Multi frequency components generation using cascaded time lenses based on space-time duality

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Abstract: We simulate and experimentally demonstrate a cascaded time lenses scheme to generate multi frequency components with frequency spacing tunable. The two time lenses are used to implement frequency-to-time mapping and spectral Talbot effect, respectively.
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1. Introduction

Multi coherent frequency components play an important role in optical communications and measurements due to its wide applications [1]. According to the space-time duality, we propose to use cascaded time lenses to generate multi frequency components and further control the frequency spacing.

In this study, we demonstrate a scheme to realize frequency components reproduction from a single tone based on cascaded time lenses. The first time lens along with a spool of single mode fiber (SMF) is used to implement frequency-to-time mapping to generate original frequency components. To reproduce more frequency components, spectral Talbot effect is employed to divide the frequency spacing based on the second time lens. The frequency spacing division factor is an integer and has been realized from 2 to 10 in the experiment.
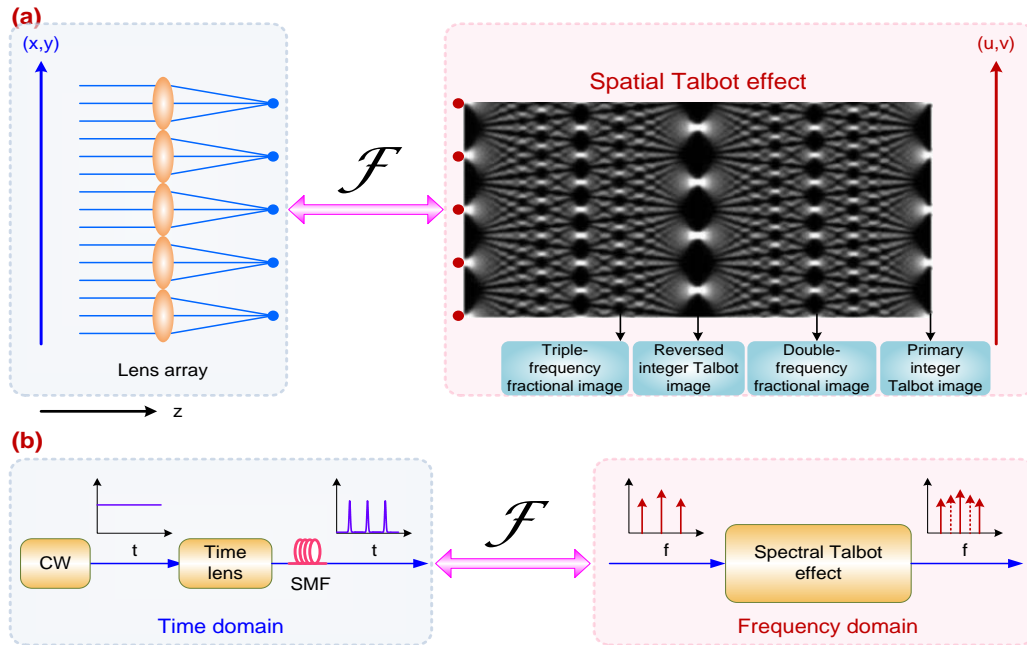
2. Operating principle and simulation results

Fig. 1. (a), Spatial equivalent of the experimental setup, resulting in multiple of spots. (b), Schematic of the frequency reproduction configuration.

A schematic of the frequency components reproduction based on space-time duality is presented in Fig. 1. As shown in Fig. 1(a), a monochromatic planar wave, coordinated by (x, y) is incident from the left. After passing through a lens array, periodical spatial spots are generated at the focal plane of the lens array. Then the angular spectra can be obtained with spatial Fourier transformation, which is coordinated by (u, v). As we know, the angular spectra of these periodical spots are also periodical spots. According to the classical scalar diffraction theory, exact or multiple imaging of these periodical spots in plane (u, v) will be formed at specific distances, which is known as integer Talbot effect or fractional Talbot effect [2]. In this way, one can obtain multi spots from the original plane wave.

Similarly, Fig. 1(b) shows a time domain equivalent of the spatial schematic. According to the space-time duality, the spatial lens array can be replaced by a time lens, which is a periodical parabolic phase modulation, and spatial

paraxial diffraction of beams is mathematically equivalent to the first-order temporal dispersion of optical pulses [3]. Thus, a continuous wave (CW) light is firstly parabolic phase modulated, then passes through a dispersive medium, i.e., a spool of SMF, resulting in a short pulse train. In this process, the output optical field can be expressed as

$$E_{out}(t) = f(t) \cdot \exp(jat^2) * \exp(jbt^2)$$
$$= \int f(\tau) \cdot \exp(ja\tau^2 + jb\tau^2 - jb2t\tau) d\tau \cdot \exp(jbt^2) \quad (1)$$

where $f(t)$ represents CW light and its Fourier transformer is written as $F(\omega)$, $b = -1/2\beta_2 L$, β_2 represents the SMF group velocity dispersion and L is the length of the SMF. When $a = -b$, Eq. (1) can be simplified with

$$E_{out}(t) = \int f(\tau) \cdot \exp(-jb2t\tau) d\tau \cdot \exp(jbt^2)$$
$$= F(2bt) \cdot \exp(jbt^2) \quad (2)$$

