X-ray Imaging

BME/EECS 516

X-ray Lecture #2

Announcements

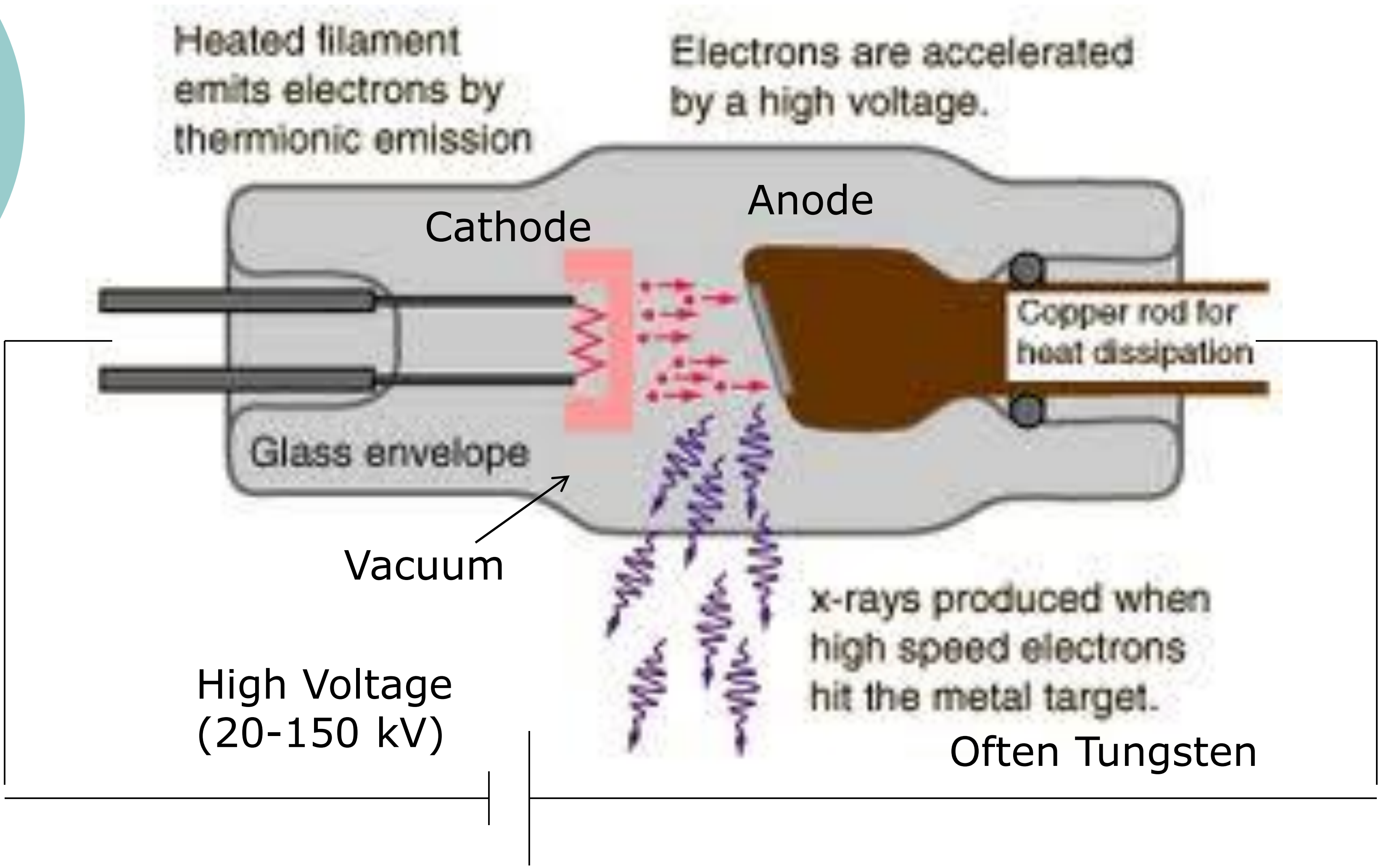
- HW #5 due today
- MRI Project due Tuesday 11/21
- Tuesday, 11/14 – guest lectures from local medical imaging industry
  - David Sarment from Xoran Technologies
  - John Seamans (UM) formerly with Delphinus Medical and GE Healthcare
- US and MRI Demos – 11/21 during class time. More info to come…

Physics - Radiation

- a process in which energetic streams of particles or photons travel through a medium or space
- X-ray photon - appropriate amount of interaction for imaging
- Behavior of Radiation Along a Line:

$$N(x) = N(0)\exp\left(-\int_0^x \mu(x')\,dx'\right)$$

X-ray Tube

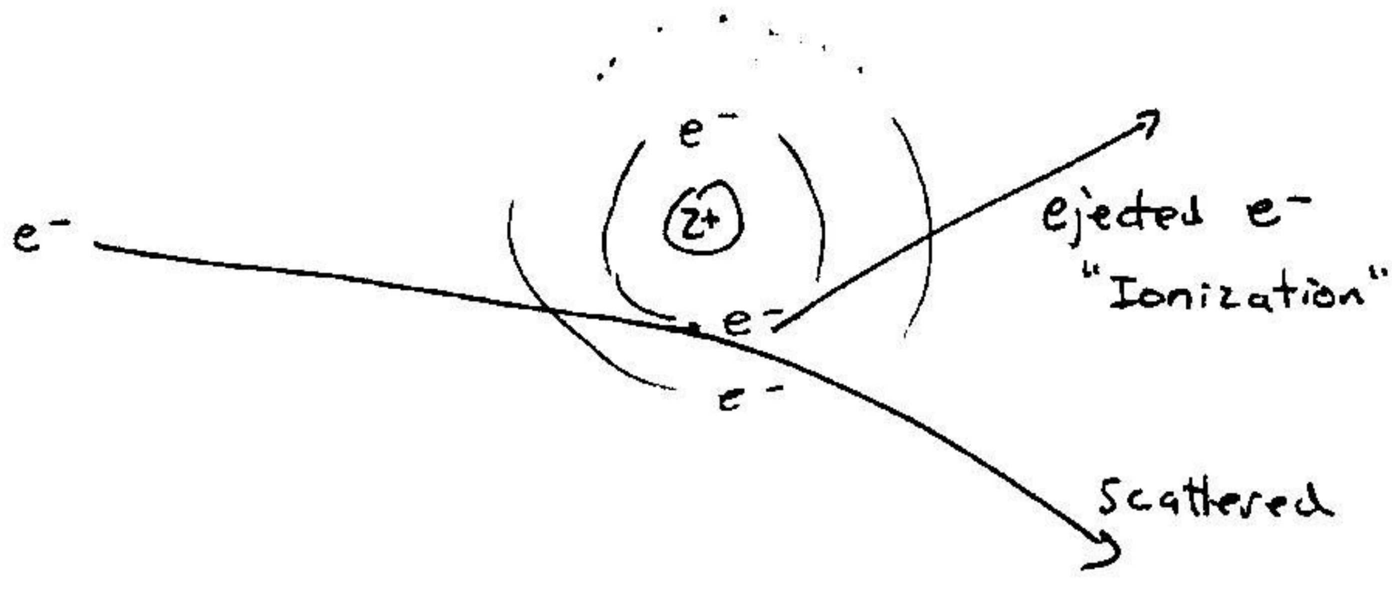


http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/imgqua/xtube.gif

Generation of x-rays: electron interactions

1. Inelastic (energy absorbing) scattering with atomic electrons
   - An electron with enough energy can eject an orbital electron out of the inner shell of a metal atom.
   - Electrons from higher energy level fill up the vacancy
   - X-ray photon are emitted from **spontaneous energy state transitions**.

Generation of x-rays: electron interactions

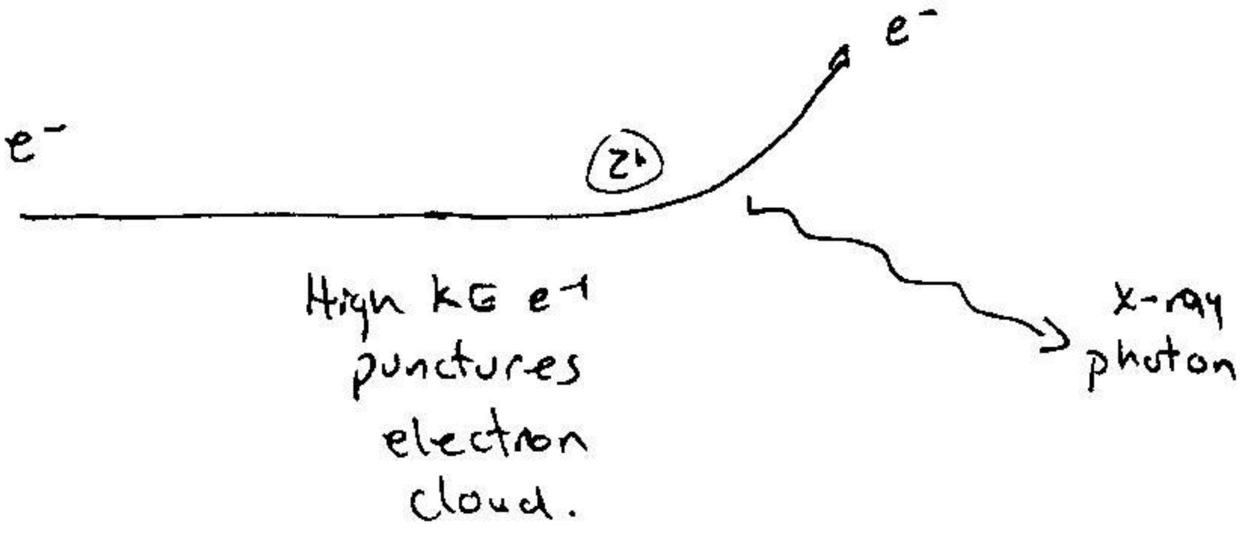


Generation of x-rays: electron interactions

- The process produce discrete emission spectrum –spectral lines. “Characteristic” x-ray energies for W
  - 58.5 keV
  - Any combination of shell transition energies (e.g. 3.2 and 61.7 keV).
- Bohr model accounts for absorption/generation of discrete valued energies.

