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Market-Implied Spread for Earthquake CAT Bonds: Financial Implications of Engineering Decisions

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In the event of natural and man-made disasters, owners of large-scale infrastructure facilities (assets) need contingency plans to effectively restore the operations within the acceptable timescales. Traditionally, the insurance sector provides the coverage against potential losses. However, there are many problems associated with this traditional approach to risk transfer including counterparty risk and litigation. Recently, a number of innovative risk mitigation methods, termed alternative risk transfer (ART) methods, have been introduced to address these problems. One of the most important ART methods is catastrophe (CAT) bonds. The objective of this article is to develop an integrative model that links engineering design parameters with financial indicators including spread and bond rating. The developed framework is based on a four-step structural loss model and transformed survival model to determine expected excess returns. We illustrate the framework for a seismically designed bridge using two unique CAT bond contracts. The results show a nonlinear relationship between engineering design parameters and market-implied spread.

KEY WORDS: Alternative risk transfer methods; CAT bonds; engineering design; risk management

1. INTRODUCTION

Natural disasters are examples of societal failure to integrate the natural and the built environment. Poor planning and engineering choices have devastating effects on the communities as they attempt to recover and rebuild. Although it is inherently difficult to quantify the cost of human life, interruption in business operations, damage to the properties or the cost of reconstruction (often measured in terms of tens of billions of dollars), it is absolutely critical to develop plans and mitigation strategies to allow for fast recovery.

Following the recent catastrophes in California and Florida (e.g., Northridge Earthquake and Hurricane Andrew), the insurance industry has initiated a number of studies to estimate financial exposure given the occurrence of a major natural disaster. The results showed a grim scenario in which the expected losses exceeded \$100 billion.(1) These numbers are even more of a concern given the fact that the balance sheets of the U.S. property liability insurers have a cumulative operating surplus of approximately \$300 billion.(2) Clearly, financial losses at such magnitudes can severely drain the capital capacity of the current insurance industry and influence the credit risk of many reinsurers.

The limited capacity of the insurance industry to absorb large financial losses led researchers and economists to develop alternative risk mitigation strategies, also known as alternative risk transfer techniques (ARTs). These alternative techniques are based on innovative ways to transfer insurer liability to the entities (e.g., capital markets) that have ability to absorb potentially excessive losses.

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Meanwhile the exposure to natural disasters does not bear any systemic risk and hence the securitized insurance instruments help the investors reduce portfolio risks.

Insurance-linked securitization (ILS) is one of the most popular ARTs. In the most general form, securitization can be defined as pooling and packaging the risks into securities and selling it to the capital markets. Even though this process comes with high transaction cost, the associated benefits make ILS a valuable risk transfer instrument.[3]

Catastrophe bonds or CAT bonds are a unique class of ILS. CAT bonds are typically structured as coupon paying bonds with a default linked to the occurrence of the trigger event (e.g., when losses after a devastating earthquake exceed a prespecified level). The financial market variables such as interest rate, firm-specific volatility, managerial decisions, economic downturn, or aggregate consumption have no impact on the default risk of CAT bonds.[4]

CAT bonds can effectively be used to reduce the total cost of financing of infrastructure projects in emerging as well as in developing economies. Insurers, reinsurers, private entities, or even governments may issue CAT bonds to obtain an immediate inflow of cash right after a catastrophe when the repair and reconstruction funds are needed the most. The immediate access to contingent capital reduces the project disruption risk if the emergency funds are not readily available. Considering the devastating impact of recent catastrophes, even if the emergency funds are available, they may become insufficient.[5]

As the occurrence of catastrophes is largely unpredictable, valuation of CAT bonds represents a challenge. Unlike other tradable securities, the valuation of CAT bonds is based on a "black box" approach in which the expected losses are estimated using complex simulation models validated by third-party engineering consulting firms. As a result of this lack of transparency, the investors have showed hesitation in investing in CAT bonds that followed up with larger premium. The objective of this article is to develop a more transparent approach to valuating CAT bonds for large constructed facilities that play a central role in the owner's business operations. Examples of these facilities include tolled bridges, power plants, airports, high-rise commercial buildings, etc. This study proposes a valuation methodology based on an engineering loss model that provides a closed-form solution for computation of the potential financial losses of engineered structures exposed to seismic risks. A simplified four-step loss model is introduced to link observable engineering design parameters of the constructed facilities and the financial parameters of CAT bonds.

2. BACKGROUND

Structure of a CAT bond typically involves a ceding party (e.g., cedant), who seeks to transfer the risk, and investors, who accept the risk for a premium. The cedant can be insurer, reinsurer, or the owner of a facility. The transfer of the risk to the capital markets is achieved by creating a special purpose vehicle (SPV) that provides coverage to the cedant and issues the securities for the investors. The cedant pays a premium in exchange for the coverage against a prespecified event, whereas SPV sells bonds to investors and collects the capital. Raised capital and insurance premium are deposited in a trust account that receives a risk-free interest. The returns generated from this account are swapped for London interbank offered rate (LIBOR) returns that are supplied by a highly rated swap counterparty. Through this swap mechanism, the bond becomes a floating rate note from which interest rate risk is largely removed.[6]

If the trigger event does not occur during the term of the CAT bond, investors receive promised coupons and the principal. However, if a catastrophic event occurs and triggers defined default parameter(s), then the raised capital residing at SPV accounts is transferred to the ceding company as promised in the bond contract. This results in a partial or total loss of principal to the investors.[6,7] Fig. 1 shows the relationship among the stakeholders when structuring CAT bonds. In this study, the cedant is assumed to be the owner of the facility.

Naturally, defining the default-trigger event plays an essential role in structuring CAT bonds. This event must be measurable and easily understood. If the trigger event is based on the level of actual monetary losses suffered by the cedant; the contract is called as an "indemnity-based" contract. Such contract type is subject to moral hazard risk. This phenomenon occurs when the cedant no longer tries to limit its potential losses as the risk is transferred to investors. Thus, moral hazard occurs due to inadequate loss control efforts by the cedant.[8] While suffering from moral hazard risk, indemnity-based contracts eliminate basis risk by offering indemnity against modeled perils.[9] However, the advantage of eliminating basis risk comes at a price. Structuring and selling indemnity-triggered CAT bonds has been rather difficult due to the lack of transparency. As

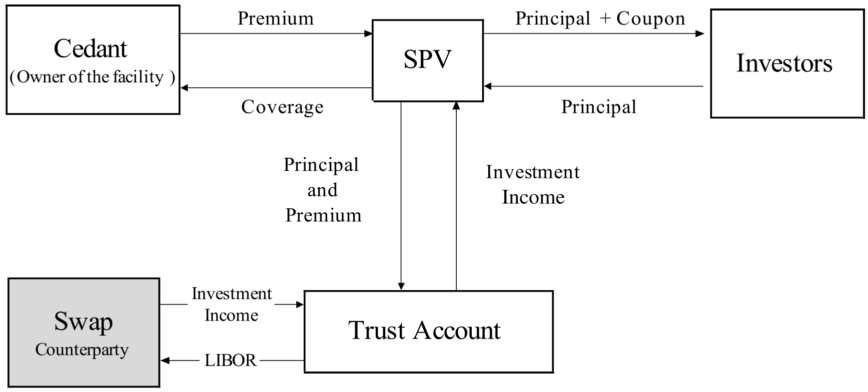


Fig. 1. CAT bond structure.

the catastrophe modeling techniques become more transparent, it is expected that the market will become more receptive.

Another option to relate the trigger event with the actual losses is to specify a loss related index (i.e., total industry loss). "Index-based" contracts help the cedant in avoiding detailed information disclosure to the competitors. However, index-based contracts are subject to basis risk as the cedant's losses may differ from the industry losses. Here, the basis risk relates to the mismatch between the index and the cedant's losses. The quality and hence the benefits of the CAT bond hedge decreases with a decrease in correlation between the cedent's losses and the contract payoff.(9) An alternative approach is to define a trigger event based on some physical parameters such as peak ground acceleration for earthquakes, or wind speed for hurricanes. This is often referred to as a "parametric-based" CAT bond contract. Even though a parametric-based CAT bond is subject to basis risk in a similar manner as the index-based contracts, it provides a transparent setting for investors to assess the risks while having a significantly shorter development period compared to indemnity-based contracts.(10)

Under indemnity contracts, the cedent reports the actual losses. This creates a situation in which the cedent is incentivized to overreport the losses, so that the trigger event is initiated. However, under both index- and parametric-based contracts, the cedant has limited to no capability in overreporting the losses. If the trigger loss is based on an industry index then the cedant's ability to initiate the trigger is proportional to its share in the index. For the parametric-based contracts, the cedant has no ability to influence the trigger event as the trigger is based on physical parameters such as location or magnitude.(11) For such triggers, the basis risk can be substantially reduced by appropriately defining the location where the event is measured. Previous studies show that industry loss indices based on narrowly defined geographical areas have less basis risk than those based on wider areas.(12) In summary, selection of the bond triggering event involves a tradeoff between the moral hazard and basis risk.

