Magnetic Resonance Imaging (MRI)

BME/EECS 516
Lecture #6 – 3rd Dim, Noise
Notes on MRI, Part 3

Announcements

- US Project Due today.
- HW #4 due Tuesday, 10/31
- HW #5 due Tuesday, 10/31
- MRI Project to be assigned next week

Last Lecture

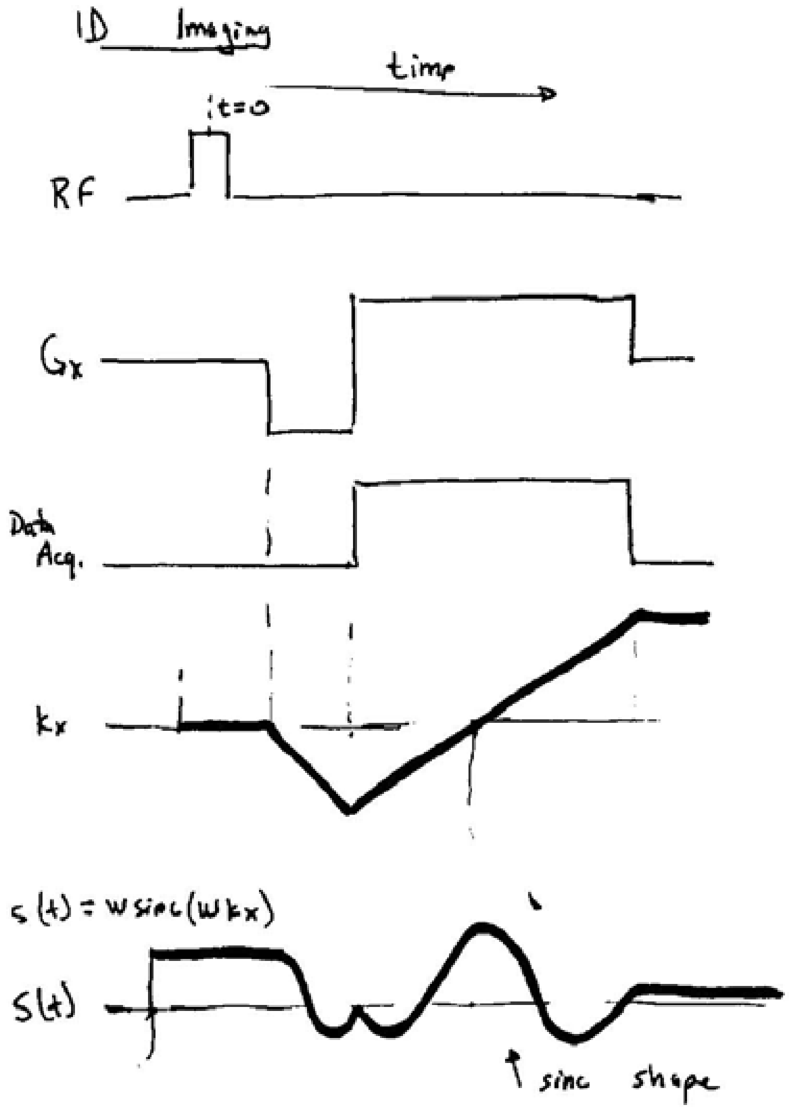
- 1D Imaging
- 2D Imaging using projections
- 2D spin-warp imaging
  - Sampling in K-space
  - Point Spread Function
  - Resolution of FFT
  - Resolution, Object, Sampling in MRI

