X-ray Imaging

BME/EECS 516

X-ray Lecture #3

Announcements

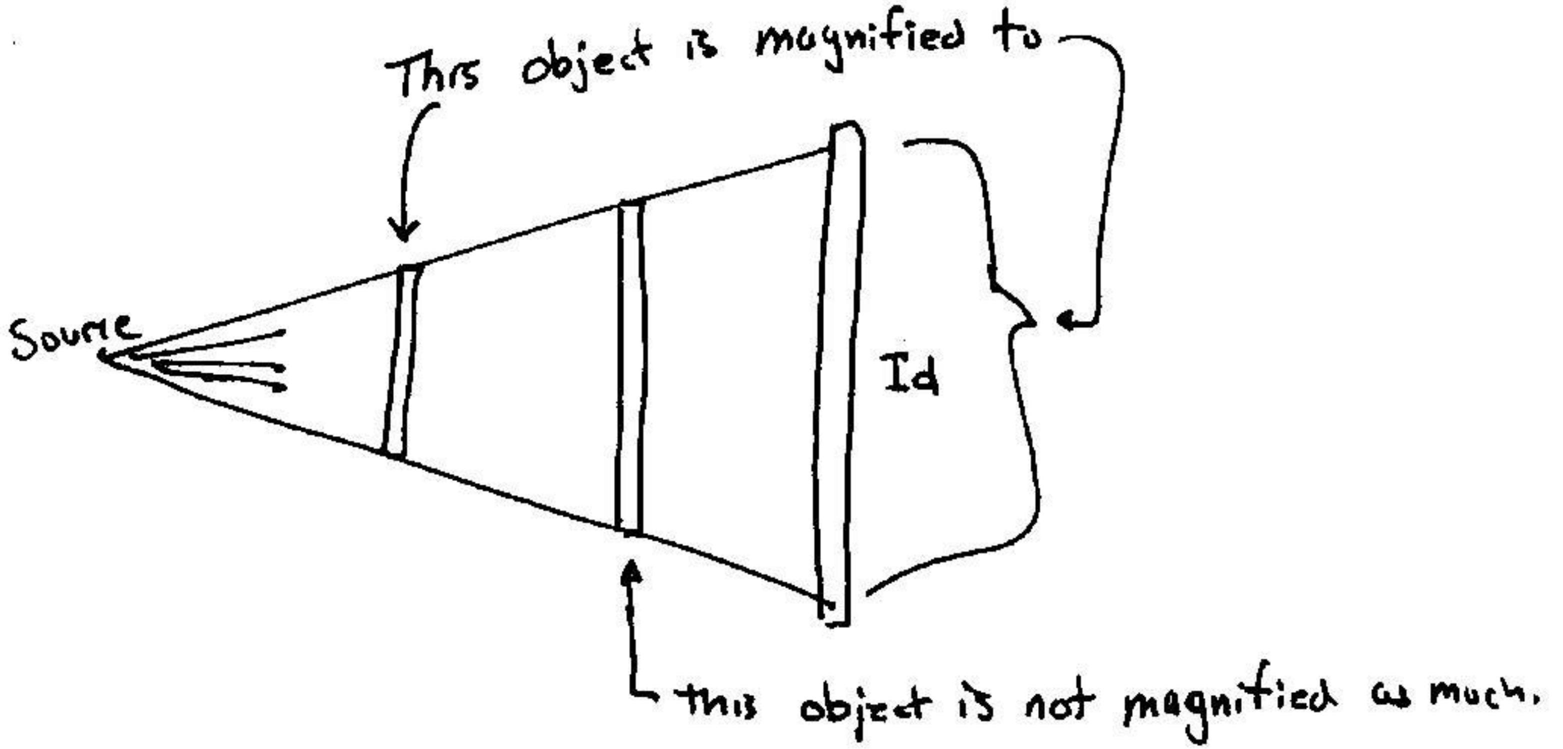
- MRI Project due Tuesday 11/21
- Tuesday, 11/14 – guest lectures from local medical imaging industry
  - David Sarment from Xoran Technologies
  - John Seamans (UM) formerly with Delphinus Medical and GE Healthcare
- US and MRI Demos – 11/21 during class time. More info to come…

Last Lecture

- Source Issues
  - Point Source Geometry
  - Intensity Variations
  - Finite (large) source size
- Overall System Response
- X-ray Detection
- Noise in X-ray Systems
  - SNR of a Poisson Measurement

