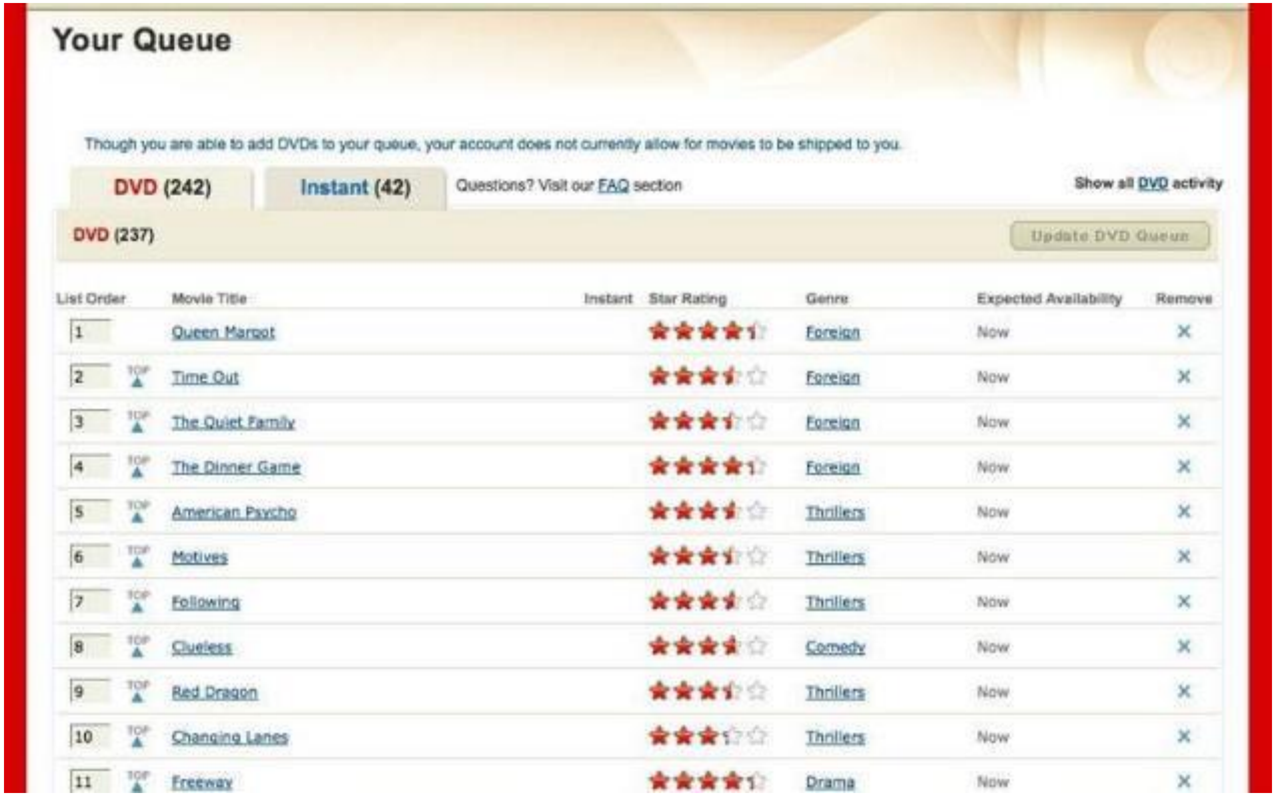
Lecture 2
Stacks and Queues



EECS 281: Data Structures & Algorithms

Data Structures and Abstract Data Types

Data Structures & Algorithms

Data Structures and ADTs

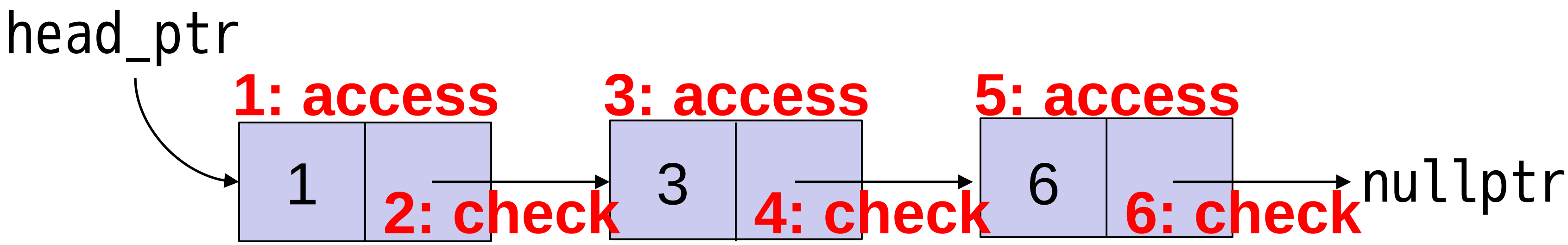
- Need a way to store and organize data in order to facilitate access and modifications
- An **abstract data type (ADT)** combines data with valid operations and their behaviors on stored data
  - e.g., insert, delete, access
  - ADTs define an interface
- A **data structure** provides a concrete implementation of an ADT

Measuring Performance

- Several design choices for implementing ADTs
  - Contiguous data (arrays or vectors)
  - Connected data (pointers or linked lists/trees)
- Runtime speed and size of data structure
  - How much <u>time</u> is needed to perform an operation? (count number of steps)
  - How much <u>space</u> is needed to perform an operation? (count size of data and pointers/metadata)
  - How does size/number of inputs affect these results? (constant, linear, exponential, etc.)
- We formalize performance measurements with **complexity analysis**

Analysis Example

- How many operations are needed to insert a value at the end of this singly-linked list?



