



X-ray Imaging

BME/EECS 516

X-ray Lecture #1



Announcements

- HW #5 due Tuesday, 11/7
- MRI Project due Tuesday 11/21
- Tuesday, 11/14 – guest lectures from local medical imaging industry
 - David Sarment from Xoran Technologies
 - John Seamans (UM) formerly with Delphinus Medical and GE Healthcare

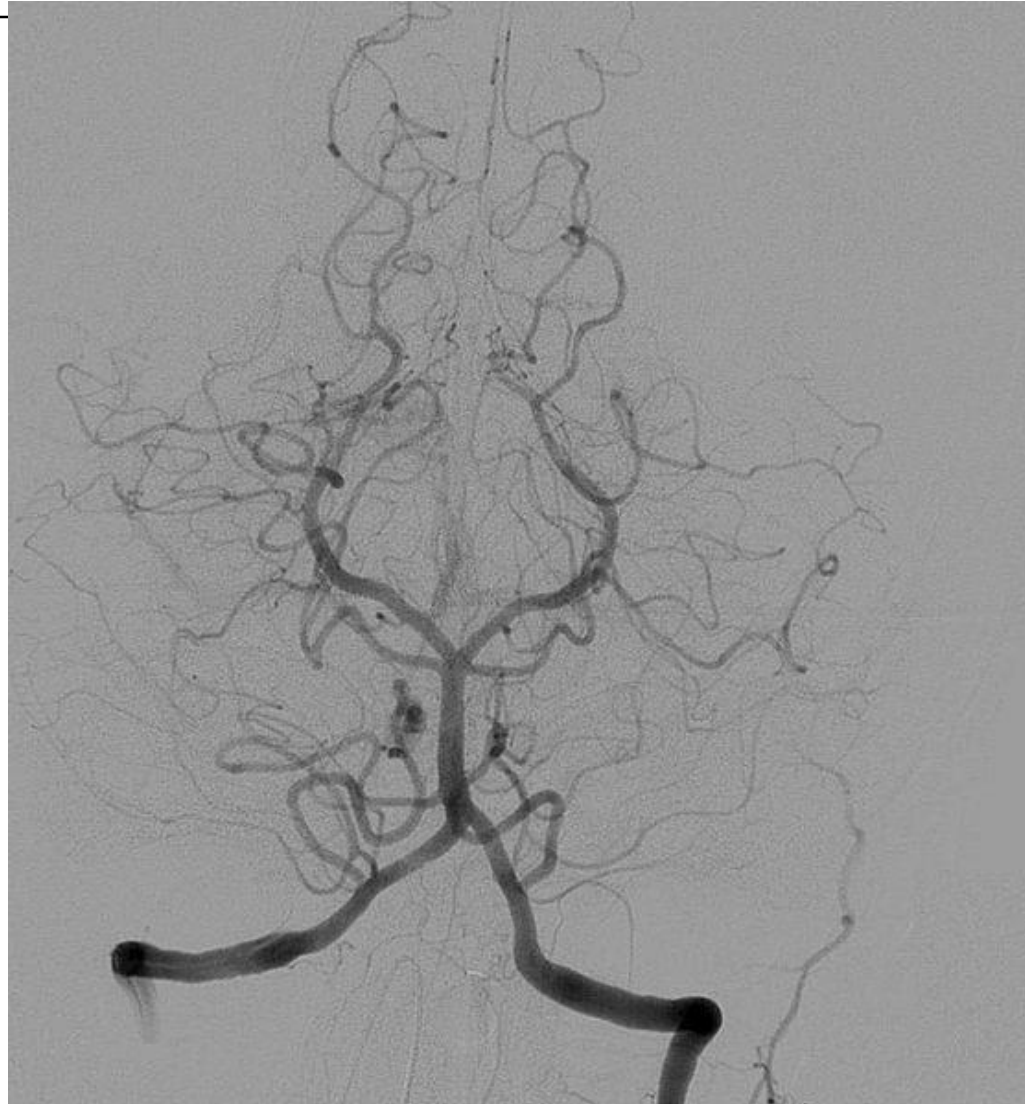
X-ray Imaging

- X-ray images are characterized by the interaction of x-ray photons and tissue.
- Applications for diagnostic imaging
 - Detection of pathology of the skeletal system (chest x-ray or dental x-ray)
 - Lung diseases (lung cancer, pulmonary edema)
 - Kidney stones,
 - Basis for CT, fluoroscopy (with contrast agent for vessel and brain imaging, e.g., angiography)

X-ray Images



Angiography





This Lecture

- Physics – Radiation
- Basic X-ray Imaging System
- Generation of x-rays
- Attenuation Coefficient
- Beam Hardening



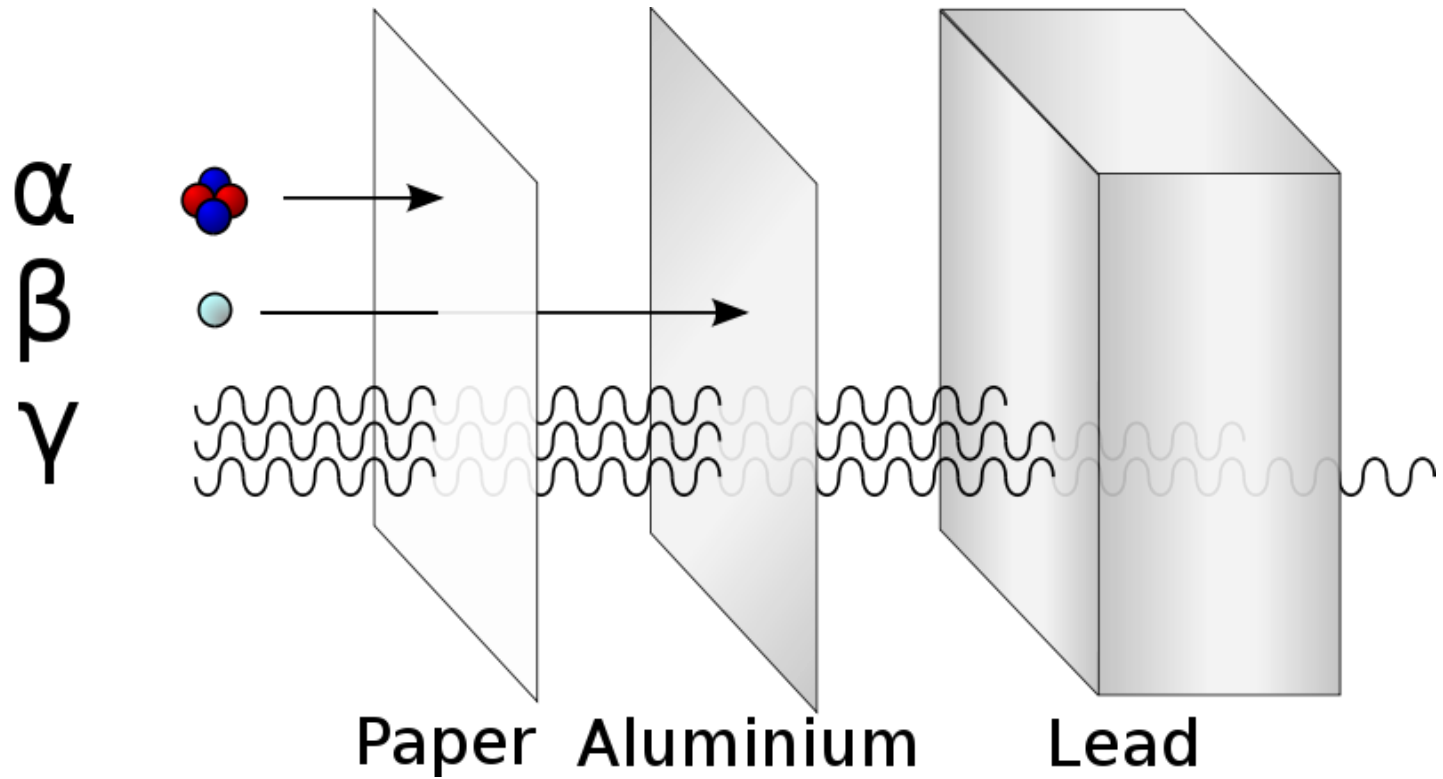
Physics

Radiation

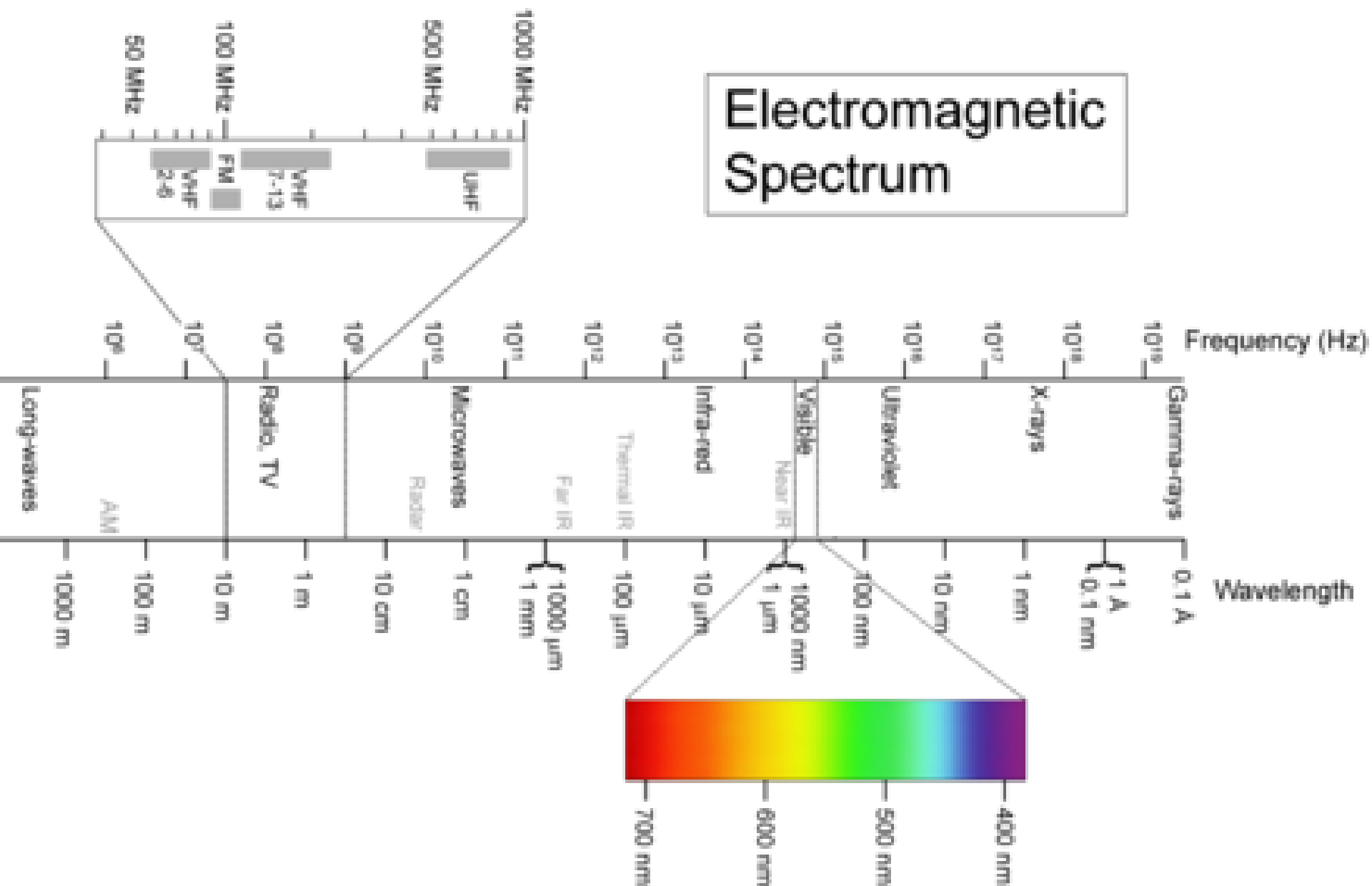
Radiation

- Definition: a process in which energetic streams of particles or photons travel through a medium or space.
- Particles: α (${}^2_2\text{He}$), e^- (electrons), β^- (electrons emitted from nuclei), β^+ (positrons), p^+ (proton), n^0 (neutrons)
- Photons: x-ray, γ -ray, annihilation photons, etc.

