sigma_{\max}^2(E_c) \le S_1+S_2, \tag{38}$$

where $E_c = [E_1, E_2, \ldots E_m]$. Then, there exists a state feedback controller $u(i,j) = K_{(0,0)}x(i,j)$ with $K = US^{-1}$ such that system (11) is asymptotically stable.

Remark 2. Theorems 3 and 4 give sufficient conditions for stabilize system (9) with delayed perturbation (4) and (5). But inequalities (27) and (33) are not linear matrix inequalities. Corollaries 3 and 4 present sufficient conditions for stabilization of system (9) with delayed perturbation (4) and (5) in a strict linear matrix inequality form.

3.2. H_∞ performance analysis

Next, we study the robust H_∞ control problem for 2-D discrete system (7)–(8) under the zero initial boundary condition.

Firstly, let $u(i,j)=0$, we consider H_∞ performance of the following system:

$$x_{(1,0)}(i,j) = Ax_{(0,0)}(i,j)+E(i,j)x_{(-d,0)}(i,j)+B_0\omega(i,j), \tag{39}$$

$$z(i,j) = C_0x_{(0,0)}(i,j)+C_1\omega(i,j). \tag{40}$$

We state the H_∞ performance of system (39)–(40) with delayed perturbation (4) under the zero initial condition. The following theorem presents a result.

Theorem 5. *Consider system (39)–(40) with delayed perturbation (4) under the zero initial boundary condition, given a positive scalar γ, if there exists a positive symmetric matrix $P = diag\{P_h, P_v\} > 0$, and scalars $\beta_1 > 0, \beta_2 > 0, \lambda > 0$ satisfying*

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ * & \Theta_{22} \end{bmatrix} < 0, \tag{41}$$

$$0 < P < \lambda I, \tag{42}$$

and

$$\eta \le \sqrt{(\beta_1+\beta_2)/\lambda}, \tag{43}$$

where

$$\Theta_{11} = A^T[P+P(\lambda I-P)^{-1}P]A-P+\beta_1 I+d\beta_2 I+C_0^TC_0,$$
$$\Theta_{12} = C_0^TC_1+A^T[P+P(\lambda I-P)^{-1}P]B_0,$$
$$\Theta_{22} = C_1^TC_1-\gamma^2 I+B_0^T[P+P(\lambda I-P)^{-1}P]B_0,$$

then the system is asymptotically stable and has H_∞ performance.

Proof. Define a Lyapunov function candidate as

$$V(i,j) = V_1(i,j)+V_2(i,j)+V_3(i,j), \tag{44}$$

with

$$V_1(i,j) = x_{(0,0)}^T(i,j)Px_{(0,0)}(i,j),$$
$$V_2(i,j) = \beta_1 \sum_{\theta=-d}^{-1} x_{(\theta,0)}^T(i,j)x_{(\theta,0)}(i,j),$$
$$V_3(i,j) = \beta_2 \sum_{\theta=1}^{d}\sum_{\tau=-\theta}^{-1} x_{(\tau,0)}^T(i,j)x_{(\tau,0)}(i,j),$$

where $P = diag\{P_h, P_v\} > 0$, $\beta_1 > 0, \beta_2 > 0$. For establishing the H_∞ performance, we introduce

$$J = \|z(i,j)\|_2^2 - \gamma^2\|\omega(i,j)\|_2^2 + \Delta V(i,j),$$

Using Eq. (40), we have

$$J = x_{(0,0)}^T(i,j)C_0^TC_0x_{(0,0)}(i,j)+\omega^T(i,j)(C_1^TC_1-\gamma^2 I)\omega(i,j)$$
$$+2x_{(0,0)}^T(i,j)C_0^TC_1\omega(i,j)+\Delta V(i.j).$$

Then, we evaluate $\Delta V(i,j)$ to yield

$$\Delta V(i,j) = \Delta V_1(i,j)+\Delta V_2(i,j)+\Delta V_3(i,j),$$

where

$$\begin{aligned}\Delta V_1(i,j) = {} & [Ax_{(0,0)}(i,j)+B_0\omega(i,j)]^TP[Ax_{(0,0)}(i,j)+B_0\omega(i,j)] \\ & +2[Ax_{(0,0)}(i,j)+B_0\omega(i,j)]^TPE(i,j)x_{(-d,0)}(i,j) \\ & -x_{(-d,0)}^T(i,j)E^T(i,j)(\lambda I-P)E(i,j)x_{(-d,0)}(i,j) \\ & +\lambda x_{(-d,0)}^T(i,j)E^T(i,j)E(i,j)x_{(-d,0)}(i,j)-x_{(0,0)}^T(i,j)Px_{(0,0)}(i,j)\end{aligned}$$