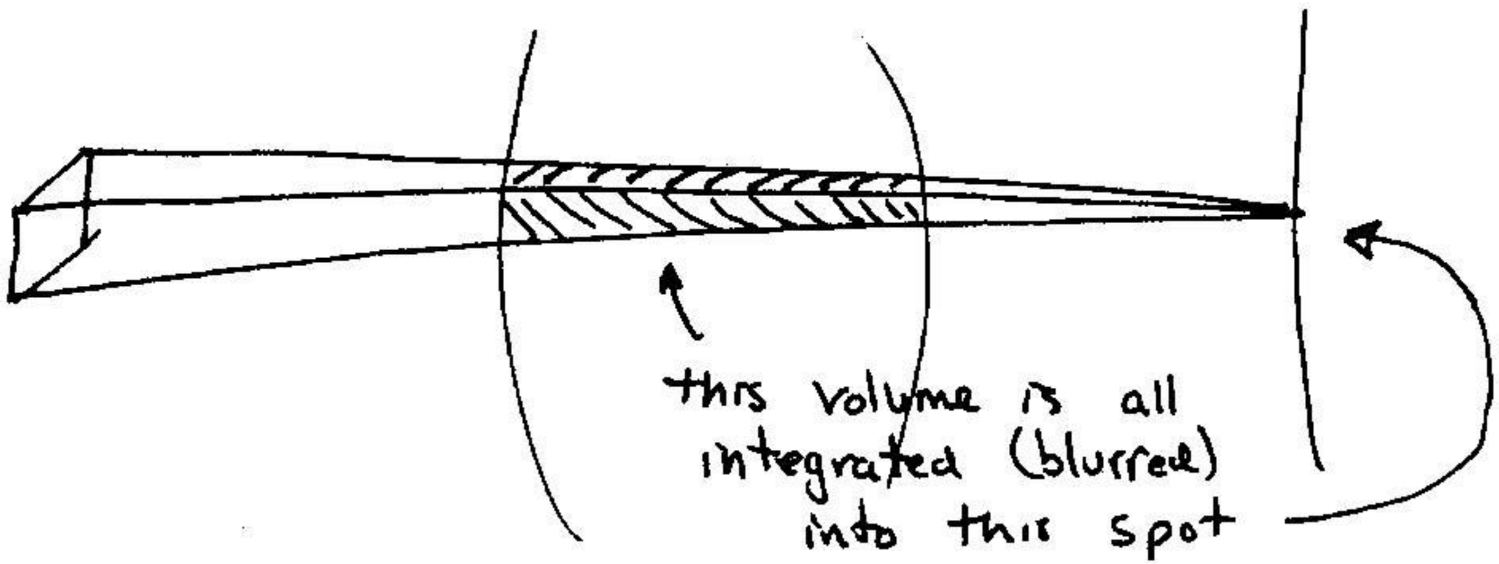
Practical X-ray Sources – two main issues

1. Geometric distortions due to point geometry – "depth dependent magnification."



Practical X-ray Sources – two main issues

2. Resolution loss (blurring) due to finite (large) source sizes



Intensity Variations

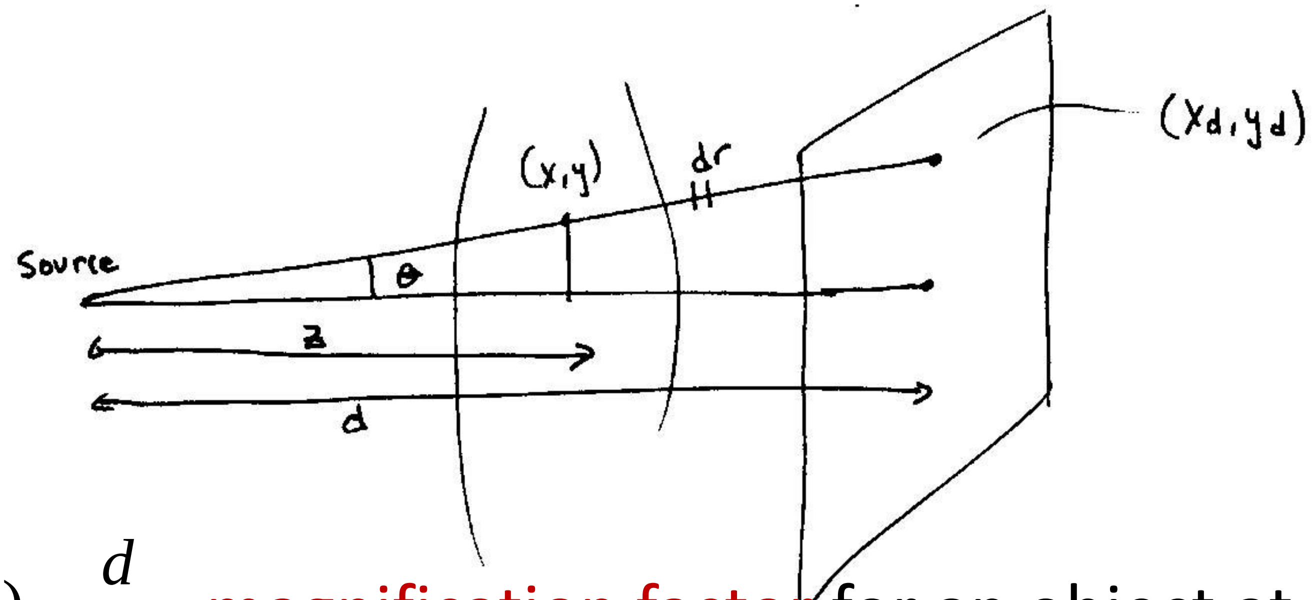
$$I_i = I_0 \cos^3\theta = I_0\left(\frac{d}{r}\right)^3 \qquad \cos\theta = \frac{d}{r}$$

- $\cos^3\theta$ (or its representation) - *incident intensity obliquity term* - has two components
  - $\cos^2\theta$ term - increase in distance from the source to the detector
  - $\cos\theta$ term - rays obliquely striking the detector

$$I_i(x_d, y_d) = I_0\left(\frac{d}{\sqrt{d^2 + r_d^2}}\right)^3 = I_0 \frac{1}{\left(1+\left(\frac{r_d}{d}\right)^2\right)^{3/2}} \qquad r^2 = d^2 + r_d^2$$

Point Source

- A point in the object (x,y) at depth z - it will strike the detector at a position $(x_d, y_d) = \left(x\frac{d}{z}, y\frac{d}{z} \right)$



$M(z) = \frac{d}{z}$ - magnification factor for an object at depth z.

