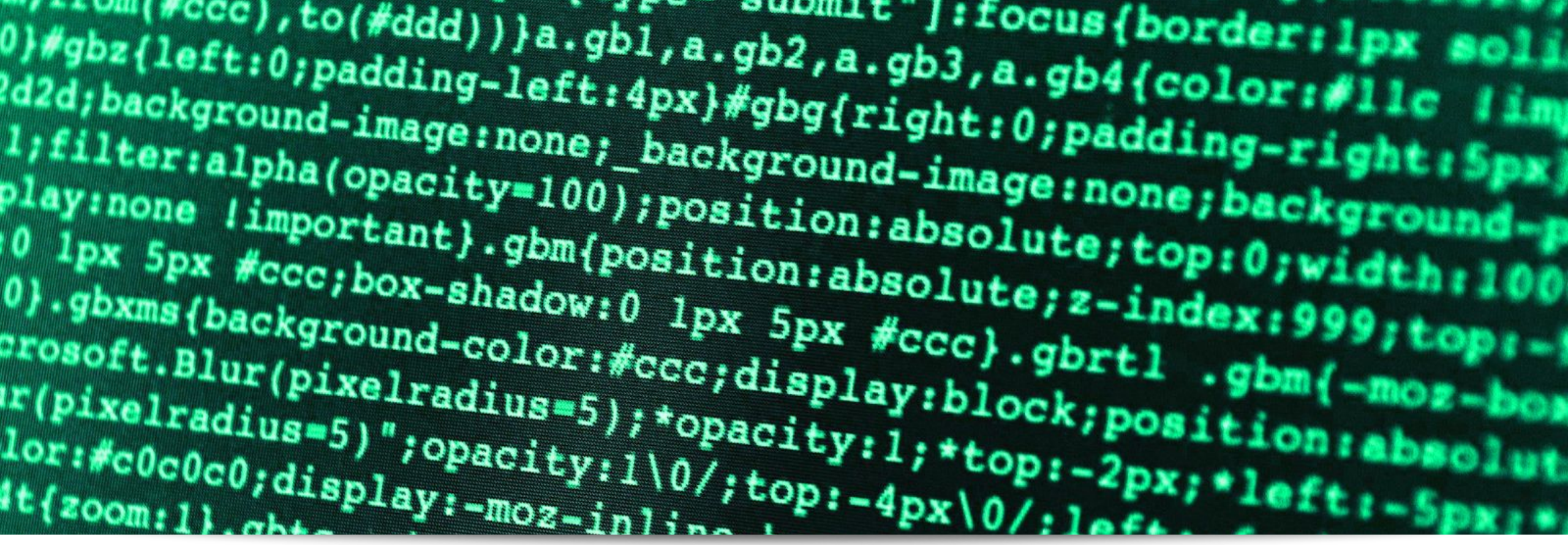


05

February 13-19, 2024

Container Types, Sorting Algorithms, Binary Search

Announcements

- **Project 2 is due on February 22nd at 11:59 PM**
- Lab 4 autograder and quiz are due on February 19th at 11:59 PM
- Lab 5 handwritten is due IN LAB by February 19th. DO NOT CALL SORT()!!!
- Lab 5 quiz is due on February 26th at 11:59 PM
  - No autograder portion (quiz is worth 15 points)

- **Midterm: Tuesday, March 5th from 7:00 PM to 9:00 PM**
  - If you need an alternate time, or SSD accommodations, you must complete the alternate exam request form as soon as possible!

- **No labs until March 12th!**

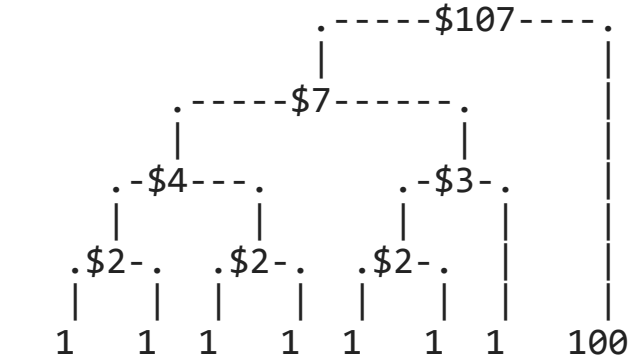
Agenda

- Midterm Topics
- Container Types
- Sorting Algorithms
- Binary Search
- Handwritten Problem

Handwritten Problem Review

Lab 4 Written Problem: Connecting Ropes

- Find the minimum cost of connecting ropes:
  - for example, if we had seven ropes of length 1 and one rope of length 100...
    - we would keep on connecting the ropes of length 1 until it becomes one rope
      - this combined rope would have a length of 7
    - we then combine this rope of length 7 with the rope of length 100
  - notice that we connect the smallest ropes first... what is a good data structure for this?




```
Sum: 1+1+1+1+1+1+1+100      = 107
Total Cost: 107+7+4+3+2+2+2 = 127
```

Lab 4 Written Problem: Connecting Ropes

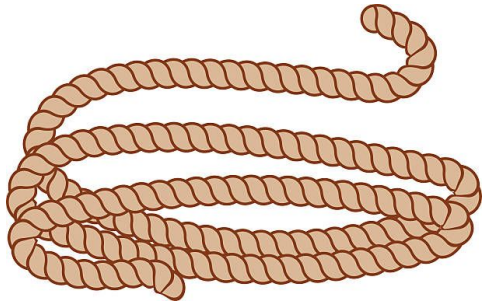
- Find the minimum cost of connecting ropes:
  - minimize the sum of lengths, so connect the shortest ropes ⟶ priority queue!

```
// calculate minimum cost required to join n ropes
int join_ropes(const vector<int>& ropes) {
    priority_queue<int, vector<int>, greater<int>> pq(ropes.begin(), ropes.end());
    int cost = 0;
    while (pq.size() > 1) {
        int rope1 = pq.top();
        pq.pop();
        int rope2 = pq.top();
        pq.pop();
        pq.push(rope1 + rope2);
        cost += (rope1 + rope2);
    }
    return cost;
}
```

we want a min PQ, so we use `greater<int>` here

connect the two shortest ropes you have until you only have one rope left, incrementing cost by the combined lengths of the ropes along the way

if you want to efficiently convert data in a container into a heap (PQ), consider using a range constructor



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    }
    return cost;
}
```

Why not just use a sorted array???

use greater<int> here

to efficiently convert data in a container into a heap (PQ), consider using a range constructor

connect the two shortest ropes you have until you only have one rope left, incrementing cost by the combined lengths of the ropes along the way

Lab 4 Written Problem: Connecting Ropes

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```

Why not just use a sorted array???

Insertion is O(n) in an array, compared to O(log n) in a priority queue!

Heapify is O(n) while sorting is O(n log n).

Midterm Topics

Midterm Topics to Review

- (Asymptotic) Complexity and Runtime Analysis, Math Foundations
- Recursion and the Master Theorem
- Container Data Structures and Array-Based Containers
- The Standard Template Library (STL)
- Stacks, Queues, and Priority Queue ADTs
- Ordered Arrays and Related Algorithms
- Set Operations (Union/Intersection) and Union-Find
- Elementary Sorts and Library-Implemented Sorts
- Quicksort and Mergesort
- Heaps and Heapsort
- Linear-Time Sorting Algorithms

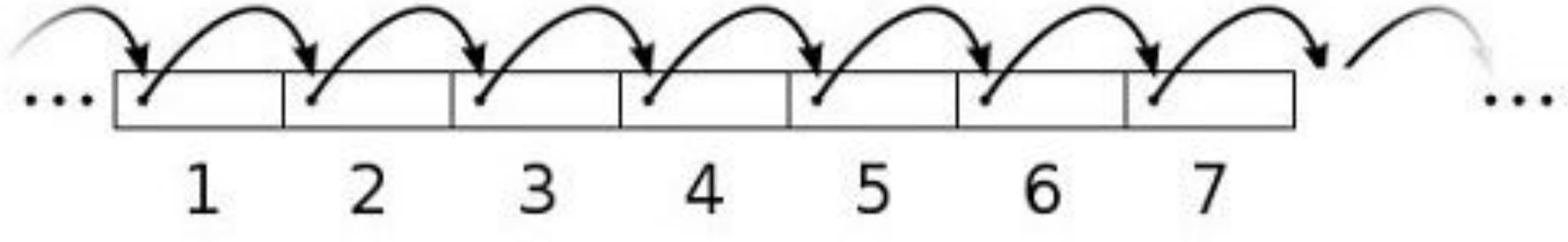
Additional Reminders

- Review all your lecture and lab notes!
- Understand how to do all the lab coding exercises and projects!
- Check out Piazza @1137 for additional (leetcode-style) FRQ practice
  - Midterm topics include sorting, binary search, union-find, stacks/queues, linked list, and array
  - Not required for success, simply a resource that's available
- Bring a pencil, eraser, MCard, and an 8.5"x11" note sheet (both sides OK)
- No calculators or other electronic devices

Types of Containers

Types of Containers

- Searchable
  - containers that support `find()`
  - searchable: vector, deque, list; not searchable: stack, queue
- Sequential
  - allows iteration over elements in some order
  - sequential: vector, deque, list; not sequential: stack, queue



Types of Containers

- Sorted
  - elements are stored in some pre-defined order based on a sorting criteria
  - does not support `insert()` in any location
  - convenient if you want to search the container
  - examples: map, set
- Ordered
  - maintains current order
  - supports `insert()` at any location
  - insertion and deletion do not disrupt the relative ordering of other elements in the container
  - examples: vector, deque, list

Types of Containers

Ordered Containers

Data Structure	insertAfter()	operator[]	delete()
Array	O(n)	O(1)	O(n)
Linked-list	O(1)	O(n)	O(1)

Sorted Containers

Data Structure	find()	operator[]	delete()
Array	O(log n)	O(1)	O(n)
Linked-list	O(n)	O(n)	O(1)

Types of Containers

- Midterm Exam Question (Fall 2018)

20. Binary Heaps

Which one of the following statements is **TRUE** about the contents of the underlying data vector used to implement a binary heap?

A) the binary heap's underlying data vector is an ordered and sorted container

B) the binary heap's underlying data vector is an ordered container, but it is not a sorted container

C) the binary heap's underlying data vector is a sorted container, but it is not an ordered container

D) the binary heap's underlying data vector is neither an ordered nor sorted container

E) none of the above

Types of Containers

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E) none of the above

Types of Containers

- Why is the answer (D)?

  The binary heap's data vector is used to represent a binary tree. The relationship between the "nodes" is only that no parent is lower in priority than any of its descendants. There are multiple array representations of the exact same binary heap.

