Magnetic Resonant Imaging (MRI)

BME/EECS 516

Lecture #7 – Spin-echoes, Noise, Parallel Imaging

Notes on MRI, Parts 3 & 4

Announcements

- HW #4 due Tuesday, 10/31 (Today!)
- HW #5 due Tuesday, 11/7
- MRI Project due Tuesday 11/21

- Tuesday, 11/14 – guest lectures from local medical imaging industry
  - David Sarment from Xoran Technologies
  - John Seamans (UM) formerly with Delphinus Medical and GE Healthcare

Last Lecture

- The 3rd Dimension – Z
  - Extend 2D Spin-Warp Imaging to 3D
- Slice Selective Excitation - common approach for dealing with the 3rd (z) dimension
  - Small Tip Angle Approximation
  - Putting Slice Selection with the Signal Equation
  - Larger Flip Angles
  - Multi-slice Imaging

Review on Slice Selective Excitation

- Step 1: Apply a z-gradient so that the resonance frequency varies in the z-direction. Within a slice of thickness Δz:

$$B_z = B_0 + G_z \cdot z$$

$$\omega(z) = \gamma(B_0 + G_z \cdot z)$$

$$\omega_{eff}(z) = \gamma G_z \cdot z$$

Slice Selective Excitation

- Step 2: Apply a bandpass RF pulse (B_1) with bandwidth (BW in Hz)

- RF pulse excite only the those spins whose resonant frequency lies within the band.
  - $[\omega_{RF} - BW * 2\pi/2, \omega_{RF} + BW * 2\pi/2] = [\gamma(B_0 + G_z z_1), \gamma(B_0 + G_z z_2)]$ to excite spins only within slice $[z_1, z_2]$

Small Tip Angle Approximation

- With small tip angle approximation:

$$m_{xy,rot}(z,\tau)=i\gamma m_0 e^{-i\gamma G_z z\tau/2}F^{-1}\left\{B_1(s+\tau/2)\right\}\Big|_{s=\frac{\gamma}{2\pi}G_z z}$$

- apply a negative G_z for a period $\tau/2$ – often called a slice rephasing pulse – to remove $e^{-i\gamma G_z z\tau/2}$

$$m_{xy,rot}(z,3\tau/2)=i\gamma m_0 F\left\{B_1(s+\tau/2)\right\}\Big|_{s=\frac{\gamma}{2\pi}G_z z}$$

Putting Slice Selection with the Signal Equation

- Go back to 3D distribution of magnetization by substituting $im_0 = m(x,y,z)$ and putting it into the signal equation (again the RF coil integrates across the object):

$$s(t) = \iiint m(x,y,z)\,p(z)\exp\left(-i2\pi\left(xk_x(t) + yk_y(t)\right)\right)dx\,dy\,dz$$

- Here we are performing 2D imaging while integrating across the slice profile

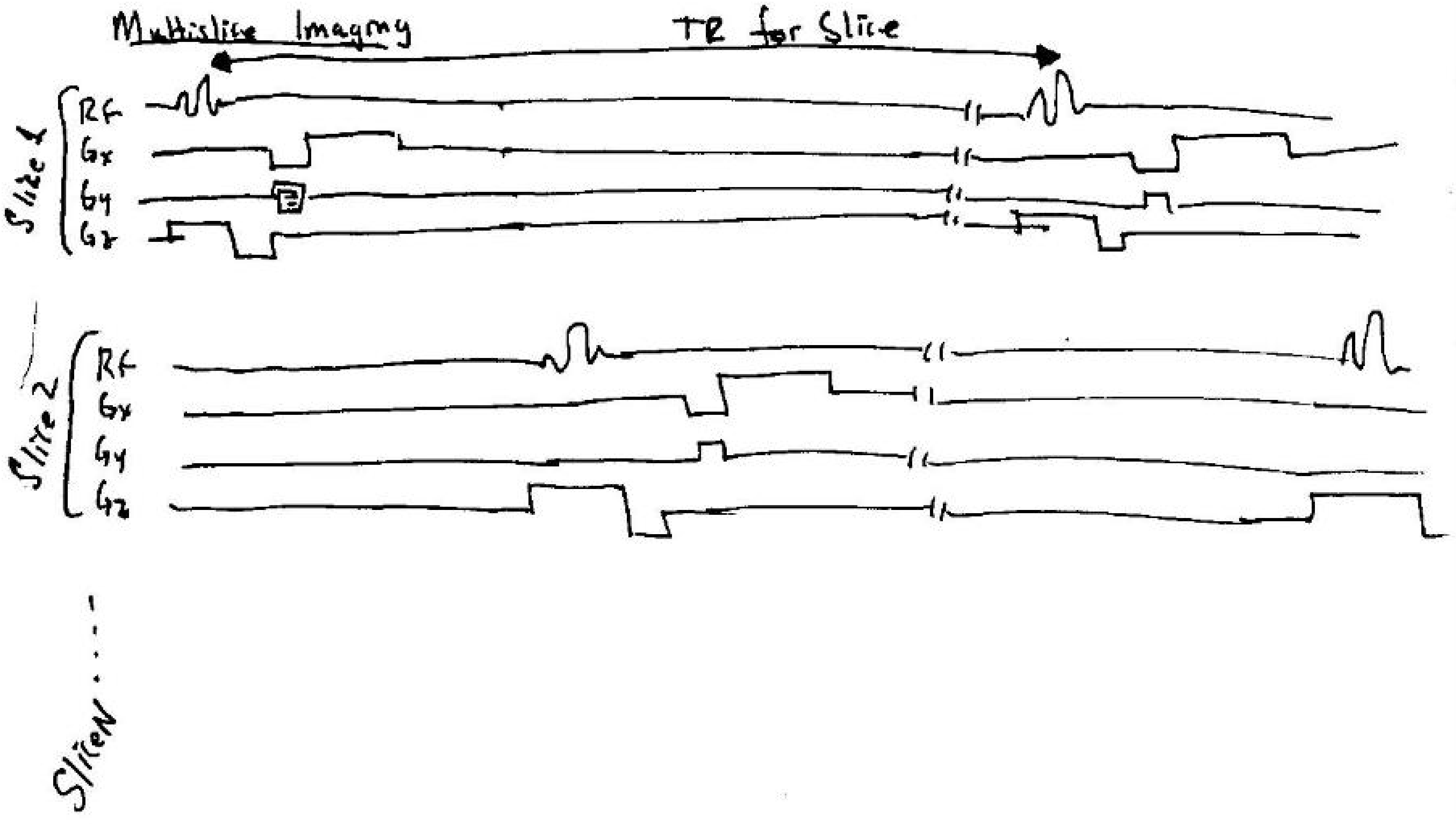
Larger Flip Angles

- Analysis using small flip angle (e.g. $\alpha < \pi/6$) approximation turns out perform well for large flip angles (e.g. $\pi/2$) as well.

- Here, a better approximation for the transverse component of the magnetization is:

$$m_{xy,rot}(z,3t\ /\ 2) = im_0 \sin\left(p(z)\right)$$

Mutli-slice Imaging



True or False

- The selection of the time between RF excitation pulses of the same slice affects T1 contrast.

- The selection of the time between RF excitation pulses of two adjacent slices affects T1 contrast.