CAT bonds also differ in the level of principal protection and the recovery rates they offer. CAT bonds are commonly offered in "principal-at-risk" and "principal-protected" tranches. If the bond is principal-at-risk type, then in the event of a catastrophe all the capital raised from the investors will be used to cover the cedant's losses. Hence, investors are subject to the risk of losing full principal amount. Coupons can be either protected at the minimum recovery value, or fully lost, much like the principal. The principal-protected tranche on the other hand is structured to attract risk-averse investors. In principal-protected CAT bonds, the whole principal amount or a prespecified portion of it is paid back to investors even if the bond defaults. Recent evidence shows that demand for principal-protected CAT bonds has significantly decreased as the investors become more familiar with this new asset class.(7)

As there are a number of differently structured CAT bonds on the market, we now define the key terms and the structure of bonds we consider in the analysis. The CAT bonds we consider (CAT1 and CAT2) are earthquake triggered and indemnity based, but not identical to those offered currently on the market. Nevertheless, they have the same key features such as: default-trigger event, coupons, principal, and contract specifications that relate the trigger event with recovery rate and payments to the cedant.

Definition 1. CAT1 is an indemnity-based principal-at-risk type CAT bond with the attachment point **a.** When the losses to the cedant exceed the attachment point **a**, the CAT1 defaults.

Definition 2. CAT2 is an indemnity-based partially principal-protected type CAT bond with the attachment point **a**, and the exhaustion point **e**. As the losses exceed attachment point, the investors in CAT2 lose their principal proportionally to the incurred losses to the cedant. The principal is fully lost when the losses reach the exhaustion point **e**.

Although previous studies relied on property claims services loss processes[13] or modeling catastrophe loss distributions and loss severity using independently identical lognormal distributions,[14] this study contributes to the literature by setting up a new asset-specific framework that considers a joint mechanism for loss arrival and intensity process. This approach fills the gap between the damage potential of the underlying asset and required risk premium of the CAT bond. The next section discusses the methodology for estimating the arrival and intensity of structural losses.

3. MODELING STRUCTURAL LOSS

When the CAT bond contract is tied to a specific constructed facility, the connection between the engineering design features and financial measures such as losses is highly visible. Although engineers typically follow design codes based on the expected severity of the hazards in the area (e.g., 100-year flood) and understand very little about the impact of design parameters on potential economic losses, financial analysts have limited knowledge in determining financial implications of engineering decisions. In fact, this disconnect is a key problem in developing more integrated project life-cycle models. To overcome this problem, a transparent and easily understood loss model for seismically damaged buildings is developed.

The state of the art in seismic loss estimation is based on a four-step performance-based earthquake engineering procedure through a direct loss estimation process.[15] Financial losses resulting from seismically damaged structures can be estimated using this approach, which can be subdivided into four sequential tasks: (1) hazard analysis; (2) structural analysis; (3) damage and hence repair-cost analysis; and (4) loss estimation. The hazard analysis requires evaluation of seismic hazard at the constructed facility site, and generates intensity measures (IMs) representative of the varying local hazard levels. The structural analysis involves predicting the response of the facility to increasing levels of ground shaking in terms of engineering demand parameters (EDP). The damage analysis uses EDPs to determine damage measures to the facility components from which repair costs can be estimated. Finally, the loss analysis involves determination of direct financial losses to the structure and its contents. Using this approach it is possible to assess loss ratios (L) for various seismic scenarios. Loss ratio is defined as the repair cost or the reinstatement cost to the cost of a new facility built under normal conditions.

Fig. 2 illustrates a set of four interrelated graphs plotted on a log–log scale. Each graph relates a linear function (in log–log space) with one of the key sets of coordinates; each relationship shown here is referenced to the design basis earthquake (DBE). Fig. 2(a) illustrates the hazard intensity, Fig. 2(b) illustrates the hazard intensity-seismic demand, and Fig. 2(c) illustrates a parametric loss model used to assess damage to constructed facilities. Structural capacities (characterized in terms of interstory drifts) are related to damage states (quantified in terms of loss ratios) of the structure. Each relationship, from location, seismic demand versus capacity, and capacity versus loss, involves uncertainty and must be treated probabilistically. As it can be observed from the figure, hazard is mapped into response, response into damage, and finally damage into losses. The interrelationship between the four steps can be represented using the composite equation:

$$\frac{L}{L_{\mathrm{DBE}}} = \left|\frac{\theta}{\theta_{\mathrm{DBE}}}\right|^{c} = \left|\frac{S_a}{S_{a_{\mathrm{DBE}}}}\right|^{bc} = \left|\frac{f}{f_{\mathrm{DBE}}}\right|^{\frac{bc}{-k}}, \quad (1)$$

where L = loss ratio; θ = interstory drift (an EDP); f = annual frequency of occurrence; k, b, c are exponents (slopes of curves in log–log space shown in Figs. 2(a)–(c), respectively); S_a = spectral acceleration (an IM); and the subscript DBE refers to the reference parameters for the DBE. For the list of parameters used to develop the structural loss model, see Table I.

This parametric loss model is based on the capacity-side fragility curves, where parameters are estimated from a structure-specific nonlinear model, and discrete damage states adopted from Hazards United States.[16] A key benefit of this model is no

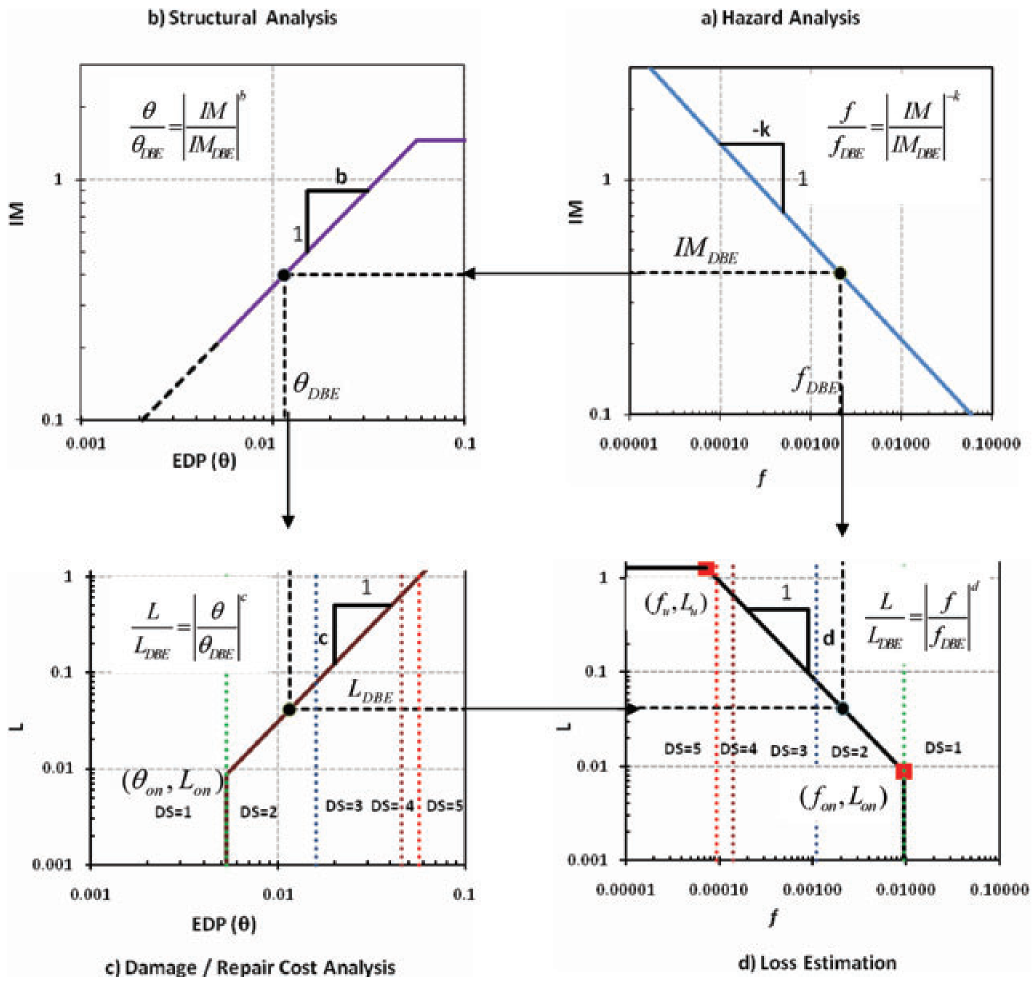


Fig. 2. Four-step loss estimation approach.

need for custom demand-side fragility curves. In fact, the relationship between IMs and EDPs is used to define the demand model, which is compared with the capacity obtained by conducting a nonlinear analysis. As reported in literature,[17] this can be expressed through a relationship relating losses with EDPs (e.g., interstory drifts) as follows:

$$\frac{L}{L_c} = \left|\frac{\theta}{\theta_c}\right|^c \quad \text{and;} \quad L_{\text{on}} \leq L \leq L_u = 1.3, \tag{2}$$

where L = loss ratio; L_c = unit cost (normally taken as $L_c = 1$ for comparative studies); L_{on} = onset of damage (when $L < L_{\text{on}}$, $L = 0$) and L_u = loss at collapse stage with a restriction of $L_u \leq 1.3$ (to allow for price surge); c = an empirically calibrated power; θ = column (or interstory) lateral drift (the EDP); and θ_c = the critical drift value for complete damage (toppling or collapse). Finally the hazard, response, and loss models are convoluted to obtain the loss-frequency curve shown in Fig. 2(d). The corresponding *annual loss function* can be mathematically specified as:

$$L = L_{\text{DBE}} \left|\frac{f}{f_{\text{DBE}}}\right|^d, \tag{3}$$

where L is a median estimate of annual loss (in terms of loss ratio) and $d = bc/-k$.

Equation (3) represents a function of the annual frequency of occurrence (f) and estimated model parameters that can be treated as random variables. Note that the developed loss model represents a framework for modeling losses for unique structures rather than a portfolio of structures (i.e., seismically or conventionally designed bridges, buildings, or other constructed facilities).

