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Robust stabilization for uncertain switched neutral systems with interval time-varying mixed delays

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ABSTRACT

This paper deals with the problems of exponential stability and stabilization of uncertain switched neutral systems (USNSs) with interval time-varying mixed delays. Interval time-varying delay exists in the state, derivatives of the state (neutral), and the output. This research emphasizes the cases where uncertainties are norm-bounded time-varying in the model. First, sufficient conditions are proposed in terms of a set of linear matrix inequalities (LMIs) to guarantee exponential stability using the average dwell time (ADT) approach and the piecewise Lyapunov function technique. Then, the corresponding conditions are obtained for the stabilization via a dynamic output feedback (DOF) controller. The problem of uncertainty in the system model is solved by designing the DOF controller and applying the Yakubovich lemma. Since the conditions obtained are not represented by LMI form, decoupling between the Lyapunov function and the system matrices is generated using the proposed slack matrix variable, and a new condition is obtained. Finally, numerical examples are given to determine the effectiveness of the proposed theorem.

1. Introduction

The delay phenomenon exists in many practical and engineering systems such as chemical processes, nuclear reactors, electrical systems, and biological systems [1–3]. It is a well-known fact that the presence of delay may cause systems to perform poorly and to even be unstable [4]. Many researchers have extensively studied the fundamental theoretical and practical problem of stability analysis of time-delay systems (TDSs) (see [5,6] and the references cited therein).

In some practical systems, the delay exists not simply in the state, but also in the derivatives of the state. This class of TDSs are referred to as neutral systems (NSs). Neutral delays usually arise when state-derivative feedback or output-derivative feedback is provided in systems which have delay in the input. The presence of this type of delay can improve the performance of the closed-loop system. Sometimes, achieving better performance requires that the delay be considered in the derivatives of the state of open-loop model [7]. NSs have a variety of applications. Examples can be found in the areas of population model [8], transmission line oscillator [9], partial element equivalent circuits [10], DC–DC converters [11], and drilling systems [7].

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Many attempts have been made in the previous research to analyze NSs with constant time-delays [12–15]. However, in many practical NSs, time-delays are usually time-varying, which can even result in massive changes to the dynamics of NSs in some cases. Lately, the NSs with time-varying delays have been the subject of study in [2–4,16–20]. Interval time-varying delay is a time delay which varies at an interval where the lower bound is not limited to zero. Networked control systems (NCSs) constitute a prime example of dynamical systems with interval time-varying delay [21,22]. Obviously, it is more difficult to analyze and synthesize systems with time-varying delays, especially when such delays are interval and the output consists of time-varying delay functions. Presence of time-varying delays in the outputs and states of the system makes the problem even more complicated.

This paper considers interval time-varying delay in the state, the derivatives of the state (neutral), and the output of the system. On the other hand, switched systems have been widely used in modeling various dynamical systems such as chemical processes [23], communication networks [24], electrical systems [25], and mechanical systems [26].

Switched systems are a class of hybrid systems which consist of a family of subsystems and which are controlled by switching laws [27]. The previous research on switched systems has generally focused on two major problems. First, from a practical point of view, a large class of physical systems are naturally multi-model whose behavior is represented by several dynamic models. Second, from a control perspective, for complex systems and systems with large uncertainties, designing a multi-controller is better than a controller [12,28].

Delays also appear in switched systems; hence the term switched time-delay systems (STDSs). There are numerous applications for such systems, including water quality control, electric power systems, productive manufacturing systems, and cold steel rolling mills [29].

Switched neutral systems (SNSs) constitute a multidisciplinary research area borrowing ideas from many diverse fields. For example, stability and stabilization problems were studied in [2,12,13,16], H_∞ control problem was discussed in [14], and state feedback (SF) control was addressed in [12,13,15]. In practice, it is often not possible to obtain full information on the state variables to use them for feedback control. This makes it necessary to study the stabilization problem of the dynamic output feedback (DOF) for this case [15].

Another point is that the application of any output feedback controller to STDSs and SNSs would result in a closed-loop system with interval time-varying delays, and this will cause problems when an attempt is made to derive stabilizability conditions for the purpose of finding controller parameters. For example, [30] proposed an output feedback controller for stabilization and H_∞ control of switched linear systems with time-varying uncertainties. Study [31] was concerned with the problem of H_∞ control for switched linear systems by DOF controller in terms of linear matrix inequalities (LMIs).

In most of the studies into SNSs, stability analysis is considered as a major control problem. For stability analysis of switched delay systems under arbitrary switching, often the common Lyapunov function (CLF) is used, but it should be noted that this may lead to conservatism. Furthermore, arbitrary switching has limited applicability because it requires all the subsystems to be stable. The multiple Lyapunov function (MLF) [15], the dwell time (DT) approach [4,15,17], and the average dwell time (ADT) approach [2,14] have been proposed as effective tools for reducing conservatism in the cases where some subsystems are unstable.

The studies into switched systems with time-delay (whether neutral or non-neutral) have proposed some other popular approaches for this purpose: the model transformation method [5], the free-weighting-matrix method [15,17,18, 32], Leibniz–Newton formula [15,16], the slack matrix [33], and choosing the appropriate Lyapunov–Krasovskii function [4,28,32,34–36].

Uncertain factors such as environment noise, uncertain parameters, and disturbance are commonly encountered in various practical and engineering systems, and this makes it very difficult to develop an exact mathematical model. Also, it has been shown that the existence of uncertainty invariably causes poor performance and even instability of control systems. Therefore, robust control of uncertain switched neutral systems (USNSs) is very important in theory and application (see [34,37] and the references cited therein). However, the stability and stabilization problem of USNSs has been minimally considered in the literature. Study [12] investigated the stabilizing SF controller for robust stabilization of a class of USNSs with constant mixed delays and time-varying structured uncertainties based on the CLF and formulated stability conditions in terms of LMIs. The LMI approach is an efficient and popular tool proposed by [38] to solve stability analysis.

In [16], the problem of exponential stability was studied for USNSs with time-varying structured uncertainties. In the study mentioned, the neutral delay was considered constant based on the Razumikhin-like approach under arbitrary switching. Sufficient conditions were obtained in [4] for exponential stability for SNSs with time-varying state delay, constant neutral delay, and norm-bounded uncertainty based on the ADT approach and the Lyapunov–Krasovskii functions. The authors in [2], proposed exponential stability criteria for USNSs with norm-bounded time-varying uncertainties, nonlinear perturbations, and interval neutral time-varying delay by using the ADT approach and the piecewise Lyapunov functional technique.

Besides, switched nonlinear systems or nonlinear systems with neutral delay have been regarded as SNSs or NSs under additive nonlinear perturbations which have been the subject of investigation in many previous studies (e.g., [2,7,18–20]).

To the best of our knowledge, the problem of stability and stabilization of SNSs via DOF controllers with uncertainties has received little attention in the previous research, and this motivated the present research.

In this paper, we consider the problems of exponential stability and stabilization of USNSs with interval time-varying mixed delays that exist in the state, derivatives of the state (neutral), and the output. Using the ADT approach and the piecewise Lyapunov function technique, sufficient conditions are suggested to guarantee the exponential stability. Then, the

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corresponding conditions are obtained for stabilization via a DOF controller. The LMI approach is used to solve the problem of stabilization.

We believe that this study is innovative in several ways. First, we studied USNSs with interval time-varying delays that exist in the state, derivatives of the state (neutral), and the output. Second, we considered time-varying uncertainty in the system model and explored uncertainty in the matrix of the derivatives of the state. And finally, we explored the problems of exponential stability and stabilization in the proposed system via DOF controller using the linearization method.

The remainder of the paper is organized as follows. Definitions, lemmas, and description of the system are given in Section 2. Section 3 summarizes the main results. Numerical examples are provided in Section 4 to illustrate the effectiveness of the proposed approach. Finally, the conclusions are given in Section 5.

The notations used throughout this paper are as follows. $\mathbb{R}^n$ denotes the n-dimensional real space. $\mathbb{R}^{m\times n}$ denotes the set of all real m by n matrices. x^T or A^T denotes the transpose of vector x (or matrix A). The notations $P > 0$ and $P < 0$ mean that matrix P is symmetric positive definite and symmetric negative definite, respectively. $P \geq 0$ and $P \leq 0$ mean that matrix P is symmetric positive semi-definite and symmetric negative semi-definite, in that order. The symbol $*$ denotes the elements below the main diagonal of a symmetric matrix. $\lambda_{max}(P)$ and $\lambda_{min}(P)$ denote the maximum and minimum eigenvalues of matrix P, respectively. I denotes the unit matrix of appropriate dimensions. $diag\,[X_1, \ldots, X_m]$ denotes the block-diagonal matrix with $X_1, \ldots, X_m$ as the diagonal elements.

2. Problem formulation

We consider the class of USNSs given by Eq. (1):

$$\begin{aligned}
&\dot{x}(t) - A_{1,\sigma(t)}(t)\,\dot{x}(t-h(t)) = A_{\sigma(t)}(t)\,x(t) + D_{\sigma(t)}(t)\,x(t-d(t)) + B_{\sigma(t)}(t)u(t)\\
&y(t) = C_{\sigma(t)}x(t) + C_{1,\sigma(t)}x(t-d(t))\\
&x(t) = \varphi(t), \quad \forall t \in [-H, 0], \qquad H = max\{d_2, h_2\}
\end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state system, $u(t) \in \mathbb{R}^m$ is the control input, $y(t) \in \mathbb{R}^p$ is the measurement output, $C_{\sigma(t)}$, and $C_{1,\sigma(t)} \in \mathbb{R}^{p\times n}$ are known constant matrices of appropriate dimensions, $\varphi(.)$ is the continuous vector valued function specifying the initial state of the system, and the function $\sigma(t) : [0, +\infty) \to L = \{1, 2, \ldots, l\}$ is a switching signal which is deterministic, piecewise constant, and right continuous. The corresponding switching sequence is $\{(t_0, \sigma(t_0)), (t_1, \sigma(t_1)), \ldots (t_k, \sigma(t_k)), \ldots\}$ ($k = 1, 2, \ldots$), where t_0 is the initial time, and t_k denotes the kth switching instant. We assume that the value of $\sigma(t)$ is unknown, but its instantaneous value is available in real-time, and the initial value $\sigma(0)$ is known in advance. The delay $h(t)$ is time-varying neutral delay satisfying

$$0 \leq h_1 \leq h(t) \leq h_2 < \infty, \qquad \dot{h}(t) \leq \mu_h < 1 \tag{2}$$

and discrete delay $d(t)$ meeting

$$0 \leq d_1 \leq d(t) \leq d_2 < \infty, \qquad \dot{d}(t) \leq \mu_d < 1. \tag{3}$$

$A_{\sigma(t)}(t), A_{1,\sigma(t)}(t), D_{\sigma(t)}(t) \in \mathbb{R}^{n\times n}$, and $B_{\sigma(t)}(t) \in \mathbb{R}^{n\times m}$ are system matrices with time-varying uncertainties.