- Can you generalize this for a list with ***n*** elements?

7: Create Node
8: Insert Value
9: Update Pointer

2*n* + 3

<u>Linear</u> function is important—coefficients and constants don't matter much

Choosing a Data Structure for a Given Application

- What to look for
  - The right operations (e.g., add_elt, remove_elt)
  - The right behavior (e.g., push_back, pop_back)
  - The right trade-offs for runtime complexities
  - Memory overhead
- Potential concern
  - Limiting interface to avoid problems (e.g., no insert_mid)
- Examples
  - **Order tracking at a fast-food drive-through** (pipeline)
  - **Interrupted phone calls to a receptionist**
  - **Your TODO list**

Data Structures and Abstract Data Types

Data Structures & Algorithms

Basic Containers: Stack

Data Structures & Algorithms

Stack ADT: Interface

- Supports insertion/removal in LIFO order
  - Last In, First Out

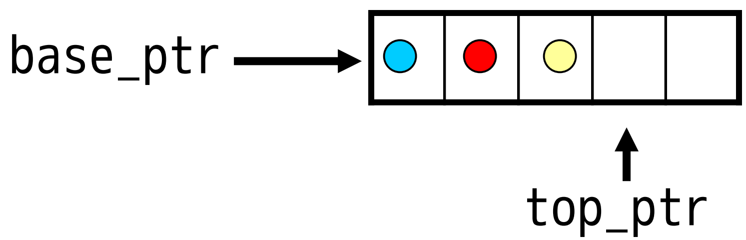
Method	Description
`push(object)`	Add object to top of the stack
`pop()`	Remove top element
`object &top()`	Return a reference to top element
`size()`	Number of elements in stack
`empty()`	Checks if stack has no elements

Examples

- Web browser's "back" feature
- Text editor's "Undo" feature
- Function calls in C++

Stack: Implementation – Array/Vector

Keep a pointer (`top_ptr`) just past the last element

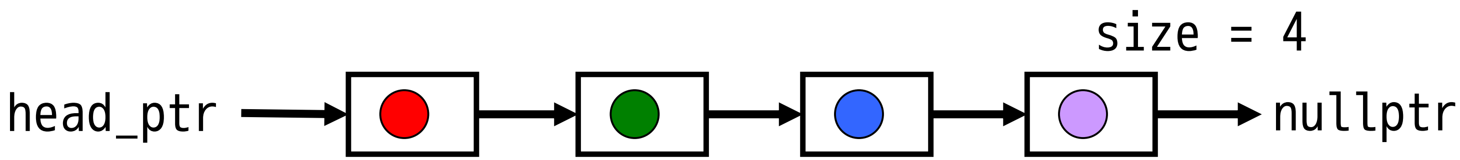


Method	Implementation
`push(object)`	1. If needed, allocate a bigger array and copy data 2. Add new element at top_ptr, increment top_ptr
`pop()`	Decrement `top_ptr`
`object &top()`	Dereference `top_ptr - 1`
`size()`	Subtract `base_ptr` from `top_ptr` pointer
`empty()`	Check if `base_ptr == top_ptr`

How many steps/operations for each method?

Stack: Implementation – Linked List

Singly-linked is sufficient



Method	Implementation
`push(object)`	Insert new node at `head_ptr`, increment size
`pop()`	Delete node at `head_ptr`, decrement size
`object &top()`	Dereference `head_ptr`
`size()`	Return `size`
`empty()`	Check if `size == 0` or `head_ptr == nullptr`

*Alternative approach: eliminate `size`, count nodes each time

How many steps/operations for each method?

Is an array or linked list more efficient for stacks?

Stack: Which Implementation?

Method	Array/Vector	Linked List
`push(object)`	Constant (linear when resizing vector)*	Constant
`pop()`	Constant	Constant
`object &top()`	Constant	Constant
`size()`	Constant	Constant (with tracked size)
`empty()`	Constant	Constant

*Averages out to constant with many pushes (amortized constant)

- The asymptotic complexities of each are similar
- The constant factor attached to the complexity is lower for vector
  - Constant number of operations, but there is "less" to do
  - The linked list must allocate memory for each node individually!
- The linked list also has higher memory overhead
  - i.e. Pointers between nodes as well as the actual data payload

STL Stacks: std::stack<>

- Code: `#include <stack>`
- You can choose the underlying container
- All operations are implemented generically on top of the given container
  - No specialized code based on given container

	Stack
Default Underlying Container	`std::deque<>`
Optional Underlying Container	`std::list<>` † `std::vector<>`