Particle and Photons



Electromagnetic Spectrum



X-ray

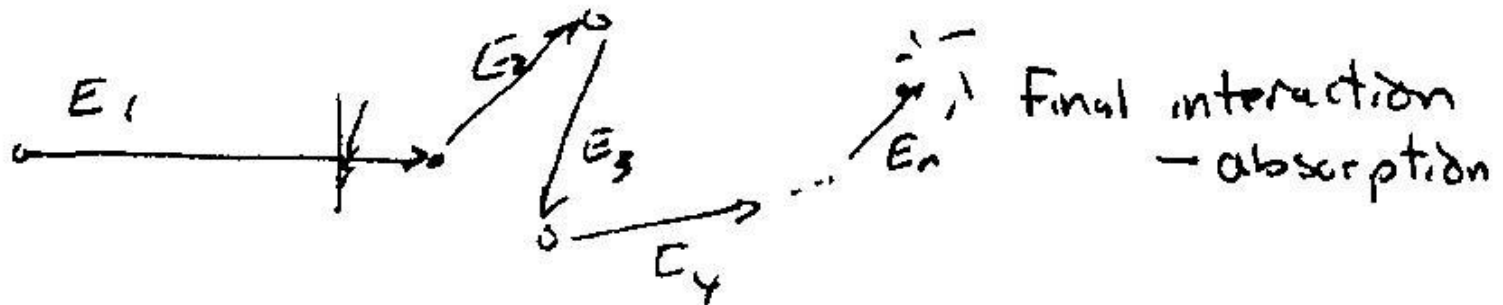
- Wavelength: 0.01 - 10 nm
- Frequency: 3×10^{16} - 3×10^{19} Hz
- Energy: 120 eV - 120 keV (1eV = 1.602×10^{-19} J)
 - eV –the amount of kinetic energy gained by a single unbound electron when it accelerates through an electric potential difference of one volt.
- X-rays and γ -rays are distinguished by their origin
 - X-rays -emitted by electrons outside the nucleus
 - Gamma rays - emitted by the nucleus.

Models for interaction of radiation and matter

- Absorption (generally low kinetic energy (KE))

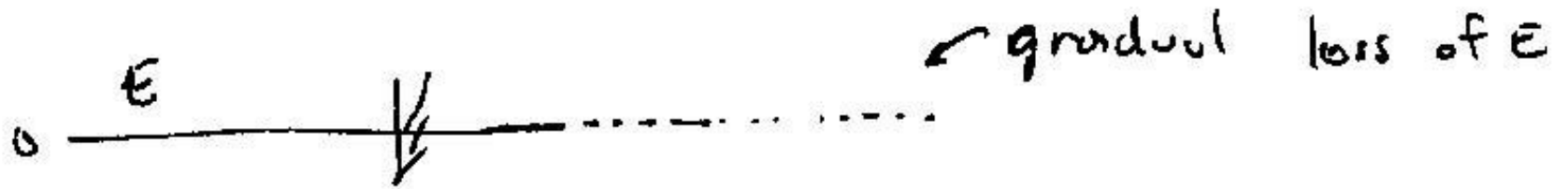


- Scattering



Models for interaction of radiation and matter

- Not a typical interaction – a gradual loss of energy



Models for interaction of radiation and matter

- The charged particles (α , e^- , β , β^+ , p^+) interact very strongly with tissue and typically do not pass completely through the human body - cannot be used for imaging.
- Photons (including x-ray) and neutrons(n^0) pass through the body with an appropriate amount of interaction for imaging (too little interaction is also bad).

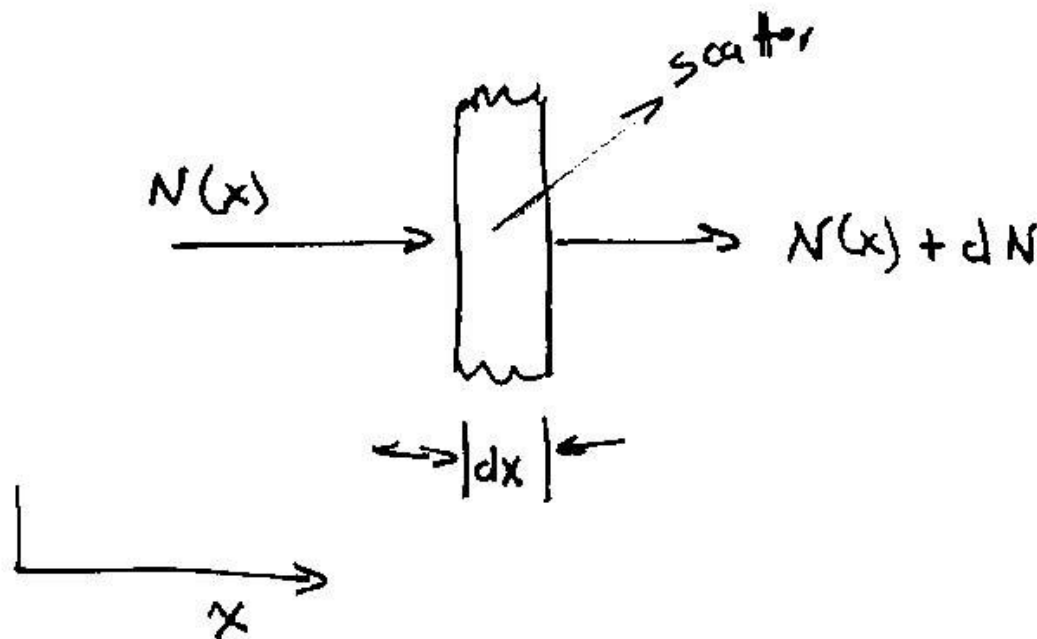


Behavior of Radiation Along a Line: Assumptions

- Matter in the pathway consists of discrete particles separated by distances that are large compared to the size of the particles.
- For a given path length along a line, an x-ray photon either interacts (with prob. p) or it doesn't.
- All interactions are independent.

Behavior of Radiation Along a Line: Assumptions

- Scattered photons scatter at a different angle and don't contribute to the continuing flux of photons along the line.



Behavior of Radiation Along a Line: Assumptions

- The change in the number of photons is:

$$dN \propto -N(x)dx$$

$$dN = -\mu N(x)dx$$

$$\frac{dN}{dx} = -\mu N(x)$$

$$N(x) = N(0) \exp\left(-\int_0^x \mu(x')dx'\right)$$

- μ is the “linear attenuation coefficient” and has units $(\text{distance})^{-1}$. For a constant μ :

$$N(x) = N(0) \exp(-\mu x)$$



Questions?