$$\begin{aligned}
&A_i(t) = A_i + \Delta A_i(t), \qquad A_{1i}(t) = A_{1,i} + \Delta A_{1,i}(t),\\
&B_i(t) = B_i + \Delta B_i(t), \qquad D_i(t) = D_i + \Delta D_i(t),
\end{aligned}$$

where A_i, A_{1i}, $D_i \in \mathbb{R}^{n\times n}$, and $B_i \in \mathbb{R}^{n\times m}$ are given constant matrices, and the time-varying matrices $\Delta A_i(t)$, $\Delta B_i(t)$, $\Delta A_{1i}(t)$, and $\Delta D_i(t)$ are assumed to be norm-bounded with appropriate dimensions satisfying the following condition:

$$\begin{bmatrix}\Delta A_i & \Delta B_i & \Delta A_{1,i} & \Delta D_i\end{bmatrix} = E_i F(t)\begin{bmatrix}N_{0,i} & N_{1,i} & N_{2,i} & N_{3,i}\end{bmatrix}, \tag{4}$$

where $E_i \in \mathbb{R}^{n\times\alpha}$ and $N_{0,i}$, $N_{2,i}$, $N_{3,i} \in \mathbb{R}^{\beta\times n}$, and $N_{1,i} \in \mathbb{R}^{\beta\times m}$ are constant matrices, and $F(t) \in \mathbb{R}^{\alpha\times\beta}$ is the unknown continuous time-varying matrix function with Lebesgue measurable elements, satisfying:

$$F^T(t)\,F(t) \leq I. \tag{5}$$

The proposed DOF controllers are as follows:

$$\begin{aligned}
&\dot{x}_c(t) = A_{c,\sigma(t)}x_c(t) + B_{c,\sigma(t)}y(t),\\
&u(t) = C_{c,\sigma(t)}x_c(t), \qquad x_c(0) = 0
\end{aligned} \tag{6}$$

where $x_c(t) \in \mathbb{R}^n$ is the controller state vector, and $A_{c,\sigma(t)}$, $B_{c,\sigma(t)}$, and $C_{c,\sigma(t)}$ are gain matrices with appropriate dimensions. Applying the DOF controller (6) to the USNS (1) gives us the closed-loop system below:

$$\begin{aligned}
&\dot{\eta}(t) = \overline{A}_{\sigma(t)}(t)\eta(t) + \overline{D}_{\sigma(t)}(t)K\eta(t-d(t)) + \overline{A}_{1,\sigma(t)}(t)K\dot{\eta}(t-h(t)),\\
&\eta(t) = \overline{\varphi}(t), \quad \forall\,\theta \in [-H, 0],
\end{aligned} \tag{7}$$

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where $\eta(t) = \begin{bmatrix} x^T(t) & x_c^T(t) \end{bmatrix}^T$, $\overline{\varphi}(t) = \begin{bmatrix} \varphi^T(t) & 0 \end{bmatrix}^T$, $K = \begin{bmatrix} I & 0 \end{bmatrix}$ and

$$\overline{A}_\sigma(t) = \begin{bmatrix} A_\sigma + \Delta A_\sigma(t) & (B_\sigma + \Delta B_\sigma(t))C_{c,\sigma(t)} \\ B_{c,\sigma(t)}C_{\sigma(t)} & A_{c,\sigma(t)} \end{bmatrix}, \qquad \overline{D}_\sigma(t) = \begin{bmatrix} D_\sigma + \Delta D_\sigma(t) \\ B_{c,\sigma(t)}C_{1,\sigma(t)} \end{bmatrix},$$
$$\overline{A}_{1\sigma}(t) = \begin{bmatrix} A_{1\sigma} + \Delta A_{1\sigma}(t) \\ 0 \end{bmatrix}, \qquad \overline{A}_\sigma = \begin{bmatrix} A_\sigma & B_\sigma C_{c,\sigma(t)} \\ B_{c,\sigma(t)}C_{\sigma(t)} & A_{c,\sigma(t)} \end{bmatrix},$$
$$\overline{D}_{\sigma(t)} = \begin{bmatrix} D_\sigma \\ B_{c,\sigma(t)}C_{1,\sigma(t)} \end{bmatrix}, \qquad \overline{A}_{1,\sigma(t)} = \begin{bmatrix} A_{1,\sigma(t)} \\ 0 \end{bmatrix}. \tag{8}$$

Before presenting the main results, we will give the following definitions and lemmas.

Definition 1 ([4]). The USNS (1) is said to be robust and exponentially stable under $\sigma(t)$, if there exist constants $M \geq 0$ and $\lambda > 0$ such that

$$\|x(t,\varphi)\| \leq Me^{-\lambda(t-t_0)}\|\varphi\|, \quad \forall t \geq t_0, \tag{9}$$

for all admissible uncertainties $F^T(t)F(t) \leq I$, where $x(t_0+\theta) = \varphi(\theta)$.

Definition 2 ([1,2]). For any $T_2 > T_1 \geq 0$, let $N_\sigma(T_1, T_2)$ denote the number of switching $\sigma(t)$ at an interval (T_1, T_2). If $N_\sigma(T_1, T_2) \leq N_0 + \frac{T_1 - T_2}{\tau_a}$ holds for any given $N_0 \geq 0$, $\tau_a \geq 0$, then the constant τ_a is called the ADT. Following the common practice in the literature, we consider $N_0 = 0$.

Lemma 1 (*Yakubovich Lemma [39]*). *Let $\Omega_0(x)$ and $\Omega_1(x)$ be two quadratic matrix functions over $\mathbb{R}^n$, and $\Omega_1(x) \leq 0$ for all $x(t) \in \mathbb{R}^n - \{0\}$. Then $\Omega_0(x) < 0$ holds for all $x(t) \in \mathbb{R}^n - \{0\}$ if and only if there exists the constant $\varepsilon \geq 0$ such that*

$$\Omega_0(x) - \varepsilon\Omega_1(x) < 0, \quad \forall x(t) \in \mathbb{R}^n - \{0\}. \tag{10}$$

Lemma 2 (*Schur complement [38]*). *Let V, S, and G be given matrices such that $G > 0$. Then*

$$\begin{bmatrix} V(x) & S(x) \\ S^T(x) & -G(x) \end{bmatrix} < 0 \leftrightarrow V(x) + S(x)G^{-1}(x)S^T(x) < 0. \tag{11}$$

3. Exponential stability analysis

In this section, we present a sufficient condition for the exponential stability of the USNS in Eq. (12) with interval time-varying mixed delays using the ADT approach, the piecewise Lyapunov function technique, and the lemmas presented above.

$$\begin{aligned} &\dot{x}(t) - (A_{1\sigma} + \Delta A_{1,\sigma}(t))\dot{x}(t - h(t)) = (A_\sigma + \Delta A_\sigma(t))x(t) + s(D_\sigma + \Delta D_\sigma(t))x(t - d(t)) \\ &x(t) = \varphi(t), \quad \forall t \in [-H, 0], \qquad H = \max\{d_2, h_2\}. \end{aligned} \tag{12}$$

Theorem 1. *For the constant $\alpha > 0$, if there exist positive-definite matrices P_i, Q_i, and R_i such that for $i \in L$*

$$\begin{bmatrix} \Pi_{11,i} & P_iD_i & P_iA_{1,i} & A_i^TR_i & N_{0i}^T & 0 & 0 & \Lambda & \Lambda & \Lambda \\ * & -(1-\mu_d)e^{-\alpha d_2}Q_i & 0 & D_i^TR_i & 0 & N_{3i}^T & 0 & 0 & 0 & 0 \\ * & * & -(1-\mu_h)e^{-\alpha h_2}R_i & A_{1,i}^TR_i & 0 & 0 & N_{2i}^T & 0 & 0 & 0 \\ * & * & * & -R_i & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{13}$$

where $\Pi_{11,i} = P_iA_i + A_i^TP_i + \alpha P_i + Q_i$ and $\Lambda = P_iE_i$, then the USNS (12) is exponentially stable for any switching signal with ADT:

$$\tau_a > \tau_a^* = \frac{\ln\mu}{\alpha}, \tag{14}$$

where $\mu \geq 1$ satisfies

$$P_i = \mu P_j, \qquad Q_i = \mu Q_j, \qquad R_i = \mu R_j, \quad \forall i, j \in L. \tag{15}$$

Moreover, the estimate of the state decay is given by

$$\|x(t)\| \leq \sqrt{\frac{b}{a}}\,\|x_{t_0}\|_{c_1} e^{-\lambda(t-t_0)}, \qquad \sqrt{\frac{b}{a}} \geq 1 \tag{16}$$

where

$$\begin{aligned}&\lambda = \frac{1}{2}\left(\alpha - \frac{\ln\mu}{\tau_a}\right), \qquad a = \min_{\forall i\in L}\lambda_{max}(P_i),\\ &b = \max_{\forall i\in L}\lambda_{max}(P_i) + d_2\max_{\forall i\in L}\lambda_{max}(Q_i) + h_2\max_{\forall i\in L}\lambda_{max}(R_i).\end{aligned} \tag{17}$$

Proof. See Appendix A.

Remark 1. When $\mu = 1$, and given $\tau_a > \tau_a^* = \frac{\ln\mu}{\alpha}$, we will have $\tau_a > \tau_a^* = 0$, which means that the switching signal σ can be arbitrary. Thus, Eq. (15) takes the following form:

$$P_i = P_j, \qquad Q_i = Q_j, \qquad R_i = R_j, \quad \forall i, j \in L. \tag{18}$$

It is possible to consider Eq. (18) in the form of Eq. (19):

$$P_i = P_j = P, \qquad R_i = R_j = R, \qquad Q_i = Q_j = Q. \tag{19}$$

This indicates that a common Lyapunov function is needed for all subsystems.

Remark 2. The obtained conditions are delay-dependent given that they are dependent on the upper bound of both discrete and neutral delays and also the upper bound of derivative of discrete and neutral delays. Furthermore, Refs. [2,16,17] regarded the conditions they derived as dependent on the lower or upper bound of discrete and neutral delays and the respective derivatives.

4. Stabilization via the DOF controller

In this section, we extend Theorem 1 to give the sufficient conditions for the existence of the DOF controller (6) for stabilization of the USNSs with interval time-varying mixed delays (1).

Theorem 2. *For the given constant $\alpha > 0$, if there exist positive-definite matrices P_i, Q_i, and R_i such that for $i \in L$*

$$\begin{bmatrix}
\overline{\Pi}_{11,i} & P_i\overline{D}_i & P_i\overline{A}_{1,i} & \overline{A}_i^T K^T R_i & \Phi_i^T & 0 & 0 & \Lambda & \Lambda & \Lambda\\
* & -(1-\mu_d)e^{-\alpha d_2}Q_i & 0 & \overline{D}_i^T K^T R_i & 0 & \Phi_{1,i}^T & 0 & 0 & 0 & 0\\
* & * & -(1-\mu_h)e^{-\alpha h_2}R_i & \overline{A}_{1,i}^T K^T R_i & 0 & 0 & \Phi_{2,i}^T & 0 & 0 & 0\\
* & * & * & -R_i & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & -I & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & * & -I & 0 & 0 & 0 & 0\\
* & * & * & * & * & * & -I & 0 & 0 & 0\\
* & * & * & * & * & * & * & -I & 0 & 0\\
* & * & * & * & * & * & * & * & -I & 0\\
* & * & * & * & * & * & * & * & * & -I
\end{bmatrix} < 0, \tag{20}$$

where

$$\overline{\Pi}_{11i} = P_i\overline{A}_i + \overline{A}_i^T P_i + \alpha P_i + K^T Q_i K, \qquad K = \begin{bmatrix} I & 0\end{bmatrix}, \qquad \Lambda = P_i\begin{bmatrix} E_i & 0\\ 0 & I\end{bmatrix},$$

$$\Phi_i = \begin{bmatrix} N_{0,i} & N_{1,i}C_{c,i}\\ 0 & 0\end{bmatrix}, \qquad \Phi_{1,i} = \begin{bmatrix} N_{3,i}\\ 0\end{bmatrix}K, \qquad \Phi_{2,i} = \begin{bmatrix} N_{2,i}\\ 0\end{bmatrix}K.$$

Then, the closed-loop system (7) is exponentially stable for any switching signal with ADT*:*

$$\tau_a > \tau_a^* = \frac{\ln\mu}{\alpha}, \tag{21}$$

where $\mu \geq 1$ satisfies

$$P_i = \mu P_j, \qquad Q_i = \mu Q_j, \qquad R_i = \mu R_j, \quad \forall i, j \in L. \tag{22}$$

Moreover, the estimate of state decay is given by

$$\|\eta(t)\| \leq \sqrt{\frac{b}{a}}\, \left\|\eta_{t_0}\right\|_{c_1} e^{-\lambda(t-t_0)}, \qquad \sqrt{\frac{b}{a}} \geq 1 \tag{23}$$

where

$$\begin{aligned} &\lambda = \frac{1}{2}\left(\alpha - \frac{\ln \mu}{\tau_a}\right), \quad a = \min_{\forall i \in L} \lambda_{max}\left(P_i\right), \\ &b = \max_{\forall i \in L} \lambda_{max}(P_i) \, + d_2 \max_{\forall i \in L} \lambda_{max}(Q_i) \, + h_2 \max_{\forall i \in L} \lambda_{max}(R_i). \end{aligned} \tag{24}$$

Proof. See Appendix B.

