Named Arguments as Intersections, Optional Arguments as Unions

Yaozhu Sun

September 10, 2024

Syntax of λ_{iu} (target)

Types	$A, B ::= \top \mid \bot \mid \mathbf{Null} \mid \mathbb{Z} \mid A \to B \mid \{\ell : A\} \mid A \wedge B \mid A \vee B$
Expressions	$e ::= \{\} \mid \mathbf{null} \mid n \mid x \mid \lambda x : A.\ e : B \mid e_1\ e_2 \mid \{\ell = e\} \mid e.\ell \mid e_1 \,,\, e_2 \mid$ $\mathbf{switch}\ e_0\ \mathbf{as}\ x\ \mathbf{case}\ A \Rightarrow e_1\ \mathbf{case}\ B \Rightarrow e_2 \mid \mathit{letin}\ e$
Let-in bindings	$\mathit{letin} ::= \mathbf{let}\ x = e\ \mathbf{in} \mid \mathit{letin}_1 \circ \mathit{letin}_2 \mid \mathbf{id}$

Syntax of UAENA[1] (source)

$\mathit{styp},\ \mathcal{A},\ \mathcal{B}$	$::=$		source types
	$\mid$	$\mathbb{Z}$	integer type
	$\mid$	$\mathcal{A} \to \mathcal{B}$	function type
	$\mid$	$\{\mathcal{P}\} \to \mathcal{B}$	function type with named parameters
$\mathit{nptyp},\ \mathcal{P}$	$::=$		named parameter types
	$\mid$	$\cdot$	empty
	$\mid$	$\ell : \mathcal{A}; \mathcal{P}$	required parameter
	$\mid$	$\ell? : \mathcal{A}; \mathcal{P}$	optional parameter
$\mathit{sexp},\ \epsilon$	$::=$		source expressions
	$\mid$	n	integer literal
	$\mid$	x	variable
	$\mid$	$\lambda(x : \mathcal{A}).\ \epsilon$	abstraction
	$\mid$	$\epsilon_1\ \epsilon_2$	application
	$\mid$	$\lambda\{\rho\}.\ \epsilon$	abstraction with named parameters
	$\mid$	$\epsilon\ \{\alpha\}$	application to named arguments
$\mathit{npexp},\ \rho$	$::=$		named parameters
	$\mid$	$\cdot$	empty
	$\mid$	$\ell : \mathcal{A}; \rho$	required parameter
	$\mid$	$\ell = \epsilon; \rho$	optional parameter
$\mathit{narg},\ \alpha$	$::=$		named arguments
	$\mid$	$\cdot$	empty
	$\mid$	$\ell = \epsilon; \alpha$	field

[1] UAENA = Unnamed Arguments Encoded Named Arguments

From Uaena to λ_{iu}

$\boxed{\Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e}$ *(Elaboration)*

$$\textsc{Ela-Int}\quad \frac{}{\Delta \vdash n : \mathbb{Z} \rightsquigarrow n}$$

$$\textsc{Ela-Var}\quad \frac{x : \mathcal{A} \in \Delta}{\Delta \vdash x : \mathcal{A} \rightsquigarrow x}$$

$$\textsc{Ela-Abs}\quad \frac{\begin{array}{c}\Delta,\, x : \mathcal{A} \vdash \epsilon : \mathcal{B} \rightsquigarrow e \\ |\mathcal{A}| = A \qquad |\mathcal{B}| = B\end{array}}{\Delta \vdash \lambda(x{:}\mathcal{A}).\ \epsilon : \mathcal{A} \to \mathcal{B} \rightsquigarrow \lambda x{:}A.\ e{:}B}$$

$$\textsc{Ela-App}\quad \frac{\begin{array}{c}\Delta \vdash \epsilon_1 : \mathcal{A} \to \mathcal{B} \rightsquigarrow e_1 \\ \Delta \vdash \epsilon_2 : \mathcal{A} \rightsquigarrow e_2\end{array}}{\Delta \vdash \epsilon_1\, \epsilon_2 : \mathcal{B} \rightsquigarrow e_1\, e_2}$$

$$\textsc{Ela-NAbs}\quad \frac{\begin{array}{c}\textbf{fresh}\ x \\ \Delta; x \vdash \rho : \mathcal{P} \rightsquigarrow \mathit{letin} \dashv \Delta' \\ \Delta' \vdash \epsilon : \mathcal{B} \rightsquigarrow e \\ |\mathcal{P}| = A \qquad |\mathcal{B}| = B\end{array}}{\Delta \vdash \lambda\{\rho\}.\ \epsilon : \{\mathcal{P}\} \to \mathcal{B} \rightsquigarrow \lambda x{:}A.\ \mathit{letin}\ e{:}B}$$

$$\textsc{Ela-NApp}\quad \frac{\begin{array}{c}\Delta \vdash \epsilon : \{\mathcal{P}\} \to \mathcal{B} \rightsquigarrow e \\ \Delta \vdash \mathcal{P} \diamond \alpha \rightsquigarrow e'\end{array}}{\Delta \vdash \epsilon\,\{\alpha\} : \mathcal{B} \rightsquigarrow e\, e'}$$