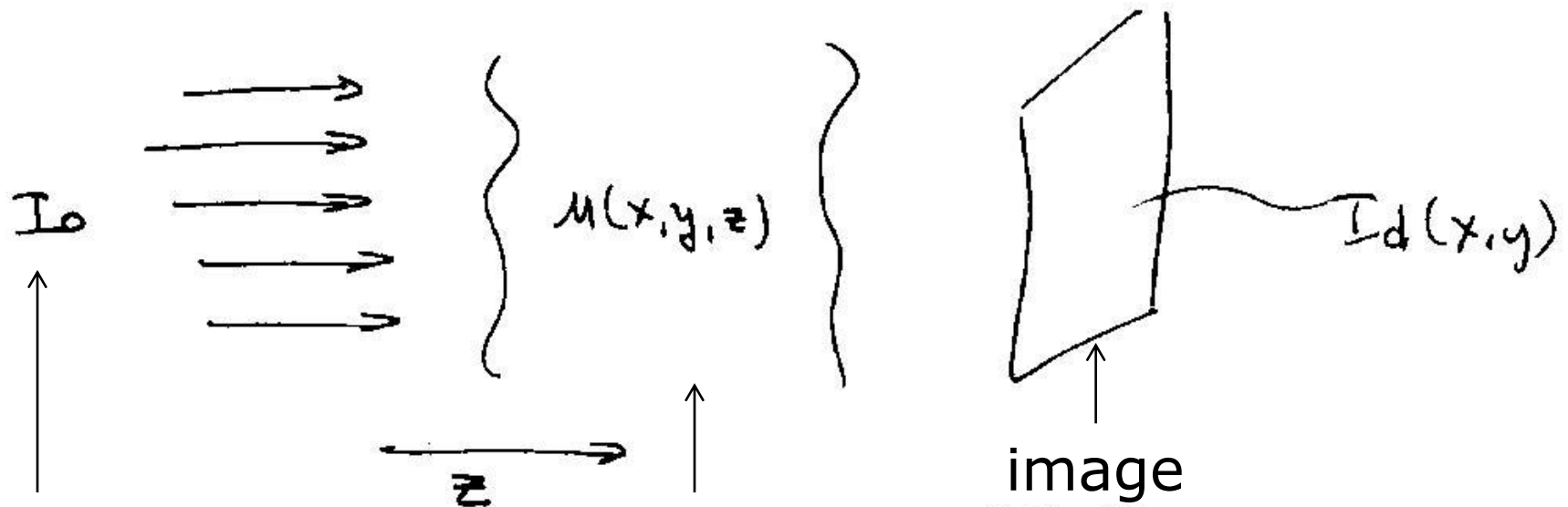
Physics - Radiation



The Basic X-ray Imaging System

Basic X-ray Imaging System

- Consider a parallel x-ray flux



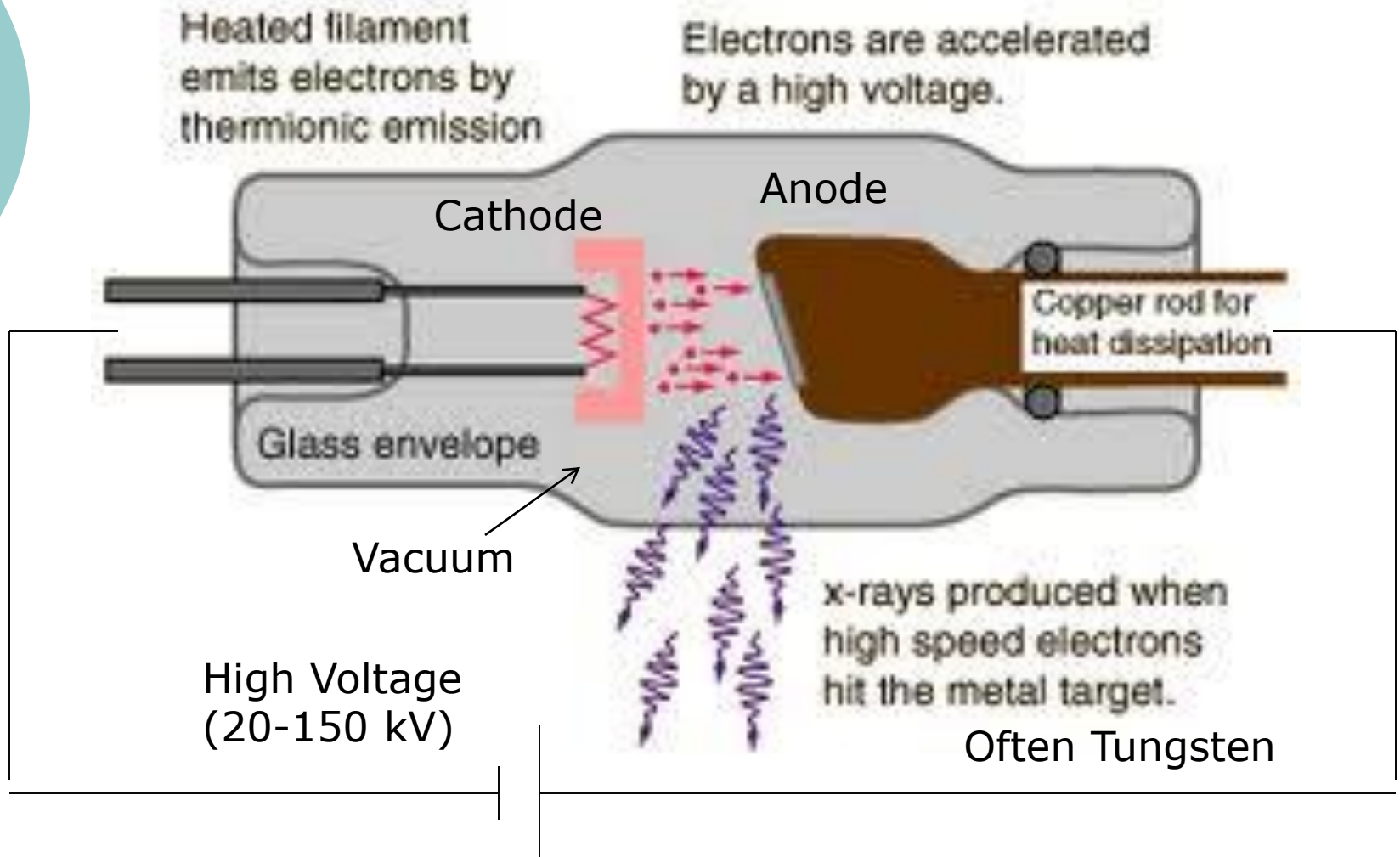
$$I_d(x, y) = I_0 \exp\left(-\int \mu(x, y, z) dz\right)$$



Basic X-ray Imaging System

- Source - Generation of x-ray
- Detector

X-ray Tube



X-ray Tube

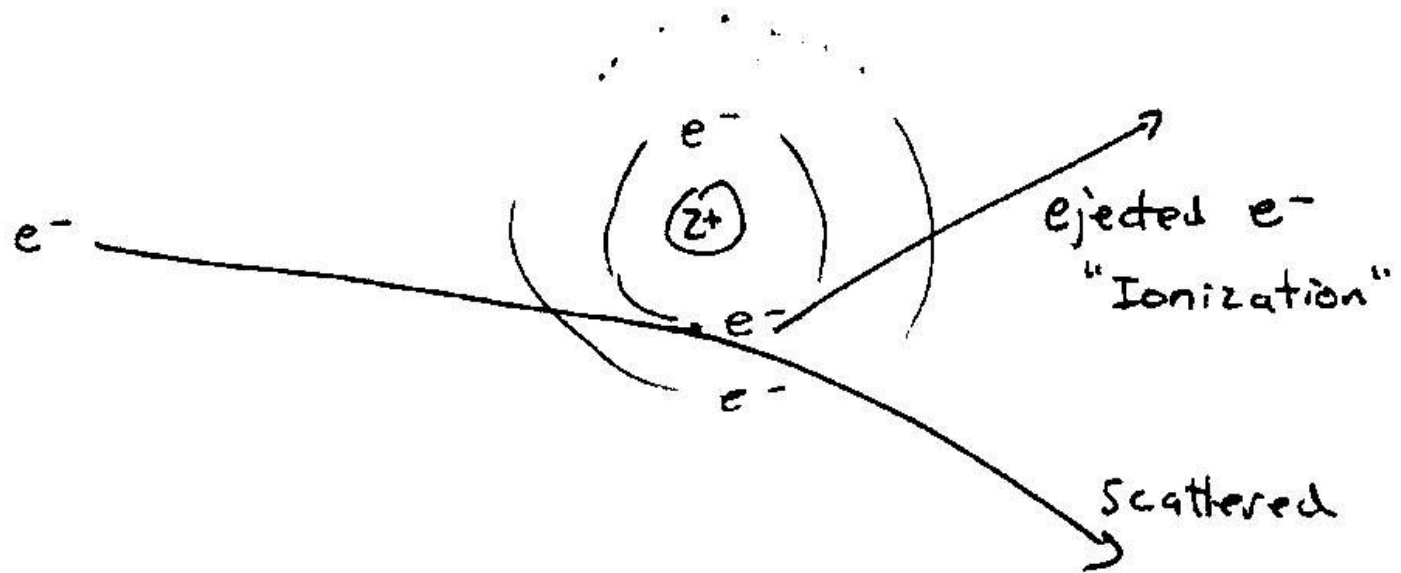




Generation of x-rays: electron interactions

1. Inelastic (energy absorbing) scattering with atomic electrons
 - An electron with enough energy can eject an orbital electron out of the inner shell of a metal atom.
 - Electrons from higher energy level fill up the vacancy
 - X-ray photon are emitted from **spontaneous energy state transitions.**

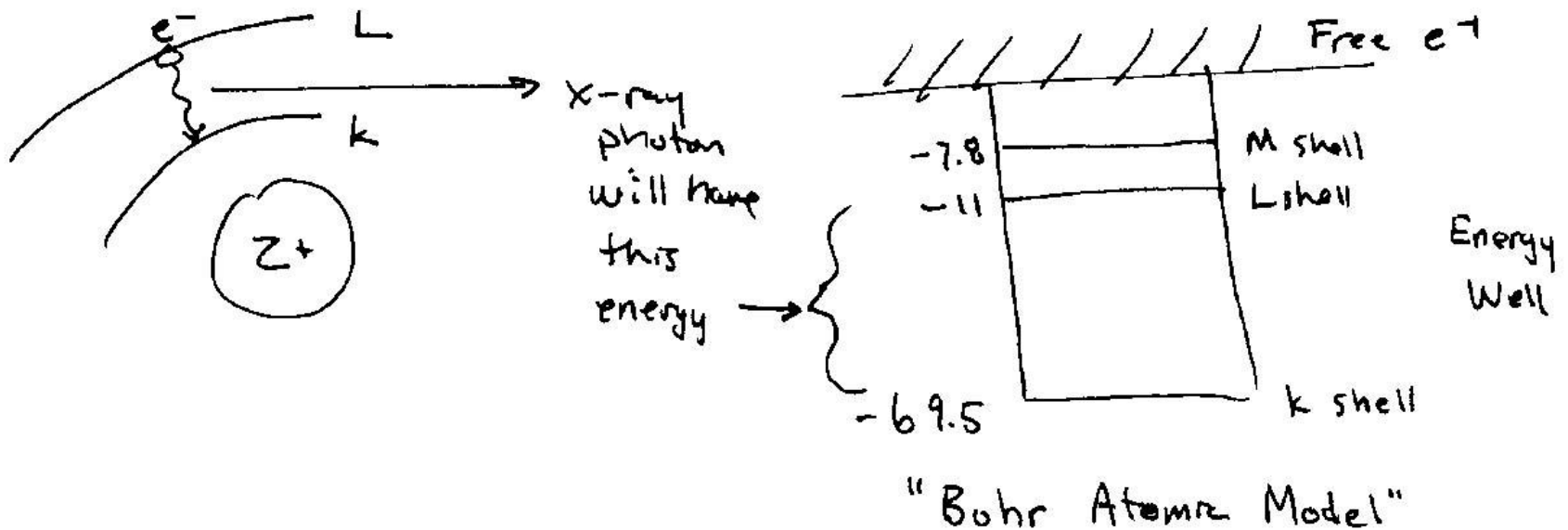
Generation of x-rays: electron interactions



Generation of x-rays: electron interactions

- The process produce discrete emission spectrum –spectral lines.
“Characteristic” x-ray energies for W
 - 58.5 keV
 - Any combination of shell transition energies (e.g. 3.2 and 61.7 keV).
- Bohr model accounts for absorption/generation of discrete valued energies.

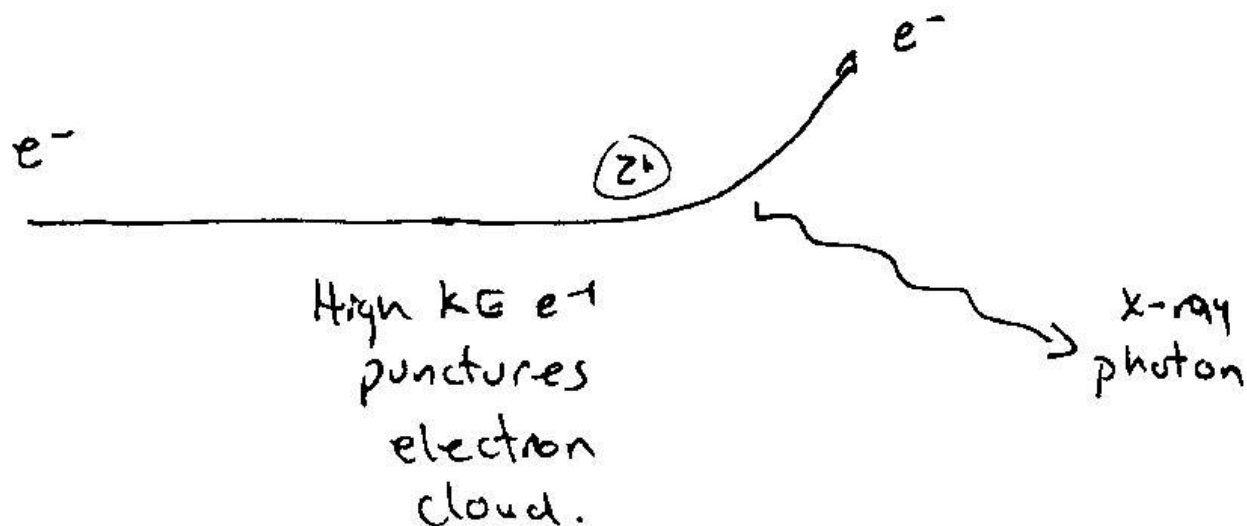
Generation of x-rays: electron interactions



