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Stability and Stabilization of Switched Linear Systems With Mode-Dependent Average Dwell Time

Xudong Zhao, Lixian Zhang, Peng Shi, and Ming Liu

***Abstract*—In this paper, the stability and stabilization problems for a class of switched linear systems with mode-dependent average dwell time (MDADT) are investigated in both continuous-time and discrete-time contexts. The proposed switching law is more applicable in practice than the average dwell time (ADT) switching in which each mode in the underlying system has its own ADT. The stability criteria for switched systems with MDADT in nonlinear setting are firstly derived, by which the conditions for stability and stabilization for linear systems are also presented. A numerical example is given to show the validity and potential of the developed techniques.**

***Index Terms*—Mode-dependent average dwell time, stability, stabilization, switched linear systems.**

I. INTRODUCTION

Switched systems, which provide an unified framework for mathematical modeling of many physical or man-made systems displaying

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switching features such as power electronics [20], flight control systems [14], network control systems [18], have been widely studied in the past decades [5], [6], [9], [16], [23], [24], [30]. The systems consists of a collection of indexed differential or difference equations and a switching signal governing the switching among them. The various switching signals differentiate switched systems from the general time-varying systems, since the solutions of the former are dependent on not only the system's initial conditions but also the switching signals.

In certain sense, the switching events in systems and control could be classified into autonomous (uncontrolled) or controlled ones [2], [8], [9], [11], [29], which, respectively, result from the system itself and the designers' intervention [12], [15]. The stability problem of the switched systems with any category of switching signals, has always been the hottest topic in the filed. Relatively, plenty of systematic results have been available for the switched systems under the uncontrolled switching, in both continuous-time domain [10], [19], and discrete-time domain [1], [3]. However, for the switched systems under controlled switching signals, the corresponding stability problem is somewhat complicated in finding suitable switching signals for ensuring system stability and improving system performance [9].

In practice, a class of controlled switching signals with restrictions on switching instants are frequently encountered, and considerable attention have been drawn to such a type of switching that we could call slow switching [9], [10], [18]. One way to specify slow switching is to introduce a scalar τ and restrict the switching signals with a property that the switching times $t_1, t_2, \ldots$ satisfy $t_{i+1} - t_i \geq \tau$ for all i belongs to the set of positive integers. This scalar τ is coined as the dwell time in the literature. By using multiple Lyapunov functions, it has been proved in [13] that the switched linear systems with stable subsystems are exponentially stable if the dwell time τ is sufficiently large. Also, some results have appeared in recent works to compute lower bounds of the dwell time for assuring the system stability [2], [4]. Yet, it is noted that, developing a dwell time switching is restrictive in some circumstances. In [7], the concept of "dwell time" is extended to the concept of "average dwell time (ADT)". It has been recognized that ADT is more flexible and efficient in system stability analysis [21], [22], [26], [28], since the ADT switching strategy may contain signals that occasionally have consecutive discontinuities separated by less than a constant τ_a (it should be compensated by switching sufficiently slowly later). Correspondingly, the stability analysis and control synthesis for the switched systems with ADT have been also reported in both the continuous-time and discrete-time domains, see for example [17], [25]–[27].

However, the property in the ADT switching that the average time interval between any two consecutive switching is at least τ_a [6], [7], [10], which is independent of the system modes, is probably still not anticipated. Besides, it has been well shown in the literature that, the minimum of admissible ADT is computed by two mode-independent parameters, i.e., the increase coefficient of the Lyapunov-like function at switching instants and the decay rate of the Lyapunov-like function during the running time of subsystems. It is straightforward that such a setup of the two *common* parameters for all subsystems in a mode-independent manner will give rise to a certain conservativeness. Therefore, how to extend the existing studies on the switched systems with ADT by providing two mode-dependent parameters, which will lead to a mode-dependent ADT accordingly, is worthwhile to proceed, which inspires us for this study.

In this paper, the problems of stability and stabilization for switched linear systems with a new class of switching signals in both continuous-time and discrete-time contexts will be investigated. The main contribution of this paper lies in that a novel notion of mode-dependent average dwell time (MDADT) is proposed which will release the restrictions of ADT. The stability and stabilization conditions of the corresponding switched linear systems with MDADT switching are also derived and formulated in terms of a set of linear matrix inequalities. The remainder of the paper is organized as follows. Section II reviews necessary definitions and lemmas on the stability analysis of general switched systems and recalls the corresponding stability criterion for the switched systems with ADT switching. In Section III, the concept of MDADT which characterizes a different set of switching signals from ADT is firstly formulated, then, the stability criteria for switched systems with MDADT in nonlinear setting are derived, upon which the conditions for stability and stabilization of switched linear systems are further developed in both continuous-time and discrete-time cases. A numerical example is presented in Section IV to demonstrate the feasibility and effectiveness of the proposed techniques.

Notations

In this paper, the notations used are fairly standard. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; the notation $\|\cdot\|$ refers to the Euclidean vector norm. $\mathbb{C}^1$ denotes the space of continuously differentiable functions, and a function $\beta : [0, \infty) \to [0, \infty)$ is said to be of class $\mathcal{K}_\infty$ if it is continuous, strictly increasing, unbounded, and $\beta(0) = 0$. In addition, the notation $P > 0\ (\geq 0)$ means that P is real symmetric and positive definite (semi-positive definite).

