Lab 5 Network Models

Due: Midnight, October 16th

In this lab, we will

1. **explore the properties of real-world social networks**
2. **learn how to simulate random graph, small world model graph and preferrential attachment model graph**
3. **explore the properties of the simulated graphs**
4. **extend the preferrential attachment model to powerlaw cluster graph**

Save Your Notebook!

- Click on File (upper left corner), Select “Save” or press Ctrl+S.
- Important: You may loose your modification to a notebook if you do not Save it explicitly.
- Advice: Save often.

Submission

- Please follow the instructions and finish the exercises.
- After you finish the lab, please Click on File, Select “Download .ipynb”
- After download is complete, Click on File, Select “Print”, and and Choose ``Save as PDF"
- Submit both the Notebook file and the PDF File as your submission for Lab 5
- Please also submit the report for Lab 5

1. Propertities of Real World Social Networks

In the class, we discussed three key measurements of real-world social networks, including degree distribution, clustering coefficients and average shortest path length. Since it takes a long time to calcualte average shortest path length for large graphs, in this lab, we will mainly focus on degree distribution and clustering coefficients.

1.1 Load the graph

Please download the file **RO_edges.csv** from Canvas and upload to DS420 in Google Drive. The file contains the edgelist of a Friendships network of users from a European country on music streaming service Deezer. Each line of the file is one edge of the network in the format of *Source_node,Target_node*. It is time consuing to visualize a large graph. Thus, we will not visualize it.

```
import networkx as nx
import matplotlib.pyplot as plt
import collections

# The following code will mount the drive
from google.colab import drive
drive.mount('/content/gdrive')
```

```
Mounted at /content/gdrive
```

```
# load the graph
G = nx.read_edgelist(path="/content/gdrive/My Drive/DS420/RO_edges.csv", delimiter=','
num_edges = G.number_of_edges()
num_nodes = G.number_of_nodes()
print('number of nodes: {}, number of edges: {}'.format(num_nodes, num_edges))
```

```
number of nodes: 41773, number of edges: 125826
```