† `std::list<>` is a doubly-linked list

Basic Containers: Stack

Data Structures & Algorithms

Basic Containers: Queue

Data Structures & Algorithms

Queue ADT: Interface

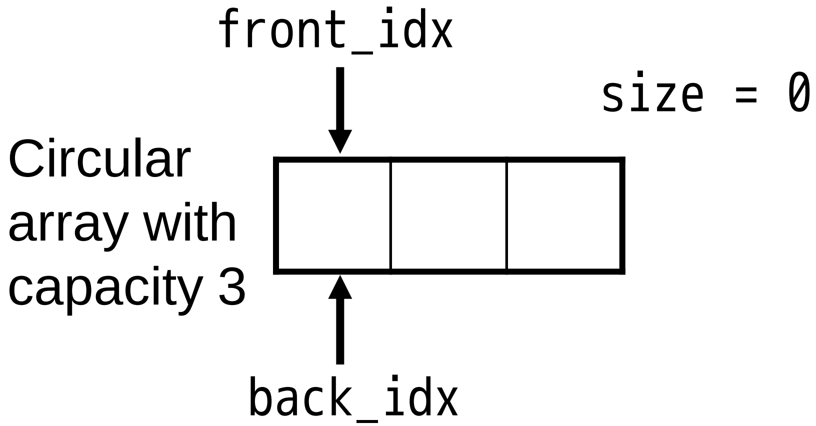
- Supports insertion/removal in FIFO order
  - First In, First Out

Method	Description
`push(object)`	Add `object` to back of queue
`pop()`	Remove element at front of queue
`object &front()`	Return reference to element at front of queue
`size()`	Number of elements in queue
`empty()`	Checks if queue has no elements