True or False

- The selection of the time between RF excitation pulses of the same slice affects T1 contrast.
  - True – TR is the excitation-excitation time within a slice
  - $m_z(TR) = \left(1 - e^{-\frac{TR}{T1}}\right)$
- The selection of the time between RF excitation pulses of two adjacent slices affects T1 contrast.
  - False – neighboring slices do not affect T1 recovery

Question

- Slice-selective, 2D spin-warp acquisition overall acquisition time is:
- A) $N_y * N_y * TR$
- B) $N_y * TR$
- C) $N_z * TR$

Question

- Slice-selective, 2D spin-warp acquisition overall acquisition time is:
- A) $N_y * N_z * TR$
- **B)** $N_y * TR$
- *C)* $N_z * TR$

- Within TR, we will collected kx for all slices along z direction (sketch).

Mutli-slice Imaging

- Slice-selective, 2D spin-warp acquisition overall acquisition time: $N_y * TR$

- For example, acquire a T1-weighted image
  - Nz - 20 slices
  - TR - 500 ms
  - 128 phase encoding lines in k-space
  - Total acquisition time - $N_y * TR$ = ~ 1 minute

Questions?

Selective Slice Excitation

3D Spin-Warp Imaging

This Lecture

- Spin Echo Pulses
- Spin-echo Spin-warp Pulse Sequence
- Why do spin-echo pulses?
- Noise in MRI
- Signal to Noise Ratio
- Array coils and parallel imaging

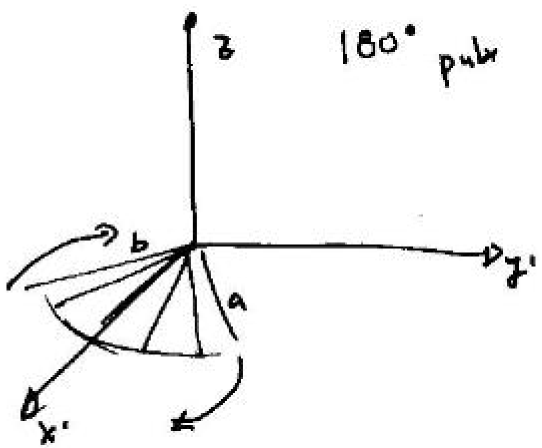
Spin Echo Pulses

refocusing of precessing nuclear spin magnetisation by a 180° pulse

Very commonly used in MRI

Spin Echo Pulses

- Earlier we described 180 degree RF pulses for purposes of inverting the m_z magnetization.
- Consider a 180 degree B1 pulse applied to the x' axis in the rotating frame



Spins are flip around x' axis like a "pancake flipper"

Spin Echo Pulses

- 180 degree B1 pulse applied to the x' axis in the rotating frame
  - t_{180-} - time just before the 180 degree pulse
  - t_{180+} - time after the 180 degree pulse

  - $m_{x,rot}(t_{180+}) = m_{x,rot}(t_{180-})$
  - $m_{y,rot}(t_{180+}) = -m_{y,rot}(t_{180-})$
  - $m_{z,rot}(t_{180+}) = -m_{z,rot}(t_{180-})$

Question

- What is the duration of the 180 degree pulse.
- A) $\pi / 2\gamma B_1$
- B) $\pi / \gamma B_1$
- C) $2\pi / \gamma B_1$

Question

- What is the duration of the 180 degree pulse.
- A) $\pi / 2\gamma B_1$
- **B)** $\boldsymbol{\pi / \gamma B_1}$
- C) $2\pi / \gamma B_1$

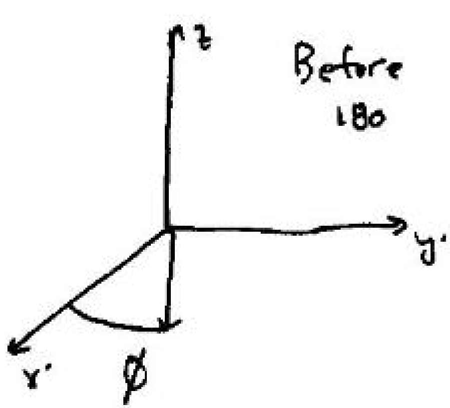
(angle/rotation rate)

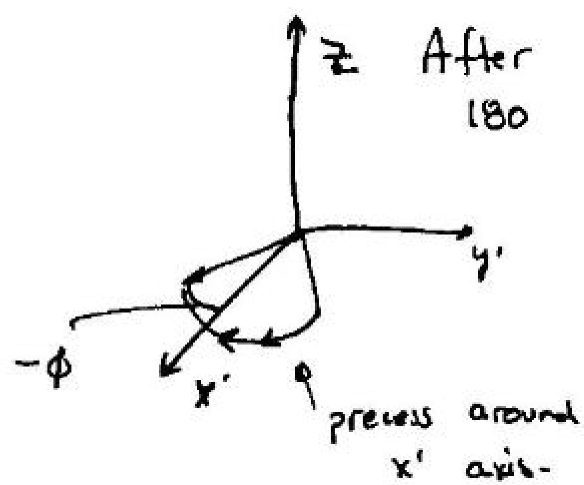
Spin Echo Pulses

- Suppose the vector is lying in the transverse plane ($m_z = 0$). Assume positioned on the x' axis and rotation phase $\Phi(\mathbf{r}, t)$ has accumulated due to ΔB terms.

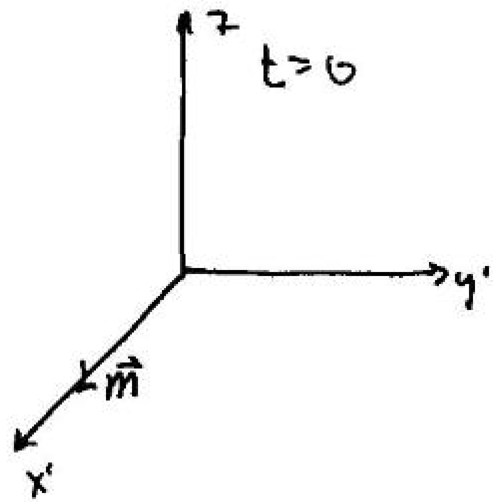
$$\Phi(\mathbf{r}, t_{180+}) = -\Phi(\mathbf{r}, t_{180-})$$

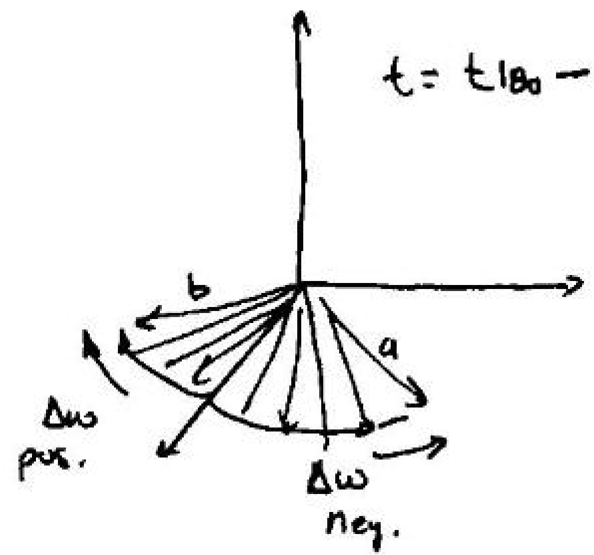
or $$m_{xy,rot}(t_{180+}) = m_{xy,rot}(t_{180-})^*$$





Why do spin-echo pulses?