Applying Lemma 1 and Eq. (42), we have

$$\begin{aligned}\Delta V_1(i,j) \le {} & [Ax_{(0,0)}(i,j)+B_0\omega(i,j)]^TP[Ax_{(0,0)}(i,j)+B_0\omega(i,j)] \\ & +[Ax_{(0,0)}(i,j)+B_0\omega(i,j)]^TP(\lambda I-P)^{-1}P[Ax_{(0,0)}(i,j)+B_0\omega(i,j)] \\ & +\lambda x_{(-d,0)}^T(i,j)E^T(i,j)E(i,j)x_{(-d,0)}(i,j)-x_{(0,0)}^T(i,j)Px_{(0,0)}(i,j)\end{aligned}$$

and obtain

$$\Delta V_2(i,j) = \beta_1 x_{(0,0)}^T(i,j)x_{(0,0)}(i,j)-\beta_1 x_{(-d,0)}^T(i,j)x_{(-d,0)}(i,j),$$
$$\Delta V_3(i,j) \le d\beta_2 x_{(0,0)}^T(i,j)x_{(0,0)}(i,j)-\beta_2 x_{(-d,0)}^T(i,j)x_{(-d,0)}.$$

Then, we have

$$J \le \zeta^T(i,j)\Theta\zeta(i,j)+x_{(-d,0)}^T(i,j)(\lambda\eta^2-\beta_1-\beta_2)x_{(-d,0)}(i,j)$$

where

$$\zeta(i,j) = \begin{bmatrix} x_{(0,0)}(i,j) \\ \omega(i,j) \end{bmatrix}.$$

By Eqs. (41) and (43), we obtain $J \le 0$. Applying the Schur complement, we have $\Delta V(i,j) \le 0$. Thus, (39)–(40) is asymptotically stable. we can get $\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\Delta V(i,j) \ge 0$, then obtain $\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}(\|z(i,j)\|^2 - \|\omega(i,j)|^2) < 0$, which implies that $\|z(i,j)\|_2 < \gamma\|\omega(i,j)\|_2$. This completes the proof. □

In Theorem 5, Eq. (41) is nonlinear. Next, we present conditions in terms of LMIs.

Corollary 5. *Consider system (39)–(40) with delayed perturbation (4) under the zero initial boundary condition, given a positive scalar γ, if there exists a positive symmetric matrix $P = diag\{P_h, P_v\} > 0$, and scalars $\beta_1 > 0, \beta_2 > 0, \lambda > 0$ satisfying*

$$\begin{bmatrix} \beta_1 I+d\beta_2 I+C_0^TC_0-P & C_0^TC_1 & A^TP & A^TP \\ * & C_1^TC_1-\gamma^2 I & B_0^TP & B_0^TP \\ * & * & -P & 0 \\ * & * & * & -\lambda I+P \end{bmatrix} < 0, \tag{45}$$

$$0 < P < \lambda I, \tag{46}$$

and

$$\eta \le \sqrt{(\beta_1+\beta_2)/\lambda}, \tag{47}$$

then the system is asymptotically stable and has H_∞ performance.

Proof. By applying the Schur complements to (45), we can easily obtain Corollary 5. This completes the proof. □

Next, we present H_∞ performance of system (39)–(40) with delayed perturbation (5). The following theorem presents a result. We omit the process of proof for brevity.

Theorem 6. *Consider system (39)–(40) with delayed perturbation (5) under the zero initial boundary condition, given a positive scalar γ, if there exists a positive symmetric matrix $P=diag\{P_h,P_v\}>0$, and scalars $\beta_1>0,\beta_2>0,\lambda>0$ satisfying*

$$\Theta=\begin{bmatrix}\Theta_{11} & \Theta_{12}\\ * & \Theta_{22}\end{bmatrix}<0, \tag{48}$$

$$0<P<\lambda I, \tag{49}$$

and

$$\sum_{\alpha=1}^{m} q_\alpha^2(i,j)\le\frac{\beta_1+\beta_2}{\lambda\sigma_{\max}^2(E_c)}, \tag{50}$$

where

$$\Theta_{11}=A^T[P+P(\lambda I-P)^{-1}P]A-P+\beta_1 I+d\beta_2 I+C_0^TC_0,$$
$$\Theta_{12}=C_0^TC_1+A^T[P+P(\lambda I-P)^{-1}P]B_0,$$
$$\Theta_{22}=C_1^TC_1-\gamma^2 I+B_0^T[P+P(\lambda I-P)^{-1}P]B_0,$$
$$E_c=[E_1,E_2,\cdots E_m],$$

then the system is asymptotically stable and has H_∞ performance.