Below, we will present a solution to the DOF control problem (6) for the USNS (1). It is worth noting that because the product term exists in Eq. (20) between the parameter-dependent Lyapunov matrix and the dynamics of the system matrix, solving the inequality will be difficult. We will generate a decoupling between the Lyapunov function and system matrices using the slack matrix variable M proposed in [40] and will obtain a new condition in Theorem 3.

Theorem 3. *For the given constant $\alpha > 0$, if there exist matrices $\mathcal{P}_{1,i} > 0$, $\mathcal{P}_{3,i} > 0$, $\mathcal{L}_{1,i} > 0$, $\mathcal{L}_{3i} > 0$, $\mathcal{R}_i > 0$, $R_i > 0$, $\mathcal{L}_{2,i}$, $\mathcal{A}_{c,i}$, $\mathcal{B}_{c,i}$, $\mathcal{C}_{c,i}$, $\mathcal{M}$, $\mathcal{G}$, and $\mathcal{K}$ such that for $i \in L$:*

$$\begin{bmatrix} \mathbb{A} & \mathbb{B} \\ * & -I \end{bmatrix} < 0, \tag{25}$$

$$\mathcal{P}_i = \begin{bmatrix} \mathcal{P}_{1,i} & \mathcal{P}_{2,i} \\ * & \mathcal{P}_{3,i} \end{bmatrix} > 0, \tag{26}$$

$$\mathcal{L}_i = \begin{bmatrix} \mathcal{L}_{1,i} & \mathcal{L}_{2,i} \\ * & \mathcal{L}_{3,i} \end{bmatrix} > 0, \tag{27}$$

$$R_i \mathcal{R}_i = I, \tag{28}$$

where

$$\mathbb{A} = \begin{bmatrix} \Sigma_{11,i} & \Sigma_{12,i} & \Sigma_{13,i} & \Sigma_{14,i} & \Sigma_{15,i} & \mathcal{M}^T A_{1,i} & A_i^T \\ * & \Sigma_{22,i} & \Sigma_{23,i} & \Sigma_{24,i} & D_i & A_{1,i} & \Sigma_{27,i}^T \\ * & * & -\mathcal{M} - \mathcal{M}^T & -\mathcal{K} - I & \Sigma_{15,i} & \mathcal{M}^T A_{1,i} & 0 \\ * & * & * & -\mathcal{G} - \mathcal{G}^T & D_i & A_{1,i} & 0 \\ * & * & * & * & \Sigma_{55,i} & 0 & D_i^T \\ * & * & * & * & * & \Sigma_{66,i} & A_{1,i}^T \\ * & * & * & * & * & * & -\mathcal{R}_i \end{bmatrix},$$

$$\mathbb{B} = \begin{bmatrix} N_{0i}^T F^T(t) & 0 & 0 & 0 & \Sigma_{1,12,i} & \Sigma_{1,13,i} & \Sigma_{1,12,i} & \Sigma_{1,13,i} & \Sigma_{1,12,i} & \Sigma_{1,13,i} \\ \Sigma_{28,i} & 0 & 0 & 0 & \Sigma_{2,12,i} & \Sigma_{2,13,i} & \Sigma_{2,12,i} & \Sigma_{2,13,i} & \Sigma_{2,12,i} & \Sigma_{2,13,i} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Phi_{1,i}^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Phi_{2,i}^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Sigma_{11,i} = \mathcal{M}^T A_i + A_i^T \mathcal{M} + \mathcal{B}_{c,i} C_i + C_i^T \mathcal{B}_{c,i}^T + \alpha \mathcal{P}_{1,i} + \mathcal{L}_{1,i},$$

$$\Sigma_{12,i} = \mathcal{A}_{c,i} + A_i^T + \alpha \mathcal{P}_{2,i} + \mathcal{L}_{2,i},$$

$$\Sigma_{22,i} = A_i \mathcal{G} + \mathcal{G}^T A_i^T + B_i \mathcal{C}_{c,i} + \mathcal{C}_{c,i}^T B_i^T + \alpha \mathcal{P}_{3,i} + \mathcal{L}_{3,i},$$

$$\Sigma_{13,i} = \mathcal{P}_{1,i} - \mathcal{M}^T + A_i^T \mathcal{M} + C_i^T \mathcal{B}_{c,i}^T,$$

$$\Sigma_{23,i} = \mathcal{P}_{2,i}^T - I + \mathcal{A}_{c,i}^T,$$

$$\Sigma_{14,i} = \mathcal{P}_{2,i} - \mathcal{K} + A_i^T,$$

$$\Sigma_{24,i} = \mathcal{P}_{3,i} - \mathcal{G} + \mathcal{G}^T A_i^T + \mathcal{C}_{c,i}^T B_i^T,$$

$$\Sigma_{15,i} = \mathcal{M}^T D_i + \mathcal{B}_{c,i} C_{1,i},$$

$$\Sigma_{27,i} = A_i\mathcal{G} + B_i\mathcal{C}_{c,i},$$
$$\Sigma_{28,i} = \mathcal{G}^T N_{0,i}^T + \mathcal{C}_{c,i}^T N_{1,i}^T,$$
$$\Sigma_{55,i} = -(1-\mu_d)\, e^{-\alpha d_2}\mathcal{L}_{1,i},$$
$$\Sigma_{66,i} = -(1-\mu_h)\, e^{-\alpha h_2} R_i,$$
$$\mathcal{M}_G^T \Lambda = \begin{bmatrix} \Sigma_{1,12,i} & \Sigma_{1,13,i} \\ \Sigma_{2,12,i} & \Sigma_{2,13,i} \end{bmatrix} = \begin{bmatrix} \mathcal{P}_{1,i}E_i & \hbar_{1,i} \\ \mathcal{P}_{2,i}^T E_i & \hbar_{2,i} \end{bmatrix},$$

then, the closed-loop system (7) is exponentially stable for any switching signal with ADT*:*

$$\tau_a > \tau_a^* = \frac{\ln\mu}{\alpha}, \tag{29}$$

where $\mu \geq 1$ satisfies

$$\mathcal{P}_i = \mu\mathcal{P}_j, \qquad \mathcal{L}_i = \mu\mathcal{L}_j, \qquad \mathcal{R}_i = \mu\mathcal{R}_j, \quad \forall\, i, j \in L. \tag{30}$$

Moreover, if the conditions above are feasible, then a desired DOF controller realization is given by

$$\mathcal{A}_{c,i} = \mathcal{M}^T A_i\mathcal{G} + M_4^T B_{c,i} C_i \mathcal{G} + \mathcal{M}^T B_i C_{c,i} G_4 + M_4^T A_{c,i} G_4,$$
$$\mathcal{B}_{c,i} = M_4^T B_{c,i}, \qquad \mathcal{C}_{c,i} = C_{c,i} G_4. \tag{31}$$

Proof. See Appendix C.

Remark 3. The problems of stability and stabilization are based on the conservative Lyapunov technique. The theories based on the Lyapunov functions guarantee the sufficient conditions rather than the necessary and sufficient ones. However, we used the multiple Lyapunov function as it is less conservative than, say, the common Lyapunov function. To further reduce the conservativeness, we employed the ADT approach. Furthermore, design proposed in this paper is dependent on the upper bound of both discrete and neutral delays and also the upper bound of derivatives of discrete and neutral delays, which makes the treatment more general with less conservatism compared with most existing results in the literature which are independent of a constant neutral delay.

Remark 4. It is worth noting that because of the existence of the product term in Eq. (20) between the parameter-dependent Lyapunov matrix and the dynamics of system matrix, it will be difficult to solve the inequality. Using the slack matrix variable M proposed in [40], we will generate a decoupling between the Lyapunov function and system matrices and will gain a new condition in Eq. (25). The proposed method is conservative, possibly due to the introduction of a common matrix M is conservative.

It is noteworthy that in Theorem 3 the inequalities are not LMI, and this is because of Eq. (28). The DOF control problem is very difficult to solve because it has a non-convex surface. To solve this problem, we use the following minimization algorithm which involves LMI conditions, proposed by [33,41].

DOF USNS Problem (Dynamic Output Feedback of Uncertain Switched Neutral Systems)

$$\min trace\left(\sum_{i\in L} R_i\mathcal{R}_i\right). \tag{32}$$

Subject to (25)–(27), (30) and

$$\begin{bmatrix} R_i & I \\ * & \mathcal{R}_i \end{bmatrix} \geq 0, \quad \forall i \in L \tag{33}$$

with E and F being given constant matrices with appropriate dimensions.

In the equations above, $\mathcal{P}_{1,i} > 0$, $\mathcal{P}_{3,i} > 0$, $\mathcal{L}_{1,i} > 0$, $\mathcal{L}_{3,i} > 0$, $\mathcal{R}_i > 0$, $R_i > 0$, $\mathcal{P}_{2,i}$, $\mathcal{A}_{c,i}$, $\mathcal{B}_{c,i}$, $\mathcal{C}_{c,i}$, $\mathcal{M}$, $\mathcal{G}$, $\mathcal{K}$ and scalar $\varepsilon > 0$.

This problem is used as an iterative technique to solve the DOF control problem. Through the linearization method [41], we can use an iterative algorithm (presented below), in which j and n denote the number of iterations and state variables, respectively.

DOF USNS Algorithm

Step 1. Find a feasible set ($\mathcal{P}_{1,i}^0$, $\mathcal{P}_{3,i}^0$, $\mathcal{L}_{1,i}^0$, $\mathcal{L}_{3,i}^0$, $\mathcal{R}_i^0$, R_i^0, $\mathcal{P}_{2,i}^0$, $\mathcal{A}_{c,i}^0$, $\mathcal{B}_{c,i}^0$, $\mathcal{C}_{c,i}^0$, $\mathcal{M}^0$, $\mathcal{G}^0$, $\mathcal{K}^0$ and scalar ε^0) satisfying (25)–(27), (30), (33). Set $j = 0$.

Step 2. Solve the following LMI problem for the variables ($\mathcal{R}_i$ and R_i):

$$\mathbb{e}^* = \min trace\left(\sum_{i\in L} R_i^j\mathcal{R}_i + R_i\mathcal{R}_i^j\right).$$

Subject to (25)–(27), (30), (33)) and denotes $\mathbb{e}^*$ be the optimized value.

Step 3. If the solution to the above minimization problem ($\mathbb{e}^*$) is approximately equal to $2n$, then ($\mathcal{P}_{1,i}$, $\mathcal{P}_{3,i}$, $\mathcal{L}_{1,i}$, $\mathcal{R}_i$, R_i, $\mathcal{P}_{2,i}$, $\mathcal{A}_{c,i}$, $\mathcal{B}_{c,i}$, $\mathcal{C}_{c,i}$, $\mathcal{M}$, $\mathcal{G}$, $\mathcal{K}$ and ε) are a feasible solution and exit.

Step 4. If $j > q$, where q is the maximum number of iterations, then the given Iterative LMI may not be feasible and exit. Otherwise, set $j = j + 1$, and go to step 2.

Remark 5. We expect the theorems presented in this paper to be extendable, using appropriate lemmas, to switched nonlinear systems with neutral type delay regarded as switched neutral linear systems under additive nonlinear perturbations. However, since the theorems are extensive and also because the focus of the present research is on switched time-delay linear systems, the question of extension will be presented in our future works.

