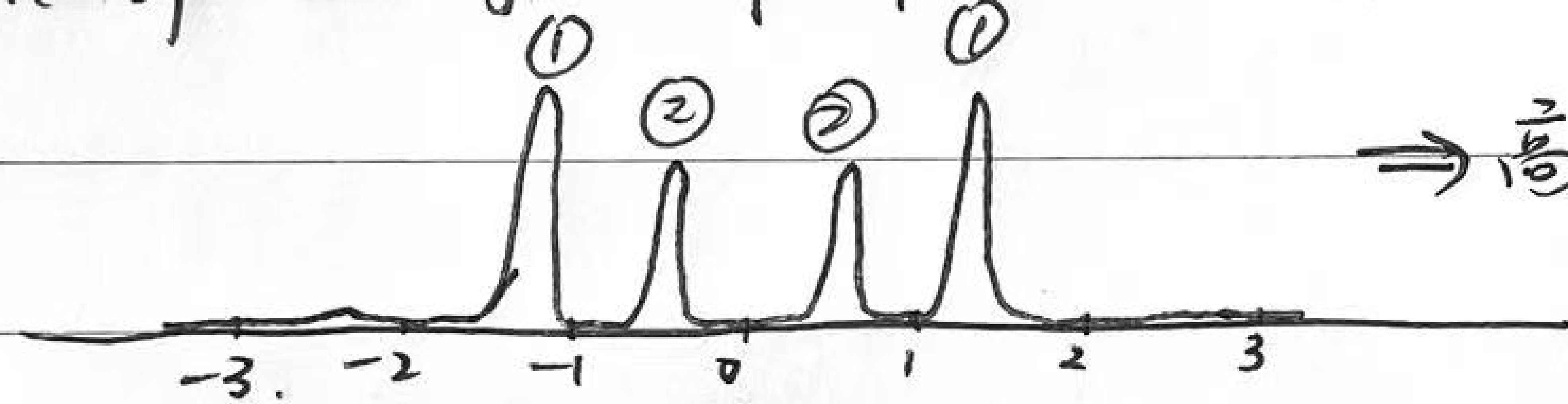
Q1. $Y(e^{j\omega}) = X(e^{j\omega}) \cdot H(e^{j\omega}) \Rightarrow |Y(e^{j\omega})| = |X(e^{j\omega})||H(e^{j\omega})|$