$\boxed{\Delta; x \vdash \rho : \mathcal{P} \rightsquigarrow \mathit{letin} \dashv \Delta'}$ *(Parameter elaboration)*

$$\textsc{PEla-Empty}\quad \frac{}{\Delta; x \vdash \cdot : \cdot \rightsquigarrow \mathbf{id} \dashv \Delta}$$

$$\textsc{PEla-Required}\quad \frac{\Delta; x \vdash \rho : \mathcal{P} \rightsquigarrow \mathit{letin} \dashv \Delta'}{\Delta; x \vdash (\ell : \mathcal{A}; \rho) : (\ell : \mathcal{A}; \mathcal{P}) \rightsquigarrow \mathbf{let}\, \ell = x.\ell\, \mathbf{in} \circ \mathit{letin} \dashv \Delta',\, \ell : \mathcal{A}}$$

$$\textsc{PEla-Optional}\quad \frac{\begin{array}{c}\Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e \qquad |\mathcal{A}| = A \\ \Delta; x \vdash \rho : \mathcal{P} \rightsquigarrow \mathit{letin} \dashv \Delta'\end{array}}{\Delta; x \vdash (\ell = \epsilon; \rho) : (\ell? : \mathcal{A}; \mathcal{P}) \rightsquigarrow \mathbf{let}\, \ell = \mathbf{switch}\, x.\ell\, \mathbf{as}\, y\, \mathbf{case}\, A \Rightarrow y\, \mathbf{case}\, \mathbf{Null} \Rightarrow e\, \mathbf{in} \circ \mathit{letin} \dashv \Delta',\, \ell : \mathcal{A}}$$

$\boxed{\Delta \vdash \mathcal{P} \diamond \alpha \rightsquigarrow e}$ *(Parameter matching)*

$$\textsc{PMat-Empty}\quad \frac{}{\Delta \vdash \cdot \diamond \cdot \rightsquigarrow \{\}}$$

$$\textsc{PMat-Extra}\quad \frac{\begin{array}{c}\Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e \\ \Delta \vdash \cdot \diamond \alpha \rightsquigarrow e'\end{array}}{\Delta \vdash \cdot \diamond (\ell = \epsilon; \alpha) \rightsquigarrow \{\ell = e\} \fatsemi e'}$$

$$\textsc{PMat-Present}\quad \frac{\begin{array}{c}\alpha.\ell \Rightarrow \epsilon \qquad \Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e \\ \Delta \vdash \mathcal{P} \diamond \alpha \backslash \ell \rightsquigarrow e'\end{array}}{\Delta \vdash (\ell : \mathcal{A}; \mathcal{P}) \diamond \alpha \rightsquigarrow \{\ell = e\} \fatsemi e'}$$

$$\textsc{PMat-Absent}\quad \frac{\alpha.\ell \not\Rightarrow \qquad \Delta \vdash \mathcal{P} \diamond \alpha \rightsquigarrow e'}{\Delta \vdash (\ell : \mathcal{A}; \mathcal{P}) \diamond \alpha \rightsquigarrow \{\ell = \mathbf{null}\} \fatsemi e'}$$

$\boxed{\alpha.\ell \Rightarrow \epsilon}$ *(Successful lookup)*

$$\textsc{LU-Field}\quad \frac{\alpha.\ell \not\Rightarrow}{(\ell = \epsilon; \alpha').\ell \Rightarrow \epsilon}$$

$\boxed{\alpha.\ell \not\Rightarrow}$ *(Failed lookup)*

$$\textsc{LD-Empty}\quad \frac{}{\cdot.\ell \not\Rightarrow}$$

$$\textsc{LD-Field}\quad \frac{\ell' \neq \ell \qquad \alpha.\ell \not\Rightarrow}{(\ell' = \epsilon; \alpha).\ell \not\Rightarrow}$$

$\boxed{|\mathcal{A}|}$ Type translation

$$
\begin{aligned}
&|\mathbb{Z}| \equiv \mathbb{Z} \\
&|\mathcal{A} \to \mathcal{B}| \equiv |\mathcal{A}| \to |\mathcal{B}| \\
&|\{\mathcal{P}\} \to \mathcal{B}| \equiv |\mathcal{P}| \to |\mathcal{B}|
\end{aligned}
$$

