# 1

$f(t)=f(t+T)$ $\omega_0=\frac{2\pi}{T}$

## 1.1

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$$f(t)=a_0+\sum_{n=1}^{\infty}[a_n\cos(n\omega_0 t)+b_n\sin(n\omega_0 t)]$$

-

$$f(t)=\sum_{n=-\infty}^{\infty}c_n e^{jn\omega_0 t}$$

$$c_n=\frac{1}{T}\int_T f(t)e^{-jn\omega_0 t}dt$$

## 1.2

- $f(t)$

$$c_0=a_0,\quad c_n=\frac{a_n-jb_n}{2},\quad c_{-n}=c_n^*$$

- $af(t)+bg(t)\leftrightarrow ac_n+bd_n$ - $f(t-t_0)\leftrightarrow c_n e^{-jn\omega_0 t_0}$ - $f(t)$ $\Rightarrow$ $c_{-n}=c_n^*$ -

$$\frac{1}{T}\int_T|f(t)|^2dt=\sum_{n=-\infty}^{\infty}|c_n|^2$$

# 2

## 2.1

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$$F(\omega)=\mathcal{F}[f(t)]=\int_{-\infty}^{\infty}f(t)e^{-j\omega t}dt$$

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$$f(t)=\mathcal{F}^{-1}[F(\omega)]=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)e^{j\omega t}d\omega$$

## 2.2

1. $\mathcal{F}[af(t)+bg(t)]=aF(\omega)+bG(\omega)$ 2. $\mathcal{F}[f(t-t_0)]=F(\omega)e^{-j\omega t_0}$ 3. $\mathcal{F}[f(t)e^{j\omega_0 t}]=F(\omega-\omega_0)$ 4. $\mathcal{F}[f(at)]=\frac{1}{|a|}F\left(\frac{\omega}{a}\right)$ 5. $\mathcal{F}[F(t)]=2\pi f(-\omega)$ 6. $\mathcal{F}\left[\frac{d^n f(t)}{dt^n}\right]=(j\omega)^n F(\omega)$ $f(\pm\infty)=0$ 7. $\mathcal{F}[t^n f(t)]=j^n\frac{d^n F(\omega)}{d\omega^n}$ 8. $\mathcal{F}\left[\int_{-\infty}^{t}f(\tau)d\tau\right]=\frac{F(\omega)}{j\omega}+\pi F(0)\delta(\omega)$ 9. $\mathcal{F}[f(t)*g(t)]=F(\omega)G(\omega)$ 10. $\mathcal{F}[f(t)g(t)]=\frac{1}{2\pi}F(\omega)*G(\omega)$ 11.

$$\int_{-\infty}^{\infty}|f(t)|^2dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}|F(\omega)|^2d\omega$$

12. $f(t)$ $F(-\omega)=F^*(\omega)$

## 2.3

- $f(t)$ $\Rightarrow F(\omega)$ - $f(t)$ $\Rightarrow F(\omega)$ - $f(t)$ $\Rightarrow F(\omega)$ - $f(t)$ $\Rightarrow F(\omega)$

## 2.4

- $f_e(t)=\frac{f(t)+f(-t)}{2}\Leftrightarrow F_e(\omega)=\mathrm{Re}\{F(\omega)\}$ - $f_o(t)=\frac{f(t)-f(-t)}{2}\Leftrightarrow F_o(\omega)=j\mathrm{Im}\{F(\omega)\}$ - $f(t)=f_e(t)+f_o(t)$ $F(\omega)=F_e(\omega)+F_o(\omega)$

## 2.5

$E=\int_{-\infty}^{\infty}|f(t)|^2dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}|F(\omega)|^2d\omega$ - $P=\frac{1}{T}\int_T|f(t)|^2dt=\sum_{n=-\infty}^{\infty}|c_n|^2$ $c_n$ - $P=\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}|f(t)|^2dt$