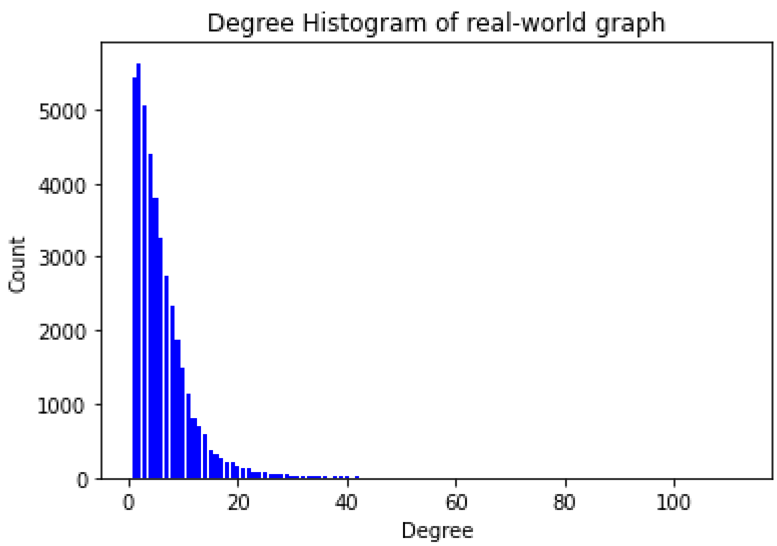
1.2 Degree Distribtuion

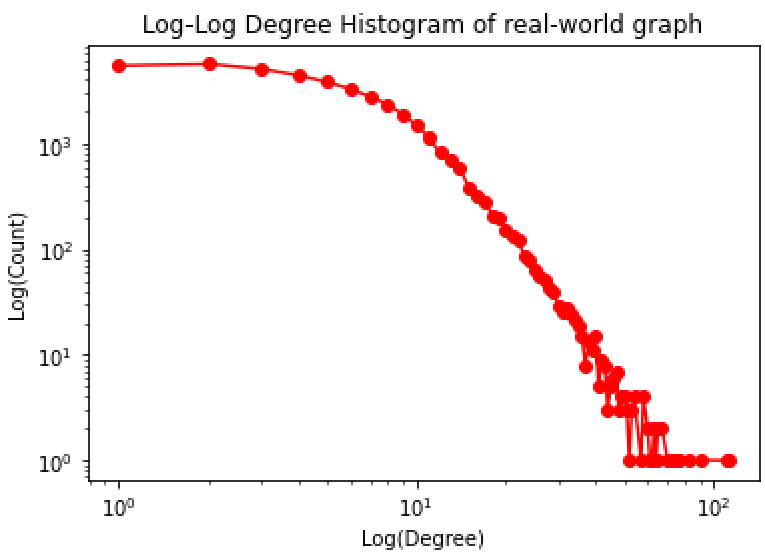
```
def plot_degree_histogram(G, title_of_figure):
    """
    This function plot the degree histogram of a graph G
    :param G: the input graph
    :param title_of_figure: the title of the figure
    :return:
    """
    degree_sequence = sorted([d for n, d in G.degree()], reverse=True)  # degree seque
    degreeCount = collections.Counter(degree_sequence)
    deg, cnt = zip(*degreeCount.items())
    fig, ax = plt.subplots()
    plt.bar(deg, cnt, width=0.80, color='b')
    plt.title("Degree Histogram of "+title_of_figure)
    plt.ylabel("Count")
    plt.xlabel("Degree")
    #ax.set_xticks([d + 0.4 for d in deg])
    #ax.set_xticklabels(deg)
    plt.show(block=False)

    ## log-log scale
    fig, ax = plt.subplots()
    plt.loglog(deg, cnt,'ro-')
    plt.title("Log-Log Degree Histogram of "+title_of_figure)
```

```
    plt.ylabel("Log(Count)")
    plt.xlabel("Log(Degree)")
    plt.show(block=False)


# Call the above function to draw degree histogram
plot_degree_histogram(G, 'real-world graph')
```





Fit the distribution using power function and straight line

```
import numpy as np
from scipy.optimize import curve_fit

def powlaw(x, a, b) :
    return a * np.power(x, b)

def linlaw(x, a, b):
    return a + x * b
```

```
# get deg and count
degree_sequence = sorted([d for n, d in G.degree()], reverse=True)  # degree sequence
degreeCount = collections.Counter(degree_sequence)
deg, cnt = zip(*degreeCount.items())

# fit the power function
popt, pcov = curve_fit(powlaw, deg, cnt)
print(popt)

fig, ax = plt.subplots()
plt.plot(deg, powlaw(deg, *popt), '--', color='g')
plt.bar(deg, cnt, width=0.80, color='b')
plt.legend(['power function','degree distribution'])
plt.show()

# fit the linear funciton in log-log scale
popt_log, pcov_log = curve_fit(linlaw, np.log10(deg), np.log10(cnt))
print(popt_log)

plt.plot(np.log10(deg), linlaw(np.log10(deg), *popt_log), '--')
plt.plot(np.log10(deg), np.log10(cnt), 'ro')
plt.legend(['linear function', 'log-log scale degree distribution'])
plt.show()
```

```
/usr/local/lib/python3.7/dist-packages/ipykernel_launcher.py:5: RuntimeWarning: (
  """
[ 7.54709456e+03 -7.97943487e-01]
```

1.3 Average Clustering Coefficients

We can call average_clustering function to calculate the average local clustering of a network. For more details, please refer to: https://networkx.github.io/documentation/stable/reference/algorithms/generated/networkx.algorithms.cluster.average_clustering.html

```
print('average clustering coefficients is {}'.format(nx.average_clustering(G)))
```

```
average clustering coefficients is 0.0912123373083256
```

2. Random Graph

We will gerante a random graph with the same number of nodes and similar number of edges with the real-world graphd and analyze its degree distribution and clustering coefficients.