$|Y(e^{j\omega})|$: DFT magnitude of output

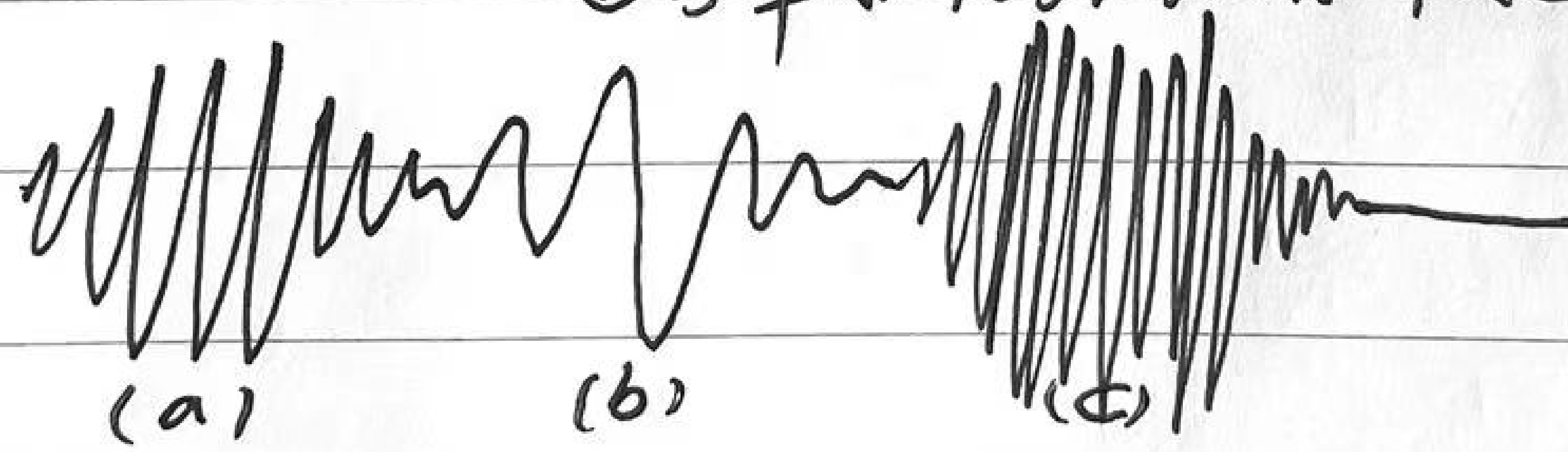


$\Rightarrow$ 高频分量被

根据Group delay图像可以得到，②号峰对应信号有约140的群延迟。

①号峰对应信号几乎没有群延迟。

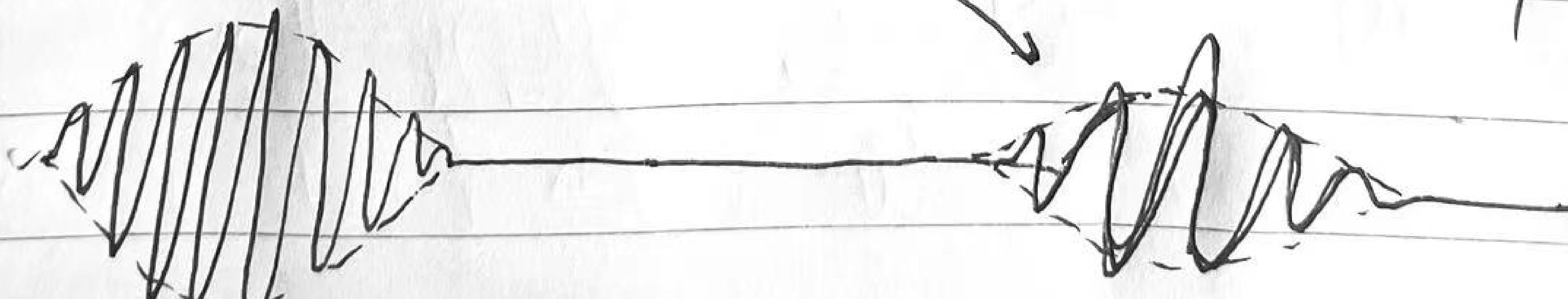
原信号中：



根据频率可知，①对应(a)，②对应(b)。

故y[n]为

延时150左右，幅值低于左边的峰



Q2. (a) $H(z) = \dfrac{1-z^{-1}}{1-\frac{5}{4}z^{-1}-\frac{3}{2}z^{-2}} = \dfrac{Y(z)}{X(z)}$

$\Rightarrow (1-z^{-1})X(z) = (1-\frac{5}{4}z^{-1}-\frac{3}{2}z^{-2})Y(z)$

$\overset{Z^{-1}}{\Longrightarrow} x[n]-x[n-1] = y[n]-\frac{5}{4}y[n-1]-\frac{3}{2}y[n-2]$

(b) When $\omega=\frac{\pi}{2}$, $x[n]-x[n-1] = \cos(\frac{\pi}{2}n)-\cos(\frac{\pi}{2}(n-1)) = \cos(\frac{\pi}{2}n)-\sin(\frac{\pi}{2}n) = \begin{cases} 1, & n\equiv 0,3 \pmod 4 \\ -1, & n\equiv 1,2 \pmod 4 \end{cases}$

$y[n]-\frac{5}{4}y[n-1]-\frac{3}{2}y[n-2] = A\left(\cos(\frac{\pi}{2}n+\phi)-\frac{5}{4}\cos(\frac{\pi}{2}(n-1)+\phi)-\frac{3}{2}\cos(\frac{\pi}{2}(n-2)+\phi)\right)$

$= A\left(\frac{5}{2}\cos(\frac{\pi}{2}n+\phi)-\frac{5}{4}\sin(\frac{\pi}{2}n+\phi)\right) = \frac{5A}{4}\left(2\cos(\frac{\pi}{2}n+\phi)-\sin(\frac{\pi}{2}n+\phi)\right)$

when $n=0$, $1 = \frac{5A}{4}(2\cos\phi-\sin\phi)$ ······ ①

$n=1$, $-1 = \frac{5A}{4}(-2\sin\phi-\cos\phi)$ ······ ②

$n=2$, $-1 = \frac{5A}{4}(-2\cos\phi+\sin\phi)$ ······ ③

$n=3$, $1 = \frac{5A}{4}(2\sin\phi+\cos\phi)$ ······ ④.

①③, ②④ $\Rightarrow \begin{cases} 2\cos\phi-\sin\phi = \frac{4}{5A} \text{ ······ ⑤} \\ 2\sin\phi+\cos\phi = \frac{4}{5A} \text{ ······ ⑥} \end{cases}$.

