Named Arguments as Intersections, Optional Arguments as Unions

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August 26, 2024

Syntax of λ_{iu} (target)

Types	$A, B ::= \top \mid \bot \mid \textbf{Null} \mid \mathbb{Z} \mid A \to B \mid \{\ell : A\} \mid A \wedge B \mid A \vee B$
Expressions	$e ::= \{\} \mid \textbf{null} \mid n \mid x \mid \lambda x\!:\!A.\ e\!:\!B \mid e_1\ e_2 \mid \{\ell = e\} \mid e.\ell \mid e_1 \,,\, e_2 \mid$ $\textbf{switch}\ e_0\ \textbf{as}\ x\ \textbf{case}\ A \Rightarrow e_1\ \textbf{case}\ B \Rightarrow e_2 \mid \textbf{let}\ x = e_1\ \textbf{in}\ e_2$

Syntax of UAENA[1] (source)

styp, $\mathcal{A}$, $\mathcal{B}$	$::=$		source types
	$\mid$	$\mathbb{Z}$	integer type
	$\mid$	$\mathcal{A} \to \mathcal{B}$	function type
	$\mid$	$\{\mathcal{P}\} \to \mathcal{B}$	function type with named parameters
nptyp, $\mathcal{P}$	$::=$		named parameter types
	$\mid$	$\cdot$	empty
	$\mid$	$\ell : \mathcal{A}; \mathcal{P}$	required parameter
	$\mid$	$\ell? : \mathcal{A}; \mathcal{P}$	optional parameter
sexp, ϵ	$::=$		source expressions
	$\mid$	n	integer literal
	$\mid$	x	variable
	$\mid$	$\lambda(x\!:\!\mathcal{A}).\ \epsilon$	abstraction
	$\mid$	$\epsilon_1\ \epsilon_2$	application
	$\mid$	$\lambda\{\rho\}.\ \epsilon$	abstraction with named parameters
	$\mid$	$\epsilon\ \{\alpha\}$	application to named arguments
npexp, ρ	$::=$		named parameters
	$\mid$	$\cdot$	empty
	$\mid$	$\ell : \mathcal{A}; \rho$	required parameter
	$\mid$	$\ell = \epsilon; \rho$	optional parameter
narg, α	$::=$		named arguments
	$\mid$	$\cdot$	empty
	$\mid$	$\ell = \epsilon; \alpha$	field
	$\mid$	$\alpha \backslash \ell$	removal

[1] UAENA = Unnamed Arguments Encoded Named Arguments

From UAENA to λ_{iu}

$\boxed{\|\mathcal{A}\| = A}$ *(Type translation)*

$$\textsc{Tr-Int}\quad \frac{}{\|\mathbb{Z}\| = \mathbb{Z}} \qquad \textsc{Tr-Arrow}\quad \frac{\|\mathcal{A}\| = A \qquad \|\mathcal{B}\| = B}{\|\mathcal{A} \to \mathcal{B}\| = A \to B} \qquad \textsc{Tr-NArrow}\quad \frac{\|\mathcal{P}\| = A \qquad \|\mathcal{B}\| = B}{\|\{\mathcal{P}\} \to \mathcal{B}\| = A \to B}$$

$\boxed{\|\mathcal{P}\| = A}$ *(Parameter type translation)*

$$\textsc{PTr-Empty}\quad \frac{}{\|\cdot\| = \top} \qquad \textsc{PTr-Required}\quad \frac{\|\mathcal{P}\| = B}{\|\ell : \mathcal{A}; \mathcal{P}\| = \{\ell : A\} \wedge B} \qquad \textsc{PTr-Optional}\quad \frac{\|\mathcal{P}\| = B}{\|\ell? : \mathcal{A}; \mathcal{P}\| = \{\ell : A \vee \mathbf{Null}\} \wedge B}$$

