

Named Arguments as Intersections, Optional Arguments as Unions

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September 28, 2024

Syntax of λ_{iu} (target)

Types	$A, B ::= \top \mid \perp \mid \mathbf{Null} \mid \mathbb{Z} \mid A \rightarrow B \mid \{\ell : A\} \mid A \wedge B \mid A \vee B$
Expressions	$e ::= \{\} \mid \mathbf{null} \mid n \mid x \mid \lambda x : A. e : B \mid e_1 e_2 \mid \{\ell = e\} \mid e.\ell \mid e_1, e_2 \mid \mathbf{switch} e_0 \mathbf{as} x \mathbf{case} A \Rightarrow e_1 \mathbf{case} B \Rightarrow e_2 \mid \mathbf{letin} e$
Let-in bindings	$\mathbf{letin} ::= \mathbf{let} x = e \mathbf{in} \mid \mathbf{letin}_1 \circ \mathbf{letin}_2 \mid \mathbf{id}$

Syntax of UAENA¹ (source)

$styp, \mathcal{A}, \mathcal{B}$	$::=$	source types
	\mathbb{Z}	integer type
	$\mathcal{A} \rightarrow \mathcal{B}$	function type
	$\{\mathcal{P}\} \rightarrow \mathcal{B}$	function type with named parameters
$nptyp, \mathcal{P}$	$::=$	named parameter types
	\cdot	empty
	$\mathcal{P}; \ell : \mathcal{A}$	required parameter
	$\mathcal{P}; \ell? : \mathcal{A}$	optional parameter
$sexp, \epsilon$	$::=$	source expressions
	n	integer literal
	x	variable
	$\lambda(x : \mathcal{A}). \epsilon$	abstraction
	$\epsilon_1 \epsilon_2$	application
	$\lambda\{\rho\}. \epsilon$	abstraction with named parameters
	$\epsilon \{\alpha\}$	application to named arguments
$nexp, \rho$	$::=$	named parameters
	\cdot	empty
	$\rho; \ell : \mathcal{A}$	required parameter
	$\rho; \ell = \epsilon$	optional parameter
$narg, \alpha$	$::=$	named arguments
	\cdot	empty
	$\alpha; \ell = \epsilon$	field

¹UAENA = Unnamed Arguments Encoded Named Arguments

From UAENA to λ_{iu}

$$\boxed{\Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e}$$

(Elaboration)

$$\begin{array}{c}
\text{ELA-INT} \\
\frac{}{\Delta \vdash n : \mathbb{Z} \rightsquigarrow n} \\
\\
\text{ELA-APP} \\
\frac{\Delta \vdash \epsilon_1 : \mathcal{A} \rightarrow \mathcal{B} \rightsquigarrow e_1 \quad \Delta \vdash \epsilon_2 : \mathcal{A} \rightsquigarrow e_2}{\Delta \vdash \epsilon_1 \epsilon_2 : \mathcal{B} \rightsquigarrow e_1 e_2} \\
\\
\text{ELA-VAR} \\
\frac{x : \mathcal{A} \in \Delta}{\Delta \vdash x : \mathcal{A} \rightsquigarrow x} \\
\\
\text{ELA-ABS} \\
\frac{\Delta, x : \mathcal{A} \vdash \epsilon : \mathcal{B} \rightsquigarrow e}{\Delta \vdash \lambda(x : \mathcal{A}). \epsilon : \mathcal{A} \rightarrow \mathcal{B} \rightsquigarrow \lambda x : |\mathcal{A}|. e : |\mathcal{B}|} \\
\\
\text{ELA-NABS} \\
\frac{\Delta \vdash_x \rho : \mathcal{P} \rightsquigarrow \text{letin} \dashv \Delta' \quad \Delta' \vdash \epsilon : \mathcal{B} \rightsquigarrow e}{\Delta \vdash \lambda\{\rho\}. \epsilon : \{\mathcal{P}\} \rightarrow \mathcal{B} \rightsquigarrow \lambda x : |\mathcal{P}|. \text{letin } e : |\mathcal{B}|} \\
\\
\text{ELA-NAPP} \\
\frac{\Delta \vdash \epsilon : \{\mathcal{P}\} \rightarrow \mathcal{B} \rightsquigarrow e \quad \Delta \vdash \mathcal{P} \diamond \alpha \rightsquigarrow e'}{\Delta \vdash \epsilon \{\alpha\} : \mathcal{B} \rightsquigarrow e e'}
\end{array}$$

$$\boxed{\Delta \vdash_x \rho : \mathcal{P} \rightsquigarrow \text{letin} \dashv \Delta'}$$