Finite (Large) Sources

- $M = M(z_0) = d / z_0$ - is the *object magnification factor*
- "Thin" object

$$\mu(x, y, z) = \tau(x, y)\delta(z - z_0)$$

- Including the I_i term:

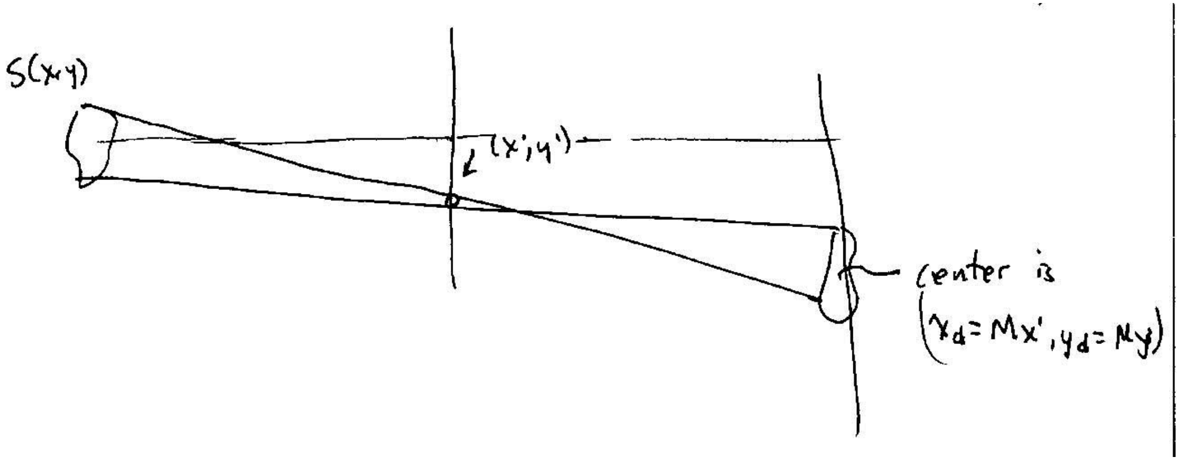
$$I_d(x_d, y_d) = I_i \exp\left(-\tau\left(\frac{x_d}{M}, \frac{y_d}{M}\right)\right) = I_i t\left(\frac{x_d}{M}, \frac{y_d}{M}\right)$$

where $t = \exp(-\tau)$ is the *transmission function*

Finite (Large) Sources

- Let the pinhole be at position (x', y')

$$t(x, y) = \delta(x - x', y - y')$$



- The impulse response function is:

$$h(x_d, y_d; x', y') = I_d(x_d, y_d) = \frac{1}{4\pi z^2 m^2} s\left(\frac{x_d - Mx'}{m}, \frac{y_d - My'}{m}\right)$$

Finite (Large) Sources

$$I_d(x_d, y_d) = \iint t(x'y')h(x_d, y_d; x', y')dx'dy'$$

$$= \frac{1}{4\pi z^2 m^2} \iint t(x'y')s\left(\frac{x_d - Mx'}{m}, \frac{y_d - My'}{m}\right)dx'dy' \text{ and sub } Mx' = x$$

$$= \frac{1}{4\pi z^2 m^2 M^2} \iint t\left(\frac{x}{M}, \frac{y}{M}\right)s\left(\frac{x_d - x}{m}, \frac{y_d - y}{m}\right)dxdy$$

$$= \frac{1}{4\pi d^2 m^2} s\left(\frac{x_d}{m}, \frac{y_d}{m}\right) ** t\left(\frac{x_d}{M}, \frac{y_d}{M}\right)$$

Overall System Response

- The detector response – $h(r_d)$
- Add the detector response to the other system elements. The overall system response is:

$$I_d(x_d, y_d) = \frac{1}{4\pi d^2 m^2} s\left(\frac{x_d}{m}, \frac{y_d}{m}\right) ** t\left(\frac{x_d}{M}, \frac{y_d}{M}\right) ** h(r_d)$$

- $M = \frac{d}{z}$ is the *object magnification*
- $m = -\frac{(d-z)}{z}$ is the source *magnification*

Overall System Response

- The impulse response function

$$h(x_d, y_d) = \frac{1}{4\pi d^2 m^2} s\left(\frac{x_d}{m}, \frac{y_d}{m}\right) ** h(r_d)$$

- For a circularly symmetric source function:

$$h(x_d, y_d) = \frac{1}{4\pi d^2 m^2} s\left(\frac{r_d}{m}\right) ** h(r_d)$$

Noise in X-ray systems

- X-ray images are created from intensity values that are related to the number of photons(N) that strike a detector element in a finite period of time (from intensity I)