Generation of x-rays: electron interactions

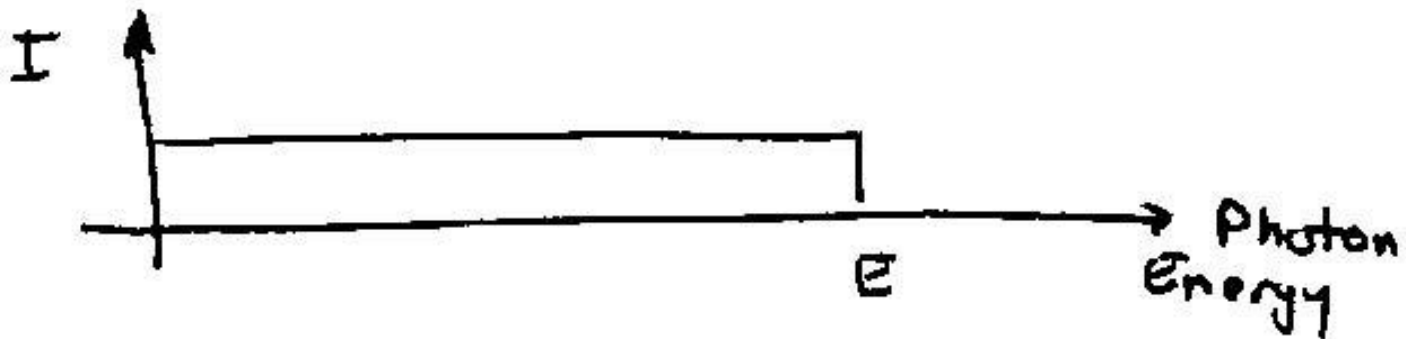
2. Bremsstrahlung "Braking"

Radiation – accelerated electrons scattered by a strong electric field near the high-Z nuclei. X-rays have continuous spectrum



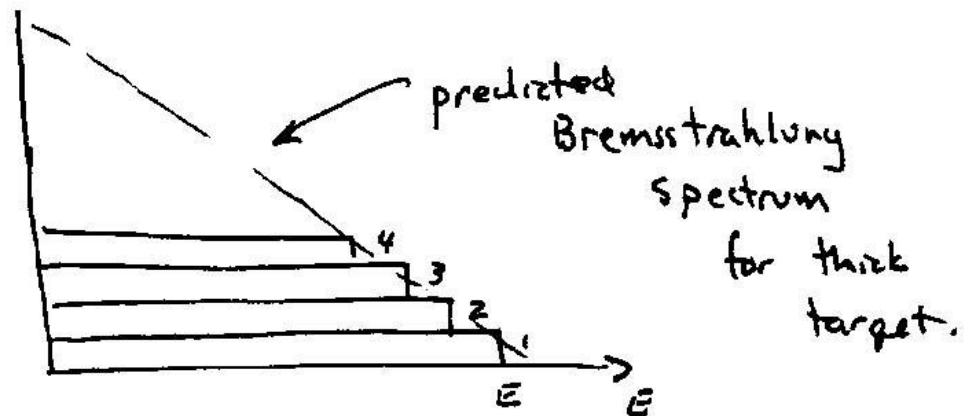
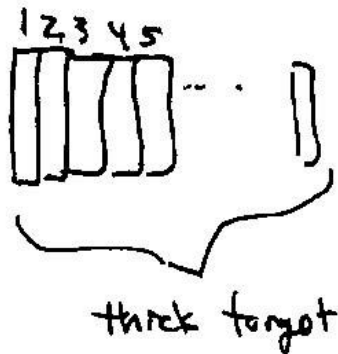
Generation of x-rays: electron interactions

- Thin target (anode) - for electrons of a particular energy, E , striking an infinitely thin target, Bremsstrahlung radiation will have a uniform distribution of energy



Generation of x-rays: electron interactions

- Thick target (anode)
 - Modeled as a series of thin targets.
 - Each thin target produces a new uniform spectrum, but with a lower peak energy.
 - Resultant spectrum is approximately linear from a peak at 0 keV to 0 at E .

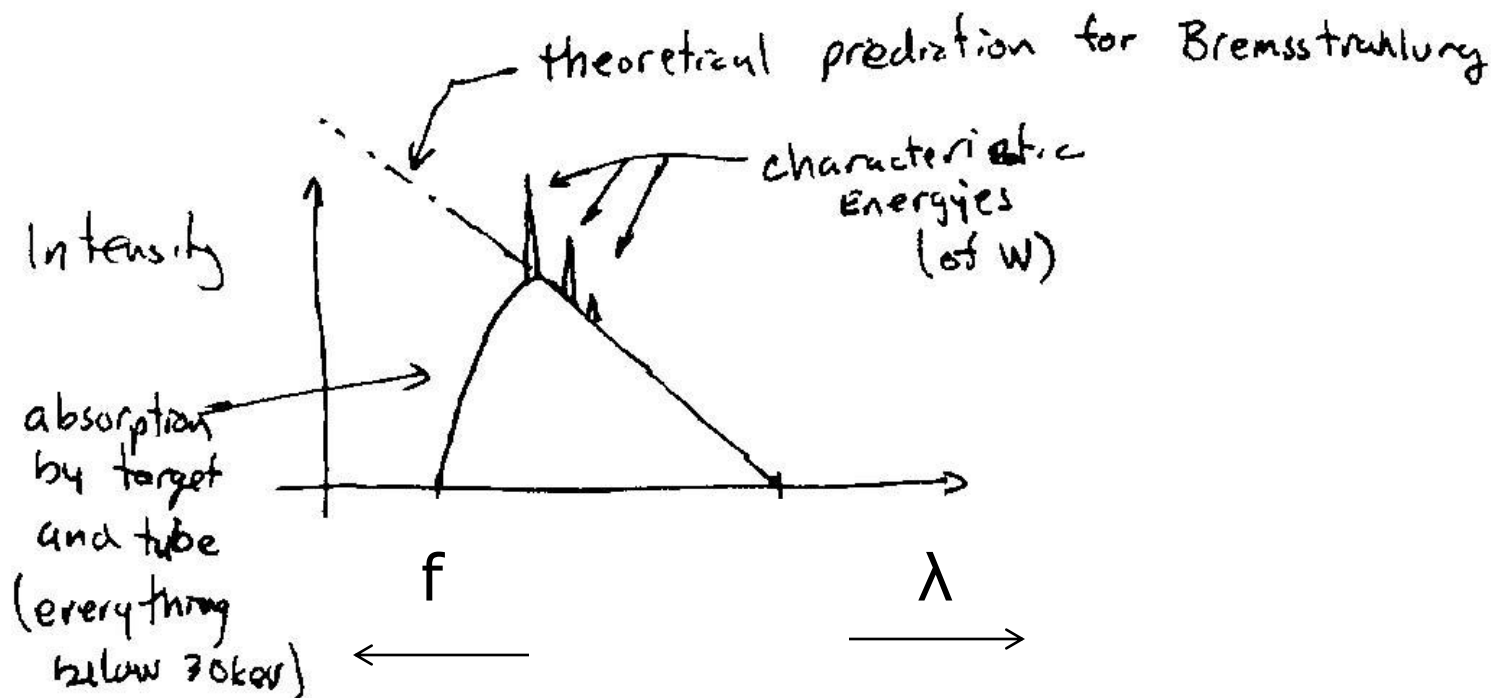


The x-ray Spectrum

- For electrons with energy E , the maximum x-ray photon energy is E .
 - 80kV x-ray tube cannot generate photon with energy (electron charge \times voltage field) higher than 80keV
- Very low energy photons are absorbed by the target and by the glass in the x-ray tube.

The x-ray Spectrum

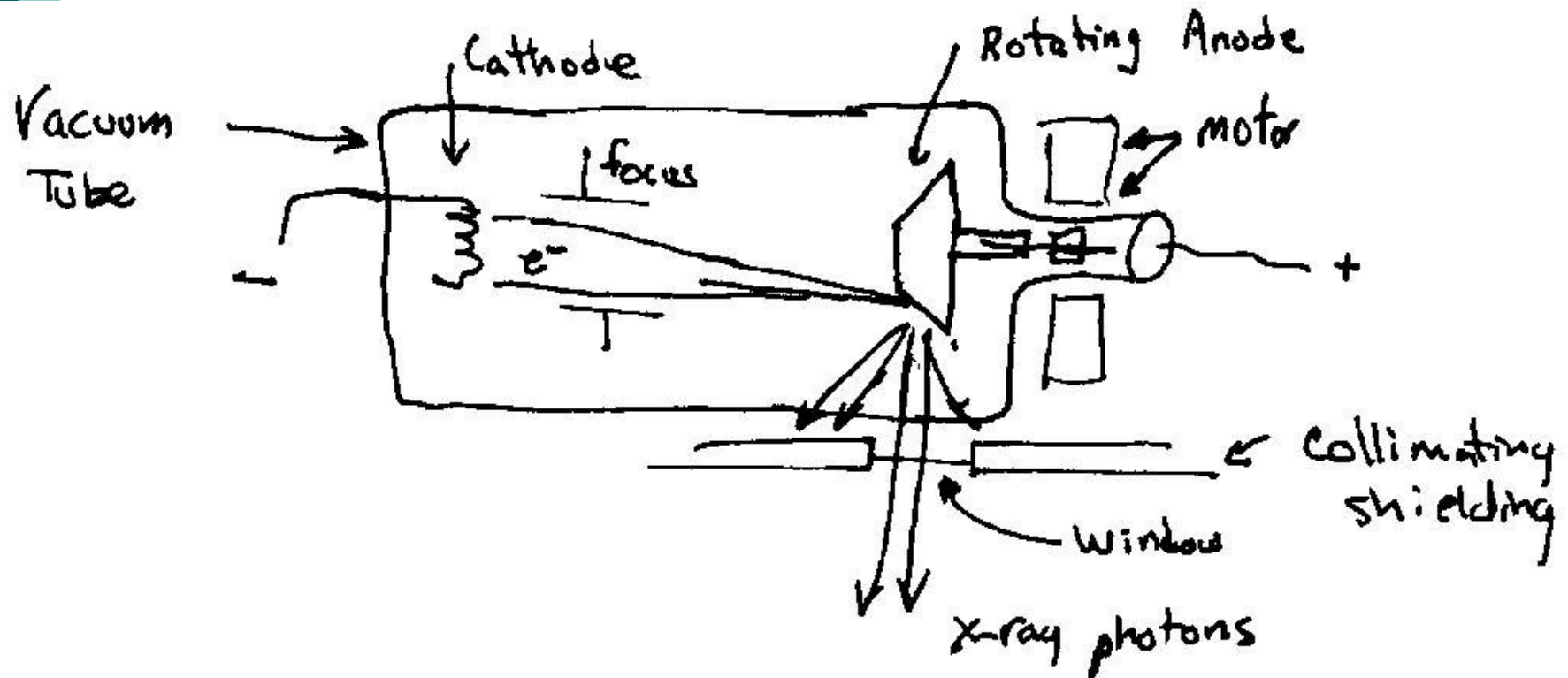
- Spectrum will have a combination of Bohr (discrete) energies and Bremsstrahlung



