

The Second Report

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Shao's Method

(1)The Balanced Incomplete CV(n_v) Method

Notations:

Sample size in model fitting: n_c

Sample size in prediction: n_v

So $n_c + n_v = n$

Category I: At least one nonzero component of β is not in β_α

Category II: β_α contains all nonzero components of β

Dataset splits:

We don't need to enumerate all of the subsets that contain n_v samples. We only need part of them. Let B denote the collection of all b subsets that have size n_v . B satisfies:

- (a) every i , $1 \leq i \leq n$, appears in the same number of subsets in B ;
- (b) every pair (i, j) , $1 \leq i < j \leq n$, appears in the same number of subsets in B .

BICV(n_v):

The BICV(n_v) selects a model by minimizing

$$\hat{\Gamma}_{\alpha, n}^{BICV} = \frac{1}{n_v b} \sum_{s \in B} \|y_s - \hat{y}_{\alpha, s^c}\|^2$$

over all $\alpha \in A$. Each α here represents a kind of model.

Important results:

- (1) Under appropriate conditions, BICV(n_v) is asymptotically correct if $n_c \rightarrow \infty$ and $n_v/n \rightarrow 1$.
- (2) Why BICV(n_v) can beat CV(1): large n_c will lead to a flat cross validation error over all the models in Category II, where

$$\Gamma_{\alpha, n_c} = \sigma^2 + n_c^{-1} d_\alpha \sigma^2$$

. So it is difficult to find a smallest Γ_{α, n_c} . While with relatively small n_c in BICV(n_v), this problem could be solved. (Why, here n_c is still large, only is relatively small compared with n_v .)

Besides, can $n_v/n \rightarrow 1$ be relaxed under most cases? (Not all cases, cause he provides a counterexample.) Even though we should improve $n_v = 1$, but is $n_v/n \rightarrow 1$ is too strong compared with $n_v = 1$?

(2)Other CV(n_v) Methods:

Monte Carlo CV(n_v) Randomly draw

Analytic Approximate CV(n_v)