1D Imaging

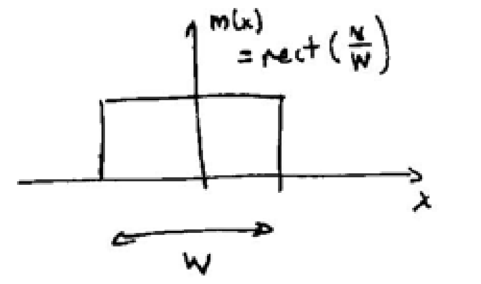
pulse sequence

k-space values

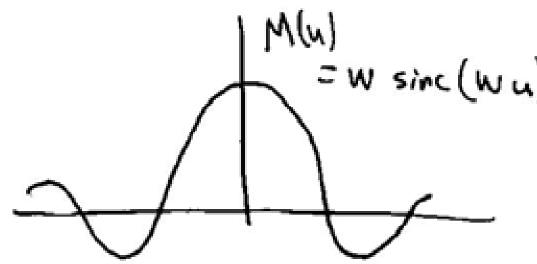
received signal



Object

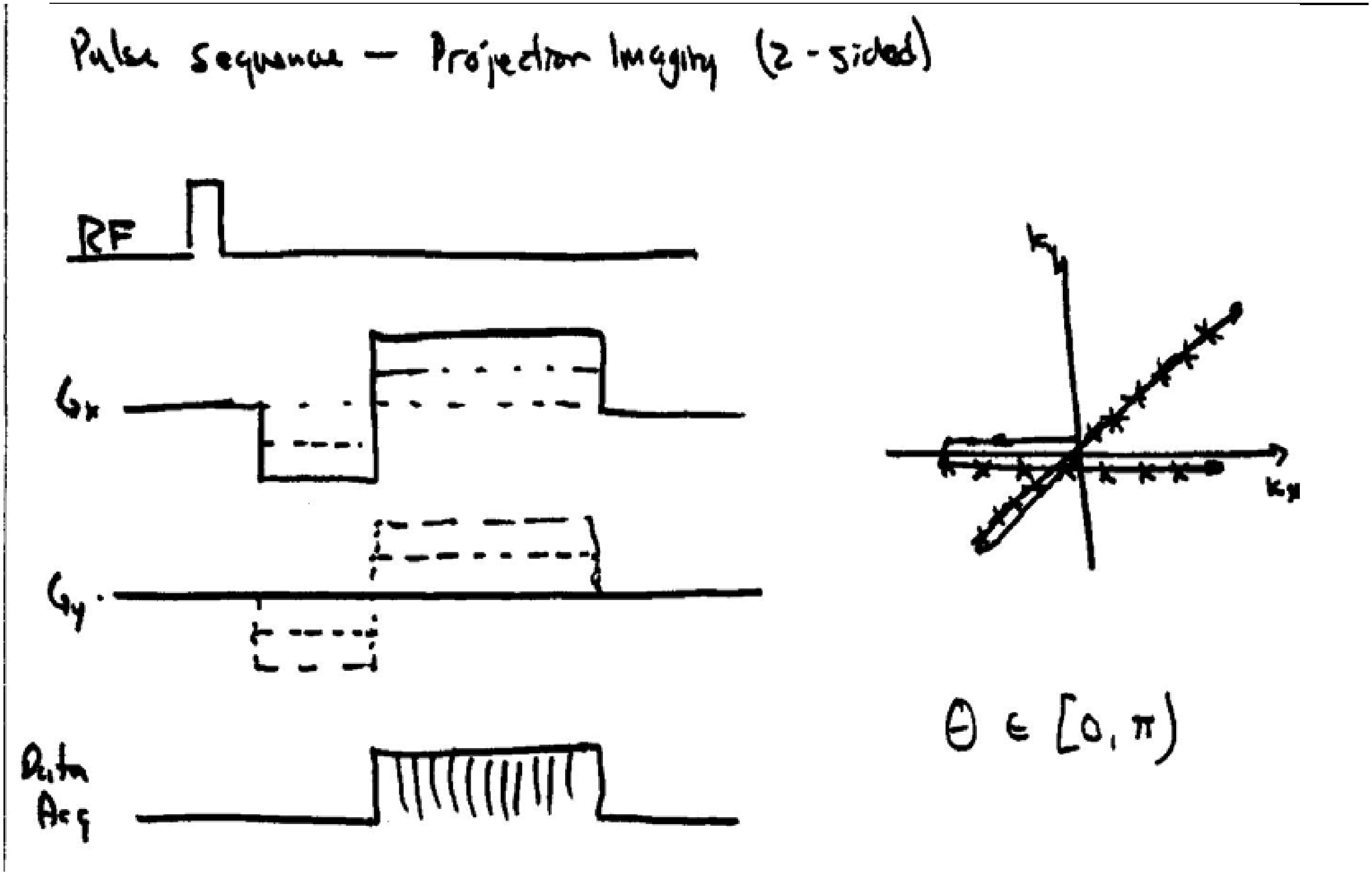


FT of Object

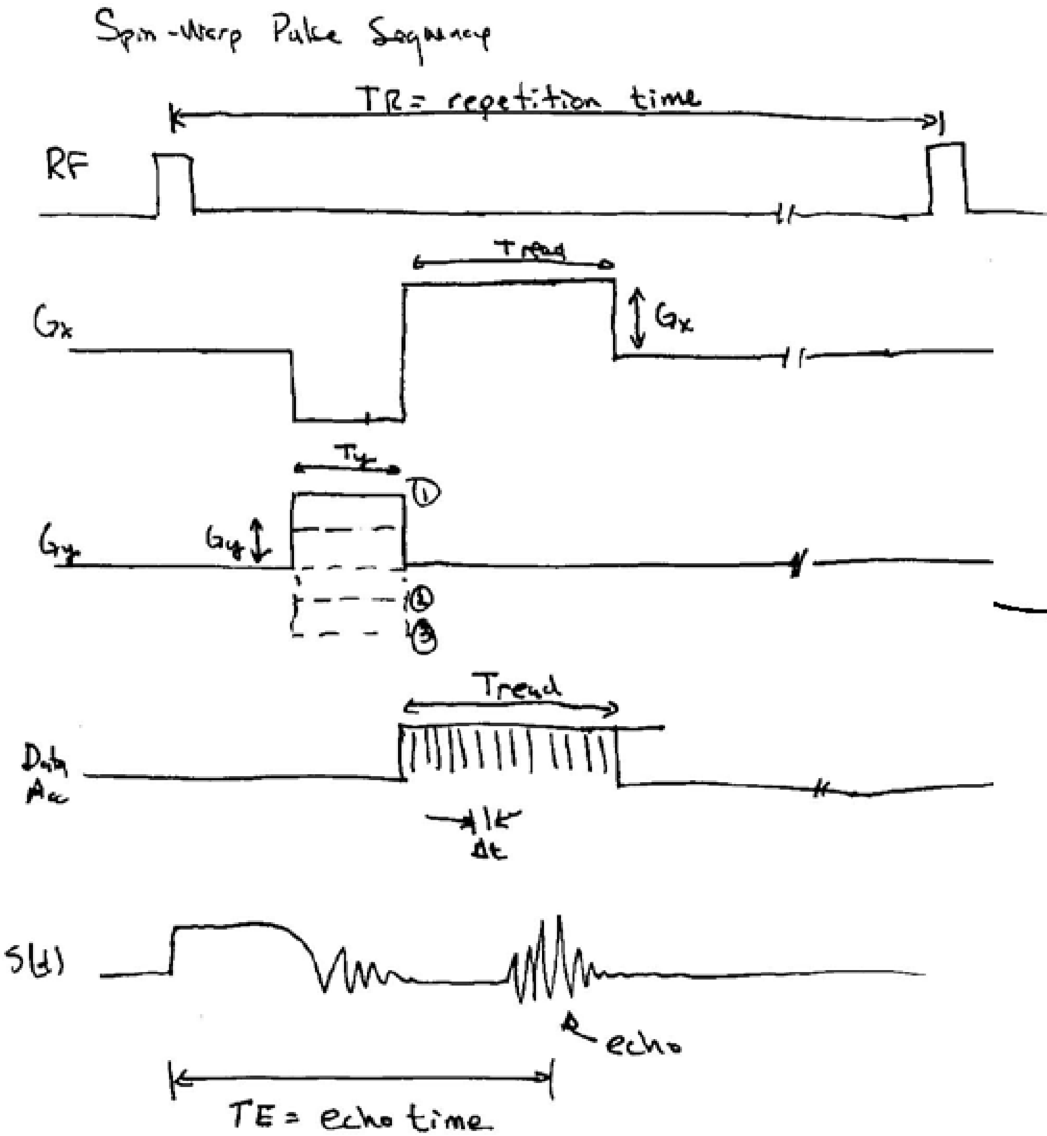


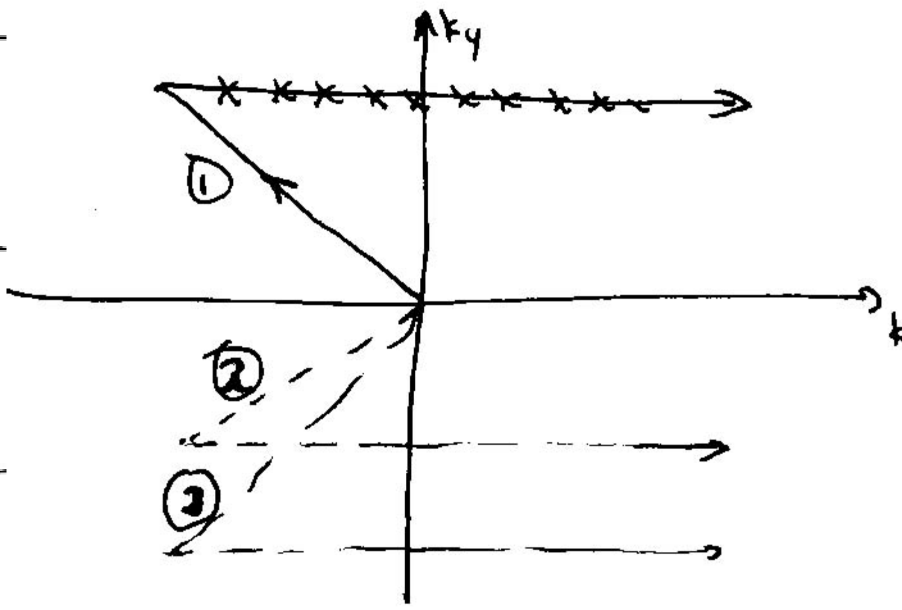
$$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)\,d\tau$$

2D Imaging using Projections



2D Spin-Warp Imaging Pulse Sequence





2D Spin-Warp Imaging - Resolution and Object and Sample Spacing in MRI

$$\mathrm{FOV}_x = 1/\Delta k_x \qquad \mathrm{FOV}_y = 1/\Delta k_y$$

$$\Delta x = 1/W_{kx} \qquad \Delta y = 1/W_{ky}$$

$$\Delta k_x = \frac{\gamma}{2\pi} G_x \Delta t \qquad W_{kx} = N_x \Delta k_x = \frac{\gamma}{2\pi} G_x T_{read}$$

$$\Delta k_y = \frac{\gamma}{2\pi} \Delta G_y T_y \qquad W_{ky} = N_y \Delta k_y = \frac{\gamma}{2\pi} 2 G_{y,\max} T_y$$

Question

- To achieve a better spatial resolution for a MR image in the x direction, you can
  - A) Increase the duration of Gx (Tread)
  - B) Increase the duration of Gy (Ty)
  - C) Increase the number of samples while maintaining time between samples (Δt)
  - D) Increase Gx amplitude

Question

- To increase of the field of view in y direction, you can
- A) Decrease the time between samples (Δt)
- B) Decrease the number of samples while maintaining time between samples (Δt)
- C) Decrease the duration of G_y (T_y)
- D) Decrease ΔG_y

This Lecture

- The 3rd Dimension – Z
  - Extend 2D Spin-Warp Imaging to 3D
- Slice Selective Excitation - common approach for dealing with the 3rd (z) dimension
  - Small Tip Angle Approximation
  - Putting Slice Selection with the Signal Equation
  - Larger Flip Angles
  - Multi-slice Imaging

The 3rd Dimension - Z

- Recall the MR signal is equal to the Fourier transform of the initial magnetization evaluated at locations defined by k_x and k_y.

$$s(t) = \iint m(x,y) \exp\left(-i2\pi\left(xk_x(t) + yk_y(t)\right)\right) dxdy$$

$$= F_{2D}\{m(x,y)\}\big|_{u=k_x(t),v=k_y(t)} = M\left(k_x(t), k_y(t)\right)$$

$$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)d\tau, \quad k_y(t) = \frac{\gamma}{2\pi}\int_0^t G_y(\tau)d\tau$$

The 3rd Dimension - Z

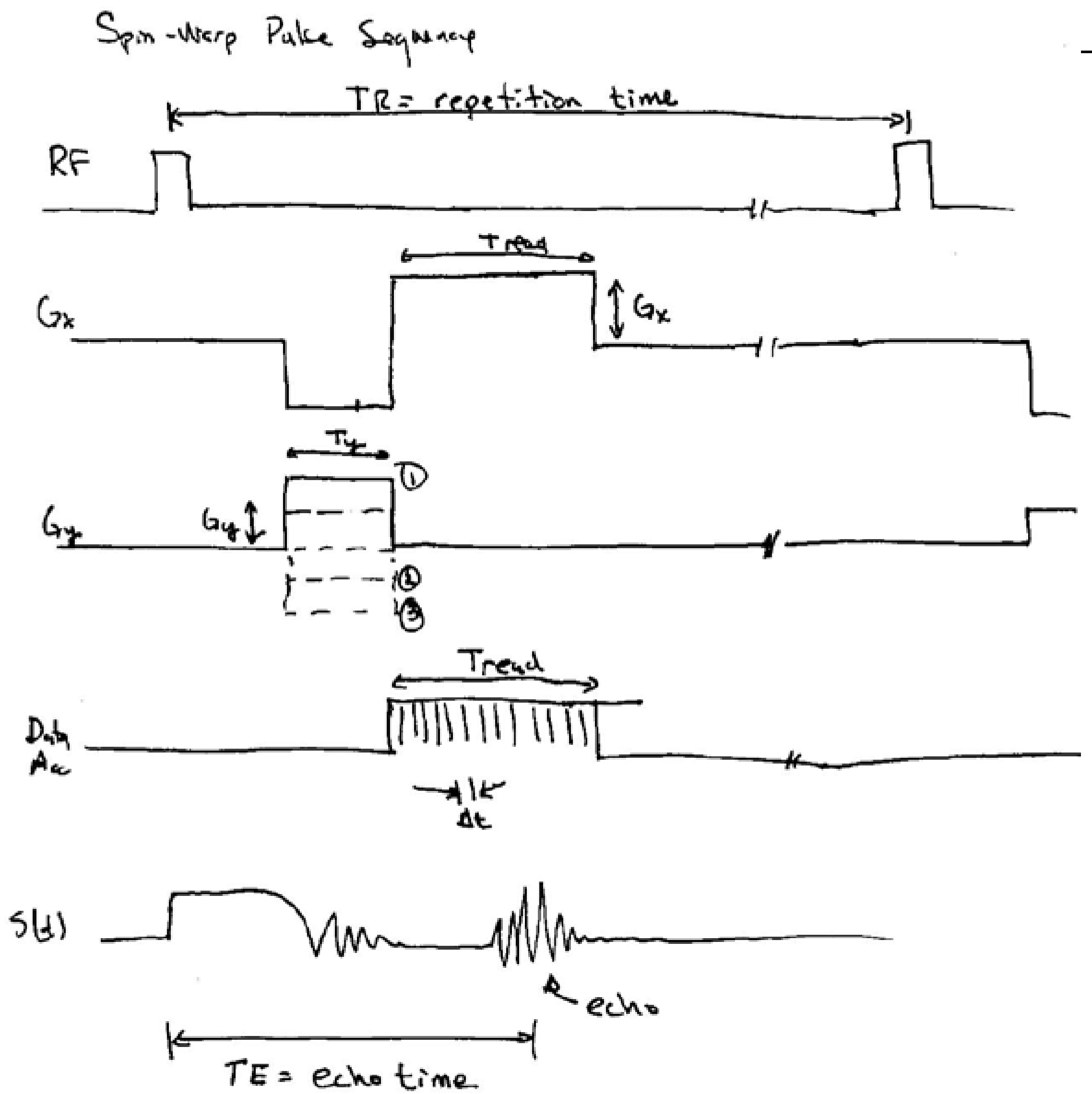
- The 3D signal equation can be written as:

$$s(t) = \iiint m(x,y,z)\exp(-i2\pi(k_x(t)x + k_y(t)y + k_z(t)z)dxdydz$$

$$= M(u,v,w)\,\Big|_{u=k_x(t),v=k_y(t),w=k_z(t),}$$

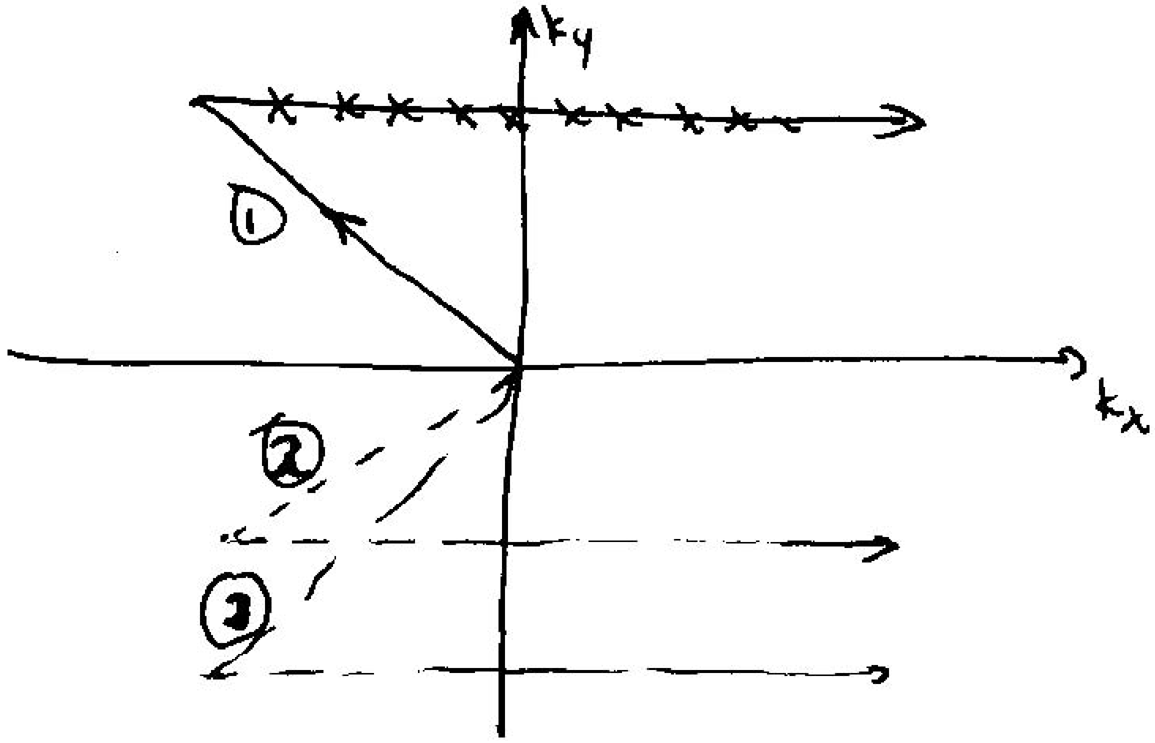
$M(u,v,w)$ is the 3D FT of $m(x,y,z)$

Recall 2D Spin-Warp Imaging Pulse Sequence

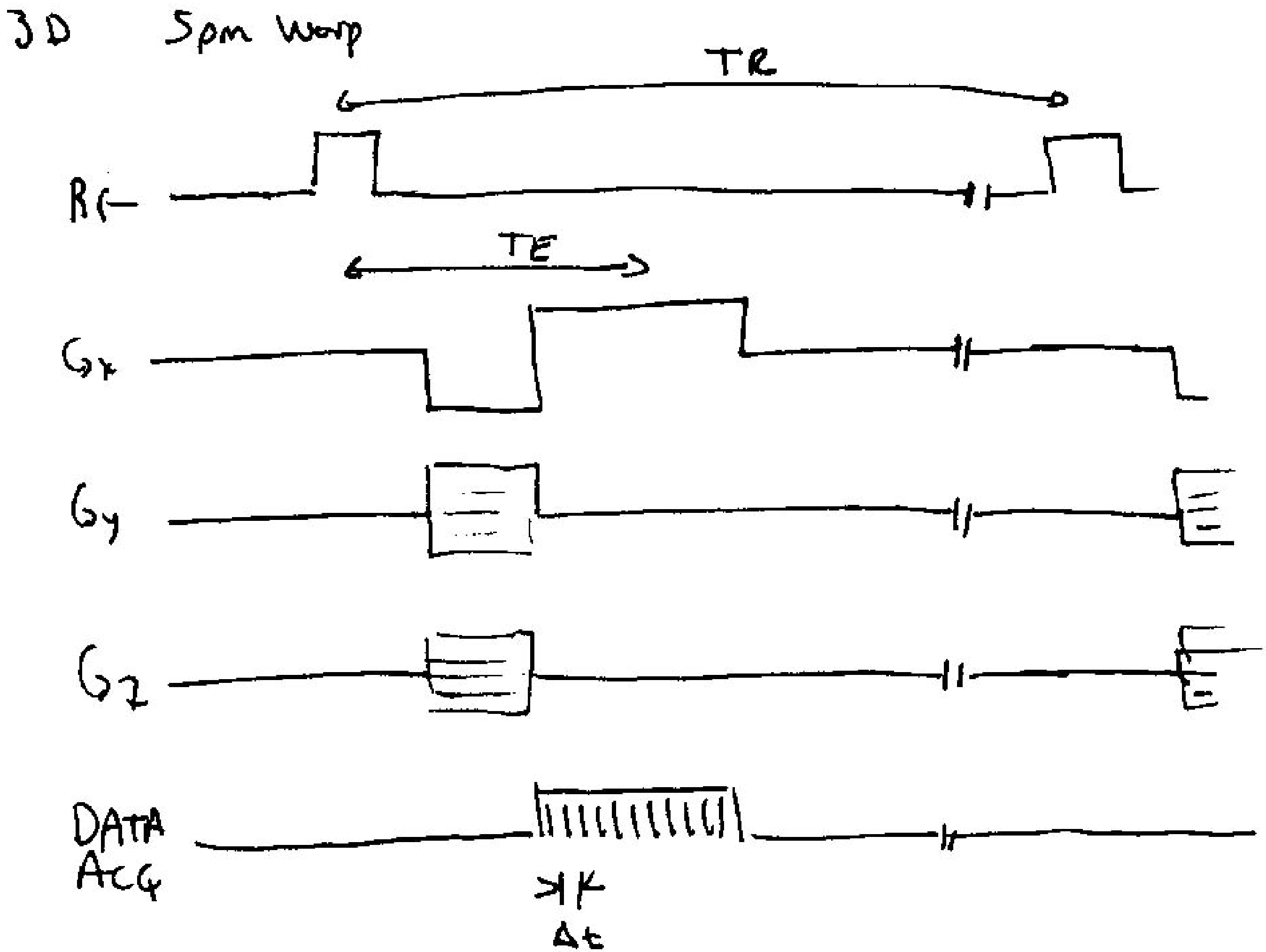


Recall 2D Spin-Warp Imaging

- One line at a time is acquired in the 2D Fourier domain (or k-space).