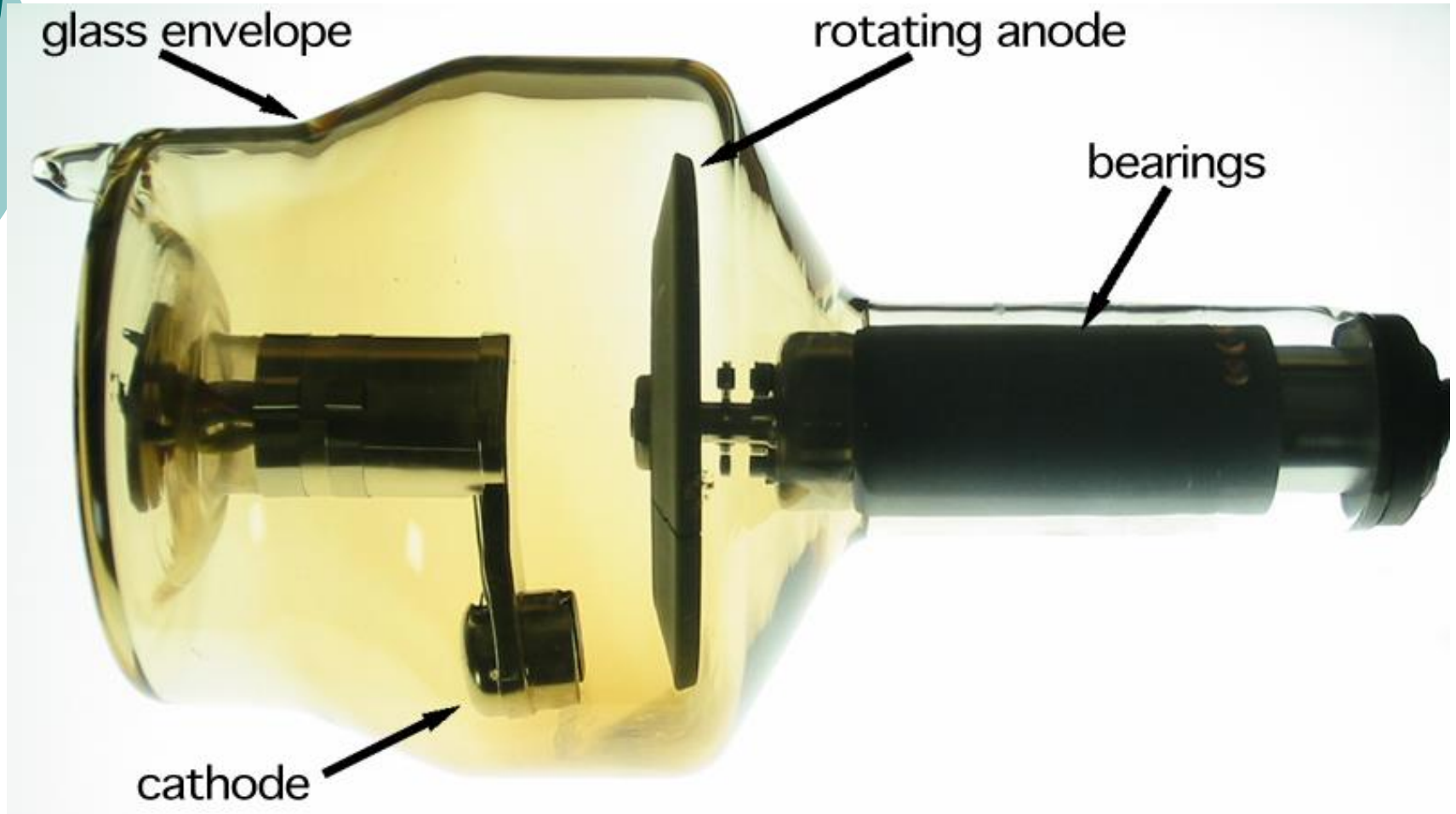
The x-ray Spectrum

- The x-ray spectrum is function of photon energy: $I_o = I_o(E)$
- I represents energy/unit time/unit area or power/unit area.

Practical x-ray tube



Practical X-ray Tube





Practical x-ray tube

- Metal target, anode, is usually made of Tungsten. Why Tungsten?
 - x-ray spectrum in desired range
 - High Z (high efficiency in stopping electrons)
 - High melting point (3300 deg. C)



Question

- For a x-ray tube connected to 100kV power supply, what is the highest cutoff energy of the x-ray photons emitted?



Question

- To increase the energy level of the x-ray photons, you can
- A) Increase the temperature to the cathode
- B) increase the voltage between the cathode and anode.



Question

- To increase the energy level of the x-ray photons, you can
 - A) Increase the temperature to the cathode
 - **B)** increase the voltage between the cathode and anode – $E = mv$
- Heat up the cathode can increase the amount of x-ray photons emitted.



Questions?

Generation of x-ray



The Attenuation Coefficient

Attenuation Coefficient

- The x-ray spectrum is a function of photon energy E : $I_0 = I_0(E)$
- The attenuation function is also a function of E : $\mu = \mu(x, y, z, E)$.

- The intensity at the output to form image:

$$I_d(x, y) = \int_E I_0(E) \exp\left(-\int \mu(x, y, z, E) dz\right) dE$$

I_d tells us nothing about z or E – it only gives us x, y information.

Attenuation Coefficient

- Two important material properties affecting μ : tissue density ρ and the atomic number Z
 - Because most x-ray photon/tissue interactions are photon/electron interactions.
- Attenuation is caused by x-ray photon interaction with the matter

X-ray Photon Interactions

- **Rayleigh-Thompson Scattering** – spontaneous, very low energy phenomenon
- **Photoelectric Absorption** – low energy phenomenon
- **Compton Scattering** – mid energy phenomenon
- **Pair Production** – high energy phenomenon

- In general, attenuation coefficient constituents:

$$\mu(E) = \mu_{rt}(E) + \mu_{pe}(E) + \mu_{cs}(E) + \mu_{pp}(E)$$



Rayleigh-Thompson Scattering

- Atomic absorption with spontaneous emission at the same energy E .
- Very low energy – a few keV
- Rarely important in the diagnostic energy range (50-200 keV)

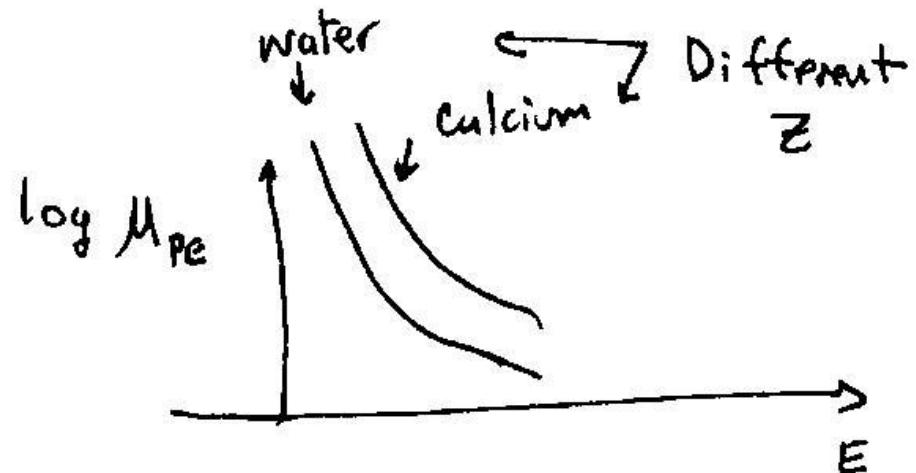
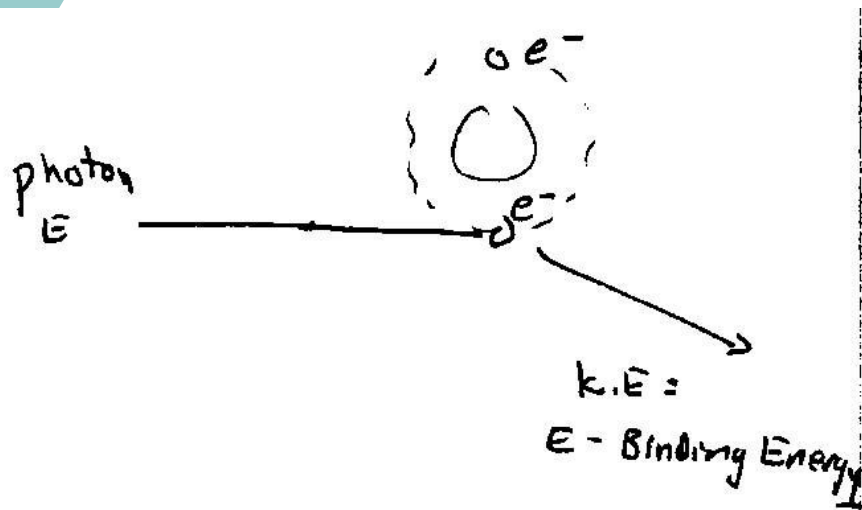


Photoelectric Absorption

- Absorption of photon by interacting with a tightly bound electron
- Leads to ejection of an electron – vacancy filled by an electron falling into it from the next shell
- If the ejected electron kinetic energy (= Absorbed photon energy) is less than binding energy of the electron, the electron is unable to escape the material.

Photoelectric Absorption

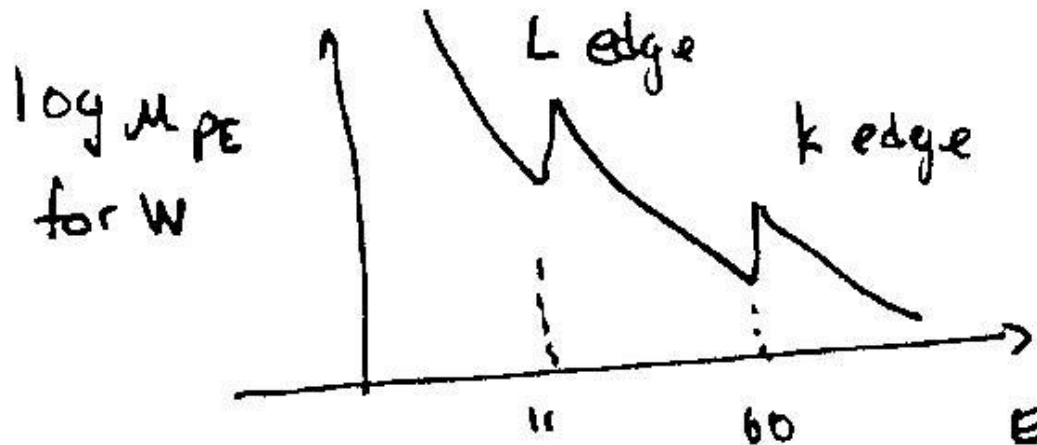
- photoelectric effect increases rapidly with atomic number, Z and with decreasing photon energy .



- Dominates μ in the lower energy part of the diagnostic spectrum (< 50 keV)

Photoelectric Absorption

- For high Z materials (e.g. Lead, Iodine, Tungsten), shell energy boundaries are evident in μ vs. E plots.



- When the energy gets high enough to make that shell's electrons available to the PE effect (when E exceeds the binding energy), then the probability of a PE interaction increases.