2.1 Generate a Random Graph

Assume the real-world graph has n nodes and m edges. To simulate a random graph of n nodes and approximately m edges with G(n,p) model, we set

$$p = \frac{m}{\binom{n}{2}}$$

where $\binom{n}{2} = \frac{n\cdot(n-1)}{2}$. With p defined above, we can use the nx.fast_gnp_random_graph(n, p) to generate a random graph with n nodes and approximately m edges. For details of fast_gnp_random_graph(n, p, seed=None, directed=False), please visit: https://pelegm-networkx.readthedocs.io/en/latest/reference/generated/networkx.generators.random_graphs.fast_gnp_random_graph.html

Exercise 1

Please generate a random graph with the same number of nodes and approxmatly the same number of edges as the real-graph in Section 1.1.

```
# TODO: please fill in following code
import math
p =  num_edges / ((num_nodes*(num_nodes-1))/2)  # probability of forming an edge, use
gnp_graph = nx.fast_gnp_random_graph(n=num_nodes, p=p)
print('number of nodes: {}, number of edges: {}'.format(gnp_graph.number_of_nodes(), g
```

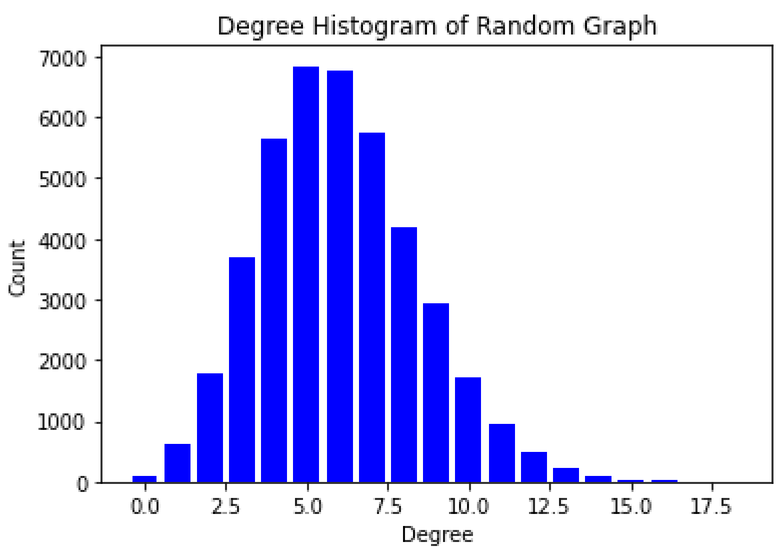
```
number of nodes: 41773, number of edges: 125488
```

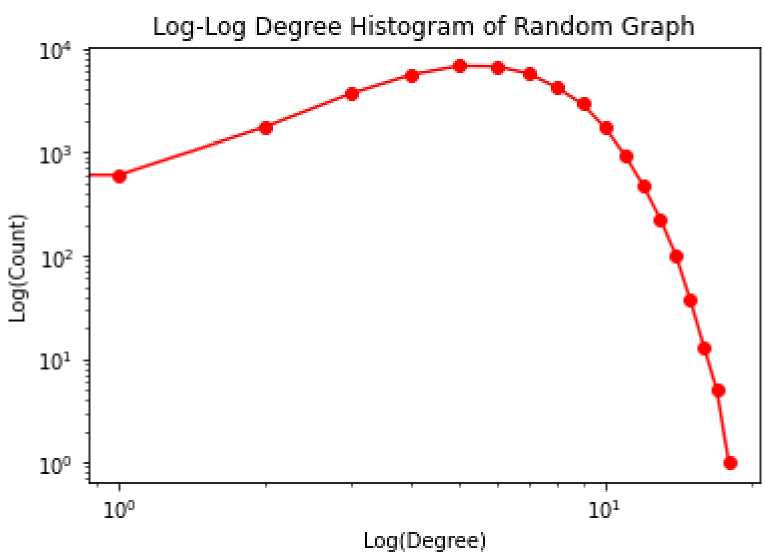
2.2 Degree Distribution of Random Graph

Exercise 2

Please call plot_degree_histogram to calculate the degree distribution

```
# TODO: call the function to plot the degree distribution
plot_degree_histogram(gnp_graph,'Random Graph')
```





2.3 Clustering Coefficients

```
print('average clustering coefficients is {}'.format(nx.average_clustering(gnp_graph))

    average clustering coefficients is 6.730702036282193e-05
```

Small-World Model

We will gerante a small world model with the same number of nodes, similar number of edges and similar clustering coefficients as the real-world graph in Section 1.1. We will then visualize its degree distribution and calculate its clustering coefficients. The function we use is nx.watts_strogatz_graph(n, k, p), where

- n: The number of nodes,
- k: Each node is connected to k nearest neighbors in ring topology (degree of the lattice),
- p: The probability of rewiring each edge

For more details, please refer to: in https://networkx.github.io/documentation/networkx-1.9/reference/generated/networkx.generators.random_graphs.watts_strogatz_graph.html.