Table I. Loss Model Parameters for Different Bridge Types

Step	Parameters	CALTRANS	Japan	NZ
(a)	Hazard IM_{DBE}	0.4	0.4	0.4
	f_{DBE}	0.0021	0.0021	0.0021
	k	3.45	2.4	3
(b)	$\tilde{\theta}_{DBE}$	0.0117	0.0115	0.0163
	b	1.25	1.23	1.27
(c)	$\tilde{\theta}_{on}$	0.0053	0.0053	0.0062
	θ_{cr}	0.0616	0.0566	0.0564
	f	1	1	1
	c	1.8	1.7	1.9
	$d = bc/-k$	−0.6522	−0.8713	−0.8043
	$\tilde{L}_{DBE}$	0.050	0.066	0.095
(d)	$\tilde{L}_u$	1.3	1.3	1.3
	$\tilde{L}_{on}$	0.012	0.018	0.015
	$\tilde{f}_u$	0.0000142	0.0000686	0.0000809
	$\tilde{f}_{on}$	0.0187282	0.0095442	0.0206514
	β_{RD}	0.42	0.4	0.43
	β_{RC}	0.30	0.30	0.30
	β_{UL}	0.35	0.35	0.35
	$\beta_{L_{on}\|\theta}$	0.35	0.35	0.35
	$\beta_{f_{on}\|\theta}$	1.425	0.976	1.239
	$\beta_{f\|L}$	1.522	1.055	1.313
	$\bar{L}_u$	1.38	1.38	1.38
	$\bar{L}_{on}$	0.013	0.019	0.016
	$\bar{f}_u$	0.0000450	0.0001196	0.0001916
	$\bar{f}_{on}$	0.051661	0.015361	0.044467

3.1. Variability in Annual Loss Function

To compute the mean value of annual loss function it is essential to account for variability associated with the model parameters. Due to the multiplicative (power) nature of the loss function and the assumption of normality of the estimated parameters, annual losses are log normally distributed. Thus, the relationship between the mean loss $\bar{L}$ and the median $L_{50\%}$ is given by:

$$\bar{L} = L_{50\%} \exp\left(\frac{1}{2}\beta_{TL}^2\right), \tag{4}$$

similarly, for other fractiles we have,

$$L_{\alpha\%} = L_{50\%} \exp(Z_\alpha \beta_{TL}), \tag{5}$$

where Z_α represents the standardized Gaussian random variable for a given percentile value with a mean value of zero and standard deviation of one, and β_{TL} the lognormal standard deviation of the loss function, often referred to as the dispersion factor.

It is assumed that the randomness in structural capacity and demand as well as the uncertainty in loss estimation are log normally distributed with logarithmic standard deviations (dispersions) of β_{RC}, β_{RD}, and β_{UL}, respectively. Then, based on the approach outlined by Kennedy *et al.*,[18] the total dispersion β_{TL} can be expressed as:

$$\beta_{TL} = \sqrt{\beta_{UL}^2 + c^2\beta_{RS}^2}, \tag{6}$$

where the parameter β_{RS} denotes the total variability associated with the structure and is given by:

$$\beta_{RS} = \sqrt{\beta_{RD}^2 + \beta_{RC}^2}. \tag{7}$$

However, it is now important to change the perspective of the variabilities from drift (in Fig. 3(c)) to annual frequency in Fig. 3(d). Thus the variability of $f$on for a given drift is given by:

$$\beta_{f_{on}|\theta} = \frac{k}{b}\beta_{RS} = \frac{k}{b}\sqrt{\beta_{RD}^2 + \beta_{RC}^2}. \tag{8}$$

Using the above mentioned dispersion factors in conjunction with the median coordinates $(\tilde{f}_{on}, \tilde{L}_{on})$ and $(\tilde{f}_u, \tilde{L}_u)$, it is possible to construct the mean loss curve plotted with the dashed line in Fig. 3(d) in terms of the coordinates $(\bar{f}_{on}, \bar{L}_{on})$ and $(\bar{f}_u, \bar{L}_u)$. This curve is essential because when plotted in a natural scale, by calculating the area beneath the entire loss curve, the expected annual losses $E(L)$ is found. This integration can be expressed as:

$$E(L) = \int_0^{\bar{f}_{on}} L\,df = \bar{L}_u \bar{f}_u + \frac{\bar{L}_{DBE}}{\bar{f}_{on}\bar{f}_{DBE}^d}\int_{\bar{f}_u}^{\bar{f}_{on}} f^d\,df = \frac{\bar{L}_{on}\bar{f}_{on} + d\bar{L}_u\bar{f}_u}{1+d}. \tag{9}$$

Here, the mean loss for the reference DBE is found by applying Equation (4) as:

$$\bar{L}_{DBE} = \tilde{L}_{DBE} \exp\left(\frac{1}{2}\beta_{TL}^2\right). \tag{10}$$

Again, applying Equations (4) and (8) we get the mean value of the annual frequency at the onset of damage as:

$$\bar{f}_{on} = \tilde{f}_{on} \exp\left(\frac{1}{2}\beta_{f_{on}|\theta}^2\right). \tag{11}$$

From Equation (1) it follows that the magnitude of the mean value of the loss at the mean frequency for the onset of damage is:

$$\bar{L}_{on} = \bar{L}_{DBE}\left|\frac{\bar{f}_{on}}{f_{DBE}}\right|^d \tag{12}$$

and similarly for the mean values for the ultimate frequency and loss:

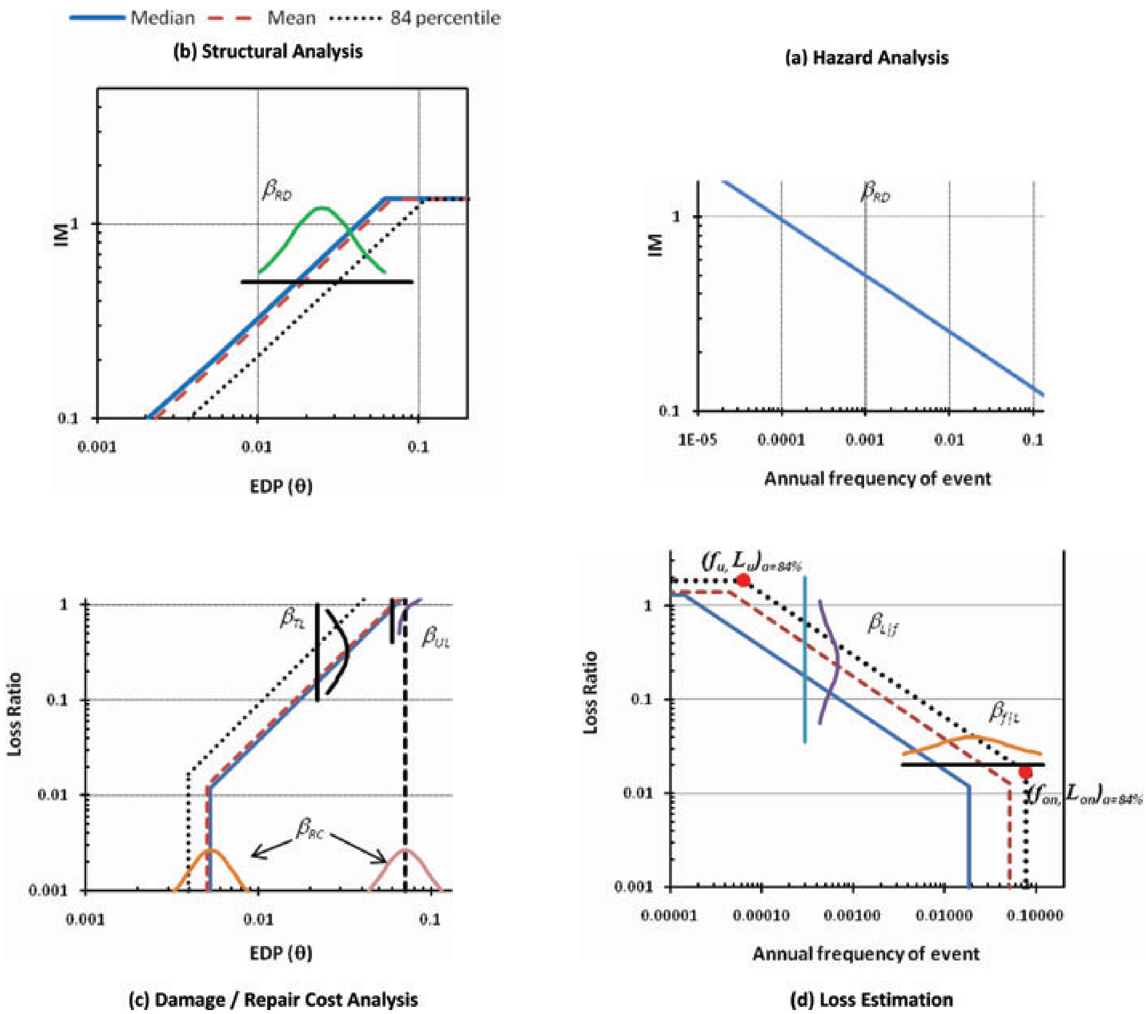


Fig. 3. Summary of the four step approach after incorporating aleotoric and epistemic uncertainties.

$$\bar{f}_u = f_{\mathrm{DBE}} \left| \frac{\bar{L}_u}{L_{\mathrm{DBE}}} \right|^{1/d}, \tag{13}$$

$$\bar{L}_u = \tilde{L}_u \exp\left(\frac{1}{2}\beta_{\mathrm{UL}}^2\right). \tag{14}$$

In the following sections we present a risk pricing model that is based on proportional hazard (P-H) transformation of annual loss function, as defined in Equation (3). Finally, we use numerical examples to illustrate the findings.

