Unit 3 Homework

W203 Statistics for Data Science

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Section Number: 05

1. Gas Station Analytics

(a) What is the probability that the next customer will request regular gas and fill the tank?

Set Event X to be the event that the next customer will request regular gas and fill the tank

$$\begin{aligned} P(X) &= P(R \cap F) \\ &= P(F|R)P(R) \\ &= 0.3 * 0.4 \\ &= 0.12 \end{aligned}$$

(b) What is the probability that the next customer will fill the tank?

Event F being the event that the next customer will fill the tank

$$P(F) = P\Big[(R \cap F) \cup (M \cap F) \cup (P \cap F)\Big]$$

Since Events $R \cap F, M \cap F, and P \cap F$ are exclusive, the above equation can be written as

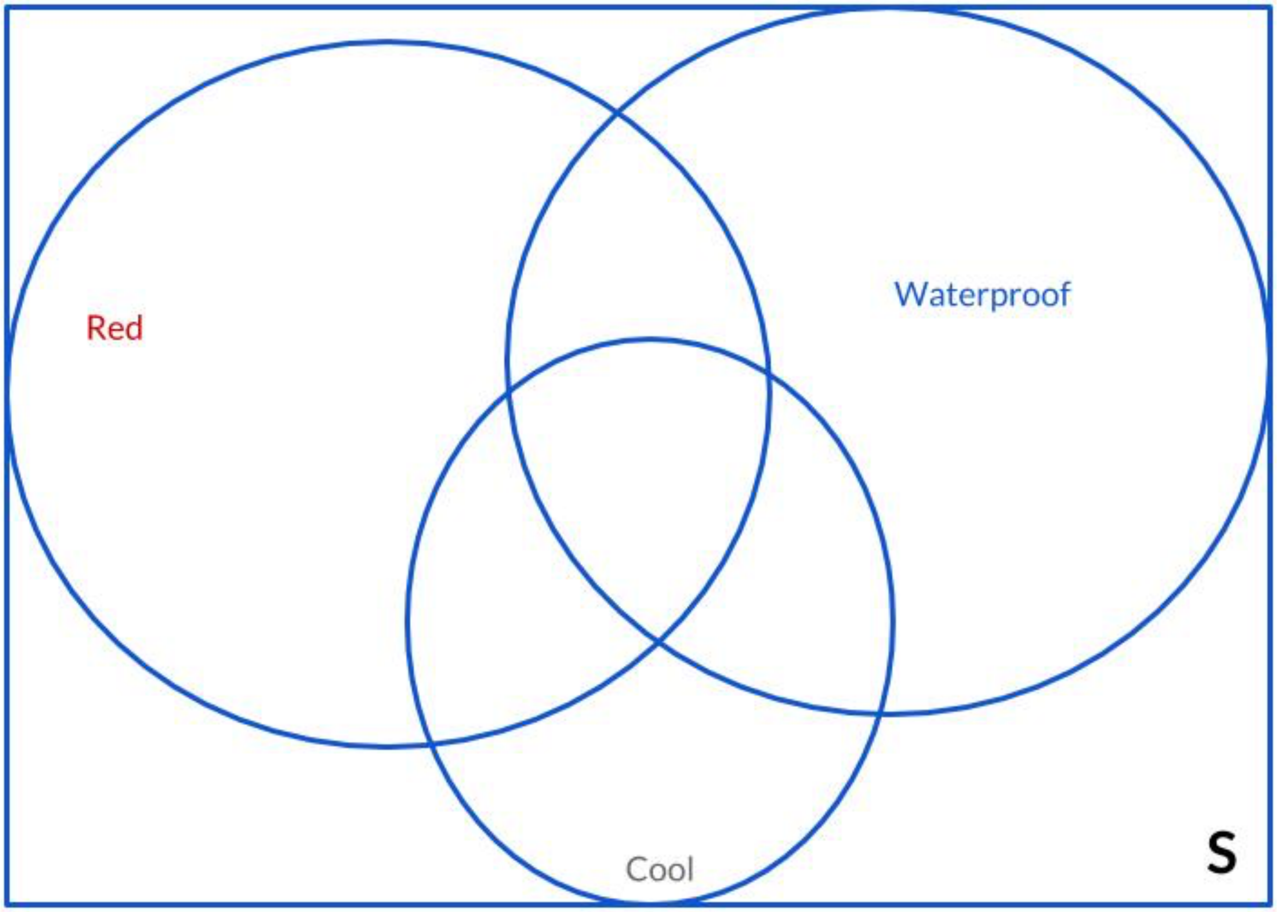
$$\begin{aligned} P(F) &= P(R \cap F) + P(M \cap F) + P(P \cap F) \\ &= P(F|R)P(R) + P(F|M)P(M) + P(F|P)P(P) \\ &= 0.3 * 0.4 + 0.6 * 0.35 + 0.5 * 0.25 \\ &= 0.455 \end{aligned}$$

