Named Arguments as Intersections, Optional Arguments as Unions

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Syntax of λ_{iu} (target)

Types	$A, B ::= \top \mid \bot \mid \mathbf{Null} \mid \mathbb{Z} \mid A \to B \mid \{\ell : A\} \mid A \wedge B \mid A \vee B$
Expressions	$e ::= \{\} \mid \mathbf{null} \mid n \mid x \mid \lambda x : A.\ e : B \mid e_1\ e_2 \mid \{\ell = e\} \mid e.\ell \mid e_1 \,,\, e_2 \mid$ $\mathbf{switch}\ e_0\ \mathbf{as}\ x\ \mathbf{case}\ A \Rightarrow e_1\ \mathbf{case}\ B \Rightarrow e_2 \mid \mathbf{let}\ x = e_1\ \mathbf{in}\ e_2$

Syntax of Uaena[1] (source)

$styp,\ \mathcal{A},\ \mathcal{B}$	$::=$		source types
	$\mid$	$\mathbb{Z}$	integer type
	$\mid$	$\mathcal{A} \to \mathcal{B}$	function type
	$\mid$	$\{\mathcal{P}\} \to \mathcal{B}$	function type with named parameters
$nptyp,\ \mathcal{P}$	$::=$		named parameter types
	$\mid$	$\cdot$	empty
	$\mid$	$\ell : \mathcal{A}; \mathcal{P}$	required parameter
	$\mid$	$\ell? : \mathcal{A}; \mathcal{P}$	optional parameter
$sexp,\ \epsilon$	$::=$		source expressions
	$\mid$	n	integer literal
	$\mid$	x	variable
	$\mid$	$\lambda(x : \mathcal{A}).\ \epsilon$	abstraction
	$\mid$	$\epsilon_1\ \epsilon_2$	application
	$\mid$	$\lambda\{\rho\}.\ \epsilon$	abstraction with named parameters
	$\mid$	$\epsilon\ \{\alpha\}$	application to named arguments
$npexp,\ \rho$	$::=$		named parameters
	$\mid$	$\cdot$	empty
	$\mid$	$\ell : \mathcal{A}; \rho$	required parameter
	$\mid$	$\ell = \epsilon; \rho$	optional parameter
$narg,\ \alpha$	$::=$		named arguments
	$\mid$	$\cdot$	empty
	$\mid$	$\ell = \epsilon; \alpha$	field
	$\mid$	$\alpha \backslash \ell$	removal

[1] Uaena = Unnamed Arguments Encoded Named Arguments

From $\textsc{Uaena}$ to λ_{iu}

$\boxed{\|\mathcal{A}\| = A}$ *(Type translation)*

$$\textsc{Tr-Int}\quad \frac{}{\|\mathbb{Z}\| = \mathbb{Z}} \qquad \textsc{Tr-Arrow}\quad \frac{\|\mathcal{A}\| = A \qquad \|\mathcal{B}\| = B}{\|\mathcal{A} \to \mathcal{B}\| = A \to B} \qquad \textsc{Tr-NArrow}\quad \frac{\|\mathcal{P}\| = A \qquad \|\mathcal{B}\| = B}{\|\{\mathcal{P}\} \to \mathcal{B}\| = A \to B}$$

$\boxed{\|\mathcal{P}\| = A}$ *(Parameter type translation)*

$$\textsc{PTr-Empty}\quad \frac{}{\|\cdot\| = \top} \qquad \textsc{PTr-Required}\quad \frac{\|\mathcal{P}\| = B}{\|\ell : \mathcal{A}; \mathcal{P}\| = \{\ell : A\} \wedge B} \qquad \textsc{PTr-Optional}\quad \frac{\|\mathcal{P}\| = B}{\|\ell? : \mathcal{A}; \mathcal{P}\| = \{\ell : A \vee \mathbf{Null}\} \wedge B}$$

$\boxed{\|\Delta\| = \Gamma}$ *(Context translation)*

$$\textsc{GTr-Nil}\quad \frac{}{\|\cdot\| = \cdot} \qquad \textsc{GTr-Cons}\quad \frac{\|\Delta\| = \Gamma \qquad \|\mathcal{A}\| = A}{\|\Delta,\, x : \mathcal{A}\| = \Gamma,\, x : A}$$