4. METHODOLOGY

To determine the value of a CAT bond one needs to estimate the risk of losses for the insured property. Here, the parameters such as the expected loss and loss distribution are of great importance. The focus of this article is on indemnity-based CAT bonds; hence the trigger event is based only on specified actual monetary losses to the cedant. Based on the loss-frequency curve illustrated in Fig. 2(d), it is possible to estimate the losses of a constructed facility that is insured with an indemnity-based CAT bond for a specified loss trigger.

We assume that the probability of occurrence (or annual occurrence frequency) of a catastrophe in T years is q and the risk free rate on U.S. treasury bills per time period T is r_f. For simplicity, we consider a single period case ($T = 1$ year) where coupons (C) are paid annually and the bond is sold at par. Because the considered assets are large constructed facilities, it is assumed that SPV is created by the cedant. To be able to pay the promised coverage (L_C) in case of a catastrophe, SPV raises the sufficient capital from the capital market by issuing the CAT bonds. If B_p

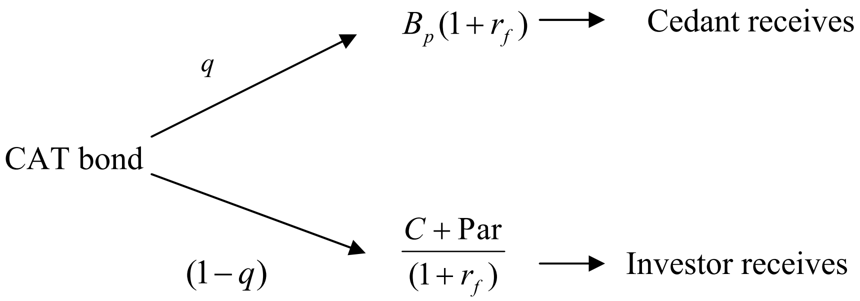


Fig. 4. CAT bond payoff diagram.

denotes the raised capital from bond issuing, the required condition for the issuer can be written mathematically as:

$$B_p(1+r_f) = L_C. \tag{15}$$

If no catastrophe occurs during the term of the bond, investors get their principal back and the promised coupon. For such cases unit fair price of the bond in terms of expected discounted cash flow can be expressed as:

$$B_p = (1-q)\frac{C+\text{Par}}{(1+r_f)}. \tag{16}$$

Fig. 4 shows the payoff of the CAT bond to both investors and cedant depending on the occurrence of prespecified catastrophe. Recall that both Equations (15) and (16) are written for $T = 1$ year. A more general formula for N periods of T years can be defined as:

$$B_p = C\sum_{n=1}^{N}\frac{(1-q)^n}{(1+r)^n} + \frac{(1-q)^T}{(1+r)^T}\text{Par}. \tag{17}$$

As discussed in the previous section, the annual occurrence probability of a loss variable exceeding a predefined threshold value due to an earthquake can be obtained from the proposed structural loss model. If the bond is priced at par, the coupon of a bond is defined by a risk premium (spread) added to the risk free rate (LIBOR). Spread (S) is defined as an interest that compensates the investors for taking on additional risk (investing in a potentially defaulting entity). As the risk increases, so does the spread.[19] Therefore, the key factor in determining the value of CAT bonds is finding the spread value for the underlying risk.

Wang[20] stated that in the absence of systematic risk, the spread corresponds to the market-implied value of the expected losses. Based on this approach, well known Wang P-H transforms are used to transform annual loss function and calculate the spread values of CAT bonds. By doing so, a direct link between the structural design parameters and the value of spread is created. The P-H transform methods are used to adjust the best-estimate distribution with respect to the varying levels of uncertainty, portfolio diversification, and market competition.[20]

The losses from catastrophic events are represented by the loss exceedance (survival) curves. For the loss variable X, the survival function $S(x)$ is given by:

$$S(x) = P\{X > x\}. \tag{18}$$

Here $S(x)$ refers the probability of loss X will exceed amount \$$x$. Clearly, the relation between the survival function and cumulative distribution function $F(x)$ can be constructed as:

$$S(x) = 1 - P\{X \le x\} = 1 - F(x). \tag{19}$$

It can be verified that, for nonnegative random variables, the mean value of the loss variable X, $E(X)$, is obtained by integration of the survival curve over the range from zero to infinity.

$$E(X) = \int_0^\infty S(x)\,dx. \tag{20}$$

However, the current market prices imply quite higher loss estimates. In fact, the investors do not account the risk of high losses due to a relatively low likelihood of catastrophe event equivalent to a small loss resulting event with a high chance of occurrence. The expectation of losses does not accentuate the very nature of catastrophes, their likelihood, and consequences.[21] To account for this, Wang[20] showed that the market in fact implies a direct transform of the objective loss exceedance curve $S(x)$ and defined the P-H risk adjusted premium of such contracts with potential losses as the mean of the transformed distribution as:

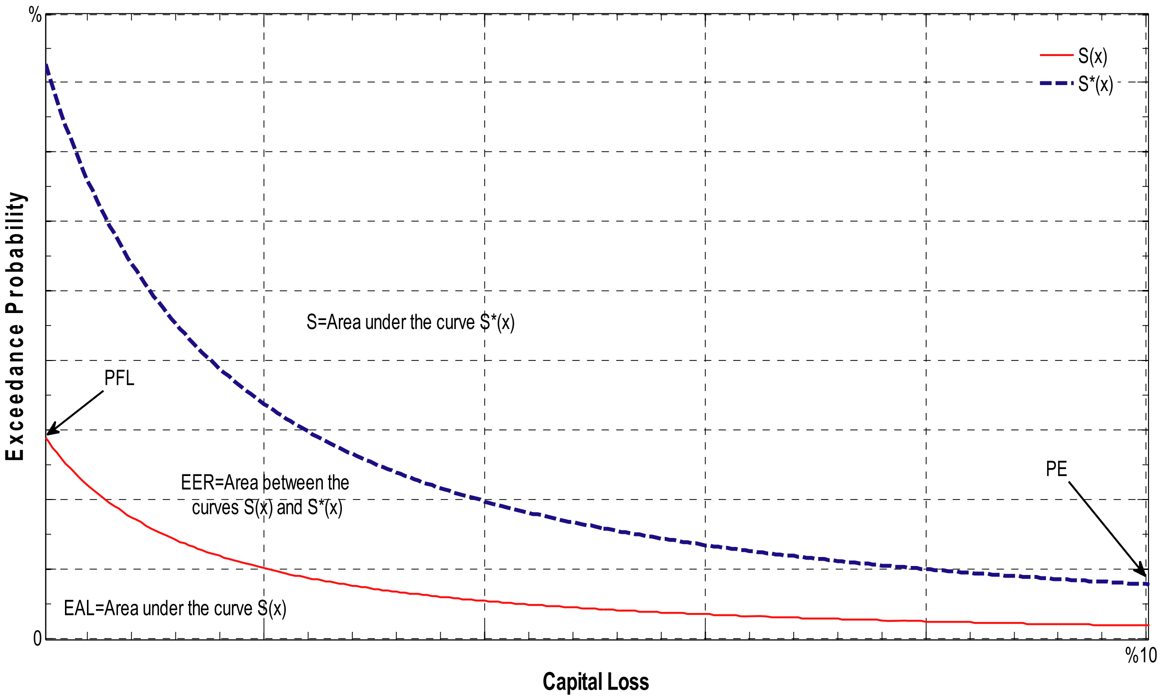


Fig. 5. Survival function and its transform.

$$\prod_{\rho}(X) = E\left[\prod_{\rho}(X)\right] = \int_0^{\infty} S^*(x)\,dx$$
$$= \int_0^{\infty} (S(x))^{1/\rho}\,dx. \tag{21}$$

The mapping $\prod_\rho : S(x) \to (S(x))^{1/\rho}$ referred to as the P-H transform and $\prod_\rho (X)$ is the risk adjusted premium or risk adjusted spread at risk aversion level $\rho \geq 1$. This model was further extended to include the Sharpe ratio for risks with skewed distribution:

$$S^*(x) = \Phi(\Phi^{-1}(S(x)) + \lambda), \tag{22}$$

where Φ is the standard normal cumulative distribution and $\lambda = (E[R] - r)/\sigma[R])$ is the Sharpe ratio.

Fig. 5 shows a typical survival function of a loss variable and its transform. The area under the survival curve represents $E(L)$, and the area under the transformed curve $S^*(x)$ gives the spread. Accordingly, the area between the two curves is simply the additional risk loading required by investors for taking on the risk. This additional risk loading is also known as expected excess return (EER). In the case of CAT bonds the estimates of probability of first loss (PFL) and the probability of cover exhaustion (PE) have vital importance in determining the spread values. Once the trigger loss value and required coverage is set, both of these probability values can be computed easily with the proposed loss model.