5. Examples

Example 1. In this section, we consider the USNS (1) which consists of two unstable subsystems.

$$
\begin{aligned}
&A_1 = \begin{bmatrix} -1.9 & 0 & 0.1 \\ 0.2 & -2.1 & 0 \\ 0 & 0.1 & 0.3 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0.2 & 0 & 0.1 \\ 0.1 & 0.1 & 0.1 \\ 0 & 0.1 & 0.2 \end{bmatrix}, \quad A_{1,1} = \begin{bmatrix} 0.1 & 0 & 0.1 \\ 0 & 0.2 & 0.1 \\ 0.1 & 0 & 0.1 \end{bmatrix}, \\
&B_1 = \begin{bmatrix} 1 \\ 0.5 \\ 1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1.2 & 1 & 1.4 \end{bmatrix}, \quad C_{1,1} = \begin{bmatrix} 0.3 & 0.1 & 0.2 \end{bmatrix}, \\
&A_2 = \begin{bmatrix} -1.8 & -0.1 & 0 \\ 0.2 & -2.3 & 0.1 \\ 0 & 0.1 & 0.2 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.2 & 0.1 & 0 \\ 0.1 & 0.2 & 0.1 \\ 0 & 0.1 & 0.1 \end{bmatrix}, \quad A_{2,1} = \begin{bmatrix} 0.2 & 0.1 & 0 \\ 0.1 & 0.1 & 0.1 \\ 0.1 & 0 & 0.1 \end{bmatrix}, \\
&B_2 = \begin{bmatrix} 0.5 \\ 0.6 \\ 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1.3 & 1.2 & 1.5 \end{bmatrix}, \quad C_{2,1} = \begin{bmatrix} 0.1 & 0.3 & 0.2 \end{bmatrix}, \\
&N_{0,1} = N_{0,2} = \begin{bmatrix} 0.09 & 0 & 0 \\ 0 & 0.03 & -0.03 \\ 0 & -0.03 & 0 \end{bmatrix}, \quad N_{1,1} = N_{1,2} = \begin{bmatrix} 0.3 \\ 0 \\ 0.3 \end{bmatrix}, \\
&N_{2,1} = N_{2,2} = \begin{bmatrix} 0 & 0.03 & 0 \\ 0 & 0.03 & -0.06 \\ 0.03 & -0.03 & 0.03 \end{bmatrix}, \quad N_{3,1} = N_{3,2} = \begin{bmatrix} 0.03 & 0 & 0.06 \\ 0 & 0.06 & -0.03 \\ 0.03 & -0.03 & 0 \end{bmatrix}, \\
&E_1 = E_2 = \begin{bmatrix} 0.03 & 0 & 0 \\ 0 & 0.09 & 0 \\ 0 & 0 & 0.03 \end{bmatrix}, \quad F(t) = \begin{bmatrix} \sin(0.1t) & 0 & 0 \\ 0 & \sin(0.1t) & 0 \\ 0 & 0 & \sin(0.1t) \end{bmatrix}.
\end{aligned}
\tag{34}
$$

Considering $d(t) = h(t) = 0.7 + 0.3\sin(t)$ and $\alpha = 0.8$, we obtain $\mu_d = 0.3$, $\mu_h = 0.3$, $d_2 = 1$, and $h_2 = 1$. The above-mentioned system at $u(t) = 0$ is unstable for the switching signal where the ADT is $\tau_a = 0.05$ (Fig. 1). Fig. 2 depicts the states of the open-loop system with the initial condition being $\varphi(t) = \begin{bmatrix} 2 & 3 & 1 \end{bmatrix}^T$, $t = [-1, 0]$.

Considering this, we aim to design a robust DOF controller $u(t)$ in the form of Eq. (6) such that the closed-loop system reaches exponential stability. Using MATLAB toolbox YALMIP, we obtained the following solutions to LMIs (25)–(28) in Theorem 3 for the DOF USNS problem by setting $\mu = 1.01$ (thus $\tau_a > \tau_a^* = \frac{\ln\mu}{\alpha} = 0.0124$) and with $i_0 = 1$:

$$
\begin{aligned}
&\mathcal{M} = \begin{bmatrix} 5.8080 & -3.0042 & -3.4084 \\ -1.4381 & 8.2550 & -0.9734 \\ 0.6992 & 0.2533 & 3.6112 \end{bmatrix}, \quad \mathcal{K} = \begin{bmatrix} 0.5503 & 0.1465 & 0.4893 \\ 0.0708 & 0.4562 & -0.2044 \\ -0.9975 & -0.5938 & -0.4777 \end{bmatrix}, \\
&\mathcal{G} = \begin{bmatrix} 0.2923 & -0.0171 & -0.1040 \\ 0.0002 & 0.2630 & -0.1235 \\ -0.0544 & 0.0561 & 0.3831 \end{bmatrix}, \quad \mathcal{A}_{c,1} = \begin{bmatrix} -1.0963 & 0.1912 & -0.5901 \\ 0.1060 & -1.0421 & 0.3184 \\ 0.8771 & 0.3920 & 0.5747 \end{bmatrix}, \\
&\mathcal{A}_{c,2} = \begin{bmatrix} -0.7143 & -0.2028 & -0.7029 \\ -0.3374 & -1.1906 & -0.4080 \\ 0.3553 & 0.2472 & 0.2140 \end{bmatrix}, \quad \mathcal{B}_{c,1} = \begin{bmatrix} -5.6986 \\ -5.3579 \\ -8.6933 \end{bmatrix}, \quad \mathcal{B}_{c,2} = \begin{bmatrix} -2.5353 \\ -4.9581 \\ -5.6857 \end{bmatrix}, \\
&\mathcal{C}_{c,1} = \begin{bmatrix} -0.2090 & -0.1391 & -0.5068 \end{bmatrix}, \quad \mathcal{C}_{c,2} = \begin{bmatrix} -0.1013 & -0.1454 & -0.5547 \end{bmatrix}.
\end{aligned}
$$

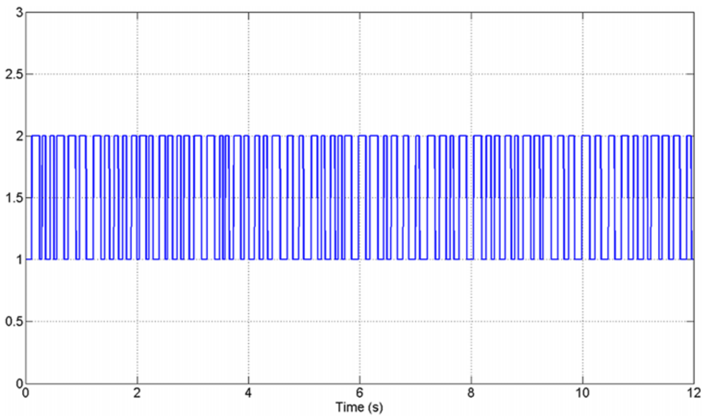

Fig. 1. The switching signal with the ADT satisfying $\tau_a = 0.05$.

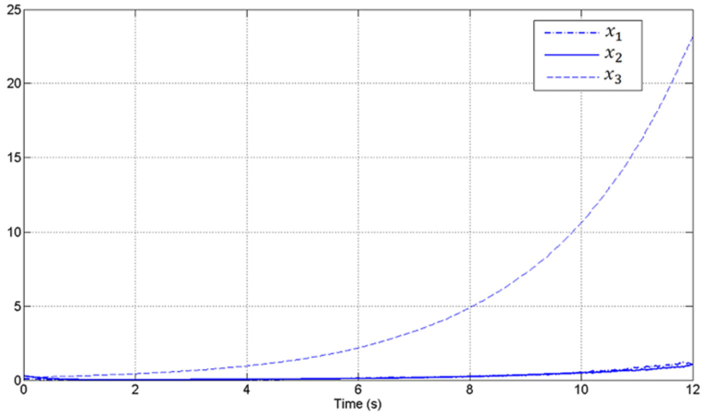

Fig. 2. State trajectories of the open-loop system.

Also, we will obtain $\mathbb{e}^* = 12.0017$ from DOF USNS algorithm. Considering $M_4 = I$, we will obtain $G_4 = \mathcal{K} - \mathcal{M}^T\mathcal{G}$ from Eq. (C.8) in Appendix C (derived from $\mathcal{K} = M_1^T G_1 + M_4^T G_4$). Therefore, we can obtain the DOF controller from Eq. (31).

$$A_{c,1} = \begin{bmatrix} -6.9800 & -2.1005 & -3.5362 \\ -3.0901 & -3.4643 & -2.0036 \\ -3.1004 & -1.5665 & -2.6734 \end{bmatrix}, \qquad A_{c,2} = \begin{bmatrix} -4.2485 & -0.9613 & -1.6814 \\ -3.2605 & -3.8532 & -2.0805 \\ -1.5635 & -0.8896 & -2.0203 \end{bmatrix},$$
$$B_{c,1} = \begin{bmatrix} -5.6986 \\ -5.3579 \\ -8.6933 \end{bmatrix}, \qquad B_{c,2} = \begin{bmatrix} -2.5353 \\ -4.9581 \\ -5.6857 \end{bmatrix},$$
$$C_{c,1} = \begin{bmatrix} 0.3073 & 0.0731 & 0.3149 \end{bmatrix}, \qquad C_{c,2} = \begin{bmatrix} 0.1791 & 0.0415 & 0.2944 \end{bmatrix}. \tag{35}$$

The states of the closed-loop system considering (35) are shown in Fig. 3. It can be seen that the closed-loop system is exponentially stable with $\lambda = 0.3$. The control input is illustrated in Fig. 4.

Example 2. In this section, we consider a water-quality dynamic model for the Nile River [22,29] in two modes of operation:

$$A_1 = \begin{bmatrix} -1.0 & 0.0 \\ -3.0 & -2.0 \end{bmatrix}, \qquad D_1 = \begin{bmatrix} -0.55 & 0.70 \\ -0.25 & -0.30 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 1.4 & 0.0 \\ 0.0 & 1.5 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 1.0 & 1.0 \\ 1.0 & 2.0 \end{bmatrix}, \qquad C_{1,1} = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.2 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 1.0 & 0.0 \\ -3.0 & -2.0 \end{bmatrix}, \qquad D_2 = \begin{bmatrix} -0.45 & -0.50 \\ -0.15 & -0.10 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 1.2 & 0.0 \\ 0.0 & 1.4 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 1.0 & 1.4 \\ 1.5 & 1.0 \end{bmatrix}, \qquad C_{2,1} = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.1 \end{bmatrix},$$

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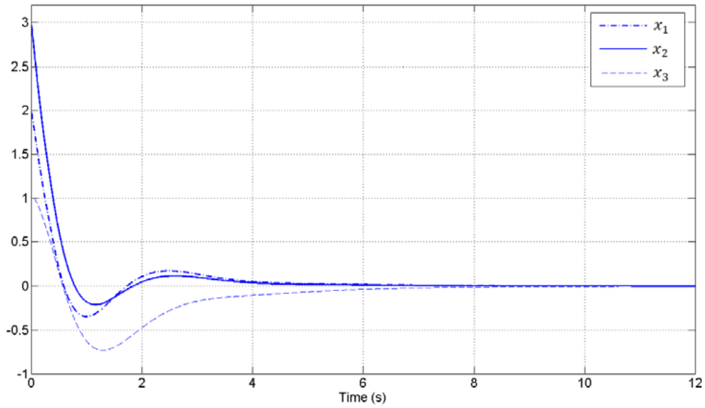

Fig. 3a. State trajectories of the system having a controller with $\varphi(t) = \begin{bmatrix} 2 & 3 & 1 \end{bmatrix}^T$.

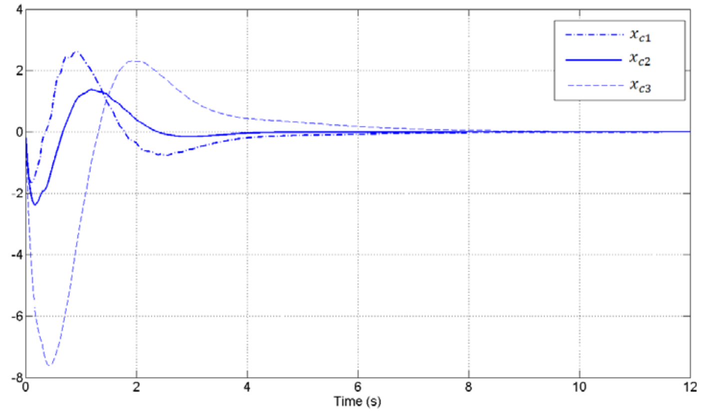

Fig. 3b. State trajectories of the controller with $x_c(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$.

