Exercise Sheet 17

Discrete Mathematics, 2021.11.30

1. Given a rooted tree G, u, v, w are vertices in G and u, v are ancestors of w. Prove that either (i) u is v's ancestor, (ii) u is v's descendent, or (iii) $u = v$.

2. Suppose G is a rooted tree such that every internal vertex has at least two children. Prove that G has more leaves than internal vertices.

3. ([R], Page 797, Exercise 46) Use Exercise 43 to prove that if G is a connected, simple graph with n vertices and G does not contain a simple path of length k, then it contains at most $(k-1)n$ edges.

4. Given a connected, undirected graph $G = (V, E)$ and $u, v, w \in V$. Let T be the spanning tree generated by a DFS process of G. Prove that if u is v's ancestor in T and there exists an edge from v to w in G, then either (i) w is u's ancestor in T, (ii) w is u's descendent in T, or (iii) $w = u$.

5. Given a connected, undirected graph $G = (V, E)$ and $u, v \in V$. Let T be the spanning tree generated by a DFS process of G. Prove that if u is v's ancestor in T then the first time visiting u happens before the first time visiting v in the DFS process.

6. Given a connected, undirected graph $G = (V, E)$ and $u, v, w \in V$. In a DFS process of G, suppose that there are two distinct forward moves, one from u to v and the other from u to w. Prove that any simple path in G from v to w either passes through u or passes through at least one u's ancestor in the tree generated by this DFS process.