However, it is very important to characterize the entire distribution as humans make decisions not only on the information about a distribution's tails and expectation, but also on full information about an event's probability space.[(21)] The estimates of probability distributions are based on the limited data and hence subject to parameter uncertainty. To adjust the parameter uncertainty, Wang[(24)] suggested empirically estimated probability distribution $F(x)$:

$$F^*(x) = Q(\Phi^{-1}(F(x))), \tag{23}$$

where, Q is the Student-t distribution with degree of freedom υ. Equation (23) can be rewritten in terms of survival function where $S(x) = 1 - F(x)$ as:

$$S^*(x) = Q(\Phi^{-1}(S(x))). \tag{24}$$

Combining the transform in Equation (22) and the parameter adjustment in Equation (14), the following two-factor model is obtained:

$$S^*(x) = Q(\Phi^{-1}(S(x)) + \lambda), \tag{25}$$

and hence, the risk adjusted premium of a survival function for a loss variable X becomes:

$$\prod_{\lambda}(X) = E\left[\prod_{\lambda}(X)\right] = \int_0^\infty Q(\Phi^{-1}(S(x)) + \lambda)\, dx. \tag{26}$$

Once the transformation parameters are set, it is critical to select the survival function that gives the most accurate potential loss information for a specific type of asset. It is possible to use the developed annual loss function (frequency-loss curve shown in Fig. 2(d)) as the survival function for computing the spread values of indemnity-based CAT bond contracts. Thus the survival function of an indemnity-based CAT bond can be defined as:

$$S(L) = f_L = f_{\mathrm{DBE}} \left| \frac{L}{L_{\mathrm{DBE}}} \right|^d. \tag{27}$$

The spread value for indemnity-based CAT bonds for a particular type of underlying structural asset can be calculated as:

$$\prod_{\lambda}(L) = E\left[\prod_{\lambda}(L)\right] = \int_0^\infty Q\left(\Phi^{-1}\left(f_{\mathrm{DBE}}\left|\frac{L}{L_{\mathrm{DBE}}}\right|^d\right) + \lambda\right) dL. \tag{28}$$

4.1. Losses Given Default

A closed-form solution for the loss distribution of investors can be derived by using risk adjusted survival function $S^*(x)$. Recall the relationship between the cumulative distribution function and the survival function, $S^*(x) = 1 - F^*(x)$, now the probability density function of losses given default, $f^*(x)$, is a derivative of $F^*(x)$. However, finding the derivative of $F^*(x)$ can be a challenge with the two-factor model. By using the simpler P-H transformation in Equation (21) ($S(x) \to (S(x))^{1/\rho}$) one can get the following relationship:

$$f^*(L) = \frac{-\left(f_{\mathrm{DBE}} \times \left|\frac{L}{L_{\mathrm{DBE}}}\right|^{1/d}\right)^{1/\rho}}{L\rho d}; \quad \text{for } L > L_T, \tag{29}$$

where L_T refers to the default trigger loss.

5. MARKET PRICE OF RISK

The CAT bond market has significantly increased in variety and number of investors over the last decade. The attractive spreads offered by these bonds have been considered as the biggest drivers of this increasing trend. Due to the weak appetite of investors for CAT bonds and unfamiliarity with this new asset class, the issuers had to offer significantly higher yields than for the similar class of corporate bonds. However, the CAT bond market has developed an increasing "know-how," which has helped both issuers and investors to move along the learning curve. Table II summarizes the PFL, PE, and annual spread values of ILS issued between 2000 and 2003.

The ILS data presented in Table II are used to estimate the two-factor model parameters defined in Equation (23). Based on minimizing the mean squared error by using the genetic algorithm (GA), the best-fit two-factor model parameters λ and υ are estimated to be 0.75 and 15, respectively.

A simple linear model, as shown in Equation (30), was fitted to the presented data to observe the sensitivity of offered spread values to PFL and PE (R^2 adjusted = 0.888). The statistical results for transactions between 2000 and 2003 indicate that the spread values are highly sensitive to the probability of occurrence (e.g., PFL) of the default event, which is an important implication in structuring CAT bond contracts.

$$S = 5.15(PFL) + 0.90(PE). \tag{30}$$

6. NUMERICAL ANALYSIS

Assume that the cedant (owner of the facility) requires coverage against financial losses resulting from a potential earthquake for a revenue-generating structural asset (e.g., bridge). This bridge is designed under the current California Department of Transportation (CALTRANS) seismic design codes. The proposed loss model for such type of an asset is illustrated with the parameters estimated to be: $f_{\mathrm{DBE}} = 0.0021$, $L_{\mathrm{DBE}} = 0.05$, $d = -0.6522$, and $k = 3.45$. These parameters are typical for a seismically designed bridge structure located in California.

Example CAT1: The cedant requires coverage if the actual monetary losses to the underlying asset exceed 10% of the replacement cost ($a = 10\%$) and issues CAT1. Losses up to the attachment level are covered by either primary insurance or private funds. This region is also known as the deductible and shown with the shaded area, A, in Fig. 6.

Because investors of CAT1 lose the whole principal in case of default, investors' expected annual

Table II. Data for ILS Issued Between 2000 and 2003

SPV	PFL	PE	S	SPV	PFL	PE	S
	to March 2000			Atlas Re II Class B	1.33%	0.53%	6.84%
Mosaic 2A	1.15%	0.04%	4.08%	Redwood Capital I	0.72%	0.34%	5.58%
Mosaic 2B	5.25%	1.15%	8.36%	Redwood Capital II	0.31%	0.14%	3.04%
Halyard Re	0.84%	0.45%	4.56%	Residential Re 2001	1.12%	0.41%	5.06%
Domestic Re	0.58%	0.44%	3.74%	St. Agatha Re	1.55%	0.87%	6.84%
Concentric Re	0.64%	0.00%	3.14%	Trinom Class A-1	2.42%	0.39%	8.11%
Juno Re	0.60%	0.33%	4.26%	Trinom Class A-2 (Pre)	1.01%	0.43%	4.06%
Residential Re	0.78%	0.26%	3.71%	Redwood Capital I	0.72%	0.72%	7.10%
Kelvin 1stE	12.10%	0.50%	10.97%	Trinom (Pre)	3.11%	3.11%	10.14%
Kelvin 2ndE	1.58%	0.07%	4.82%		Apr 2002–Mar 2003		
Golden Eagle B	0.17%	0.17%	2.99%	Fujiyama	0.88%	0.42%	4.06%
Golden Eagle A	0.78%	0.49%	5.48%	Pioneer A Jun-02	1.59%	0.97%	6.08%
Namazu Re	1.00%	0.32%	4.56%	Pioneer A Dec-02	1.59%	0.97%	5.32%
Atlas Re A	0.19%	0.05%	2.74%	Pioneer A Mar-03	1.59%	0.97%	5.58%
Atlas Re B	0.29%	0.19%	3.75%	Pioneer B Jun-02	1.59%	1.05%	5.07%
Atlas Re C	5.47%	1.90%	14.19%	Pioneer B Sep-02	1.59%	1.05%	5.32%
Seismic Ltd	1.13%	0.47%	4.56%	Pioneer B Dec-02	1.59%	1.05%	5.32%
	Apr 2000–Mar 2001			Pioneer B Mar-03	1.59%	1.05%	4.82%
Alpha Wind FRN	0.99%	0.38%	4.62%	Pioneer C Jun-02	1.59%	0.98%	6.08%
Alpha Wind Prefs	2.08%	0.99%	7.10%	Pioneer C Sep-02	1.59%	0.98%	6.08%
Residential Re	0.95%	0.31%	4.16%	Pioneer C Dec-02	1.59%	0.98%	6.08%
NeHi	0.87%	0.56%	4.16%	Pioneer C Mar-03	1.59%	0.98%	6.08%
MedRe Class A	0.28%	0.17%	2.64%	Pioneer D Jun-02	0.27%	0.19%	1.77%
MedRe Class B	1.47%	0.93%	5.93%	Pioneer D Sep-02	0.27%	0.19%	1.77%
PRIME Hurricane	1.46%	1.08%	6.59%	Pioneer D Dec-02	0.27%	0.19%	1.77%
PRIME EQEW	1.69%	1.07%	7.60%	Pioneer D Mar-03	0.27%	0.19%	1.77%
Western Capital	0.82%	0.34%	5.17%	Pioneer E Jun-02	1.59%	1.01%	4.31%
Halyard Re	0.84%	0.04%	5.58%	Pioneer E Dec-02	1.59%	1.01%	4.82%
SR Wind ClA-1	1.07%	0.44%	5.83%	Pioneer F Jun-02	1.60%	1.02%	7.60%
SR Wind Cl A-2	1.13%	0.53%	5.32%	Pioneer F Dec-02	1.60%	1.02%	7.60%
NeHi	1.00%	0.87%	4.56%	Pioneer F Mar-03	1.60%	1.02%	7.60%
Gold Eagle 2001	1.18%	1.18%	7.10%	Residential Re 2002	1.12%	0.40%	4.97%
SR Wind Cl B-2	1.13%	1.13%	6.59%	Studio Re Ltd.	1.38%	0.22%	5.17%

[*] All ILS data from Lane (2000, 2001, 2002)—http://www.lanefinancialllc.com

losses ($E(L)_I^{CAT1}$) per invested principal is simply the probability value that losses exceed attachment point (10%); thus $E(L)_I^{CAT1} = 0.504\%$ (Fig. 6). To compute the spread value, the market adjusted survival curve, $S^*(x)$, for CAT1 investors is then constructed and shown in Fig. 7.

Recall that all probability values used in the analyses are the exceedance probabilities ($P\{X > x\}$). By using the risk adjusted survival curve ($S^*(x)$), $P\{L > 10\%\}$ is calculated as 4.4%. This value is related to the perceived risk, or in case of the CAT1 bond, its default probability implied by the market and hence the market-implied risk premium (spread) is estimated to be $S_{CAT1} = 4.4\%$ or 440 basis points.

Example CAT2: Now consider the case where the cedant issues CAT2 for the same coverage required in the CAT1 example with same attachment point ($a = 10\%$) and $e = 100\%$ (exhaustion point where the investors lose entire principal). Fig. 8 shows the survival curve of the cedant, $S(x)$, and the risk adjusted transform, $S^*(x)$, for investors.