Corollary 6. *Consider system (39)–(40) with delayed perturbation (5) under the zero initial boundary condition, given a positive scalar γ, if there exists a positive symmetric matrix $P=diag\{P_h,P_v\}>0$, and scalars $\beta_1>0,\beta_2>0,\lambda>0$ satisfying*

$$\begin{bmatrix}\beta_1 I+d\beta_2 I+C_0^TC_0-P & C_0^TC_1 & A^TP & A^TP\\ * & C_1^TC_1-\gamma^2 I & B_0^TP & B_0^TP\\ * & * & -P & 0\\ * & * & * & -\lambda I+P\end{bmatrix}<0, \tag{51}$$

$$0<P<\lambda I, \tag{52}$$

and

$$\sum_{\alpha=1}^{m} q_\alpha^2(i,j)\le\frac{\beta_1+\beta_2}{\lambda\sigma_{\max}^2(E_c)}, \tag{53}$$

where $E_c=[E_1,E_2,\ldots E_m]$. Then, the system is asymptotically stable and has H_∞ performance.

3.3. Robust H_∞ controller design

In the following, we design a state feedback controller $u(i,j)=Kx_{(0,0)}(i,j)$ such that 2-D discrete systems (7)–(8) with delayed perturbation have H_∞ performance. Applying Theorems 5, 6 and the Schur complement, results are proposed. We omit the process of proof.

Theorem 7. *Consider system (7)–(8) with delayed perturbation (4) under the zero initial boundary condition, given a positive scalar γ, if there exist positive symmetric matrices $S=diag\{S_h,S_v\}$, $S_1=diag\{S_{h1},S_{v1}\}$, $S_2=diag\{S_{h2},S_{v2}\}$, $S_3=diag\{S_{h3},S_{v3}\}$, and a matrix U satisfying*

$$\begin{bmatrix}S_1+dS_2I-S & 0 & SA^T+U^TB_1^T & SA^T+U^TB_1^T & SC_0^T+U^TC_2^T\\ * & -\gamma^2 I & B_0^T & B_0^T & C_1^T\\ * & * & -S & 0 & 0\\ * & * & * & -S_3+S & 0\\ * & * & * & * & -I\end{bmatrix}<0, \tag{54}$$

$$S<S_3, \tag{55}$$

and

$$\eta^2 S_3\le S_1+S_2, \tag{56}$$

then the system is stabilizable and has H_∞ performance, and a suitable H_∞ state feedback controller gain is given by $K=US^{-1}$.

Theorem 8. *Consider system (7)–(8) with delayed perturbation (5) under the zero initial boundary condition, given a positive scalar γ, if there exist positive symmetric matrices $S=diag\{S_h,S_v\}$, $S_1=diag\{S_{h1},S_{v1}\}$, $S_2=diag\{S_{h2},S_{v2}\}$, $S_3=diag\{S_{h3},S_{v3}\}$, and a matrix U satisfying*

$$\begin{bmatrix}S_1+dS_2I-S & 0 & SA^T+U^TB_1^T & SA^T+U^TB_1^T & SC_0^T+U^TC_2^T\\ * & -\gamma^2 I & B_0^T & B_0^T & C_1^T\\ * & * & -S & 0 & 0\\ * & * & * & -S_3+S & 0\\ * & * & * & * & -I\end{bmatrix}<0, \tag{57}$$

$$S<S_3, \tag{58}$$

and

$$S_3\sigma_{\max}^2(E_c)\sum_{\alpha=1}^{m} q_\alpha^2(i,j)\le S_1+S_2, \tag{59}$$

where $E_c=[E_1,E_2,\cdots E_m]$. Then, then the system is stabilizable and has H_∞ performance, and a suitable H_∞ state feedback controller gain is given by $K=US^{-1}$.

Remark 3. Theorems 7 and 8 give sufficient conditions in terms of LMIs for stabilize system (7)–(8) with delayed perturbation (4) and (5) and have H_∞ performance.