- Photon counts (N) is Poisson distributed, and

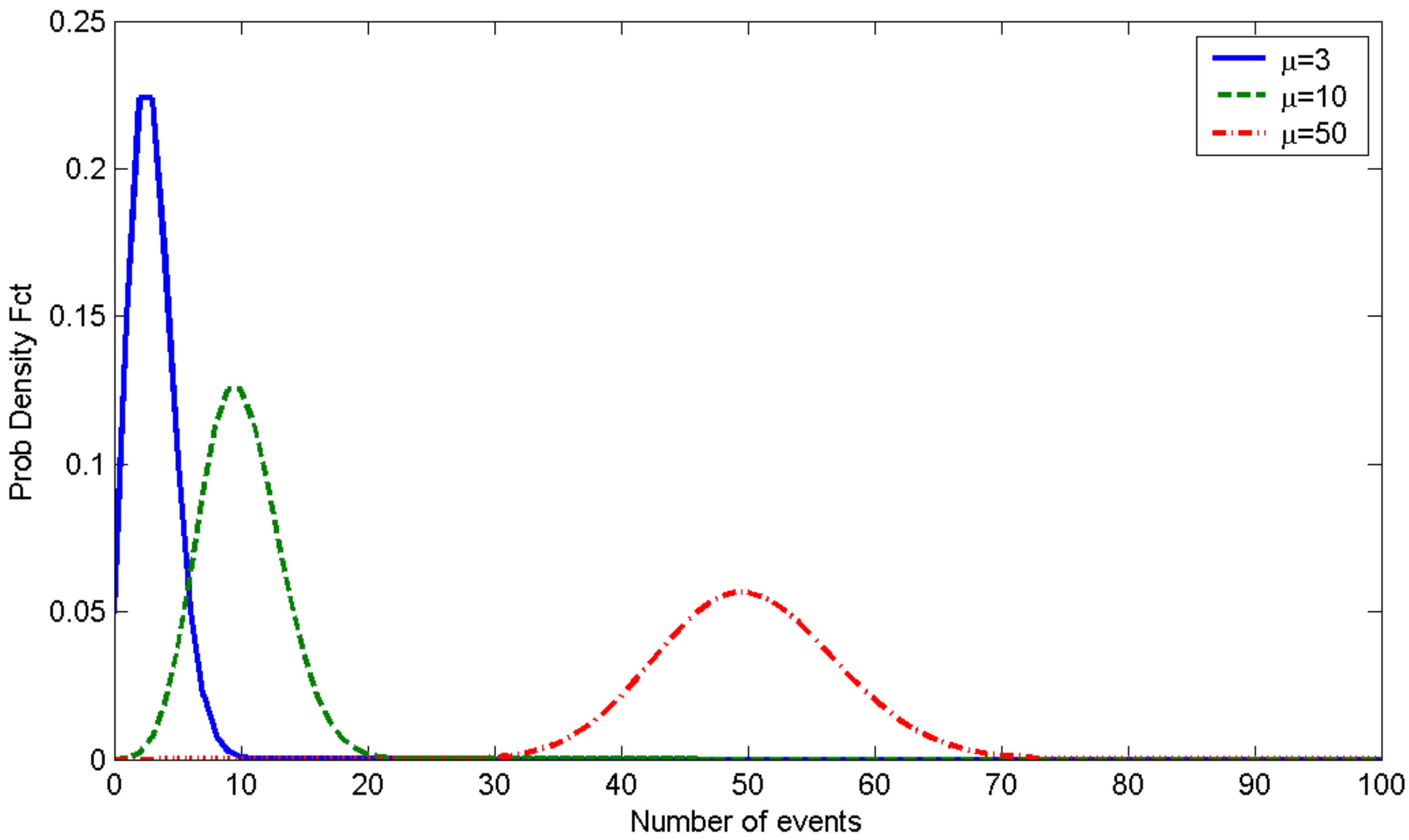
$$SNR = \frac{\bar{S}}{\sigma_S} = \frac{N}{\sqrt{N}} = \sqrt{N}$$

SNR of a Poisson Measurement

- SNR increases with the square root of the number of photons, N.
- $SNR \propto (patient\ dose)$
- If pixel size is doubled for a fixed I_0, then N, will also double and SNR will increase by $\sqrt{2}$