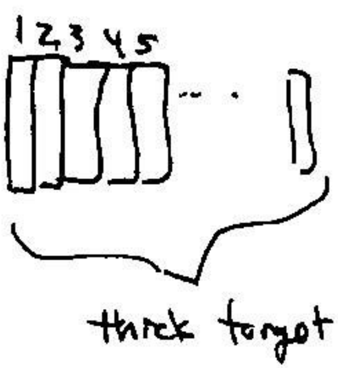
Generation of x-rays: electron interactions

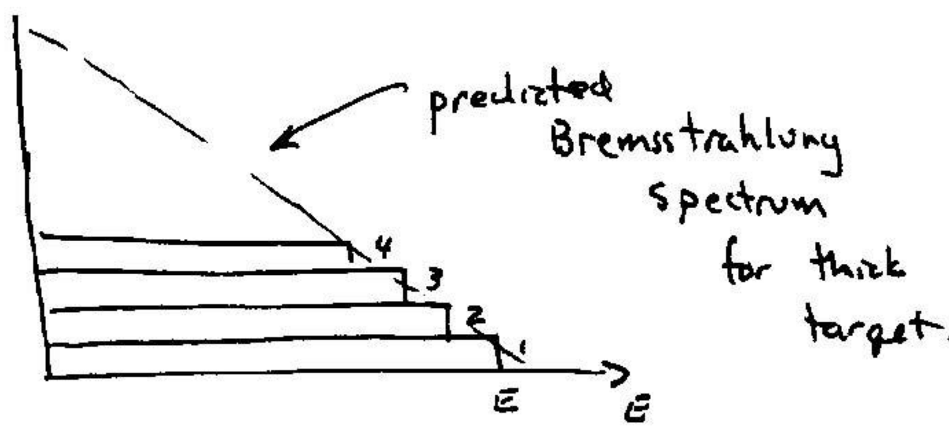
2. Bremsstrahlung “Braking” Radiation – accelerated electrons scattered by a strong electric field near the high-Z nuclei. X-rays have continuous spectrum



Generation of x-rays: electron interactions

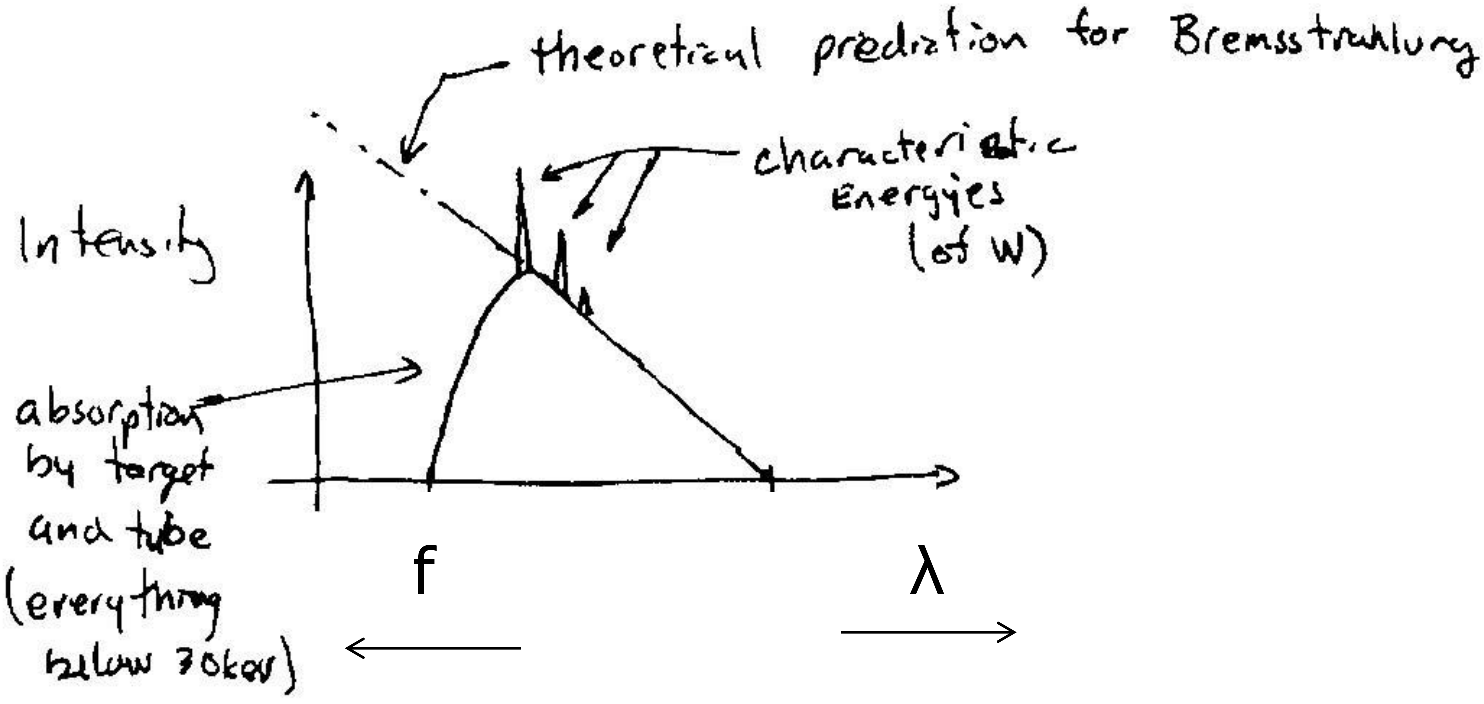
- Thick target (anode)
  - Modeled as a series of thin targets.
  - Each thin target produces a new uniform spectrum, but with a lower peak energy.
  - Resultant spectrum is approximately linear from a peak at 0 keV to 0 at E.





The x-ray Spectrum

- Spectrum will have a combination of Bohr (discrete) energies and Bremsstrahlung



The x-ray Spectrum

- The x-ray spectrum is function of photon energy: $I_0 = I_0(E)$

- I represents energy/unit time/unit area or power/unit area.

Attenuation Coefficient

- The x-ray spectrum is a function of photon energy E: $I_0 = I_0(\mathrm{E})$
- The attenuation function is also a function of E: $\mu = \mu(x,y,z,\mathrm{E})$.
- The intensity at the output to form image:

$$I_d(x,y) = \int_E I_0(E)\exp\left(-\int \mu(x,y,z,E)\,dz\right)dE$$

I_d tells us nothing about z or E – it only gives us x,y information.

Attenuation Coefficient

- Two important material properties affecting μ: tissue density ρ and the atomic number Z
  - Because most x-ray photon/tissue interactions are photon/electron interactions.
- Attenuation is caused by x-ray photon interaction with the matter

X-ray Photon Interactions

- **Rayleigh-Thompson Scattering** – spontaneous, very low energy phenomenon
- **Photoelectric Absorption** – low energy phenomenon
- **Compton Scattering** – mid energy phenomenon
- **Pair Production** – high energy phenomenon

- In general, attenuation coefficient constituents:

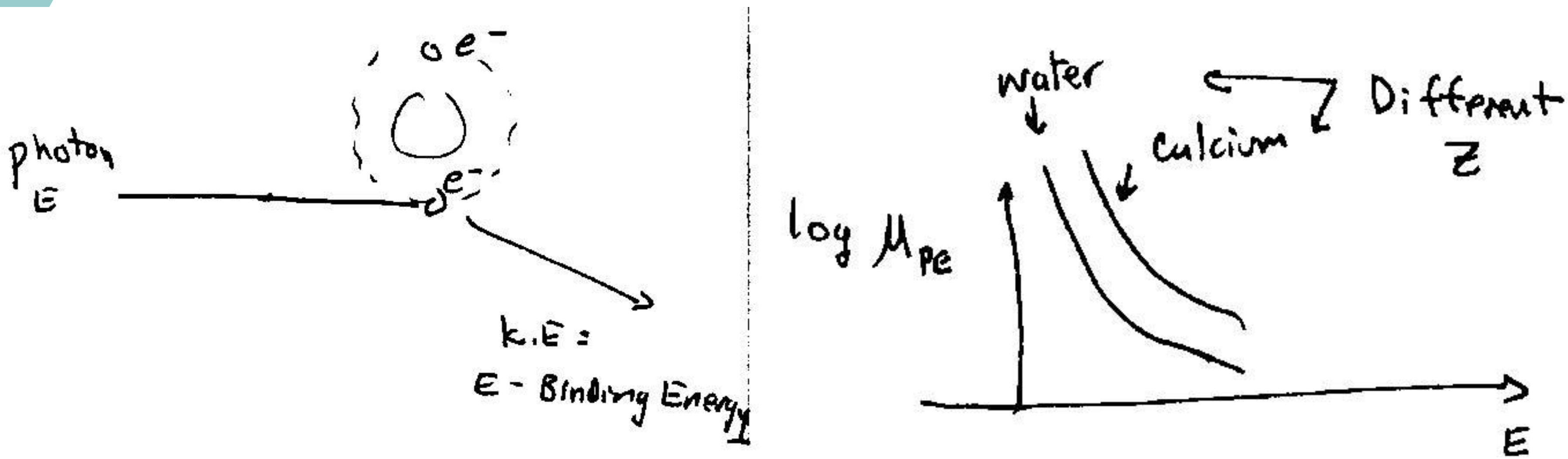
$$\mu(E) = \mu_{rt}(E) + \boxed{\mu_{pe}(E) + \mu_{cs}(E)} + \mu_{pp}(E)$$

Photoelectric Absorption

- Absorption of photon by interacting with a tightly bound electron
- Leads to ejection of an electron – vacancy filled by an electron falling into it from the next shell
- If the ejected electron kinetic energy (= Absorbed photon energy) is less than binding energy of the electron, the electron is unable to escape the material.

Photoelectric Absorption

- photoelectric effect increases rapidly with atomic number, Z and with decreasing photon energy .



- Dominates μ in the lower energy part of the diagnostic spectrum (<50 keV)

Compton Scattering

- Scatting of photons by an elastic collision with a free electron in the outer shell.
- Part of the x-ray energy is transferred to the electron; the rest taken by the scattered “degraded” photon.
- Elastic collisions preserve *E* and momentum (*p*).