Examples

- Waiting in line for lunch
- Adding songs to the end of a playlist

Queue: Implementation – Circular (Ring) Buffer

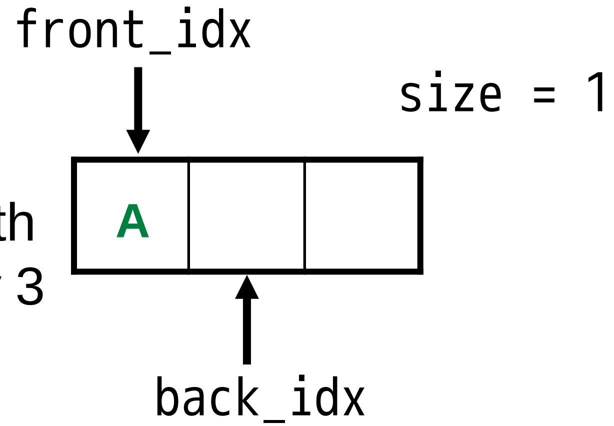


Event Sequence

1. `back_idx == front_idx` since array is empty

Queue: Implementation – Circular Buffer

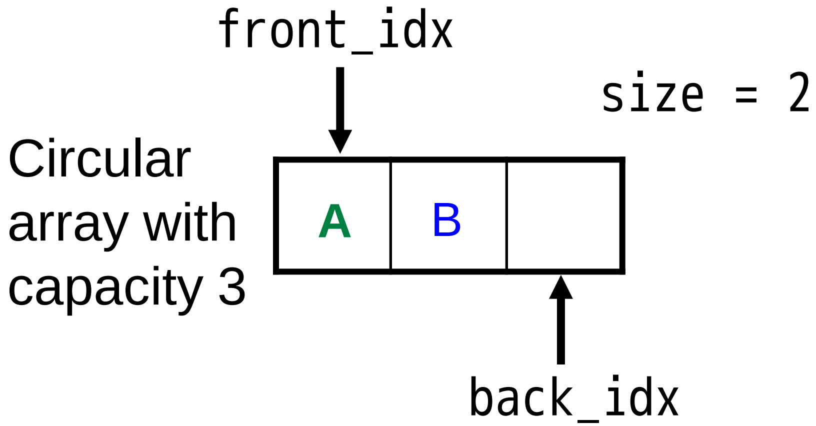
Circular array with capacity 3



Event Sequence

1. `back_idx == front_idx` since array is empty
2. push A

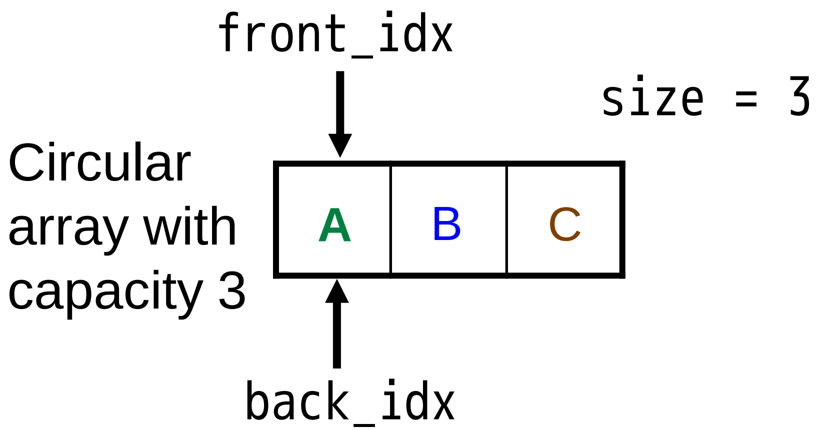
Queue: Implementation – Circular Buffer



Event Sequence

1. `back_idx == front_idx` since array is empty
2. push A
3. push B

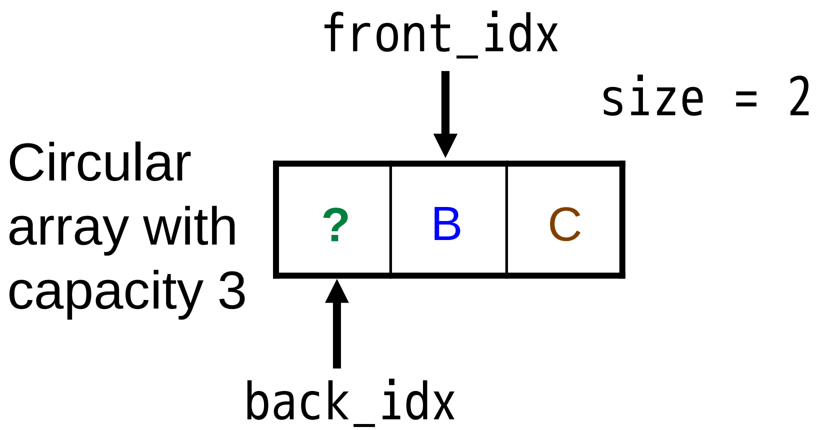
Queue: Implementation – Circular Buffer



Event Sequence

1. `back_idx == front_idx` since array is empty
2. push A
3. push B
4. push C

Queue: Implementation – Circular Buffer

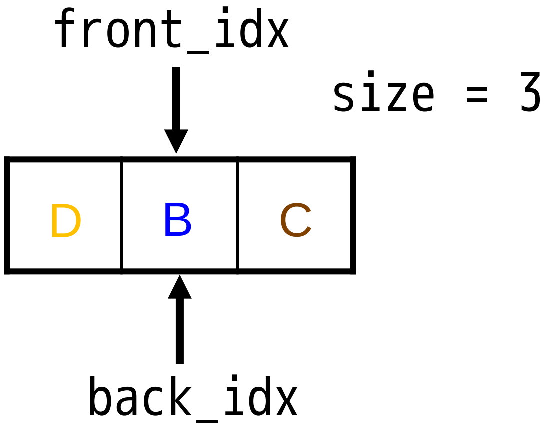


Event Sequence

1. `back_idx == front_idx` since array is empty
2. push A
3. push B
4. push C
5. pop (A)

Queue: Implementation – Circular Buffer

Circular array with capacity 3

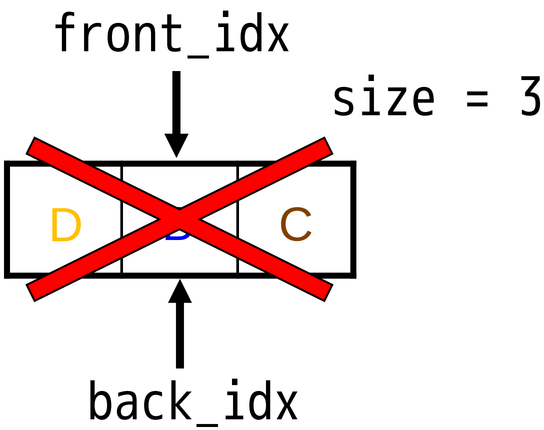


Event Sequence

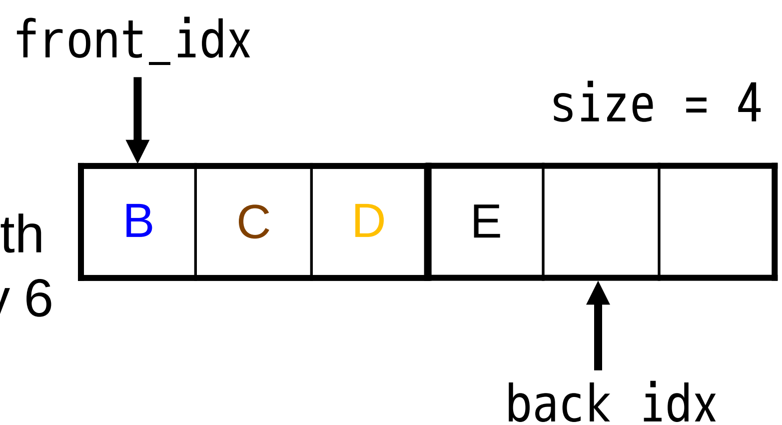
1. `back_idx == front_idx` since array is empty
2. push A
3. push B
4. push C
5. pop (A)
6. push D

Queue: Implementation – Circular Buffer

Circular array with capacity 3



Circular array with capacity 6



Event Sequence

1. `back_idx == front_idx` since array is empty
2. push A
3. push B
4. push C
5. pop (A)
6. push D
7. push E (need to allocate more memory)*

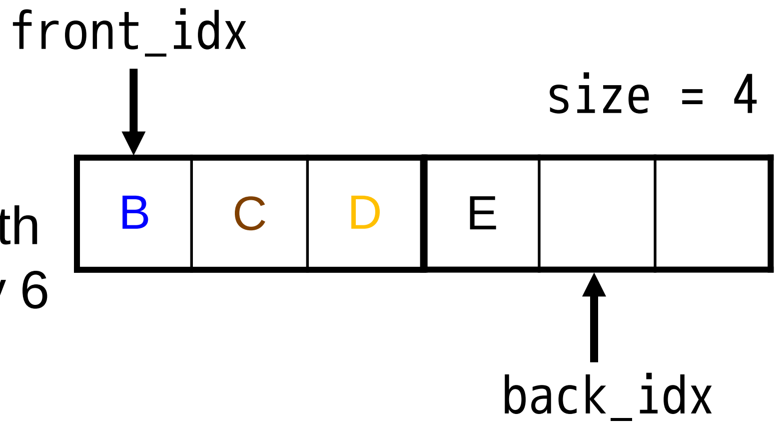
* When allocating more memory, it is common to double memory