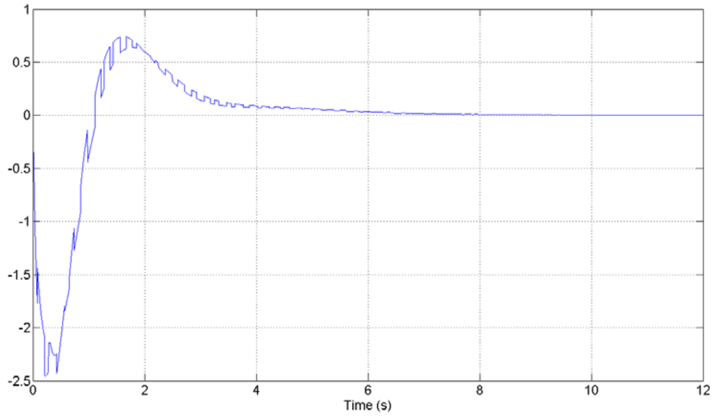

Fig. 4. DOF control trajectory (control input).

$$N_{0,1} = \begin{bmatrix} 0.5 & -0.3 \\ -0.2 & 0.8 \end{bmatrix}, \qquad N_{1,1} = \begin{bmatrix} -0.2 & 0.0 \\ 0.0 & -0.1 \end{bmatrix}, \qquad N_{3,1} = \begin{bmatrix} -0.2 & 0.1 \\ 0.1 & -0.2 \end{bmatrix},$$
$$N_{0,2} = \begin{bmatrix} 0.7 & -0.3 \\ -0.6 & 0.7 \end{bmatrix}, \qquad N_{1,2} = \begin{bmatrix} -0.1 & 0.1 \\ 0.0 & -0.1 \end{bmatrix}, \qquad N_{3,2} = \begin{bmatrix} -0.2 & 0.1 \\ 0.1 & -0.1 \end{bmatrix},$$
$$E_1 = E_2 = 0.1 \times \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, \qquad F(t) = \begin{bmatrix} \sin(0.1t) & 0 \\ 0 & \sin(0.1t) \end{bmatrix}. \tag{36}$$

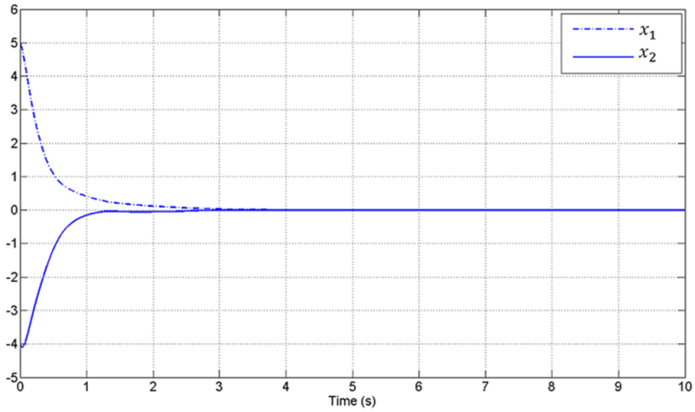

Fig. 5a. State trajectories of the system having a controller with $\varphi(t) = \begin{bmatrix} 5 & -4 \end{bmatrix}^T$, $t = [-0.5, 0]$.

Given $A_{1,1} = A_{1,2} = 0$, $d(t) = h(t) = 0.3 + 0.2\sin(t)$, and $\alpha = 0.5$, we obtain $\mu_d = 0.2$, $\mu_h = 0.2$, $d_2 = 0.5$, and $h_2 = 0.5$. The above-said system at $u(t) = 0$ is unstable for the switching signal where the ADT is $\tau_a = 1$. Regarding this, we seek to design a robust DOF controller $u(t)$ in the form of Eq. (6) such that the closed-loop system reaches exponential stability. Using MATLAB toolbox YALMIP, we came up with the following solutions to the LMIs (25)–(28) in Theorem 3 for the DOF USNS problem by setting $\mu = 1.1$ (thus $\tau_a > \tau_a^* = \frac{\ln\mu}{\alpha} = 0.1906$) and with $i_0 = 1$:

$$\mathcal{M} = 10^3 \times \begin{bmatrix} 0.1841 & -0.2485 \\ -0.0818 & 1.4359 \end{bmatrix}, \quad \mathcal{K} = \begin{bmatrix} -0.0764 & 0.1912 \\ 0.1229 & -0.6769 \end{bmatrix}, \quad \mathcal{G} = \begin{bmatrix} 0.2419 & 0.1111 \\ 0.0970 & 0.3667 \end{bmatrix},$$
$$\mathcal{A}_{c,1} = \begin{bmatrix} 1.0464 & -2.1743 \\ 0.0866 & 0.2001 \end{bmatrix}, \quad \mathcal{A}_{c,2} = \begin{bmatrix} 2.0980 & -1.8825 \\ 0.1231 & 0.2488 \end{bmatrix},$$
$$\mathcal{B}_{c,1} = 10^3 \times \begin{bmatrix} -4.8112 & 2.4125 \\ 7.5333 & -3.1883 \end{bmatrix}, \quad \mathcal{B}_{c,2} = 10^3 \times \begin{bmatrix} 1.9770 & -2.8022 \\ -2.8054 & 5.0740 \end{bmatrix},$$
$$\mathcal{C}_{c,1} = \begin{bmatrix} -0.4217 & 0.0767 \\ 0.6193 & 0.1747 \end{bmatrix}, \quad \mathcal{C}_{c,2} = \begin{bmatrix} -0.8638 & -0.1064 \\ 0.6424 & 0.1770 \end{bmatrix}.$$

Also, we will obtain $\mathbb{e}^* = 8.0004$ from DOF USNS algorithm. Considering $M_4 = I$, we will obtain $G_4 = \mathcal{K} - \mathcal{M}^T\mathcal{G}$ from Eq. (C.8) in Appendix C (derived from $\mathcal{K} = M_1^T G_1 + M_4^T G_4$). Therefore, we can obtain the DOF controller from Eq. (31).

$$A_{c,1} = \begin{bmatrix} -18.4008 & -0.7475 \\ 37.2201 & 0.2307 \end{bmatrix}, \quad A_{c,2} = \begin{bmatrix} -17.2153 & -0.7245 \\ 38.6284 & 0.3185 \end{bmatrix},$$
$$B_{c,1} = 10^3 \times \begin{bmatrix} -4.8112 & 2.4125 \\ 7.5333 & -3.1883 \end{bmatrix}, \quad B_{c,2} = 10^3 \times \begin{bmatrix} 1.9770 & -2.8022 \\ -2.8054 & 5.0740 \end{bmatrix},$$
$$C_{c,1} = \begin{bmatrix} 0.0114 & 0.0001 \\ -0.0155 & -0.0007 \end{bmatrix}, \quad C_{c,2} = \begin{bmatrix} 0.0222 & 0.0006 \\ -0.0161 & -0.0007 \end{bmatrix}. \tag{37}$$

The states of the closed-loop system considering (37) are shown in Fig. 5. The control inputs and outputs are illustrated in Figs. 6 and 7, respectively.

6. Conclusion

This paper is concerned with the problems of stability and stabilization of USNSs with interval time-varying mixed delays. The emphasis of the paper is on the cases where uncertainties are norm-bounded time-varying and exist in the matrix of the derivatives of the state. To guarantee exponential stability, sufficient conditions were derived in terms of a set of LMIs using the ADT approach and the piecewise Lyapunov function technique. Then, we obtained the corresponding conditions for stabilization via a DOF controller. We solved the problem of uncertainty in USNSs by designing a DOF controller and employing the Yakubovich lemma. Using the proposed slack matrix variable, we generated a decoupling between the Lyapunov function and system matrices in order to gain a new condition. The effectiveness of the proposed DOF controller was verified by means of examples. In most studies, the stabilization of SNSs is discussed without considering uncertainty and time-varying delays. It seems that it is more practical to consider uncertainty in system parameters and interval time-varying delays in the state and derivatives of the state. The obtained results may help us explore other problems for SNSs

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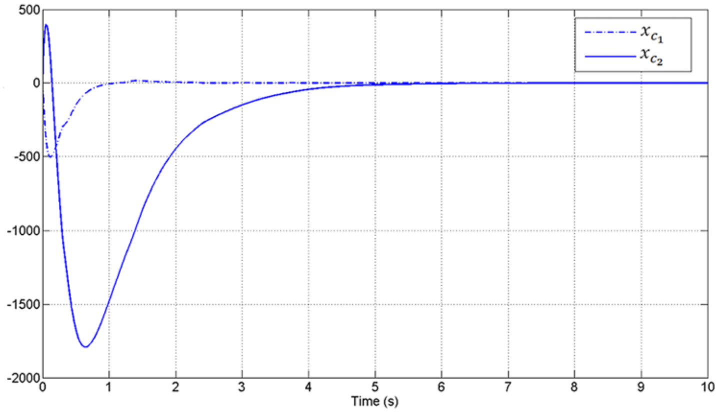

Fig. 5b. State trajectories of the controller with $x_c(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$.

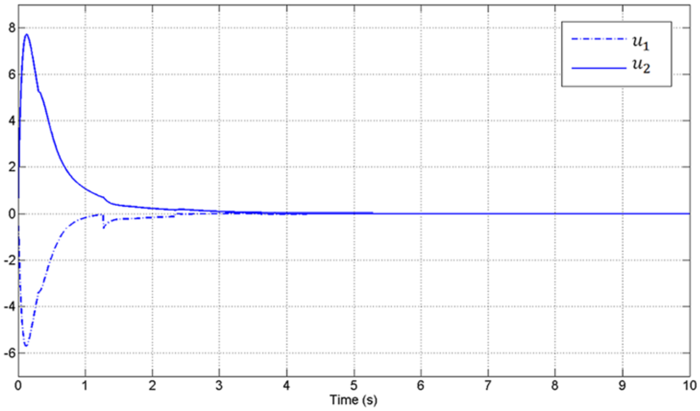

Fig. 6. DOF control trajectory (control input).

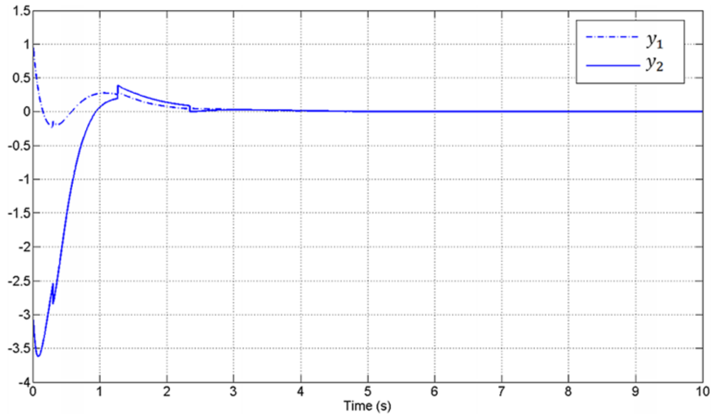

Fig. 7. Trajectories of the system with the controller.

such as guaranteed cost control, where neutral delay is supposed to be time-varying and the derivative of the discrete delay is less than one. To further the present research, we are currently working on the restrictions on the control signal within the framework of guaranteed cost control. Also, we will extend the presented theorems to give sufficient conditions for the existence of the DOF controller for stabilization of the switched nonlinear system with neutral type delay.