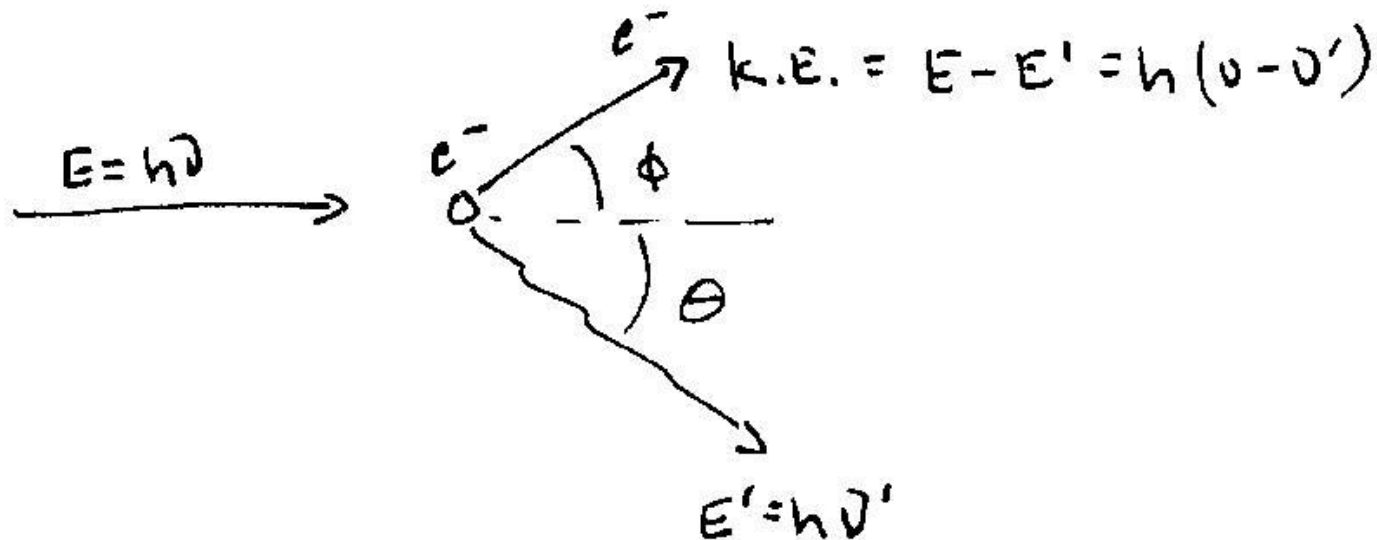


Compton Scattering

- Scattering of photons by an elastic collision with a free electron in the outer shell.
- Part of the x-ray energy is transferred to the electron; the rest taken by the scattered “degraded” photon.
- Elastic collisions preserve E and momentum (p).

Compton Scattering

- For loosely bound electrons or very high energy photons, the equations for free electrons hold reasonably



Compton Scattering

- Conservation of energy:

$$\text{K.E.} = E - E' = (m - m_0)c^2$$

m_0 – mass of the rest electron

c – speed of light

$m = \frac{m_0}{\sqrt{1 - v^2 / c^2}}$ mass of the moving electron

E – photon energy before collision

E' – photon energy after collision

Compton Scattering

- Conservation of momentum in x and y directions:

$$\frac{E}{c} = \frac{E'}{c} \cos \theta + mv \cos \phi$$

$$\frac{E'}{c} \sin \theta = mv \sin \phi$$

Compton Scattering

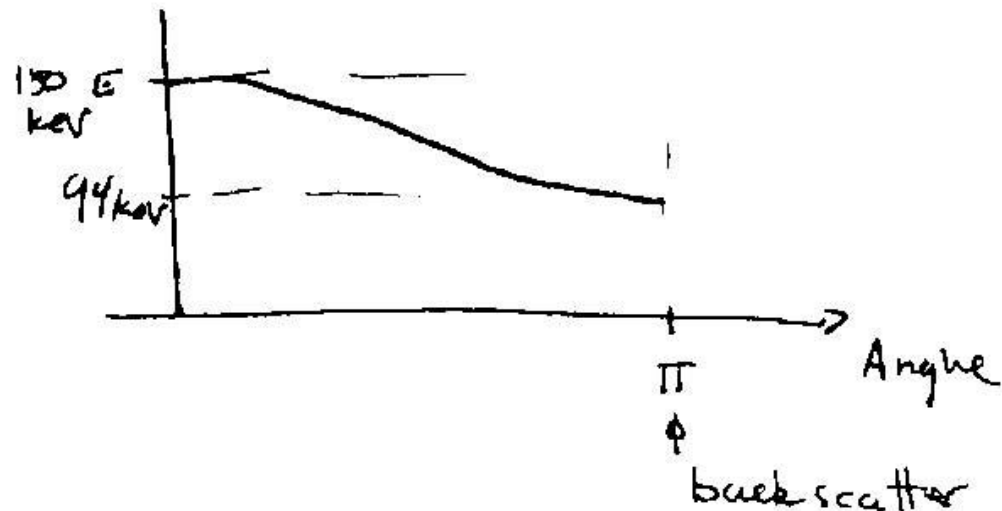
- solving these equations → energy of the scattered photon:

$$E' = \frac{E}{1 + \frac{E}{E_e} (1 - \cos \theta)}$$

where $E_e = m_0 c^2 = 511 \text{ keV}$, the rest energy of an electron.

Compton Scattering

- For $E \ll E_e$, there is very little change in energy with angle.
- For higher E :





Compton Scattering

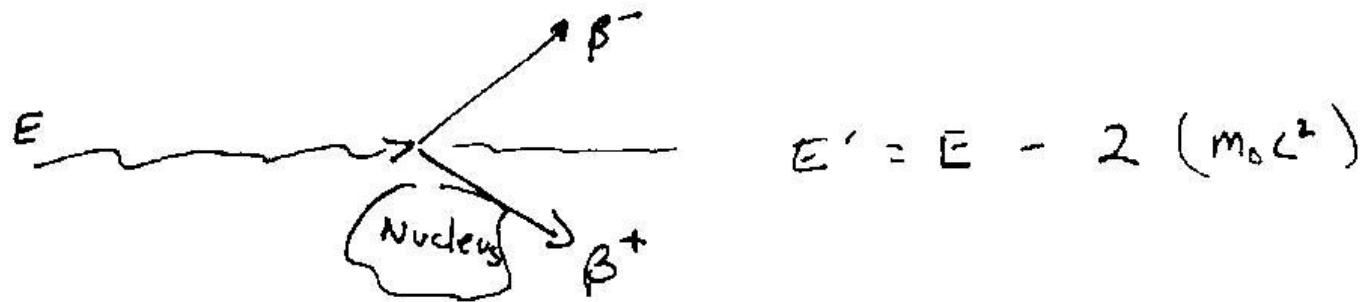
- For low E , scatter is essentially isotropic in angle
- For higher E , scatter is preferentially forward scattered (little change in photon E).
- Very hard to discriminate between forward scattered photons and unimpeded photons based on energy.

Compton Scattering

- μ_{cs} is nearly constant across diagnostic spectrum
- Compton scatter comes mostly from atomic electrons (μ_{cs} is proportional to ρ)
- At higher E , Compton scatter dominates over the PE effect.

Pair Production

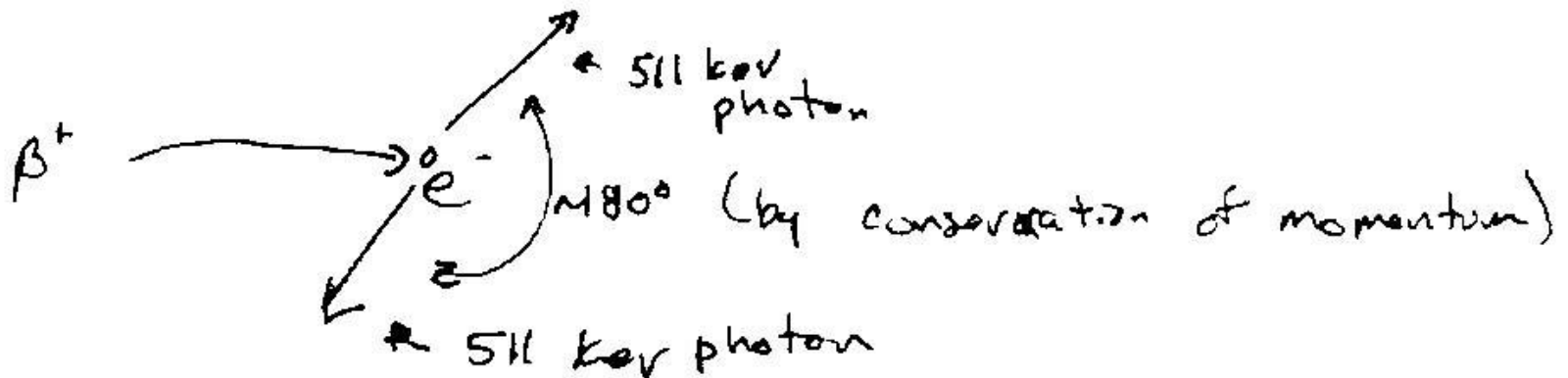
- Spontaneous creation of an electron/positron pair



- Photon energy is transferred to mass energy in the electron and positron.

Pair Production

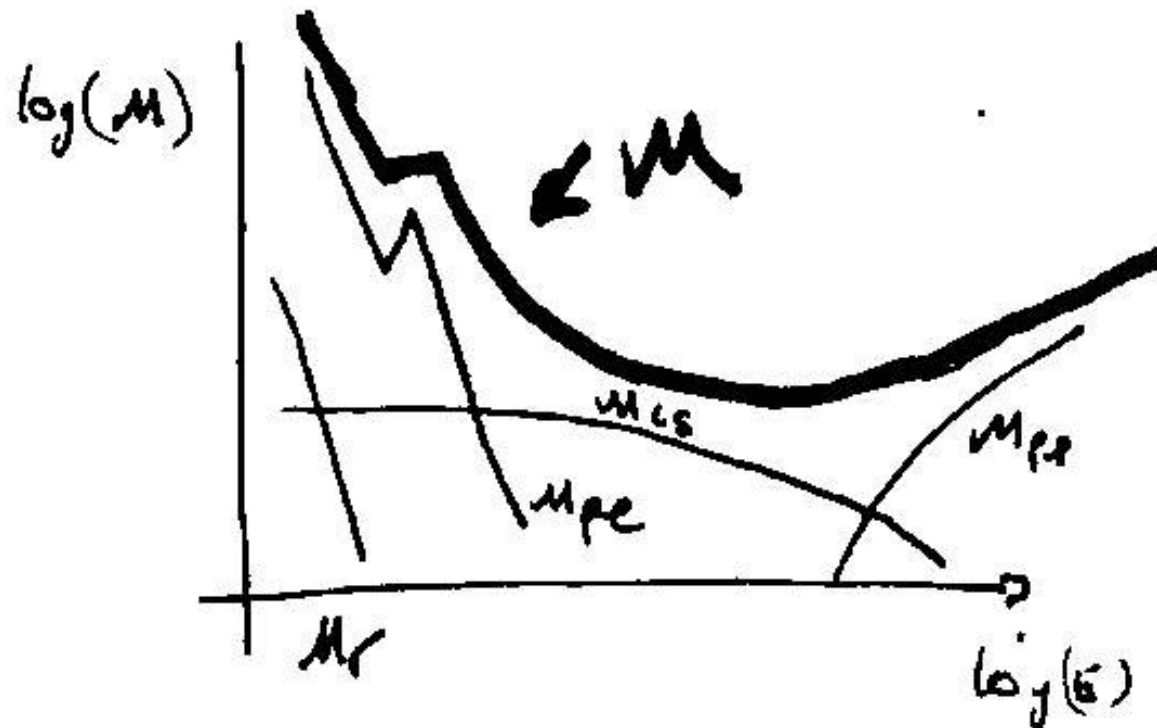
- Only occur for x-ray photons $> 1022 \text{ keV}$ (not in the diagnostic spectrum)
 - Since the rest energy of electron and positron is 511 keV



Total Linear Attenuation Coefficient for Photons

- The combined coefficient

$$\mu(E) = \mu_{rt}(E) + \mu_{pe}(E) + \mu_{cs}(E) + \mu_{pp}(E)$$



Total Linear Attenuation Coefficient for Photons

- An alternate to linear attenuation coefficient is the “mass attenuation coefficient” $\tau = \mu / \rho$
(units: cm²/gm)
- convenient when describing the behavior of composite materials with N constituent components:

$$\tau = \frac{1}{M} \sum_{i=1}^N m_i \tau_i$$

m_i are the masses of the components and M is the total mass.

