Named Arguments as Intersections, Optional Arguments as Unions

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Syntax of λ_{iu} (target)

Types	$A, B ::= \top \mid \bot \mid \mathbf{Null} \mid \mathbb{Z} \mid A \to B \mid \{\ell : A\} \mid A \wedge B \mid A \vee B$
Expressions	$e ::= \{\} \mid \mathbf{null} \mid n \mid x \mid \lambda x : A.\ e : B \mid e_1\ e_2 \mid \{\ell = e\} \mid e.\ell \mid e_1 \,,\, e_2 \mid$
	$\mathbf{switch}\ e_0\ \mathbf{as}\ x\ \mathbf{case}\ A \Rightarrow e_1\ \mathbf{case}\ B \Rightarrow e_2 \mid \mathit{letin}\ e$
Let-in bindings	$\mathit{letin} ::= \mathbf{let}\ x = e\ \mathbf{in} \mid \mathit{letin}_1 \circ \mathit{letin}_2 \mid \mathbf{id}$

Syntax of Uaena[1] (source)

$\mathit{styp},\ \mathcal{A},\ \mathcal{B}$	$::=$		source types
	$\mid$	$\mathbb{Z}$	integer type
	$\mid$	$\mathcal{A} \to \mathcal{B}$	function type
	$\mid$	$\{\mathcal{P}\} \to \mathcal{B}$	function type with named parameters
$\mathit{nptyp},\ \mathcal{P}$	$::=$		named parameter types
	$\mid$	$\cdot$	empty
	$\mid$	$\mathcal{P};\ \ell : \mathcal{A}$	required parameter
	$\mid$	$\mathcal{P};\ \ell? : \mathcal{A}$	optional parameter
$\mathit{sexp},\ \epsilon$	$::=$		source expressions
	$\mid$	n	integer literal
	$\mid$	x	variable
	$\mid$	$\lambda(x : \mathcal{A}).\ \epsilon$	abstraction
	$\mid$	$\epsilon_1\ \epsilon_2$	application
	$\mid$	$\lambda\{\rho\}.\ \epsilon$	abstraction with named parameters
	$\mid$	$\epsilon\ \{\alpha\}$	application to named arguments
$\mathit{npexp},\ \rho$	$::=$		named parameters
	$\mid$	$\cdot$	empty
	$\mid$	$\rho;\ \ell : \mathcal{A}$	required parameter
	$\mid$	$\rho;\ \ell = \epsilon$	optional parameter
$\mathit{narg},\ \alpha$	$::=$		named arguments
	$\mid$	$\cdot$	empty
	$\mid$	$\alpha;\ \ell = \epsilon$	field

[1] Uaena = Unnamed Arguments Encoded Named Arguments

From Uaena to λ_{iu}

$\boxed{\Delta \vdash \epsilon : \mathcal{A} \leadsto e}$ *(Elaboration)*

$$\textsc{Ela-Int}\quad \frac{}{\Delta \vdash n : \mathbb{Z} \leadsto n}$$

$$\textsc{Ela-Var}\quad \frac{x : \mathcal{A} \in \Delta}{\Delta \vdash x : \mathcal{A} \leadsto x}$$

$$\textsc{Ela-Abs}\quad \frac{\Delta, x : \mathcal{A} \vdash \epsilon : \mathcal{B} \leadsto e \qquad |\mathcal{A}| = A \qquad |\mathcal{B}| = B}{\Delta \vdash \lambda(x{:}\mathcal{A}).\ \epsilon : \mathcal{A} \to \mathcal{B} \leadsto \lambda x{:}A.\ e{:}B}$$

$$\textsc{Ela-App}\quad \frac{\Delta \vdash \epsilon_1 : \mathcal{A} \to \mathcal{B} \leadsto e_1 \qquad \Delta \vdash \epsilon_2 : \mathcal{A} \leadsto e_2}{\Delta \vdash \epsilon_1\, \epsilon_2 : \mathcal{B} \leadsto e_1\, e_2}$$