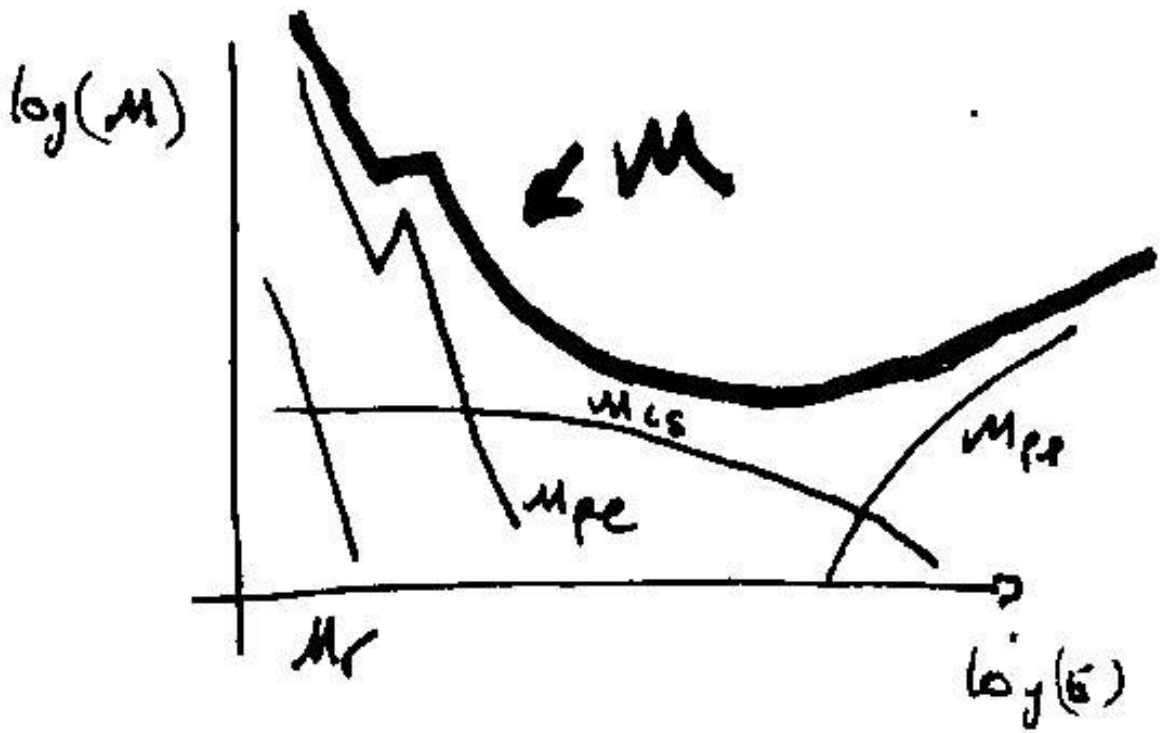
Compton Scattering

- μ_{cs} is nearly constant across diagnostic spectrum
- Compton scatter comes mostly from atomic electrons (μ_{cs} is proportional to ρ)
- At higher *E*, Compton scatter dominates over the PE effect.

Total Linear Attenuation Coefficient for Photons

- The combined coefficient

$$\mu(E) = \mu_{rt}(E) + \mu_{pe}(E) + \mu_{cs}(E) + \mu_{pp}(E)$$



X-ray Attenuation Coefficients for muscle, fat, bone

Note - mass attenuation coefficient: μ/ρ

Macovski – Fig. 3.7

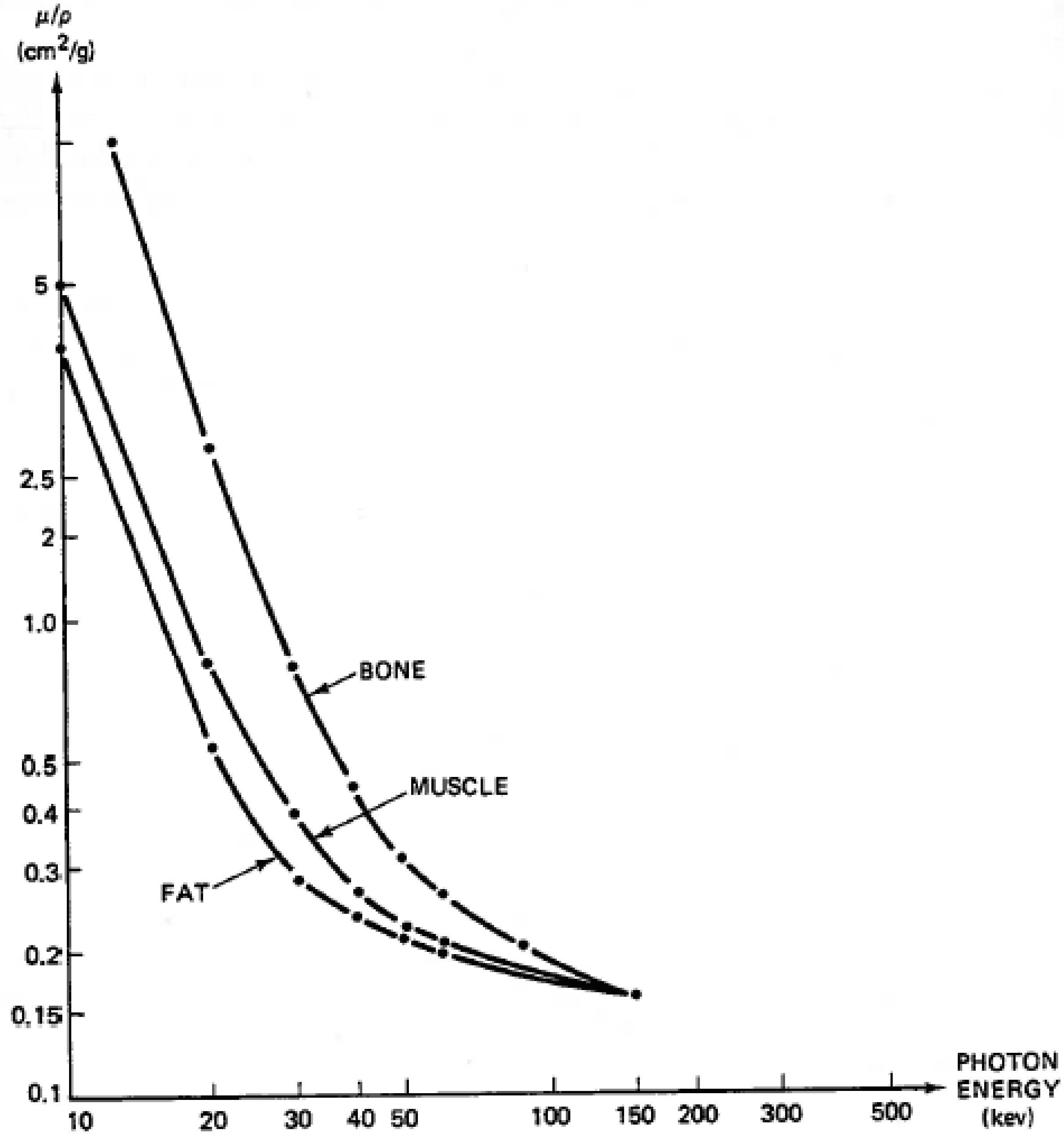


FIG. 3.7 X-ray attenuation coefficients for muscle, fat, and bone, as a function of photon energy.

Topics for Today

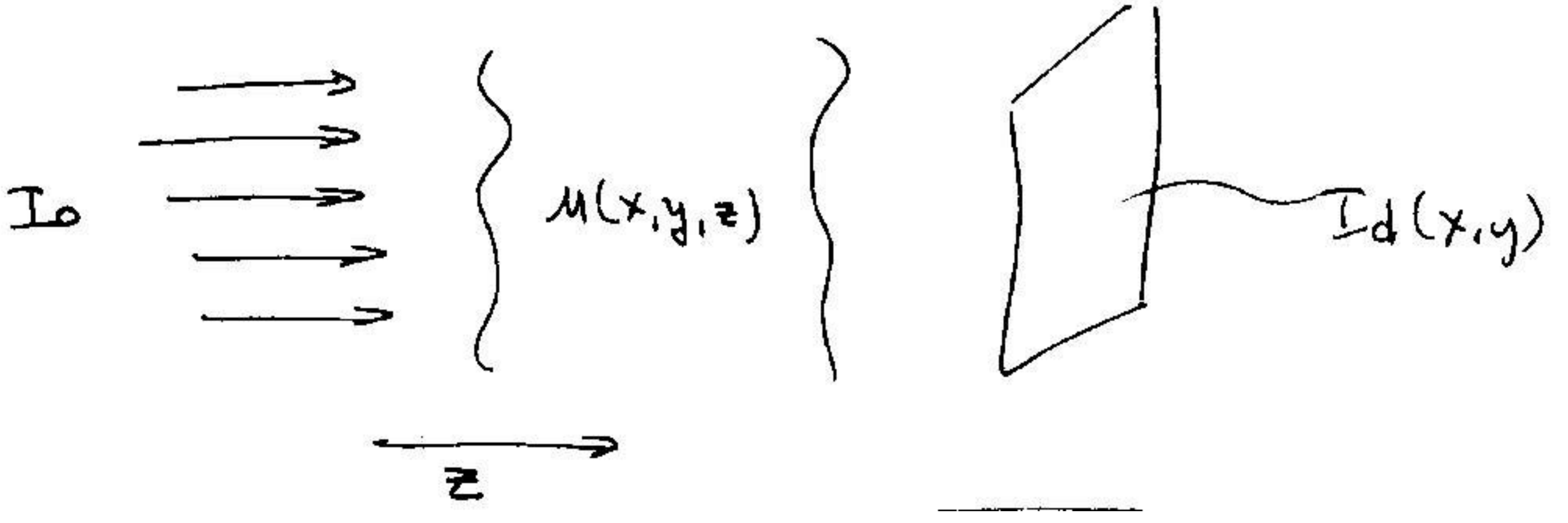
- Source geometry and magnification
- Finite source sizes

Source Issues

Parallel X-ray Imaging System

Practical X-ray Sources

Parallel X-ray Imaging System

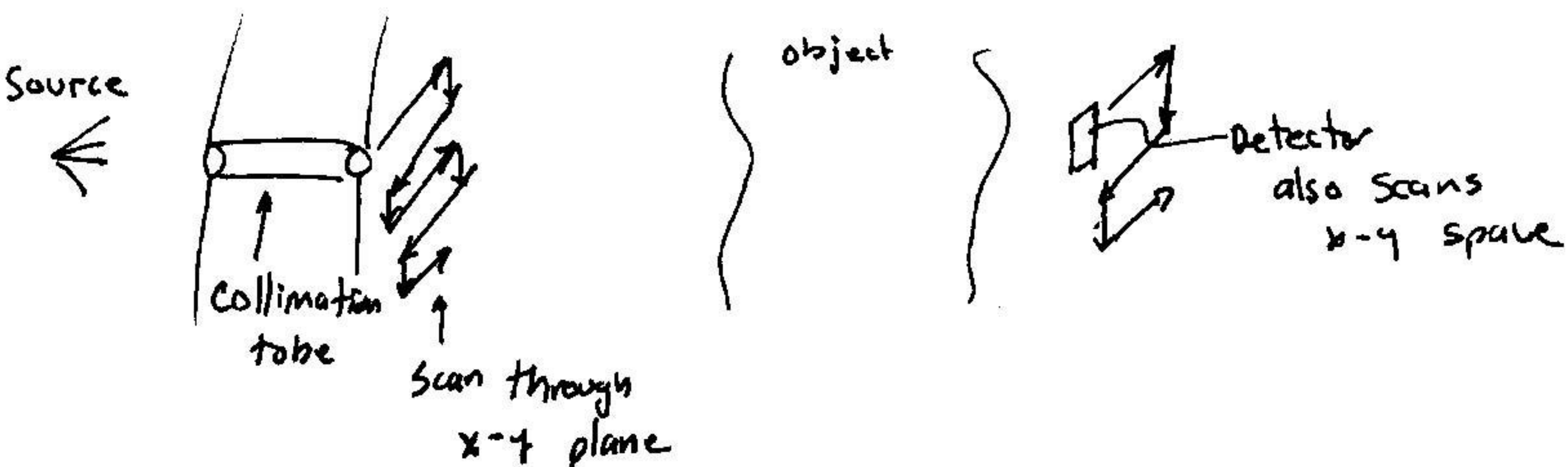


$$I_d(x, y) = I_0 \exp\left(-\int \mu(x, y, z)dz\right)$$

However, there are essentially no practical medical projection x-ray systems where the source has parallel rays.