$\boxed{\Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e}$ *(Elaboration)*

$$\textsc{Ela-Int}\quad \frac{}{\Delta \vdash n : \mathbb{Z} \rightsquigarrow n} \qquad \textsc{Ela-Var}\quad \frac{x : \mathcal{A} \in \Delta}{\Delta \vdash x : \mathcal{A} \rightsquigarrow x} \qquad \textsc{Ela-Abs}\quad \frac{\begin{array}{c}\Delta,\, x : \mathcal{A} \vdash \epsilon : \mathcal{B} \rightsquigarrow e \\ \|\mathcal{A}\| = A \qquad \|\mathcal{B}\| = B\end{array}}{\Delta \vdash \lambda(x{:}\mathcal{A}).\, \epsilon : \mathcal{A} \to \mathcal{B} \rightsquigarrow \lambda x{:}A.\, e{:}B}$$

$$\textsc{Ela-App}\quad \frac{\begin{array}{c}\Delta \vdash \epsilon_1 : \mathcal{A} \to \mathcal{B} \rightsquigarrow e_1 \\ \Delta \vdash \epsilon_2 : \mathcal{A} \rightsquigarrow e_2\end{array}}{\Delta \vdash \epsilon_1\, \epsilon_2 : \mathcal{B} \rightsquigarrow e_1\, e_2} \qquad \textsc{Ela-NAbs}\quad \frac{\begin{array}{c}\mathbf{fresh}\, x \\ \Delta; x \vdash \rho : \mathcal{P} \rightsquigarrow \mathit{letin} \dashv \Delta' \\ \Delta' \vdash \epsilon : \mathcal{B} \rightsquigarrow e \\ \|\mathcal{P}\| = A \qquad \|\mathcal{B}\| = B\end{array}}{\Delta \vdash \lambda\{\rho\}.\, \epsilon : \{\mathcal{P}\} \to \mathcal{B} \rightsquigarrow \lambda x{:}A.\, \mathit{letin}\, e{:}B} \qquad \textsc{Ela-NApp}\quad \frac{\begin{array}{c}\Delta \vdash \epsilon : \{\mathcal{P}\} \to \mathcal{B} \rightsquigarrow e \\ \Delta \vdash \mathcal{P} \diamond \alpha \rightsquigarrow e'\end{array}}{\Delta \vdash \epsilon\, \{\alpha\} : \mathcal{B} \rightsquigarrow e\, e'}$$

$\boxed{\Delta; x \vdash \rho : \mathcal{P} \rightsquigarrow \mathit{letin} \dashv \Delta'}$ *(Parameter elaboration)*

$$\textsc{PEla-Empty}\quad \frac{}{\Delta; x \vdash \cdot : \cdot \rightsquigarrow \mathbf{id} \dashv \Delta} \qquad \textsc{PEla-Required}\quad \frac{\Delta; x \vdash \rho : \mathcal{P} \rightsquigarrow \mathit{letin} \dashv \Delta'}{\Delta; x \vdash (\ell : \mathcal{A}; \rho) : (\ell : \mathcal{A}; \mathcal{P}) \rightsquigarrow \mathbf{let}\, \ell = x.\ell\, \mathbf{in} \circ \mathit{letin} \dashv \Delta',\, \ell : \mathcal{A}}$$

$$\textsc{PEla-Optional}\quad \frac{\begin{array}{c}\Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e \qquad \|\mathcal{A}\| = A \\ \Delta; x \vdash \rho : \mathcal{P} \rightsquigarrow \mathit{letin} \dashv \Delta'\end{array}}{\Delta; x \vdash (\ell = \epsilon; \rho) : (\ell? : \mathcal{A}; \mathcal{P}) \rightsquigarrow \mathbf{let}\, \ell = \mathbf{switch}\, x.\ell\, \mathbf{as}\, y\, \mathbf{case}\, A \Rightarrow y\, \mathbf{case}\, \mathbf{Null} \Rightarrow e\, \mathbf{in} \circ \mathit{letin} \dashv \Delta',\, \ell : \mathcal{A}}$$

