The University of Michigan
Electrical Engineering & Computer Science
EECS 281: Data Structures and Algorithms
Winter 2024

FINAL EXAM
KEY 1 – ANSWERS
Thursday, April 25, 2024
8:00AM – 10:00AM

Uniqname: ____________ **Student ID:** ____________

Name: ____________

Uniqname of person to your left: ____________

Uniqname of person to your right: ____________

Honor Pledge:
"I have neither given nor received unauthorized aid on this examination, nor have I concealed any violations of the Honor Code."

Signature: ____________

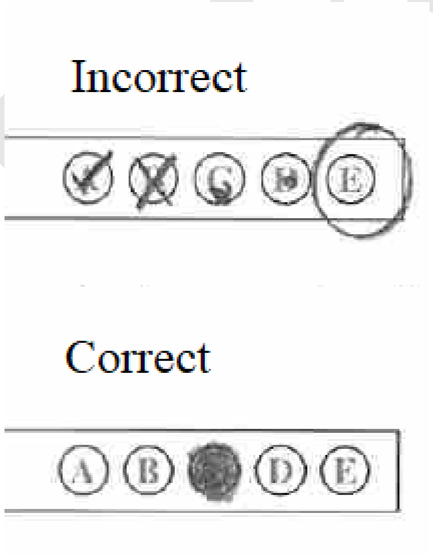
INSTRUCTIONS:

- The exam is closed book and notes except for one 8.5"x11" sheet of handwritten notes (both sides). All electronic device use is prohibited during the exam (calculators, phones, laptops, etc.).
- Print your name, student ID number, and uniqname **LEGIBLY**. Sign the Honor Pledge; this pledge applies to both the written and multiple choice sections. Your exam will not be graded without your signature.
- Record the **UNIQNAME** of the students to both your left and right. Write down `nullptr` if you are at the end of a row and there is no one on a given side.
- Record your **UNIQNAME** at the top of each odd-numbered page in the space provided. This will allow us to identify your exam if pages become separated during grading.
- This exam booklet contains **BOTH** the multiple choice and written portions of the exam. Instructions for each portion are provided at the beginning of their respective sections.
- **DO NOT** detach any pages from this booklet.
- There should be 22 pages in this book; if you are missing any pages, please let an instructor know.

Multiple Choice Portion [60 points]

MULTIPLE CHOICE INSTRUCTIONS:

- Select only **ONE** answer per multiple choice question.
- There are 24 multiple choice questions worth 2.5 points each.
- Record your choices for all multiple choice questions using the circles in the boxed area next to each question. Use a number 2 pencil, not a pen (so that you may change your answer). Fill the circle completely. There is no partial credit for multiple-choice questions. Make sure you bubble in the correct letter. See the example below for how to select your answer.



- There is no penalty for wrong answers: it is to your benefit to record an answer to each multiple-choice question, even if you're not sure what the correct answer is.
- The questions vary in difficulty — try not to spend too much time on any one question. Use your time wisely.
- When asked about memory complexity, consider all possible sources (stack and heap).

Record your answers in the bubbles next to each question.

1. Collision Resolution

Ⓐ Ⓑ [✓ Ⓒ] Ⓓ

You are given a hash table with size $M = 10$ and hash function $H(n) = (2n + 3) \mod M$. Collisions are resolved using **quadratic** probing. What would the table look like (with an X denoting an empty location) after inserting the following elements in the order given?

4, 10, 12, 7, 2

A) [X, 4, X, 10, X, X, X, 12, 7, 2]

B) [X, 4, 2, 10, X, X, X, 12, 7, X]

C) [X, 4, X, 10, X, X, 2, 12, 7, X]

D) [X, 4, X, 10, X, X, 2, 12, X, 7]

4 $\rightarrow$ index 1; 10 $\rightarrow$ 3; 12 $\rightarrow$ 7; 7 $\rightarrow$ 7, then 8; 2 $\rightarrow$ 7, then 8, 1, finally 6

2. Hash Table Applications

[✓ Ⓐ] Ⓑ Ⓒ Ⓓ

In which one of the following situations would you want to use a hash table?

A) You have a classroom full of students each with a 10-digit ID number, and you want to find them by their ID number.

B) You want to find the highest priority thread to execute, each with its own assigned priority.

