+https://tu-dresden.de/ing/informatik/smt/cgv/studium/lehrveranstaltungen/ws1920/cg3
- it is about global illumination, physically based simulation
- phy. rendering
    - Rendering equation
    - Stochastic rendering
    - Realistic materials
    - Cache Based GI Methods
    - sampling techniques
    - path based rendering
    - advanced materials
- phy. simulation
    - Newtonian Physics
    - Contact Handling
    - Particle Based Simulation
    - Rigid bodies
    - Fluids
- radiance vs irradiance
    - radiance是可以积累的，能量流，irr是单位面积上的
    - irr描绘了Radiometry Power
    - radiant exitance or radiosity和irr类似，是不同方向上的，一个入社一个出

*review resources
1. ql from slides
2. ql official questions
3. ql from pic notes
4. active mode!

---keywords
- > qq
- > pp
- > tobecontinue
- > tobere

---preview
- extended know. search online, as prepare and concreate understanding
    - from interest
        - //用于提升兴趣
        - 数值方法
        - 数值优化
        - 计算流体力CFD
            - 基本没有计算流体的基础，在老板的催促下，从零基础到改代码也只用了2个月，我来从一个本科生的低端视角谈谈入门计算流体。
            - 你得搞清楚conservative form的NS equation，因为我们本科学的那种用物质导数写的动量守恒是不好解的。
            - 你会发现其实解NS方程跟wave equation也没啥大的差别。只不过，Ns里面的conserved variable是个vector.
            - 通过Riemann solver求解interface flux，再通过flux correction得到continuous flux。这几个步骤用到的数学知识也就是简单的插值
            - 看Toro那本书也就能懂黎曼求解器了
            - 一些流体力学、传热学的基础知识，几个基本方程要知道，几类边界条件得明白
            - 然后学习一下简单的数值计算，例如如何用数值方法近似计算微分、如何用矩阵的形势表示方程组等
            - 如果是打算自己编程算的话，那么就在高性能数值计算这个方向上深挖，主要就是如何进行大尺度矩阵运算
            - 大尺度矩阵运算
            - 首先最基本的控制方程，不管是欧拉还是NS，层流还是湍流，如果做aeroacoustics还有波动方程和lighthill analogy等等
            - 方保镕的《矩阵论》还是不错的
        - SVBRDF/
    - from slides detail

brdf材质
smoke simulation paper
exploits physics unique to smoke in order to design a numerical method
relatively coarse grids
as compared to the much finer grids used in the computational fluid dynamics literature
We use the inviscid Euler equations in our model
since they are usually more appropriate for gas modeling and less computationally intensive than the viscous NavierStokes equations used by others.
Our model also correctly handles the interaction of
smoke with moving objects.
interaction
smoke is highly complex and turbulent.
Visual smoke models have many obvious applications in
the industry including special effects and interactive games
both be easy to use and produce highly realistic results.
Only recently have researchers in computer graphics
started to excavate the abundant CFD literature
Unfortunately, current CG smoke models are either
too slow or suffer from too much numerical dissipation.
numerical dissipation
---detailview
---essential
pruf-re-ok- slides-intro
Physically
Physically Based Graphics
tt paper http://blog.mmacklin.com/project/flex
Newtonian Physics
Contact Handling
https://research.nvidia.com/publication/interactive-indirect-illumination-using-voxel-cone-tracing
tt papers page 5!

https://developer.download.nvidia.com/SDK/10.5/direct3d/Source/ScreenSpaceAO/doc/ScreenSpaceAO.pdf

+qq Computational fluid dynamics - Wikipedia
a branch of fluid mechanics that uses numerical analysis and data structures to analyze and solve problems that involve fluid flows.
interaction of the fluid (liquids and gases) with surfaces defined by boundary conditions.
complex simulation scenarios such as transonic or turbulent flows
qq what are typicall complicated computing cases? x2
transonic or turbulent flows
wind tunnels.
full-scale testing
aerodynamics and aerospace analysis, weather simulation
fluid flows and heat transfer, and engine and combustion analysis.
The volume occupied by the fluid is divided into discrete cells (the mesh). The mesh may be uniform or non-uniform, structured or unstructured, consisting of a combination of hexahedral, tetrahedral
prismatic, pyramidal or polyhedral elements.
During preprocessing
what we do typically when doing pre processing ?

https://en.wikipedia.org/wiki/Computational_fluid_dynamics#Turbulence_models
Methodology
Boundary conditions are defined. This involves specifying the fluid behaviour and properties
For transient problems, the initial conditions are also defined.
steady-state or transient.
postprocessor
visualization
analysis
Reynolds number turbulent flows
combustion
are recast in a conservative form
and then solved over discrete control volumes.
The finite element method (FEM) is used in structural analysis of solids
qq whats the diff. bet. FVM and FEM
    The finite volume method (FVM)
        is a common approach used in CFD codes
    structural analysis of solids, but is also applicable to fluids

    special care to ensure a conservative solution.
    much more stable than the finite volume approach
    FEM can require more memory and has
        slower solution times than the FVM.
    slower
    more stable
这是“经典”或者说标准的方法，在商用软件和研究用程序中最为常见
有限差分方法。这个方法有历史上的意义而且易于编程。现在只在特殊化的代码中使用

雷诺平均纳维－斯托克斯方程（RANS）是湍流最古老的方法
qq how can we model the fluids?
    * 对于不粘滞流体用欧拉方程，对于粘滞流体用纳维-斯托克斯方程
光滑粒子流体动力学
smoothed-particle hydrodynamics, SPH）
模拟连续介质动力学的计算方法，如固体力学和流体流动。
无网格的拉格朗日方法（即坐标系与流体一起移动）
mechanics of continuum media
拉格朗日乘数法
最优化问题中应用,寻找多元函数在其变量受到一个或多个条件的约束时的极值的方法
    一个有 n 个变量与 k 个约束条件的最优化问题转换为一个解有 n + k 个变量的方程组的解的问题
这种方法中引入了一个或一组新的未知数，
    即拉格朗日乘数，又称拉格朗日乘子，
    或拉氏乘子,比如
    https://zh.wikipedia.org/wiki/%E6%8B%89%E6%A0%BC%E6%9C%97%E6%97%A5%E4%B9%98%E6%95%B0
* 其核心就是把约束变量引入了一个方程之中，不然怎么关联？ 应该还有别的关联方法
qq what is lageridian multiplier? how it was used?
    strategy for finding the local maxima and minima of a function

        subject to equality constraints
    find the stationary points
    The great advantage of this method is that it allows the
        optimization to be solved
        without explicit parameterization in
        terms of the constraints

widely used to solve challenging constrained optimization problems

the method of Lagrange multipliers is generalized by the Karush–Kuhn–Tucker conditions, which can also take into account inequality constraints of the form {\displaystyle h(\mathbf {x})\leq c}{\displaystyle h(\mathbf {x})\leq c}.

it can handle Multiple constraints

Modern formulation

Interpretation of the Lagrange multipliers

Numerical optimization

The critical points of Lagrangians occur at saddle points, rather than at local maxima (or minima)

many numerical optimization techniques, such as hill climbing, gradient descent, some of the quasi-Newton methods,

https://en.wikipedia.org/wiki/Lagrange_multiplier#Multiple_constraints

*

a method to find the local maxima and minima

of a function subject to equality constraints

it can also be generalized to ...

it solves introducing a new variable

拉格朗日乘数法含 n 个变量和 k 个约束的情况

拉格朗日乘数法的正确性的证明

qql proof it!

qql fomula

Lagrange function (or Lagrangian or Lagrangian expression

We introduce a new variable ({\displaystyle \lambda }\lambda)

called a Lagrange multiplier

(or Lagrange undetermined multiplier)

the {\displaystyle \lambda }\lambda term may be either added or subtracted

Multiple constraints

solving {\displaystyle n+M}{\displaystyle n+M} equations in {\displaystyle n+M}{\displaystyle n+M} unknowns

Modern formulation via differentiable manifolds

tt code! IK

hydrodynamics

qq what is the diff. bet. cfd and sph?

cfd indicates computational fluid dynamics

cfd is larger concept

sph is: Smoothed-particle hydrodynamics

sph is a non-grid based method

CFD applications

结构计算方法

一般分为有限元（FEM finite element method）、

离散元（DEM discrete element method）、

还有边界元(EEM)。

FEM

finite element method 有限元方法

是将介质复杂几何区域离散为具有简单几何形状的单元通过单元集成

可以用有限的、相互关联的单元模拟无限的复杂体

无论多么复杂的几何体都能用相应的单元简化，从而建模分析计算出结果。

感觉无处下手的工程问题简单化，这是最大的优点。 有限元法采用矩阵形式表达，编程性高

大变形问题中的网格畸变问题，本质在于单元插值造成的

采用 distinct element method 是为了和连续介质力学中的 finite element method 相区别

DEM

Discrete Element Method

颗粒材料和散装固体行业
复杂散料运输工程的虚拟设计和仿真
举几个运用到离散元的例子，比如矿业中的筛分和运输，
转运站的设计，食物和药品的混合与干燥等
https://www.youtube.com/watch?v=5JcFgj2gHx8
避免了堵塞、扬尘、跑偏、撒 料，减少扬尘，才有可能完成一个完美的散料运输工程!

离散元是由 Cundall 在研究岩土颗粒时，面对岩面的不连续受力而提出的
土就是 松散颗粒的堆积物。
把一个时间点中每个颗粒的受力、颗粒的速度、加速度和位 移都计算出来
便完成了这个时间点的仿真计算
受力、颗粒的速度、加速度

qq explain following: DEM, CFD, SPH, FEM, FVM? FDM
The finite difference method (FDM) has [historical importance]
the interaction of the fluid (liquids and gases) with surfaces defined by boundary conditions
accuracy and speed of complex simulation scenarios
transonic or turbulent flows.
engineering problems in many fields of study and industries
engine and combustion analysis.
Computational magnetohydrodynamics (CMHD)
electrically conducting fluids
complexity mainly arises due to the presence of a magnetic field and its coupling with the fluid
avoid the presence of unrealistic effects, namely magnetic monopoles
model astrophysical systems
a method for representing and evaluating partial differential equations
are converted to surface integrals, using the divergence theorem
finite differences approximate
finite differences approximate the derivatives.
FDMs convert linear ordinary differential equations (ODE) or non-linear partial differential equations (PDE)
FDMs are the dominant approach to numerical solutions of PDEs
FDM is an older method than FEM that requires less computational power but is also
less accurate in some cases where higher-order accuracy is required
requires more computational power and is also more exigent on the quality of the mesh

qq if we want to simulate fluid, what kind of method can we use?
FEM, SPH, particle based, cfd methods
CFD solver based on Smoothed Particle Hydrodynamics (SPH)

qq if we want to siulate sand?
DEM

qq what if we want to similate powders?
> sugar or proteins
> bulk materials in storage silos
> granular matter, like sand;
> Blocky or jointed rock masses
> rock masses
DEM

qq compare CFD methods: FEM, FVM, and SPH

qq if we want to simulate: gases, wind
Discretization methods or SPH
key model: Turbulence models

https://www.youtube.com/watch?v=VTV6xGhyUWw

https://www.youtube.com/watch?v=hZC6ORUbLog
gases, wind, ? smoke, fire
magnets and charges
Laboratory
Stochastic Processes
Siggraph 2014: Dynamic On-Mesh Procedural Generation Control
https://www.youtube.com/watch?v=IRu-LJ3J88Q
https://www.microsoft.com/en-us/research/publication/visual-simulation-of-weathering-by-%CE%B3-ton-tracing/
lighting
explosions
&h fluids are the most complicated ones to simulate
Vorticity Refinement
retell-
//
把重要的不重要的都串联起来，给别人讲的时候很有用！有条理
考试一方面是问题，另一方面是基础的理解！
1.almost every thing around us can be simulated in our computer,
eg. weather, water, ... diff. method should be used to best simulate
them. Those methods should be classified and compared.

2.And there are many scenarios that needs a high computational complexity eg. xxx.....

3.to solve them, we apply ... to ...

4.those are numerical methods, solving odes or pdes
the goal is to simulate accurately, computational*
other methods exist for optimize* such as Lagrange multiplier
can be used for solving IK

5.
lev4* computational methods
lev3 improve *speed and *accuracy!
lev2* choose the right method, compare them
lev1* detail, explain
6.
geometry oomi
optimization
dynamic oomi
optimization
simulation oomi
computational (lower level, components of opti.)
pruf-re-ok- slides-1 CG3_01_Physically-Based-Simulation
prev.
*rendering equation
is an integral equation in which the equilibrium radiance leaving a 平衡辐射
point is given as the sum of emitted plus reflected radiance under a geometric optics approximation
used for realistic rendering techniques
λ可以认为是波长
https://en.wikipedia.org/wiki/Rendering_equation
hemisphere centered around {\displaystyle \mathbf {n} }\mathbf {n}
双向反射率分布函数可以认为是一个材质函数

- Solving the rendering equation for any given scene is the primary challenge in realistic rendering.
- One approach to solving the equation is based on finite element methods, leading to the radiosity algorithm
- Another approach using Monte Carlo methods has led to many different algorithms including path tracing,
  - photon mapping, and Metropolis light transport, among others.
- it does not include:
  - Transmission, which occurs when light is transmitted through the surface, such as when it hits a glass object or a water surface,
  - Subsurface scattering,
  - Phosphorescence,
  - Fluorescence,
  - Relativistic Doppler effect,
- 工作贡献:
  - 修改渲染方程
  - 加速渲染过程

heat
- task1
  - laplace::sum of second derivatives
  - 特征长度：用于代替不重要的量，用于 localization? p14 slides01
  - dimension less var. left in equations !....

sph-Smoothed-particle hydrodynamics
- a computational method used for simulating the mechanics of continuum media
- resolution of the method can easily be adjusted with respect to variables such as density.
- such as solid mechanics and fluid flows.
- without mesh

Computational fluid dynamics 计算流体力学
- Computational fluid dynamics (CFD) is a branch of fluid mechanics that uses numerical
  - analysis and data structures to analyze and solve problems that involve fluid flows.
- free-stream flow of the fluid
- and the interaction of the fluid (liquids and gases) with surfaces defined by boundary conditions.
- Ongoing research yields software that improves the accuracy and speed of complex simulation scenarios such as transonic or turbulent flows
- Initial validation of such software is typically performed using experimental apparatus such as wind tunnels.
- weather simulation, natural science and environmental engineering, industrial system design and analysis, biological
  - engineering and fluid flows, and engine and combustion analysis
- computational modeling of turbulent flows

FEA
- FEA (finite element analysis) is one numerical method for solving partial differential equations
- CFD (computational fluid dynamics) includes any numerical method used to solve fluid flow problems.

numerical method
- numerical method is a mathematical tool
  - designed to solve numerical problems.

The appearance of our universe strongly depends

on the values of some constants
nondimensionalization
通过一个合适的变量替代，将一个涉及物理量
的方程的部分或全部的单位移除，以求
简化实验或者计算的目的
使得方程相对简单
但是在工程中意义未必大
“自定义物理梁刚”
qq the idea of nondimensionalization?
1.partial [removal] of [physical dimensions] from an equation
suitable [substitution of variables].
2.[simplify] and parameterize problems where
measured units are involved
3.These units refer to quantities intrinsic to the system,
rather than units such as SI units

Vector operator in vector calculus
include the gradient, divergence, and curl:
https://mathinsight.org/divergence_curl_examples

http://tutorial.math.lamar.edu/Classes/CalcIII/CurlDivergence.aspx
http://blog.sina.com.cn/s/blog_4d0723b301017ivo.html
向量还是标量?
http://www.ittc.ku.edu/~jstiles/220/handouts/The%20Curl%20of%20a%20Conservative%20Field.pdf

http://www.home.uni-osnabrueck.de/mfrankland/Math241/Math241_165_ConservativeR3.pdf
The principle of least action
or the principle of stationary action
can be used to obtain the equations of motion for that system
The principle can be used to derive Newtonian, Lagrangian and Hamiltonian equations of motion, and even general relativity (see Einstein–Hilbert action).
It was historically called "least" because its solution requires finding the path that has the least value
variational calculus
later
Symplectic integrator
https://www.av8n.com/physics/symplectic-integrator.htm
later
continuity equation

https://en.wikipedia.org/wiki/Continuity_equation#Definition_of_flux
https://en.wikipedia.org/wiki/Navier%E2%80%93Stokes_equations
later
ql
qq what is the equation of harmonic oscillator? p4
qq solve?
qq rewrite with state vector
qq when using numeric method, there will be err,
> how can we know that?
> how to solve? x2 p5
qq physical quantities and their relationship? -large
qq how derived units can be written?? in terms of base units x7+1 p7
qq What unit has damping constant? p8

qq how units can be used? x2 p8
> derive units of physical constants
qq how can we validate physical formulas??
qq the case of a sphere? p9
qq solid angle def.?
qq the unit of solid angle?? p9
qq extended units and their expression with standard units p11
steradian? newton?
Pascal? Joule?
qq drive nondimensionalization of damped harmonic oscillator p14
首先找出谁是变量，对变量做代换，引入一个常量（和变量同单位的）
这样一除就可以得到一个无量纲的
> why damping ratio is unit free?
qq a few characteristic unit free parameters, why are they useful?
qq how density is defined? trible derivative? op18
> convert from and to density and mass?
qq what is Eulerian and Lagragian view of a sys.? p19
1.discretized specification, based on grids or meshes
typically FEM can be used
2.fields are easy to implement

1.in form of particles that move in space,
a particle simulation usually performed
2.conceptually simple
set of particles
qq convert from above one to an other? p20
interpreting density as particle probability and sampling
qq how forces can be computed in a field? p21
qq what is gradient? express with standard unit vectors
a vector whose components are the partial derivatives
pointing in the direction of the
steepest slope
qq what is Conservative Force def.??
total work done in moving a particle between two points is independent of the taken path
path independent
qq what is gradient theorem? proof it! computes the Work done with force.
work done is path independent, and just
potential difference of path end points
qq the condition that the force is conservative??
curl vanishes
qq how to compute curl? wiki
qq compute the work done by force f from x1 to x2 p22
qq the equ. of solid angle?? wrt. theta phi p23
qq &h The solid angle corresponding to all directions is 4pi
qq proof that oumiga all = 4*pi p24
qq eular Lagrange equation? describing a harmonic oscillator p29
qq Lagrangian of a whole sys. ? p28
qq action S of a sys. ? p28 ?? pp
qq what is the principle of stationary action? used to compute the what? with variational calculus?
is a variational principle that,
when applied to the action of a mechanical system,
can be used to obtain the equations of motion for that system

requires finding the path of motion in space that has the least value

even general relativity (see Einstein–Hilbert action)

The principle remains central in modern physics and mathematics

[The path taken] by the system between times t1 and t2 and configurations

q1 and q2 is the one for which the action is stationary (no change) to first order.

qq Noether theorem?

qq what are typical Conserved quantities? p32

qq diff. integration methods and their properties,

> stable? preserve energy?...

> show them shortly,and the pic

p33

qq quantities that are conserved, due to spatial and temporal symmetries in the laws of physics....

qq flux def.? p34

qq velocity field def. p34

qq function for both integral form and differential form.

qq why no source term in conserved quantities?

qq what is continuity equation used for? p35

describe temporal changes of

physical quantities inside of fluids or fields

qq diff. bet. gradiant/ divergence/ curl operators??

https://en.wikipedia.org/wiki/Del#Divergence

The divergence of a vector field

is a scalar function

qq The formula for the vector product??

r1

Hook‘s law

Harmonic Oscillator

Analytic Solution

qq formulas op04

solution

qq product rule??

https://flothesof.github.io/harmonic-oscillator-three-methods-solution.html

explicit Euler

with step width ?

Runge Kutta

Ordinary differential equation

常微分方程

https://www.youtube.com/watch?v=4qu2oRkTlVo

http://www.maths.lth.se/na/courses/FMN050/media/material/part14.pdf

The Explicit Euler Method

&p prob. with analytic solution, but not so imp.

Numerical solution using Scipy

Scipy offers a number of tools for dealing

with ordinary differential equations

qq Derivation of the Explicit Euler Method

first order approximation to the exact solution

qq Graphical Illustration of the Explicit Euler Method

each step introduces an error and ends up on a different trajectory

How do these errors accumulate?

Ignoring the second order term
Taylor series
Finally a constuction principle based on
Taylor expansion shall be explained.
Explicit Euler Method
Stability Analysis
qq proof the stability
quantify the condition on the step size.
how small depends on the differential equation
requires that the amplification factor is bounded by one
stable
satisfies the condition (5.4)
stability region of the Euler method

qq numerical solution op5
Physical Quantities
7 base units
scale ?? and 7 exponents

in terms of a base unit:
derived units
qq what is derived units, how they looks lke?
若现实中重力的强度每天都有所改变，就会违反能量守恒定律，
因为观察者可以在重力弱的那天把重物举起，然后在重力强的时
候放下来，这样就得到了比一开始输入的能量更多的能量
qq what unit can we used for measuring luminous intensity
Noether theorem
Physical units help to validate physical formulas
one cannot add quantities of different units
left and right side of an equality must have the same unit
damped harmonic oszillator
What unit has damping constant
qq what is the unit of xxx constant??
&h example op08
The seven SI base units and the interdependency of their definitions.
https://en.wikipedia.org/wiki/Nondimensionalization
Nondimensionalization
qq what is Nondimensionalization
This technique can simplify and
parameterize problems where measured units are involved.
the term scaling is used interchangeably with nondimensionalization
force constant of spring
To simplify the equation
choose a constant value for
&h define new units
&h choose constant val. for varibles and eliminate units by deviding them
nondimensionalized versions of our original variables
&h define new non-dim. varibles
substitution
https://www.math.wisc.edu/~angenent/519.2016s/notes/non-dimensionalization.htm
ct su
https://www.math.wisc.edu/~angenent/519.2016s/notes/non-dimensionalization.html
make the constant term

coefficient
both equal to 1
Note that instead of three undetermined parameters (a, b, c) it only has one parameter
only has one parameter
undetermined parameters
general numerical process often used in differential equations
qq Buckingham π theorem??
*a theorem for dimensional analysis
if there is a physically meaningful equation involving a certain
number n of physical variables,
then the original equation can be rewritten in terms of a set of
白金汉π定理可以视为是形式化的雷诺量纲分析法。
白金汉π定理是量纲分析中的重要定理，在工程、应用数学及物理中都会用到
白金汉π定理可以视为是一种无量纲化的框架，其中提供方法，从已知的物理量中找到一组无量纲的参数
在流体中运动的物体，其阻力方程中包括以下五个物理量：速度 u、流体密度 ρ、动黏滞系数 ν、 物体截面大小 A 以及阻力 F
可以将阻力方程简化为由阻力系数 CD 及雷诺数 Re 组成的方程
二个物理量是由上述物理量组合而成。
Solid angle measures directions by their covered area on the unit sphere
measured in steradian (sr)
Solid angle
qq what is the unit of Solid angle??
For steradian all base units cancel out, but we still
write it as sr to
qq why we still write sr unit??
distinguish from a pure scalar
cancel out
non standard units
luminous flux
illuminance
Newton
Hertz
Joule
Watt
SI units the most fundamental constants
Newtonian constant of gravitation
Planck constant
speed of light in vacuum
universal in nature and constant in time.
Reynolds number Re
by a simple variable substitution
parameterize different behaviors of the system
qq example of damped harmonic oszillator
Substitutions
unit free damping ratio defines system behavior
damping ratio defines system behavior
photogrammetric measurements
describe the system with unit free variables
and a few characteristic unit free parameters
spectral power
small interval
derivative
From a physical point of view
triple derivative of mass

triple integration of density
gives back mass
triple derivative of mass
yields density
respect to volume
&h
Field vs Particles
qq compare those two views
    Eulerian View
        Describe physics as fields, often discretized over grids or meshes
    Lagragian View
        Describe physics in form of particles that move in space
        particles are the discretization unit and typically do not represent single physical, but bundles
qq how can we transfer from field view to particle? and back?
    interpolation
    //
    density estimation
    reconstruction
similar to the case of: from density, we know a
    volumetric representation, and we reconstru. back to spacial view
    ---REVIEW---

    ---
gradient is
    direction of steepest ascent
qq compare the direction of force and gradient of potential?
    force points in opposite direction
potential energy ?? with respect to location
conservative force
qq how is conservative force defined? op22
gradient theorem yields that work done is just
    potential difference of path end points
? gradient theorem
The gradient theorem, also known as the
    fundamental theorem of calculus for line integrals
a line integral through a gradient field can be
    evaluated by evaluating the original scalar field at the endpoints of the curve.
The theorem is a generalization of the fundamental theorem
    of calculus to any curve in a plane or space
    (generally n-dimensional) rather than just the real line.
continuously differentiable function
theorem
https://en.wikipedia.org/wiki/Gradient_theorem
conservative forces does not depend on the path followed
    by the object, but only the end points, as the above equation shows.
Work done by conservative forces does not depend
    on the path followed by the object, but only the end points
such as the gravitational force between
    the Earth and another mass, whose work is determined
    only
qq def. of Conservative force
&h Conservative means independency somehow ...

* no work done nor necessary for cyclic paths
cyclic paths
force is conservative, iff curl of force vanishes
qq the condition of a force that is conservative
    curl vanishes
In vector calculus, the curl is a vector operator
    like gradient operator
that describes the infinitesimal rotation of a vector field
    in three-dimensional Euclidean space.
The alternative terminology rotation or rotational and alternative notations
divergence is a vector operator that operates on a vector field,
    producing a scalar field
旋度的计算里面x其实就是叉乘!
https://en.wikipedia.org/wiki/Curl_(mathematics)
The resulting vector field describing the curl would be
    uniformly going in the negative z direction.
tt vis of a vector filed!

https://upload.wikimedia.org/wikipedia/commons/thumb/8/80/Uniform_curl.svg/500px-Uniform_curl.svg.png
    consider, the posi. (0,1) and (1,0)
    we can determin direction and scale of a given point, which,
        we can draw a arrow to vis. !!
    sampling and compute
The Laplacian of any tensor field {\displaystyle \mathbf {T} }
    \mathbf {T} ("tensor" includes scalar and vector)
    is defined as the divergence of the gradient of the tensor:
named after Pierre-Simon Laplace, is a differential
    operator defined over a vector field.
Laplace operator
An example of the usage of the vector Laplacian is the
    Navier-Stokes equations for a Newtonian incompressible flow:
Another example is the wave equation for the electric field that
    can be derived from the Maxwell's equations in the absence
    of charges and currents
The Klein–Gordon equation (Klein–Fock–Gordon equation or
    sometimes Klein–Gordon–Fock equation) is a
    relativistic wave equation, related to the Schr?dinger equation.
qq express the dir with theta and phi
&h op23
The solid angle corresponding to all directions
The solid angle of a hemisphere
a lot of physical quantities are derivatives of others
integration of conservative forces along paths
    can be computed from differences in potential energy
Light travels along the shortest path with respect to time
qq what is Minimization Principle
qq what is Fermat’s Principle
    Light travels along the shortest path
    wrt. time
    根据费马原理，光线传播的路径是所需时间为极值的路径
qq what is 斯涅尔定律 Snell's law?? law of refraction
    equivalent to the ratio of phase velocities

取传播时间{\displaystyle T}T 对变数{\displaystyle x}x 的导数，设定其为零：

根据正弦函数定义，可以得到传播速度与折射角的关系式：

最短时间原理

qq derive the equ. of refraction

https://en.wikipedia.org/wiki/Snell%27s_law

https://zh.wikipedia.org/wiki/%E6%9C%80%E5%B0%8F%E4%BD%9C%E7%94%A8%E9%87%8F%E5%8E%9F%E7%90%86

Calculus of variations

https://zh.wikipedia.org/wiki/%E6%B3%9B%E5%87%BD%E5%88%86%E6%9E%90%90

variational calculus

Euler Langrange Equations

the dynamics of physical systems

can be formulated as minimization problem

* if minimization is over functions, one needs variational calculus

from Least Action Principle one can derive the Euler Lagrange Equations

generalize equations of motions

拉格朗日最小作用量原理

推广至位形空间，

拉格朗日最小作用量原理阐明

{\displaystyle \delta A=\delta \int \sum _{i}p_{i}\mathrm {d} q_{i}=0}\delta A=\delta \int \sum_i p_i \mathrm{d}q_i=0; 其中，{\displaystyle p_{i}}p_{i}是广义动量，{\displaystyle q_{i}}q_i 是广义坐标。

求作用量的稳定值，

scalar Lagrangian ?

state vector

kinetic energy

potential

两点间最短曲线为一直线

the action ?? of the system is defined as the functional

两点之间最短曲线的另一种求解

https://zh.wikipedia.org/wiki/%E6%AD%90%E6%8B%89-%E6%8B%89%E6%A0%BC%E6%9C%97%E6%97%A5%E6%96%B9%E7%A8%8B#%E7%AC%AC%E4%BA%8C%E6%96%B9%E7%A8%8B

Euler-Lagrange equation

二阶偏微分方程

提供了求泛函的临界值（平稳值）函数

泛函的定义域为函数空间

harmonic oscillator

those two examples:

mechanical systems can be completely described through the scalar Lagrangian

qq the equ. of scalar Lagrangian?? op28

kinetic energy T and the potential energy V

qq action S of the system?? op28

qq the euler langrange equ.?? op29

qq apply euler langrange equ. to harmonic oscillator!! op29

qq proof that line is the shortest distance bet. two points !!

with priciple of least action or lagrange equ.

https://zh.wikipedia.org/wiki/%E6%AD%90%E6%8B%89-%E6%8B%89%E6%A0%BC%E6%9C%97%E6%97%A5%E6%96%B9%E7%A8%8B#%E7%AC%AC%E4%BA%8C%E6%96%B9%E7%A8%8B

&h imp. is, what we want to minimize ! in phy,

we want to minimize T-V over time and in distance

calcu. we want to minimize the distance bet. two points.