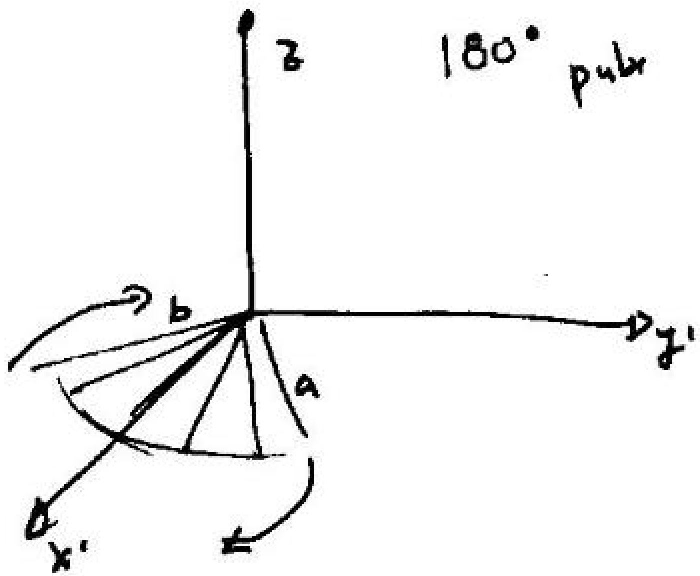


- Ignoring the k-space term ($\int_0^{t_{180-}} G(t)\cdot r\,dt$), the phase accumulation at the time of 180 degree pulse t_{180}

  $$\Phi\,(\mathbf{r},\, t_{180-}) = \Delta\omega(\mathbf{r})\,t_{180}$$

Why do spin-echo pulses?

- Apply 180 degree pulse to flip the spins

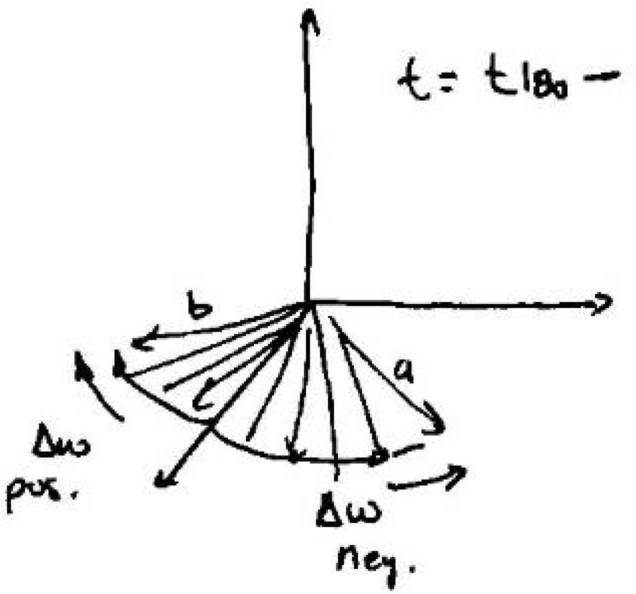


Spins are flip around x' axis like a "pancake flipper"

Why do spin-echo pulses?

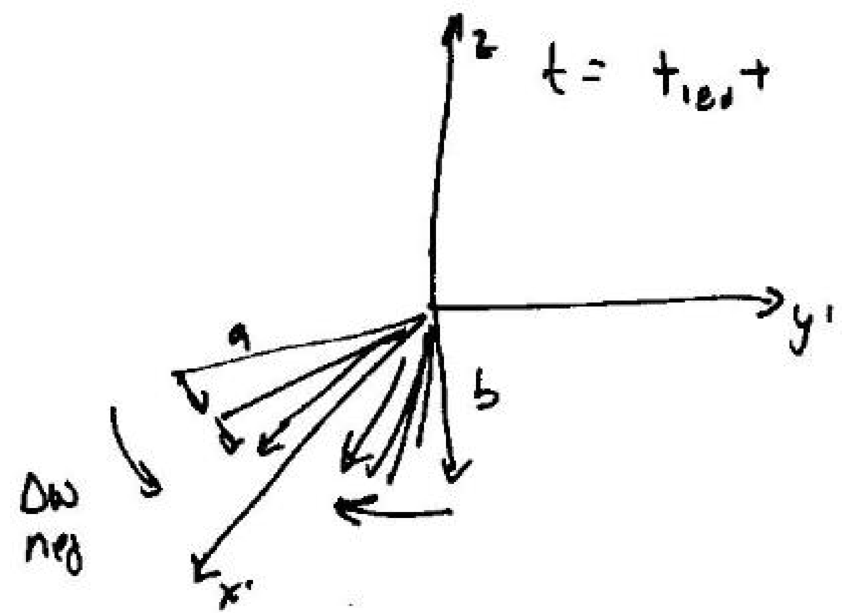
Before 180 degree pulse

$\boldsymbol{\Phi}(\mathbf{r}, t180+) = -\Delta\omega(\mathbf{r})t_{180}$



Just after 180 degree pulse

$\boldsymbol{\Phi}(\mathbf{r}, t180-) = \Delta\omega(\mathbf{r})t_{180}$



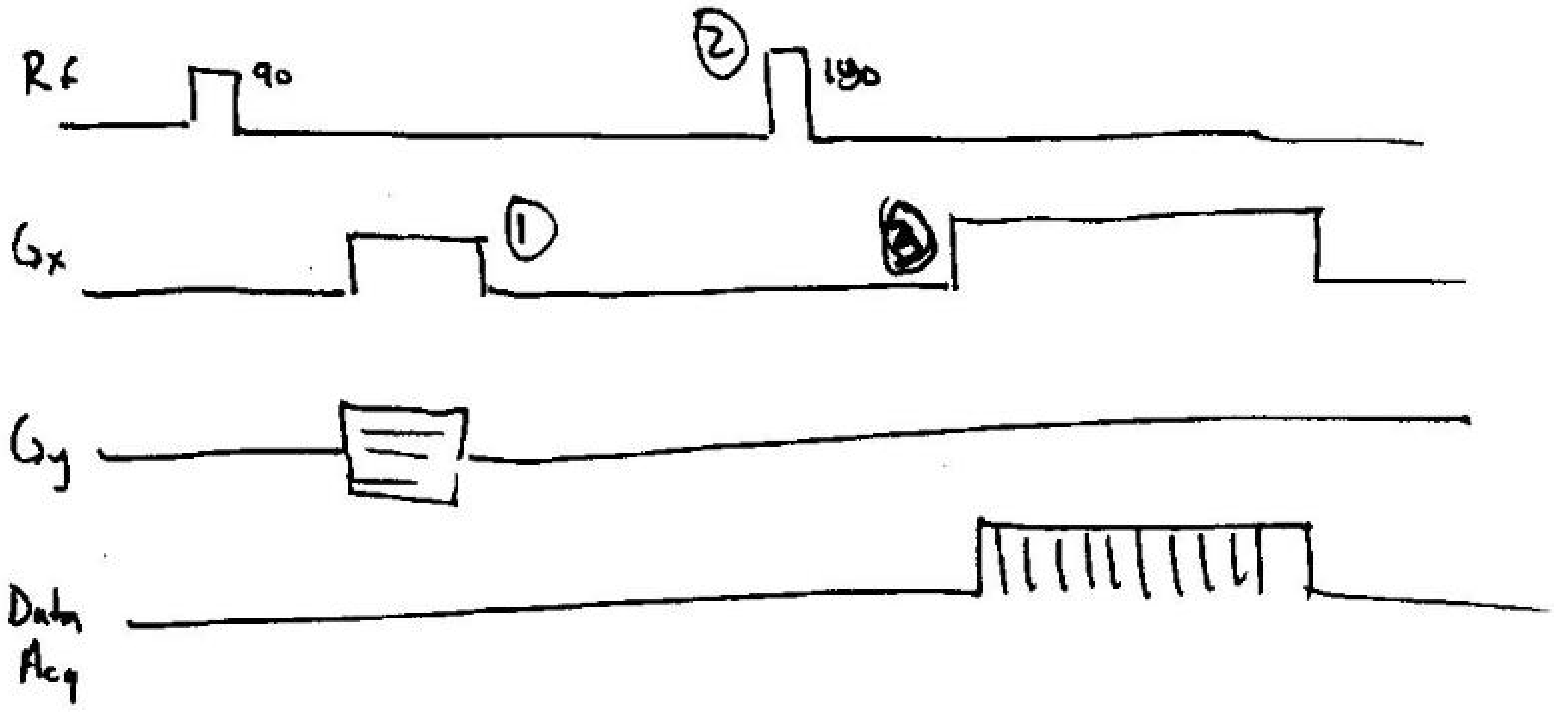
Spin Echo Pulses

- When imaging, the phase term results from gradients and can be written as:
  $\Phi(\mathbf{r}, t) = 2\pi(\mathbf{k}(t).\mathbf{r})$ à $\mathbf{k}(t_{180+}) = -\mathbf{k}(t_{180-})$

- A 180 degree pulse will invert the location in k-space.