Queue: Implementation – Circular Buffer

Event Sequence

1. `back_idx == front_idx` since array is empty
2. push A
3. push B
4. push C
5. pop (A)
6. push D
7. push E (need to allocate more memory)*

Circular array with capacity 6



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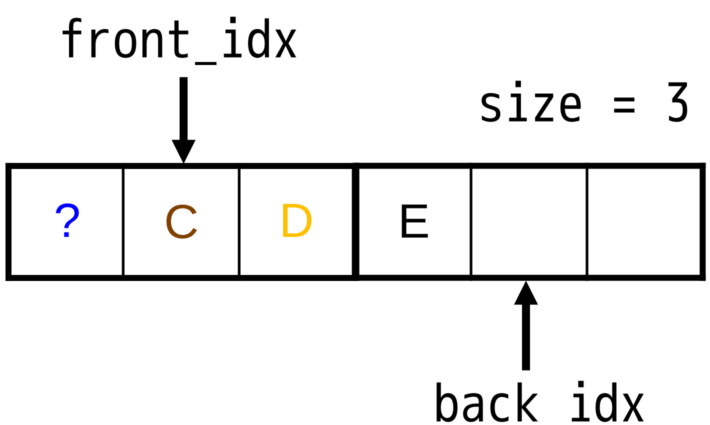
Queue: Implementation – Circular Buffer

Event Sequence

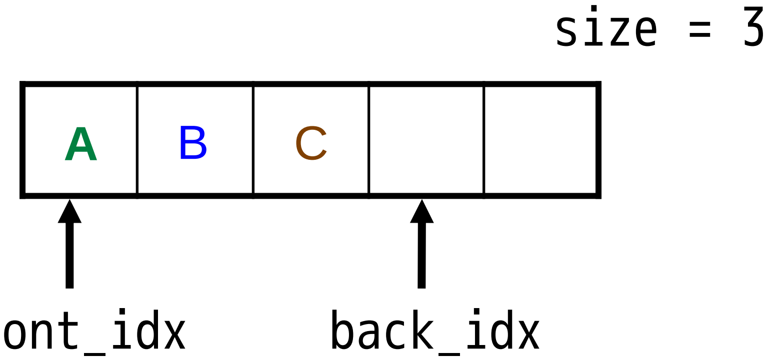
1. `back_idx == front_idx` since array is empty
2. push A
3. push B
4. push C
5. pop (A)
6. push D
7. push E (need to allocate more memory)*
8. pop (B)

Circular array with capacity 6



Queue: Implementation – Circular Buffer

Use a circular array

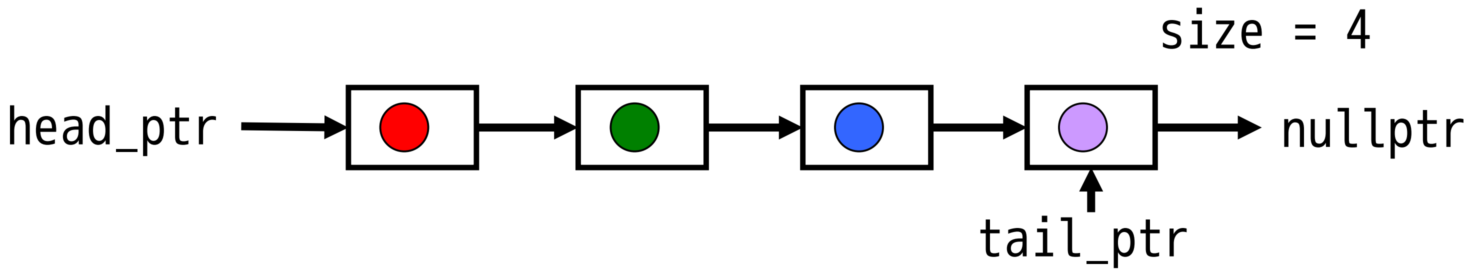


Method	Implementation
`push(object)`	1. If `size == capacity`, reallocate larger array and copy over elements, "unrolling" as you go unroll: start `front_idx` at 0, insert all elements 2. Insert value at `back_idx`, increment size and `back_idx`, wrapping around either as needed
`pop()`	Increment `front_idx`, decrement size
`object &front()`	Return reference to element at `front_idx`
`size()`	Return `size`
`empty()`	Check if `size == 0`

How many steps/operations for each method?

Queue: Implementation – Linked List

Singly-linked is sufficient, tail ptr for efficiency



Method	Implementation
`push(object)`	Append node after `tail_ptr`, increment size
`pop()`	Delete node at `head_ptr`, decrement size
`object &front()`	Deference `head_ptr`
`size()`	Return `size`
`empty()`	Return `head_ptr == nullptr`

*Alternative approach: count nodes when needed

How many steps/operations for each method?

Queue: Which Implementation?