II. Problem Formulation and Preliminaries

Consider a class of switched linear systems given by

$$\delta x(t) = A_{\sigma(t)} x(t) + B_{\sigma(t)} u(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, the symbol δ denotes the derivative operator in the continuous-time context ($\delta x(t) = (d/dt)x(t)$) and the shift forward operator in the discrete-time case ($\delta x(t) = x(t+1)$). $\sigma(t)$ is a piecewise constant function of time, called a switching signal, which takes its values in the finite set $S = \{1, \ldots, M\}$, M is the number of subsystems. Also, for a switching sequence $0 < t_1 < \cdots < t_i < t_{i+1} < \cdots$, $\sigma(t)$ is continuous from right everywhere and may be either autonomous or controlled. When $t \in (t_i, t_{i+1})$, we say the $\sigma(t_i)^{th}$ subsystem is active. The two-matrix pair (A_p, B_p), $\forall \sigma(t) = p \in S$, represents the p^{th} subsystem or p^{th} mode of (1).

Now, we present the following exponential stability definition of system (1) for later development and we denote time by k in the discrete-time case.

Definition 1: [9] The equilibrium $x = 0$ of system (1) is globally uniformly exponentially stable (GUES) under certain switching signal $\sigma(t)$ if for $u(t) = 0$ (or $u(k) = 0$) and initial conditions $x(t_0)$ (or $x(k_0)$), there exist constants $\alpha > 0$, $\delta > 0$ (respectively, $0 < \varsigma < 1$) such that the solution of the system satisfies $\|x(t)\| \leq \alpha e^{-\delta(t-t_0)} \|x(t_0)\|$, $\forall t \geq t_0$ (respectively, $\|x(k)\| \leq \alpha \varsigma^{(k-k_0)} \|x(k_0)\|$, $\forall k \geq k_0$).

The control input $u(t)$ (or $u(k)$) in (1) is used to achieve system stability or certain performances for certain switching signals. In this paper, the state feedback is considered with $u(t) = K_{\sigma(t)} x(t)$ (or $u(k) = K_{\sigma(k)} x(k)$), where K_p, $\forall \sigma(t) = p \in S$, is the controller gain to be determined. Then, the resulting closed-loop system is given by

$$\delta x(t) = \bar{A}_p x(t) \tag{2}$$

where

$$\bar{A}_p = A_p + B_p K_p. \tag{3}$$

Therefore, in this paper, we aim at finding a more general set of admissible switching signals and the corresponding state-feedback controllers, such that the resulting closed-loop system (2) is GUES. For this purpose, let us first revisit the definition of the ADT property and the stability results for switched nonlinear systems with ADT.

Definition 2: [7] For a switching signal $\sigma(t)$ and each $t_2 \geq t_1 \geq 0$, let $N_\sigma(t_2, t_1)$ denote the number of discontinuities of $\sigma(t)$ in the open interval (t_1, t_2). We say that $\sigma(t)$ has an average dwell time τ_a if there exist two positive numbers N_0 (we call N_0 the chatter bound here) and τ_a such that

$$N_\sigma(t_2, t_1) \leq N_0 + \frac{t_2 - t_1}{\tau_a}, \quad \forall t_2 \geq t_1 \geq 0.$$

Remark 1: Definition 2 means that if there exists a positive number τ_a such that a switching signal has the ADT property, the ADT between any two consecutive switching is no smaller than a common constant τ_a for all system modes.

Lemma 1: [7] Consider the continuous-time switched system $\dot{x}(t) = f_{\sigma(t)}(x(t)),\ \sigma(t) \in S$ and let $\lambda > 0,\ \mu > 1$ be given constants. Suppose that there exist $\mathbb{C}^1$ functions $V_{\sigma(t)} : \mathbb{R}^n \to \mathbb{R}$, and two class $\mathcal{K}_\infty$ functions κ_1, κ_2 such that, $\forall p \in S$

$$\kappa_1(\|x(t)\|) \leq V_p(x(t)) \leq \kappa_2(\|x(t)\|) \tag{4}$$
$$\dot{V}_p(x(t)) \leq -\lambda V_p(x(t)) \tag{5}$$

and $\forall (\sigma(t_i) = p, \sigma(t_i^-) = q) \in S \times S, p \neq q$

$$V_p(x(t_i)) \leq \mu V_q(x(t_i)) \tag{6}$$

then the system is globally uniformly asymptotically stable (GUAS) for any switching signal with ADT

$$\tau_a \geq \tau_a^* = \frac{\ln \mu}{\lambda}. \tag{7}$$

Lemma 2: [28] Consider the discrete-time switched system $x(k+1) = f_{\sigma(k)}(x(k)), \sigma(k) \in S$ and let $0 < \lambda < 1$ and $\mu > 0, \forall p \in S$ be given constants. Suppose that there exists positive definite $\mathbb{C}^1$ functions $V_{\sigma(k)} : \mathbb{R}^n \to \mathbb{R}$, $\sigma(k) \in S$ and two class $\mathcal{K}_\infty$ functions κ_1, κ_2 such that

$$\kappa_1(\|x(k)\|) \leq V_p(x_k) \leq \kappa_2(\|x(k)\|) \tag{8}$$
$$\Delta V_p(x(k)) \leq -\lambda V_p(x(k)) \tag{9}$$

and $\forall (\sigma(k_i) = p, \sigma(k_{i-1}) = q) \in S \times S, p \neq q$

$$V_p(x(k_i)) \leq \mu V_q(x(k_i)) \tag{10}$$

then the system is GUAS for any switching signal with ADT

$$\tau_a > \tau_a^* = -\frac{\ln \mu}{\ln(1-\lambda)}. \tag{11}$$

III. Main Results

For the purpose of this paper, the definition of the MDADT property which we use to restrict a new class of switching signals is first given in the following.