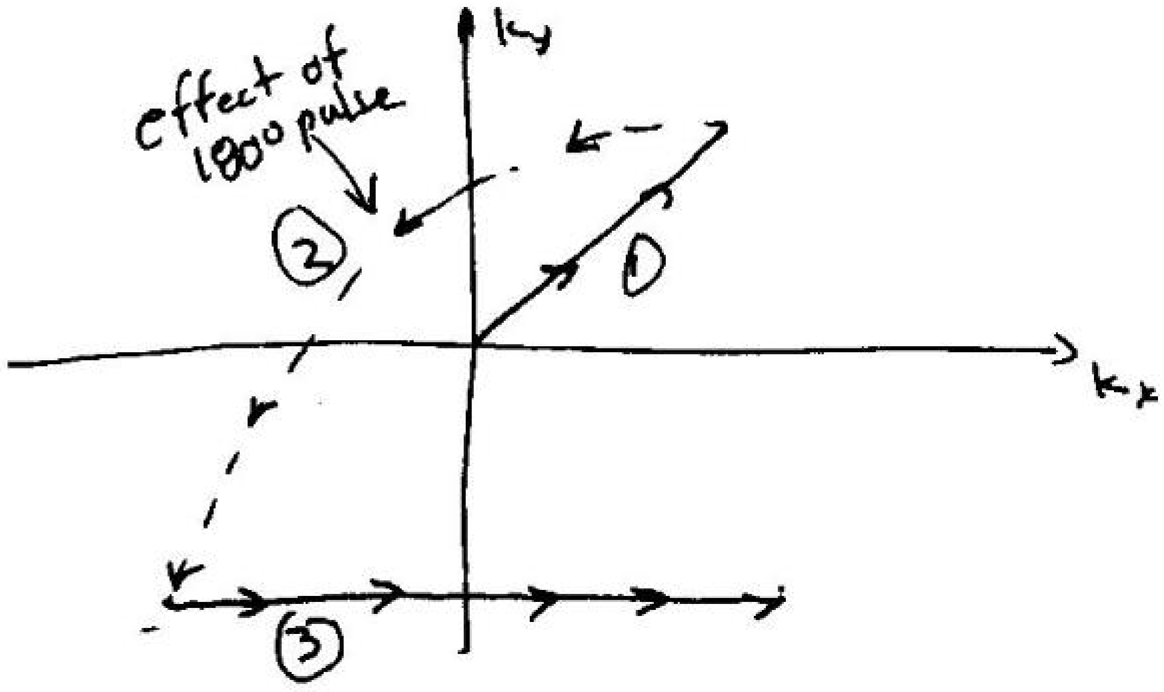
- Spin-echo 180 degree pulses are at times known as “phase reversal” or “time reversal” pulses.

Spin-echo Spin-warp Pulse Sequence



Spin-echo Spin-warp Pulse Sequence

- 180 degree pulse inverts the position in k-space:



Questions?

Spin Echo Pulses

Spin-echo Spin-warp Pulse Sequence

Why do spin-echo pulses?

- Magnetic field inhomogeneity can results in intra-voxel signal dephasing.
  - Due to many reasons – various chemical or magnetic filed distribution (e.g., deoxygenated homoglobin variation)

Why do spin-echo pulses?

- Consider a magnetic field inhomogeneity function $\Delta B(\mathbf{r})$. Effective magnetic field (rotating frame):

  $$B_{z,\mathrm{eff}} = \mathbf{G}(t).\mathbf{r} + \Delta B(\mathbf{r})$$

- Corresponding phase function:

  $$\Phi(\mathbf{r}, t) = 2\pi(\mathbf{k}(t).\mathbf{r}) + \Delta\omega(\mathbf{r})t$$

- **when integrating $\Delta\omega(\mathbf{r})t$ across a voxel some signal may be lost.** The spin-echo pulse brings this phase back together again.

Spin Echo Pulses in Spin Warp sequences

https://www.youtube.com/watch?v=GDElT6Tz7_Q

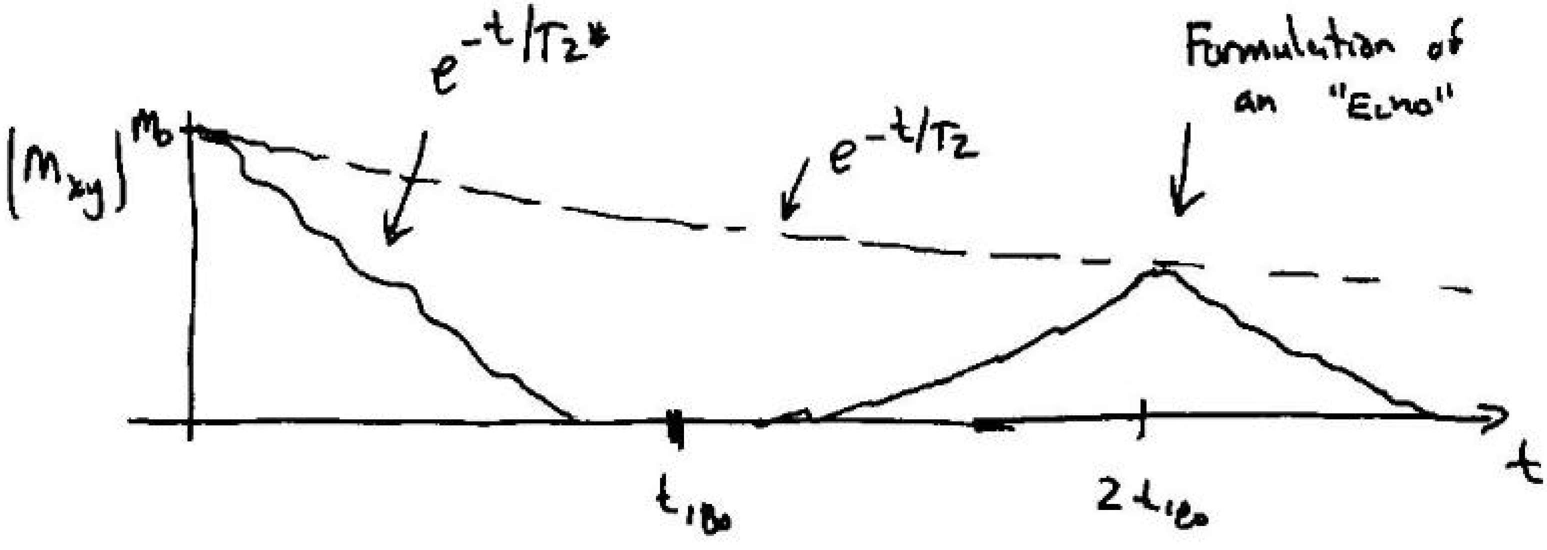
Why do spin-echo pulses?

