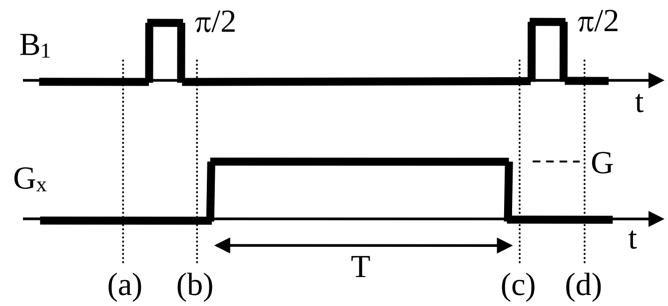
Homework #5

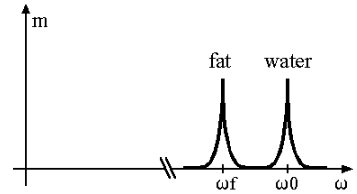
Due: 11/7/23

1. Consider an object with initial magnetization $m_0(x,y)$ = rect(x/X, y/Y). (6 pts each)
   a. Determine $M_0(u,v)$ - the 2D Fourier transform of $m_0(x,y)$.
   b. For gradient waveforms $G_x(t) = A$ and $G_y(t) = 0$, determine and sketch the k-space path and give an expression for the received signal, $s(t)$.
   c. For gradient waveforms $G_x(t) = a/X$ and $G_y(t) = a/Y$, determine and sketch the k-space path and give an expression for the received signal, $s(t)$. (a>0)
   d. For gradient waveforms $G_x(t) = 0$ and $G_y(t) = \frac{a}{Y} rect\left(\frac{t - T/2}{T}\right)$, where $T = \frac{4\pi}{a\gamma}$, determine and sketch the k-space path and give an expression for the received signal, $s(t)$. (a, T > 0)

2. Special pulse sequences can be used to generate usual patterns in the transverse or longitudinal magnetization across the image. We examine one such pulse sequence here. Using a 1D object $m_0(x) = 1$ that is initially in the equilibrium state, describe the longitudinal, $m_z(x)$, and transverse magnetization, $m_{xy,rot}(x)$, as functions of x at the points labeled (a)-(d). You may neglect relaxation and assume the duration of the gradient is T. (Hint - describe m_z(x) and $m_{xy,rot}(x)$, as a function of x; neglect relaxation here which means the amplitude of M remains m_0). This is related to HW#4, problem 3. (6 pts each)

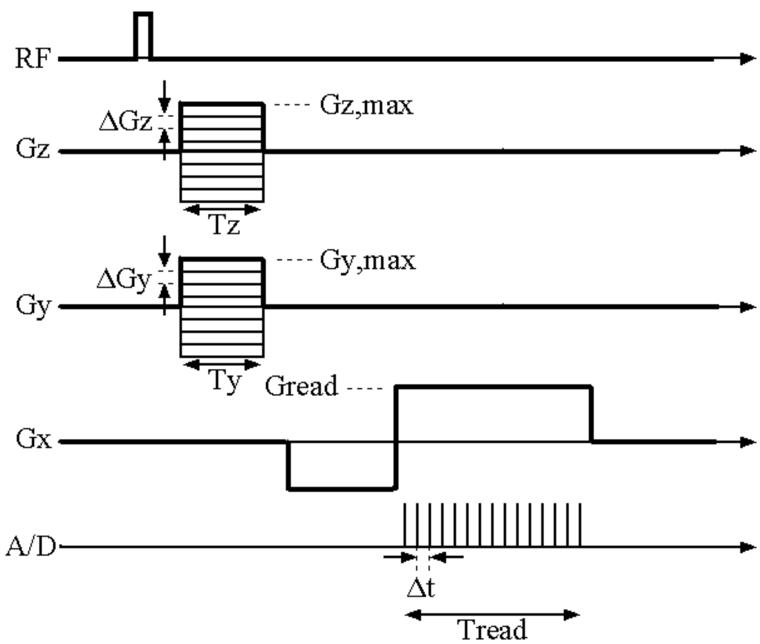


3. In class, we excited a particular slice by applying a gradient in the z-direction applying a band-limited RF pulse with a particular frequency of excitation and a particular bandwidth. Let look at a similar situation – excitation of particular chemical species, for example, water and fat. Different chemical species have different resonant frequencies based on a phenomenon known as chemical shift, in which surrounding electron clouds influence the strength of the B field seen by the nucleus. If we consider water is at some frequency $\omega_0 = \gamma B_0$, fat is at $\omega_f = \gamma B_0(1 - \delta_f)$, where δ_f is the chemical shift of fat relative to water (3.5×10^{-6}). (5 pts each)



a. For B_0 = 1.5 T, find (f_0 - f_f) Assume we are imaging ^{1}H (For protons (^{1}H), $\gamma/2\pi$ = 42.58 MHz/T).
b. Describe an RF pulse in time domain (pulse envelope shape and parameters, amplitude, carrier frequency, etc.) that has the appropriate characteristics for a 90 degree excitation of water but no excitation of fat. One common approach is the make the spectrum of the RF pulse symmetrical around the water resonance. (To calculate A, use approximation from the example in the notes/lecture - flip angle = gamma*A*T; center of the RF pulse here can start at -τ/2, therefore you don't have to calculate the τ here.)

4. Consider a 3D spin-warp pulse sequence as shown in the sketch. Let $T_y = T_z$ = 5 ms, and T_{read} = 20 ms. Suppose our desired field of views are $FOV_x = FOV_y = FOV_z$ = 20 cm and Δx = 1 mm, Δy = 2 mm, and Δz = 5 mm. Assume γ for protons. Determine the following parameters: (5 pts each)
a. ΔG_z
b. $G_{z,max}$
c. ΔG_y
d. $G_{y,max}$
e. G_{read}
f. Δt



5. Consider a volume coil and a surface coil. Let the volume coil have sensitivity, $S_v(x) = 1$, and the surface coil have the following sensitivity pattern (as a function of distance from the coil):

$$S_s(x) = \frac{1}{\left(1+\left(\frac{x}{a}\right)^2\right)^{3/2}}$$, where a is the coil radius.

Let the noise variance of the volume coil be $\sigma_v^2 = 1$ and the noise variance of the surface coil be $\sigma_s^2 = 0.001 \cdot a^3$, where a is assumed to be in units of cm. (Question 5: Since we assume everything other than the coil is different, for comparison purpose SNR = S(x)/σ.)
(6 pts each)

a. For a = 5 cm, determine for which distance from the object surface it is advantageous (from a signal to noise ratio standpoint) to use the surface coil over the volume coil (and vice versa). SNR = (signal magnitude)/σ, where σ is the noise standard deviation.
b. For a = 10 cm, determine for which distance from the object surface it is advantageous to use the surface coil over the volume coil (and vice versa).