Definition 3: For a switching signal $\sigma(t)$ and any $T \geq t \geq 0$, let $N_{\sigma p}(T, t)$ be the switching numbers that the p^{th} subsystem is activated over the interval $[t, T]$ and $T_p(T, t)$ denote the total running time of the p^{th} subsystem over the interval $[t, T]$, $p \in S$. We say that $\sigma(t)$ has a mode-dependent average dwell time τ_{ap} if there exist positive numbers N_{0p} (we call N_{0p} the mode-dependent chatter bounds here) and τ_{ap} such that

$$N_{\sigma p}(T, t) \leq N_{0p} + \frac{T_p(T, t)}{\tau_{ap}}, \quad \forall T \geq t \geq 0.$$

Remark 2: Definition 3 constructs a novel set of switching signals with a MDADT property, it means that if there exist positive numbers $\tau_{ap}, p \in S$ such that a switching signal has the MDADT property, we only require the average time among the intervals associated with the p^{th} subsystem is larger than τ_{ap} (Note that, the intervals here are not adjacent).

Now, the following lemmas present the stability results for the switched nonlinear systems with MDADT.

Lemma 3: (Continuous-time Version) Consider the continuous-time switched system

$$\dot{x}(t) = f_{\sigma(t)}(x(t)),\ \sigma(t) \in S \tag{12}$$

and let $\lambda_p > 0, \mu_p > 1, p \in S$ be given constants. Suppose that there exist $\mathbb{C}^1$ functions $V_{\sigma(t)} : \mathbb{R}^n \to \mathbb{R}$, and class $\mathcal{K}_\infty$ functions κ_{1p}, κ_{2p}, $p \in S$ such that, $\forall p \in S$,

$$\kappa_{1p}(\|x(t)\|) \leq V_p(x(t)) \leq \kappa_{2p}(\|x(t)\|) \tag{13}$$
$$\dot{V}_p(x(t)) \leq -\lambda_p V_p(x(t)) \tag{14}$$

and $\forall (\sigma(t_i) = p, \sigma(t_i^-) = q) \in S \times S, p \neq q$

$$V_p(x(t_i)) \leq \mu_p V_q(x(t_i)) \tag{15}$$

then the system is GUAS for any switching signal with MDADT

$$\tau_{ap} \geq \tau_{ap}^* = \frac{\ln \mu_p}{\lambda_p}. \tag{16}$$

Proof: For any $T > 0$, let $t_0 = 0$ and denote $t_1, t_2 \ldots t_i, t_{i+1}, \ldots t_{N_\sigma(T,0)}$ the switching times on the interval $[0, T]$, where $N_\sigma(T, 0) = \sum_{p=1}^{M} N_{\sigma p}(T, 0)$.

Then, we set

$$\phi(t) := e^{\lambda_{\sigma(t)} t} V_{\sigma(t)}(x(t)). \tag{17}$$

Function (17) is piecewise differentiable along solution (12). For any $t \in [t_i, t_{i+1}]$, we have:

$$\dot{\phi}(t) = \lambda_{\sigma(t_i)} \phi(t) + e^{\lambda_{\sigma(t_i)} t} \dot{V}_{\sigma(t_i)}(x(t)).$$

By (14), we obtain that $\dot{\phi}(t) \leq 0$. This, together with (15) and (17), implies

$$\begin{aligned}
&\phi(t_{i+1}) \\
&= e^{\lambda_{\sigma(t_{i+1})} t_{i+1}} V_{\sigma(t_{i+1})}(x(t_{i+1})) \\
&\leq \mu_{\sigma(t_{i+1})} e^{\lambda_{\sigma(t_{i+1})} t_{i+1}} V_{\sigma(t_i)}(x(t_{i+1})) \\
&= \mu_{\sigma(t_{i+1})} e^{\lambda_{\sigma(t_{i+1})} t_{i+1} - \lambda_{\sigma(t_i)} t_{i+1}} \phi(t_{i+1}^-) \\
&\leq \mu_{\sigma(t_{i+1})} e^{(\lambda_{\sigma(t_{i+1})} - \lambda_{\sigma(t_i)}) t_{i+1}} \phi(t_i) \leq \mu_{\sigma(t_i)} \mu_{\sigma(t_{i+1})} \\
&\quad \cdot \exp\{(\lambda_{\sigma(t_{i+1})} - \lambda_{\sigma(t_i)}) t_{i+1} + (\lambda_{\sigma(t_i)} - \lambda_{\sigma(t_{i-1})}) t_i\} \phi(t_{i-1}) \\
&\quad \cdots \\
&\leq \prod_{j=0}^{i} \mu_{\sigma(t_{j+1})} e^{\sum_{j=0}^{i} (\lambda_{\sigma(t_{j+1})} - \lambda_{\sigma(t_j)}) t_{j+1}} \phi(t_0).
\end{aligned}$$