Exercise 3

Please fill in the following code to generate a small world model with the same number of nodes, similar number of edges and similar clustering coefficients with the real-world graph in Section 1.1.

Obviously, we should set *n* as the number of nodes of the real graph in Section 1.1.

k should be an integer close to the average degree of the real-world graph.

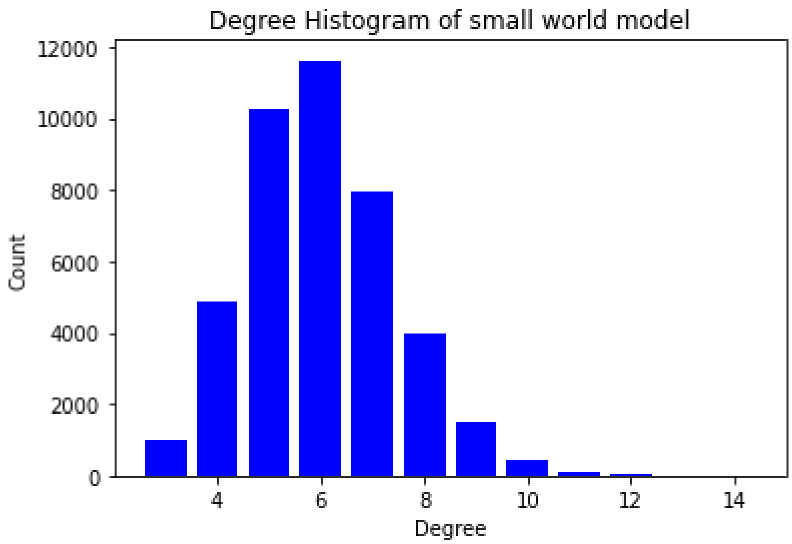
For simplicity, we set *p = 0.45*

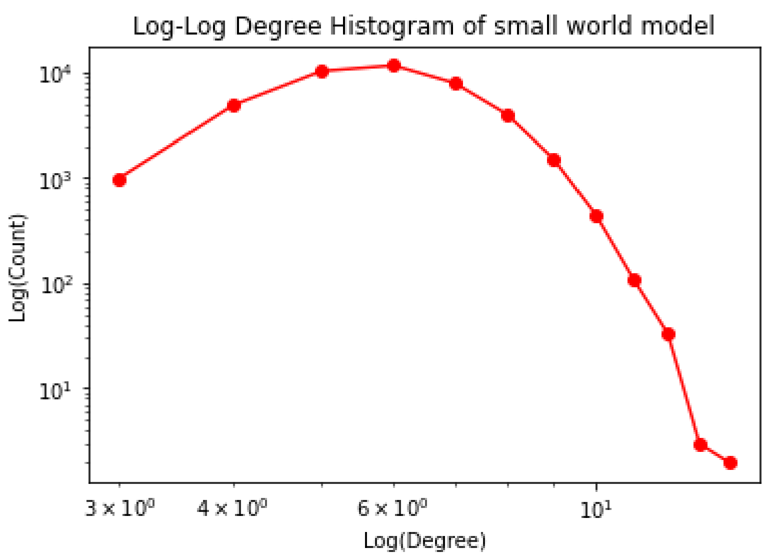
```
# TODO: please fill in the following codes
average_degree = sum([d for n, d in G.degree()])/num_nodes
small_world = nx.watts_strogatz_graph(n=num_nodes, k=round(average_degree), p=0.45)
print('number of nodes: {}, number of edges: {}'.format(small_world.number_of_nodes(),

    number of nodes: 41773, number of edges: 125319
```

