Assignment 2

Algorithm Design and Analysis
October 14, 2022

Notice:

1. You can choose ***three*** from problems 1-6.

2. For problems 1-6, you should do at least the following things:

   (a) Describe your algorithm in natural language **AND** pseudo-code;
   (b) Describe the optimal substructure and DP equation;
   (c) Prove the correctness of your algorithm;
   (d) Analyze the complexity of your algorithm.

1 Money Robbing

A robber is planning to rob houses along a street. Each house has a certain amount of money stashed, the only constraint stopping you from robbing each of them is that adjacent houses have security system connected and it will automatically contact the police if two adjacent houses were broken into on the same night.

1. Given a list of non-negative integers representing the amount of money of each house, determine the maximum amount of money you can rob tonight without alerting the police.

2. What if all houses are arranged in a circle?

2 Ugly Number

An ugly number is a positive integer whose prime factors are limited to 2, 3, and 5.

Given an integer n, return the nth ugly number.

3 Largest Divisible Subset

Given a set of distinct positive integers, find the largest subset such that every pair (S_i, S_j) of elements in this subset satisfies: $S_i \% S_j = 0$ or $S_j \% S_i = 0$. Please return the largest size of the subset.

Note: $S_i \% S_j = 0$ means that S_i is divisible by S_j.

4 Target Sum

You are given an integer array nums and an integer target.

You want to build an expression out of nums by adding one of the symbols '+' and '-' before each integer in nums and then concatenate all the integers.

For example, if nums = [2, 1], you can add a '+' before 2 and a '-' before 1 and concatenate them to build the expression "+2-1".

Return the number of different expressions that you can build, which evaluates to target.

Example:
Input: nums = [1,1,1,1,1], target = 3
Output: 5
Explanation: There are 5 ways to assign symbols to make the sum of nums be target 3.
-1 + 1 + 1 + 1 + 1 = 3
+1 - 1 + 1 + 1 + 1 = 3
+1 + 1 - 1 + 1 + 1 = 3
+1 + 1 + 1 - 1 + 1 = 3
+1 + 1 + 1 + 1 - 1 = 3

5 Best Time to Buy and Sell Stock with Cooldown

You are given an array **prices** where **prices**$[i]$ is the price of a given stock on the ith day.

Find the maximum profit you can achieve. You may complete as many transactions as you like (i.e., buy one and sell one share of the stock multiple times) with the following restrictions:

After you sell your stock, you cannot buy stock on the next day (i.e., cooldown one day).

Note: You may not engage in multiple transactions simultaneously (i.e., you must sell the stock before you buy again).

Example:
Input: prices = [1,2,3,0,2]
Output: 3
Explanation: transactions = [buy, sell, cooldown, buy, sell]

6 Edit distance

Given two strings word1 and word2, return the minimum number of operations required to convert word1 to word2.

You have the following three operations permitted on a word:

Insert a character
Delete a character
Replace a character
Example:
Input: word1 = "horse", word2 = "ros"
Output: 3
Explanation:
horse -> rorse (replace 'h' with 'r')
rorse -> rose (remove 'r')
rose -> ros (remove 'e')