Exercise Sheet 7

Discrete Mathematics, 2021.10.14

1. Prove that $\exists x.(P(x) \to \forall y.P(y))$ is true on any interpretation.

2. (P67, Ex.32, [R]) Consider the first order language with symbol set $S = \{P, Q, T\}$ in which P, Q represent two binary predicates and T represents a ternary predicate. Express the negations of each of these statements so that all negation symbols immediately precede predicates.
a) $\exists z.\forall y.\forall x.T(x, y, z)$
b) $(\exists x.\exists y.P(x, y)) \land (\forall x.\forall y.Q(x, y))$
d) $\forall y.\exists x.\exists z.\,((T(x, y, z) \lor Q(x, y))$

3. a) ([R], Page 125, Exercise 1(c)) List the members of this set:

$$\{x|x \text{ is the square of an integer and } x < 100\}$$

   b) List all subsets of $\{\emptyset, \{\emptyset\}\}$.

4. Are the following statements correct? Explain why (informally).

   a) ([R], Page 125, Exercise 10(c)) $\{\emptyset\} \in \{\emptyset\}$
   b) ([R], Page 125, Exercise 10(d)) $\{\emptyset\} \in \{\{\emptyset\}\}$
   c) For any set A, $A \subseteq \mathcal{P}(A)$.
   d) For any set A, $A \in \mathcal{P}(A)$.

5. ([R], Page 126, Exercise 18) Find two sets A and B such that $A \in B$ and $A \subseteq B$.

6. Find three sets A, B and C, such that $A \in B$, $A \subseteq B$, $B \in C$ and $B \subseteq C$.