```
{1,2,3,4,5,6} // a valid min-heap
{1,3,2,6,5,4} // an identical heap
```

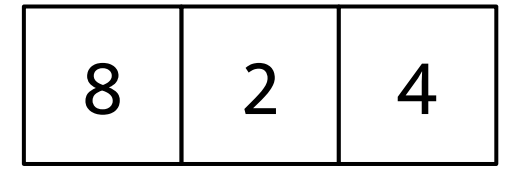
  The variations above show that the binary heap **is not sorted**, as the data in the vector does not have to be in a specific order in order for the binary heap to be valid.

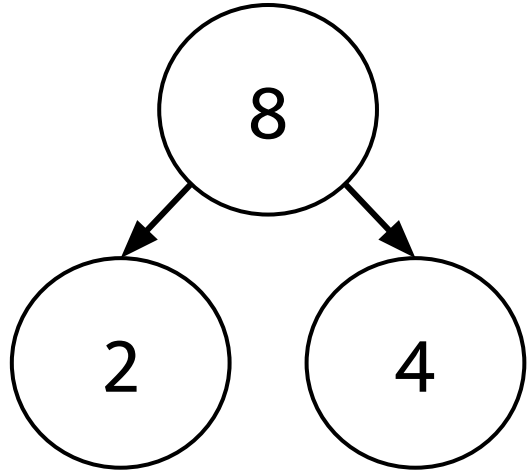
  To show that it is not ordered, we have to determine a situation where adding an element changes the relative ordering of elements already in the heap.

Types of Containers

- Why is the answer (D)?

  Current array representation: `{8,2,4}`



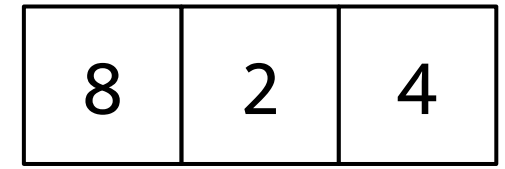


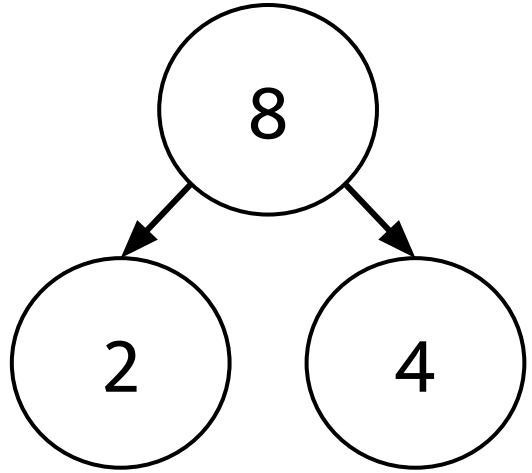
Types of Containers

- Why is the answer (D)?

  Current array representation: `{8,2,4}`

  Let's add the element `9` to this binary heap!





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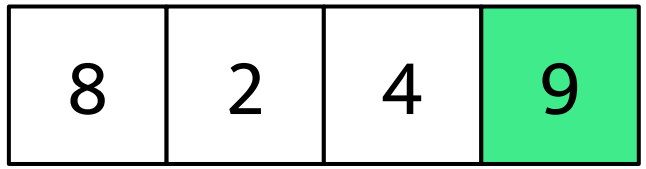

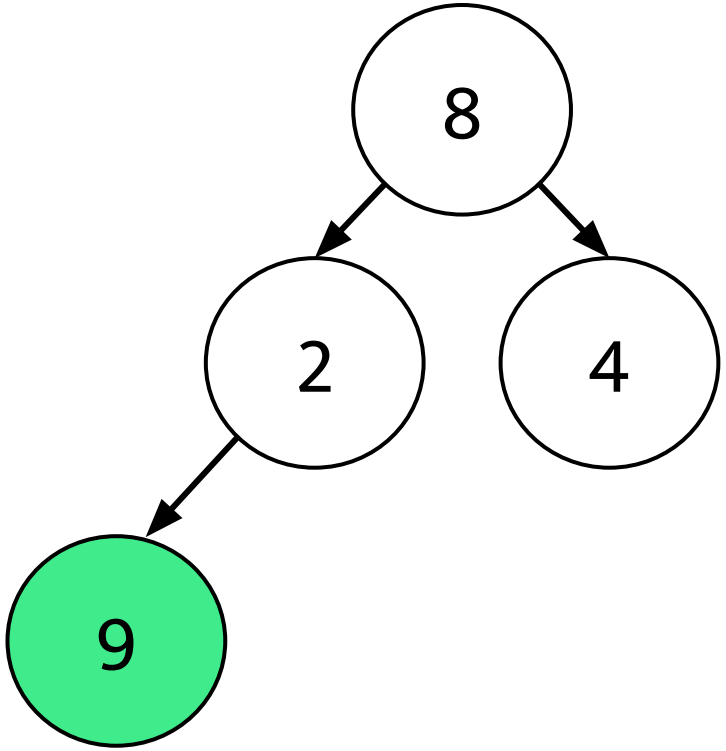

Types of Containers

- Why is the answer (D)?

  Current array representation: `{8,2,4}`

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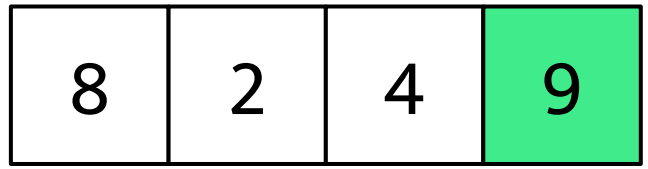
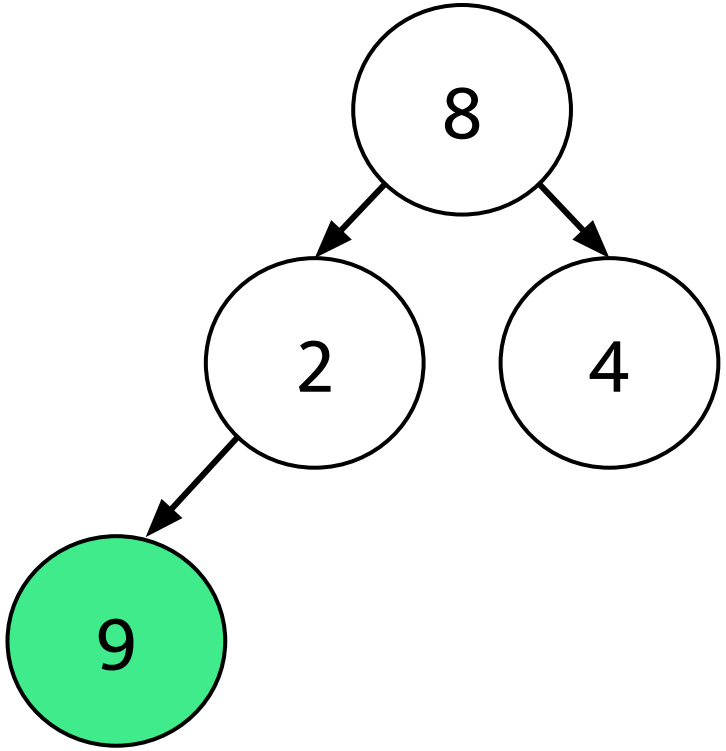
  Fix the heap invariant!





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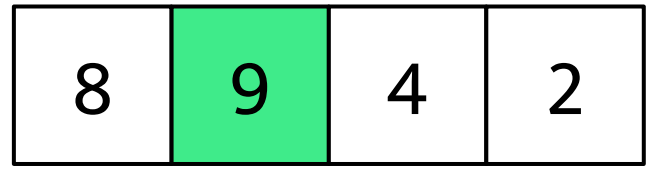

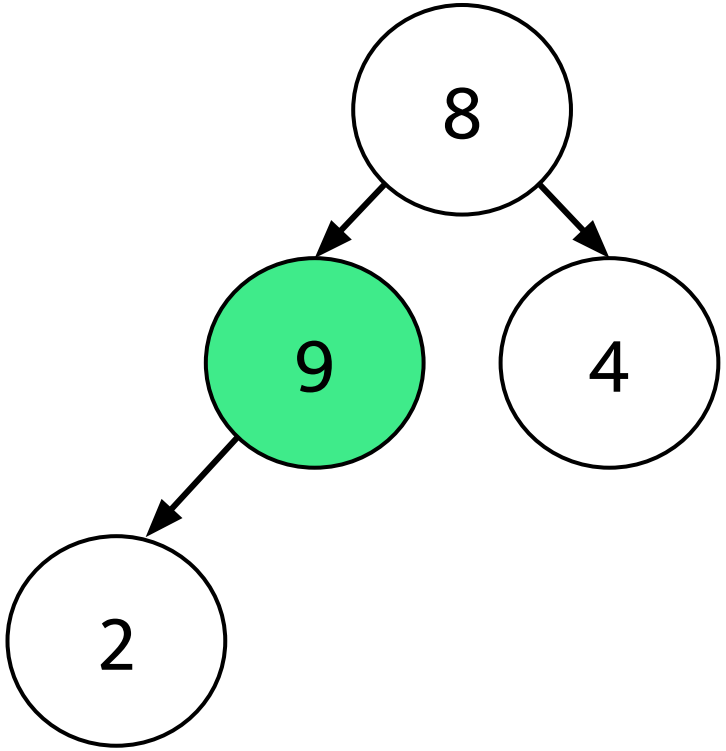

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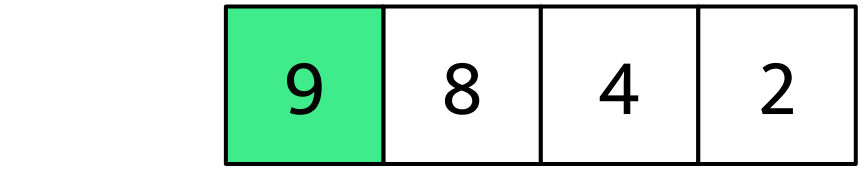
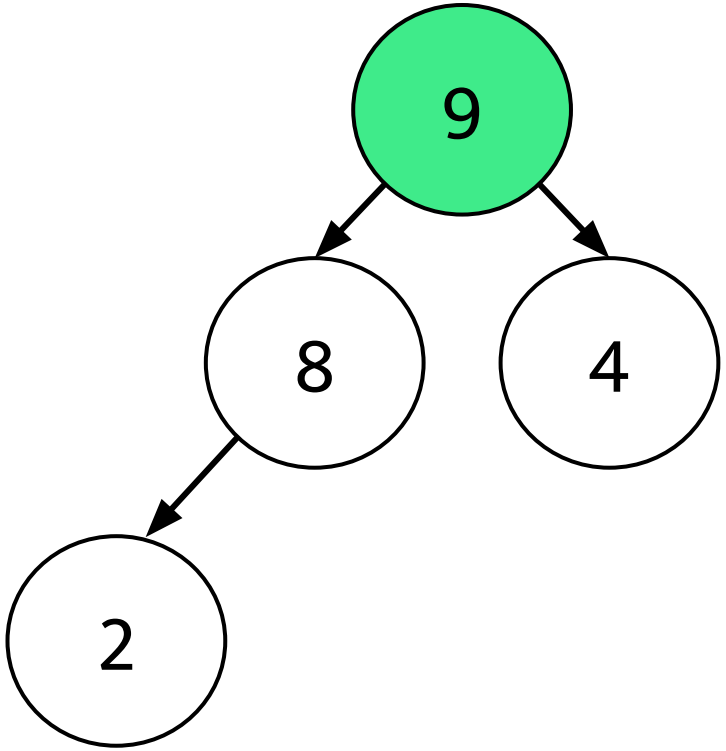




Types of Containers

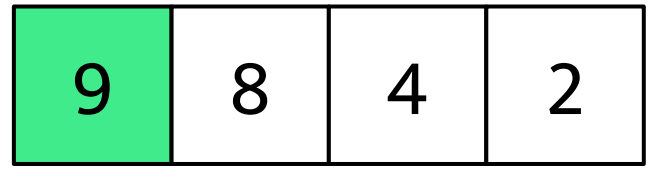
- Why is the answer (D)?

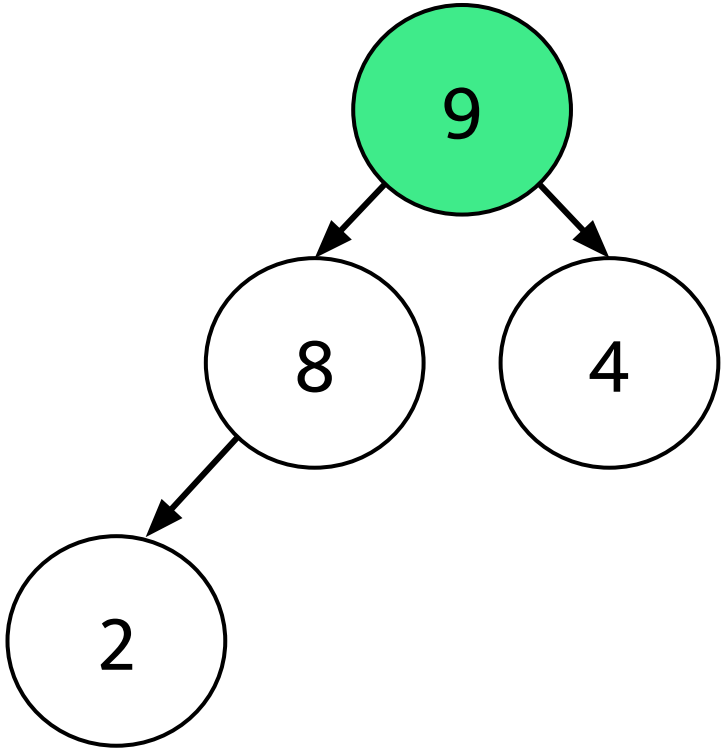
  Current array representation: `{8,2,4}`

  Let's add the element `9` to this binary heap!

  Fix the heap invariant!

  New array representation: `{9,8,4,2}`





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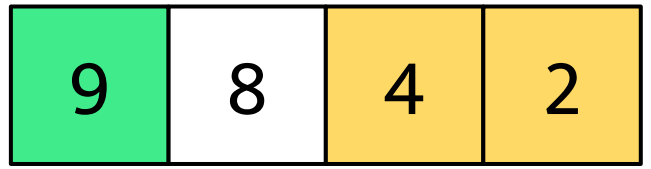
  Current array representation: `{8,2,4}`

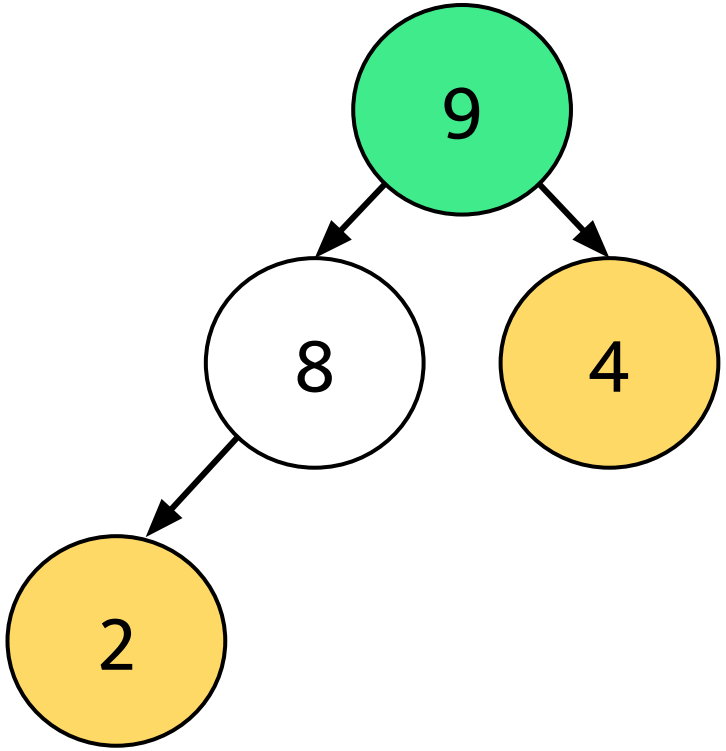
  Let's add the element `9` to this binary heap!

  Fix the heap invariant!

  New array representation: `{9,8,4,2}`

  Notice how adding `9` to this binary heap changed the relative ordering of the other elements! Before the insertion, `2` was before `4` in the array. However, after `9` was added, `2` is now after `4` in the array! This shows that the binary heap **is not ordered**.





Types of Iterators

- Understand the differences between different types of iterators
- Further reading here: https://ajzhou.gitlab.io/eecs281/notes/chapter11/

Iterators in the STL can be placed into five different categories based on the operations they support. These five categories are:

- *Input iterators:* read only, forward moving (single pass only)
- *Output iterators:* write only, forward moving (single pass only)
- *Forward iterators:* forward moving (multiple passes allowed)
- *Bidirectional iterators:* forward and backward moving
- *Random access iterators:* provides random access

	Input	Output	Forward	Bidirect.	Random
Supports dereference (⋆) and read	✓		✓	✓	✓
Supports dereference (⋆) and write		✓	✓	✓	✓
Supports forward movement (++)	✓	✓	✓	✓	✓
Supports backward movement (−−)				✓	✓
Supports multiple passes			✓	✓	✓
Supports == and !=	✓		✓	✓	✓
Supports pointer arithmetic (+, −, etc.)					✓
Supports pointer comparison (<, >, etc.)					✓

Sorting Algorithms

Sorting Algorithms

- What to consider?
  - time complexity
  - space complexity
  - recursion
    - is it tail recursive?
    - if not, what is the stack depth?
  - stability
    - does it preserve the original order of elements that are equal?
  - adaptability
    - does it perform optimizations based on the order the elements are already in?

Overview of Sorts

- Elementary sorts:
  - bubble sort
  - selection sort
  - insertion sort
- More advanced sorts:
  - mergesort
  - quicksort
  - heapsort
  - counting sort
- The following website is a great resource for visualizing these sorts!

  https://visualgo.net/en

Bubble Sort

- If you're ever asked a sorting question during an interview...

Bubble Sort

Bubble Sort

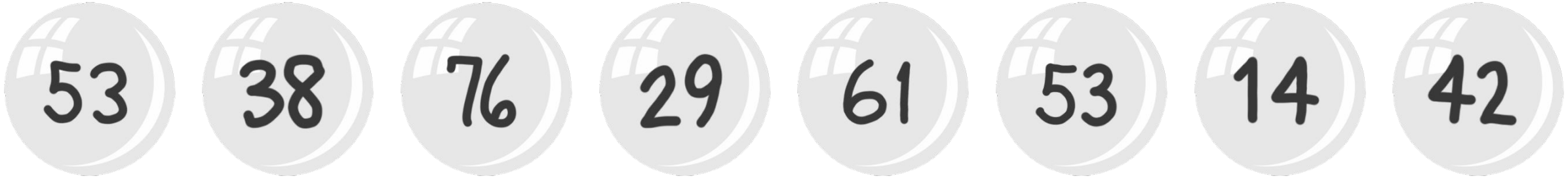
- Characteristics of bubble sort:

Best Case	Average Case	Worst Case	Memory	Stable?
$\Omega(n)$	$\Theta(n^2)$	$O(n^2)$	$O(1)$	Yes

  - Not used in practice because it is inefficient for large datasets.