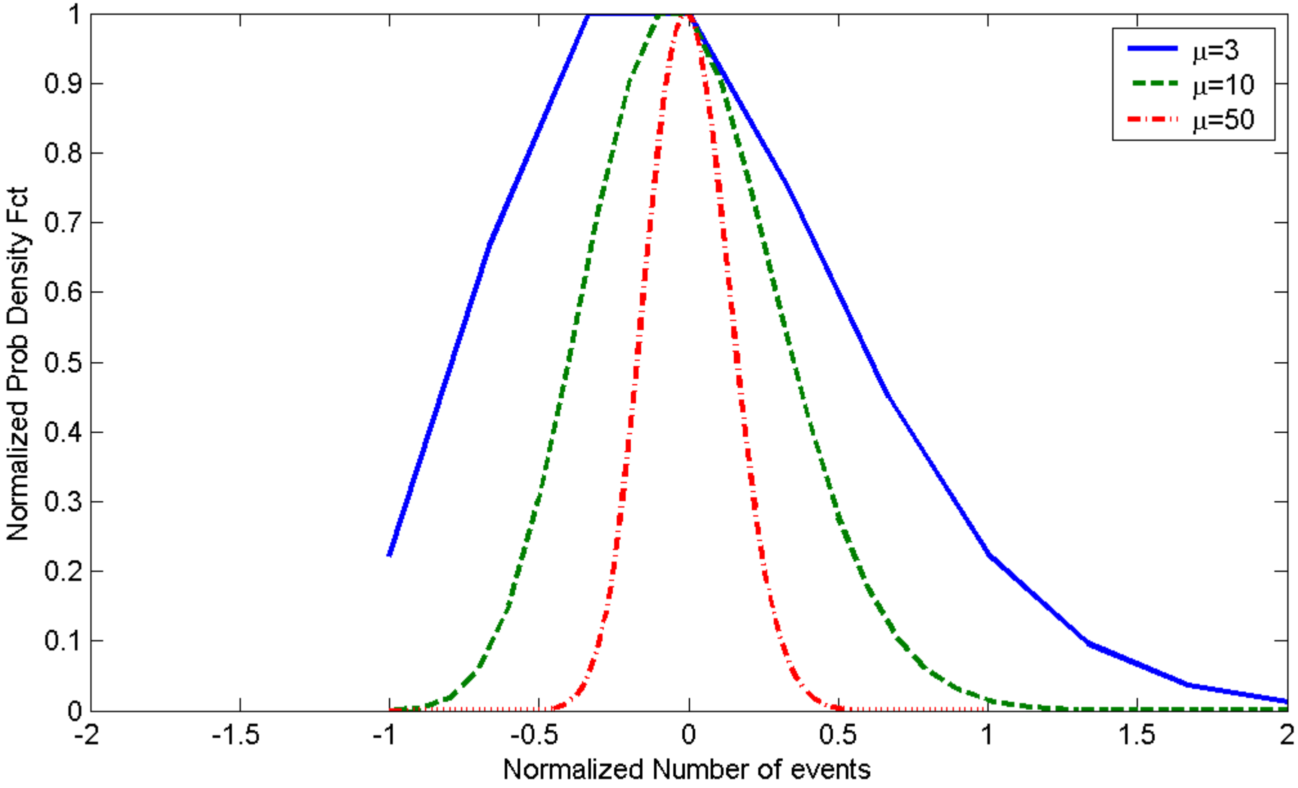
SNR of a Poisson Measurement

Poisson distributions as the mean increases from 3 to 50



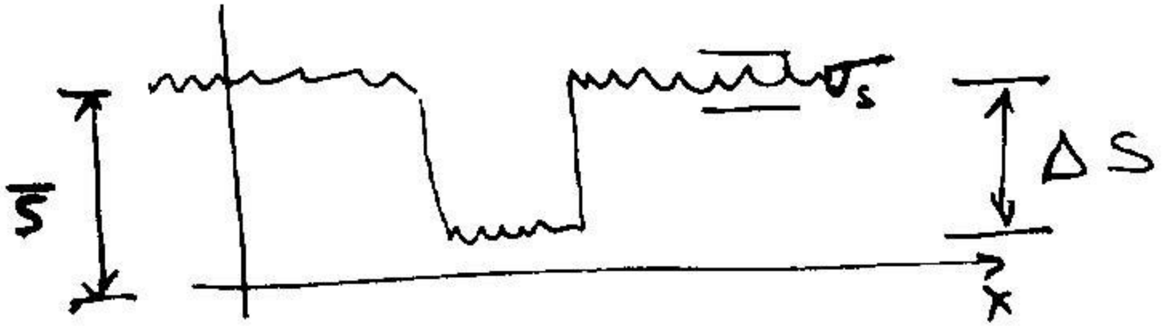
SNR of a Poisson Measurement

Poisson distributions (subtract the mean and divide the x-axis by the mean)



Noise in Detectors

- Consider an output to an x-ray



- We define a number of quantities

Noise in Detectors

We define a number of quantities

- Contrast: $C = \Delta S / \bar{S}$
- Signal to Noise Ratio: $SNR = \bar{S} / \sigma_s$
- Contrast to Noise Ratio:

$$CNR = \Delta S / \sigma_s = C \cdot SNR$$

Noise in Detectors

- Suppose the incident x-ray photons arriving at the detector are Poisson(N) and that the detector has detection efficiency η (that is η is the probability that any photon is detected

$$SNR_{\text{det}} = \frac{\eta N}{\sqrt{\eta N}} = \sqrt{\eta N}$$

Noise in Detectors

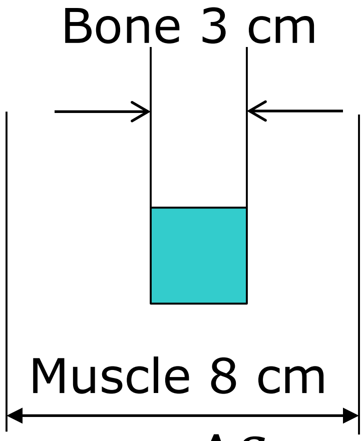
- Suppose the incident x-ray photons arriving at the detector are Poisson(N) and that the detector has detection efficiency η
  - η is the probability that any photon is detected
- Detected photons $\sim$ Poisson(ηN)

$$SNR_{\text{det}} = \frac{\eta N}{\sqrt{\eta N}} = \sqrt{\eta N}$$

Example - Contrast

Photon energy = 90 keV

- Bone - $\mu_b = 0.4\ cm^{-1}$
- Muscle - $\mu_m = 0.2\ cm^{-1}$



- Calculate contrast $C = \frac{\Delta S}{\bar{S}}$. To simplify, use parallel x-ray beams.

