Named Arguments as Intersections, Optional Arguments as Unions

Yaozhu Sun

September 9, 2024

Syntax of λ_{iu} (target)

Types	$A, B ::= \top \mid \bot \mid \mathbf{Null} \mid \mathbb{Z} \mid A \to B \mid \{\ell : A\} \mid A \wedge B \mid A \vee B$
Expressions	$e ::= \{\} \mid \mathbf{null} \mid n \mid x \mid \lambda x : A.\ e : B \mid e_1\ e_2 \mid \{\ell = e\} \mid e.\ell \mid e_1 \,,\, e_2 \mid$ $\mathbf{switch}\ e_0\ \mathbf{as}\ x\ \mathbf{case}\ A \Rightarrow e_1\ \mathbf{case}\ B \Rightarrow e_2 \mid \mathbf{let}\ x = e_1\ \mathbf{in}\ e_2$

Syntax of Uaena[1] (source)

styp, $\mathcal{A}$, $\mathcal{B}$	$::=$		source types
	$\mid$	$\mathbb{Z}$	integer type
	$\mid$	$\mathcal{A} \to \mathcal{B}$	function type
	$\mid$	$\{\mathcal{P}\} \to \mathcal{B}$	function type with named parameters
nptyp, $\mathcal{P}$	$::=$		named parameter types
	$\mid$	$\cdot$	empty
	$\mid$	$\ell : \mathcal{A}; \mathcal{P}$	required parameter
	$\mid$	$\ell? : \mathcal{A}; \mathcal{P}$	optional parameter
sexp, ϵ	$::=$		source expressions
	$\mid$	n	integer literal
	$\mid$	x	variable
	$\mid$	$\lambda(x : \mathcal{A}).\ \epsilon$	abstraction
	$\mid$	$\epsilon_1\ \epsilon_2$	application
	$\mid$	$\lambda\{\rho\}.\ \epsilon$	abstraction with named parameters
	$\mid$	$\epsilon\ \{\alpha\}$	application to named arguments
npexp, ρ	$::=$		named parameters
	$\mid$	$\cdot$	empty
	$\mid$	$\ell : \mathcal{A}; \rho$	required parameter
	$\mid$	$\ell = \epsilon; \rho$	optional parameter
narg, α	$::=$		named arguments
	$\mid$	$\cdot$	empty
	$\mid$	$\ell = \epsilon; \alpha$	field
	$\mid$	$\alpha \backslash \ell$	removal

[1] Uaena = Unnamed Arguments Encoded Named Arguments

From Uaena to λ_{iu}

$\boxed{\Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e}$ *(Elaboration)*

$$\textsc{Ela-Int}\quad \frac{}{\Delta \vdash n : \mathbb{Z} \rightsquigarrow n}$$

$$\textsc{Ela-Var}\quad \frac{x : \mathcal{A} \in \Delta}{\Delta \vdash x : \mathcal{A} \rightsquigarrow x}$$

$$\textsc{Ela-Abs}\quad \frac{\Delta,\, x : \mathcal{A} \vdash \epsilon : \mathcal{B} \rightsquigarrow e \qquad |\mathcal{A}| = A \qquad |\mathcal{B}| = B}{\Delta \vdash \lambda(x{:}\mathcal{A}).\, \epsilon : \mathcal{A} \to \mathcal{B} \rightsquigarrow \lambda x{:}A.\, e{:}B}$$

$$\textsc{Ela-App}\quad \frac{\Delta \vdash \epsilon_1 : \mathcal{A} \to \mathcal{B} \rightsquigarrow e_1 \qquad \Delta \vdash \epsilon_2 : \mathcal{A} \rightsquigarrow e_2}{\Delta \vdash \epsilon_1\, \epsilon_2 : \mathcal{B} \rightsquigarrow e_1\, e_2}$$