$\boxed{\Delta \vdash \mathcal{P} \diamond \alpha \rightsquigarrow e}$ *(Parameter matching)*

$$\textsc{PMat-Empty}\quad \frac{}{\Delta \vdash \cdot \diamond \cdot \rightsquigarrow \{\}} \qquad \textsc{PMat-Extra}\quad \frac{\begin{array}{c}\Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e \\ \Delta \vdash \cdot \diamond \alpha \rightsquigarrow e'\end{array}}{\Delta \vdash \cdot \diamond (\ell = \epsilon; \alpha) \rightsquigarrow \{\ell = e\} \,,\, e'} \qquad \textsc{PMat-Present}\quad \frac{\begin{array}{c}\alpha.\ell \Rightarrow \epsilon \qquad \Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e \\ \Delta \vdash \mathcal{P} \diamond \alpha \backslash \ell \rightsquigarrow e'\end{array}}{\Delta \vdash (\ell : \mathcal{A}; \mathcal{P}) \diamond \alpha \rightsquigarrow \{\ell = e\} \,,\, e'}$$

$$\textsc{PMat-Absent}\quad \frac{\alpha.\ell \not\Rightarrow \qquad \Delta \vdash \mathcal{P} \diamond \alpha \rightsquigarrow e'}{\Delta \vdash (\ell : \mathcal{A}; \mathcal{P}) \diamond \alpha \rightsquigarrow \{\ell = \mathbf{null}\} \,,\, e'}$$

Example 1

$$\mathit{letin}_1 = \mathbf{let}\, x = z.x\, \mathbf{in} \qquad \mathit{letin}_2 = \mathbf{let}\, y = \mathbf{switch}\, z.y\, \mathbf{as}\, y\, \mathbf{case}\, \mathbb{Z} \Rightarrow y\, \mathbf{case}\, \mathbf{Null} \Rightarrow 0\, \mathbf{in} \qquad e_0 = \mathit{letin}_1 \circ \mathit{letin}_2\ (x + y)$$

$$\dfrac{\cdot, y : \mathbb{Z}, x : \mathbb{Z} \vdash x + y : \mathbb{Z} \rightsquigarrow x + y \qquad \dfrac{\dfrac{\cdot \vdash 0 : \mathbb{Z} \rightsquigarrow 0 \qquad \dfrac{}{\cdot; z \vdash \cdot : \cdot \rightsquigarrow \mathbf{id} \dashv \cdot}\,\textit{PE}\textsc{la-Empty}}{\cdot; z \vdash (y = 0; \cdot) : (y\,?: \mathbb{Z}; \cdot) \rightsquigarrow \mathit{letin}_2 \dashv \cdot, y : \mathbb{Z}}\,\textit{PE}\textsc{la-Optional}}{\cdot; z \vdash (x : \mathbb{Z}; y = 0; \cdot) : (x : \mathbb{Z}; y\,?: \mathbb{Z}; \cdot) \rightsquigarrow \mathit{letin}_1 \circ \mathit{letin}_2 \dashv \cdot, y : \mathbb{Z}, x : \mathbb{Z}}\,\textit{PE}\textsc{la-Required}}{\cdot \vdash (\lambda\{x : \mathbb{Z}; y = 0; \cdot\}.\ x + y) : \{x : \mathbb{Z}; y\,?: \mathbb{Z}; \cdot\} \to \mathbb{Z} \rightsquigarrow \lambda z{:}\{x : \mathbb{Z}\} \wedge \{y : \mathbb{Z} \vee \mathbf{Null}\}.\ e_0 : \mathbb{Z}}\,\textit{E}\textsc{la-NPAbs}$$

$$\dfrac{\cdots \textit{E}\textsc{la-NAbs} \cdots \qquad \dfrac{\cdot \vdash 1 : \mathbb{Z} \rightsquigarrow 1 \qquad \dfrac{\dfrac{\cdot \vdash 2 : \mathbb{Z} \rightsquigarrow 2 \qquad \dfrac{}{\cdot \vdash \cdot \diamond \cdot \rightsquigarrow \{\}}\,\textit{PM}\textsc{at-Empty}}{\cdot \vdash \cdot \diamond (z = 2; \cdot) \rightsquigarrow \{z = 2\}}\,\textit{PM}\textsc{at-Extra}}{\cdot \vdash (y\,?: \mathbb{Z}; \cdot) \diamond (z = 2; \cdot) \rightsquigarrow \{y = \mathbf{null}\} \,,\, \{z = 2\}}\,\textit{PM}\textsc{at-Absent}}{\cdot \vdash (x : \mathbb{Z}; y\,?: \mathbb{Z}; \cdot) \diamond (x = 1; z = 2; \cdot) \rightsquigarrow \{x = 1\} \,,\, \{y = \mathbf{null}\} \,,\, \{z = 2\}}\,\textit{PM}\textsc{at-Present}}{\cdot \vdash (\lambda\{x : \mathbb{Z}; y = 0; \cdot\}.\ x + y)\,\{x = 1; z = 2; \cdot\} : \mathbb{Z} \rightsquigarrow (\lambda z{:}\{x : \mathbb{Z}\} \wedge \{y : \mathbb{Z} \vee \mathbf{Null}\}.\ e_0 : \mathbb{Z})\,(\{x = 1\} \,,\, \{y = \mathbf{null}\} \,,\, \{z = 2\})}\,\textit{E}\textsc{la-NApp}$$