From Eq. (2) we can see that, the output optical field is a scaled replica of the input spectrum, which is the well-known frequency-to-time mapping. Since the CW light has an impulse function in the frequency domain, the output temporal signal after the SMF is a short pulse train, whose spectrum consists of multi frequency components.

Similar to the spatial method, to achieve more frequency components in frequency domain, we employ spectral Talbot effect. Spectral Talbot effect occurs when a coherent periodic frequency comb is temporally phase modulated, which can be expressed as $h(t) = \exp(i\varphi t^2 / 2)$ and φ is the linear chirp coefficient of this time lens process [4]. The spectral Talbot effect condition can be expressed as

$$\varphi = \pm \frac{s}{m} \frac{\omega_r^2}{4\pi} \quad (3)$$

where s and m are mutually prime integer numbers. Under this parabolic phase modulation condition, the frequency spacing of the frequency comb is divided by a division factor m.

Figure 2 shows the simulated results. A CW light is first phase modulated by a parabolic radio frequency (RF) signal (green dot line in Fig. 2(a)), which results in multi frequency components as shown in Fig. 2(b). Figures 2(c) and 2(d) are the output waveform and spectrum of the proposed scheme with SMF in the circuit, while Figs. 2(e) and 2(f) are the output waveform and spectrum of the proposed scheme without SMF in the circuit. From Figs. 2(c)-2(f), we can see that although the output waveforms are different, the output spectra have both the frequency spacing division with a factor of 2. Moreover, calculated from Eq. (2), the length of the SMF should be around 740.12 km. In practical implementation, it is too long and will lead large loss and the scheme is bulky. Thus we should omit SMF in the experiment.
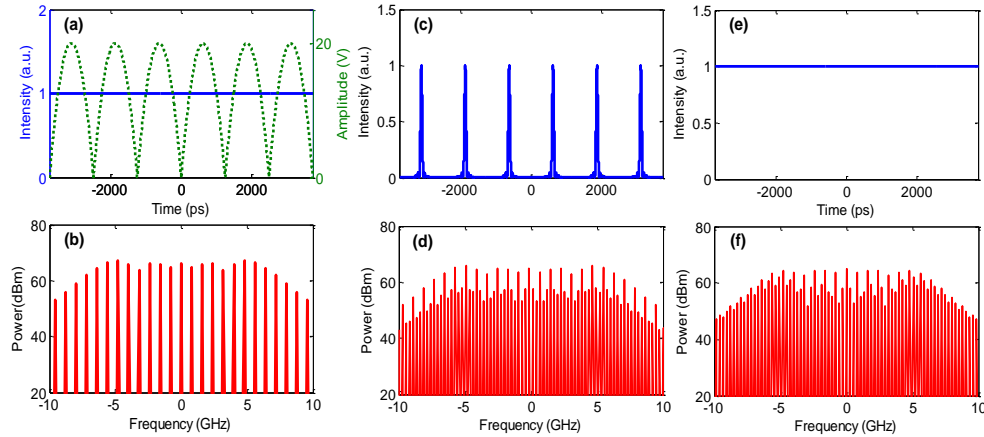

Fig. 2. Simulation results. (a) is the waveforms of input CW light (blue solid line) and parabolic phase modulation signal (green dot line). (b) is the generated multi frequency components after the first time lens. (c) and (d) are the generated waveform and spectrum of the second time lens with SMF in the scheme. (e) and (f) are the generated waveform and spectrum of the second time lens without SMF in the scheme.

3. Experimental setup and results

Figure 3 shows the experimental setup of the proposed frequency reproduction scheme. A CW light is emitted by a tunable laser source (TLS). And then the CW light is split into two parts by a 50:50 coupler. One acts as signal light to generate multi frequency components by cascaded time lenses and another is used as local oscillator (LO). The signal light is emitted to cascaded phase modulators (PMs). An electrical arbitrary waveform generator (AWG) is employed to produce the required periodic parabolic RF signals to be applied on the PMs to realize time lenses process. In order to obtain the corresponding suitable parabolic phase modulation, the two channels of AWG need to

be properly synchronized. So an optical delay line (ODL) and a RF attunes (ATT) are used to temporally align each period with the corresponding modulation time slot and get a suitable modulation amplitude, respectively. The generated multi frequency components finally are mixed with LO in the balanced photo-detector (BPD) for frequency down-conversion. Multiple intermediate frequencies (IFs) are generated between them and recorded by an electrical spectrum analyzer (ESA). The BPD along with ESA is just a method to observe the generated spectrum, and only half of the spectrum is shown in the ESA.
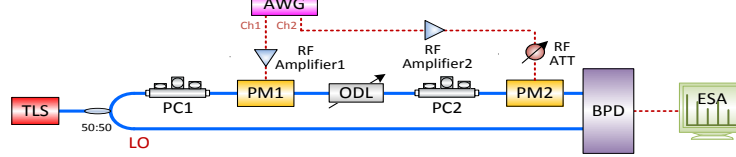