- Phase continues to accumulate
- If we look at time 2 t_{180}, we will have a total phase accumulation of:

$$\Phi(\mathbf{r}, 2\ t_{180}) = \Phi(\mathbf{r}, t_{180+}) + \Delta\omega(\mathbf{r})t_{180}$$
$$= -\Delta\omega(\mathbf{r})t_{180} + \Delta\omega(\mathbf{r})t_{180} = 0$$

Why do spin-echo pulses?

- The signal in the transverse plane ($|m_{xy}|$) will look like this:



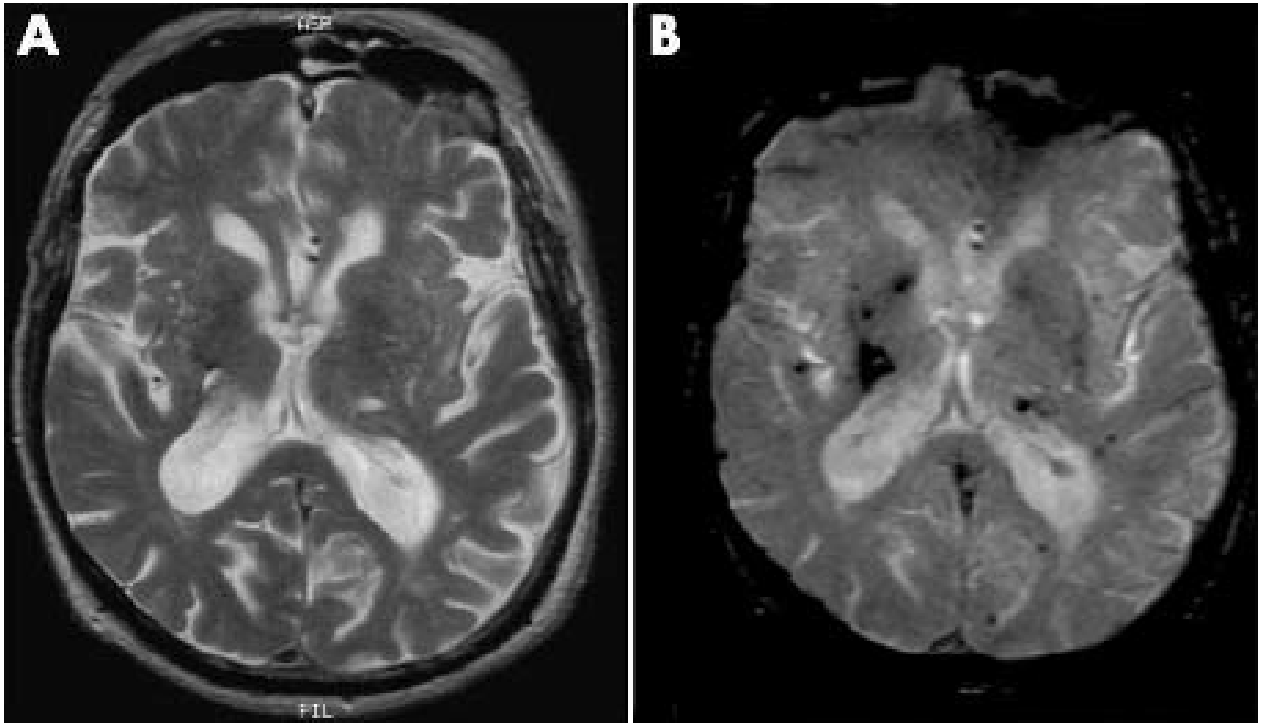
T2 vs. T2* weighted images

- T2* is shorter than T2 because the decay induced by the signal dephasing from the inhomogenity effect, when integrated by the coil, result in reduced signal.

- When all dephasing from inhomogenity in a volume tissue is cancelled by the spin-echo, T2 decay still remains.

- If use T2* instead of T2 – may lose detailed info.

T2- vs. T2*-weighted images



Symms, M & Jäger, H.R. & Schmierer, Klaus & Yousry, T.A.. (2004). A Review of Structural Magnetic Resonance Neuroimaging. Journal of neurology, neurosurgery, and psychiatry. 75. 1235-44. 10.1136/jnnp.2003.032714.

T2* weighted scans

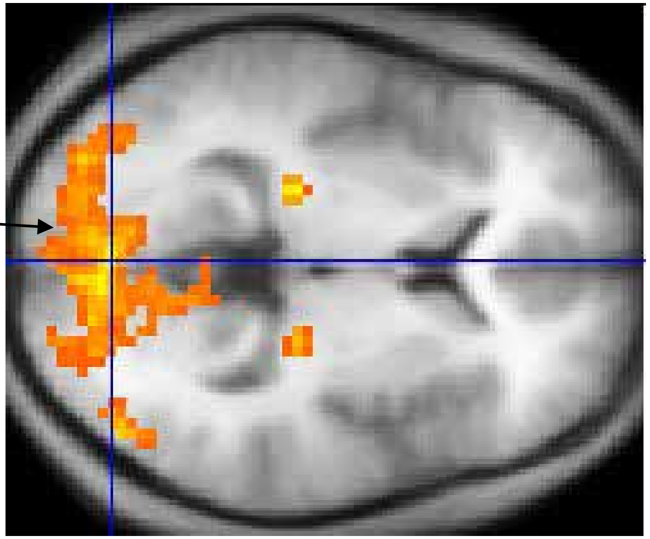
- No spin echo is used so it is subject to additional losses above the normal *T*2 decay.

- More prone to susceptibility losses at air/tissue boundaries, but can increase contrast for certain types of tissue, such as venous blood – deoxygenated homoglobin (substantial magnetic field inhomogenity).

T2* weighted scans

- Used in fMRI to detect
  - Change in blood oxygenation level
  - Neural activity in brain through BOLD (blood-oxygen-level dependent)

regions of neural activation



Questions?

Why do spin-echo pulses?

Noise and Signal to Noise Ratio in MRI

Noise in MRI

- Sources of noise in MRI
  - Thermal noise from body – thermal vibration of ions, electrons, etc. (Dominant source of noise in most MRI systems)
  - Quantization noise in the A/D devices
  - Preamp/electronic noise
  - Thermal noise in RF coil

Thermal noise

- Not related to the NMR
  - Present with or without B_0, RF, Gradients
- Uniform spectral density (near ω_0) – white noise
- Comes from the whole body – amount of noise depends on the amount of the body to which the receive coil is sensitive

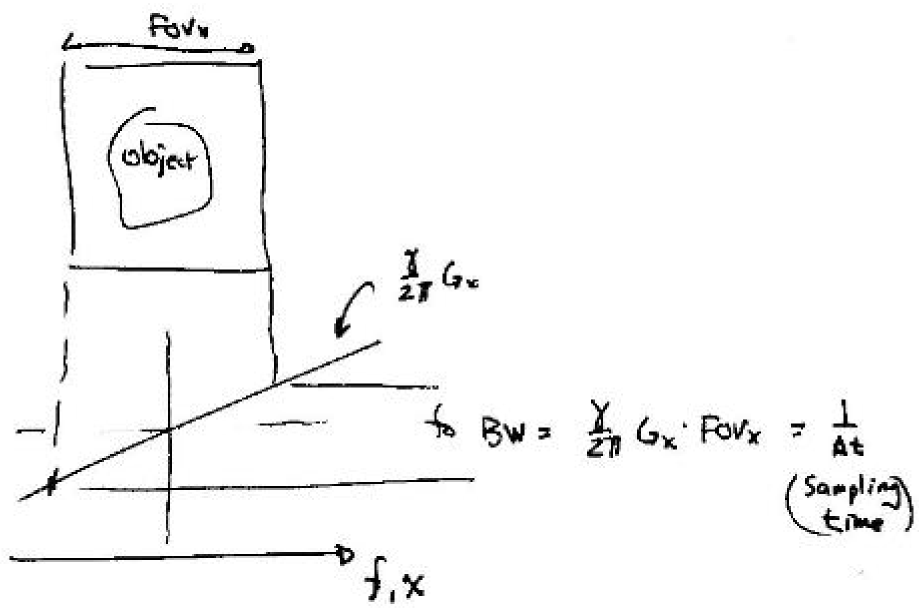
Noise Characteristics

- Both MR signal (FT of the image) and) 2D image reconstructed by a N_x by N_y inverse 2D DFT of the MR signal - Resultant noise will be zero-mean, additive, independent, bi-variate Gaussian noise.

