CS 317 Algorithms
Spring 2025
Mid-Term Exam
18/3/2024
Time Limit: 120 Minutes

Name: _____

Student ID: _____

This exam contains 7 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You are allowed one sheet of handwritten notes.

You are required to show your work for each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by logical reasoning, explanation, or algebraic work will receive no credit if incorrect; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

Do not write in the table to the right.

1. **Let us get started!**

   (a) (5 points) An improvement to the multiplication method given in class involves splitting each n-bit number into *three* pieces of $n/3$ bits each (i.e., write X as $2^{2n/3}A + 2^{n/3}B + C$ and write Y as $2^{2n/3}D + 2^{n/3}E + F$). A straightforward product would now involve 9 multiplications of $n/3$-bit numbers, but it is possible to rearrange terms and reduce this to 5 multiplications of $n/3$-bit numbers, plus a constant number of additions and shifts. Write down the resulting recurrence and solve it.

(b) (5 points) Show that it is possible to find the median of 5 elements using at most 6 comparisons.

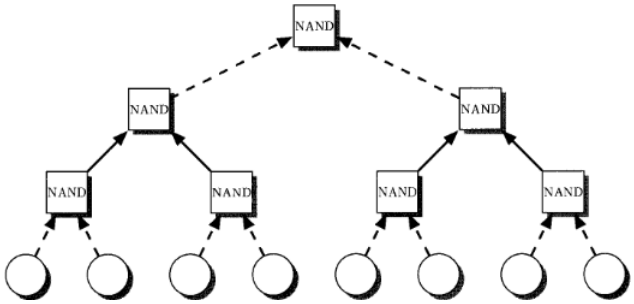
2. One drawback of our policy to double the amount of memory allocated for an array when it gets full is wasted space. For example, an array of 33 elements would require a memory allocation of 64 elements.

   One possible way to reduce the wasted space is to allocate memory in sizes that are perfect squares. At any time, an array of size n would be stored in a block of size b^2, where $b = \lceil \sqrt{n} \rceil$. If adding an element results in $n + 1 > b^2$, then a new array is allocated of size $(b + 1)^2$. We copy the n elements to the new array and add the new element. The cost of this is n.

   (a) (5 points) Prove that the amount of wasted space for an array of size n would be $O(\sqrt{n})$.

   (b) (5 points) Determine the amortized cost for a sequence of n insert operations.

3. (10 points) The smallest possible proof of the value of a $NAND$ tree (shown below) consists of a single $False$ child of each $True$ node, and both children of each $False$ node, till we have a set of leaves that determine the value of the root. Show that in any $NAND$ tree of depth k and $N = 2^k$ leaves, this proof consists of exactly $2^{k/2}$ leaves if k is even.



4. (10 points) Consider a pattern matching problem, where we have a long string T of length n and a pattern string P of length m. The goal is to output the location of all occurrences of the pattern P inside the text T. For example, if $T = abracadabra$ and $P = ab$ then the output should be $\{0, 7\}$. Moving forward, assume the strings are binary and we would like to apply ideas of the string matching hashing algorithm considered in class. Our algorithm is allowed to be incorrect with a small probability. For full credit give an $O(m + n)$ hashing algorithm that satisfies the above criteria.

5. Consider the Minimum Edit Distance problem for two string $x[1 \cdots m]$ and $y[1 \cdots n]$. In addition to insert and delete, we are now also allowed a 'substitute' operation. So, to convert 'SNOWY' into 'SUNNY', we have three edits: insert U, substitute O by N and delete W.

   (a) (5 points) Fill in the entries $E(i, j)$ of the following table which represents the minimum edit distance between prefix of the string $x[1 \cdots i]$ and $y[\cdots j]$.

		P	O	L	Y
	0	1	2	3	4
E	1				
X	2				
P	3				
O	4				

   (b) (5 points) Give the definition of $E(i, j)$ in general, assuming the base case entries are already filled in.