And this minimal should be form as a integral!
&h first, we def. the action S to be minimized. then,
def. a functional L, which to be integrated.
Then, we can write a lagrange equ. based on L.
By solving that equ, we know how L looks like!!
qq how can we apply functional Analysis/ priciple of least action to a problem?
def. action S -> scalar Lagrangian L -> Euler Langrange Equation -> solve
minimization is over functions, one needs variational calculus
variational calculus
Galilean transformation
used to transform between the coordinates of two reference frames which differ only by constant relative motion
within the constructs of Newtonian physics
Newtonian physics.
Galilean geometry.
Lorentz transformations
Poincaré transformations
Galilean symmetries
written as the composition of a rotation, a translation and a uniform motion of spacetime.
https://zh.wikipedia.org/wiki/%E7%BE%A4%E8%AE%BA
上述例子中的群分别对应着伽利略群，洛伦兹群和{\displaystyle U(1)}U(1)群
对称群为连续群和分立群的情形分别被称为“连续对称性”（continuous symmetry)和“离散对称性”（discrete symmetry）
1950年代杨振宁和米尔斯意识到规范对称性可以完全决定一个理论的拉格朗日量的形式，并构造了核作用的{\displaystyle SU(2)}SU(2)规范理论
从此，规范对称性
This theorem only applies to continuous and smooth symmetries over physical space.
qq what is Noether's theorem
Any [differentiable] [symmetry of the action]
of a physical system has a
corresponding [conservation law].
time symmetry
energy is conserved
linear momentum
differentiable symmetry
qq what are typical diff. symmetry and their corresponding conservation laws??
time...location...orientation symmetry
&h *
Noether's theorem也是一种广泛的理论，不仅仅应用在经典力学
Principle of Least / Stationary Action也是
tt tt legrange force, phy. formation + functional
泛函分析（英语：Functional Analysis）
是现代数学分析的一个分支
其研究的主要对象是函数构成的函数空间
泛函分析历史根源是由对函数空间的研究和对函数的变换（如傅立叶变换等）的性质的研究
这种观点被证明是对微分方程和积分方程的研究中特别有用
变分法是处理泛函的数学领域，和处理函数的普通微积分相对。
譬如，这样的泛函可以通过未知函数的积分和它的导数来构造。
变分法最终寻求的是极值函数
变分法的关键定理是欧拉－拉格朗日方程。它对应于泛函的临界点
变分法在理论物理中非常重要：在拉格朗日力学中，

以及在最小作用量原理在量子力学的应用中
它们也在材料学中研究材料平衡中大量使用。而在纯数学中的例子有，
黎曼在调和函数中使用狄利克雷原理。
变分一词用于所有极值泛函问题。微分几何中的测地线
的研究是很显然的变分性质的领域。
***这意味着，一个函数的参数是函数。这个名词首次被雅克·
阿达马在 1910 年使用于这个课题的书中
topology, vector calculus, functional analysis, ...
Hilbert spaces
Explicit numerical integration
like the explicit Euler add energy to the system
implicit integration techniques are stable but
unnaturally damp the system and remove energy
damp
Symplectic integrators
Symplectic integrators conserve energy as good as possible but are not stable for stiff systems.
not stable for stiff systems
Hamilton's equations, a system of ordinary
differential equations that arises in classical mechanics.
https://zh.wikipedia.org/wiki/%E5%BE%AE%E5%88%86%E5%87%A0%E4%BD%95
辛拓扑源于经典力学的哈密顿表述，其中特定经典系统的相空间有辛流形的结构。
https://en.wikipedia.org/wiki/Symplectic_integrator
qq the equ. for numerical integration? x3!! op33
err?? sec. deriv. of x
qq compare them!
explicit Euler
implicit Euler
semi-implicit
symplectic Euler
A continuity equation keeps book on all changes in ?
qq what is dS vector here??
乃是描述守恒量传输行为的偏微分方程。由于在各自适当条件下，
质量、能量、动量、电荷等等，都是守恒量
很多种传输行为都可以用连续性方程来描述
传输行为
qq what is the continuity equation? def. ?
* idea: 在任意区域内某种守恒量总量的改变，等于从边界进入或离去的数量
应用散度定理，可以从微分形式推导出积分形式
高斯通量理论（Gauss' flux theorem）、散度定理（Divergence Theorem）、
高斯散度定理（Gauss's Divergence
一个把向量场通过闭合曲面的流动（即通量）与曲面内部的向量场的
表现联系起来的定理。该定理与斯托克斯定理（Stokes' Theorem）是向量中两大重要定理
直观地，所有源点的和减去所有汇点的和，就是流出这区域的净流量
在一维，它等价于微积分基本定理；
在二维，它等价于格林公式。
***这个定理是更一般的斯托克斯公式的特殊情形比较简单的形式
所有源点的和减去所有汇点的和，就是流出这区域的净流量
density source ??:
Continuity equation
qq integral and differential form of Continuity equation! x2
and their relationship, how can be converted to each other
https://en.wikipedia.org/wiki/Continuity_equation#Integral_form
The flux of q is a vector field
If there is a velocity field u which describes the relevant flow

density times the velocity field:
the flux is by definition equal to the density times the velocity field:
Particle View
quantities that are conserved in physical systems like energy
due to spatial and temporal symmetries in the laws of physics
qq what is the reason of conservation law in phy? in depth
Symplectic numerical integration methods target
for energy preservation
Continuity equations describe temporal
changes of physical quantities inside of fluids or fields.
qq For conserved quantities they do not have a source term??
qq what is source term?
temporal changes of physical quantities
qq how can we describe the temporal changes of physical quantities in a phy system? fluid...or other quan, mass....
-->use continuity equ. to describe
qq what is divergence theorem?? the basic of continuity equ.
a result that [relates] the flux of a vector field through a
closed surface to the divergence of the
field in the volume enclosed.
&h continuity equ. is the simplified version of Stokes equ. ??
&p pp
addiq:
qq gradient theorem, proof op22
qq what is the def. of conservative force??
** negated gradient of potential energy wrt. location
// the description!
tt futher derivation of motion equ. of diff. phy systems!
the theorem behind them! phy stuffs!
tt futher use numerical methods to solve them! code
pruf-re-ok- slides-8 CG3_08_DGLs(Differentialgleichung)
single variable
ordinary differential equation
qq what is differential equation?
describes a relation between a function
in one or several variables
qq what is the diff. bet. ode and pde??
several variables arise
unique solution
boundary conditions
qq what is boundary conditions??
Wave Eq., Maxwell Eg., Schr緩inger Eq.,
Einsteinean Field Eq., Navier-Stokes-Eq
highest derivative
qq implicit representation of an ode of order n?and explicit representation?
In this case the ansatz will yield an (n-1)-th order
equation for {\displaystyle v}v
&p pp op4 1 should be n-1??
qq what is Order reduction?
DE of order n can be transformed into
[a system of] DE of order 1.
&h maybe 1 right...
combined with derivatives up to (?? - 1) to a ??-d
qq what is phase space, the case in physical system of 2nd order?
we have learned?

uniquely defines time evolution
defined through position and velocity
phase space
state
qq 自治（驻定）的系统?
在数学中，一个动力系统被称为自治（驻定）的，当且仅当这个系统
由一组常微分方程组成，并且这些方程的表达式与动力系统的自变量无关。
一个动力系统被称为自治（驻定）的
在有关物理的动力系统中，自变量通常是时间。
这时自治系统通常表示其中的物理规律不随时间变化的系统，
这时自治系统通常表示其中的物理规律不随时间变化的系统，
也就是说空间中每一点的性质在过去、现在和将来都是一样的。
理论上说，所有的动力系统都可以转化为自治系统。
物理上来说，这表示空间中一点的性质不仅取决于它的位置，
还取决于时间：在不同的时间，经过此一点的质点或粒子会受到不同的影响
does not depend explicitly on t,
qq the case of Harmonic Oscillator, phase space?? what is that equ.??
qq what is the time evoluiton func. of Harmonic Oscillator?? p6
vector field on the simulation domain
qq what is autonomous DE?? how can it be used? why we need to know this?
if the time evoluiton funcction does not depend explicitly on t
In case of autonomous
DE, the time evolution correspond to streamlines of the vector field
qq what is Picard–Lindelöf theorem
* gives a set of Lipschitz conditions under which
an initial value problem has a unique solution.
on time evolution,
https://en.wikipedia.org/wiki/Picard%E2%80%93Lindel%C3%B6f_theorem
qq does Picard–Lindelöf theorem always true?
* not for pdes
pdes are typically harder prob. we can not say it has unique soluiton
* In case of Kollisions
cannot be fulfill due to instantaneous
changes in the velocity
对于偏微分是不适用的，对于碰撞是不适用的
The Picard–Lindel?f theorem guarantees a unique solution
on some interval containing
converges toward a local solution
柯西-利普希茨定理（Cauchy-Lipschitz Theorem），又称皮卡
-林德勒夫定理（Picard-Lindel?f Theorem）
这个定理有点像物理学中的决定论思想：当我们知道了一个系统的特性（
微分方程）和在某一时刻系统的情况（{\displaystyle x(t
_{0})=x_{0}}{\displaystyle x(t_{0})=x_{0}}）时，下
一刻的情况是唯一确定的。
在数学中，特别是实分析，利普希茨连续（Lipschitz continuity）
以德国数学家鲁道夫·利普希茨命名，是一个
比通常连续更强的光滑性条件
* 直觉上，利普希茨连续函数限制了函数改变的速度，符合利普希
茨条件的函数的斜率，必小于一个称为利普希茨
常数的实数（该常数依函数而定）。
K 称为 {\displaystyle f}f 的利普希茨常数。
若{\displaystyle K<1}K<1，{\displaystyle f}f 称为收缩映射。
双李普希茨(bi-Lipschitz)的。
Lipschitz condition

differentiable function
solves
intial value problem
not similar result an the existence and uniqueness of a solution for pdes
pdes
&h pdes are typically harder prob.
qq 初值问题边值问题与柯西问题
diff. bet. initial prob., boundary prob. and cauchy prob. ??!!
边值问题
是一个微分方程和一组称之为边界条件的约束条件。
边值问题的解通常是符合约束条件的微分方程的解
*其实就是已知变直而不是t=0值的求解过程
第一类边值条件: 也称为狄利克雷边界条件
第二类边值条件: 也称为诺伊曼边界条件
第三类边值条件: 物理系统边界上物理量与垂直边界导数的线性组合
初值问题
是一个涉及微分方程式与一些*初始条件的问题
*初值问题就是根据方程和初始条件求函数解析式的问题!
柯西问题
由初值问题推广而来，与边值问题相对
法国数学家奥古斯丁·路易·柯西的名字命名
(n-1)维的光滑流形S
他们的区别就是不同的初始条件
https://zh.wikipedia.org/wiki/%E6%9F%AF%E8%A5%BF%E9%97%AE%E9%A2%98
qq describe the initial val. problem! of odes
qq what is Lipschitz condition? **
&h
The derivative of order zero means that the function itself is specified.
collectively known as the Cauchy data
asks for the solution of a partial differential equation
that satisfies certain conditions
Existence and uniqueness of solutions
In case of Kollisions the Lipschitz-condition cannot be fulfill due to instantaneous changes in the velocity.
Lipschitz-condition
qq re- what is Lipschitz-condition, bi-Lipschitz-condition??
qq explain: * Every function that has bounded first derivatives is Lipschitz continuous
&h
最小的常数
strong form of uniform continuity for functions
the smallest such bound is called the Lipschitz constant of the function

bounded first derivatives is Lipschitz continuous
* guarantees the existence and uniqueness of the solution
to an initial value problem
A special type of Lipschitz continuity, called contraction,
is used in the Banach fixed-point theore
Continuously differentiable ? Lipschitz continuous
? α-H?lder continuous ? uniformly continuous
derivative f'(x) exists and is itself a continuous function
class C2 if the first and second derivative of the function
both exist and are continuous
exist and are continuous
function is smooth or equivalently, of class C∞.
Theorem of Picard / Lindel?f

qq can we solve sys. with collisions with closed form?? n why?? op08
qq how can we archieve Analytic Solutions of odes??
Maple is a symbolic and numeric computing environment
as well as a multi-paradigm programming language
such as symbolic mathematics, numerical analysis, data processing
Use algebra program like Maple
Collisions and other effects make system of DEs
too complicated for analytic solutions
Lipschitz-condition cannot be fulfill
qq how can we solve initial value problem numerically?? p10
Integral on right side is determined numerically
Numeric Integration
qq re- how conditions like initial value be used??
qq formulate a numerical solution form of a DE
qq why not a typical integration? how can we solve it? op10
unknown Functions appears on right side too
Approximate with polygonal line
qq use explicit euler to solve this??
accumulated error
qq what is the bkg image p13? what field?
a vector field
qq error with respect to what?
analytic solution
Decreasing the size of the time step decreases the approximation error
large time steps
can deviate significantly from solution between integration points
step error is proportional to h 2
Approximation Order
the accumulated error grows with O(h)
accumulated error grows with O(h)
qq expression of the accu. error?? op15
qq what is the appro. order of explict euler method??? error estimation? op15
Taylor series
&h
an initial value problem is composed of a DE and an initial value
ordinary DEs only depend on single variable
order ?? DE can be transformed to linear system of order 1 DEs
linear system of
can be characterized by their approximation order
One calls k the approximation order of the method
approximation order
k the approximation order
qq but the accumulated error grows with O(h)?? not h^2 ??
qq what is the approximation order of a method whose has error pt. h^2 ??
qq what is the approximation order of explicit Euler?
1
&h
implicit methods are stable but lead to
system of non-linear equations to be solve
most efficient scheme depends on application
//
qq what Numeric Integration methods do you know? x5
//Verlet

qq how does Verlet Method get a higher order?
Exploit Taylor series addind +h and –h
*eliminates some componnets to achieve a higher order
qq estimate velocity from Numeric Integration, which difference scheme?
centeral difference
have best approximation order
backward difference
forward difference
Second-order central
https://en.wikipedia.org/wiki/Finite_difference
qq whats the approxi. order of Verlet method??
approximation order 3:(higher order)
qq the fomr. of verlet method, limitations of it?
specifically designed for physical systems of order 2
qq the formula/ proof of 4 methods -large
&h
qq formula of diff. interpolation schemes x3 which one has better approxi. order??
// Euler-Cromer Methode
qq does Euler-Cromer Methode have approx. order ?
qq why is Euler-Cromer Methode more stable than others of the same order?
semi-implicit, symplectic
more stable than other methods of approximation order 1.
// Runge-Kutta
qq what is the App. ord. of Runge-Kutta Methods??
qq the idea of RK? why have higher order?
Family
App. ord. n builds on n function evaluations
Very popular is RK4
kernel style?
//
qq explain A-stable op21
generated approximations converge to
qq what test case does A-stable use?
a simple test case
&h we are talking about numeric integration method
qq what means of a DE is stiff??
explicit euler not converge all the time
explicit methods do not always converge.
qq does implicit Euler Method is A-stable?
qq proof that! op22 large
&h
non-linear equations in the components of ????+1
Typically a Newton-Iteration is used to solve the system of equations
Newton-Iteration
pays off as larger steps can be taken
is it recommand to use implicit from?? y can paid off
qq f ! explicit and implicit case! for Midpoint methods
qq implicit midpoint is also symplectic??
red chord is approximately parallel to the tangent line at the midpoint
Both methods are of approximation order 2
implicit midpoint is also symplectic
qq what feature does symplectic mean?
energy preservation
more stable

&h we can proof that implicit midpoint is symplectic, but may in a hard way
qq let step be a kth app.ord. method. how can we adapt stepwidth? op26
qq how can we estimate step error??
if ? is larger than toleranz ?
op27
qq why use damped harmonic oscillator as a demo?
the analytic solution is known
&h
Analysis of numerical methods for the simulation of deformable models
implicit methods
Newton-Iteration
implicit midpoint method
qq does most efficient scheme depends on application?
pruf-re-ok- slides-4 CG3_04_Sampling-Techniques
ql
---REVIEW---

// transfer to the pdf file in this folder , start from slide 4

qq how does nrook sampling work? p3
qq how one can gen. random shaffed vector? p3
qq what means caustic ?
qq how to gen. uniformly distributed random points with shuffe in c++ p4
qq what means sampling in regular grids ? p5
..it is possi. when you get used to it, it will be faster
https://onedrive.live.com/view.aspx?resid=9712B8A082E369A%211407&id=documents&wd=target%28%E5%BF%AB%E9%80%9F%E7%AC%94%E8%AE%B0.one%7C0BE32A9B-AC05-4328-847D-6AF52075C99B%2F%E6%97%A0%E6%A0%87%E9%A2%98%E9%A1%B5%7C2FB0ABD7-9762-455D-9478-7562C513D45F%2F%29
onenote:https://d.docs.live.net/09712b8a082e369a/zhongyuan%20的笔记本/快速笔记.one#section-id={0BE32A9B-AC05-4328-847D-6AF52075C99B}&page-id={2FB0ABD7-9762-455D-9478-7562C513D45F}&end
---r1
qq which Sampling Techniques do you know?
Random Samplings
Rejection Sampling
Importance Sampling
Transformation Sampling
N-Rooks Sampling
Multiple Importance Sampling
qq what means sth. follow a specific random variable distribution, draw to illu.
uniform, Normal or Binomia?
Problem: We fixed the 1D projections, but ruined the
2D distribution! It is no better than that of random sampling:
The Mersenne Twister is a pseudorandom number generator (PRNG).
was developed in 1997
Mersenne Twister algorithm is based on the Mersenne prime 219937-1
uses a 32-bit word length.
There is another implementation (with five variants[3]) that uses a 64-bit word length

MT19937-64; it generates a different sequence.
uniform_real_distribution
the <algorithm> header
qq with which func. can we gen. random var.?? op03
with Generators std::mt19937 and some Distribution functions
qq bring a vector into random order?
std::random_shuffle() from the <algorithm> header
std::random_shuffle (myvector.begin(), myvector.end());
std::mt19937
qq how can we use std::mt19937 to gen. uniform distributed 2d samples??

> it is only a generator, must act as a parameter
qq what is sample clustering?
qq what is the problem of such a graph?(above)
qq what is a good sampling ??
avoids sample clustering
qq what is Possion disk sampling, idea?
avoid sample clustering, with rejection test
qq compare Stratified grid sampling and N-rooks sampling
nrooks covers each dimension much better in high dimensions

Stratified grid sampling samples with uniform sampling on each grid cell

n-rook performs stratified grid sampling in 1D per dimension
qq how can we sample according to a distribution: function_f/ integrate(f)?

we can achieve this reject the samples that are larger then desired value
qq what's the problem with Rejection Sampling?
1.no access to the probability density
2.reject some samples, not so effecient
will not work when [the accept rate] too low
the area too small when compared to the whole interval

qq how Rejection Sampling can be used explain the pic! op06
qq how can we Sampling any Shape? in a simple manner?
rejection sampling
transformation sampling
qq how can we sample uniformly inside a given shape?
rejection sampling
transformation sampling
Rejection sampling can be trivially combined with other sampling strategies
There is no access to the probability density in rejection sampling
qq the idea and the goal of Transformation Sampling??
sample xi according to some distribution px over the interval []

sampling uniformly, and then transfer to the desired distribution

suitable function
cumulative density function
their cdf are the same! op134 drafts on ipad
* sample unit square first and transfer to the desired shape
qq what is marginalization?
qq what is cumulative density function CDF

the probability that X will take a value less than or equal to x
qq derive the equ. used in tranformation sampling, the key condition connecting both?
qq how can we understand transformation?? g(ita)
qq compute an example as found in picnotes
qq we know how to gen. uniform distributed val.
> in a [0,1] interval, but how can we transform it to arbitry interval??
qq can we change the interval with transformation sampling? y
hh inverse function in 1d marginalization in 2d
qq which relations do we have in 2d?
qq the results! op11 down side
qq and folow arbitry distribution at the same time??
Transformation Sampling
marginalization over ?
qq why do we compute conditional probability pdfs and cdfs??
we want to relate cumulative distribution functions of both

qq how can we slove this in 2d??
equating cumulative distributions
conditional probability pdfs
the sampling procedure
qq the sampling procedure in 2d, example op13!!
compute equalities and then, inverse!
draft on ipad op137
qq what is equating cumulative distributions, inverse function??
marginal distribution
uniform distribution
qq how can we get uniform distribution over disk??
> from a square(in unifrom distrib.) x2 techniques
transformation sampling
rejection sampling
&err op13
&h a r need to be added..
tt in 3d case??
qq the idea of transformation sampling? can we do this?
> sampling of unit triangle and then transformed to arbitrary triangle
qq what we have to calcu. for a transformation sampling??
the transformation function !!!!
qq how can we sample manifolds?
sample in the embedding space
and then project the samples onto the manifold
qq what is manifold and its embedding space?? draw to illustrate!
qq how can we get uniform circle sampling?? op15
sample unit disk with rejection or transformation sampling
then, do projection
qq compare rejection and transformation sampling
qq how can we uniformly sample the sphere surface??
1. transformation or rejection sampling for a unit sphere
2. project the samples onto the surface!
normalize sample
yields uniform sampling of circle
**qq why Double Coverage is a kind of transformation sampling technique?
transformation sampling: sampling some where uniformly,

and then, forming a new shape!
qq what is Double Coverage?
sometimes, it is more convenient to [sample a shape of twice the size]
**qq how can we sample a triangle uniformly??
sample a shape of twice the size and then
project the samples from wrong side back to the right side
hh [implicit] transformational sampling??
extend triangle to unit square
**qq how can we sample normals on a hemisphere??
> example hemisphere for given normal ?
sample on unit square, negate the negative normals
reflect samples from upper right triangle back into the triangle
transformation from unit triangle to arbitrary triangle
qq how can we let probability density p(x,x) be proportional to a constant brdf?
use transformation sampling and marginalization approach:
---tobecontinue
qq what is marginalization approach?
probability density
qq what is the probability density for a hemisphere??
normalization of density from integration over hemisphere
qq what is Transformation Sampling a Diffuse
> BRDF in angle parameterization??
ambient reflection constant
https://en.wikipedia.org/wiki/Phong_reflection_model
specular reflection constant
diffuse reflection constant
the ratio of reflection of the diffuse term of incoming light
ambient reflection constant,
qq what is a Phong model?
Phong reflection is an [empirical model] of [local illumination]
It describes the way a surface reflects light
It is based on Phong's informal observation
The model also includes an ambient term to account for the small amount of light that is scattered
for physical plausible issue, we can further modify the phong model!
*by adding the diffuse cosine to the specular part
sum of contributions from all light sources could be computed
it has Computationally more efficient alterations
When this constant is large the specular highlight is small.
which is a shininess constant for this material,
which is larger for surfaces that are smoother and more mirror-like
which is the direction pointing towards the viewer
(such as a virtual camera)
Computationally more efficient
Inverse Phong reflection model
Visual illustration of the Phong equation:
here the light is white, the ambient and diffuse colors are
both blue, and the specular color is white
**ambient component is uniform (independent of direction).

qq how can we make the phong model more physical plausible?
*by adding the diffuse cosine to the specular part
qq how can we sample a Phong Lobe?
qq what is the probability density of the Phong Lobe?
how can we consider the condition here?
by rejection sampling
&p pp 是用Lobe代替hemisphere? 在Phong模型中依然可以重要性采样，但是为什么?
hh https://zhuanlan.zhihu.com/p/21376124
Approximate Transformation Sampling a Phong Lobe - Pseudocode

&?problem is that sample in direction given out dir.
north pole
符号[公式]表示按向量的分量相乘，因为[公式]和[公式]都包含RGB三个分量。

&h thta is, tensor product. the same as cov.??
Reflectance Equation
上式称为反射方程（Reflectance Equation），用来计算表面反射辐射率
对于点光源、方向光等理想化的精准光源（Punctual Light）
微表面理论（Microfacet Theory）认为我们看到的表面上的一点是由很多朝向各异且光学平的微小表面组成。

https://pic4.zhimg.com/80/85e93632f88e18f46cc9b73729c31a93_1440w.jpg
法线分布函数（Normal Distribution Function，简写为NDF）
向NDF输入一个朝向[公式]，NDF会返回朝向是[公式]的微表面数占微表面总数的比例

比如有1%的微表面朝向是[公式]，那么就有1%的微表面可能将光线反射到[公式]方向

并不是所有微表面都能收到接受到光线
Shadowing
也不是所有反射光线都能到达眼睛，下面中间的图，一部分反射光线被遮挡住了，这种现象称为Masking
Masking
光线在微表面之间还会互相反射，如下面右边的图，这可能也是一部分漫射光的来源，在建模高光时忽略掉这部分光线
光学平面并不会将所有光线都反射掉，而是一部分被反射，一部分被折射，反射比例符合菲涅尔方程（Fresnel Equations）
Torrance-Sparrow基于微表面理论，用上述三个函数建立了高光BRDF模型
F/D/G函数的约束条件和选择
有很多论文基于上式给出不同的F/D/G函数，从而实现不同效果。关于F/D/G函数的比较，可以参考Background:
Physics and Math of Shading
Sampling a Discrete Distribution
qq how can we sample discreate distribution?? op21
search primitive i with
measured on discrete sample locations
spectrum
qq what discreate sampling techniques do you know as example?
light sources sampling
a point light source can be measured
on a discrete sample of directions
BRDF parts (D/S/M) can be sampled
in a discreate manner
---Multiple importance sampling (MIS)
qq MIS-estimator? expression?
can be used when several sampling strategies are available
qq why the integrant is often a product of several functions??
ray path!
qq why MIS minimize variance?

weights the strategies to minimize overall variance
qq the idea of MIS multiple improtance sampling
exploits advantages of different strategies and combine them together

The weights need to be normalized
qq give a equ. to show what unbaised mean!!!op26
> the MIS estimator is unbiased??
qq how can we choose the weight so that MIS estimator is unbiased??
qq how can we compute average density?? op27
qq what is Balance Heuristic? formula op28
Choose weights proportional to n * p
qq what is power heuristic?
Choose weights proportional to (n * p) ^ beta
qq what is the relationship bet. them?
power heuristic can help to reduce variance further
the power heuristic close to the optimal MIS weighting
qq two conditions the MIS estimator??
the power heuristic can help to reduce variance further
Veach proves a bound for the power heuristic
close to the optimal MIS weighting
qq how can we compute optimal MIS weighting?? which heuristic
qq how can we combine direct light sampling and brdf sampling techniques
> to reduce variance?
to evaluate an integral
qq what is a good choice for the exponent ??
In numerical experiments he finds a good choice for the exponent: ?? = 2.
pp
qq distinguish brdf sampling and direct light sampling
brdf should sample near light source
Weights vis for MIS! tt
given pdf there are a multitude of different sampling strategies
from which one can choose
Rejection sampling works even for unnormalized pdfs
does not provide any means to evaluate the pdf at a sample
unnormalized pdfs
qq compare rejection sampling with transformational sampling
Transformation sampling allows to use uniformly
distributed samples and can be applied when
the pdf is integrable and the cdf invertible.
but can also done implicitly
&h uniform distrib. are transformed somehow
Pdfs resulting from the product of functions can be
handled with multiple importance sampling, which
samples each function and additionally weights the
samples based on the function values.
samples each function
a product of several functions
&p pp why we need a product of several functions?? wha about just one func.??
PhD thesis
It is based on the idea of using more than one
sampling technique to evaluate an integral,
where each technique is designed to sample some feature of the

integrand that might otherwise lead to high variance

Our key results are on how to combine the samples: we present combination
strategies that are provably close to optimal
This leads to low-variance estimators that are useful in a
variety of problems in graphics
qq tt what are low-variance estimators??
distribution ray tracing, multi-pass radiosity algorithms,
and bidirectional path tracing
http://graphics.stanford.edu/papers/veach_thesis
tt !!!! have to read
&h full read!!
pruf-re-ok- slides-6 CG3_06_Advanced_Materials
The reflection type can vary a lot from purely diffuse to mirror reflection
the specular reflection does not introduce additional color
Empirical
qq what is Physical Plausible
Torrance Sparrow
Lafortune
Plastik
Roughness and Anisotropy
qq which Empirical Shading Models do you know?? x3
Lambertian model (diffuse only)
Phong model
Blinn-Phong: similar to Phong with variant in specular term
scalar BRDFs are defined implicitly
When looking towards light source above a reflecting sources
Phong model is unrealistically
spectral coefficient times scalar BRDF
qq compare diffuse, glossy and mirror reflaction with a graph
qq what is the diff, bet. glossy and specular reflaction?
qq computing reflected radiance from incoming radiance
> for the following model: Lambertian, phong model, blinn-phong model
> op07 large
qq what is v here?
visibility check
qq how can we compute omega_half?
qq when is Phong model is unrealistically? why pp
looking towards light source above a reflecting
qq compare white-sky albedo and black-sky albedo, large
integrating over all outgoing direction yields blacksky-albedo
0, corresponding to a black body that absorbs all incident radiation, to
1, corresponding to a body that reflects all incident radiation.
the actual albedo {\displaystyle {\alpha }}{\alpha }
(also called blue-sky albedo) can then be given as a linear combination of the both
albedo
qq what is blue-sky albedo?
measure of the diffuse reflection of solar radiation out of the total
solar radiation and measured on a scale from 0
absorbs all incident radiation,
corresponding to a body that reflects all incident radiation.
Surface albedo is defined as the ratio of radiosity to the
irradiance (flux per unit area) received by a surface
The proportion reflected is not only determined by properties of

the
surface itself, but also by the spectral and angular
distribution
of solar radiation reaching the Earth's surface
Directional-hemispherical reflectance is the reflectance of a surface under direct illumination (with no diffuse component)
over all viewing directions.
It is sometimes called "black-sky albedo"
https://journals.ametsoc.org/doi/pdf/10.1175/1525-7541%282004%29005%3C0003%3AUMBAAD%3E2.0.CO%3B2
Black-sky albedo (abs) is defined as the albedo in the
absence of a diffuse component
is a function of solar zenith angle.
dual to directional hemispherical reflectance
directional
in the absence of a direct component
independent of solar zenith angle
These two extremes can be combined as a function of the diffuse skylight
fraction (S) for a representation of an actual albedo
diffuse skylight fraction (S)
black
directional
bi-hemispherical reflectance
directional hemispherical reflectance
bi-
directional
spectral coefficient
times a scalar BRDF
Any physical plausible BRDF-modell must fulfill the following two properties
https://en.wikipedia.org/wiki/Helmholtz_reciprocity
Helmholtz reciprocity
Any physical plausible BRDF-modell must fulfill
qq 2 properties that Any physical plausible BRDF-modell must fulfill?? x2 op10
Helmholtz-Reciprocity (HR):
Energy Preservation (EP):
http://www.thetenthplanet.de/archives/255
The Blinn-Phong Normalization Zoo
Wet and dry parts on an asphalt surface, in backlight
Gloss Blinn-Phong exponent, a measure of surface smoothness
http://www.thetenthplanet.de/archives/255
tt must read
normalization constants
qq compute the normalization constants!! op11
qq how can we make Phong model more realistic?
by modifing its equ. with physical constrains!
Helmholtz-Reciprocity (HR):
Energy Preservation (EP):
qq what is normal distribution function (NDF)
??????(Normal Distribution Function,???NDF
法向分布函数
向 NDF 输入一个朝向 omega，NDF 会返回朝向 omega 的微表面数占微表面总数的比例
https://zhuanlan.zhihu.com/p/21376124
laser microstructured steel surface
red beech wood
Wear surface of chalk-filled polypropylene
Microstructure of pearlite

carbon nanotubes bundles
Self-occlusion and self-shadowing
Cook and Torrance
reflection on micro facets
geometry term covering self occlusion and self
    shadowing of V-shaped grooves
distribution of microfacet normals that correspond to half vectors in Blinn-Phong model
V-shaped grooves
https://inst.cs.berkeley.edu/~cs283/sp13/lectures/cookpaper.pdf
Microfacet Models
&p pp op21
qq cook torrance brdf equ.? write down!
qq the physical meaning of each term!
    for fresnel equ. it computes a term to control
        bet. ideal mirror reflection and ideal refraction
qq how can we simulate/approximate materials with multiple layers
    affine combinations of several distributions
qq what can be included in the geometry term? x2
    self shadowing + self occlusion should be considered
    in the cook-Torrance microfacet model
qq what is self shadowing and self occlusion ??
qq how can we cpmpute geometry term G?
    Filter approach, the min value of those three cases?
qq describe the filter approach in more detail
    fully illuminated
    occlusion
    shadowing
qq compare cook-torrance and oren nayer?
qq how can we compute the Fresnel term more effeciently?? why we have to approximate it?
+
    https://zhuanlan.zhihu.com/p/57032810
        Normal Distribution Function) 法线分布函数
        (Fresnel Equation) 菲涅尔方程: 菲涅尔方程描述的是在不同的表面角下表面所反射的光线所占的比率。
        业界方案一般都采用的 Schlick 的 Fresnel 近似
        G: UE4的方案采用的 Schlick-GGX
        G 和 F 都是用来对光线进行衰减
        F 考虑的是菲涅尔效应
        为什么不再考虑下同时对函数 F 和 G 进行重要性采样呢?
            第一，他们本身不是概率密度函数，那么我们首先要得到一个概率密度函数，要求其在半球面上的积分值为1
    many different options for our specular BRDF
        http://graphicrants.blogspot.com/2013/08/specular-brdf-reference.html
        Another weak point in existing models appears
    qq https://blog.selfshadow.com/publications/s2013-shading-course/
        various radiometric quantities
        radiance
        quantify the magnitude of light along a single ray
        qq what is a spectral quantity?
            the amount varies as a function of wavelength
        for production (film and game) rendering, RGB triples are used instead.
        it is convenient to have all vectors point away from the surface
        *The surface’s response to light is quantified by a

function called the BRDF
*brdf is the surface’s response to light
Each direction (incoming and outgoing) can be parameterized with two numbers (e.g. polar coordinates),
so the overall dimensionality of the BRDF is four
qq what are isotropic BRDFs?
rotating the [light and view directions] around [the surface normal]
does not affect the BRDF.
parameter becomes three in the case of isotropic BRDFs
t is a tangent vector defining a preferred direction over the surface
(this is only used for anisotropic BRDFs,
* Avoiding back-facing light directions
is straightforward
qq how can we set the light contributions from any back-facing direction to zero?
can be avoided by clamping n·v to 0
* over all directions above the surface
continuous weighted average.
qq why we use component-wise vector multiplication here? in computing reflection?
both BRDF and light color are spectral (RGB) vectors
the BRDF gives the relative contribution of light from each incoming direction to the outgoing light.
swapped
Energy conservation refers to the fact that a surface cannot reflect more than 100% of incoming light energy.
must not exceed 1.
The vector halfway between l and v is called the half-vector or half-angle vector ; we will denote it as h.
the denominator 4(n · l)(n · v) is * a correction factor
macrosurface
we can think of this value (which we will denote F0) as the characteristic specular reflectance of the material.
Limitations of the Microfacet Model
This formulation of microfacet theory is quite powerful and flexible
If these are important, then modifications or extensions to the model may be required.
The microfacet model does not take account of pronounced wave optics effects such as diffraction and interference.
the microfacet model is based on a relatively limited model of the surface microgeometry, with several unstated assumptions.
the definition of the normal distribution function
This is equivalent to assuming that microgeometry height and normal are uncorrelated
However, this assumption is not always true
A surface with a strong correlation between height and orientation
is the best-understood and most successful tool
qq explain the terms in cook Torrance bsdf! op19
qq what is the relationship bet. cook Torrance bsdf and Fresnel equation?
qq how can we compute the Geometry Term? which three cases are there? draw to illustrate!
qq standerd solutions in applications?
Schlick 的 Fresnel 近似
UE4 的方案采用的 Schlick-GGX