Therefore

$$\begin{aligned}
\phi(T^-) &\leq \phi(t_{N_\sigma}) \\
&\leq \prod_{j=0}^{N_\sigma - 1} \mu_{\sigma(t_{j+1})} e^{\sum_{j=0}^{N_\sigma - 1} (\lambda_{\sigma(t_{j+1})} - \lambda_{\sigma(t_j)}) t_{j+1}} \phi(0).
\end{aligned}$$

It then follows from (17) that

$$\exp(\lambda_{\sigma(T-)}T)V_{\sigma(T-)}(x(T)) \le \prod_{j=0}^{N_\sigma-1}\mu_{\sigma(t_{j+1})}e^{\sum_{j=0}^{N_\sigma-1}(\lambda_{\sigma(t_{j+1})}-\lambda_{\sigma(t_j)})t_{j+1}}V_{\sigma(0)}(x(0)).$$

This implies

$$\begin{aligned}
&V_{\sigma(T-)}(x(T))\\
&\le \prod_{j=0}^{N_\sigma-1}\mu_{\sigma(t_{j+1})}\exp\left\{\sum_{j=0}^{N_\sigma-1}(\lambda_{\sigma(t_{j+1})}-\lambda_{\sigma(t_j)})t_{j+1}-\right.\\
&\quad\left.\lambda_{\sigma(t_{N_\sigma})}T+\lambda_{\sigma(t_0)}t_0\right\}V_{\sigma(0)}(x(0))\\
&\le \prod_{p=1}^{M}\mu_p^{N_{\sigma p}}\exp\left\{-\sum_{p=1}^{M}[\lambda_p\sum_{s\in\psi(p)}(t_{s+1}-t_s)]-\right.\\
&\quad\left.\lambda_{\sigma(t_{N_\sigma})}(T-t_{N_\sigma})\right\}V_{\sigma(0)}(x(0))\\
&\le \exp\left\{\sum_{p=1}^{M}N_{0p}\ln\mu_p\right\}\exp\left\{\sum_{p=1}^{M}\frac{T_p}{\tau_{ap}}\ln\mu_p-\right.\\
&\quad\left.\sum_{p=1}^{M}\lambda_pT_p\right\}V_{\sigma(0)}(x(0))\\
&=\exp\left\{\sum_{p=1}^{M}N_{0p}\ln\mu_p\right\}\exp\left\{\sum_{p=1}^{M}(\frac{\ln\mu_p}{\tau_{ap}}-\lambda_p)T_p\right\}V_{\sigma(0)}(x(0))
\end{aligned}$$

where $\psi(p)$ denotes the set of s satisfying $\sigma(t_s) = p, t_s \in \{t_0, t_1 \ldots t_i, t_{i+1}, \ldots t_{N_\sigma-1}\}$. Therefore, if there exist constants τ_{ap}, $p \in S$ satisfying (16), we have

$$V_{\sigma(T-)}(x(T)) \le e^{\sum_{p=1}^{M}N_{0p}\ln\mu_p}e^{\max_{p\in S}((\ln\mu_p/\tau_{ap})-\lambda_p)T}V_{\sigma(0)}(x(0)).$$

Therefore, we conclude that $V_{\sigma(T-)}(x(T))$ convergences to zero as $T \to \infty$ if the MDADT satisfies (16), then, the asymptotic stability can be deduced with the aid of (13). $\square$

Lemma 4: (Discrete-time Version) Consider the discrete-time switched system

$$x(k+1) = f_{\sigma(k)}(x(k)),\ \sigma(k) \in S \tag{18}$$

and let $0 < \lambda_p < 1$ and $\mu_p \ge 1$, $p \in S$ be given constants. Suppose that there exist $\mathbb{C}^1$ functions $V_{\sigma(k)} : \mathbb{R}^n \to \mathbb{R}$, $\sigma(k) \in S$, and class $\mathcal{K}_\infty$ functions κ_{1p} and κ_{2p}, $p \in S$, such that $\forall\sigma(k) = p \in S$

$$\kappa_{1p}(\|x(k)\|) \le V_p(x_k) \le \kappa_{2p}(\|x(k)\|) \tag{19}$$

$$\Delta V_p(x(k)) \le -\lambda_pV_p(x(k)) \tag{20}$$

and $\forall(\sigma(k_i) = p, \sigma(k_{i-1}) = q) \in S \times S, p \ne q$,

$$V_p(x(k_i)) \le \mu_pV_q(x(k_i)) \tag{21}$$

then the system is GUAS for any switching signal with MDADT

$$\tau_{ap} > \tau_{ap}^* = -\frac{\ln\mu_p}{\ln(1-\lambda_p)}. \tag{22}$$

Proof: The proof for the above Lemma can be obtained using the similar techniques in Lemma 3. We omit it here due to the space limitation. $\square$

