

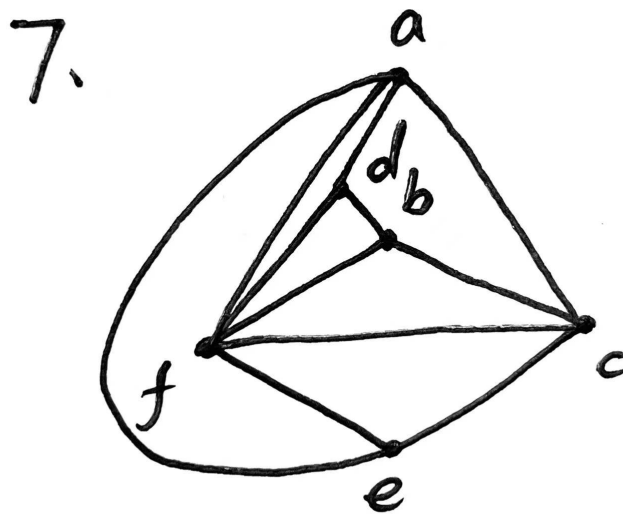
Homework 20

1.

Because $2|E| = \sum \deg(R) \geq 5r$, $r \leq \frac{2}{5}|E|$. By Euler's law, $r = |E| - |V| + 2$, so $|E| \leq \frac{5}{3}|V| - \frac{10}{3}$.

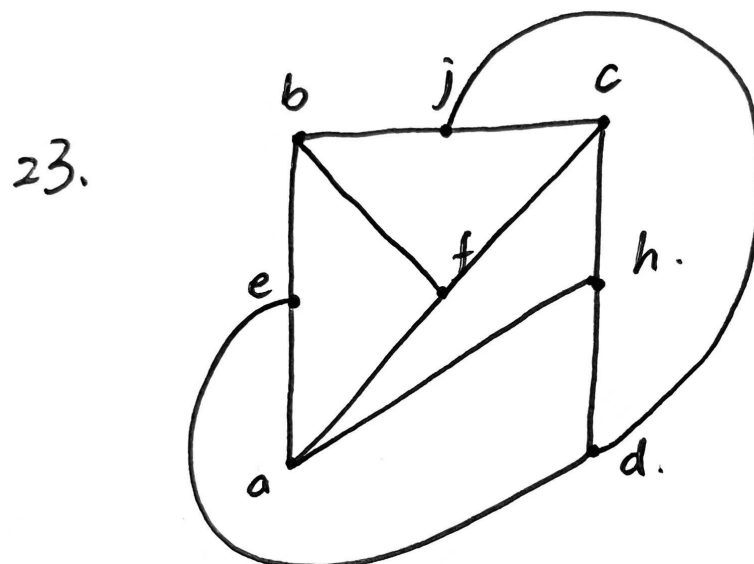
2.

7. It's a planar graph.



8. It isn't a planar graph, for it has a subgraph $G' = (V', E')$ $V' = \{a, b, d, e, f, h, i\}$, which is homeomorphic to $K_{3,3}$. (We can divide these seven vertices into three parts $\{a, e, f\}$, $\{b, d, i\}$, $\{h\}$, and the vertex h is used in the path connects e and i . We can easily find that there is a edge connects a vertex in $\{a, e, f\}$ and a vertex in $\{b, d, i\}$ in G except the pair of e and i .)

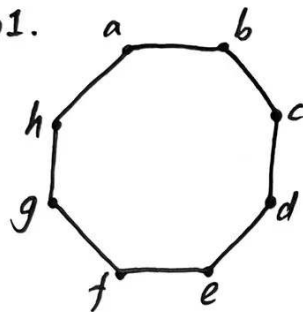
9. It's a planar graph.



25. It isn't a planar graph, for it has a subgraph $G' = (V', E')$ $V' = \{a, b, c, d, e, g\}$, which is homeomorphic to K_5 . (We can divide these six vertices into two parts $\{a, b, c, d, g\}$ and $\{e\}$, and the vertex e is used in the path connects a and d . We can easily find that there is a edge connects a vertex in $\{a, b, c, d, g\}$ and another in G except the pair of a and d .)

3.

Step1.



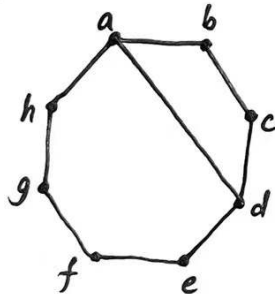
edge-connected component:

①ad ②bg ③cf

④be ⑤af ⑥eh

Both have 2 choices.

Step2. Add ①ad.



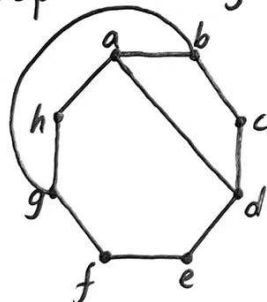
Then

②bg ③cf

④be have 1 choice.

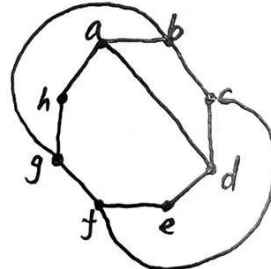
⑤af ⑥eh have 2 choices.

Step3. Add ②bg.



⑥eh ⑤af ③cf ④be have 1 choice.

Step4. Add ③cf



④be have no choice.

So the graph 8 isn't a planar graph.

(④be has no choice because b and e are not in a region in the graph in step 4.)