Example : Contrast

- Muscle only:

$$I_m = I_0 e^{-\mu_m \times 8} = I_0 e^{-0.2 \times 8} = 0.202 I_0$$

- Muscle + bone:

$$I_{m+b} = I_0 e^{-\mu_m \times (8-3) - \mu_b \times 3} = I_0 e^{-0.2 \times 5 - 0.4 \times 3} = 0.111 I_0$$

- Calculate contrast:

$$C = \frac{\Delta S}{S} = \frac{I_m - I_{m+b}}{\mathrm{I_m}} = \frac{0.202 I_0 - 0.111 I_0}{0.202 I_0} = 0.45$$

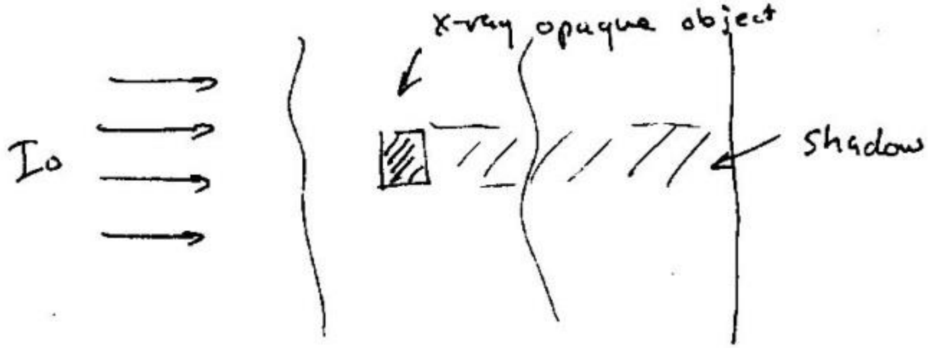
Questions

SNR and noise

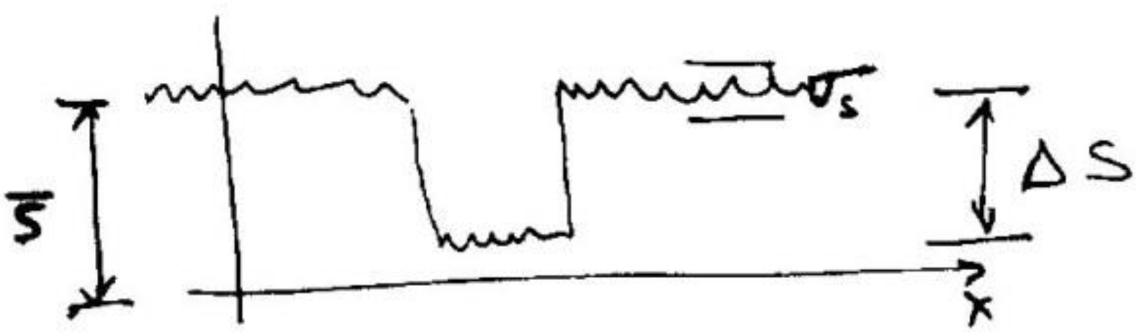
Impact of Scatter on Noise

Scatter in an x-ray system

Consider an x-ray system

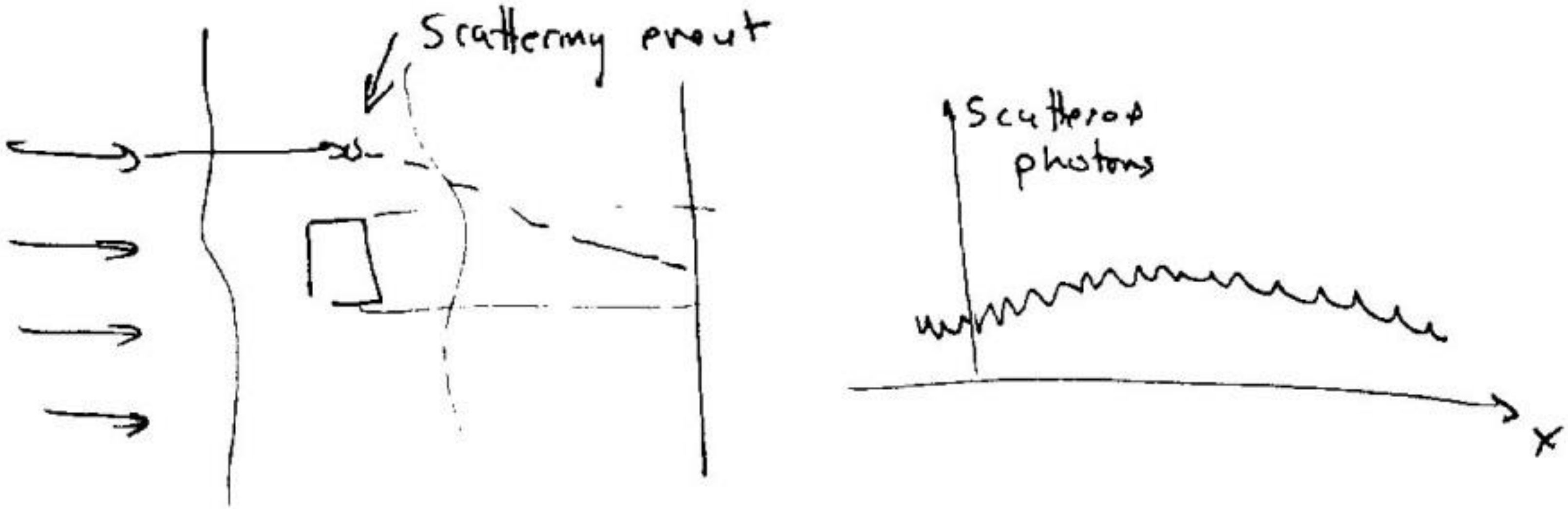


And the image might look like (in 1D):



