

Exercise Sheet 6

Discrete Mathematics, 2021.10.12

1. (P56, Ex.43, [R]) Consider the first order language with symbol set $S = \{P, Q\}$ in which P and Q represent two unary predicates. Determine whether $\forall x. (P(x) \rightarrow Q(x))$ and $\forall x. P(x) \rightarrow \forall x. Q(x)$ are logically equivalent. Justify your answer.
2. Is $\neg\forall x. (\phi \rightarrow \psi)$ logically equivalent to $\exists x. (\phi \wedge \neg\psi)$? You do not need to give a formal proof, but try to explain the intuition behind your answer.
3. Suppose R is a binary predicate symbol. Prove that $\exists x. \forall y. R(x, y) \models \exists x. R(x, x)$.
4.
 - a) Prove that if $\phi \models \psi$ then $\forall x. \phi \models \forall x. \psi$.
 - b) Prove that if $\Phi, \phi \models \psi$ and x does not freely occur in Φ then $\Phi, \forall x. \phi \models \forall x. \psi$.
 - c) Demonstrate an example in which
 - $\Phi, \phi \models \psi$
 - x does freely occur in Φ
 - $\Phi, \forall x. \phi \not\models \forall x. \psi$.