Exercise 4: Degree Distribution of Small World Model

```
# TODO: Please fill in the following codes
plot_degree_histogram(small_world, 'small world model')
```





Exervise 5: Clustering Coefficients of Small World Model

```
# TODO: please calcualte the average clustering coefficient of the small world model
print('average clustering coefficients is {}'.format(nx.average_clustering(small_worl
```

```
average clustering coefficients is 0.10268819081927555
```

4 Preferrential Attachment Model

We will generate a scale-free graph with the same number of nodes and simialr number of edges with the real-world graph. We will then analyze its degree distribution.

4.1 Generate Scale-Free Graph with Preferrential Attachment Model

We can call barabasi_albert_graph(n, m, seed=None) to generate a scale-free graph, where

- n: The number of nodes,
- m: Number of edges to attach from a new node to existing nodes
- seed : int, optional, Seed for random number generator (default=None).

For more details of barabasi_albert_graph(n, m, seed=None), please visit: https://networkx.github.io/documentation/networkx-1.9/reference/generated/networkx.generators.random_graphs.barabasi_albert_graph.html

Since we want to generate a graph with the same number of nodes as the real graph in Section 1.1, and simialr number of edges with the real graph, we should set

- n to be the number of nodes of the real graph
- m as
$$m \approx \frac{k}{n}$$
so that the number of edges will be $m \times n \approx k$, where k is the number of edges of the real world graph

Exercise 6: Preferrential Attachment Model

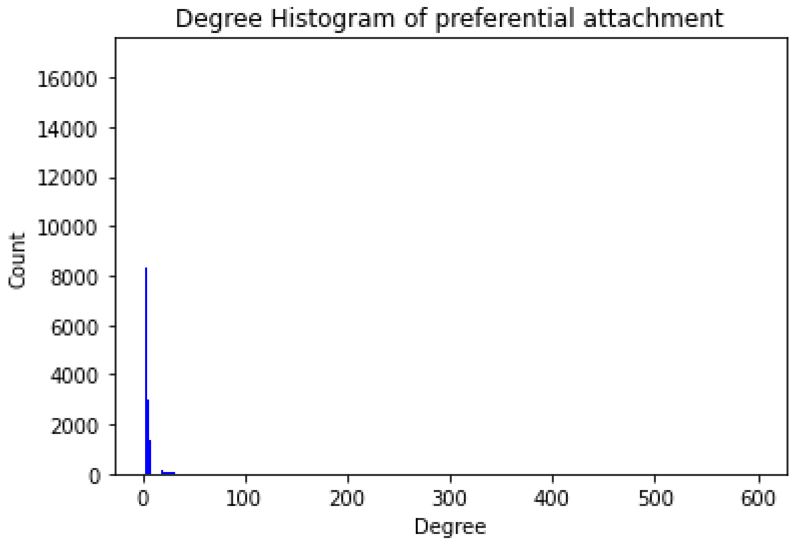
Please call nx.barabasi_albert_graph to generate a preferrential attachment graph with the same number of nodes and similar number of edges with the real-world graph in Section 1.1.

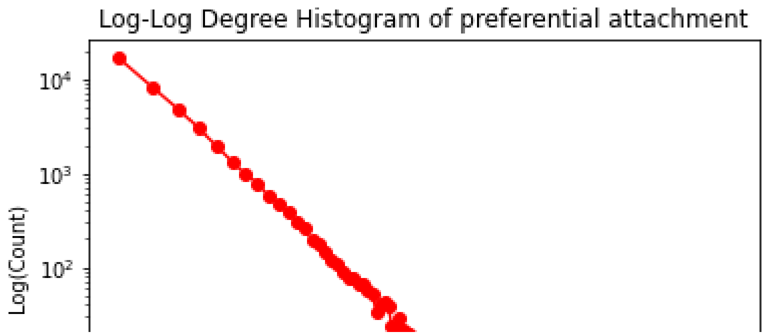
```
# TODO: Please fill in the code here
preferential_attachment = nx.barabasi_albert_graph(n=num_nodes, m=round((num_edges/nur
print('number of nodes: {}, number of edges: {}'.format(preferential_attachment.numbe
```

```
number of nodes: 41773, number of edges: 125310
```

