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Optimization of Electric Vehicle Charging Station Location Based on Game Theory

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Abstract—**The industrialization process of electric vehicles depends largely on a reasonable solution of battery charging. There are huge search spaces and multiple objectives for the location optimization of charging station, and many factors are difficult to be quantified. In this paper, on the basis of analyzing the traditional site selection evaluation method, game theory is proposed to optimize the electric vehicle charging station location. The optimization game model is built and its algorithm steps are given. The results indicate that this optimization method based on game theory can make charging station location more rational and scientific.**

Keywords-electric vehicle; charging station location; optimization; game theory

I. INTRODUCTION

In the background of low-carbon economy, electric vehicle has become the future direction of the automobile. With strong government support, China has become fertile soil for the development of new energy vehicles. Since 2009, 13 cities of China have started lead the way to popularize the energy-saving and new energy vehicle, and the advancement of the electric vehicle technology is very great in china [1].

However, the industrialization process of electric vehicles depends largely on a reasonable solution of battery charging [2][3][4]. In addition to the electric vehicle charging technology, the construction of related facilities must be considered in advance. Development of electric vehicles and construction of charging station form the so-called bilateral network of market in economics. The electric vehicle's popularization depends on whether charging station has large scale, while the large-scale construction of charging station depends on enough electric vehicle consumers on market. Two factors are essential before the whole market is mature. Automobile companies can not start push the electric vehicle to market until the charging station has been built. The two parts should go ahead in hand.

For the charging stations' construction, location problem should be solved firstly [5][6][7]. Only appropriate station location can bring user convenience and attract more users to buy electric vehicles, at the same time improve benefit of the charging station. It is also important for the investment in infrastructure, charging station's quality, safety and economy. In 2010, the government of China proposed promoting the charging station construction at 27 provinces or cities. So it's urgently needed to research the charging station location problem of electric vehicle.

II. CHARGING STATION LOCATION PROBLEM DESCRIPTION

How to choose the station location should consider many factors, such as government planning, views of surrounding residents, distribution of electric vehicles around, land use situation, traffic conditions, geographical conditions, weather conditions, fire-proof and explosion-proof conditions, station harmonic pollution problem, electricity network situation, station load and charging pattern, total investment and management costs, and so on. There are huge search spaces and multiple objectives for the location optimization of charging station, and many factors are difficult to be quantified. Traditional modeling methods can not effectively solve the charging station location optimization problem.

Game theory is the mathematical analysis of any situation involving a conflict of interest, with the intent of indicating the optimal choices that, under given conditions will lead to a desired outcome. So game theory is very available to resolve the charging station location problem. Based on Nash equilibrium and combining mixed strategy in game theory, this paper builds charging station layout optimization model and carry out instance analysis.

III. CHARGING STATION LOCATION OPTIMIZATION MODELING BY GAME THEORY

A. The players

There are too many factors influencing the station location. In order to get the optimal location project, the main factors gotten by hierarchy analysis and evaluation method are selected as the players set of the station location game.

B. The strategy

Decision-makers select the final project from all options. So those location projects should be the game's strategy set *S*. In the strategy set *S*, the effective strategy *Si* of every player i is set *A*, and all effective strategies make up the set *S*. Suppose there are *n* main factors of station location project and *m* location available projects, then $A=\{a_1,a_2,\cdots,a_m\}$, $S_i=A(i=1,2,\cdots m)$, $S=\{a|(a_{ij})\}_{n\times m}$.

C. The payoff functions

It is by analyzing effect strength on different projects caused by various factors to determine the final project. So the player payoff function *u* should use the effect strength *C* of the factors on every project. Payoff function *u* accords with player's rational judgments: player *i* corresponding to different location options *a*, *b*, if the magnitude of value $C_a \geq C_b$, then $u(C_a) \geq u(C_b)$. Different factors and payoff functions of location project make up the payoff function set $U = \{u|u_i(c)i \in N\}_{n\times m}$.

D. Game modeling

The optimization of charging station location is a non-cooperative static completely information game model. Strategy set *S* made up of m projects, *n* factors and payoff function set *U* compose the game model: $G = \{N, S, U\}$, where *N* is the main factor set, $S = \{a|(a_i)i \in N\}_{n\times m}$ is the strategy set, $U = \{u|u_i(c)i \in N\}_{n\times m}$ is the payoff function set.