Remark 3: It can be seen from Lemma 1 and Lemma 2 that the parameters λ and μ are same for all subsystems, i.e., mode-independent. However, the parameters λ_p, μ_p prescribed in Lemma 3 and Lemma 4 are mode-dependent, therefore, we can conclude that $\tau_{ap}^* \le \tau_a^*$, $\forall p \in S$ from (5)–(7) and (14)–(16), and the mode-dependent features would reduce the conservativeness existed in Lemma 1 and Lemma 2. In fact, we also note that if $\tau_a = \tau_{ap}$, $\forall p \in S$, one readily knows from Definition 3 that

$$\sum_{p\in S}N_{\sigma p}(T,t) \le \sum_{p\in S}N_{0p} + \sum_{p\in S}\frac{T_p}{\tau_a}, \quad \forall T \ge t \ge 0.$$

Thus, there exist positive numbers $N_0 = \sum_{p\in S}N_{0p}$ and $\tau_a = \tau_{ap}$ such that

$$N_\sigma(T,\ t) \le N_0 + \frac{T-t}{\tau_a}, \quad \forall T \ge t \ge 0.$$

That is, a switching signal with bounded MDADT τ_{ap}^* also has bounded ADT $\tau_a^* \equiv \tau_{ap}^*$, $\forall p \in S$ in the special case of $\lambda \equiv \lambda_p$, $\mu \equiv \mu_p$, $\forall p \in S$. Therefore, it is clear that Lemma 3 (or Lemma 4 in the discrete-time case) presents a more general stability criterion than Lemma 1 (respectively, Lemma 2) which corresponds to the special case of $\lambda = \lambda_p$, $\mu = \mu_p$, $\tau_a = \tau_{ap}$, $\forall p \in S$. From this, together with Remark 1 and Remark 2, we can conclude that the MDADT switching has the advantage of flexibility for a switched system where the switching is able to or need be designed.

Next, based on the results obtained above, we give the stability conditions for system (1) with MDADT.

Theorem 1: (Continuous-time Case) Consider the switched linear system (1) when $u(t) \equiv 0$ and let $\lambda_p > 0$, $\mu_p > 1$, $p \in S$ be given constants. If there exist matrices $P_p > 0$, $\forall p \in S$, such that, $\forall(p,q) \in S \times S$, $p \ne q$,

$$A_p^TP_p + P_pA_p + \lambda_pP_p \le 0 \tag{23}$$

$$P_p - \mu_pP_q \le 0 \tag{24}$$

then, the switched linear systems (1) is GUES with MDADT satisfying (16).

Proof: Here, we choose the Lyapunov function as follows:

$$V_p(x(t)) = x^T(t)P_px(t), \quad \forall\sigma(t) = p \in S \tag{25}$$

where P_p, $\forall p \in S$ is a positive definite matrix satisfying (23) and (24). Then, from (1), (14), (15) and (25), we have, $\forall(p,q) \in S \times S$, $p \ne q$,

$$\begin{aligned}
&\dot{V}_p(x(t)) + \lambda_pV_p(x(t))\\
&\quad= \lambda_px^T(t)P_px(t) + x^T(t)P_pA_px(t) + x^T(t)A_p^TP_px(t)\\
&V_p(x(t_i)) - \mu_pV_q(x(t_i))\\
&\quad= x^T(t_i)P_px(t_i) - \mu_px^T(t_i)P_qx(t_i).
\end{aligned}$$

Thus, if (23) and (24) hold, system (1) is GUAS for any switching signal with MDADT (16) by Lemma 3. In addition, by denoting $\delta = -(1/2)[\max_{p\in S}((\ln\mu_p/\tau_{ap})-\lambda_p)]$, we can obtain from (13) and (25) that the system state satisfies $\|x(t)\| \le \alpha e^{-\delta(t-t_0)}\|x(t_0)\|$, $\forall t \ge t_0$ for a certain $\alpha > 0$, i.e., the underlying system is GUES. $\square$

Theorem 2: (Discrete-time Case) Consider the switched linear system (1) when $u(t) \equiv 0$ and let $0 < \lambda_p < 1$ and $\mu_p \ge 1$, $p \in S$ be given constants. If there exist matrices $P_p > 0$, $\forall p \in S$, such that, $\forall(p,q) \in S \times S$, $p \ne q$

$$A_p^TP_pA_p + \lambda_pP_p - P_p \le 0 \tag{26}$$

$$P_p - \mu_pP_q \le 0 \tag{27}$$

then, the switched linear systems (1) is GUES with MDADT satisfying (22).

TABLE I
COMPUTATION RESULTS FOR THE SYSTEM UNDER TWO DIFFERENT SWITCHING SCHEMES

Switching schemes	ADT switching	MDADT switching
Criteria for controller design	Corollary 1 in [26]	Theorem 3 in the paper
Controller gains	$\Gamma_1: \begin{cases} K_1 = \begin{bmatrix} 73.66 & 66.14 \end{bmatrix} \\ K_2 = \begin{bmatrix} -19.94 & -2.75 \end{bmatrix} \\ K_3 = \begin{bmatrix} 3.25 & -15.24 \end{bmatrix} \end{cases}$	$\Gamma_2: \begin{cases} K_1 = \begin{bmatrix} 93.79 & 69.75 \end{bmatrix} \\ K_2 = \begin{bmatrix} -59.81 & -34.25 \end{bmatrix} \\ K_3 = \begin{bmatrix} -53.91 & -63.58 \end{bmatrix} \end{cases}$
Switching signals	$\tau_{\mathrm{a}}^* = 0.99$ $(\mu = 2, \lambda \leq 0.7)$	$\tau_{\mathrm{a}1}^* = 0.22, \tau_{\mathrm{a}2}^* = 0.49, \tau_{\mathrm{a}3}^* = 0.99$ $(\mu_1 = \mu_2 = \mu_3 = 2,$ $\lambda_1 \leq 3.1, \lambda_2 \leq 1.4, \lambda_3 \leq 0.7)$

Proof: The proof of Theorem 2 is similar to the one for Theorem 1, and thus omitted here. □

Now, we are in a position to give the existence conditions of a set of stabilizing controllers for system (1) with the MDADT switching.

