Named Arguments as Intersections, Optional Arguments as Unions

Yaozhu Sun

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Syntax of λ_{iu} (target)

Types	$A, B ::= \top \mid \bot \mid \mathbf{Null} \mid \mathbb{Z} \mid A \to B \mid \{\ell : A\} \mid A \wedge B \mid A \vee B$
Expressions	$e ::= \{\} \mid \mathbf{null} \mid n \mid x \mid \lambda x\!:\!A.\ e\!:\!B \mid e_1\ e_2 \mid \{\ell : A = e\} \mid e.\ell$
	$\mid e_1 \,,\, e_2 \mid \mathbf{switch}\ e_0\ \mathbf{as}\ x\ \mathbf{case}\ A \Rightarrow e_1\ \mathbf{case}\ B \Rightarrow e_2 \mid \mathit{letin}\ e$
Let-in bindings	$\mathit{letin} ::= \mathbf{let}\ x = e\ \mathbf{in} \mid \mathit{letin}_1 \circ \mathit{letin}_2 \mid \mathbf{id}$

Syntax of Uaena[1] (source)

Types	$\mathcal{A}, \mathcal{B} ::= \mathbb{Z} \mid \mathcal{A} \to \mathcal{B} \mid \{\mathcal{P}\} \to \mathcal{B} \mid \{\mathcal{K}\}$
Named parameter types	$\mathcal{P} ::= \cdot \mid \mathcal{P};\ \ell : \mathcal{A} \mid \mathcal{P};\ \ell? : \mathcal{A}$
Named argument types	$\mathcal{K} ::= \cdot \mid \mathcal{K};\ \ell : \mathcal{A}$
Expressions	$\epsilon ::= n \mid x \mid \lambda(x\!:\!\mathcal{A}).\ \epsilon \mid \lambda\{\rho\}.\ \epsilon \mid \epsilon_1\ \epsilon_2 \mid \{\kappa\}$
Named parameters	$\rho ::= \cdot \mid \rho;\ \ell : \mathcal{A} \mid \rho;\ \ell = \epsilon$
Named arguments	$\kappa ::= \cdot \mid \kappa;\ \ell = \epsilon$

[1] Uaena = Unnamed Arguments Encoded Named Arguments

From Uaena to λ_{iu}

$\boxed{\Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e}$ *(Elaboration)*

Ela-Int
$$\frac{}{\Delta \vdash n : \mathbb{Z} \rightsquigarrow n}$$

Ela-Var
$$\frac{x : \mathcal{A} \in \Delta}{\Delta \vdash x : \mathcal{A} \rightsquigarrow x}$$

Ela-Abs
$$\frac{\Delta, x : \mathcal{A} \vdash \epsilon : \mathcal{B} \rightsquigarrow e}{\Delta \vdash \lambda(x{:}\mathcal{A}).\ \epsilon : \mathcal{A} \rightarrow \mathcal{B} \rightsquigarrow \lambda x{:}|\mathcal{A}|.\ e{:}|\mathcal{B}|}$$

Ela-NAbs
$$\frac{\begin{array}{c}\Delta \vdash_x \rho : \mathcal{P} \rightsquigarrow \mathit{letin} \dashv \Delta' \\ \Delta' \vdash \epsilon : \mathcal{B} \rightsquigarrow e\end{array}}{\Delta \vdash \lambda\{\rho\}.\ \epsilon : \{\mathcal{P}\} \rightarrow \mathcal{B} \rightsquigarrow \lambda x{:}|\mathcal{P}|.\ \mathit{letin}\ e{:}|\mathcal{B}|}$$

Ela-App
$$\frac{\begin{array}{c}\Delta \vdash \epsilon_1 : \mathcal{A} \rightarrow \mathcal{B} \rightsquigarrow e_1 \\ \Delta \vdash \epsilon_2 : \mathcal{A} \rightsquigarrow e_2\end{array}}{\Delta \vdash \epsilon_1\, \epsilon_2 : \mathcal{B} \rightsquigarrow e_1\, e_2}$$