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Appendix A

Proof of Theorem 1. A Lyapunov functional candidate is defined in the following form:

$$V\left(x\left(t\right),i\right)\triangleq V_1\left(x\left(t\right),i\right)+V_2\left(x\left(t\right),i\right)+V_3\left(x\left(t\right),i\right) \tag{A.1}$$

where

$$\begin{aligned}
V_1\left(x\left(t\right),i\right)&=x^T\left(t\right)P_ix\left(t\right)\\
V_2\left(x\left(t\right),i\right)&=\int_{t-d(t)}^{t}e^{\alpha(s-t)}x^T\left(s\right)Q_ix\left(s\right)ds\\
V_3\left(x\left(t\right),i\right)&=\int_{t-h(t)}^{t}e^{\alpha(s-t)}\dot{x}^T\left(s\right)R_i\dot{x}\left(s\right)ds,
\end{aligned}$$

and $P_i=P_i^T>0$, $Q_i>0$, and $R_i>0$ are to be determined. Taking the derivative of $V(x(t),i)$ with respect to t along the trajectory of the USNS (12) and using Eqs. (2)–(4) and (8) and adding $\alpha V\left(x\left(t\right),i\right)$, we will get

$$\begin{aligned}
\dot{V}\left(x\left(t\right),i\right)+\alpha V\left(x\left(t\right),i\right)\le{}&\alpha x^T\left(t\right)P_ix\left(t\right)+x^T\left(t\right)\left[P_iA_i+A_i^TP_i\right]x\left(t\right)\\
&+x^T\left(t\right)\left[P_i\left[E_iF(t)N_{0,i}\right]+\left[E_iF(t)N_{0,i}\right]^TP_i\right]x\left(t\right)\\
&+x^T\left(t\right)\left[P_iD_i\right]x\left(t-d\left(t\right)\right)+x^T\left(t-d\left(t\right)\right)\left[D_i^TP_i\right]x(t)\\
&+x^T\left(t\right)\left[P_i\left[E_iF(t)N_{3,i}\right]\right]x\left(t-d\left(t\right)\right)\\
&+x^T\left(t-d\left(t\right)\right)\left[E_iF\left(t\right)N_{3i}\right]^TP_ix(t)\\
&+x^T\left(t\right)\left[P_iA_{1,i}\right]\dot{x}\left(t-h\left(t\right)\right)+\dot{x}^T\left(t-d\left(t\right)\right)\left[A_{1,i}^TP_i\right]x(t)\\
&+x^T\left(t\right)\left[P_i\left[E_iF\left(t\right)N_{2,i}\right]\dot{x}\left(t-h\left(t\right)\right)\right]\\
&+\dot{x}^T\left(t-d\left(t\right)\right)\left[E_iF(t)N_{2,i}\right]^TP_ix(t)\\
&+x^T\left(t\right)Q_ix\left(t\right)-\left(1-\mu_d\right)e^{-\alpha d_2}x^T\left(t-d\left(t\right)\right)Q_ix\left(t-d\left(t\right)\right)\\
&+\dot{x}^T\left(t\right)R_i\dot{x}\left(t\right)-\left(1-\mu_h\right)e^{-\alpha h_2}\dot{x}^T\left(t-h(t)\right)R_i\dot{x}\left(t-h(t)\right).
\end{aligned} \tag{A.2}$$

So (A.2) is rewritten as the following linear inequality:

$$\dot{V}\left(x\left(t\right),i\right)+\alpha V\left(x\left(t\right),i\right)\le\Sigma^T(t)\Pi_i\Sigma(t) \tag{A.3}$$

where $\Sigma(t)\triangleq\left[x^T\left(t\right)\quad x^T\left(t-d\left(t\right)\right)\quad\dot{x}^T\left(t-h\left(t\right)\right)\quad\Gamma^T\left(i,t\right)\quad\Gamma_1^T\left(i,t\right)\quad\Gamma_2^T\left(i,t\right)\right]^T$, and

$$\begin{aligned}
&\Gamma\left(i,t\right)=\left[F\left(t\right)N_{0,i}\right]x\left(t\right),\qquad\Gamma_1\left(i,t\right)=\left[F\left(t\right)N_{3,i}\right]x\left(t-d\left(t\right)\right),\\
&\Gamma_2\left(i,t\right)=\left[F(t)N_{2,i}\right]\dot{x}\left(t-h\left(t\right)\right),
\end{aligned} \tag{A.4}$$

$$\begin{aligned}
\Pi_i\triangleq{}&\begin{bmatrix}\Pi_{11,i}&P_iD_i&P_iA_{1,i}&\Lambda&\Lambda&\Lambda\\ *&-\left(1-\mu_d\right)e^{-\alpha d_2}Q_i&0&0&0&0\\ *&*&-\left(1-\mu_h\right)e^{-\alpha h_2}R_i&0&0&0\\ *&*&*&0&0&0\\ *&*&*&*&0&0\\ *&*&*&*&*&0\end{bmatrix}\\
&+\begin{bmatrix}\begin{bmatrix}A_i^T\\D_i^T\\A_{1,i}^T\end{bmatrix}R_i\begin{bmatrix}A_i^T\\D_i^T\\A_{1,i}^T\end{bmatrix}^T&\begin{matrix}0&0&0\\0&0&0\\0&0&0\end{matrix}\\ \begin{matrix}*&*&*\\ *&*&*\\ *&*&*\end{matrix}&\begin{matrix}0&0&0\\ *&0&0\\ *&*&0\end{matrix}\end{bmatrix},
\end{aligned} \tag{A.5}$$

$$\Pi_{11i}=P_iA_i+A_i^TP_i+\alpha P_i+Q_i.$$

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However, if we can prove that $\Pi_i < 0$, then the exponential stability of the system is guaranteed, meaning that

$$\dot{V}\left(x\left(t\right), i\right) + \alpha V\left(x\left(t\right), i\right) \leq 0. \tag{A.6}$$

According to the Schur complement lemma, condition (A.5) is equivalent to

$$\mathbb{T}_i = \begin{bmatrix} \Pi_{11,i} & P_iD_i & P_iA_{1,i} & A_i^TR_i & \Lambda & \Lambda & \Lambda \\ * & -(1-\mu_d)e^{-\alpha d_2}Q_i & 0 & D_i^TR_i & 0 & 0 & 0 \\ * & * & -\left(1-\mu_h\right)e^{-\alpha h_2}R_i & A_{1,i}^TR_i & 0 & 0 & 0 \\ * & * & * & -R_i & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & 0 \end{bmatrix}. \tag{A.7}$$

Thus,

$$\dot{V}\left(x\left(t\right), i\right) + \alpha V\left(x\left(t\right), i\right) \leq \Sigma^T\left(t\right).\mathbb{T}_i.\Sigma(t). \tag{A.8}$$

Also, if we can prove that $\mathbb{T}_i < 0$, which implies $\dot{V}\left(x\left(t\right), i\right) + \alpha V\left(\eta\left(t\right), i\right) \leq 0$, then exponential stability of the system is guaranteed. However, due to the existence of zero on the main diagonal matrix $\mathbb{T}_i$, we cannot simply conclude $\mathbb{T}_i < 0$. To solve this problem which is caused by uncertainties, we apply Lemma 1 (Yakubovich lemma). We can easily obtain the following results using (5) and (A.4)

$$\begin{aligned} \Gamma^T\left(i,t\right).\Gamma\left(i,t\right) &= x^T\left(t\right)N_{0,i}^TF^T\left(t\right).F\left(t\right)N_{0,i}x\left(t\right) \leq x^T\left(t\right)N_{0,i}^TN_{0,i}x\left(t\right) \\ \Gamma_1^T\left(i,t\right).\Gamma_1\left(i,t\right) &= x^T\left(t-d\left(t\right)\right)N_{3,i}^TF^T\left(t\right).F\left(t\right)N_{3,i}x\left(t-d\left(t\right)\right) \\ &\leq x^T\left(t-d\left(t\right)\right)N_{3,i}^TN_{3,i}x\left(t-d\left(t\right)\right) \\ \Gamma_2^T\left(i,t\right).\Gamma_2\left(i,t\right) &= \dot{x}^T\left(t-h\left(t\right)\right)N_{2,i}^TF^T\left(t\right).F\left(t\right)N_{2,i}\dot{x}\left(t-h\left(t\right)\right) \\ &\leq \dot{x}^T\left(t-h\left(t\right)\right)N_{2,i}^TN_{2,i}\dot{x}\left(t-h\left(t\right)\right). \end{aligned} \tag{A.9}$$

Now, according to Lemma 1, if

$$\dot{V}\left(x\left(t\right), i\right) + \alpha V\left(x\left(t\right), i\right) \leq \Sigma^T\left(t\right).\mathbb{T}.\Sigma(t) < H, \tag{A.10}$$

where

$$\begin{aligned} \mathcal{H} = &\left(\Gamma^T\left(i,t\right).\Gamma\left(i,t\right) - x^T\left(t\right)N_{0,i}^TN_{0,i}x\left(t\right)\right) \\ &+ \left(\Gamma_1^T\left(i,t\right).\Gamma_1\left(i,t\right) - x^T\left(t-d\left(t\right)\right)N_{3,i}^TN_{3,i}x\left(t-d\left(t\right)\right)\right) \\ &+ \left(\Gamma_2^T\left(i,t\right).\Gamma_2\left(i,t\right) - \dot{x}^T\left(t-h\left(t\right)\right)N_{2,i}^TN_{2,i}\dot{x}\left(t-h\left(t\right)\right)\right) \end{aligned}$$

then system (12) is exponentially stable. So (A.10) is rewritten as the following inequality:

$$\dot{V}\left(x\left(t\right), i\right) + \alpha V\left(x\left(t\right), i\right) - \mathcal{H} < 0. \tag{A.11}$$

Now, according to Lemma 1, given

$$\Omega_1|_{\varepsilon=1} = \mathcal{H}, \tag{A.12}$$

$$\Omega_0\left(x\right) = \dot{V}\left(x\left(t\right), i\right) + \alpha V\left(x\left(t\right), i\right) < 0 \quad \text{holds if and only if}$$

$$\Omega_0(x) - \varepsilon\Omega_1(x) < 0. \tag{A.13}$$

It can be seen that (A.13) is equivalent to (A.11). Writing (A.11) in the matrix form, we will obtain inequality (A.14), which will guarantee the stability of the system.

$$\begin{bmatrix} (1,1) & (1,2) \\ * & (2,2) \end{bmatrix} < 0 \tag{A.14}$$

where

$$(1,1) = \begin{bmatrix} \Pi_{11i} + N_{0,i}^TN_{0,i} & P_iD_i & P_iA_{1,i} & A_i^TR_i \\ * & -\left(1-\mu_d\right)e^{-\alpha d_2}Q_i + N_{3,i}^TN_{3,i} & 0 & D_i^TR_i \\ * & * & -\left(1-\mu_h\right)e^{-\alpha h_2}R_i + N_{2,i}^TN_{2,i} & A_{1,i}^TR_i \\ * & * & * & -R_i \end{bmatrix},$$

$$(1,2)=\begin{bmatrix}\Lambda & \Lambda & \Lambda\\ 0&0&0\\ 0&0&0\\ 0&0&0\end{bmatrix},\qquad (2,2)=\begin{bmatrix}-I&0&0\\ *&-I&0\\ *&*&-I\end{bmatrix}.$$

According to the Schur complement lemma, condition (A.14) is equivalent to

$$\begin{bmatrix} & & & & N_{0,i}^T & 0 & 0 & \Lambda & \Lambda & \Lambda\\ & \Delta_i & & & 0 & N_{3,i}^T & 0 & 0 & 0 & 0\\ & & & & 0 & 0 & N_{2,i}^T & 0 & 0 & 0\\ & & & & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & -I & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & -I & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & -I & 0 & 0 & 0\\ * & * & * & * & * & * & * & -I & 0 & 0\\ * & * & * & * & * & * & * & * & -I & 0\\ * & * & * & * & * & * & * & * & * & -I\end{bmatrix}<0 \tag{A.15}$$

where

$$\Delta_i=\begin{bmatrix}\Pi_{11i} & P_iD_i & P_iA_{1,i} & A_i^TR_i\\ * & -(1-\mu_d)e^{-\alpha d_2}Q_i & 0 & D_i^TR_i\\ * & * & -(1-\mu_h)\,e^{-\alpha h_2}R_i & A_{1,i}^TR_i\\ * & * & * & -R_i\end{bmatrix}.$$