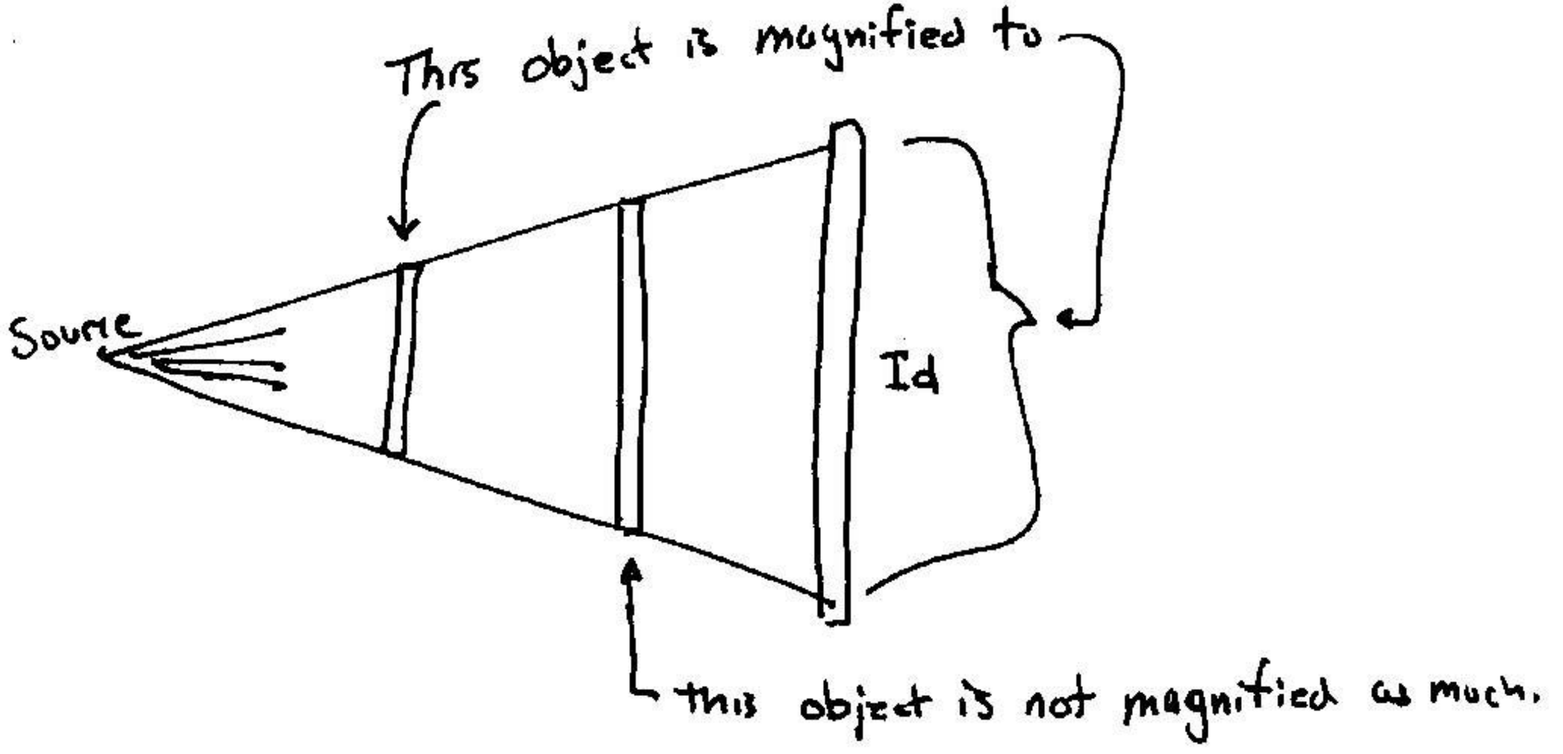
Parallel X-ray Imaging System

- Some scanning systems may achieve parallel rays - appropriate for industrial inspection operations, but too slow for medical applications.



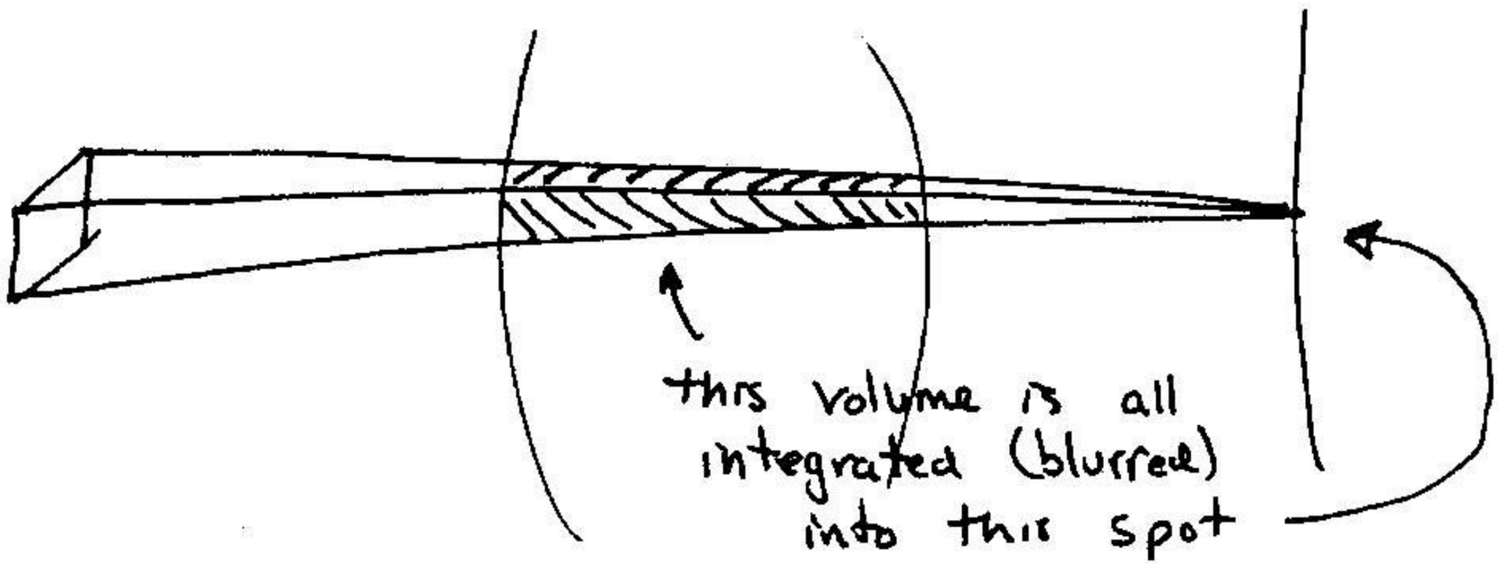
Practical X-ray Sources – two main issues

1. Geometric distortions due to point geometry – “depth dependent magnification.”



Practical X-ray Sources – two main issues

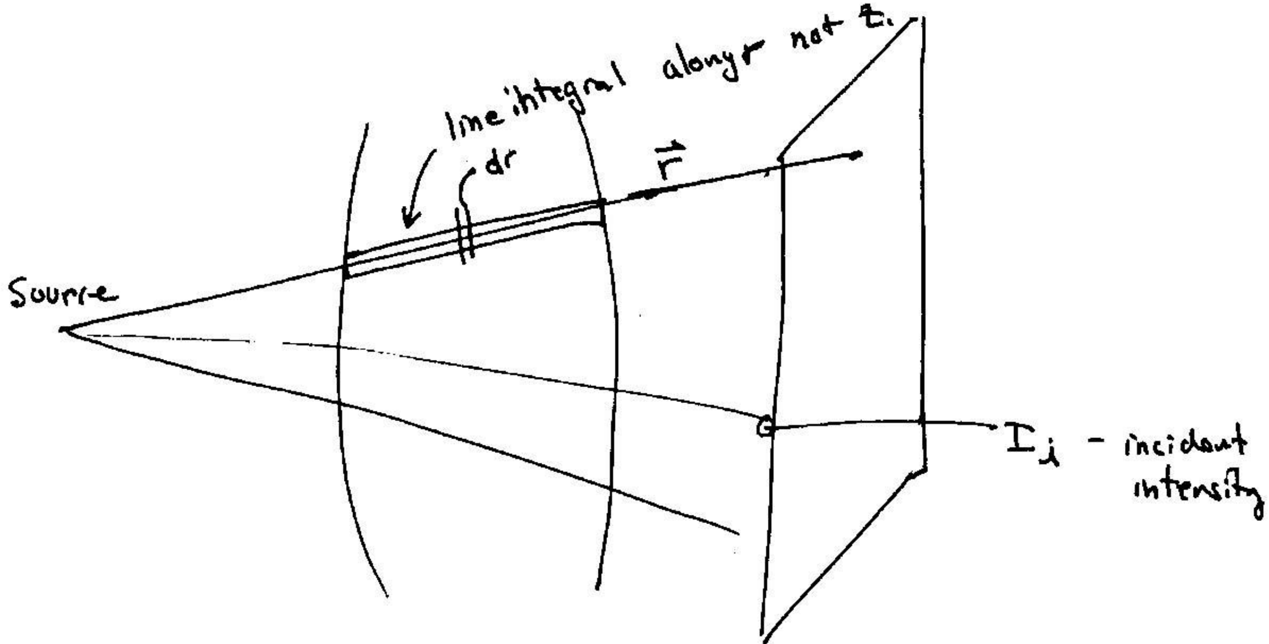
2. Resolution loss (blurring) due to finite (large) source sizes



