Exercise Sheet 2

Discrete Mathematics, 2021.9.16

Note:The following exercises involve a new logical operators $\oplus$. Its truth table is as follows:

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

1. a) Prove: $p \vee \neg q, q \vee \neg r \vDash p \vee \neg r$.
   b) Prove: $p \vDash p \vee \neg p$.

2. a) Prove: if $\phi \vDash \psi$ then $\phi \wedge \psi \equiv \phi$ and $\phi \vee \psi \equiv \psi$.
   b) Use the conclusion above to prove absorption laws that we learnt in class.

3. a) Prove that $\neg(p \wedge \neg q) \equiv \neg p \vee q$.
   b) Prove that $\neg(p \oplus q) \equiv (\neg p) \oplus q$.
   c) Prove that $p \oplus (\neg p) \oplus q \equiv \neg q$.

4. Consider the compound proposition $\phi = \neg(p \wedge \neg(q \oplus r))$ where p, q, r are propositional variables. Give a disjunctive normal form of ϕ.