业界较为主流的法线分布函数 GGX (Trowbridge-Reitz)
qq how can we do Improtance sampling in cook torrance model?
improtance sampling according to D
因为其本身就是一个概率密度函数，那么我们在实现重要性采样的时候就方便了许多。
由于以上两点，在重要性采样上难以同时利用上函数 D、F、G
综合考虑之下，只对函数 D 进行重要性采样。
参数 albedo、metallic、roughness 和 ambient occlusion 都可以用贴图来提供

https://inst.eecs.berkeley.edu//~cs294-13/fa09/lectures/cookpaper.pdf
http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.50.2297&rep=rep1&type=pdf
Derivation of Frensnel Equations
qq Derivation of Frensnel Equations
tt must read
Fresnel Equations for Dielectric
https://zhuanlan.zhihu.com/p/21376124
tt must read detail derive
Air to Glass example
Fresnel
what is an anisotropic brdf, what an isotropic?
Fresnel Equations for Metals
qq Fresnel Equations for Metals equ?
"Fresnel term approximations for metals.?(2005)
https://dspace5.zcu.cz/bitstream/11025/11214/1/Lazanyi.pdf
https://dspace5.zcu.cz/bitstream/11025/11214/1/Lazanyi.pdf
qq which fresnel term approximations exists for metals??
Schlick 的 Fresnel 近似
UE4 的方案采用的 Schlick-GGX
qq Oren-Nayar is a brdf model for diffuse approximation??
qq 迪士尼原则--brdf 的深入研究 https://zhuanlan.zhihu.com/p/60977923
迪士尼原则的 BRDF 与 BSDF 相关总结
Disney Principled BRDF
由于其高度的易用性以及方便的工作流，已经被电影和游戏业界广泛使用，
并成为了次时代高品质渲染技术的代名词
SIGGRAPH 2012
2015 年提出的“迪士尼 BSDF（Disney BSDF）”
一方面是因为硬件性能的限制，另一方面，则是因为早期的基于物理的渲染
模型包含大量复杂而晦涩的物理参数
不利于美术人员的理解、使用和快速产出。
在创作电影《无敌破坏王（Wreck-It Ralph）》期间，迪士尼动画工作
室对基于物理的渲染进行了系统的研究
迪士尼动画电影《无敌破坏王》（2012）
主流游戏引擎都开始从传统的渲染工作流转移到基于物理的渲染工作流。
【GDC 2014】 Unity：《Physically Based Shading in Unity》
qq 提出了三个方面的工具与资源
MERL 100 BRDF 材质库
BRDF Explorer。迪士尼为分析、比较和新开发 BRDF 模型而开发的可视化工具
最主要的工作便是对材质数据库的观察与进行理论分析
Diffuse 项的观察结论　Specular D 项的观察结论　Specular F
项的观察结论　Specular G 项的观察结论　布料（Fabric）材质的观察结论　彩虹色（Iridescence）的观察结论
qq 各个模型的年代？具体的时间要记住！
Hanrahan-Krueger 模型（1993）
Oren-Nayar 模型（1995）
微观分布函数 D
绝大多数 MERL 材质都有镜面波瓣（specular lobes）
从本质上而言，Disney Principled BRDF 模型是金属和非金属的混合

型模型，最终结果是基于金属度（metallice）在金属BRDF和非金属BRDF之间进行线性插值。
应使用直观的参数，而不是物理类的晦涩参数。
参数应尽可能少。
参数在其合理范围内应该为0到1。
qq 什么是艺术导向（Art Directable）
在2012年迪士尼提出，他们的着色模型是Art Directable
而不一定要是完全物理正确(physically correct) 的
并且对微平面BRDF的各项都进行了严谨的调查，并提出了清晰明确而简单的解决方案。

原则性”的易用模型，而不是严格的物理模型
艺术导向
可以仅通过少量的参数来涵盖自然界中绝大多数的材质，并可以得到非常逼真的渲染品质。
非常逼真的渲染品质
固有色（baseColor）贴图才会同时包含金属和非金属的材质数据
Disney采用了通用的microfacet Cook-Torrance BRDF着色模型
tt must read
qq Overview over BRDF models
Graphical overview of BRDF models
BRDF MEASUREMENT
https://zhuanlan.zhihu.com/p/21376124
is a reflectivity model for diffuse reflection from rough surfaces.
It has been shown to accurately predict the appearance
of a wide range of natural surfaces, such as concrete, plaster, sand
Reflectance is a physical property of a material that describes

how it reflects incident light.
reflectance properties
*Most reflectance models can be broadly classified into two categories:
diffuse and specular
the diffuse component is often assumed to be Lambertian.
*A surface that obeys Lambert's Law appears equally
bright from all viewing directions.
This model for diffuse reflection was proposed by
Johann Heinrich Lambert in 1760
has been perhaps the most widely used reflectance model
in computer vision and graphics
however, the Lambertian model is an inadequate
approximation of the diffuse component.
qq why Lambertian model inadequate ?
This is primarily because the Lambertian model
*does not take the [roughness of the surface] into account.
Rough surfaces can be modelled as a set of facets
with different slopes
Analysis of this phenomenon has a long history and can be traced back almost a century
fit experimental data
The Oren脖ayar reflectance model, developed by Michael Oren and Shree K. Nayar in 1993
The model takes into account complex physical phenomena such as masking, shadowing and interreflections between points on the surface facets. I
It can be viewed as a generalization of Lambert抯 law
important implications for human vision and computer vision problems, such as shape from shading, photometric stereo, etc.

roughness
qq who introduced microfacet model first to computer graphics?
proposed by Torrance and Sparrow
be composed of long symmetric V-cavities.
Each cavity consists of two planar facets
In particular, the Gaussian distribution is often used,
and thus the variance of the Gaussian distribution
the variance of the Gaussian distribution,
{\displaystyle \sigma ^{2}}\sigma ^{2}, is a measure of the

roughness of the surfaces.
The standard deviation of the facet slopes
(gradient of the surface elevation)
*In the Oren-Nayar reflectance model,
each facet is assumed to be Lambertian in reflectance
In the case of {\displaystyle \sigma =0}\sigma =0
(i.e., all facets in the same plane),
we have {\displaystyle A=1}A=1,
and {\displaystyle B=0}B=0, and thus
the Oren-Nayar model simplifies to the Lambertian model:
simplifies to the Lambertian model
qq what is the relationship bet. Oren-Nayer model and Lambertian model?
qq the idea of oren nayer model?
generlization of Lambertian
more accurately than the Lambertian model.
corresponding to different surface roughnesses (
retro-reflective materials
Oren-Nayar
Fresnel Equations
https://zhuanlan.zhihu.com/p/21376124
Fresnel
反射和折射的比例由菲涅尔方程（Fresnel Equations）给出，菲涅尔方程比较复杂，
图形学里一般使用近似公式计算。
coefficients
The Fresnel equations (or Fresnel coefficients)
describe the reflection and transmission of light
(or electromagnetic radiation in general)
when incident on an interface between different optical media.
(or Fresnel coefficients)
They were deduced by Augustin-Jean Fresnel (/fre?'n?l/)
who was the first to understand that light is a transverse wave,
even though no one realized that the "vibrations" of
the wave were electric and magnetic fields.
transmittance factor:
reflection factor:
orthogonal
parallel
qq explain how to use the frensel equ.
insert into Cook Torrance model, when computing brdf
Oren-Nayar
geometry factor and visibility
Anisotropic APS-BRDF
https://dl.acm.org/doi/pdf/10.1145/344779.344814
tt must read
*Most materials are Isotropic:

(you can rotate about normal without changing reflections)
brushed metal etc. preferred tangential direction
qq what is a Anisotropic material?
the reflections will change when we rotate about normal
direction-dependent mediums
the mechanical properties of anisotropic materials depend on the orientation of the material’s body.
qq what is APS brdf?
Ashikmin, Premoze, Shirley
qq how to To support anisotropy??
one needs a tangent vector ?? ? pointing
in x-direction within tangent space
use anisotropic Gaussian distribution in the brdf computing equ.
just modify the distribution function to a desired form!
qq how can we handle that the energy preservation will not hold?
corrected diffuse BRDF
with [bi-]hemispherical reflectance
qq what meterials are typically Anisotropic??

qq what is Torrance-Sparrow model?
也被称为Cook-Torrance模型
Assume the surface is made up grooves at the microscopic leve
? Assume the faces of these grooves (called microfacets)
are perfect reflectors.
Take into account 3 phenomena
Torrance-Sparrow Result Sparrow Result
Fresnel term:
Geometric Attenuation:
Distribution: distribution function determines what percentage of microfacets are oriented to reflect in
How much of the macroscopic surface is visible to the light source
How much of the macroscopic surface is visible to the viewer
https://digibug.ugr.es/bitstream/handle/10481/19751/rmontes_LSI-2012-001TR.pdf;jsessionid=97878771AD4E28B65B909823A8B0C1F2?sequence=1
tt must read
qq what does color mean when vis. brdf as Lobe?
distributions of out going strength for all directions!
sum over to one
qq what means a brdf is phy plausible? x2 +
derived from laws of physics
non negative, symmetric, energy conservation
efficient importance sampling possible
https://www.disneyanimation.com/technology/brdf.html
tt must read
tt lobe related!!!!! must read
http://www1.cs.columbia.edu/~cs4160/html08spr/slides/lecture20.pdf
---Messung von BRDFs tobecontinue
https://digibug.ugr.es/bitstream/handle/10481/19751/rmontes_LSI-2012-001TR.pdf;jsessionid=81B56BD1DB6156AD61574AC2151786F5?sequence=1

qq measures reflectance for combinations of?? what are inputs of the measuring process?
sensorand light source position
qq how can we reduce number of necessary measurements? x2
Helmholtz reciprocity and isotropy of BRDF can help
* measurement of interchanged dots yields the same value
* For isotropic BRDFs, all measurememts around should have the

same value
can move sensor or rotate sample
Helmholz Reciprocity implies?
qq how can a single BRDF sample can be illustated?
by two dots
qq For isotropic BRDFs all measurements with the dots rotated around the normal
direction yield the same value, why?
qq how can we get the brdf result? completely measure a BRDF
sample all combinations of light and sensor locations.
qq how can we measure a brdf with minimal effort ?
> sensor locations can be restricted to?
a 1D half arc
Bildbasierte BRDF Messung
qq which devices can be used to measure brdf?
Gonioreflektometer
ccd camera
qq Three elemental components that can be used to model a variety of light-surface interactions??
> draw a graph to show the results
wiki
https://en.wikipedia.org/wiki/Bidirectional_reflectance_distribution_function
Aufnahme vieler Samples mit einem Bild einer CCD Kamera
Geometrie & Kalibrierung
Aufnahmeausr黶tung
qq what pre requirements are needed?? op48
dark room
calibration targets (checkerboard, metal spheres)
Lafortune Model
qq describe the image-based BRDF measurement system??
> what is the idea behind? x2
acquire samples with ccd camera from diff. directions
do interpolation
qq idea of Lafortune model? the formula?
[Generalization] of Phong’s reflectance model
Replaces dot product with [weighted dot product]
Fitted Lafortune model,
a generalization of Phong with multiple specular lobes,
and intended for parametric fits of measured data
suitable for brdf acquiration
Addition mehrerer Loben

https://www.cs.drexel.edu/~david/Classes/CS431/Lectures/BRDF.pdf
tt must read .
&h

https://www.sciencedirect.com/science/article/pii/S0263224113003072#:~:text=Measurement%20and%20modeling%20of%20Bidirectional%20Reflectance%20Distribution%20Function%20(BRDF)%20is,spectrometer%20and%20three%2Ddimensional%20turntables.
Measurement
optical scattering characteristics of the material surface
http://graphics.stanford.edu/~smr/cs348c/surveypaper.html
the BRDF is a function of four dimensions
there are problems with light source and camera stability
Ward built an imaging reflectometer that uses the
two degrees of freedom inherent in a camera
imaging system to reduce the number of moving parts
the system captures the light reflected into all directions at once

Since Ward fit his data to a model with a small number of free parameters,
these were not major problems. They confirm, however,
how difficult it is to measure BRDFs, even with careful attention to the measuring apparatus.
http://graphics.stanford.edu/~smr/cs348c/img6.gif
BRDF是关于入射光方向和反射光方向的四维实值函数，
它等于反射方向的光亮度和沿入射方向的入射光的辉度之比：
BRDF的可逆性源自于Helmholtz光路可逆性
交换入射光和反射光的角色，并不会改变BRDF的值
能量守恒性质
可以写成不同角度入射光的光亮度乘BRDF的积分
Phong模型缺乏物理解释
并且对于某些金属材质，它并不十分准确
Phong模型是在Lambert漫反射模型的基础上，添加了镜面反射项，以表达反射角上的镜面反射效果
Phong模型的扩展
BRDF描述了入射光在物体表面某一点反射后出射光的分布情况
分布情况
qq three types of brdf? compare them!
经验模型（Empirical Models）
Phong model/ Blinn-Phong model.... old ones
基于物理的模型（Physical-based Models）
computed from analytical models
数据表达的模型（Data-driven Models）
direct measurement of BRDFs, captures
https://www.xzbu.com/1/view-11108267.htm
qq the idea of Lafortune Model
qq how many parameters should be computed for a one-Lobe model? two-Lobe model?
3 * (1 + 3i), i = 1,2...
12 and 21
Parameter (12 fur ein ein-Lob Modell)
---Bidirectional texture function (BTF)
qq what is BTF
Bidirectional texture function
a 6-dimensional function
textures can be computed from BTF
can acquire more infos than just texture
depending on planar texture coordinates (x,y) as well as on view and illumination spherical angles.
for each pair of directions acquire image of ?? × ?? reflectance samples
In practice this function is obtained as a set of several thousand color images of material sample
* It is an image-based representation, since the geometry of the surface is unknown and not measured.
qq idea of spatial BRDF?? sBRDF
similar to BTF: high dimentional function, contains many useful information!
hh BTF is first introduced in 1999, and many similar concepts are introduced since then
qq the form of a btf database?
BTF measurements are collections of images
qq two representation of BTF? draw a graph to illustrate!
we can transfer to each other by rearranging BTF
texture representation + abrdf representation
qq applications of BTF? x2 typical applications! best suitable applications!

photorealistic material rendering of objects
in virtual reality systems and for visual scene analysis
recognition of skin texture
tt to read must!
https://dl.acm.org/doi/pdf/10.1145/300776.300778
https://cg.cs.uni-bonn.de/de/projekte/btfdbb/
http://library.utia.cas.cz/separaty/2009/RO/filip-bidirectional%20texture%20function%20modeling%20state%20of%20the%20art%20survey.pdf
hh
https://www.cs.columbia.edu/CAVE/software/curet/index.php
tt database!!!
205 measurements per sample
https://cg.cs.uni-bonn.de/de/projekte/btfdbb/download/
OBJECTS2012 Datasets
提出了一种快速的 BTF 图像超分辨率重建的方法。通过奇异值分解将已
采集到的低分辨率 BTF 数据分解为本征纹理矩阵和特征 ABRDF 矩阵,
然后采用图像超分辨率重建算法,提升本征纹理矩阵的分辨率。
最后,高分辨率的 BTF 图像可以通过高分辨率本征纹理矩阵和低
分辨率的特征 ABRDF 矩阵相乘获得
需要采集真实物体表面在不同视角方向和光照方向下的纹理图像
这些纹理图像的集合
可以被称为双向纹理函数(Bidirectional Texture Function,BTF)数据
由于 BTF 数据维度较高,现实采集中存在采集时间过长、
采集数据量庞大等问题,给其实际应用带来了困难
BRDF was measured by recording images of the sample under 205 different combinations of viewing and illumination directions.

https://www1.cs.columbia.edu/CAVE//exclude/curet/html/brdfm.html
tt must read
qq what is BSSRDF
Bidirectional [Subsurface Scattering] Reflection Distribution Function
qq the idea of BSSRDF
scattering inside of material is also considered
BSSRDF much more [natural light distribution] on skin
qq unit of a bssrdf? and its dimention?
1/(m2 * sr)
8-dimensional parameter space
hh tt
Subsurface Scattering

https://graphics.stanford.edu/courses/cs448-05-winter/papers/nicodemus-brdf-nist.pdf
四）- 次表面散射
物理着色（四）- 次表面散射
继续写之前的基于物理着色系列
https://zhuanlan.zhihu.com/p/21247702
次表面散射是现实中一种非常常见的材质外观，所有有着半透明外观的物体
例如玉石，大理石，蜡烛，可乐，苹果，牛奶（见下图）包括人类的皮肤，等等。
半透明
下图则是电影《功夫熊猫 3》中的一幕，画面中的玉石也是次表面散射材质的典型例子之一。
次表面散射现象的模拟也比在前三篇文章里介绍过的一般的表面反射复杂很多，因为要正确的模拟这种现象，光线不止再物体的表面发生散射，而是会先折射到物体内部
对于次表面散射性质的材质来说，光线出射的位置和入射的位置是不一样的，而且每一点的亮度取决于物体表面所有其他位置的亮度
物体的形状，厚度等
BSDF 只能用于描述物体表面某一点的散射性质

所以它无法描述像次表面散射这种现象。
要模拟这样的现象，最简单，最精确，也是计算量最大，最慢的方法，就是直接在物体内部的空间求解带有 Participating Media 的渲染方程
例如最简单的 Volumetric Path Tracing，Volumetric Photon Mapping 等等。
[2]是最简单的 Volumetric Path Tracing 的 tutorial，[3]是 State of the art 的 Participating media estimator。因为话题很大，这里不再展开，以后有时间会再专门写写相关的内容。
BSSRDF 可以指定不同的光线入射位置和出射的位置。Jensen 在 2001 年的论文[4]可以说那是次表面材质建模最重要的一篇论文，推导了许多重要的物理公式
BSSRDF 重要性采样
计算模型，渲染时的参数转换，以及测量了许多生活中常见材质的散射系数等等
大部分后来的论文都是在基于这篇文章中的理论的一些提升。不过这篇论文中有大量的数学以及物理公式
https://graphics.stanford.edu/papers/bssrdf/bssrdf.pdf
BSSRDF 的意义在于快速的近似。对于许多吸收系数特别低，散射系数特别高的材质来说（大部分浑浊的半透明物体例如牛奶，大理石，苹果，肉等等）
https://link.zhihu.com/?target=https%3A//www.mitsuba-renderer.org/releases/current/documentation.pdf
https://zhuanlan.zhihu.com/p/20091064

https://pic4.zhimg.com/80/992488f76560cd99204fba77ef47fcb7_1440w.jpg
当然实际渲染中这种平面不存在，所有模型都不是无限厚度的，也都是有各种形状的，

现实中所有物体内部的散射系数也都肯定不会是均匀的
这也是 BSSRDF 近似渲染的最主要的错误来源。
EDXRay 里用两种完全不同的方法渲染的颜色
材质以及介质参数完全一样的两个佛像。左边的佛像是基于 BSSRDF，右边的则是无偏的 Participating Media
BSSRDF 因为假设所有表面都的厚度都是无限的，在实际模型中如果有厚度比较薄的地方，误差会更为明显。
这也是为什么 BSSRDF 渲染的结果相比 Participating Media 要暗淡一些。除了这些误差以外，BSSRDF 也没办法渲染出焦散
Left: BRDF ?hard“ light distributio
right: BSSRDF describes light transport and scattering inside of material.
scattering inside of material
qq how bssrdf is used?
describe scattering inside of material!!!!
internal color bleeding in shadowed region under nose.
[much more natural light distribution on skin.]
pruf-re-ok- slides-3 CG3_03_Monte-Carlo-Techniques
prev.
Numerical_integration
https://en.wikipedia.org/wiki/Numerical_integration
brick rule, simpson rule
Quadrature rules based(二次的)
can be derived by constructing interpolating functions that are easy to integrate
Typically these interpolating functions are polynomials.
only polynomials of low degree are used, typically linear and quadratic.
midpoint rule or rectangle rule
This is called a composite rule, extended rule, or iterated rule.
For example, the composite trapezoidal rule
Simpson's rule, which is based on a polynomial of order 2, is also a Newton–Cotes formula.

概率密度函数pdf
pdf更加有用一些
累积分布函数CDF 是那个大F
PDF：概率密度函数（probability density function），在数学中，连续型随机变量的概率密度函数
（在不至于混淆时可以简称为密度函数）
是一个描述这个随机变量的输出值，在某个确定的取值点附近的可能性的函数。
PMF : 概率质量函数（probability mass function)，在概率论中，概率质量函数是离散随机变
量在各特定取值上的概率。
蒙特·卡罗积分
是对理想积分的近似。
核心就是两个字：采样(Sampling)
如果我们先验地知道函数的形状，那我们就可以针对性地生成非均匀分布的随机样本，这样能够在相同样本数量的情况下对目标积分得到一个更准确的估计
https://www.qiujiawei.com/monte-carlo/
采样样本越多，就越逼近真实的积分结果，这是蒙特·卡罗积分的最核心特性
每个样本的出现概率都'可以'是各不相同的。
轮盘赌
十九世纪俄罗斯，由监狱的狱卒强迫囚犯进行
*用于递归深度的控制，一句话就ok
used to control the depth of recur. of the ray traveling through the scene
ql
qq the operator form of the rendering equ. ? op4, i wrote
qq what is the meaning of each compon. ? s3op6, i wrote
qq trace-reflection operator? s2p18
qq splitting of the BRDF s2p18
qq what is measurement operator? p4
qq explain the dimensional explosion p6
qq how can we solve a integral numerically? p6 x4 methods... roughly
qq what is a quadrature rule
qq why single sample estimator funcs? p9
qq what is importance sampling ? p9
qq how does Monte Carlo Techniques solved the problem of the curse of dimensionality? p11
qq why we use importance sampling?
qq what is the MC technique applying to reflaction integral? p17
qq how can one compute Lin recur.in p17? code online
qq why not use fixed termination depth? p18
qq proof that the exp. value will stay the same! p18
qq how to implement russian roulette? p21 returns the estimate?
qq explain the given code p19-21
qq why sampling? p27
qq explain code p29
qq how to sample when there are a lot of light sources? p30
qq what is path tracing p31
qq the monte carlo case of path tracing p32 formula
---r1
qq measurement equ in operator form and rendering equ. op4
measurement operator
spectral power
its output? its dimention?
multi-dimensional nested integrals
can also be written as nested integrals

nested integrals
qq why nested integrals?? op5 formulation
qq how can we compute spectual power with help of nasted integral?? op5
qq what is the dimension of the nasted integral??
evaluate nested integrals
qq quadrature rule
brick-rule, trapezoidal-rule, Simpson-rule
qq error estimation in brick-rule?? 1d,2d,d dimention!
qq why exponential to d?
qq explain dimensional explosion or the curse of dimensionality
[required number of samples]
to get the error below
&h too many number of samples are needed to
let the error below a certain level.
qq error estimation for d dimention??
qq what is the idea of Monte-Carlo Quadrature
cast integration as expected value problem
does not suffer from the curse of dimensionality
variance is independent of dimention, decrease with square root of N
uniform distribution
qq the expected val of func. f in uniform distribution case ?? op07
qq the error analysis of Monte-Carlo estimator,
> how it decreases with increasing number of samples
take a look at the standard deviation!
the standard deviation (measure for statistical error)
estimator decreases with the square root of number of samples:
qq the most imp. part of MC estimator is that Independent of the dimension
qq variance of the expected val??
number of samples
qq the expression of the estimator?? op7
qq how fast error changes with the number of sampels?? compute?
compute the actual integral of the function
independent of the sampling distribution
qq a modified estimator? why we do the modification??
The free choice of the sampling distribution
it significantly influence the variance of the estimate
qq what is the new expected value of estimator and variation?? op09
qq what simplifacation will be done for nested integrals
> along light transport paths
just sample once
reduce to single sample estimator
qq what is Importance sampling, why good for estimating ??
choose px proportional to fx, reduce variance
qq why can we reduce variance in this way??
assume p = c * f...
qq what can we do to reduce variance with MC estimator??
estimate the distribution of f and make p proportional to f !! op10
qq ofcourse we can do every thing with uniform distribution
> instead of Importance Sampling? but with a high variance!

infinite dimensional integral
Numeric approximation of this integral with standard

quadrature approaches
suffers from the curse of dimensionality
qq the idea of MC integration?
Monte Carlo techniques cast the numeric integration problem as an expected value estimation problem
averaging estimates from samples drawn from a distribution
Standard deviation (sqrt of variance)
Standard deviation (sqrt of variance) of the estimate corresponds to the approximation
A good choice of ??(??) can significantly reduce variance
variance
---MONTE CARLO GLOBAL ILLUMINATION
contributions to a pixel depend on parameters for light measurement and for the light path
qq why can we sample independently?
can be multiplied
yields product of individual terms
Distribution Raytracing
qq how can we summ over all contributions of rays ??
> single path Monte Carlo estimator looks like?
> from all diff. paths
Sampling the spectrum
qq how can we Sampling the spectrum??
To reuse Monte Carlo samples one samples ??
from the [sum] of the efficiency curves:
spectral quantities
---lense equation
qq what is focal points? not always pass through a focal point!
qq give the names in blank places!
qq lens equ.! large
the aperture size ?? (measured as area of circle)
qq how can we simulate depth of field (DoF) in GI??
enhance your photos.
simulate with help of a thin lens (draw)
the zone of acceptable sharpness within a photo that will appear in focus.
Some images may have very small zones of focus which is called shallow depth of field.
shallow depth of field.
qq Three main factors that will affect your control of the Dof???
aperture (f-stop),
distance from the subject to the camera,
focal length of the lens on your camera.
qq How does aperture control depth of field? addi
hole through which light enters the camera) controls the amount of light entering your lens.
Large aperture = Small f-number = Shallow (small) depth of field
aperture
&h Dof can be controled by aperture
To simulate depth of field
qq how can we simulate Dof in rendering ??
simulate with help of a thin lens (draw)
&h with help of apertur
Focal Length refers to the capability of a lens to magnify the image of a distant subject.
qq the longer you set your focal length the shallower the depth of field?

&h simulate the Dof effect by introducing a
thin lens and adjest its para.
---Spatial and Temporal Filtering
qq what is box filter when doing spacial filtering??
qq what is optimal sinc-filter ?
Mitchell-Netravali-Filter
qq what is box filter
tensor product
qq what is tensor product
Spatial
qq how can we do Spatial filtering ??
> temporal filtering for a blur effect?
Temporal filtering gives motion blurr and can be done in the same way for a time interval.
qq how can we achieve motion blurr?
with time space filtering, similar to spatial space filtering
qq how can we extend 1d filtering to 2d? when applying spatial and time space filtering?
performing tensor product
qq why uniform sampling of the pixel area corresponds to a box filter??
qq what is the filter kernel looks like for box filter?
> and its relationship with gaussian filter?
qq what is theoretically optimal sinc-filter??
a sinc filter is an idealized filter that removes all frequency components
above a given cutoff frequency, without affecting [lower frequencies]
Mitchell-Netravali-Filter
die zum Beispiel beim Antialiasing oder bei der
> Skalierung von Rastergrafiken verwendet werden k鯟nen
werden sie auch als bikubische Filter bezeichnet
da sie zu den kubischen Splines z鋒len
Die Mitchell-Netravali-Filter wurden im Rahmen einer
Untersuchung zu Artefakten von Rekonstruktionsfiltern entworfen
qq how can we extend 1d filter to 2d?? op17
qq how can we approximate the reflect term of RE?? approximate the directional form?? op18
> using MC integration, the equ! fully understanding!
How to choose p for efficient importance sampling
qq when to terminate?
Russian Roulette
qq the diea of Russian Roulette?
terminates with a probability (decreases when ray travels in the scene)
returns the estimate
qq the form of expectation in RR?? and why it looks like this? op19
normalize Monte Carlo estimates, why should we normalize it?
qq why rr need a binary random varible b
qq why the expectation remains the same
qq why we have to make sure that the expectation remains the same??
The variance is increased
*&h we can stochastically terminate the recursion
in the rendering equation without changing the expectation

value.
without changing the expectation value.
qq why not terminate in a fixed way?? op19
A fixed termination depth would introduce bias
qq will the success probability decrease?? pp
qq how can we do sample_hemisphere()? what should be feed in, what do we get from that?
N-rook sampling of primary rays
construct ray from aperture sample through pixel sample
L_in_? = incoming_radiance(scene, ray, ?[k], potential);
&h how can we express the term in our equ. when coding ?? op22
努塞尔数
split incoming radiance in direct and indirect illumination to support direct light sampling
Nusselt’s Analogon
the cosine weighted solid angle corresponds to its projection onto the unit disk
qq what is Nusselt's Analogon?? how can be used to improtance sampling?? op25
can be used to perform importance sampling of the cosine term
qq how can we do this? projection back? the formula?
qq we sampled according to what?? op25
qq draw a graph to illus.
qq what is Direct Light Sampling??
directly illuminated part, no reflection on ray path
ray is traced to the light
qq the expression of L_direct and L_indirect ?? op26
one splits integral into sum over light sources
point light sources emit spectral intensity which is spectral light power per solid angle
directional lights must be written in the directional form
we need to sample a number ???? of points ???? on the light source
and estimate the direct light contribution
Sampling of area light sources can have high variance
spherical light source is half invisible to any scene point
projection is hard to do and hard to
sample and we still need visibility check.
qq which light sources are there? x3
pnt, dir, area light source
qq how can we sample light source in this picture?? op28 draw!
half invisible
spans a direction cone
Indirect light typically comes from all directions
Refined Implementation for Direct Light Sampling
qq explain the code
Dealing with Large Number of Lights
qq how to Dealing with Large Number of Lights with help of the idea of RR??
one defines a probability parea|pnt|dir for each light source
such that summation of probabilities is one./ with normalized probability
Then one chooses one (or more) light source[s] according to the assigned probabilities
best to assign the probabilities proportional to the emitted
spectral power of the light sources

returns the estimate over probability p
qq how can we define the probability for each light source,
> when we have multiple sources ?? op31
proportional to the emitted spectral power of the light sources
qq how can we reduce variance in Path Tracing?
Sampling a large number of primary rays is always
necessary to reduce variance sufficiently.
The support of colors, spatio temporal filtering, and depth of field yields a large number of
parameters to integrate over resulting in a large number of primary rays
qq Importance sampling is introduced to?
sampling the Hemisphere
Cosine term with Nusselt's analogon
BRDF into diffuse and specular parts
direct and indirect illumination
qq explain the process of path tracing (later)
---particle/smoke sim
pruf-re-ok- slides-9 CG3_09_Particles
Generalizations of Point Mass
3d Particle Grid
Class-Hierarchy-Physics Particle
qq what properties does a Particle have? x4+
•state
•mass
•color
•density
Radius
Transparency
Age (used to destruct and re-birth old particles)
qq how can we gen. particles, randomly? which sampling?
transformation sampling
One can use transformation sampling to
generate random particle from
uniformly distributed random variables
Flock of Birds, Herds, Schools
Boids
qq what is boids, how diff. from particles ??
*Extend particles with
orientation and 3D modell
*can be used to model Flock of Birds, Herds, Schools
& op08
with orientation and 3D modell
qq what should we cosider when imp. such a sys.?? x4 what cases should be avoid?
avoid collisions
avoid obstacles
synchronize directions
stay close to others
qq how can we do Visual Simulation of Weathering?
qq the idea of gamma-ton Tracing
g.can be seen as an extend to photon tracing
simulate Weathering with particles
1.gammaton [shooting]
emit from the outer hemisphere of the scene
Each particle stores weathering information
Each particle is assigned propabilities for being one of 4 states

2.[propergation]
follow physical and other laws
surfaces are sampled into point clouds
When particle collides with surface, the movement probabilities
change according to surface reflectance
3.[perform] Gamma-transport until final [equilibrium/ until Settlement]
https://www.idi.ntnu.no/emner/tdt03/Presentations2013/Reiten%20-%20gammaton-tracing.pdf
&h
?-tons are generated on the outer hemisphere of the scene
Particle Systems
surfaces are sampled into point clouds states
qq Movement Probabilities for Each particle? possible movement??
Linear Motion (in case of high energy)
Parabolic Curve (modelling particle with carriers)
Flowing along surface (particles with low energy)
Settlement (particles with very low energy)
qq when does the movement probabilities change?
according to surface reflectance
qq where does γ-ton reflectance stores initially??
point clouds, sampled from surface
?-tons that carry water increase Patina fraction of surface
qq Patina case? how can we achieve this??
blend between two materials A and B with a fraction
qq what does a particle carry in this case?? op15
fraction

