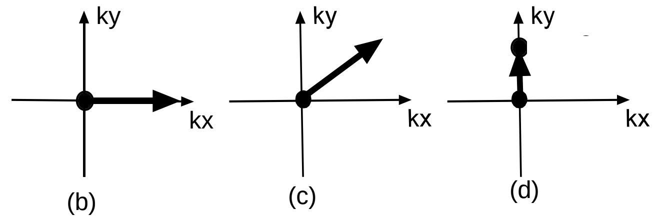
Solutions, Homework #5

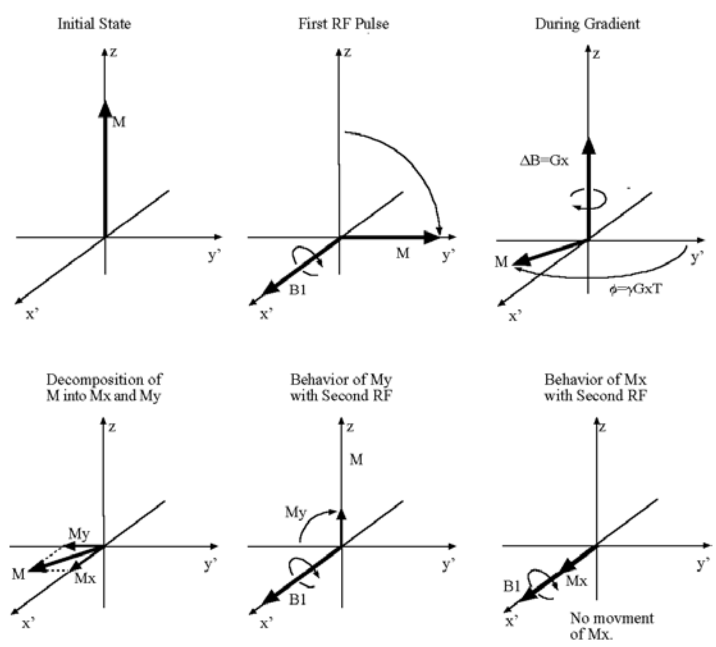
1.
  a. $M_0(u,v) = XY\,\mathrm{sinc}(Xu)\,\mathrm{sinc}(Yv)$
  b. $k_x(t) = \gamma At/(2\pi)$, $k_y(t) = 0$. Assuming that the scaling constant is 1, then $s(t) = XY\,\mathrm{sinc}(X\gamma At/(2\pi))$
  c. $k_x(t) = \gamma at/(2\pi X)$, $k_y(t) = \gamma at/(2\pi Y)$. $s(t) = XY\,\mathrm{sinc}^2(\gamma at/(2\pi))$.
  d. For $t \in [0,T]$, $k_x(t) = 0$, $k_y(t) = \gamma at/(2\pi Y)$. $s(t) = XY\,\mathrm{sinc}(\gamma at/(2\pi))$

     For $t > T$, $k_x(t) = 0$, $k_y(t) = \gamma aT/(2\pi Y) = 2/Y$.

     $s(t) = XY\,\mathrm{sinc}(0)\,\mathrm{sinc}(2) = 0$.



2. First of all, $m_0(x) = 1$ implies that the magnetization is uniform everywhere. The first RF pulse tips the magnetization into the transverse plane. Looking in the rotating frame, **M** lies along the y'-axis. We then apply a gradient that makes the resonant frequency vary as a function of spatial position $\omega(x) = \omega_0 + \gamma Gx$. In the rotating frame, the resonant frequency is $\omega_{rot}(x) = \gamma Gx$. Assuming that the gradient is turned on for length of time, T, then the phase that accumulates (as a function of spatial position) is $\phi_{rot}(x) = \gamma GxT$. We can decompose the magnetization vector into two components $M_{x,rot} = \sin(\phi)$ and $M_{y,rot} = \cos(\phi)$. During the second RF pulse $-M_{y,rot}$ is rotated up to M_z and $M_{x,rot}$ does not change. Ultimately, we end up with a sinusoidal variation with x of the x- and z-components of the magnetization.

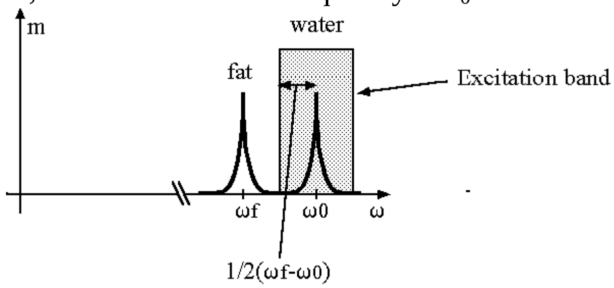


a) $m_{xy,rot}(x) = 0$; $m_z(x) = m_0$
b) $m_{xy,rot}(x) = i\, m_0$; $m_z(x) = 0$
c) $m_{xy,rot}(x) = i\, m_0 \exp(-i\,\gamma GxT) = i\, m_0 \cos(\gamma GxT) + m_0 \sin(\gamma GxT)$; $m_z(x) = 0$
d) $m_{xy,rot}(x) = m_0 \sin(\gamma GxT)$; $m_z(x) = -m_0 \cos(\gamma GxT)$

