Homework #3
Due Date: Nov. 5, 2023

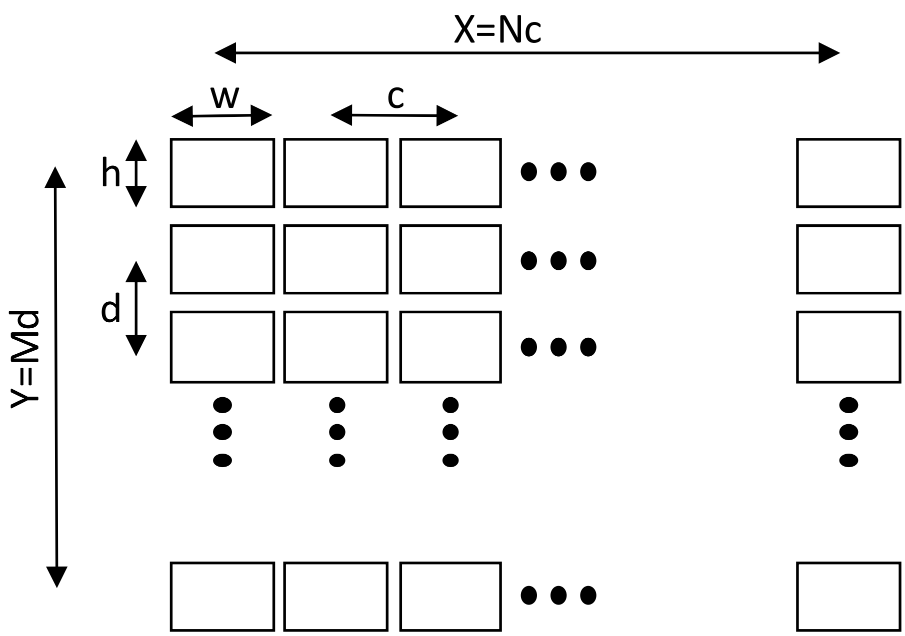

Figure for Problem 1

1. 2D US Array Systems. In class, all of our analysis of phased-array ultrasound imaging systems involved one-dimensional arrays. Now, we will look at a two-dimensional phased-array, pulse-echo system of size N x M elements (X x Y). This system will be capable of imaging a 3D volume of reflectors. Let the operating frequency f_0 = 3 MHz, the speed of sound speed c = 1.5 mm/μs, and the overall and the total aperture size is 10 mm (in x) x 5 mm (in y). Assume Fresnel approximation.
   a. Determine element spacing (c and d) and the total number of detectors (N x M) so that there is no aliasing. 5 pts
   b. Determine the steady state transmit pressure pattern in x_z and y_z (i.e., the x-y coordinate system at depth z) at some focal depth z for an on-axis beam given that each transducer is rectangular having dimension w by h. 10 pts
   c. Assuming the array is used to transmit and receive, determine the combined pressure pattern in terms of your result from part b. 5 pts
   d. What is the beam spacing in x and y ($\Delta x_z/z$ and $\Delta y_z/z$) for this system (described in part c). 10 pts
   e. At depth z=100 mm, we wish to sample a 100 mm x 100 mm region of the object. Determine the minimum number of beams to sweep this space (e.g. a pyramid shaped space). 5 pts
   f. What is the time delay, τ_i, for the i^{th} element at position (x_i , y_i) as a function of depth (z) and beam offsets x_z and y_z? As in the 1D array, assume that the center of the array is the origin of the coordinate system. Give the Fresnel and Fraunhoffer approximations. 6 pts
   g. For this part, use the number of beams and beam spacing from part e. Assuming that we wish to receive reflections from a maximum depth of interest is 150 mm, determine the

maximum frame rate (frequency at which the whole volume is acquired) for 3D mode. If we operate the same device in 2D mode (scanning through x_z/z positions for a fixed y_z/z), what is the maximum frame rate? 9 pts

2. Beamforming in Oceanography. You've just received a contract from the NOAA to build a hydrophone array system to track migrating whales. The whales make several noises that are of interest to you: groans (20-50 Hz), coos (200-500 Hz) and whistles (1000-1500 Hz). Your budget will only allow you to buy 64 hydrophones and the computers to do the beamforming (e.g. you have a 64 element 1D array). You will consider two different designs:
   i. An array system capable of beamforming all of these sounds.
   ii. An array system capable of beamforming just the groans and coos.

   Assume that the speed of sound in sea water is 1500 m/s. 5pts/question

   a. What is the element spacing and total traducer aperture ($D = 2a$) for both of the above systems necessary to prevent aliasing (grating lobes)?
   b. At what approximate distance does the "far-field" or Fraunhoffer zone begin for both systems for a 50 Hz groan?
   c. At what distance does the "far-field" begin for both systems for a 500 Hz coo?
   d. At what distance does the "far-field" begin for both systems for a 1500 Hz whistle?
   e. Suppose there is a whale 50 km away. Is this the "far field"? If it groans at 50Hz, with what accuracy can you tell its lateral (not range) position for both arrays?
   f. If it coos at 500Hz, with what accuracy can you tell its lateral position for both arrays?
   g. If it whistles at 1500Hz, with what accuracy can you tell its lateral position for both arrays?
   h. For a whistles at 1500Hz, at what angular position is the location of the first grating lobe for array ii assuming the main beam is steered to $\theta = 0$?
   i. What is the propagation delay for a whale to an array 50 km away? With our present system, can we say anything about the range or depth of the whale from the array?
   j. Suppose we were given funds to build a second array, how might we use that system to improve our range resolution?