IV. MODEL SOLVING

Based on Nash equilibrium [8]: In a normal game with a finite number of players, if each player can choose from finite strategies, there is a Nash equilibrium at least. In the game of station location optimization, the finite main factors compose the finite player set *N* and the finite available location projects compose the finite game strategy set *S*. Hence, this game taking the main factors as players has one Nash equilibrium solution.

A. method of solution

Though at least one Nash equilibrium exists $S' = (a'_{ij})_n$ (a'_{ij} denotes the project a'_j corresponding to the factor i), only when various factors of Nash equilibrium select the same project, the optimal option can be determined. As each project carries out initial demonstration, different strategies with the same factor don't have obvious disparity, leading that the algorithms can't run to exclude strict bad strategies. In this case we adopt mixed strategy algorithm [8][9] to transform the original problem into a linear programming problem:

$$X = \min \sum_{i=1}^{m} x_i$$

$$s.t. \begin{cases} 1 - \sum_{i=1}^{m} a_{ij} x_i \leq 0; j = 1,2,\cdots n \\ x_i \geq 0 \end{cases}$$

Where x_i is any mixed strategy based on S_i.

Using the primal-dual path following method to solve the linear programming above, then

$$\min c^T x$$

$$s.t. \begin{cases} Ax - l = 0 \\ x - v = 0 \\ x, l, v \geq 0 \\ (c, x, v) \in R^m, l \in R^n, A \in R^{n\times m} \end{cases}$$

Where *c* and *v* are $1\times m$ matrixes, *l* is a $n\times 1$ matrix, *A* is a $n\times m$ matrix.

By introducing slack variable vectors *l* and *v*, inequality constraints are transformed into equality constraints. Then the equality constraints are dealt with by Lagrange and slack factors are dealt with by uniform perturbed factor μ to get the Lagrange function:

$$L(x, l, y, z) = C^T x - y^T (Ax - l) - z^T (x - v)$$
$$- \mu \sum_{j=1}^{n} Inl_j - \mu \sum_{j=1}^{n} Inv_j$$

When the first-order partial derivative of the Lagrange function to each variable vectors is zero, we can derive the following KKT equations [10]:

$$\begin{cases} L_x = c - A^T y - z = 0 \\ L_y = Ax - l = 0 \\ L_z = x - v = 0 \\ L_l = diag(l) diag(y) e - \mu e = 0 \\ L_v = diag(v) diag(z) e - \mu e = 0 \end{cases}$$

Solving corresponding correction equations by Newton's method:

$$\begin{bmatrix} 0 & -A^T & -I & 0 & 0 \\ A & 0 & 0 & -I & 0 \\ I & 0 & 0 & 0 & -I \\ 0 & L & 0 & Y & 0 \\ 0 & 0 & V & 0 & Z \end{bmatrix} \times \begin{bmatrix} \Delta v \\ \Delta y \\ \Delta z \\ \Delta l \\ \Delta v \end{bmatrix} = - \begin{bmatrix} L_x \\ L_y \\ L_z \\ L_l \\ L_v \end{bmatrix}$$

Where *I* is an identity matrix, *L* = diag (*l*) , *Y* =diag (*y*) , *V* = diag (*v*) , *Z* = diag (*z*).

B. Algorithm steps

- Step 1: Initialization; Choose x(0), l(0), v(0)>0, y(0), Z(0)>0. Set centering parameter $\sigma \in (0,1)$, tolerance $\varepsilon = 10^{-6}$, k=0, *k*max=50.
- Step 2: Compute complementary gap:

  $$G_{ap}^{(k)} = l^{(k)} y^{(k)} + v^{(k)} z^{(k)}$$

  If $G_{ap}^{(k)} < \varepsilon$, then output optimal solution and stop, otherwise continue.
- Step 3: Compute the perturbed factor $u^{(k)} = \dfrac{\sigma \times G_{ap}^{(k)}}{m + n}$ and solve the correction equations for Δx, Δy, Δl, Δv and Δz.
- Step 4: Determine the maximum step length in the primal and dual space,

  $$stepP = \min\left\{ \min(\frac{-l_i}{\Delta l_i} : \Delta l_i < 0, i = 1,\cdots,n), \min(\frac{-v_i}{\Delta v_i} : \Delta v_i < 0, i = 1,\cdots,m), 1 \right\}$$

$$stepD = \min\left\{ \min(\frac{-y_i}{\Delta y_i} : \Delta y_i < 0, i = 1,\cdots,n) \right.$$
$$\left. ,\min(\frac{-z_i}{\Delta z_i} : \Delta z_i < 0, i = 1,\cdots,m), 1 \right\}$$
$$step = 0.995 \min\{stepP, stepD\}.$$