$$\textsc{Ela-NAbs}\quad \frac{\textbf{fresh}\, x \qquad \Delta \vdash_x \rho : \mathcal{P} \leadsto \mathit{letin} \dashv \Delta' \qquad \Delta' \vdash \epsilon : \mathcal{B} \leadsto e \qquad |\mathcal{P}| = A \qquad |\mathcal{B}| = B}{\Delta \vdash \lambda\{\rho\}.\ \epsilon : \{\mathcal{P}\} \to \mathcal{B} \leadsto \lambda x{:}A.\ \mathit{letin}\, e{:}B}$$

$$\textsc{Ela-NApp}\quad \frac{\Delta \vdash \epsilon : \{\mathcal{P}\} \to \mathcal{B} \leadsto e \qquad \Delta \vdash \mathcal{P} \diamond \alpha \leadsto e'}{\Delta \vdash \epsilon\,\{\alpha\} : \mathcal{B} \leadsto e\, e'}$$

$\boxed{\Delta \vdash_x \rho : \mathcal{P} \leadsto \mathit{letin} \dashv \Delta'}$ *(Parameter elaboration)*

$$\textsc{PEla-Empty}\quad \frac{}{\Delta \vdash_x \cdot : \cdot \leadsto \textbf{id} \dashv \Delta}$$

$$\textsc{PEla-Required}\quad \frac{\Delta \vdash_x \rho : \mathcal{P} \leadsto \mathit{letin} \dashv \Delta'}{\Delta \vdash_x (\rho;\ \ell : \mathcal{A}) : (\mathcal{P};\ \ell : \mathcal{A}) \leadsto \mathit{letin} \circ \textbf{let}\, \ell = x.\ell\, \textbf{in} \dashv \Delta', \ell : \mathcal{A}}$$

$$\textsc{PEla-Optional}\quad \frac{\textbf{fresh}\, y \qquad x \neq y \qquad \Delta \vdash_x \rho : \mathcal{P} \leadsto \mathit{letin} \dashv \Delta' \qquad \Delta' \vdash \epsilon : \mathcal{A} \leadsto e \qquad |\mathcal{A}| = A}{\Delta \vdash_x (\rho;\ \ell = \epsilon) : (\mathcal{P};\ \ell? : \mathcal{A}) \leadsto \mathit{letin} \circ \textbf{let}\, \ell = \textbf{switch}\, x.\ell\, \textbf{as}\, y\, \textbf{case}\, A \Rightarrow y\, \textbf{case}\, \textbf{Null} \Rightarrow e\, \textbf{in} \dashv \Delta', \ell : \mathcal{A}}$$

$\boxed{\Delta \vdash \mathcal{P} \diamond \alpha \leadsto e}$ *(Parameter matching)*

$$\textsc{PMat-Empty}\quad \frac{}{\Delta \vdash \cdot \diamond \cdot \leadsto \{\}}$$

$$\textsc{PMat-Extra}\quad \frac{\Delta \vdash \epsilon : \mathcal{A} \leadsto e \qquad \Delta \vdash \cdot \diamond \alpha \leadsto e'}{\Delta \vdash \cdot \diamond (\alpha;\ \ell = \epsilon) \leadsto e' \,,\, \{\ell = e\}}$$

$$\textsc{PMat-Required}\quad \frac{\alpha.\ell \Rightarrow \epsilon \qquad \Delta \vdash \epsilon : \mathcal{A} \leadsto e \qquad \Delta \vdash \mathcal{P} \diamond \alpha\backslash\ell \leadsto e'}{\Delta \vdash (\mathcal{P};\ \ell : \mathcal{A}) \diamond \alpha \leadsto e' \,,\, \{\ell = e\}}$$