Method	Array/Vector	Linked List
`push(object)`	Constant (linear when resizing vector)*	Constant
`pop()`	Constant	Constant
`object &front()`	Constant	Constant
`size()`	Constant	Constant (with tracked size)
`empty()`	Constant	Constant

*Averages out to constant with many pushes (amortized constant)

- The asymptotic complexities of each are similar
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- The linked list also has higher memory overhead
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STL Queues: `std::queue<>`

- Code: `#include <queue>`
- You can choose the underlying container
- All operations are implemented generically on top of the given container
  - No specialized code based on given container

	Queue
Default Underlying Container	`std::deque<>`
Optional Underlying Container	`std::list<>`

Basic Containers: Queue

Data Structures & Algorithms

Basic Containers: Deque

Data Structures & Algorithms

Deque Terminology Clarification

- "Deque" is an abbreviation of Double-Ended Queue.
  Pronounced "deck"
- "Dequeue" is another name for removing something from a queue.
  Pronounced "dee-queue"
- The STL includes `std::deque<>`, which is an implementation of a Deque, and is usually based on a growable collection of fixed-sized arrays.

Deque ADT: a queue and stack in one

(Double-ended Queue)

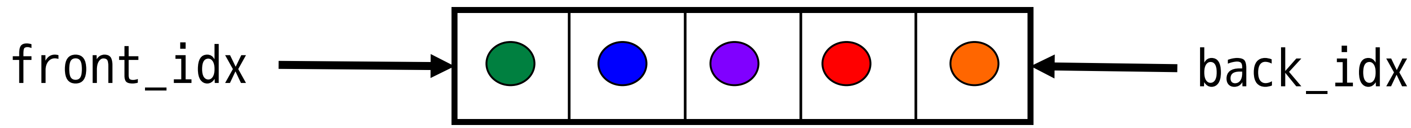
- ADT that allows efficient insertion and removal from the front and the back
- 6 major methods
  - `push_front(), pop_front(), front()`
  - `push_back(), pop_back(), back()`
- Minor methods
  - `size(), empty()`
- Can traverse using iterator

- STL incudes constant time `operator[]()`

Simple Deque Implementation

Circular Buffer

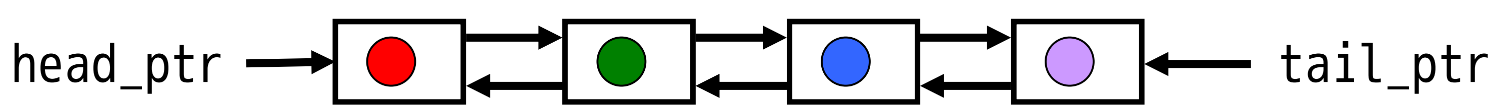
– `front_idx` and `back_idx` both get incremented/decremented



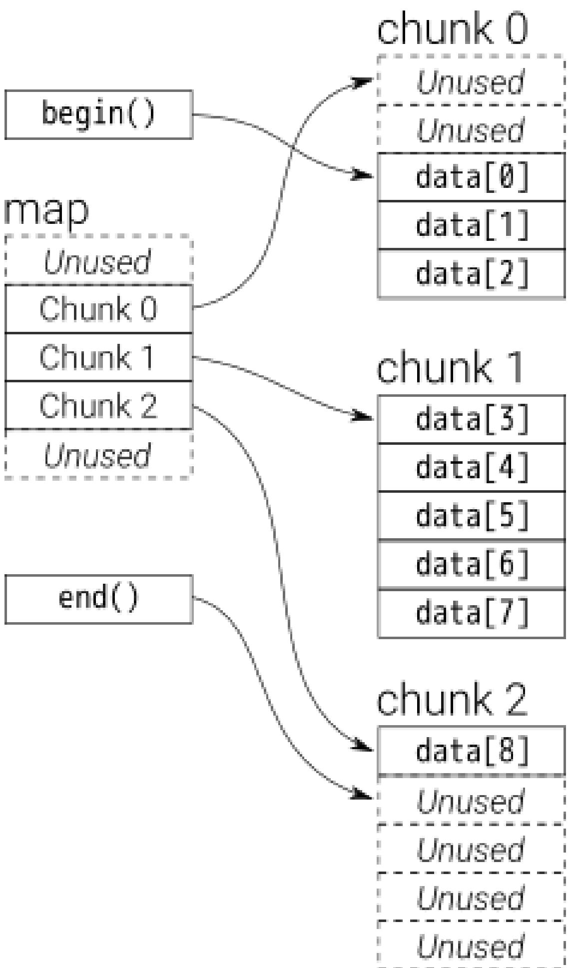
Doubly-linked list

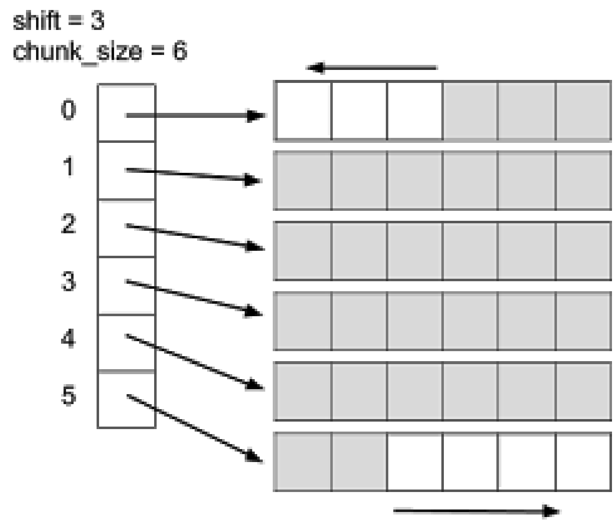
– Singly-linked doesn’t support efficient removal

– Other operations map directly to doubly-linked list operations



STL Deque, Two Internal Views





Taken from: https://stackoverflow.com/questions/6292332/what-really-is-a-deque-in-stl
and http://cpp-tip-of-the-day.blogspot.com/2013/11/how-is-stddeque-implemented.html

STL Deques: `std::deque<>`

- Code: `#include <deque>`
- Stack/Queue-like behavior at both ends
- Random access with `[]` or `.at()`

Basic Containers: Deque

Data Structures & Algorithms

Customizable Containers: Priority Queue

Data Structures & Algorithms

What is a Priority Queue?