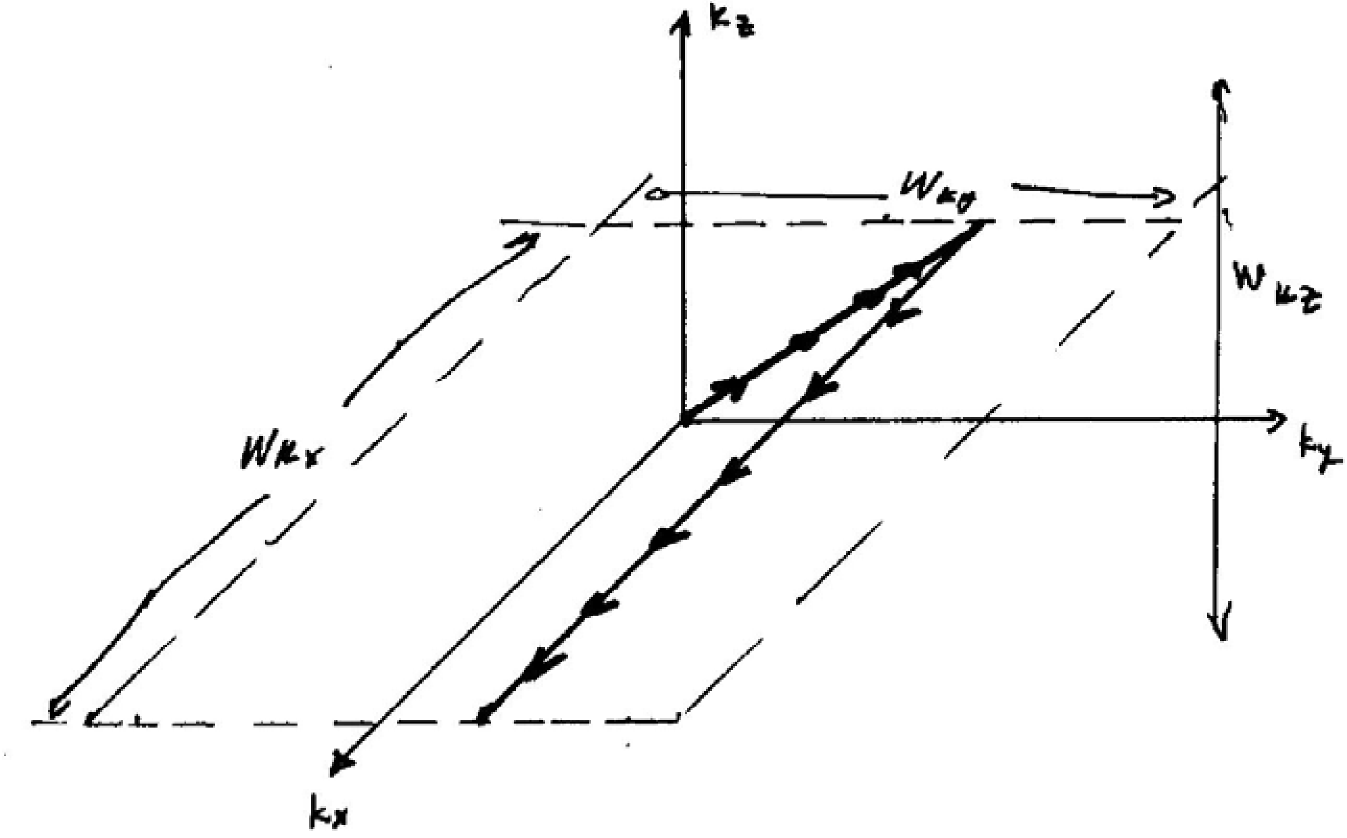


3D Spin-Warp Imaging



3D Spin-Warp Imaging

Acquire a cubic data set one line at a time:



The 3rd Dimension - Z

- In 2D field:

  $FOV_x = 1/\Delta k_x$ $\quad\quad$ $FOV_y = 1/\Delta k_y$

  $\Delta x = 1/W_{kx}$ $\quad\quad$ $\Delta y = 1/W_{ky}$

- Add the Z dimension:

  $FOV_z = 1/\Delta k_z$ $\quad\quad$ $\Delta z = 1/W_{kz}$

The 3rd Dimension - Z

- Samples in the y direction: N_y
  Samples in the z direction: N_z
  Total time to acquire 3D volume: $N_y * N_z * TR$

- For example, $N_y = N_z = 128$ and TR = 33 ms, Image acquisition time = 128 x 128 x 33 min = 9 min - Too long!

Question

- Can we use a short TR (<1ms) to reduce the image acquisition time and obtain a high frame rate?

Question

- Can we use a short TR (<1ms) to reduce the image acquisition time and obtain a high frame rate?
- Not easily...
  - Too small a TR results in low signal intensity - recall signal intensity
  - Limitation on how squeezing too many things in a very short time (RF pulses, gradients, etc.) - bioeffects
  - Poor SNR (more on this later) from short readouts

Question

- Δt of several μs is typically used in MRI acquisition. Would using a short Δt (sub-μs) improve the imaging?

Question

- Δt of several μs is typically used in MRI acquisition. Would using a short Δt (sub-μs) improve the imaging?
- No.
  - Main issues is reduced signal to noise ratio (more on this later)
  - Field of view is very large, but this isn't necessarily a problem

Questions?

3D Spin-Warp Imaging

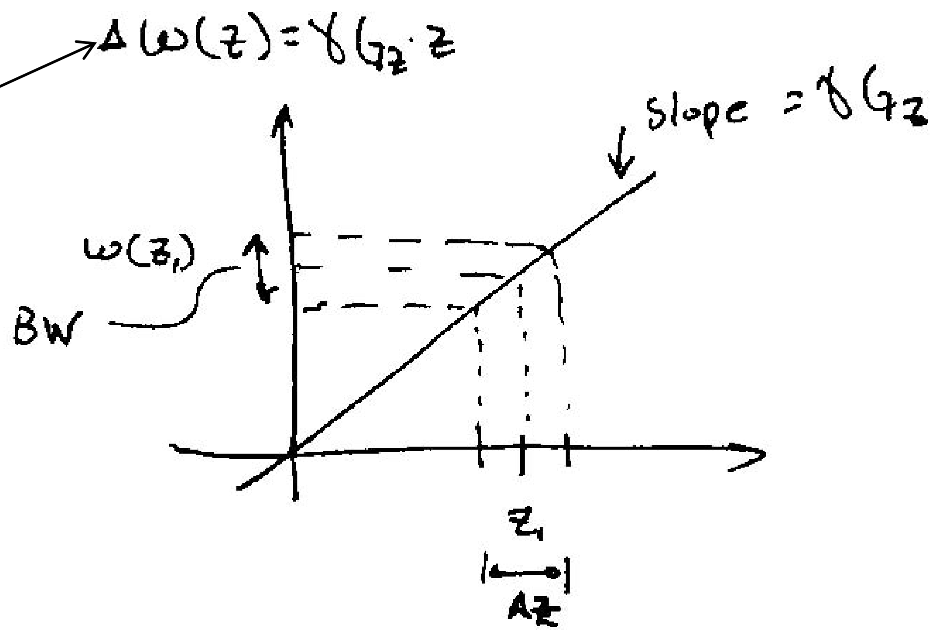
Slice Selective Excitation

The most common approach for dealing with the 3rd (z) dimension

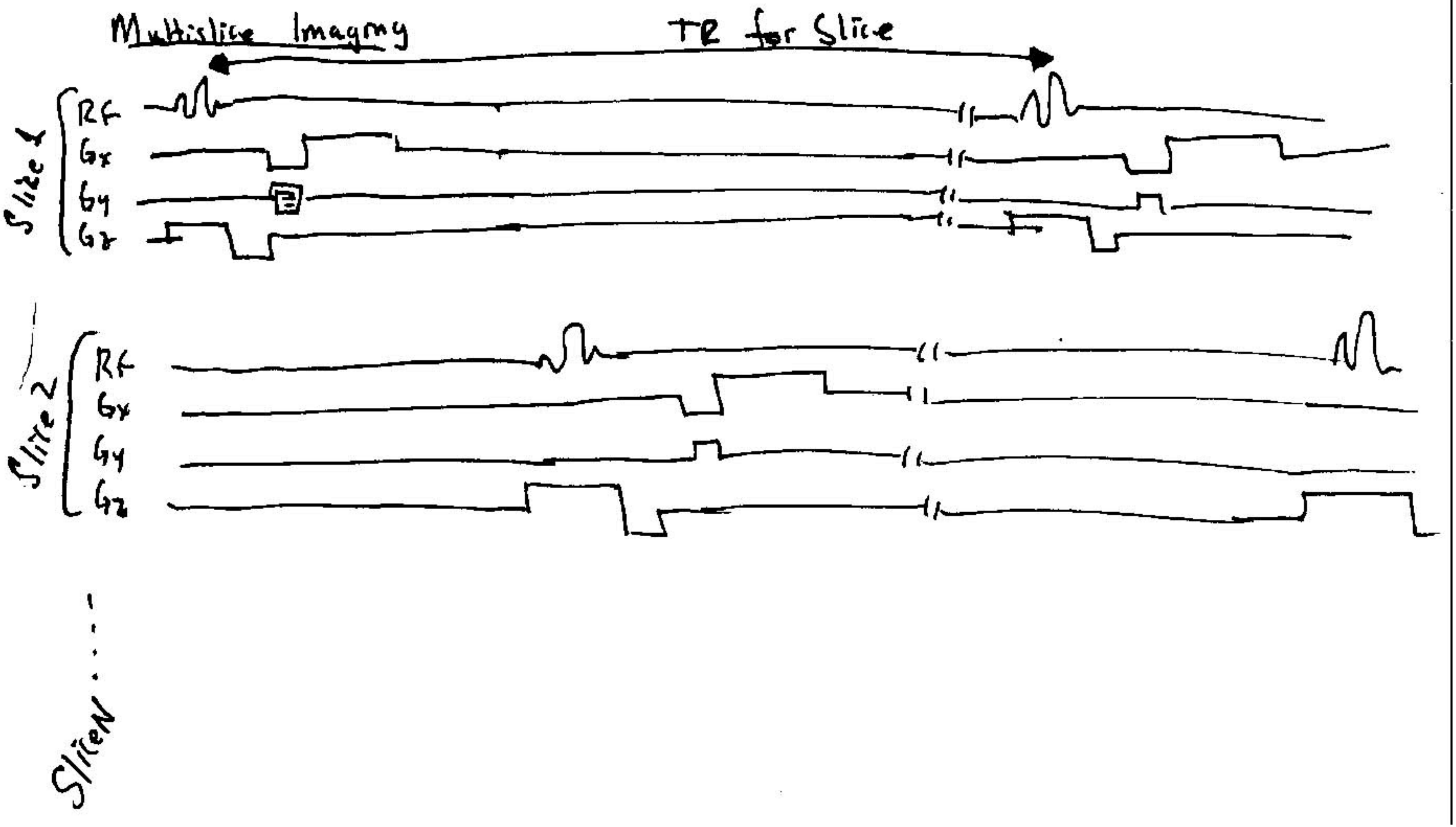
Slice Selective Excitation

- Apply a z-gradient so that the resonance frequency varies in the z-direction
- Apply a bandpass RF pulse to excite only the those spins whose resonant frequency lies within the band:

In addition to ω_0 from B_0 at z direction



Slice Selective Excitation



Slice Selective Excitation

- Step 1: Apply a z-gradient so that the resonance frequency varies in the z-direction. Within a slice of thickness Δz:

$$B_z = B_0 + G_z \cdot z$$

$$\omega(z) = \gamma(B_0 + G_z \cdot z)$$

$$\omega_{eff}(z) = \gamma G_z \cdot z$$

Slice Selective Excitation

- Step 2: Apply a bandpass RF pulse (B_1) with bandwidth (BW in Hz)

- RF pulse excite only the those spins whose resonant frequency lies within the band.
  - $[\omega_{RF} - BW * 2\pi/2, \omega_{RF} + BW * 2\pi/2] = [\gamma(B_0 + Gzz_1), \gamma(B_0 + Gzz_2)]$ to excite spins only within slice $[z_1, z_2]$

Slice Selective Excitation

- How to determine G_z for a slice thickness = Δz?
  - Bandwidth of the RF pulse – BW
  - Bandwidth of the nuclei spin frequency in the magnetic field – $G_z \Delta z \frac{\gamma}{2\pi}$

- $$G_z = \frac{BW}{\left(\frac{\gamma}{2\pi}\right)\Delta z}$$

True of False

- When exciting different locations/slices, BW and G_z remains the same using the same slice thickness (Δz).

- When exciting different locations/slices, different RF excitation pulses with different center frequencies will be used.

Slice Selective Excitation

- When exciting different locations/slices, BW and G_z remains the same using the same slice thickness (Δz).

$$G_z = \frac{BW}{\left(\frac{\gamma}{2\pi}\right)\Delta z}$$

- ω_{RF} is changed for each slice, depending on the center location of each slice.

$$\omega_{RF} = \omega_0 + \gamma G_z z_{slice} = \gamma B_0 + \gamma G_z z_{slice}$$

Question

- How does RF pulse look like in the frequency domain for a rectangular slice profile?
  - A) $\mathrm{rect}((f-f_{RF})/BW)$
  - B) $\mathrm{sinc}((f-f_{RF})/BW)$
  - C) $\exp(-\pi(f-f_{RF})^2/BW^2)$
  - D) $\delta(f-f_0)$

Slice Selective Excitation

- How does RF pulse look like in the time domain?
  - Recall: $F^{-1}\{G(f - f_0)\} = g(t)e^{i2\pi f_0 t}$

- Example : $B1(f) = \mathrm{rect}\left(\frac{f-f_{RF}}{BW}\right)$

$$B_1(t) = BW\ \mathrm{sinc}(t \cdot BW)e^{i\gamma G_z z_{slice} t}$$

Typical MRI values

- Typical RF pulse – on the order of kW power and 5-10 ms in length.
- Typical gradient amplitude – 4 Gauss/cm = 40 mT/m.
- Typical Resolution – 1 mm.

Questions?

Slice Selective Excitation –
Bloch equation

Small Tip Angle Approximation

Solve Bloch Equation for
Slice Selective Excitation

Slice Selective Excitation

- Solve the Bloch equations for this specific case, Let
  - B_1- time-varying magnetic field rotating at ω_0
  - Rotating frame be at $\omega_{frame} = \omega_0$

$$\mathbf{B_1}(t) = B_1(t)(\cos W_0 t\mathbf{i} + \sin W_0 t\mathbf{j})$$

$$\mathbf{B_{1,\mathit{eff}}}(t) = B_1(t)\mathbf{i}'$$

- Recall B_1 is in x-y plane

Slice Selective Excitation

- A z-gradient is applied, so the component in the z-direction is:

$$\mathbf{B}_{\mathbf{z}}(z) = (B_0 + G_z \times z)\mathbf{k}$$

$$\mathbf{B}_{\mathbf{z}.,eff}(z) = (G_z \times z)\mathbf{k}$$

- The net effective applied field is:

$$\mathbf{B}_{eff} = B_1(t)\mathbf{i}' + (G_z \times z)\mathbf{k}$$

Slice Selective Excitation

- Bloch equation for this case reduces to

$$\frac{d\mathbf{M}_{\text{rot}}}{dt} = \mathbf{M}_{\text{rot}} \times \gamma \mathbf{B}_{\text{eff}} = \begin{bmatrix} 0 & \gamma G_z z & 0 \\ -\gamma G_z z & 0 & \gamma B_1(t) \\ 0 & -\gamma B_1(t) & 0 \end{bmatrix} \mathbf{M}_{\text{rot}}$$

- What we would like to know is how $\mathbf{M}_{\text{rot}}$, varies vs. z following the application of B_1. This is a very difficult equation to solve because it is non-linear.

Small Tip Angle Approximation

- To solve the above Bloch equation, use small tip angle approximation. Assume the B_1 produces a small net rotation angle:

$$\int \gamma B_1(t)\,dt < \frac{\pi}{6} \quad (30^\circ)$$

- In this case, we can assume $m_z \approx m_0$ during the RF pulse. Essentially, we are saying that:

$$\cos\left(\int_0^t \gamma B_1(t)\,dt\right) \approx 1$$

Small Tip Angle Approximation

- Under this assumption, $\frac{dm_{z(t)}}{dt} = 0 \rightarrow m_z(t) = m_0$.

- The above Bloch equation can then be rewritten as:

$$\frac{d\mathbf{M}_{\mathbf{rot}}}{dt} = \begin{bmatrix} 0 & \gamma G_z z & 0 \\ -\gamma G_z z & 0 & \gamma B_1(t) \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} m_{x,rot} \\ m_{y,rot} \\ m_0 \end{bmatrix}$$

Small Tip Angle Approximation

- Now solve for

$$m_{xy,rot}(z,t) = m_{x,rot}(z,t) + i m_{y,rot}(z,t).$$

- Write a differential equation using for the transverse component:

$$\frac{dm_{xy,rot}}{dt} = \frac{dm_{x,rot}}{dt} + i\frac{dm_{y,rot}}{dt}$$

$$= \gamma G_z z m_{y,rot} - i\gamma G_z z m_{x,rot} + i\gamma B_1(t) m_0$$

$$= -i\gamma G_z z m_{xy,rot} + i\gamma B_1(t) m_0$$

Small Tip Angle Approximation

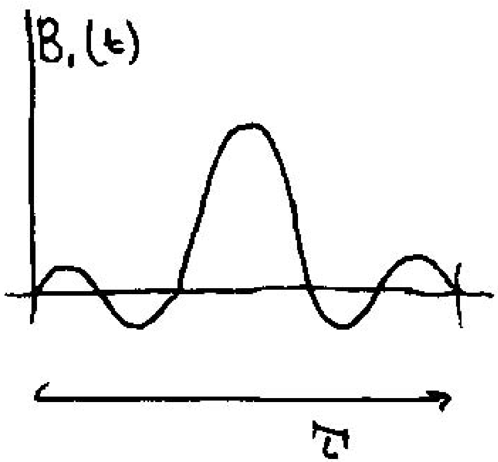
- It is a first order differential equation
  - $i\gamma G_z z$ is a constant with respect to time
  - Driving function $i\gamma\, B_1(t) m_0$
  - Initial condition, $m_{xy,rot}(z,t) = 0$

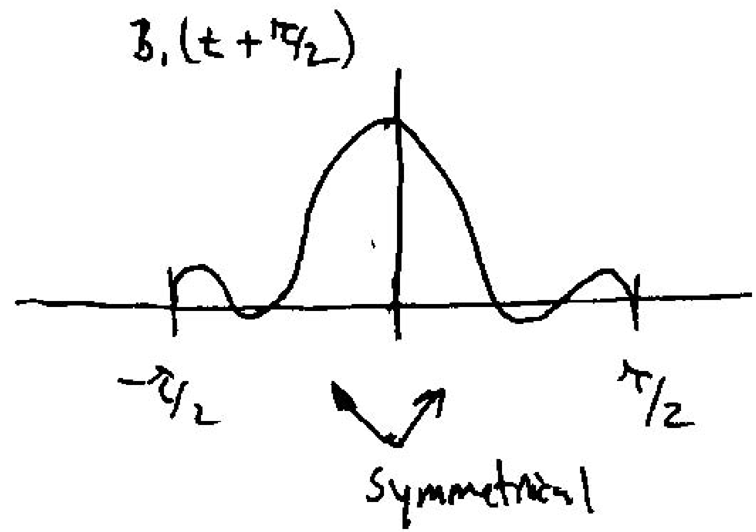
- Solution to this differential equation

$$m_{xy,rot}(z,t) = i m_0 e^{-i\gamma G_z z t} \int_0^t \exp(i\gamma G_z z \xi)\gamma B_1(\xi) d\xi$$