C) You have a rectangular floor composed of square tiles, and you want to know the color of a tile given a set of coordinates.

D) You are given a set of names, and want to print them out in alphabetical order.

(B) there's no way to find the highest value in a hash table; (C) needs two indices (not 1; use a 2D vector); (D) there's no way to get alphabetical order from a hash table

Record your answers in the bubbles next to each question.

3. Tree ADT　　A B C [✓ D] E

What does the following function return?

```
int count(Node* curNode) {
  if (curNode == nullptr)
    return 0;
  else if (curNode->parent == nullptr)
    return 1 + count(curNode->left) + count(curNode->right);
  else if (curNode->left == nullptr && curNode->right == nullptr)
    return 1;
  else
    return count(curNode->left) + count(curNode->right);
} // count()
```

A) The number of nodes in the tree

B) The number of internal nodes

C) The number of external (leaf) nodes

D) The number of external (leaf) nodes plus the root

E) The depth of the tree

nullptr is 0, the root (no parent) is 1 + recursive, leaf nodes count as 1, and everything else is the sum of the children recursively

4. *baugh Tree　　A B C [✓ D] E

Suppose there existed something called a *baugh-Tree. A *baugh-Tree can be implemented similar to the **array**-based binary tree covered in lecture, except that its internal nodes can have up to six children. What are the best- and worst-case **space** complexities, respectively, of a *baugh-Tree with n nodes?

A) $\Theta(1)$, $\Theta(6n)$

B) $\Theta(n)$, $\Theta(2^n)$

C) $\Theta(2^n)$, $\Theta(6^n)$

D) $\Theta(n)$, $\Theta(6^n)$

E) $\Theta(6^n)$, $\Theta(6^n)$

If the tree is complete, it fits in the array exactly ($\Theta(n)$); a 'stick' going right would need 1 element at depth 1, 6 elements at depth 2, 36 elements at depth 3, etc.

Record your answers in the bubbles next to each question.

5. Tree Traversal Ⓐ [✓ B] Ⓒ Ⓓ Ⓔ

Given the following preorder and inorder traversals of a binary tree, what is the postorder traversal of this same binary tree?

- Preorder: 11, 19, 31, 7, 10
- Inorder: 31, 19, 11, 10, 7

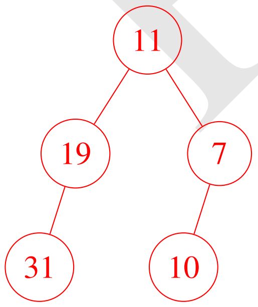
A) 11, 19, 7, 31, 10

B) 31, 19, 10, 7, 11

C) 19, 31, 10, 7, 11

D) 31, 10, 7, 19, 11

E) 10, 7, 31, 19, 11



Reconstruct the tree, then perform the postorder traversal

6. Binary Search Tree Complexity Ⓐ Ⓑ Ⓒ [✓ D] Ⓔ

Suppose you have a binary search tree, where h represents the tree's height and n represents the number of nodes. What is the worst-case complexity of a search operation on this tree in terms of h and/or n?

A) $\Theta(n \log h)$

B) $\Theta(n^2)$

C) $\Theta(\log n)$

D) $\Theta(h)$

E) $\Theta(n \log_n h)$

The worst-case complexity of search, insert and remove is always the height of the tree, regardless of whether it is balanced

Record your answers in the bubbles next to each question.

7. Binary Search Tree vs. Binary Tree [✓ A] B C D E

Given **pointer**-based representations of both, what **can** be said about a binary search tree that **cannot** be said about a binary tree?

A) The inorder traversal of a tree is always a sorted list.

B) The worst-case space complexity of a tree is $O(2^n)$.

C) A tree balances itself when a single branch becomes uneven.

D) The worst-case time complexity of inserting an element is $O(n)$.

E) It is always a complete binary tree.

(A) is true only of BST; (B) is only true for an array representation; (C) is only AVL trees; (D) can be tree whether it is a binary tree or a BST; (E) depends on the structure, not the rule about where to place values

8. Inorder Successor [✓ A] B C D E

Suppose that in a binary search tree, when a node with two children is deleted, it is replaced by its inorder successor. Given a pointer to the node to be deleted, what is the time complexity of finding the inorder successor in the average case, if the tree contains n nodes?