4. Numerical examples

In this section, we will give numerical examples to demonstrate the effectiveness of the main results.

Example 1. Consider delayed system (7)–(8) with unstructured perturbation (4) given by

$$A=\begin{bmatrix}0.8 & 0.2\\ 0.1 & 0.9\end{bmatrix},\quad B_0=\begin{bmatrix}0.2 & 0.1\\ 0.5 & 0.1\end{bmatrix},\quad B_1=\begin{bmatrix}0.1 & 0.3\\ 0.4 & 0.2\end{bmatrix},$$

$$C_0=\begin{bmatrix}0.7 & 0.2\\ 0.1 & 0.3\end{bmatrix},\quad C_1=\begin{bmatrix}0.3 & 0.2\\ 0.3 & 0.1\end{bmatrix},\quad C_2=\begin{bmatrix}0.2 & 0.1\\ 0.1 & 0.3\end{bmatrix},$$

$$d=3,\quad \eta=0.4,\quad \gamma=0.8,\quad E(i,j)=0.4\sin(i+j).$$

By Theorem 7, inequalities (54)–(56) are feasible, and the matrices are

$$S=\begin{bmatrix}0.4118 & 0\\ 0 & 3.4586\end{bmatrix},\quad S_1=\begin{bmatrix}0.1628 & 0\\ 0 & 1.1690\end{bmatrix},$$

$$S_2=\begin{bmatrix}0.0142 & 0\\ 0 & 0.0634\end{bmatrix},\quad S_3=\begin{bmatrix}0.9334 & 0\\ 0 & 6.8138\end{bmatrix},$$

$$U=\begin{bmatrix}-0.7232 & -7.4393\\ -0.5773 & 1.7440\end{bmatrix},\quad K=\begin{bmatrix}-1.7563 & -2.1510\\ -1.4020 & 0.5043\end{bmatrix}.$$

Then, there exists a state feedback controller $u(i,j)=Kx_{(0,0)}(i,j)$ such that system (7)–(8) with structured perturbation (5) is asymptotically stable and has H_∞ performance.
When

$$u(i,j)=Kx_{(0,0)}(i,j),$$

let

$$\omega(i,j)=\begin{bmatrix}(i+j+1)^{-3}\\ (i+j+1)^{-3}\end{bmatrix},$$

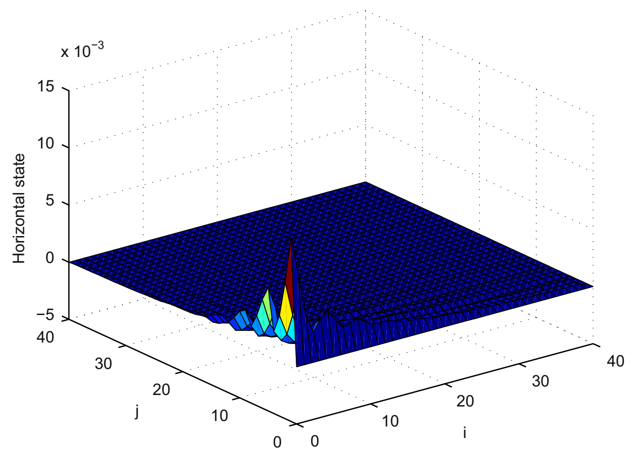

Fig. 1. Horizontal state response of closed-loop system (13)–(14) in Example 1.

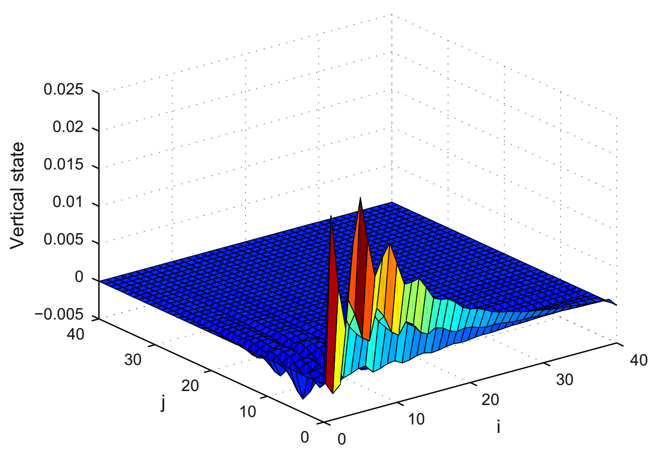

Fig. 2. Vertical state response of closed-loop system (13)–(14) in Example 1.

we can see graphics simulation of closed-loop system (13)–(14) in Figs. 1 and 2. We note that closed loop system (13)–(14) is asymptotically stable.