Point Source Geometry Intensity Variations

Point Source Geometry

- Find expressions for the image intensity, $I_d(x_d, y_d)$, for a point source geometry



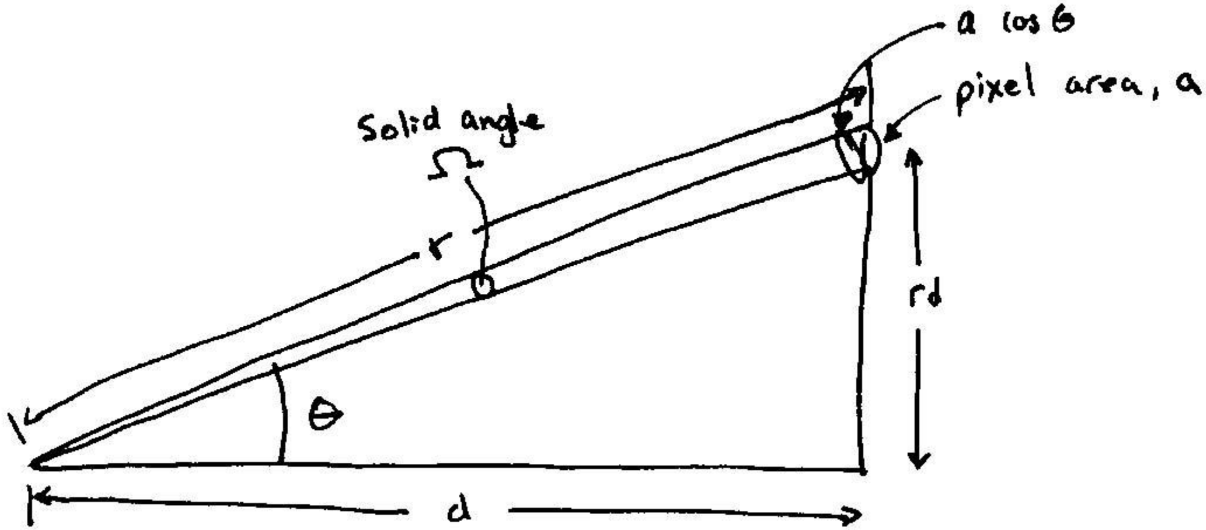
$$I_d(x_d, y_d) = I_i(x_d, y_d)\exp\left(-\int \mu(x,y,z)dr\right)$$

Point Source Geometry

- (x_d, y_d) - coordinate system in the output detector plane.
- (x, y, z) - coordinate system of the object.
- $I_i(x_d, y_d)$ - spatially variant incident intensity replaces I_o.
- The integration is along some path r with variable of integration dr.
- $I_d(x_d, y_d)$ – image intensity

Intensity Variations

- Incident intensity - maximal at the center and falls off towards the edges.
  - Increases in distance from the source
  - Rays obliquely striking the detector.



Intensity Variations

- Intensity - power/unit area - expression for the intensity I_i:

$$I_i = \frac{(\text{photons})(\text{mean photon E})}{(\text{unit area})(\text{exposure time})} = \frac{kN}{a}\frac{\Omega}{4\pi}$$

  - k - scaling coefficient
  - N - number of photon emitted during the observation time
  - $\Omega/4\pi$ - fraction of the surface of a sphere that is subtended by pixel area *a*.

Intensity Variations

- Ω – solid angle

- For a pixel of area a at some position angle θ away from the origin, the part of a sphere covered will be $a \cos\theta$. Thus:

$$\frac{\Omega}{4\pi} = \frac{a\cos\theta}{4\pi r^2} \quad \text{or} \quad \Omega = \frac{a\cos\theta}{r^2}$$

Intensity Variations

- Intensity at the origin of the detector - $I_0 = I_i(0,0)$.
  - At the origin, $\theta = 0$
  - Distance from the source to the detector - $r = d$

$\Omega = a/d^2$ and $I_0 = I_i(0,0) = \dfrac{kN}{4\pi d^2}$

- Intensity, I_0, falls off with $1/d^2$ as the detector moves away from the source.

Intensity Variations

$$I_0 = \frac{kN}{4\pi d^2} \Rightarrow k = I_0 \frac{4\pi d^2}{N}$$

$$I_i = \frac{kN}{a}\frac{\Omega}{4\pi} = I_0 d^2 \frac{\cos\theta}{r^2} \qquad \cos\theta = \frac{d}{r}$$

$$I_i = I_0 \cos^3\theta = I_0 \left(\frac{d}{r}\right)^3$$

- $\cos^3\theta$ (or its representation) - *incident intensity obliquity term* - has two components
  - $\cos^2\theta$ term - increase in distance from the source to the detector
  - $\cos\theta$ term - rays obliquely striking the detector

Intensity Variations

- Put this expression in the coordinate system of the detector using and

$$r_d^2 = x_d^2 + y_d^2 \qquad r^2 = d^2 + r_d^2$$

$$I_i(x_d, y_d) = I_0 \left(\frac{d}{\sqrt{d^2 + r_d^2}} \right)^3 = I_0 \frac{1}{\left(1 + \left(\frac{r_d}{d} \right)^2 \right)^{3/2}}$$

Question

- A chest x-ray system has a detector plane of 40x40 cm. The distance between the x-ray tube and the detector plane is 20 cm. At the edge of the detector plane, the x-ray intensity is ___ of the intensity at the center.
  - A) sqrt(1/2)
  - B) sqrt(1/8)
  - C) sqrt(1/27)
  - D) sqrt(1/125)

Question

- A chest x-ray system has a detector plane of 40x40 cm. The distance between the x-ray tube and the detector plane is 20 cm. At the edge of the detector plane, the x-ray intensity is ____ of the intensity at the center.
  - A) sqrt(1/2)
  - B) sqrt(1/8)
  - C) sqrt(1/27)
  - D) sqrt(1/125)

  - Answer B) - $\cos^3\frac{\pi}{4}$ =0.35

Questions?

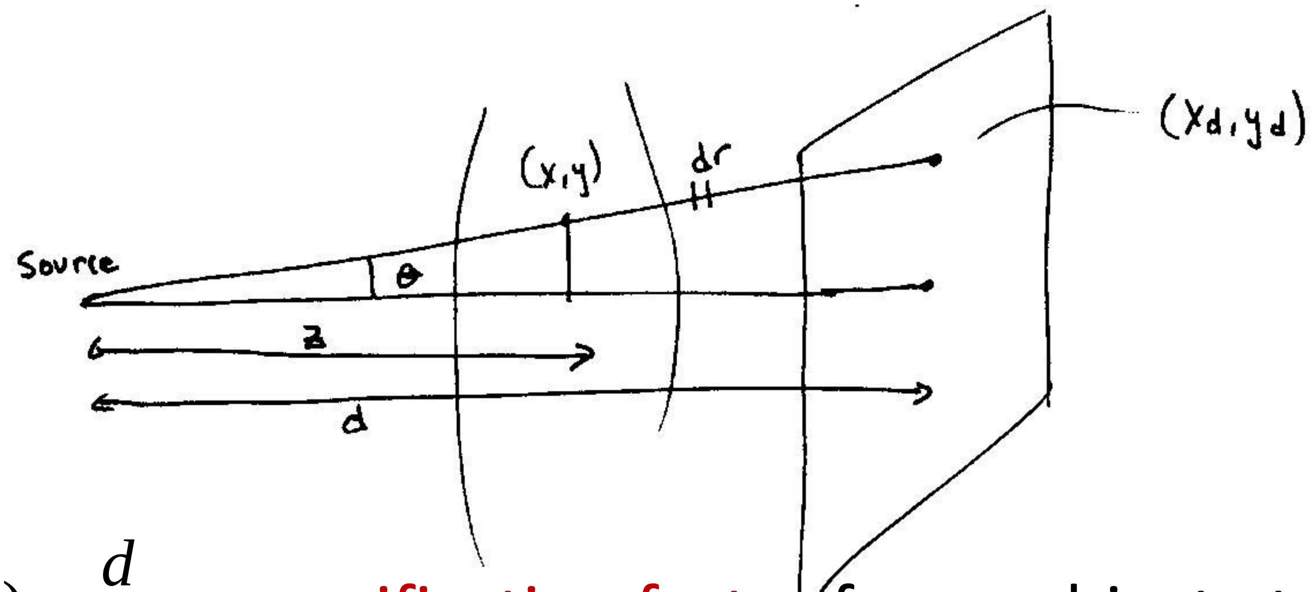
Point Source Geometry
Intensity Variations

Point Geometry Magnificataion

Practical X-ray Sources

Point Source

- A point in the object (x,y) at depth z - it will strike the detector at a position $(x_d, y_d) = \left(x\frac{d}{z}, y\frac{d}{z}\right)$



$M(z) = \frac{d}{z}$ - magnification factor for an object at depth z.

