Exercise Sheet 3

Discrete Mathematics, 2021.9.26

Note:The following exercises involve a new logical operators $\oplus$. Its truth table is as follows:

p	q	$p \to q$
T	**T**	**T**
T	**F**	**F**
F	**T**	**T**
F	**F**	**T**

1. a) Prove that $p \to (q \to p)$ is a tautology.
   b) Prove that $(p \to q \to r) \to (p \to q) \to (p \to r)$ is a tautology.
   c) Prove that $p \to q \to r \equiv (p \wedge q) \to r$.
   Here, $\to$ is right associative, i.e. $\phi \to \psi \to \chi$ means $\phi \to (\psi \to \chi)$.

2. (P36, Ex.52, [R]) In this exercise we will show that $\{|\}$ is a functionally complete collection of logical operators. (**Note**: $p|q$ means p NAND q. The proposition p NAND q is true when either p or q, or both, are false; and it is false when both p and q are true. The operators $|$ is called the Sheffer stroke after H. M. Sheffer)

   a) Show that $\phi|\phi$ is logically equivalent to $\neg\phi$.

   b) Show that $(\phi|\psi)|(\phi|\psi)$ is logically equivalent to $\phi \wedge \psi$.

   c) Show that $\{|\}$ is a functionally complete collection of logical operators based on the results above.