$\boxed{|\mathcal{P}|}$ Parameter type translation

$$
\begin{aligned}
&| \cdot | \equiv \top \\
&|\ell : \mathcal{A}; \mathcal{P}| \equiv \{\ell : |\mathcal{A}|\} \wedge |\mathcal{P}| \\
&|\ell? : \mathcal{A}; \mathcal{P}| \equiv \{\ell : |\mathcal{A}| \vee \mathbf{Null}\} \wedge |\mathcal{P}|
\end{aligned}
$$

$\boxed{|\Delta|}$ Typing context translation

$$
\begin{aligned}
&| \cdot | \equiv \cdot \\
&|\Delta, x : \mathcal{A}| \equiv |\Delta|, x : |\mathcal{A}|
\end{aligned}
$$

Theorem 1 (Elaboration soundness) *If* $\Delta \vdash \epsilon : \mathcal{A} \rightsquigarrow e$, *then* $|\Delta| \vdash e : |\mathcal{A}|$.

Example 1

$$\mathit{letin}_1 = \textbf{let}\, x = z.x\, \textbf{in} \qquad \mathit{letin}_2 = \textbf{let}\, y = \textbf{switch}\, z.y\, \textbf{as}\, y\, \textbf{case}\, \mathbb{Z} \Rightarrow y\, \textbf{case}\, \textbf{Null} \Rightarrow 0\, \textbf{in} \qquad e_0 = \mathit{letin}_1 \circ \mathit{letin}_2\ (x+y)$$

$$\dfrac{\cdot,\, y : \mathbb{Z},\, x : \mathbb{Z} \vdash x + y : \mathbb{Z} \rightsquigarrow x + y \qquad \dfrac{\dfrac{\cdot \vdash 0 : \mathbb{Z} \rightsquigarrow 0 \qquad \dfrac{}{\cdot; z \vdash \cdot : \cdot \rightsquigarrow \textbf{id} \dashv \cdot}\ \textit{PEla-Empty}}{\cdot; z \vdash (y = 0; \cdot) : (y\,?: \mathbb{Z}; \cdot) \rightsquigarrow \mathit{letin}_2 \dashv \cdot,\, y : \mathbb{Z}}\ \textit{PEla-Optional}}{\cdot; z \vdash (x : \mathbb{Z}; y = 0; \cdot) : (x : \mathbb{Z}; y\,?: \mathbb{Z}; \cdot) \rightsquigarrow \mathit{letin}_1 \circ \mathit{letin}_2 \dashv \cdot,\, y : \mathbb{Z},\, x : \mathbb{Z}}\ \textit{PEla-Required}}{\cdot \vdash (\lambda\{x : \mathbb{Z}; y = 0; \cdot\}.\ x + y) : \{x : \mathbb{Z}; y\,?: \mathbb{Z}; \cdot\} \to \mathbb{Z} \rightsquigarrow \lambda z{:}\{x : \mathbb{Z}\} \wedge \{y : \mathbb{Z} \vee \textbf{Null}\}.\ e_0 {:} \mathbb{Z}}\ \textit{Ela-NAbs}$$

$$\dfrac{\cdots \textit{Ela-NAbs} \cdots \qquad \dfrac{\cdot \vdash 1 : \mathbb{Z} \rightsquigarrow 1 \qquad \dfrac{\dfrac{\cdot \vdash 2 : \mathbb{Z} \rightsquigarrow 2 \qquad \dfrac{}{\cdot \vdash \cdot \diamond \cdot \rightsquigarrow \{\}}\ \textit{PMat-Empty}}{\cdot \vdash \cdot \diamond (z = 2; \cdot) \rightsquigarrow \{z = 2\}}\ \textit{PMat-Extra}}{\cdot \vdash (y\,?: \mathbb{Z}; \cdot) \diamond (z = 2; \cdot) \rightsquigarrow \{y = \textbf{null}\} , \{z = 2\}}\ \textit{PMat-Absent}}{\cdot \vdash (x : \mathbb{Z}; y\,?: \mathbb{Z}; \cdot) \diamond (x = 1; z = 2; \cdot) \rightsquigarrow \{x = 1\} , \{y = \textbf{null}\} , \{z = 2\}}\ \textit{PMat-Present}}{\cdot \vdash (\lambda\{x : \mathbb{Z}; y = 0; \cdot\}.\ x + y)\,\{x = 1; z = 2; \cdot\} : \mathbb{Z} \rightsquigarrow (\lambda z{:}\{x : \mathbb{Z}\} \wedge \{y : \mathbb{Z} \vee \textbf{Null}\}.\ e_0 {:} \mathbb{Z})\,(\{x = 1\} , \{y = \textbf{null}\} , \{z = 2\})}\ \textit{Ela-NApp}$$