  - Algorithm: Repeatedly step through list to be sorted, comparing each pair of adjacent items and swapping if they are in the wrong order. Each time pass through a smaller section of the list until no more swaps are needed.
  - Stable because swaps only happen for elements with different values.
  - Adaptive bubble sort accomplishes best-case $\Omega(n)$ runtime: keep a boolean "swap" flag to indicate if any swaps were made during each pass of the array - if not, then the array is already sorted!

Bubble Sort

- Bubble sort in action:



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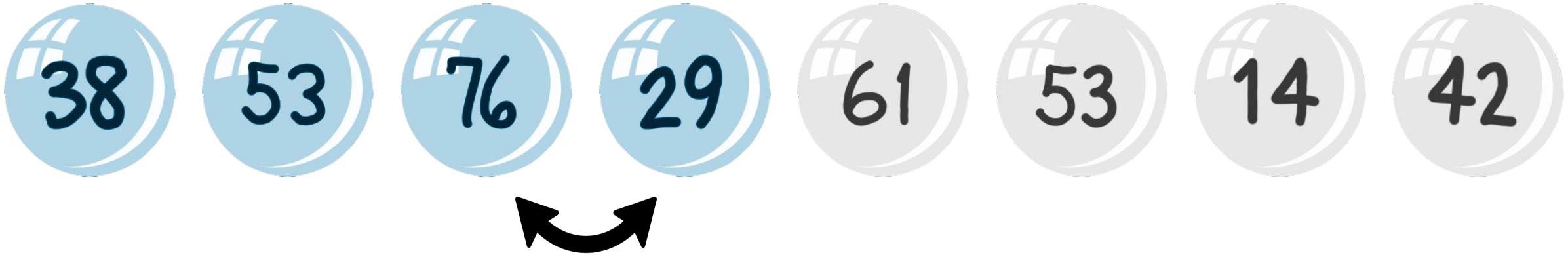

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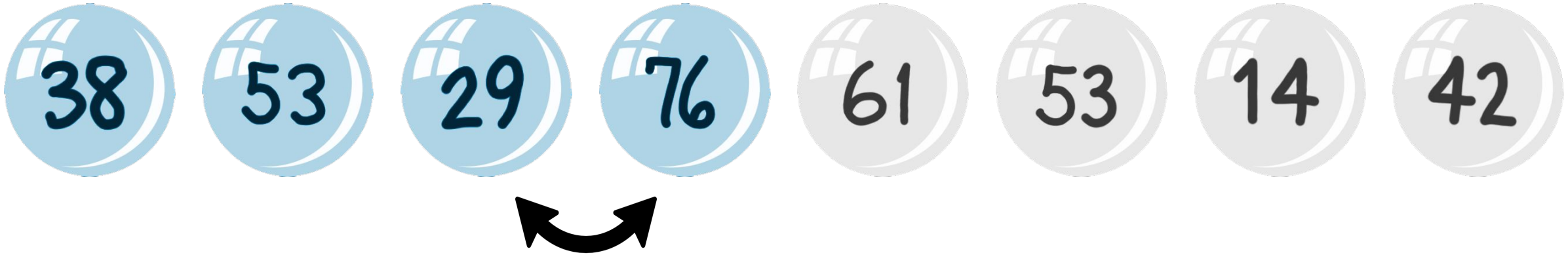

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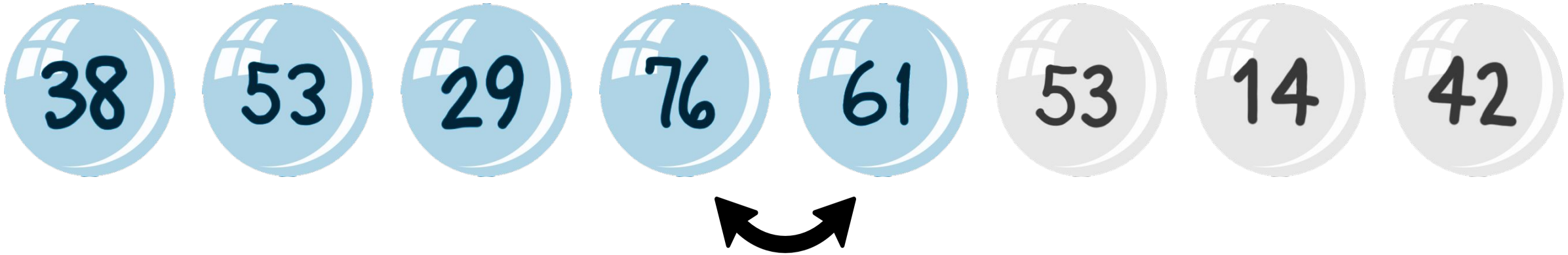

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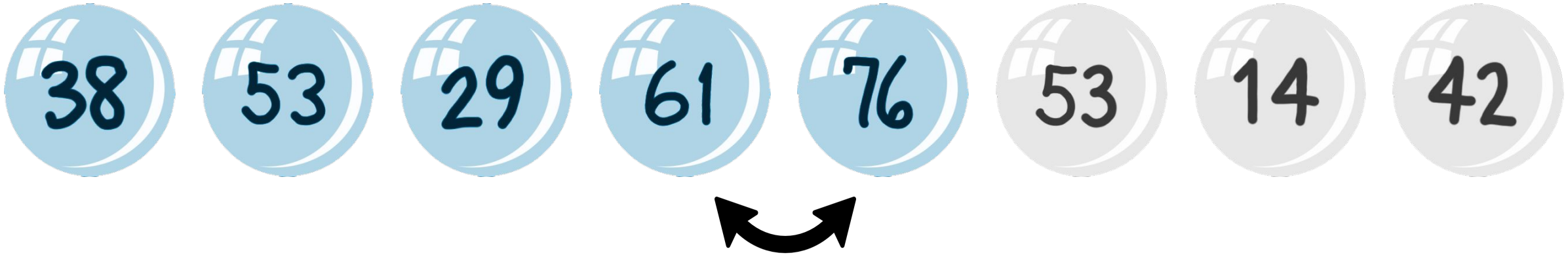

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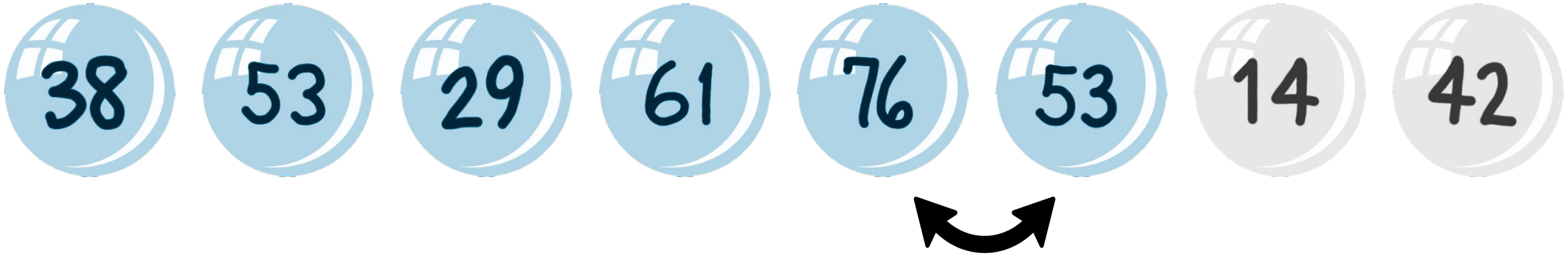

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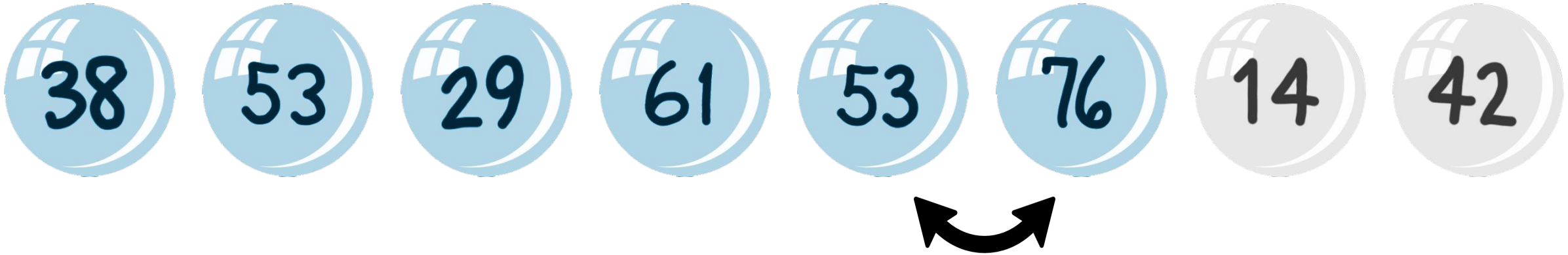

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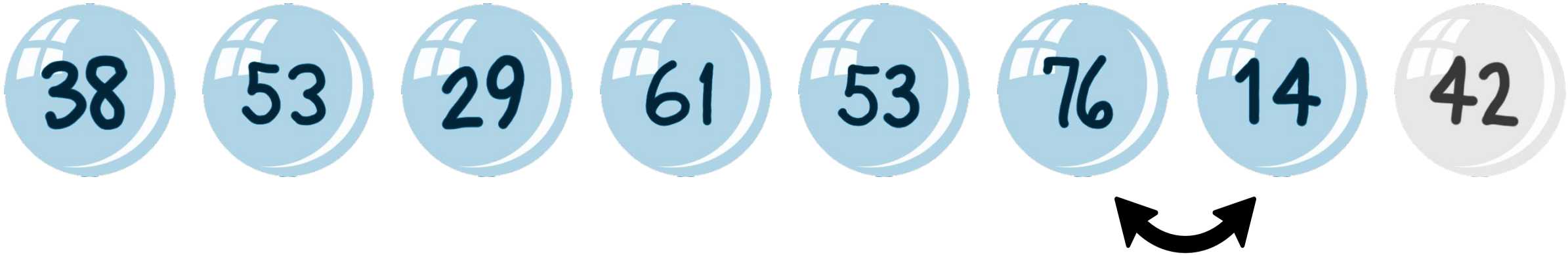

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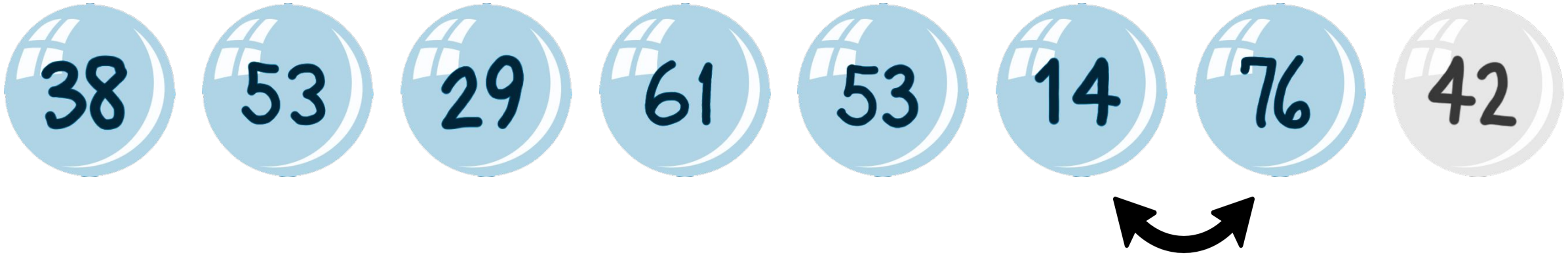

Bubble Sort

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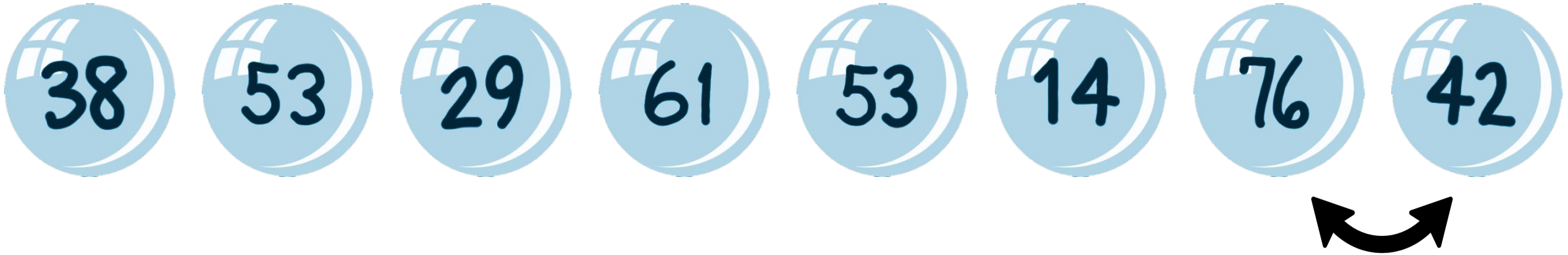
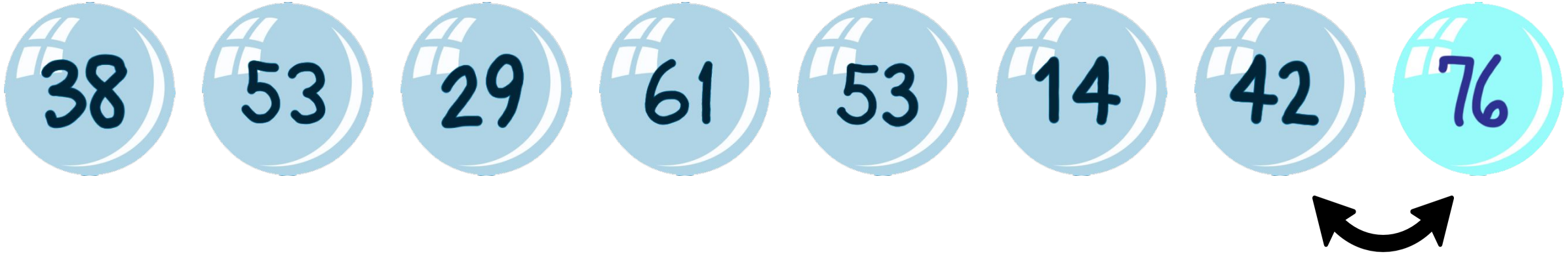


Bubble Sort

- Bubble sort in action:



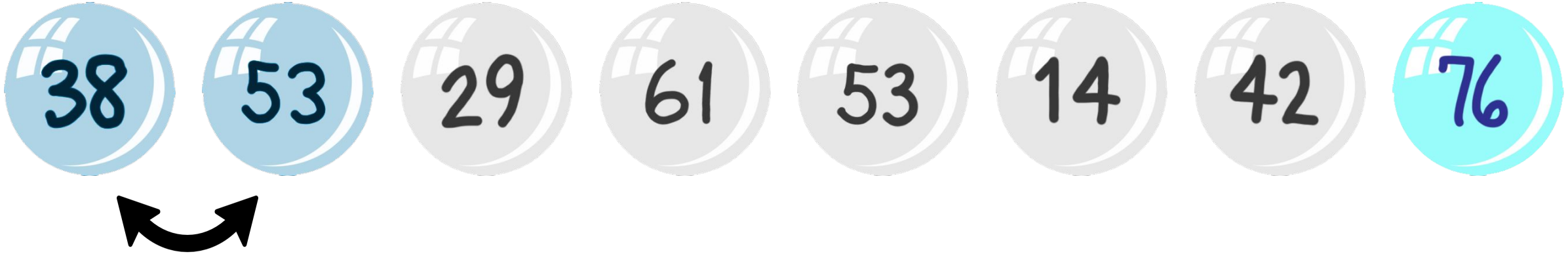
in this version of bubble sort, larger elements “bubble” to the top!

Bubble Sort

- Bubble sort in action:

Bubble Sort

- Bubble sort in action:



Bubble Sort

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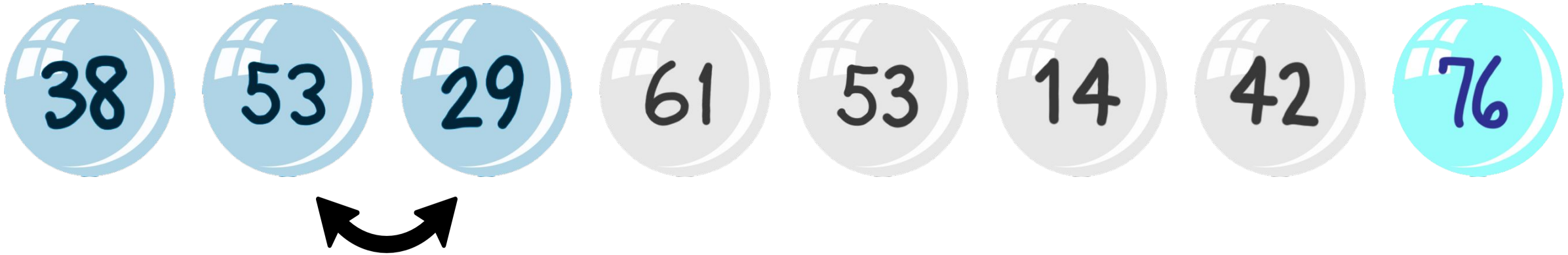

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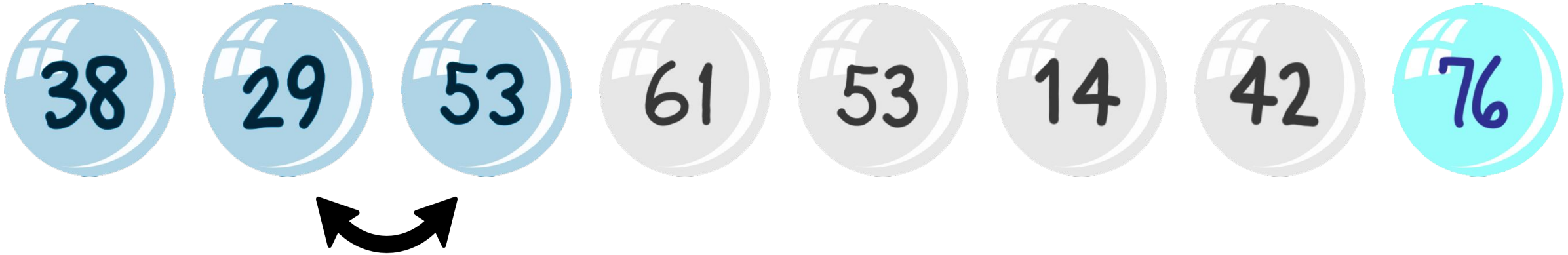

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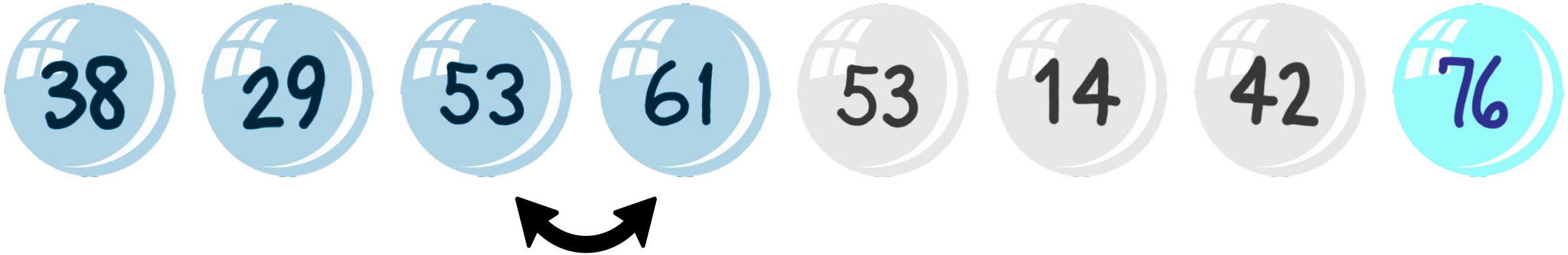

Bubble Sort

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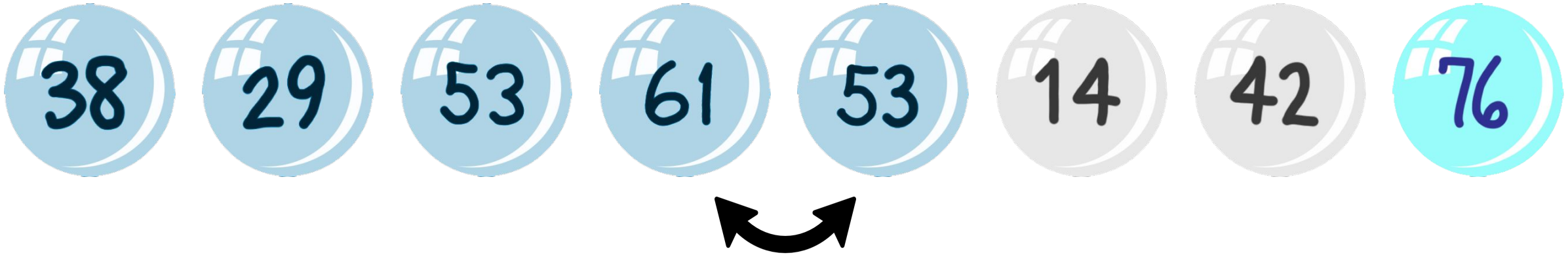

Bubble Sort

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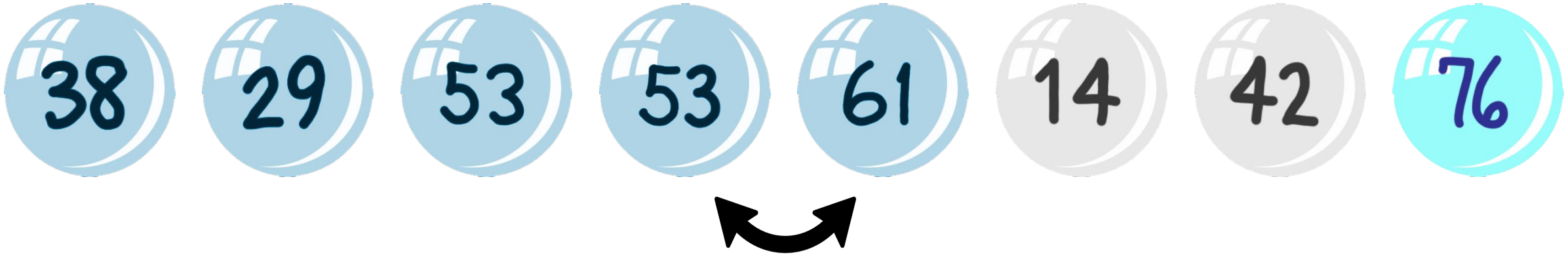

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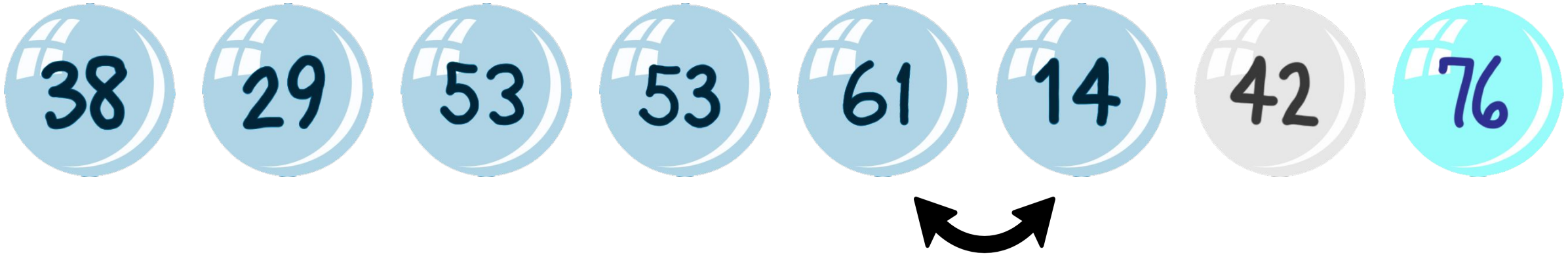

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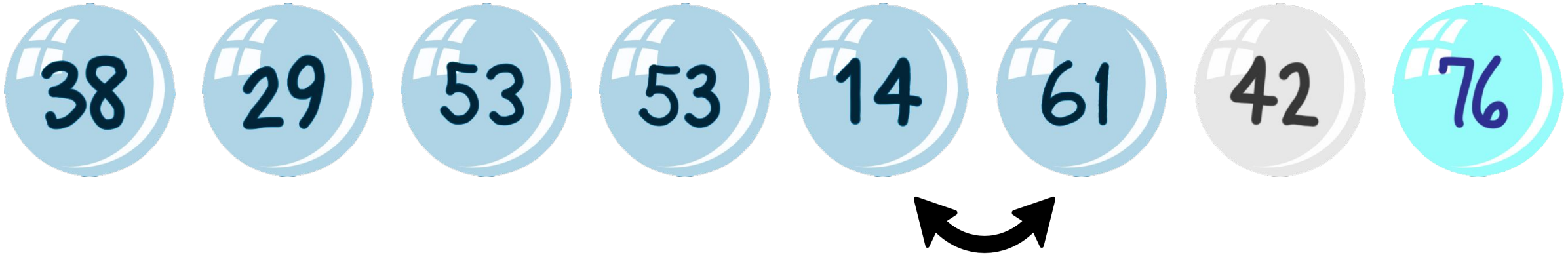

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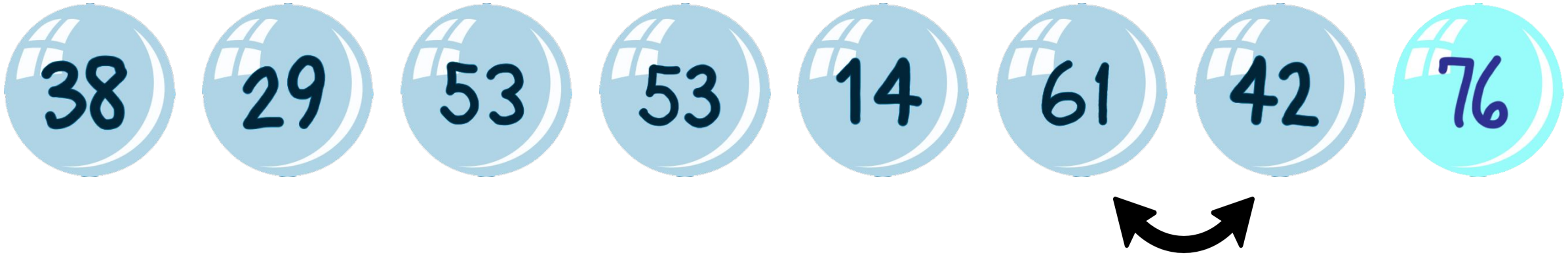

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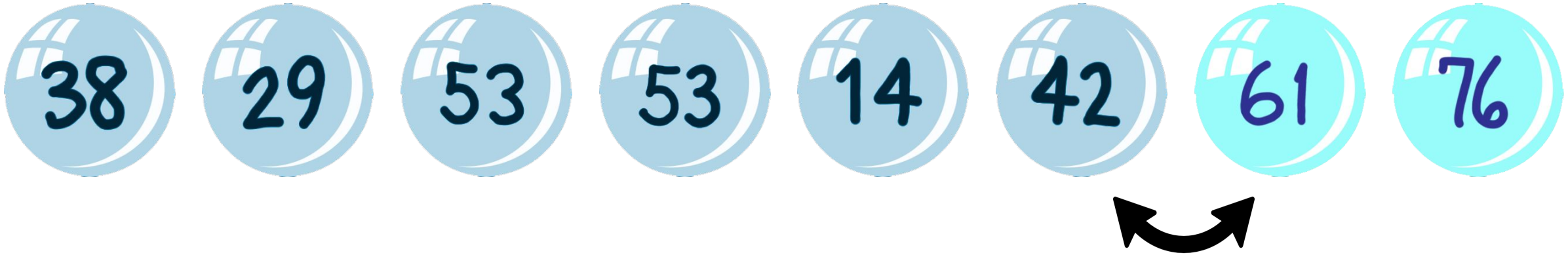

Bubble Sort

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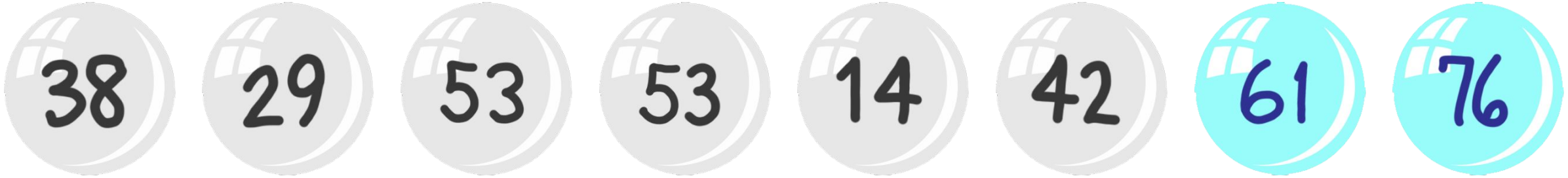

Bubble Sort

- Bubble sort in action:

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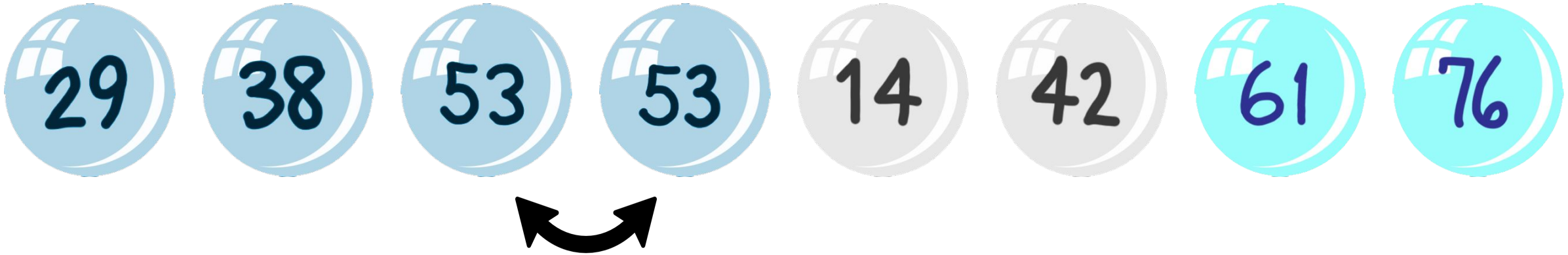

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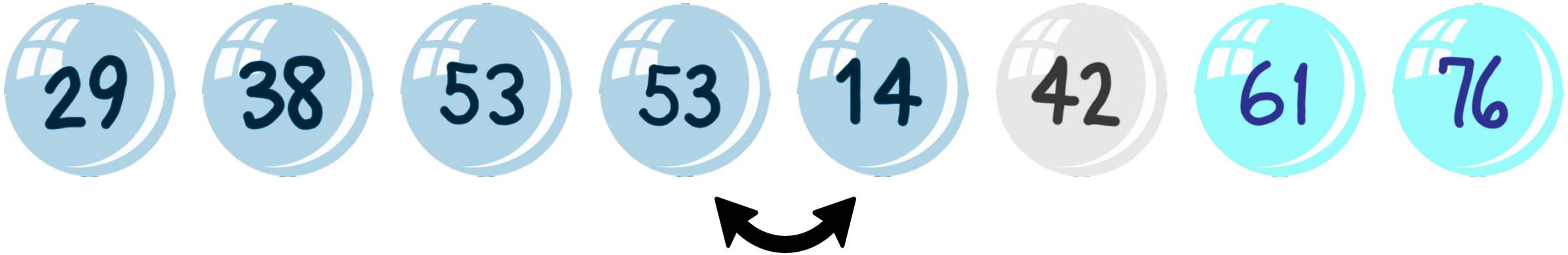

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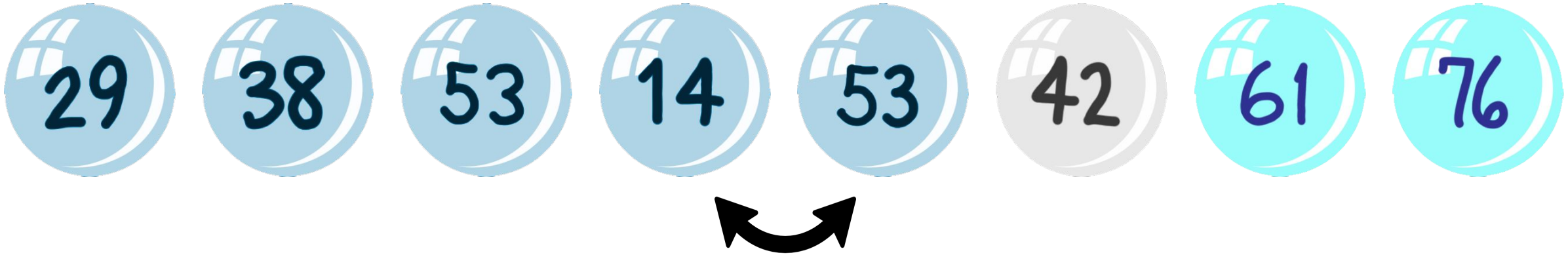

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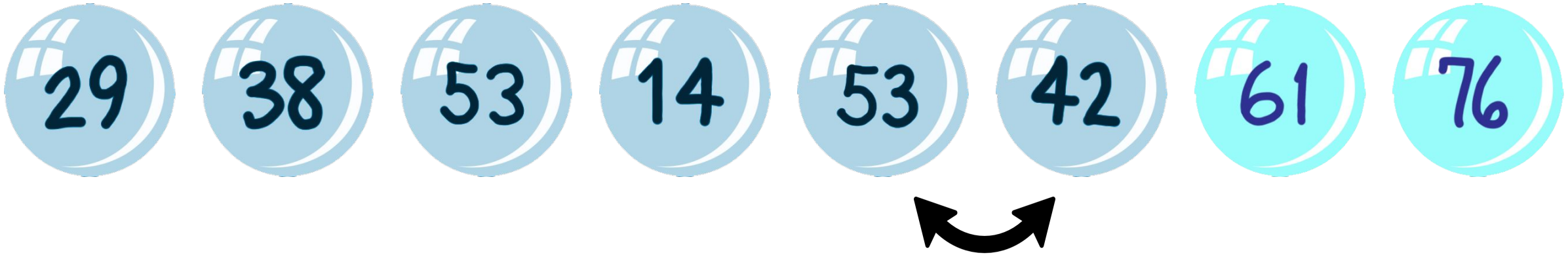

Bubble Sort

- Bubble sort in action:

notice that the second 53 reached the end before the first 53... this is because bubble sort is stable!

Bubble Sort

- Bubble sort in action:

Bubble Sort

- Bubble sort in action:

Bubble Sort

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Bubble Sort

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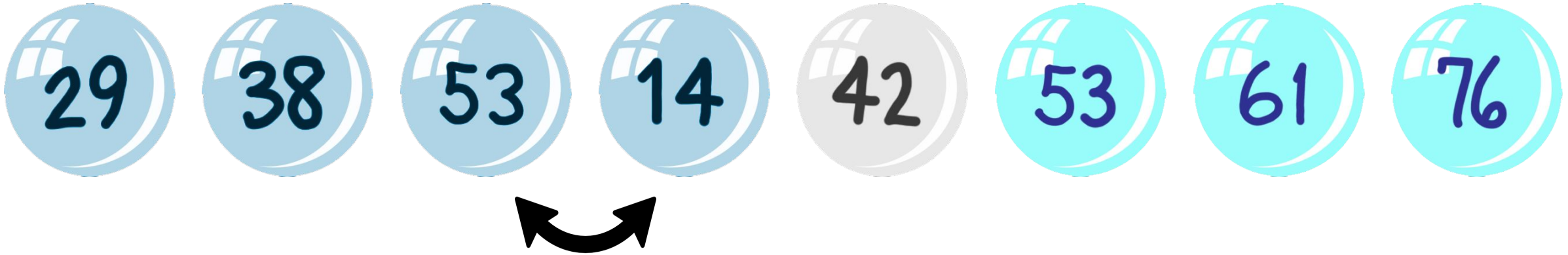

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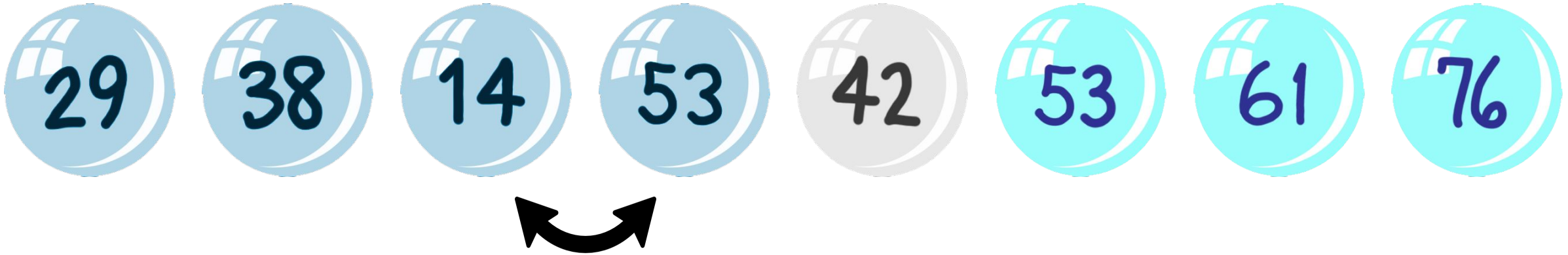

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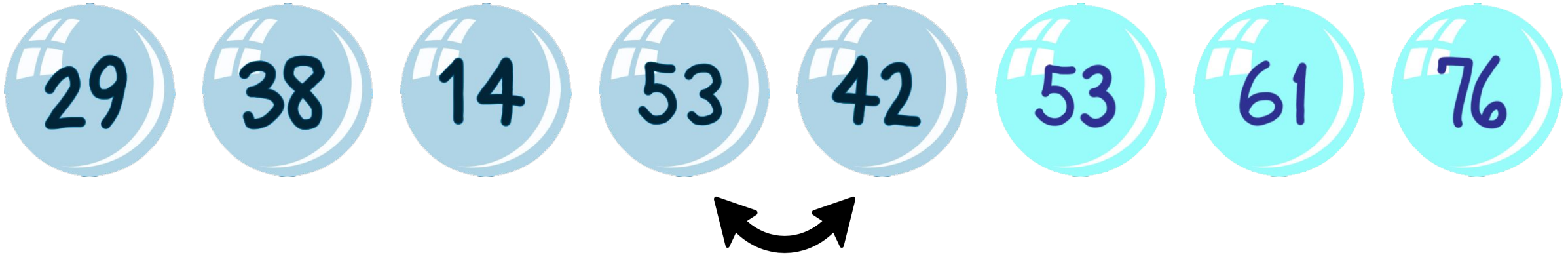

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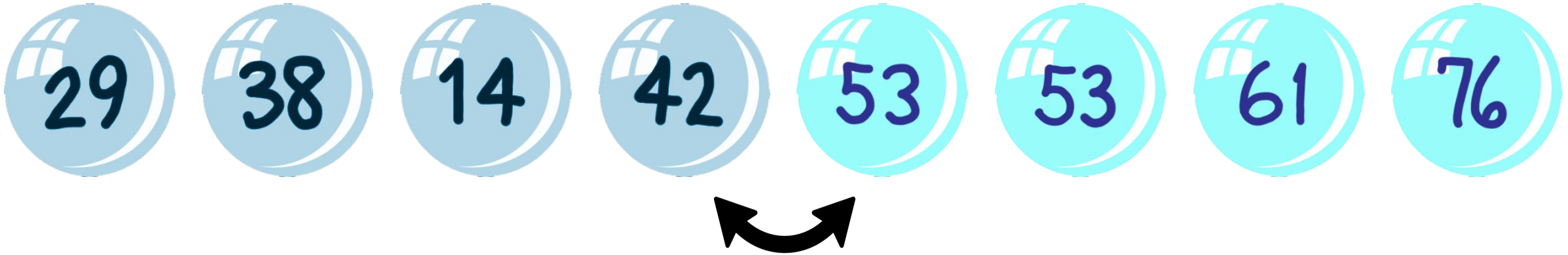

Bubble Sort

- Bubble sort in action:

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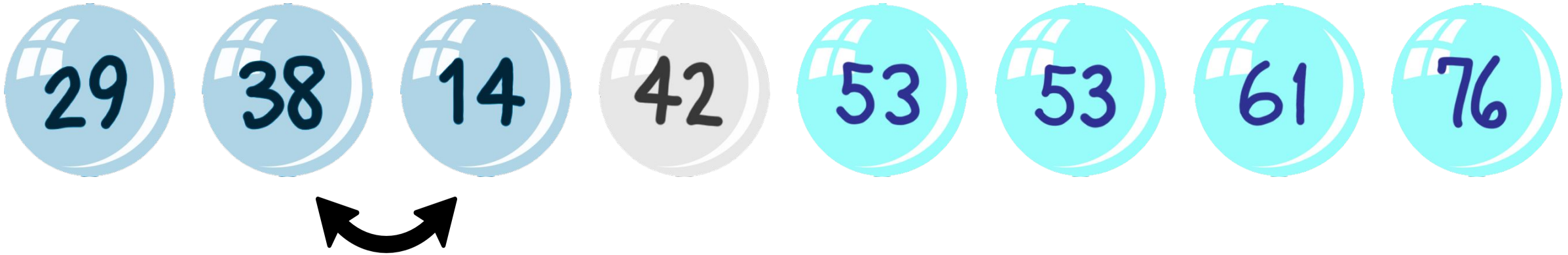

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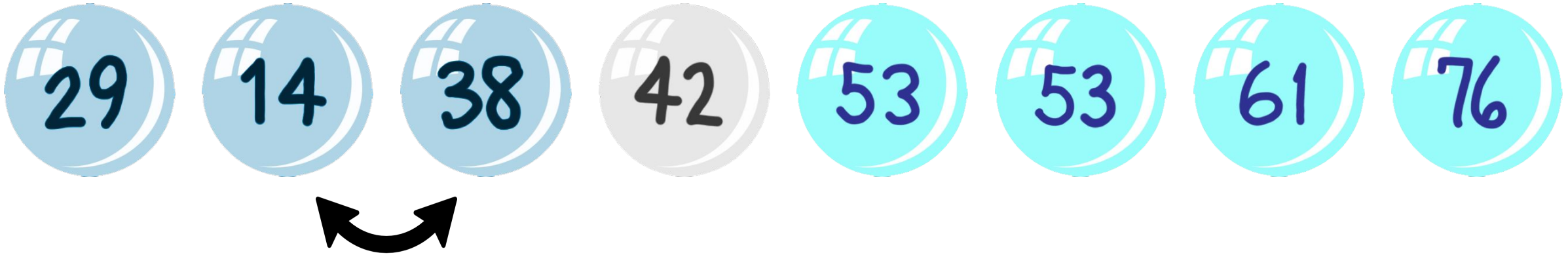

Bubble Sort

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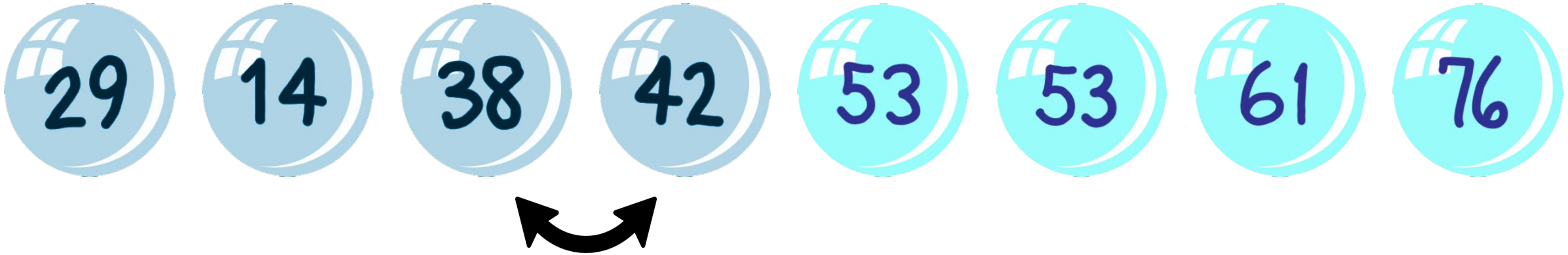

Bubble Sort

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- Bubble sort in action:

Bubble Sort

- Bubble sort in action:

Bubble Sort

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Bubble Sort

- Bubble sort in action:

Selection Sort

Selection Sort

- Characteristics of selection sort:

Best Case	Average Case	Worst Case	Memory	Stable?