$$\textsc{PMat-Present}\quad \frac{\alpha.\ell \Rightarrow \epsilon \qquad \Delta \vdash \epsilon : \mathcal{A} \leadsto e \qquad \Delta \vdash \mathcal{P} \diamond \alpha\backslash\ell \leadsto e'}{\Delta \vdash (\mathcal{P};\ \ell? : \mathcal{A}) \diamond \alpha \leadsto e' \,,\, \{\ell = e\}}$$

$$\textsc{PMat-Absent}\quad \frac{\alpha.\ell \not\Rightarrow \qquad \Delta \vdash \mathcal{P} \diamond \alpha \leadsto e'}{\Delta \vdash (\mathcal{P};\ \ell? : \mathcal{A}) \diamond \alpha \leadsto e' \,,\, \{\ell = \textbf{null}\}}$$

$\boxed{\alpha.\ell \Rightarrow \epsilon}$ *(Successful lookup)*

$$\textsc{LU-Present}\quad \frac{\alpha.\ell \not\Rightarrow}{(\alpha;\ \ell = \epsilon).\ell \Rightarrow \epsilon}$$

$$\textsc{LU-Absent}\quad \frac{\ell' \neq \ell \qquad \alpha.\ell \Rightarrow \epsilon}{(\alpha;\ \ell' = \epsilon').\ell \Rightarrow \epsilon}$$

$\boxed{\alpha.\ell \not\Rightarrow}$ *(Failed lookup)*

$$\textsc{LD-Empty}\quad \frac{}{\cdot.\ell \not\Rightarrow}$$

$$\textsc{LD-Absent}\quad \frac{\ell' \neq \ell \qquad \alpha.\ell \not\Rightarrow}{(\alpha;\ \ell' = \epsilon).\ell \not\Rightarrow}$$

$\boxed{|\mathcal{A}|}$ Type translation

$$
\begin{aligned}
&|\mathbb{Z}| \equiv \mathbb{Z} \\
&|\mathcal{A} \rightarrow \mathcal{B}| \equiv |\mathcal{A}| \rightarrow |\mathcal{B}| \\
&|\{\mathcal{P}\} \rightarrow \mathcal{B}| \equiv |\mathcal{P}| \rightarrow |\mathcal{B}|
\end{aligned}
$$

$\boxed{|\mathcal{P}|}$ Parameter type translation

$$
\begin{aligned}
&|\cdot| \equiv \top \\
&|\mathcal{P};\, \ell : \mathcal{A}| \equiv |\mathcal{P}| \wedge \{\ell : |\mathcal{A}|\} \\
&|\mathcal{P};\, \ell? : \mathcal{A}| \equiv |\mathcal{P}| \wedge \{\ell : |\mathcal{A}| \vee \mathbf{Null}\}
\end{aligned}
$$

$\boxed{|\Delta|}$ Typing context translation

$$
\begin{aligned}
&|\cdot| \equiv \cdot \\
&|\Delta,\, x : \mathcal{A}| \equiv |\Delta|,\, x : |\mathcal{A}|
\end{aligned}
$$

Theorem 1 (Elaboration soundness) *If* $\Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e$, *then* $|\Delta| \vdash e : |\mathcal{A}|$.

Example 1

$$\mathit{letin}_1 \equiv \mathbf{let}\, x = z.x\, \mathbf{in} \qquad \mathit{letin}_2 \equiv \mathbf{let}\, y = \mathbf{switch}\, z.y\, \mathbf{as}\, y\, \mathbf{case}\, \mathbb{Z} \Rightarrow y\, \mathbf{case}\, \mathbf{Null} \Rightarrow x+1\, \mathbf{in} \qquad e_0 \equiv \mathit{letin}_1 \circ \mathit{letin}_2\ (x+y)$$