Theorem 3: (Continuous-time Case) Consider the switched linear systems (2) and let $\lambda_p > 0, \mu_p > 1, p \in S$ be given constants. If there exist matrices $U_p > 0$, and $T_p, \forall p \in S$, such that, $\forall (p, q) \in S \times S$, $p \neq q$

$$A_pU_p + B_pT_p + U_pA_p^T + T_p^TB_p^T + \lambda_pU_p \leq 0 \tag{28}$$

$$U_q \leq \mu_pU_p \tag{29}$$

then there exists a set of stabilizing controllers such that system (2) is GUES for any switching signal with MDADT satisfying (16). Moreover, if (28) and (29) have a solution, the controller gains can be given by

$$K_p = T_pU_p^{-1}. \tag{30}$$

Proof: Theorem 1 implies that if

$$\bar{A}_p^TP_p + P_p\bar{A}_p + \lambda_pP_p \leq 0 \tag{31}$$

$$P_p - \mu_pP_q \leq 0 \tag{32}$$

system (2) is GUES for any switching signal with MDADT (16). Replacing $\bar{A}_p$ in (31) by (3), setting $U_p = P_p^{-1}$ and $T_p = K_pP_p^{-1}$, we can see that, if (28) holds, (31) is satisfied. Moreover, if (29) holds, we can obtain that $U_q - \mu_pU_p \leq 0$. By Schur complement, we note that $U_q - \mu_pU_p \leq 0$ is equivalent to

$$\Lambda = \begin{bmatrix} -\mu_pU_p & I \\ I & -U_q^{-1} \end{bmatrix} \leq 0.$$

Furthermore, we have that $\Lambda \leq 0$ is equivalent to $-U_q^{-1} - I^T(\mu_pU_p)^{-1}I \leq 0$ by Schur complement, i.e., (32) holds. Additionally, if the inequalities (28) and (29) have feasible solutions, the admissible controller gains can be given by (30) since $T_p = K_pP_p^{-1}$, this ends the proof. □

Theorem 4: (Discrete-time Case) Consider the switched linear systems (2) and let $0 < \lambda_p < 1$ and $\mu_p \geq 1, p \in S$ be given constants. If there exist matrices $U_p > 0$, and $T_p, \forall p \in S$, such that, $\forall (p, q) \in S \times S, p \neq q$

$$\begin{bmatrix} -U_p & A_pU_p + B_pT_p \\ * & -(1-\lambda_p)U_p \end{bmatrix} \leq 0 \tag{33}$$

$$U_q \leq \mu_pU_p \tag{34}$$

then there exists a set of controllers such that system (2) is GUES for any switching signal with MDADT satisfying (22). Moreover, if (33) and (34) have a solution, the admissible controllers can be given by (33).

Proof: Due to space limitation, we omit the proof of Theorem 4 that can be carried out by refering to the standard techniques used in Theorem 3. □

Remark 4: Theorem 3 and 4 are extensions of [25, Corollary 1 and Corollary 2], respectively, which concern the stabilization problems for switched linear systems with ADT switching. Analogous to the discussions in Remark 3, the Theorems 3 and 4 obtained above are LMIs for given λ_p and μ_p, that would reduce the conservativeness to find feasible controllers in the scheme of MDADT switching for the system than using λ and μ corresponding to the scheme of ADT switching.

IV. NUMERICAL EXAMPLE

In this section, a numerical example in continuous-time domain will be presented to demonstrate the potential and validity of the results obtained above. The verification for the discrete-time cases can be carried out in a same vein.

1) Example 1: Consider the switched linear systems consisting of three subsystems described by

$$A_1 = \begin{bmatrix} 3.9 & 1.5 \\ 2.5 & 2.3 \end{bmatrix}, B_1 = \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix}, A_2 = \begin{bmatrix} 1.4 & 0.3 \\ 1 & -2.7 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, A_3 = \begin{bmatrix} -2.2 & 0.1 \\ -2 & -0.4 \end{bmatrix}, B_3 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}.$$

Our purpose here is to design a set of mode-dependent stabilizing controllers and find the admissible switching signals with MDADT such that the resulting closed-loop system is stable.

To illustrate the advantages of the proposed MDADT switching, we shall also present the design results of both controllers and switching signals for the systems with ADT switching for the sake of comparison. By different approaches and setting the relevant parameters appropriately, the computation results for the system with two different switching schemes are listed in Table I.

It can be seen from Table I that the minimal MDADT are reduced to $\tau_{a1}^* = 0.22, \tau_{a2}^* = 0.49, \tau_{a3}^* = 0.99$, for given $\mu = \mu_1 = \mu_2 = \mu_3 = 2$, and one special case of MDADT switching is $\tau_a^* = \tau_{a1}^* = \tau_{a2}^* = \tau_{a3}^* = 0.99$ by setting $\lambda = \lambda_1 = \lambda_2 = \lambda_3 = 0.7$, which is the ADT switching, i.e., the designed MDADT switching is more general.