## 2.6

$$\delta(t)\leftrightarrow 1$$
$$1\leftrightarrow 2\pi\delta(\omega)$$
$$e^{j\omega_0 t}\leftrightarrow 2\pi\delta(\omega-\omega_0)$$
$$\cos(\omega_0 t)\leftrightarrow \pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$$
$$\sin(\omega_0 t)\leftrightarrow j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$$
$$e^{-at}u(t)\leftrightarrow \frac{1}{a+j\omega}\quad (a>0)$$
$$te^{-at}u(t)\leftrightarrow \frac{1}{(a+j\omega)^2}\quad (a>0)$$
$$e^{-a|t|}\leftrightarrow \frac{2a}{a^2+\omega^2}\quad (a>0)$$
$$u(t)\leftrightarrow \pi\delta(\omega)+\frac{1}{j\omega}$$
$$\mathrm{rect}\left(\frac{t}{\tau}\right)\leftrightarrow \tau\cdot\frac{\sin(\omega\tau/2)}{\omega\tau/2}$$
$$\mathrm{sinc}(t)=\frac{\sin(\pi t)}{\pi t}\leftrightarrow \mathrm{rect}\left(\frac{\omega}{2\pi}\right)$$
$$\mathrm{tri}\left(\frac{t}{\tau}\right)\leftrightarrow \tau\cdot\frac{\sin^2(\omega\tau/2)}{(\omega\tau/2)^2}$$
$$\sum_{n=-\infty}^{\infty}\delta(t-nT)\leftrightarrow \frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta(\omega-k\omega_0)$$

## 2.7

- $f(t)=\sum_{n=-\infty}^{\infty}c_n e^{jn\omega_0 t}$

$$F(\omega)=2\pi\sum_{n=-\infty}^{\infty}c_n\delta(\omega-n\omega_0)$$

- $f(t)=\sum_{k=-\infty}^{\infty}f_0(t-kT)$

$$F(\omega)=F_0(\omega)\cdot\frac{2\pi}{T}\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$$

$$c_n=\frac{1}{T}F_0(n\omega_0)$$

# 3 (DTFT)

## 3.1

$$X(e^{j\omega})=\mathcal{F}_d[x[n]]=\sum_{n=-\infty}^{\infty}x[n]e^{-j\omega n}$$

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$$x[n]=\mathcal{F}_d^{-1}[X(e^{j\omega})]=\frac{1}{2\pi}\int_{-\pi}^{\pi}X(e^{j\omega})e^{j\omega n}d\omega$$

## 3.2

- $\mathcal{F}_d[ax[n]+by[n]]=aX(e^{j\omega})+bY(e^{j\omega})$ - $\mathcal{F}_d[x[n-n_0]]=e^{-j\omega n_0}X(e^{j\omega})$ - $\mathcal{F}_d[e^{j\omega_0 n}x[n]]=X(e^{j(\omega-\omega_0)})$ - $X(e^{j(\omega+2\pi)})=X(e^{j\omega})$ - $x[n]$ $X(e^{-j\omega})=X^*(e^{j\omega})$ - $\mathcal{F}_d[nx[n]]=j\frac{dX(e^{j\omega})}{d\omega}$ - $x_k[n]=\begin{cases}x[n/k] & k|n\\ 0\end{cases}$ $X_k(e^{j\omega})=X(e^{jk\omega})$ - $\mathcal{F}_d[x[n]*y[n]]=X(e^{j\omega})Y(e^{j\omega})$ - $\mathcal{F}_d[x[n]y[n]]=\frac{1}{2\pi}X(e^{j\omega})*Y(e^{j\omega})$

## 3.3

- $x[n]$ $\Rightarrow X(e^{j\omega})$ - $x[n]$ $\Rightarrow X(e^{j\omega})$ - $x[n]$ $\Rightarrow X(e^{j\omega})$ - $x[n]$ $\Rightarrow X(e^{j\omega})$

## 3.4 DTFT

$$\delta[n]\leftrightarrow 1$$
$$\delta[n-n_0]\leftrightarrow e^{-j\omega n_0}$$
$$a^n u[n]\leftrightarrow \frac{1}{1-ae^{-j\omega}}\quad (|a|<1)$$
$$u[n]\leftrightarrow \frac{1}{1-e^{-j\omega}}+\pi\sum_{k=-\infty}^{\infty}\delta(\omega-2\pi$$
$$\mathrm{rect}_N[n]\leftrightarrow \frac{\sin(\omega N/2)}{\sin(\omega/2)}e^{-j\omega(N-1)/2}$$

# 4

## 4.1

$$F(s)=\mathcal{L}[f(t)]=\int_{-\infty}^{\infty}f(t)e^{-st}dt,\quad s=$$

## 4.2

- $\mathcal{L}[af(t)+bg(t)]=aF(s)+bG(s)$ROC $R_1\cap R_2$ - $\mathcal{L}[f(t-t_0)]=e^{-st_0}F(s)$ - $\mathcal{L}[e^{-at}f(t)]=F(s+a)$ROC$\mathrm{Re}(s+a)\in R$ - $\mathcal{L}[f(at)]=\frac{1}{|a|}F\left(\frac{s}{a}\right)$ROC$\frac{s}{a}\in R$ - $\mathcal{L}[f'(t)]=sF(s)$ROC $R$ - $s$ $\mathcal{L}[tf(t)]=-\frac{dF(s)}{ds}$ROC$R$ - $\mathcal{L}\left[\int_{-\infty}^{t}f(\tau)d\tau\right]=\frac{F(s)}{s}$ROC $R\cap\{\mathrm{Re}(s)>0\}$ - $\mathcal{L}[f(t)*g(t)]=F(s)G(s)$ROC $R_1\cap R_2$ - $f(0^+)=\lim_{s\to\infty}sF(s)$ - $\lim_{t\to\infty}f(t)=\lim_{s\to 0}sF(s)$