A) $\Theta(\log n)$

B) $\Theta(n)$

C) $\Theta(n \log n)$

D) $\Theta(n^2)$

E) $\Theta(n!)$

In the average case the tree is somewhat balanced (not a stick), so the height of the tree is $\Theta(\log n)$, and the time to find the inorder succcessor is at worst the tree height

Record your answers in the bubbles next to each question.

9. AVL Trees [✓ A] B C D E

Which of the following statements about AVL trees is **TRUE**?

A) Optimally re-balancing an AVL tree after a deletion can take more than $O(1)$ time.

B) For an insert operation, up to three rotations are needed.

C) Searching an AVL tree can take longer than $O(\log n)$.

D) Given an n-node binary search tree that has never been balanced, the BST can be made into a balanced AVL tree using $O(\log n)$ rotations.

E) A tree is AVL balanced if and only if the absolute difference in heights of the root's children is less than 2.

One rotation (single or double) takes $O(1)$ time, BUT you have to traverse each level of the tree from the point of the deletion to the root (which could be $O(\log n)$ levels away).

10. Sorting A B C [✓ D] E

Given an arbitrary array of data elements, we can use an AVL tree to sort them. What is the complexity of a sorting algorithm implemented using an AVL tree?

A) $\Theta(n)$ best case, $\Theta(n \log n)$ worst case

B) $\Theta(n \log n)$ best case, $\Theta(n^2)$ worst case

C) $\Theta(n)$ in all cases

D) $\Theta(n \log n)$ in all cases

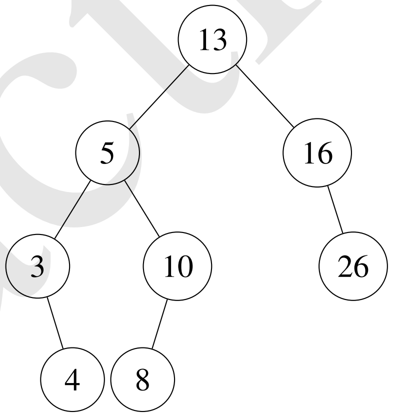
E) None of the above

Every insertion into an AVL tree takes $\Theta(\log n)$ time (once going downward to find the insertion point, and then once more going back upwards to find the rotation point, if any), and there are n insertions needed.

Record your answers in the bubbles next to each question.

11. Insertion/Deletion Ⓐ [✓ B] Ⓒ Ⓓ Ⓔ

Given the following AVL tree:



Perform the following operations:

1. Delete 13 (replace with its inorder successor)
2. Insert 9

What are the left and right children of “16” after performing the operations?

A) 8, 26

B) 9, 26

C) 10, 26

D) `nullptr`, 26

E) `nullptr`, `nullptr`

Deleting 13 (and replacing it with 16) makes the tree unbalanced, requiring RR(16) to fix. Inserting 9 (as the right child of 8) forces a double rotation: RL(8) followed by RR(10).

Record your answers in the bubbles next to each question.

12. Algorithm Families – Password Trouble Ⓐ Ⓑ Ⓒ [✓ Ⓓ]

You have forgotten your Wolverine Access password. Unfortunately, you used a random generator to create your password, so you have no idea what the password could be. The only feedback you get is whether what you typed is right or wrong. Which algorithm family would be best suited for finding your password?

A) Branch and Bound

B) Dynamic Programming

C) Divide and Conquer

D) Brute Force

A false result on one password gives you no information about what to try next

13. Graphs Ⓐ [✓ Ⓑ] Ⓒ Ⓓ

Use the adjacency matrix given below. Starting from V1, determine a possible DFS sequence.

$$\begin{array}{c|cccccc|}
 & \textbf{V1} & \textbf{V2} & \textbf{V3} & \textbf{V4} & \textbf{V5} & \textbf{V6} \\
\textbf{V1} & 0 & 1 & 1 & 0 & 1 & 0 \\
\textbf{V2} & 1 & 0 & 0 & 1 & 0 & 0 \\
\textbf{V3} & 1 & 0 & 0 & 0 & 1 & 1 \\
\textbf{V4} & 0 & 1 & 0 & 0 & 1 & 0 \\
\textbf{V5} & 1 & 0 & 1 & 1 & 0 & 1 \\
\textbf{V6} & 0 & 0 & 1 & 0 & 1 & 0
\end{array}$$

Note: the order that adjacent vertices is visited is unspecified, you must determine which order is possible.

