BME/EECS-516, Medical Imaging Systems
Solution Homework # 3

Problem # 1

a) Conditions to avoid aliasing: $d \le \dfrac{\lambda}{2}$, $c \le \dfrac{\lambda}{2}$

$\lambda = \dfrac{C}{f_0}$, C = 1.5 mm/μs, fo = 3 MHz, → λ = 0.5 mm

→ Critical Spacing = c = d = $\dfrac{\lambda}{2}$ = 0.25 mm

→ $\boxed{c = 0.25 \text{ mm} \qquad d = 0.25 \text{ mm}}$

$N = \dfrac{D}{d} + 1$, D = 10 mm → $\boxed{N = 41}$

$M = \dfrac{L}{c} + 1$, L = 5 mm → $\boxed{M = 21}$

$\boxed{\text{Total number of elements} = M \text{ x } N = 861}$

Note: N = 40 and M= 20 is also an acceptable solution.

b) Using the x_z - z formulation (vs. sinθ - r) of diffraction we get:

$$p(x_z,y_z) = \frac{e^{ikz} e^{ik\frac{x_z^2+y_z^2}{2z}}}{z} \mathcal{F}_{2D}\left\{ |S(x,y)| \right\} \Bigg|_{u=\frac{x_z}{\lambda \cdot z}, v=\frac{y_z}{\lambda \cdot z}}$$

The aperture function could be expressed in the following form:

$$S(x,y) = \left[\frac{1}{dc} \text{comb}\left(\frac{x}{d}, \frac{y}{c}\right) ** \text{rect}\left(\frac{x}{w}, \frac{y}{h}\right) \right] \cdot \text{rect}\left(\frac{x}{D}, \frac{y}{L}\right)$$

Where D = (N-1)·d and L = (M-1)·c

$$S(u,v) = \frac{Dw}{d}\frac{Lh}{c} \text{Sinc}(wu, hv) \cdot \sum_{m,n=-\infty}^{\infty} \text{Sinc}\left(D\left(u - \frac{n}{d}\right)\right) \cdot \text{Sinc}\left(L\left(v - \frac{m}{c}\right)\right)$$

$$\rightarrow \boxed{p(x_z,y_z) = \frac{e^{ikz} e^{ik\frac{x_z^2+y_z^2}{2z}}}{z} \frac{Dw}{d}\frac{Lh}{c} \text{Sinc}\left(w\frac{x_z}{\lambda \cdot z}, h\frac{y_z}{\lambda \cdot z}\right) \cdot \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \text{Sinc}\left(D\left(\frac{x_z}{\lambda \cdot z} - \frac{n}{d}\right)\right) \cdot \text{Sinc}\left(L\left(\frac{y_z}{\lambda \cdot z} - \frac{m}{c}\right)\right)}$$

c) Combined beam pattern = $\boxed{B(x_z,y_z) = \left(p(x_z, y_z) \right)^2}$

d) The main lobe (assuming c = d = λ/2 is approximately:

$$\sim \mathrm{Sinc}^2\left(D\frac{x_z}{\lambda \cdot z}\right)\cdot \mathrm{Sinc}^2\left(L\frac{y_z}{\lambda z}\right)$$

thus the effective aperture function is:

$$S(x,y) = \Lambda\left(\frac{x}{D}\right)\cdot\Lambda\left(\frac{y}{L}\right) \qquad \text{(triangle function)}$$

Therefore $\frac{\Delta x_z}{\lambda \cdot z} \le \frac{1}{2D}$ and $\frac{\Delta y_z}{\lambda \cdot z} \le \frac{1}{2L}$

Then $\boxed{\frac{\Delta x_z}{z} \le 0.025 \text{ and } \frac{\Delta y_z}{z} \le 0.05}$

e) Number of beams in x = $\frac{100}{\Delta x_z} = \frac{100 \cdot 2 \cdot D}{\lambda \cdot z} = \frac{100 \cdot 2 \cdot 10}{0.5 \cdot 100} = 40$

Number of beams in y = $\frac{100}{\Delta y_z} = \frac{100 \cdot 2 \cdot L}{\lambda \cdot z} = \frac{100 \cdot 2 \cdot 5}{0.5 \cdot 100} = 20$

→ $\boxed{\text{Total number of beams = 20 x 40 = 800}}$

f) $$\tau_i = \frac{\sqrt{(x_z - x_i)^2 + (y_z - y_i)^2 + z^2}}{c}$$

$$\boxed{\text{Fresnel approximation: } \tau_i = \frac{z}{c} + \frac{1}{2 \cdot z \cdot c}\cdot\left[(x_z - x_i)^2 + (y_x - y_i)^2\right]}$$

$$\boxed{\text{Fraunhoffer approximation: } \tau_i = \frac{z}{c} + \frac{x_z^2 + y_z^2}{2 \cdot z \cdot c} - \frac{x_i x_z + y_i y_z}{z \cdot c}}$$

g) Round trip time = $\frac{2 \cdot 150}{1.5} \frac{\text{mm}}{\text{mm/ sec}}$ = 200 μs/beam