$\Omega(n^2)$	$\Theta(n^2)$	$O(n^2)$	O(1)	No*

  - Good when auxiliary memory is limited.
  - Algorithm: divide input list into sub-list of sorted items and sub-list of unsorted items. Add the smallest non-selected item to the back of the sorted portion (usually by swapping by whatever was there). This increases the sorted portion's size. Repeat until the whole array is sorted.
    - in other words, find the smallest element in the list and swap with the first position, find the second smallest element and swap with second position...
  - Additional memory can make the sort stable (break ties using original index)

Selection Sort

- Selection sort in action:

Selection Sort

- Selection sort in action:

the gray shading indicates
the minimum value seen
so far for each traversal

Selection Sort

- Selection sort in action:

Selection Sort

- Selection sort in action:

Selection Sort

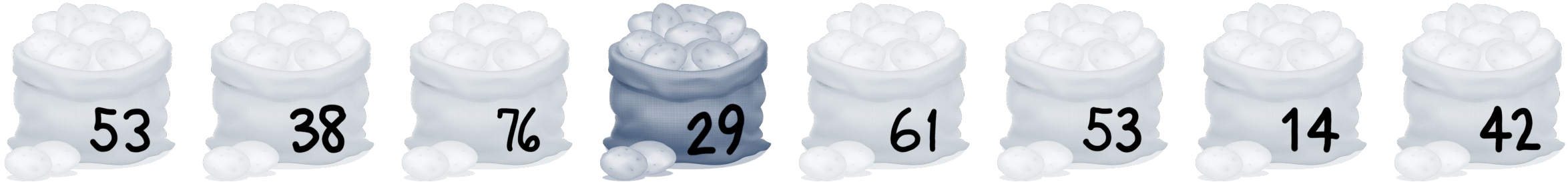
- Selection sort in action:

Selection Sort

- Selection sort in action:

Selection Sort

- Selection sort in action:



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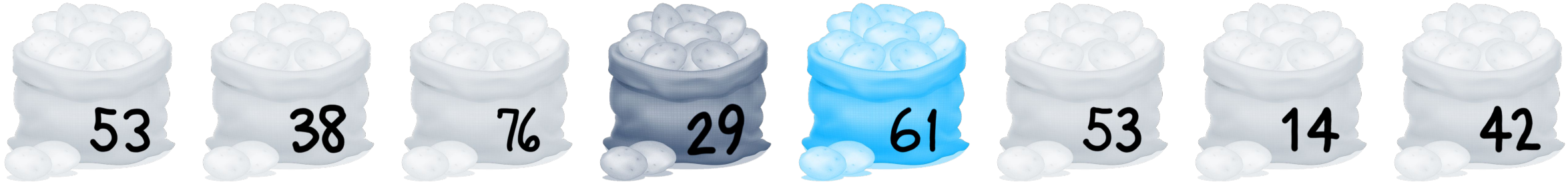

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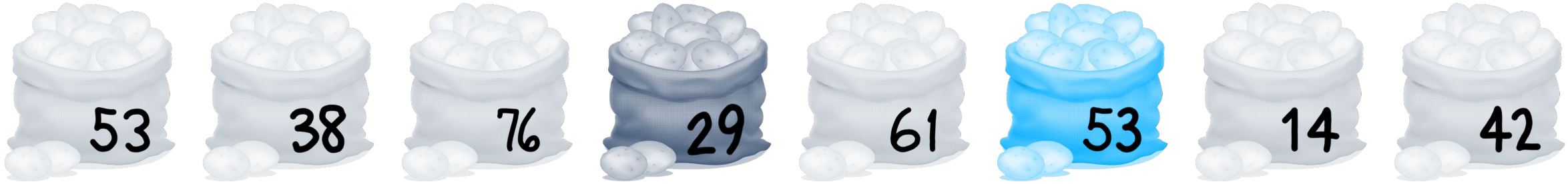

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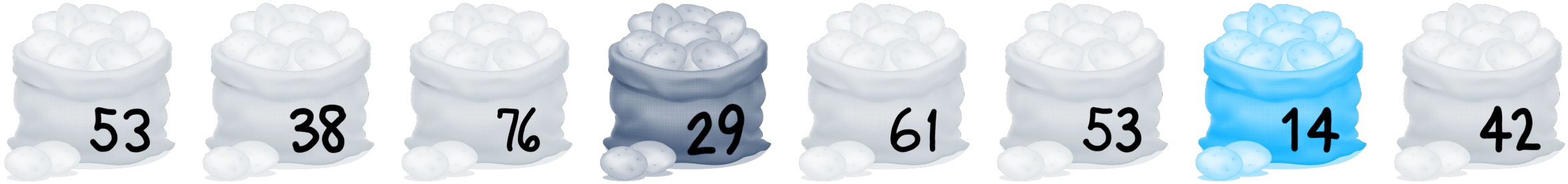

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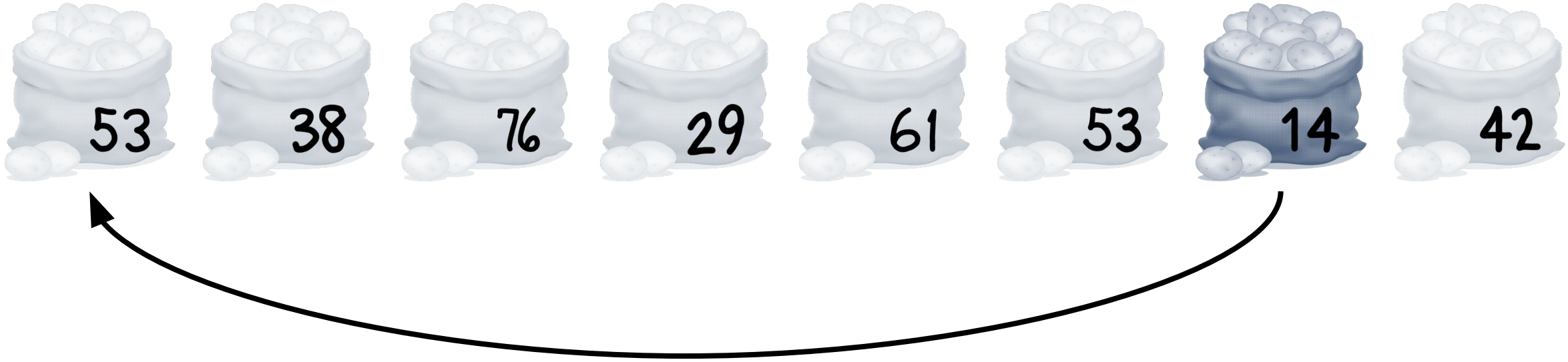

find smallest and send to first index, find second smallest and send to second index, etc.

Selection Sort

- Selection sort in action:

Selection Sort

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Selection Sort

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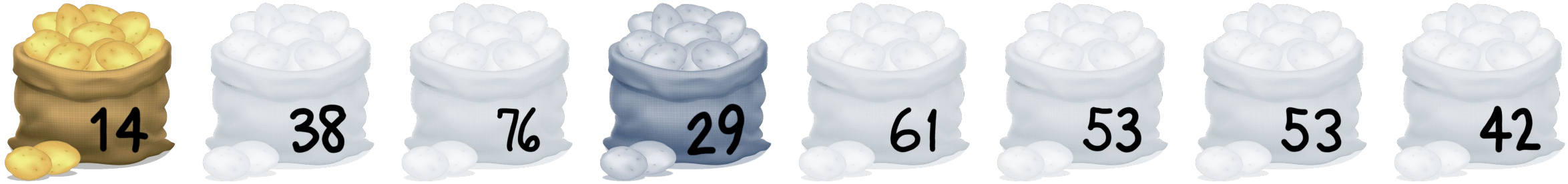

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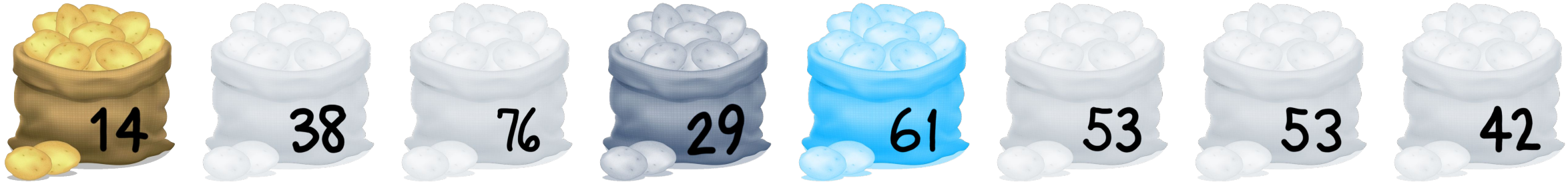

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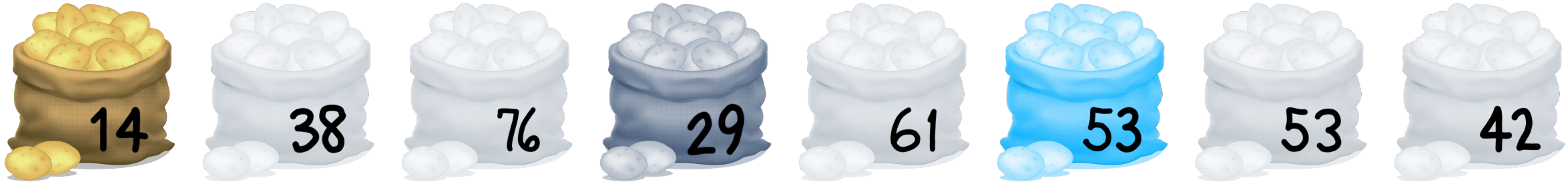

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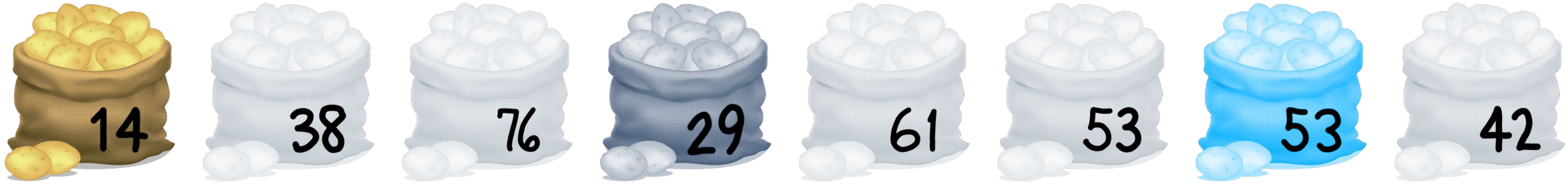

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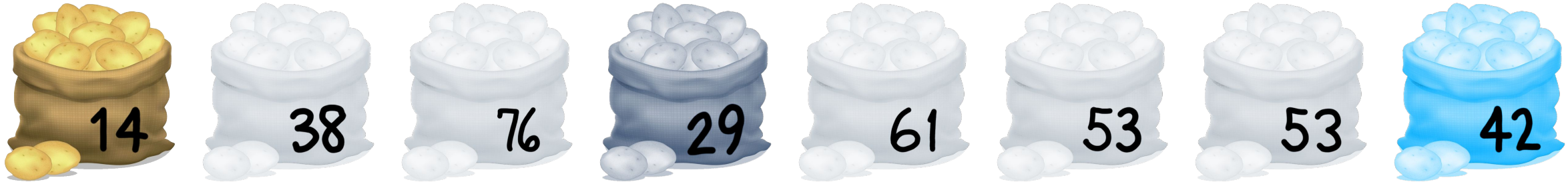

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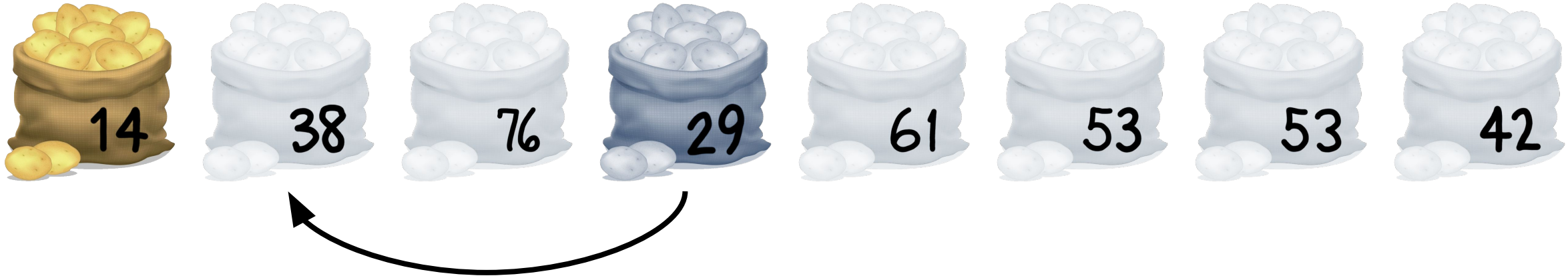

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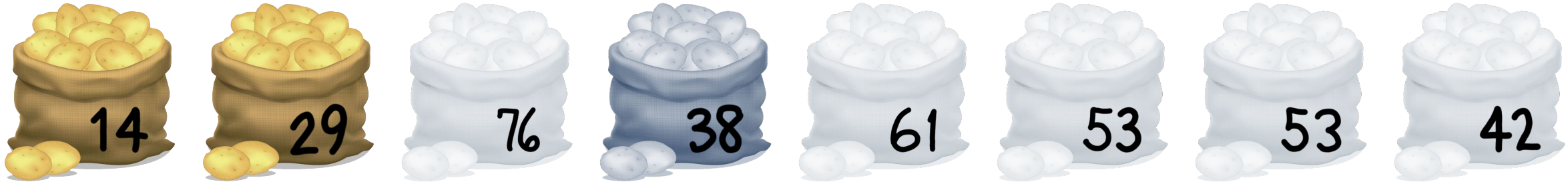

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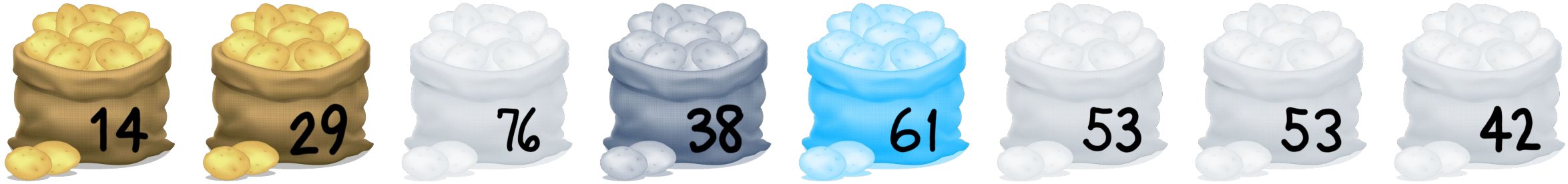

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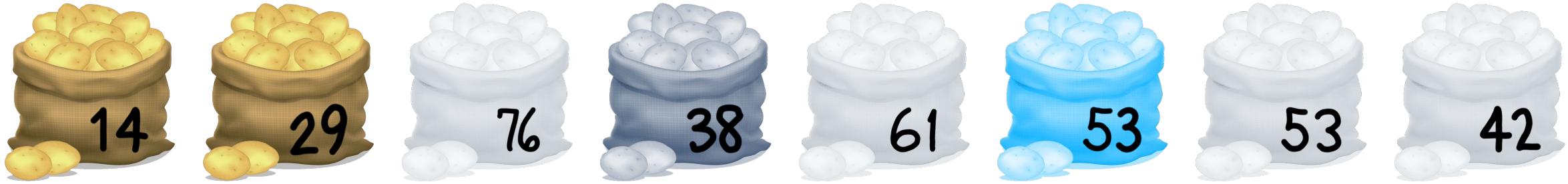

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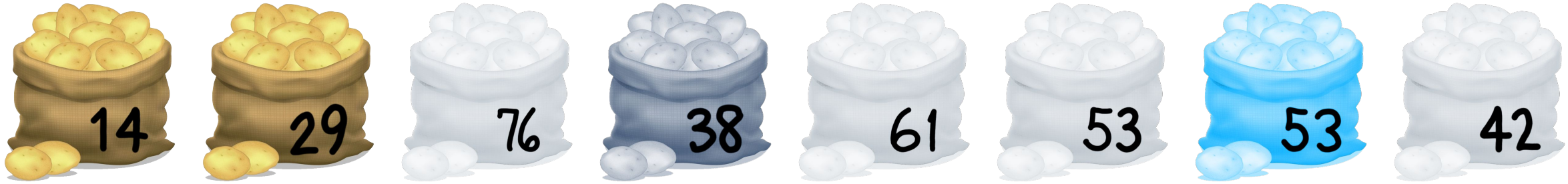

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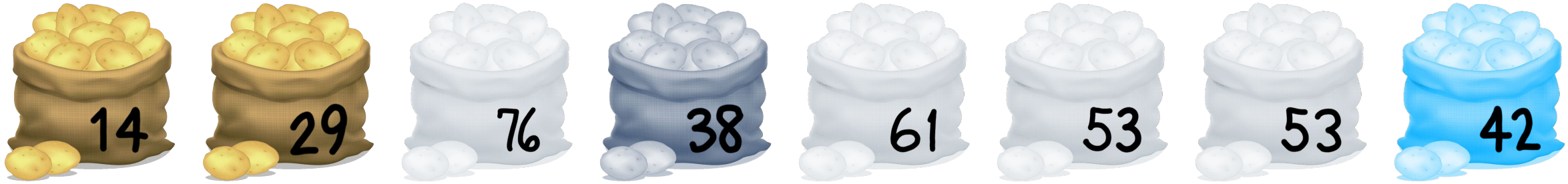

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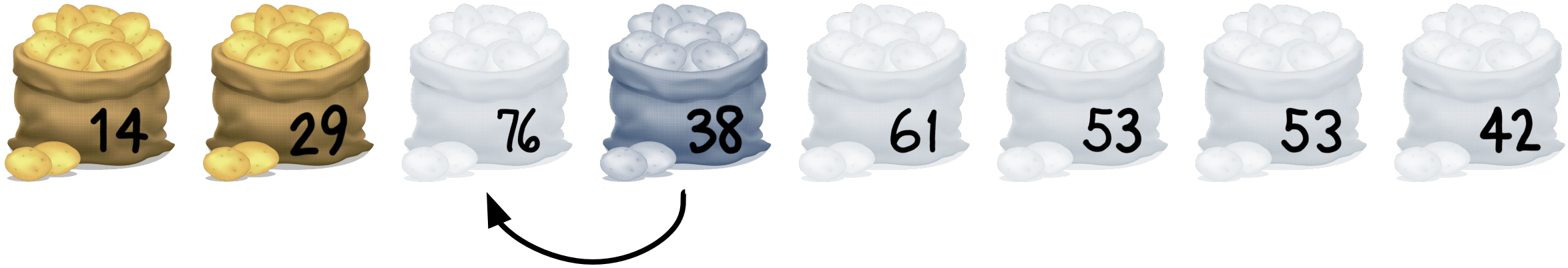

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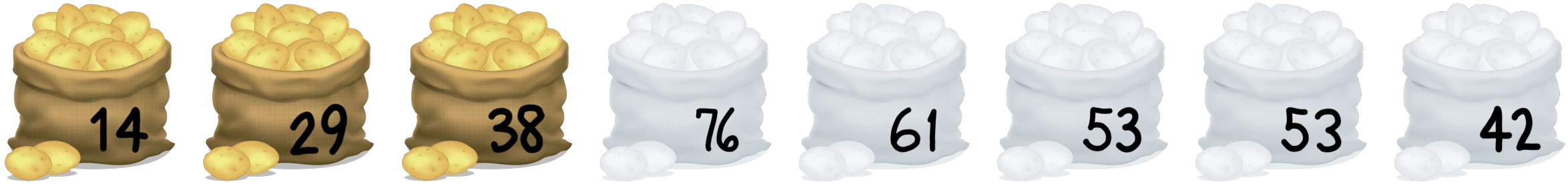

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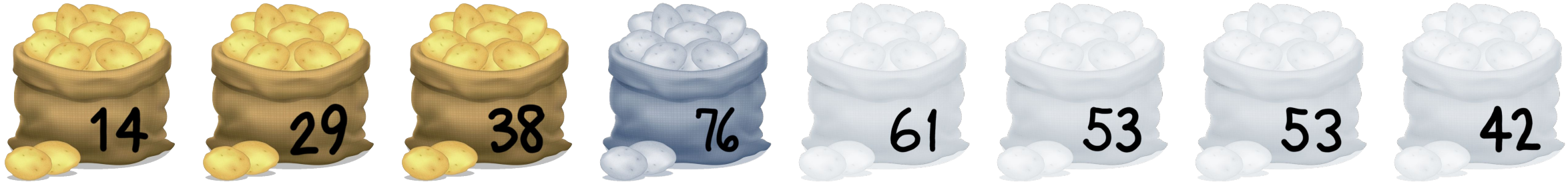

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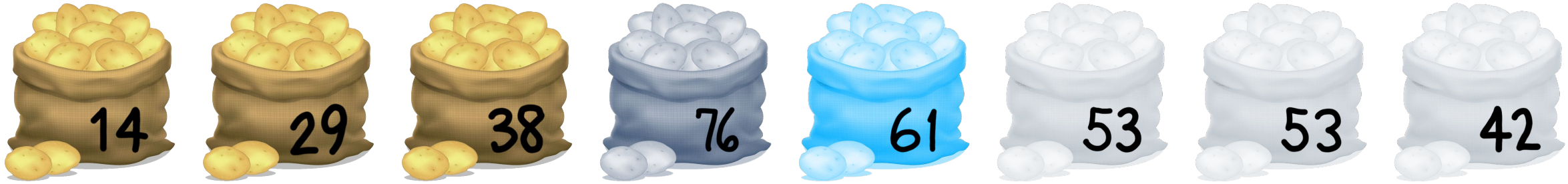

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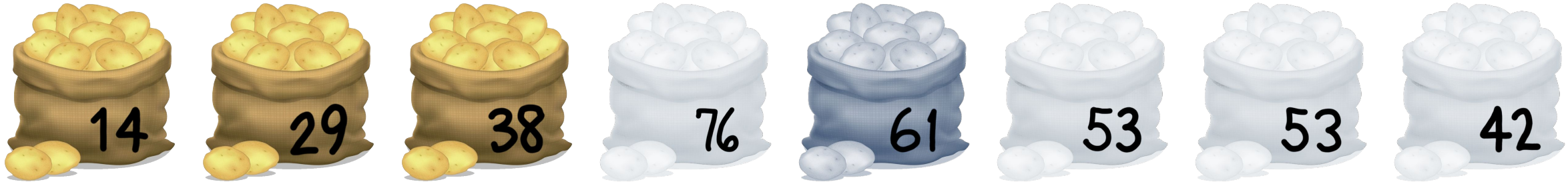

Selection Sort

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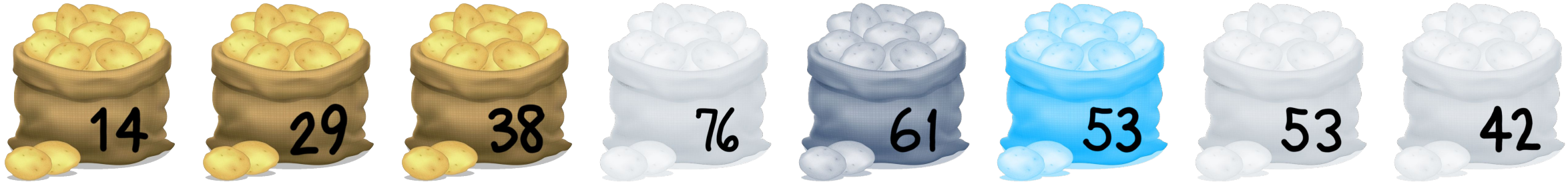

Selection Sort

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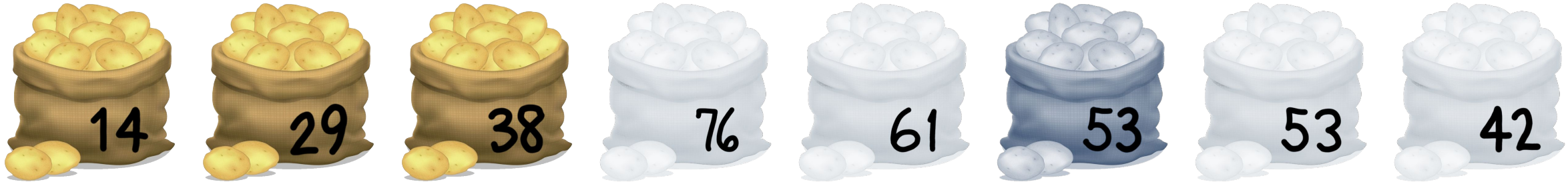

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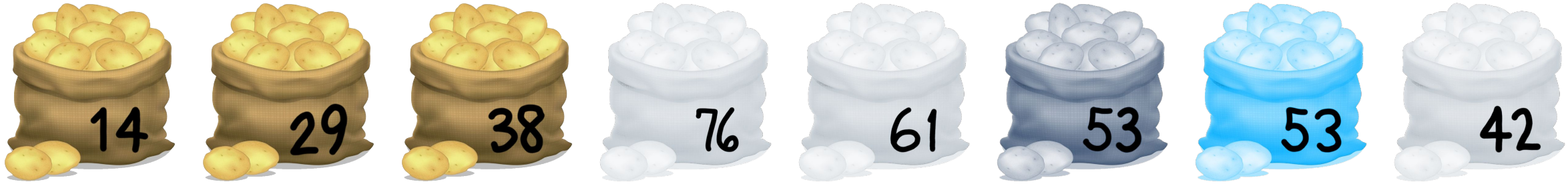

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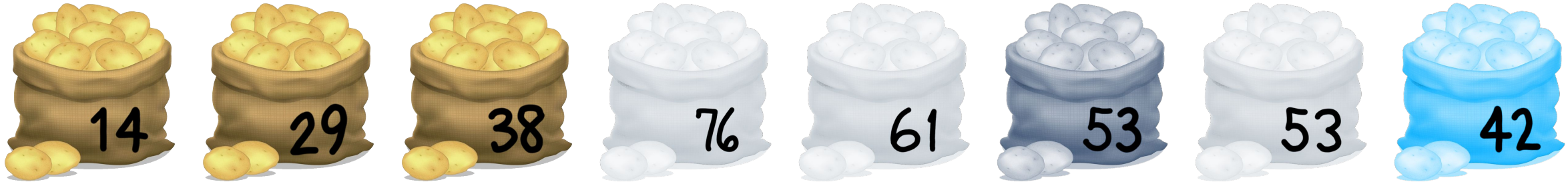

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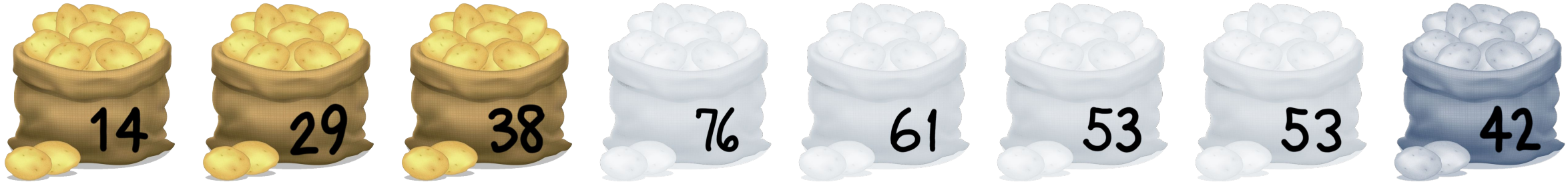

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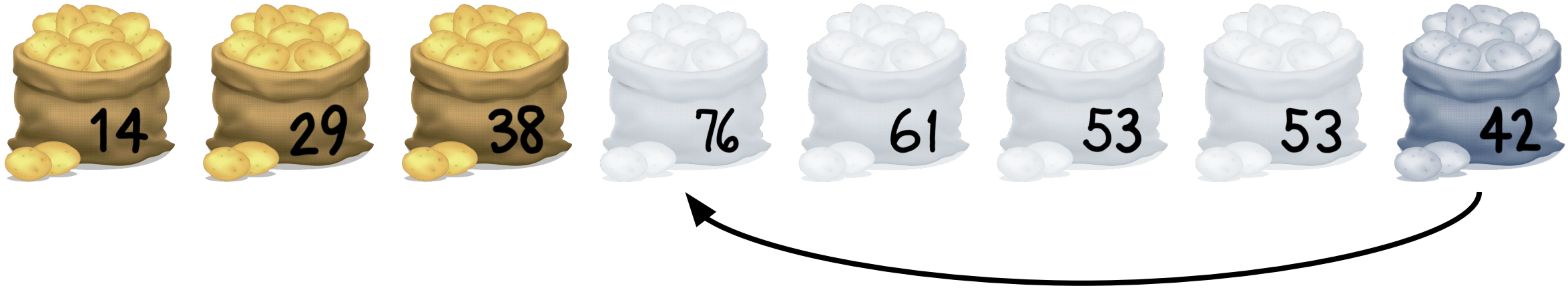

Selection Sort

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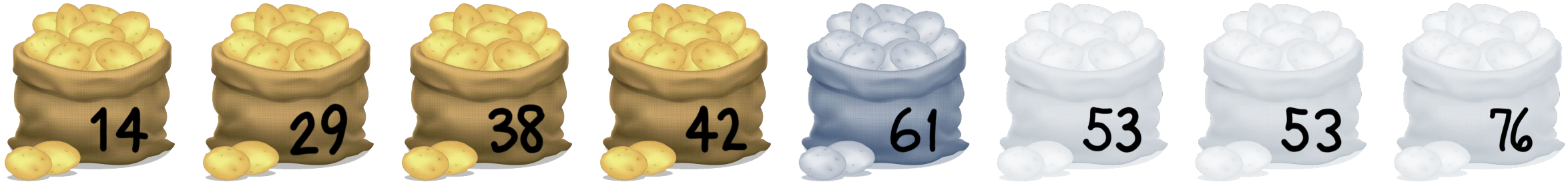

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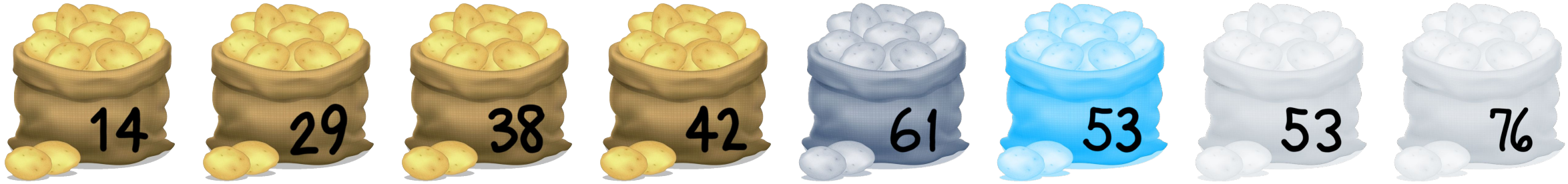

Selection Sort

- Selection sort in action:

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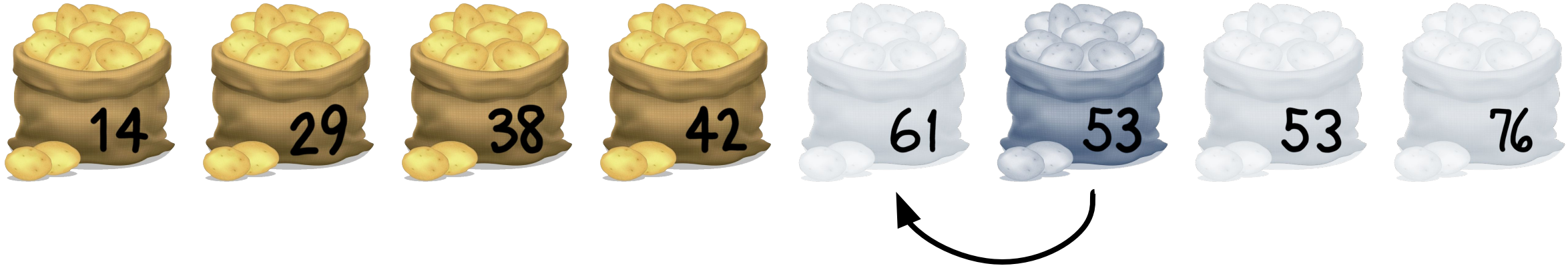
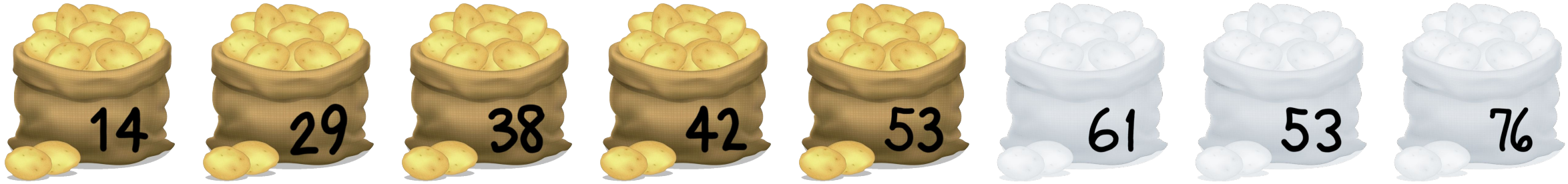


NOTE: here, selection sort is not stable! The current 53 was actually **after** the other 53 prior to sorting, but after the sort, it will come **before** the other 53.

Selection Sort

- Selection sort in action:



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Selection Sort

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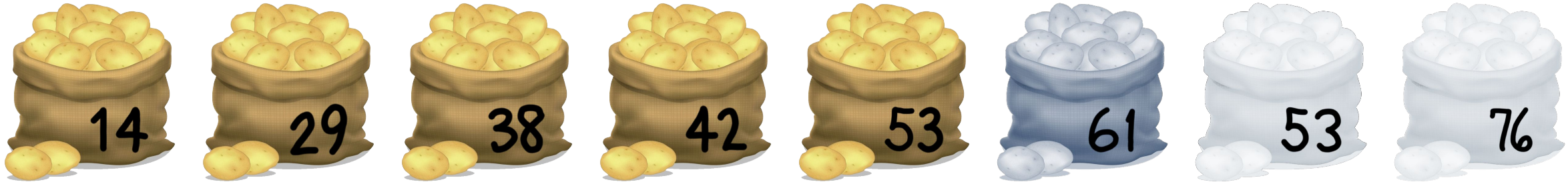

Selection Sort

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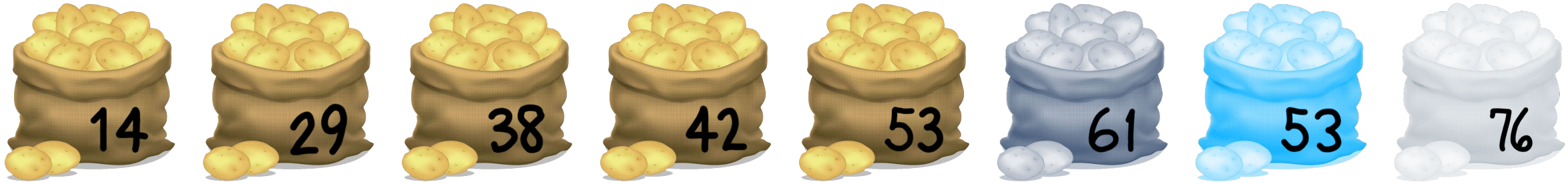

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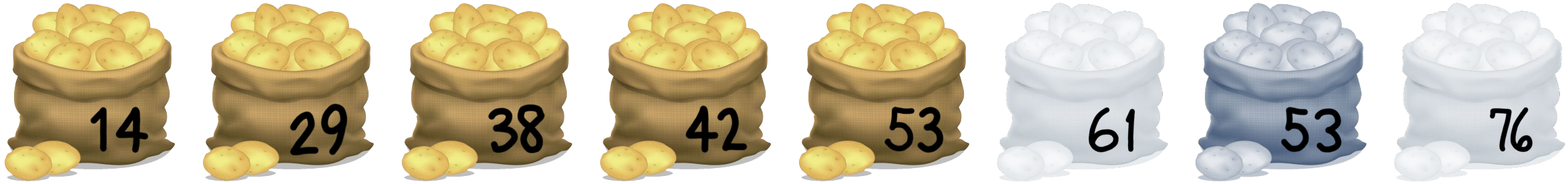

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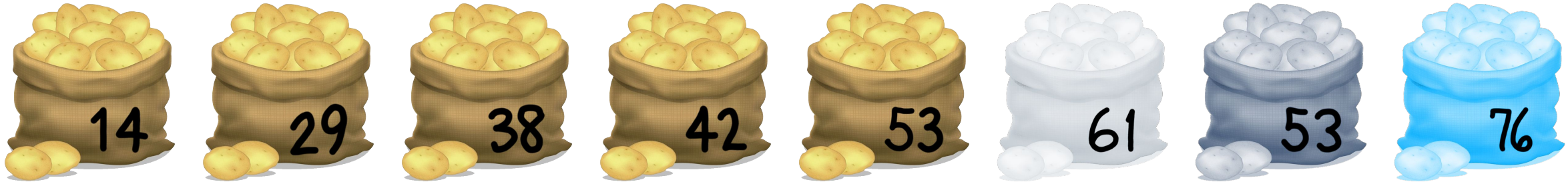

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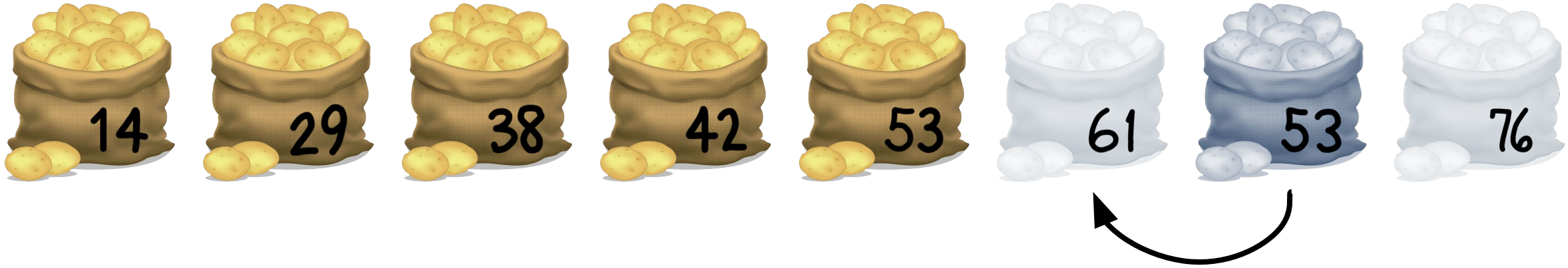

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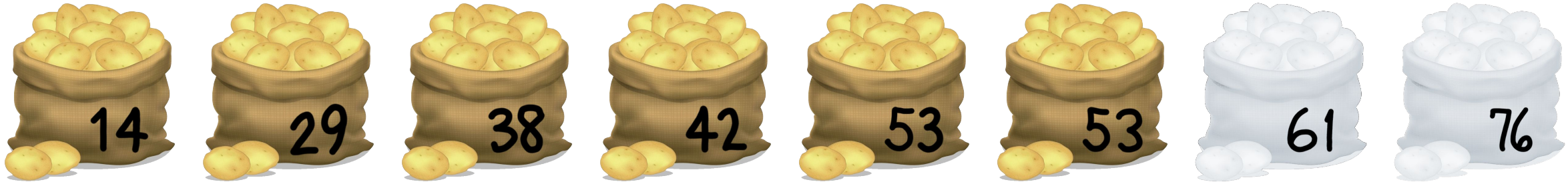

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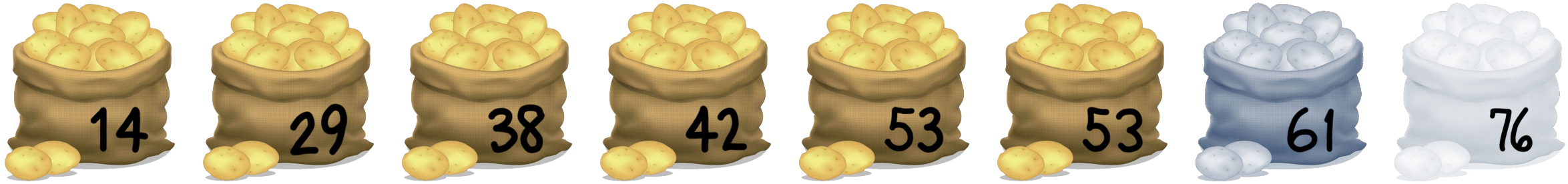

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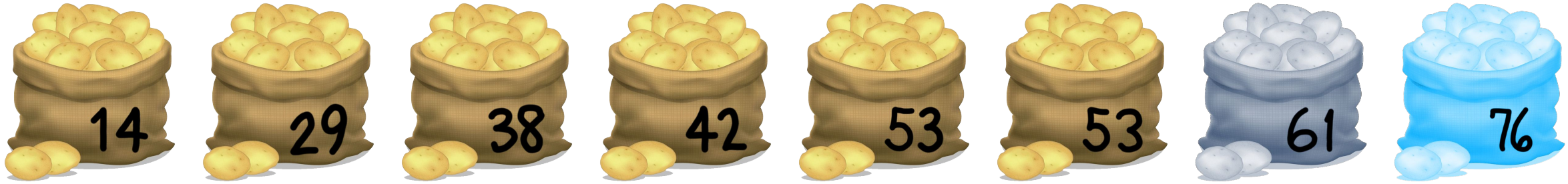

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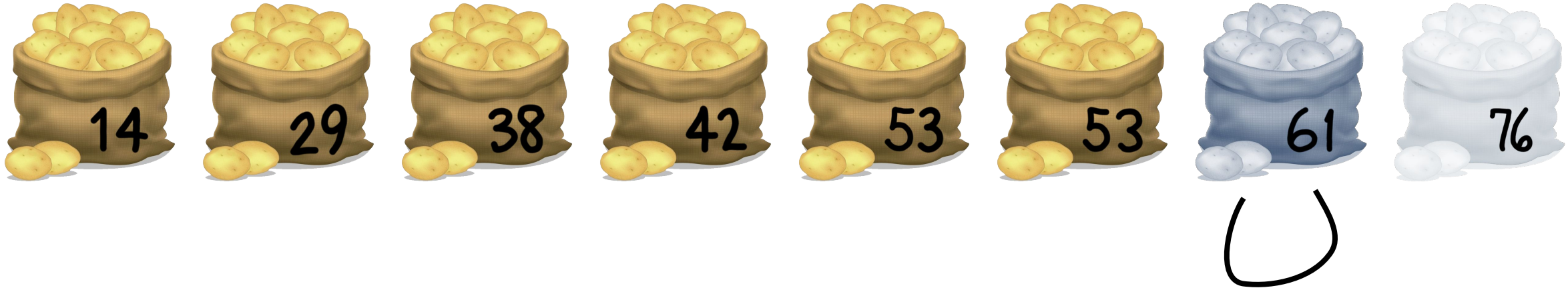

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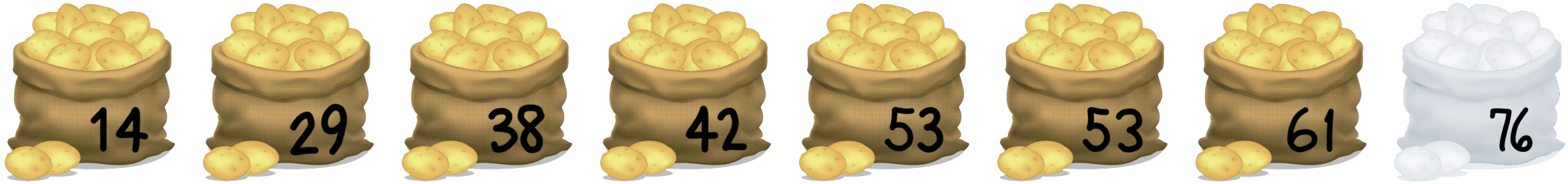

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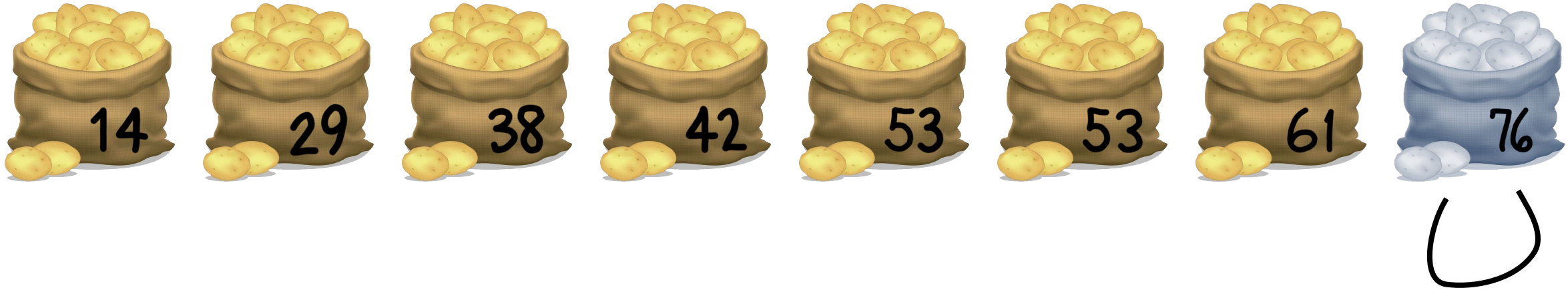

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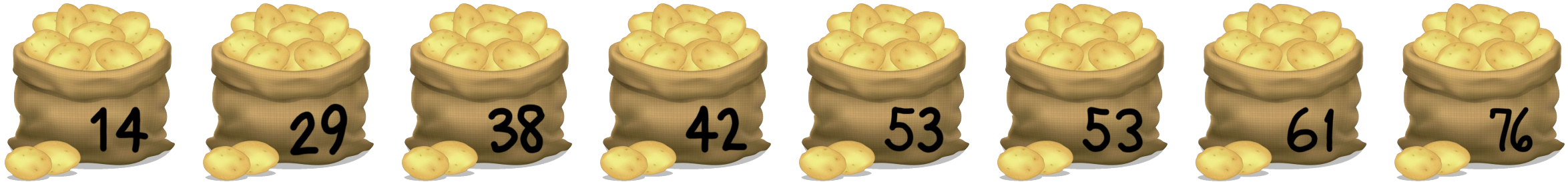


Insertion Sort

Insertion Sort

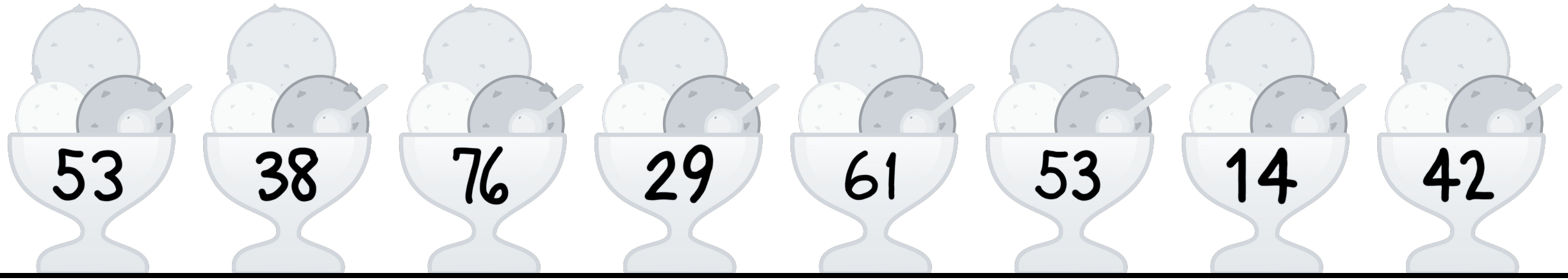
- Characteristics of insertion sort:

Best Case	Average Case	Worst Case	Memory	Stable?
$\Omega(n)$	$\Theta(n^2)$	$O(n^2)$	$O(1)$	Yes

  - Very efficient on “almost” sorted list!
  - One of the fastest sorting algorithms on small input sizes.
  - Algorithm: take each item and “insert” it into the correct location when only considering all elements before it (i.e. swap left until it is where it belongs).

Insertion Sort

- Insertion sort in action:



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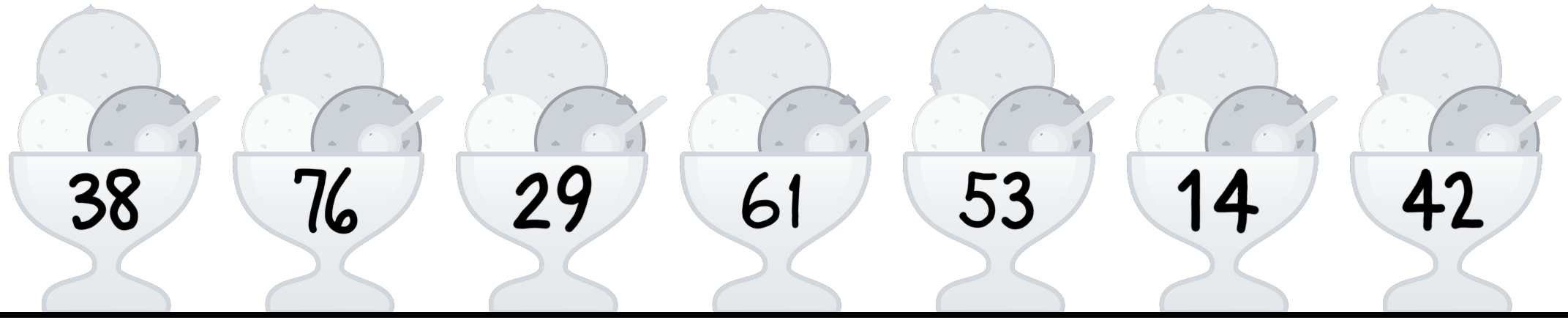

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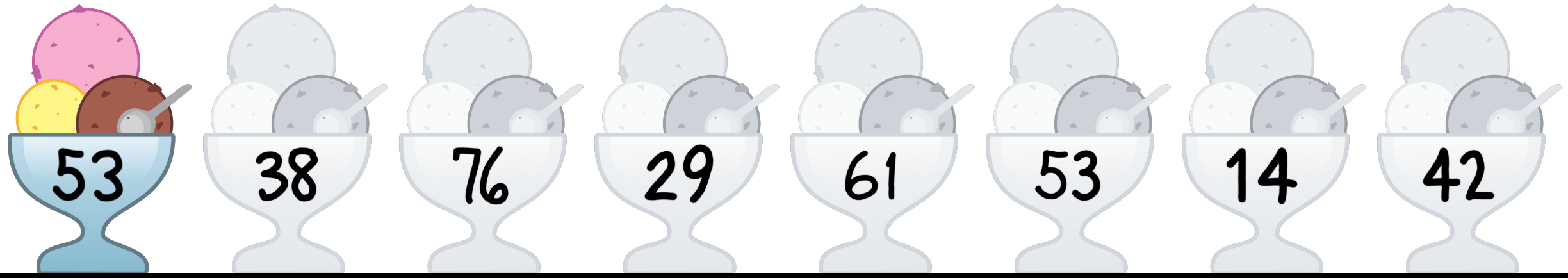

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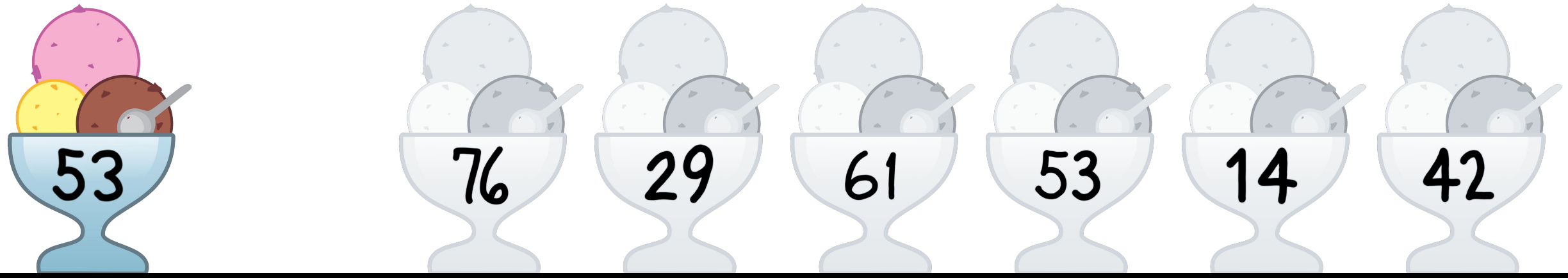

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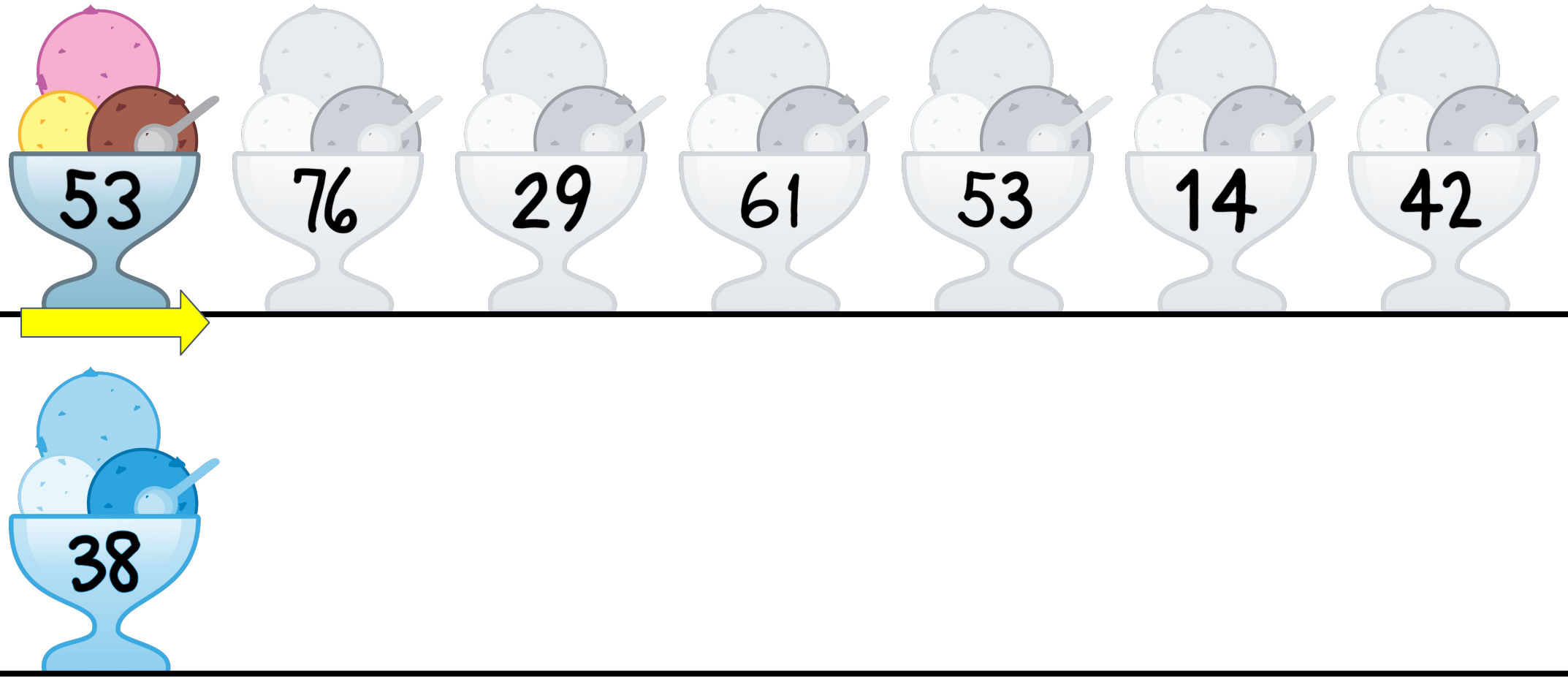

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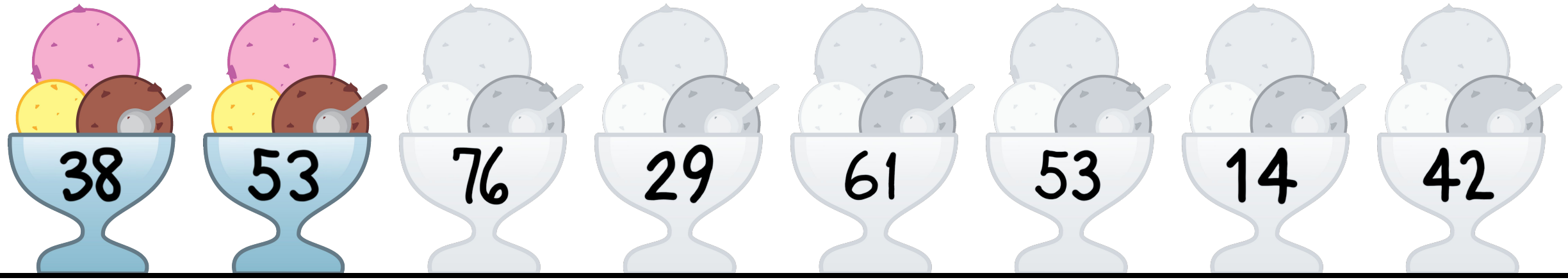

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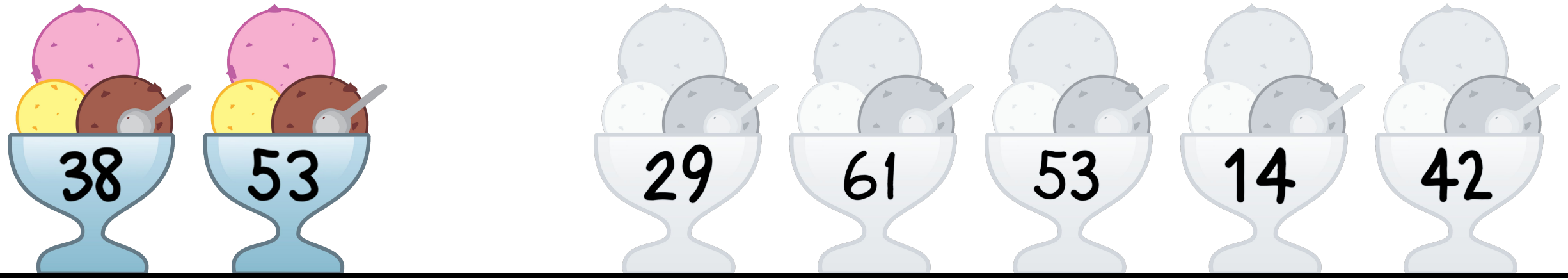

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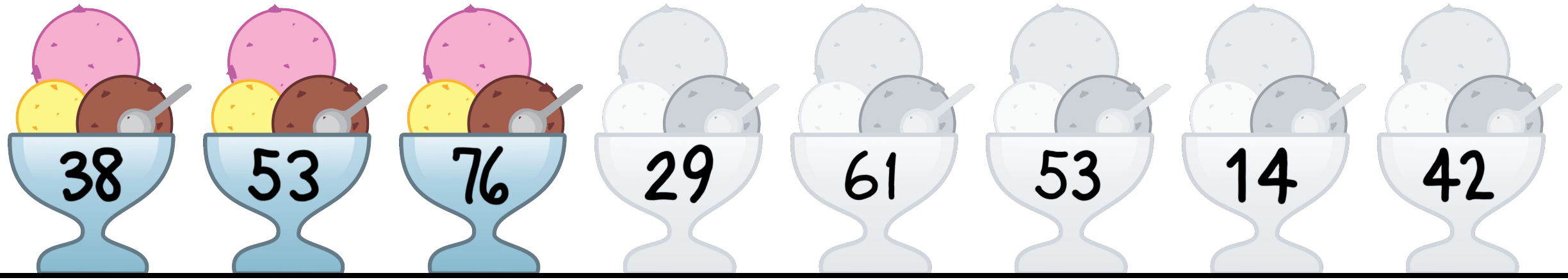

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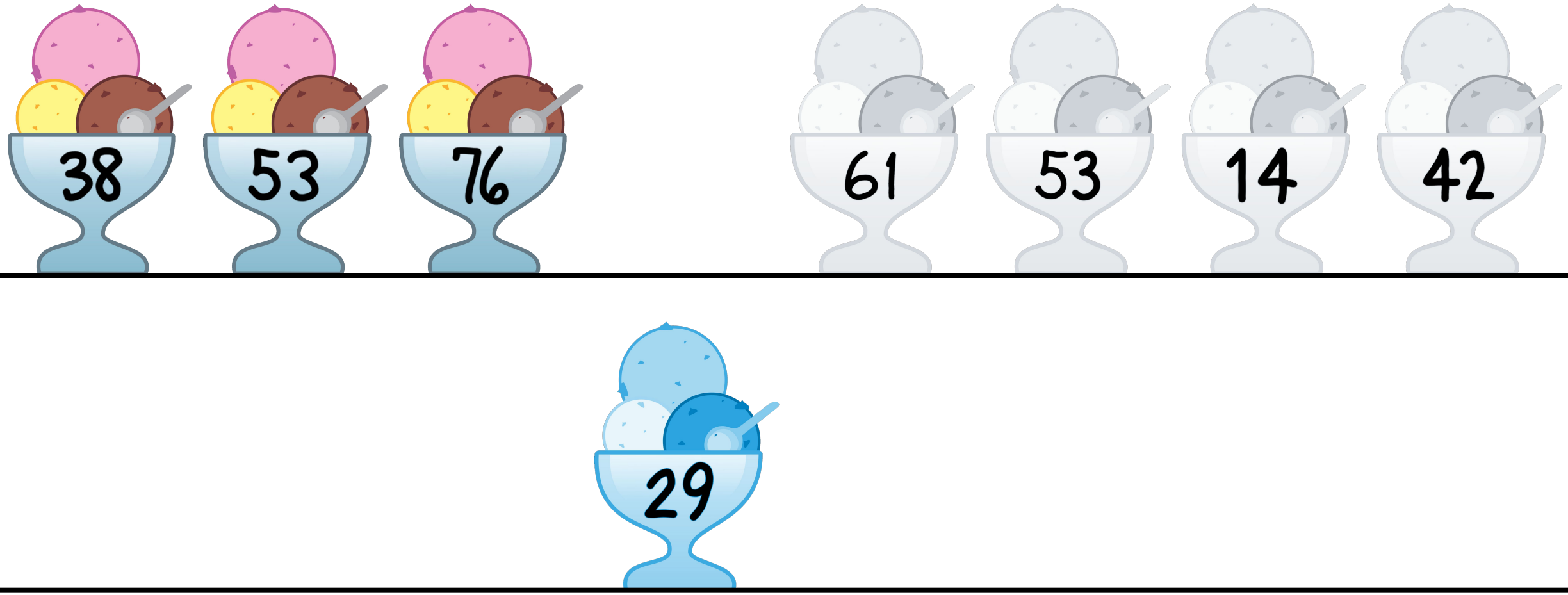

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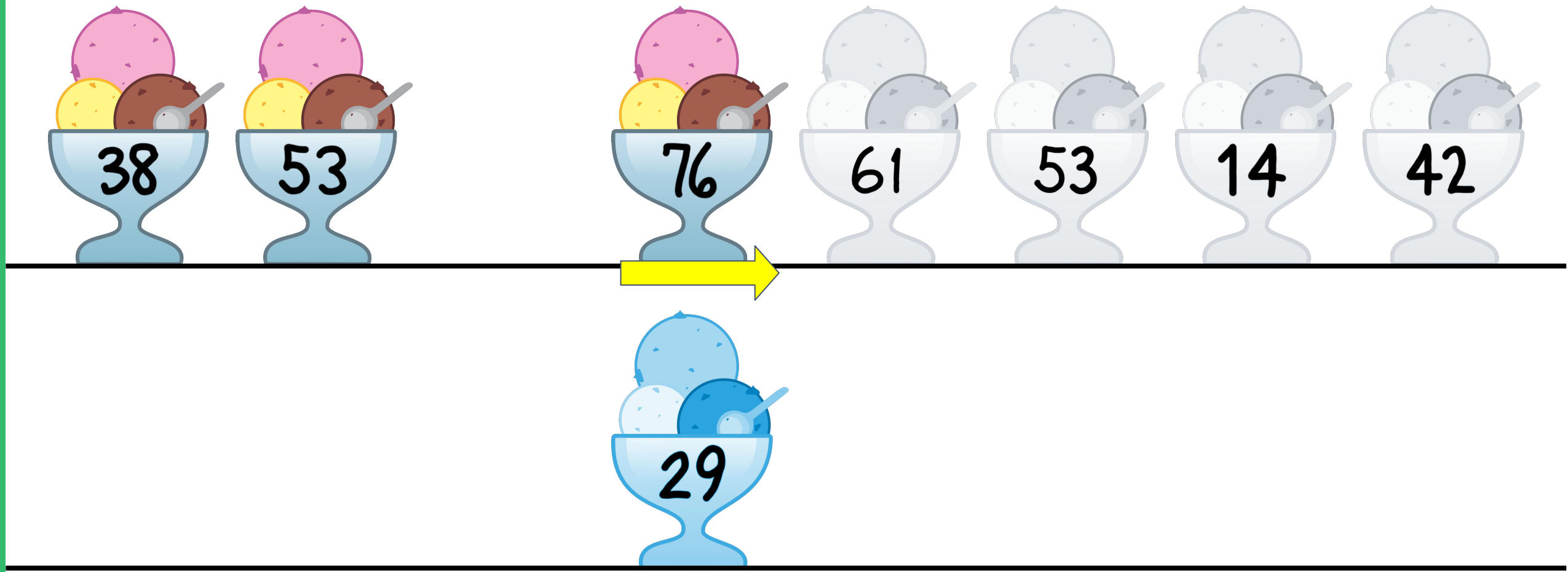

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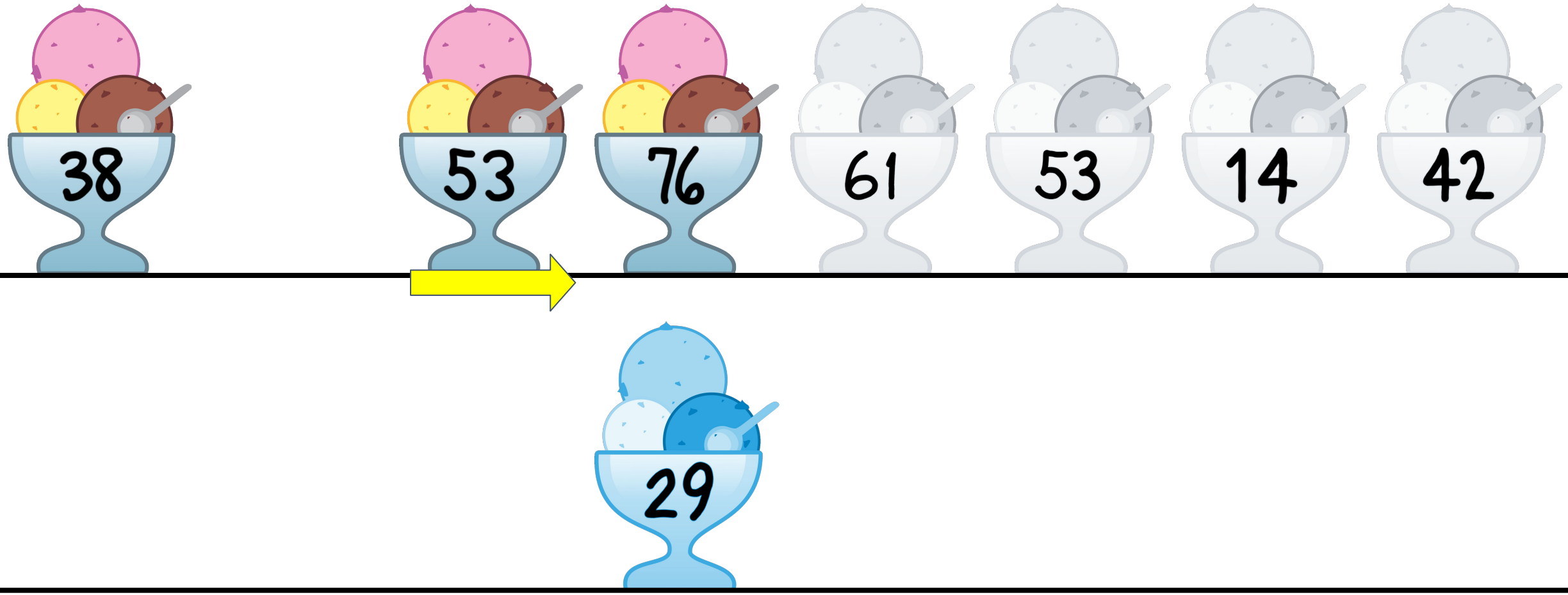

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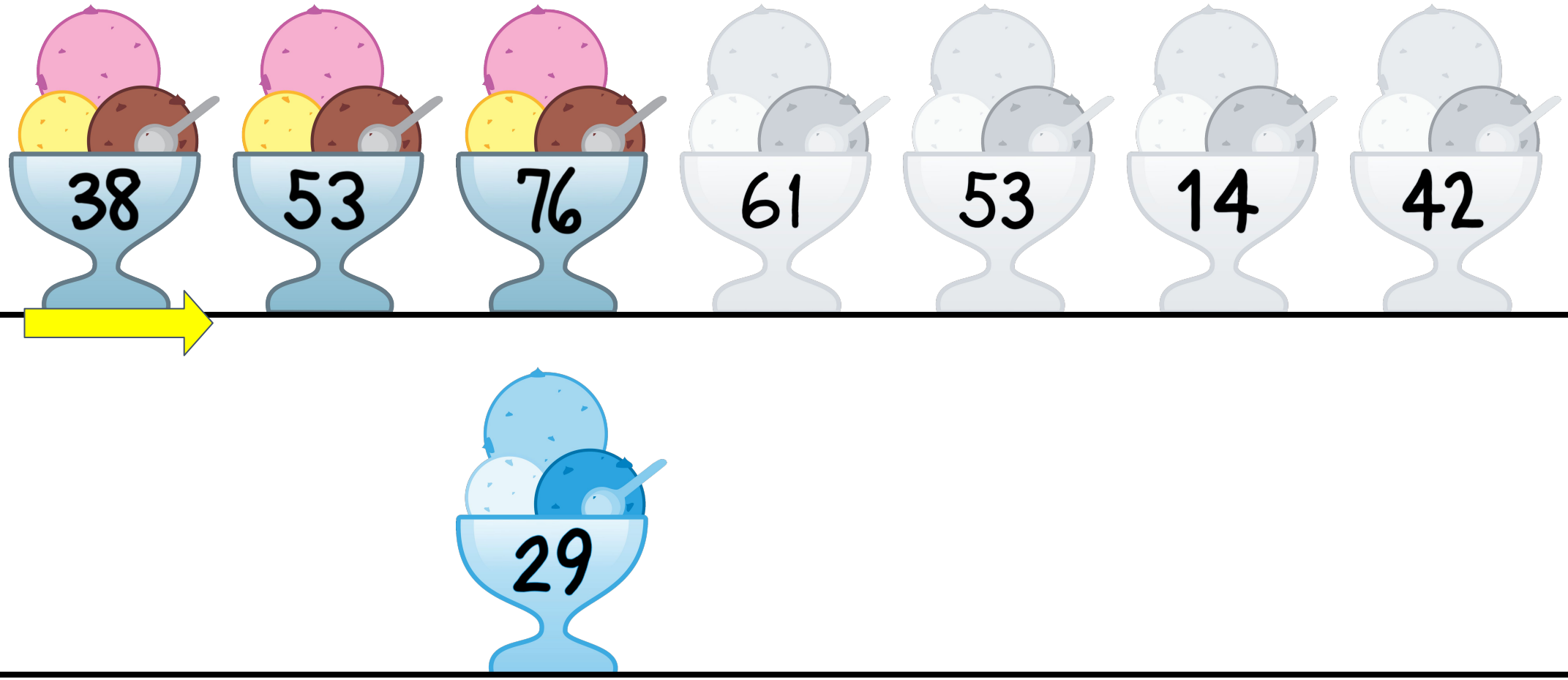

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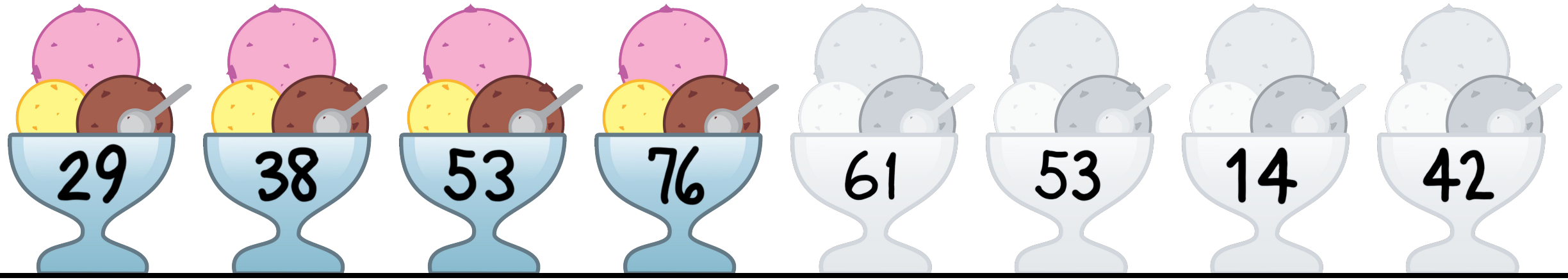

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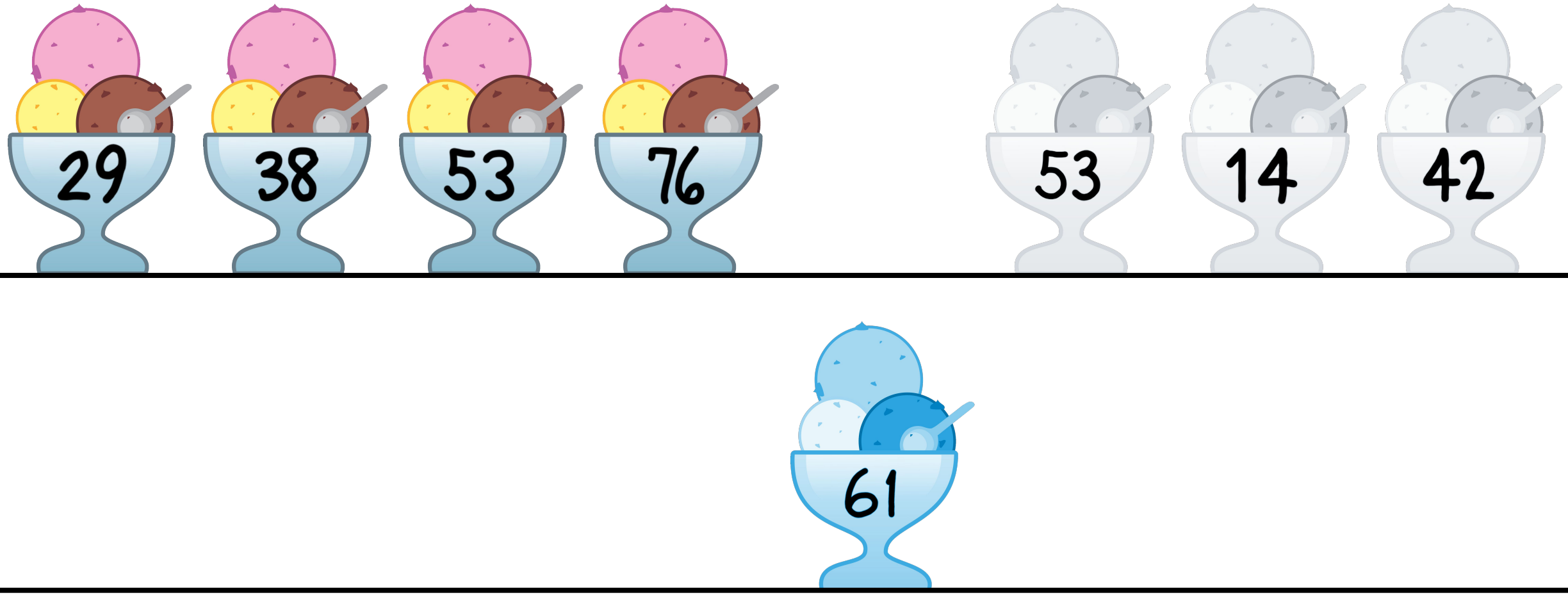

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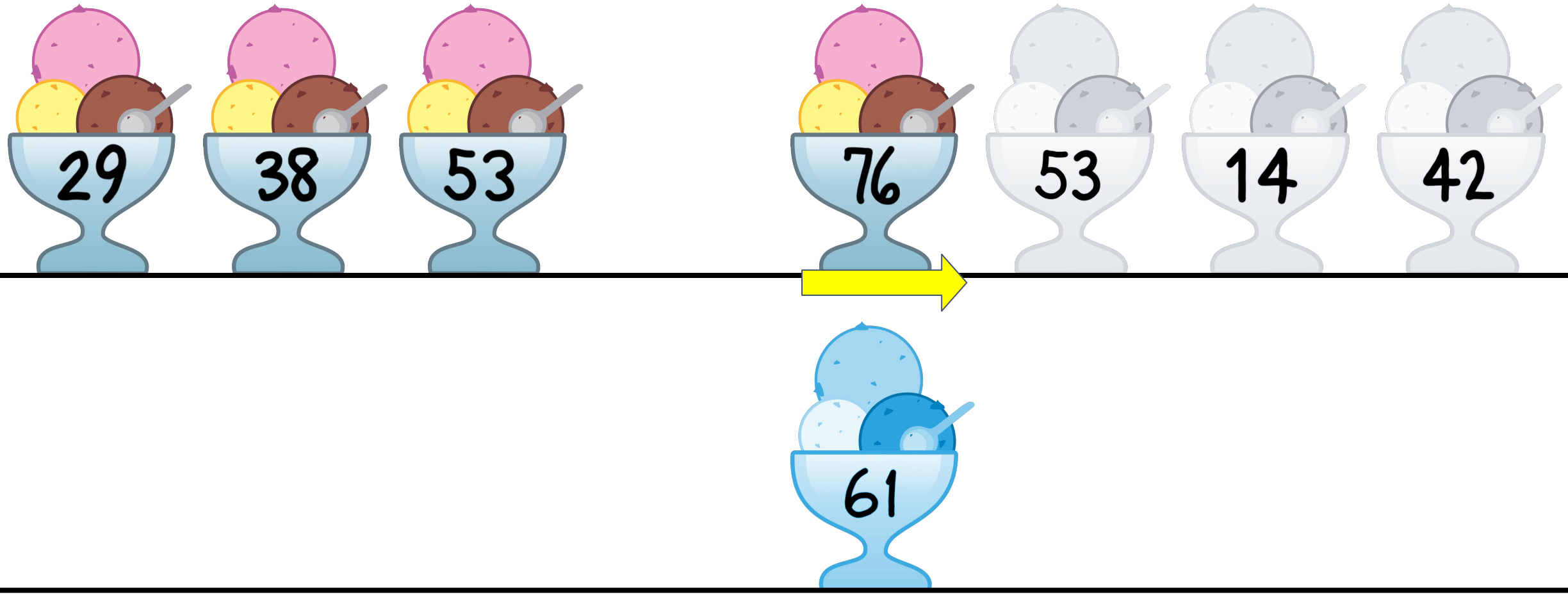

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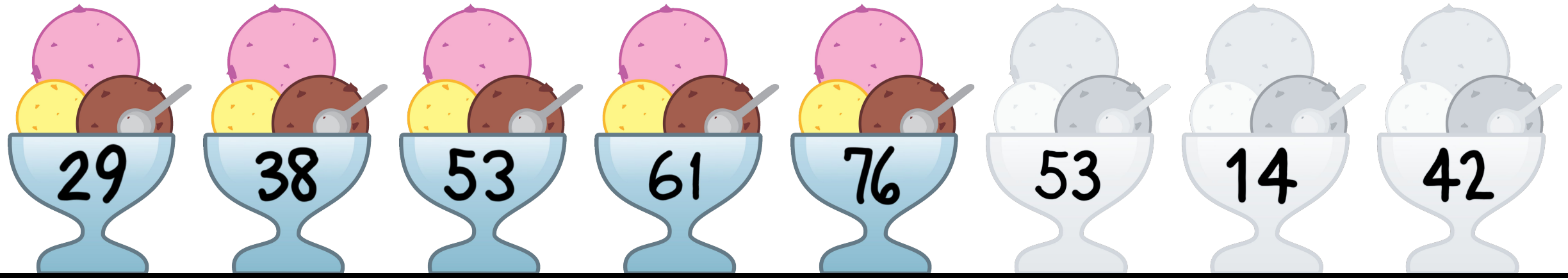

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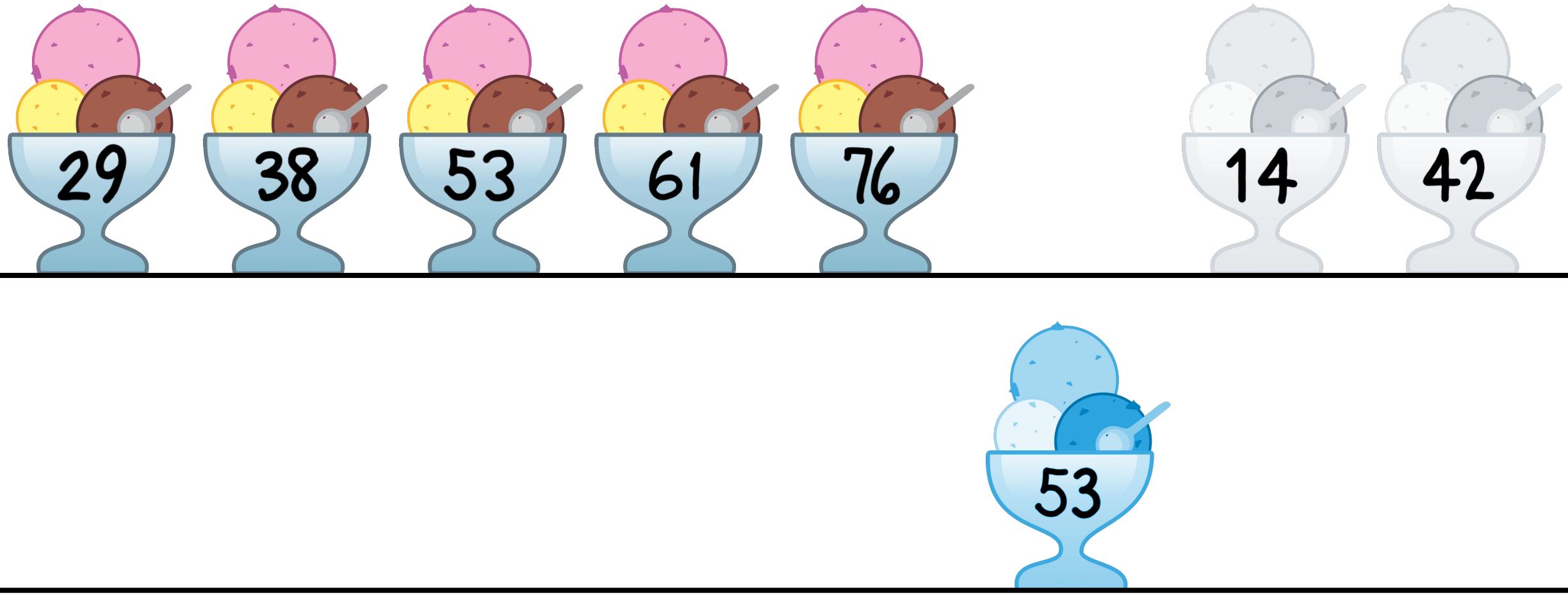

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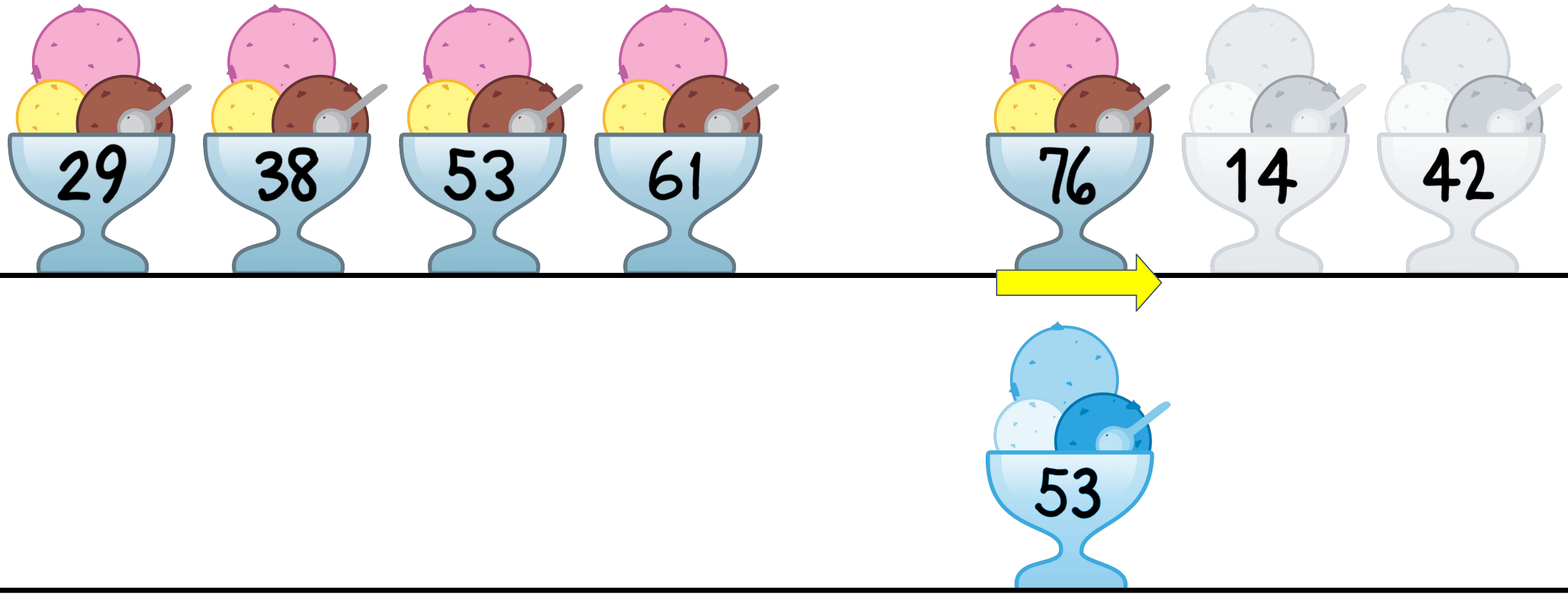

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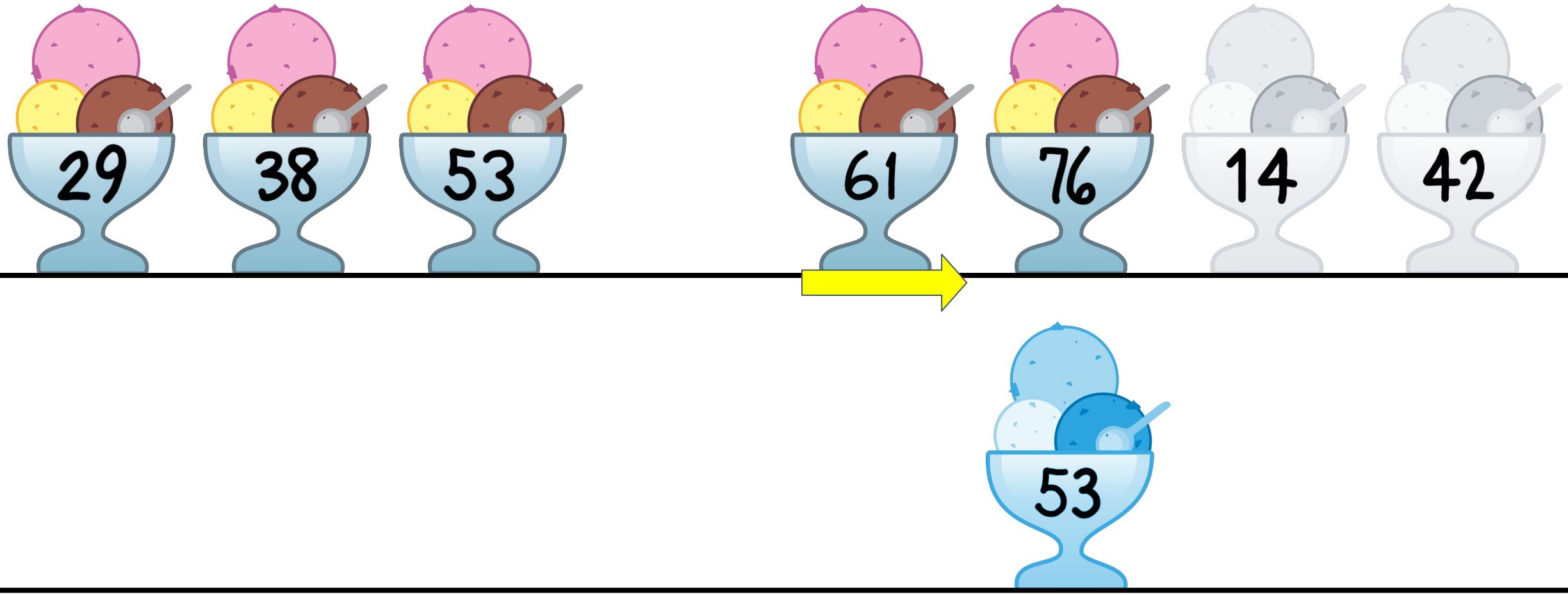

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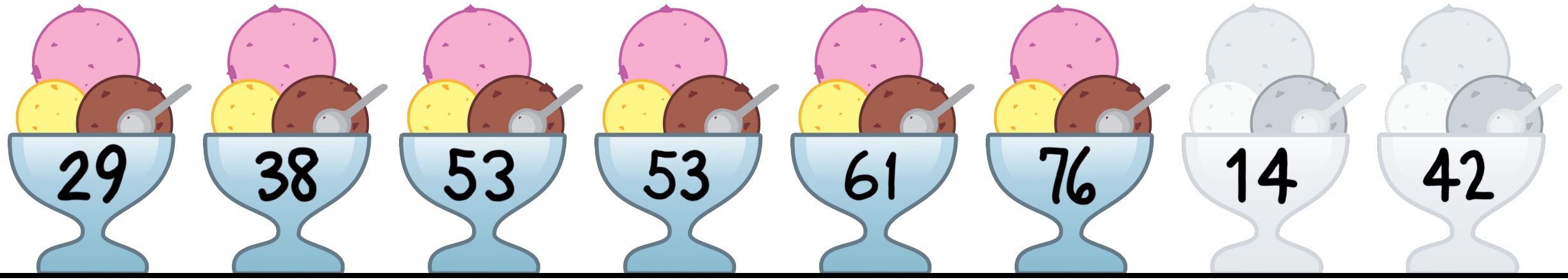

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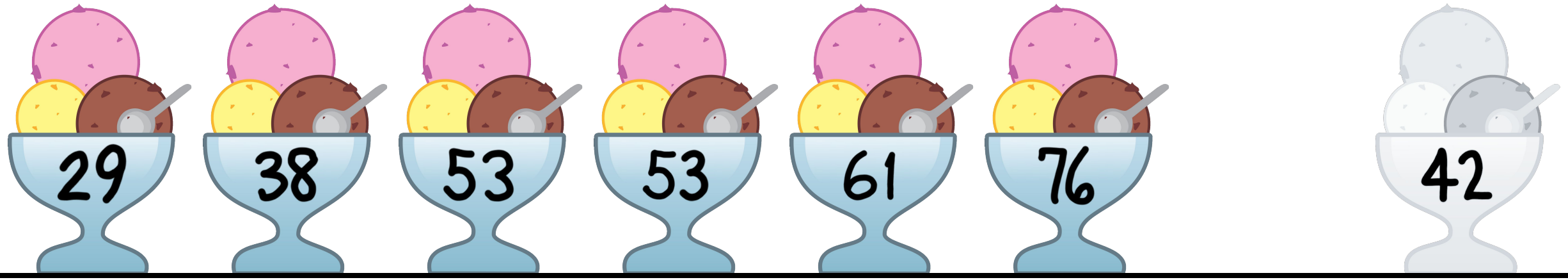

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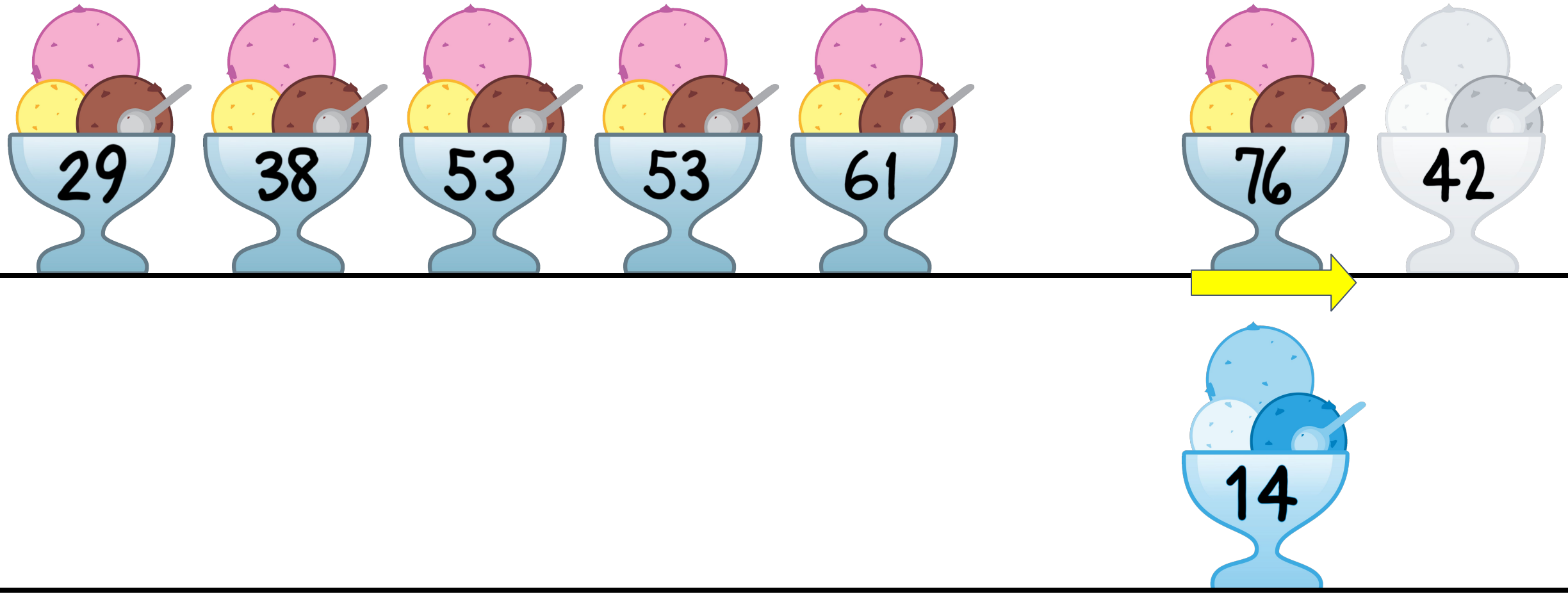

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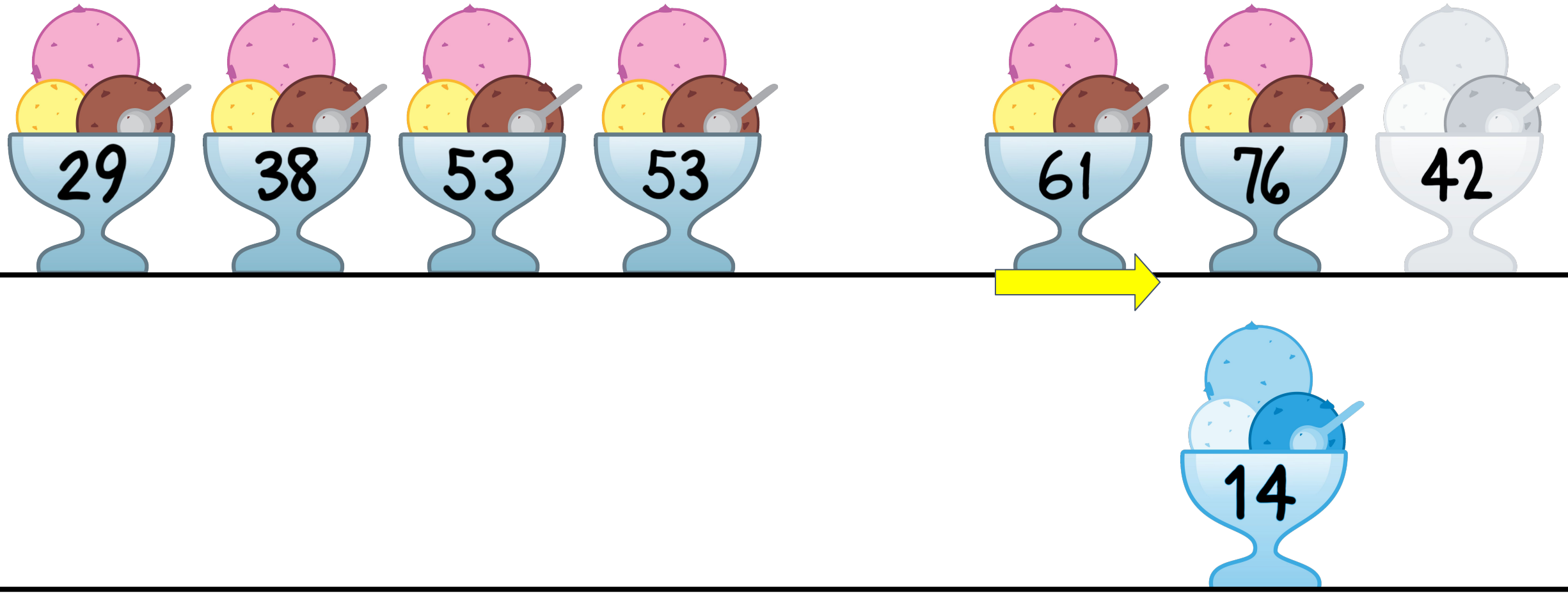

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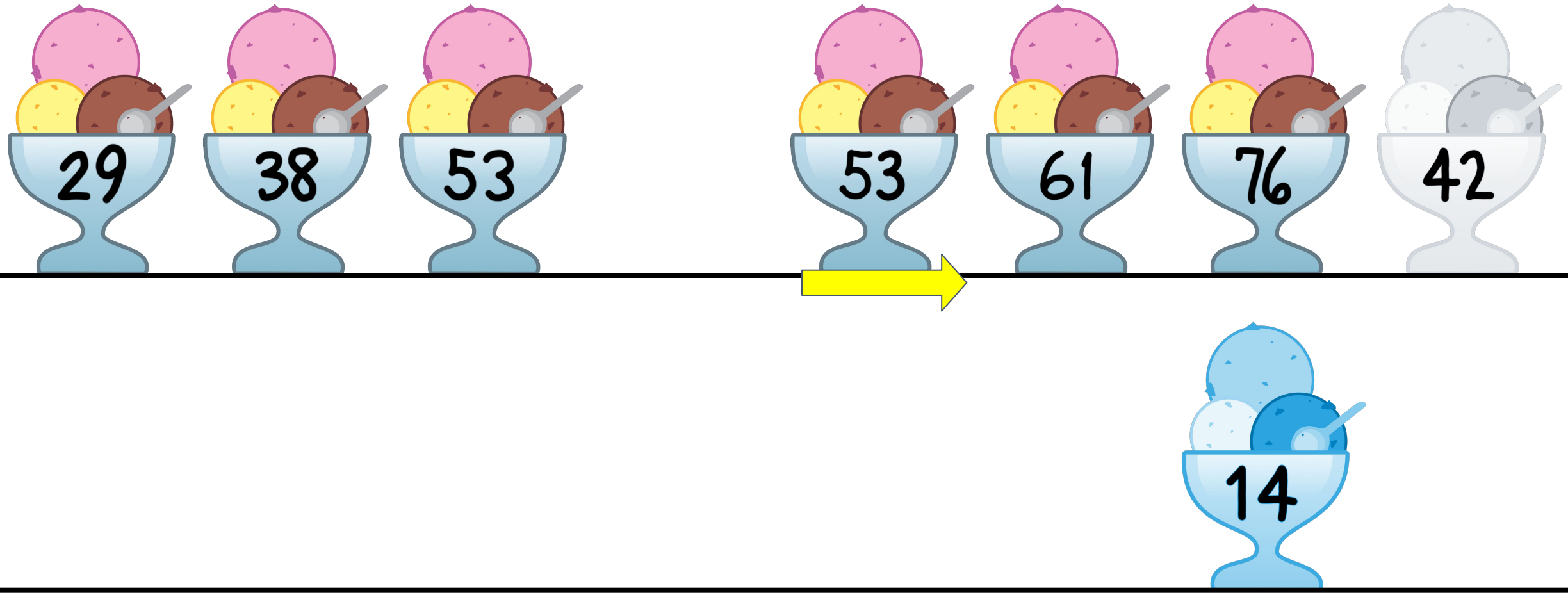

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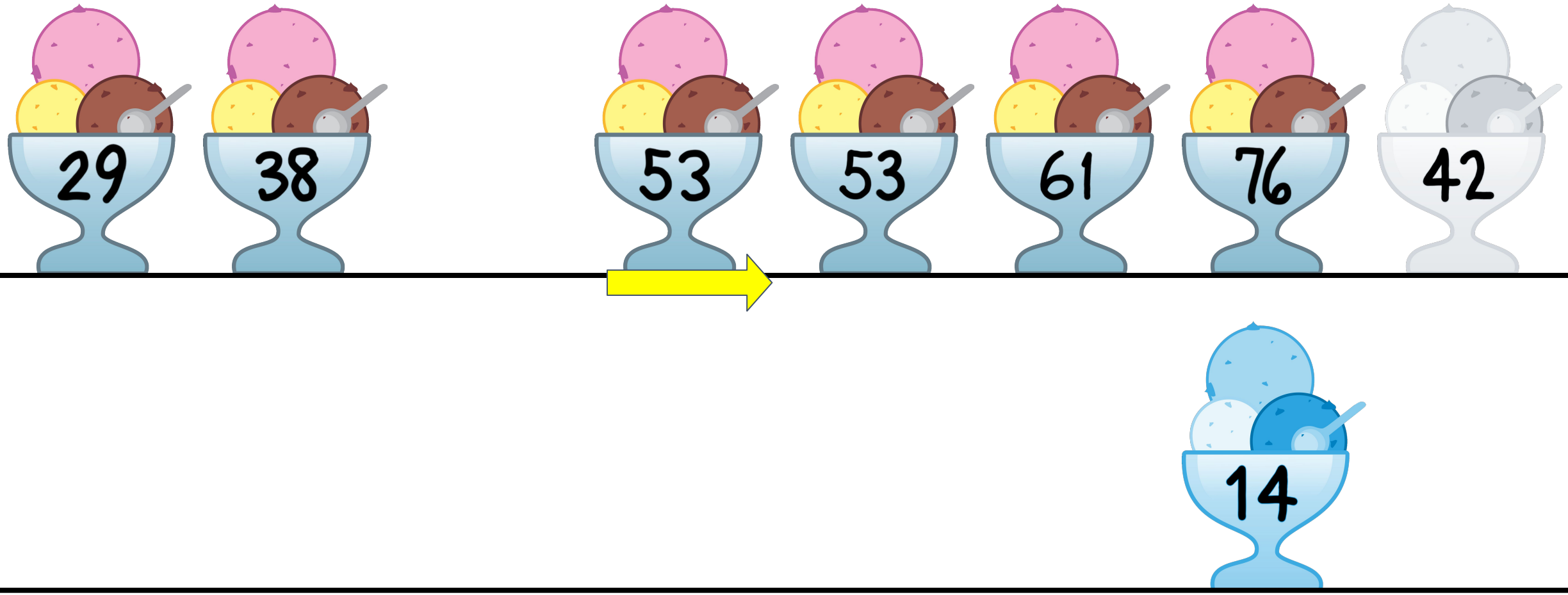

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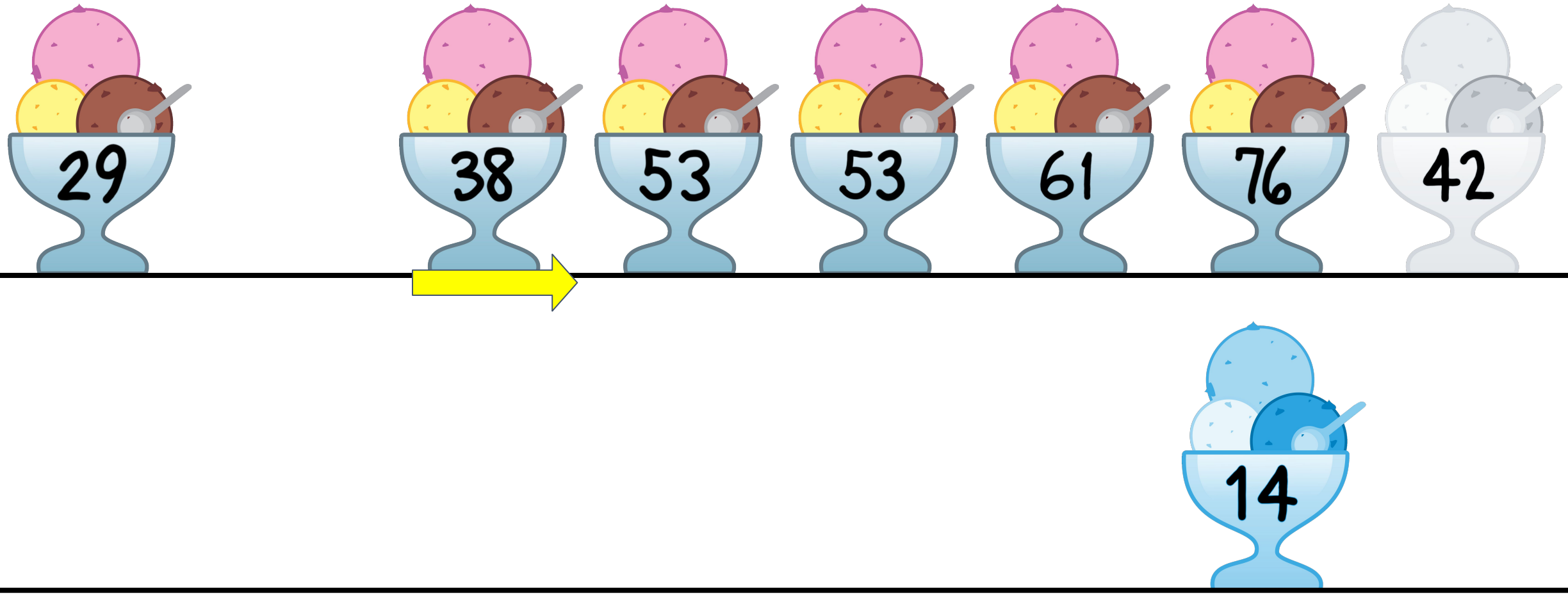
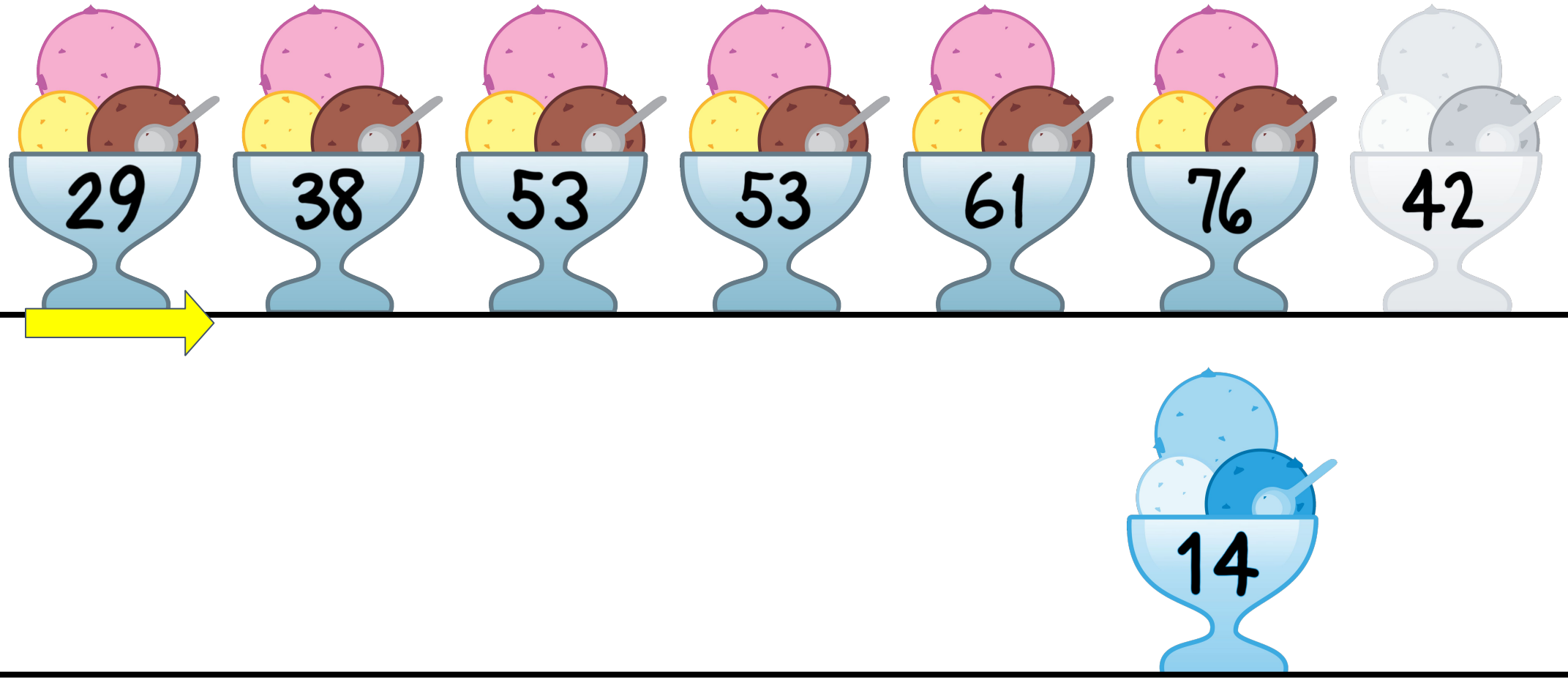


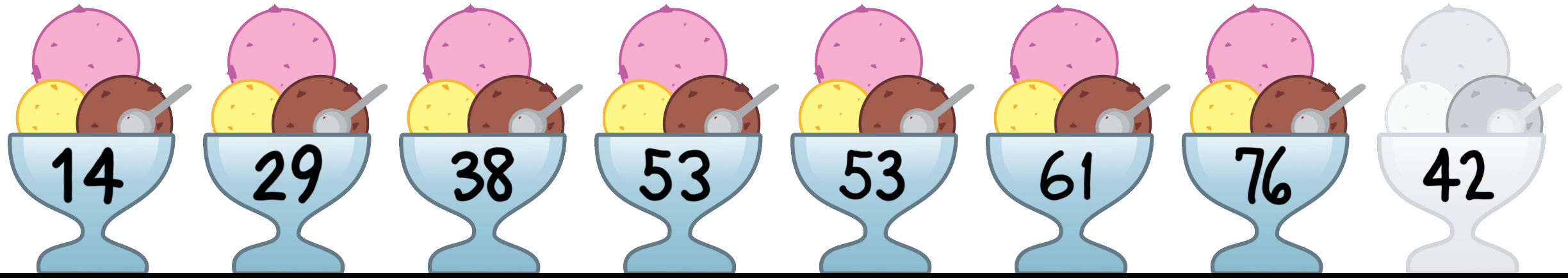
Insertion Sort

- Insertion sort in action:



Insertion Sort

- Insertion sort in action:

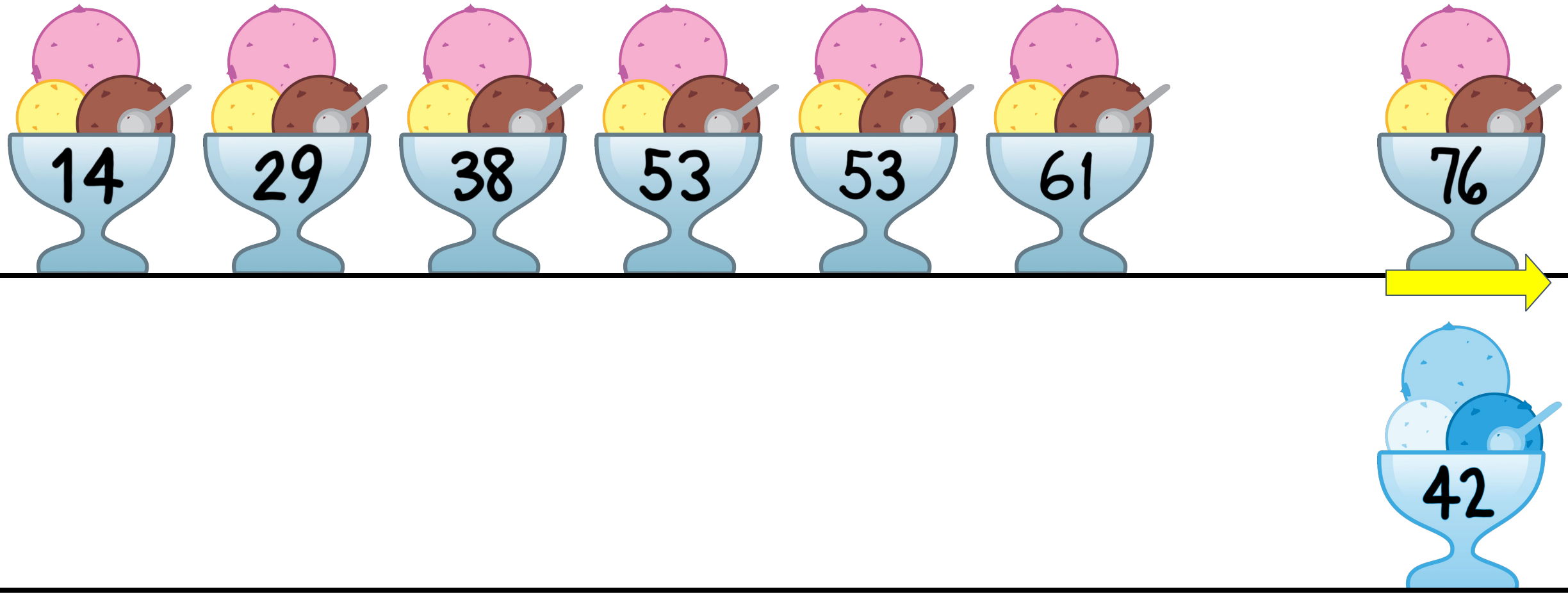


Insertion Sort

- Insertion sort in action:

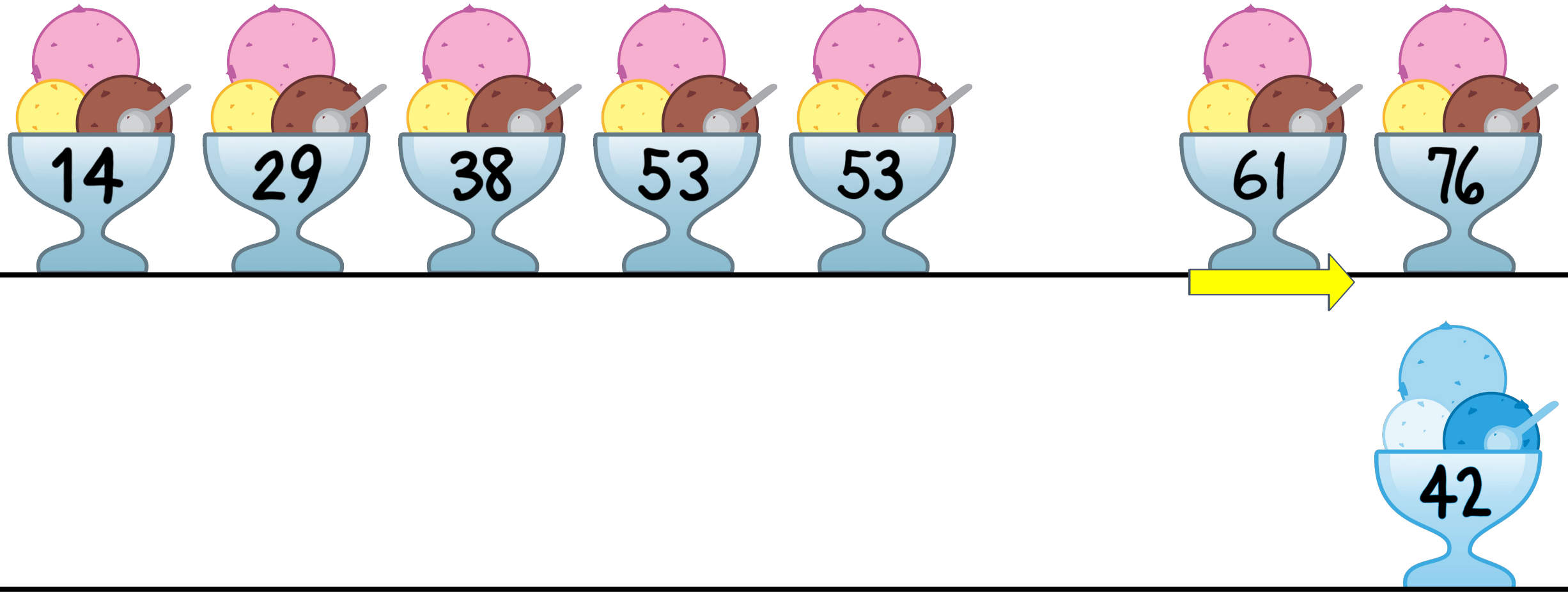
Insertion Sort

- Insertion sort in action:



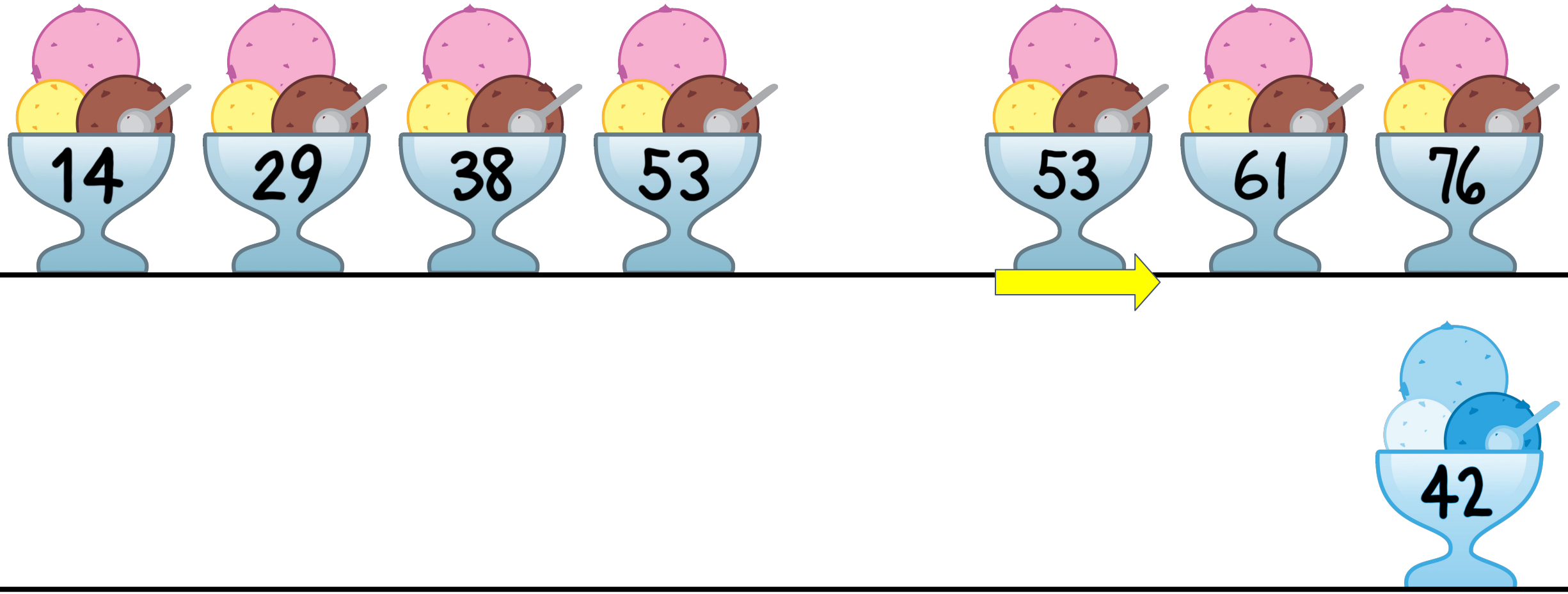
Insertion Sort

- Insertion sort in action:



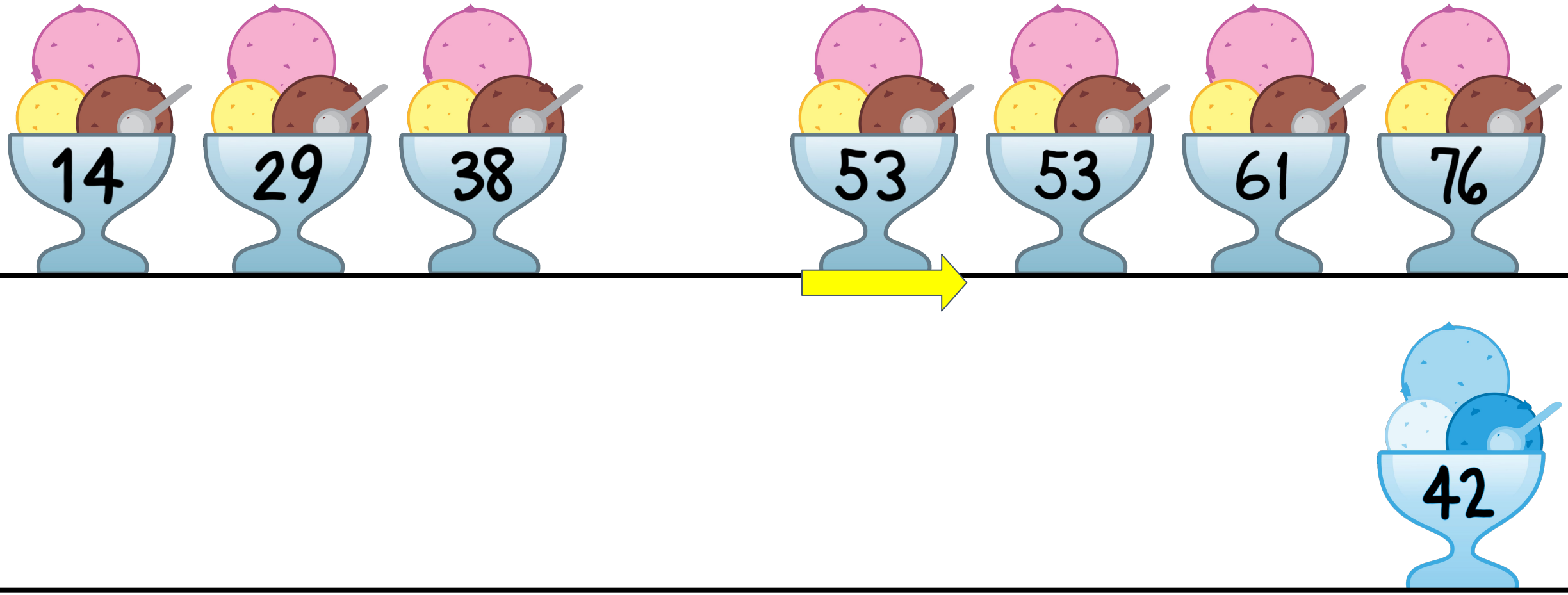
Insertion Sort

- Insertion sort in action:



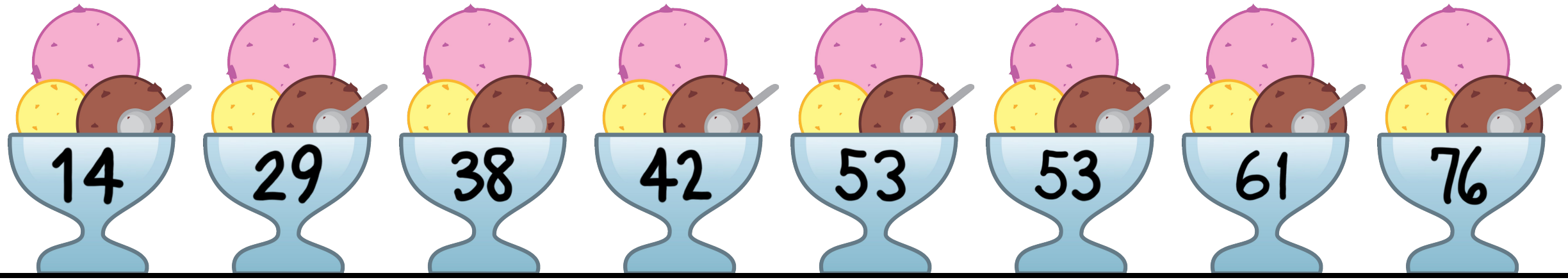
Insertion Sort

- Insertion sort in action:



Insertion Sort

- Insertion sort in action:



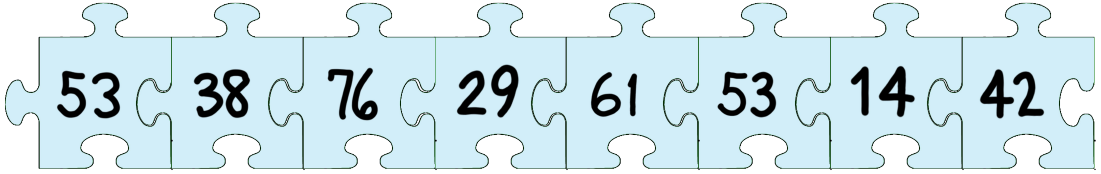
Merge Sort

Merge Sort (Top Down)

- Take a list, split it in half
  - Recursively sort both halves
- Combine halves with merge function
- Base case occurs when halves have length 1

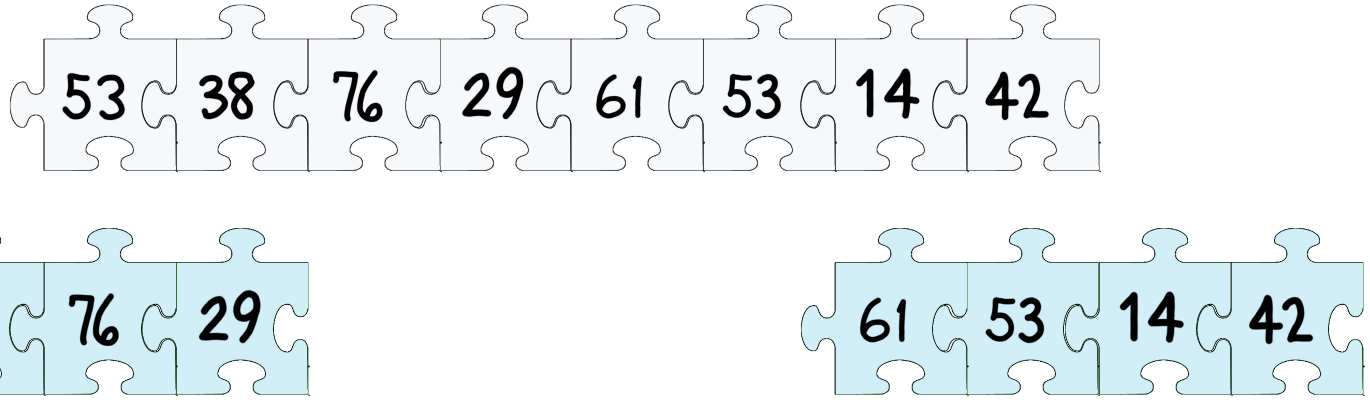
Merge Sort (Top Down)

- Merge sort in action:



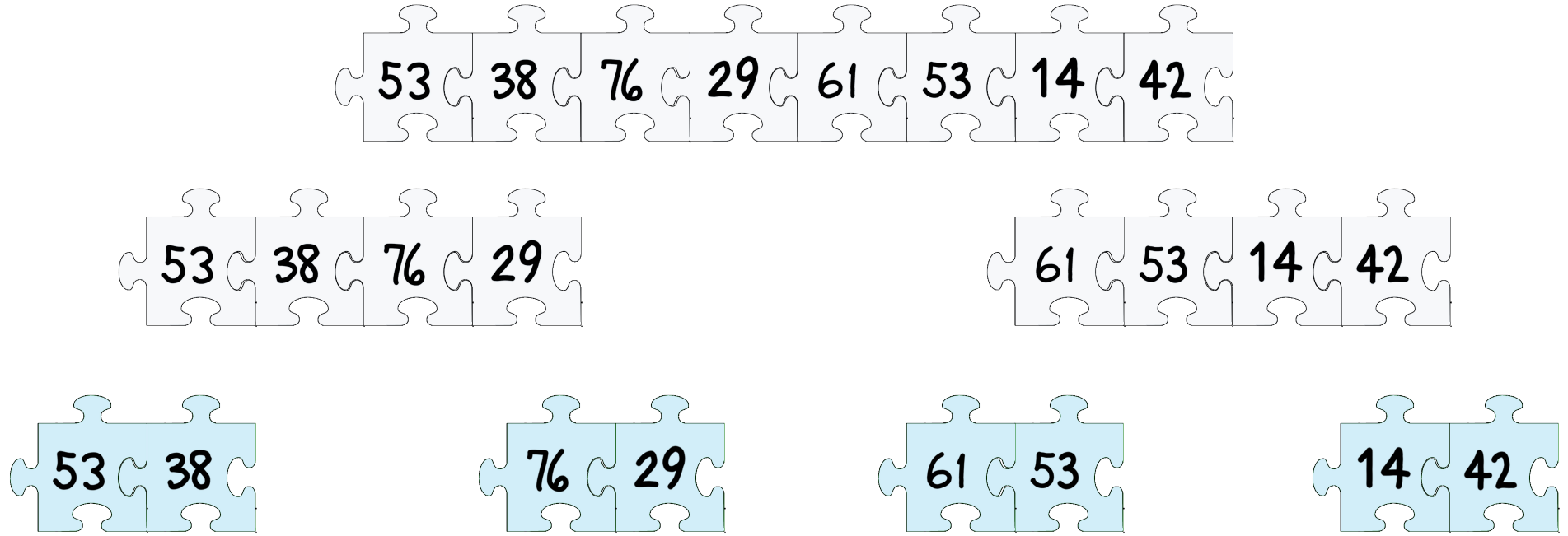
Merge Sort (Top Down)

- Merge sort in action:



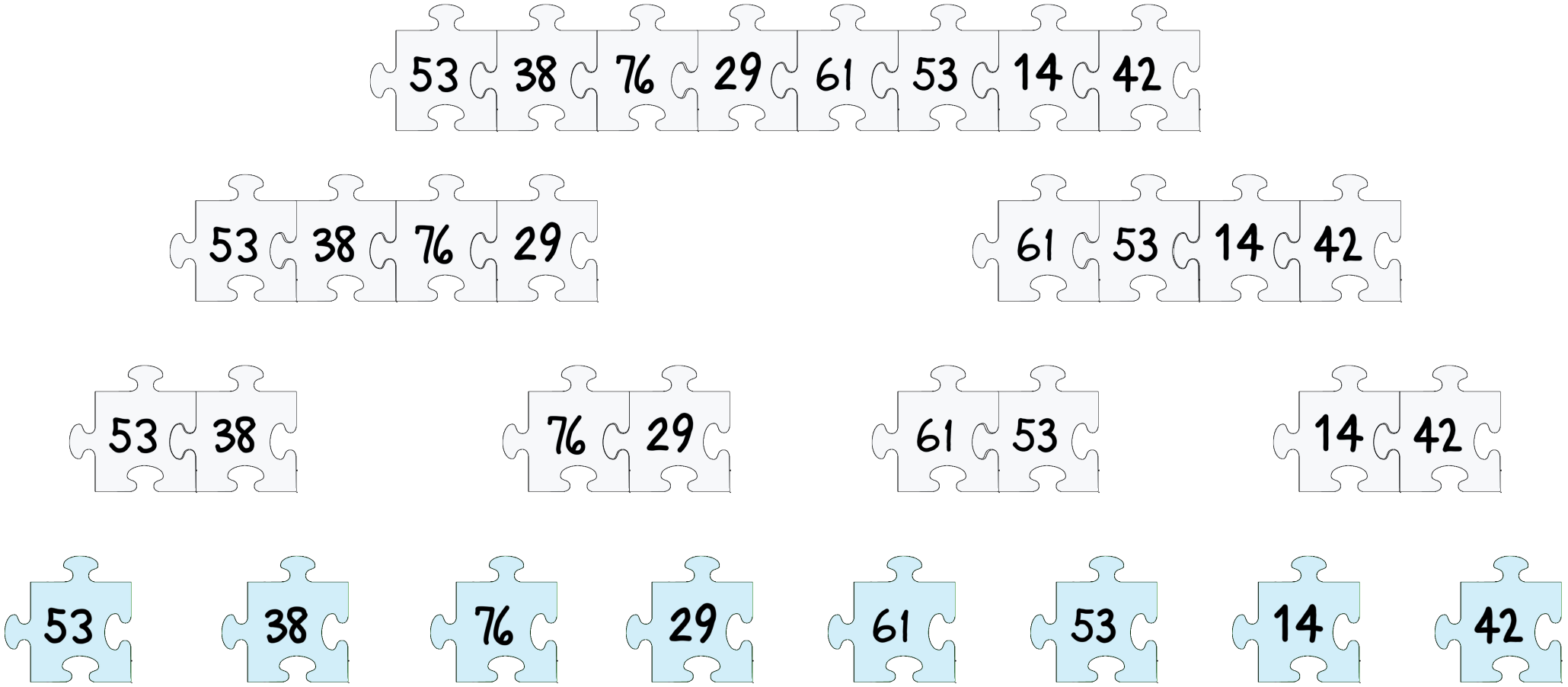
Merge Sort (Top Down)

- Merge sort in action:



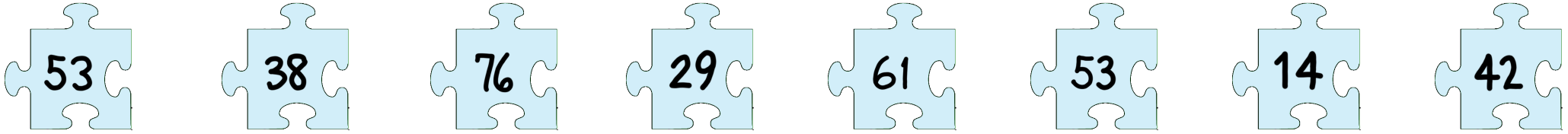
Merge Sort (Top Down)

- Merge sort in action:



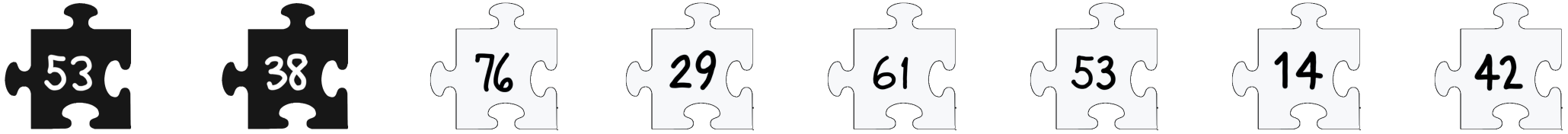
Merge Sort (Top Down)

- Merge sort in action:



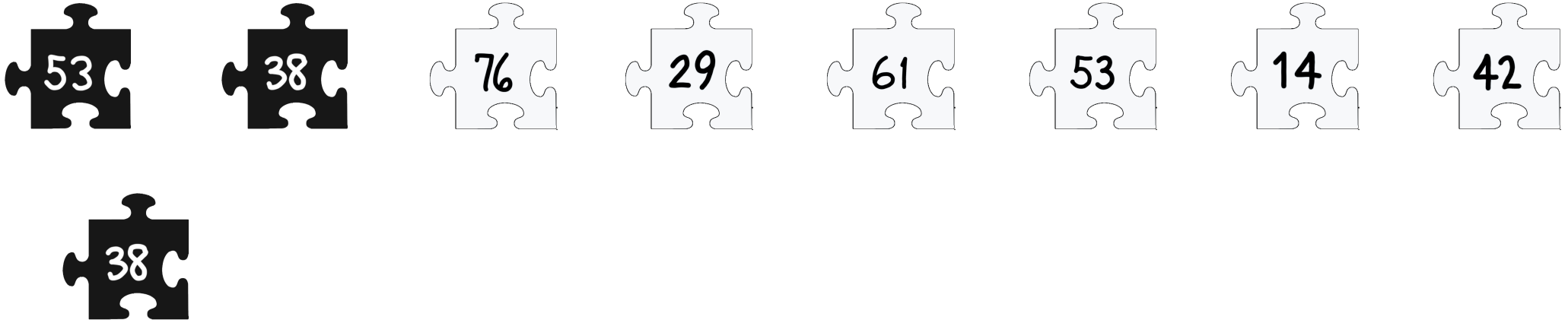
Merge Sort (Top Down)

- Merge sort in action:



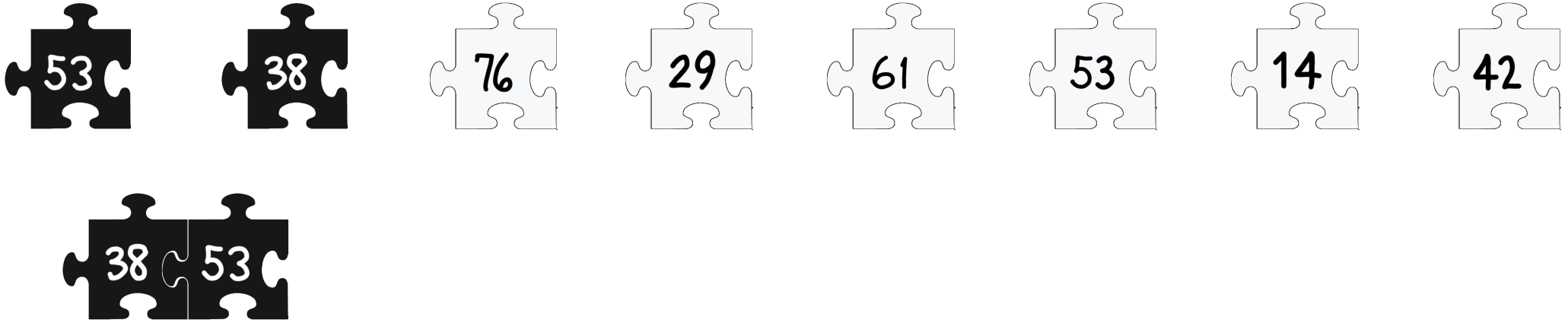
Merge Sort (Top Down)

- Merge sort in action:



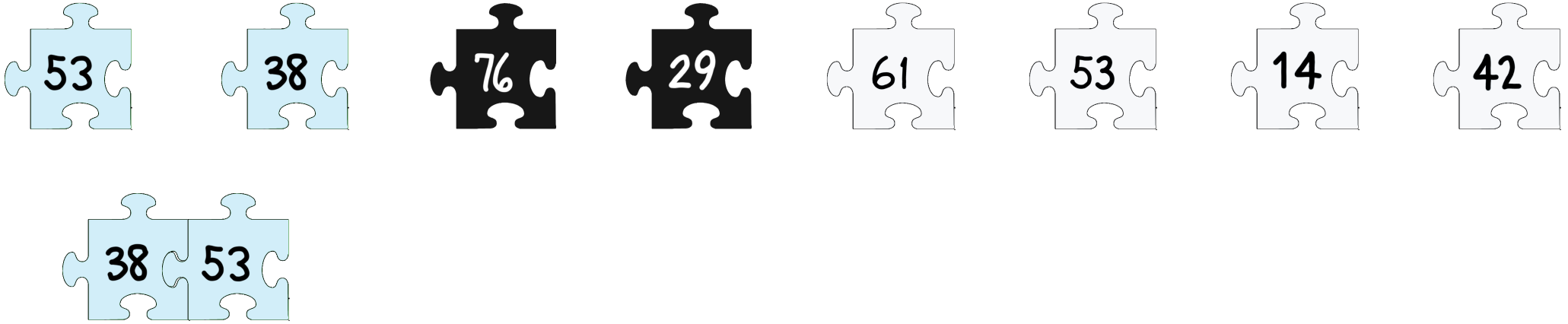
Merge Sort (Top Down)

- Merge sort in action:



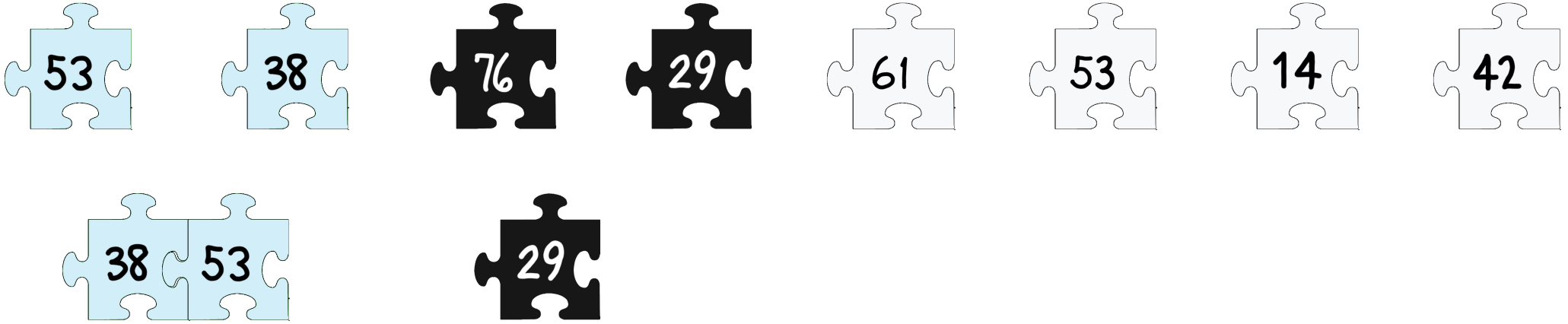
Merge Sort (Top Down)

- Merge sort in action:



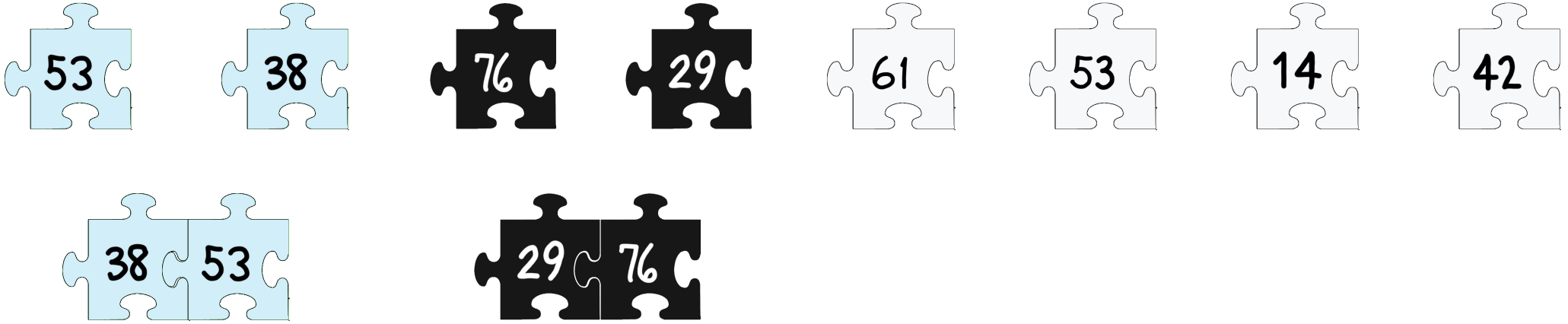
Merge Sort (Top Down)

- Merge sort in action:



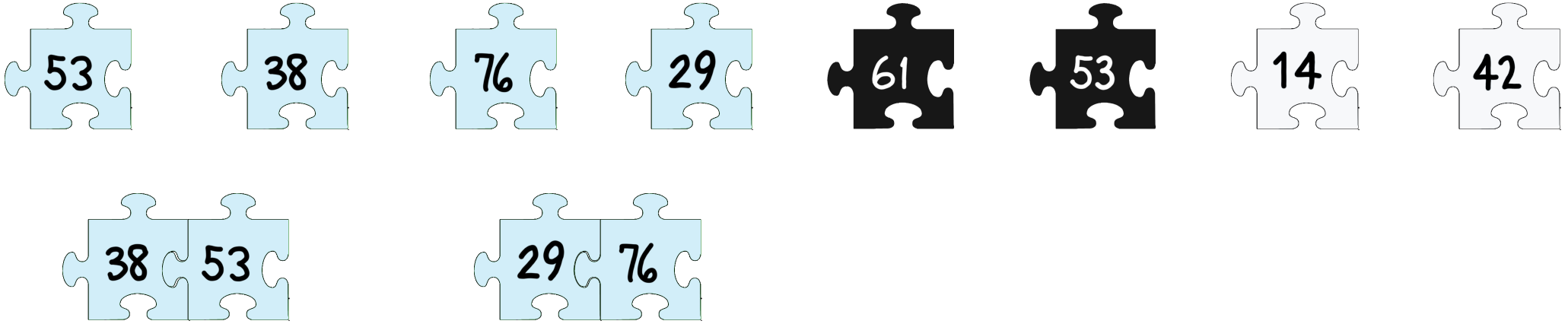
Merge Sort (Top Down)

- Merge sort in action:



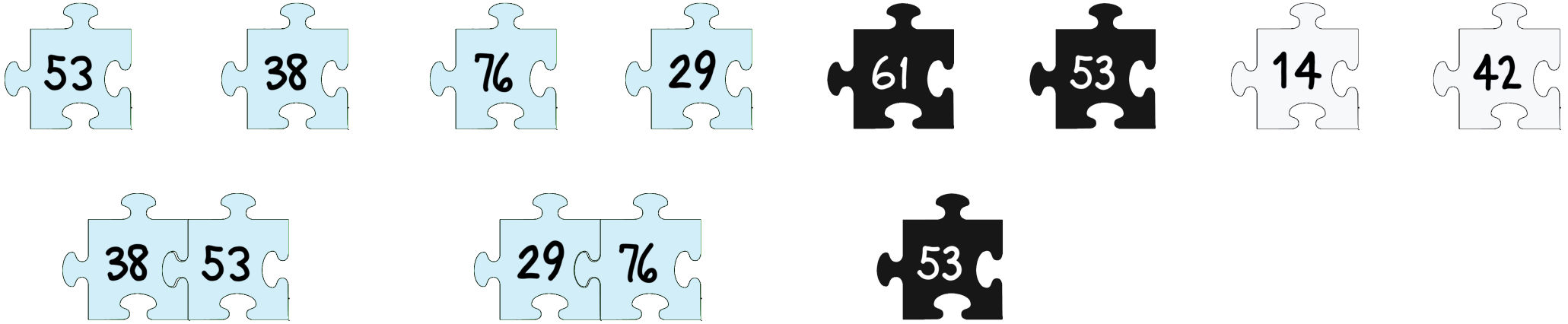
Merge Sort (Top Down)

- Merge sort in action:



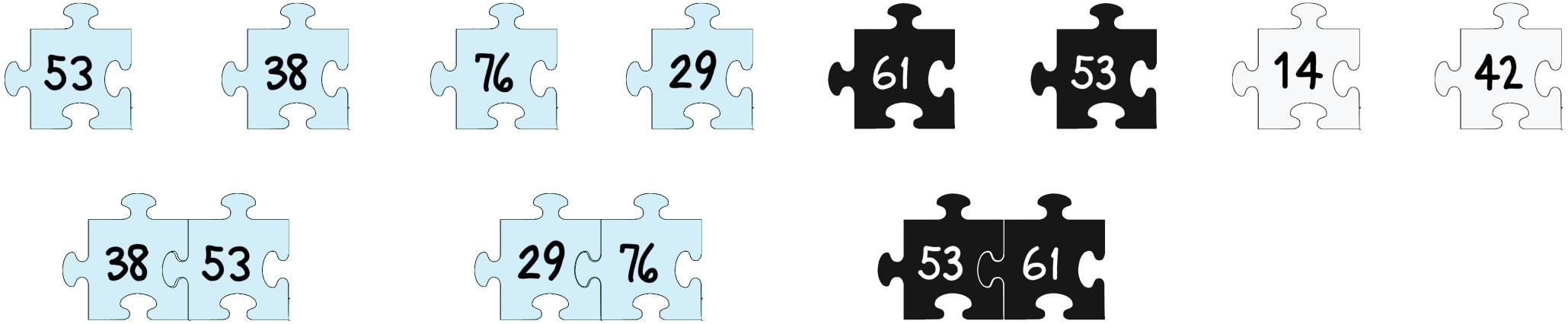
Merge Sort (Top Down)

- Merge sort in action:



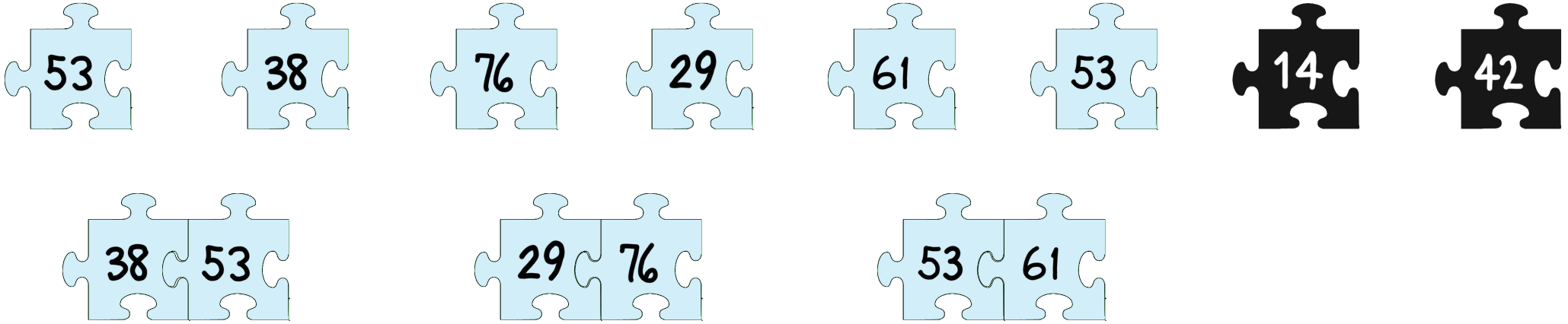
Merge Sort (Top Down)

- Merge sort in action:



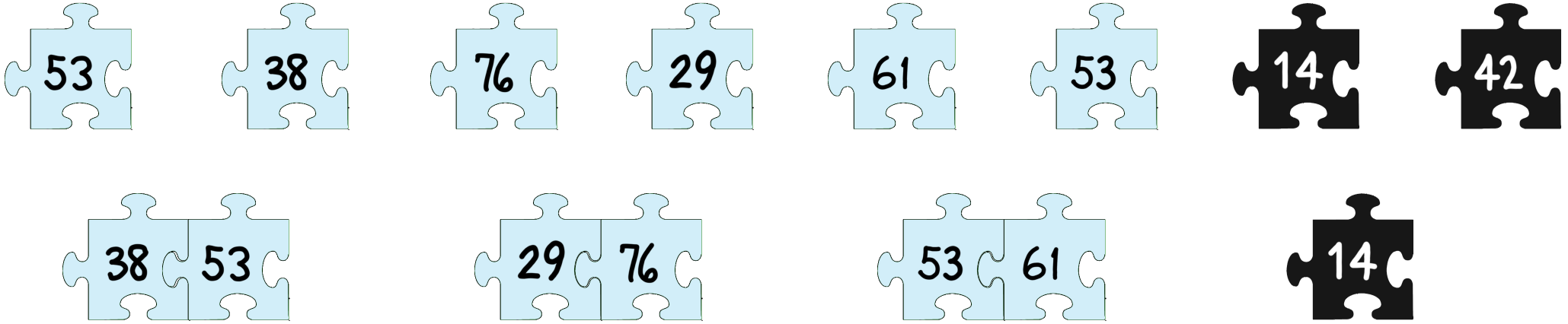
Merge Sort (Top Down)

- Merge sort in action:



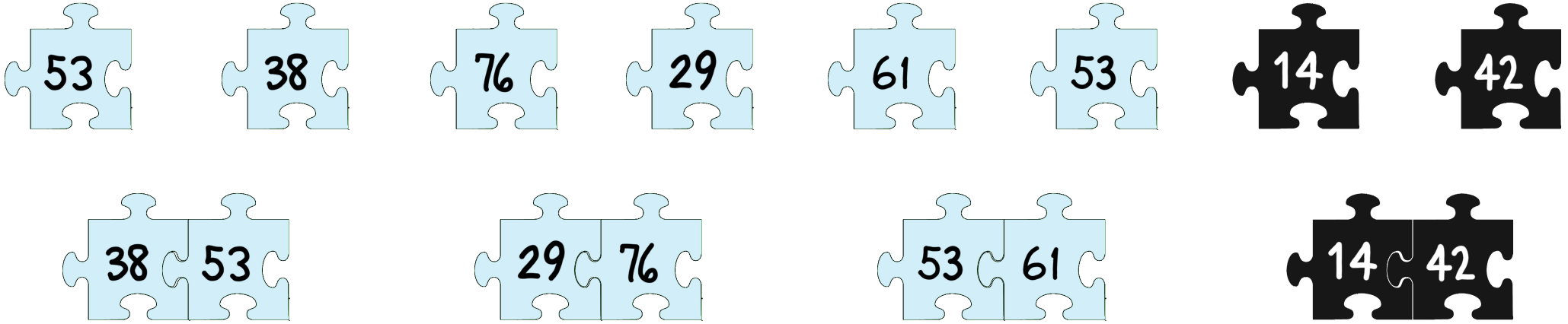
Merge Sort (Top Down)

- Merge sort in action:



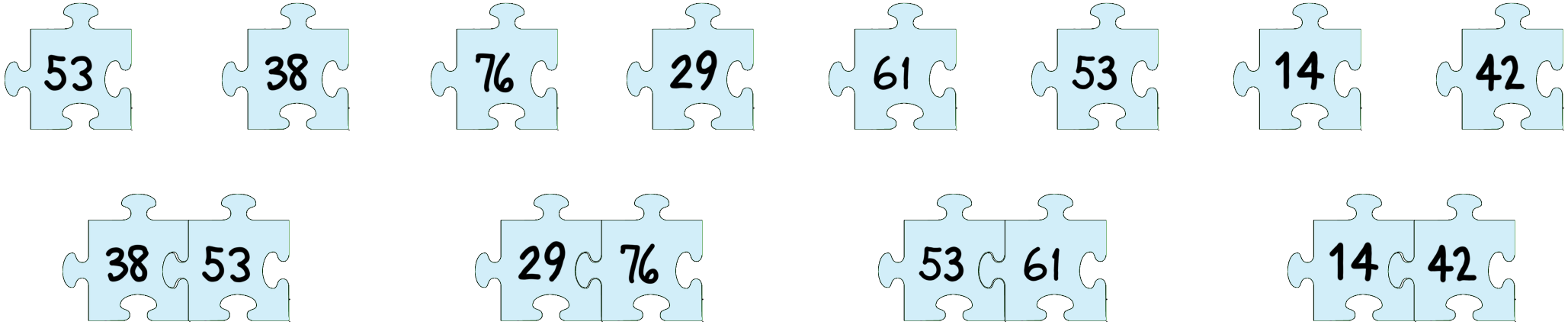
Merge Sort (Top Down)

- Merge sort in action:



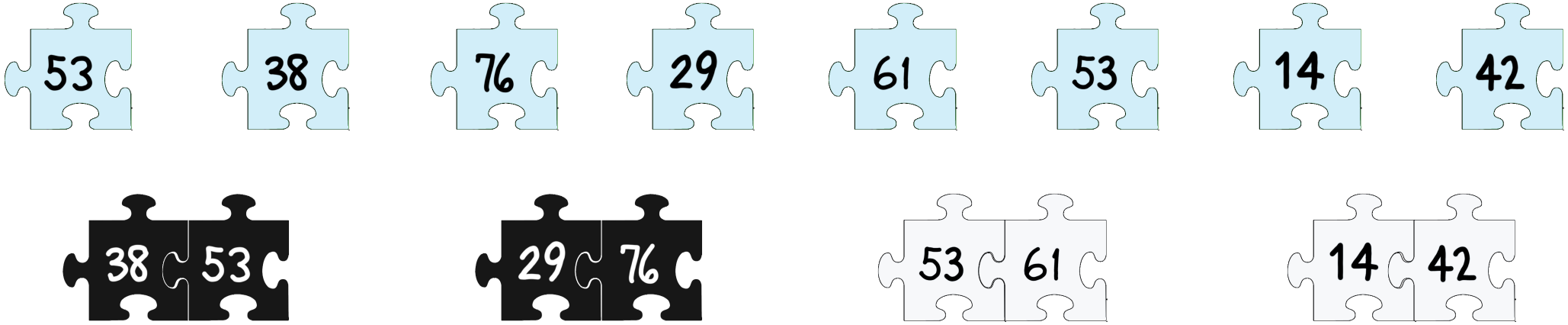
Merge Sort (Top Down)

- Merge sort in action:



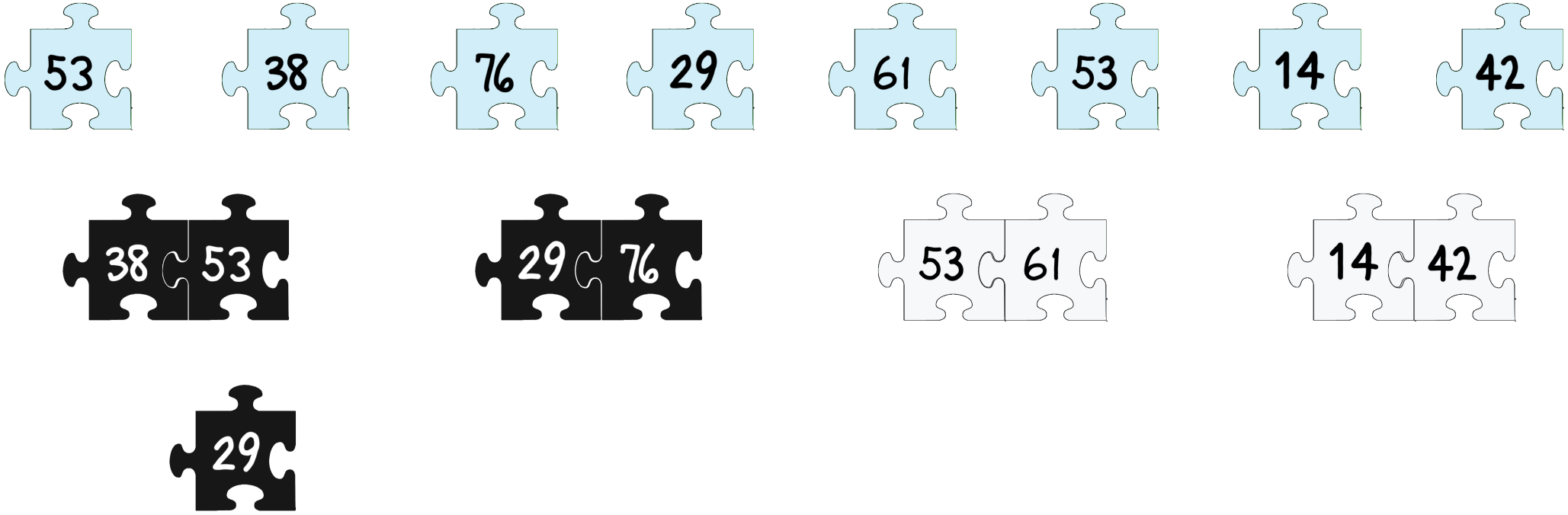
Merge Sort (Top Down)

- Merge sort in action:



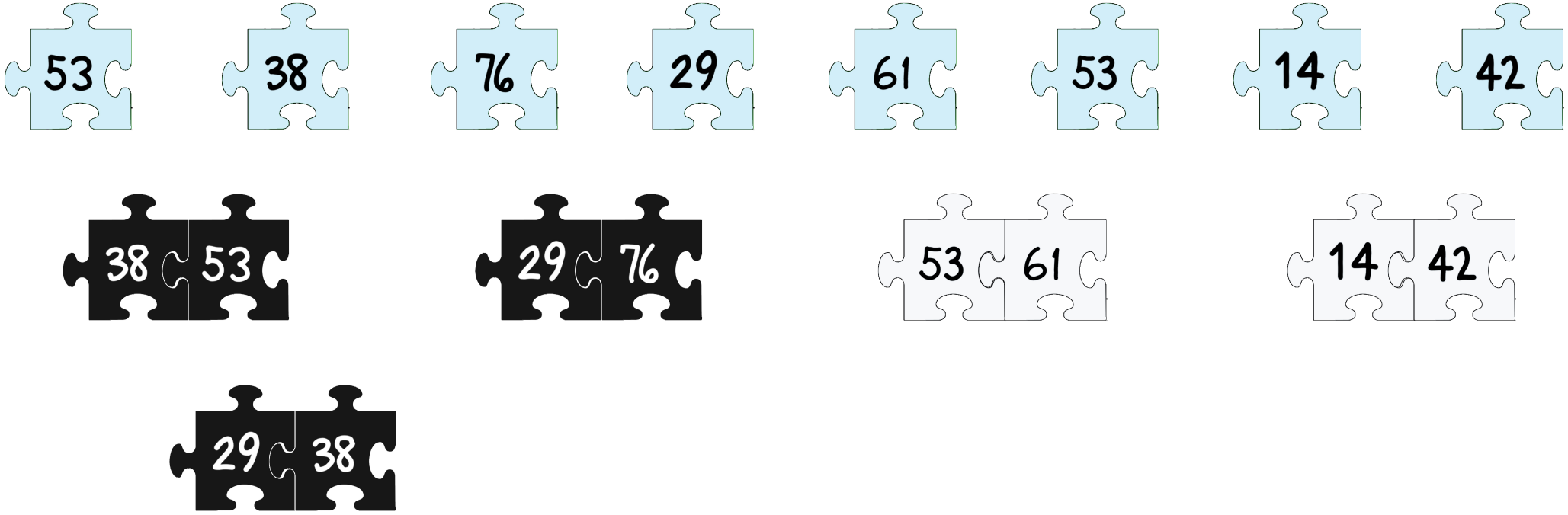
Merge Sort (Top Down)

- Merge sort in action:



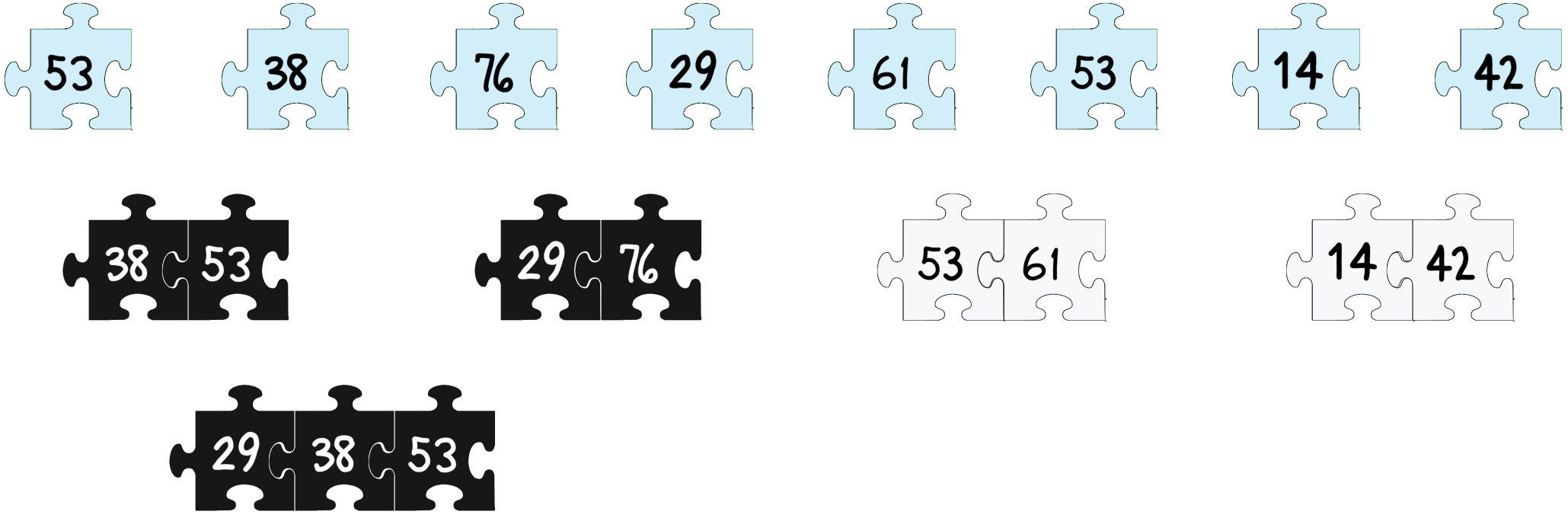
Merge Sort (Top Down)

- Merge sort in action:



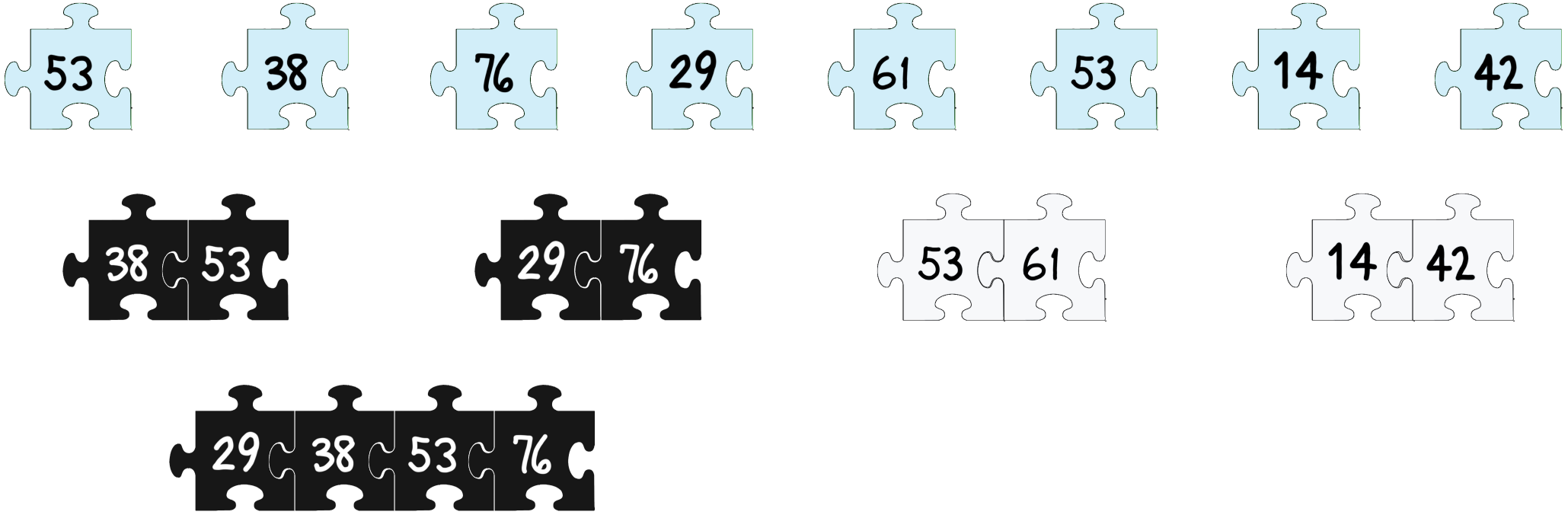
Merge Sort (Top Down)

- Merge sort in action:



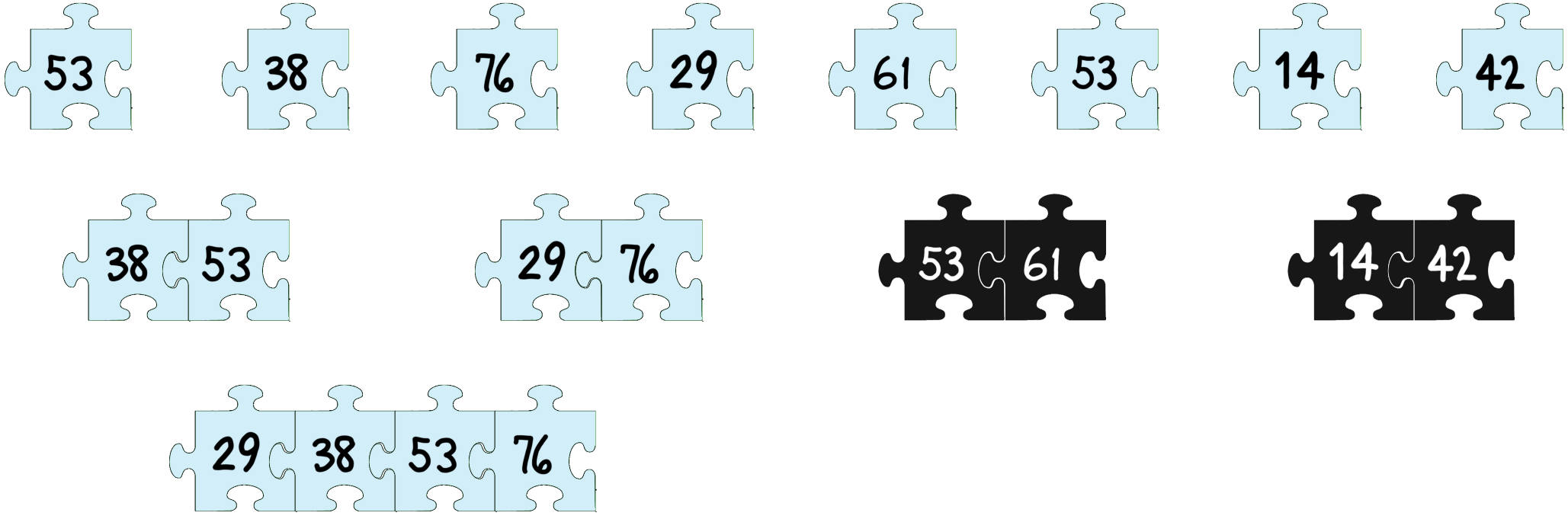
Merge Sort (Top Down)

- Merge sort in action:



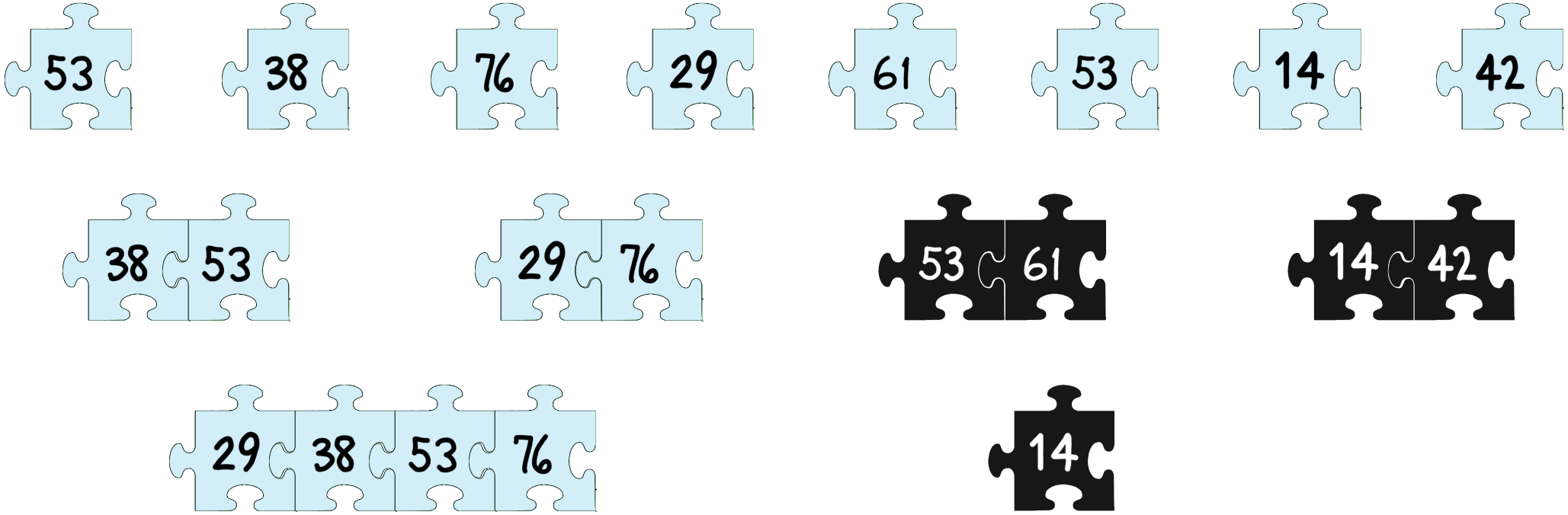
Merge Sort (Top Down)

- Merge sort in action:



Merge Sort (Top Down)

- Merge sort in action:



Merge Sort (Top Down)

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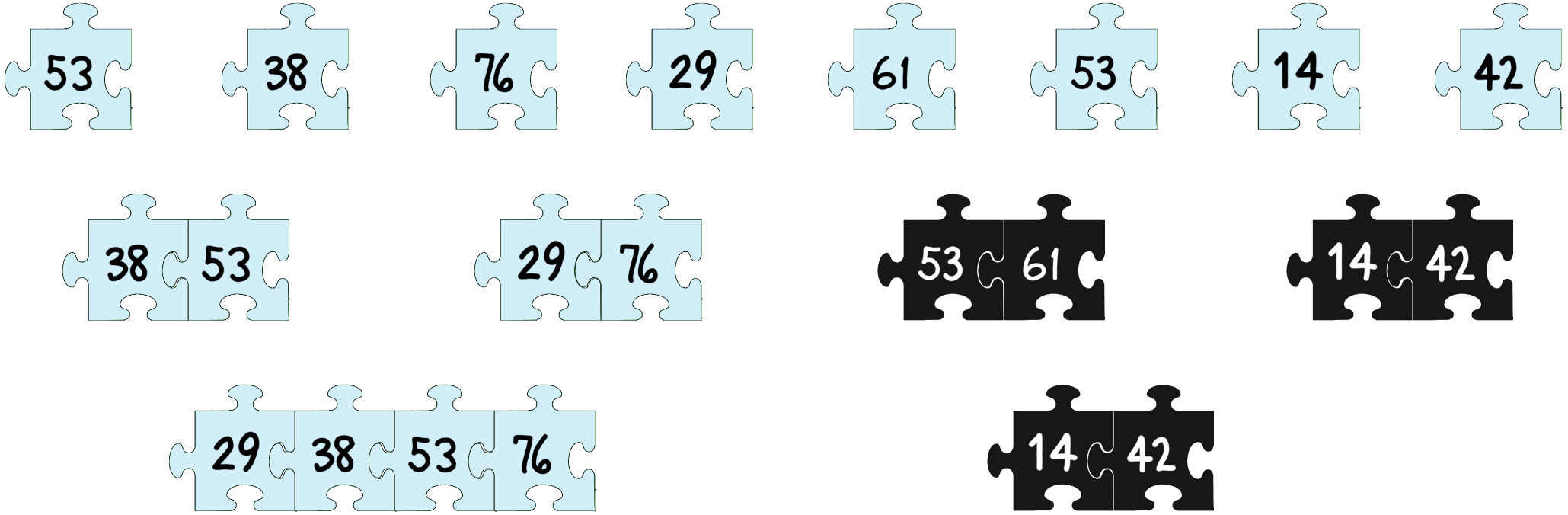
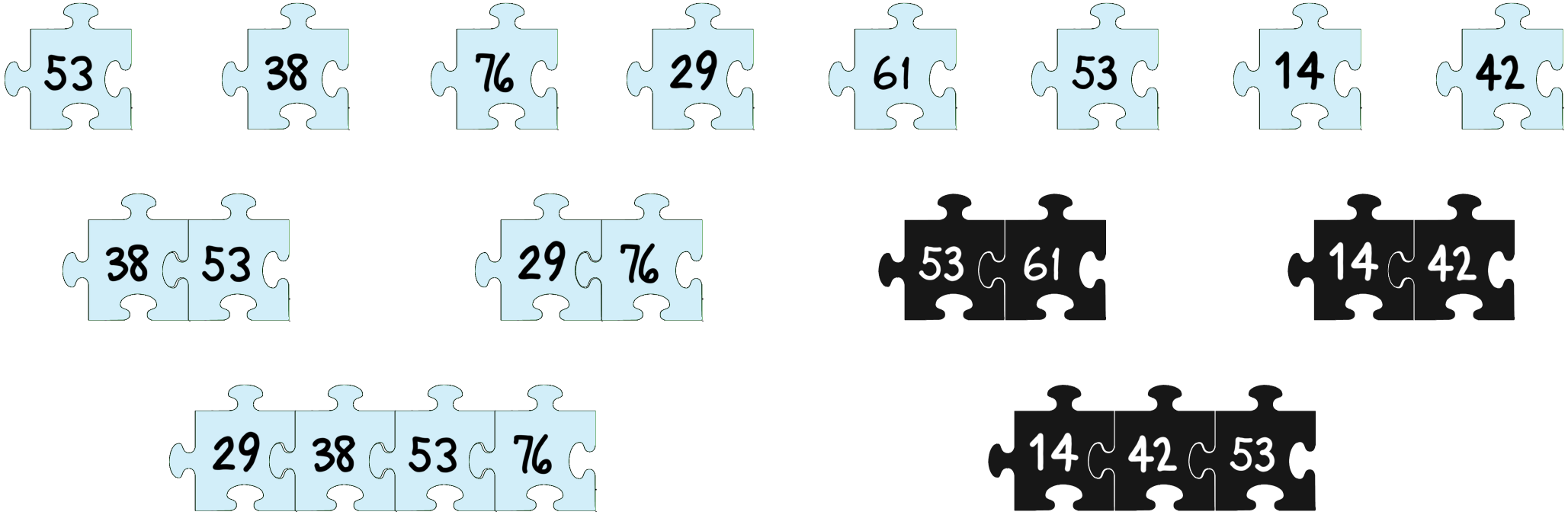


Merge Sort (Top Down)

- Merge sort in action:



Merge Sort (Top Down)

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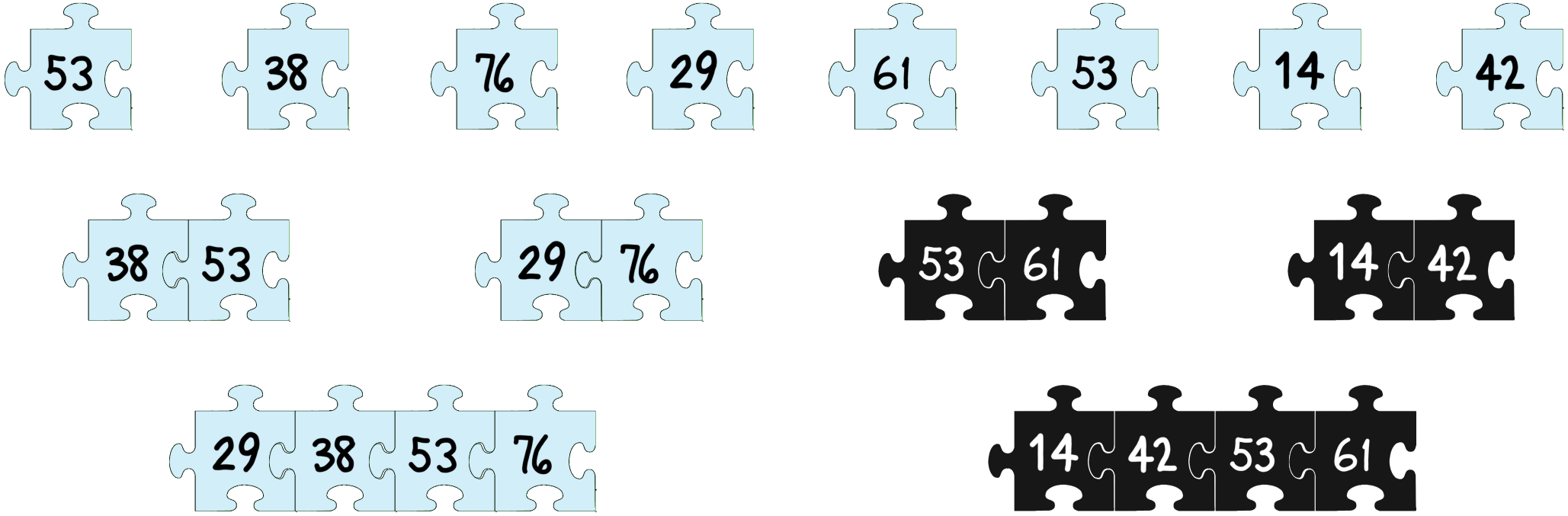
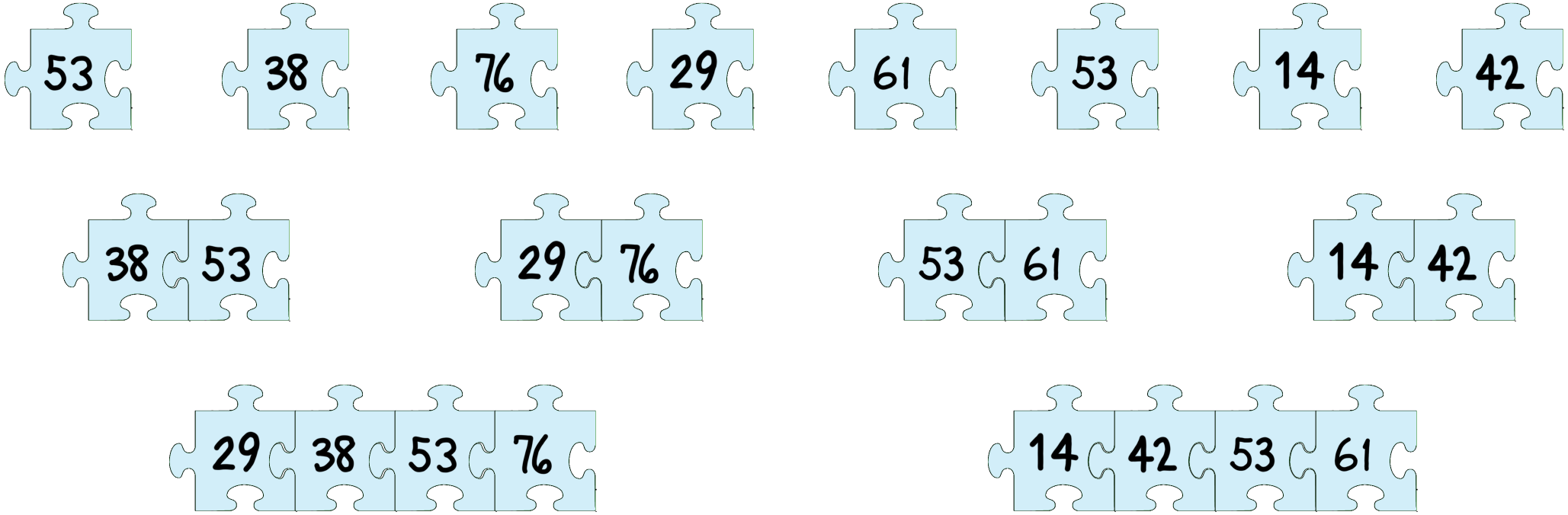


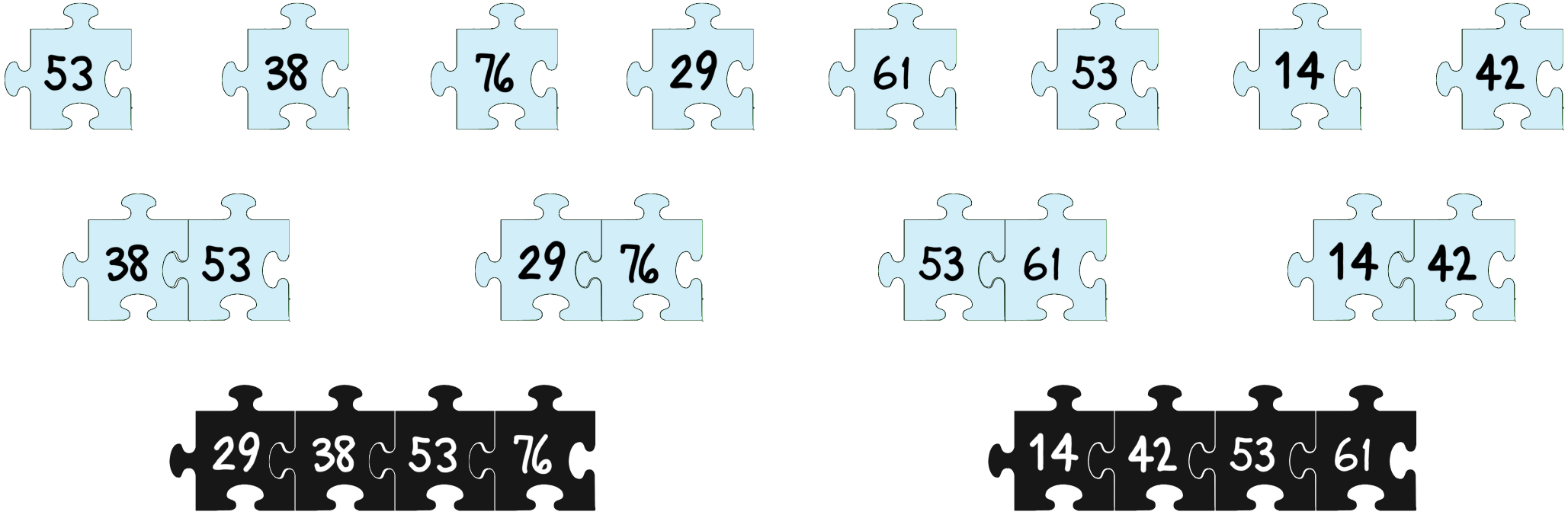
Merge Sort (Top Down)

- Merge sort in action:



Merge Sort (Top Down)

- Merge sort in action:



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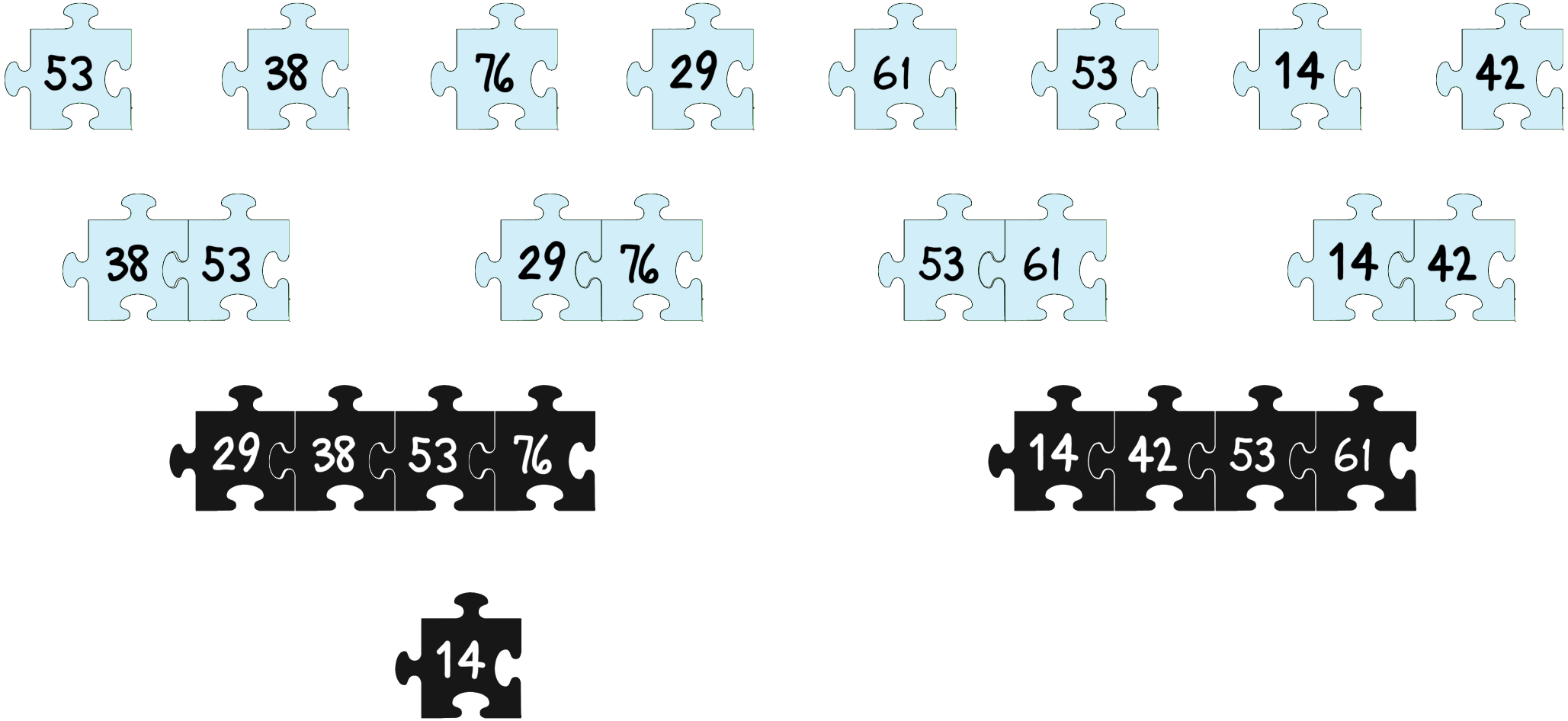
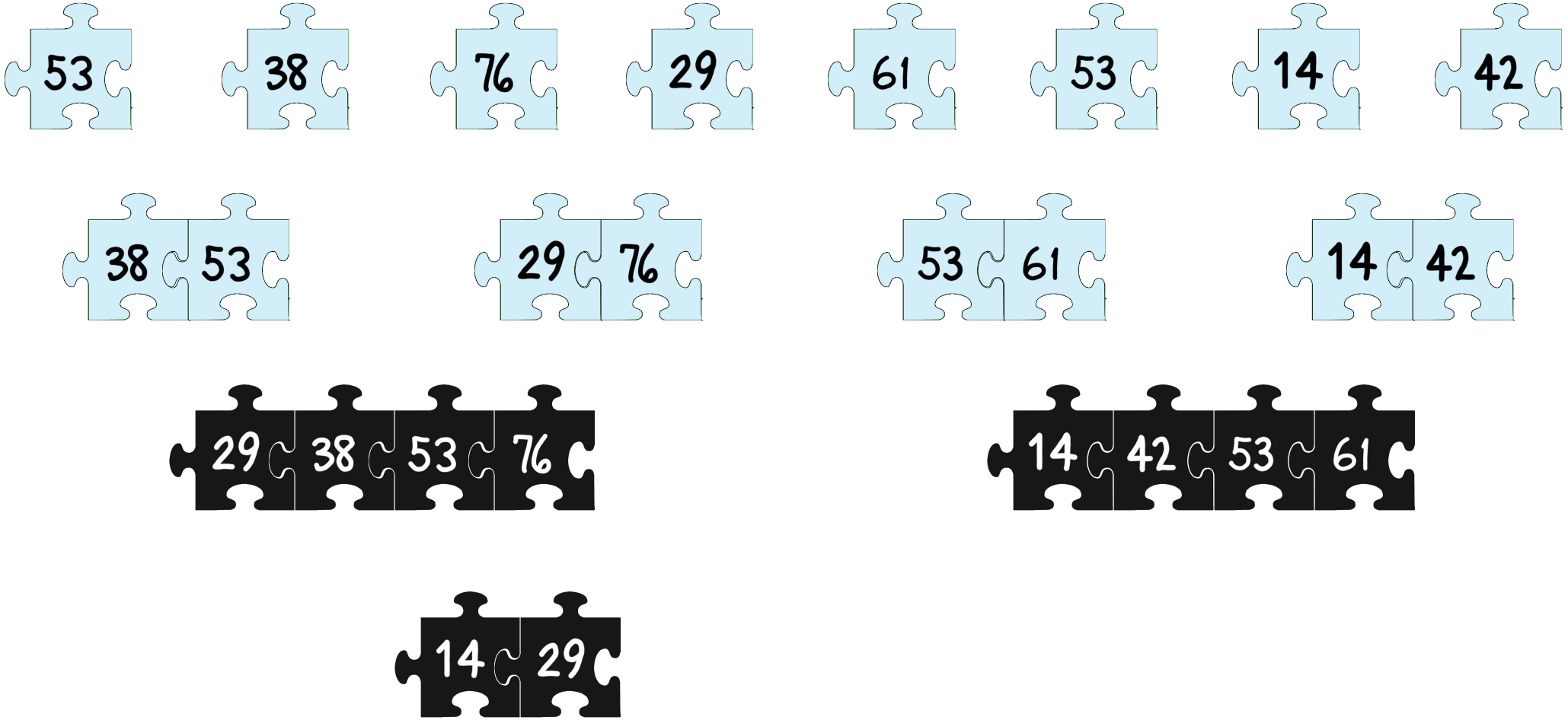


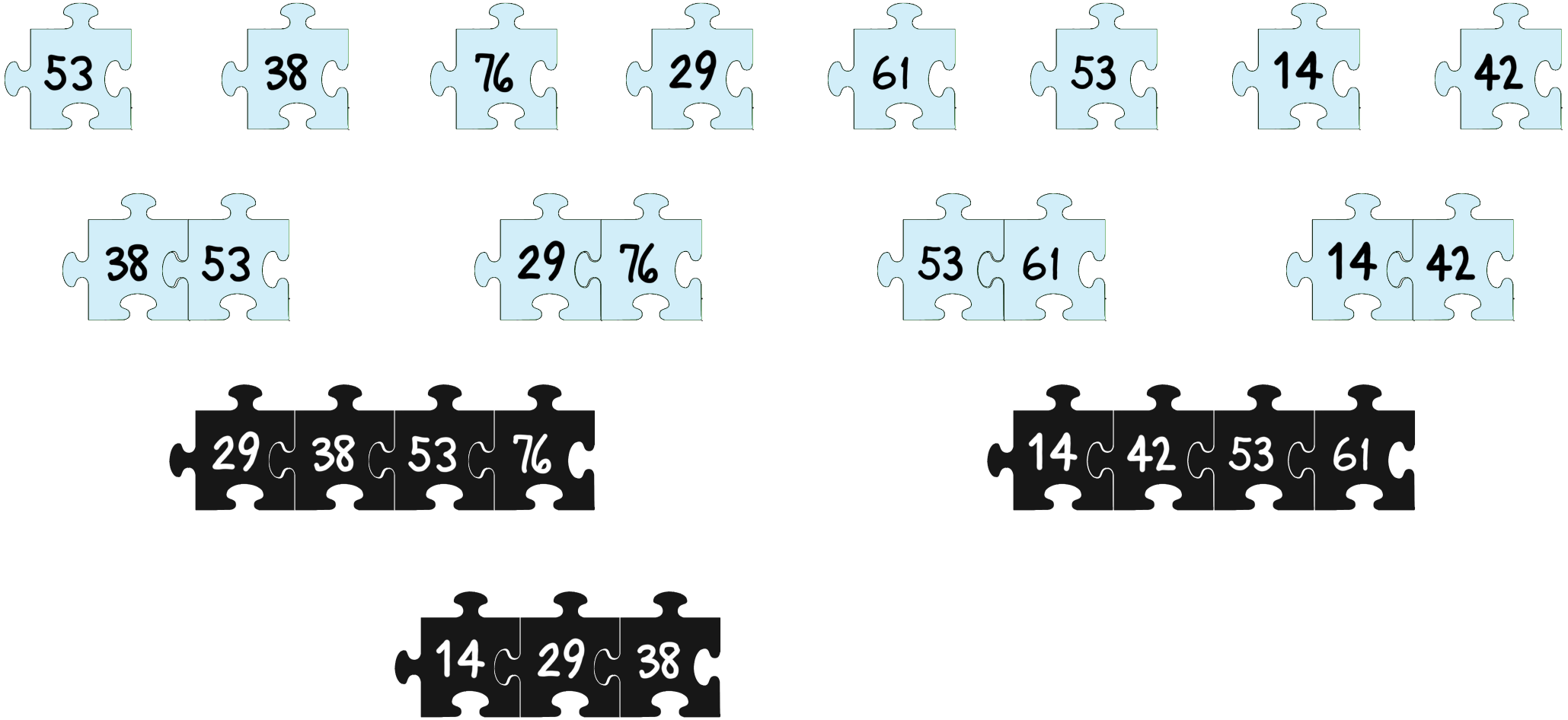
Merge Sort (Top Down)

- Merge sort in action:



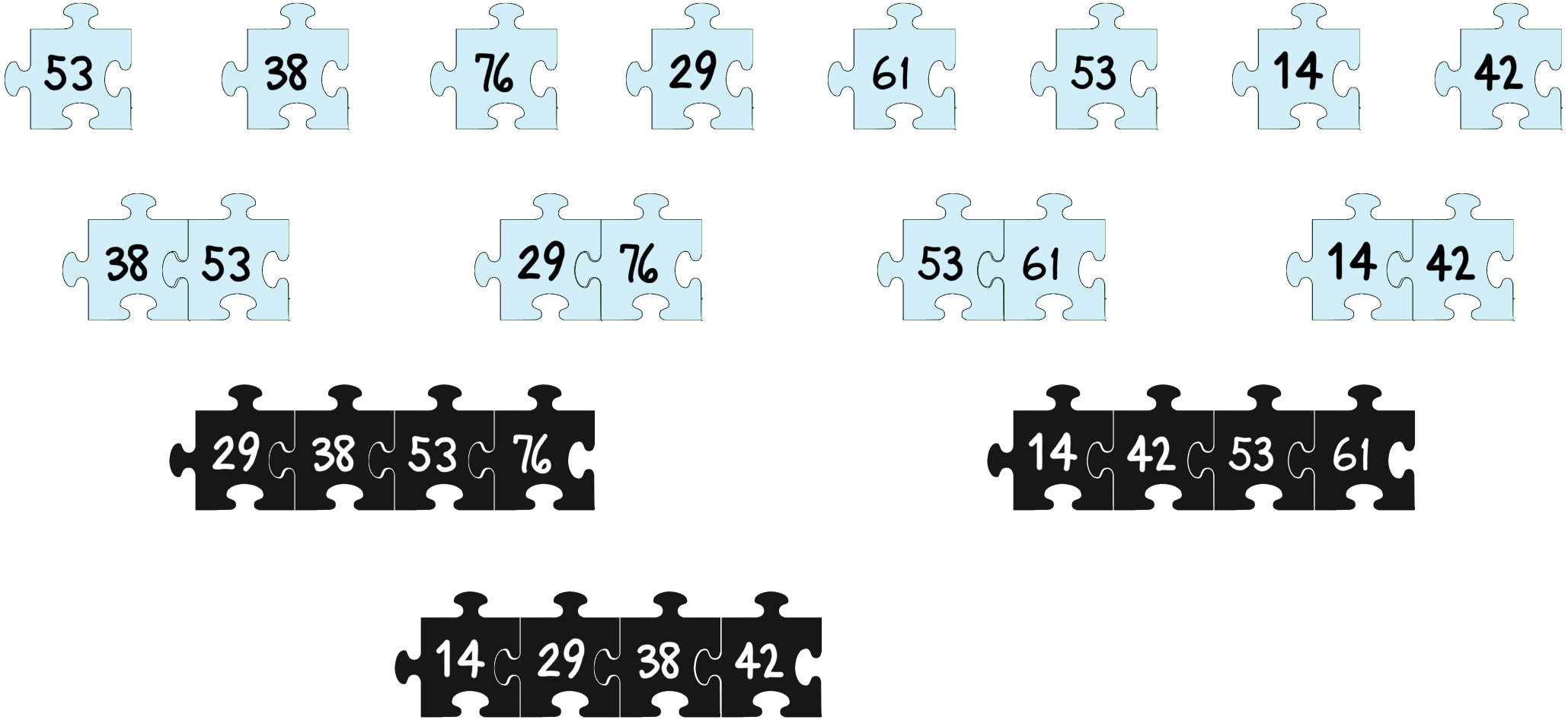
Merge Sort (Top Down)

- Merge sort in action:



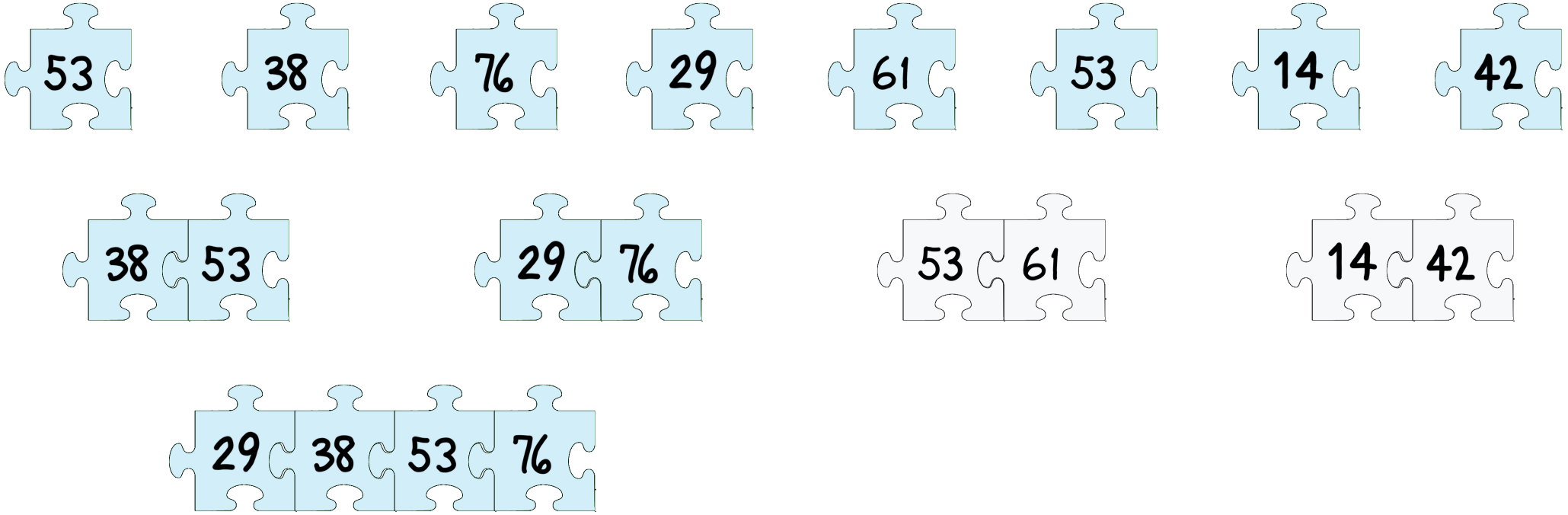
Merge Sort (Top Down)

- Merge sort in action:



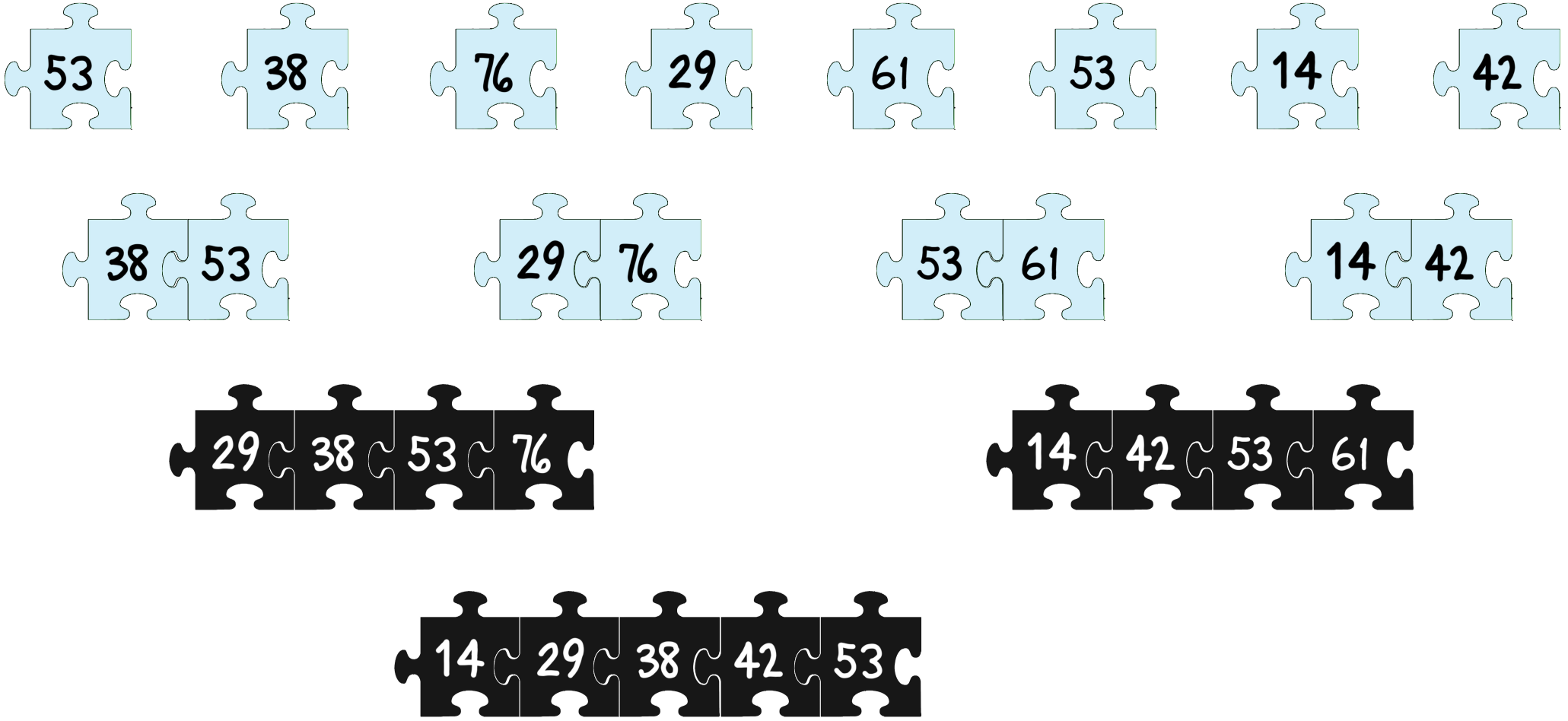
Merge Sort (Top Down)

- Merge sort in action:



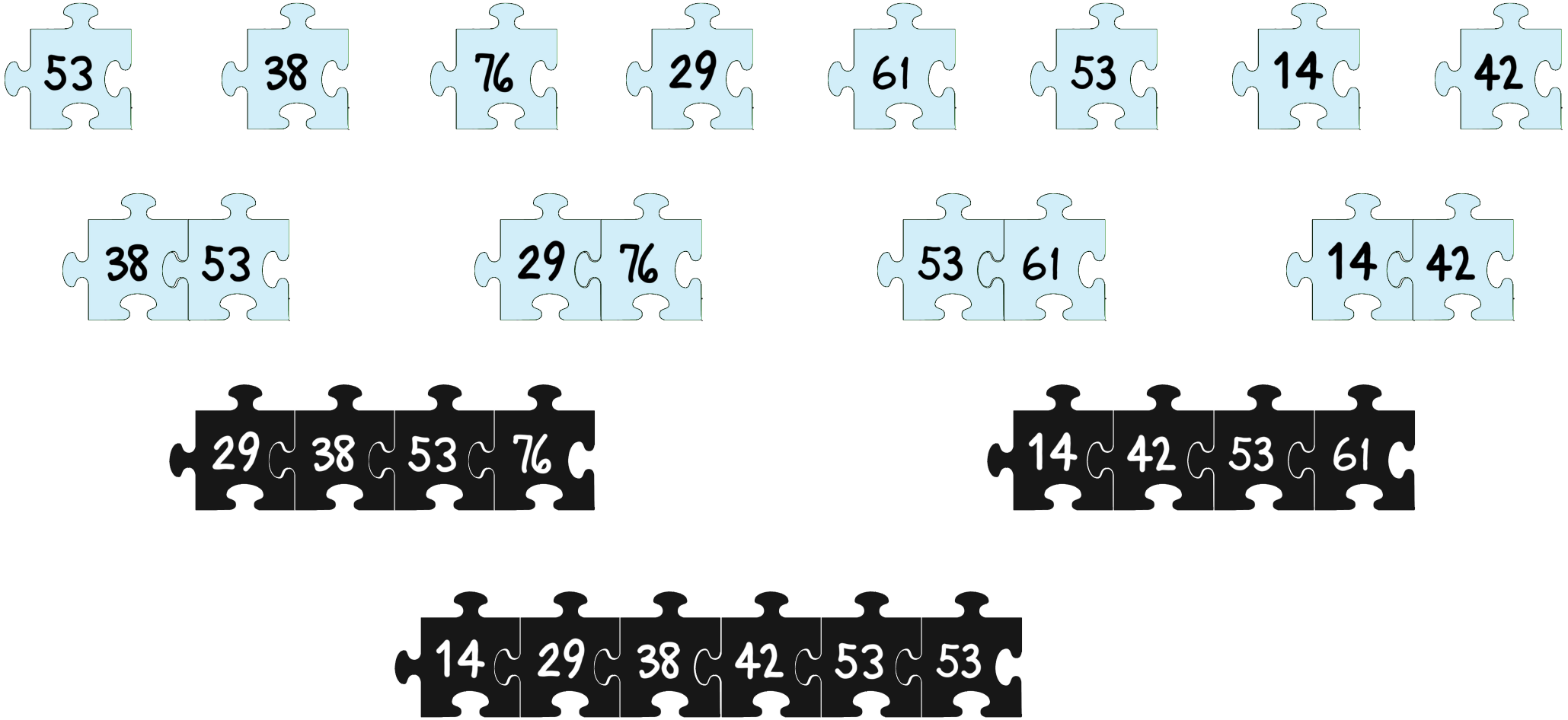
Merge Sort (Top Down)

- Merge sort in action:



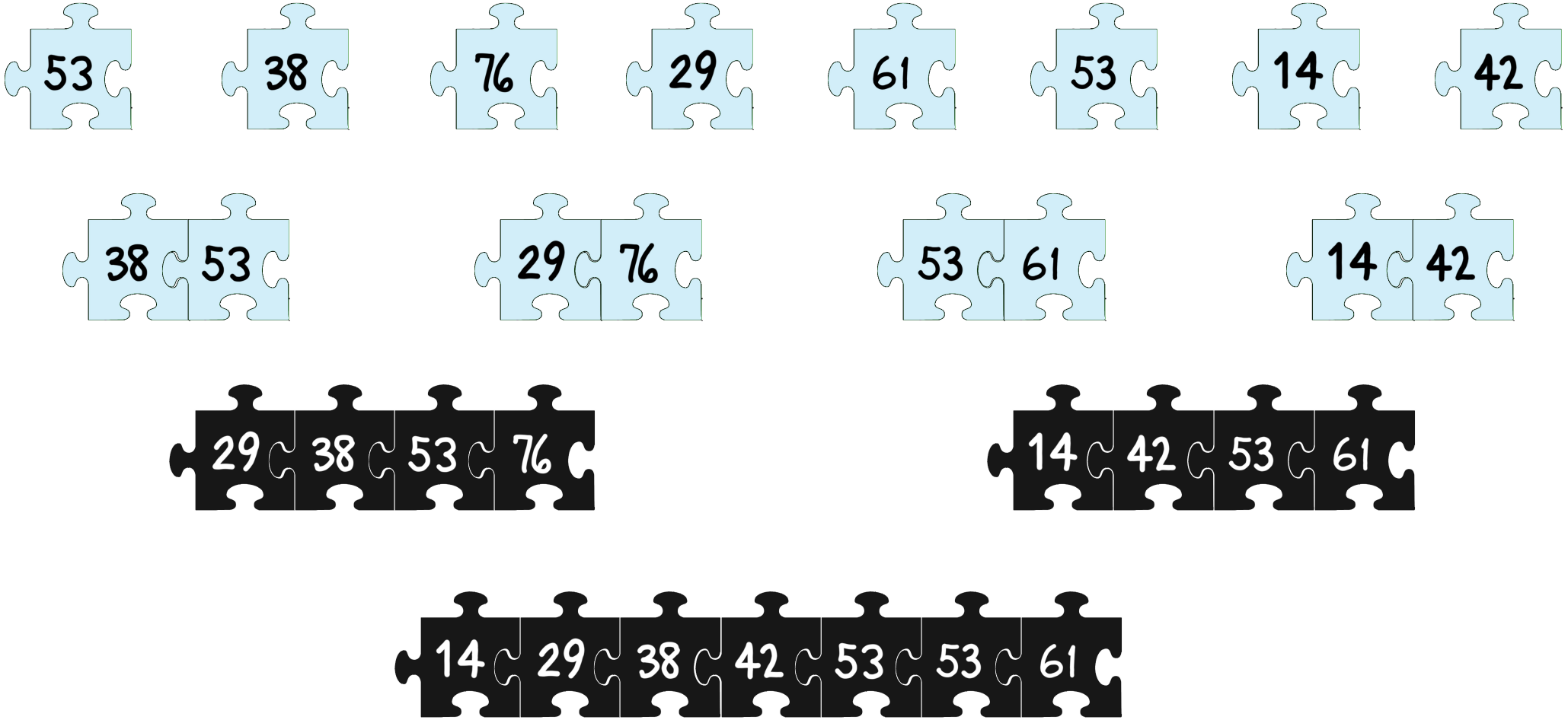
Merge Sort (Top Down)

- Merge sort in action:



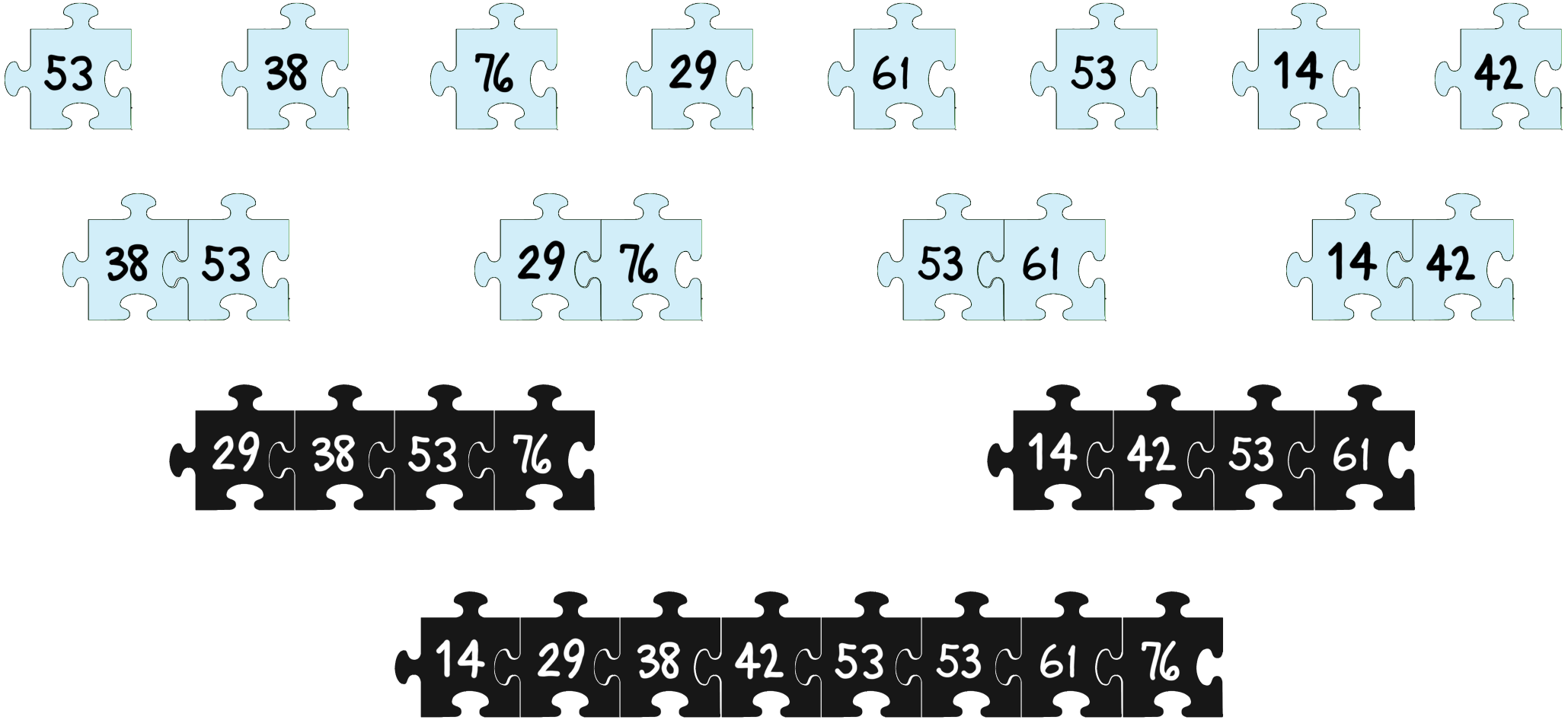
Merge Sort (Top Down)

- Merge sort in action:



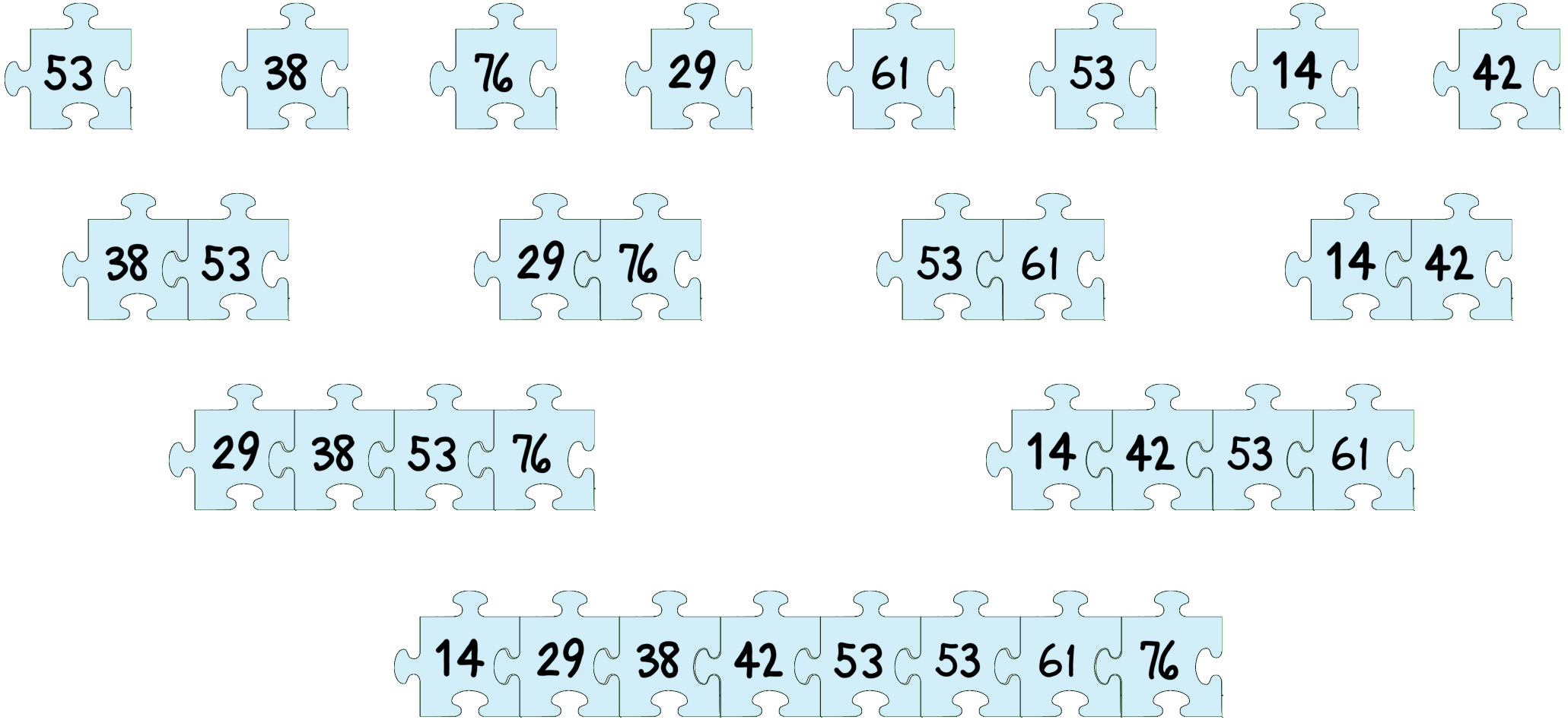
Merge Sort (Top Down)

- Merge sort in action:



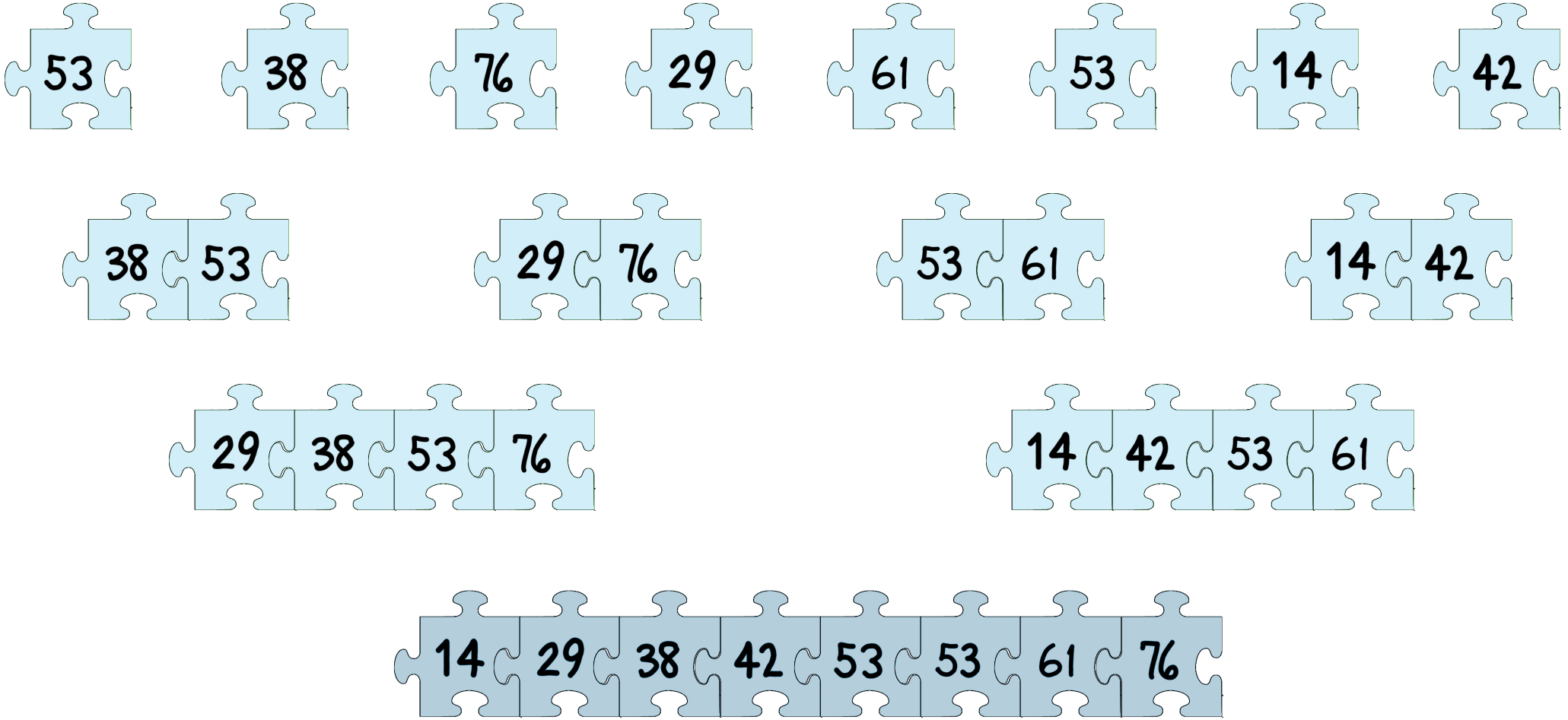
Merge Sort (Top Down)

- Merge sort in action:



Merge Sort (Top Down)

- Merge sort in action:



Merge Sort Complexity Analysis