A) V1, V2, V3, V4, V5, V6

B) V1, V2, V4, V5, V6, V3

C) V1, V3, V5, V2, V4, V6

D) V1, V3, V4, V6, V5, V2

Starting at V1, you can go to V2, V3, or V5 next. If you go to V2, the only unvisited vertex it leads to is V4, which eliminates (A), and V1, V5 doesn't match any options, leaving V2. Our current potential sequence is V1, V2, V4. From V4, you can reach V2 (already visited) or V5. From V5, you can visit V3 or V6. Since V6 leads to V3, choice (B) is possible.

Record your answers in the bubbles next to each question.

14. MST Ⓐ [✓ Ⓑ] Ⓒ Ⓓ Ⓔ

Given the distance matrix below, what is the total weight of its minimum spanning tree?

$$\begin{bmatrix} 0 & 4 & 10 & 3 & 2 \\ 4 & 0 & 9 & 5 & 6 \\ 10 & 9 & 0 & 8 & 7 \\ 3 & 5 & 8 & 0 & 1 \\ 2 & 6 & 7 & 1 & 0 \end{bmatrix}$$

A) 13

B) 14

C) 15

D) 16

E) 18

Let's call the vertices V1 through V5, as in the previous problem, and solve this via Kruskal. When you pick an edge, you could write down your running total of edge length, and cross out both copies of that value. When you find an edge that would produce a cycle, cross out both copies but don't add to the running total. You can solve this greedily by choosing edge length 1, which connects V4 to V5 (total cost 1). Then pick length 2, which connects V1 to V5 (total cost 3). Edge length 3 would connect V1 to V4, but that would produce a cycle. Edge length 4 connects V1 to V2 (total cost 7). Length 5 would connect V2 to V4 (cycle). Length 6 would connect V2 to V5 (cycle). Length 7 connects V3 to V5 (total cost 14).

15. Algorithm Families – Cookies Ⓐ [✓ Ⓑ] Ⓒ Ⓓ Ⓔ

As you leave the dining hall, you decide to grab some cookies to go. You have conveniently brought some containers for storing cookies. Dining hall cookies come in three sizes: 3 cm, 4 cm, and 5 cm (in radius). All cookies have the same thickness. Your containers are cylindrical, so you can only store cookies by stacking them on top of each other, and each container has a radius of 5 cm or less. What is the best method for taking cookies in order to maximize the total volume of cookies taken?

A) Brute Force

B) Greedy

C) Dynamic Programming

D) Backtracking

E) Branch and Bound

Since you can never place any cookies side by side, you must stack them on top of each other. Taking a 5 cm cookie doesn't prevent you from taking more cookies, so the 5 cm cookie will have the largest volume. Take as many of those as you can fit, then proceed to 4 cm cookies, and finally 3 cm cookies. Stop when your containers are full, or there are no more cookies.

Record your answers in the bubbles next to each question.

NOTE: Both questions on this page use the following information.

Suppose you are given a maze represented as a graph with a start node, an end node, and a set of "black" (inaccessible) and "white" (accessible) nodes, with edges between them. The goal is to find a simple path from the start to end, going through only accessible nodes.

NOTE: This actual question is slightly poor because it's under-specified. Is the graph sparse or dense? Do we have an adjacency matrix or adjacency list? We should make the same assumptions for both questions. Let's assume worst case, which means approaching completely connected, which implies an adjacency matrix. Let's also assume we don't write a terrible solution, and never visit a node more than once.

16. Backtracking

Ⓐ [✓ B] Ⓒ Ⓓ

Which of the following represents the tightest-bound complexity of solving this problem using a recursive backtracking algorithm, where n is the number of nodes?

A) $\Theta(n)$

B) $\Theta(n^2)$

C) $\Theta(n^3)$

D) $\Theta(n!)$

This is basically Project 1 with a stack (recursive uses a call stack). Since every node might be connected to up to n other nodes, and we consider every node once, we reach $O(n^2)$.

17. Searching

Ⓐ [✓ B] Ⓒ Ⓓ

Which of the following represents the tightest-bound complexity of solving this problem using BFS, where n is the number of nodes?

