Notes on MRI, Part 4

Parallel Imaging

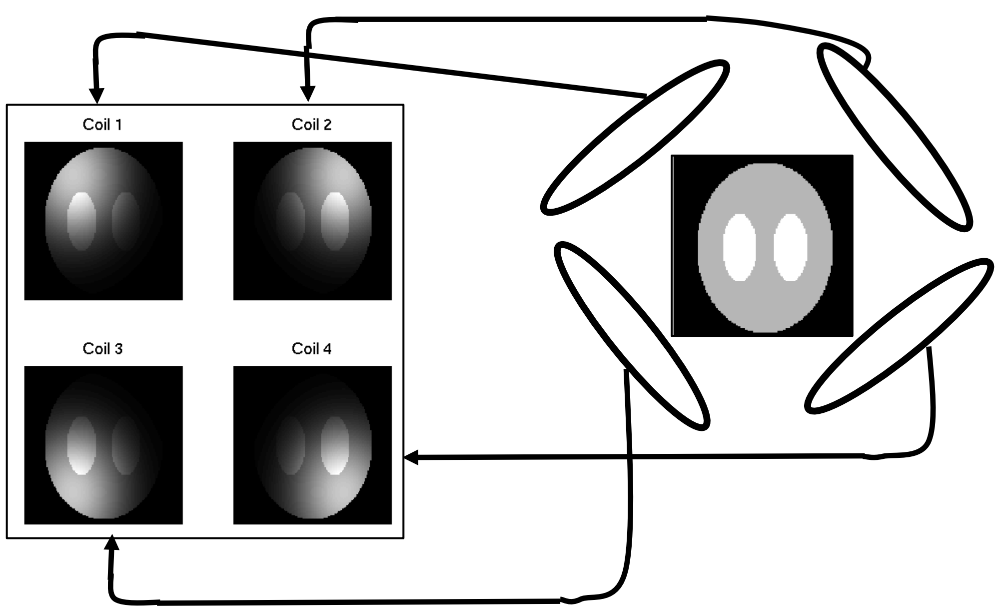
In MRI Notes 2, we noted that the receive coil sensitivity need not be constant ($\mathbf{C}(\mathbf{r})$), but could have a shape dictated by the geometry of the coil windings. Indeed, it is very common to have multiple receiver channels, each with a different pattern (($\mathbf{C}_l(\mathbf{r})$). There are two compelling reasons to do this:

1) SNR: Small coils are sensitive to the signal of interest but only receive noise from the volume directly in front of a coil. By combining signals from array of coils, one achieve the benefit of a large region of coverage but the SNR advantage of small coils.
2) Imaging from reduced (aliased) data sets: This is usually done to increase the speed and/or spatial resolution of an image acquisition. The basic idea is that by sampling multiple coils, one can use the additional measurements to eliminate aliasing.

The signal equation can now be represented as follows (in 2D):

$$s_l(t) = \iint C_l(x,y)m(x,y)\exp\left(-i2\pi\left(k_x(t)x + k_y(t)y\right)dxdy\right.$$

where $C_l(x,y)$ is the complex coil sensitivity pattern for the l^{th} receive coil. Note that each $s_l(t)$ is received separately and each goes through its own complex demodulator, so for L coils, there are L channels of signals and k-space data samples, which can be reconstructed into the L images $m_l(x,y) = [C_l(x,y)m(x,y)]$.



SNR

It can be shown that for independent noise in the L channels, an SNR optimal image reconstruction can be represented as:

$$\widehat{m}(x,y) = \frac{\sum_l C_l^*(x,y) m_l(x,y)}{\sum_l |C_l(x,y)|^2}$$

which requires knowledge of $C(x,y)$. There are multiple ways of determining the coil sensitivity, but a common way is to collect images of each small array coil and divide by a uniform by suboptimal SNR coil image (commonly a very large coil known as the body coil).

A general solution for correlated noise channels is:

$$\widehat{m}(x,y) = (\mathbf{C}^H \Psi^{-1} \mathbf{C})^{-1} \mathbf{C}^H \Psi^{-1} \mathbf{M}$$

where **C** is the vector of coil sensitives at (x,y) and **M** is the vector coil image values at (x,y) and Ψ is the covariance matrix for coil noise.