Fig. 3. Schematic diagram of the experimental setup.

The measured output spectra are depicted in Fig. 4. And the predicted spectral Talbot effects are observed, leading to the expected frequency spacing division processes by the corresponding frequency spacing division factor m. Figure 4(a) shows the output periodic frequency components of PM1, which acts as the original multi frequency components with 0.8 GHz spacing to implement spectral Talbot effect. And Figs. 4(b)-4(j) are the corresponding output spectra when the factor m equals to 2-10, respectively. From Figs. 4(b)-4(j), we can see that with the factor m increasing, the number of spectral lines increases and the energy of each original spectral line decreases. It can be seen as the energy redistribution of the original frequency components shown in Fig. 4(a).
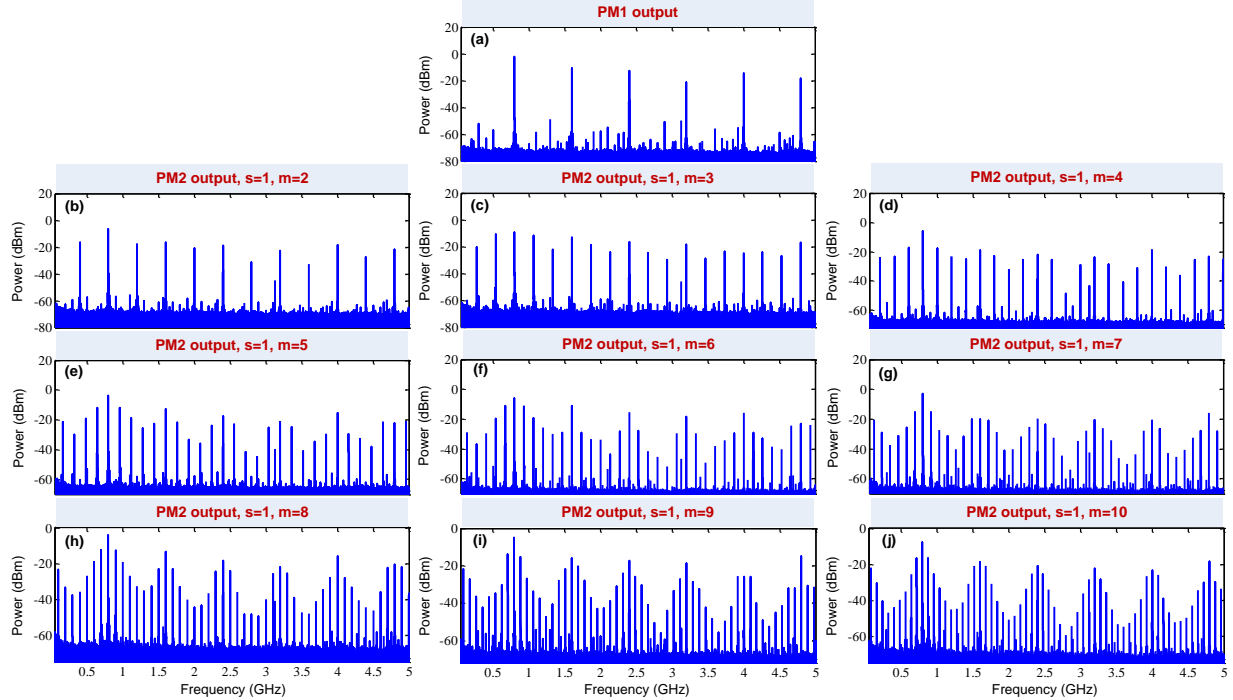

Fig. 4. Experimental results. (a) is the output spectrum of the first PM. (b)-(j) are the output spectra of PM2 with m=2-10, respectively.

4. Conclusions

In this paper, we proposed a cascaded lenses scheme to generate multi frequency components from a single tone with frequency spacing tunable. The two time lenses are used to implement frequency-to-time mapping and spectral Talbot effect, respectively. The two time lenses in our scheme are implemented by phase modulators driven with parabolic radio frequency signals generated by an electrical arbitrary waveform generator.

5. References

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