A) $\Theta(n)$

B) $\Theta(n^2)$

C) $\Theta(n^3)$

D) $\Theta(n!)$

BFS will find the minimum path length (in terms of number of edges traversed), but the amount of work done is not different from DFS.

Record your answers in the bubbles next to each question.

18. Computing Sequence A B [✓ C] D

Consider the following recurrence relation:

- $F(0, k) = k$
- $F(n, k) = F(n-1, k+1) + F(n-1, k)$

Using dynamic programming, what is the best time complexity for computing $F(n, n)$?

A) $\Theta(\log n)$

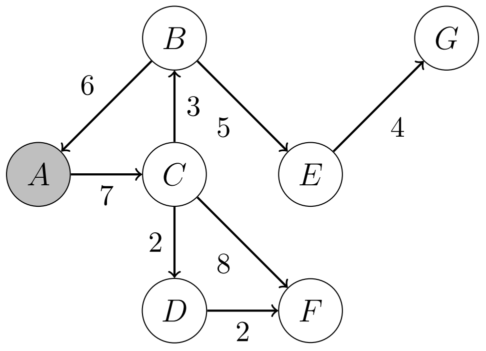
B) $\Theta(n)$

C) $\Theta(n^2)$

D) $\Theta(2^n)$

Notice that we need a memo of size n by n, and at most we need to fill in every value in that 2D memo, calculating each location exactly once.

19. Dijkstra's Shortest Path A [✓ B] C D E



Starting at vertex A and trying to find the shortest path to G, which of the following progressions shows how the vertices are added to our set?

A) A, B, D, F, B, E, G

B) A, C, D, B, F, E, G

C) A, C, B, D, F, E, G

D) A, C, D, B, E, F, G

E) A, C, B, E, G

Starting at A, the only outgoing edge leads to C (total cost 7); this eliminates choice (A). We could next visit D (total cost 9) or B (total cost 10). Choosing the lower cost, D is next; this eliminates choices (C) and (E), leaving only (B) and (D) as possible answers. Our next choice could be F at a total cost of 11, or B at a total cost of 10; we pick the smaller, which is consistent with both remaining possible answers. Next we can choose E (total cost 11) or E (total cost 15); we choose the lower cost, which eliminates option (D), leaving (B) as the only possible valid answer.

Record your answers in the bubbles next to each question.

20. Algorithm Families – Procrastination Ⓐ Ⓑ [✓ Ⓒ] Ⓓ Ⓔ

Your programming project is due in 4 hours, and you haven't even started! You have a few options: writing a test case gives you 2 points and takes between 10 and 15 minutes, writing a function gives you 7 points and takes between 30 and 50 minutes, working on the driver gives you 20 points and takes 2 hours. There are 6 functions and 10 test cases to write, and the times vary based on the specific test case or function. Given that starting early is no longer an option, what is the best approach for optimizing your project score?

A) Brute Force

B) Greedy

C) Dynamic Programming

D) Backtracking

E) Branch and Bound

This is basically the knapsack problem, with time representing the limiting factor, and points being the value. It's a 0-1 knapsack because getting something like a test file half written isn't valid. We know that greedy isn't guaranteed to find an optional solution, so dynamic programming is the solution.

21. Counting Paths Ⓐ [✓ Ⓑ] Ⓒ Ⓓ

In a certain game played on a grid, a game piece can either move 1 cell to the **right**, 1 cell **up**, or 1 cell **diagonally** (up and right) on a turn. The bottom-left corner is location (0, 0). For example, from cell (0, 0) to cell (2, 1) there are 5 paths: RRU, RUR, URR, DR, and RD (U = up, R = right, and D = diagonal). Using dynamic programming, what is the time complexity of counting the number of paths from $(0, 0)$ to (x, y)?

A) $\Theta(x + y)$

B) $\Theta(xy)$

C) $\Theta(x^2y^2)$

D) $\Theta(3^{x+y})$

The memo will be of size x by y, and every location is calculated exactly once.

Record your answers in the bubbles next to each question.

22. Complexity and Preprocessing Ⓐ Ⓑ Ⓒ [✓ D]

Suppose you're given an array such as {40, 20, 10, 30} and allowed to do $\Theta(n)$ preprocessing time with $\Theta(n)$ memory. After the preprocessing step, you will receive q queries asking for the sum of the elements in some interval, such as [1, 4) whose sum in the above example is 60. What is the worst-case **time** complexity of the best algorithm executing q queries (**not** including the time to do preprocessing)?