Cloth Simulation
qq Woven fabrics typically has which two distinguished directions?
warp and weft (Kett und Schuss)
influence the physical properties of the fabrics.
qq how can we model this?
rectangular grid with vertices

http://www-labs.iro.umontreal.ca/~bernhard/PDF/Thomasze08Asynchronous.pdf
Provot
Provot 1995
discretization of a piece of fabrics with a regular grid of particles
qq how can we simulate internal forces? x3
simulated by springs
shear springs
structural springs
flexion springs
qq what If spring constant is chosen too small for easier integration
too elastic
qq solution to this problem? x2 how can we simulate clothes more stable and stiff? x2
use small steps for explicit integration or implicit integrating
limit maximum elongation
qq what is Kawabata-Measurements? x3 test in three cases!
&h
Add constraint on maximum edge length
one measures the three different forces over the elongation
Kawabata-Measurements

```
                qq which three forces are used?
                    structural
                    shear
                    flexion forces
                qq what is Hysteresis effect? how can we simulate?
                    needs to add friction forces wrt.
                        stretching, shearing and bending velocities
                    or use Kelvin-Voigt material, also called a
                        Voigt material, is a viscoelastic material
                        having the properties both of elasticity and viscosity.
                        Kelvin-Voigt-SpringModu
                        allows to model the hysteresis effect by a damper element
                        acts like a memory element that records the history of the
movement
                        memory element t
                        Kelvin-Voigt-Spring Modul
                Particle systems are the simplest possibility to model a
                    variety of effects like fire, herds and weathering in an adhoc
                forming a grid like structures with *additional connections
                qq what possible additional connections are there?
        ---rbody sim
            pruf-re-ok- slides-10 CG3_10_RigidBody
                https://www.cs.cmu.edu/~baraff/sigcourse/
                qq what are Dynamics and Kinematics
                    *Kinematics- Deals with study of motion
                        of material objects [without] taking into
                            account the factors which causes motion
                    *Dynamics is based on concept of force.
                    Mechanics involves forces and displacements,
                        typically on objects with mass
                    problems involving forces
                    How "much does the spring stretch? " is classified as
"dynamics."
                Equarions of Motion
                qq short notation that projections from one onto the other vector
op3
                Euclidean transformation, i.e.
                one defines a local object coordinate system per rigid body
                Euclidean transformation
                qq what is Euclidean transformation
                    a rotation and a translation
                    A rigid body can be positioned in space with it
                qq how can we define natural origin and rotation of a rigid body?
in 2d and 3d
                    center of gravity ??
                qq what is the natural origin of an object?
                Discrete Case and contineous case!
                affine combination
                center of gravity ?
                qq how can we compute center of gravity in descrate case and
continus case??
                    *average position weighted with the point mass
                    summations become integrals
                qq what is affine combination
                ?? and ?? stand for world and object coordinate system
                For base vectors, the additional superscripts
                indicate which coordinate system is spanned by the base.
                base vectors
                qq draw the base vectors on a given graph!
```

qq transform from object corrdi. to world coordi. *** and back. formula!!x2
two contribu - tions to the velocity
Angular velocity
qq derive velocity equ.!! both angular and linear velocity!
    op07 can not understand what prof. is doing
    http://www.kwon3d.com/theory/moi/iten.html
    https://www.cs.cmu.edu/~baraff/sigcourse/notesd1.pdf
As with kinematics, dynamics can be split into linear and angular motion
applied to the center of gravity and added to the total force
equivalent to the one for point masses
force needs to be transported to centroid also
    orthogonal to force action lines
qq what is the diff. bet. kinematics and dynamics??
    in dynamics, forces are considered, included
Torque ?? is the rotational equivalent to force
lever action
qq what is the dir. of Torque?? tobere
    Torque points along the rotation axis
length
component of the force orthogonal to ???? .
qq how to understand this? :force acts twice once for linear and once the angular dynamics
&p Fges?? ges stands for? op8 pp
qq expression of the torque? in 3d case. vec3 calcu. op10
    http://www.kwon3d.com/theory/moi/iten.html
qq the dimention of a torque in 3d case?
    should be a vec3 varible!
total torque
qq what is moment of inertia
    angular acceleration extracted, drived from torque expression
    The proportionality constant is called the
        moment of inertia and is the equivalent to the mass in linear dynamics
    proportionality constant is called the moment of inertia
    is the equivalent to the mass in linear dynamics.
    the equivalent to the mass in linear dynamics.
    &h is the similar thing compared to mass in linear dynamis,
        if only linear dynamics are present, I will not exist
    grows quadratically in the distance to the axis of rotation
qq continuous case, the inertia tensor results from? the formula in continus case?
qq calcu. of Inertia in both continus or descrate case *
* cross product is anticommutative
* Vector triple product
* matrix multiplication
qq derive the fromula of Inertia tensor
    &h possible: 在dot的时候可以把前面的转置，(rw)r 可以轮转成 rr^T w wrong!
    matrix form!! a* a^T
    https://www.cs.cmu.edu/~baraff/sigcourse/notesd1.pdf
    http://www.kwon3d.com/theory/moi/iten.html
qq why use a* a^T?? matrix form??
qq inertia tensor fromula op11 both descrete case and continus case! In 2d and 3d! x4
qq * give the formula in 3d case! op12 calcu. Inertia in Example of circular disk
qq how can we express inertia tensor in program?

mat3
qq what is the dimention of inertia tensor??
> and inv_mass? and angular velocities?position? orientation?
mat3, float, vec3
qq give the calculation formula of computing the I of Cuboid! op13
> and the result that we can directly use later!
&h
Rotation matrices are square matrices, with real entries
that is, a square matrix R is a rotation matrix if and only
if RT = R-1 and det R = 1
Rotation matrices can either pre-multiply column
vectors (Rv), or post-multiply row vectors (wR). H
However, Rv produces a rotation in the opposite direction with respect to wR. T
rotations produced on column vectors are described by means of a
pre-multiplication
column vectors
To obtain exactly the same rotation (i.e. the same final coordinates of point P), the row vector must be post-multiplied by the transpose of R (i.e. wRT).
The rotation matrix of the orientation will be multiplied from the left and transposed from the right
he dot product of two column vectors is the matrix product
is the row vector obtained by transposing {\displaystyle \mathbf {x} }
\mathbf {x} and the resulting 1? matrix is identified with

its unique entry.
look into the topic of change of basis and matrix representations
of linear transformations.
Saying two matrices are similar amounts to them being
two representations of the same linear transformation with
respect to (possibly) different ordered bases
of the same linear transformation
change of basis
qq how can we compute inertia tensor in world coordinate system?
> transform from local coordi to global coordi. op14
&p ok-op14 why this? have we imp. this before?? yes
qq how can we change bases from one to an other??.... matrix calcu. later!!! tt
linear momentum and angular momentum
qq what is the relation between force and linear momentum? tobere
> Torque and angular momentum??
Angular momentum ?? is a vectorial conserved quantity
calculate_mass_and_inertia
qq Inertia Tensor are changing during simulation?
have to be calculated in world coordi. sys.!
?Inertia Tensor
qq what kind of spinning tops are there?? !
prolate oblate
two kinds
qq will angular momentum and angular velosity
> pointing at the same direction?
maybe not
qq say, what’s the dir of angular velocity and the one of torque!?
qq when will they pointing to the same dir?
if both point along a main axis! op15

qq where are spinning tops??
qq what kind of spinning tops are there?? x2
The state of the rigid body is uniquely defined
qq how a state of a rbody is defined?? x4 by four components?? position, ori. and?
qq why only reciprocal values are needed?
&h
only reciprocal values of
mass and inertia tensor are
needed.
qq rotation must be orthogonalized after each integration step? we did not do that ?
can be done with polar decomposition.
qq what is the The time evolution function as motion equ.?? op17 *** research
qq what does it mean if a 0 is stored in reciprocal mass or tensor of inertia???
tt https://en.wikipedia.org/wiki/Tensor_algebra#Coalgebra
by multiplying each element of u by each element of v:[1][2]
in index notation:
qq Normal component of mass matrix compute!! task5!! description..
tt dyadic product
https://en.wikipedia.org/wiki/Talk%3ADyadic_product
for each collision, save both normal and tangential impulses
pruf-re-ok- slides-11 CG3_11_Collision
// r2 only: op19/59 later- fixed! finished!
Friction
Collision Handling
qq what is coll. handling??
determine new object velocities / momenta
without a change of positions
qq what is free motions
qq what is penetration depth??
How deeply did the objects penetrate each other?
which direction is the connection of the shortest distance?
Simulation Loop
qq the sub tasks of a rbody simulator?? which components are there? op07 large
> explain their functionalities!
qq Filtering of contact points, what kind of contact pairs can be ignored??
qq why we filtering
qq main idea/ gen. idea/ which phy. laws are used in computing collisions?
Use energy and momentum conservation for a consistent
recalculation of momenta
qq when and how we have to do an integration step?
we compute each step and this can be summed up to a integral
broad phase: quickly find all collision pairs
quickly
narrow phase
analyze found intersections
Elastic vs. Inelastic impacts:
persistent contacts with slipping under friction can occur.
? In both elastic and inelastic impacts, we assume that
speeds change instantaneously.
Impact types
qq what impact types are there?? x3
elastic

partical elastic
inelastic
collision time must be determined precisely
in order to simulate the collision correctly.
Elastic Impact Details
local surroundings of elastic bodies become springs
following Hertz’s force law
qq what is Hertz’s force law?
similar to Hook‘s law, but higher order, as there is an exponent part
to the displacement
qq which two phases are there?
compression phase
restitution phase
qq what is local influence area
constant ?? depends on
qq the Hertz’s constant can be calculated with a formula, for spec. shape?
depends on local parameters.
illustration for shear modulus
qq compare Rigid Body Impact and inelastic impact
both bodies coalesce and unite into one large body
energy is lost in deformation
//
elastic collision the bodies touch each other
briefly and bounce off each other without loss of energy
//
momentum is conserved in both cases
Central Case
two bodies of mass ??1 and ??2 meet centrally
deformation work
qq deformation work and velocity after collesion? -noneed
> for both cases: elastic and inelastic cases
qq the velocities and work can be computed? op13
qq nearly all materials are partially elastic?
qq how can we quantify the elastic property if a material? formula? tobere
Coefficient of restitution
¬ impact inverts direction of relative velocity
qq what is COR? coeffi. of restitution??
qq what is the direction of relative velocity before and after collision?
impact inverts direction of relative velocity
velocities after impact and the deformation energy
qq the velocities can be computed with epsilon! op14
qq is the rel. velocity reversed after collision??
qq If both masses are the same, we have a trivial solution?
This simply corresponds to the bodies exchanging
their initial velocities to each other.
exchanging
Sliding friction
Rolling friction
qq which non conservative forces do you know?? p15
Dry Friction Forces
Sliding friction, Rolling friction,
Viscouse Friction Forces
Stokes‘ friction., Air friction
qq how is the relationship bet. Stokes‘ friction and Air friction and velocity? how to determin the coef.?

properties of a contact of two object and not properties of an object or of a material.
qq is COR and coeffi. of friction properties of an object or material??
n two!
qq Typically used values can be found online? which key word?
Contact_Table

http://atc.sjf.stuba.sk/files/mechanika_vms_ADAMS/Contact_Table.pdf
contact normal, which indicates the direction
perpendicular to the contact, is important for the treatment
qq which contact types are there for polyhedral?
> 2 non-degenerated contact types?
vertex-face
edge-edge
> degenerated cases
Vertex-vertex, vertex-edge and edge-edge (id)
less likely occour case
Edge-face and face-face
extended contacts
qq hwo can we calcu. contect normal in edge-edge case?? op18
cross product of the two edges
vertex-face
qq what is a trival solution?
solution in special cases, easy cases typically
qq what is a degenerate case?
less likely to happen
multiple contact normal directions, normal can not be computed directly
NP hard contact resolution problems
qq how can we handle multiple contact normal directions?? avoid NP hardness
solve iter.
choosing an arbitrary contact normal
qq what is indeterminate contact?
degenerated case, normal can not be computed directly
qq what are degenerate types typically? x5
qq how can we handle face overleap?
contacts are replaced by
multiple point contacts one for each corner.
&h multiple posint contact!
qq how can Rectangle intersection problem be solved? algo! the complexity? op21
Sort...enter event...exit event...
nlogn+k
qq the idea of BVH collision test
the collision test only
descents down (in detail level) only when the root nodes intersect
qq what do we need to test collections in near phase?
triangle-triangle intersection tests
qq tt how can we do xxx intersection tests?
qq Contact Extraction algo? large x3+ tobere
[body movement]
[Broad phase] enumerates potential contact pairs
[Near phase] filters out actual contact pairs
compute [exact contact information] for list of contact pairs
positional vertex
normal

penetration depth
qq Near Phase with SAT for OOBB, how many axis to be tested??
qq the idea of SAT theorem??
no intersection between two bodies
if [one separating axis exist] where 1D projections do not intersect
//Collision Handling
&h
impulse transfer
we can treat multiple contact collisions through a sequence of single contact impact
Interpenetration from discrete time stepping can be cured
by the introduction of bias velocities
bias velocities
joint constraints allow for update of external
forces through solution of LCP
A numerically robust LCP solver for simulating articulated
rigid bodies in contact

https://pdfs.semanticscholar.org/f6c9/47008716ad5cde2bae42732277cf4a7036ce.pdf
http://www.roboticsproceedings.org/rss04/p12.pdf
2008
animation.rwth-aachen.de/media/papers/2012-EG-STAR_Rigid_Body_Dynamics.pdf
*https://animation.rwth-aachen.de/media/papers/2012-EG-STAR_Rigid_Body_Dynamics.pdf
https://www.cs.cmu.edu/~baraff/sigcourse/notesd2.pdf

*https://box2d.org/files/ErinCatto_SequentialImpulses_GDC2006.pdf
qq we have to express conservation law in normal direction! ??!
3rd Newton‘s law
qq how can we simulate joints? ensure the joint constraint?
ensure the velocity after collisions is zero
qq what is cross product matrix?
qq explain the idea of inverse mass matrix! K-matrix!
relate the change in velocity to the force impact
qq tt Derivation of K-Matrix!
qq tt compute the Change in Total Energy, derivation and the result?
qq how can we compute impulse? how can we update velocities of the two? pp with code??
> how can we compute impulse in: inelastic/ elastic/ partial elastic case?
> (based on ennergy conservation)
(Frictionless Impact and with frictions)
qq how can friction force be visualized?
friction cone
qq what does it indicates if a spin of the rigid body is observed?
friction force/ impulse imp.
qq what is normal_mass and tangent_mass? what are their difference?

> and how are they used??

```
mat3 m1 = c.body1->inv_inertia*(dot(r1, r1) * I - dyad(r1, r1));
mat3 m2 = c.body2->inv_inertia*(dot(r2, r2) * I - dyad(r2, r2));

float inv_mass_sum = c.body1->inv_mass + c.body2->inv_mass;
```

```
                    c.normal_mass = 1.0f / (inv_mass_sum + dot(c.normal, m1 *
c.normal) + dot(c.normal, m2 * c.normal));
                qq how can we compute/ include friction forces?
                    compute Tangential impulse similarly to normal impulse!
                qq * how can we decide whether to apply static or sliding
friction??
                    we compute static friction first, check if it is smaller than
                    the value of sliding friction, if not smaller, we apply sliding
friction
                    or this can also be done with the help of math::climp()
function
                qq draw to illu. parallel and orthogonal force impact! op36
                qq how can we compute tangential part of K matrix??
                qq impulse transfer?
                    compute/ update velocity in simulation with the help of impulse
                qq how can we handle Multiple Contacts? op39
                    update for each each body i and each contact point j
                qq * for the Newton’s Cradle, our current knowledge is not
sufficient! why?
                    it is not a dispersion-free system.!
                    [impact propagation time] can not be ignored!
                qq how to handle this case? we may can not simulate Cradle
                    collisions are dispersion-free and propagate fast
                qq describe the Sequential Impulses Algorithm(loop) detail  op46
                    1.compute list of contact points
                    // the v-p-v approach //
                    2.compute relative velocities
                    3.iteratively apply impulses with constrains (impulse
accumulation)
                        and add de-penetration bias velocity
                    4.update linear velocities and angular velocities
                qq how can we avoid penetration between diff. bodies?
                    add de-penetration velocities op47
                qq what is the pre-condition of impulses computation?
                    list of contact points & normals extracted
                qq impulses computation: op47
                    inelastic normal impulse, friction impulse, de-penetration
impulse
                qq how can we inprove the stability of the Sequential impulse loop?
** x2
                    key1. compute iteratively, k=10-50,
                    key2. permute order of contact list continuously
                qq examples of Contact Constraints (for contact forces)
                    The contact forces can "push" but not "pull".
                    once two bodies have separated at a contact point, there is no
                        force between them at that contact point.
                qq what is contact forces vector? can we apply constrains on it?
                &h LCP problem
                    一般是*不等式
                    LCP vs LP: linear complementary problem has unequal objective
function
                    arises in computational mechanics field
                    encompasses the well-known quadratic programming as a special
case.
                    can be solved by simplex algorithm
                qq how can we solve LCP problem arises in computational mechanics?
                    express/ classify contact points to vanishing and non-vanishing

                        and then solve with linear programming
```

qq how can we deal with Interpenetration from discrete time
stepping ?
** with bias velocities
qq how can we handle in games? e.g. chains of bodies?
//geometrical constrains arised in computational mechanics!
*https://www.cs.cmu.edu/~baraff/papers/sig89.pdf
solving linear systems of inequalities
calculate the forces that would naturally arises
The bodies may also be constrained to satisfy certain
geometrical relationships
however, it is not a valid analytical model of forces
between bodies in resting contact
Penalty Methods vs. Analytical Methods
several advantages over penalty methods.
Penalty methods for rigid bodies are often computationally
expensive
the differential equations that arise using penalty
methods may be "stiff"
In contrast, analytical methods for rigid bodies give exact
answers and produce differential equations
require far fewer time steps during simulation
analytical solutions are based directly on the laws of
Newtonian dynamics.
Analytical methods however are much more complex to derive
and implement
Simulation using Analytical Methods
arbitrary holonomic geometric constraints can be achieved
with this paper!
use Analytical methods instead of the one we used before
but it is relative hard to implement
---fluid sim
pruf-re-ok- slides-12 CG3_12_Fluid
https://www.cs.ubc.ca/~rbridson/fluidsimulation/
http://david.li/fluid/
mit verschiedenen Parametern f黵 Gase und Fl黶sigkeiten
angewendet werden.
Fire (Fedwik 2002)
Momentum Equation
qq Incompressible Navier-Stoke’s op7 equ.
qq “Incompressibility condition”?
stream lines diverge (div ?? >0)
stream lines
qq what is divergence of a vector field u?? op08 the pic!
qq does div depends on time??
no, only on spatial derivatives
Divergence does not depend on the time
derivative but only on spatial derivatives
&h divergence does not dep. on time
perform variable substitution
characteristic values in dimension-less constants
define dimension-less force
dimension-less Navier-Stoke甜equations
asterix
explain a given rendering equation.
do not have to learn the rendering equation by heart
The same holds for rigid boy collision and Navier Stokes equation.
component form
Reynold甜 Number
qq what is Reynolds Number

it is the ratio between [inertia force and frictional force]
*determines the behavior of the fluid
Re determines the depth of the vortex cascade.
for large Re, small vortices are still relevant.
thus the simulation in such situation is computational expensive!
(for gases typically large and for
fluids a broad range [water ?honey] possible
qq what is Re<1, Re>40, Re>2000??
stationary flow
periodic behavior
individual vortices
turbulent, chaotic, vortical
qq what are particularly hard to simulate?
small vortices
qq what is Existence- and Uniqueness of „physically plausible “ (energy-restricted)
> solution of Navier-Stoke‘s-Equations in 3 dimensions? tt
qq what are our goals in cg?
[fast] approximate solutions
with a good [visual impression]
without [suppressing vortices].
qq what happends to Reynold when simulating gases??
qq explain the behavier of Re number
Kolmogorov-Cascade of turbulent Fluids
Re determines the depth of the vortex cascade
The accurate simulation of turbulent fluids is computational expensive
Then a high grid resolution and small time steps are necessary
qq how Re affacts turbulent fluids??
large vortices
刺hysically plausible ?(energy-restricted) solution of Navier-Stoke甜-Equations in 3 dimensions
Millennium Problems
Precise simulation of air resistence of cars,
buoyancy of airplanes, water resistance of ships,
spray nozzles, weather forecasts
computationally complex
good visual impression
fast approximate solutions
Momentum Equation of fluid
qq what is material derivative??
qq What forces act on the blob? x4
gravity: mg
pressure: integrating [pressure gradient] over the blob’s volume Vdelta(p)
Viscosity: blurred diffused, yields [Laplacian]
body forces: k added by user
qq the momentum equation?? Navier–Stokes momentum equation!
> explain it! derive! op19
qq we define a quantity in Eulerian view, how can we compute the derivative
> in Lagrangian view?
> derive the material derivative!`
(take use of the chain rule to derive, and rearrange it)
qq what is advection?
if q moves with the fluid particles and no external forces !
The 擁ormal?contact force is pressure (force/area)
frictional part of contact force

simple model is that we want velocity to be blurred/diffused/
Blurring in PDE form comes from the Laplacian
heat equation
qq the expression of heat equ. op22
standard eq抧
? Label each speck of matter, track where it goes (how fast, acceleration, etc.)
Measure stuff as it flows past
Lagrangian viewpoint
Eulerian viewpoint:
qq compare those two viewpoitns
boils down
It all boils down to the chain rule
How fast is ?? of that blob of fluid changing in the Lagrangian viewpoint?
Material Derivative
qq what is Material Derivative? can we tell how
> fast this quantity changes?? op26
acceleration
material derivative
qq why equals to zero?? in "advection" equ??!!
how fast ?? is changing at [a fixed point] in space
holds even if the vector field is velocity itself:
dot-product and gradient
our fluid has a colour variable
qq what is the phy. meaning of xxx op27, how can we compute the changing in
> a fixed point?
qq what do we know if the fluid is advection fluid? op27 !
we have some equ. derived from that!
qq what if our fluid is color ... even if the vector field is velocity itself
> how can we handle this situation?
componentwise!
component-by-component
that is dot-product!
qq Advection Equation
liquids change their volume as well as gases,
otherwise there would be no sound underwater
qq Real fluids are compressible why we assume liquids are incompressible
> in fliud simulation?
Compressibility is nearly irrelevant for animation
has a very little effect when simulating
qq derive the incompressible condition op31
fix your eyes on any volume Ω of space...
qq why irrelevant for animation?? op30
Acoustic waves usually have little effect on visible fluid motion
qq the reason that div of u is zero everywhere?? for incompressible fluids.
fix your eyes on any volume, assume fluid is incompressible,
we know volume of
fluid in = volume of fluid out
divergence theorem
constrained dynamics
qq why velocity field is divergence free for incompressible fluids?

qq how can we simulate this div free system? x2

1.constrained dynamics
2.artificial compressibility
solve for a pressure that makes our fluid incompressible
*constrained dynamics
may have to slove linear system
pressure is the matching Lagrange multiplier
Track density, make pressure proportional to density variation
*soft-constraint
artificial compressibility
&h
make the velocity field divergence free
s a constraint, and
To avoid having to solve linear systems, can turn this into a soft-constraint
Track density, make pressure proportional to density variation
easier to accelerate a linear solver?if the linear system is well-posed
numerical methods for solving incompressible Navier Stokes equations
how can the advection step be approached in a semi-Lagragian manner
不可压缩流
不可压缩流是流速的
散度等于零的流动，更精确地称为等容流
这理想流动可以用来简化理论分析
所有的物质多多少少都是可压缩的。
“等容”这一术语指的是流动性质，不是物质性质
*由于做了不可压缩这假设，物质流动的主导方程能够极大地简化。
随体导数(material derivative)表达
也就是说，随着物质元素的移动，质量密度是常数
质量密度是常数
这些方程建立了流体的粒子动量的改变率（力）和作用在液体内部的压力的变化和耗散粘滞力（类似于摩擦力）以及重力之间的关系。
这些粘滞力产生于分子的相互作用，能告诉我们液体有多粘
任意给定区域的力的动态平衡
它们可以用于模拟天气，洋流，管道中的水流，星系中恒星的运动，翼型周围的气流
它们也可以用于飞行器和车辆的设计，血液循环的研究，电站的设计，污染效应的分析，等等。
依赖微分方程来描述流体的运动。不同于代数方程，这些方程不寻求建立所研究的变量
（譬如速度和压力）的关系，而寻求建立这些量的变化率或通量之间的关系
变化率或通量之间的关系
用数学术语来讲，这些变化率对应于变量的导数
实用上，也只有最简单的情况才能用这种方法获得已知解。这些情况通常涉及稳定态（流场不随时间变化）的非紊流，
中流体的粘滞系数很大或者其速度很小（低雷诺数）
对于更复杂的情形，例如厄尔尼诺这样的全球性气象系统或机翼的升力，
纳维-斯托克斯方程的解必须借助计算机才能求得。这个科学领域称为计算流体力学
21 世纪流体力学领域的重要技术之一
目前有多种商业 CFD 软件问世，比如 FLUENT、CFD-ACE+（CFDRC）、Phoenics、CFX、Star-cd 等。
CFD 最基本的考虑是如何把连续流体在计算机上用离散的方式处理。一个方法是把空间区域离散化成小胞腔
应用合适的算法来解运动方程（对于不粘滞流体用欧拉方程，对于粘滞流体用纳维-斯托克斯方程)
*不粘滞流体用欧拉方程，对于粘滞流体用纳维-斯托克斯方程
欧拉方程的数值解法非常倚赖特征线法。
光滑粒子流体动力学，求解流体问题的拉格朗日方法，

*如果选择不使用基于网格的方法，也有一些可选的替代
谱方法，把方程映射到像球谐函数和切比雪夫多项式等正交函数上的技术
格子波尔兹曼方法（Lattice Boltzmann Methods），它在直角正交格点上模拟一个等价的中尺度系统，而不是求解宏观系统（也不是真正的微观物理）
Lattice Boltzmann Methods
很多实例中，其他方程和纳维-斯托克斯方程要同时被求解。这些其他的方程可能包括描述种类浓度，化学反应，热传导，等等。很多高级的代码允许更复杂的情形的模拟，涉及到多相流（例如，液/气，固/气，液/固）或者非牛顿流体（例如血液
其他方程和纳维-斯托克斯方程要同时被求解
大涡流模拟（LES）是一种技术，其中更小的涡流被滤掉并用亚格点尺度模型来建模，而更大的能量的涡流则被模拟出来。该方法通常比 RANS 模型需要更细化的网格，但是比 DNS 解所需要的网格粗的多。
离散化完成后要进行的方程系统的基本求解由很多数值线性代数的为人熟知的许多算法完成。可以使用静态的迭代方法，譬如对称高斯-赛得尔（Gauss-Seidel）或者渐进松弛（successive overrelaxation），或者克雷洛夫子空间法（Krylov subspaces）。在后者中，解的余数在非线性算子的一个子空间的正交基上最小化
这些技术广泛用于工程设计或者分析和流体相互作用的设备，例如车辆，飞行器，泵，化学装置和通风系统等等。该技术也用于计算机图形学，因为动画家不能像控制固态人物那样简单的处理流体，因而必须使用 CFD 技术结合交互工具，达成动画影片或游戏中的流体模拟
研发了基于天河高效能计算机的“高精度数值风洞”和“高超声速 CFD 软件平台”
对称守恒网格导数计算原则
http://www.xys.org/
**有一个总的正确的模拟的物理公式，但是模拟的时候是针对情景的不同程度的简化优化

qq besides grid based methods, what other methods do you know?? web
Inviscid Fluids
&h not viscid
In most scenarios, viscosity term is much smaller
Zero viscosity = “inviscid”
Inviscid Navier-Stokes = “Euler equations”
qq what is numerical dissipation??
在计算物理学中，数值耗散（numerical dissipation）也称为数值扩散（numerical diffusion）
是指一种出现在微分方程数值解中的副作用。当一个没有耗散的纯平流方程式利用
数值分析方式求解时，其初始波的能量会依类似耗散过程的方式减少！*
不过有时为了提升数值解的数值稳定性，会特别加入人工耗散（artificial dissipation）
can be seen as an artifact
耗散是出现在非匀相热力学系统中不可逆过程的结果。
耗散过程是指能量（内能、动能或势能）由一种形式转换到另一种形式，
而且后者可以作的功少于前者
例如将能量转换为热是一种耗散过程
热力学的耗散过程在本质上就是不可逆的，此过程以固定的速率产生熵。
不可逆过程
一些特定情形下的耗散过程无法用单一的哈密顿力学方程来描述
人工耗散（artificial dissipation
numerical dissipation
qq what is numerical dissipation? how can be solved ??
side-effects that may occur as a result of a numerical solution
will cause the loss of energy
"artificial dissipation" is intentionally added to improve the numerical stability
viscosity of air
viscosity of water
Dynamic viscosity
qq what is kinematic viscosity
qq is air more viscous than water?
air is 10 times more viscous than water
qq incompressible Euler equations? and explain!

We know what’s going on inside the fluid: what about at the surface
qq which Three types of surface can be found with fluid??
https://www.youtube.com/watch?v=kphzE0KXuNw
? Solid wall: fluid is adjacent to a solid body
water is so much denser than air
Free surface
Other fluid: possibly discontinuous jump in quantities (density, …)
Boundary Conditions
qq how can we model solid wall? No fluid can enter or come out of!
“no-stick” condition
let fluid slip past tangentially
qq mathematical expression of this? op38
qq what is "no-stick" condition
qq what is “no-slip” condition?
For viscous fluids only, can additionally impose “no-slip” condition
qq how can we model Free Surface?
with help of gradient of pressure,
we can set pressure to be 0
qq what if surface tension is impro. ??If surface tension is important
pressure is instead related to mean curvature of surface
qq how about the pressure in this case??
mean curvature of surface is considered
qq how can we model surface tension in Free Surface?? op39
qq when simulating two fluids interacting with each other,
> do we know Density jump?
> what about the Normal velocity jump??
> what about pressure jump?
> (take the previous video as example!)
&h
At fluid-fluid boundaries, the trick is to determine “jump conditions”
With no surface tension, pressure jump [??] = 0 vanishes
Normal velocity jump must be zero
one-dimensional flow in fluids or a one-dimensional deformation in solids.
Rankine–Hugoniot conditions

https://wikimedia.org/api/rest_v1/media/math/render/svg/e32e53864fd40d3dfe7ac8cf8ab59ddcddfd24d9
&h those jump condiitons is somehow related to conservations
fluid-fluid boundaries
how can we determain fluid-fluid boundaries??
https://en.wikipedia.org/wiki/Rankine%E2%80%93Hugoniot_conditions
Normal velocity jump must be zero
&p pp detail?? tt
Numerical Simulation Overview
momentum equation
pain to handle them all simultaneously
Splitting Example
qq hwo can we split a differential equation?? op43 pic given
a separate solution module for each term
qq can we split any equ. by solving them saparately? how? op43
> F and G function
> and in First order accurate in time!

qq which component can we Split Momentum equ? x4
    advection
    gravity
    diffusion
    pressure update
qq How can we make the fluid incompressible?
Eulerian grid
qq compare simulation strategies: Eulerian grid and SPH based in nutshell
qq op45 x3x1
    +Trivial to [set up] easy to define grids
    +easy [spatial derivatives] computation
    +Particularly good for the effect of [pressure]
    -advection doesn’t work so well
qq what is A Simple Grid Disaster??
    numerical solver may give other solutions
qq how can we solve such Simple Grid Disaster?
    estimate divergence at stagger the grid, instead of a grid point
    Staggered Grids
    we estimate divergence at a grid point as
    put velocities halfway between grid points
qq what is The MAC Grid? pros and cons for it?
    From the Marker-and-Cell (MAC) method [Harlow&Welch’65]
    particular staggering of variables
qq compute pressure gradient at velocity sampling point?
qq where are the velocity sampling points? and pressure sampling points?
//works well for incompressible fluids:
qq where is the velocity??
    velocity ????+1/2,??,?? in middle
    x-part of velocity
    y-part of velocity
    ?? in middle of x-face
    pressure gradient
qq how can we compute velocity and pressure gradient in this pic.?? op48
qq where are we going to compute pressure gradient??
qq what are downside of MAC grid? And array storage??
    translate indices in code, and thus make interpolaiton annoying

qq Downside of this MC grid based strategy, possible solution??
    switch back and forth, and averaging later
Robert Bridson’s philosophy: avoid averaging as much as possible!
We’re on an Eulerian grid, though,
    so the result will be called “semi-Lagrangian”
    (introduced to graphics by [Stam’99])
qq what is Semi-Lagrangian Advection??
    use the Lagrangian notion of advection directly
    but solve it within grids, thus, Eulerian grids
qq the idea of Semi-Lagrangian Advection??
    trace backwards through the velocity field, q doesn’t change
qq why Chief interesting aspect of fluid motion is vortices??
qq compute the x_old with help of x_new? in explicit euler?
    Forward Euler does not
    Instability, obvious spiral artifacts
qq compute u at staggered location?
qq what is the particular requirements of a stable ODE integrator?
    handle rotation well, without artifacts

qq does Forward Euler handle vortices well? why?
qq can Forward Euler handle vortices correctly?? op53
qq which ODE integrator can be used here with a better result?
Runge-Kutta 2 is simplest half-decent method
Runge-Kutta 2
qq how can we get the velocity and compute u(xyz) vector?? based on MAC grid?
by averaging the appropriate staggered components:
at (??,??, ??)
qq op54
we only want to advect quantities in an incompressible velocity field
qq why advection step should be the first in the split integration?? op55
use the old incompressible values stored in the grid
and do not have to update them
when we trace back from a grid point to x old
we won't land on a grid point
qq how can we get q(x_old) when the x_old position is not on grid?
Solution: interpolate from nearby grid points
Simplest method: bi/trilinear interpolation
be careful to get it right for staggered quantities
staggered quantities
we're interpolating from is not in the fluid?
qq What if x old isn't in the fluid? what should we do in this case?
Extrapolate before advection, to all grid points in the
domain that aren't fluid
use extrapolated velocity to update, to interpolate
qq whta happends when we are beyond the original observation range
qq what is the diff. bet. interpolation and extrapolation??
the range is diff!
extrapolation is subject to greater uncertainty and
a higher risk of producing meaningless results.
interpolation is down between known observations
qq region of interest (ROI)
interpolation focuses on an area known as the region of interest (ROI)
be careful about what's known and unknown on the grid
&h
trickiest bits of code
与内插类似，但其所得的结果意义更小，而且更加受不确定性影响。
此公式与线性内插是一样的。只是线性内插时
extrapolation is a type of estimation, beyond
the value of a variable on the basis of its relationship
with another variable
It is similar to interpolation, which produces estimates between
known observations
interior reconstruction can be applied to dental or cardiac CT images, the concept is not limited to CT.
A sound choice of which extrapolation method to apply relies on a prior knowledge of the process that created the existing data points.
Crucial questions are, for example, if the data can be
assumed to be continuous, smooth, possibly periodic etc.
A conic section can be created using five points near
the end of the known data
conic sections template (on paper) or with a computer.
the inviscid "no-stick" boundary condition

qq only constraint the normal components of u when facing solids?
constraint the normal components
we need to extrapolate into solids
can’t use the solid’s own velocity
qq why cannot can't use the solid's own
bad numerical viscosity
qq what is volumetric animator forces?
qq how can we add such a force?
qq we have to solve pressure before advection?
https://github.com/tunabrain/incremental-fluids/blob/master/1-matrixless/Documentation.md
computational fluid dynamics
closely follows Robert Bridson's book, "Fluid Simulation for Computer Graphics",
The solvers in this project come in a large variety
ranging from minimalistic to complex. All solvers are Eulerian in
nature and run on a staggered Marker-and-Cell grid.
The number of the solver defines a progression
codes with higher number build on codes with lower number,
either adding on features or replacing methods with better ones.
not in any of the successing solvers to avoid clutter.
save individual frames.
qq “Poisson equation” for pressure?? op62
The discrete pressure update on the MAC grid:
qq what is the incompressibility condition on the new velocity on grid ijk??
divergence free
*lets voxelize the geometry
each grid cell is either fluid, solid, or empty
qq what are possible grid point types?? x3
qq explain this pic
qq why we update pressure first?
we can compute the “ghost” solid pressure at boundaries and then, inside
after the first iterate that all pressure has been computed, we can apply
advection, fluid simulation, update velocities! (pp ?? may prob.)
qq compute the ghost pressure for the simplist case!
qq linear equations for pressure due to div free?
End up with a sparse set of linear equations to solve for pressure
qq do we have apply iterative computing process here? why?
Iterate until the divergence is small enough (pp ?? may prob.)
qq *why Voxelization is Suboptimal?? drawbacks of grid based solution? x2
Free surface artifacts
may not grid-aligned
Solid wall artifacts
boundaries may not grid-aligned
Slopes are turned into stairs
SPH – Algorithm Variants
SPH – Pressure Projection
&h paper rel. start from op71
only solve for ?? in Fluid cells
empty cells: ?? = 0 (free surface)
qq how can we compute the "ghost" solid pressure in this case??