Example 2. Consider delayed system (7)–(8) with structured perturbation (5) given by

$$A=\begin{bmatrix}0.7 & 0.02\\ 0.1 & 0.9\end{bmatrix},\quad B_0=\begin{bmatrix}0.02 & 0.2\\ 0.4 & 0.1\end{bmatrix},\quad B_1=\begin{bmatrix}0.2 & 0.4\\ 0.5 & 0.1\end{bmatrix},$$

$$C_0=\begin{bmatrix}0.6 & 0.1\\ 0.1 & 0.4\end{bmatrix},\quad C_1=\begin{bmatrix}0.3 & 0.4\\ 0.1 & 0.1\end{bmatrix},\quad C_2=\begin{bmatrix}0.1 & 0.1\\ 0.1 & 0.2\end{bmatrix},$$

$$E_1=\begin{bmatrix}0.04 & 0.03\\ 0.05 & 0.02\end{bmatrix},\quad E_2=\begin{bmatrix}0.01 & 0\\ 0.02 & 0.05\end{bmatrix},\quad E_3=\begin{bmatrix}0.08 & 0\\ 0.03 & 0.04\end{bmatrix},$$

$$d=4,\quad m=3,\quad \gamma=0.7,\quad q_1=\sin(i+j),\quad q_2=\cos(i+j),\quad q_3=0.1.$$

By Theorem 8, inequalities (57)–(59) are feasible, and we can obtain

$$S=\begin{bmatrix}1.2834 & 0\\ 0 & 2.1780\end{bmatrix},\quad S_1=\begin{bmatrix}0.2599 & 0\\ 0 & 0.5472\end{bmatrix},$$

$$S_2=\begin{bmatrix}0.0548 & 0\\ 0 & 0.1194\end{bmatrix},\quad S_3=\begin{bmatrix}3.5989 & 0\\ 0 & 4.5033\end{bmatrix},$$

$$U=\begin{bmatrix}-0.5402 & -4.2244\\ -2.3138 & 1.2641\end{bmatrix},\quad K=\begin{bmatrix}-0.4209 & -1.9396\\ -1.8028 & 0.5804\end{bmatrix}.$$

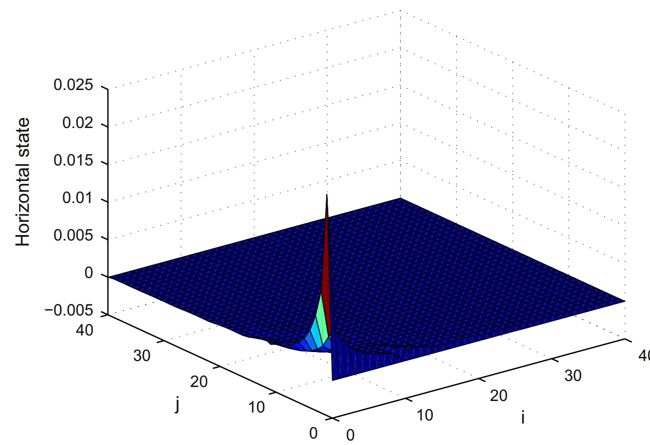

Fig. 3. Horizontal state response of closed-loop system (13)–(14) in Example 2.

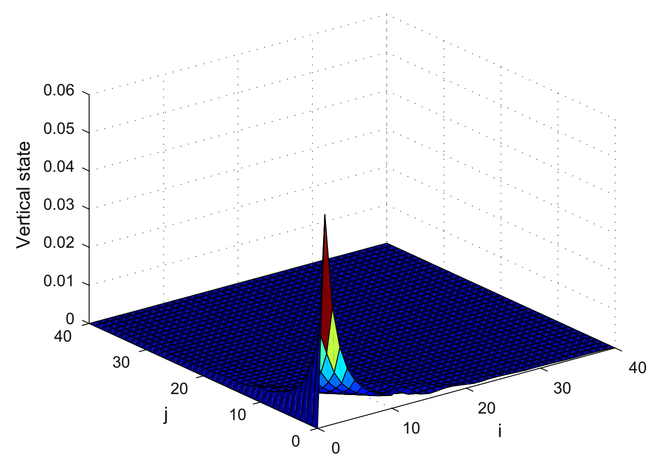

Fig. 4. Vertical state response of closed-loop system (13)–(14) in Example 2.

