1.

Proof of a)

Suppose Σ is a set of p, q, and r. Given Σ and $J : \Sigma \to \{$ T, F $\}$. By definition, we need to prove, for any J, if $[[p \vee \neg q]] = [[q \vee \neg r]] =$ T, then $[[p \vee \neg r]] =$ T.

Case 1. $[[\text{q}]]_J =$ T, then $p \vee \neg q \equiv p \vee \text{F} \equiv p \equiv \text{T}$, which means the true value of p is true. Thus the true value of $p \vee \neg r$ is true.

Case 2. $[[\text{q}]]_J =$ F, then $q \vee \neg r \equiv \text{F} \vee \neg r \equiv \neg r \equiv \text{T}$, which means the true value of $\neg r$ is true. Thus the true value of $p \vee \neg r$ is true.

Thus by definition, $p \vee \neg r$ is a consequence of $p \vee \neg q$, $q \vee \neg r$. *QED*.

Proof of b)

In fact, $p \vee \neg p$ is a tautology. If the true value of p is true, the true value of $p \vee \neg p$ is true. Thus by definition, $p \vee \neg p$ is a consequence of p. *QED*.

2.

Proof of a)

- If $[[\phi]]_J =$ T, then $[[\psi]]_J =$ T, which is because $\phi \vDash \psi$. In this case,

$$\phi \wedge \psi \equiv \text{T} \wedge \text{T} \equiv \text{T} \equiv \phi,$$
$$\phi \vee \psi \equiv \text{T} \vee \text{T} \equiv \text{T} \equiv \psi.$$

- If $[[\phi]]_J =$ F, then

$$\phi \wedge \psi \equiv \text{F} \wedge \psi \equiv \text{F} \equiv \phi,$$
$$\phi \vee \psi \equiv \text{F} \vee \psi \equiv \psi.$$

QED.

Proof of b)

We need to prove $p \vee (p \wedge q) \equiv p$ (Absorption Laws). Using the conclusion above (if $\phi \vDash \psi$, then $\phi \vee \psi \equiv \psi$), we need only to prove p is a consequence of $p \wedge q$. If the true value of $p \wedge q$ is true, the true value of p is true. Thus by definition, p is a consequence of $p \wedge q$. *QED*.

3.

Proof of a)

Using De Morgan's Laws and double negation law, we can find that $\neg(p \wedge \neg q) \equiv \neg p \vee \neg\neg q \equiv \neg p \vee q$. *QED*.

Proof of b)

Here is the truth table of $(\neg p) \oplus$ q.

p	q	¬p	$(\neg p) \oplus$q
T	T	F	T
T	F	F	F
F	T	T	F
F	F	T	T

Here is the truth table of $\neg(p \oplus q)$.

p	q	$p \oplus q$	$\neg(p \oplus q)$
T	T	F	T
T	F	T	F
F	T	T	F
F	F	F	T

So $\neg(p \oplus q)$ and $(\neg p) \oplus q$ are logically equivalent with each other. *QED*.

Proof of c)
Here is the truth table of $p \oplus (\neg p) \oplus q$.

p	q	$\neg$p	$p \oplus (\neg p)$	$p \oplus (\neg p) \oplus q$
T	T	F	T	F
T	F	F	T	T
F	T	T	T	F
F	F	T	T	T

So $p \oplus (\neg p) \oplus q$ and $\neg q$ are logically equivalent with each other. *QED*.

4.
Here is the truth table of ϕ.

p	q	r	$q \oplus r$	$\neg(q \oplus r)$	$p \wedge \neg(q \oplus r)$	ϕ
T	T	T	F	T	T	F
T	T	F	T	F	F	T
T	F	T	T	F	F	T
T	F	F	F	T	T	F
F	T	T	F	T	F	T
F	T	F	T	F	F	T
F	F	T	T	F	F	T
F	F	F	F	T	F	T

From the truth table, we can find that $\neg p \vee (q \wedge \neg r) \vee (\neg q \wedge r)$ is a disjunctive normal form of ϕ.