Noise in MRI

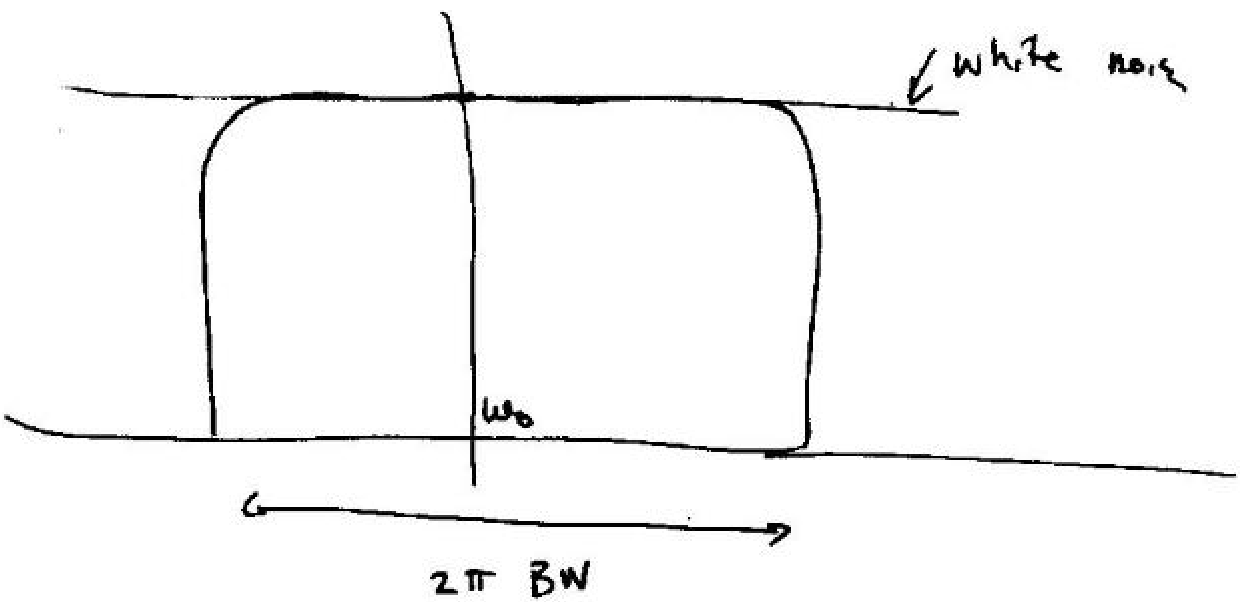
- Noise in Spin-warp imaging
  - frequency encoding gradient - Gx
  - field of view - FOVx
  - bandwidth of the receiver will be:

$$BW = \frac{\gamma}{2\pi} G_x FOV_x = \frac{1}{\Delta t}$$



Noise Characteristics

- Noise variance (σ_n^2) for each sample is proportional to the presampling bandwidth :



Signal to Noise Ratio

- **Noise**/pixel in a 2D image will be

$$\frac{\sigma_n^2}{N_x N_y} \propto \frac{BW}{N_x N_y} = \frac{1}{N_x N_y \Delta t} = \frac{1}{T_{A/D}}$$

where $T_{A/D}$ is the total time the A/D is sampling

Signal to Noise Ratio

- The **signal** ($x_{i,j}$) represents the total amount of magnetization in a particular voxel

$$\sum\sum x_{i,j} = X_{0,0} = M(0,0)$$

- The signal is proportional to $m_0 V$
  - $V = \Delta x \Delta y \Delta z$ -“voxel” volume
  - Δz - slice thickness

Signal to Noise Ratio

- Signal to noise ratio:

$$SNR \propto \frac{signal}{\sigma_n} = m_0 V \sqrt{T_{A/D}}$$

m_0 is proportional to ρ - the concentration the nucleus of interest, B_0, and γ.

Example 1

- An image is too noisy, so we average together neighboring pixels to achieve $\Delta y' = \Delta y*2$ (all other dimensions remain the same and Δt hasn't changed either)

- Question: How does the SNR change. i.e. $SNR' = ??\ SNR_{orig}$?

Example 1

- An image is too noisy, so we average together neighboring pixels to achieve $\Delta y' = \Delta y*2$ (all other dimensions remain the same and Δt hasn’t changed either)

- Answer: By averaging in image domain, we effectively are discarding samples in k-space ($N_y' = N_y/2$): $T_{A/D}' = T_{A/D}/2$

$$SNR' = 2\Delta x\Delta y\sqrt{\frac{T_{A/D}}{2}} = \sqrt{2}SNR_{orig}$$

The SNR is improved by sqrt(2).

Example 2

- SNR of an image was too noisy. We compensated by acquiring a lower resolution $\Delta y' = \Delta y * 2$ (all other dimensions remain the same) but we've compensated so as to preserve the original acquisition time $T_{A/D}' = T_{A/D}$ (e.g., by changing Δt)

- Question: How does the SNR change. i.e. $SNR' = ?? \, SNR_{orig}$?

Example 2

- SNR of an image was too noisy. We compensated by acquiring a lower resolution $\Delta y' = \Delta y * 2$ (all other dimensions remain the same) but we've compensated so as to preserve the original acquisition time $T_{A/D}' = T_{A/D}$ (e.g., by changing Δt)

- Answer:

$$SNR' = 2\Delta x \Delta y \sqrt{T_{A/D}} = 2SNR_{orig}$$

The SNR is improved by 2 times.

Lessons learned

- It is preferable to anticipate the SNR that is necessary for a given image and set the acquisition accordingly. We don’t achieve as good of an SNR by smoothing the image after it is acquired than if we had acquired at the appropriate resolution originally.

Question

- Suppose we increase our image acquisition time by a factor of 2. The resolution is the same (i.e., Δx and Δy remain the same) and $T_{A/D}' = 2\ T_{A/D}$. The new SNR change is changed to:
- A) 2 SNR_{orig}
- B) $\sqrt{2}$ SNR_{orig}
- C) SNR_{orig}
- D) $SNR_{orig}/\sqrt{2}$

Question

- Suppose we increase our image acquisition time by a factor of 2. The resolution is the same (i.e., Δx and Δy remain the same) and $T_{A/D}' = 2\,T_{A/D}$

- Answer:

$$SNR' = \Delta x \Delta y \sqrt{2T_{A/D}} = \sqrt{2}\,SNR_{orig}$$

Averaging increases the SNR by sqrt(*N*) where *N* is the number of averages

Question

- Suppose we increased the field strength by a factor of 2, $B_0' = 2\,B_0$. Everything else remain the same. The new SNR change is changed to:
- A) 2 SNR_{orig}
- B) sqrt(2) SNR_{orig}
- C) SNR_{orig}
- D) SNR_{orig}/sqrt(2)

Question

- Suppose we increased the field strength by a factor of 2, $B_0' = 2\ B_0$.