op66
qq re- when solving pressure, how can we handle Boundary conditions?? op65 x2
&h the pressure should be zero but be a
ghost val. to simulate correctly
pressure update formula into discrete divergence
qq how can we conpute pressure for solid cells??
&p drop to 5??op68
qq how to update pressure and velocity?? op68
&h
Discrete Pressure Equations
End up with a sparse set of linear equations to solve for pressure
sparse set of linear equations
Matrix is symmetric positive (semi-)definite)
正定、半正定矩阵的直觉代表一个向量经过它的变化后的向量与其本身的夹角小于等于90度。
更为直观的理解是从几何图形的角度去看，正定矩阵就是一个椭球。
等价于，矩阵A(A为对称矩阵)的特征值全大于0，特征值全大于0，以其对应的特征向量为基时，曲面相当于往上翻的，因为都是拉伸变换
等价于，存在一个可逆矩阵B，满足A=BTB。(这实际间接说明了A的全部特征值大于0，因为负负得正…)
可逆矩阵B
正定矩阵
Matrix is symmetric positive (semi-)definite
qq what about the pressure matrix?
qq the linear equ. of pressure can be solved effeciently?
Instead use Preconditioned Conjugate Gradient,
with Incomplete Cholesky preconditioner
&p pp
Residual is how much divergence there is in
Iterate until satisfied it’s small enough
divergence
qq why we iterate?
divergence
Free surface artifacts
Solid wall artifacts:
Left with strangely textured surface
If boundary not grid-aligned, O(1) error
Slopes are turned into stairs,
water will pool
*ref. https://github.com/tunabrain/incremental-fluids
soot particles
qq how can we simulate smoke? phy model?
composed of soot particles suspended in air
Smoke is composed of soot particles suspended in air
track concentration of soot on the grid: ??(??,??)
Evolution equation:
qq equ. for soot concentration on grid
Usually smoke is moving because the air is hot
pressure vs. gravity ends up in buoyancy (hot air rising)
Need to track temperature
qq why and how can we track temprature?? the Evolution equation!
&h Need to track temperature
qq what is conduction??
qq how can we handle Boundary conditions??
extrapolation
The Boussinesq approximation is applied
to problems where the fluid varies in

temperature from one place to another,
driving a flow of fluid and heat transfer.
The fluid satisfies conservation of mass, conservation of momentum and conservation of energy.
qq what is Boussinesq approximation, what may be its advantage?
Density variation due to temperature or soot concentration is very small
use a ghost force to simulate buoyancy!
computational effecient
qq how can we add external buoyancy force in momentum equation??
qq which dir has buoyancy force? what are alpha and beta in such equ.?
Constants
&h
ariations in fluid properties other than density ρ are ignored
In the Boussinesq approximation, variations in fluid properties other than density ρ are ignored
continuity equation for conservation of mass i
general expression for conservation of momentum of an incompressible, Newtonian fluid (the Navier–Stokes equations) is
where α is the coefficient of thermal expansion
conservation of momentum
conservation of mass
convection equations
Density variation due to temperature or soot concentration is very small
Use the “Boussinesq approximation”: fix density=constant
but add external buoyancy force in momentum equation:
“Boussinesq approximation
Constants ?? and ?? can be tuned
qq what If no soot and no temperature difference??
qq bouy. force is a vector force?
qq what boundary condition we have to apply when facing open boundaries?
free surface (p = 0)
explicitly zero velocity on far boundaries
We let air blow in or out as it wants
explicitly zero velocity on far boundaries
Vorticity Confinement
vortices live on all scales and have high visual impact
qq why vortices disolves too fast?
numerical dissipation disolves vortices too fast
qq how can we overcome it?/ how can we preserve the vortices?? x2
[Fedkiw 2001] add force that keeps vortices alive
[Fedkiw 2005] trace vortex marker particles and apply forces
couple them with forces to grid simulation
trace vortex marker particles
Fedkiw
qq what is vorticity?
vortex strength and axis of rotation
qq what is gradient vorticity?? which direction?
axis of rotation/ gradient vorticity/ vorticity confinement force
qq how can we determain vortex strength and axis of rotation?? op77
gradient vorticity
points towards vortex core line
vorticity confinement force
qq what is the equ. of vorticity confinement force?? op77

qq what is the direction of vorticity confinement force??
qq why we need delta x here?
ensures independence of grid resolution
qq mark omega, ita and force dir in this pic.!op77
numerical dissipation of soot density and temperature counteracts the gains of vorticity confinement
counteracts
qq what is monotonic Hermite Interpolation, how it benefits? solves overshooting??
Fedkiw [2001] propose to use a modified Hermite interpolation
ensure the monotonicity of the resulting Hermite spline.
qq how can we do vorticity confinement ??
qq why we would apply monotonic Hermite Interpolation instead of
> other higher order functions?
fights back numerical dissipation and do not have
the problem of overshooting
qq draw example curve for Hermite interpolaiton and the monotonic one!
Monotonic Cubic Interpolation for smoke simulation
monotone cubic interpolation is a variant of cubic interpolation
that preserves monotonicity of the data set being interpolated.
preserves monotonicity of the data set being interpolated.
Monotonicity is preserved by linear interpolation but not guaranteed
by cubic interpolation.
tt use c++ and python mixed
modified Hermite interpolation that is monotonic in the data.
numerical dissipation
qq how can we deal with numerical dissipation?
with interpolation
qq but can not use higher order interpolaiton!
beacase of overshooting
qq why we typically use SPH approach for water simulation?
voxelization does not working well when simulating water air surafce
voxelization approach does not sufficiently reproduce water surfaces
&h water and sph approach left
qq how can we sampling the water-air surface correctly?
> two better ways than grid based ones
Marker particles, render with meta balls
Level Sets
qq compare voxelization approach and Marker particles
qq the idea of Marker particles!
sample fluid volume with [particles]
define [cell state] to be fluid if it contains
marker particles, empty or solid otherwise
[move particles] in incompressible grid velocity field
[render surface] via meta balls
[resample] resulting iso-surface on simulation
qq idea of Level Sets method
[represent interface] by signed distance level set function
[advect level set] according to
[recompute] distant values by fast marching method
&p pp
tt papers!!!
** tt Felzenszwalb, P., & Huttenlocher, D. (2004).

Distance transforms of sampled functions. Cornell University.
http://www.cs.cornell.edu/~dph/papers/dt.pdf
Fast Distance Function
&pp err? op81 ! x - x' ==> (x,y) - (x',y) ^ 2 ??
qq what is Fast Distance Function,
> how can it be used to levelset methods?? op81
nD-case can be reduced to (n-1)D-case,
nDcase can be computed in linear time!
qq the computing example of Fast Distance Function on op82
qq 3 ways to define a surface when simnulating
nD-case can be reduced to (n-1)D-case
3D transform can also be computed in linear time
Felzenszwalb, P., & Huttenlocher, D. (2004). Distance transforms of sampled functions. Cornell University
tt paper!!!!!
Then 1D distance transform is height of lower envelop over all parabolae
qq how lower envelop looks like??
list of ?? parabola parts
lower envelop over all parabolae
all parabolae
compute lower envelop of parabolae
height of lower envelop over all parabolae
qq how can we compute the intersection of two evolops?
qq how Evelop can be built, description? and the code?
Evelop is built in sweep from left to right,
adding one parabola at a time
horizontal range of parabola part ?
qq the Extrapolation process??
qq Extrapolation initialization?
extract and [store interface points]
[store for each grid point] closest point on interface
[interpolate] fluid quantities onto interface points
// compute fluid quantities on each grid cell point
qq Extrapolation methods for surface computation?? x2
1. copy values from closest point on interface
2. align gradient of distance points parallel to ray
get information from points with smaller distance
signed distance simplifies extrapolation significantly
store for each grid point closest point on interface
solve the PDE with finite differences (Careful “upwind” scheme needed
based on Fast Distance Function
qq the algo. to compute Fast Distance Function??! op84
&h the algo is grid based so, we use q from 0 to n-1 to iterate all grid cells!
signed distance simplifies extrapolation significantly
Careful “upwind” scheme needed
&p scheme??
tt read from code!!!
qq what means Adaptive Discretization?
level set method was extended to octrees
taking use of octrees
//SPH
http://physbam.stanford.edu/~fedkiw/papers/stanford2004-07.pdf
qq what is SPH, and why we use it instead of grid based methods?
Smoothed Particle Hydrodynamics
there is a conclusion that sph provide best realism at same computational speed.

Ihmsen, M., Orthmann, J., Solenthaler, B., Kolb, A., & Teschner, M. SPH fluids in Computer Graphics. Eurographics STAR, 2014
provide best realism at same computational speed.

***https://cg.informatik.uni-freiburg.de/publications/2014_EG_SPH_STAR.pdf
https://ge.in.tum.de/publications/2017-sig-um/
comparation
https://www.youtube.com/watch?v=chnS24QfgNY
https://www.youtube.com/watch?v=coCqA4vGziU
youtube.com/watch?v=coCqA4vGziU
https://www.youtube.com/watch?v=coCqA4vGziU
Infinite continuous adaptivity for incompressible SPH
realistic kinematic viscosity for water
qq what is the sph formulation of the momentum equ.?
with kernel ??, smoothing length ? and number of dimensions ?
Interpolation
qq how can quantities be interpolated in sph method? op91
from closest particles
cubic kernel function or quintic spline
gaussian kernel
qq which interpolation are we going to use in practice?
quintic spline interpolaiton
qq which 2 methods to interpolate exists?
qq which mehtod is the best one theoretically?? why it is probabilistic in practice?
for theory Gaussian is best but in practice problematic
due to [infinite support]
qq what is the prob. with gaussian kernel? why we do not use it?? op91
&p pp infinite support ? particles that very far(infinite ones) are affected??
standard approach
qq how can we compute partial derivatives of the desired quantity?? Ai,
> in a simple way? what may be the prob. of that?
has numerical problems
qq how can we compute a robust approx. for gradient and lap. of Ai??
uses the more robust approximations, derived in ori paper!
&p pp derive??
qq how can we compute pressure?
qq how can we update pressure and viscosity? what is the updating eqution?
> Basic Algorithm op93
further variations at fluid-fluid and fluid-solid boundaries
Spatial derivatives: Spatial derivatives can be computed in various ways
various alternatives have been investigated and currently, the following
approximations are preferred
[Mon92,MFZ97]:
&h see paper for detail! he just put it here without expain..........
fluid has near constant density
qq how can we compute, estimate time step width Δ?? smaller than a threshold?
> with safty term
is computed from Courant-Friedrich-Levy(CFL)
can move no more than 0.4? per time step

SPH – Basic Algorithm
qq SPH - Basic Algorithm op93, detail see paper!
qq how time step width t? is computed is defined?? computed??
qq when interpolating, how can we do Neighbor Search effeciently??
    > what may be the benefits?
    typically a regular and potentially hashed grid is used
    better locality by using Z-index Sorting of grid cells
    *Z-index Sorting
    large number of GPU based approaches available
    tt https://en.wikipedia.org/wiki/Z-order_curve
    provide more locality
    *knn or other neighbor search approaches exist
qq compare Z-Index Sort and KNN algo.?
    Z-index is better for gpu based parallism, knn runs on cpu
    Z-index has the best GPU performance when reading neighbors
qq why Z-order_curve can provide more locality?
    suitable for pgu parallel??
    suitable for the gpu mem. layout
    &h ***Some GPUs store texture maps in Z-order to increase spatial locality of
        reference during texture mapped rasterization.
    This allows cache lines to represent rectangular tiles
    increasing the probability that nearby accesses are in the cache
    decreases the probability of costly, so called, "page breaks"
    This is important because 3d rendering involves arbitrary transformations
        (rotations, scaling, perspective, and distortion by animated surfaces)
&h
    Morton space filling curve,[1] Morton order or Morton code
    map multidimensional data to one dimension while preserving locality of the data points
    Coordinate values

https://upload.wikimedia.org/wikipedia/commons/thumb/d/da/Lebesgue-3d-step3.png/300px-Lebesgue-3d-step3.png
    Interleaving the binary coordinate values yields binary z-values as shown
    Connecting the z-values in their numerical order produces the recursively Z-shaped curve.
    Two-dimensional Z-values are also called as quadkey ones.
    https://upload.wikimedia.org/wikipedia/commons/thumb/3/30/Z-curve.svg/800px-Z-curve.svg.png
    The Z-ordering can be used to efficiently build a quadtree for a set of points.
    building a pointer based quadtree
    https://en.wikipedia.org/wiki/Hilbert_curve
    &h many related curve exists
    3d rendering involves arbitrary transformations
    These formats are often referred to as swizzled textures or twidled textures.
    UB-tree
    Space-filling curve
    https://en.wikipedia.org/wiki/Locality-sensitive_hashing#Locality-preserving_hashing
    https://en.wikipedia.org/wiki/Geohash
    Neighbor Search
qq why it has a z shape?

发出的，

**Use with one-dimensional data structures for range searching
qq incompressible condition for sph approach?
divergence free, low density error, can be done iteratively
qq what search strategies can be used for Neighbor Search?? op94
can be used to make velocity field divergence free
different choices for stiffness constant
qq what sph Algorithm Variants do you know?
with splitting trick, computing intermediate values
or, EOS solver
without pressure force and in a second step do the pressure update.
Incompressibility Update
qq how can we make Incompressibility Update in sph simulation ? divergence free?
use perr in formula (9)
sparse linear solver needed
qq how can we do Pressure Projection in sph?
Pressure Projection means computing pressure with alternatives
tt detail see paper!
and their combinations can be used
qq what is Multi-Phase Fluids?
simulating multiple fluids
qq artefacts at the fluidfluid interface? what can be the reason? how can we solve it? x2
difference in fluid densities leads to
number density
interpolation schame
Solid Body Coupling
sample solids along boundary with particles
Solid Body Coupling
qq what is Solid Body Coupling op99 how can we archieve this?
add constraint forces that avoid interpenetration
compute forces and torques of fluid on solids
qq how can we render in sph simulation?? given a set of particles op100
define iso-surface from particles
render with volume rendering techniques
lots of works with particle visualization techniques exists!
can be implemented on GPU (see scivis lecture on particle visualization)
*Versatile Rigid-Fluid Coupling for Incompressible SPH (SIGGRAPH 2012)
*Infinite continuous adaptivity for incompressible SPH (SIGGRAPH 2017)
Simple Grid Disaster
&p pp numerical solver has other solutions
> how to understand this??
ok
qq why not use regular grids? when will cause prob.? in which cases?? op46
---ray sim
pruf-re-ok- slides-2 CG3_02_Rendering-Equation
prev.
ray tracing
现在游戏基本都没有应用光源追踪技术，光线都是由你能看到的亮光的物体自身发出的，
电脑也不会计算每个光源从哪里来，
到哪里去，更不会计算这些光源的相互叠加。
只是通过及时演算物体阴影和控制光线的强弱来“模拟”人眼看到的真实情

况

海中没有椰子树倒影这一点就很不真实。实际上，在游戏中使用的光源越多

画面在越貌似华丽的同时，破绽也会越多，唯一的解决办法就是采用光源跟踪技术。

原因很简单，使用光线追踪技术的运算量异常庞大，这么多年来的历代显卡都无法胜任这项工作

现在的光源追踪技术也远非完美。计算出正确的反射和折射角度也不代表就能达到完全真实的视觉效果

游戏开发人员试着在《雷神之锤III》中加入了光线追踪效果，悬浮的奖励道具在墙上的投影就是通过光学追踪计算出来的，使得光源的真实感大大提高

种基于光栅化的渲染系统，往往只支持局部照明(Local Illumination)

局部照明在渲染几何图形的一个像素时，光照计算只能取得该像素的信息，而不能访问其他几何图形的信息。

https://www.jianshu.com/p/0375389e6a3e

该图片出自《孤岛惊魂》，尽管看似水面显示出了远处山峰的倒影，却不能渲染植被、船骸等细节。

理论上，阴影(Shadow)、反射(Reflection)、折射(Refraction)均为全局照明(Global Illumination)效果，

实际应用中，栅格化渲染系统可以使用预处理(如阴影贴图(shadow mapping)、环境贴图(environment mapping))去模拟这些效果

Rasterization 和 ray tracing 其实是相对的

把物体表面间对光的反射和折射成为间接光、间接反射、间接折射。光线在物体之间的传播方式是光线跟踪算法的基础。

只有很少部分可以进入人的眼睛，因此实际光线跟踪算法的跟踪方向与光传播的方向是相反的（反向光线跟踪）

由视点与像素(x,y)发出一根射线

光栅化渲染，简单地说，就是把大量三角形画到屏幕上。当中会采用深度缓冲(depth buffer, z-buffer)，来解决多个三角形重叠时的前后问题

光线追踪除了容易支持一些全局光照效果外，亦不局限于三角形作为几何图形的单位。任何几何图形，能与一束光线计算交点(intersection point)，就能支持

没有三角形化这个过程!

要计算一点是否在阴影之内，也只须发射一束光线到光源，检测中间是否存在障碍物。

本例代码尝试使用基于物件(object-based)的方式编写

主要计算量在求交点?

https://zhuanlan.zhihu.com/p/41269520

光栅化渲染作为相对的两个概念

以物体为单位划分为若干个子任务，每个物体由若干三角面组成

下一层就会失去对上一层全局信息的了

比如拆分成物体后，我们就不知道场景里其他物体的存在了，拆分成三角面后，我们就无法得知其他三角面的信息

拆解任务可以让渲染过程高度并行化，所以非常的快，但是同时因为全局信息的丢失，我们很难实现一些需要全局信息的渲染效果。

全局照明效果的，譬如环境光遮蔽，间接反射，漫射等。这些技术五花八门，实现方法完全没有一个统一的框架可以遵循

这些光线彼此不知道对方，但却知道整个场景的信息。每条光线会和场景并行地求交，根据交点位置获取表面的材质、纹理等信息，并结合光源信息计算光照

一个三角面/一根光线的整个绘制过程都可以划分为若干个阶段，这些阶段合在一起就是我们通常意义上说的管线（Pipeline）

可编程管线则允许用户使用自定义的着色器（Shader）对数据（顶点，面，像素）进行处理

在原有光栅化管线的基础上，也就引申出了计算管线（Compute Pipeline）

DXR API 实际上就是定义了一种适用于光线追踪的 Raytrace Pipeline。

brdf

https://zhuanlan.zhihu.com/p/41269520

BRDF 描述的是表面本身的性质，比如它的光滑程度，导电程度等等

一部分是表面的自发光（上述公式中的 [公式] ）

另一部分是其他表面的射向该表面的光线（上述公式中的 [公式] ）BRDF（上述公式中的 [公式] ）作用后的结果

BRDF 描述的是表面本身的性质，比如它的光滑程度，导电程度等等。由于四面八方的光线都会作用在这个表面，所以我们需要对所有方向进行积分，也就是一个球面上的积分

只有位于正半空间的方向才会对最终积分有贡献，所以最后这个球面的积分就变成了一个半球的积分

bidirectional reflectance distribution function

双向反射率分布函数

蒙特卡洛积分

本身只是一种数值计算的方法，它和光线追踪本身无关

蒙特卡洛积分就是这样一种方法。它的思路很简单：为了估计某个函数在一个定义域内的积分，我只需要在这个定义域内随机地找一些采样点，

计算采样点所在位置的函数值，把所有采样点的函数值平均即可得到该积分的估计值：

对一些高维的难以求解的积分有非常好的效果，而渲染方程就是这样的一类积分，所以它常常被用在求解渲染方程上

如果我们先验地知道函数的形状，那我们就可以针对性地生成非均匀分布的随机样本，这样能够在相同样本数量的情况下对目标积分得到一个更准确的估计

重要性采样

和它的名字一样，就是尽量采样积分定义域内重要的点，少采样不重要的点。

路径追踪

渲染方程本身在绝大多数情况下是无法直接求解的

人们把渲染方程用不同的数学等价形式改写，然后对新的方程形式进行近似求解

以它的法线为中心向半球内发射若干条光线，求出每条光线和场景的交点，要进一步以交点的法线为中心发射若干条光线

极难求解的递归过程

而路径追踪的方法把某个点的颜色看作是若干条光路（Path）合起来的贡献。一条光路可以认为是若干表面点连接而成的一条线段，为了计算某个点的着色，

我们只需要以该点为起点，构建若干条路径，并将每条路径上的光照贡献叠加即可。

我们可以认为路径追踪就是把光线以路径的形式重新组织了起来

*全局光照 GI

https://www.cnblogs.com/machong8183/p/7543724.html

(Global Illumination,简称 GI), 或被称为 Indirect Illumination

既考虑场景中直接来自光源的光照（Direct Light）又考虑经过场景中其他物体反射后的光照（Indirect Light）的一种渲染技术。

使用全局光照能够有效地增强场景的真实感。

在直接光源（阳光）照射不到的地方，得到了更好的亮度和细节表现，从而使整张渲染效果更具真实感。

*仅仅使用直接光照的渲染中往往有黑色不真实阴影，因为光源没法直接照射到--区分

*镜面反射、折射、阴影一般不需要进行复杂的光照方程求解，也不需要进行迭代的计算。因此，这些部分的算法已经十分高效，甚至可以做到实时

光的漫反射表面反弹时的方向是近似“随机”，因此不能用简单的光线跟踪得到反射的结果，

***往往需要利用多种方法进行多次迭代，直到光能分布达到一个基本平衡的状态

全局光照现今已有多种实现方向

以及各种实现派别

如光线追踪（Ray Tracing）派系，其实就是一个框架，符合条件的都可称为光线追踪，

其又分为递归 式光线追踪（Whitted-style Ray Tracing），

分布式光线追踪（Distribution Ray Tracing），

蒙特卡洛光线追踪（Monte Carlo Ray Tracing）等。

而路径追踪（Path tracing）派系，又分为蒙特卡洛路径追踪（Monte Carlo Path Tracing），

双向路径追踪（BidirectionalPath Tracing），

能量再分配路径追踪（Energy Redistribution Path Tracing）等。

其中有些派系又相互关联，如路径追踪，就是基于光线追踪，
结合了蒙特卡洛方法而成的一种新的派系
编年
ray casting
从每一个像素射出一条射线，然后找到最接近的物体挡住射线的路径
光线追踪 Ray Tracing [1979]
如果该点处表面是镜面或折射面，则继续向反射或折射方向跟踪另一条光线，如此递归下去，直到光线逃逸出场景或达到设定的最大递归深度。
**渲染方程 The Rendering Equation [1986]
场景中光能传输达到稳定状态以后，物体表面某个点在某个方向上的辐射率（Radiance）与入射辐射亮度等的关系。
可以将渲染方程理解为全局光照算法的基础，Kajiya 在 1986 年第一次将渲染方程引入图形学后，随后出现的很多全局光照的算法，都是以渲染方程为基础，对其进行简化的求解，以达到优化性能的目的
渲染方程根据光的物理学原理，以及能量守恒定律，完美地描述了光能在场景中的传播。很多真实感渲染技术都是对它的一个近似
是物理上对于光线的一个完美描述
Path Tracing [1986]
根据渲染方程， Kajiya 提出的路径追踪方法是第一个无偏（Unbiased）的渲染方法
双向路径追踪（Bidirectional Path Tracing）的基本思想是同时从视点、光源打出射线，经过若干次反弹后，
将视点子路径（ eye path） 和光源子路径（ light path） 上的顶点连接起来（连接时需要测试可见性），
以快速产生很多路径
梅特波利斯光照传输 Metropolis Light Transport [1997]
怎样去尽可能多的采样一些贡献大的路径，而该方法可以自适应的生成贡献大的路径，简单来说它会避开贡献小的路径
传统的光线跟踪并不一定是真实效果图像，只有在非常近似或者完全实现渲染方程的时候才能实现真正的真实效果图像
环境光遮蔽（Ambient Occlusion，简称 AO）是全局光照明的一种近似替代品
可以产生重要的视觉明暗效果，通过描绘物体之间由于遮挡而产生的阴影， 能够更好地捕捉到场景中的细节
可以解决漏光，阴影漂浮等问题，改善场景中角落、锯齿、裂缝等细小物体阴影不清
AO 特效在直观上给玩家的主要感觉体现在画面的明暗程度上
局部的细节画面尤其是暗部阴影会更加明显一些
SSAO-Screen space ambient occlusion
SSDO-Screen space directional occlusion
HDAO-High Definition Ambient Occlusion
HBAO+-Horizon Based Ambient Occlusion+
AAO-Alchemy Ambient Occlusion
ABAO-Angle Based Ambient Occlusion
PBAO
VXAO-Voxel Accelerated Ambient Occlusion
一般而言，Ambient Occlusion 最常用方法是 SSAO，如 Unreal Engine 4 中的 AO，即是用 SSAO 实现
渲染方程以及变种
measurement equation
是否可以被感知?
spectral Importance Function
flows from the sensor to the light sources
measures the influence of the radiance onto the sensor
就算被感知，是否重要?
photon mapping
radiometry 渲染方程的物理原理与方程推导!
https://zhuanlan.zhihu.com/p/56020885
qq 一个波长为 lamda 的光子所携带的能量?

qq 渲染的稳态假设?
qq flux?
单位时间内通过某一个表面或者区域的能量
光释放能量的过程实际上是离散的
qq what is irradiance and radiant exitance?? their unit?
qq what is Lambert’s law?
qq 平面角? projection? unit of plane angle?
Radians
qq intensity vs irradiance?? pp
强度描述了光关于方向的分布，但是仅仅在讨论点光源时有意义。
irradiance 意义更广泛
qq 辐射率（Radiance）? 用的最多! L(p,w)
同时考虑方向和面积的分布
光线在真空中行进时其辐射率是保持不变
通过对面积、方向和时间进行积分可以计算出这里介绍的其它所有物理量
qq 这里 dA 是和谁垂直?
方向
qq what is the parameter of the radiance function? L? L(p,w)
qq radiance depends on position and solid angle, irradiance deps. only on posi.!
E(p,n)....L(p,w)....
qq compute energy from L, radiance! by integrating multiple times!
qq what is theta?
the angle bet. w and normal direction!
可以不写 costheta，换成投影立体角（Projected solid angle），这样做是好编程的
qq parametric representation of flux, with radiance known!
柠檬表皮吸收蓝光并反射红光和绿光，所以当柠檬被白光照射时我们看到的颜色是黄色
这是考虑了光线反射的光谱分布
&h
光线在诸如皮肤、树叶、蜡和流体等特殊材质中都展现出了次表面传输（subsurface light transport）的特性
BRDF 和 BSSRDF 是常见的两种描述光反射机制的抽象方法
BRDF 描述了物体表面某一点的反射。因为 BRDF 忽略了次表面散射，对于大部分
没有次表面散射现象或者该现象不太明显的材质
这样的简化能够在引入极少错误的情况下极大提高效率。BSSRDF 则是 BRDF 的推广，
详细的描述了光在半透明材质中的散射情况。
bidirectional reflectance [distribution function]
qq brdf defined? parameter of brdf?
output radiance over input irradiance!
not both radiance! hard to measure input radiance!
qq formulation to show that brdf has: 互易性（reciprocity）and 能量守恒（energy conservation）
BTDF 不满足互易性（reciprocity）
并将其称为 BSDF（bidirectional scattering distribution function，双向散射分布函数）。
qq what is bsdf?
fr + ft
qq should we always compute absolute value for cos term?
depends on which direction the normal points to! the same or not
qq what is the diff. bet. scattering equation and reflection equation?
描述物体次表面散射现象的数学形式

考虑了入射点以及出射点不同的情况
qq derive the reflaction/transmission equation with brdf and bssrdf??
ql
qq why we need Ambient Occlusion?
ambient occlusion is a shading and rendering technique
to calculate [how exposed] each point in a scene is to ambient lighting.
the interior of a tube is typically more occluded (and hence darker) than the exposed outer surfaces
darker the deeper inside
accessibility value that is calculated for each surface point
qq basic rendering equation p10(transport equation, directional form)
> explain the meaning of them p12
qq rendering equation II, area formulation. with visibility check p16
qq operator form p17
> what means double reflaction of emitted light? in equation. p17
qq refined with D,S,M... p18
qq mark them p19
qq measurement equation calculating brightness of a pixel? p20
> what is spectral efficiency?
> spatial power?
qq spectral importance function? p21
the influence of the radiance
measurememt of improtance
qq speed and energy of a tiny photon
qq calcu. of photon energy p25
qq Spectral Energy Density of diff. light source judge. p26
qq def. of radiance and irradiance p27 p28
qq how comes the integration part in rendering equation? p29
qq def. of BRDF p29
qq diffuse brdf? p30
qq Without volumetric scattering, the radiance is constant along rays?
qq brdf relates incoming irradiance with outgoing radiance?
qq BRDFs can be analyzed by integrating over one directional argument. Integration over outgoing
> directions yields directional hemispherical reflections that must be ≤ 1.
> why? p32
qq how can we Solve the Rendering Equation? x2 p34
1. form a linear sys.
2. sample ray paths
---REVIEW---

---
qq the steps of Bidirectional Path-Tracing p35
---r1
basic radiometry and hemispherical reflectance
Kajiya, J. T. (1986). The Rendering Equation.

https://www.researchgate.net/profile/Michael_Cohen12/publication/220720201_A_Radiosity_Method_for_Non-Diffuse_Environments/links/00b49520a6f1482f85000000/A-Radiosity-Method-for-Non-Diffuse-Environments.pdf

Ambient Occlusion
qq what is Ambient Occlusion
shading and rendering technique used to calculate
how exposed each point in a scene is to ambient lighting.
show more occluded (and hence darker) ones clearer
can be seen as accessibility value that is calculated for each surface point.
often used as a post-processing effect.
ambient occlusion is a global method
qq what is global illumination?
the illumination at each point
is a function of other geometry in the scene
Scattering
Multiple Scattering
qq what is color bleeding
objects or surfaces are colored
by [reflection of colored light]
from nearby surfaces.
qq what is radiosity
a GI algorithm
can not handle all kinds of paths, just a few, a fixed type

illumination arriving on a surface comes not just
directly from the light sources,
but also from other surfaces reflecting light
*mirror reflections and soft shadows
caustic
Mirror Reflection and Caustics
mirror reflections
qq what is mirror reflections
reflecting, providing a mirror image.
&h
People commonly think of the reflection as being reversed left to right;
however, this is a misconception.
The reflection of light rays is one of the major
aspects of [geometric optics];
tt geometric optics
refraction, or the bending of light rays
bending of light rays
it will bounce off at a 30-degree angle to the right
if the surface of the mirror is curved, the angles of
reflection are different at different points on the surface
tt spherical mirror
outside rearview mirrors on cars
under surveillance in stores.
concave, or curved inward
single location known as the focal point
produces a [magnifying effect]
The radius of curvature of a mirror determines
its magnification factor and its focal length.
amateur astronomers
tt Newtonian reflecting telescope
passes from one transparent medium to another, such as
from air into glass.
tt In a vacuum, the speed of light, denoted as "c," is constant.