$\boxed{\Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e}$ *(Elaboration)*

$$\textsc{Ela-Int}\quad \frac{}{\Delta \vdash n : \mathbb{Z} \rightsquigarrow n} \qquad \textsc{Ela-Var}\quad \frac{x : \mathcal{A} \in \Delta}{\Delta \vdash x : \mathcal{A} \rightsquigarrow x} \qquad \textsc{Ela-Abs}\quad \frac{\Delta, x : \mathcal{A} \vdash \epsilon : \mathcal{B} \rightsquigarrow e \qquad \|\mathcal{A}\| = A \qquad \|\mathcal{B}\| = B}{\Delta \vdash \lambda(x{:}\mathcal{A}).\ \epsilon : \mathcal{A} \to \mathcal{B} \rightsquigarrow \lambda x{:}A.\ e{:}B}$$

$$\textsc{Ela-App}\quad \frac{\Delta \vdash \epsilon_1 : \mathcal{A} \to \mathcal{B} \rightsquigarrow e_1 \qquad \Delta \vdash \epsilon_2 : \mathcal{A} \rightsquigarrow e_2}{\Delta \vdash \epsilon_1\, \epsilon_2 : \mathcal{B} \rightsquigarrow e_1\, e_2}$$

$$\textsc{Ela-NAbs}\quad \frac{\mathbf{fresh}\ x \qquad \Delta; x \vdash \rho : \mathcal{P} \rightsquigarrow \mathit{letin} \dashv \Delta' \qquad \Delta' \vdash \epsilon : \mathcal{B} \rightsquigarrow e \qquad \|\mathcal{P}\| = A \qquad \|\mathcal{B}\| = B}{\Delta \vdash \lambda\{\rho\}.\ \epsilon : \{\mathcal{P}\} \to \mathcal{B} \rightsquigarrow \lambda x{:}A.\ \mathit{letin}\ e{:}B}$$

$$\textsc{Ela-NApp}\quad \frac{\Delta \vdash \epsilon : \{\mathcal{P}\} \to \mathcal{B} \rightsquigarrow e \qquad \Delta \vdash \mathcal{P} \diamond \alpha \rightsquigarrow e'}{\Delta \vdash \epsilon\,\{\alpha\} : \mathcal{B} \rightsquigarrow e\, e'}$$

$\boxed{\Delta; x \vdash \rho : \mathcal{P} \rightsquigarrow \mathit{letin} \dashv \Delta'}$ *(Parameter elaboration)*

$$\textsc{PEla-Empty}\quad \frac{}{\Delta; x \vdash \cdot : \cdot \rightsquigarrow \mathbf{id} \dashv \Delta} \qquad \textsc{PEla-Required}\quad \frac{\Delta; x \vdash \rho : \mathcal{P} \rightsquigarrow \mathit{letin} \dashv \Delta'}{\Delta; x \vdash (\ell : \mathcal{A}; \rho) : (\ell : \mathcal{A}; \mathcal{P}) \rightsquigarrow \mathbf{let}\, \ell = x.\ell\, \mathbf{in} \circ \mathit{letin} \dashv \Delta', \ell : \mathcal{A}}$$

$$\textsc{PEla-Optional}\quad \frac{\Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e \qquad \|\mathcal{A}\| = A \qquad \Delta; x \vdash \rho : \mathcal{P} \rightsquigarrow \mathit{letin} \dashv \Delta'}{\Delta; x \vdash (\ell = \epsilon; \rho) : (\ell? : \mathcal{A}; \mathcal{P}) \rightsquigarrow \mathbf{let}\, \ell = \mathbf{switch}\, x.\ell\, \mathbf{as}\, y\, \mathbf{case}\, A \Rightarrow y\, \mathbf{case}\, \mathbf{Null} \Rightarrow e\, \mathbf{in} \circ \mathit{letin} \dashv \Delta', \ell : \mathcal{A}}$$