$$\textsc{Ela-NAbs}\quad \frac{\textbf{fresh}\, x \qquad \Delta; x \vdash \rho : \mathcal{P} \rightsquigarrow \mathit{letin} \dashv \Delta' \qquad \Delta' \vdash \epsilon : \mathcal{B} \rightsquigarrow e \qquad |\mathcal{P}| = A \qquad |\mathcal{B}| = B}{\Delta \vdash \lambda\{\rho\}.\, \epsilon : \{\mathcal{P}\} \to \mathcal{B} \rightsquigarrow \lambda x{:}A.\, \mathit{letin}\, e{:}B}$$

$$\textsc{Ela-NApp}\quad \frac{\Delta \vdash \epsilon : \{\mathcal{P}\} \to \mathcal{B} \rightsquigarrow e \qquad \Delta \vdash \mathcal{P} \diamond \alpha \rightsquigarrow e'}{\Delta \vdash \epsilon\, \{\alpha\} : \mathcal{B} \rightsquigarrow e\, e'}$$

$\boxed{\Delta; x \vdash \rho : \mathcal{P} \rightsquigarrow \mathit{letin} \dashv \Delta'}$ *(Parameter elaboration)*

$$\textsc{PEla-Empty}\quad \frac{}{\Delta; x \vdash \cdot : \cdot \rightsquigarrow \textbf{id} \dashv \Delta}$$

$$\textsc{PEla-Required}\quad \frac{\Delta; x \vdash \rho : \mathcal{P} \rightsquigarrow \mathit{letin} \dashv \Delta'}{\Delta; x \vdash (\ell : \mathcal{A}; \rho) : (\ell : \mathcal{A}; \mathcal{P}) \rightsquigarrow \textbf{let}\, \ell = x.\ell\, \textbf{in} \circ \mathit{letin} \dashv \Delta',\, \ell : \mathcal{A}}$$

$$\textsc{PEla-Optional}\quad \frac{\Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e \qquad |\mathcal{A}| = A \qquad \Delta; x \vdash \rho : \mathcal{P} \rightsquigarrow \mathit{letin} \dashv \Delta'}{\Delta; x \vdash (\ell = \epsilon; \rho) : (\ell? : \mathcal{A}; \mathcal{P}) \rightsquigarrow \textbf{let}\, \ell = \textbf{switch}\, x.\ell\, \textbf{as}\, y\, \textbf{case}\, A \Rightarrow y\, \textbf{case}\, \textbf{Null} \Rightarrow e\, \textbf{in} \circ \mathit{letin} \dashv \Delta',\, \ell : \mathcal{A}}$$

$\boxed{\Delta \vdash \mathcal{P} \diamond \alpha \rightsquigarrow e}$ *(Parameter matching)*

$$\textsc{PMat-Empty}\quad \frac{}{\Delta \vdash \cdot \diamond \cdot \rightsquigarrow \{\}}$$

$$\textsc{PMat-Extra}\quad \frac{\Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e \qquad \Delta \vdash \cdot \diamond \alpha \rightsquigarrow e'}{\Delta \vdash \cdot \diamond (\ell = \epsilon; \alpha) \rightsquigarrow \{\ell = e\} \,,\, e'}$$

$$\textsc{PMat-Present}\quad \frac{\alpha.\ell \Rightarrow \epsilon \qquad \Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e \qquad \Delta \vdash \mathcal{P} \diamond \alpha \backslash \ell \rightsquigarrow e'}{\Delta \vdash (\ell : \mathcal{A}; \mathcal{P}) \diamond \alpha \rightsquigarrow \{\ell = e\} \,,\, e'}$$

$$\textsc{PMat-Absent}\quad \frac{\alpha.\ell \not\Rightarrow \qquad \Delta \vdash \mathcal{P} \diamond \alpha \rightsquigarrow e'}{\Delta \vdash (\ell : \mathcal{A}; \mathcal{P}) \diamond \alpha \rightsquigarrow \{\ell = \textbf{null}\} \,,\, e'}$$

$\boxed{|\mathcal{A}|}$ Type translation

$$\begin{aligned}
&|\mathbb{Z}| \equiv \mathbb{Z}\\
&|\mathcal{A} \to \mathcal{B}| \equiv |\mathcal{A}| \to |\mathcal{B}|\\
&|\{\mathcal{P}\} \to \mathcal{B}| \equiv |\mathcal{P}| \to |\mathcal{B}|
\end{aligned}$$