material's refractive index
tt refractive index
qq Crepuscular Rays are volumetric scattering effects
Realtime volumetric scattering in game engine,
qq what is volumetric scattering
* radiance may not constant along rays between surfaces
scattering from particles in the atmosphere makes the sky blue and sunsets red.
how light is affected as it passes through participating media
large numbers of very small particles are considered
casts a volumetric shadow
we made the assumption that scenes are made of
collections of surfaces in a vacuum
but this may not always hold
simulate
fog and smoke attenuate and scatter light
qq compare volumetric Subsurface scattering and volumetric scattering!
consider the scattering inside the objects and the
qq describe the light transporting process
qq basic observation about the light transport ??
light is reflected between mutually visible surfaces
objects reflect and become emitters too
dynamic energy equilibrium
qq which Simplifications will be made to simulate the light transport ?? x2
no volumetric scattering
ignore polarization
qq raytracing operator:
qq what is the equ. in this light path??
total outgoing light is emission plus reflection
link outgoing to incoming light with trace
directional formulation of rendering equation
qq describe which parts are there in transport equation
emission + reflection
materials + ray-scene-intersection
qq formula of RE !! [Immel et al 1986]
outer hemisphere of surface is integration domain for incoming directions
radiance
qq Fredholm integral equation
solution is known as the Liouville脰eumann series.
Fredholm equations arise naturally in the theory of
signal processing, for example as the famous spectral
concentration problem popularized by David Slepian.
qq why we use Radiance? &h
it is useful because it indicates [how much of the power emitted,
reflected, transmitted or received by a surface will be

received by an optical system looking at that surface
from a specified angle of view
radiance and luminance are both sometimes called "brightness"
indicates how much of the power emitted
? is the partial derivative symbol;
A cos ? is the projected area.

qq The SI unit of radiance??
    watt per steradian per square metre (W穩r-1機-2),
emitted radiance
qq how can we parameterize the direction omega? op13
qq what is brdf
    bidirectional reflectance distribution function
    modeling materials
qq explain the rendering equ. each para. op11
https://en.wikipedia.org/wiki/Fredholm_integral_equation
qq how total outgoing radiance can be solved in math view ??
吨ow to parametrize and integrate directions ???
integration of solid angle yields double integral over ?? and ?
double integral
Parametrization & Integration
qq what if is constant C?
    result in pi * C !
Connection to Surface Patches
qq RE in Surface Patches formulation, with integration over scene area and
    > visibility check, explain!op15
    plugging into rendering equation yields new form
    rendering equation, with integration over [scene area] and [visibility check]
qq mark omega y->x in this pic
qq transfrom the RE to the form of Lout!
qq explain this equ. in a new form rendering equ.
Area Formulation [Kajiya et al 1986]
qq how can we do visibility test ?
qq what is geometry factor ?
    visibility test and cosin terms and solid angle term
&h map radiance functions onto radiance functions: use Lout, no Lin!
qq write RE in operators formulation! gain insight on a higher level!
    > the operator form of the rendering equation
    with R and T operator applying one after an other
qq write down those operators !! op17
qq solving it? how can we? what is the meaning of each part?
    with Taylor expansion!
    emitted light, single reflaction, double reflection... sum over of them
    the radiance function can be computed as the sum over emitted light
    single reflection of emitted light, double reflection of emitted light and so on
    single reflection of emitted light
    solving for ???? out formally
    &p pp derive?? ok
    adding up contributions by all different kinds of ray paths
qq RT can be refined to L-DSM-E, what is the meaning?
    Diffuse, specular, mirror
    from source (L) to eye (E)
qq we can write a regular expression for that?
qq write the path expression with a given pic!
qq whats the regular expression of all paths? in a scene! op19
Taylor expansion
qq compute Lout op17
Operator Formulation

ray tracing and reflection as functionals
operators
reflection
splitting of the BRDF into
qq splitting of the BRDF into?? x3 parts!!!
    diffuse (??), specular (??) and mirror reflecting (??)
    ???? = ?? + ?? + ?
solved by adding up contributions by all different kinds of ray paths.
in order of light transport from light source (??) to eye (??)
qq explain the regular expression syntax
to capture sets of rays
qq how can we reperesent diff. kinds of rays??
    > use a regular expression syntax?? how op18
qq give the name of the paths! in this pic!! op19
Specular reflection
In this process, each incident ray is reflected at the same angle to the surface normal as the incident ray
The law of reflection states that for each incident ray the angle of incidence equals the angle of reflection,
and the incident, normal, and reflected directions are coplanar.
scattered away from the surface in a range of directions rather than just one
qq definition of diffuse, specular and mirror, difference??
properties of the material, the wavelength of the light, and the angle of incidence
specular reflection is a mirror
&p diff. bet. S and M????op19
lighting solution is the sum of
Neumann Series
Monte Carlo methods compute these integrals probabilistically
– light reflected once TE,
– light reflected twice TTE, etc.
TTTE = Second Indirect Bounce
Global Illumination (GI): A concept that represent all the lighting of a scene that is not coming from a direct light source.
Image Based Lighting (IBL): A technique that uses an image as a light source
Irradiance map : Precomputed environment map that contains diffuse lighting data of the environment.
Importance Sampling : A math technique to approximate the result of an integral.
Split Sum Approximation : A way,used in Unreal Engine 4, to transform the specular radiance integral into 2 sums that can be easily baked into prefiltered textures.
https://books.google.de/books?id=mSJB1KLY7woC&pg=PA56&lpg=PA56&dq=heckbert+90+path+gramma&source=bl&ots=5P104l12wp&sig=ACfU3U39IdXt9jQV11wHLxLJrfpzQp2nZA&hl=en&sa=X&ved=2ahUKEwjOgZ6-6OrpAhWBM-wKHRM6BYIQ6AEwAHoECAsQAQ#v=onepage&q=heckbert%2090%20path%20gramma&f=false
&p pp mirror reflaction???? brdf??? op18
Wavelength can be defined as the distance between two successive crests
crests or troughs of a wave.
It is measured in the direction of the wave.
electromagnetic wave
In the simplest case efficiency is one for rays that hit the sensor and zero otherwise.
qq what is Sensor Efficiency?

not all rays hit the camera are accepted!
the effeciency defines the improtance!
qq how can we compute the brightness of a pixel
The brightness of the pixel is spectral power
which can be computed though the measurement equation as integral
over all incoming rays hitting the sensor
weighted radiance over all allowed direction
on aperture of area ?? and solid angle of incoming
directions that hit sample pixel ?
qq describe the simplified camera model, where is aperture? pixels?
qq given cam. model. explain this measurememt equ. componentwise ! op20
incoming and outgoing importance function
(also called potential or response function)
qq describe rendering equation with spectral importance function
qq for each sampling point in the scene, there is a corresponding weight? two
qq the weights flow from the sensor to the light sources
qq illu with a graph!
measures the influence of the radiance ???? in/???? out onto the sensor (often restricted to one pixel)
Importance Function
qq what is spectral importance function
qq how can you describe the RE, what phy. meaning does it have?
equilibrium of light emission and absorption processes
qq two parameterizations of the rendering equation
over directions and over surface Area
The operator notation allows to formally solve the
rendering equation by adding contributions of all types of light paths
Measurements are integrated over ray space and measured in units of power
&h are used to combine all rays that hit the sensor and make
contributes to pixel val.
qq ray direction, which not accepted, will not make contributions to sensor? pixel value? y
Importance function is used to measure influence of
radiance along an arbitrary ray in the scene
&h read BRDF first!!!!
basic radiometry and
qq what is hemispherical reflectance
light is composed of photons
qq each photon has characteristic?? x2
frequency ? and wavelength ?
qq what is the relationship bet. freq./wavelength and light speed? op24
qq each photon carries a tiny amount of energy q?
visualization of a light photon
qq what is typical wavelength for lights?
400nm - 800nm
qq how can we know the energy given a graph? spectural graph!
directed transport of energy
qq how can we compute the energy of a photon?? hv
qq range of wavelength??
qq Spectral Energy Density
qq what is emission spectrum of different light sources??

exanple!

spectral power
irradiance
radiant exitance or radiosity
ncoming and outgoing radiance ????
    in|out are densities of Φ?? in|out over the direction
direction ?? of transport and the area ??⊥ orthogonal to ??.
Irradiance
spectral power
Radiance
qq what is the diff. bet. irradiance and radiosity?? op27
qq the area orthogonal to what?
    omega, direction!
relating to surface area needs additional cosine term:
surface reflection distributes incoming light
    over all outgoing directions
density of reflected radiance ???? reflect with respect to irradiance
qq how brdf is defined?? op29
integration over hemisphere yields total reflected radiance
directional hemispherical reflectance ???? or albedo
qq what is total reflected radiance, how to compute?? op30
qq what is directional hemispherical reflectance? its range??
    albedo
qq what if for a constant (diffuse) BRDF?
    R = pi * pdiff.
qq is radiance constant along rays?
    Without volumetric scattering, the radiance is constant along rays.
&h
    辐照度表示各种频率辐射的总量
    假设一个点光源均匀地朝着所有方向传播光波，则辐照度按照平方反比定律递减。

    每单位立体角每单位投射表面的辐射通量。
    辐射率（Radiance）
    辐射能（Radiant energy）
    焦耳
    &h energy are expressed with j
    &h but we need to know per sr energy transformed
    光谱辐射率（Spectral radiance
    光谱辐照度（Spectral irradiance）
    https://dl.acm.org/doi/pdf/10.1145/3197517.3201399
    Diffuse BDRF & Reflectance
    Diffuse BDRF
    albedo
    directional hemispherical reflectance ???
    outgoing light
    fraction
    albedo
qq how is albedo/ directional hemispherical reflectance defined?? op30
BRDF defined as the outgoing radiance divided by the irradiance
qq The units of the BRDF ??
    inverse steradians
A BRDF describes the relation between the
    incoming irradiance and outgoing radiances at a given point P on the surface

https://web.cs.wpi.edu/~emmanuel/courses/cs563/write_ups/chuckm/chuckm_BRDFs_overvi

ew.html#:~:text=The%20BRDF%2C%20referred%20to%20as,point%20P%20on%20the%20surface.
almost indistinguishable from photographs of real environments.
simulate the behavior of light, from the source,
interacting with the scene
Global illumination algorithms try to collect the
contributions of all parts of the environment
which are illuminating a given point of the scene
qq what is bsdf
compute the transformation that occurs at this point
between incoming and outgoing light
is a tool for describing the distribution of reflected light at a surface.
how much of that light ray will be reflected in a
particular outgoing direction.
&h how to understand the shape of the brdf??
BRDF is an approximation of the BSSRDF,
The BRDF ignores sub-surface scattering
qq what is bssrdf
bi-directional sub-surface scattering reflectance distribution function
sub-surface scattering
Acquiring BRDFs
Gonioreflectometer
Representing BRDFs
*tt
https://web.cs.wpi.edu/~emmanuel/courses/cs563/write_ups/chuckm/chuckm_BRDFs_overvi
ew.html#:~:text=The%20BRDF%2C%20referred%20to%20as,point%20P%20on%20the%20surface.
...
In radiometry, when considering a point on the surface
The solid angle is used to refer to some small surface area on the hemisphere.
qq how many steradians for a hemisphere??
2pi in a unit hemisphere.
how much light hits a point from all incoming directions
watts per meter squared
Incident light contributes to irradiance
while reflected light, or exitant light
contributes to radiant exitance
qq radiance describes?? phy meaning of that??
[how much light] is arriving at a point from a specific direction.
qq relations bet. radiance and irradiance
watts per steradian meters squared
Full derivation
directional hemispherical reflectance
radiometric quantities are derived from photon energy per time (power)
most important is radiance
Without volumetric scattering, the radiance is constant along rays.
qq what is the fact of volumetric scattering??
radiance is not constant any more
BRDF relates incoming irradiance with outgoing
radiance and is integrated over hemisphere relative to surface normal.
qq expression of brdf?
qq why lower then 1 ?
qq when Integration over outgoing directionsq ??
---

qq the idea of numerical solution strategies
*estimate the RE with sampling and predict its integral parts
Monte-Carlo Integration
integrate pairwise geometry terms over patches
qq the case of Radiosity can be baked? why?
view independent
Assumption: only diffuse emission and reflection
limited light path
qq the baken process of radiosity??
[discretize surface] into patches
integrate pairwise
derive linear equ.!
solve for vector of radiosities
qq derive system of linear equations in case of Radiosity baken!
qq why double integral in 3d not trible? pp
qq what is domega? paramiterization , omega can be para. to two angles !
dphi * sintheta * dtheta
solution is independent of view point and can be baked
Solving the Rendering Equation
qq how can we solve the RE numerically?? the algo in course level?? op34
pairwise geometry terms
system of linear equations:
Bidirectional Path-Tracing
Monte-Carlo Integration
interconnect paths
*qq describe the process of bi-directional path tracing op35
contineous radiance fields
qq the baken process?
light bounces between visible surfaces until [equilibrium]
bounces are described by BRDF (material property)
*qq how can we describe and scan material property?
rendering equation is integral equation in outgoing radiance
y integration of radiance over the rays hitting the sensor
linear equations or integration? integrated linear system !!
!or sys. with integration/ summ
qq monte-carlo integration techniques
an integration technique
qq how monte-carlo integration techniques be used?? later, highlev
diffuse emission and reflection
reduces problem to a system of linear equations
pure diffuse emission
pruf-re-ok- slides-5 CG3_05_Path-Based-Rendering
---REVIEW---

Metropolis Light Transport
PATH BASED APPROACHES
Light Tracing ?caustics
reflecting and refracting
caustics
qq what is caustics, draw to illustrate! envelop? incident rays? reflacted rays?
typically a light pattern on the ground

lighting effect brings a scense of realistic
crutial tasks for realistic rendering
different approximation techniques
connects them to the eye by direct eye sampling
after Russian Roulette terminated a path
qq illu. Path Tracing with Direct Lighting and Light Tracing with Eye Rays
op06
qq how light sampling can be connected to eye??
> directly hit the eye position?? n
connects to the eye point or chooses a point [on the lense]
qq always connect to the eye point?? n
pixels get different number of samples or even none
for BPT an additional sample buffer
qq Light Tracing – Algorithm op08 large!
qq how can we render caustics with lower variance?
> how to choose the best renderer?
caustics are reproduced better with light tracing
but all other effects are worse than with path tracing
qq is light tracing optimal?
qq what benefits does light tracing have? x1
qq compare light tracing and path tracing, good in which effect?

light tracing should be only used in combined approaches
qq how can a lower variance be achieved? what if we sample the same rays many times?
it is important that individual methods do not
sample the same rays several times
qq should we reuse the light ray several times?? n
qq what is the key point of a cache based method?
intermediate results can be reused futher, thus have a lower variance
interconnects path from eye with path from light source
qq the idea of Bidirectional Path Tracing?
start sampling from both side
use Roussian Roulette twice for termination
all possible connections between subpaths
compute whether connection is blocked, drop the whole path maybe
multiply probabilities of subpaths
Irradiance Mapping
Roussian Roulette

http://www.pbr-book.org/3ed-2018/Light_Transport_III_Bidirectional_Methods/Bidirectional_Path_Tracing.html
16.3.4 Multiple Importance Sampling
qq what is BPT in rendering
qq show a ray path with x w notation! op14
edge lengths ?? = 1
qq what is Overall average length?
qq what are possible sampling techniques?
Implementation of BPT without multiple importance
sampling has additional artefacts
qq for BPT, there are still problems with which paths?
LSDSE-paths
to solve this we need cache based methods
Metropolis Light Transport, Eric Veach and Leonidas J. Guibas, SIGGRAPH 97
https://graphics.stanford.edu/papers/metro/metro.pdf

tt must read
//from chapter 8 op245
a measure on this space of paths
measurement contribution function
it allows general-purpose integration methods to be applied.
an integration method that allows several different sampling strategies to be efficiently combined.
path integral model
integral equations
These new techniques can only be properly understood within the path integral framework.
e three-point form of the light transport equations
These measures have natural physical interpretations whose meanings are described
eliminate the directional variables
he arrow notat
light flow
s the union of all scene surfaces, A is the area measure on M
e mutually visib
use the path integral framework in Monte Carlo algorithm
#k is a product measur
much simpler structure
path integral formulation
integral equation approach requires two equations (the light transport and measurement equations)
defined recursively
Measurements are defined and computed directly, by organizing the calculations around a geometric primitive (the path)
Each path specifies the emission, scattering, and measurement events along a complete photon trajectory
along a complete photon trajectory
starting with a vertex in the middle, and building the path outwards in both directions
With the path integral approach, on the other hand, it is possible to construct paths in arbitrary ways
construct paths in arbitrary ways
computing probability densities on paths
we must be able to evaluate the functions fj and p for the given path X# .
n how this path was generated
depends on all of the random choices made during this proces
Local path samplin
generate vertices one at a time, based on local information at existing vertices (such as the BSDF
For example, this is what happens when the BSDF at an existing vertex is sampled
once a vertex on a light source has been chosen
combining these three simple techniques, it is possible to sample paths in a great variety of ways
general enough to accommodate virtually all path sampling techniques that are used in practice
compare the probabilities with which a given path is sampled by different techniques.
Heckbert抯 regular expression notation for paths
Heckbert [1990] introduced a useful notation
classifying paths by means of regular expressions.
L denotes the first vertex of the path
E denotes the last vertex (the camera position or 揺ye?
scattering event at each vertex

not the type of the surface

surface itself is allowed to be a combination of specular and diffuse

Multiple Importance Sampling

using more than one sampling technique to evaluate a given integral

Our motivation is that most numerical integration problems in computer graphics are 揹ifficult?

we would like to design a sampling strategy that gives a low-variance estimate of the integral.

We do not construct new sampling techniques ?we assume that these are given to us

by computing weighted combinations of the sample values

can be parameterized by a set of weighting functions

Our goal is to find an estimator with minimum variance, by choosing these weighting functions appropriately

we can take several potentially good techniques and combine them so that the strengths of each are preserved.

This chapter is organized as follows.

variance reduction techniques

multi-pass algorithms

The glossy highlights problem

Smooth surfaces (r = 0) correspond to highly polished, mirror-like reflections

while rough surfaces (r = 1) correspond to diffuse reflection

It is possible to simulate a variety of surface finishes by using intermediate roughness values in the range 0 < r < 1.

different light source sizes and surface finishes

then examine why each one has high variance in some situations.

sampling the BSDF and sampling the light source

samples were chosen according to the density

subtended by the light source at the current poin

One of these sampling strategies can have a much lower variance than the other, depending on the size of the light source and the surface roughness paramete

For example, if the light source is small and the material is relatively diffuse, then sampling the light source gives far better results than sampling the BSD

polishe

n sampling the BSDF is far superio

high variance is caused by inadequate sampling where the integrand is larg

e ideal density function for sampling would be proportional to the product of all of these factor

neither sampling strategy takes all of these factors into accoun

Dry Erase: Infinite VR Whiteboard

y, so that we can express the two sampling strategies as different probability distributions on the same domai

many problems in graphics that are similar to the glossy highlights exampl

large number of integrals of a specific form must be evaluate

depend on various parameters of the scene mode

This makes it difficult to design an adequate sampling strategy, since the parameter values are not known in advance.

The main issue is that we would like low-variance results for the entire range of parameter value

too complicated to sample from directl

unconsidered factors has a large effec

Our main concern
how these samples can be automatically combined to obtain low-variance results over the entire range of surface roughness and light source parameters.
Monte Carlo integration can be made more robust by using more than one sampling technique to evaluate the same integral
can be represented as a set of weighting functions
balance heuristic
This image was rendered using both the BSDF sampling strategy and the light source sampling strategy
The samples are exactly the same as those for Figure 9.2(a) and (b), except that here the two kinds of samples are combined using the balance heuristic.
effective over the entire range of glossy surfaces
These images show close-ups of the glossy highlights test scene
it does not work quite as well as the other
We now present two families of combination strategies
maximum heuristic.
power heuristic
Variance bounds
The final gather problem
we consider a simple test case motivated by multi-pass light transport algorithms.
combination strategies
Notice that the sampling technique in Figure 9.12(a) does not work well for points near the light source, since this technique does not take into account the 1=r2 distance term of the scattering equation
It extends importance sampling to the case where more than one sampling technique is used
integrand is a sum of several quantities
A good example in graphics is the BSDF, which is often written as a sum of diffuse, glossy, and specular components
The process of taking one or more samples from each component is essentially a form of stratified sampling
the sample allocation is not as important as choosing a good combination strateg
can improve the variance by at most a factor of n, plus a small additive ter
the optimal sample allocation is irrelevant
sophisticated sampling strategies are generally designed for a specific light source geometry
y (e.g. the light source must be a triangle or a sphere)
Second, they are often expensive: for example, taking a sample may involve numerical inversion of a function
Thus, combining samples from two or more techniques
make direct lighting calculations more robust
additional cost is small
have shown strong bounds on their performance relative to all other combination strategies
the balance heuristic is an excellent choice for a combination strategy: it has the best theoretical bounds
and is the simplest to implement.
the given integral is a low-variance proble
better performance is most noticeable
Direct lighting calculations are a good example of where this optimization is useful
useful
provides a new viewpoint on Monte Carlo integration
o find a single “perfect” sampling technique, here the goal is

to find a set of techniques that cover the important features of the integrand
It does not matter if there are a few bad sampling techniques as well
the results will not be significantly affected.
whenever there is some situation that is not handled well, then we can simply add another sampling technique designed for that situation alone
We believe that there are many applications that could benefit from this approach, both in computer graphics and elsewhere.
By varying the number of vertices generated from each side, we obtain a family of sampling techniques for paths of all lengths
Samples from all of these techniques are then combined using multiple importance sampling
efficiencyoptimized Russian roulette
area-product measure
measurement contribution function
Samples from all of these techniques are then combined using multiple importance sampling.
detailed mathematical description
derive explicit formulas for the sample contributions
summarizes our conclusions
The multi-sample estimator
Recall that according to the path integral framework of Chapter 8,
each measurement can be written in the form
artefacts
incremental
1). It is important to note that there is more than one sampling technique for each path le
th: in fact, for a given length k it is easy to see that there are k + 2 different sampling techniques (by letting s = 0; : : : ; k +
concatenating two separately generated pieces.
: The four bidirectional sampling techniques for paths of lengthk = 2.
(b) Monte Carlo path tracing with a direct lighting calculation,
(d) tracing photons and recording an image sample only when photons hit the camera lens.
finite-aperture lens
different probability distributions on the space of paths, which makes them useful for sampling different kinds of effects
sampling different kinds of effects
(b) works well under most circumstances (for paths of length two), technique (a) can be superior if the table is very glossy or specular
Similarly, techniques (c) or (d) can have the lowest variance if the light source is highly directional.
correspond to different density functions ps;t on the space of paths
All of these density functions are good candidates for importance sampling
because they take into account different factors of the measurement contribution function fj
a wide variety of scenes and lighting effects can be handled well.
h s + t vertices and k = s + t 1 ed
the connecting ver
the connecting
op328
Average path length
qq calcu. Average path length

Path Measure
tt read ori!! must read
qq which sampling techs do you know related to calculating path pdf? [open question]
    pixel filter sampling, Roussian Roulette,
    splitting of BRDF in components, cosine term sampling,
    BRDF sampling, direct light sampling, or MIS
    splitting of BRDF in components
    cosine term sampling
qq how can we compute path pdf, which varients are there?? xn ,
    many sampling techniques exists
&p pp op18 --> veach thesis!!!
qq what is Image formation, how can we do this? algo.! op19
qq show that The same path can be generated in several different ways illu.
qq and this may intro. bais?? y
qq explain the given picture/ image--pic notes
Path Throughput
dependency on preceding path vertices.
again we have significant dependencies on preceding path vertices.
the visibility test
direct light sampling.
qq how can we compute path pdfs?? op21 Path pdfs
    the dependency on preceding path vertices
qq what does the last pdf mean? x2
Area sampling is used only for initial light source
    samples ??0 and direct light sampling
Area sampling
to support sampling of directions
qq how can we sample directions??
qq to support sampling of directions?
    direction pdfs need to be converted to area pdfs
    with geometry factor
qq why there is no visibility check in direct sampling?? op22
    it is implicit, as the pdf will be 0
qq the idea of BPT?
    connects light and path tracing by generating one path
        in each direction and interconnecting them
    light path
    reversed eye path
    connect paths
    connecting edge? vertices?
qq how the path looks like with zy formation??
qq what is s and t?
qq how can we compute k from s and t? k is the number of edges
    ---REVIEW---

    ---
here ?? and ?? are the lengths (number verts)
of the light and eye paths
connected edge length ?? = ?? + ?? - 1
?? and ?? are sampled independently
each tuple ?? = ??,?? w
defines different sampling strategy ???? of path ?? with the special cases
?? = 0 ?no light path
qq special cases!
    > what does it mean if s=0? path tracing

> and s = 1? direct light sampling
> t=0? t=1?
light tracing, improtant to caustics
t=1: direct eye sampling
qq can we do MIS on one path? y what do we need as pre-requires?
need to be able also to compute all other probabilities
weights the strategies to minimize overall variance
Weight Computations Example
MIS products lower variance
the lenth of a path is not fixed
*for a fixed length of path, fixed number of vertices
we can apply different sampling strategies
those strategies has different probability functions used
so can be combined together with MIS
wide variety of scenes and lighting effects can be handled
can taken into account!
---tobecontinue
**!pp MIS is for single path or multiple paths?? ok
fixed number of vertices, physically different paths
qq additional artefacts for BPT without MIS? why?
where faces meet!
the geometry term can become large and not considered with the
original sampling technique
---
qq why we set 1 to the first few vertices?
to avoid any extra variance on short subpaths
qq how can we compute weights increamentally?? op29
rearrange the weight computation
qq how can we do Russian Roulette in BPT?
similar way ! op28
qq Efficient Sampling with path groups, how can we do this?
with a table!!
Efficient Sampling with path groups
qq are there Efficient Computation of Contribution? idea? x2
incrementally during generation of all path samples
cancel out some factors......
tt paper to read
cancel out
qq what problems can bpt arise?
still problems with LSDSE-paths
qq what is the most app. advantages for caching??
the same calculations can be reused several times by a second
method.
reused
---METROPOLIS LIGHT TRANSPORT
qq idea of Metropolis Sampling?
compared to russian rolate
define Markov-Chain on omega
MLT算法依赖于非常好的突变策略，使得一些非常困难的路径可以更轻易被采样到
*sampling the secondary paths with help of [Markov Chain Monte
Carlo]
applies a sequence of [random mutations]
[handle a variety of difficult lighting] situations
*instead of RR, we can think that the sampling is done in
[unit-hypercube]
but MTL [not take use of improtance sampling]
some interesting applications exits:
We could also use MLT to render a sequences of images (as
in animation), by sampling the the entire space-time of

paths
at once
*inspired by the [Metropolis sampling method in computational physics]
qq which scenes are best suitable for MLT?
a.brightly lit room next to a dark room containing the camera,
with a door slightly ajar between them
The problem is that for some environments, most
paths do not contribute significantly to the image
Naive path tracing will be very inefficient,
because it will have difficulty generating paths
Metropolis Light Transport 的论文就是用了一个光源在一个
微微打开的门缝后面的场景当的例子
Metropolis Sampling 主要就是用来渲染一些采样路径特别复杂的场景的
一般室外大太阳的场景用不着付出额外的计算去用这么 fancy 的采样方法
大多数路径都是刚好能有效连接光源和 camera 的
这些所有路径中只有一部分路径对最终成像有比较大的贡献的
（一个光线照不到的桌子底下反射 100 次也是没有什么贡献的）
b.Metroplis Sampling 也可以比较高效的计算 Specular-Diffuse-Specular 这种路径
qq why markov process can be assumed?
首先光线传递的随机性（光子抵达物体表面材质后被吸收或随机反射）仅由当前状态决定，
和历史无关，满足马可夫过程定义
it has Markov property: *memoryless property of a stochastic process.
qq what is Markov property
conditional probability distribution
depends only upon the present state
not on the sequence of events that preceded it.
qq what is Markov process
A process with Markov property is called a Markov process.
qq what is Markov chain.
discrete-time stochastic process satisfying
the Markov property is known as a Markov chain
光线传播本身是 time-dependent 模型
hh https://zhuanlan.zhihu.com/p/28345852
suggested extensions!
马尔可夫蒙特卡洛方法（MCMC，Markov Chain Monte Carlo）算法被引入到渲染领域
这个被称为影响 20 世纪科技发展最重要的
与传统的蒙特卡洛方法（例如路径追踪）不同的是
它通过利用样本之间的相关性来更好地探索一个分布函数
相关性
使得一些非常困难的路径可以更轻易被采样到，然而 MLT 算法依赖于非常好的突变策略
2002 年，Csaba Kelemen 提出了 PSSMLT 算法
（Simple and Robust Mutation Strategy for Metropolis Light Transport Algorithm）
这个采样的过程实际上是对多个[0,1]的随机数的采样过程，
这些[0,1]随机数构成一个高维的单位超立方体（unit-hypercube），
这称为原采样空间（Primary sample space
所以如果直接对该空间采样，也能够得到一条路径。
另外最重要的是因为使用重要性采样，原采样空间的被积函数为 f/p 接近为一个常数，

这使得原采样空间的被积函数更平坦，采样的方差更低，
考虑传统的 MLT 算法并没有使用重要性采样
Markov Chain Monte Carlo
qq idea of PSSMLT

高维的单位超立方体（unit-hypercube）

它将“路径采样”当做一个黑盒子：它直观给你一些随机数，不管你使用什么方法给我一条路径就可以

某个顶点一个微小的突变可能导致后续路径发生较大的变换，从而使种子路径和突变路径差异太大而丧失相关性

MLT 算法被提出来后最重大的一个进展要数 2014 年 Toshiya Hachisuka 提出的 MMLT 算法（Multiplexed Metropolis Light Transport）

路径除了在每个原采样空间内部执行突变，还可以在不同采样技术之间执行突变。

这是一种采样技术，叫 Metropolis Sampling。

Metropolis Sampling 只需要你能求得 f(x)的值，就可以自动生成一组和 f 的大小成比例的样本（就是下图中的红点，f 越大红点越密集）

这些所有路径中只有一部分路径对最终成像有比较大的贡献的 （一个光线照不到的桌子底下反射 100 次也是没有什么贡献的）

不在没有贡献的路径上面浪费计算量

绝大多数路径都没有贡献因为他们被强挡住了，所有求交和 brdf 采样求值的计算都浪费了。

如果用 markov chain 来生成样本，则得到的路径是这样的。 大多数路径都是刚好能有效连接光源和 camera 的。

Metropolis Sampling 主要就是用来渲染一些采样路径特别复杂的场景的

一般室外大太阳的场景用不着付出额外的计算去用这么 fancy 的采样方法

Metropolis Light Transport 的论文就是用了一个光源在一个微微打开的门缝后面的场景当的例子

Metroplis Sampling 也可以比较高效的计算 Specular-Diffuse-Specular 这种路径

Specular-Diffuse-Specular

high level 的说了一下为什么渲染可以用到 markov chain。 具体的实现也不麻烦，基于一个叫 Detailed Balance 的方法

首先光线传递的随机性（光子抵达物体表面材质后被吸收或随机反射）仅由当前状态决定， 和历史无关，满足马可夫过程定义

满足马可夫过程定义

第二，现实场景的物体材质不存在全部是完美镜面的情况，也就是系统中存在扩散/耗散项（非 Dirac 分布的 BRDF，以及吸收项），所以能量分布会趋于稳定，收敛到一个稳态

光线传播本身是 time-dependent 模型，但是渲染里假定了能量瞬间完成重分布， 所以需要证明稳态方程解的存在唯一性。

MMLT 的重要意义在于它将统计中的模拟回火（simulated tempering） 的思路引入到了 MLT 算法中

模拟退火本来用于解决最优质问题，由于物体加温之后分子状态之间的转移更活跃， 因此状态更易于在全局转移

而随着温度逐渐降低，状态之间的转移就逐渐减弱。所以为了求最优值

所以为了求最优值，如下图中的最大值，我们首先将温度升高， 然后让其慢慢冷却，在其过程中不断寻找最大值， 随着温度降低，最终将停留在最大值处。

在 MMLT 算法中，温度即是采样技术，不同的采样技术将原采样空间划分为多个子空间。

当采样技术发生改变时，虽然所有的[0,1]随机数并没有发生变化， 但是由于使用了不同的采样技术，这些随机数被使用的方式也完全不同了

理想情况下，我们希望在改变采样技术时，能够保持路径不变。 因此， Benedikt Bitterli 于今年提出的 RJMCMC （Reversible Jump Metropolis Light Transport using Inverse Mappings）

使用一个反向映射方法，当从采样技术 1 向采样技术 2 转移时， 它将状态 1 对应的原采样空间的路径在对应路径空间中的形式， 通过一个反向映射转换到采样技术 2 对应的原采样空间， 这个过程是决定性（deterministic）计算的

Jacopo Pantaleon 的 CMLT（Charted Metropolis Light Transport） 几乎也与上述思路类似，但是强调更泛化的概念

HISANARI OTSU 的 Fusing State Spaces for Markov Chain Monte Carlo Rendering 在算法上稍微不同于上述两篇论