A) $O(qn)$

B) $O(q \log n)$

C) $O(qn^2)$

D) $O(q)$

In preprocessing, form an array of cumulative sums, for instance $\{40, 60, 70, 100\}$. Then for each query, simply subtract one cumulative sum from another (such as $100 - 40 = 60$). Thus each query is $O(1)$, making q queries $O(q)$.

Record your answers in the bubbles next to each question.

23. Dijkstra's Algorithm Ⓐ Ⓑ [✓ Ⓒ] Ⓓ

Which of the following is **FALSE** about Dijkstra's algorithm?

A) Dijkstra's algorithm uses a Greedy approach to solving its problem.

B) Dijkstra's algorithm cannot be used on an input graph with negative edge weights.

C) Dijkstra's algorithm only finds the shortest path between two input vertices.

D) Dijkstra's algorithm is capable of achieving a time complexity $O(|E|\log|V|)$.

Dijkstra's Algoritm solves the "Single-Source Shortest Path" problem. You may form the question asking about a start and end vertex, but whichever vertex you set to a distance of 0, you find shortest paths involving all possible other vertices with that distance 0 vertex.

24. Greedy Algorithms Ⓐ [✓ Ⓑ] Ⓒ Ⓓ

You are writing room scheduling software for the University. Of course, everyone's favorite room is 1670 BBB, so all instructors want to hold their classes there. Each of the classes has a firm start and end time. Your algorithm needs to find the maximum number of classes that can be held in the room without any conflicts between classes. Which of the following algorithms produces the optimal solution?

Note: classes can start any time and have any length.

Hint: create some examples.

A) Greedily choose classes with the earliest start time.

B) Greedily choose classes with the earliest end time.

C) Greedily choose classes with the fewest conflicts with other classes.

D) Greedily choose classes that take the shortest time (`endTime - startTime`).

The only way to really see this is to run through some examples. This is a well-known optimization problem, and you should learn to recognize it.

Written Portion [40 points]

WRITTEN PORTION INSTRUCTIONS:

- Please write your code legibly in the space provided. Solutions that are difficult to read may receive fewer points.
- Use the back of the question page to work out solutions for each problem and then rewrite your final answer in the space provided.
- The directions for each problem will specify which STL algorithms/functions may be used in your solution.
- Solutions that exceed the allowed line count will lose one (1) point per line over the limit.
- Partial credit will be awarded for partial solutions.
- Errors in program logic or syntax may result in a reduced score.
- Be sure to consider all possible sources of memory (stack and heap) and time complexity.

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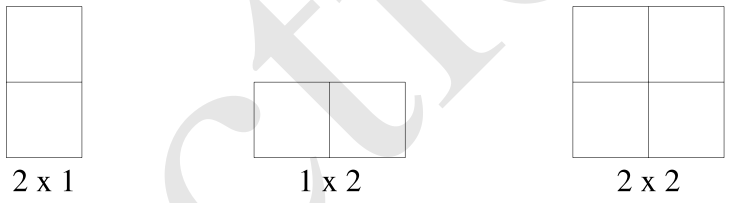
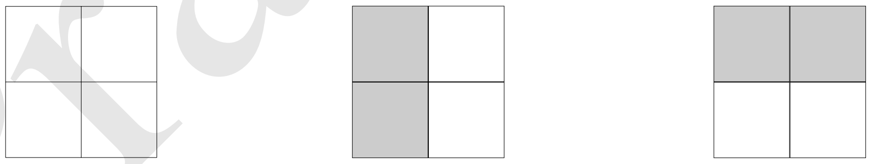
25. Dynamic Programming: Tiling [15 Points]

You are given a 2 x n board ($n > 0$) You are interested in the number of **distinct** ways to tile the given board using tiles of dimensions (2 x 1), (1 x 2) and (2 x 2) as shown below:



For example, for a 2 x 2 (i.e., $n = 2$) board, there are 3 ways:


Using one 2x2 tile Using two 2x1 tiles Using two 1x2 tiles

Requirements: Your solution must be $O(n)$ time. You may use up to $O(n)$ auxiliary space.