X-ray Attenuation Coefficients from Muscle, Fat and bone

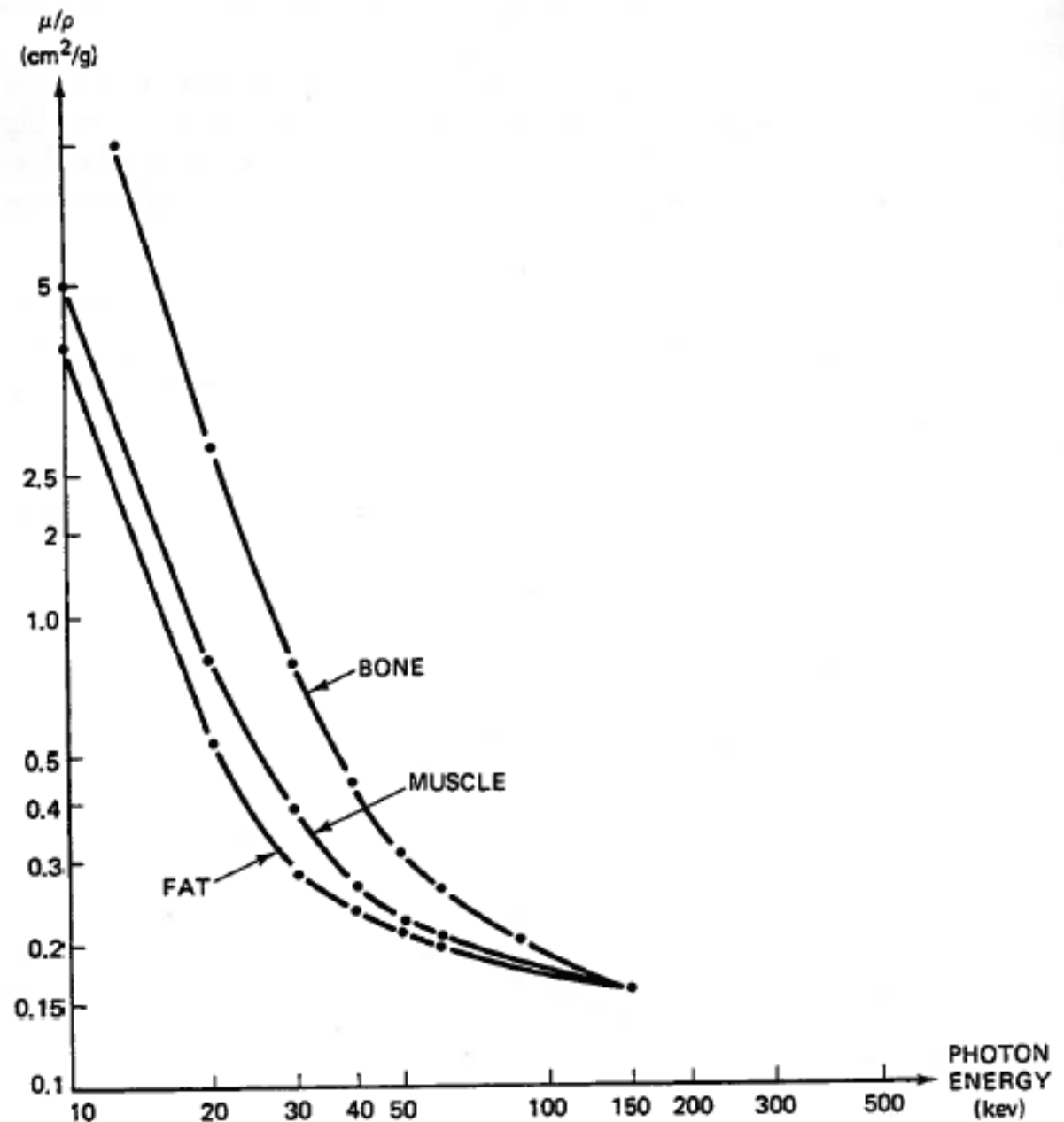


FIG. 3.7 X-ray attenuation coefficients for muscle, fat, and bone, as a function of photon energy.



True or False

- Attenuation of clinical x-ray imaging primary come from Compton scattering and photoelectric absorption.



Compton Scattering

- Which of the following X-ray photon property decreases as it goes through through Compton Scattering.
 - A) Energy
 - B) Speed
 - C) Frequency
 - D) Wavelength



Compton Scattering

- Which of the following X-ray photon property decreases as it goes through through Compton Scattering.
 - **A)** Energy
 - B) Speed
 - **C)** Frequency
 - D) Wavelength

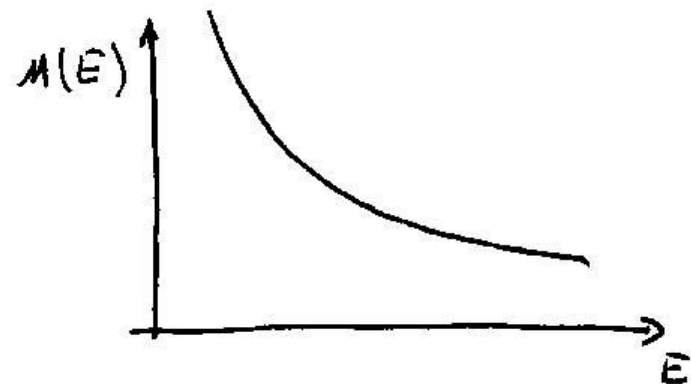


Questions?

Attenuation Coefficient
X-ray Photon interaction
Total Linear Attenuation
Coefficient for Photons

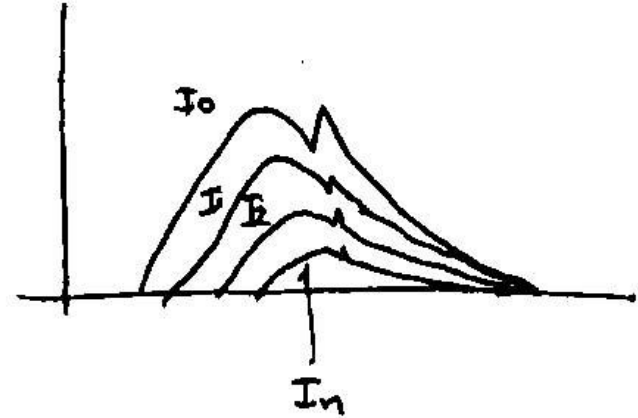
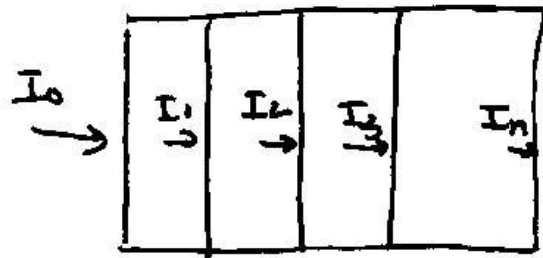
Beam Hardening

- Because the attenuation spectrum is not uniform across the diagnostic energy spectrum, the output spectrum will have a different intensity distribution than the input spectrum, $I_0(E)$.



Beam Hardening

- If we split an object into several smaller parts, and look at then energy spectrum at for each part:



Beam Hardening

- The mean energy:

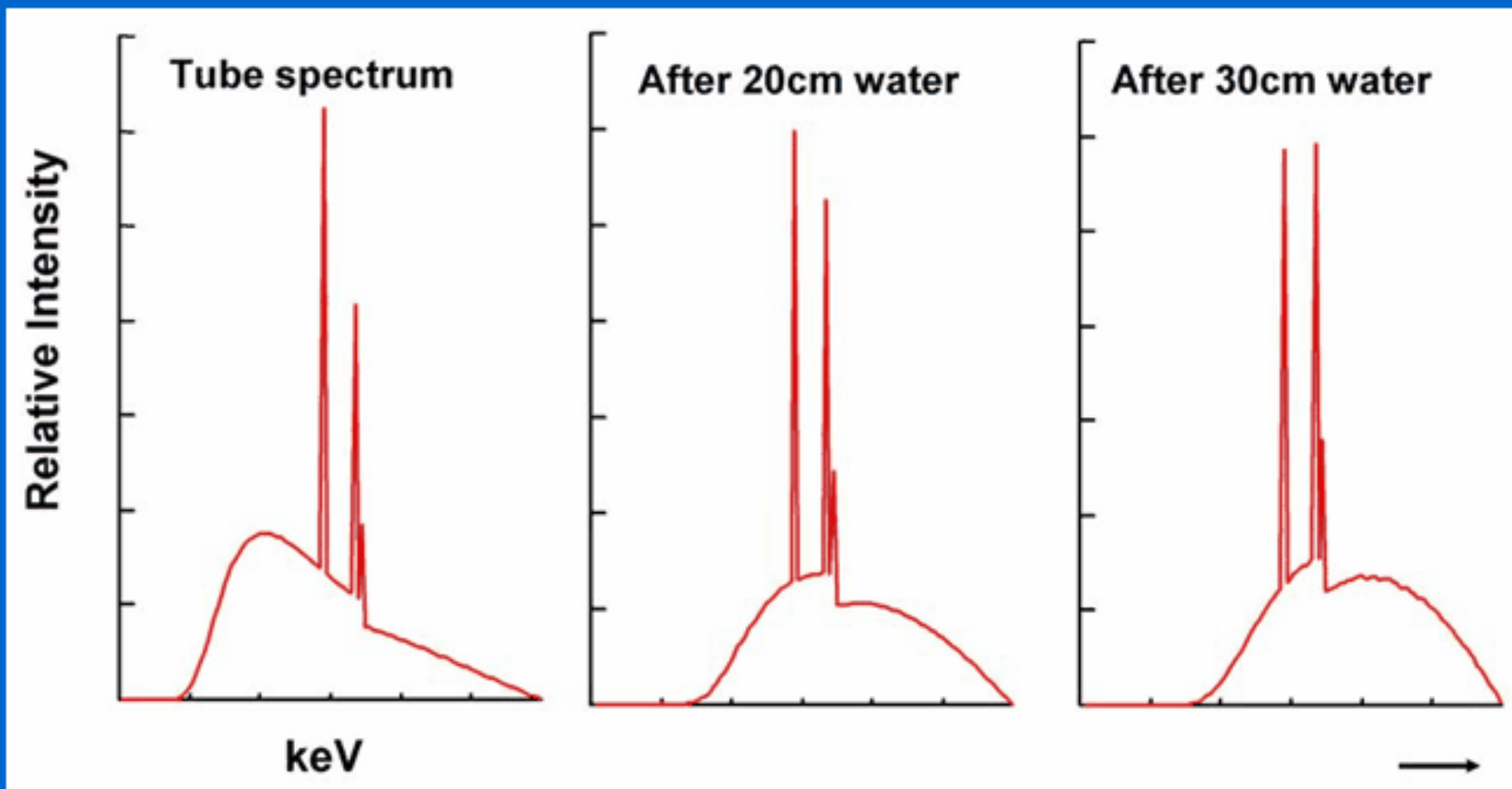
$$\bar{E} = \frac{\int EI(E)dE}{\int I(E)dE}$$

