Named Arguments as Intersections, Optional Arguments as Unions

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Syntax of λ_{iu} (target)

$$
\begin{array}{lrl}
\text{Types} & A, B ::= & \top \mid \bot \mid \mathbf{Null} \mid \mathbb{Z} \mid A \to B \mid \{\ell : A\} \mid A \wedge B \mid A \vee B \\
\text{Expressions} & e ::= & \{\} \mid \mathbf{null} \mid n \mid x \mid \lambda x : A.\ e : B \mid e_1\ e_2 \mid \{\ell : A = e\} \mid e.\ell \\
 & & \mid e_1 \,,\, e_2 \mid \mathbf{switch}\ e_0\ \mathbf{as}\ x\ \mathbf{case}\ A \Rightarrow e_1\ \mathbf{case}\ B \Rightarrow e_2 \mid \mathit{letin}\ e \\
\text{Let-in bindings} & \mathit{letin} ::= & \mathbf{let}\ x = e\ \mathbf{in} \mid \mathit{letin}_1 \circ \mathit{letin}_2 \mid \mathbf{id}
\end{array}
$$

Syntax of UAENA[1] (source)

$$
\begin{array}{lrl}
\text{Types} & \mathcal{A}, \mathcal{B} ::= & \mathbb{Z} \mid (\mathcal{A}) \to \mathcal{B} \mid \{\mathcal{P}\} \to \mathcal{B} \mid \{\mathcal{K}\} \\
\text{Named parameter types} & \mathcal{P} ::= & \cdot \mid \mathcal{P};\, \ell : \mathcal{A} \mid \mathcal{P};\, \ell ? : \mathcal{A} \\
\text{Named argument types} & \mathcal{K} ::= & \cdot \mid \mathcal{K};\, \ell : \mathcal{A} \\
\text{Expressions} & \epsilon ::= & n \mid x \mid \lambda(x : \mathcal{A}).\ \epsilon \mid \lambda\{\rho\}.\ \epsilon \mid \epsilon_1\ \epsilon_2 \mid \{\kappa\} \\
\text{Named parameters} & \rho ::= & \cdot \mid \rho;\, \ell : \mathcal{A} \mid \rho;\, \ell = \epsilon \\
\text{Named arguments} & \kappa ::= & \cdot \mid \kappa;\, \ell = \epsilon
\end{array}
$$

[1] UAENA = Unnamed Arguments Extended with Named Arguments

From $\textsc{Uaena}$ to λ_{iu}

$\boxed{\Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e}$ *(Elaboration)*

$$\textsc{Ela-Int}\quad \frac{}{\Delta \vdash n : \mathbb{Z} \rightsquigarrow n}$$

$$\textsc{Ela-Var}\quad \frac{x : \mathcal{A} \in \Delta}{\Delta \vdash x : \mathcal{A} \rightsquigarrow x}$$

$$\textsc{Ela-Abs}\quad \frac{\Delta,\, x : \mathcal{A} \vdash \epsilon : \mathcal{B} \rightsquigarrow e}{\Delta \vdash \lambda(x\!:\!\mathcal{A}).\ \epsilon : (\mathcal{A}) \to \mathcal{B} \rightsquigarrow \lambda x\!:\!|\mathcal{A}|.\ e\!:\!|\mathcal{B}|}$$

$$\textsc{Ela-NAbs}\quad \frac{\begin{array}{c}\Delta \vdash_x \rho : \mathcal{P} \rightsquigarrow \mathit{letin} \dashv \Delta' \\ \Delta' \vdash \epsilon : \mathcal{B} \rightsquigarrow e\end{array}}{\Delta \vdash \lambda\{\rho\}.\ \epsilon : \{\mathcal{P}\} \to \mathcal{B} \rightsquigarrow \lambda x\!:\!|\mathcal{P}|.\ \mathit{letin}\ e\!:\!|\mathcal{B}|}$$

$$\textsc{Ela-App}\quad \frac{\begin{array}{c}\Delta \vdash \epsilon_1 : (\mathcal{A}) \to \mathcal{B} \rightsquigarrow e_1 \\ \Delta \vdash \epsilon_2 : \mathcal{A} \rightsquigarrow e_2\end{array}}{\Delta \vdash \epsilon_1\, \epsilon_2 : \mathcal{B} \rightsquigarrow e_1\, e_2}$$