Small Tip Angle Approximation

- We want the solution to this differential equation at the time of the end of the RF pulse - τ, $m_{xy,rot}(z,\tau)$:





Small Tip Angle Approximation

- Substitute $s = x - \tau/2$
- Assume RF pulse is symmetrical (even) around $\tau/2$ and is zero outside of the interval $[0,\tau]$.

$$m_{xy,rot}(z,\tau) = i m_0 e^{-i\gamma G_z z\tau} \int_{-\tau/2}^{\tau/2} \exp(i\gamma G_z z(s+\tau/2))\gamma B_1(s+\tau/2)ds$$

$$= i\gamma m_0 e^{-i\gamma G_z z\tau/2} \int_{-\tau/2}^{\tau/2} \exp(i\gamma G_z zs) B_1(s+\tau/2)ds$$

$$= i\gamma m_0 e^{-i\gamma G_z z\tau/2} \int_{-\infty}^{\infty} B_1(s+\tau/2)\exp\left(i2\pi\left(\frac{\gamma}{2\pi}G_z z\right)s\right)ds$$

$$= i\gamma m_0 e^{-i\gamma G_z z\tau/2} F^{-1}\left\{B_1(s+\tau/2)\right\}\Big|_{x=\frac{\gamma}{2\pi}G_z z}$$

Small Tip Angle Approximation

- For symmetrical RF pulses, the forward and inverse FT are the same.

$$F\{g(x)\} = \int_{-\infty}^{\infty} g(x) e^{-i2\pi xs} dx = \int_{-\infty}^{\infty} g(-x) e^{i2\pi xs} dx = F^{-1}\{g(x)\}$$

Small Tip Angle Approximation

- Thus, under the small tip angle approximation, the slice profile (variation of $m_{xy,rot}$ with z) is related to the spectrum of the RF pulse:

$$\left|m_{xy,rot}(z,t)\right| \mu \left|F\left\{B_1(t)\right\}\right|_{f=\frac{g}{2p}G_z z}$$

$$m_{xy,rot}(z,\tau)=i\gamma m_0 e^{-i\gamma G_z z\tau/2}F\left\{B_1(t+\tau/2)\right\}\Big|_{f=\frac{\gamma}{2\pi}G_z z}$$

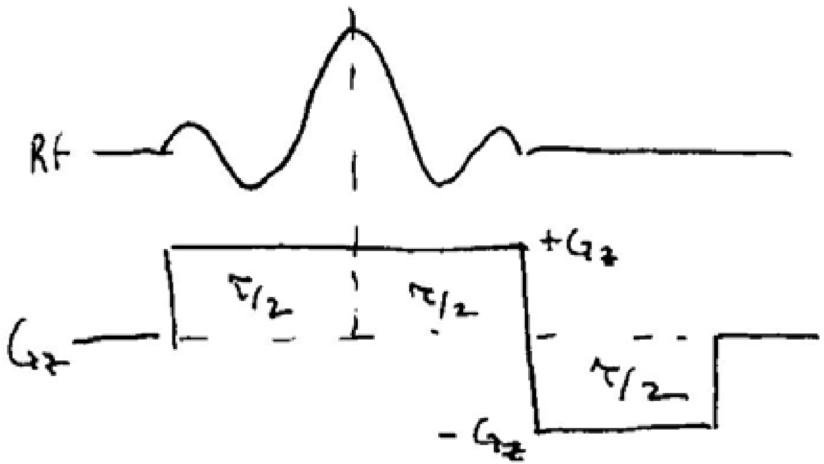
Small Tip Angle Approximation

- We're almost there, but still have some undesired phase variation in the z-direction - $\exp(-i\gamma G_z z\tau / 2)$
  - can lead to undesired phase destruction when integrated by the RF coil.

$$m_{xy,rot}(z,\tau) = i\gamma m_0 e^{-i\gamma G_z z\tau/2} F\{B_1(t+\tau/2)\}\Big|_{f=\frac{\gamma}{2\pi}G_z z}$$

Small Tip Angle Approximation

- How is this fixed?
  Simply apply a negative G_z for a period $\tau/2$ – often called a slice rephasing pulse.



Small Tip Angle Approximation

- Slice rephasing pulse results in phase accumulation of:

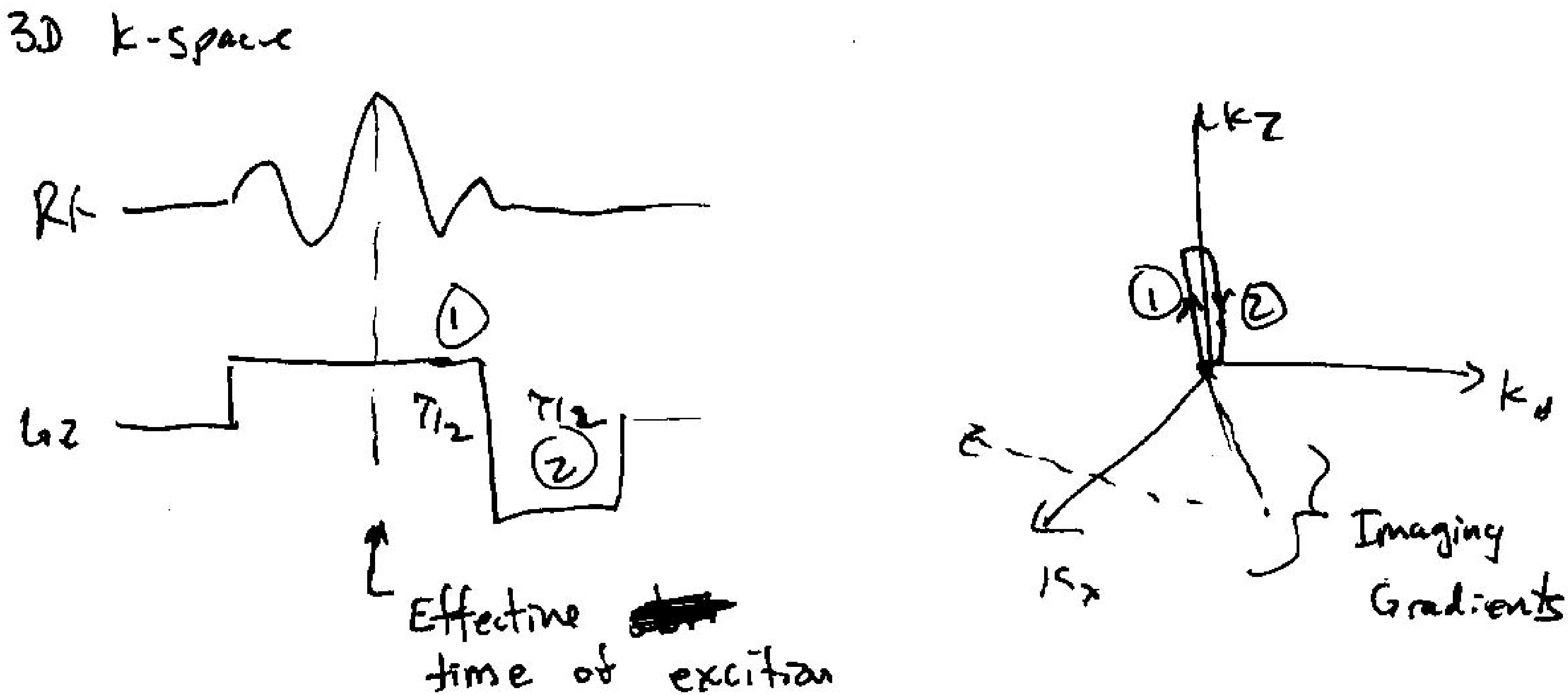
$$\exp\left(-i\gamma\int_{\tau}^{3\tau/2}\Delta B(z,t)\,dt\right)=\exp\left(i\gamma G_z z\tau/2\right)$$

thus:

$$m_{xy,rot}(z,\tau)=i\gamma m_0 F\left\{B_1(t+\tau/2)\right\}\Big|_{f=\frac{\gamma}{2\pi}G_z z}$$

Small Tip Angle Approximation

k-space picture



Small Tip Angle Approximation

- RF pulse applied along real (**i'**) axis à magnetization end up along imaginary (**j'**)axis.

$$m_{xy,rot}(z,3\tau/2)=i\gamma m_0 F\{B_1(t+\tau/2)\}\Big|_{f=\frac{\gamma}{2\pi}G_z z}$$

- The flip angle at the center of the slice is:

$$\alpha=\int_{-\tau/2}^{\tau/2}\gamma B_1(s+\tau/2)ds=\gamma F\{B_1(s+\tau/2)\}\ \Big|_{z=0}$$

  - After an α pulse, $|m_{xy,\ rot}|\ =\ m_0\sin\alpha\sim m_0\alpha$ (for small α, $\sin\alpha\sim\alpha$).

RF pulse - MRI project

- $B_1(t) = A*RF\_shape(t)$
  - A - amplitude of RF pulse
  - RF_shape(t) - shape of RF pulse

- For a 90° pulse (flip angle = 90°), if RF_shape(t) is known, how to calculate A?