- Answer:

$$SNR' = 2SNR_{orig}$$

- Keeping resolution constant, we can use this additional SNR to reduce the number of averages (and thus overall imaging time) by a factor of 4! – time is money in imaging!

Question

- Suppose we wanted to halve all 3 voxel dimensions $(\Delta x, \Delta y, \Delta z)$ but keep the SNR the same. How much longer will the acquisition time need to be?
- A) 2
- B) $\sqrt{8}$
- C) 8
- D) $8^2 = 64$

Question

- Suppose we wanted to halve all 3 voxel dimensions $(\Delta x, \Delta y, \Delta z)$ but keep the SNR the same. How much longer will the acquisition time need to be?

D) $8^2 = 64$ (unfortunately)

- In $SNR \propto V\sqrt{T_{A/D}}$, $V \downarrow 8$, so $T_{A/D} \uparrow 64$

How to increase Signal to Noise Ratio?

- Averaging in image domain
  - SNR improved by sqrt(N)
  - Can be compensated by increasing acquisition time to further increase SNR

- Increase acquisition time by N times
  - SNR improved by sqrt(N)

- Increased the field strength B_0
  - SNR improved by N times

Questions?

Signal to Noise Ratio

Noise Characteristics

- Zero-mean and additive

$$s(t) = M(k_x(t), k_y(t)) + n$$

- Samples are independent (due to the whiteness of the spectrum)

- Gaussian distributed (it results from large numbers of vibrating particles – Law of Large Numbers)

Noise Characteristics

- Bi-variate - independent noise in real + imaginary (the channels of the complex demodulator are orthogonal) ($n = n_i + i\, n_q$)

Noise Characteristics

- A 1D image is reconstructed by an N point DFT:

$$x_n = \frac{1}{N}\sum_{m=0}^{N-1} X_m \exp\left(i2\pi\frac{nm}{N}\right)$$

- X_m - independent noise having variance .
- Noise in image pixels x_n will also be zero-mean, additive, independent, bi-variate Gaussian noise, but with variance

Noise Characteristics

- Derivation of the variance is:

$$\mathrm{var}(x_n) = E\left[x_n x_n^*\right] = E\left[\frac{1}{N}\sum_{m=0}^{N-1} X_m e^{i2\pi\frac{nm}{N}} \frac{1}{N}\sum_{l=0}^{N-1} X_l e^{-i2\pi\frac{nl}{N}}\right]$$

$$= \frac{1}{N^2}\left(E\left[\sum_{m=0}^{N-1} X_m^2\right] + E\left[\sum_{m\neq l} X_m X_l e^{i2\pi\frac{n(m-l)}{N}}\right]\right)$$

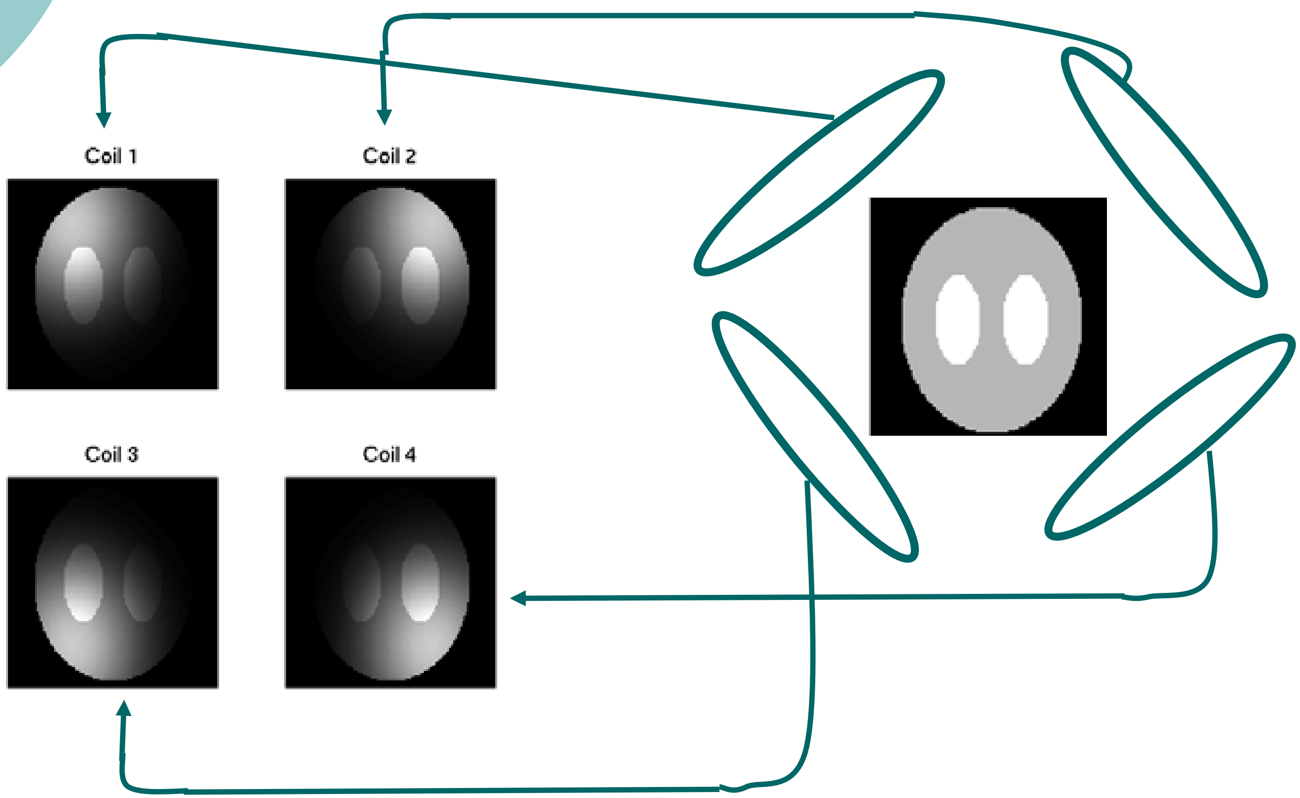
$$= \frac{1}{N^2}\sum_{m=0}^{N-1} E\left[X_m^2\right] = \frac{1}{N^2} N\sigma_n^2 = \frac{\sigma_n^2}{N}$$

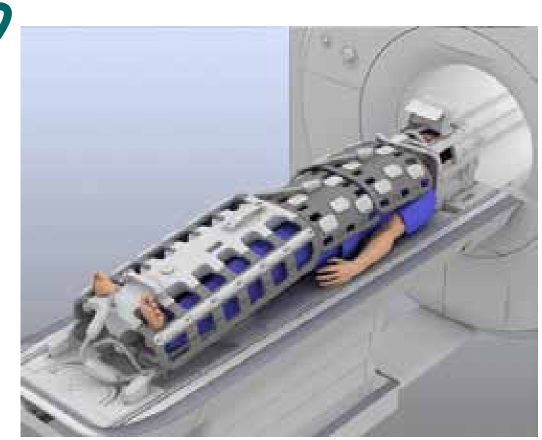
Array Coils and Parallel Imaging

Signals Weighted by Coil Sensitivity

$$s_l(t) = \iint C_l(x, y)m(x, y)\exp(-i2\pi\big(k_x(t)x + k_y(t)y\big)dxdy$$

$m_l(x, y) = C_l(x, y)m(x, y)$ for coil # l





Siemens Healthineers

SNR Advantages

- SNR:
  - Small coils are sensitive to the signal of interest but only receive noise from the volume directly in front of a coil.
  - Better SNR, poor coverage
- Combining signals from array of coils, one achieve the benefit of a large region of coverage but the SNR advantage of small coils.