$$\textsc{Ela-NApp}\quad \frac{\begin{array}{c}\Delta \vdash \epsilon_1 : \{\mathcal{P}\} \to \mathcal{B} \rightsquigarrow e_1 \\ \Delta \vdash \epsilon_2 : \{\mathcal{K}\} \rightsquigarrow e_2 \\ \Delta \vdash_{e_2} \mathcal{P} \diamond \mathcal{K} \rightsquigarrow e_2'\end{array}}{\Delta \vdash \epsilon_1\, \epsilon_2 : \mathcal{B} \rightsquigarrow e_1\, e_2'}$$

$$\textsc{Ela-NEmpty}\quad \frac{}{\Delta \vdash \{\cdot\} : \{\cdot\} \rightsquigarrow \{\}}$$

$$\textsc{Ela-NField}\quad \frac{\begin{array}{c}\Delta \vdash \{\kappa\} : \{\mathcal{K}\} \rightsquigarrow e' \\ \Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e\end{array}}{\Delta \vdash \{\kappa;\ \ell = \epsilon\} : \{\mathcal{K};\ \ell : \mathcal{A}\} \rightsquigarrow e' \,,\, \{\ell : |\mathcal{A}| = e\}}$$

$\boxed{\Delta \vdash_x \rho : \mathcal{P} \rightsquigarrow \mathit{letin} \dashv \Delta'}$ *(Named parameter elaboration)*

$$\textsc{PEla-Empty}\quad \frac{}{\Delta \vdash_x \cdot : \cdot \rightsquigarrow \mathbf{id} \dashv \Delta}$$

$$\textsc{PEla-Required}\quad \frac{\Delta \vdash_x \rho : \mathcal{P} \rightsquigarrow \mathit{letin} \dashv \Delta'}{\Delta \vdash_x (\rho;\ \ell : \mathcal{A}) : (\mathcal{P};\ \ell : \mathcal{A}) \rightsquigarrow \mathit{letin} \circ \mathbf{let}\, \ell = x.\ell\, \mathbf{in} \dashv \Delta',\, \ell : \mathcal{A}}$$

$$\textsc{PEla-Optional}\quad \frac{\begin{array}{c}\Delta \vdash_x \rho : \mathcal{P} \rightsquigarrow \mathit{letin} \dashv \Delta' \\ \Delta' \vdash \epsilon : \mathcal{A} \rightsquigarrow e\end{array}}{\Delta \vdash_x (\rho;\ \ell = \epsilon) : (\mathcal{P};\ \ell? : \mathcal{A}) \rightsquigarrow \mathit{letin} \circ \mathbf{let}\, \ell = \mathbf{switch}\, x.\ell\, \mathbf{as}\, y\, \mathbf{case}\, |\mathcal{A}| \Rightarrow y\, \mathbf{case}\, \mathbf{Null} \Rightarrow e\, \mathbf{in} \dashv \Delta',\, \ell : \mathcal{A}}$$

$\boxed{\Delta \vdash_e \mathcal{P} \diamond \mathcal{K} \rightsquigarrow e'}$ *(Call site rewriting)*

$$\textsc{PMat-Empty}\quad \frac{}{\Delta \vdash_e \cdot \diamond \mathcal{K} \rightsquigarrow \{\}}$$

$$\textsc{PMat-Required}\quad \frac{\begin{array}{c}\mathcal{K} :: \ell \Rightarrow \mathcal{A} \\ \Delta \vdash_e \mathcal{P} \diamond \mathcal{K} \rightsquigarrow e'\end{array}}{\Delta \vdash_e (\mathcal{P};\ \ell : \mathcal{A}) \diamond \mathcal{K} \rightsquigarrow e' \,,\, \{\ell : |\mathcal{A}| = e.\ell\}}$$