$$\dfrac{\cdot, x:\mathbb{Z}, y:\mathbb{Z} \vdash x+y:\mathbb{Z} \rightsquigarrow x+y \qquad \dfrac{\cdot, x:\mathbb{Z} \vdash x+1:\mathbb{Z} \rightsquigarrow x+1 \qquad \dfrac{\cdot \vdash 0:\mathbb{Z} \rightsquigarrow 0 \qquad \dfrac{}{\cdot \vdash_z \cdot : \cdot \rightsquigarrow \mathbf{id} \dashv \cdot}\ \textit{PEla-Empty}}{\cdot \vdash_z (\cdot;\, x:\mathbb{Z}) : (\cdot;\, x:\mathbb{Z}) \rightsquigarrow \mathit{letin}_1 \dashv \cdot, x:\mathbb{Z}}\ \textit{PEla-Required}}{\cdot \vdash_z (\cdot;\, x:\mathbb{Z};\, y = x+1) : (\cdot;\, x:\mathbb{Z};\, y?:\mathbb{Z}) \rightsquigarrow \mathit{letin}_1 \circ \mathit{letin}_2 \dashv \cdot, x:\mathbb{Z}, y:\mathbb{Z}}\ \textit{PEla-Optional}}{\cdot \vdash (\lambda\{\cdot;\, x:\mathbb{Z};\, y = x+1\}.\, x+y) : \{\cdot;\, x:\mathbb{Z};\, y?:\mathbb{Z}\} \to \mathbb{Z} \rightsquigarrow \lambda z{:}\{x:\mathbb{Z}\} \wedge \{y:\mathbb{Z} \vee \mathbf{Null}\}.\, e_0{:}\mathbb{Z}}\ \textit{Ela-NAbs}$$

$$\dfrac{\cdots \textit{Ela-NAbs} \cdots \qquad \dfrac{\cdot \vdash 1:\mathbb{Z} \rightsquigarrow 1 \qquad \dfrac{\dfrac{\cdot \vdash 2:\mathbb{Z} \rightsquigarrow 2 \qquad \dfrac{}{\cdot \vdash \cdot \diamond \cdot \rightsquigarrow \{\}}\ \textit{PMat-Empty}}{\cdot \vdash \cdot \diamond (\cdot;\, z = 2) \rightsquigarrow \{z = 2\}}\ \textit{PMat-Extra}}{\cdot \vdash (\cdot;\, x:\mathbb{Z}) \diamond (\cdot;\, x = 1;\, z = 2) \rightsquigarrow \{z = 2\} ,\!, \{x = 1\}}\ \textit{PMat-Required}}{\cdot \vdash (\cdot;\, x:\mathbb{Z};\, y?:\mathbb{Z}) \diamond (\cdot;\, x = 1;\, z = 2) \rightsquigarrow \{z = 2\} ,\!, \{x = 1\} ,\!, \{y = \mathbf{null}\}}\ \textit{PMat-Absent}}{\cdot \vdash (\lambda\{\cdot;\, x:\mathbb{Z};\, y = x+1\}.\, x+y)\,\{\cdot;\, x = 1;\, z = 2\} : \mathbb{Z} \rightsquigarrow (\lambda z{:}\{x:\mathbb{Z}\} \wedge \{y:\mathbb{Z} \vee \mathbf{Null}\}.\, e_0{:}\mathbb{Z})\,(\{x = 1\} ,\!, \{y = \mathbf{null}\} ,\!, \{z = 2\})}\ \textit{Ela-NApp}$$