Point Source Magnification

- Attenuation coefficient at location (x_d, y_d) in output coordinates

$$I_d(x_d, y_d) = I_i(x_d, y_d)\exp\left(-\int \mu(x, y, z)dr\right)$$

$$\mu(x, y, z) = \mu\left(\frac{x_d}{M(z)}, \frac{y_d}{M(z)}, z\right)$$

$$I_i(x_d, y_d) = I_0 \frac{1}{\left(1+\left(\frac{r_d}{d}\right)^2\right)^{3/2}}$$

Point Source Magnification

- Attenuation coefficient at location (x_d, y_d) in output coordinates

$$I_d(x_d, y_d) = \frac{I_0}{\left(1 + \left(\frac{r_d}{d}\right)^2\right)^{3/2}} e^{-\int \mu\left(\frac{x_d}{M(z)}, \frac{x_d}{M(z)}, z\right) dr}$$

Finite (Large) Sources

Not a point source anymore

Finite (Large) Sources

- First consider a "thin" object. Let the attenuation coefficient be:

$$\mu(x, y, z) = \tau(x, y)\delta(z - z_0)$$

$$I_d(x_d, y_d) = I_0 \exp\left(-\int \tau\left(\frac{x_d}{M(z)}, \frac{y_d}{M(z)}\right)\delta(z - z_0)dz\right)$$

$$= I_0 \exp\left(-\tau\left(\frac{x_d}{M(z_0)}, \frac{y_d}{M(z_0)}\right)\right)$$

Finite (Large) Sources

- $M = M(z_0) = d / z_0$ - is the *object magnification factor*

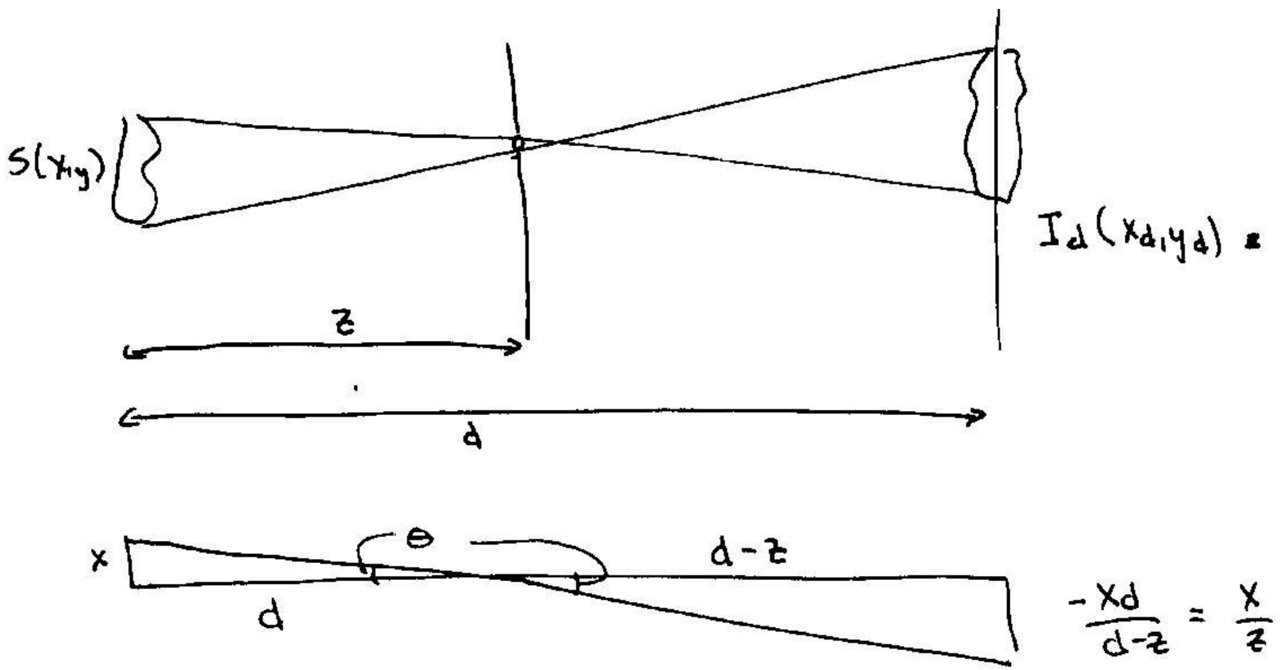
- Including the I_i term:

$$I_d(x_d, y_d) = I_i \exp\left(-\tau\left(\frac{x_d}{M}, \frac{y_d}{M}\right)\right) = I_i t\left(\frac{x_d}{M}, \frac{y_d}{M}\right)$$

where $t = \exp(-\tau)$ is the *transmission function*

Finite (Large) Sources

- Consider a finite source function $s(x,y)$ and a very small pinhole transmission function:



$$\frac{-x_d}{d-z} = \frac{x}{z}$$

$$x_d = -\left(\frac{d-z}{z}\right)\cdot x$$

$$= m(z)\cdot x$$

Finite (Large) Sources

- Now the image is an image of the source with the *source magnification factor*,

$$I_d(x_d, y_d) = ks\left(\frac{x_d}{m}, \frac{y_d}{m}\right) \qquad m = m(z) = -\frac{d - z}{z}$$

  - *k* - scaling factor that is proportional to the area of the pinhole
  - If I_d is to represent the impulse response of the system

Finite (Large) Sources

- The area of the pinhole: $\iint \delta(x, y)dxdy = 1$

- Capture efficiency of the pinhole

$$\eta = \frac{\text{pinhole area}}{4\pi z^2} = \frac{1}{4\pi z^2}$$

- Total number of photon emitted

$$N = \iint s(x, y)dxdy$$

Finite (Large) Sources

- Total number of photons to get through the pinhole $N\eta = \frac{N}{4\pi z^2}$
- The same number at the detector

$$\iint ks\left(\frac{x_d}{m}, \frac{y_d}{m}\right) dx_d dy_d = kNm^2 = \frac{N}{4\pi z^2}$$

- Hence, the scaling coefficient is

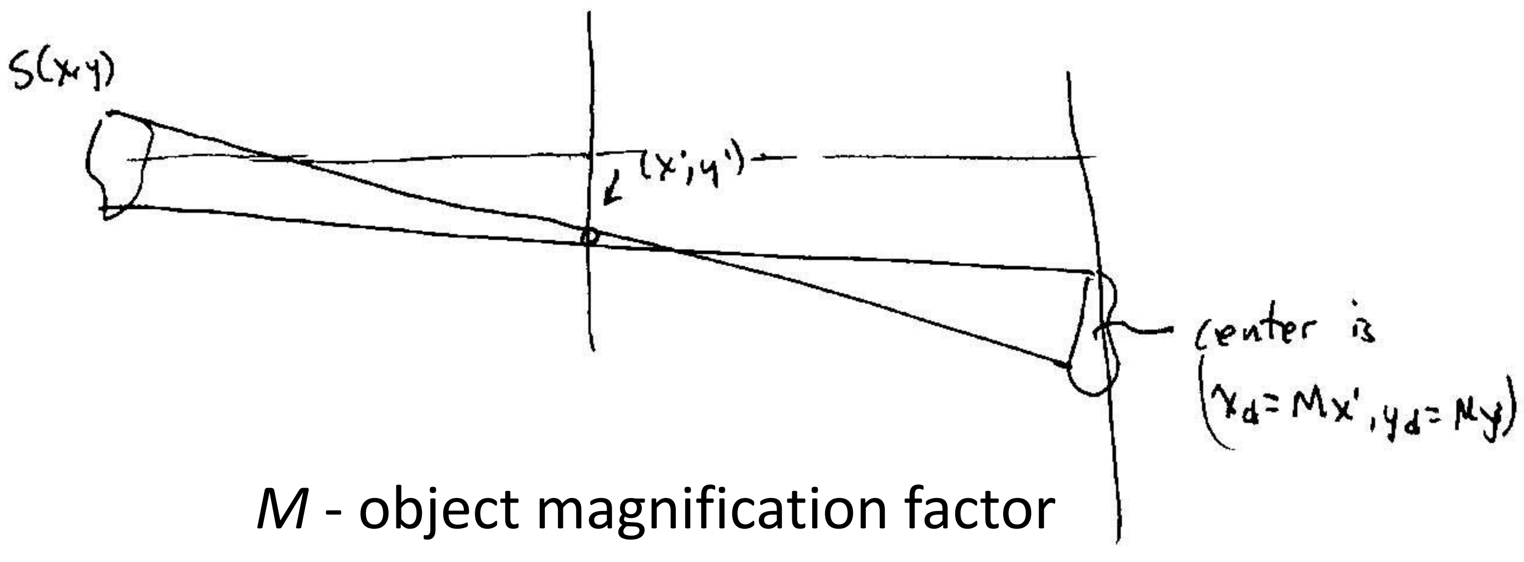
$$k = \frac{1}{4\pi z^2 m^2}$$

Finite (Large) Sources

$$I_d(x_d, y_d) = \frac{1}{4\pi z^2 m^2} s\left(\frac{x_d}{m}, \frac{y_d}{m}\right)$$

- let the pinhole be at position (x′,y′)

  $t(x,y) = \delta(x-x', y-y')$



M - object magnification factor

Finite (Large) Sources

- The image of the source is now located at (x_d=Mx', y_d=My')

- The impulse response function is:

$$h(x_d, y_d; x', y') = I_d(x_d, y_d) = \frac{1}{4\pi z^2 m^2} s\left(\frac{x_d - Mx'}{m}, \frac{y_d - My'}{m}\right)$$

- Now we can calculate the image for an arbitrary transmission function using the superposition integral – recall linear system in FT lectures.

Finite (Large) Sources

$$I_d(x_d, y_d) = \iint t(x'y')h(x_d, y_d; x', y')dx'dy'$$

$$= \frac{1}{4\pi z^2 m^2} \iint t(x'y')s\left(\frac{x_d - Mx'}{m}, \frac{y_d - My'}{m}\right)dx'dy' \text{ and sub } Mx' = x$$

$$= \frac{1}{4\pi z^2 m^2 M^2} \iint t\left(\frac{x}{M}, \frac{y}{M}\right)s\left(\frac{x_d - x}{m}, \frac{y_d - y}{m}\right)dxdy$$

$$= \frac{1}{4\pi d^2 m^2} s\left(\frac{x_d}{m}, \frac{y_d}{m}\right) ** t\left(\frac{x_d}{M}, \frac{y_d}{M}\right)$$

Finite (Large) Sources

- The final image is equal to the convolution of the magnified source and the magnified object.
  - **The object is blurred by the source function.**

- The frequency domain equivalent:

$$F_{2D}\{I_d(x_d, y_d)\} = \frac{1}{4\pi z^2} S(mu, mv) T(Mu, Mv)$$

Finite (Large) Sources

- The object is magnified by $M(z_0)$ and blurred by $m(z_0)$
  - $m(z_0) = -(d-z_0)/z_0 = 1-d/z_0$
  - $M(z_0) = d/z_0$

- If z_0 = 1/3 d, object is magnified by ? and blurred by ?

Finite (Large) Sources

- The object is magnified by $M(z_0)$ and blurred by $m(z_0)$
  - $m(z_0) = -(d-z_0)/z_0 = 1-d/z_0$
  - $M(z_0) = d/z_0$

- If $z_0 = 1/3\ d$, object is magnified by 3 times and blurred by -2 magnified source

Finite (Large) Sources

- The object is **magnified by $M(z_0)$** and **blurred by $m(z_0)$**
  - $m(z_0) = -(d-z_0)/z_0 = 1-d/z_0$
  - $M(z_0) = d/z_0$

- If $z_0 = 1/3\ d$, object is magnified by ? and blurred by ?

- If $z_0 = 2/3\ d$, object is magnified by ? and blurred by ?