- will increase (get harder) as we move through the object:

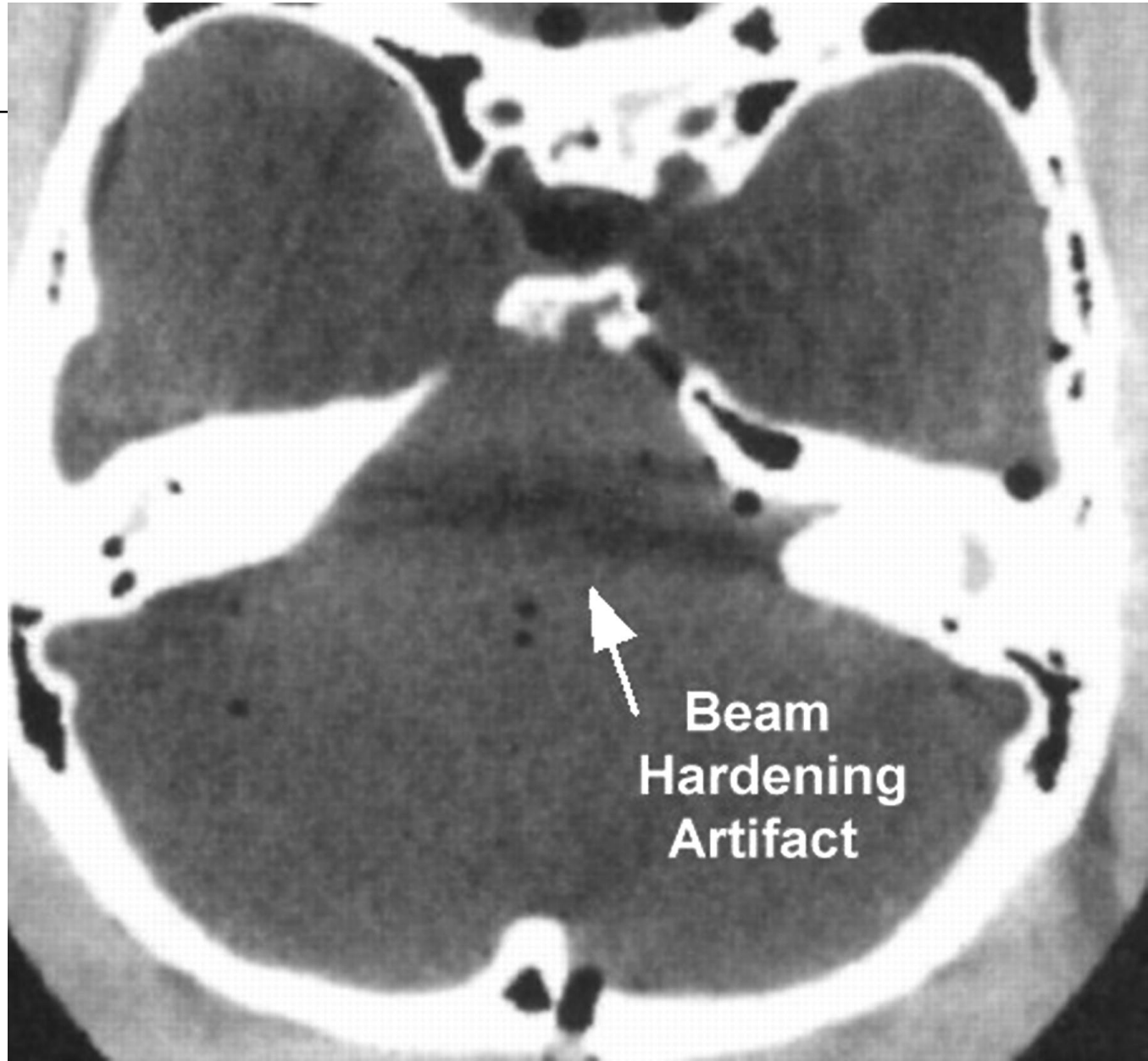
$$\bar{E}_0 < \bar{E}_1 < \bar{E}_2 < \dots < \bar{E}_n$$

Beam hardening

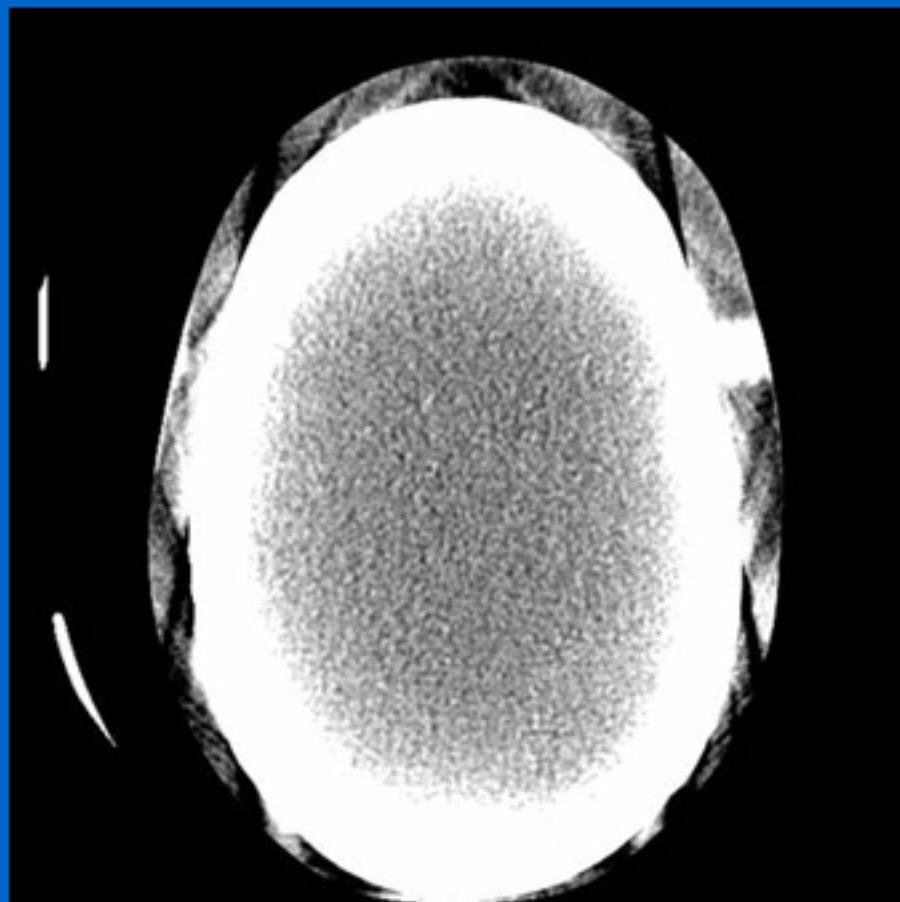
As an x-ray beam passes through an object, its mean energy increases



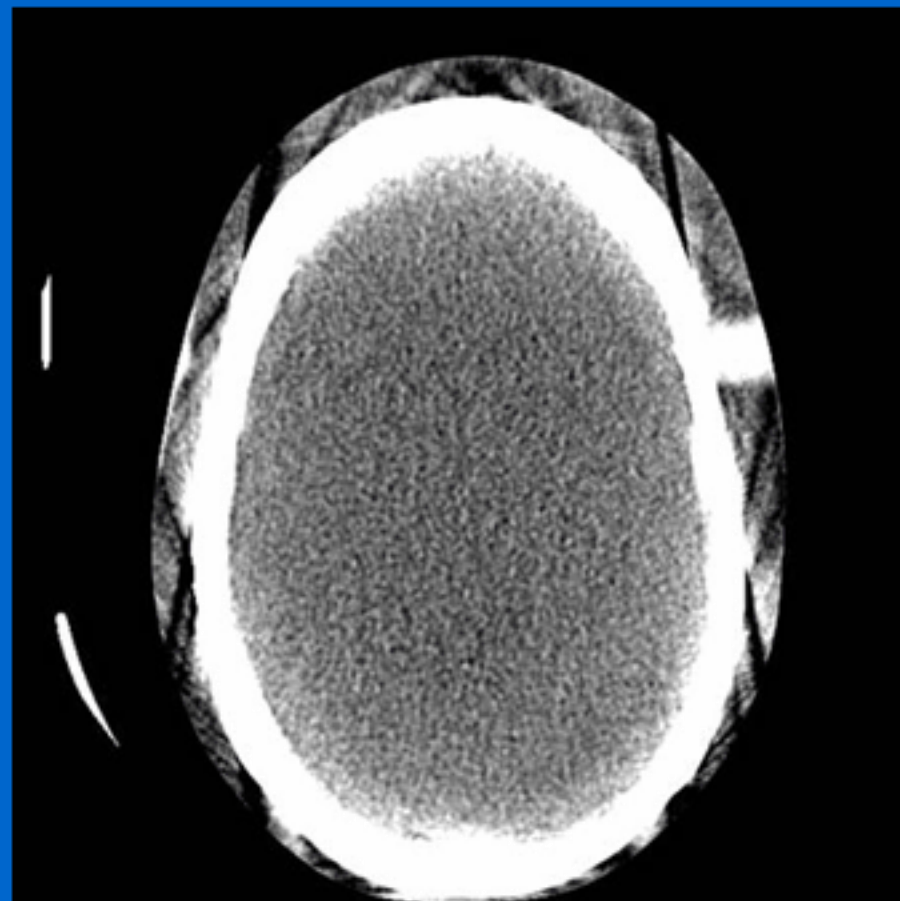
Beam Hardening Artifact



Beam hardening correction software



Raw image



After beam hardening
correction



True or False

- As the X-ray photons move deeper into the object, the apparent attenuation coefficient will increase.

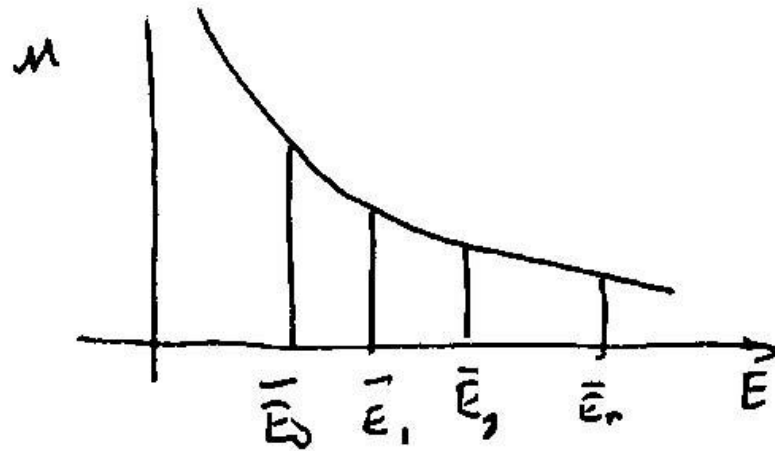


True or False

- As the X-ray photons move deeper into the object, the apparent attenuation coefficient will increase.
- False – As the X-ray photons move deeper, less lower energy photons (with higher attenuation coefficient).

Beam Hardening

- A particular tissue type will have a μ that changes as a function of position along the path.



- As we move deeper into the object, there is less attenuation than expected, given the initial spectrum, $I_0(E)$.



Beam Hardening

- One solution is to make the beam “hard” to begin with - often accomplished by filtering out the low E photons with a thin metal plate (often use aluminum).



Questions?

Beam Hardening?