Ela-NApp
$$\frac{\begin{array}{c}\Delta \vdash \epsilon_1 : \{\mathcal{P}\} \rightarrow \mathcal{B} \rightsquigarrow e_1 \\ \Delta \vdash \epsilon_2 : \{\mathcal{K}\} \rightsquigarrow e_2 \\ \Delta \vdash_{e_2} \mathcal{P} \diamond \mathcal{K} \rightsquigarrow e_2'\end{array}}{\Delta \vdash \epsilon_1\, \epsilon_2 : \mathcal{B} \rightsquigarrow e_1\, e_2'}$$

Ela-NEmpty
$$\frac{}{\Delta \vdash \{\cdot\} : \{\cdot\} \rightsquigarrow \{\}}$$

Ela-NField
$$\frac{\begin{array}{c}\Delta \vdash \{\kappa\} : \{\mathcal{K}\} \rightsquigarrow e' \\ \Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e\end{array}}{\Delta \vdash \{\kappa;\ \ell = \epsilon\} : \{\mathcal{K};\ \ell : \mathcal{A}\} \rightsquigarrow e' \,,\, \{\ell : |\mathcal{A}| = e\}}$$

$\boxed{\Delta \vdash_x \rho : \mathcal{P} \rightsquigarrow \mathit{letin} \dashv \Delta'}$ *(Parameter elaboration)*

PEla-Empty
$$\frac{}{\Delta \vdash_x \cdot : \cdot \rightsquigarrow \mathbf{id} \dashv \Delta}$$

PEla-Required
$$\frac{\Delta \vdash_x \rho : \mathcal{P} \rightsquigarrow \mathit{letin} \dashv \Delta'}{\Delta \vdash_x (\rho;\ \ell : \mathcal{A}) : (\mathcal{P};\ \ell : \mathcal{A}) \rightsquigarrow \mathit{letin} \circ \mathbf{let}\, \ell = x.\ell\, \mathbf{in} \dashv \Delta', \ell : \mathcal{A}}$$

PEla-Optional
$$\frac{\begin{array}{c}\Delta \vdash_x \rho : \mathcal{P} \rightsquigarrow \mathit{letin} \dashv \Delta' \\ \Delta' \vdash \epsilon : \mathcal{A} \rightsquigarrow e\end{array}}{\Delta \vdash_x (\rho;\ \ell = \epsilon) : (\mathcal{P};\ \ell? : \mathcal{A}) \rightsquigarrow \mathit{letin} \circ \mathbf{let}\, \ell = \mathbf{switch}\, x.\ell\, \mathbf{as}\, y\, \mathbf{case}\, |\mathcal{A}| \Rightarrow y\, \mathbf{case}\, \mathbf{Null} \Rightarrow e\, \mathbf{in} \dashv \Delta', \ell : \mathcal{A}}$$

$\boxed{\Delta \vdash_e \mathcal{P} \diamond \mathcal{K} \rightsquigarrow e'}$ *(Parameter matching)*

PMat-Empty
$$\frac{}{\Delta \vdash_e \cdot \diamond \mathcal{K} \rightsquigarrow \{\}}$$

PMat-Required
$$\frac{\begin{array}{c}\mathcal{K} :: \ell \Rightarrow \mathcal{A} \\ \Delta \vdash_e \mathcal{P} \diamond \mathcal{K} \rightsquigarrow e'\end{array}}{\Delta \vdash_e (\mathcal{P};\ \ell : \mathcal{A}) \diamond \mathcal{K} \rightsquigarrow e' \,,\, \{\ell : |\mathcal{A}| = e.\ell\}}$$

PMat-Present
$$\frac{\begin{array}{c}\mathcal{K} :: \ell \Rightarrow \mathcal{A} \\ \Delta \vdash_e \mathcal{P} \diamond \mathcal{K} \rightsquigarrow e'\end{array}}{\Delta \vdash_e (\mathcal{P};\ \ell? : \mathcal{A}) \diamond \mathcal{K} \rightsquigarrow e' \,,\, \{\ell : |\mathcal{A}| \vee \mathbf{Null} = e.\ell\}}$$