RF pulse - MRI project

- $B_1(t)$ = A*RF_shape(t)
  - A - amplitude of RF pulse
  - RF_shape(t) - shape of RF pulse
- For a 90° pulse

$$\gamma \int_0^t B_1(\tau)\, d\tau = \frac{\pi}{2}$$

$$B_1(t) = A \cdot RF\_shape(t)$$

$$\gamma \int_0^t A \cdot RF\_shape(\tau)\, d\tau = \frac{\pi}{2}$$

MRI project

- For a slice thickness Δz, calculate G_z:
  - $BW * 2\pi = \gamma G_z \Delta z \Rightarrow G_z = BW / (\gamma / 2\pi) \Delta z$

- In time domain, the RF pulse is:

$$RF_{shif}(t) = RF(t) e^{i\omega_{shift} t}$$

$$\omega_{shift} = \gamma G_z z_{slice}$$

MRI project

Rotating Frame

$$B_x = RF(t)\cos(\omega_{shift}t)$$

$$B_y = RF(t)\sin(\omega_{shift}t)$$

$$B_z = G_x x + G_y y + G_z z$$

Questions?

Small Tip Angle Approximation

Putting Slice Selection with the Signal Equation

Putting Slice Selection with the Signal Equation

- Define our slice profile function:

$$p(z) = gF\left\{B_1(t+t/2)\right\}\Big|_{f=\frac{g}{2p}G_z z}$$

- Magnetization:

$$m_{xy,rot}(z,3\tau/2) = i\gamma m_0 F\left\{B_1(t+\tau/2)\right\}\Big|_{f=\frac{\gamma}{2\pi}G_z z}$$

$$m_{xy,rot}(z,3t/2) = im_0\, p(z)$$

Putting Slice Selection with the Signal Equation

- 3D distribution of magnetization by substituting $im_0 = m(x,y,z)$ and putting it into the signal equation :

$$s(t) = \iiint m(x,y,z)\,p(z)\exp\left(-i2\pi\left(xk_x(t) + yk_y(t)\right)\right)dxdydz$$

- Here we are performing 2D imaging while integrating across the slice profile

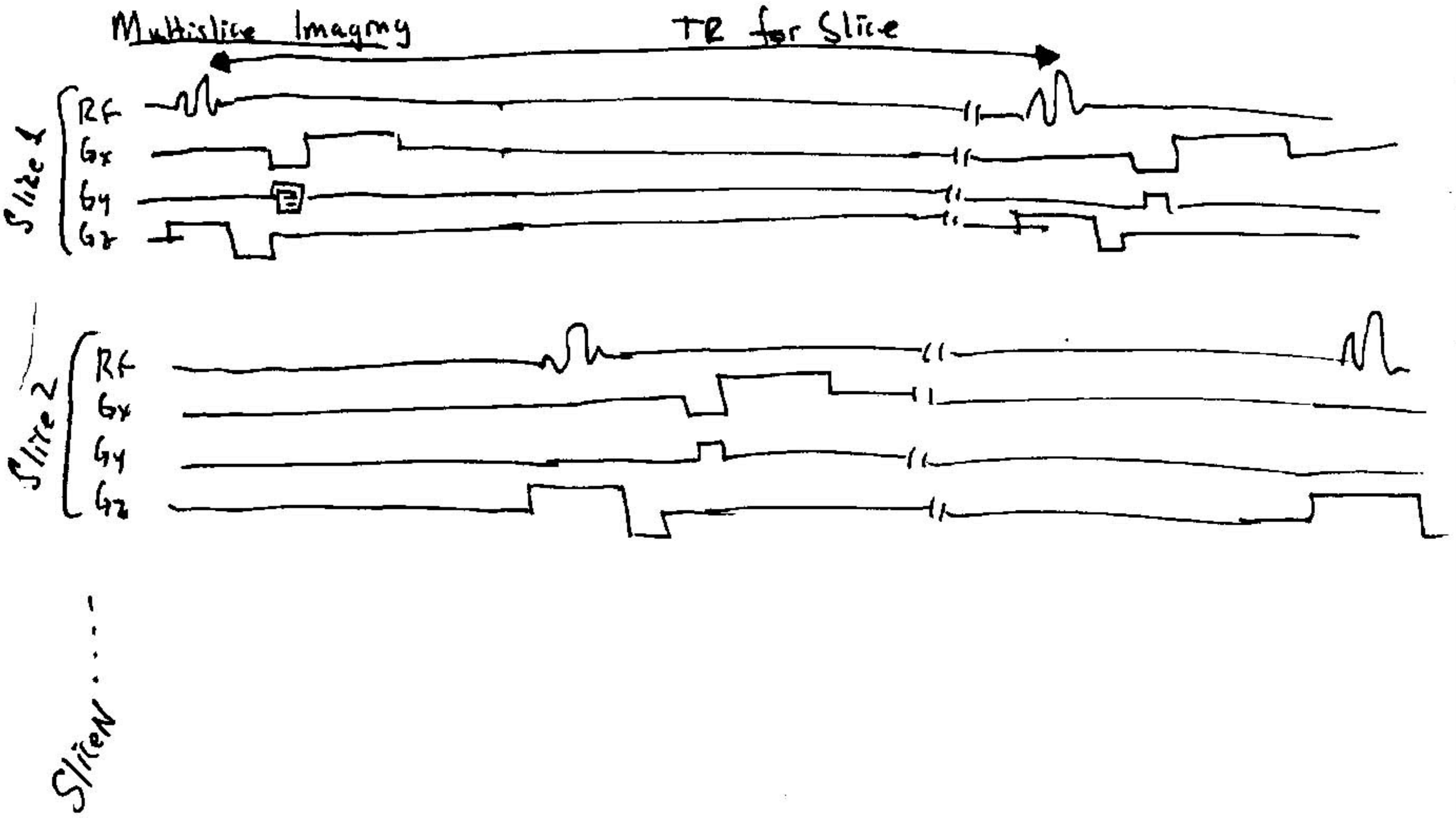
Larger Flip Angles

- Analysis using small flip angle (e.g. $\alpha < \pi/6$) approximation turns out perform well for large flip angles (e.g. $\pi/2$) as well.

- Here, a better approximation for the transverse component of the magnetization is:

$$m_{xy,rot}(z,3t\ /\ 2) = im_0 \sin\big(p(z)\big)$$

Mutli-slice Imaging



Mutli-slice Imaging

- Slice-selective, 2D spin-warp acquisition overall acquisition time is:
- A) $N_y * N_y * TR$
- B) $N_y * TR$
- C) $N_z * TR$

Mutli-slice Imaging

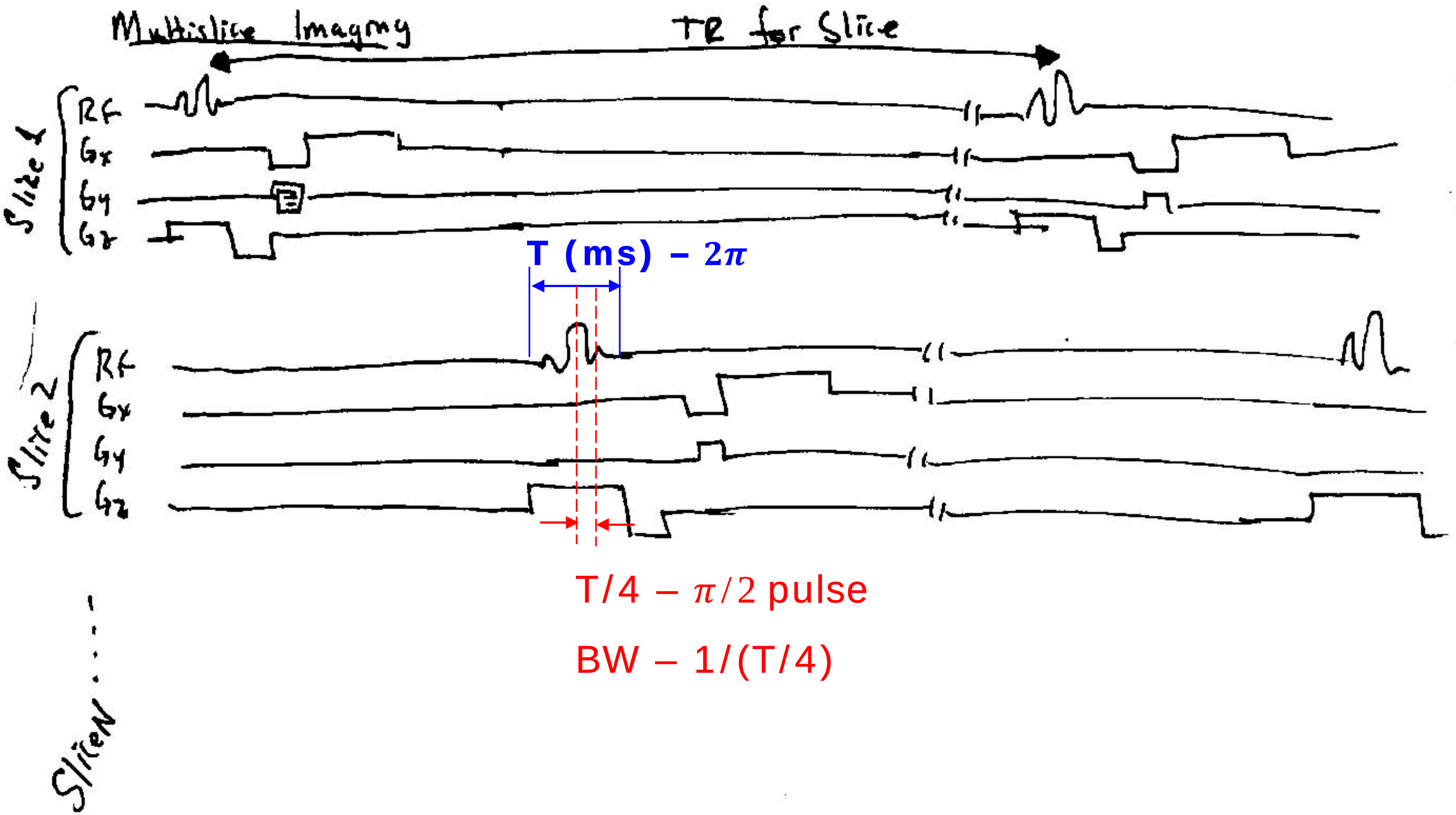
- Slice-selective, 2D spin-warp acquisition overall acquisition time: $N_y * TR$

- For example, acquire a T1-weighted image
  - Ny - 20 slices
  - TR - 500 ms
  - 128 phase encoding lines in k-space
  - Total acquisition time - $N_y * TR$ = ~ 1 minute

Question?

