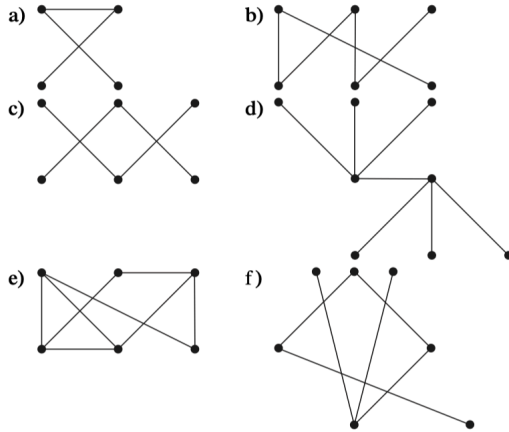# Exercise Sheet 15-16
Discrete Mathematics, 2021.11.24-25

1. ([R], Page 755, Exercise 2(a)(b)(c)(d)(e)(f)) Which of these graphs are trees?



2. In a rooted tree $G = (V, E)$, we use $d(u, v)$ to represent the length of the unique simple path connecting $u$ and $v$, for any $u, v \in V$. Prove that the unique simple path connecting $u$ and $v$ will pass through $w$ if and only if $d(u, v) = d(u, w) + d(w, v)$.

3. Given a rooted tree $G$, define relation $R_1 = \{(u, v) \mid u \text{ is } v\text{'s ancestor in } G\}$, and relation $R_2 = \{(u, v) \mid u\text{'s level is (strictly) smaller than } v\text{'s level in } G\}$. (a) Prove that $R_1 \subseteq R_2$. (b) Find a rooted tree $G$ such that $R_1 = R_2$. (c) Find a rooted tree such that $R_1 \neq R_2$.

4. Prove that in a rooted tree $G$, if $v$ is a descendant of $u$, then the unique simple path from $v$ to $u$ only passes through $u$'s descendants and $u$, (i.e. every vertex on this path is either $u$'s descendant or $u$ itself).