PMat-Absent
$$\frac{\begin{array}{c}\mathcal{K} :: \ell \not\Rightarrow \\ \Delta \vdash_e \mathcal{P} \diamond \mathcal{K} \rightsquigarrow e'\end{array}}{\Delta \vdash_e (\mathcal{P};\ \ell? : \mathcal{A}) \diamond \mathcal{K} \rightsquigarrow e' \,,\, \{\ell : |\mathcal{A}| \vee \mathbf{Null} = \mathbf{null}\}}$$

$\boxed{\mathcal{K} :: \ell \Rightarrow \mathcal{A}}$ *(Successful lookup)*

LU-Present
$$\frac{\mathcal{K} :: \ell \not\Rightarrow}{(\mathcal{K};\ \ell : \mathcal{A}) :: \ell \Rightarrow \mathcal{A}}$$

LU-Absent
$$\frac{\ell' \neq \ell \qquad \mathcal{K} :: \ell \Rightarrow \mathcal{A}}{(\mathcal{K};\ \ell' : \mathcal{B}) :: \ell \Rightarrow \mathcal{A}}$$

$\boxed{\mathcal{K} :: \ell \not\Rightarrow}$ *(Failed lookup)*

$$\frac{\textsc{LD-Empty}}{\cdot :: \ell \not\Rightarrow} \qquad \textsc{LD-Absent}\ \frac{\ell' \neq \ell \qquad \mathcal{K} :: \ell \not\Rightarrow}{(\mathcal{K};\, \ell' : \mathcal{A}) :: \ell \not\Rightarrow}$$

$\boxed{|\mathcal{A}|}$ Type translation

$$\begin{aligned}
&|\mathbb{Z}| \equiv \mathbb{Z}\\
&|\mathcal{A} \to \mathcal{B}| \equiv |\mathcal{A}| \to |\mathcal{B}|\\
&|\{\mathcal{P}\} \to \mathcal{B}| \equiv |\mathcal{P}| \to |\mathcal{B}|\\
&|\{\mathcal{K}\}| \equiv |\mathcal{K}|
\end{aligned}$$

$\boxed{|\mathcal{P}|}$ Parameter type translation

$$\begin{aligned}
&|\cdot| \equiv \top\\
&|\mathcal{P};\, \ell : \mathcal{A}| \equiv |\mathcal{P}| \wedge \{\ell : |\mathcal{A}|\}\\
&|\mathcal{P};\, \ell? : \mathcal{A}| \equiv |\mathcal{P}| \wedge \{\ell : |\mathcal{A}| \vee \mathbf{Null}\}
\end{aligned}$$

$\boxed{|\mathcal{K}|}$ Argument type translation

$$\begin{aligned}
&|\cdot| \equiv \top\\
&|\mathcal{K};\, \ell : \mathcal{A}| \equiv |\mathcal{K}| \wedge \{\ell : |\mathcal{A}|\}
\end{aligned}$$

$\boxed{|\Delta|}$ Typing context translation

$$\begin{aligned}
&|\cdot| \equiv \cdot\\
&|\Delta,\, x : \mathcal{A}| \equiv |\Delta|,\, x : |\mathcal{A}|
\end{aligned}$$

Theorem 1 (Elaboration soundness) *If* $\Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e$, *then* $|\Delta| \vdash e : |\mathcal{A}|$.