The following are numerical findings for CAT2 investors: $E(L)_I^{CAT2} = 0.0668\%$ and $S_{CAT2} = 1.197\% = 119.7$ basis points. Notice the significant decrease in $E(L)_I^{CAT2}$ compared to $E(L)_I^{CAT1}$. This is the consequence of increased recovery rate (level of principal protection) provided to CAT2 investors.

7. MANAGERIAL IMPLICATIONS

CAT bonds are highly attractive securities for investors in terms of returns. Using the results obtained from the numerical examples presented in the

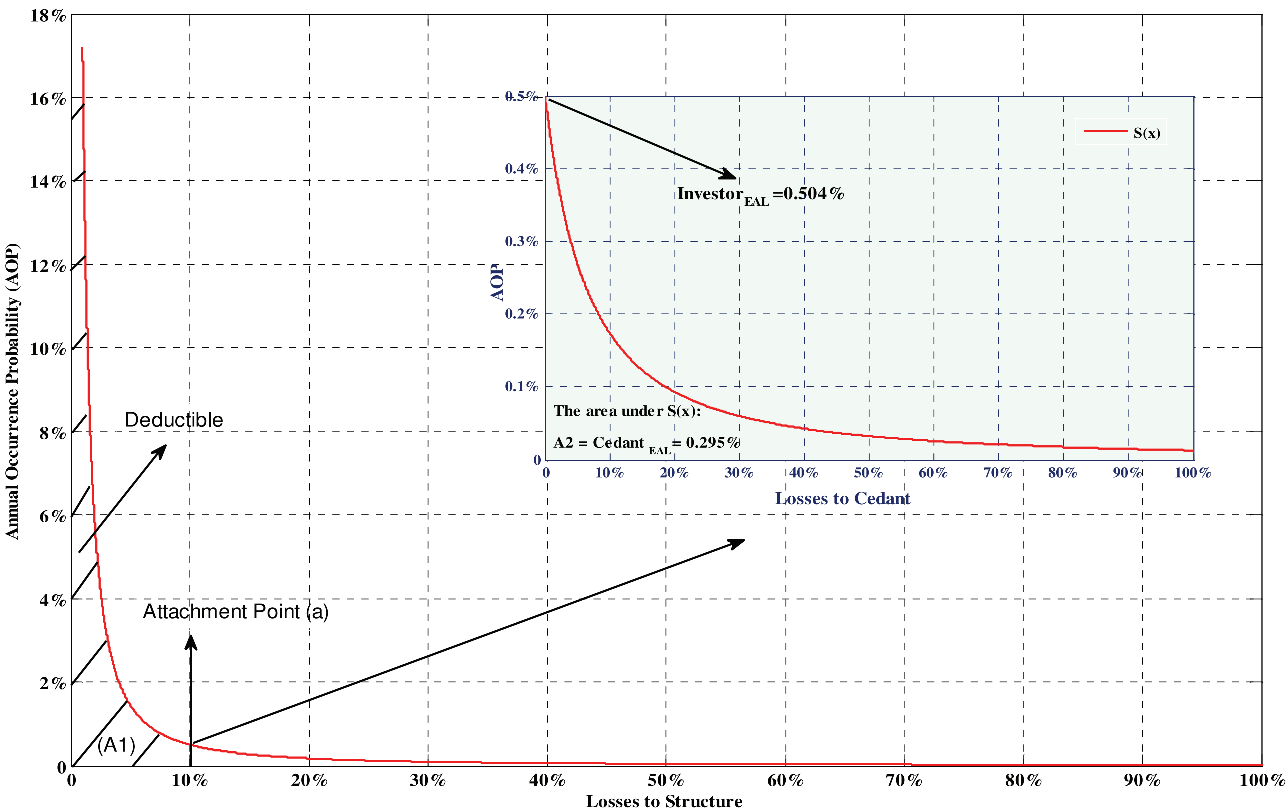


Fig. 6. Losses to cedant and $E(L)$ to investors with CAT1.

previous section, it is possible to compare spreads of analyzed CAT bonds with corporate bond spreads with respect to the underlying risk. Fig. 9 illustrates $E(L)$ and implied spreads for both corporate and CAT bonds.

Fig. 9 clearly indicates that both CAT1 and CAT2 offer higher spreads than similar graded corporate bonds relative to the underlying risk. In fact, the structural loss analysis shows that the estimated expected losses do not support the rating. The hypothesis of the study is that this discrepancy can be eliminated if there is more information and transparency in how engineering parameters are mapped into the expected losses. The current “black box” approach results in inflated risks and makes this class of ILS less liquid.

A key component governing both $E(L)$ and the spread estimates is the value of the default trigger loss, or the attachment point. This point can act as a deductible limit to adjust the risk profile of the bond. In fact, the issuers can adjust it based on needed coverage and available funds. Fig. 10 presents the sensitivity of $E(L)$ and S to default trigger losses (attachment point). It can be observed from Fig. 10 that both $E(L)$ and S decrease as we increase the attachment point. For a 100% increase in the trigger loss point (from 10% to 20%), $E(L)$ decreases by 40% and S decreases by 25%.

Further, we have considered the impact of time to maturity (duration of CAT bonds) on market-implied spread values. Fig. 11 illustrates the theoretical term structure of CAT2 bond. When compared to term structure of corporate bonds, CAT2 bonds show similar behavior.

Fig. 12 illustrates the comparison of $E(L)$ of corporate bonds for given bond ratings and $E(L)$ of CAT2 over the 5-year range (for corporate bond data, see Hamilton[23]). Evidently, from Fig. 12, CAT2 bonds should be rated as either A or Baa whole letter grade according to Moody’s rating standard. It is notable that the marginal increase in $E(L)$ over time is very similar to higher investment grade bonds (e.g., Aaa, Aa, and A rated corporate bonds) rather than the Baa class.

Finally, one of the most important aspects of this study is determining how engineering parameters

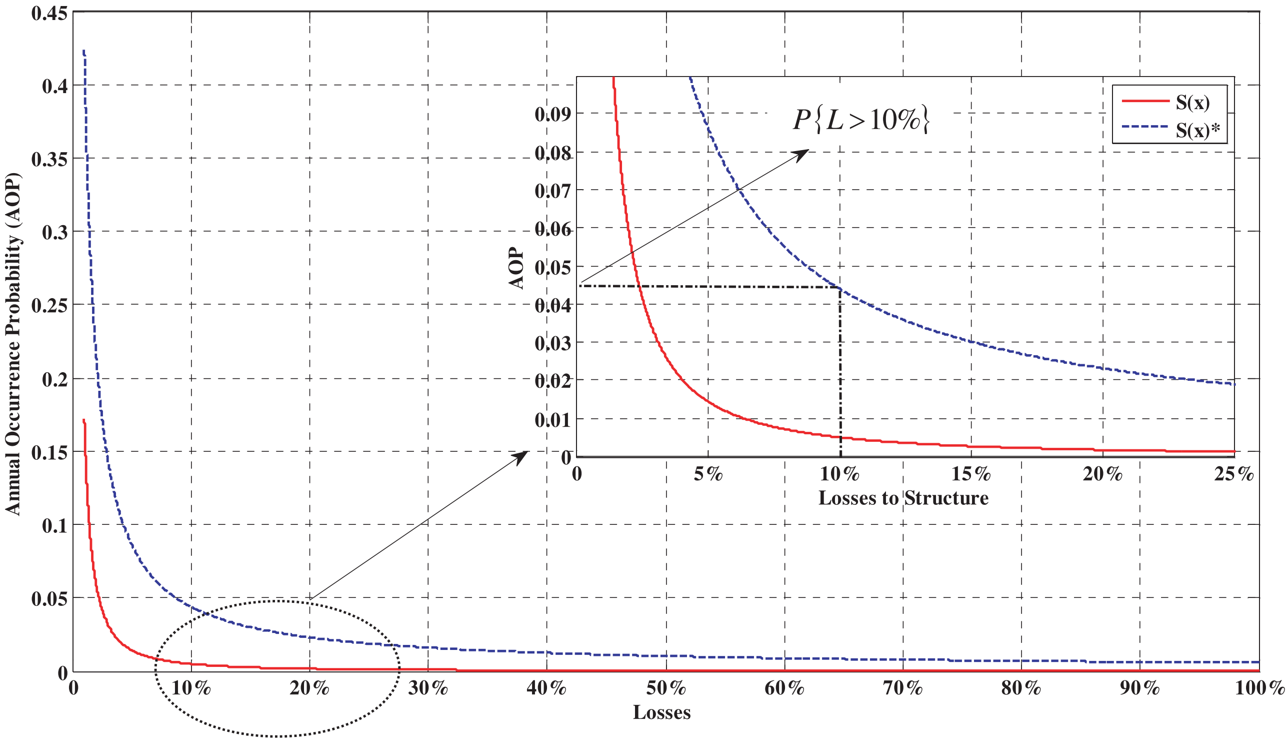


Fig. 7. $S(x)$ and $S^*(x)$.

affect CAT bond risk loading (e.g., spread). Traditional seismic design philosophy puts little emphasis on design solutions that go beyond legal and code-imposed requirements. However, as insurance cost can be a significant component of life-cycle costs for the facilities in hazard-prone areas (e.g., California, Indonesia, etc.), seismic design philosophy must be changed to consider optimal level of design variables that minimize not only the first (i.e., construction) cost, but also the total life-cycle cost including insurance cost. In Fig. 13, we show the impact of changes in engineering design variables over the spread of CAT bonds issued for the bridge structure. Six parameters showed changes markedly higher than the 10% variation, namely; IM_{DBE}, b, k, c, θ_{DBE}, and θ_c. These parameters can be grouped to represent seismic hazard demand (IM_{DBE}, k), structural response demand (b, θ_{DBE}), and structural damage capacity (c, θ_c). Thus, it is critical to have dependable local hazard data and specific structure behavioral models to accurately predict the expected losses and market-implied spread.