```
mergesort(int [] a, int left, int right) {
   if (right > left) {
      middle = left + (right - left)/2;
      mergesort(a, left, middle);
      mergesort(a, middle+1, right);
      merge(a, left, middle, right);
   }
   return;
}
```

$$T(n) \in \begin{cases} \Theta(n^{log_b a}), & \text{if } a > b^c \\ \Theta(n^c log_2 n), & \text{if } a = b^c \\ \Theta(n^c), & \text{if } a < b^c \end{cases}$$

T(n) = time to sort left half + time to sort right half + time to merge halves

= T(n/2) + T(n/2) + n

Therefore, we have: T(n) = 2 * T(n/2) + n, T(1) = 1

Merge Sort Complexity Analysis

```
mergesort(int [] a, int left, int right) {
   if (right > left) {
      middle = left + (right - left)/2;
      mergesort(a, left, middle);
      mergesort(a, middle+1, right);
      merge(a, left, middle, right);
   }
   return;
}
```

By the Master Theorem, the complexity is Θ(*n* log *n*).

$$T(n) \in \begin{cases} \Theta(n^{log_b a}), & \text{if } a > b^c \\ \Theta(n^c log_2 n), & \text{if } a = b^c \\ \Theta(n^c), & \text{if } a < b^c \end{cases}$$

T(n) = time to sort left half + time to sort right half + time to merge halves

= T(n/2) + T(n/2) + n

Therefore, we have: T(n) = 2 * T(n/2) + n, T(1) = 1

Merge Sort

- Characteristics of merge sort:

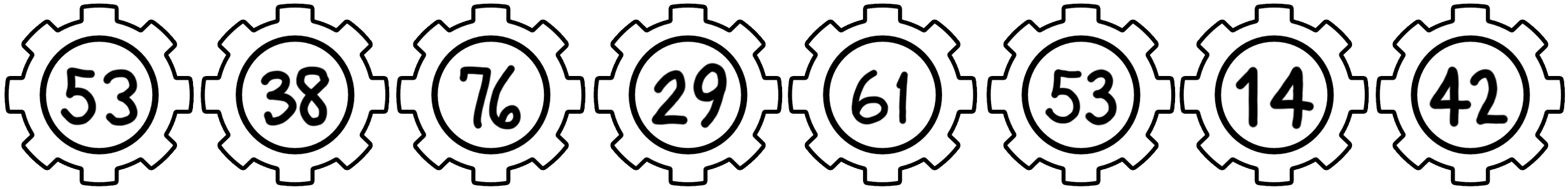
Best Case	Average Case	Worst Case	Memory	Stable?
Ω(n log n)	Θ(n log n)	O(n log n)	O(n)	Yes

  - Comparison-based, divide-and-conquer algorithm.
  - Bottom-up and top-down algorithms available.
  - Algorithm: divide unsorted list into *n* sublists, each containing 1 element. Repeatedly merge sublists to produce new sublists until there is only one sublist (the final sorted list) left.
  - Stable as long as merge is stable. Not adaptive.
  - Memory is O(n) worst-case because most implementations merge into a separate container and then copy over to the original container.

Quicksort

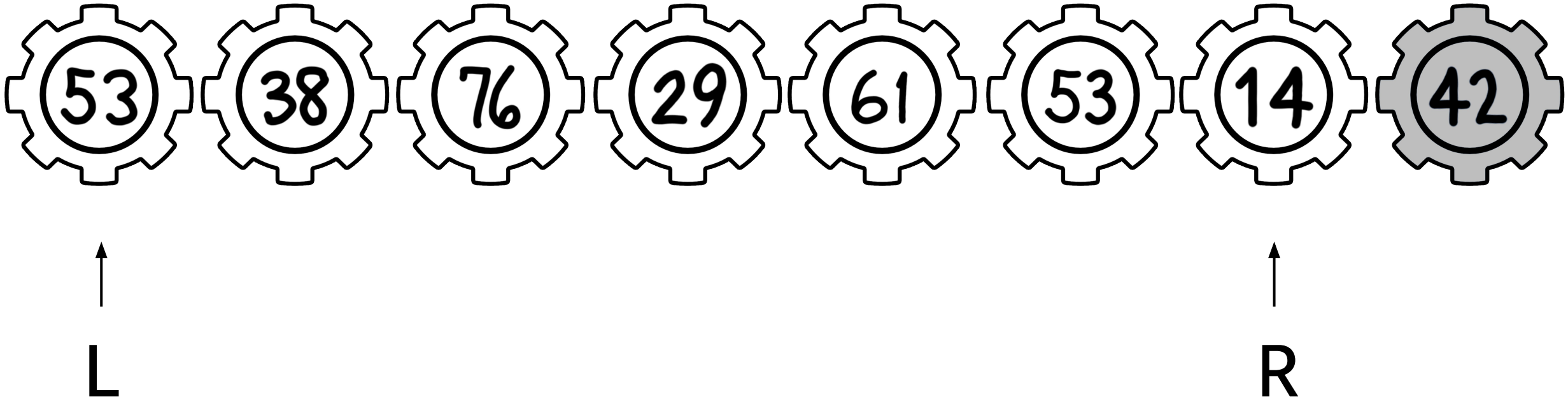
Quicksort

- Quicksort in action (with last element as pivot):



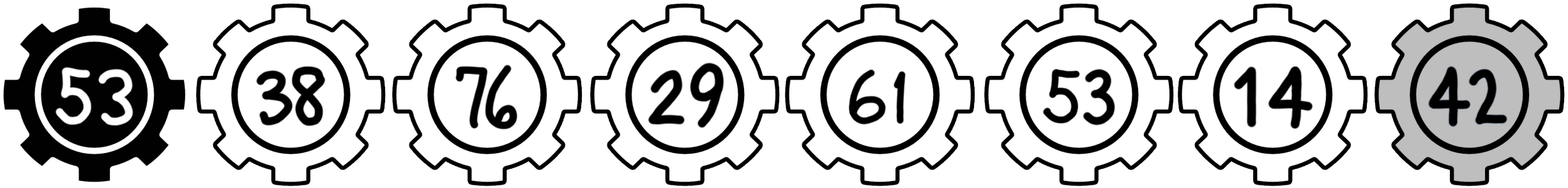
Quicksort

- Quicksort in action (with last element as pivot):



Quicksort

- Quicksort in action (with last element as pivot):



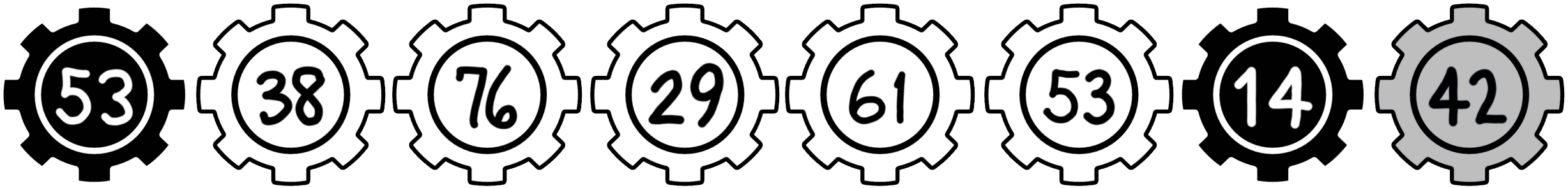
L

R

> 42

Quicksort

- Quicksort in action (with last element as pivot):



L

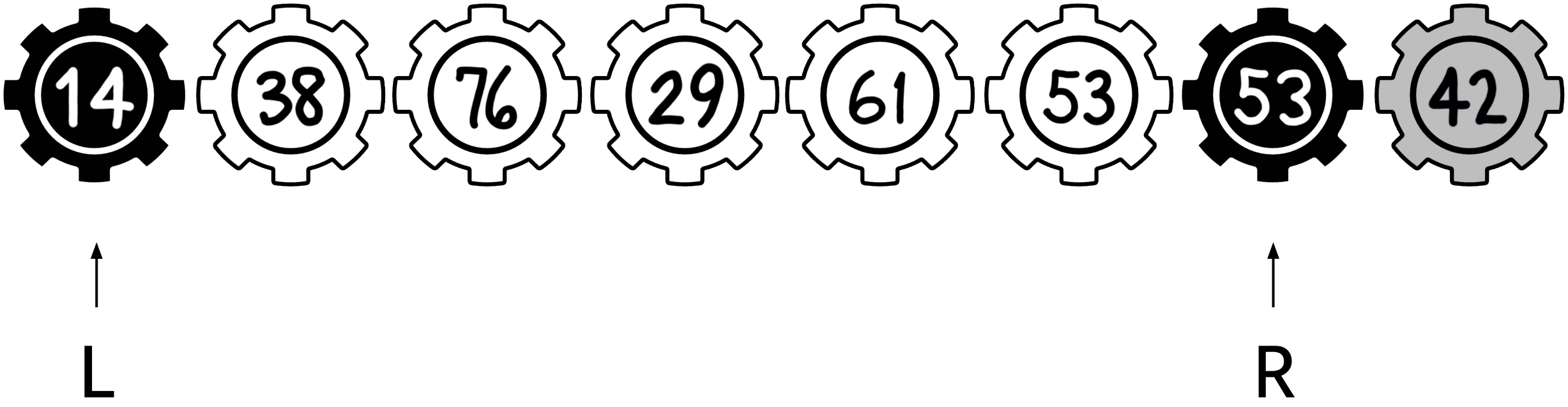
R

> 42

< 42

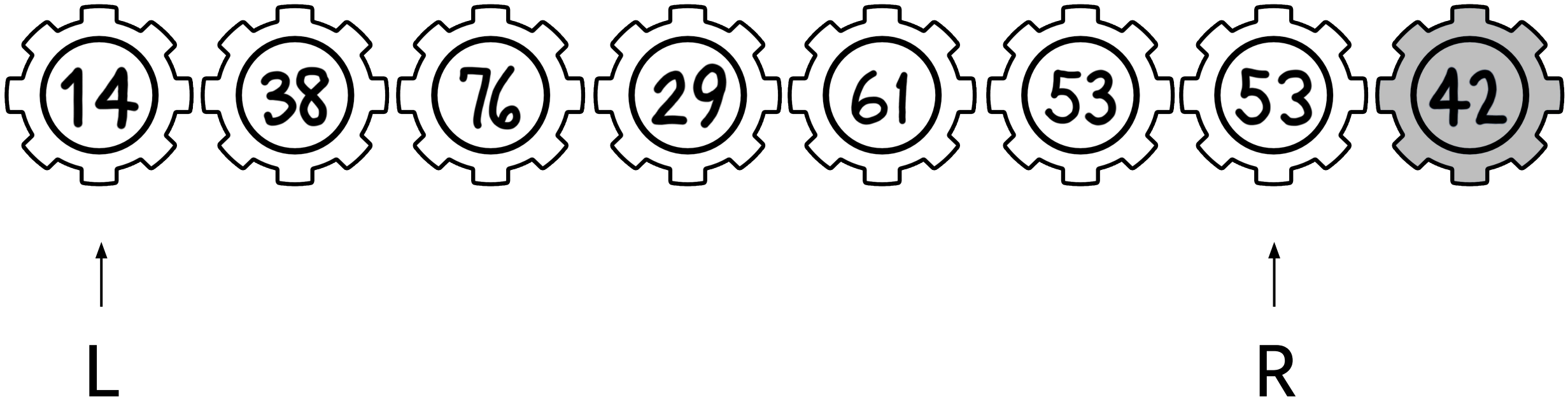
Quicksort

- Quicksort in action (with last element as pivot):



Quicksort

- Quicksort in action (with last element as pivot):



Quicksort

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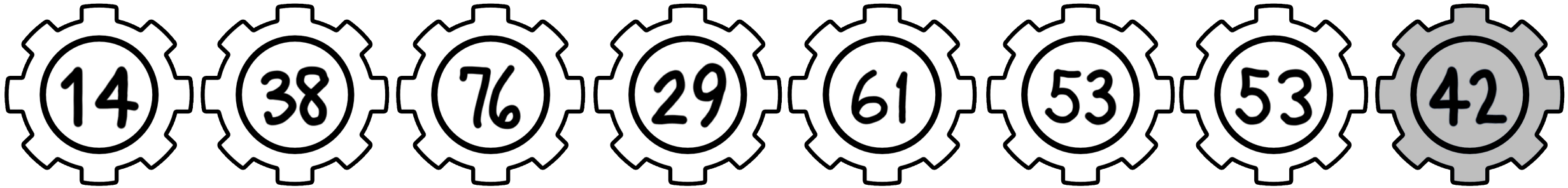

L

R

< 42

Quicksort

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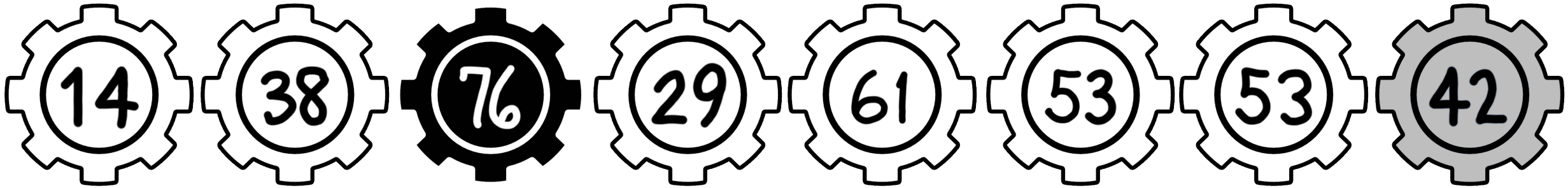

L

R

> 42

Quicksort

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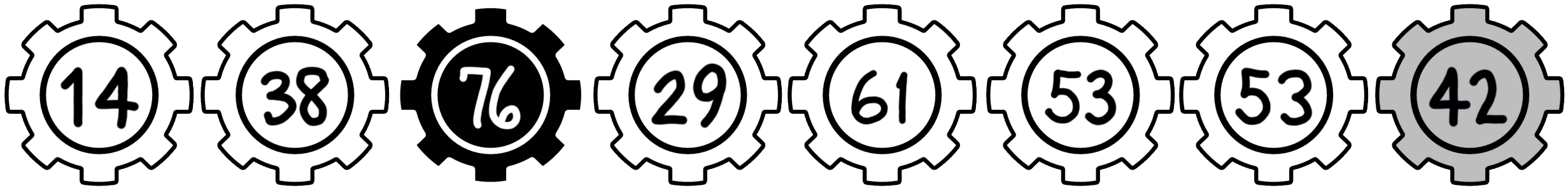

L

> 42

R

> 42

Quicksort

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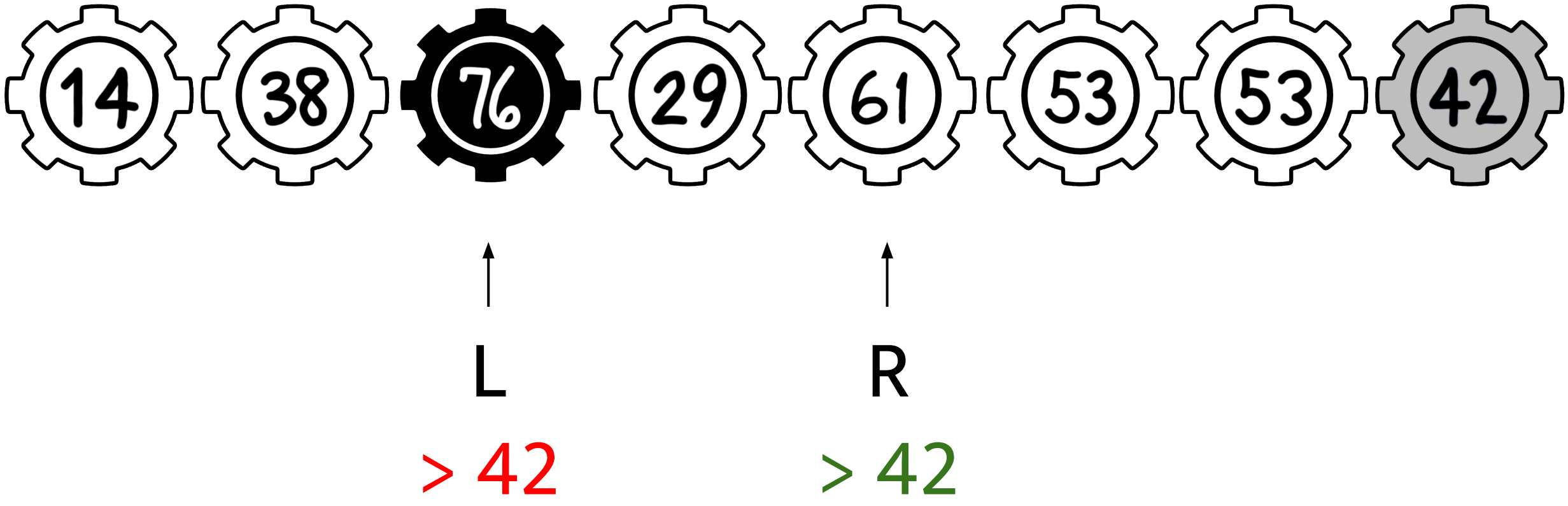

Quicksort

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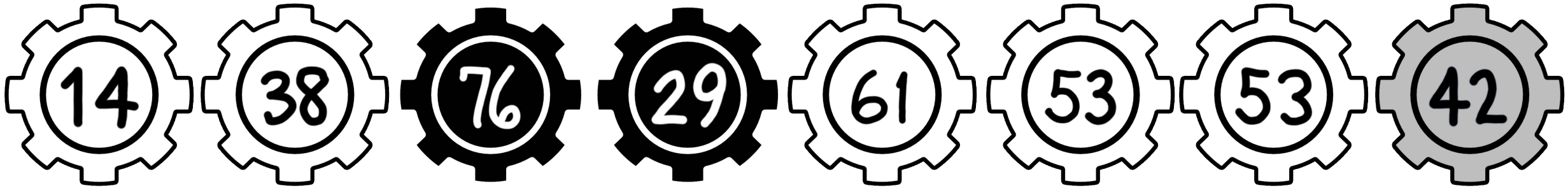
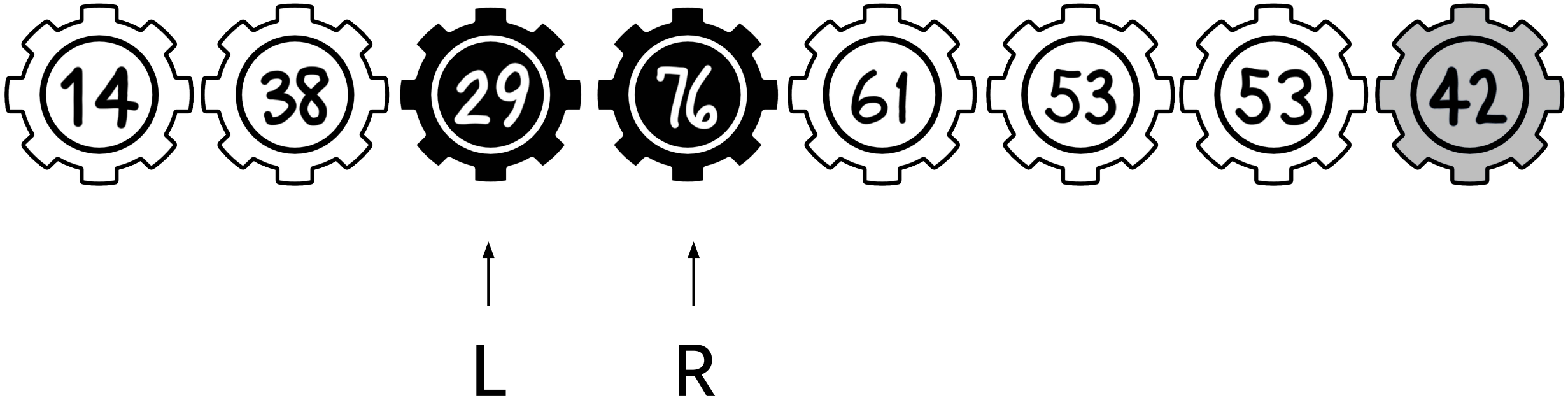


L R

> 42 < 42

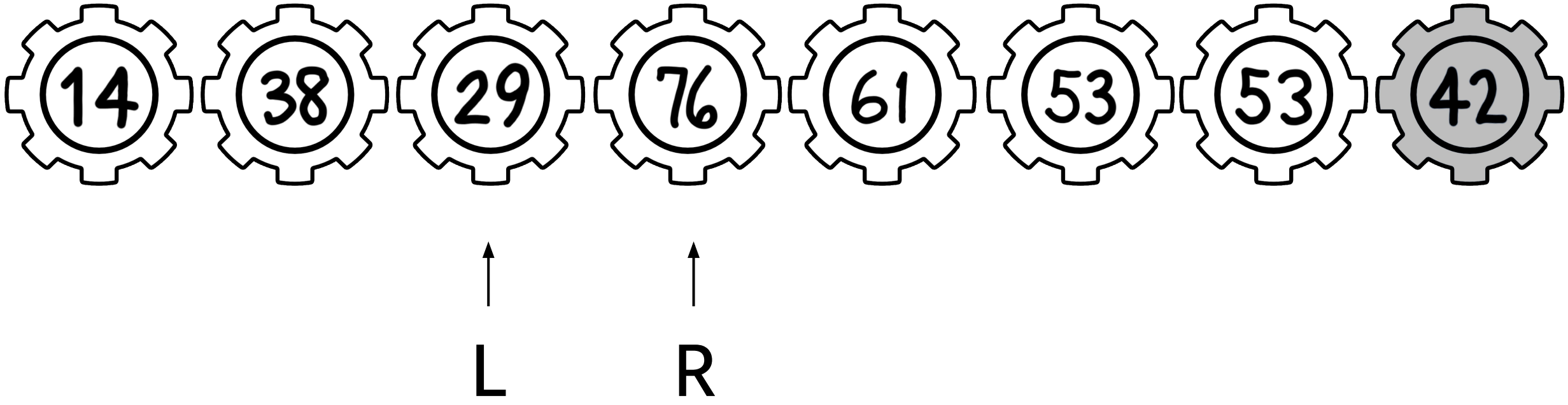
Quicksort

- Quicksort in action (with last element as pivot):



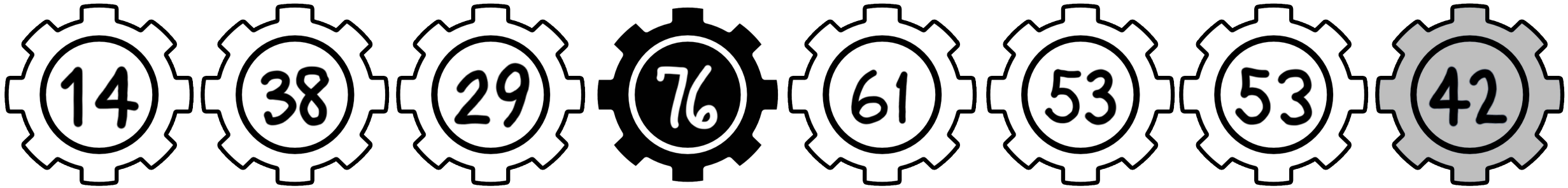
Quicksort

- Quicksort in action (with last element as pivot):



Quicksort

- Quicksort in action (with last element as pivot):



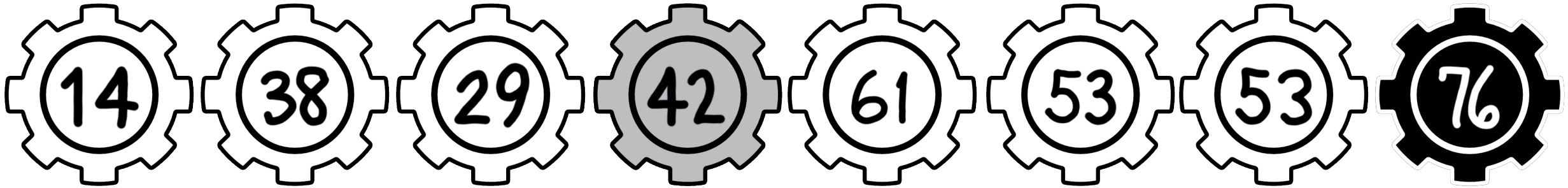
R

L

left and right are equal...
swap with the pivot!

Quicksort

- Quicksort in action (with last element as pivot):



Quicksort

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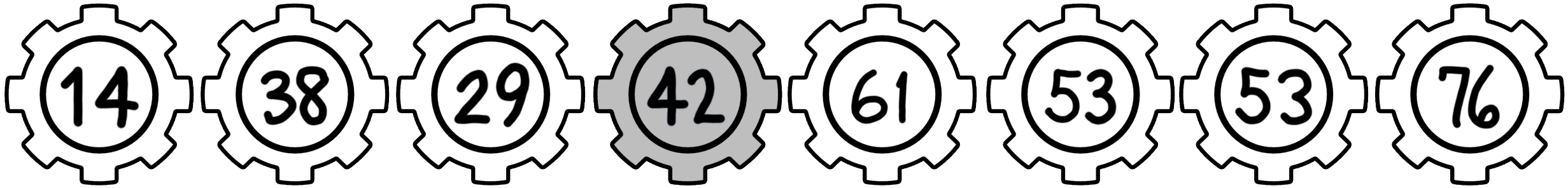

Quicksort

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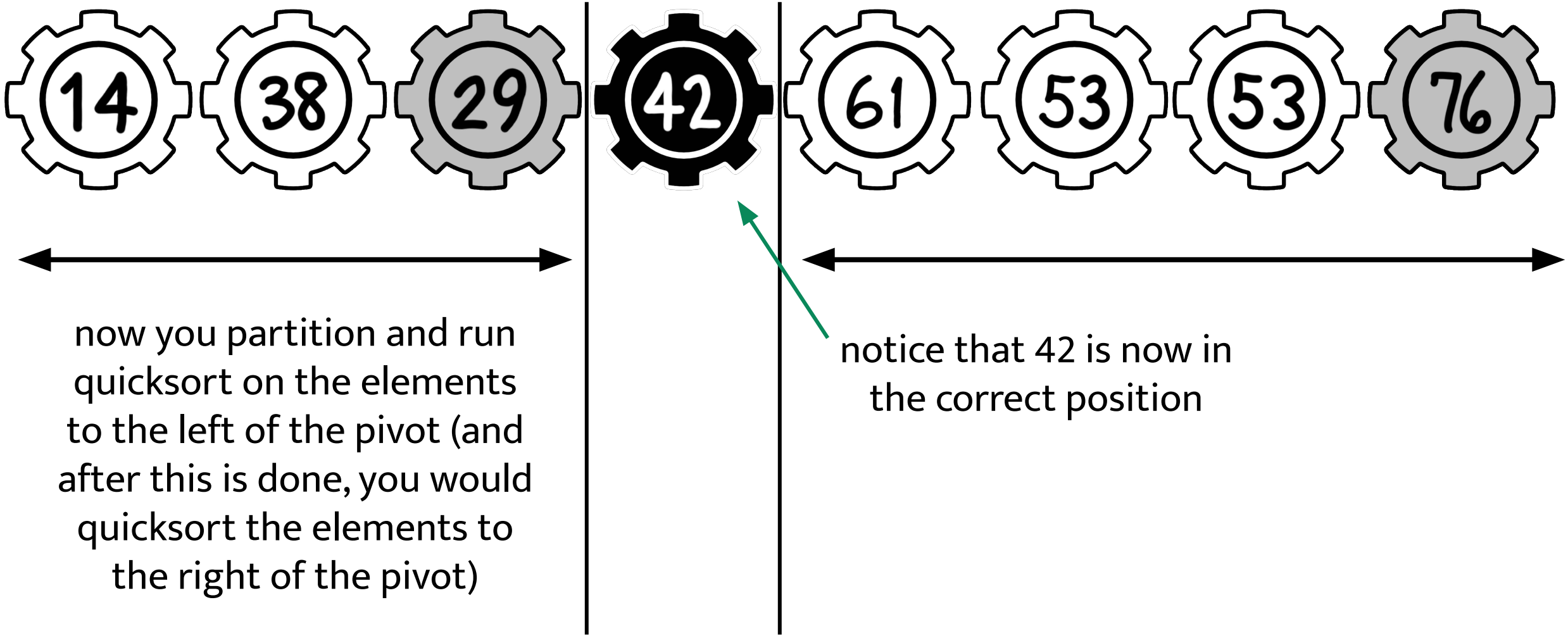
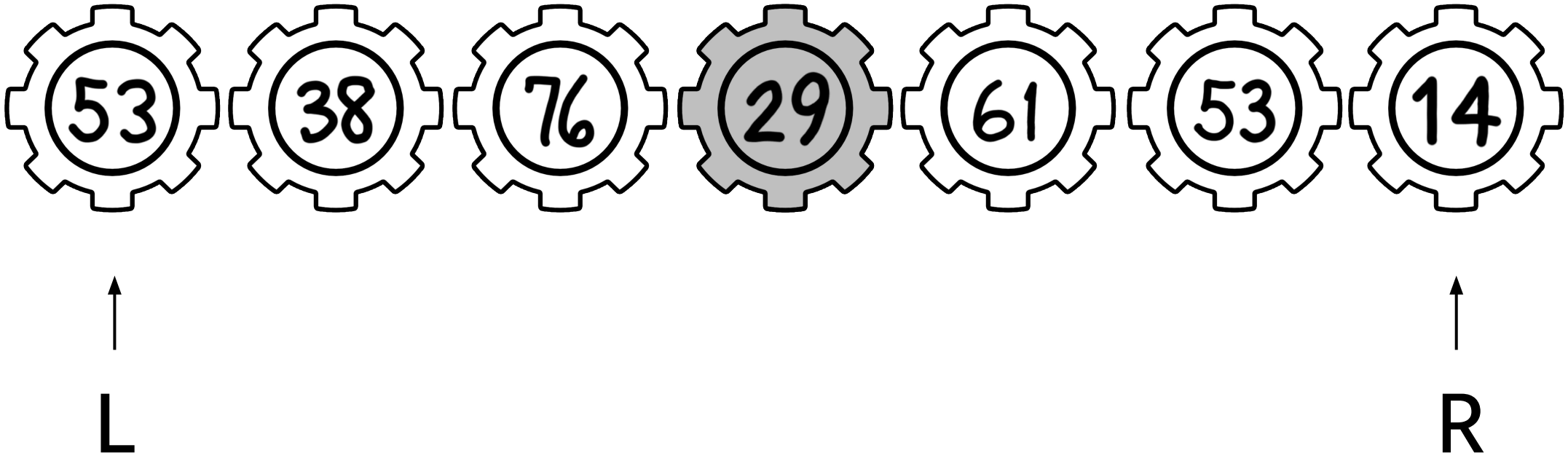


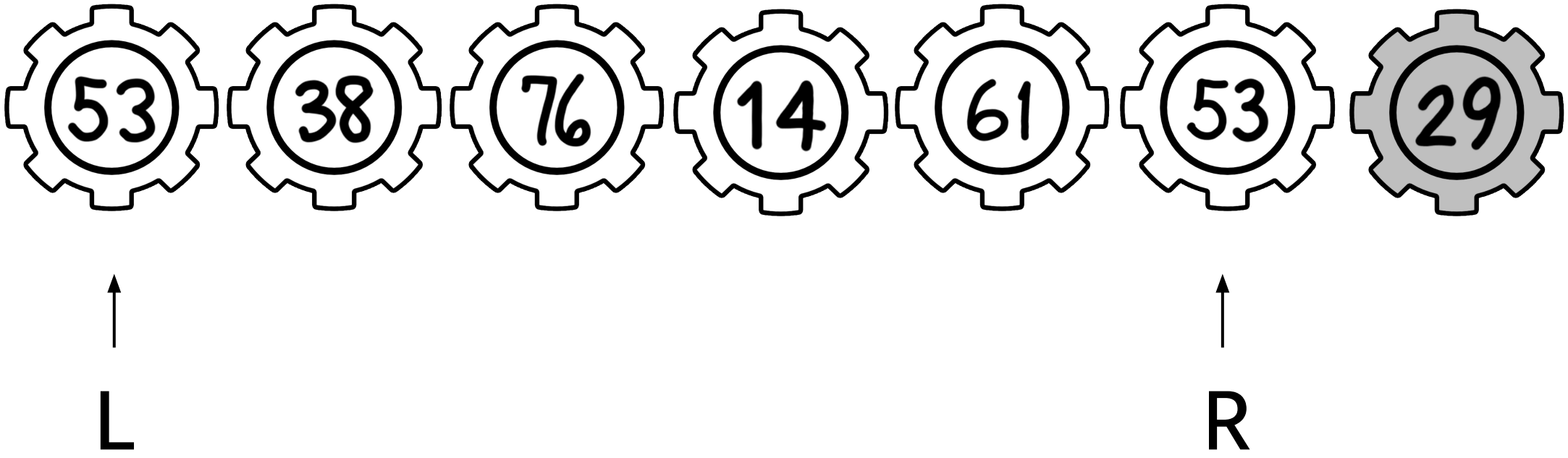
Quicksort — Choosing a different initial pivot

- Suppose we wanted to pick a different pivot that more evenly partitions the array, such as the middle.
- How can we incorporate this new pivot into our algorithm?



Quicksort — Choosing a different initial pivot

- Suppose we wanted to pick a different pivot that more evenly partitions the array.
- How can we incorporate this new pivot into our algorithm?
  - **Swap our new pivot with the last element, and proceed as before!**



Quicksort

- Characteristics of quicksort:

Best Case	Average Case	Worst Case	Memory	Stable?
$\Omega(n \log n)$	$\Theta(n \log n)$	$O(n^2)$	$O(\log n)$	No*

  - Comparison-based sorting algorithm.
  - Most implementations are unstable, except for complex in-place partitioning.
  - Often faster in practice than other $n \log n$ sorting algorithms.
  - Algorithm: pick a pivot from the list. Reorder list so that elements with values less than the pivot come before the pivot, while elements with values greater than or equal to the pivot come after it. After partitioning, the pivot is in the final position. Recursively sort the sub-list of elements with smaller values and the sub-list of elements with greater values.

Quicksort

- What causes the worst-case $O(n^2)$ behavior?
- What is the ideal pivot choice?
- Why can't we do this? What can we do instead?

Quicksort

- What causes the worst-case $O(n^2)$ behavior?

  When the smallest or largest element is chosen as the pivot every time (the more uneven the partition, the worse the performance).

- What is the ideal pivot choice?

- Why can't we do this? What can we do instead?

Quicksort

- What causes the worst-case $O(n^2)$ behavior?

  When the smallest or largest element is chosen as the pivot every time (the more uneven the partition, the worse the performance).

- What is the ideal pivot choice?

  The median element (most evenly partitions the array - roughly equal number of elements to both left and right of pivot).

- Why can't we do this? What can we do instead?

Quicksort

- What causes the worst-case $O(n^2)$ behavior?

  When the smallest or largest element is chosen as the pivot every time (the more uneven the partition, the worse the performance).

- What is the ideal pivot choice?

  The median element (most evenly partitions the array - roughly equal number of elements to both left and right of pivot.

- Why can't we do this? What can we do instead?

  The median is often hard to find. To solve this, we can use median sampling instead (e.g. take the median of a sample).

Summary of Sorts

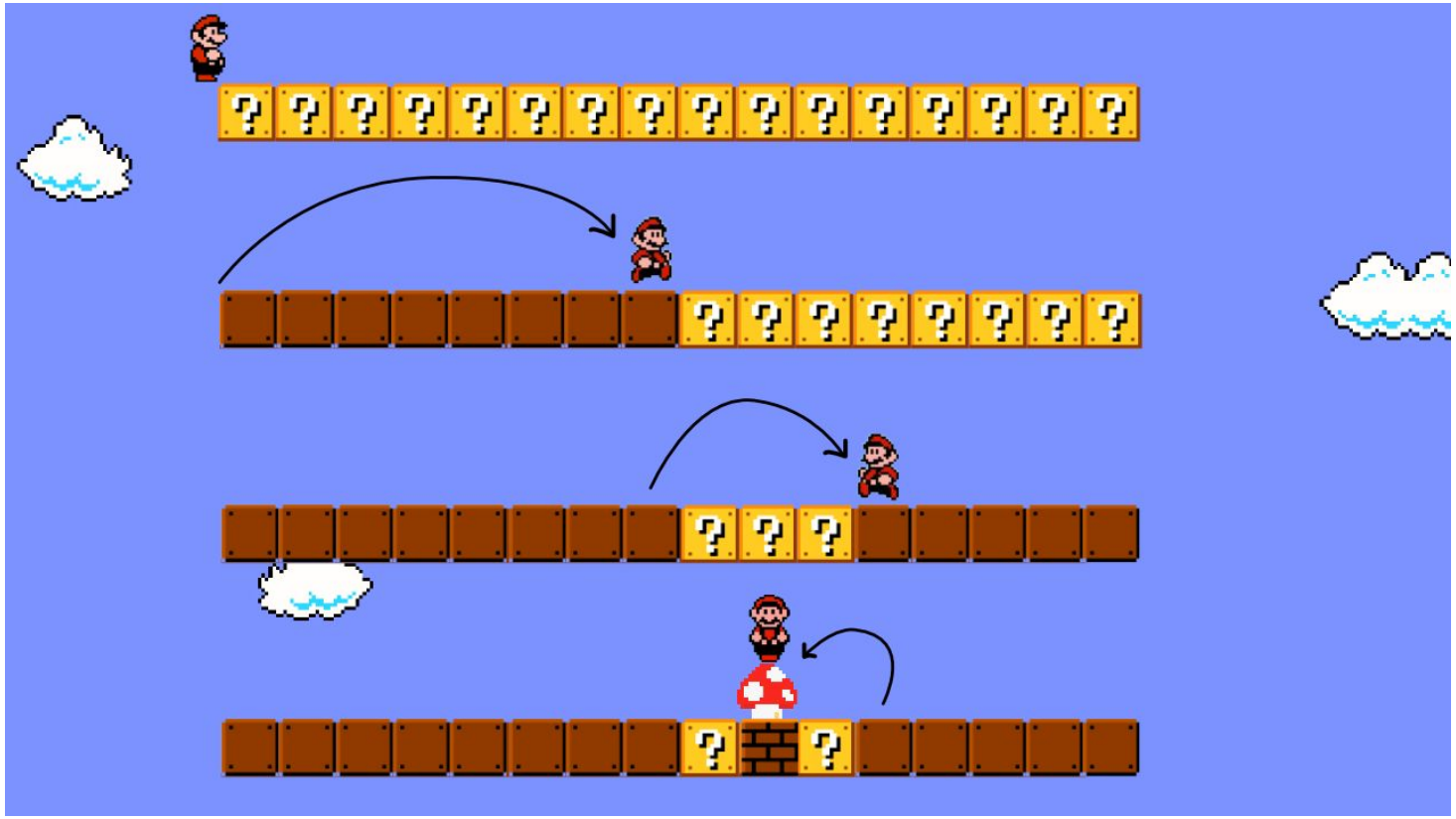
Summary

Sort	Best	Average	Worst	Memory	Stable?	Adaptive?
Bubble	$\Omega(n)$	$\Theta(n^2)$	$O(n^2)$	$O(1)$	Yes	Yes
Selection	$\Omega(n^2)$	$\Theta(n^2)$	$O(n^2)$	$O(1)$	No	No
Insertion	$\Omega(n)$	$\Theta(n^2)$	$O(n^2)$	$O(1)$	Yes	Yes
Heap	$\Omega(n \log n)$ (distinct keys)	$\Theta(n \log n)$	$O(n \log n)$	$O(1)$	No	No
Merge	$\Omega(n \log n)$	$\Theta(n \log n)$	$O(n \log n)$	$O(n)$	Yes (if merge is stable)	No
Quick	$\Omega(n \log n)$	$\Theta(n \log n)$	$O(n^2)$	$O(\log n)$	No	No

Binary Search

Binary Search Interview Problems

- Binary search is a classic topic for interview problems.
- Let's look at a few problems that can be solved using binary search!



Binary Search Question #1

- You are given an array that is first increasing and then decreasing. Find the maximum value of this array.