$$\textsc{PMat-Present}\quad \frac{\begin{array}{c}\mathcal{K} :: \ell \Rightarrow \mathcal{A} \\ \Delta \vdash_e \mathcal{P} \diamond \mathcal{K} \rightsquigarrow e'\end{array}}{\Delta \vdash_e (\mathcal{P};\ \ell? : \mathcal{A}) \diamond \mathcal{K} \rightsquigarrow e' \,,\, \{\ell : |\mathcal{A}| \vee \mathbf{Null} = e.\ell\}}$$

$$\textsc{PMat-Absent}\quad \frac{\begin{array}{c}\mathcal{K} :: \ell \not\Rightarrow \\ \Delta \vdash_e \mathcal{P} \diamond \mathcal{K} \rightsquigarrow e'\end{array}}{\Delta \vdash_e (\mathcal{P};\ \ell? : \mathcal{A}) \diamond \mathcal{K} \rightsquigarrow e' \,,\, \{\ell : |\mathcal{A}| \vee \mathbf{Null} = \mathbf{null}\}}$$

$\boxed{\mathcal{K} :: \ell \Rightarrow \mathcal{A}}$ *(Successful lookup)*

$$\textsc{LU-Present}\quad \frac{\mathcal{K} :: \ell \not\Rightarrow}{(\mathcal{K};\ \ell : \mathcal{A}) :: \ell \Rightarrow \mathcal{A}}$$

$$\textsc{LU-Absent}\quad \frac{\ell' \neq \ell \qquad \mathcal{K} :: \ell \Rightarrow \mathcal{A}}{(\mathcal{K};\ \ell' : \mathcal{B}) :: \ell \Rightarrow \mathcal{A}}$$

$\boxed{\mathcal{K} :: \ell \not\Rightarrow}$ *(Failed lookup)*

$$\frac{}{\cdot :: \ell \not\Rightarrow}\ \textsc{LD-Empty} \qquad \frac{\ell' \neq \ell \qquad \mathcal{K} :: \ell \not\Rightarrow}{(\mathcal{K};\, \ell' : \mathcal{A}) :: \ell \not\Rightarrow}\ \textsc{LD-Absent}$$

$\boxed{|\mathcal{A}|}$ Type translation

$$\begin{aligned}
&|\mathbb{Z}| \equiv \mathbb{Z}\\
&|(\mathcal{A}) \to \mathcal{B}| \equiv |\mathcal{A}| \to |\mathcal{B}|\\
&|\{\mathcal{P}\} \to \mathcal{B}| \equiv |\mathcal{P}| \to |\mathcal{B}|\\
&|\{\mathcal{K}\}| \equiv |\mathcal{K}|
\end{aligned}$$

$\boxed{|\mathcal{P}|}$ Parameter type translation

$$\begin{aligned}
&|\cdot| \equiv \top\\
&|\mathcal{P};\, \ell : \mathcal{A}| \equiv |\mathcal{P}| \wedge \{\ell : |\mathcal{A}|\}\\
&|\mathcal{P};\, \ell? : \mathcal{A}| \equiv |\mathcal{P}| \wedge \{\ell : |\mathcal{A}| \vee \mathbf{Null}\}
\end{aligned}$$

$\boxed{|\mathcal{K}|}$ Argument type translation

$$\begin{aligned}
&|\cdot| \equiv \top\\
&|\mathcal{K};\, \ell : \mathcal{A}| \equiv |\mathcal{K}| \wedge \{\ell : |\mathcal{A}|\}
\end{aligned}$$

$\boxed{|\Delta|}$ Typing context translation

$$\begin{aligned}
&|\cdot| \equiv \cdot\\
&|\Delta,\, x : \mathcal{A}| \equiv |\Delta|,\, x : |\mathcal{A}|
\end{aligned}$$

Theorem 1 (Elaboration soundness) *If* $\Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e$, *then* $|\Delta| \vdash e : |\mathcal{A}|$.

