Exercise 4

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1.

a) $(\neg p \vee \neg q \vee \neg r) \wedge (\neg p \vee q \vee r)$

b) For any J, let $[[p_1]]_J \equiv [[q \oplus r]]_J$, $[[p_2]]_J \equiv [[p \rightarrow (q \oplus r)]]_J$.
Then ϕ is satisfiable iff $\psi = ((q \oplus r) \leftrightarrow p_1) \wedge ((p \rightarrow p_1) \leftrightarrow p_2) \wedge p_2$ is satisfiable.

2.

a) Pick: $p_1 \mapsto$T, $p_3 \mapsto$F.
UnitPro: $p_2 \mapsto$T, $p_5 \mapsto$T, $p_4 \mapsto$T, $p_6 \mapsto$T.
There is no conflict in this total truth assignment.

b) Pick: $p_3 \mapsto$F.
UnitPro: $p_2 \mapsto$T, $p_4 \mapsto$T.
Pick: $p_1 \mapsto$T.
UnitPro: $p_5 \mapsto$T, $p_6 \mapsto$T.
There is no conflict in this total truth assignment.

c) Pick: $p_3 \mapsto$T, $p_5 \mapsto$T.
UnitPro: $p_2 \mapsto$T, $p_4 \mapsto$T.
Then the true value of $\neg p_3 \vee \neg p_4$ must be false, which means the true value of the compound proposition is false. So $UnitPro(J_3)$ causes a conflict on the compound proposition.

3.

a) $p_1 \wedge p_5 \wedge \neg p_2$ causes the conflict. $p_3 \wedge p_5$will be added.

b) Unpick: p_1, p_5.

c) UnitPro: $p_5 \mapsto$T, $p_6 \mapsto$T.
Pick: $p_1 \mapsto$T.
There is no conflict in this total truth assignment.

4.

a) UnitPro: $p_9 \mapsto$F, $p_6 \mapsto$F, $p_{10} \mapsto$T.

b) UnitPro: $p_5 \mapsto$T, $p_4 \mapsto$T.
Conflict!

c) $p_1 \wedge p_5$causes the conflict. $p_1 \wedge p_4 \wedge \neg p_7$will be added.

d) Unpick: p_1, p_5, p_7.

e) UnitPro: $p_1 \mapsto$T.

5.

a) $\exists x(C(x) \wedge D(x) \wedge F(x))$

b) $\forall x(C(x) \vee D(x) \vee F(x))$

c) $\exists x(C(x) \wedge \neg D(x) \wedge F(x))$

d) $\forall x(\neg C(x) \vee \neg D(x) \vee \neg F(x))$ or $\neg\exists x(C(x) \wedge D(x) \wedge F(x))$

e) $(\exists x C(x)) \wedge (\exists x D(x)) \wedge (\exists x F(x))$