Then, there exists a state feedback controller $u(i,j)=Kx_{(0,0)}(i,j)$ such that system (7)–(8) with structured perturbation (5) is asymptotically stable and has H_∞ performance. When $u(i,j)=Kx_{(0,0)}(i,j)$, let

$$\omega(i,j)=\begin{bmatrix}(i+j+1)^{-1}\\ (i+j+1)^{-1}\end{bmatrix},$$

we can see graphics simulation of closed-loop system (13)–(14) in Figs. 3 and 4. We note that closed loop system (13)–(14) is asymptotically stable.

The above examples have shown the effectiveness of the proposed approach for robust H_∞ control for 2-D discrete systems with delayed perturbations.

5. Conclusion

The main contribution of this paper is that a new method has been presented for robust H_∞ control for 2-D discrete systems with delayed perturbations described by unstructured and structured in the RM firstly. The results are delay-dependent and expressed in terms of LMIs. Further development on robust control for such systems will be required to reduce conservative and obtain more relaxed criteria.

Please cite this article as: Ye S, et al. Robust. ISA Transactions (2014), http://dx.doi.org/10.1016/j.isatra.2013.12.017

Acknowledgment

This work is jointly supported by the National Natural Science Foundation of China under Grant 60874078, 61074006, 61174137, 61203024, 61304249, 51307074, Natural Science Fundamental Research Project of Jiangsu Colleges and Universities under Grant 12KJB120001, Natural Science Foundation of Jiangsu Province Grant BK20130466, and Jiangsu University Superiority Discipline Construction Project Subsidization Project.