Example 1

$$letin_1 \equiv \mathbf{let}\, x = r.x\, \mathbf{in} \qquad letin_2 \equiv \mathbf{let}\, y = \mathbf{switch}\, r.y\, \mathbf{as}\, y\, \mathbf{case}\, \mathbb{Z} \Rightarrow y\, \mathbf{case}\, \mathbf{Null} \Rightarrow x + 1\, \mathbf{in} \qquad e_0 \equiv letin_1 \circ letin_2\ (x + y)$$

$$\dfrac{\cdot, x : \mathbb{Z}, y : \mathbb{Z} \vdash x + y : \mathbb{Z} \rightsquigarrow x + y \qquad \dfrac{\cdot, x : \mathbb{Z} \vdash x + 1 : \mathbb{Z} \rightsquigarrow x + 1 \qquad \dfrac{\cdot \vdash 0 : \mathbb{Z} \rightsquigarrow 0 \qquad \dfrac{}{\cdot \vdash_r \cdot : \cdot \rightsquigarrow \mathbf{id} \dashv \cdot}\ \textit{PEla-Empty}}{\cdot \vdash_r (\cdot; x : \mathbb{Z}) : (\cdot; x : \mathbb{Z}) \rightsquigarrow letin_1 \dashv \cdot, x : \mathbb{Z}}\ \textit{PEla-Required}}{\cdot \vdash_r (\cdot; x : \mathbb{Z}; y = x + 1) : (\cdot; x : \mathbb{Z}; y? : \mathbb{Z}) \rightsquigarrow letin_1 \circ letin_2 \dashv \cdot, x : \mathbb{Z}, y : \mathbb{Z}}\ \textit{PEla-Optional}}{\cdot \vdash (\lambda\{\cdot; x : \mathbb{Z}; y = x + 1\}.\ x + y) : \{\cdot; x : \mathbb{Z}; y? : \mathbb{Z}\} \to \mathbb{Z} \rightsquigarrow \lambda r{:}\{x : \mathbb{Z}\} \wedge \{y : \mathbb{Z} \vee \mathbf{Null}\}.\ e_0{:}\mathbb{Z}}\ \textit{Ela-NAbs}$$

$$\dfrac{\begin{array}{c}\cdots \textit{Ela-NAbs} \cdots \\ \cdot \vdash \{\cdot; x = 1; z = 2\} : \{\cdot; x : \mathbb{Z}; z : \mathbb{Z}\} \rightsquigarrow e_1 \qquad \dfrac{\dfrac{\dfrac{}{\cdot \vdash_{e_1} \cdot \diamond (\cdot; x : \mathbb{Z}; z : \mathbb{Z}) \rightsquigarrow \{\}}\ \textit{PMat-Empty}}{\cdot \vdash_{e_1} (\cdot; x : \mathbb{Z}) \diamond (\cdot; x : \mathbb{Z}; z : \mathbb{Z}) \rightsquigarrow \{\} , \{x : \mathbb{Z} = e_1.x\}}\ \textit{PMat-Required}}{\cdot \vdash_{e_1} (\cdot; x : \mathbb{Z}; y? : \mathbb{Z}) \diamond (\cdot; x : \mathbb{Z}; z : \mathbb{Z}) \rightsquigarrow \{\} , \{x : \mathbb{Z} = e_1.x\} , \{y : \mathbf{Null} = \mathbf{null}\}}\ \textit{PMat-Absent}\end{array}}{\cdot \vdash (\lambda\{\cdot; x : \mathbb{Z}; y = x + 1\}.\ x + y)\, \{\cdot; x = 1; z = 2\} : \mathbb{Z} \rightsquigarrow (\lambda r{:}\{x : \mathbb{Z}\} \wedge \{y : \mathbb{Z} \vee \mathbf{Null}\}.\ e_0{:}\mathbb{Z})\, (\{\} , \{x : \mathbb{Z} = e_1.x\} , \{y : \mathbf{Null} = \mathbf{null}\})}\ \textit{Ela-NApp}$$

$$e_1 \equiv \{\cdot; x = 1; z = 2\}$$

Appendix: Semantics of λ_{iu}

$\boxed{A <: B}$ *(Subtyping)*

$$\frac{}{A <: \top}\ \textsc{Sub-Top} \qquad \frac{}{\bot <: A}\ \textsc{Sub-Bot} \qquad \frac{}{\mathbf{Null} <: \mathbf{Null}}\ \textsc{Sub-Null} \qquad \frac{}{\mathbb{Z} <: \mathbb{Z}}\ \textsc{Sub-Int} \qquad \frac{B_1 <: A_1 \qquad A_2 <: B_2}{A_1 \to A_2 <: B_1 \to B_2}\ \textsc{Sub-Arrow}$$