文
今年这几篇论文质量还是可以的，并且更是有
Wojciech Jarosz，Wenzel Jakob, Carsten Dachsbacher, Anton S.
Kaplanyan 等这些保驾护航。
qq how Markov Chain can be used for MLT??
qq how simulate tempering can be used for MLT??
hh
这种采样技术之间的转移太高效了。
秦春林，从事游戏开发已有 7 个年头，担任游戏开发工程师，
主要研究方向是引擎开发和图形渲染。物理专业出身的我属于半路出家
https://item.taobao.com/item.htm?spm=a1z10.1-c.w4004-17761944037.2.27fe5c3cX900DC&id=569868401235
不像 PBRT 聚焦于比较经典的路径追踪算法实现，《全局光照技术》覆盖了截止 2017 年大量的行业中最前沿的渲染技术
例如 VCM/UPS，降噪技术，频率域分析，微分流形/几何，梯度域渲染，基于光束的参与媒介渲染技术等等
这些都是近几年离线渲染领域最前沿的方向和技术，被广泛运用于最近皮克斯，迪士尼等动画电影当中，使读者能够了解到最前沿的行业动态和趋势
而 《Physically Based Rendering: From the Theory to Implementation》聚焦于离线渲染
实时和离线渲染同属于计算机图形学领域， 它们只是在当前硬件水平下针对实时性需求的划分
很多实时的全局光照模型都是从离线渲染模型优化，改进而来
全局光照技术》主要是按照各种全局光照技术为主线进行组织的，本书基本上覆盖了当前行业中最重要的约十种全局光照技术，例如针对离线渲染领域的路径追踪，梅特波利斯光照传输，光子映射，针对交互式需求的辐射度方法
为了加速光线追踪计算，与 Path tracing 直接从物体复杂的几何表述中获取光照信息不同，
Photon mapping 方法则将光照信息存储为一个独立的数据结构(称为 Photon map)
这大大加速了光线追踪的计算。
形成了一本 270 多页的英文稿子（如下视频），虽然该稿子仅仅是出于学习的目的，因此内容不是很严谨，但是它依然能够帮助我建立起一个 3D 渲染技术的宏观视图，它可以说是现在这本《全局光照技术》的重要基础，并且我相信很少有人在学习的过程中能够形成这种级别的笔记稿子。
在这个学习的过程中，我感觉到了知识系统性的重要性，尤其对于图形学这种涉及大量数学知识，以及各种各样算法实现的学科，因此我尝试去总结各种知识的分类及其之间的联系
于是我在完成第一章之后就将其大约 80 多页的内容放到网络上供大家下载试读，我希望借助这种方式能够实时收集到读者的反馈和建议
然后印刷上市后才能收到读者的反馈，然而这些反馈基本上对图书质量没有用处，因为它必须要等到再版时才能根据这些反馈作出调整，而很多技术图书甚至没有再版的机会。
《全局光照技术》试读章节的电子文件在各个论坛，QQ 群以及 TheGIBook 官网已经被下载超过 2000 人次
qq the idea of Metropolis Light Transport
qq MLT is best suitable for ? which kind of scene ??
Compute different error norms (L1 , L2 and L?)
with respect to reference image
error values of other methods (PT and BPT) over error value of MLT:
qq how to explain those numbers? later
pruf-re-ok- slides-7 CG3_07_Cache-Based-GI
---REVIEW---

---
qq how radius update work in ppm?
N is number of photons in previous iter. and M is number of current iter.
user adjustable fraction alpha to adjest how much of M will be

used in current estimation
the radius can be updated according to?...
qq Bidirectional Path Tracing is very bad for?? good for??
*caustics in mirror
caustics in water
LSDSE path
qq idea of Photon mapping: op08 x4
simpler:
[generate] large number of [light paths] first,
[trace] light paths trough scene
[store] photons on diffuse or glossy surfaces
[estimate] reflected radiance with help of closeby photons
1.1
generate large number of [light paths] first
cache them and find for each eye path the matching ones
[emit large number of photons]
1.2
trace light paths trough purely mirroring or refracting surfaces
Russian Roulette on diffuse or glossy surfaces
1.3
store photons on diffuse or glossy surfaces in a kd-tree
+ glossy = Specular not equ. to mirror, partially mirror reflection pp ?
2.1
generate per pixel one or several eye paths
use closeby photons to estimate reflected radiance
qq where do we store photons in the scene?
diffuse or glossy surfaces, Store photons only at surfaces where the BRDF is not a delta function
do not store on mirror surfaces, we trace further when such a hit found
qq when do we store a photon to caustic photon map? what is the criteria? and when global photon map?
Store photons in the caustic photon map, if the photon path
[had a delta BRDF as first hit]
qq how can we enforce the caustic probability?
rejection sampling
qq when geathering, which hits are accepts?
non-delta, bkg, light
radiance reconstruction from
[caustic photon map] and a one-sample Monte Carlo estimator of the
incoming radiance exploiting the [global photon map]
qq what is a photon filter?
a photon filter can filter out photons with very different normals.
qq Radiance Estimation? draw a graph to illustrate op11
reflected radiance
qq how can we estimate radiance in photon mapping?? op11
simplest approach and others
qq describe the process of Photon Emission op12
> how can we compute photon power??
> location and direction of each photon emitted?
qq does each photon distributes the complete light into the scene?
qq illu. the radiance estimate process with a graph
qq why normalize?? op13
qq how can we support several light sources?? op13
> compute total light emitted from the light source, photon

power in this case!
sampling of a light source index
to support several light sources
qq what is pixel estimates in PM?
qq how can we sample reflected direction??
> how can we update the power of each photon after reflaction?? op14
if surface BRDF has diffuse or glossy part, store photon in photon map
terminate if Russian Roulette with success probability ?? fails
sample reflected direction ???? ' with pdf ????' . according to BRDF ?? and recurs wit
qq describe Photon Tracing process (part of algorithm, detail view) op14
with the information necessary for later radiance estimates
photons are stored in a list
some optimizations need storage of surface normal, previous photon on light path, light source index, and flags
Photon Storage
qq what kind info. is stored in photon stru.?? op15 basic terms x3
position
power, cosine weighted power
incoming direction
qq how to accelerate the collection of photons
kd-tree can be built in inplace to
avoid storage of child pointers
&p pp what does it mean????
proved most simple and flexible over surface subdivision or texture maps
Jensen proposes in 1996 (Global illumination using photon maps)
qq what is cone filter ? why use it??
in radiance estimate
in collecting process
http://graphics.ucsd.edu/~henrik/papers/photon_map/
tt must read
Illumination splitting
based on BRDF
incoming radiance
direct light sampling
caustic photon map
Approximations
Different Approximations
qq which Different Approximations can be used in rendering pass
> of photon mapping?? op17
direct light sampling
Monte-Carlo Ray Tracing with multiple importance sampling
make use of caustic photon map
make use of global photon map
qq which Two Photon Maps?? diff?
caustic:
terminate paths at diffuse or glossy surface
define pdf for directions such that specular objects
have very high probability
*large number of photons needed
global
*fewer photon density needed
*support for very long paths
*do not store first diffuse surface hit photon
as accounted for by direct light sampling

and caustic photon map
qq why do not store first diffuse
their are dealed with by caustic photon map, redundent, variance!
qq compare those two photon maps:
> caustic photon map and global photon map! the diff.??
qq why we need caustic photon map??
emit more photons and thus to get a lower variance!
qq why not store first diffuse in global photon??
should sample diff. ray path to decrease variance
qq how can we do importance sampling in both cases?
> caustic photon mapping and global photon mapping?? !
qq when we terminate paths in both maps?
diffuse or glossy surfaces
by russian r, RR
qq Photon Mapping Discussion, advantages of photon mapping x4x1
+allows to find light paths to find LSDSE paths,
which are [hard to solve for BPT]
+cached paths can be [reused] better computation
+simple to be generalized to [volumetric effects]
+[additional optimization] potential
shadow photons
use photon map for importance sampling
-density estimation introduces bias
qq pro cons for photon mapping when compared to bpt
1996 Jensen: Global illumination using photon maps
tt must read
hh
Photon Mapping GI Engine 光子贴图GI引擎
Irradiance Cache GI Engine 辐射缓存GI引擎
GI（Global Illumination）全局照明模式就是尝试模拟这种光子反弹式的物理照明过程

这种光子模拟过程为渲染增加了真实感，帮助数字艺术家得到更为生动、真实的画面。

GI也可以营造出极高质量的效果。
当光子碰撞到一个粗糙的表面，它就会被 随机地向各个方向散射出去，
这就是所谓的Diffuse Global Illumination(漫反射全局照明)
当光子击中一个强烈反射或者折射的表面（如镜面或者玻璃），
https://pic2.zhimg.com/80/v2-2e2c27b5acbb305dc191f0b0f9dea799_1440w.jpg
In Redshift, all of these techniques are called “GI Engines”
Each GI Engine has its pros and cons.
为什么要有分别独立的Primary和Secondary的GI引擎呢?
Primary GI 的效果对于相机是直接可见的，
所以需要尽可能地提高渲染品质级别。另一方面，
Secondary GI照明往往对最终光影影响很小，
所以可以用稍低的质量（可以想像成“虚化”或者“噪点”）而不会影响最终渲染质量。

For reasonable numbers of photons, it renders fast
唯一可以渲染Caustic(焦散)效果的方式。
*Photon mapping is an outdated technique
Photons have to be stored in GPU memory so
too many photons can be prohibitive in terms of memory usage
有一些光子会最终因处于摄像机外面不可见，所以系统计算处理
他们花的时间以及存储它们都会是一种成的浪费
除非在每个像素上都使用大量采样射线，否则会产生颗粒感的图像
---inversed PM
qq the computation time of PM? Inversed PM??

see pic notes for answer
qq the computational demand for both cases??
in high quality GI there are many more final gathering rays needed than photon
A speedup of a factor up to 6 can be achieved by first tracing eye paths to diffuse or glossy surfaces and storing them as so called reverse photons:
see pic notes for answer
qq the idea of reverse photon mapping?
the fact is that more final gathering rays needed than photons
use reverse photons to acc. the rendering process up to factor 6
qq reverse photon mapping algo? x4
eye pass
for each pixel [generate eye paths] and store reverse photons
build kd tree
light pass
[generate photons]
[density estimation]
[radiance estimation], distribute energy to reverse photons

3 nearest neighbor photons define density estimation radius in forward photon mapping
qq how to estimate photon influence radii in reversed pm?? op23
first distributing all photons
Photons are iterated again
qq how can we measure the influences of reverse photons??
qq the complexity for this?
&h see pic
tt must read
Havran et al., Fast final gathering via reverse photon mapping, 2005
Fast final gathering via reverse photon mapping, 2005
where reverse photon mapping achieved speedups of factors up to 6
qq Pseudocode of the inversed photon mapping algorithm op24
hh
一开始出现的是Photon Mapping，后来有了Progressive Photon Mapping，后来才有了Stochastic Progressive Photon Mapping。
SPPM是PPM的改版（或者说“升级版”）。SPPM的优势是没有内存的限制。
能轻易捕捉到各种光照效果(比如Monte Carlo ray tracing不易捕捉到的SDS路径)
这主要表现为灰暗的墙角，错误的颜色辉映以及漏光等现象。
photon ray gathering，需要建立复杂的数据结构比如ray map。
紧接着，在07年由Herzog等人提出了photon ray splatting以代替photon ray gathering
该方法将整个光子映射算法流程倒了过来，即先进行eye tracing,再进行photon tracing。
这种由于密度估计范围r的不趋于零而存在的一种偏差，要解决这一问题，就必须想办法将光子数量扩大到无穷大
实际上，借用photon ray splatting中倒转光子映射算法流程的思想，可以破除光子映射中内存的限制，原因很简单，因为屏幕像素有限，所以要存下eye sample到内存是很轻松的事情
那么光子图根本就不需要存储，这样要多少光子就可以发射多少光子。
08年，由Hachisuka等人提出了渐进式光子映射(Progressive Photon Mapping)
通过第一步将eye sample组织成一个kd-tree结构，而后一轮轮地发射光子，每一轮将r减小一次，每一轮渲染一张图片。
由于倒转了PM算法的步骤顺序，PPM算法的光子搜集方法又从KNN变回到了直方图估

计

但是由于 PPM 算法本身已经没有了光子数量的限制，所以这点劣势很容易以大量的光子数量扳回

有了 PPM，我们甚至可以不需要 final gathering，因为 final gathering 本身也极其耗时，且不能消除偏差

如果是像反走样这样简单的活还可以依靠在 eye sample kd-tree 中存下每个像素点的所有超采样点来完成的话，那么要模拟反射模糊，景深，运动模糊就更麻烦了

要模拟反射模糊，景深，运动模糊就更麻烦了

09 年，还是 Hachisuka，在 PPM 算法稍作改进，提出了 SPPM(Stochastic Progressive Photon Mapping)，该算法与 PPM 不同之处在于，在每一轮光子发射完毕后重新进行一次 eye tracing

并且每次 eye tracing 都带上一个随机的扰动，让每次构建的 eye sample kd-tree 中的数据都不一样，下图展示了这两个算法的区别:

这样做使得 SPPM 效率上略低于 PPM，但是换来的是更加健壮和灵活，实际上 SPPM 在处理 gloss 反射的时候效果远好于 PPM

11 年 Claude Knaus 等人就证明了这种方式和 PPM 算法具有同样的鲁棒性，由于和标准 PM 算法的极其相似，所以之前在标准 PM 算法中可用的扩展

正向跟踪和逆向跟踪两种思想

两个方向相互对称，而它们每个都具有另一个所没有的优点，bidirectional pathtracing 连接了正向与逆向两个路径

photon mapping 同样利用了正向与逆向两个方向的信息，不仅如此，它还重用了信息(光子图)，所以一般情况下能比 bidirectional pathtracing 收敛更快

从古老的正向跟踪算法 Monte Carlo Light Tracing 中，可以看到正向跟踪普遍存在的问题，当场景空间很大，但是能被视点看到的部分却很小的时候，会有大量的采样点对最终图像额颜色值贡献很小甚至毫无贡献

受到 Metropolis light transport 的启发，Fan Shaohua 等人于 05 年将 Metropolis 准则引入了 PM 算法中的 photon tracing 中

11 年，Chen jiating 等人在 SPPM 基础上加入 Metropolis 准则，同样提高了 SPPM 在这类场景的收敛速率。

In limit for infinitely many photons and zero KDE radius method becomes unbiased

```
    Such convergent methods  are called consistent
    Photon mapping (PM) is biased
---
qq why Photon mapping (PM) is biased??
qq what is KDE?? why can introduce bias in PM
qq PM is not a convergent method??
qq the idea of PPM? Progressive Photon Mapping
    iterates PM with
        [decreasing density estimation radius] such that error
        converges to zero in limit
qq which three important PPM approaches are there?
    2008 Hachisuka: Progressive Photon Mapping (PPM)
    2009 Hachisuka: Stochastic PPM (SPPM)
    2011 Knaus: PPM - A Probabilistic Approach (probPPM
qq such three methods have the same radius update? op28
    with  decreasing density estimation radius
        error  converges to zero in limit
qq How to store KDE radius that can be updated per iteration??
    stored per reverse photon
        user adjustable fraction
qq How to combine the results of photon mapping iterations?? op29
```

divide summed pixel contributions by $i \cdot M$

```
        M is the number of photons generated in current iteration
qq how can we reduce low frequency noise in realistic rendering?
// Holoportation: Virtual 3D Teleportation in Real-time
hh
    use reverse photons to  store KDE radius
    reverse photons
```

```
                    store KDE radius
                    update KDE radii
                    update KDE radi
                    per photon estimate influence radius from KDE radius of
                        closest  reverse photons
                    add photon contribution to accumulated  radiances in influenced
photons
                    *ppm introduces lower variance
                    reverse photons
                qq does ppm use reverse photons?? y
                the same rendering time
                tt must read
                    https://www.ci.i.u-tokyo.ac.jp/~hachisuka/ppm.pdf

https://cg.ivd.kit.edu/publications/p2012/APPM_Kaplanyan_2012/APPM_Kaplanyan_2012.p
df
                qq draw a graph to illu. the PPM algorithm
                qq draw a graph to show SPPM algorithm
                qq the diff. bet. PPM and SPPM?? number of passes can be written as
an equ.?
                    a complicated updating pass
                    [distributed ray tracing pass]
                    ori:
                        ray tracing pass + n * photon tracing pass
                    sppm:
                        ray tracing pass + n * (photon tracing pass + distributed
ray tracing pass)
                qq what is randomly gen. hit point? how it affects the rendering
result??
                    lower RMS error
                qq in what kind of scene is best suitable for SPPM??
                    gloss 反射
                qq the pipeline for each method -re
                qq hwo can we measure rendering quality?? use
                    relative error metric
                tt must read !!!
                    https://www.ci.i.u-tokyo.ac.jp/~hachisuka/sppm.pdf
                qq year of SPPM, ProbPPM?
                    2009, 2011
                ---
                Progressive Photon Mapping: A Probabilistic Approach
                https://www.cs.umd.edu/~zwicker/publications/PPMProbabilistic-
TOG11.pdf
                tt must read
                qq major new contribution of probPPM?
                    *based on a probabilistic derivation
                        a new formulation of progressive photon mapping
                        proof of consistency (convergence)
                    *allows for arbitrary kernels
                        without local photon statictics
                hh
                    幸而在 08 年提出的 Progressive Photon Mapping 将这个空间无限的要求剔除,
                        其核心思想是颠倒了光子发射和光线跟踪计算的顺序, 逐步求精计算结果
                    Finally, our approach is readily applicable to volumetric
photon mapping
                    We compare our algorithm to previous progressive
                        photon mapping approaches and show that we
                        achieve the same convergence to unbiased results,
                        even without local photon statistics.
```

https://www.cs.umd.edu/~zwicker/publications/PPMProbabilistic-TOG11.pdf

We fixed the initial radius globally

using a box kernel (top row) and a Gaussian kernel (bottom row).

numerically approximate solutions of the rendering equation [Kajiya 1986].

at equal computational cost, it can often produce images with less noise than other Monte Carlo algorithms

the expected error of any approximation with a limited number of samples is non-zero.

biased

The reason for the computational efficiency of photon mapping is that it caches and reuses Monte Carlo samples.

This approximation, however, acts similarly to a low-pass filter on the cached samples.

It always returns an overly smooth approximation of the true radiance, and hence causes the non-zero expected error, or bias, of the solution.

This bias only vanishes in theory, if it were possible to cache an infinite number of photons.

a simple strategy that breaks this memory bottleneck.

They incrementally update a sequence of photon mapping results,

where each step in the sequence uses a limited number of photons.

The key is to reduce the radius such that, in the limit, the incremental updates converge to an exact, unbiased solution of the rendering equation

The statistics include for example the number of photons collected in the region

the regions are generalized to render effects such as glossy reflections or depth of field.

we introduce a probabilistic derivation of progressive photon mapping

The key property of our approach is that it does not require the maintenance of local statistics.

memoryless progressive photon mapping

As a benefit, we can compute each step in parallel or with a standard photon mapper used as a black box

a standard photon mapper used as a black box

allows for arbitrary kernels in the radiance estimate

we demonstrate that it is readily applicable to volumetric photon mapping

we achieve the same convergence to unbiased results, even without local statistics.

In summary, we make the following contributions:

a memoryless algorithm that does not require the maintenance of statistics,

and is trivial to extend to volumetric photon mapping.

Volumetric Progressive Photon Mapping. One of the main advantages of our derivation is that its extension to volumetric progressive photon mapping is trivial

qq the idea of probPPM -re

qq what is the main contribution of it?? probPPM! -re

globally const or per pixel KDE radius

&h donnot use local statictics any more and can be easily adapted to volumetric case

. There is no need to store local statistics

The iterations of the main loop are independent and can be

performed in parallel.
all pixels
box kerne
The Gaussian kernel leads to slightly smoother results with similar bias.
In other words, our approach includes the stochastic PPM scenario
using a conventional photon mapper used as a black box.
ProbPPM Comparison of Kernels
&p hard to fully understand, we have to read PPM first! PPM->SPPM->probPPM
*main contribution of probPPM is parallel
---VERTEX CONNECTION AND MERGING (VCM)
qq idea of Vertex Connection and Merging x2
combines stochastic PPM with bidirectional path tracing
such that advantages of both methods are exploited
-- SPPM + BPT
first PPM is reformulated in formalism of BPT and then
integrated via multiple importance sampling -- MIS
*https://cgg.mff.cuni.cz/~jaroslav/papers/2012-vcm/2012-vcm-paper.pdf
tt must read !!
while PPM has difficulties in handling the illumination coming
from the room seen in the mirror
stochastic PPM
bidirectional path tracing
BPT ? vertex Connection (VC)
PM ? Vertex Merging (VM)
vertex Connection
vcm is a reformulation of photon mapping as a bidirectional path sampling technique
*The benefit of our new formulation is twofold
*BPT is one of the most versatile light transport algorithms.
not efficient for transport
paths with specular-diffuse-specular (SDS) configurations
widely acknowledged that BPT is not efficient
paths with specular-diffuse-specular (SDS) configurations
an interior of a car or a building
sample SDS paths with low probability density
which may even go to zero if point light sources and pinhole cameras are allowed
the problem of insufficient techniques
Efficient handling of SDS paths
Our new VCM algorithm automatically
computes a good mixture of sampling techniques from BPT
and PPM to robustly capture the entire illumination in the scene
The rightmost column
shows the relative contributions of the BPT and PPM techniques to the VCM image in false color.
BPT [Veach and Guibas 1997] and PM/PPM [Fan et al. 2005;
Hachisuka and Jensen 2011].
has long been demonstrated with photon mapping (PM)
A progressive variant has recently drawn attention with its ability to converge with a bounded memory footprint
Moreover, an adaptation of these heuristics to glossy reflectance is not obvious
//only be replicated using an anisotropic model.

http://resources.mpi-inf.mpg.de/departments/d4/teaching/ws200708/cg/slides/CG07-Brdf+Texture.pdf

qq the MIS formulation of VC and VM
&h many ways exits when computing a ray path from
> light to eye!!! so use MIS in between!
---REAL-TIME PM
https://dl.acm.org/doi/pdf/10.1145/1572769.1572783
tt must read
http://graphics.ucsd.edu/~henrik/papers/fur_bssrdf.pdf
tt must read
qq bsdf = ? + ?
BSDF是普遍意义上的说法，反映光入射与出射的强度对应关系。
可以通过BxDF以及入射，出射角度计算光的强度
BSDF可以认为包含了BRDF和BTDF，BRDF是反射而BTDF是透射
BSDF = BRDF+BTDF,
下图直观上描述了BSDF,BRDF的意义以及二者关系

https://pic3.zhimg.com/80/8c60f94b8b6f430fc5dcb41068770454_1440w.jpg
BRDF
qq what is the parameter of brdf?
qq what is the diff. bet. brdf and btdf?
---ql pruf-official--with answer
//you should be able to explain a [given] rendering/ dynamic equation
<> Explain algorithmic techniques
qq path, light and bidirectional path tracing x4 + x4 + x2
path tracing samples paths from the eye,
and connects them to the light by [direct light sampling]
after [Russian Roulette] terminated a path
//
sample, tracing/ the ray bounces around the scene,
connect
[light path in improtant
light tracing samples paths from the [light sources]
and connects them to the eye by [direct eye sampling]
after [Russian Roulette] terminated a path
BPT connects light and path tracing by [generating one path in each direction]
and [interconnecting them]
//
generate, interconnect

+++
path tracing:
[the ray path] generated from the eye,
bounces around the scene,
terminates with RR,
perform direct light sampling
light tracing:
the ray path generated from the light,
bounces around the scene,
terminates with RR,
perform direct eye sampling
bpt:
generating one path in each direction,
interconnect them in the middle
qq idea of multiple importance sampling according to brdf and L_in x2
pic

```
                qq what is MIS? +igen
                    it will be used when [several sampling strategies] are
available,
                        it  [weights] the strategies to minimize overall variance
                qq idea of multiple importance sampling in BPT
                    several sampling strategies exists for a given ray length,
                        we can combine them to reduce the bias
                qq direct and indirect light sampling
                    directly illuminated part, [no reflection on ray path ]
                qq final gathering x2
                    used in the cache based rendering technique,
                        we [cache] results from one method, the same calculations
can be reused several times by a second method.
                    the second method is called final gathering
                    //
                    the process of collecting photons
                qq photon mapping x4
                    simpler:
                        [generate] large number of photon from light source,
                        [trace] them trough the scene
                        [store] photons on diffuse or glossy surfaces
                        reflected radiance can be [estimated] with help of closeby
photons
                qq reverse photon mapping x4
                    eye pass
                        for each pixel [generate eye paths] and store reverse
photons
                            build kd tree
                    light pass
                        [generate photons ]
                        [density estimation ]
                        [radiance estimation], distribute energy to reverse photons

                qq progressive photon mapping x1
                    iterates PM with [decreasing density estimation radius]
                        such that error converges to zero in limit
                qq vertex merging
                    combines stochastic PPM with bidirectional path tracing
                        such that advantages of both methods are exploited
                        (SPPM abd BPT)
                        + with help of MIS
                    vertex connecting and merging
                qq general idea of gamma-ton tracing x3
                    g.can be seen as an extend to photon tracing
                        simulate Weathering with particles
                    1.gammaton [shooting]
                        emit from the outer hemisphere of the scene
                            Each particle stores weathering information
                        Each particle is assigned propabilities for being one of 4
states
                    2.[propergation]
                        follow physical and other laws
                        surfaces are sampled into point clouds
                        When particle collides with surface, the movement
probabilities
                            change according to surface reflectance
                    3.perform [Gamma-transport] until final [equilibrium/ until
Settlement]
                qq idea behind boids
```

1.[Extend] particles with orientation and 3D modell
2.can be [used to] model Flock of Birds, Herds, Schools
qq how to simulate a piece of cloth with a particle system?
visually: use a rectangular grid with vertices
forces: simulated by springs, internal forces
qq Why do stiff springs cause problems?
if spring constant is chosen too small for easier integration,
there will be a super elastic effect
two solution strategies exists
qq How to model hysteresis in cloth simulation? x2
1.add friction forces that depend on
stretching, shearing and bending velocities
2.Kelvin-Voigt-Spring, damping effect
qq How can one avoid collisions in a rigid body simulator?
additional de-penetration inpulse, force
qq What is contact point and what kind of non-degenerate contacts exist for polygonal meshes?
> 2 non-degenerated contact types?
vertex-face
edge-edge
> degenerated cases
Vertex-vertex, vertex-edge and edge-edge (id)
less likely occour case
Edge-face and face-face
extended contacts
:less likely to happen and special care should be taken
<> Explain math
qq integration over solid angle
pic
qq Monte Carlo (MC) estimation of integral
pic
qq variance, bias
a property to an estimator, in statistics
can be measured by [standard deviation]
qq why do we need variance in solving rendering equ.? + igen
1.to solve rendering equ., we have to solve multi-dimensional nested integrals
2.MC technique cast integration as [expected value problem], by gen. samples
thus, lower variance means better approximation
qq when is MC better than Rieman Integration?
evaluating multidimensional, infinite dimensional integral
qq MC estimation of reflection integral
pic
qq idea of importance sampling
choose px proportional to fx
a technique to refuce variance
qq Russian Roulette
terminates the traversal of a ray with a probability
(decreases when ray travels in the scene)
qq Nusselt’s analogon
the cosine weighted solid angle corresponds to its projection onto the unit disk
such we can perform importance sampling of the cosine term
qq uniform, rejection, N-rooks, transformation sampling
+toread a branch of sampling theory
uniform: randomly choose a value within a given range
rejection: reject samples that not expected
rejection test

N-rooks: stratified grid sampling with random shuffle
1.n samples are placed in an n x n grid
2.there is a single sample, or rook, in each row and each column.
//
step1: Randomly place one sample in each cell [along the main diagonal] of the grid
step2: Randomly shuffle
benefits in high dimention, covers each dimension much better.
transformation sampling: sampling uniformly and then transformed to arbitrary shape
qq What parts of the reflection integral in directional form can be importance sampled?
brdf and L_in and cosin term x3
qq what is a differential equation? + igen
a relation between a function in one or several variables and the [partial] derivatives of it
qq ordinary and partial differential equations
In case of a single variable, ODE
if derivatives of several variables arise, PDE
qq give some examples of ODE and PDEs?
ode: oscillator with frictions
pde: Wave Eq., Maxwell Eg
qq reduction to order one system of ODEs, order reduction in phy?
pic
qq explicit, implicit, symplectic Euler
pic
qq what is time evoluiton funciton?
can be derived from equ. of motion, used to compute state of the system in an arbitry time t
fully describe the state of the system
function describe the state of the system when time evolves
systems with internal state (also called stateful systems).
qq A-stabel
iff generated approximations converge to 0 for any step width and arbitrary long time
qq what means stiff for a DE?
explicit methods do not always converge.
qq general idea behind step width adaptation
comparing to two steps with half stepwidth, threadhold method, larger than toleranz...
re-estimate h with a funciton
qq how to find the best integrator for a given problem?
depends on application, how fast are expected to run, real time application?
qq what is numerical dissipation and how does it influence numerical methods for solving incompressible
> Navier Stokes equations?
Numerical simulation typically makes errors
it is a kind of problem arises in numerical computing, energy will be reduced in the system
disolves vortices too fast
qq what is the splitting trick when solving ODEs?
split a differential equation in diff. parts, update varibles Sequentially
exploits intermediate varibles
pic
+integrate diff. parts saparately

qq how can the advection step be approached in a semi-Lagragian manner? x3
solve “the advection equation” for any grid quantity q
use the Lagrangian notion of advection directly to compute
interpolate from nearby grid points to get the result
+++
solve “the advection equation” for any grid quantity q
trace with Lagrangian notion
compute q(x_old) with interpolation from nearby grids
qq how does the pressure update work for incompressible fluids?
an additional divergence constraint to ensure the incompressiblility
pic
star means intermediate varibles !
qq what are the different boundary conditions used in fluid simulation?
x3
fluid-fluid boundaries
fluid solid boundaries
fluid air/ free space boundaries
“ghost” solid pressure
qq how do we interpolate attributes in SPH simulation?
cubic kernel function or quintic spline
for theory Gaussian is best, not used in practice due to infinite support
<> Explain physics
qq unit, idea of non-dimensionalization
SI units
non-dimensionalization
1.partial [removal] of [physical dimensions] from an equation
suitable [substitution of variables].
2.[simplify] and parameterize problems where
measured units are involved
3.These units refer to quantities intrinsic to the system,
rather than units such as SI units
qq Noether's theorem
Any differentiable symmetry of the [action] of a physical
system has a corresponding conservation law
high lev understanding of the phy. system
qq radiometry: energy, power, radiosity and irradiance, radiance (incoming vs outgoing), brdf, albedo(Diffuse BDRF),
> potential, spectral quantities
power in area A, power per area and solid angle
brdf: defines how light is reflected at an opaque surface
It is employed in the optics of real-world light
Diffuse BDRF: given incoming direction, integrate over all outgoing direction
qq rendering equation in directional, area formulation and operator form, light path regular expressions
pic
qq geometry factor and visibility
pic explain it!
qq emitted radiance of light source, sensor efficiency and measurement equation
what is W here? spectral efficiency
qq brdf properties: Helmholtz reciprocity, energy conservation
pic write them down
qq how to make Phong and Blinn-Phong brdfs physically plausible?