3.

a. $\gamma/2\pi$ = 42.58 MHz/T, so f_0 = (42.58 MHz/T)(1.5 T) = 63.87 MHz and
$\Delta f = f_0 - f_f = \gamma/2\pi\, B_0\, \delta_f$ = (42.58 MHz/T)(1.5 T)(3.5 x 10^{-6}) = 223.5 Hz.

b. As shown in the drawing below, one possible excitation pulse will be centered at the frequency of water, f_0, and will have a rectangular frequency profile with a half-bandwidth of ½ of the difference in water and fat frequencies. The water excitation pulse, therefore, will have a center frequency of f_0 and a bandwidth of $\Delta f = f_0 - f_f$.



which has a spectrum rect$((f-f_0)/\Delta f)$. The RF pulse will then be similar to the IFT of this function – $B_{1,rot}(t) = A\,\mathrm{sinc}(t/T)$ (perhaps truncated with a rect or hanning window), where $1/T = \Delta f$ = 223.5 Hz or T = 4.47 ms. The integrated areas under this RF pulse is γAT, which for a 90 degree flip angle, should be set equal to $\pi/2$. Thus, $A = \pi/(2\gamma T)$. The carrier frequency at which it is applied will be f_0 = 63.87 MHz. The RF pulse will now look like: $B_{1,xy}(t) = A\,\mathrm{sinc}(t/T)\exp(-i2\pi f_0)$. (Recall that spins in MRI go in the negative frequency direction.)

4.

a. $FOV_z = 1/\Delta k_z$ and $\Delta k_z = \gamma/2\pi\, T_z\, \Delta G_z$, thus $\Delta G_z = 2\pi/(\gamma\, T_z\, FOV_z)$ = 0.023 mT/m.

b. $\Delta z = 1/(2k_{z,max})$ and $k_{z,max} = \gamma/2\pi\, T_z\, G_{z,max}$, thus $G_{z,max} = 2\pi/(\gamma\, T_z\, 2\, \Delta z)$ = 0.47 mT/m.

c. $FOV_y = 1/\Delta k_y$ and $\Delta k_y = \gamma/2\pi\, T_y\, \Delta G_y$, thus $\Delta G_y = 2\pi/(\gamma\, T_y\, FOV_y)$ = 0.023 mT/m.

d. $\Delta y = 1/(2k_{y,max})$ and $k_{y,max} = \gamma/2\pi\, T_y\, G_{y,max}$, thus $G_{y,max} = 2\pi/(\gamma\, T_y\, 2\, \Delta y)$ = 1.17 mT/m.

e. $\Delta x = 1/W_{kx}$ and $W_{kx} = \gamma/2\pi\, T_{read}\, G_{read}$, thus $G_{read} = 2\pi/(\gamma\, T_{read}\, \Delta x)$ = 1.17 mT/m.

f. $FOV_x = 1/\Delta k_x$ and $\Delta k_x = \gamma/2\pi\, \Delta t\, G_{read}$ and $G_{read} = 2\pi/(\gamma\, T_{read}\, \Delta x)$, thus
$\Delta t = T_{read}\, \Delta x / FOV_x$ = 0.1 ms (100 μs).

5. Let the signal strength (as a function of x) is equal to sensitivity $S(x)$ and the noise is equal to σ. The signal to noise ratio is then $S(x)/\sigma$. For the volume coil $S_v(x) = 1$ and $\sigma_v = 1$, therefore $SNR_v = 1$. For the surface coil, the SNR is

$$SNR_s = \frac{1}{(0.1 \cdot a)^{3/2}\left(1+\left(\frac{x}{a}\right)^2\right)^{3/2}} = \frac{1}{\left((0.1 \cdot a)\left(1+\left(\frac{x}{a}\right)^2\right)\right)^{3/2}}$$

To find the region where $SNR_s > SNR_v$, we merely need to find for which x that $SNR_s > 1$.

a. $a = 5$ cm, $SNR_s > SNR_v$, for $x < 5$ cm, that is if we are interested in a structure closer to the coil than 5 cm, it is preferred (from the SNR standpoint) to use the surface coil, otherwise the volume coil is better.

$$SNR_S = \frac{1}{\left(0.5\left(1+\left(\frac{x}{5}\right)^2\right)\right)^{3/2}} > 1$$

$$0.5\left(1+\left(\frac{x}{5}\right)^2\right) < 1$$

$$\left(\frac{x}{5}\right)^2 < 1$$

$$x < 5$$

b. $a = 10$ cm, $SNR_v > SNR_s$, for all non-zero values of x, therefore, the volume will always have better SNR. (No values of x satisfy the below relationship.)

$$SNR_S = \frac{1}{\left(1\left(1+\left(\frac{x}{10}\right)^2\right)\right)^{3/2}} > 1$$

$$\left(1+\left(\frac{x}{5}\right)^2\right) < 1$$

$$\left(\frac{x}{10}\right)^2 < 0$$

$$x^2 < 0$$