Implementation: Use the back of this page as a working area, then rewrite **neatly** on the front. Limit: 12 lines of code (points deducted if longer). You may use any C, C++, or STL function, algorithm, or data structure that you wish.

```
int number_of_tilings(int n) {
    // n is guaranteed to be > 0
    // This is the bottom-up solution
    vector<int> memo(n + 1);
    memo[0] = 1;  // Base case: one way to not pick any tiles
    memo[1] = 1;  // Base case: one way to tile a size 1 board
    for (int i = 2; i <= n; i++)
        memo[i] = memo[i - 1] + 2 * memo[i - 2];

    return memo[n];
}  // O(n) Time. O(1) space if only previous two results are maintained
   // We could ask to find answer for all i <=n and return a vector.

Recurrence (write this on the back BEFORE coding):
Let f(i) = number of ways to tile the first i columns
f(i) = f(i-1) + 2 * f(i-2)
    Take all solutions till i-1 and add one 2x1 tile (vertical) on ith column
    Take all solutions till i-2 and add two 1x2 tiles (horizontal) across i-1 & i
    Take all solutions till i-2 and add one 2x2 tile across i-1 & i

// Top-down helper
int tilings_helper(vector<int> &memo, int i) {
    if (i <= 1)      // Base cases
        return 1;
    if (memo[i] > 0) // Value is already known
        return memo[i];
    memo[i] = tilings_helper(memo, i - 1) + 2 * tilings_helper(memo, i - 2);
    return memo[i];
}
```

```
// Top-down starter function
int number_of_tilings_TD(int n) {
    // n is guaranteed to be > 0
    vector<int> memo(n + 1);

    return tilings_helper(memo, n);
}
```

26. Programming: Searching in a Tree [10 Points]

A *set* is a data structure that can efficiently (less than $O(n)$ time) see if it contains a given element. Using the following definition of a binary tree node:

```
struct Node {
  int val;
  Node *left;
  Node *right;
};
```

Implement the `exists()` function on a BST-based set.

Requirements: In the average case, your solution must be $O(\log n)$, and you may use up to $O(\log n)$ auxiliary space.

Implementation: Use the back of this page as a working area, then rewrite **neatly** on the front. Limit: 15 lines of code (points deducted if longer). You may use any C, C++, or STL function/algorithm that you wish.

```
bool exists(Node *node, int val) {
    // Recursive solution
    if (node == nullptr)
        return false;
    if (node->val == val)
        return true;
    if (val < node->val)
        return exists(node->left, val);
    else
        return exists(node->right, val);
    return false;  // not strictly needed

    // Iterative solution
    Node *cur = node; // Could just use node
    while (cur != nullptr) {
        if (val == cur->val)
            return true;
        if (val < cur->val)
            cur = cur->left;
        else
            cur = cur->right;
    } // while
    return false;
} // exists()
```

This page is intentionally left blank.
You may use this page as working space.

27. Programming: Searching in a Hash Table [15 Points]

A *set* is a data structure that can efficiently (less than $O(n)$ time) see if it contains a given element. Given the following definition of a hash table structure:

```
enum class Status { Empty, Occupied, Deleted };
struct HashTable {
  std::vector<int> buckets;
  std::vector<Status> status;
};
```

Suppose this hash table implements the quadratic probing mechanism for collision resolution. When hashing an integer value `val`, the hash function is simply `val % M`, where `M` represents the size of the hash table. Implement the `exists()` function for a set based on a hash table.

Requirements: Your solution must implement quadratic probing. Your solution must be $O(n)$ time. You may use $O(1)$ auxiliary space.

Implementation: Use the back of this page as a working area, then rewrite **neatly** on the front. Limit: 16 lines of code (points deducted if longer). You may use any C, C++, or STL function/algorithm that you wish.

```
bool exists(const HashTable &tbl, int val) {
    const size_t M = tbl.buckets.size(); // not needed, but easier to type
    size_t start_index = val % M;

    for (size_t j = 0; j < M; ++j) {
        size_t index = (start_index + j * j) % M;
        if (    tbl.status[index] == Status::Occupied
             && val == tbl.buckets[index])
            return true;

        if (tbl.status[index] == Status::Empty)
            return false;
    } // for

    return false;
} // exists()
```

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You may use this page as working space.