Exercise Sheet 4

Discrete Mathematics, 2020.9.27

1. Consider the compound proposition $\phi = p \to (q \oplus r)$ where p, q, r are propositional variables.

   (a) Find a compound proposition ψ in CNF such that $\phi \equiv \psi$.

   (b) Use the algorithm that we learned in class to construct a compound proposition ψ in CNF such that ϕ is satisfiable if and only if ψ is satisfiable.

2. Consider potential process of determining whether

   $$(\neg p_1 \vee p_2) \wedge (\neg p_1 \vee p_3 \vee p_5) \wedge (\neg p_2 \vee p_4) \wedge (\neg p_3 \vee \neg p_4) \wedge (p_1 \vee p_5 \vee \neg p_2) \wedge (p_2 \vee p_3) \wedge (p_2 \vee \neg p_3) \wedge (p_6 \vee \neg p_5)$$

   is SAT or UNSAT.

   a) Calculate UnitPro($\mathcal{J}_1$) where $\mathcal{J}_1 = [p_1 \mapsto \mathbf{T}, p_3 \mapsto \mathbf{F}]$.

   b) Calculate UnitPro($\mathcal{J}_2$) where $\mathcal{J}_2 = [p_3 \mapsto \mathbf{F}]$.

   c) Calculate UnitPro($\mathcal{J}_3$) where $\mathcal{J}_3 = [p_3 \mapsto \mathbf{T}, p_5 \mapsto \mathbf{T}]$.

3. Consider potential process of using CDCL to determine whether

   $$\begin{array}{l}(\neg p_1 \vee p_2) \wedge \\ (\neg p_1 \vee p_3 \vee p_5) \wedge \\ (\neg p_2 \vee p_4) \wedge \\ (\neg p_3 \vee \neg p_4) \wedge \\ (p_1 \vee p_5 \vee \neg p_2) \wedge \\ (p_2 \vee p_3) \wedge \\ (p_2 \vee \neg p_3) \wedge \\ (p_6 \vee \neg p_5)\end{array}$$

   is SAT or UNSAT. After "Pick $p_3 \mapsto \mathbf{F}$; UnitPro $p_2 \mapsto \mathbf{T}$; UnitPro $p_4 \mapsto \mathbf{T}$; Pick $p_5 \mapsto \mathbf{F}$; UnitPro $p_1 \mapsto \mathbf{F}$; Conflict",

   a) Which conflict clause will be generated?

   b) Which propositional variables will be unpicked?

   c) What will be the next unit propagation result?

4. Consider potential process of CDCL to determine whether

   $$\begin{array}{l}(p_1 \vee p_4 \vee \neg p_5 \vee \neg p_7) \wedge \\ (p_1 \vee p_5) \wedge \\ (p_1 \vee p_7) \wedge \\ (p_2 \vee p_4 \vee \neg p_9) \wedge \\ (\neg p_2 \vee p_9 \vee \neg p_{10}) \wedge \\ (\neg p_3 \vee \neg p_8) \wedge \\ (\neg p_6 \vee p_9) \wedge \\ (p_6 \vee p_{10}) \wedge \\ (\neg p_7 \vee p_8 \vee \neg p_9 \vee p_{10}) \wedge \\ (\neg p_9 \vee \neg p_{10})\end{array}$$

   is SAT or UNSAT. After "Pick $p_2 \mapsto \mathbf{F}$; Pick $p_4 \mapsto \mathbf{F}$",

a) What's the result of unit propagation?

After that, if we further do "Pick $p_7 \mapsto \mathbf{T}$; Pick $p_1 \mapsto \mathbf{F}$",

b) What's the result of unit propagation?

c) Which conflict clause will be generated?

d) Which propositional variables will be unpicked?

e) What will be the next unit propagation result?

5. (P53, Ex.10, [R]) Let $C(x)$ be the statement "x has a cat", let $D(x)$ be the statement "x has a dog", and let $F(x)$ be the statement "x has a ferret". Express each of these statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.

   a) A student in your class has a cat, a dog, and a ferret.

   b) All students in your class have a cat, a dog, or a ferret.

   c) Some student in your class has a cat and a ferret, but not a dog.

   d) No student in your class has a cat, a dog, and a ferret.

   e) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.