Coil Combination

- For coils with independent, equal variance noise:

$$\hat{m}(x,y) = \frac{\sum_l C_l^*(x,y)\, m_l(x,y)}{\sum_l |C_l(x,y)|^2}$$

- Pretty good and easy method (don't need to know C_l:

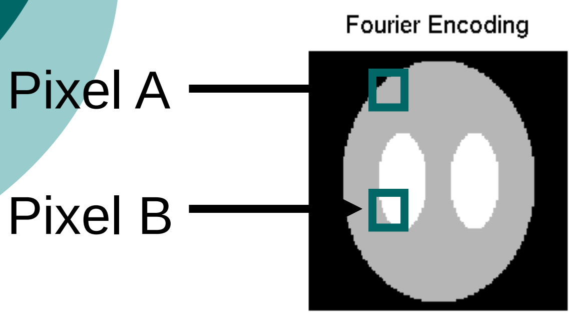
$$\hat{m}(x,y) = \sqrt{\sum_l |C_l(x,y) m(x,y)|^2}$$

Parallel Imaging

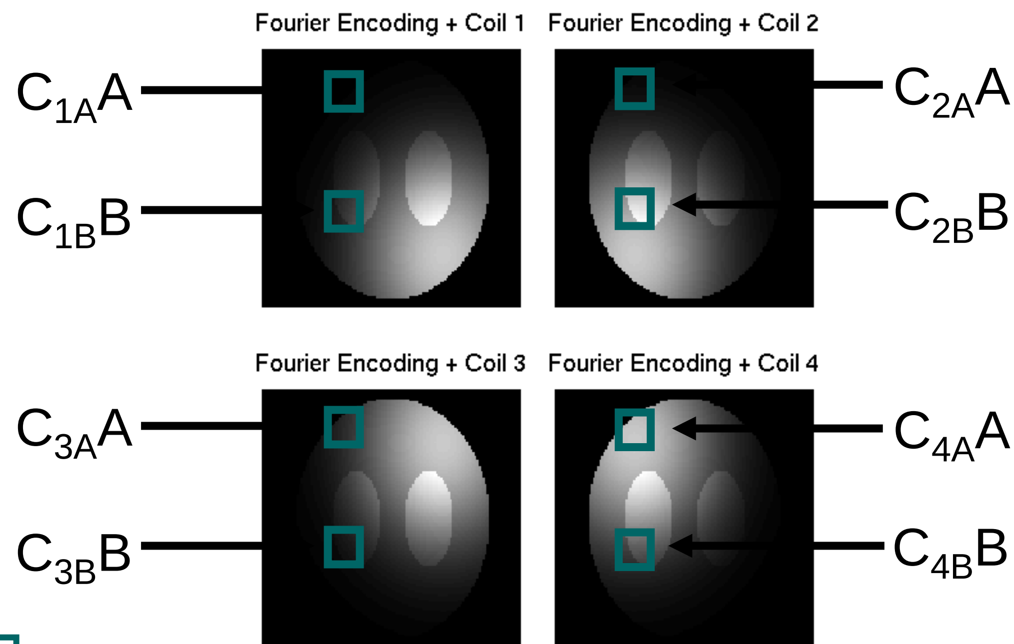
- Use coils coarse coil patterns to reduce aliaising
- Speed acquisition/increase spatial resolution
- Basic idea: by sampling multiple coils, one can use the additional measurements to eliminate aliasing

SENSE Imaging – An Example

Full Fourier Encoding
Volume Coil



Full Fourier Encoding
Array Coil



Unknown Pixel Values A & B

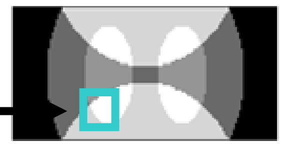
Known Sensitivity Info S_{1A}, S_{1B},…

SENSE Imaging – An Example

Reduced Fourier – Speed-Up R=2
Volume Coil

Insufficient Data
To Determine A & B

Reduced Fourier Encoding

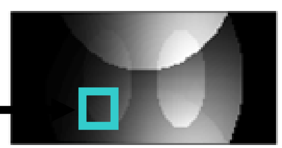


A+B

Reduced Fourier – Speed-Up R=2
Array Coil

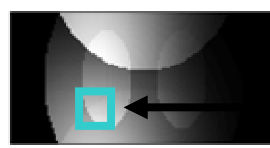
Extra Coil
Measurements
Allow Determination
of A & B

Reduced Fourier + Coil 1



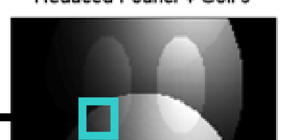
$C_{1A}A+C_{1B}B$

Reduced Fourier + Coil 2



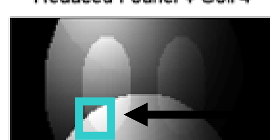
$C_{2A}A+C_{2B}B$

Reduced Fourier + Coil 3



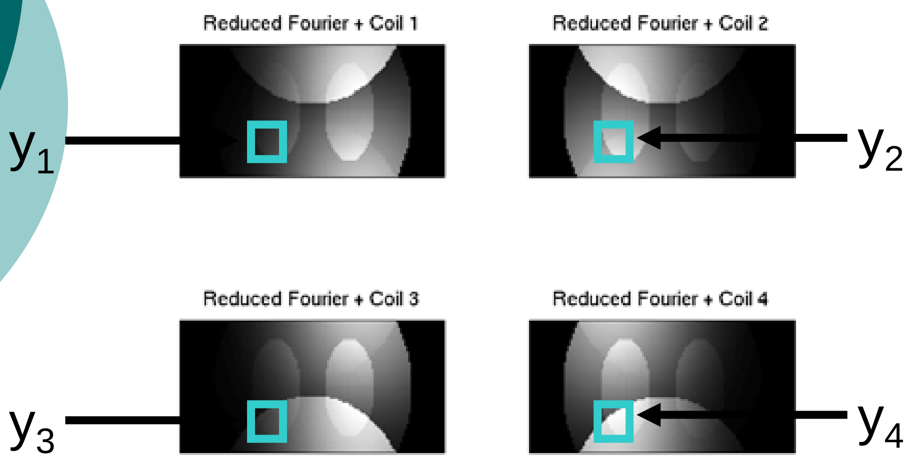
$C_{3A}A+C_{3B}B$

Reduced Fourier + Coil 4



$C_{4A}A+C_{4B}B$

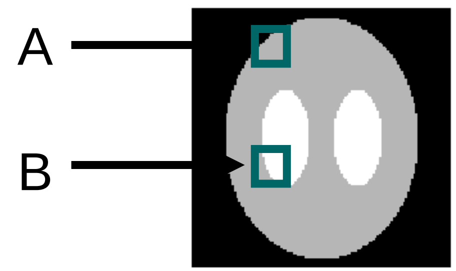
SENSE Imaging – An Example



$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} C_{1A} & C_{1B} \\ C_{2A} & C_{2B} \\ C_{3A} & C_{3B} \\ C_{4A} & C_{4B} \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

$$\begin{bmatrix} A \\ B \end{bmatrix} = \mathbf{C}^{+} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

Solving this matrix equation leads to A & B and the unaliased image



Parallel Imaging

- Pros:
  - Much faster acquisition
  - Higher spatial resolution
  - Some reduction of distortions
- Cons:
  - Lower SNR (partially compensated by SNR improvement from coil arrays)