To further show the merit of MDADT switching, let us now consider the resulting closed-loop system performances. Applying the obtained controller, under the scheme of ADT switching and MDADT switching, respectively, we can obtain the state responses for each closed-loop subsystem as shown in Fig. 1. For each closed-loop subsystem $\bar{A}_p$, one can obtain by (31) that $\bar{A}_p^TP_p + P_p\bar{A}_p + \lambda_pP_p \leq 0$, i.e., $(\bar{A}_p + 0.5\lambda_pI)^TP + P(\bar{A}_p + 0.5\lambda_pI) \leq 0$. Then, it yields that $\bar{A}_p + 0.5\lambda_pI$ is Hurwitz matrix, which implies that the maximum real part of the eigenvalues of $\bar{A}_p$ is smaller than $-0.5\lambda_p$. That is to say, by MDADT, the lower bound of the decay rate of the states for the closed-loop subsystem 1 can be increased to $0.5\lambda_1 = 1.55$, which is much bigger than $0.5\lambda = 0.35$ obtained by ADT. Therefore, it is shown that there is a better transient behavior in the state response of closed-loop subsystem 1 by Γ_2, which potentially improves the state response.

Then, generating one possible switching sequences with the ADT property and the MDADT property, one can obtain the corresponding

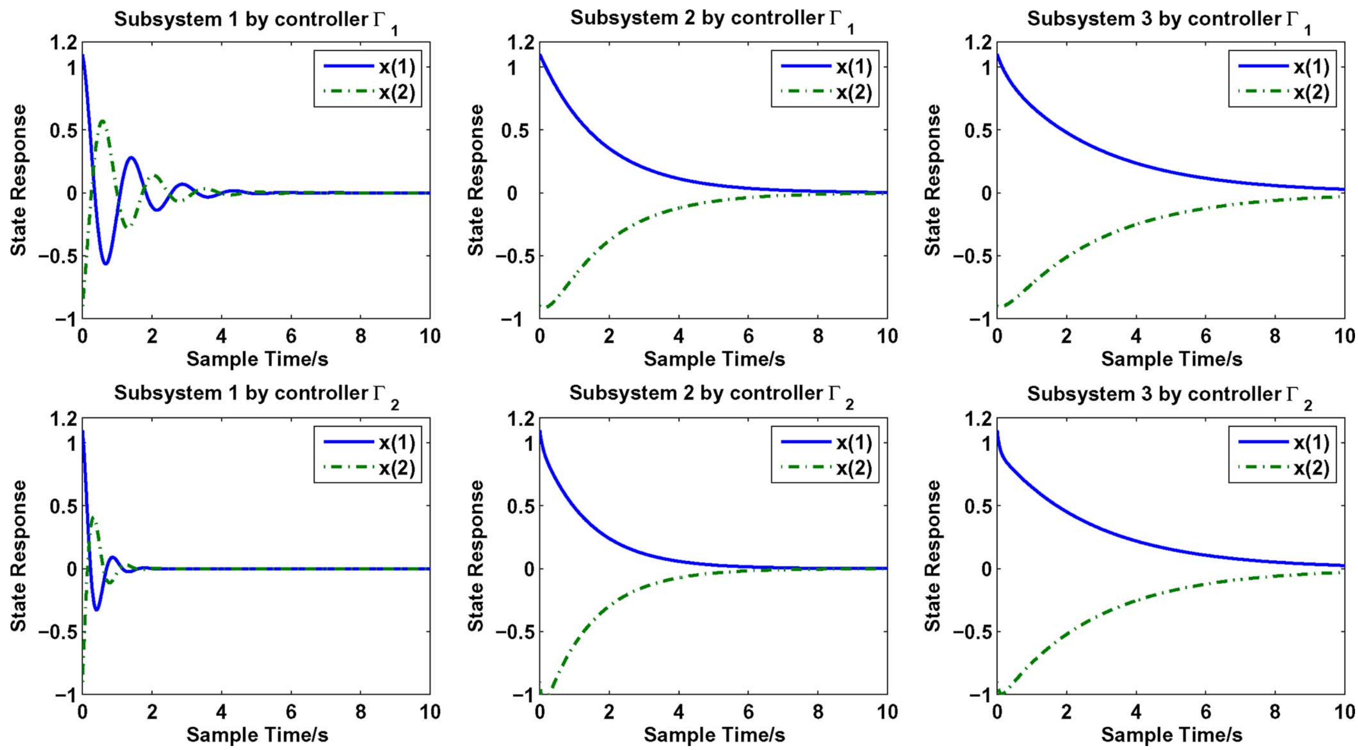

Fig. 1. The state response comparisions of the closed-loop subsystems by controllers Γ_1 and Γ_2.