### 4.3

$$\begin{aligned}
\delta(t) &\leftrightarrow 1 \quad (\ s\ ) \\
u(t) &\leftrightarrow \frac{1}{s} \quad (\mathrm{Re}(s) > 0) \\
-u(-t) &\leftrightarrow \frac{1}{s} \quad (\mathrm{Re}(s) < 0) \\
e^{-at}u(t) &\leftrightarrow \frac{1}{s+a} \quad (\mathrm{Re}(s) > -a) \\
-e^{-at}u(-t) &\leftrightarrow \frac{1}{s+a} \quad (\mathrm{Re}(s) < -a) \\
te^{-at}u(t) &\leftrightarrow \frac{1}{(s+a)^2} \quad (\mathrm{Re}(s) > -a) \\
-te^{-at}u(-t) &\leftrightarrow \frac{1}{(s+a)^2} \quad (\mathrm{Re}(s) < -a) \\
e^{-a|t|} &\leftrightarrow \frac{2a}{s^2-a^2} \quad (-a < \mathrm{Re}(s) < a) \\
t^n u(t) &\leftrightarrow \frac{n!}{s^{n+1}} \quad (\mathrm{Re}(s) > 0)
\end{aligned}$$

## 5 z

### 5.1

$$X(z) = \mathcal{Z}[x[n]] = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

### 5.2

- $\mathcal{Z}[ax[n] + by[n]] = aX(z) + bY(z)$ROC $R_1 \cap R_2$ - $\mathcal{Z}[x[n-n_0]] = z^{-n_0}X(z)$ROC$R$ $z = 0$ $z = \infty$ - $\mathcal{Z}[a^n x[n]] = X\left(\frac{z}{a}\right)$ROC$|z/a| \in R$ - z $\mathcal{Z}[nx[n]] = -z\frac{dX(z)}{dz}$ROC$R$ - $\mathcal{Z}[x[n] * y[n]] = X(z)Y(z)$ROC $R_1 \cap R_2$ - $\mathcal{Z}[x[n] - x[n-1]] = (1 - z^{-1})X(z)$ROC $R \cap \{z \neq 0\}$ - $\mathcal{Z}\left[\sum_{k=-\infty}^{n} x[k]\right] = \frac{X(z)}{1-z^{-1}}$ROC $R \cap \{|z| > 1\}$ - $x[0] = \lim_{z\to\infty} X(z)$ - $\lim_{n\to\infty} x[n] = \lim_{z\to 1}(z-1)X(z)$ $z = 1$

### 5.3 z

$$\begin{aligned}
\delta[n] &\leftrightarrow 1 \quad (\ z\ ) \\
u[n] &\leftrightarrow \frac{z}{z-1} \quad (|z| > 1) \\
-u[-n-1] &\leftrightarrow \frac{z}{z-1} \quad (|z| < 1) \\
a^n u[n] &\leftrightarrow \frac{z}{z-a} \quad (|z| > |a|) \\
-a^n u[-n-1] &\leftrightarrow \frac{z}{z-a} \quad (|z| << |a|) \\
na^n u[n] &\leftrightarrow \frac{az}{(z-a)^2} \quad (|z| > |a|) \\
-na^n u[-n-1] &\leftrightarrow \frac{az}{(z-a)^2} \quad (|z| << |a|) \\
\cos(\omega_0 n)u[n] &\leftrightarrow \frac{z(z-\cos\omega_0)}{z^2 - 2z\cos\omega_0 + 1} \\
\sin(\omega_0 n)u[n] &\leftrightarrow \frac{z\sin\omega_0}{z^2 - 2z\cos\omega_0 + 1}
\end{aligned}$$