$$\frac{A <: B}{\{\ell : A\} <: \{\ell : B\}}\ \textsc{Sub-Rcd} \qquad \frac{A <: B \qquad A <: C}{A <: B \wedge C}\ \textsc{Sub-And} \qquad \frac{A <: C}{A \wedge B <: C}\ \textsc{Sub-AndL} \qquad \frac{B <: C}{A \wedge B <: C}\ \textsc{Sub-AndR} \qquad \frac{A <: C \qquad B <: C}{A \vee B <: C}\ \textsc{Sub-Or}$$

$$\frac{A <: B}{A <: B \vee C}\ \textsc{Sub-OrL} \qquad \frac{A <: C}{A <: B \vee C}\ \textsc{Sub-OrR}$$

$\boxed{\Gamma \vdash e : A}$ *(Typing)*

$$\frac{}{\Gamma \vdash \{\} : \top}\ \textsc{Typ-Top} \qquad \frac{}{\Gamma \vdash \mathbf{null} : \mathbf{Null}}\ \textsc{Typ-Null} \qquad \frac{}{\Gamma \vdash n : \mathbb{Z}}\ \textsc{Typ-Int} \qquad \frac{x : A \in \Gamma}{\Gamma \vdash x : A}\ \textsc{Typ-Var} \qquad \frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash (\lambda x{:}A.\ e{:}B) : A \to B}\ \textsc{Typ-Abs}$$

$$\frac{\Gamma \vdash e_1 : A \to B \qquad \Gamma \vdash e_2 : A}{\Gamma \vdash e_1\ e_2 : B}\ \textsc{Typ-App} \qquad \frac{\Gamma \vdash e : A}{\Gamma \vdash \{\ell : A = e\} : \{\ell : A\}}\ \textsc{Typ-Rcd} \qquad \frac{\Gamma \vdash e : \{\ell : A\}}{\Gamma \vdash e.\ell : A}\ \textsc{Typ-Prj}$$

$$\frac{\Gamma \vdash e_1 : A \qquad \Gamma \vdash e_2 : B}{\Gamma \vdash e_1 \,,\, e_2 : A \wedge B}\ \textsc{Typ-Merge} \qquad \frac{\begin{array}{c}\Gamma \vdash e_0 : A \vee B \\ \Gamma, x : A \vdash e_1 : C \\ \Gamma, x : B \vdash e_2 : C\end{array}}{\Gamma \vdash \mathbf{switch}\ e_0\ \mathbf{as}\ x\ \mathbf{case}\ A \Rightarrow e_1\ \mathbf{case}\ B \Rightarrow e_2 : C}\ \textsc{Typ-Switch}$$

$$\frac{\Gamma \vdash \mathit{letin} \dashv \Gamma' \qquad \Gamma' \vdash e : A}{\Gamma \vdash \mathit{letin}\ e : A}\ \textsc{Typ-Let} \qquad \frac{\Gamma \vdash e : A \qquad A <: B}{\Gamma \vdash e : B}\ \textsc{Typ-Sub}$$

$\boxed{\Gamma \vdash \mathit{letin} \dashv \Gamma'}$ *(Let-in binding)*

$$\frac{\Gamma \vdash e : A}{\Gamma \vdash \mathbf{let}\ x = e\ \mathbf{in} \dashv \Gamma, x : A}\ \textsc{LB-Let} \qquad \frac{\begin{array}{c}\Gamma \vdash \mathit{letin}_1 \dashv \Gamma' \\ \Gamma' \vdash \mathit{letin}_2 \dashv \Gamma''\end{array}}{\Gamma \vdash \mathit{letin}_1 \circ \mathit{letin}_2 \dashv \Gamma''}\ \textsc{LB-Comp} \qquad \frac{}{\Gamma \vdash \mathbf{id} \dashv \Gamma}\ \textsc{LB-Id}$$