apply those two physically plausible conditions: Helmholtz Reciprocity and
energy conservation to the calssical model
make the terms symmetric, skip the denominator terms
qq explain microfacette models on a coarse level, what are the individual parts?
x3 The BRDF results from
Distribution of the orientation of the microfacets
Properties of Planar Reflection
Self-occlusion and self-shadowing
microfacets modeled as planar reflectors
qq what do the Fresnel coefficients describe?
reflection on micro facets
describe the reflection and transmission of light
(or electromagnetic radiation in general)
when incident on an interface between different optical media.
+toread Metal Fresnel Term Approximations
qq what is an anisotropic brdf, what an isotropic?
when the incoming direction changes, outgoing direction varies pp
brushed metal
Most materials are isotropic
qq how to measure a brdf?
[measures reflectance] for [combinations] of sensor omega_out and light source omega_in
Helmholtz reciprocity and isotropy of BRDF help to reduce samples needed
one can move sensor or rotate sample
qq how to represent a measured brdf efficiently?
with two dots on a hemi-sphere drawn from above with the normal direction in the center
qq What is a bssrdf?
Subsurface Scattering inside of material is considered
BSSRDF much more natural [light distribution] on given object
qq quantities: mass, acceleration, momentum, (conservative) force. energy (kinetic, potential)
pic
qq what different forces do you know? x4 x2
Contact force:
Frictional Force, Tension Force, Spring force, Resistance Force (viscosity)
Action-at-a-distance forces:
Gravitational Force, Magnetic Force, Electrical Force
qq inertia moment, inertia tensor, vector representation of angular velocity,
> angular acceleration, torque,
> rotational kinetic energy
derivative from time to time
pic
qq the idea of the Sequential impulse loop/ algorithm + igen, iwrote
1.compute list of contact points
// the v-p-v approach
2.compute relative [velocities]
3.iteratively apply impulses with constrains (impulse accumulation)
and add de-penetration bias velocity
4.update linear [velocities and angular velocities]

qq how can pose of rigid body be described
pic
position, orientation, linear and angular momentum
qq how does pose evolve over time without friction?
pic
time evolution funciton
qq how does orientation change in dependence of angular velocity?
the same, above, time evolution funciton
qq how do forces applied to a point on a rigid body act on its linear and angular velocities?
forces can be transfered to impulses (with direction information),
and thus can be used to update linear and angular velocity
qq what is a “Kraftstoß” and how does it help to compute the change in momentum during a rigid body simulation
Impulsänderung
Jeder Kraftstoß ist mit einer Impulsänderung verbunden
Der Kraftstoß auf einen Körper ist gleich der Änderung seines Impulses.
it is the change of impulse, can be used to update velocities, momentums
qq what are kinetic and dynamic viscosity?
dynamic viscosity gives force instead of acceleration(kinetic viscosity)
they are used in diff. equ. : momentum equ. and force equ.
qq what are incompressible fluids?
incompressible condition
pic
divergence of u is zero. u is velocity with components u,v,w
divergence depends only on spatial derivatives
---+ql picture notes!
---story mode
---
cg3
rendering equ./ ray simulation/ ray tracing/ particle tracing
// derive equ.
derive: phy view
operator form, nested integral form, directional, area formulation, light path regular expressions
geometry factor and visibility
radiance computing: emitted radiance of light source, sensor efficiency and measurement equation
how to make Phong and Blinn-Phong brdfs physically plausible? derive a new equ follow phy rules
explain microfacette models on a coarse level, what are the individual parts? more realistic brdf!
what do the Fresnel coefficients describe?
brdf properties: Helmholtz reciprocity, energy conservation
evaluate: math view
MC method, other integrator?
formula in MC method? why should we use MC integrator?
reduce variance? MIS
integration over solid angle
variance, bias, why do we need variance in solving rendering equ.?
MC estimation of reflection integral
Nusselt’s analogon
uniform, rejection, N-rooks, transformation sampling

            - what is an anisotropic brdf, what an isotropic?
            - how to measure a brdf?
            - how to represent a measured brdf efficiently?
            - what is a bssrdf?
    - // inteprete above equ. in codes
        - sample the rays: coder view/ geo. view
            - basic:
                - path, light and bidirectional path tracing
            - cache-based method:
                - final gathering, photon mapping, reverse photon mapping, ppm, vcm, sppm, probPPM
        - particle tracing
            - gamma-ton tracing, boids
        - +tracing other quantities
- differental equ./ ode, pde, physical simulation
    - essential
        - // solving DE
            - what is a differential equation?
            - ordinary and partial differential equations
            - explicit, implicit, symplectic Euler
            - what is time evoluiton funciton?
            - A-stabel
            - what means stiff for a DE?
            - general idea behind step width adaptation
            - how to find the best integrator for a given problem?
        - // quantities
            - unit, idea of non-dimensionalization
            - Noether theorem
            - radiometry: energy, power, radiosity and irradiance, radiance
            - quantities: mass, acceleration, momentum, (conservative) force. energy (kinetic, potential)
            - what forces do you know?
            - inertia moment, inertia tensor, vector representation of angular velocity,
                - > angular acceleration, torque,
                - > rotational kinetic energy
    - rbody simualtion
        - // derive equ.
            - the simulation loop!
            - the idea of the Sequential impulse loop/ algorithm + igen, iwrote
        - // inteprete
            - // diff. parts of the equ./ algo:
                - how can pose of rigid body be described
                - how does pose evolve over time without friction?
                - how does orientation change in dependence of angular velocity?
                - what is a “Kraftstoß” and how does it help to compute the change in momentum during a rigid body simulation
                - avoid collisions in a rigid body simulator?
        - // addi. simulation requirements
            - contact point and what kind of non-degenerate contacts exist?
                - handle them better!
    - fluid simulation
        - // derive
            - what are incompressible fluids?
            - the idea, the simualtion loop? derive the simulation

```
equ.
                            what are kinetic and dynamic viscosity?
                            splitting trick when solving ODEs
                        // inteprete
                            // diff. parts of the simulation equ./ algo.
                                how can the advection step be approached in a semi-
Lagragian manner?
                                how does the pressure update work for
incompressible fluids?
                        // addi. simulation requirements
                            how do we interpolate attributes in SPH simulation?
                            what are the different boundary conditions used in
fluid simulation?
                            what is numerical dissipation and how does it influence
numerical methods for solving incompressible
                                Navier Stokes equations?
                    cloth simulation
                        // derive
                            the idea, the simualtion loop?
                            how to model a piece of cloth
                        // inteprete
                            // addi. simulation requirements
                                stiff spring? hysteresis?
            ---
            cg3 (山川河流人物光线)
                essential
                rbody sim
                fliud sim
                cloth sim
                ray sim
    ---summ. active mode
        idea, hlev understanding
            take away knowledge:
                lev4* computational methods
                lev3 improve *speed and *accuracy!
                lev2* choose the right method, compare them
                lev1* detail, explain
            about my oomi framework
                geometry oomi
                    optimization
                dynamic oomi
                    optimization
                simulation oomi
                        computational (lower level, components of opti.)
    ---problems
        is VR = high frame rate rendering ?
        what is V meaning ? s2p15.... p16
        why not M p18
        why 2+2+2.... s3p5
        s2p10 should be x'?
        err? s3p5 2*(2+2+2...)
        potential 的计算，为啥和radiance一样麻烦？ p19
        sample的过程？ 有一定pdf函数？ 完全随机？
        not understood p26
    ---extended, from tt slides ref papers
        // papers
            // essential
                http://physbam.stanford.edu/~fedkiw/papers/cam2000-08.pdf
                http://physbam.stanford.edu/~fedkiw/papers/stanford2004-07.pdf
```

http://www.cs.cornell.edu/~dph/papers/dt.pdf

http://web.stanford.edu/class/cs277/resources/papers/Moller1997b.pdf
// other mixed
Online Optical Marker-based Hand Tracking with Deep Labels
https://dl.acm.org/doi/pdf/10.1145/3197517.3201399
Singularity-Robust Inverse
Kinematics Using Lagrange
Multiplier for Redundant
Manipulators

https://pdfs.semanticscholar.org/815c/f5781791ea6e8ecfef42a5252f12455f9cbb.pdf
// rbody sim
https://www.cs.cmu.edu/~baraff/papers/sig89.pdf
https://animation.rwth-aachen.de/media/papers/2012-EG-STAR_Rigid_Body_Dynamics.pdf
http://www.roboticsproceedings.org/rss04/p12.pdf
http://blog.mmacklin.com/project/flex
A Numerically Robust LCP Solver for Simulating Articulated Rigid Bodies in Contact
http://www.roboticsproceedings.org/rss04/p12.pdf
numerically robust algorithm for solving linear complementarity problems (LCPs)
articulated rigid bodies
Constraint-based approaches often employ linear complementarity problem (LCP)
Most models can be categorized into penalty- and constraint-based methods
Modeling collisions and contacts
contact forces are easily computed from penetration depths and relative velocities
the approach suffers from numerical instability problem due to impulsive forces
linear complementarity problem (LCP) [1] to formulate the constraints.
LCPs can be solved by either iterative or pivot-based approach
numerical root-finding techniques such as Newton’s method to find the equilibrium.
Pivot-based approaches are theoretically guaranteed to find a solution with finite number of trials
for general problems
However, it is known that pivot-based approaches often suffer from numerical problem especially for large-scale and/or ill-conditioned problems
Jourdan et al. [4] applied an iterative LCP solver similar to Gauss-Seidel algorithm
proved convergence in most practical cases.
free rigid bodies, in which case M is generally sparse and the LCP is likely to be relatively easily solved by both iterative and pivot-based approaches
iterative methods do not guarantee convergence to a solution
The main contribution of this paper is improvement of Lemke Algorithm to deal with large-scale and ill-conditioned LCPs derived from frictional contacts between articulated rigid bodies of arbitrary geometry
Unified Particle Physics for Real-Time Applications http://blog.mmacklin.com/project/flex/
Bunnies parachute into a pool of water.
Cloth, rigid bodies and fluids coupled through constraints interact seamlessly in our framework.
Unified Particle Physics

Unified Particle Physics for Real-Time Applications
* treat contact and collisions in a unified manner
* model gases, liquids, deformable solids, rigid bodies and clothing with two-way interactions are modeled with particles and constrains between them

parallel constraint solver based on position based dynamics that is efficient enough for real-time applications.

SIGGRAPH 2014

Creates redundant work

Everything is a set of particles connected by constraints

recreate these packages in real-time

Melting, phase-changes

Position-Based Dynamics

Using particles connected by constraints as our fundamental building block allows us to treat contact and collisions in a unified manner

+https://d2f99xq7vri1nk.cloudfront.net/legacy_app_files/pdf/nucleus.pdf

+https://matthias-research.github.io/pages/publications/posBasedDyn.pdf

// fluid sim

https://cg.informatik.uni-freiburg.de/intern/seminar/gridFluids_GPU_Gems.pdf

https://cg.informatik.uni-freiburg.de/publications/2014_EG_SPH_STAR.pdf

https://cg.informatik.uni-freiburg.de/

https://ge.in.tum.de/publications/2017-sig-um/

Perceptual Evaluation of Liquid Simulation Methods

https://ge.in.tum.de/publications/2017-sig-um/

visual accuracy metric

perceptual evaluation, liquid simulation

smoothed particle hydrodynamics

suitable one among various methods for a given task

This paper targets numerical simulations of liquids

Versatile Rigid-Fluid Coupling for Incompressible SPH

https://cg.informatik.uni-freiburg.de/publications/2012_SIGGRAPH_rigidFluidCoupling.pdf

Infinite Continuous Adaptivity for Incompressible SPH

http://s2017.siggraph.org/sites/default/files/firstpages_part_08.pdf

SPH Fluids in Computer Graphics

https://cg.informatik.uni-freiburg.de/publications/2014_EG_SPH_STAR.pdf

Fluids in Games

https://www.cs.ubc.ca/~rbridson/fluidsimulation/

http://physbam.stanford.edu/~fedkiw/

// cloth sim

http://www-labs.iro.umontreal.ca/~bernhard/PDF/Thomasze08Asynchronous.pdf

View-Dependent Adaptive Cloth Simulation

http://graphics.berkeley.edu/papers/Koh-VDA-2014-07/

Given a prescribed camera motion, the method adjusts the criteria controlling refinement to account for visibility and apparent size in the camera’s view

dynamically adaptive mesh refinement

coarsening

camera’s view frustum

Characters completely outside the view frustum are simulated at very low resolution

a simple way of achieving computational savings for cloth simulation
The main limitation of our method is that it requires an adaptive framework
We also believe that our approach would scale very well to massive scenes with thousands of actors,
scale very well
where it would produce even larger savings.
there would be additional challenges in applying it to interactive animation.
it may not be possible to apply it in interactive settings where camera motions are not predetermined.
virtual characters wearing simulated clothing is now widespread
cloth simulation remains computationally expensive
fine details will not be visible to the viewer and work spent computing those details is wasted
For closeup shots where only part of a character is in frame
// smoke sim
https://www.diva-portal.org/smash/get/diva2:676008/FULLTEXT01.pdf
Visual Simulation of Smoke
http://physbam.stanford.edu/~fedkiw/papers/stanford2001-01.pdf
// fire sim
Animating Fire with Sound
https://research.cs.cornell.edu/Sound/fire/
https://research.cs.cornell.edu/Sound/fire/FireSound2011.pdf
Not necessarily physically accurate!
a large artistic component
visual effects
5 Tips For Good Looking Fluid Sims
Used in Harry Potter film
Our method produces the familiar sound of roaring flames synchronized with an
underlying low-frequency physically based flame simulation
Additional mid- to high-frequency sound content is synthesized using methods based on spectral bandwidth extension
synthesizing plausible fire sounds that are synchronized with physically based fire animations
combustion sounds
data-driven texture synthesis to synthesize high-frequency content based on input flame sound recordings
https://on-demand.gputechconf.com/gtc/2012/presentations/S0102-Flame-on-RT-Fire-Simulation-for-Video-Games.pdf
// Magnetic sim
http://www.dartmouth.edu/~boolzhu/papers/ferrofluid.pdf
// spec. materials sim
http://tiantianliu.cn/papers/liu17quasi/liu17quasi.html
Visual Simulation of Weathering By γ-ton Tracing
https://www.microsoft.com/en-us/research/publication/visual-simulation-of-weathering-by-%CE%B3-ton-tracing/

https://www.microsoft.com/en-us/research/wp-content/uploads/2016/02/gamma_ton.pdf
A weathering sequence generated by our system
this paper presents a visual simulation technique
that works well for a wide variety of weathering phenomena
// rendering and brdf, ray sim
bsdf

https://diglib.eg.org/bitstream/handle/10.2312/egs20011003/short16.pdf?sequence=1&isAllowed=y

https://zhuanlan.zhihu.com/p/60977923
https://zhuanlan.zhihu.com/p/21376124
https://dspace5.zcu.cz/bitstream/11025/11214/1/Lazanyi.pdf
https://rgl.epfl.ch/publications

https://www.mpi-inf.mpg.de/departments/computer-graphics/publications
https://dl.acm.org/doi/pdf/10.1145/1572769.1572783
http://graphics.ucsd.edu/~henrik/papers/fur_bssrdf.pdf
http://www.apsipa.org/proceedings_2013/papers/380_A-Datadriven-Song-2943205.pdf
https://web.cs.wpi.edu/~matt/courses/cs563/talks/radiosity.html

https://www1.cs.columbia.edu/CAVE/publications/pdfs/Oren_SIGGRAPH94.pdf

https://web.cs.wpi.edu/~emmanuel/courses/cs563/write_ups/chuckm/chuckm_BRDFs_overview.html#:~:text=The%20BRDF%2C%20referred%20to%20as,point%20P%20on%20the%20surface.
http://www.thetenthplanet.de/archives/255

https://digibug.ugr.es/bitstream/handle/10481/19751/rmontes_LSI-2012-001TR.pdf;jsessionid=97878771AD4E28B65B909823A8B0C1F2?sequence=1

https://www.cs.drexel.edu/~david/Classes/CS431/Lectures/BRDF.pdf
https://dl.acm.org/doi/pdf/10.1145/300776.300778
https://cg.cs.uni-bonn.de/de/projekte/btfdbb/
http://library.utia.cas.cz/separaty/2009/RO/filip-bidirectional%20texture%20function%20modeling%20state%20of%20the%20art%20survey.pdf
https://graphics.stanford.edu/papers/bssrdf/bssrdf.pdf
https://cgl.ethz.ch/publications/papers/papers.php
A Radiosity Method for Non-Diffuse Environments
https://cg.cs.uni-bonn.de/de/projekte/btfdbb/

+https://cg.cs.uni-bonn.de/aigaion2root/attachments/sattler-2003-efficient.pdf
classical GI
Robust Monte Carlo Methods for Light Transport Simulation
http://graphics.stanford.edu/papers/veach_thesis
bidirectional path tracing

https://graphics.cs.kuleuven.be/publications/BDPT/BDPT_paper.pdf
ined as a generalisation of the wellknown path tracing algo
forms signicantly better for typical indoor scenes where indirect lighting is imp
RELATED WORK An important milestone in the development of the global illumination theory for computer graphics was the introduction of the radios ity method
is based on the e
s the scene to be discretised into patches or elements and as such it is a nit
ele ment method The radiosity solution i
ce from the viewpoint In #Ka jiya# # Ka jiya presented
ender ing equation and introduced path tracing as a Monte Car
#Cook et al### Shirley### Monte Carlo techniques are capable of handling the most general class of lighting e   ects but are
hit points on the respective particle paths are then connected using shadow rays
and the appropriate contributions are added to the ux of the pix

+https://www.cg.tuwien.ac.at/research/publications/2017/dodik-2017-pcbpt/

Global Illumination using Photon Maps

http://graphics.ucsd.edu/~henrik/papers/ewr7/global_illumination_using_photon_maps_egwr96.pdf

http://graphics.ucsd.edu/~henrik/papers/photon_map/

It represents a significant improvement of a previously described approach both with respect to speed, accuracy and versatility

packets of energy (photons)

one high resolution caustics photon map to render caustics

distribution ray tracing algorithm

radiosity has been extended with directional capabilities

qq what is a radiosity algorithm?

radiosity is an application of the finite element method to solving the rendering equation

for scenes with surfaces that reflect light diffusely.

Unlike rendering methods that use Monte Carlo algorithms (such as path tracing),

which handle all types of light paths, typical radiosity only account for paths

(represented by the code "LD*E")

Radiosity is a global illumination algorithm in the sense that the illumination

arriving on a surface comes not just directly from the light sources,

but also from other surfaces reflecting light.

Radiosity is viewpoint independent

the scene can be baked

linear system can be driven

qq which three classes of gi algorithms are there?

* radiosity implementations and ray tracing implementations before cache based imp.

which increases the calculations involved

Radiosity methods were first developed in about 1950 in the engineering field of heat transfer.

Notable commercial radiosity engines are Enlighten by Geomerics

accounted for diffuse indirect lighting.

In this context, radiosity is the total radiative flux (both reflected and re-radiated) leaving a surface

as radiant exitance.

Calculation of radiosity, rather than surface temperatures, is a key aspect of the radiosity method that permits linear matrix methods to be applied to the problem.

multiple bounces are computed

Individual patches are visible as squares on the walls and floor.

lends an added element of realism to the finished scene

mimics real-world phenomena

Consider a simple room scene.

Difference between standard direct illumination without shadow umbra, and radiosity with shadow umbra

subtle lighting effects are noticeable around the room

bled onto the grey walls

model soft indirect illumination

radiosity algorithm is performed on a simple geometric approximation of the original model.

a two pass method in which we simplify the representation of the illumination instead of simplifying the geometry

we store incoming flux (photons)
The use of photons allows us to estimate surface radiance at surfaces with arbitrary BRDF’s
situations where we need an accurate computation and situations in which an approximate estimate can be applied
For highly glossy surfaces we do however trace additional sample rays since reasonable radiance estimates for these surfaces require a large number of photons.
using a separate caustics photon map which has a high density of photons.
The caustics photon map is used only to store photons corresponding to caustics
created by emitting photons towards the specular objects in the scene and storing these as they hit diffuse surfaces.
this requires a high density of photons.
The global photon map is used as a rough approximation of the light/flux within the scene
and at the following intersection points we store shadow photons.
improved both the speed, reduced the memory requirements and improved the accuracy of the method
The photons are stored in a balanced kd-tree
The fact that the tree is balanced guarantees that the time it takes to locate M photons in a tree with N photons is $O(M \cdot \log_2(N))$
The final image is rendered using Monte Carlo ray tracing
while the specular part are highly glossy and ideal specular reflection models
L_r, depends on the radiance values in the rest of the scene and it can be solved directly using Monte Carlo techniques like path tracing.
We distinguish between two different evaluations of the integrals: An accurate and an approximate
(it contributes only little to the pixel radiance
The approximate evaluation is simply the radiance estimate obtained from the global photon map
qq how can be evaluate the direct illumination part in both cases? approximate and accurate
* The rendering equation (1) can be split into a sum of several components.
  and the reflection part can be splited to four parts
  the method used to evaluate each part depends on wheather we want to evaluate it in accurate
  way or approximate take a look at the direct light part,
  radiance estimate can be taken directly from global photon map(approximation) or
  perform light source evaluations!

qq how can evaluate Caustics part of the reflected integral?
represents caustics on diffuse and slightly glossy surfaces
evaluate this term using the information in the caustics photon map
the radiance estimate based on the caustics photon map is visualized directly
number of photons in the caustics photon map must be high.
The information in the photon map can be used to compute the radiance leaving a

surface in a given direction
An alternative could be using a sphere of a fixed size and use all the photons within this sphere
considered a number of adaptive strategies for computing the necessary size of the sphere based on the local photon density.
cone-filter to the estimate.
too low the radiance estimation strategy can give blurry results.
a general two-pass global illumination method
Comparisons with existing global illumination techniques indicate that
the photon map provides an efficient environment for global illumination.
qq the photon map structure is completely separated from the geometric representation? how to understand this?
The photon map code can be provided in a separate module that contains the necessary
functions (e.g. a function that given a position and a surface definition returns the
radiance in a given direction).
Schlick's reflection model
colour bleeding effect between the walls
metallic teapot
using Ward's anisotropic model
Metropolis Light Transport
https://graphics.stanford.edu/papers/metro/metro.pdf
Progressive Photon Mapping
https://www.ci.i.u-tokyo.ac.jp/~hachisuka/ppm.pdf
This type of illumination is difficult to simulate with Monte Carlo ray tracing methods such as path tracing,
qq three main types of GI?
radiosity methods, Monte Carlo ray tracing methods, cache based methods
Monte Carlo ray tracing methods includes: path tracing, bidirectional path tracing, and Metropolis light transport.
Photon mapping is significantly better at capturing the caustics lighting seen through the lamp
it is cache based method!
qq what is the prob. of this pic? it lacks the fine detail in the illumination
Sampling and Reconstruction, Density Estimation
algorithms based on Monte Carlo ray tracing are capable of solving the rendering equation without any approximations [Dutre et al. 2006].
light being transported along a specular to diffuse to specular path (SDS path) before being seen by the eye
this type of illumination is very common.
unbiased Monte Carlo ray tracing methods such as path tracing
Progressive photon mapping uses multiple photon tracing steps to compute an
accurate solution without maintaining every photons from each iteration.
qq what is the biggest advantage of progressive photon mapping??
* Arbitry accuracy can be achieved by a limited memory consumption, when it compared to traditional photon mapping methods
Each photon tracing pass results in an increasingly accurate global illumination solution
any desired accuracy can be reached using a limited

amount of memory.
qq what is Progressive Radiance Estimate
To address the shortcomings of BDPT Veach and Guibas [1997] proposed the Metropolis light transport algorithm
qq In MLT, each path is generated based on?
the mutation (perturbation) of a previous path.
illumination coming through a slight opening of door.
such paths are difficult to generate by mutating existing paths.
improvement to MLT called Energy Redistribution Path Tracing (ERPT)
it shares the same weakness in the context of mirror reflections of caustics.
Path space is the space of all possible light transport paths in a scene
Note that path tracing, BDPT, MLT, ERPT and PMC-ER are all unbiased path-space based methods
Photon mapping is a two-pass global illumination algorithm developed by Jensen [1996].
two-pass global illumination algorithm
qq what is nearest neighbor density estimation
qq why the density estimation process can be considered as a way of loosely
> connecting paths from the eye to the light?
photon mapping is very effective at rendering SDS paths
qq the final quality of PM is often limited by?
the maximum number of photons
Havran et al. introduced the concept of reverse photon mapping [Havran et al. 2005].
ray tracing in the first pass and photon tracing in the second pass.
The motivation for this approach is to reduce the complexity and improve the performance of photon mapping
when a large number of rays
adaptive image filtering algorithm
Our method also uses the number of photons to reduce the search radius of the radiance estimate
With progressive photon mapping we can use an unlimited number of photons, since we do not need to store all the photons and we retain all the advantages of the standard photon mapping method
being able to handle non-Lambertian surfaces.
a new radiance estimate
radius update
qq two conditions to ensure the convergence of the PM method?
use an infinite number of photons in the photon map
the radius should converge to zero.
obtain a solution with arbitrary precision
Photon mapping with 20 million photons results in a noisy and blurry image
Stochastic Progressive Photon Mapping
https://www.ci.i.u-tokyo.ac.jp/~hachisuka/sppm.pdf
Progressive photon mapping [Hachisuka et al. 2008] removed the memory bound of photon mapping,
which makes the results converge to the correct solutions
(i.e., bias goes to zero in the limit).
* bias goes to zero in the limit
The application of Markov Chain Monte Carlo have been shown

to improve the efficiency of the path construction
progressive refinement of photon statistics at a point where the radiance value is computed
a new progressive density estimation technique. P
with bounded memory consumption
still retains the robustness of photon mapping
improves its robustness to an even wider class of scene settings.
We are not aware of existing work in density estimation literatures outside graphics
Time dependent photon mapping [Cammarano and Jensen 2002]
qq how can we model motion blur and depth of field in GI algos? x2
motion blur requires computing the average radiance value over a
visible part of a scene for a given shutter time,
and depth-of-field needs the average radiance value over a part of scene that is visible through a lens
the original progressive radiance estimate is restricted to computing the correct radiance value at a point ~x.
shared statistics over a region that we would like to compute the average radiance value for
over the region
Figure 2 summarizes the difference between the algorithms of progressive photon mapping and our algorithm
the modification is simple
In order to explain our new formulation
Progressive Photon Mapping: A Probabilistic Approach

https://www.cs.umd.edu/~zwicker/publications/PPMProbabilistic-TOG11.pdf
main contribution: proof of convergence
Adaptive Progressive Photon Mapping

https://cg.ivd.kit.edu/publications/p2012/APPM_Kaplanyan_2012/APPM_Kaplanyan_2012.pdf
Fast Final Gathering via Reverse Photon Mapping
http://people.mpi-inf.mpg.de/alumni/d4/2016/rherzog/Papers/reversePMeg05.pdf
Light Transport Simulation with Vertex Connection and Merging
https://cgg.mff.cuni.cz/~jaroslav/papers/2012-vcm/2012-vcm-paper.pdf
main idea: combine bpt and sppm with help of MIS
Unifying points, beams, and paths in volumetric light transport simulation
http://www.smallupbp.com/
fancy renderer
voxel-cone-tracing
https://research.nvidia.com/publication/interactive-indirect-illumination-using-voxel-cone-tracing
Voxel Cone Tracing
realistic image synthesis
Indirect illumination
off-line computation and pre-baking can be acceptable
many applications (games, simulators, etc.) require real-time or interactive approaches
evaluate indirect illumination
costly precomputation
voxel octree representation generated and updated on the fly from a regular

scene mesh coupled with an approximate voxel cone tracing
scene-independent performance and can handle complex scenes with dynamic content
integration of 2-bounce illumination
drastically improves the realism of a rendered scene
complex scenes are challenging to illuminate
qq Global illumination is computationally expensive for several reasons?
computing visibility between arbitrary points in the 3D scene, which is difficult with rasterization based rendering
it requires integrating lighting information over a large number of directions for each shaded point.
millions of triangles
We reach real-time frame rates even for highly detailed environments and
produce plausible indirect illumination (see Teaser).
Teaser
pre-filtered hierarchical voxel representation of the scene geometry
stored on the GPU in the form of a dynamic sparse voxel octree
efficiently exploits the GPU rasterization pipeline.
d is then updated interactively with moving objects
dynamic modifications
The main contributions of our work are the following
a fast GPU-based mesh voxelization and octree-building algorithm
An efficient approximate cone-tracing integration
supports diffuse and glossy reflections
Recent GPU implementations of photon mapping [Hac05,WZPB09] use clustering and exploit the spatial coherence of illumination
High-Quality Global Illumination Rendering Using Rasterization
can be seen in the accompanying video
But this solution is not fast enough to provide real-time performances
sparse voxel octree structure storing geometry and direct lighting information
pp can not classified properly
https://www.eecs.yorku.ca/~amana/research/cones.pdf
Global illumination using photon maps
anti-aliasing
image synthesis
what level of detail is sufficient in a texture map or in a procedural or hierarchical model of an object when ray tracing
exploit area-sampling techniques to avoid aliasing artifacts
not enough information associated with the ray to perform anti-aliasing
There is no way of knowing or calculating what else is visible in the neighborhood surrounding the sample point
The only way to anti-alias within standard ray tracing is to go to higher resolution. Whitted proposed adaptive supersampling and it is now almost universally used
to modify the definition of "ray"
The pixel should represent not a point but an area of the screen

pyramid with the apex at the eye and the base defined by the four planes that cut the borders of the pixel
only one ray per pixel is sufficient regardless of scene complexity.
Voxel Cone Tracing
modern GPU rasterization pipeline acceleration
http://www.jp.square-enix.com/tech/library/pdf/Real-Time%20Bidirectional%20Path%20Tracing%20via%20Rasterization%20(preprint).pdf
perceptually important for interactive applications such as video games, lighting design or virtual reality systems.
high rendering cost.
Many techniques including shaders have been developed for rasterization pipeline
on a modern commodity GPU
Monte Carlo integration to calculate indirect illumination
This paper proposes a new real-time rendering system
an approximate bidirectional path tracing in a simplistic way.
implemented with a modern GPU rasterization
Global ray-bundles were often used for off-line rendering to accelerate visibility test.
Global Ray-Bundles
we create global ray-bundles using per pixel linked-list construction on a DirectX 11 GPU
Real-Time Bidirectional Path Tracing via Rasterization
Efficient Simulation of Light Transport in Scenes with Participating Media using Photon Maps, Henrik Wann Jensen
http://graphics.ucsd.edu/~henrik/papers/sig98.html
http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.118.6575&rep=rep1&type=pdf
Fast, Flexible, Physically-Based Volumetric Light Scattering

https://developer.nvidia.com/sites/default/files/akamai/gameworks/downloads/papers/NVVL/Fast_Flexible_Physically-Based_Volumetric_Light_Scattering.pdf
A Practical Analytic Single Scattering Model for Real Time Rendering

https://cseweb.ucsd.edu/~ravir/papers/singlescat/scattering.pdf
EFFICIENTLY RENDERING SHADOWS USING THE PHOTON MAP
https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.47.2582&rep=rep1&type=pdf
Efficient Caustic Rendering with Lightweight Photon Mapping
https://graphics.cg.uni-saarland.de/papers/grittmann-2018-lwpm-paper.pdf
https://www.ci.i.u-tokyo.ac.jp/~hachisuka/mmlt.pdf
https://developer.download.nvidia.com/SDK/10.5/direct3d/Source/ScreenSpaceAO/doc/ScreenSpaceAO.pdf
// non-papers
vector calculus
functional analysis
computational mechanics
computational fluids
黎曼求解器
矩阵论
Newtonian Physics
meta ball
level set method, heat diffusion
reconstruction methods

photon mapping papers
MLT paper and more recently/ advanced technique
position estimation methods MCMC
recent realistic rendering papers
sppm/ ppm/ pm/ ... advanced methods
vcm/
渲染材质
icp algos
https://www.cs.cmu.edu/~baraff/sigcourse/
https://box2d.org/files/ErinCatto_SequentialImpulses_GDC2006.pdf
Symplectic integrator
https://www.av8n.com/physics/symplectic-integrator.htm
continuity equation

https://en.wikipedia.org/wiki/Continuity_equation#Definition_of_flux
https://en.wikipedia.org/wiki/Navier%E2%80%93Stokes_equations
later
Vector operator and vector calculus
include the gradient, divergence, and curl:
https://mathinsight.org/divergence_curl_examples
http://tutorial.math.lamar.edu/Classes/CalcIII/CurlDivergence.aspx
http://blog.sina.com.cn/s/blog_4d0723b301017ivo.html
向量还是标量?
http://www.ittc.ku.edu/~jstiles/220/handouts/The%20Curl%20of%20a
%20Conservative%20Field.pdf

http://www.home.uni-osnabrueck.de/mfrankland/Math241/Math241_165_ConservativeR3.pdf

http://www.pbr-book.org/3ed-2018/Light_Transport_III_Bidirectional_Methods/
Metropolis_Light_Transport.html
https://en.wikipedia.org/wiki/Z-order_curve
https://en.wikipedia.org/wiki/Tensor_algebra#Coalgebra
https://en.wikipedia.org/wiki/Talk%3ADyadic_product
https://people.math.ethz.ch/~salamon/PREPRINTS/funcana.pdf

http://ramanujan.math.trinity.edu/wtrench/texts/TRENCH_LAGRANGE_METHOD.PDF

https://www.whitman.edu/mathematics/multivariable/multivariable_16_Vector_Calculus.
pdf
https://www.math.ust.hk/~machas/vector-calculus-for-engineers.pdf
http://www.mecmath.net/calc3book.pdf
https://www.math.uwaterloo.ca/~hwolkowi/matrixcookbook.pdf
https://ccrma.stanford.edu/~dattorro/matrixcalc.pdf
http://www.doc.ic.ac.uk/~ahanda/referencepdfs/MatrixCalculus.pdf
http://www.ams.sunysb.edu/~zhu/ams571/matrixvector.pdf
http://www.personal.rdg.ac.uk/~sis01xh/teaching/CY4C9/ANN3.pdf
http://personal.lse.ac.uk/sasane/ma412.pdf
---re-考试复习
注意事项
带证件
考前保持清醒，注意饮食作息:
在家吃饭，超市买一些红牛，早睡早起
考前保持规律复习，从不同的角度复习效果更好，lower variance
复习过程
前期阶段: 解决所有问题，形成问题列表。
想象自己是讲师，不断重复讲解，深化知识点
背的时候不停回看， 这样背诵效率最高，不要怕麻烦
把困难的任务先完成，不要退缩
后期重复，最后几天每日 iter: 1h，每日至少 5h

```
//
    刷基础，顺序刷，随机刷
    刷错误，经常错的问题反复训练 tobere
//
    刷意识，加快反应速度，顺序，FULL
    刷口语，完全用语言描述, FULL
//
    刷扩展，论文阅读计算, HUGE
    刷讲解，active mode
...
```