```
INPUT: arr[] = {8, 10, 20, 80, 100, 200, 400, 500, 3, 2, 1}
OUTPUT: 500

INPUT: arr[] = {1, 3, 50, 10, 9, 7, 6}
OUTPUT: 50

INPUT: arr[] = {10, 20, 30, 40, 50}
OUTPUT: 50

INPUT: arr[] = {120, 100, 80, 20, 0}
OUTPUT: 120
```

Binary Search Question #1

- You are given an array that is first increasing and then decreasing. Find the maximum value of this array.
- Solution:
  - perform a binary search
  - if middle element is greater than both of its adjacent elements, it is the maximum
  - if middle is greater than element to its right and smaller than element to its left, then max element lies to the left of mid (we are on decreasing side of the array)
  - if middle element is smaller than element to its right and greater than element to its left, then max element lies to the right of mid (we are on increasing side)

Binary Search Question #2

- You are given a sorted array that is rotated at a certain position. Return the minimum element.

```
INPUT: arr[] = {5, 6, 1, 2, 3, 4}
OUTPUT: 1

INPUT: arr[] = {1, 2, 3, 4}
OUTPUT: 1

INPUT: arr[] = {2, 1}
OUTPUT: 1

INPUT: arr[] = {2, 3, 4, 5, 6, 1}
OUTPUT: 1
```

Binary Search Question #2

- You are given a sorted array that is rotated at a certain position. Return the minimum element.
- Solution:
  - perform a binary search
  - the minimum element is the only element whose previous element is greater than it
  - check the middle element - if the element to its left is greater than it, mid is the min
  - if the element to its left is less than or equal, it is not the minimum; the minimum lies either in its left or right half
  - if the middle element is smaller than the last element, the minimum lies in the left half; else, it lies in the right half
  - When you have narrowed the minimum down to two elements, you can compare their values to find the min

Example:

{5, 6, 1, 2, 3, 4}

Mid element is 2. The element to the left is not greater than it. Hence, it is not the minimum.

Since 2 is smaller than 4 (the last element), it means the rotation point lies before mid. Therefore, we check the left half.

Binary Search Question #3

- You are given a sorted array where every number occurs twice except for one number. Find this number that only appears once.

```
INPUT: arr[] = {1, 1, 6, 6, 9, 9, 22, 22, 26, 30, 30, 40, 40}
OUTPUT: 26

INPUT: arr[] = {1, 2, 2, 3, 3, 4, 4}
OUTPUT: 1

INPUT: arr[] = {1, 1, 2, 2, 3, 3, 4}
OUTPUT: 4

INPUT: arr[] = {281}
OUTPUT: 281
```

Binary Search Question #3

- You are given a sorted array where every number occurs twice except for one number. Find this number that only appears once.
- Key observation: if there were no odd element out, the first occurence of each number would be found at an even index, and the second occurrence would be found at an odd index.
  - This is true until we reach the odd one out! Once we reach this point, the opposite will be true (the first occurrence would be found at an odd index, the second at an even index). We will do a binary search to find the index of the odd one out.
  - If mid is even, check if a[mid] == a[mid + 1]
  - if mid is odd, check if a[mid] == a[mid - 1]
    - if these conditions hold, the odd one out must be found to the right, else it must be found to the left!

Bonus Interview Brain Teaser

Brain Teaser (Exercise for Home)

- There are 2810 light bulbs lined up in a row in a long room. Each bulb has its own switch, and all switches are switched OFF at the beginning. The bulbs are numbered consecutively from 1 to 2810.

  There are 2810 people lined up outside the entry door. Person 1 enters the room, flips the switch on every bulb, and exits. After person 1 leaves the room, person 2 enters and flips the switch on every second bulb (turning off bulbs 2, 4, 6...). After person 2 leaves, person 3 enters and flips the switch of every third bulb (changing the state on bulbs 3, 6, 9...). This continues until all 2810 people have passed through the room, where the 2810th person only flips the switch of the 2810th bulb.

  After all 2810 people have exited the room, how many light bulbs are turned ON?

  Suppose you have the ability to remove bulbs from the room in reverse order, first removing bulb 2810, then 2809, etc. What is the minimum number of bulbs you would have to remove for the number of bulbs that are turned ON to change?

Handwritten Problem

Handwritten Problem

- Given a vector with *n* elements with values of either 0, 1, or 2, devise an O(n) algorithm to sort this vector. You must do this in a **single pass** of the vector. You may **NOT** copy items, create arrays or strings, or do any other memory allocation. Make sure your algorithm works for all cases.
- You may use `std::swap` to swap two items in the vector.

```
BEFORE: arr[] = {2, 1, 0, 0, 2, 1, 2, 2, 0, 1, 1, 1, 0}
AFTER: arr[] = {0, 0, 0, 0, 1, 1, 1, 1, 1, 2, 2, 2, 2}
```

```
// sort a vector of 0s, 1s, and 2s in linear time
void sort012(vector<int>& nums);
```

Good Luck on the Exam!