- Each datum paired with a priority value
  - Priority values are usually numbers
  - Should be able to compare priority values (<)
- Supports insertion of data and inspection
- Supports removal of datum with highest priority
  - ”Most important” determined by given ordering

Like a group of bikers where the fastest ones exit the race first

Priority Queue Example: Emergency Call Center

- Operators receive calls and assign levels of urgency
- Lower numbers indicate more urgent calls
- Calls are dispatched (or not dispatched) by computer to police squads based on urgency

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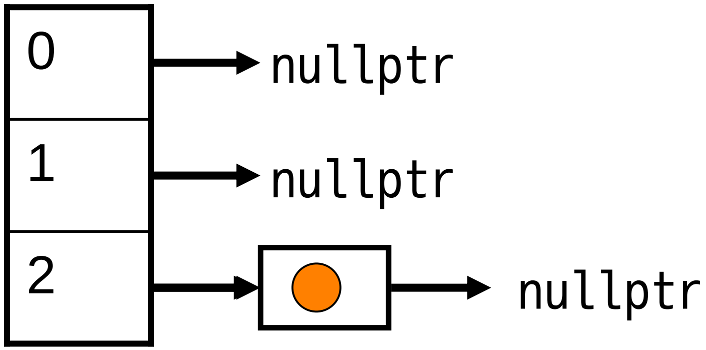
1. Level 2 call comes in



Priority Queue Example: Emergency Call Center

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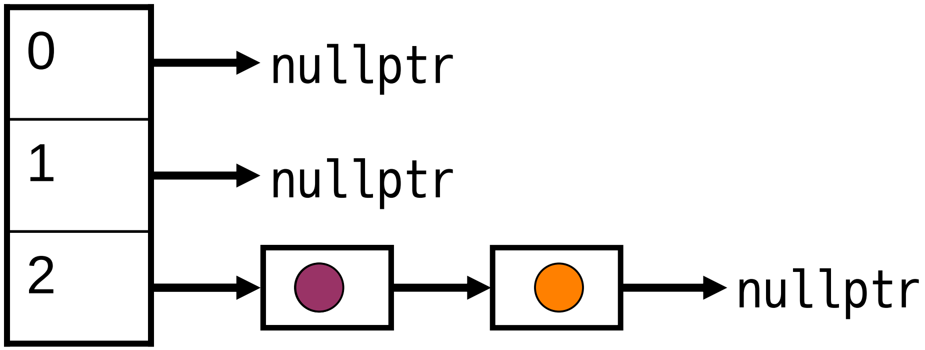
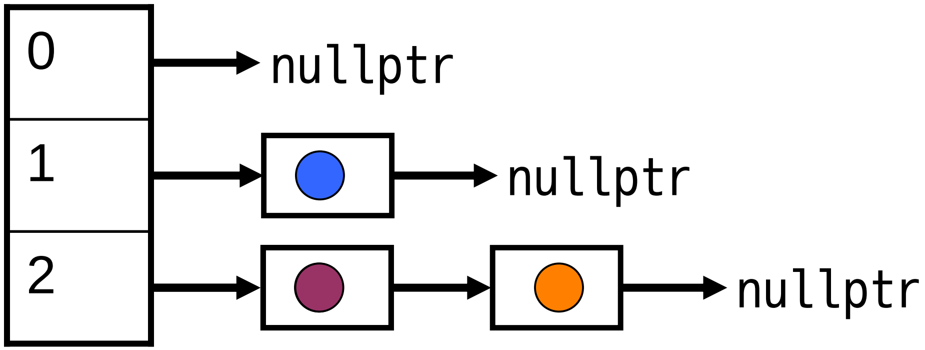
1. Level 2 call comes in
2. Level 2 call comes in



Priority Queue Example: Emergency Call Center

- Operators receive calls and assign levels of urgency
- Lower numbers indicate more urgent calls
- Calls are dispatched (or not dispatched) by computer to police squads based on urgency

1. Level 2 call comes in
2. Level 2 call comes in
3. Level 1 call comes in



Priority Queue Example: Emergency Call Center

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- Lower numbers indicate more urgent calls
- Calls are dispatched (or not dispatched) by computer to police squads based on urgency