⑤^2+⑥^{2}得 $5 = \frac{32}{25A^2} \Rightarrow A^2 = \frac{32}{125} \Rightarrow A = \frac{4\sqrt{10}}{25}$

联立⑤⑥得 $\tan\phi = \frac{1}{3} \Rightarrow \phi = \arctan\frac{1}{3}$

Q4. (a) $\arg(H_a(e^{j\omega})) = \arctan \frac{b\sin(\omega)}{a+b\cos\omega}$

$$grd(H_a(e^{j\omega})) = -\frac{1}{\left(\frac{b\sin\omega}{a+b\cos\omega}\right)^2+1} \frac{-b\cos\omega(a+b\cos\omega) - b^2\sin^2\omega}{(a+b\cos\omega)^2} = \frac{b^2+ab\cos\omega}{(b\sin\omega)^2+(a+b\cos\omega)^2} = \frac{b^2+ab\cos\omega}{b^2+2ab\cos\omega+a^2}$$

(b) $\arg(H_b(e^{j\omega})) = \arctan \frac{c\sin\omega}{1+c\cos\omega}$

$$grd(H_b(e^{j\omega})) = -\frac{1}{\left(\frac{c\sin\omega}{1+c\cos\omega}\right)^2+1} \frac{c\cos\omega(1+c\cos\omega)+c^2\sin^2\omega}{(1+c\cos\omega)^2} = -\frac{c^2+c\cos\omega}{c^2+2c\cos\omega+1}$$

(c) $\arg(H_c(e^{j\omega})) = \arg(H_a(e^{j\omega})) + \arg(H_b(e^{j\omega}))$

$$grd(H_c(e^{j\omega})) = grd(H_a(e^{j\omega})) + grd(H_b(e^{j\omega})) = \frac{b^2+ab\cos\omega}{b^2+2ab\cos\omega+a^2} - \frac{c^2+c\cos\omega}{c^2+2c\cos\omega+1}$$

(d) $H_b'(e^{j\omega}) \triangleq \frac{1}{1+de^{-j\omega}}$, $grd(H_b'(e^{j\omega})) = -\frac{d^2+d\cos\omega}{d^2+2d\cos\omega+1}$

$$\arg(H_d(e^{j\omega})) = \arg(H_b(e^{j\omega})) + \arg(H_b'(e^{j\omega}))$$

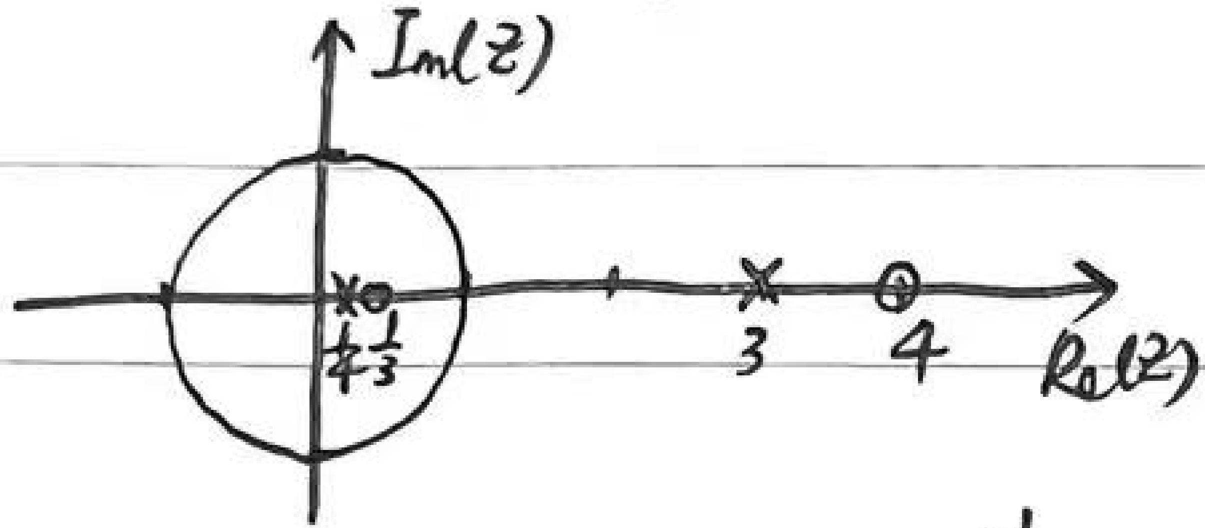
$$grd(H_d(e^{j\omega})) = grd(H_b(e^{j\omega})) + grd(H_b'(e^{j\omega})) = -\frac{c^2+c\cos\omega}{c^2+2c\cos\omega+1} - \frac{d^2+d\cos\omega}{d^2+2d\cos\omega+1}$$