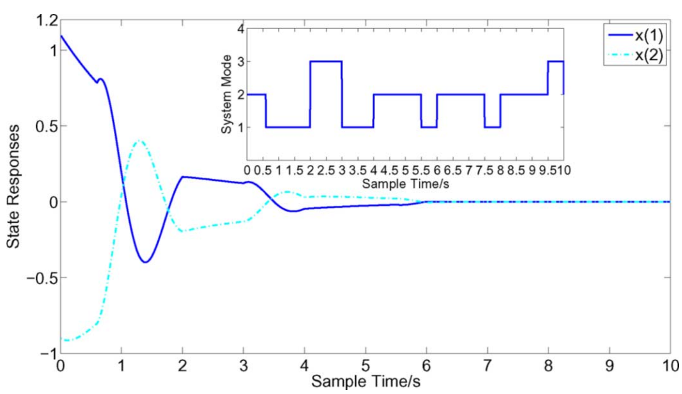

Fig. 2. State response of the closed-loop system by controllers Γ_1 under switching signal σ with $\tau_a = 1.0$.

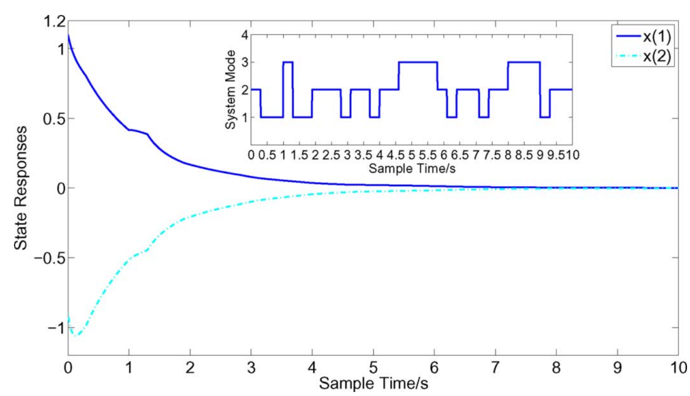

Fig. 3. State response of the closed-loop system by controllers Γ_2 under switching signal σ with $\tau_{a1} = 0.3$, $\tau_{a2} = 0.6$, $\tau_{a3} = 1.0$.

state responses of the closed-loop system as shown in Figs. 2 and 3, respectively, for the same initial state condition. It can be seen from the curves that the state response of closed-loop system is fluctuated under the ADT switching scheme, but is smooth under the MDADT switching scheme. To present the reason more clearly, we denote the running time of the p^{th} subsystem at the l^{th} working as $t_{p,l}, \forall p \in S$, $l \in \mathbb{N}^+$, and use $t^A_{p,l}$ and $t^M_{p,l}$ to represent the running time of the subsystem under ADT and MDADT switching schemes, respectively. It can be observed that the state responses both begin with subsystem 2 and in Fig. 2, $0.5 < t^A_{2,1} < 1$ and in Fig. 3, $t^M_{2,1} < 0.5$. Then, due to the constraint of ADT switching ($\tau_a = 1$ in Fig. 2), we need $t^A_{1,1} > 1$. But for the case of MDADT switching, we can remove the constraint on $t^M_{1,1}$ since we only need $\tau_{a1} \geq 0.3$ which is nothing to do with $t^M_{1,1}$ (can be actually less than 0.3). The comparison of the switching signals in Figs. 2 and 3 shows that even for $t^A_{2,1} > t^M_{2,1}$ (we want $t^A_{1,1}$ to be a little shorter), we can attain $t^M_{1,1} < t^A_{1,1}$. This will better the state response because of the shorter running time needed on the subsequent subsystem 1, which has high-frequency dynamics as shown in Fig. 1.

Thus, from the demonstrations above, we conclude that it will be more flexible in practice to design a MDADT switching to perfect or improve the system performances with fewer constraints.

V. Conclusion

The stability and stabilization problems for switched systems with a class of MDADT switching are studied in this paper. The proposed MDADT switching is less restrict than the traditional ADT switching in practice. The stability results for the switched systems with MDADT are derived in both linear and nonlinear contexts. Moreover, the minimal MDADT for admissible switching signals and the corresponding state feedback controller are designed for the systems in both continuous-time and discrete-time cases. Finally, a numerical example is included to illustrate the effectiveness and efficiency of the theoretic results obtained.

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Infinite-Horizon Switched LQR Problems in Discrete Time: A Suboptimal Algorithm With Performance Analysis

Wei Zhang, Jianghai Hu, and Alessandro Abate

***Abstract*—This paper studies the quadratic regulation problem for discrete-time switched linear systems (DSLQR problem) on an infinite time horizon. A general relaxation framework is developed to simplify the computation of the value iterations. Based on this framework, an efficient algorithm is developed to solve the infinite-horizon DSLQR problem with guaranteed closed-loop stability and suboptimal performance. Due to these guarantees, the proposed algorithm can be used as a general controller synthesis tool for switched linear systems.**

***Index Terms*—Hybrid systems, optimal control, switched LQR, switched systems.**

This paper studies an extension of the classical LQR problem to switched linear systems (SLS), which will be referred to as the Discrete-Time Switched LQR (DSLQR) Problem. The goal is to find both the continuous-control and switching-control strategies to minimize a quadratic cost functional over an infinite time horizon. The problem is expected to play a fundamental role in the study of switched and hybrid systems as the classical LQR problem does for linear systems.

In our earlier work [11], we have proved some analytical properties for the finite-horizon DSLQR problem. Due to the discrete nature of the switching control sequence, the exact solution to the DSLQR problem

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