3D mode → 800 beams x 200 μs/beam = 160 ms frame time

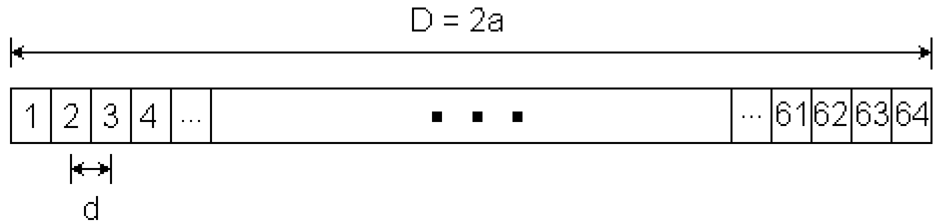
$\boxed{\text{3D mode} \rightarrow \text{6.25 Hz frame rate}}$

2D mode → 40 beams x 200 μs/beam = 8 ms frame time

$\boxed{\text{2D mode} \rightarrow \text{125 Hz frame rate}}$

Problem # 2

a)



To avoid aliasing: $d \le \lambda/2$

$\lambda = C/fo$ and $D = N \cdot d$ (Some people used $D = d\cdot(N-1)$ which is O.K. and the results change a little bit)

For the different range of frequencies we have the following conditions:

Sound	f [Hz]	d [m]	D [m]	System 1	System 2
Groans	20	37.5	2400	✓	✓
	50	15	960	✓	✓
Coos	200	3.75	240	✓	✓
	500	1.5	96	✓	✓
Whistles	1000	0.75	48	✓	x
	1500	0.5	32	✓	x

As can be seen:

$\boxed{\text{System 1} \rightarrow d = 0.5\text{m}, D = 32\text{ m}}$

$\boxed{\text{System 2} \rightarrow d = 1.5\text{m}, D = 96\text{ m}}$

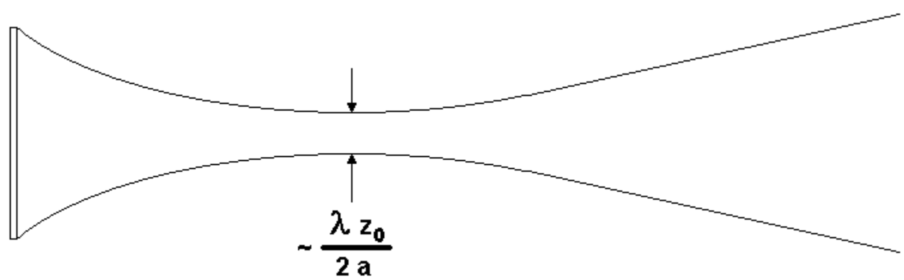
(b), (c), and (d)

Fraunhoffer zone → $z > \frac{(2a)^2}{\lambda}$

	f [Hz]	System 1 z far-field [m]	System 2 z far-field [m]
(b)	50	34.13	307.2
(c)	500	341.33	3072
(d)	1500	1024	9216

(e), (f), and (g)

z = 50Km → Far-field for both systems



	f [Hz]	System 1 Lateral resolution at z = 50 Km [m]	System 2 Lateral resolution at z = 50 Km [m]
(e)	50	46875	15625
(f)	500	4687.5	1562.5
(g)	1500	1562.5	520.83 *

* = With this system, at 1500 Hz we have aliasing

1500 Hz → 500 Hz → Lateral resolution = 1562.5 m h)

If $\theta_0 = 0$ then:

$$p(r,\theta) = K\cos\theta \cdot \mathrm{Sinc}\left(w\frac{\sin\theta}{\lambda}\right) \cdot \sum_{-\infty}^{\infty} \mathrm{Sinc}\left[2a\left(\frac{\sin\theta}{\lambda} - \frac{n}{d}\right)\right]$$

first grating lobe → n = 1

f = 1500 Hz → λ = 1m

System 2 → d = 1.5 m

We want to locate the maximum of the first grating lobe so:

$$\frac{\sin\theta}{\lambda} - \frac{n}{d} = 0 \quad @\ n = 1, \lambda = 1, d = 1.5$$

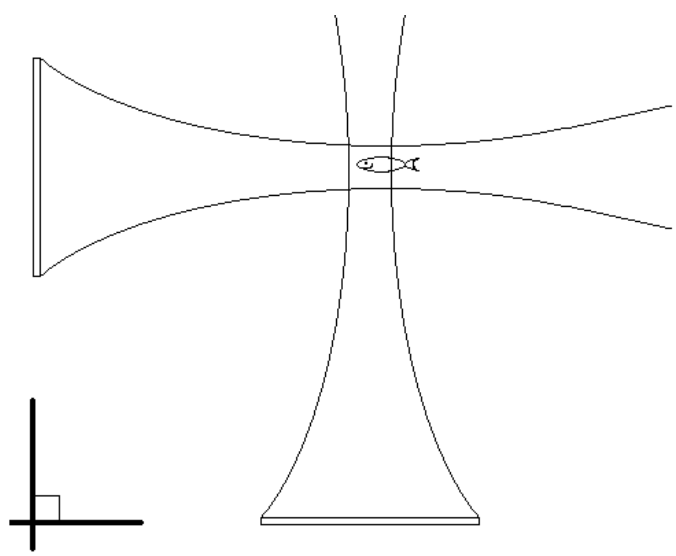
$\boxed{\rightarrow \quad \theta = 41.81^\circ}$

i)

C = 1500 m/s, d = 50 Km → t = d/C → t = 33.3 sec.

With this system (receive only) we cannot say anything about the range or depth of the whale from the array.

j) With a second array, we can use triangulation to improve our range resolution. One way is using the array in the following configuration (One array is used for the x-dimension and the other one for the y-dimension):



We can also use some kind of synchronization between the two arrays located at different position and use the time delay information from each array to estimate the position of the whale.