Finite (Large) Sources

- The object is magnified by $M(z_0)$ and blurred by $m(z_0)$
  - $m(z_0) = -(d-z_0)/z_0 = 1-d/z_0$
  - $M(z_0) = d/z_0$

- If $z_0 = 1/3\ d$, object is magnified by 3 times and blurred by -2 magnified source
- If $z_0 = 2/3\ d$, object is magnified by 1.5 times and blurred by -0.5 magnified source

Finite (Large) Sources

- To minimize source blurring, we should
  - A) position the subject as close to the source as possible
  - B) position the subject right in the middle between the source and the detector
  - C) position the subject immediately next to or on top of the detector

Finite (Large) Sources

- To minimize the blurring, we should
  - A) position the subject as close to the source as possible
  - B) position the subject right in the middle between the source and the detector
  - C) position the subject immediately next to or on top of the detector

  - Answer: C)

Finite (Large) Sources

- To minimize the magnification of the object on the image, we should
  - A) position the subject as close to the source as possible
  - B) position the subject right in the middle between the source and the detector
  - C) position the subject immediately next to or on top of the detector

Finite (Large) Sources

- To minimize the magnification of the object on the image, we should
  - A) position the subject as close to the source as possible
  - B) position the subject right in the middle between the source and the detector
  - C) position the subject immediately next to or on top of the detector

  - Answer: C)

Finite (Large) Sources

- When $|m|$ is made small, i.e., the depth plane as far from the source as possible: $z_0 \rightarrow d$
  - *The least blurring*
  - Reduces geometric distortions
  - Common practice for x-ray imaging is to position the subject immediately next to or on top of the detector

Finite (Large) Sources

- If the thickness of the body is not a limiting factor, then let *d*, *z* → ∞ .
  - Make the system close to a parallel ray geometry with |*m*| = → 0 and M → 1.
  - Main problem: $I_0 \propto 1/d^2$ → 0 and SNR $\propto \sqrt{I_0}$ → 0.

- Make *s*(*x*,*y*) as small as possible to reduce blurring, but might reduce the number of photons created and thus reduce SNR.

Finite (Large) Sources

- For a complex object, make

$$\mu(x,y,z) = \sum \tau_i(x,y)\delta(z - z_i)$$

and each plane will have its own magnification factors.

  - gives you some idea of how blurring and magnification might affect different parts of a real object differently.

Questions?

Finite (Large) Sources

Overall System Response

Object Blurring

Overall System Response

- The detector response – $h(r_d)$
- Add the detector response to the other system elements. The overall system response is:

$$I_d(x_d, y_d) = \frac{1}{4\pi d^2 m^2} s\left(\frac{x_d}{m}, \frac{y_d}{m}\right) ** t\left(\frac{x_d}{M}, \frac{y_d}{M}\right) ** h(r_d)$$

Overall System Response

- The impulse response function

$$h(x_d, y_d) = \frac{1}{4\pi d^2 m^2} s\left(\frac{x_d}{m}, \frac{y_d}{m}\right) ** h(r_d)$$

- For a circularly symmetric source function:

$$h(x_d, y_d) = \frac{1}{4\pi d^2 m^2} s\left(\frac{r_d}{m}\right) ** h(r_d)$$

True or False

- The detector blurs the magnified object, not the source

- The magnified source also blurs the magnified object

- Source and object have the same magnification factors

True or False

- The detector blurs the magnified object, not the source True
- The magnified source also blurs the magnified object True
- Source and object have the same magnification factors False

Object Blurring

- Examine the response in the coordinate system of the object (x,y) rather than the detector (x_d,y_d):

$$I(x,y) = ks\left(\frac{Mx}{m},\frac{My}{m}\right)**t\left(x,y\right)**h(Mr_d)$$

- Effective magnification of source: $\left|\frac{m}{M}\right| = \frac{d-z}{d}$

- Effective magnification of the detector response $\frac{1}{M} = \frac{z}{d}$

Object Blurring

- The two magnified blurring effects are in competition:
  - To make the source blurring (d-z)/d small, make $z \rightarrow d$
  - To make the detector blurring z/d small, make $z \rightarrow 0$

Object Blurring

- Comments:
  - For most x-ray systems, the detector response is very small and the source is almost always bigger. Therefore, we would like to make $z \rightarrow d$.
  - For other kinds of systems, e.g. digital fluoroscopy systems, the detector resolution is a bit larger and for these systems an intermediate z may be appropriate.

Questions?

Over all System Response

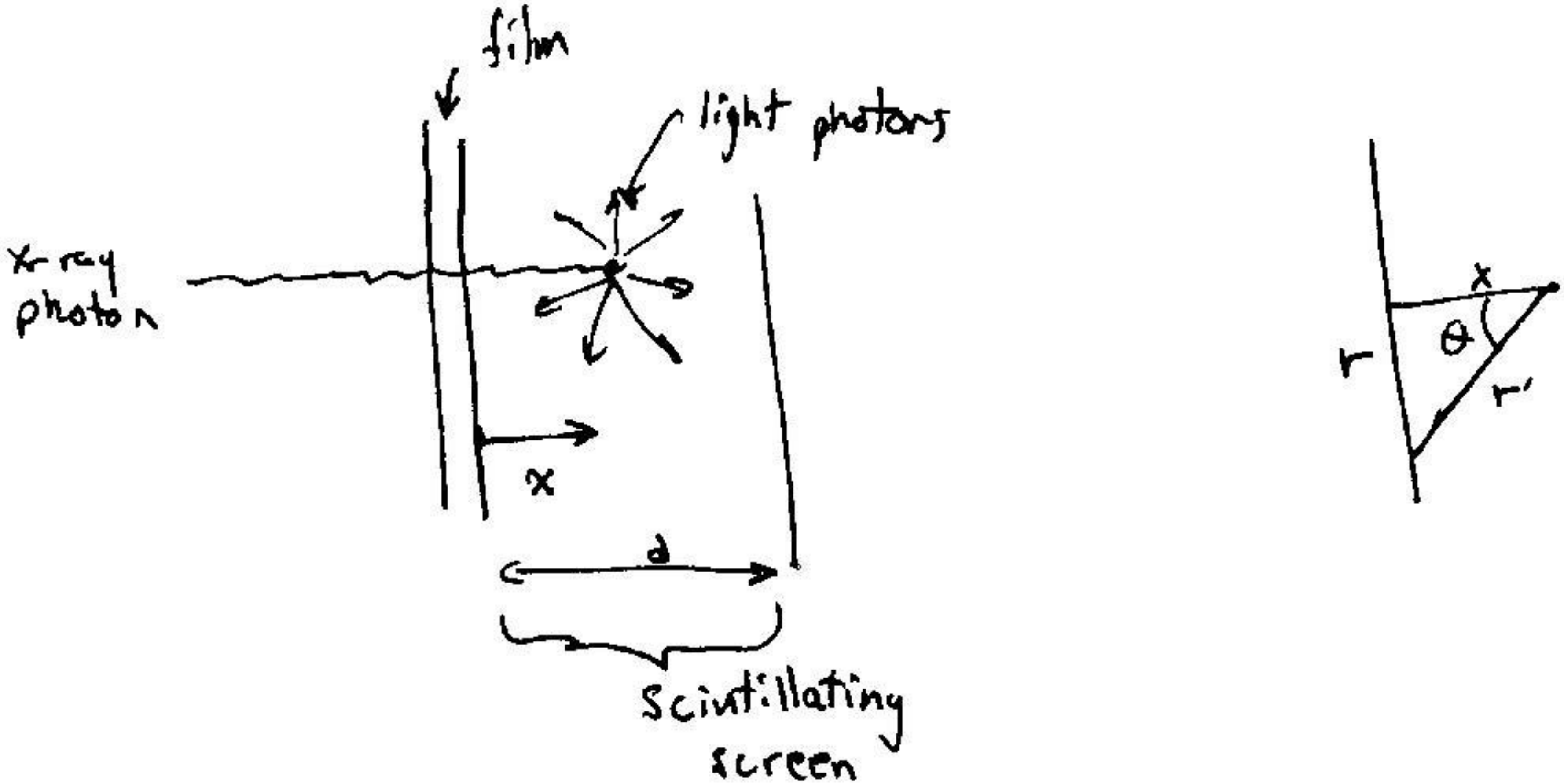
Object Blurring

X-ray Detection

X-ray Detection – Photographic Film

- Traditional x-ray detector – film based
  - Not used anymore
- Films are generally not very sensitive to x-rays, so x-rays must first be converted to visible light by a scintillator
- Materials such as sodium iodide (NaI) can "convert" an X-ray photon to a visible photon

X-ray Detection – Photographic Film



X-ray Detection – Image Plate

- Photo-stimulable phosphors (PSPs) are sensitive to X-rays (often called image plates)
  - Can store weak radioactive signal in a phosphor plate
  - Can be stimulated with visible light and produce a luminescent signal (image readout with a laser)
  - require higher X-ray exposure – needs intensifying screens.
  - Film X-ray equipment requires no modification to use them.

Flat Panel Detector

- Scintillator layer of gadolinium oxysulfide or cesium iodide converts x-rays to light.
- A silicon-on glass detector array (mix between LCD and digital camera sensor chip) is behind the scintillator layer.
- Each pixel on the glass detector contains a photodiode – generates electronic signal for producing a digital image.
- Typical pitch: 100 μm, 2-4k pixels/side

Flat Panel Detector

- Advantages
  - Electronic storage
  - More sensitive - allows a lower dose of radiation
  - Faster and cheaper than film.
  - Lighter, more durable, smaller in volume, more accurate, and have much less image distortion than image plates.
  - Can be produced in larger sizes

Flat Panel Detectors

https://landing1.gehealthcare.com/rs/005-SHS-767/images/Discovery%20XR656HD%20Product%20Datasheet.pdf

Philips C-arm

http://www.healthcare.philips.com/main/products/xray/products/c_arms/veradiusneo/specifications.wpd#FlatdetectorSection

Detector Issues

- In selecting detector characteristics, we will have a resolution/SNR trade-off
  - The thicker detectors have better SNR, but a larger impulse response (i.e. worse resolution)

Questions?

