

Q1. (a) $\tilde{X}(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}nk}$

$$\tilde{X}_3(k) = \sum_{n=0}^{3N-1} x(n) e^{-j\frac{2\pi}{3N}nk}$$

$$= \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{3N}nk} (1 + e^{-j\frac{2\pi}{3}k} + e^{-j\frac{4\pi}{3}k})$$

$$= \begin{cases} 0, & 3 \nmid k \\ 3\tilde{X}(\frac{k}{3}), & 3|k \end{cases}$$

(b) $\tilde{X}(k) = \sum_{n=0}^1 x(n) e^{-j\pi nk} = 1 + 2e^{-j\pi k} = \begin{cases} -1, & 2 \nmid k \\ 3, & 2|k \end{cases}$

$$\tilde{X}_3(k) = \sum_{n=0}^5 x(n) e^{-j\frac{\pi}{3}nk} = 1 + e^{-j\frac{2\pi}{3}k} + e^{-j\frac{4\pi}{3}k} + 2(e^{-j\frac{\pi}{3}k} + e^{-j\pi k} + e^{-j\frac{5\pi}{3}k})$$

$$= \begin{cases} 9, & k \equiv 0 \pmod{6} \\ -3, & k \equiv 3 \pmod{6} \\ 0, & \text{otherwise} \end{cases} = \begin{cases} 0, & 3 \nmid k \\ 3\tilde{X}(\frac{k}{3}), & 3|k \end{cases}$$

Q2. (a) $x(n) = \alpha^n u(n)$

$$X(e^{j\omega}) = \frac{1}{1 - \alpha e^{-j\omega}}$$

(b) $\tilde{X}(k) = \sum_{n=0}^{N-1} \tilde{x}(n) e^{-j\frac{2\pi}{N}nk}$

$$= \sum_{n=0}^{N-1} \sum_{r=-\infty}^{+\infty} x(n+rN) e^{-j\frac{2\pi}{N}nk}$$

$$= \sum_{n=0}^{N-1} \sum_{r=0}^{+\infty} \alpha^{n+rN} e^{-j\frac{2\pi}{N}nk}$$

$$= \sum_{n=0}^{N-1} \alpha^n e^{-j\frac{2\pi}{N}nk} \left(\sum_{r=0}^{+\infty} (\alpha^N)^r \right)$$

$$= \sum_{n=0}^{N-1} (\alpha e^{-j\frac{2\pi}{N}k})^n \cdot \frac{1}{1 - \alpha^N}$$

$$= \frac{1 - (\alpha e^{-j\frac{2\pi}{N}k})^N}{1 - \alpha e^{-j\frac{2\pi}{N}k}} \cdot \frac{1}{1 - \alpha^N} = \frac{1}{1 - \alpha e^{-j\frac{2\pi}{N}k}}$$

(c) $\text{If } \omega = \frac{2\pi k}{N} \text{ then } \tilde{X}(k) = X(e^{j\omega})$