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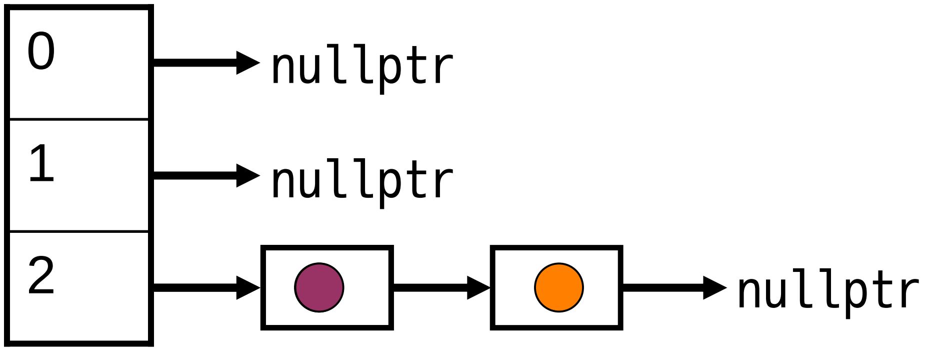
1. Level 2 call comes in
2. Level 2 call comes in
3. Level 1 call comes in
4. A call is dispatched



Priority Queue Example: Emergency Call Center

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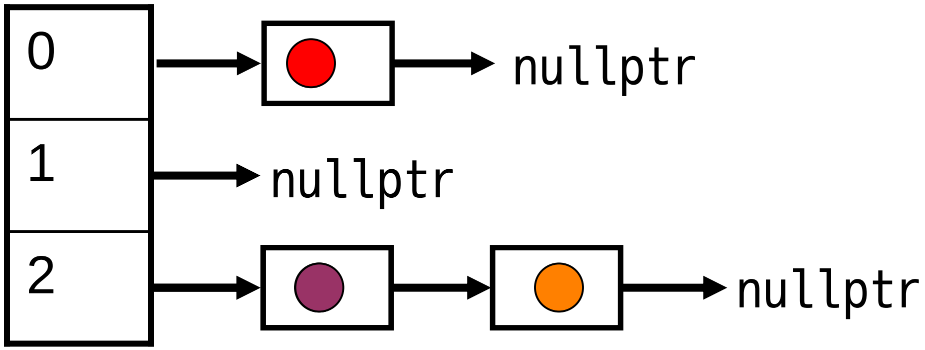
- Calls are dispatched (or not dispatched) by computer to police squads based on urgency

1. Level 2 call comes in
2. Level 2 call comes in
3. Level 1 call comes in
4. A call is dispatched
5. Level 0 call comes in



Priority Queue Example: Emergency Call Center

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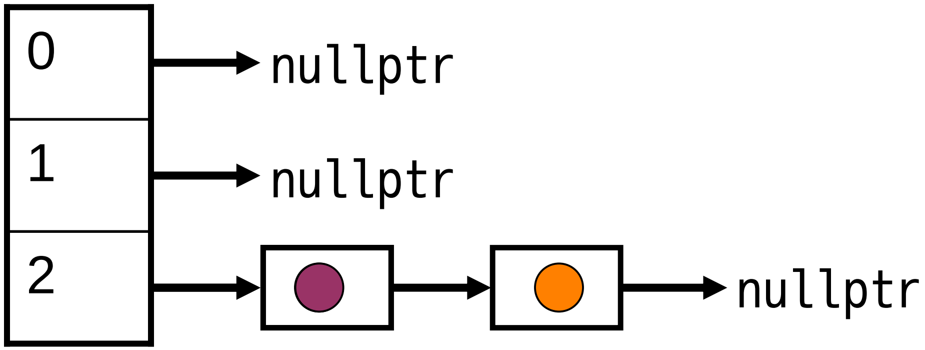
1. Level 2 call comes in
2. Level 2 call comes in
3. Level 1 call comes in
4. A call is dispatched
5. Level 0 call comes in
6. A call is dispatched



Priority Queue ADT: Interface

- Supports insertion, with removal in descending priority order

Method	Description
`push(object)`	Add `object` to the priority queue
`pop()`	Remove highest priority element
`const object &top()`	Return a reference to highest priority element
`size()`	Number of elements in priority queue
`empty()`	Checks if priority queue has no elements

Examples

- Hospital queue for arriving patients
- Load balancing on servers

Priority Queue Implementations

Underlying Implementation	Insert	Remove
Unordered sequence container	Constant	Linear
Sorted sequence container	Linear	Constant
Heap (presented in a future lecture)	Logarithmic	Logarithmic
Array of linked lists (for priorities of small integers)	Constant	Constant

A Customizable Container

- By default `std::priority_queue<>` uses `std::less<>()` to determine relative priority of two elements
- A ”default PQ” is a “max-PQ”, where the largest element has highest priority
- If a “min-PQ” is desired, customize with `std::greater<>()`, so the smallest element has highest priority
- If the PQ will hold elements that cannot be compared with `std::less<>()` or `std::greater<>()`, customize with custom comparator (function object)
- Custom comparators can work with objects, perform tie-breaks on multiple object members, and other functionality

STL PQs: `std::priority_queue<>`

- STL will maintain a Heap in any random access container
  - `#include <queue>`
- Common `std::priority_queue<>` declarations
  - "Max" PQ using `std::less<>()`
    `std::priority_queue<T> myPQ;`
  - PQ using a custom comparator type, `COMP`
    `std::priority_queue<T, vector<T>, COMP> myPQ;`
- Manual priority queue implementation with standard library functions
  - `#include <algorithm>`
  - `std::make_heap()`
  - `std::push_heap()`
  - `std::pop_heap()`

Customizable Containers: Priority Queue

Data Structures & Algorithms