The results further show that specific design types and structural material characteristics can significantly help in reducing the cost of mitigating structural losses from earthquakes. The impact of structural response and damage potential parameters (b and c) is almost as important as the hazard parameter (IM). This is an important implication for the structural designers, who should exercise special attention when evaluating designs and developing specifications.

8. IMPLEMENTATION BARRIERS

The issuers of CAT bonds often state that this securitization is very costly, and investors are reluctant to purchase catastrophe-linked securities despite the offered attractive premiums, which are sometimes more than 500 basis points over the LIBOR.[24] The lack of liquidity and relative novelty may have substantially contributed to this high premium demand. The highly risk-averse behavior raises the question whether the problem is about the current offerings or whether there are some psychological barriers associated with the risks of catastrophe-linked securities.

Froot[25] states that the global catastrophe risk distribution system fails to spread the risks of major

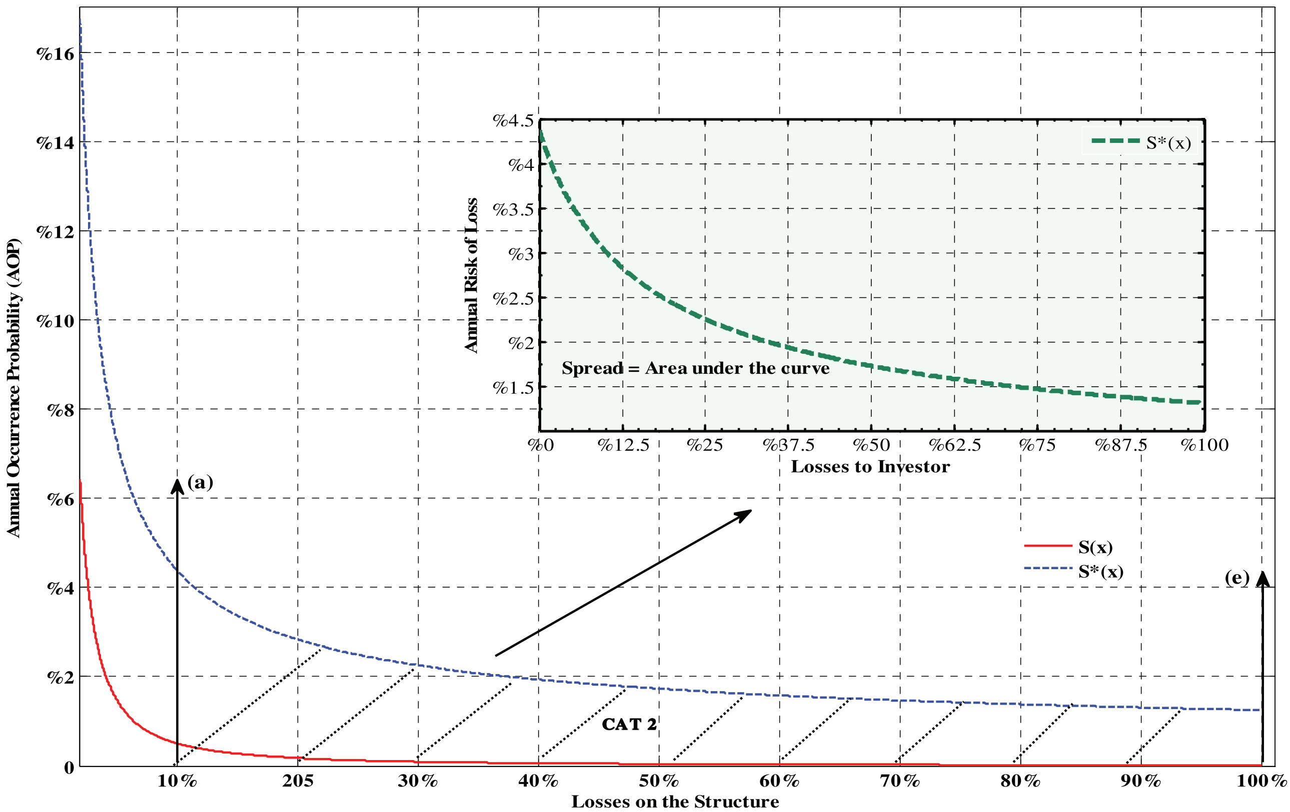
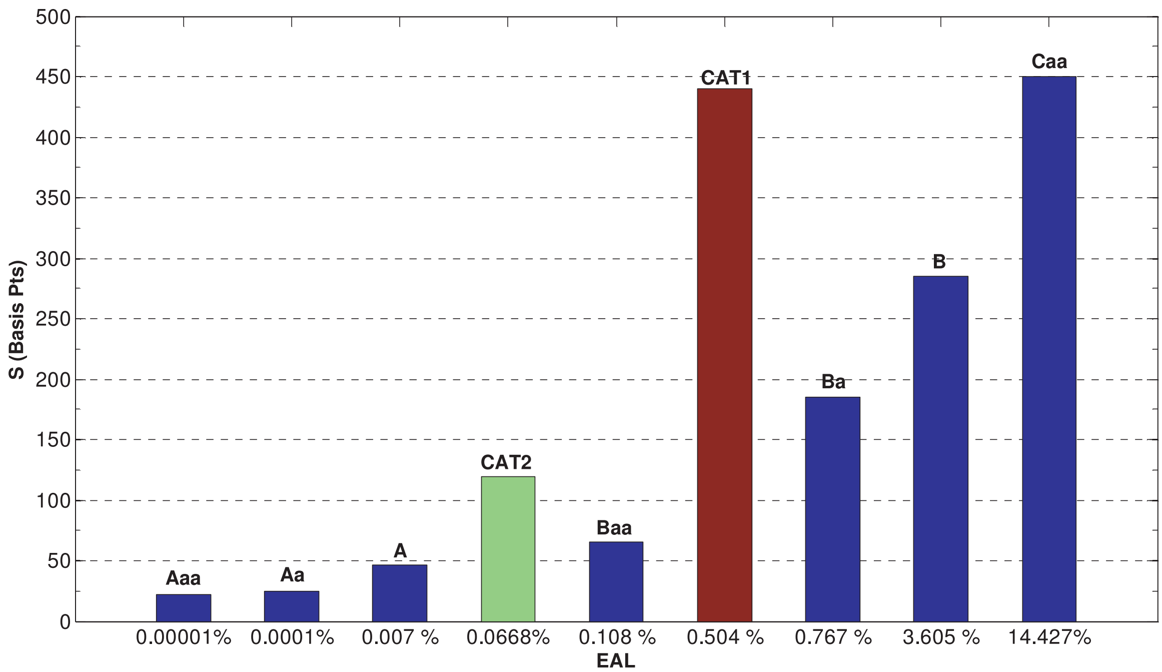


Fig. 8. $S(x)$ and $S^{*}(x)$ of bridge structure with CAT2 coverage.



*All spread data from Reuters—http://www.bondsonline.com

Fig. 9. $E(L)$ vs. spread

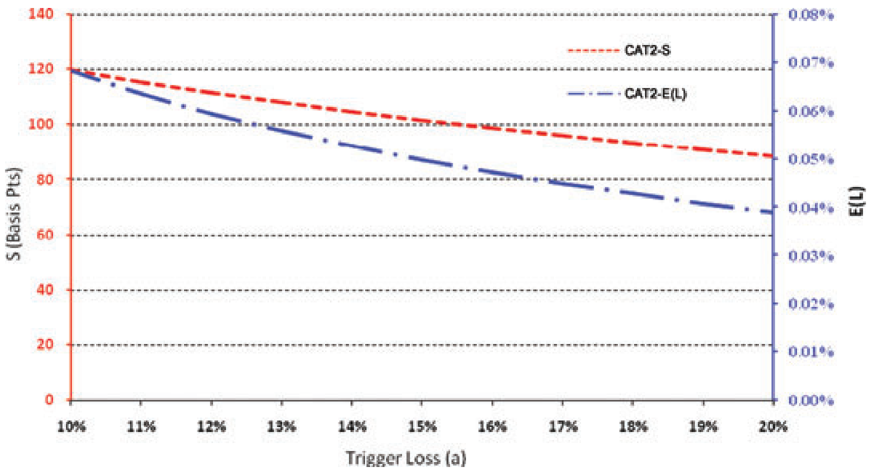


Fig. 10. CAT2 *E(L)* & S vs. trigger loss (a).