Example 1

$$letin_1 \equiv \mathbf{let}\ x = args.x\ \mathbf{in} \qquad letin_2 \equiv \mathbf{let}\ y = \mathbf{switch}\ args.y\ \mathbf{as}\ y\ \mathbf{case}\ \mathbb{Z} \Rightarrow y\ \mathbf{case}\ \mathbf{Null} \Rightarrow x+1\ \mathbf{in} \qquad e_0 \equiv letin_1 \circ letin_2\ (x+y)$$

$$\dfrac{\cdot, x:\mathbb{Z}, y:\mathbb{Z} \vdash x+y:\mathbb{Z} \leadsto x+y \qquad \dfrac{\cdot, x:\mathbb{Z} \vdash x+1:\mathbb{Z} \leadsto x+1 \qquad \dfrac{\cdot \vdash 0:\mathbb{Z} \leadsto 0 \qquad \dfrac{}{\cdot \vdash_{args} \cdot : \cdot \leadsto \mathbf{id} \dashv \cdot}\,\textit{PEla-Empty}}{\cdot \vdash_{args} (\cdot; x:\mathbb{Z}) : (\cdot; x:\mathbb{Z}) \leadsto letin_1 \dashv \cdot, x:\mathbb{Z}}\,\textit{PEla-Required}}{\cdot \vdash_{args} (\cdot; x:\mathbb{Z}; y = x+1) : (\cdot; x:\mathbb{Z}; y?:\mathbb{Z}) \leadsto letin_1 \circ letin_2 \dashv \cdot, x:\mathbb{Z}, y:\mathbb{Z}}\,\textit{PEla-Optional}}{\cdot \vdash (\lambda\{\cdot; x:\mathbb{Z}; y = x+1\}.\ x+y) : \{\cdot; x:\mathbb{Z}; y?:\mathbb{Z}\} \to \mathbb{Z} \leadsto \lambda args{:}\{x:\mathbb{Z}\} \wedge \{y:\mathbb{Z} \vee \mathbf{Null}\}.\ e_0{:}\mathbb{Z}}\,\textit{Ela-NAbs}$$

$$\dfrac{\cdots\,\textit{Ela-NAbs}\cdots \qquad \cdot \vdash \{\cdot; x=1; z=2\} : \{\cdot; x:\mathbb{Z}; z:\mathbb{Z}\} \leadsto e_1 \qquad \dfrac{\dfrac{\dfrac{}{\cdot \vdash_{e_1} \cdot \diamond (\cdot; x:\mathbb{Z}; z:\mathbb{Z}) \leadsto \{\}}\,\textit{PMat-Empty}}{\cdot \vdash_{e_1} (\cdot; x:\mathbb{Z}) \diamond (\cdot; x:\mathbb{Z}; z:\mathbb{Z}) \leadsto \{\} \mathbin{,} \{x:\mathbb{Z} = e_1.x\}}\,\textit{PMat-Required}}{\cdot \vdash_{e_1} (\cdot; x:\mathbb{Z}; y?:\mathbb{Z}) \diamond (\cdot; x:\mathbb{Z}; z:\mathbb{Z}) \leadsto \{\} \mathbin{,} \{x:\mathbb{Z} = e_1.x\} \mathbin{,} \{y:\mathbf{Null} = \mathbf{null}\}}\,\textit{PMat-Absent}}{\cdot \vdash (\lambda\{\cdot; x:\mathbb{Z}; y = x+1\}.\ x+y)\,\{\cdot; x=1; z=2\} : \mathbb{Z} \leadsto (\lambda args{:}\{x:\mathbb{Z}\} \wedge \{y:\mathbb{Z} \vee \mathbf{Null}\}.\ e_0{:}\mathbb{Z})\,(\{\} \mathbin{,} \{x:\mathbb{Z} = e_1.x\} \mathbin{,} \{y:\mathbf{Null} = \mathbf{null}\})}\,\textit{Ela-NApp}$$

$$e_1 \equiv \{\cdot; x=1; z=2\}$$

Appendix: Semantics of λ_{iu}

$\boxed{A <: B}$ *(Subtyping)*

$$\textsc{Sub-Top}\ \frac{}{A <: \top} \qquad \textsc{Sub-Bot}\ \frac{}{\bot <: A} \qquad \textsc{Sub-Null}\ \frac{}{\mathbf{Null} <: \mathbf{Null}} \qquad \textsc{Sub-Int}\ \frac{}{\mathbb{Z} <: \mathbb{Z}} \qquad \textsc{Sub-Arrow}\ \frac{A_2 <: A_1 \qquad B_1 <: B_2}{A_1 \to B_1 <: A_2 \to B_2}$$

