$Y = (y_1, y_2, \cdots, y_n)^T$ the elements of Y are independent

① Given Y from generating model M_0

$$d(M, M_0) = E_0\Big[\big(\underset{\substack{\uparrow \\ M_0}}{\overset{RV}{y^*}} - x^{*T}\underset{\substack{\uparrow \\ M}}{\overset{OLS}{\hat{\beta}_M}}\big)^2\Big],$$ Given Y providing a new data point x^*

$\Downarrow$

$d(\cdot, M_0)$

where E_0 denotes the expectation under the generating model M_0.

Model M: Linear Model $Y = X_M^T \beta_M + \varepsilon$ $\varepsilon \sim N(0, \sigma^2 I_n)$

② Unconditional on Y, the overall expectation is

$$D(M, M_0) = E_0\{E_0[(y^* - x^{*T}\hat{\beta})^2 \mid \hat{\beta} = \hat{\beta}_M]\}$$

$D(\cdot, M_0)$

$$= E_0\{E_0[(y^* - x^{*T}\hat{\beta})^2 \mid Y = Y]\}$$

SLLN:

$$\frac{1}{N}\sum_{j=1}^{N}(y_j^* - x_j^*\hat{\beta}_M)^2 \xrightarrow{a.s} d(M, M_0) \quad \text{Given } Y.$$

$$\frac{1}{M}\sum_{k=1}^{M}\left(\frac{1}{N}\sum_{j=1}^{N}(y_j^* - x_j^*\hat{\beta}_M^k)\right) \xrightarrow[?]{a.s} D(M, M_0)$$

$\Downarrow$

For each k, generate the corresponding Y_k for Model M_0 with Y_k in hand, $\hat{\beta}_M$ is also determined, name it $\hat{\beta}_M^k$.