$\boxed{|\mathcal{P}|}$ Parameter type translation

$$\begin{aligned}
&|\cdot| \equiv \top\\
&|\ell : \mathcal{A}; \mathcal{P}| \equiv \{\ell : |\mathcal{A}|\} \wedge |\mathcal{P}|\\
&|\ell? : \mathcal{A}; \mathcal{P}| \equiv \{\ell : |\mathcal{A}| \vee \textbf{Null}\} \wedge |\mathcal{P}|
\end{aligned}$$

$\boxed{|\Delta|}$ Typing context translation

$$\begin{aligned}
&|\cdot| \equiv \cdot\\
&|\Delta,\, x : \mathcal{A}| \equiv |\Delta|,\, x : |\mathcal{A}|
\end{aligned}$$

Example 1

$$letin_1 = \textbf{let}\, x = z.x\, \textbf{in} \qquad letin_2 = \textbf{let}\, y = \textbf{switch}\, z.y\, \textbf{as}\, y\, \textbf{case}\, \mathbb{Z} \Rightarrow y\, \textbf{case}\, \textbf{Null} \Rightarrow 0\, \textbf{in} \qquad e_0 = letin_1 \circ letin_2\ (x+y)$$

$$\dfrac{\cdot, y:\mathbb{Z}, x:\mathbb{Z} \vdash x+y:\mathbb{Z} \rightsquigarrow x+y \qquad \dfrac{\dfrac{\cdot \vdash 0:\mathbb{Z} \rightsquigarrow 0 \qquad \dfrac{}{\cdot; z \vdash \cdot : \cdot \rightsquigarrow \textbf{id} \dashv \cdot}\;\textit{PEla-Empty}}{\cdot; z \vdash (y=0;\cdot) : (y?:\mathbb{Z};\cdot) \rightsquigarrow letin_2 \dashv \cdot, y:\mathbb{Z}}\;\textit{PEla-Optional}}{\cdot; z \vdash (x:\mathbb{Z}; y=0;\cdot) : (x:\mathbb{Z}; y?:\mathbb{Z};\cdot) \rightsquigarrow letin_1 \circ letin_2 \dashv \cdot, y:\mathbb{Z}, x:\mathbb{Z}}\;\textit{PEla-Required}}{\cdot \vdash (\lambda\{x:\mathbb{Z}; y=0;\cdot\}.\, x+y) : \{x:\mathbb{Z}; y?:\mathbb{Z};\cdot\} \to \mathbb{Z} \rightsquigarrow \lambda z{:}\{x:\mathbb{Z}\} \wedge \{y:\mathbb{Z} \vee \textbf{Null}\}.\, e_0{:}\mathbb{Z}}\;\textit{Ela-NPAbs}$$

$$\dfrac{\cdots\textit{Ela-NAbs}\cdots \qquad \dfrac{\cdot \vdash 1:\mathbb{Z} \rightsquigarrow 1 \qquad \dfrac{\dfrac{\cdot \vdash 2:\mathbb{Z} \rightsquigarrow 2 \qquad \dfrac{}{\cdot \vdash \cdot \diamond \cdot \rightsquigarrow \{\}}\;\textit{PMat-Empty}}{\cdot \vdash \cdot \diamond (z=2;\cdot) \rightsquigarrow \{z=2\}}\;\textit{PMat-Extra}}{\cdot \vdash (y?:\mathbb{Z};\cdot) \diamond (z=2;\cdot) \rightsquigarrow \{y=\textbf{null}\} , \{z=2\}}\;\textit{PMat-Absent}}{\cdot \vdash (x:\mathbb{Z}; y?:\mathbb{Z};\cdot) \diamond (x=1; z=2;\cdot) \rightsquigarrow \{x=1\} , \{y=\textbf{null}\} , \{z=2\}}\;\textit{PMat-Present}}{\cdot \vdash (\lambda\{x:\mathbb{Z}; y=0;\cdot\}.\, x+y)\,\{x=1; z=2;\cdot\} : \mathbb{Z} \rightsquigarrow (\lambda z{:}\{x:\mathbb{Z}\} \wedge \{y:\mathbb{Z} \vee \textbf{Null}\}.\, e_0{:}\mathbb{Z})\,(\{x=1\} , \{y=\textbf{null}\} , \{z=2\})}\;\textit{Ela-NApp}$$

Theorem 1 (Elaboration soundness) *If* $\Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e$, *then* $|\Delta| \vdash e : |\mathcal{A}|$.

