

# Exercise 1

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1.

- $p \wedge q$
- $p \wedge \neg q$
- $\neg p \wedge \neg q$
- $p \vee q$

2.

a)

$p$	$q$	$\neg p$	$\neg q$	$\neg p \vee q$	$\neg q \vee p$	$(\neg p \vee q) \wedge (\neg q \vee p)$
T	T	F	F	T	T	T
T	F	F	T	F	T	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

b)

$p$	$q$	$\neg p$	$\neg q$	$\neg p \vee q$	$\neg q \vee p$	$(\neg p \vee q) \vee (\neg q \vee p)$
T	T	F	F	T	F	T
T	F	F	T	F	T	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

c)

$p$	$q$	$r$	$q \vee r$	$p \wedge (q \vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

d)

$p$	$q$	$r$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	F	F
F	T	T	F	F	F
F	T	F	F	F	F
F	F	T	F	F	F
F	F	F	F	F	F

e)

$p$	$q$	$p \wedge q$	$\neg(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

f)

$p$	$q$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

	$p$	$q$	$\neg p$	$\neg q$	$p \wedge q$	$\neg p \wedge \neg q$	$(p \wedge q) \wedge (\neg p \wedge \neg q)$
	T	T	F	F	T	F	T
g)	T	F	F	T	F	F	F
	F	T	T	F	F	F	F
	F	F	T	T	F	T	T

We can find that

a) and g) are logically equivalent with each other,

c) and d) are logically equivalent with each other,

e) and f) are logically equivalent with each other.

3.

a) *Proof:* Note that  $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$  is true when the three variables  $p$ ,  $q$ , and  $r$  have the same truth value. Hence, it is satisfiable as there is at least one assignment of truth values for  $p$ ,  $q$ , and  $r$  that makes it true. *QED.*

b) *Proof:* Note that  $\neg((p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p))$  is true when at least one of  $p$ ,  $q$ , and  $r$  is true and at least one is false. Hence, it is satisfiable as there is at least one assignment of truth values for  $p$ ,  $q$ , and  $r$  that makes it true. *QED.*