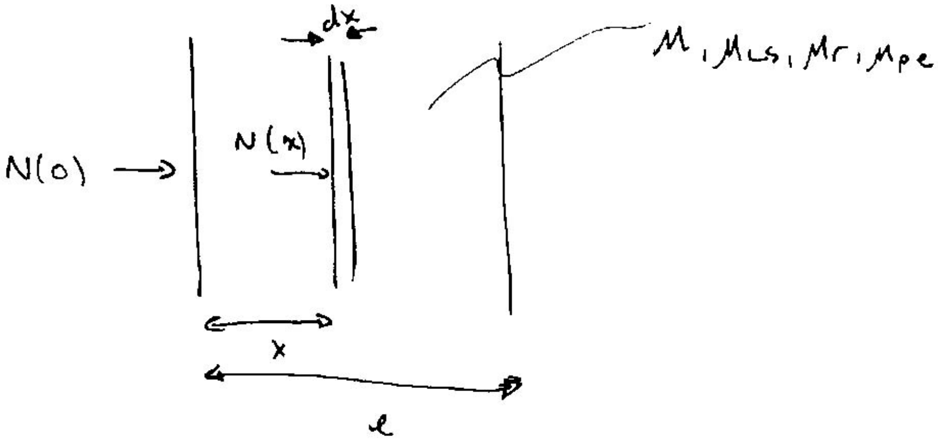
Now adding scatter

- Scanner photons go anywhere



- By increasing $\bar{S}$ and σ_s, the scattered photons will reduce both the contrast and the contrast to noise ratio.

How many photons are scattered?



- The number of scattered photons in interval dx is:

$$N_{cs}(x) = \mu_{cs} N(x) dx$$

How many photons are scattered?

$$N_{cs} = \int_0^l N_{cs}(x)dx$$

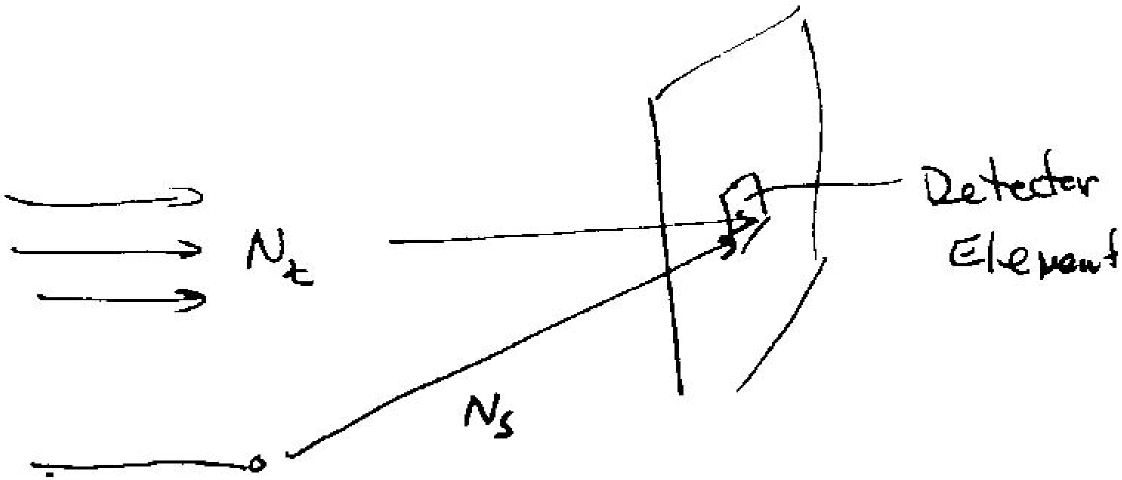
$$= \int_0^l \mu_{cs} N(x)dx$$

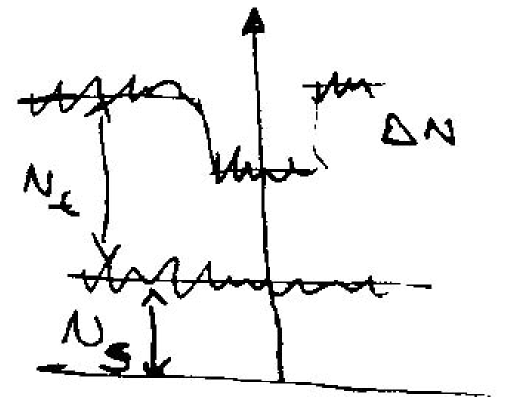
$$= \int_0^l \mu_{cs} N(0)\exp(-\mu x)dx$$

$$= \frac{\mu_{cs}}{\mu} N(0)(1-\exp(-\mu l))$$

- N_{cs} is $\frac{\mu_{cs}}{\mu}$ time the total number of interacting photons.
- More scatter for more radio-opaque objects

Scatter is Additive Noise





- Original contrast: $$C = \frac{\Delta N}{N}$$
- Reduced contrast: $$C_r = \frac{\Delta N}{N + N_s}$$