Thus, according to (13), the exponential stability of the system is guaranteed. In addition, if we assume that the ith subsystem is active when $t\in[t_k,t_{k+1})$, then the following matrix inequalities hold:

$$V\left(x\left(t\right),i\right)\le e^{-\alpha(t-t_k)}V\left(x\left(t_k\right),i(t_k)\right). \tag{A.16}$$

From (15) and (A.1), at the switching moment t_k, we will have

$$V\left(x\left(t_k\right),i(t_k)\right)\le \mu V\left(x\left(t_k^-\right),i(t_k^-)\right). \tag{A.17}$$

So from (A.16) and (A.17) for $t\in[t_k,t_{k+1})$, and according to Definition 2, we know that $\rho=N_\sigma\left(t_0,t\right)\le\frac{t-t_0}{\tau_a}$, $t_0=0$. Then,

$$\begin{aligned}V\left(x\left(t\right),i\right)&\le e^{-\alpha(t-t_k)}V\left(x\left(t_k\right),i(t_k)\right)\le \mu e^{-\alpha(t-t_k)}V\left(x\left(t_k^-\right),i(t_k^-)\right)\\ &\le \mu e^{-\alpha\left(t-t_{k-1}\right)}V\left(x\left(t_{k-1}\right),i(t_{k-1})\right)\le \mu^2e^{-\alpha\left(t-t_{k-1}\right)}V\left(x\left(t_{k-1}^-\right),i(t_{k-1}^-)\right)\\ &\le\cdots\le \mu^\rho e^{-\alpha(t-t_0)}V\left(x\left(t_0\right),i(t_0)\right)\le e^{-\left(\alpha-\frac{\ln\mu}{\tau_a}\right)t}V\left(x\left(t_0\right),i(t_0)\right).\end{aligned} \tag{A.18}$$

Furthermore, given the definition of the Lyapunov function (A.1) and its monotonousness and (A.18), we will obtain the following inequalities:

$$a\left\|x(t)\right\|^2\le V\left(x\left(t\right),i\right)\le e^{-\left(\alpha-\frac{\ln\mu}{\tau_a}\right)t}V\left(x\left(t_0\right),i(t_0)\right)\le b\left\|x\left(t_0\right)\right\|_{cl}^2 \tag{A.19}$$

where a and b are defined in (17). Then, we have

$$\left\|x(t)\right\|^2\le\frac{1}{a}V\left(x\left(t\right),i\right)\le\frac{b}{a}e^{-\left(\alpha-\frac{\ln\mu}{\tau_a}\right)t}\left\|x\left(t_0\right)\right\|_{cl}^2. \tag{A.20}$$

This means that the system is exponentially stable according to Definition 1 with $t_0=0$. The proof is complete at this point.

Appendix B

Proof of Theorem 2. A Lyapunov functional candidate is defined in the following form:

$$\begin{aligned}V\left(\eta\left(t\right),i\right)\triangleq{}&\eta^T\left(t\right)P_i\eta\left(t\right)+\int_{t-d(t)}^{t}e^{\alpha(s-t)}\eta^T\left(s\right)K^TQ_iK\eta\left(s\right)ds\\ &+\int_{t-h(t)}^{t}e^{\alpha(s-t)}\dot\eta^T\left(s\right)K^TR_iK\dot\eta\left(s\right)ds\end{aligned} \tag{B.1}$$

where $P_i > 0$, $Q_i > 0$ and $R_i > 0$ are to be determined. Taking the derivative of $V(\eta(t), i)$ with respect to t along the trajectory of the USNS (1) and using Eqs. (2)–(4) and (8) and adding $\alpha V(\eta(t), i)$, we will get

$$\begin{aligned}\dot{V}(\eta(t), i) + \alpha V(\eta(t), i) \le\ & \eta^T(t)\left[P_i\overline{A}_i + \overline{A}_i^T P_i + \alpha P_i\right]\eta(t)\\ & + \eta^T(t)\left\{P_i\begin{bmatrix} E_iF(t)N_{0,i} & E_iF(t)N_{1,i}C_{c,i}\\ 0 & 0\end{bmatrix}\right.\\ & \left. + \begin{bmatrix} E_iF(t)N_{0,i} & E_iF(t)N_{1,i}C_{c,i}\\ 0 & 0\end{bmatrix}^T P_i\right\}\eta(t)\\ & + 2\eta^T(t)P_i\overline{D}_iK\eta(t-d(t))\\ & + 2\eta^T(t)P_i\begin{bmatrix} E_iF(t)N_{3,i}\\ 0\end{bmatrix}K\eta(t-d(t))\\ & + 2\eta^T(t)P_i\overline{A}_{1,i}K\dot{\eta}(t-h(t))\\ & + 2\eta^T(t)P_i\begin{bmatrix} E_iF(t)N_{2,i}\\ 0\end{bmatrix}K\dot{\eta}(t-h(t))\\ & + \eta^T(t)K^TQ_iK\eta(t)\\ & - (1-\mu_d)e^{-\alpha d_2}\eta^T(t-d(t))K^TQ_iK\eta(t-d(t))\\ & + \dot{\eta}^T(t)K^TR_iK\dot{\eta}(t)\\ & - (1-\mu_h)e^{-\alpha h_2}\dot{\eta}^T(t-h(t))K^TR_iK\dot{\eta}(t-h(t)).\end{aligned} \tag{B.2}$$

So (B.2) is rewritten as the following linear inequality:

$$\dot{V}(\eta(t), i) + \alpha V(\eta(t), i) \le \overline{\Sigma}^T(t)\,\overline{\Pi}_i\,\overline{\Sigma}(t), \tag{B.3}$$

where $\overline{\Sigma}(t) \triangleq \left[\eta^T(t) \quad \eta^T(t-d(t))K^T \quad \dot{\eta}^T(t-h(t))K^T \quad \overline{\Gamma}^T(i,t) \quad \overline{\Gamma}_1^T(i,t) \quad \overline{\Gamma}_2^T(i,t)\right]^T$ and

$$\begin{aligned}&\overline{\Gamma}(i,t) = \begin{bmatrix} F(t)N_{0,i} & F(t)N_{1,i}C_{c,i}\\ 0 & 0\end{bmatrix}\eta(t), \qquad \overline{\Gamma}_1(i,t) = \begin{bmatrix} F(t)N_{3,i}\\ 0\end{bmatrix}K\eta(t-d(t)),\\ &\overline{\Gamma}_2(i,t) = \begin{bmatrix} F(t)N_{2,i}\\ 0\end{bmatrix}K\dot{\eta}(t-h(t)),\end{aligned} \tag{B.4}$$

$$\begin{aligned}\overline{\Pi}_i \triangleq\ & \begin{bmatrix} \overline{\Pi}_{11,i} & P_i\overline{D}_i & P_i\overline{A}_{1,i} & \Lambda & \Lambda & \Lambda\\ * & -(1-\mu_d)e^{-\alpha d_2}Q_i & 0 & 0 & 0 & 0\\ * & * & -(1-\mu_h)e^{-\alpha h_2}R_i & 0 & 0 & 0\\ * & * & * & 0 & 0 & 0\\ * & * & * & * & 0 & 0\\ * & * & * & * & * & 0\end{bmatrix}\\ & + \begin{bmatrix} \begin{bmatrix}\overline{A}_i^TK^T\\ \overline{D}_i^TK^T\\ \overline{A}_{1,i}^TK^T\end{bmatrix} R_i \begin{bmatrix}\overline{A}_i^TK^T\\ \overline{D}_i^TK^T\\ \overline{A}_{1,i}^TK^T\end{bmatrix}^T & \begin{matrix} 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{matrix}\\ \begin{matrix} * & * & *\\ * & * & *\\ * & * & *\end{matrix} & \begin{matrix} 0 & 0 & 0\\ * & 0 & 0\\ * & * & 0\end{matrix}\end{bmatrix}\end{aligned} \tag{B.5}$$

and

$$\overline{\Pi}_{11i} = P_i\overline{A}_i + \overline{A}_i^TP_i + \alpha P_i + K^TQ_iK.$$

However, if we can prove that $\overline{\Pi}_{\mathrm{i}} < 0$, then the exponential stability of the system is guaranteed, meaning that

$$\dot{V}(\eta(t), i) + \alpha V(\eta(t), i) \le \overline{\Sigma}^T(t)\overline{\Pi}_i\overline{\Sigma}(t). \tag{B.6}$$

We can easily obtain the following results using (5):

$$\begin{aligned}&\overline{\Gamma}^T(i,t).\overline{\Gamma}(i,t) \le \eta^T(t)\,\Phi_{\mathrm{i}}^{\mathrm{T}}\Phi_{\mathrm{i}}\eta(t),\\ &\overline{\Gamma}_1^T(i,t).\overline{\Gamma}_1(i,t) \le \eta^T(t-d(t))\,\Phi_{1,\mathrm{i}}^{\mathrm{T}}\Phi_{1,i}\eta(t-d(t)),\\ &\overline{\Gamma}_2^T(i,t).\overline{\Gamma}_2(i,t) \le \dot{\eta}^T(t-h(t))\,\Phi_{2,\mathrm{i}}^{\mathrm{T}}\Phi_{2,\mathrm{i}}\dot{\eta}(t-h(t)).\end{aligned} \tag{B.7}$$

Like the proof of Theorem 1 (Eqs. (A.6)–(A.13)), using Lemma 1 gives us inequality (B.8), which will make the system exponentially stable.

$$\begin{bmatrix} (1,1) & (1,2) \\ * & (2,2) \end{bmatrix} < 0 \tag{B.8}$$

where

$$(1,1) = \begin{bmatrix} \overline{\Pi}_{11,i} + \Phi_i^T\Phi_i & P_i\overline{D}_i & P_i\overline{A}_{1,i} & \overline{A}_i^T K^T R_i \\ * & -(1-\mu_d)e^{-\alpha d_2}Q_i + \Phi_{1,i}^T\Phi_{1,i} & 0 & \overline{D}_i^T K^T R_i \\ * & * & -(1-\mu_h)e^{-\alpha h_2}R_i + \Phi_{2,i}^T\Phi_{2,i} & \overline{A}_{1,i}^T K^T R_i \\ * & * & * & -R_i \end{bmatrix},$$

$$(1,2) = \begin{bmatrix} \Lambda & \Lambda & \Lambda \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad (2,2) = \begin{bmatrix} -I & 0 & 0 \\ * & -I & 0 \\ * & * & -I \end{bmatrix}.$$

According to the Schur complement lemma, condition (B.8) is equivalent to

$$\begin{bmatrix} \multicolumn{4}{c}{\multirow{4}{*}{$\overline{\Delta}_i$}} & \Phi_i^T & 0 & 0 & \Lambda & \Lambda & \Lambda \\ &&&& 0 & \Phi_{1,i}^T & 0 & 0 & 0 & 0 \\ &&&& 0 & 0 & \Phi_{2,i}^T & 0 & 0 & 0 \\ &&&& 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \tag{B.9}$$

where

$$\overline{\Delta}_i = \begin{bmatrix} \overline{\Pi}_{11i} & P_i\overline{D}_i & P_i\overline{A}_{1,i} & \overline{A}_i^T K^T R_i \\ * & -(1-\mu_d)e^{-\alpha d_2}Q_i & 0 & \overline{D}_i^T K^T R_i \\ * & * & -(1-\mu_h)\,e^{-\alpha h_2}R_i & \overline{A}_{1,i}^T K^T R_i \\ * & * & * & -R_i \end{bmatrix}.$$

Thus, according to (20), the exponential stability of the system is guaranteed. Also, if we assume that the ith subsystem is active when $t \in [t_k, t_{k+1})$, then the following matrix inequalities hold:

$$V(\eta(t), i) \le e^{-\alpha(t-t_k)} V(\eta(t_k), i(t_k)). \tag{B.10}$$

From (15) and (B.1), at the switching moment t_k, we will have

$$V(\eta(t_k), i(t_k)) \le \mu V\left(\eta\left(t_k^-\right), i(t_k^-)\right). \tag{B.11}$$

