1. Let J be any S-interpretation, where $S = \{P, Q\}$.
   Case 1. If $[[\forall x. P(x)]]_J$ =true, then for any element a in J's domain, $[[P(x)]]_{J[x\mapsto a]}$ =True and $[[\forall y. P(y)]]_J =$ true. Thus $[[P(x) \rightarrow \forall y. P(y)]]_{J[x\mapsto a]} =$ true. So $[[\exists x. \left(P(x) \rightarrow \forall y. P(y)\right)]]_J$ =true.
   Case 2. If $[[\forall x. P(x)]]_J$ =false, which means there exists at least one element a in J's domain, such that $[[P(x)]]_{J[x\mapsto a]} =$ false. Thus $[[P(x) \rightarrow \forall y. P(y)]]_{J[x\mapsto a]} =$ true. So $[[\exists x. \left(P(x) \rightarrow \forall y. P(y)\right)]]_J$ =true.
   Therefore, $[[\exists x. \left(P(x) \rightarrow \forall y. P(y)\right)]]_J$ =true is right on any interpretation J.
2. a) $\forall z. \exists y. \exists x. \neg T(x, y, z)$.
   b) $(\forall x. \forall y. \neg P(x, y)) \vee (\exists x. \exists y. \neg Q(x, y))$.
   d) $\exists y. \forall x. \forall z. (\neg T(x, y, z)) \wedge \neg Q(x, y))$.
3. a) {0,1,4,9,16,25,36,49,64,81}
   b) $\emptyset, \{\emptyset\}, \left\{\{\emptyset\}\right\}, \{\emptyset, \{\emptyset\}\}$.
4. a) False, for $\emptyset \in \{\emptyset\}$.
   b) True, for $\{\emptyset\}$ is an element of $\{\{\emptyset\}\}$.
   c) False, for $A \in P(A)$.
   d) True, for A is an element of $P(A)$.
5. We can let $A = \{0\}$, $B = \{0, \{0\}\}$ as a simple example.
6. We can let $A = \emptyset$, $B = \{\emptyset\}$, $C = \{\emptyset, \{\emptyset\}\}$ as a simple example.