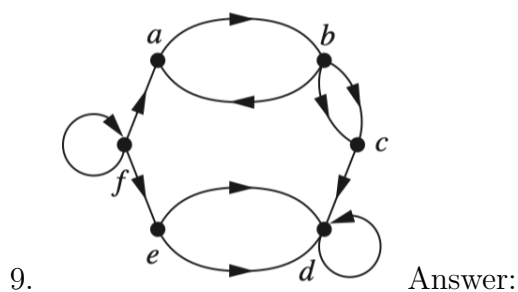
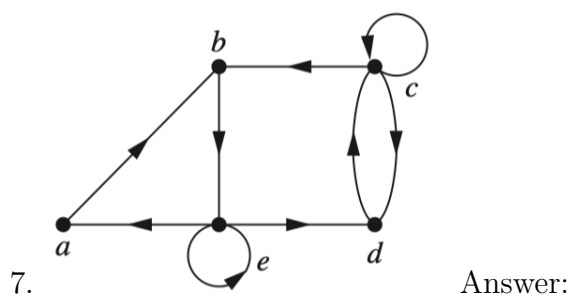
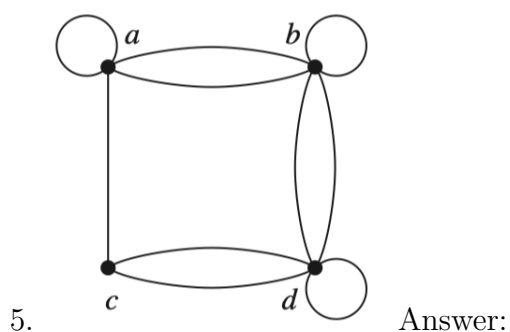
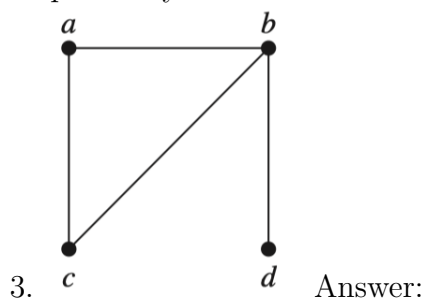


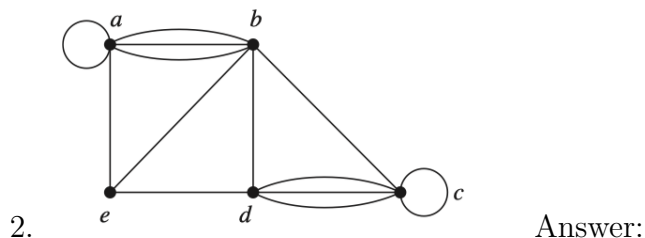
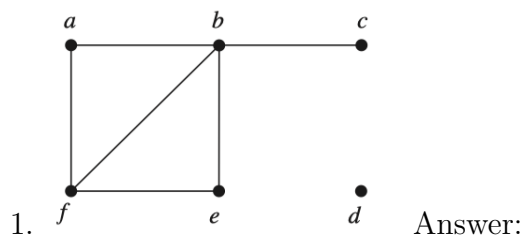
Exercise Sheet 13

Discrete Mathematics, 2021.11.11

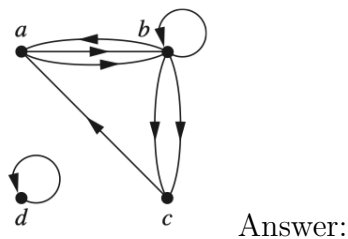
1. ([R], Page 650, Exercise 3,5,7,9) For Exercises 3–9, determine whether the graph shown has directed or undirected edges, whether it has multiple edges, and whether it has one or more loops. Use your answers to determine the type of graph in Table 1 this graph is.



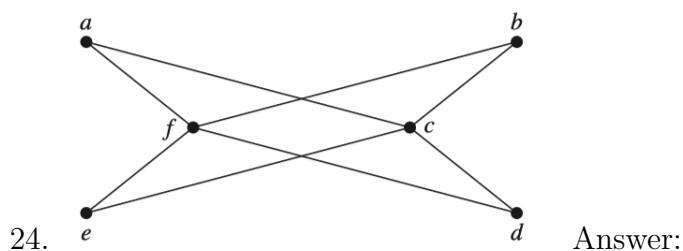
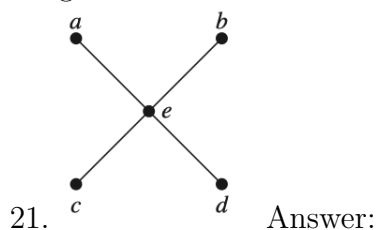
2. ([R], Page 665, Exercise 1, 2) In Exercises 1-2 find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph.



3. ([R], Page 665, Exercise 8) In Exercises 8 determine the number of vertices and edges and find the in-degree and out-degree of each vertex for the given directed multigraph.



4. ([R], Page 665, Exercise 21,24) In Exercises 21,24 determine whether the graph is bipartite. You may find it useful to apply Theorem 4 and answer the question by determining whether it is possible to assign either red or blue to each vertex so that no two adjacent vertices are assigned the same color.



5. **(Optional homework. 1 additional point.)** A function $f : \mathbb{R} \rightarrow \mathbb{R}$ is monotonically increasing if for any $x < y$, $f(x) < f(y)$. Prove that the set of monotonically increasing functions from \mathbb{R} into \mathbb{R} is equinumerous to \mathbb{R} . (选做题可以不做)
6. **(Optional homework. 1 additional points.)** We know that $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x - y \in \mathbb{Q}\}$ is an equivalence relation on \mathbb{R} . Suppose $P = \{[a]_R \mid a \in \mathbb{R}\}$. Prove that $P \approx \mathbb{R}$. (选做题可以不做)