(Parameter elaboration)

$$\begin{array}{c}
\text{PELA-EMPTY} \\
\frac{}{\Delta \vdash_x \cdot : \cdot \rightsquigarrow \text{id} \dashv \Delta} \\
\\
\text{PELA-REQUIRED} \\
\frac{\Delta \vdash_x \rho : \mathcal{P} \rightsquigarrow \text{letin} \dashv \Delta'}{\Delta \vdash_x (\rho; \ell : \mathcal{A}) : (\mathcal{P}; \ell : \mathcal{A}) \rightsquigarrow \text{letin} \circ \text{let } \ell = x.\ell \text{ in} \dashv \Delta', \ell : \mathcal{A}} \\
\\
\text{PELA-OPTIONAL} \\
\frac{\Delta \vdash_x \rho : \mathcal{P} \rightsquigarrow \text{letin} \dashv \Delta' \quad \Delta' \vdash \epsilon : \mathcal{A} \rightsquigarrow e}{\Delta \vdash_x (\rho; \ell = \epsilon) : (\mathcal{P}; \ell? : \mathcal{A}) \rightsquigarrow \text{letin} \circ \text{let } \ell = \text{switch } x.\ell \text{ as } y \text{ case } |\mathcal{A}| \Rightarrow y \text{ case Null} \Rightarrow e \text{ in} \dashv \Delta', \ell : \mathcal{A}}
\end{array}$$

$$\boxed{\Delta \vdash \mathcal{P} \diamond \alpha \rightsquigarrow e}$$

(Parameter matching)

$$\begin{array}{c}
\text{PMAT-EMPTY} \\
\frac{}{\Delta \vdash \cdot \diamond \cdot \rightsquigarrow \{\}} \\
\\
\text{PMAT-EXTRA} \\
\frac{\Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e \quad \Delta \vdash \cdot \diamond \alpha \rightsquigarrow e'}{\Delta \vdash \cdot \diamond (\alpha; \ell = \epsilon) \rightsquigarrow e', \{\ell = e\}} \\
\\
\text{PMAT-REQUIRED} \\
\frac{\alpha.\ell \Rightarrow \epsilon \quad \Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e \quad \Delta \vdash \mathcal{P} \diamond \alpha \setminus \ell \rightsquigarrow e'}{\Delta \vdash (\mathcal{P}; \ell : \mathcal{A}) \diamond \alpha \rightsquigarrow e', \{\ell = e\}} \\
\\
\text{PMAT-PRESENT} \\
\frac{\alpha.\ell \Rightarrow \epsilon \quad \Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e \quad \Delta \vdash \mathcal{P} \diamond \alpha \setminus \ell \rightsquigarrow e'}{\Delta \vdash (\mathcal{P}; \ell? : \mathcal{A}) \diamond \alpha \rightsquigarrow e', \{\ell = e\}} \\
\\
\text{PMAT-ABSENT} \\
\frac{\alpha.\ell \not\Rightarrow \quad \Delta \vdash \mathcal{P} \diamond \alpha \rightsquigarrow e'}{\Delta \vdash (\mathcal{P}; \ell? : \mathcal{A}) \diamond \alpha \rightsquigarrow e', \{\ell = \text{null}\}}
\end{array}$$

$$\boxed{\alpha.\ell \Rightarrow \epsilon}$$

(Successful lookup)

$$\begin{array}{c}
\text{LU-PRESENT} \\
\frac{\alpha.\ell \not\Rightarrow}{(\alpha; \ell = \epsilon).\ell \Rightarrow \epsilon} \\
\\
\text{LU-ABSENT} \\
\frac{\ell' \neq \ell \quad \alpha.\ell \Rightarrow \epsilon}{(\alpha; \ell' = \epsilon').\ell \Rightarrow \epsilon}
\end{array}$$

$$\boxed{\alpha.\ell \not\Rightarrow}$$

(Failed lookup)

$$\begin{array}{c}
\text{LD-EMPTY} \\
\frac{}{\cdot.\ell \not\Rightarrow} \\
\\
\text{LD-ABSENT} \\
\frac{\ell' \neq \ell \quad \alpha.\ell \not\Rightarrow}{(\alpha; \ell' = \epsilon).\ell \not\Rightarrow}
\end{array}$$

$$\boxed{|\mathcal{A}|} \quad \text{Type translation}$$

$$\begin{array}{l}
|\mathbb{Z}| \equiv \mathbb{Z} \\
|\mathcal{A} \rightarrow \mathcal{B}| \equiv |\mathcal{A}| \rightarrow |\mathcal{B}| \\
|\{\mathcal{P}\} \rightarrow \mathcal{B}| \equiv |\mathcal{P}| \rightarrow |\mathcal{B}|
\end{array}$$