Appendix: Semantics of λ_{iu}

$\boxed{A <: B}$ *(Subtyping)*

$$\textsc{Sub-Top}\quad \frac{}{A <: \top} \qquad \textsc{Sub-Bot}\quad \frac{}{\bot <: A} \qquad \textsc{Sub-Int}\quad \frac{}{\mathbb{Z} <: \mathbb{Z}}$$

$$\textsc{Sub-Arrow}\quad \frac{B_1 <: A_1 \qquad A_2 <: B_2}{A_1 \to A_2 <: B_1 \to B_2} \qquad \textsc{Sub-Rcd}\quad \frac{A <: B}{\{\ell : A\} <: \{\ell : B\}}$$

$$\textsc{Sub-And}\quad \frac{A <: B \qquad A <: C}{A <: B \wedge C} \qquad \textsc{Sub-AndL}\quad \frac{A <: C}{A \wedge B <: C} \qquad \textsc{Sub-AndR}\quad \frac{B <: C}{A \wedge B <: C}$$

$$\textsc{Sub-Or}\quad \frac{A <: C \qquad B <: C}{A \vee B <: C} \qquad \textsc{Sub-OrL}\quad \frac{A <: B}{A <: B \vee C} \qquad \textsc{Sub-OrR}\quad \frac{A <: C}{A <: B \vee C}$$

$\boxed{\Gamma \vdash e : A}$ *(Typing)*

$$\textsc{Typ-Top}\quad \frac{}{\Gamma \vdash \{\} : \top} \qquad \textsc{Typ-Int}\quad \frac{}{\Gamma \vdash n : \mathbb{Z}} \qquad \textsc{Typ-Var}\quad \frac{x : A \in \Gamma}{\Gamma \vdash x : A}$$

$$\textsc{Typ-Abs}\quad \frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash (\lambda x \!:\! A.\ e \!:\! B) : A \to B}$$

$$\textsc{Typ-App}\quad \frac{\Gamma \vdash e_1 : A \to B \qquad \Gamma \vdash e_2 : A}{\Gamma \vdash e_1\, e_2 : B} \qquad \textsc{Typ-Rcd}\quad \frac{\Gamma \vdash e : A}{\Gamma \vdash \{\ell = e\} : \{\ell : A\}} \qquad \textsc{Typ-Prj}\quad \frac{\Gamma \vdash e : \{\ell : A\}}{\Gamma \vdash e.\ell : A}$$

$$\textsc{Typ-Merge}\quad \frac{\Gamma \vdash e_1 : A \qquad \Gamma \vdash e_2 : B}{\Gamma \vdash e_1 \,,\, e_2 : A \wedge B}$$

$$\textsc{Typ-Switch}\quad \frac{\begin{array}{c}\Gamma \vdash e : A \vee B \\ \Gamma, x : A \vdash e_1 : C \\ \Gamma, x : B \vdash e_2 : C\end{array}}{\Gamma \vdash \textbf{switch}\ e_0\ \textbf{as}\ x\ \textbf{case}\ A \Rightarrow e_1\ \textbf{case}\ B \Rightarrow e_2 : C}$$

$$\textsc{Typ-Let}\quad \frac{\Gamma \vdash \mathit{letin} \dashv \Gamma' \qquad \Gamma' \vdash e : A}{\Gamma \vdash \mathit{letin}\ e : A} \qquad \textsc{Typ-Sub}\quad \frac{\Gamma \vdash e : A \qquad A <: B}{\Gamma \vdash e : B}$$