Q4. 由图得. $H(z)=K\dfrac{1-4z^{-1}}{(1-\frac{1}{2}z^{-1})(1-3z^{-1})}$

将在单位圆外的零极点配入 $H_{ap}(z)$ 中得

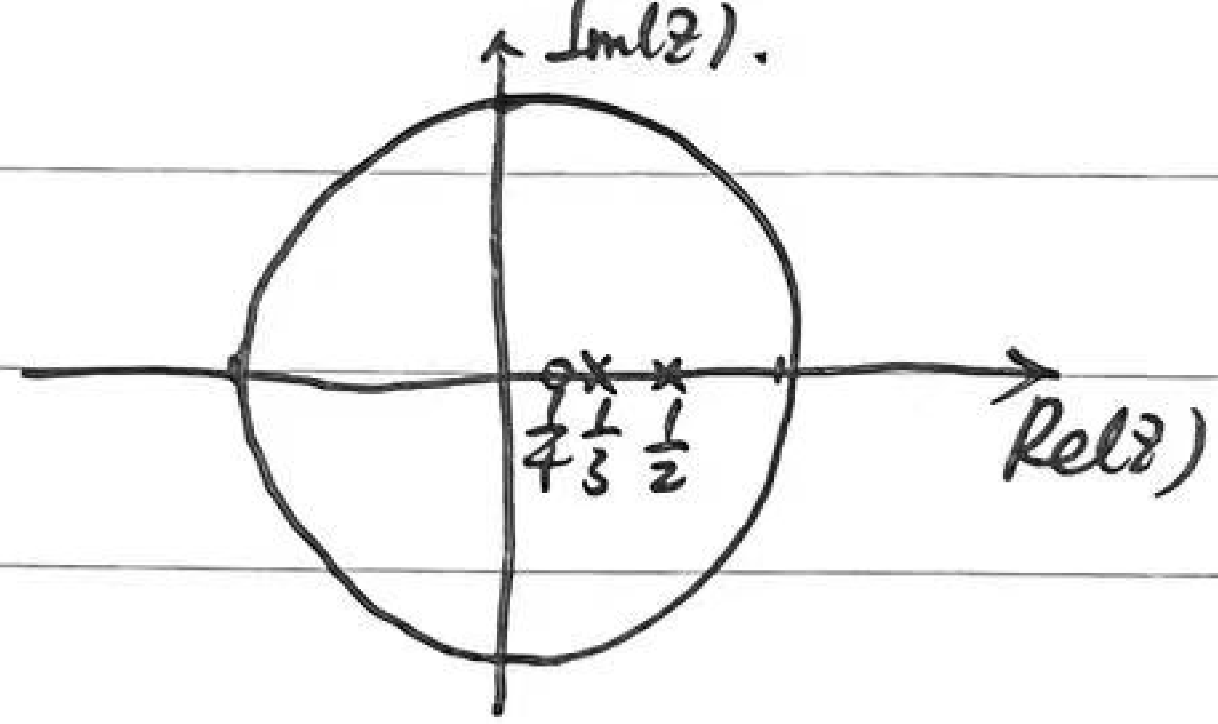
$$H_{ap}(z)=\frac{z^{-1}-3}{1-3z^{-1}}\,\frac{z^{-1}-\frac{1}{4}}{1-\frac{1}{4}z^{-1}}$$

$H_{ap}(z)$ 的零极点图如下:



$$H_{min}(z)=\frac{H(z)}{H_{ap}(z)}=K\frac{1-4z^{-1}}{(1-\frac{1}{2}z^{-1})\cancel{(1-3z^{-1})}}\,\frac{\cancel{1-3z^{-1}}}{z^{-1}-3}\,\frac{1-\frac{1}{4}z^{-1}}{z^{-1}-\frac{1}{4}}=-4K\frac{1-\frac{1}{4}z^{-1}}{(1-\frac{1}{2}z^{-1})(z^{-1}-3)}=\frac{4}{3}K\frac{1-\frac{1}{4}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{3}z^{-1})}$$