$$\textsc{Sub-Rcd}\ \frac{A <: B}{\{\ell : A\} <: \{\ell : B\}} \qquad \textsc{Sub-And}\ \frac{A <: B \qquad A <: C}{A <: B \wedge C} \qquad \textsc{Sub-AndL}\ \frac{A <: C}{A \wedge B <: C} \qquad \textsc{Sub-AndR}\ \frac{B <: C}{A \wedge B <: C} \qquad \textsc{Sub-Or}\ \frac{A <: C \qquad B <: C}{A \vee B <: C}$$

$$\textsc{Sub-OrL}\ \frac{A <: B}{A <: B \vee C} \qquad \textsc{Sub-OrR}\ \frac{A <: C}{A <: B \vee C}$$

$\boxed{\Gamma \vdash e : A}$ *(Typing)*

$$\textsc{Typ-Top}\ \frac{}{\Gamma \vdash \{\} : \top} \qquad \textsc{Typ-Null}\ \frac{}{\Gamma \vdash \mathbf{null} : \mathbf{Null}} \qquad \textsc{Typ-Int}\ \frac{}{\Gamma \vdash n : \mathbb{Z}} \qquad \textsc{Typ-Var}\ \frac{x : A \in \Gamma}{\Gamma \vdash x : A} \qquad \textsc{Typ-Abs}\ \frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash (\lambda x\!:\!A.\ e\!:\!B) : A \to B}$$

$$\textsc{Typ-App}\ \frac{\Gamma \vdash e_1 : A \to B \qquad \Gamma \vdash e_2 : A}{\Gamma \vdash e_1\ e_2 : B} \qquad \textsc{Typ-Rcd}\ \frac{\Gamma \vdash e : A}{\Gamma \vdash \{\ell : A = e\} : \{\ell : A\}} \qquad \textsc{Typ-Prj}\ \frac{\Gamma \vdash e : \{\ell : A\}}{\Gamma \vdash e.\ell : A}$$

$$\textsc{Typ-Merge}\ \frac{\Gamma \vdash e_1 : A \qquad \Gamma \vdash e_2 : B}{\Gamma \vdash e_1 \,,\, e_2 : A \wedge B} \qquad \textsc{Typ-Switch}\ \frac{\begin{array}{c}\Gamma \vdash e_0 : A \vee B \\ \Gamma, x : A \vdash e_1 : C \\ \Gamma, x : B \vdash e_2 : C\end{array}}{\Gamma \vdash \mathbf{switch}\ e_0\ \mathbf{as}\ x\ \mathbf{case}\ A \Rightarrow e_1\ \mathbf{case}\ B \Rightarrow e_2 : C}$$

$$\textsc{Typ-Let}\ \frac{\Gamma \vdash \mathit{letin} \dashv \Gamma' \qquad \Gamma' \vdash e : A}{\Gamma \vdash \mathit{letin}\ e : A} \qquad \textsc{Typ-Sub}\ \frac{\Gamma \vdash e : A \qquad A <: B}{\Gamma \vdash e : B}$$

$\boxed{\Gamma \vdash \mathit{letin} \dashv \Gamma'}$ *(Let-in binding)*

$$\textsc{LB-Let}\ \frac{\Gamma \vdash e : A}{\Gamma \vdash \mathbf{let}\ x = e\ \mathbf{in} \dashv \Gamma, x : A} \qquad \textsc{LB-Comp}\ \frac{\begin{array}{c}\Gamma \vdash \mathit{letin}_1 \dashv \Gamma' \\ \Gamma' \vdash \mathit{letin}_2 \dashv \Gamma''\end{array}}{\Gamma \vdash \mathit{letin}_1 \circ \mathit{letin}_2 \dashv \Gamma''} \qquad \textsc{LB-Id}\ \frac{}{\Gamma \vdash \mathbf{id} \dashv \Gamma}$$