4.2 Degree Distribution of Preferrential Attachment Model

```
# call the function to plot the degree distribution
plot_degree_histogram(preferential_attachment, 'preferential attachment')
```





4.3 Clustering Coefficients of Preferrential Attachment Model

```
print('average clustering coefficients is {}'.format(nx.average_clustering(preferentia
```

```
average clustering coefficients is 0.0015988403372000103
```

Fill In the Report and Submit

Now you have analyzed the real-world graph and simulated three graphs with network models, please fill in the Lab5_Network_Models_Report. Please submit Lab5_Network_Models_Report, this ipynb file and a pdf version to Canvas.

5. Powerlaw Cluster Graph (Optional, will not be graded)

Powerlaw Cluster Graph is essentially a Barabási–Albert (BA) growth model (also known as preferrential attachment model) with an extra step that each random edge is followed by a chance of making an edge to one of its neighbors too (and thus a triangle). The algorithm is given as

- Initial condition: To start with, the network consists of a small graph with m_0 vertices
- Growth: One vertex v with m edges is added at every time step.
- Preferential attachment (PA): Each edge of v is then attached to an existing vertex with the probability proportional to its degree, i.e. the probability for a vertex w to be attached to v is $P(w) = \frac{d_w}{\sum_{k \in V} d_k}$
- Triad formation (TF): If an edge between v and w was added in the previous PA step, then with probability p, add one more edge from v to a randomly chosen neighbor of w. If there remains no pair to connect, i.e., if all neighbors of w were already connected to v, do a PA step instead.

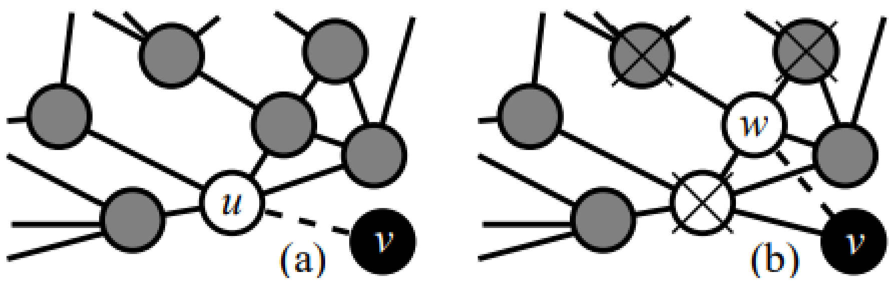


FIG. 1: *Preferential attachment and triad formation.* In the preferential attachment step (a) the new vertex v chooses a vertex u to attach to with a probability proportional to its degree. In the triad formation step (b) the new vertex v chooses a vertex w in the neighborhood of the one linked to in the previous preferential attachment step. $\times$ symbolizes "not-allowed to attach to" (either since no triad would be formed, or that an edge already exists).

This algorithm improves on BA in the sense that it enables a higher average clustering to be attained if desired.

5.1 Generate Scale-Free Graph with Powerlaw Cluster Graph

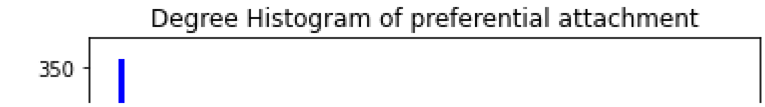
We can call powerlaw_cluster_graph(n, m, p, seed=None) to generate the powerlaw cluster graph where

- n: The number of nodes,
- m: Number of edges to attach from a new node to existing nodes
- Probability of adding a triangle after adding a random edge
- seed : int, optional, Seed for random number generator (default=None)