$\boxed{\Delta \vdash \mathcal{P} \diamond \alpha \rightsquigarrow e}$ *(Parameter matching)*

$$\textsc{PMat-Empty}\quad \frac{}{\Delta \vdash \cdot \diamond \cdot \rightsquigarrow \{\}} \qquad \textsc{PMat-Extra}\quad \frac{\Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e \qquad \Delta \vdash \cdot \diamond \alpha \rightsquigarrow e'}{\Delta \vdash \cdot \diamond (\ell = \epsilon; \alpha) \rightsquigarrow \{\ell = e\} \,,\, e'}$$

$$\textsc{PMat-Present}\quad \frac{\alpha.\ell \Rightarrow \epsilon \qquad \Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e \qquad \Delta \vdash \mathcal{P} \diamond \alpha \backslash \ell \rightsquigarrow e'}{\Delta \vdash (\ell : \mathcal{A}; \mathcal{P}) \diamond \alpha \rightsquigarrow \{\ell = e\} \,,\, e'}$$

$$\textsc{PMat-Absent}\quad \frac{\alpha.\ell \not\Rightarrow \qquad \Delta \vdash \mathcal{P} \diamond \alpha \rightsquigarrow e'}{\Delta \vdash (\ell : \mathcal{A}; \mathcal{P}) \diamond \alpha \rightsquigarrow \{\ell = \mathbf{null}\} \,,\, e'}$$

Theorem 1 (Elaboration soundness) *If $\Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e$ and $\|\mathcal{A}\| = A$, then $\Gamma \vdash e : A$.*

Example 1

$$\mathit{letin}_1 = \textbf{let}\; x = z.x\; \textbf{in} \qquad \mathit{letin}_2 = \textbf{let}\; y = \textbf{switch}\; z.y\; \textbf{as}\; y\; \textbf{case}\; \mathbb{Z} \Rightarrow y\; \textbf{case}\; \textbf{Null} \Rightarrow 0\; \textbf{in} \qquad e_0 = \mathit{letin}_1 \circ \mathit{letin}_2\ (x + y)$$

$$\dfrac{\cdot, y : \mathbb{Z}, x : \mathbb{Z} \vdash x + y : \mathbb{Z} \rightsquigarrow x + y \qquad \dfrac{\dfrac{\cdot \vdash 0 : \mathbb{Z} \rightsquigarrow 0 \qquad \dfrac{}{\cdot; z \vdash \cdot : \cdot \rightsquigarrow \mathbf{id} \dashv \cdot}\textit{PEla-Empty}}{\cdot; z \vdash (y = 0; \cdot) : (y? : \mathbb{Z}; \cdot) \rightsquigarrow \mathit{letin}_2 \dashv \cdot, y : \mathbb{Z}}\textit{PEla-Optional}}{\cdot; z \vdash (x : \mathbb{Z}; y = 0; \cdot) : (x : \mathbb{Z}; y? : \mathbb{Z}; \cdot) \rightsquigarrow \mathit{letin}_1 \circ \mathit{letin}_2 \dashv \cdot, y : \mathbb{Z}, x : \mathbb{Z}}\textit{PEla-Required}}{\cdot \vdash (\lambda\{x : \mathbb{Z}; y = 0; \cdot\}.\ x + y) : \{x : \mathbb{Z}; y? : \mathbb{Z}; \cdot\} \to \mathbb{Z} \rightsquigarrow \lambda z{:}\{x : \mathbb{Z}\} \wedge \{y : \mathbb{Z} \vee \textbf{Null}\}.\ e_0{:}\mathbb{Z}}\textit{Ela-NPAbs}$$