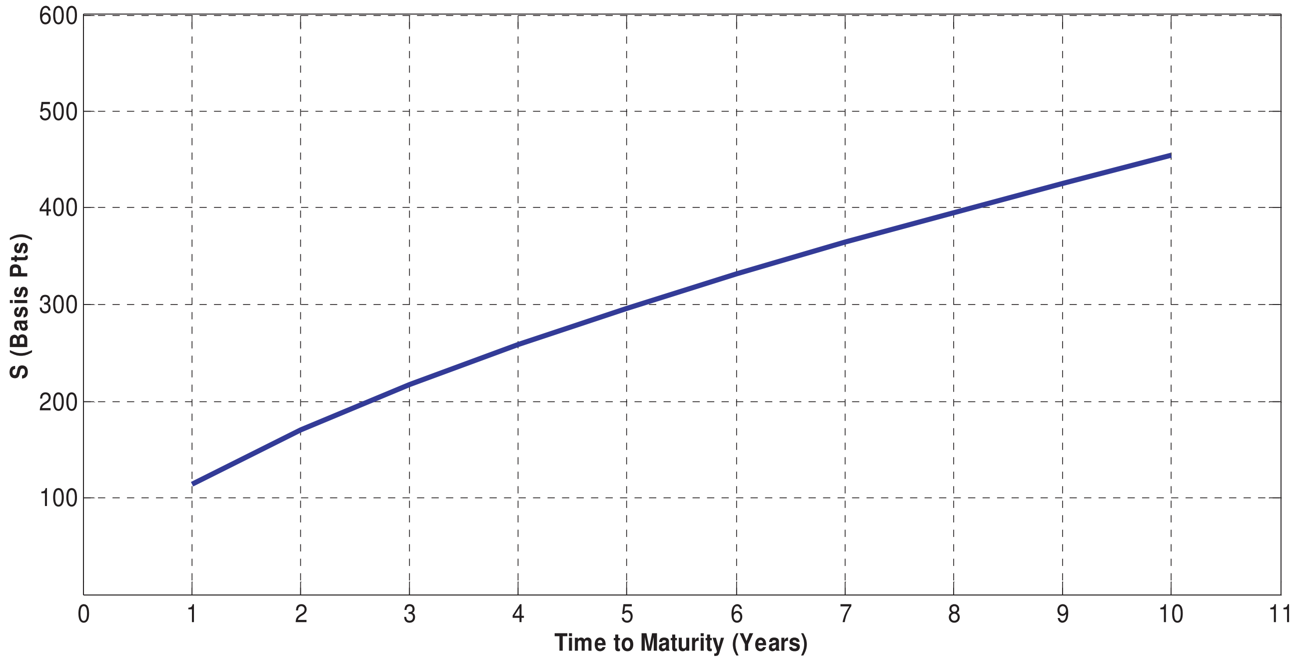


Fig. 11. Theoretical term structure of CAT2 bond.

catastrophes and hence high costs appear due to the consequent inefficient risk sharing. The major barriers that prevent risks from being properly spread can be summarized as: (1) insufficiency of capital within the global reinsurance industry, (2) inefficiency of the corporate form for reinsurance, (3) presence of moral hazard at the insurer level, and basis risk at the investor level, and (4) behavioral factors. Involving capital markets via securitization would address the first barrier. As to the second barrier, ILS may provide a lower cost of managing catastrophe risk than raising large amounts of equity capital. As to the third barrier, to the point that perceived basis risk is too large by some hedgers, the reinsurers can overcome this problem by creating a diversified portfolio of primary insurance contracts and hence hedge the residual risk in the CAT-linked derivatives markets.[12] Alternatively, the residual risk due to the difference between the estimated amount of loss and the actual claims suffered by the issuer can be either retained by the issuer or hedged with a customized reinsurance contract.[26] As far as overcoming the moral hazard risk is concerned, the straightforward and simple CAT-loss estimation models can create a

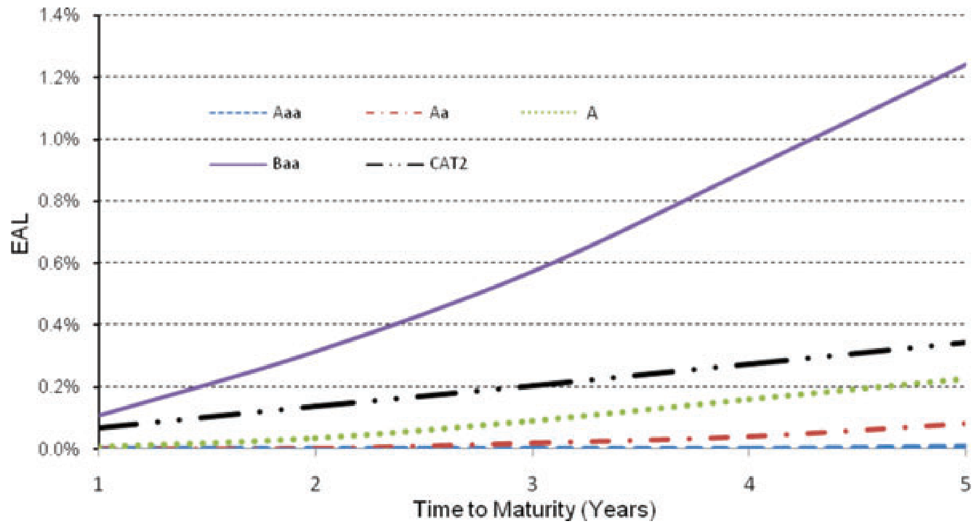


Fig. 12. $E(L)$ vs. time to maturity.

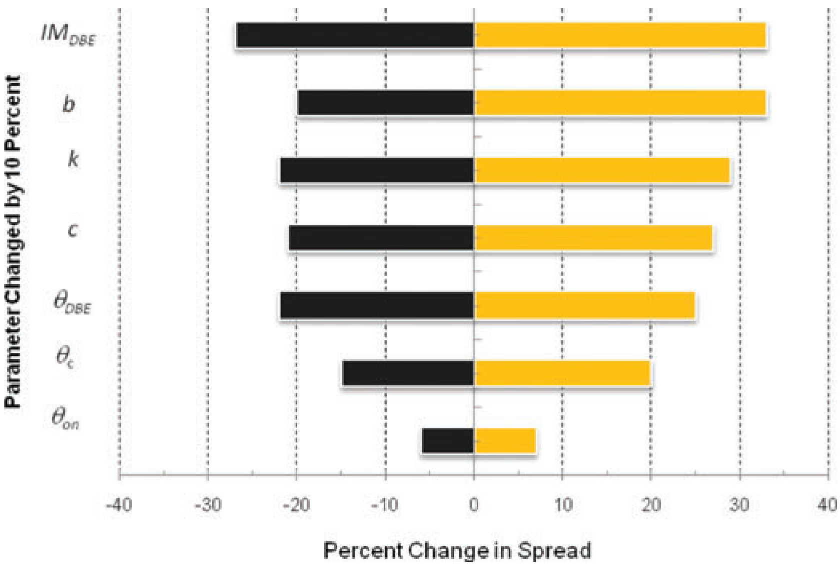


Fig. 13. Sensitivity analysis for engineering design parameters.

transparent link between the potential losses to the issuers and resulting investor risks.

However, addressing the fourth barrier is rather difficult as it requires understanding of the risk preferences of investors. The ambiguity aversion of the investors is one of the behavioral barriers that remarkably increases the cost of catastrophe-linked securitizations.[27] The investors demand higher spreads if there is significant ambiguity associated with the risk, which is often the case with the natural hazards and "black box" approaches for estimating losses. Further, investors may overweight small probabilities (i.e., statistically rare events like catastrophes). Several studies showed that for two alternative investments having the same expected loss values, preference is given to a sure small loss rather than a very small chance of a relatively large loss.[27] In a conventional pricing model a potential risk with a very small probability would result in a comparably small risk loading and hence premium; however, as the decision weighting function of the investors overweighs the small probabilities, investors demand a higher return.

Typically, investors are reluctant to invest the effort and time required to understand the potential risks of the new securities. Not surprisingly, the cost of learning increases as the complexity of the products increases. In this setting, the simplified asset-specific engineering approach presented in this article addresses this barrier as it creates a straightforward linkage between engineering design characteristics and damage potential of the underlying asset. This modeling feature enables investors to analyze the risks based on the observable physical parameters of the underlying structure.

9. CONCLUSIONS AND RECOMMENDATIONS

This article introduces an asset-specific engineering approach for determining market-implied spread of CAT bonds. The implementation of a simplified closed-form engineering model to a bond valuation process creates a transparent procedure that should increase the confidence in the estimates of potential losses and the interest in securitization of natural hazards. Further, being able to determine the value of CAT bonds for a particular structure type ensures the life-cycle design considerations and more effective management practices for the underlying asset.

In this study, we have implemented the four-step engineering loss model to compute structural and financial losses. The results indicated that the four-step model can be integrated with financial valuation methods to compute financial indicators such as spread, rating, and others. However, this integration is a two-way street. The structural engineers can use the developed model to support evaluation of design alternatives to make sound life-cycle analysis decisions, including possible risk transfer strategies for different types of assets and coverage needs. Furthermore, the model allows comparison of all available risk transfer instruments such as primary insurance, reinsurance, and others.

One of the concrete findings of this study is that well designed structures are inherently safe and provide a good degree of protection. As verified in the analyses, such well designed structures reduce the required spread values. However, the analysis showed that the current spreads are significantly higher relative to the expected annual losses and are grossly conservative.

Even though CAT bond markets have showed a growing trend, there are myriad remaining issues requiring the attention of the research community. Some of the remaining issues are: (1) from the investor's perspective, the basis risk, adverse selection, and moral hazard are important factors and should be further investigated, (2) the demand surge such as demand for building material and labors after catastrophes should be considered in the analysis for a better estimation of needed funds, (3) the initial wealth and the expected future cash flow of both investors and issuers influence the decision making process; such impacts should be modeled and included in pricing framework, and (4) although this work considers the structural component of facility losses for computing market-implied spread, it does not account for losses on nonstructural elements. The loss model presented in the article has the capability for incorporating nonstructural components and can therefore be extended. All of these are subjects for future work.

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