References

[1] Kim JH. Robust stability of linear systems with delayed perturbations. IEEE Trans Autom Control 1996;41(12):1820–2.
[2] Ni ML, Er MJ. Stability of linear systems with delayed perturbations: an LMI approach. IEEE Trans Circuits Syst I: Fundam Theory Appl 2002;49(1):108–12.
[3] Trinh H, Aldeen M. On the stability of linear systems with delayed perturbations. IEEE Trans Autom Control 1994;39(9):1948–51.
[4] Hou C, Gao F, Qian J. Improved delay time estimation of RC ladder networks. IEEE Trans Circuits Syst I: Fundam Theory Appl 2000;47(2):242–6.
[5] Park JH, Jung HY. On the exponential stability of a class of nonlinear systems including delayed perturbations. J Comput Appl Math 2003;159(2):467–71.
[6] Niamsup P, Mukdasai K, Phat VN. Improved exponential stability for time-varying systems with nonlinear delayed perturbations. Appl Math Comput 2008;204(1):490–5.
[7] Trinh H, Aldeen M. On robustness and stabilization of linear systems with delayed nonlinear perturbations. IEEE Trans Autom Control 1997;42(7):1005–7.
[8] Ooba T, Funahashi Y. Comments on Robust stability of linear systems with delayed perturbations. IEEE Trans Autom Control 1999;44(8):1582–3.
[9] Liu PL. New results on stability analysis for time-varying delay systems with non-linear perturbations. ISA Trans 2013;52(3):318–25.
[10] Wu H. Adaptive robust control of uncertain nonlinear systems with nonlinear delayed state perturbations. Automatica 2009;45(8):1979–84.
[11] Chen S, Chou J, Zheng L. Stability robustness of linear output feedback systems with both time-varying structured and unstructured parameter uncertainties as well as delayed perturbations. J Frankl Inst 2005;342(2):213–34.
[12] Kaczorek T. Two-dimensional linear systems. Berlin: Springer-Verlag; 1985.
[13] Xiao Y. 2-D algebraic test for robust stability of quasipolynomials with interval parameters. Asian J Control 2006;8(2):174–9.
[14] Lin Z, Ying JQ, Xu L. An algebraic approach to strong stabilizability of linear nD MIMO systems. IEEE Trans Autom Control 2002;47(9):1510–4.
[15] Xu S, Lam J, Galkowski K, Lin Z. An LMI approach to the computation of lower bounds for stability margins of 2D discrete systems. Dyn Contin Discrete Impuls Syst Ser B: Appl Algorithms 2006;13(2):221–36.
[16] Cui J, Li Q, Hu G, Zhu Q, Zhang X. Asymptotical stability of 2-D linear discrete stochastic systems. Digital Signal Process 2012;22(4):628–32.
[17] Ebihara Y, Ito Y, Hagiwara T. Exact stability analysis of 2-D systems using LMIs. IEEE Trans Autom Control 2006;51(9):1509–13.
[18] Yang R, Xie L, Zhang C. H_2 and mixed H_2/H_∞ control of two-dimensional systems in Roesser model. Automatica 2006;42(9):1507–14.
[19] Du C, Xie L, Zhang C. H_∞ control and robust stabilization of two-dimensional systems in Roesser models. Automatica 2001;37(2):205–11.
[20] Xu S, Lam J, Lin Z, Galkowski K. Positive real control for uncertain two-dimensional systems. IEEE Trans Circuits Syst I: Fundam Theory Appl 2002;49(11):1659–66.
[21] Yang R, Zhang C, Xie L. Linear quadratic Gaussian control of 2-dimensional systems. Multidimens Syst Signal Process 2007;18(4):273–95.
[22] Guan X, Long C, Duan G. Robust optimal guaranteed cost control for 2D discrete systems. IEE Proc Control Theory Appl 2001;148(5):355–61.
[23] Gao H, Lam J, Xu S, Wang C. Stability and stabilization of uncertain 2-D discrete systems with stochastic perturbation. Multidimens Syst Signal Process 2005;16(1):85–106.
[24] Xu H, Zou Y, Xu S. Robust H_∞ control for a class of uncertain nonlinear two-dimensional systems. Int J Innov Comput Inf Control 2005;1(2):181–91.
[25] Li X, Gao H. Robust finite frequency filtering for uncertain 2-D Roesser systems. Automatica 2012;48(6):1163–70.
[26] Kosugi N, Suyama K. A new method for solving Bezout equations over 2-D polynomial matrices from delay systems. Syst Control Lett 2012;61(6):723–9.
[27] Xu H, Lin Z, Makur A. The existence and design of functional observers for two-dimensional systems. Syst Control Lett 2012;61(2):362–8.
[28] Dhawan A, Kar H. An LMI approach to robust optimal guaranteed cost control of 2-D discrete systems described by the Roesser model. Signal Process 2010;90(9):2648–54.
[29] Paszke W, Lam J, Galkowski K, Xu S, Lin Z. Robust stability and stabilisation of 2D discrete state-delayed systems. Syst Control Lett 2004;51(3–4):277–91.
[30] Xu H, Zou Y, Lu J, Xu S. Robust H_∞ control for a class of uncertain nonlinear two-dimensional systems with state delays. J Frankl Inst 2005;42(7):877–91.
[31] Peng D, Guan X. Output feedback H_∞ control for 2-D state-delayed systems. Circuits Syst Signal Process 2009;28(1):147–67.
[32] Xu J, Nan Y, Zhang G, Ou L, Ni H. Delay-dependent H_∞ control for uncertain 2-D discrete systems with state delay in the Roesser Model. Circuits Syst Signal Process 2013;32(3).
[33] Paszke W, Lam J, Galkowski K, Xu S, Rogers E, Kummert A. Delay-dependent stability of 2D state-delayed linear systems. In: IEEE international symposium on circuits and systems, 2006. p. 2813–6.
[34] Izuta G. Observer design for 2D discrete systems with delays. In: International conference on control and automation, vol. 1, 2005. p. 224–9.
[35] Chen SF. Stability analysis for 2-D systems with interval time-varying delays and saturation nonlinearities. Signal Process 2010;90(7):2265–75.
[36] Ye S, Li J. Robust control for a class of 2-D discrete uncertain delayed systems. In: The 10th IEEE international conference on control and automation (ICCA), 2013. p. 1048–52.
[37] Wang Y, Xie L, de Souza CE. Robust control of a class of uncertain nonlinear systems. Syst Control Lett 1992;19(2):139–49.
[38] Zha W, Zhai J, Fei S. Global output feedback control for a class of high-order feedforward nonlinear systems with input delay. ISA Trans 2013;52(4):494–500.
[39] Jiang S, Fang H. H_∞ static output feedback control for nonlinear networked control systems with time delays and packet dropouts. ISA Trans 2013;52(2):215–22.
[40] Wen S, Zeng Z, Huang T, Bao G. Observer-based H_∞ control of a class of mixed delay systems with random data losses and stochastic nonlinearities. ISA Trans 2013;52(2):207–14.