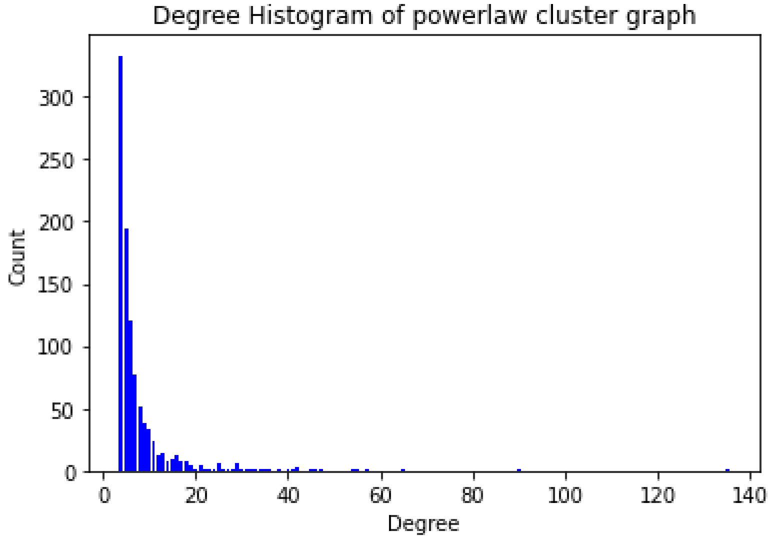
For more details, please refer to https://networkx.org/documentation/stable/reference/generated/networkx.generators.random_graphs.powerlaw_cluster_graph.html

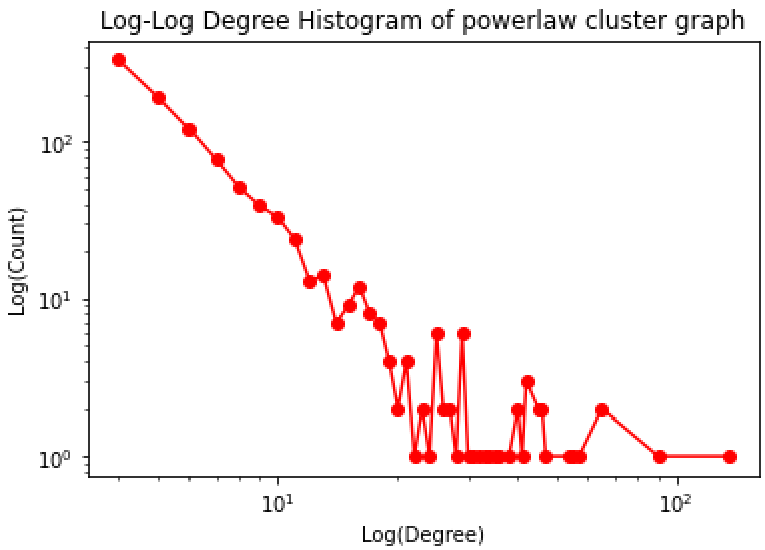
A comparison between preferrential attachment model and powerlaw cluster graph

```
# preferrential attachment model
preferential_attachment = nx.barabasi_albert_graph(n=1000, m=4)
plot_degree_histogram(preferential_attachment, 'preferential attachment')
print('average clustering coefficients is {}'.format(nx.average_clustering(preferentia
```




```
# powerlaw cluster graph
powerlaw_cluster_graph = nx.powerlaw_cluster_graph(n=1000, m=4, p=0.15)
plot_degree_histogram(powerlaw_cluster_graph, 'powerlaw cluster graph')
print('average clustering coefficients is {}'.format(nx.average_clustering(powerlaw_cl
```






```
average clustering coefficients is 0.09127439169692193
```

Exercise 7: Powerlaw Cluster Graph (Optional, will not be graded)

Please call nx.powerlaw_cluster_graph to generate a small world model with the same number of nodes, similar number of edges and similar clustering coefficient with the real-world graph in Section 1.1.

```
# TODO: Please fill in the code here
powerlaw_cluster_graph = nx.(n=?????????, m=?????????, p=??????)
```

```
print('number of nodes: {}, number of edges: {}'.format(powerlaw_cluster_graph.number_
```

5.2 Degree Distribution of Powerlaw Cluster Graph

```
# call the function to plot the degree distribution
plot_degree_histogram(powerlaw_cluster_graph, 'powerlaw_cluster_graph')
```

5.3 Clustering Coefficients of Powerlaw Cluster Graph

```
print('average clustering coefficients is {}'.format(nx.average_clustering(powerlaw_cl
```