$\boxed{|\mathcal{P}|}$ Parameter type translation

$$\begin{aligned} |\cdot| &\equiv \top \\ |\mathcal{P}; \ell : \mathcal{A}| &\equiv |\mathcal{P}| \wedge \{\ell : |\mathcal{A}|\} \\ |\mathcal{P}; \ell? : \mathcal{A}| &\equiv |\mathcal{P}| \wedge \{\ell : |\mathcal{A}| \vee \mathbf{Null}\} \end{aligned}$$

$\boxed{|\Delta|}$ Typing context translation

$$\begin{aligned} |\cdot| &\equiv \cdot \\ |\Delta, x : \mathcal{A}| &\equiv |\Delta|, x : |\mathcal{A}| \end{aligned}$$

Theorem 1 (Elaboration soundness) *If $\Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e$, then $|\Delta| \vdash e : |\mathcal{A}|$.*

Example 1

$letin_1 \equiv \mathbf{let} \ x = r.x \ \mathbf{in}$

$letin_2 \equiv \mathbf{let} \ y = \mathbf{switch} \ r.y \ \mathbf{as} \ y \ \mathbf{case} \ \mathbb{Z} \Rightarrow y \ \mathbf{case} \ \mathbf{Null} \Rightarrow x + 1 \ \mathbf{in}$

$e_0 \equiv letin_1 \circ letin_2 \ (x + y)$

$$\begin{array}{c}
\frac{\cdot, x : \mathbb{Z} \vdash x + 1 : \mathbb{Z} \rightsquigarrow x + 1 \quad \frac{\cdot \vdash 0 : \mathbb{Z} \rightsquigarrow 0 \quad \frac{}{\cdot \vdash_r \cdot : \cdot \rightsquigarrow \mathbf{id} \dashv \cdot} P_{ELA-EMPTY}}{\cdot \vdash_r (\cdot; x : \mathbb{Z}) : (\cdot; x : \mathbb{Z}) \rightsquigarrow letin_1 \dashv \cdot, x : \mathbb{Z}} P_{ELA-REQUIRED}}{\cdot, x : \mathbb{Z}, y : \mathbb{Z} \vdash x + y : \mathbb{Z} \rightsquigarrow x + y \quad \cdot \vdash_r (\cdot; x : \mathbb{Z}; y = x + 1) : (\cdot; x : \mathbb{Z}; y^? : \mathbb{Z}) \rightsquigarrow letin_1 \circ letin_2 \dashv \cdot, x : \mathbb{Z}, y : \mathbb{Z}} P_{ELA-OPTIONAL}}{\cdot \vdash (\lambda\{\cdot; x : \mathbb{Z}; y = x + 1\}. x + y) : \{\cdot; x : \mathbb{Z}; y^? : \mathbb{Z}\} \rightarrow \mathbb{Z} \rightsquigarrow \lambda r : \{x : \mathbb{Z}\} \wedge \{y : \mathbb{Z} \vee \mathbf{Null}\}. e_0 : \mathbb{Z}} ELA-NABS}
\\[20pt]
\frac{\cdot \vdash 2 : \mathbb{Z} \rightsquigarrow 2 \quad \frac{}{\cdot \vdash \cdot \diamond \cdot \rightsquigarrow \{\}} P_{MAT-EMPTY}}{\cdot \vdash \cdot \diamond (\cdot; z = 2) \rightsquigarrow \{z = 2\}} P_{MAT-EXTRA}
\\[10pt]
\frac{\cdot \vdash 1 : \mathbb{Z} \rightsquigarrow 1 \quad \cdot \vdash (\cdot; x : \mathbb{Z}) \diamond (\cdot; x = 1; z = 2) \rightsquigarrow \{z = 2\}_, \{x = 1\}}{\cdot \vdash (\cdot; x : \mathbb{Z}; y^? : \mathbb{Z}) \diamond (\cdot; x = 1; z = 2) \rightsquigarrow \{z = 2\}_, \{x = 1\}_, \{y = \mathbf{null}\}} P_{MAT-ABSENT}
\\[10pt]
\frac{\dots ELA-NABS \dots \quad \cdot \vdash (\cdot; x : \mathbb{Z}; y = x + 1) \{ \cdot; x = 1; z = 2 \} : \mathbb{Z} \rightsquigarrow (\lambda r : \{x : \mathbb{Z}\} \wedge \{y : \mathbb{Z} \vee \mathbf{Null}\}. e_0 : \mathbb{Z}) (\{z = 2\}_, \{x = 1\}_, \{y = \mathbf{null}\})}{\cdot \vdash (\lambda\{\cdot; x : \mathbb{Z}; y = x + 1\}. x + y) \{ \cdot; x = 1; z = 2 \} : \mathbb{Z} \rightsquigarrow (\lambda r : \{x : \mathbb{Z}\} \wedge \{y : \mathbb{Z} \vee \mathbf{Null}\}. e_0 : \mathbb{Z}) (\{z = 2\}_, \{x = 1\}_, \{y = \mathbf{null}\})} ELA-NAPP}
\end{array}$$