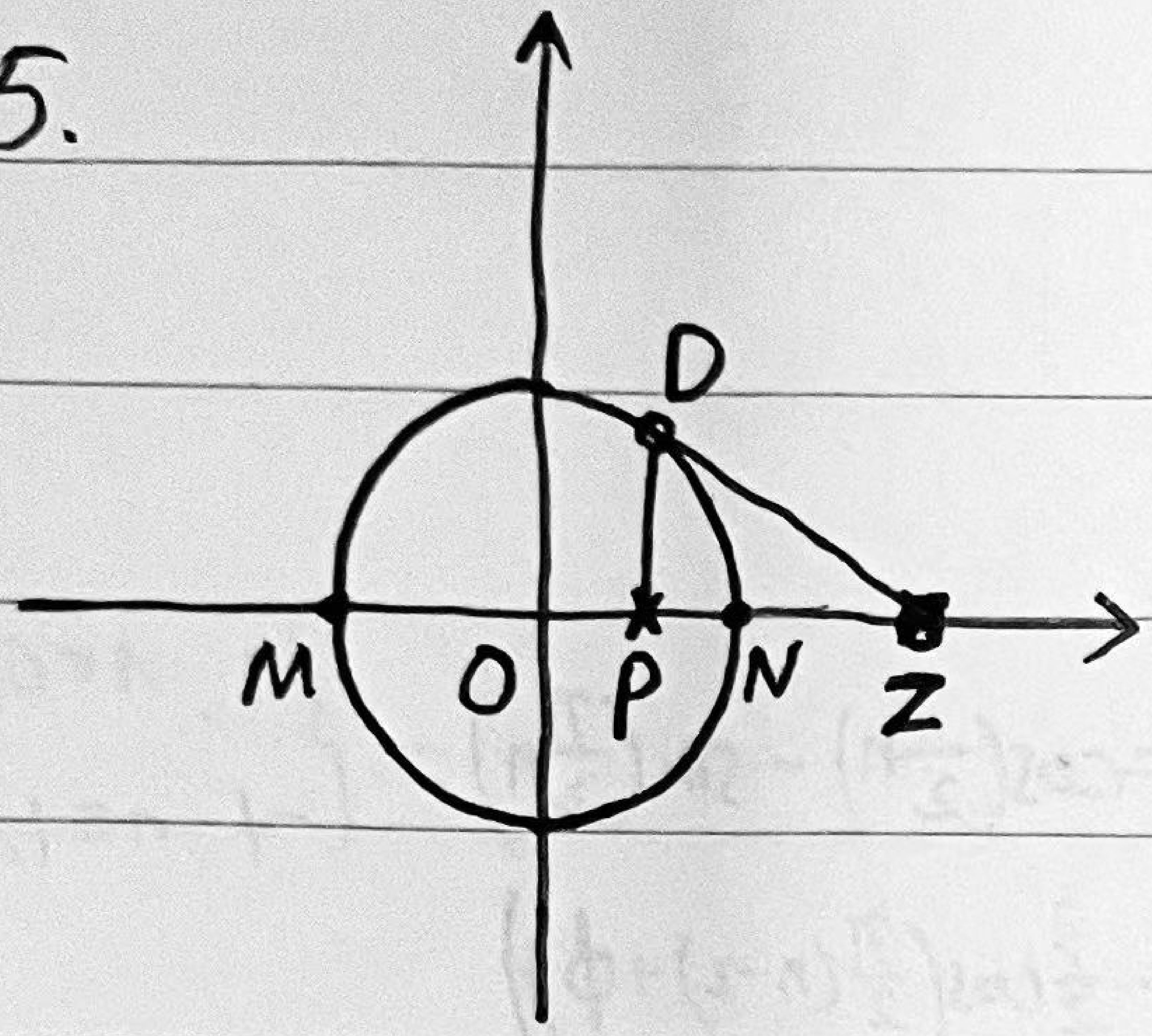
$H_{min}(z)$ 的零极点图如下:



如果给定了scale factor, 这个分解应该是唯一的, 因为无法再引入单个的非原点的零极点。

(如果引入一对零极点, 那么如果出现在Hap和Hmin中 respectively, 就还需要引入单位圆外的一对零极点, 而这一对点无法出现在Hmin中).

Q5.



对于M，N两点，$\frac{|ZN|}{|PN|}=\frac{|ZM|}{|PM|}=\alpha$.

不妨 $|OP|=x$，$|OZ|=y$.

则有 $\frac{y-1}{1-x}=\frac{y+1}{1+x}=\alpha$.

$\Rightarrow (y-1)(1+x)=(y+1)(1-x)$

$\Rightarrow xy-x+y-1=-xy-x+y+1$

$\Rightarrow 2xy=2 \Rightarrow xy=1$. $\square$.