A simple way of producing a “good” image from array coils is:

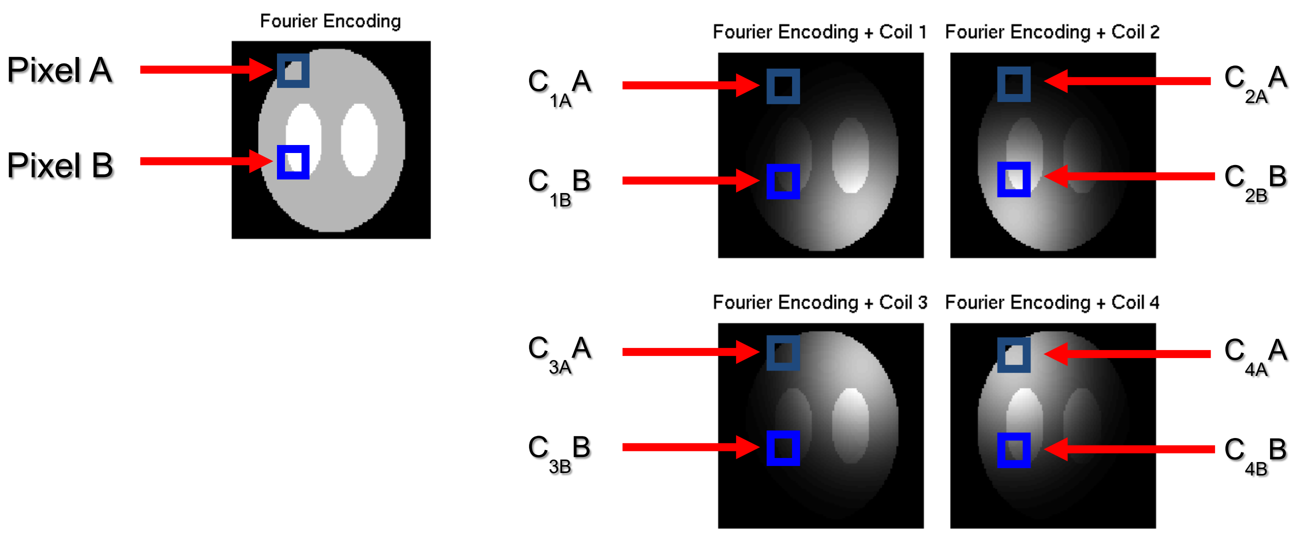
$$\widehat{m}(x,y) = \sqrt{\sum_l |C_l(x,y) m(x,y)|^2}$$

(the 2-norm of the coil images), which doesn’t require knowledge of C_l.

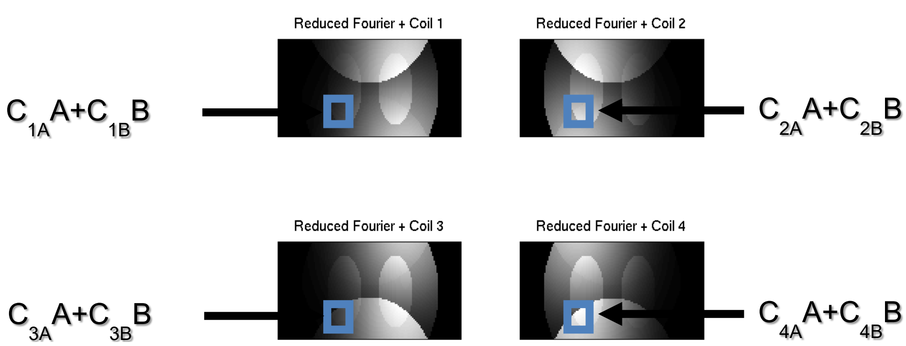
Parallel Imaging

An interesting extension of array coils is it use in parallel imaging, that is, using "parallel" channels of signals to reconstruct image from undersampled data. There are many ways of doing this reconstruction, but perhaps the most straightforward to understand is known as SENSE imaging (Pruessmann, Klaas P., et al. "SENSE: sensitivity encoding for fast MRI." *Magnetic Resonance in Medicine*, 42(5): 952-962, 1999.).

Consider 2 unknown pixels in an image which are then encoded with coil sensitivities:



Now consider a case where we have undersampling the phase encoding direction which will result in aliasing in the each of the L coil images:



With a single receive coil, one cannot separate pixel A from its alias pixel B, but with additional provided by multiple receiver channels and known sensitivity maps, one can reconstruct the unknown pixels A & B from a simple matrix equation:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} C_{1A} & C_{1B} \\ C_{2A} & C_{2B} \\ C_{3A} & C_{3B} \\ C_{4A} & C_{4B} \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

which is overdetermined and the pixel values can be found using some sort of (least squares) pseudoinverse:

$$\begin{bmatrix} A \\ B \end{bmatrix} = \mathbf{C}^{+} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$