Appendix: Semantics of λ_{iu}

$A <: B$

(Subtyping)

$\frac{\text{SUB-TOP}}{A <: \top}$	$\frac{\text{SUB-BOT}}{\perp <: A}$	$\frac{\text{SUB-NULL}}{\mathbf{Null} <: \mathbf{Null}}$	$\frac{\text{SUB-INT}}{\mathbb{Z} <: \mathbb{Z}}$	$\frac{\text{SUB-ARROW}}{B_1 <: A_1 \quad A_2 <: B_2 \quad A_1 \rightarrow A_2 <: B_1 \rightarrow B_2}$
$\frac{\text{SUB-RCD}}{A <: B \quad \{\ell : A\} <: \{\ell : B\}}$	$\frac{\text{SUB-AND}}{A <: B \quad A <: C \quad A <: B \wedge C}$	$\frac{\text{SUB-ANDL}}{A <: C \quad A \wedge B <: C}$	$\frac{\text{SUB-ANDR}}{B <: C \quad A \wedge B <: C}$	$\frac{\text{SUB-OR}}{A <: C \quad B <: C \quad A \vee B <: C}$
	$\frac{\text{SUB-ORL}}{A <: B \quad A <: B \vee C}$		$\frac{\text{SUB-ORR}}{A <: C \quad A <: B \vee C}$	

$\Gamma \vdash e : A$

(Typing)

$\frac{\text{TYP-TOP}}{\Gamma \vdash \{\} : \top}$	$\frac{\text{TYP-NULL}}{\Gamma \vdash \mathbf{null} : \mathbf{Null}}$	$\frac{\text{TYP-INT}}{\Gamma \vdash n : \mathbb{Z}}$	$\frac{\text{TYP-VAR}}{x : A \in \Gamma \quad \Gamma \vdash x : A}$	$\frac{\text{TYP-ABS}}{\Gamma, x : A \vdash e : B \quad \Gamma \vdash (\lambda x : A. e : B) : A \rightarrow B}$
$\frac{\text{TYP-APP}}{\Gamma \vdash e_1 : A \rightarrow B \quad \Gamma \vdash e_2 : A \quad \Gamma \vdash e_1 e_2 : B}$		$\frac{\text{TYP-RCD}}{\Gamma \vdash e : A \quad \Gamma \vdash \{\ell = e\} : \{\ell : A\}}$		$\frac{\text{TYP-PRJ}}{\Gamma \vdash e : \{\ell : A\} \quad \Gamma \vdash e.\ell : A}$
$\frac{\text{TYP-MERGE}}{\Gamma \vdash e_1 : A \quad \Gamma \vdash e_2 : B \quad \Gamma \vdash e_1, e_2 : A \wedge B}$		$\frac{\text{TYP-SWITCH}}{\Gamma \vdash e_0 : A \vee B \quad \Gamma, x : A \vdash e_1 : C \quad \Gamma, x : B \vdash e_2 : C \quad \Gamma \vdash \mathbf{switch} \, e_0 \, \mathbf{as} \, x \, \mathbf{case} \, A \Rightarrow e_1 \, \mathbf{case} \, B \Rightarrow e_2 : C}$		
	$\frac{\text{TYP-LET}}{\Gamma \vdash \mathbf{letin} \, \vdash \Gamma' \quad \Gamma' \vdash e : A \quad \Gamma \vdash \mathbf{letin} \, e : A}$		$\frac{\text{TYP-SUB}}{\Gamma \vdash e : A \quad A <: B \quad \Gamma \vdash e : B}$	

$\Gamma \vdash \mathbf{letin} \, \vdash \Gamma'$

(Let-in binding)

$\frac{\text{LB-LET}}{\Gamma \vdash \mathbf{let} \, x = e \, \mathbf{in} \, \vdash \Gamma, x : A}$	$\frac{\text{LB-COMP}}{\Gamma \vdash \mathbf{letin}_1 \, \vdash \Gamma' \quad \Gamma' \vdash \mathbf{letin}_2 \, \vdash \Gamma'' \quad \Gamma \vdash \mathbf{letin}_1 \circ \mathbf{letin}_2 \, \vdash \Gamma''}$	$\frac{\text{LB-ID}}{\Gamma \vdash \mathbf{id} \, \vdash \Gamma}$
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