$$\dfrac{\cdots\textit{Ela-NAbs}\cdots \qquad \dfrac{\cdot \vdash 1 : \mathbb{Z} \rightsquigarrow 1 \qquad \dfrac{\dfrac{\cdot \vdash 2 : \mathbb{Z} \rightsquigarrow 2 \qquad \dfrac{}{\cdot \vdash \cdot \diamond \cdot \rightsquigarrow \{\}}\textit{PMat-Empty}}{\cdot \vdash \cdot \diamond (z = 2; \cdot) \rightsquigarrow \{z = 2\}}\textit{PMat-Extra}}{\cdot \vdash (y? : \mathbb{Z}; \cdot) \diamond (z = 2; \cdot) \rightsquigarrow \{y = \textbf{null}\} \mathbin{,} \{z = 2\}}\textit{PMat-Absent}}{\cdot \vdash (x : \mathbb{Z}; y? : \mathbb{Z}; \cdot) \diamond (x = 1; z = 2; \cdot) \rightsquigarrow \{x = 1\} \mathbin{,} \{y = \textbf{null}\} \mathbin{,} \{z = 2\}}\textit{PMat-Present}}{\cdot \vdash (\lambda\{x : \mathbb{Z}; y = 0; \cdot\}.\ x + y)\, \{x = 1; z = 2; \cdot\} : \mathbb{Z} \rightsquigarrow (\lambda z{:}\{x : \mathbb{Z}\} \wedge \{y : \mathbb{Z} \vee \textbf{Null}\}.\ e_0{:}\mathbb{Z})\, (\{x = 1\} \mathbin{,} \{y = \textbf{null}\} \mathbin{,} \{z = 2\})}\textit{Ela-NApp}$$

Appendix: Semantics of λ_{iu}

$\boxed{A <: B}$ *(Subtyping)*

Sub-Top
$$\frac{}{A <: \top}$$

Sub-Bot
$$\frac{}{\bot <: A}$$

Sub-Int
$$\frac{}{\mathbb{Z} <: \mathbb{Z}}$$

Sub-Arrow
$$\frac{B_1 <: A_1 \qquad A_2 <: B_2}{A_1 \to A_2 <: B_1 \to B_2}$$

Sub-Rcd
$$\frac{A <: B}{\{\ell : A\} <: \{\ell : B\}}$$

Sub-And
$$\frac{A <: B \qquad A <: C}{A <: B \wedge C}$$

Sub-AndL
$$\frac{A <: C}{A \wedge B <: C}$$

Sub-AndR
$$\frac{B <: C}{A \wedge B <: C}$$

Sub-Or
$$\frac{A <: C \qquad B <: C}{A \vee B <: C}$$

Sub-OrL
$$\frac{A <: B}{A <: B \vee C}$$

Sub-OrR
$$\frac{A <: C}{A <: B \vee C}$$

$\boxed{\Gamma \vdash e : A}$ *(Typing)*

Typ-Top
$$\frac{}{\Gamma \vdash \{\} : \top}$$

Typ-Int
$$\frac{}{\Gamma \vdash n : \mathbb{Z}}$$

Typ-Var
$$\frac{x : A \in \Gamma}{\Gamma \vdash x : A}$$

Typ-Abs
$$\frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash (\lambda x{:}A.\ e{:}B) : A \to B}$$

Typ-App
$$\frac{\Gamma \vdash e_1 : A \to B \qquad \Gamma \vdash e_2 : A}{\Gamma \vdash e_1\, e_2 : B}$$

Typ-Rcd
$$\frac{\Gamma \vdash e : A}{\Gamma \vdash \{\ell = e\} : \{\ell : A\}}$$

Typ-Prj
$$\frac{\Gamma \vdash e : \{\ell : A\}}{\Gamma \vdash e.\ell : A}$$

Typ-Merge
$$\frac{\Gamma \vdash e_1 : A \qquad \Gamma \vdash e_2 : B}{\Gamma \vdash e_1 \mathbin{,,} e_2 : A \wedge B}$$

Typ-Switch
$$\frac{\begin{array}{c}\Gamma \vdash e : A \vee B \\ \Gamma, x : A \vdash e_1 : C \\ \Gamma, x : B \vdash e_2 : C\end{array}}{\Gamma \vdash \mathbf{switch}\ e_0\ \mathbf{as}\ x\ \mathbf{case}\ A \Rightarrow e_1\ \mathbf{case}\ B \Rightarrow e_2 : C}$$

