

3. If there are at least two different minimum spanning trees in a connected weighted graph and the weight of the edges in this graph are different, then we can write two different minimum spanning trees T_1 , T_2 as $T_1 = \{e_1, e_2, \dots, e_n\}$, $T_2 = \{e'_1, e'_2, \dots, e'_n\}$. And let the weights of edges in T_1 satisfy $w(e_i) < w(e_j)$ iff $i < j$, let the weights of edges in T_2 satisfy $w(e'_i) < w(e'_j)$ iff $i < j$, if $w(e_p)$ represents the weight of edge e_p . Because T_1 and T_2 are different, there exists k such that $e_k \neq e'_k$, and write the least integer k as t . Assume $w(e_t) < w(e'_t)$, then e_t will not belong to T_2 . Since T_2 is a tree, when we add e_t to T_2 , there will be a circuit which has e_t as one of its edges. And it's obvious that there are some edges not in the set $\{e'_1, \dots, e'_{t-1}, e_t\}$ in this circuit; otherwise, the circuit will in T_1 , too, which contradict that T_1 is a tree. Then delete any one edge that not in the set $\{e'_1, \dots, e'_{t-1}, e_t\}$ from $T_2 + e_t$. After this process of adding an edge and deleting an edge, the new graph is still a tree, and write it as T'_2 . And because the weight of the edge we delete is more than the weight of e_t , so the total weight of T'_2 is less than the total weight of T_2 . It causes a contradiction, because in this assumption we have found a new spanning tree whose total weight is less than the weight of the minimum spanning tree. So there is a unique minimum spanning tree in a connected weighted graph if the weights of the edges are all different.