Appendix: Semantics of λ_{iu}

$\boxed{A <: B}$ *(Subtyping)*

$$\textsc{Sub-Top}\quad \frac{}{A <: \top} \qquad \textsc{Sub-Bot}\quad \frac{}{\bot <: A} \qquad \textsc{Sub-Null}\quad \frac{}{\mathbf{Null} <: \mathbf{Null}} \qquad \textsc{Sub-Int}\quad \frac{}{\mathbb{Z} <: \mathbb{Z}} \qquad \textsc{Sub-Arrow}\quad \frac{B_1 <: A_1 \qquad A_2 <: B_2}{A_1 \to A_2 <: B_1 \to B_2}$$

$$\textsc{Sub-Rcd}\quad \frac{A <: B}{\{\ell : A\} <: \{\ell : B\}} \qquad \textsc{Sub-And}\quad \frac{A <: B \qquad A <: C}{A <: B \wedge C} \qquad \textsc{Sub-AndL}\quad \frac{A <: C}{A \wedge B <: C} \qquad \textsc{Sub-AndR}\quad \frac{B <: C}{A \wedge B <: C} \qquad \textsc{Sub-Or}\quad \frac{A <: C \qquad B <: C}{A \vee B <: C}$$

$$\textsc{Sub-OrL}\quad \frac{A <: B}{A <: B \vee C} \qquad \textsc{Sub-OrR}\quad \frac{A <: C}{A <: B \vee C}$$

$\boxed{\Gamma \vdash e : A}$ *(Typing)*

$$\textsc{Typ-Top}\quad \frac{}{\Gamma \vdash \{\} : \top} \qquad \textsc{Typ-Int}\quad \frac{}{\Gamma \vdash n : \mathbb{Z}} \qquad \textsc{Typ-Var}\quad \frac{x : A \in \Gamma}{\Gamma \vdash x : A} \qquad \textsc{Typ-Abs}\quad \frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash (\lambda x\!:\!A.\ e\!:\!B) : A \to B}$$

$$\textsc{Typ-App}\quad \frac{\Gamma \vdash e_1 : A \to B \qquad \Gamma \vdash e_2 : A}{\Gamma \vdash e_1\, e_2 : B} \qquad \textsc{Typ-Rcd}\quad \frac{\Gamma \vdash e : A}{\Gamma \vdash \{\ell = e\} : \{\ell : A\}} \qquad \textsc{Typ-Prj}\quad \frac{\Gamma \vdash e : \{\ell : A\}}{\Gamma \vdash e.\ell : A}$$

$$\textsc{Typ-Merge}\quad \frac{\Gamma \vdash e_1 : A \qquad \Gamma \vdash e_2 : B}{\Gamma \vdash e_1 \,,\, e_2 : A \wedge B} \qquad \textsc{Typ-Switch}\quad \frac{\begin{array}{c}\Gamma \vdash e_0 : A \vee B \\ \Gamma, x : A \vdash e_1 : C \\ \Gamma, x : B \vdash e_2 : C\end{array}}{\Gamma \vdash \mathbf{switch}\ e_0\ \mathbf{as}\ x\ \mathbf{case}\ A \Rightarrow e_1\ \mathbf{case}\ B \Rightarrow e_2 : C}$$

$$\textsc{Typ-Let}\quad \frac{\Gamma \vdash \mathit{letin} \dashv \Gamma' \qquad \Gamma' \vdash e : A}{\Gamma \vdash \mathit{letin}\ e : A} \qquad \textsc{Typ-Sub}\quad \frac{\Gamma \vdash e : A \qquad A <: B}{\Gamma \vdash e : B}$$

$\boxed{\Gamma \vdash \mathit{letin} \dashv \Gamma'}$ *(Let-in binding)*

$$\textsc{LB-Let}\quad \frac{\Gamma \vdash e : A}{\Gamma \vdash \mathbf{let}\ x = e\ \mathbf{in} \dashv \Gamma, x : A} \qquad \textsc{LB-Comp}\quad \frac{\begin{array}{c}\Gamma \vdash \mathit{letin}_1 \dashv \Gamma' \\ \Gamma' \vdash \mathit{letin}_2 \dashv \Gamma''\end{array}}{\Gamma \vdash \mathit{letin}_1 \circ \mathit{letin}_2 \dashv \Gamma''} \qquad \textsc{LB-Id}\quad \frac{}{\Gamma \vdash \mathbf{id} \dashv \Gamma}$$