Multi-slice imaging

Slice Selective Excitation



T/4 – $\pi/2$ pulse

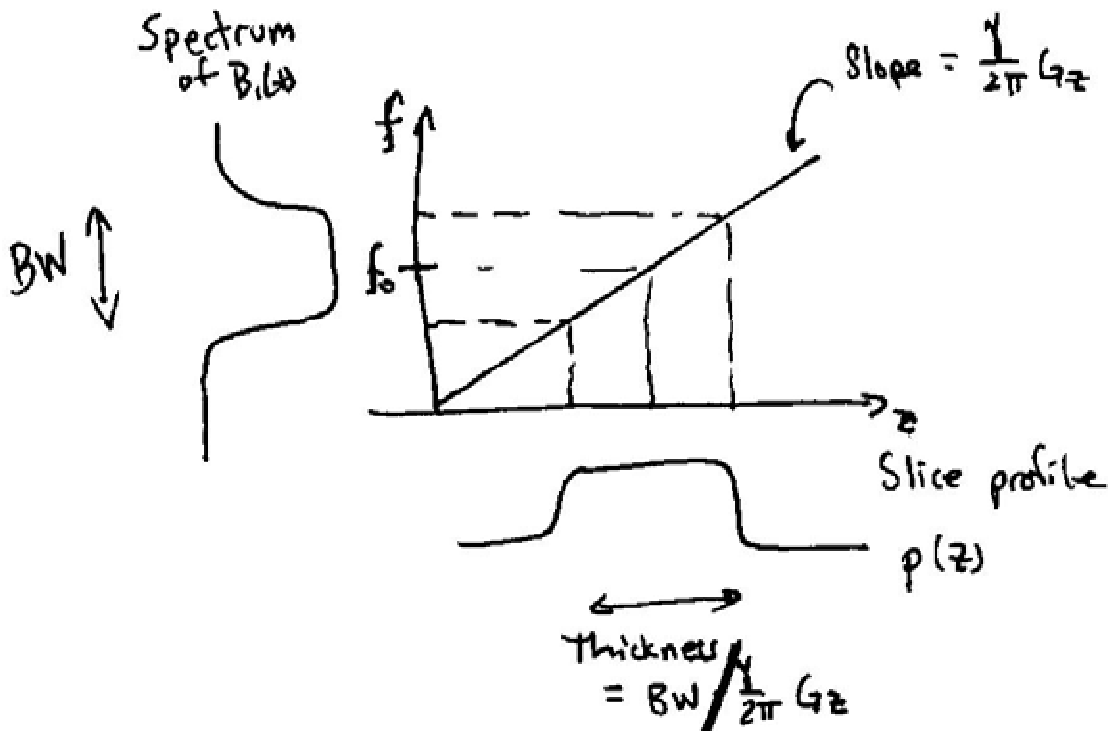
BW – 1/(T/4)

Mutli-slice Imaging

- The most common way to image 3D volumes in MRI uses interleaved slice selective excitation.
  - Slice 1 is excited and part of the k-space data are acquired, then slice 2, 3...
  - After all have been acquired, come back to slice 1 to acquire additional parts of the k-space data
  - When one slice is excited, the others are not perturbed - each slice has it's own TR.
  - Slice selection allows the efficient use of longer TR's by simultaneously acquiring many slices.

Small Tip Angle Approximation

- $f = \frac{g}{2p} G_z z$ is the conversion between spectrum and the z

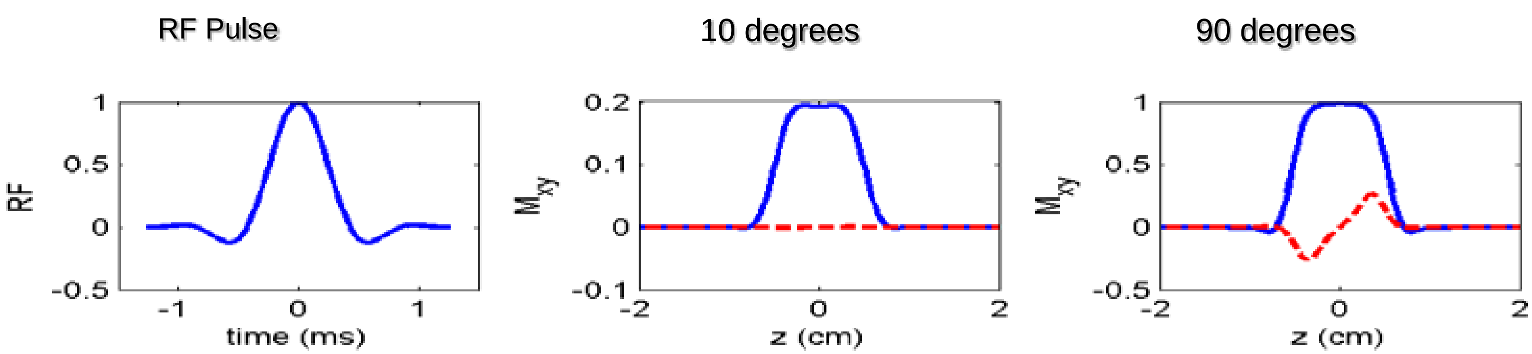


Small Tip Angle Approximation

- K-space picture for rephasing
  - Assume RF pulse occurs instantaneously a the center of the pulse (at $\tau/2$)
  - Begin accumulation area in k-z after $\tau/2$
  - By applying a negative gradient for the same duration as the last half of the pulse, the areas cancel and the k-space location in the z direction is returned to the origin.

Larger Flip Angles

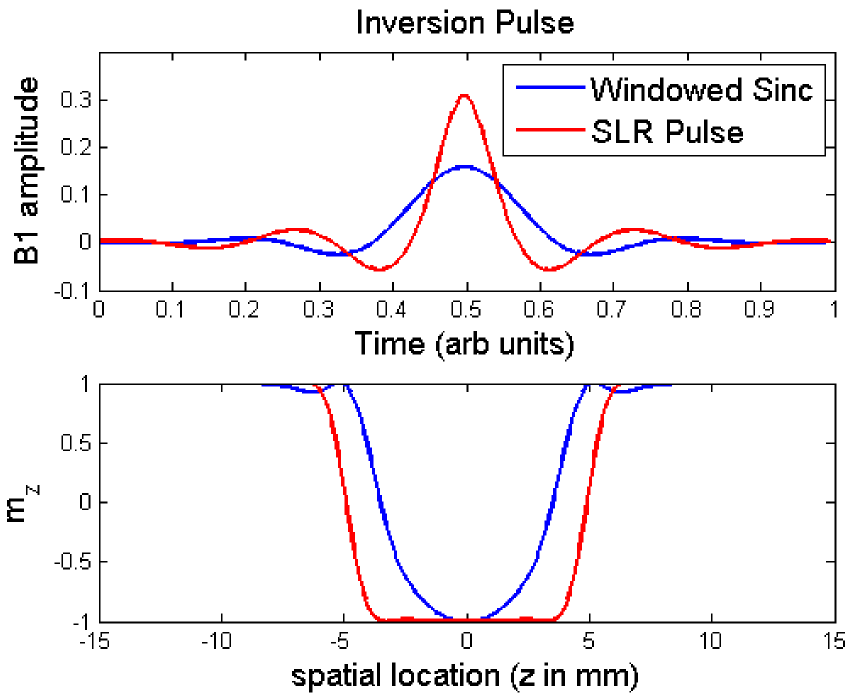
- Due to the non-linearity of the Bloch equations, an exact solution would require numeric simulation of the RF pulse.



Design of Large Flip Angle pulses

- Suppose you wanted a slice selective 180 degree pulse. “sinc” pulses are not very good.

Here is an example of a 180 degree pulse designed using Fourier (small tip) methods and a large tip angle algorithm known as the Shinnar-LeRoux (SLR) method



Example –sinc RF pulse

- Consider an RF pulse:

$$B_1(t) = A\,\mathrm{sinc}\left(\frac{t - \tau/2}{T}\right)$$

- Spectrum:

$$F\{B_1(s+\tau/2)\} = AT\,\mathrm{rect}\left(\frac{f}{BW}\right), \text{ where } BW = 1/T$$

- Magnetization:

$$m_{xy,rot}(z, 3\tau/2) = i\gamma m_0 AT\,\mathrm{rect}\left(\frac{zG_z\gamma}{2\pi BW}\right) = i\alpha m_0\,\mathrm{rect}\left(\frac{z}{\Delta z}\right)$$

- Slice thickness: $\Delta z = \dfrac{2\pi BW}{\gamma G_z}$

- Flip angle (small angle): $\alpha \cong \gamma AT$

The 3rd Dimension - Z

- Spin-Warp imaging can be easily extended to 3D by using phase encoding in two dimensions (rather than 1) and frequency encoding in the remaining dimension.