(c) Given that the next customer fills the tank, what is the conditional probability that they use regular gas?

$$
\begin{aligned}
P(R|F) &= \frac{P(R \cap F)}{P(F)} \\
&= \frac{0.12}{0.455} \\
&= 0.2637
\end{aligned}
$$

2. The Toy Bin

(a) Draw an area diagram to represent these events.



(b) What is the probability of getting a red, waterproof, cool toy?

> Set Event R as getting a red toy, Event W as getting a waterproof toy, Event C as getting a cool toy, and Event N as getting a neither red, waterproof, nor cool toy.

$$\begin{aligned} P(R \cup W \cup C) &= P(R) + P(W) + P(C) - P(R \cap W) - P(R \cap C) - P(W \cap C) + P(R \cap W \cap C) \\ &= 1 - P(N) \end{aligned}$$

Therefore,

$$\begin{aligned} P(R \cap W \cap C) &= 1 - P(N) - [P(R) + P(W) + P(C) - P(R \cap W) - P(R \cap C) - P(W \cap C)] \\ &= 1 - 1/6 - [1/2 + 1/2 + 1/3 - 1/4 - 1/6 - 1/6] \\ &= \frac{1}{12} \end{aligned}$$

(c) You pull out a toy at random and you observe only the color, noting that it is red. Conditional on just this information, what is the probability that the toy is not cool?

$$\begin{aligned} P(C'|R) &= \frac{P(C' \cap R)}{P(R)} \\ &= \frac{P(R) - P(C \cap R)}{P(R)} \\ &= \frac{1/2 - 1/6}{1/2} \\ &= \frac{2}{3} \end{aligned}$$

(d) Given that a randomly selected toy is red or waterproof, what is the probability that it is cool?

$$\begin{aligned}
P(C|(R \cup W)) &= \frac{P(C \cap (R \cup W))}{P(R \cup W)} \\
&= \frac{P(C \cap R) + P(C \cap W) - P(R \cap W \cap C)}{P(R) + P(W) - P(R \cap W)} \\
&= \frac{1/6 + 1/6 - 1/12}{1/2 + 1/2 - 1/4} \\
&= \frac{1}{3}
\end{aligned}$$

3. On the Overlap of Two Events

(a) What are the maximum and minimum possible values for P (A ∩ B)?

The maximum value is reached when A is located entirely within B in the area diagram. In this case,

$$P(A \cap B)_{max} = P(A) = \frac{1}{2}$$

Because

$$P(A) + P(B) = 1/2 + 2/3 > 1$$

It is not possible that A and B do not overlap with each other. Therefore, the minimum value is reached when A and B have the least amount of overlapped area, which means B' is located entirely within A.

$$\begin{aligned}
P(A \cap B)_{min} &= P(A) - P(B') \\
&= 1/2 - (1 - 2/3) \\
&= \frac{1}{6}
\end{aligned}$$

(b) What are the maximum and minimum possible values for P(A|B)?

Because $P(A|B) = \frac{P(A \cap B)}{P(B)}$

Therefore,

$$\begin{aligned} P(A|B)_{max} &= \frac{P(A \cap B)_{max}}{P(B)} \\ &= \frac{1/2}{2/3} \\ &= 0.75 \\ P(A|B)_{min} &= \frac{P(A \cap B)_{min}}{P(B)} \\ &= \frac{1/6}{2/3} \\ &= 0.25 \end{aligned}$$

4. Can’t Please Everyone!

Set Event L as an event that a Berkeley student likes statistics; Event W as an event that a Berkeley student completes w203. From the question we know that $P(L|W) = \frac{3}{4}$, $P(L|W') = \frac{1}{4}$, and $P(W) = 0.01$

$$\begin{aligned} P(L) &= P(L|W) * P(W) + P(L|W') * P(W') \\ &= P(W) * (3/4) + P(W') * (1/4) \end{aligned}$$

$$\begin{aligned} P(W|L) &= \frac{P(W \cap L)}{P(L)} \\ &= \frac{P(W) * (3/4)}{P(W) * (3/4) + P(W') * (1/4)} \\ &= \frac{0.01 * (3/4)}{0.01 * (3/4) + 0.99 * (1/4)} \\ &= 0.029 \end{aligned}$$