Appendix: Semantics of λ_{iu}

$\boxed{A <: B}$ *(Subtyping)*

$$\textsc{Sub-Top}\ \frac{}{A <: \top} \qquad \textsc{Sub-Bot}\ \frac{}{\bot <: A} \qquad \textsc{Sub-Int}\ \frac{}{\mathbb{Z} <: \mathbb{Z}} \qquad \textsc{Sub-Arrow}\ \frac{B_1 <: A_1 \qquad A_2 <: B_2}{A_1 \to A_2 <: B_1 \to B_2} \qquad \textsc{Sub-Rcd}\ \frac{A <: B}{\{\ell : A\} <: \{\ell : B\}}$$

$$\textsc{Sub-And}\ \frac{A <: B \qquad A <: C}{A <: B \wedge C} \qquad \textsc{Sub-AndL}\ \frac{A <: C}{A \wedge B <: C} \qquad \textsc{Sub-AndR}\ \frac{B <: C}{A \wedge B <: C} \qquad \textsc{Sub-Or}\ \frac{A <: C \qquad B <: C}{A \vee B <: C} \qquad \textsc{Sub-OrL}\ \frac{A <: B}{A <: B \vee C}$$

$$\textsc{Sub-OrR}\ \frac{A <: C}{A <: B \vee C}$$

$\boxed{\Gamma \vdash e : A}$ *(Typing)*

$$\textsc{Typ-Top}\ \frac{}{\Gamma \vdash \{\} : \top} \qquad \textsc{Typ-Int}\ \frac{}{\Gamma \vdash n : \mathbb{Z}} \qquad \textsc{Typ-Var}\ \frac{x : A \in \Gamma}{\Gamma \vdash x : A} \qquad \textsc{Typ-Abs}\ \frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash (\lambda x\!:\!A.\ e\!:\!B) : A \to B}$$

$$\textsc{Typ-App}\ \frac{\Gamma \vdash e_1 : A \to B \qquad \Gamma \vdash e_2 : A}{\Gamma \vdash e_1\, e_2 : B} \qquad \textsc{Typ-Rcd}\ \frac{\Gamma \vdash e : A}{\Gamma \vdash \{\ell = e\} : \{\ell : A\}} \qquad \textsc{Typ-Prj}\ \frac{\Gamma \vdash e : \{\ell : A\}}{\Gamma \vdash e.\ell : A}$$

$$\textsc{Typ-Merge}\ \frac{\Gamma \vdash e_1 : A \qquad \Gamma \vdash e_2 : B}{\Gamma \vdash e_1 \,,\, e_2 : A \wedge B} \qquad \textsc{Typ-Switch}\ \frac{\begin{array}{c}\Gamma \vdash e : A \vee B \\ \Gamma, x : A \vdash e_1 : C \\ \Gamma, x : B \vdash e_2 : C\end{array}}{\Gamma \vdash \mathbf{switch}\ e_0\ \mathbf{as}\ x\ \mathbf{case}\ A \Rightarrow e_1\ \mathbf{case}\ B \Rightarrow e_2 : C}$$

$$\textsc{Typ-Let}\ \frac{\Gamma \vdash \mathit{letin} \dashv \Gamma' \qquad \Gamma' \vdash e : A}{\Gamma \vdash \mathit{letin}\ e : A} \qquad \textsc{Typ-Sub}\ \frac{\Gamma \vdash e : A \qquad A <: B}{\Gamma \vdash e : B}$$

$\boxed{\Gamma \vdash \mathit{letin} \dashv \Gamma'}$ *(Let-in binding)*

$$\textsc{LB-Let}\ \frac{\Gamma \vdash e : A}{\Gamma \vdash \mathbf{let}\ x = e\ \mathbf{in} \dashv \Gamma, x : A} \qquad \textsc{LB-Comp}\ \frac{\begin{array}{c}\Gamma \vdash \mathit{letin}_1 \dashv \Gamma' \\ \Gamma' \vdash \mathit{letin}_2 \dashv \Gamma''\end{array}}{\Gamma \vdash \mathit{letin}_1 \circ \mathit{letin}_2 \dashv \Gamma''} \qquad \textsc{LB-Id}\ \frac{}{\Gamma \vdash \mathbf{id} \dashv \Gamma}$$