X-ray Detection

SNR and Noise in X-ray system

Noise in X-ray system

- X-ray images are created from intensity values that are related to the number of photons(N) that strike a detector element in a finite period of time (from intensity I)

- In X-ray, $SNR = \dfrac{\bar{S}}{\sigma_S} = \dfrac{N}{\sqrt{N}} = \sqrt{N}$

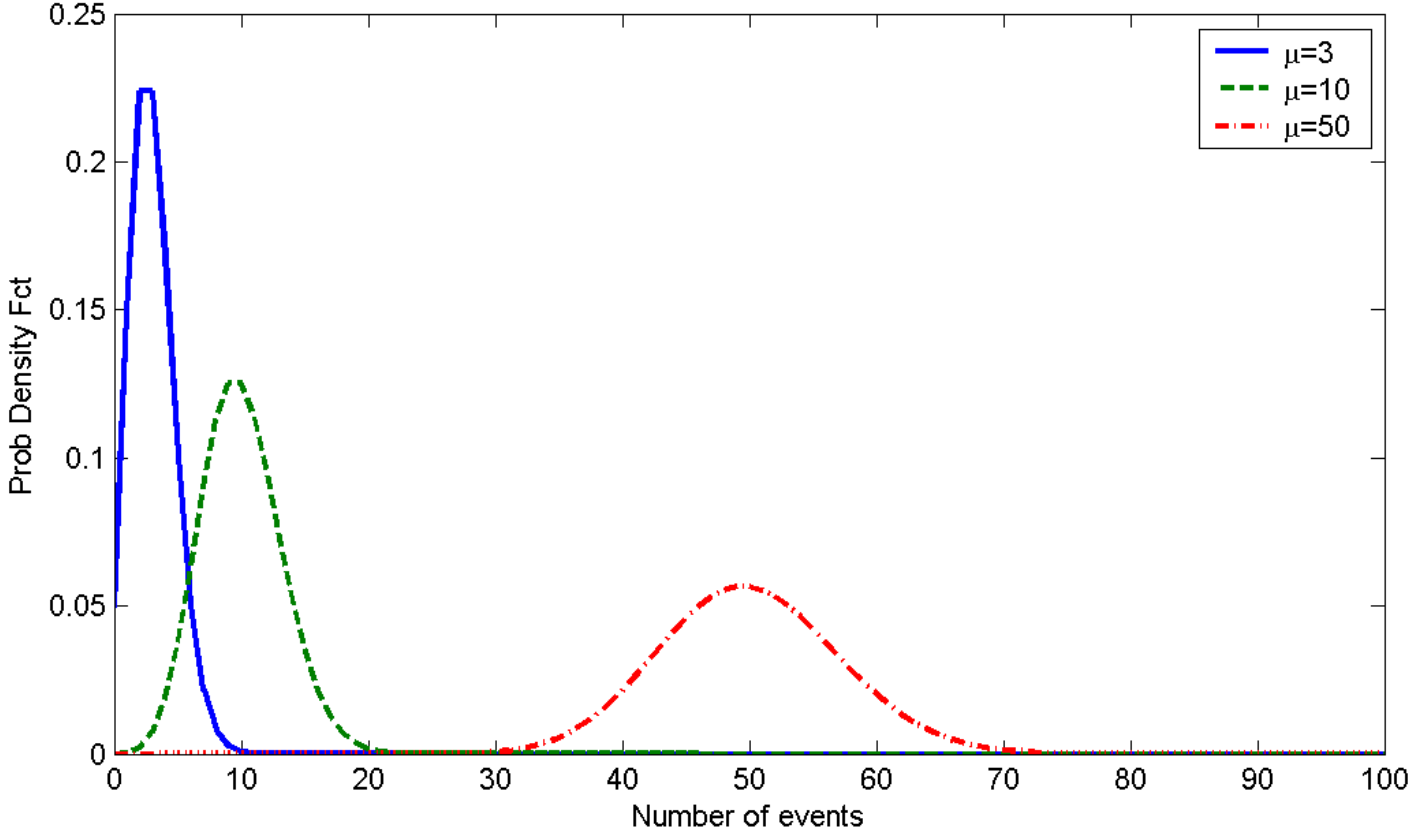
SNR of a Poisson Measurement

- SNR increases as the square root of the number of photons.
  - *SNR increases as the square root of the dose to the patient.*

- By averaging together two neighboring pixels, roughly double the photon counts and improve the SNR by $\sqrt{2}$.

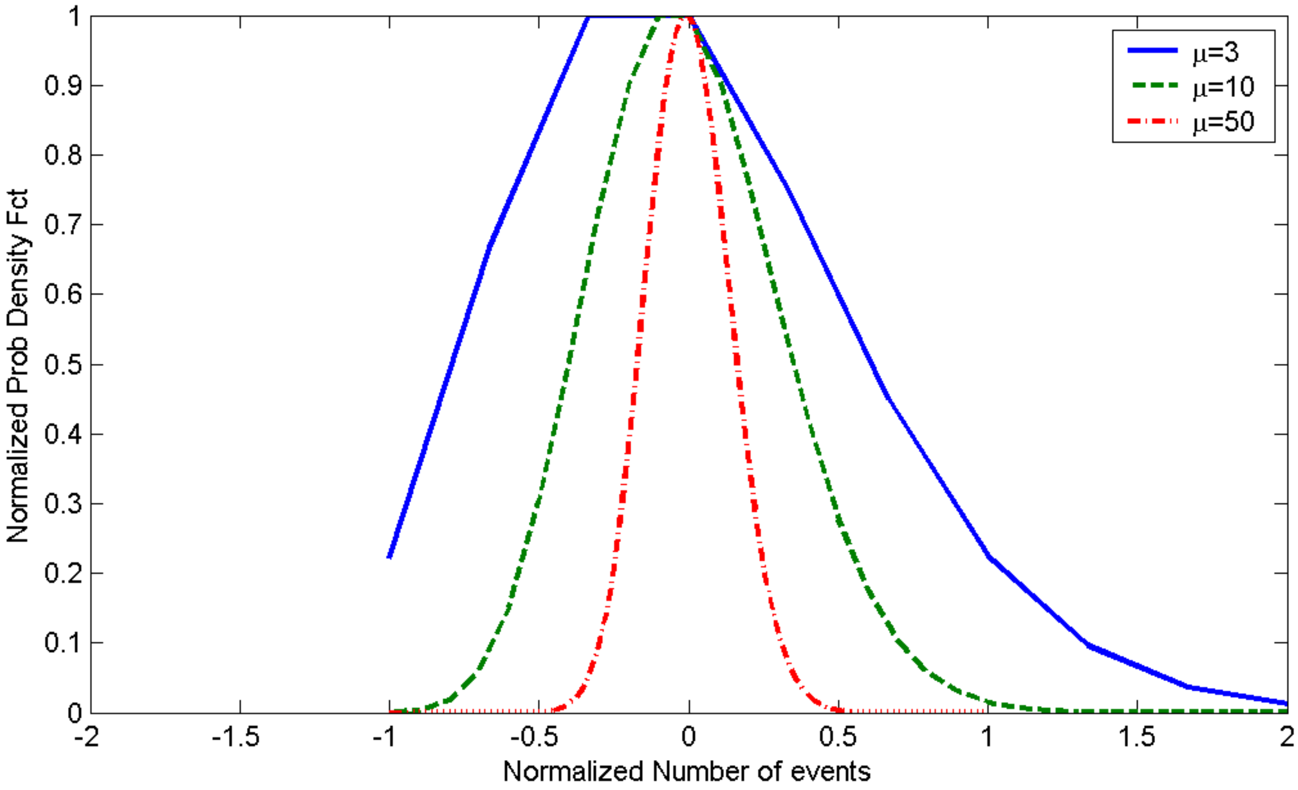
SNR of a Poisson Measurement

Poisson distributions as the mean increases from 3 to 50



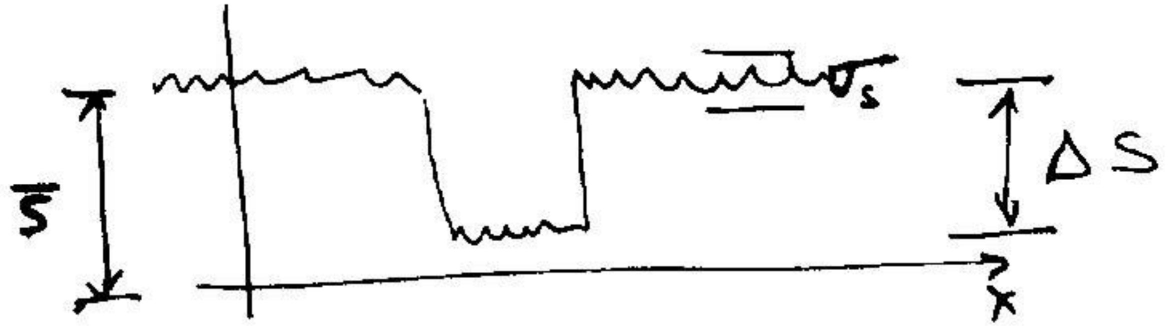
SNR of a Poisson Measurement

Poisson distributions (subtract the mean and divide the x-axis by the mean)



Noise in Detectors

- Consider an output to an x-ray



- We define a number of quantities

Noise in Detectors

We define a number of quantities

- Contrast: $C = \Delta S / \bar{S}$
- Signal to Noise Ratio: $SNR = \bar{S} / \sigma_s$
- Contrast to Noise Ratio:

$$CNR = \Delta S / \sigma_s = C \cdot SNR$$

Noise in Detectors

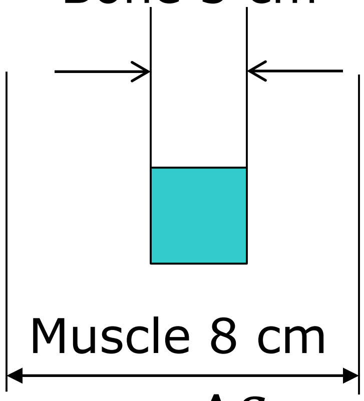
- Suppose the incident x-ray photons arriving at the detector are Poisson(N) and that the detector has efficiency η

$$SNR_{\text{det}} = \frac{\eta N}{\sqrt{\eta N}} = \sqrt{\eta N}$$

Example - Contrast

Photon energy = 90 keV

- Bone - $\mu_b = 0.4\ \text{cm}^{-1}$
- Muscle - $\mu_m = 0.2\ \text{cm}^{-1}$



- Calculate contrast $C = \frac{\Delta S}{\bar{S}}$. To simplify, use parallel x-ray beams.

Example : Contrast

- Muscle only:

$$I_m = I_0 e^{-\mu_m \cdot 8} = I_0 e^{-0.2 \cdot 8} = 0.202 I_0$$

- Muscle + bone:

$$I_{m+b} = I_0 e^{-\mu_m \cdot (8-3) - \mu_b \cdot 3} = I_0 e^{-0.2 \cdot 5 - 0.4 \cdot 3} = 0.111 I_0$$

- Calculate contrast:

$$C = \frac{\Delta S}{S} = \frac{I_m - I_{m+b}}{\mathrm{I_m}} = \frac{0.202 I_0 - 0.111 I_0}{0.202 I_0} = 0.45$$

Questions

SNR and noise