Typ-Sub
$$\frac{\Gamma \vdash e : A \qquad A <: B}{\Gamma \vdash e : B}$$

$\boxed{v \longrightarrow_A v'}$ *(Type casting)*

Cast-Top
$$\frac{}{v \longrightarrow_\top v}$$

Cast-Int
$$\frac{}{n \longrightarrow_\mathbb{Z} n}$$

Cast-Arrow
$$\frac{A_1 \to A_2 <: B_1 \to B_2}{\lambda x{:}A_1.\ e{:}A_2 \longrightarrow_{B_1 \to B_2} \lambda x{:}A_1.\ e{:}B_2}$$

Cast-Rcd
$$\frac{v \longrightarrow_A v'}{\{\ell = v\} \longrightarrow_{\{\ell : A\}} \{\ell = v'\}}$$

Cast-And
$$\frac{v \longrightarrow_A v_1 \qquad v \longrightarrow_B v_2}{v \longrightarrow_{A \wedge B} v_1 \mathbin{,,} v_2}$$

Cast-OrL
$$\frac{v \longrightarrow_A v'}{v \longrightarrow_{A \vee B} v'}$$

Cast-OrR
$$\frac{v \longrightarrow_B v'}{v \longrightarrow_{A \vee B} v'}$$

Cast-MergeL
$$\frac{v_1 \longrightarrow_A v_1'}{v_1 \mathbin{,,} v_2 \longrightarrow_A v_1'}$$

Cast-MergeR
$$\frac{v_2 \longrightarrow_A v_2'}{v_1 \mathbin{,,} v_2 \longrightarrow_A v_2'}$$

$\boxed{e \longrightarrow e'}$ *(Small-step operational semantics)*

Step-AppBeta
$$\frac{v \longrightarrow_A v'}{(\lambda x{:}A.\ e{:}B)\, v \longrightarrow ([v'/x]e) : B}$$

Step-PrjBeta
$$\frac{}{\{\ell = v\}.\ell \longrightarrow v}$$

Step-Dispatch
$$\frac{}{(v_1 \mathbin{,,} v_2)\, v_3 \longrightarrow e}$$

Step-SwitchL
$$\frac{v \longrightarrow_A v'}{\mathbf{switch}\ v\ \mathbf{as}\ x\ \mathbf{case}\ A \Rightarrow e_1\ \mathbf{case}\ B \Rightarrow e_2 \longrightarrow [v'/x]e_1}$$

Step-SwitchR
$$\frac{v \longrightarrow_B v'}{\mathbf{switch}\ v\ \mathbf{as}\ x\ \mathbf{case}\ A \Rightarrow e_1\ \mathbf{case}\ B \Rightarrow e_2 \longrightarrow [v'/x]e_2}$$

Step-AppL
$$\frac{e_1 \longrightarrow e_1'}{e_1\, e_2 \longrightarrow e_1'\, e_2}$$

Step-AppR
$$\frac{e_2 \longrightarrow e_2'}{v_1\, e_2 \longrightarrow v_1\, e_2'}$$

Step-Rcd
$$\frac{e \longrightarrow e'}{\{\ell = e\} \longrightarrow \{\ell = e'\}}$$

Step-Prj
$$\frac{e \longrightarrow e'}{e.\ell \longrightarrow e'.\ell}$$

Step-MergeL
$$\frac{e_1 \longrightarrow e_1'}{e_1 \,,\, e_2 \longrightarrow e_1' \,,\, e_2}$$

Step-MergeR
$$\frac{e_2 \longrightarrow e_2'}{v_1 \,,\, e_2 \longrightarrow v_1 \,,\, e_2'}$$

Step-Switch
$$\frac{e_0 \longrightarrow e_0'}{\textbf{switch}\ e_0\ \textbf{as}\ x\ \textbf{case}\ A \Rightarrow e_1\ \textbf{case}\ B \Rightarrow e_2 \longrightarrow \textbf{switch}\ e_0'\ \textbf{as}\ x\ \textbf{case}\ A \Rightarrow e_1\ \textbf{case}\ B \Rightarrow e_2}$$