## 6 LTI

### 6.1

$$H(s) = \frac{Y(s)}{X(s)} = \mathcal{L}[h(t)]$$

### 6.2

- BIBORe$(p_j) < 0$ - - Re$(p_j) > 0$

### 6.3

$$H(j\omega) = H(s)\Big|_{s=j\omega}$$

- $|H(j\omega)|$ - $\angle H(j\omega)$

### 6.4

$$y(t) = Kx(t - t_d)$$

- $|H(j\omega)| = K$ - $\angle H(j\omega) = -\omega t_d$ - $H(j\omega) = Ke^{-j\omega t_d}$

### 6.5

- $\tau_g(\omega) = -\frac{d\theta(\omega)}{d\omega}$$\theta(\omega) = \angle H(j\omega)$ - $\tau_p(\omega) = -\frac{\theta(\omega)}{\omega}$ - $\tau_g = \tau_p = t_d$

### 6.6

$H(s) = \frac{N(s)}{D(s)}$ $p_i$

$$H(s) = \sum_i \frac{r_i}{s - p_i}, \quad r_i = [(s - p_i)H(s)]_{s=p_i}$$

## 7 LTI

### 7.1

$$H(z) = \frac{Y(z)}{X(z)} = \mathcal{Z}[h[n]]$$

### 7.2

- BIBO$|p_j| < 1$ - - $|p_j| > 1$

### 7.3

$$H(e^{j\omega}) = H(z)\Big|_{z=e^{j\omega}}$$

### 7.4

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}}$$

## 8

### 8.1 s

- $H(s)$ ROC Re$(s) > \sigma_0$ - ROC Re$(p) < 0$ - ROC Re$(s) > \sigma_0$ $\sigma_0 < 0$

### 8.2 z

- ROC $|z| > r$ - ROC$|z| = 1$ $|p| < 1$ - ROC $|z| > r$ $r < 1$

## 9

- $H(s) = H_1(s)H_2(s)$ $H(z) = H_1(z)H_2(z)$ - $H(s) = H_1(s) + H_2(s)$ $H(z) = H_1(z) + H_2(z)$ - $H(s) = \frac{H_1(s)}{1 \pm H_1(s)H_2(s)}$ $H(z) = \frac{H_1(z)}{1 \pm H_1(z)H_2(z)}$+-

## 10 z

### 10.1

— — z — ———————————-——— ——————— — Re$(s) < 0$ — $|z| < 1$ — — — — — Re$(s) > 0$ — $|z| > 1$ —

### 10.2

$$z = e^{sT} \qquad s = \frac{1}{T}\ln z$$

- s z - s z - s z - $-\pi/T < \omega < \pi/T$

## 11

### 11.1

- $x_p(t) = x(t)\sum_{n=-\infty}^{\infty} \delta(t - nT)$ -

$$X_p(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s)), \quad \omega_s = \frac{2\pi}{T}$$

$\omega_s$

### 11.2

- $x(t)$ $\omega_M$$X(j\omega) = 0, |\omega| > \omega_M$ - $\omega_s \geq 2\omega_M$

### 11.3

-

$$H_r(j\omega) = \begin{cases} T & |\omega| < \omega_c \\ 0 & |\omega| \geq \omega_c \end{cases}, \quad \omega_M < \omega_c < \omega$$

- ZOH

$$x_0(t) = \sum_{n=-\infty}^{\infty} x(nT)\mathrm{rect}\left(\frac{t - nT - T/2}{T}\right)$$

$X_0(j\omega) = H_0(j\omega)X_p(j\omega)$ $H_0(j\omega) = T\mathrm{sinc}\left(\frac{\omega T}{2}\right)e^{-j\omega T/2}$ -

$$x_1(t) = \sum_{n=-\infty}^{\infty} x(nT)\mathrm{tri}\left(\frac{t - nT}{T}\right)$$

$X_1(j\omega) = H_1(j\omega)X_p(j\omega)$ $H_1(j\omega) = T\mathrm{sinc}^2\left(\frac{\omega T}{2}\right)$

### 11.4

1. C/D$x[n] = x_c(nT)$

$$X(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\frac{\omega - 2\pi k}{T}\right)$$

2. $Y(e^{j\omega}) = H(e^{j\omega})X(e^{j\omega})$ $\Omega = \omega T$ 3. D/C - $y_c(t) = \sum_{n=-\infty}^{\infty} y[n]\frac{\sin(\pi(t-nT)/T)}{\pi(t-nT)/T}$ - $y_c(t) = \sum_{n=-\infty}^{\infty} y[n]\mathrm{rect}\left(\frac{t-nT-T/2}{T}\right)$ 4. $H_{eff}(j\Omega) = H(e^{j\Omega/T})|\Omega| < \pi/T$

### 11.5

- $\omega_s < 2\omega_M$ "" -

### 11.6

1. / 2. 3. z