So from (B.10) and (B.11) for $t \in [t_k, t_{k+1})$, and according to Definition 2, we know $\rho = N_\sigma(t_0, t) \le \frac{t-t_0}{\tau_a}$, $t_0 = 0$. Then,

$$\begin{aligned} V(\eta(t), i) &\le e^{-\alpha(t-t_k)} V(\eta(t_k), i(t_k)) \le \mu e^{-\alpha(t-t_k)} V\left(\eta\left(t_k^-\right), i(t_k^-)\right) \\ &\le \mu e^{-\alpha\left(t-t_{k-1}\right)} V(\eta(t_{k-1}), i(t_{k-1})) \le \mu^2 e^{-\alpha\left(t-t_{k-1}\right)} V\left(\eta\left(t_{k-1}^-\right), i(t_{k-1}^-)\right) \\ &\le \cdots \le \mu^\rho e^{-\alpha(t-t_0)} V(\eta(t_0), i(t_0)) \le e^{-\left(\alpha - \frac{\ln\mu}{\tau_a}\right)t} V(\eta(t_0), i(t_0)). \end{aligned} \tag{B.12}$$

Furthermore, given the definition of the Lyapunov function (B.1) and its monotonousness and (B.12), we will have the following inequalities:

$$a\,\|\eta(t)\|^2 \le V(\eta(t), i) \le e^{-\left(\alpha - \frac{\ln\mu}{\tau_a}\right)t} V(\eta(t_0), i(t_0)) \le b\,\|\eta(t_0)\|_{cl}^2 \tag{B.13}$$

where a and b are defined in (17). Then, we have

$$\|\eta(t)\|^2 \le \frac{1}{a} V(\eta(t), i) \le \frac{b}{a} e^{-\left(\alpha - \frac{\ln\mu}{\tau_a}\right)t} \|\eta(t_0)\|_{cl}^2. \tag{B.14}$$

Hence, by Definition 1, $t_0 = 0$, and the closed-loop system (7) is exponentially stable. The proof is complete at this point.

Appendix C

Proof of Theorem 3. According to Theorem 2 in the preceding section, inequality (20) guarantees the stability of the closed loop system. By introducing a slack matrix M, the conditions for the stabilization of Theorem 2 are put in the following form:

$$\begin{bmatrix} \multicolumn{5}{c}{\multirow{5}{*}{$\acute{\Delta}_i$}} & \Phi_i^T & 0 & 0 & \Lambda & \Lambda & \Lambda \\ &&&&& 0 & 0 & 0 & 0 & 0 & 0 \\ &&&&& 0 & \Phi_{1,i}^T & 0 & 0 & 0 & 0 \\ &&&&& 0 & 0 & \Phi_{2,i}^T & 0 & 0 & 0 \\ &&&&& 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \tag{C.1}$$

where

$$\acute{\Delta}_i = \begin{bmatrix} \acute{\Pi}_{11i} & P_i - M^T + \overline{A}_i^T M & M^T \overline{D}_i & M^T \overline{A}_{1,i} & \overline{A}_i^T K^T \\ * & -M - M^T & M^T \overline{D}_i & M^T \overline{A}_{1,i} & 0 \\ * & * & -(1-\mu_d)e^{-\alpha h_2} Q_i & 0 & \overline{D}_i^T K^T \\ * & * & * & -(1-\mu_h)e^{-\alpha h_2} R_i & \overline{A}_{1,i}^T K^T \\ * & * & * & * & -R_i^{-1} \end{bmatrix} \tag{C.2}$$

and

$$\acute{\Pi}_{11i} = M^T \overline{A}_i + \overline{A}_i^T M + \alpha P_i + K^T Q_i K.$$

The inequality (C.1) can be confirmed by performing a projection transformation:

$$\begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \overline{A}_i & \overline{D}_i & \overline{A}_{1,i} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}, \tag{C.3}$$

and we can obtain condition (20). The DOF control problem will be solved using Eq. (C.1) rather than Eq. (20). Indeed, solving the DOF control problem using Eq. (20) is very complicated because there is a product term between the parameter-dependent Lyapunov matrix and the dynamics of the system matrix in it. If the condition in (C.1) is met, then matrix M is non-singular. Given the matrix M and its inverse matrix, the following partitions are considered:

$$M = \begin{bmatrix} M_1 & M_2 \\ M_4 & M_3 \end{bmatrix}, \qquad G = M^{-1} = \begin{bmatrix} G_1 & G_2 \\ G_4 & G_3 \end{bmatrix}. \tag{C.4}$$

Without losing generality, we assume that M_4 and G_4 are non-singular. The following non-singular matrices will be defined:

$$\mathcal{M}_M = \begin{bmatrix} M_1 & I \\ M_4 & 0 \end{bmatrix}, \qquad \mathcal{M}_G = \begin{bmatrix} I & G_1 \\ 0 & G_4 \end{bmatrix}. \tag{C.5}$$

Given Eqs. (C.4) and (C.5), we obtain

$$G\mathcal{M}_M = \mathcal{M}_G, \qquad M\mathcal{M}_G = \mathcal{M}_M, \qquad M_1 G_1 + M_2 G_4 = I. \tag{C.6}$$

Multiplying diag($\mathcal{M}_G$, $\mathcal{M}_G$, I, I, I, I, I, I, I, I, I), by pre- and post-Eq. (C.1), we will have:

$$\begin{bmatrix} (1,1) & (1,2) \\ * & (2,2) \end{bmatrix} < 0, \tag{C.7}$$

where

$$(1,1)=\begin{bmatrix}\widetilde{\Pi}_{11,i} & \widetilde{\Pi}_{12,i} & \mathcal{M}_G^T M^T \overline{D}_i & \mathcal{M}_G^T M^T \overline{A}_{1,i} & \mathcal{M}_G^T \overline{A}_i^T K^T \\ * & -\mathcal{M}_G^T (M^T+M)\mathcal{M}_G & \mathcal{M}_G^T M^T \overline{D}_i & \mathcal{M}_G^T M^T \overline{A}_{1,i} & 0 \\ * & * & -(1-\mu_d)e^{-\alpha h_2} Q_i & 0 & \overline{D}_i^T K^T \\ * & * & * & -(1-\mu_h)e^{-\alpha h_2} R_i & \overline{A}_{1,i}^T K^T \\ * & * & * & * & -R_i^{-1}\end{bmatrix},$$

$$(1,2)=\begin{bmatrix}\mathcal{M}_G^T \Phi_i^T & 0 & 0 & \mathcal{M}_G^T \Lambda & \mathcal{M}_G^T \Lambda & \mathcal{M}_G^T \Lambda \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \Phi_{1,i}^T & 0 & 0 & 0 & 0 \\ 0 & 0 & \Phi_{2,i}^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$(2,2)=\begin{bmatrix}-I & 0 & 0 & 0 & 0 & 0 \\ * & -I & 0 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -I\end{bmatrix},$$

$$\widetilde{\Pi}_{11,i}=\mathcal{M}_G^T\left(M^T\overline{A}_i+\overline{A}_i^T M+\alpha P_i+K^T Q_i K\right)\mathcal{M}_G,\qquad \widetilde{\Pi}_{12,i}=\mathcal{M}_G^T\left(P_i-M^T+\overline{A}_i^T M\right)\mathcal{M}_G.$$

We define the following matrices:

$$\mathcal{R}_i=R_i^{-1},\qquad \mathcal{M}=M_1,\qquad \mathcal{G}=G_1,\qquad \mathcal{K}=M_1^T G_1+M_4^T G_4,$$

$$\mathcal{P}_i=\mathcal{M}_G^T P_i \mathcal{M}_G=\begin{bmatrix}\mathcal{P}_{1,i} & \mathcal{P}_{2,i} \\ * & \mathcal{P}_{3,i}\end{bmatrix}>0,\qquad P_i=\begin{bmatrix}P_{1,i} & P_{2,i} \\ * & P_{3,i}\end{bmatrix},$$

$$\mathcal{L}_i=\mathcal{M}_G^T K^T Q_i K\mathcal{M}_G=\begin{bmatrix}I \\ \mathcal{G}^T\end{bmatrix} Q_i \begin{bmatrix}I \\ \mathcal{G}^T\end{bmatrix}^T=\begin{bmatrix}\mathcal{L}_{1,i} & \mathcal{L}_{2,i} \\ * & \mathcal{L}_{3,i}\end{bmatrix}>0,$$

$$\begin{aligned}\mathcal{M}_G^T\Lambda &= \begin{bmatrix}I & 0 \\ G_1^T & G_4^T\end{bmatrix}\begin{bmatrix}P_{1,i} & P_{2,i} \\ P_{2,i}^T & P_{3,i}\end{bmatrix}\begin{bmatrix}E_i & 0 \\ 0 & I\end{bmatrix}=\begin{bmatrix}P_{1,i}E_i & P_{2,i} \\ G_1^T P_{1,i}E_i+G_4^T P_{2,i}^T E_i & G_1^T P_{2,i}+G_4^T P_{3,i}\end{bmatrix} \\ &= \begin{bmatrix}\Sigma_{1,12,i} & \Sigma_{1,13,i} \\ \Sigma_{2,12,i} & \Sigma_{2,13,i}\end{bmatrix}=\begin{bmatrix}\mathcal{P}_{1,i}E_i & P_{2,i} \\ \mathcal{P}_{2,i}^T E_i & G_1^T P_{2,i}+G_4^T P_{3,i}\end{bmatrix}=\begin{bmatrix}\mathcal{P}_{1,i}E_i & \hbar_{1,i} \\ \mathcal{P}_{2,i}^T E_i & \hbar_{2,i}\end{bmatrix}.\end{aligned}\tag{C.8}$$

From Eq. (C.8), we can easily achieve $Q_i=\mathcal{L}_{1,i}$., $P_{1,i}=\mathcal{P}_{1,i}$. In addition, we define the matrices below:

$$\begin{aligned}&\mathcal{A}_{c,i}=M_1^T A_i G_1+M_4^T B_{c,i} C_i G_1+M_1^T B_i C_{c,i} G_4+M_4^T A_{c,i} G_4,\\ &\mathcal{B}_{c,i}=M_4^T B_{c,i},\qquad \mathcal{C}_{c,i}=C_{c,i} G_4.\end{aligned}\tag{C.9}$$

Thus, considering Eqs. (8), (C.4)–(C.5), and (C.8)–(C.9), we will have:

$$\begin{aligned}&G_1^T M_1^T+G_4^T M_2^T=I,\qquad G_1^T M_4^T+G_4^T M_3^T=0,\\ &\mathcal{M}_G^T M^T\overline{A}_i\mathcal{M}_G=\begin{bmatrix}\mathcal{M}^T A_i+\mathcal{B}_{c,i}C_i & \mathcal{A}_{ci} \\ A_i & A_i\mathcal{G}+B_i\mathcal{C}_{c,i}\end{bmatrix},\\ &\mathcal{M}_G^T M^T\overline{D}_i=\begin{bmatrix}\mathcal{M}^T D_i+\mathcal{B}_{c,i}C_{1,i} \\ D_i\end{bmatrix},\qquad \mathcal{M}_G^T M^T\overline{A}_{1i}=\begin{bmatrix}\mathcal{M}^T A_{1,i} \\ A_{1i}\end{bmatrix},\\ &\mathcal{M}_G^T M^T\mathcal{M}_G=\begin{bmatrix}\mathcal{M}^T & \mathcal{K} \\ I & \mathcal{G}\end{bmatrix},\qquad K\overline{A}_i\mathcal{M}_G=\begin{bmatrix}A_i & A_i\mathcal{G}+B_i\mathcal{C}_{c,i}\end{bmatrix}.\end{aligned}\tag{C.10}$$

Note that Eq. (C.7) is equivalent to Eq. (25). Thus, Theorem 3 is completely proved.

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