Theorem 1 (Elaboration soundness) *If* $\Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e$ *and* $\|\Delta\| = \Gamma$ *and* $\|\mathcal{A}\| = A$, *then* $\Gamma \vdash e : A$.

Appendix: Semantics of λ_{iu}

$\boxed{A <: B}$ *(Subtyping)*

$$\textsc{Sub-Top}\quad \frac{}{A <: \top} \qquad \textsc{Sub-Bot}\quad \frac{}{\bot <: A} \qquad \textsc{Sub-Int}\quad \frac{}{\mathbb{Z} <: \mathbb{Z}}$$

$$\textsc{Sub-Arrow}\quad \frac{B_1 <: A_1 \qquad A_2 <: B_2}{A_1 \to A_2 <: B_1 \to B_2} \qquad \textsc{Sub-Rcd}\quad \frac{A <: B}{\{\ell : A\} <: \{\ell : B\}}$$

$$\textsc{Sub-And}\quad \frac{A <: B \qquad A <: C}{A <: B \wedge C} \qquad \textsc{Sub-AndL}\quad \frac{A <: C}{A \wedge B <: C} \qquad \textsc{Sub-AndR}\quad \frac{B <: C}{A \wedge B <: C}$$

$$\textsc{Sub-Or}\quad \frac{A <: C \qquad B <: C}{A \vee B <: C} \qquad \textsc{Sub-OrL}\quad \frac{A <: B}{A <: B \vee C} \qquad \textsc{Sub-OrR}\quad \frac{A <: C}{A <: B \vee C}$$

$\boxed{\Gamma \vdash e : A}$ *(Typing)*

$$\textsc{Typ-Top}\quad \frac{}{\Gamma \vdash \{\} : \top} \qquad \textsc{Typ-Int}\quad \frac{}{\Gamma \vdash n : \mathbb{Z}} \qquad \textsc{Typ-Var}\quad \frac{x : A \in \Gamma}{\Gamma \vdash x : A}$$

$$\textsc{Typ-Abs}\quad \frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash (\lambda x{:}A.\ e{:}B) : A \to B}$$

$$\textsc{Typ-App}\quad \frac{\Gamma \vdash e_1 : A \to B \qquad \Gamma \vdash e_2 : A}{\Gamma \vdash e_1\, e_2 : B} \qquad \textsc{Typ-Rcd}\quad \frac{\Gamma \vdash e : A}{\Gamma \vdash \{\ell = e\} : \{\ell : A\}} \qquad \textsc{Typ-Prj}\quad \frac{\Gamma \vdash e : \{\ell : A\}}{\Gamma \vdash e.\ell : A}$$

$$\textsc{Typ-Merge}\quad \frac{\Gamma \vdash e_1 : A \qquad \Gamma \vdash e_2 : B}{\Gamma \vdash e_1 \,,\, e_2 : A \wedge B}$$

$$\textsc{Typ-Switch}\quad \frac{\begin{array}{c}\Gamma \vdash e : A \vee B \\ \Gamma, x : A \vdash e_1 : C \\ \Gamma, x : B \vdash e_2 : C\end{array}}{\Gamma \vdash \mathbf{switch}\ e_0\ \mathbf{as}\ x\ \mathbf{case}\ A \Rightarrow e_1\ \mathbf{case}\ B \Rightarrow e_2 : C}$$

$$\textsc{Typ-Let}\quad \frac{\Gamma \vdash \mathit{letin} \dashv \Gamma' \qquad \Gamma' \vdash e : A}{\Gamma \vdash \mathit{letin}\ e : A} \qquad \textsc{Typ-Sub}\quad \frac{\Gamma \vdash e : A \qquad A <: B}{\Gamma \vdash e : B}$$