- Step 5: Update the primal and dual variables by

$$\begin{bmatrix} x \\ y \\ z \\ l \\ v \end{bmatrix}^{(k+1)} = \begin{bmatrix} x \\ y \\ z \\ l \\ v \end{bmatrix}^{(k)} + step \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta l \\ \Delta v \end{bmatrix}$$

- Step 6: Counter increases by one and return step 2 into the next cycle.

From a large number of calculations we know that the selection of primal variable has great influence on convergence of the algorithm. Inequality constraints transformed into equality constraints can indeed improve the convergence of the algorithm when slack variables l and v are by other variables in the initial conditions.

V. EXAMPLE

In a city three places are going to build electric vehicle charging stations. Firstly the main factors influencing the station layout should be determined, and then the payoff function set U of every strategy through relevant data should be derived. The main factors are government planning N1, views of surrounding residents N2, distribution of electric vehicles around N3, land use situation N4, traffic conditions N5, geographical conditions N6, weather conditions N7, fire-proof and explosion-proof conditions N8, station harmonic pollution problem N9, electricity grid situation N10, station load and charging pattern N11, total investment costs N12, running cost per year N13, management and operation mode N14.

These 14 main factors above are used for game model. Strategy set are three charging stations location to be selected. The effect strength C on each project by the main factors composes the payoff function set *U* (Take the whole system's payoff function as 100. For 1, 2, 3, 8, every factor's relevant payoff function crest value is five, and the remaining 10 factors, crest value corresponding to each factor is eight.) Strategy description of the final model is shown as table I.

TABLE I. STRATEGY DESCRIPTION OF GAME MODEL ABOUT CHARGING STATION LOCATION LAYOUT OPTIMIZATION

Factor	Project 1	Project 2	Project 3
N1	1	1	3
N2	1	1	3
N3	2	2	1
N4	3	2	3
N5	1	3	4
N6	3	2	3
N7	1	3	4
N8	2	1	2
N9	3	1	4
N10	2	4	2
N11	2	4	1
N12	3	2	3
N13	5	2	1
N14	3	1	4

Convert the model to linear programming problem:

$$\min(x_1 + x_2 + x_3)$$

$$s.t.\begin{cases} x_1 + x_2 + 3x_3 \ge 1 \\ x_1 + x_2 + 3x_3 \ge 1 \\ 2x_1 + 2x_2 + x_3 \ge 1 \\ 3x_1 + 2x_2 + 3x_3 \ge 1 \\ x_1 + 3x_2 + 4x_3 \ge 1 \\ 3x_1 + 2x_2 + 3x_3 \ge 1 \\ 1x_1 + 3x_2 + 4x_3 \ge 1 \\ 2x_1 + x_2 + 2x_3 \ge 1 \\ 3x_1 + x_2 + 4x_3 \ge 1 \\ 2x_1 + 4x_2 + 2x_3 \ge 1 \\ 1x_1 + 3x_2 + 4x_3 \ge 1 \\ 3x_1 + 2x_2 + 3x_3 \ge 1 \\ 5x_1 + x_2 + 2x_3 \ge 1 \\ 3x_1 + x_2 + 4x_3 \ge 1 \\ x_1, x_2, x_3 \ge 0 \end{cases}$$

Using primal-dual path following algorithm, we can derive x1=0.2701, x2=0.1299, x3=0.2000. Project 1 is the optimal one for its higher probability. But seen from the result, the advantage of project 1 is not obvious comparing with project 3, which may spark controversy.

VI. CONCLUSION

The candidate projects of electric vehicle charging station location layout are compared by building game model. By level analysis and evaluation the main influencing factors are determined as the player of the game. This method avoids local division and achieves collaborative optimization in the system. The game model is transformed into linear programming model and solved by primal-dual path following algorithm. This way makes the calculation process simple and clear with strong feasibility and practicality.

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