Scatter and CNR

- Std. Dev. $\sigma = \sqrt{\eta N + \eta N_s}$
- CNR

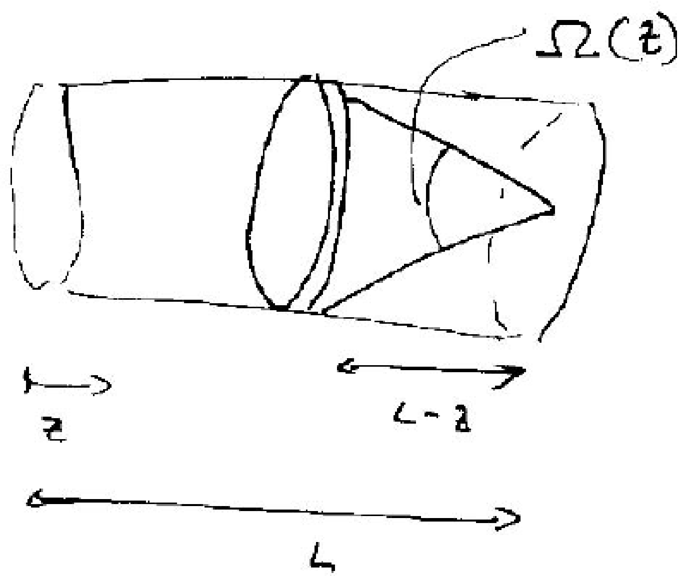
$$CNR_r = \frac{\eta \Delta N}{\sqrt{\eta N + \eta N_s}} = C \frac{\eta N}{\sqrt{\eta N + \eta N_s}} = C\sqrt{\eta N}\,\frac{1}{\sqrt{1 + N_s / N}}$$

- CNR reduction factor:

$$\sqrt{1 + N_s / N} \text{ or } \sqrt{1+\Psi}\text{, where } \Psi = \frac{N_s}{N}$$

CNR Reduction

- Example: (Details in notes)



$$F(z) = \frac{\Omega(z)}{4\pi} e^{-\mu(L-z)}$$

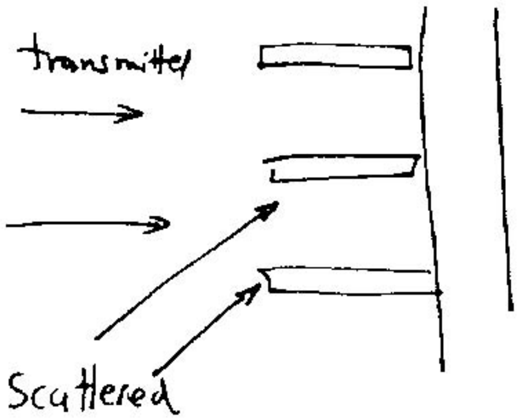
$$\Psi = 1.28$$

$$\sqrt{1+\Psi} = 1.5$$

- CNR reduction of ~50%.

Scatter Reduction Grids

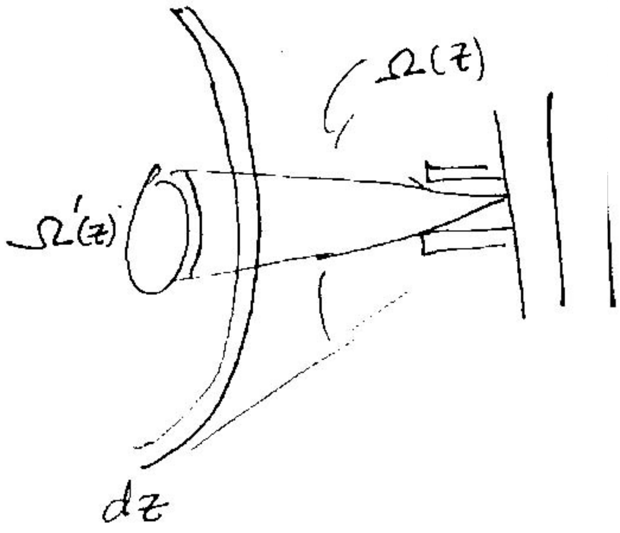
- Grids made of high μ materials



- Only allows forward scattered photons to reach detector

Scatter Reduction Grids

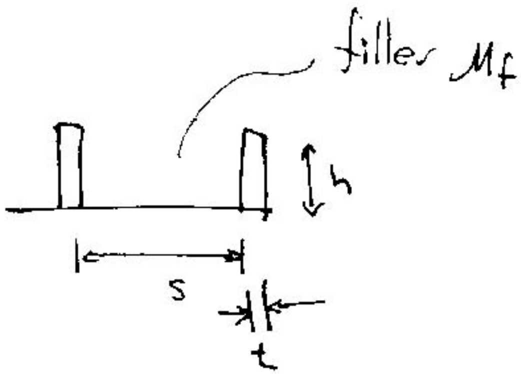
- Solid angle reaching detector



- Scatter reduction: $R_s = \dfrac{\int \Omega'(z)dz}{\int \Omega(z)dz}$

Scatter Reduction Grids

- Also reduces transmitted photons



- Grid efficiency → $$\eta_t = \frac{s-t}{s}\exp(-\mu_f h)$$

CNR with Scatter Reduction

- New CNR:

$$CNR = C\frac{\eta\eta_t N}{\sqrt{\eta\eta_t N + \eta R_s N_s}} = C\sqrt{\eta N}\frac{\sqrt{\eta_t}}{\sqrt{1 + \frac{R_s\Psi}{\eta_t}}}$$

- CNR reduction factor:

$$\sqrt{\frac{1}{\eta_t}}\sqrt{1 + \frac{R_s\Psi}{\eta_t}}$$

Questions?

Scatter and CNR

Scatter reduction