CSE 317: Design and Analysis of Algorithms – Theory homework 1

Assigned: September 9, 2025, Due: September 17, 2025 on Gradescope.

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1. (10 points) Show that following algorithm computes $2^n - 1$ for all $n \in \mathbb{N} \cup \{0\}$.

   Algorithm: $f(n)$

   1. **if** $n = 0$ **or** $n = 1$ **then return** n
   2. **else return** $(3 \cdot f(n-1) - 2 \cdot f(n-2))$

2. Consider the following modification of Algorithm MERGESORT. The algorithm first divides the input array $A[l..h]$ into four parts A_1, A_2, A_3 and A_4 instead of two. It then sorts each part recursively, and finally merges the four sorted parts to obtain the original array in sorted order.

   Assume for simplicity that n is a power of 4 (i.e., $n = 4^k$, for $k \in \mathbb{N}$).

   (a) (10 points) Write out the modified algorithm.

   (b) (10 points) Analyze its running time (and express the time complexity in O-notations.)

3. (10 points) Let $A = \langle a_1, a_2, \ldots, a_n \rangle$ be a sequence (or an array) of $n \geq 1$ integers i.e., $a_i \in \mathbb{Z}$ for $1 \leq i \leq n$.

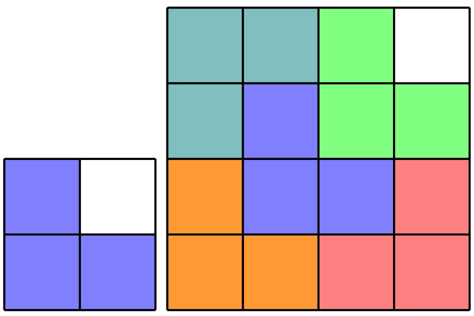
   Design an $\Theta(n)$-time algorithm to reorder the input sequence A such that all negative numbers in A precede all nonnegative numbers in A. For example if $A = \langle 2, -5, 0, -2, 3, 7, -10, 6 \rangle$ then the output should be something like $\langle -5, -2, -10, 2, 0, 3, 7, 6 \rangle$.

   [Note that there may be multiple possible correct answers however the only requirement is that all negative numbers must precede all nonnegative numbers.]

4. (10 points) You are given an $n \times n$ grid A of distinct integers. A number $A[i][j]$ is a *local minimum* if it is smaller than all its immediate neighbors (up, down, left, and right). Note that elements on the edges or corners have fewer neighbors.

   Design a divide-and-conquer algorithm to find any one local minimum in the grid. A naive search would take $O(n^2)$ time. Your algorithm must be more efficient.

5. (20 points) Let B be an $n \times n$ chessboard, where n is a power of 2 (i.e, $n = 2^k$ for some $k \in \mathbb{N}$). Use a divide-and-conquer argument to describe (in words) how to cover all squares of B except one with L-shaped tiles. For example, if $n = 2$, then there are four squares three of which can be covered by one L-shaped tile, and if $n = 4$, then there are 16 squares of which 15 can be covered by 5 L-shaped tiles.

   2×2 4×4

6. (10 points) You are given an array A of $n \geq 1$ integers such that elements of A are in $\mathbb{Z}$. Consider A is a circular array that is the last element is considered to be adjacent to the first element. Now modify the contiguous maximum subarray sum discussed in class to work for this circular array.

7. (10 points) If A and B be matrices of sizes $n \times m$ and $m \times r$, respectively. Then what is the total number of element (scalar) multiplications and total number of element (scalar) additions performed by the traditional algorithm to compute the product AB. Also describe the time complexity of traditional algorithm in O-notations.

8. (10 points) Let T_1 and T_2 be two binary trees. Design an efficient divide-and-conquer algorithms to determine whether T_1 and T_2 are *identical*.

   Note that: two binary trees are called *identical* if they:

   - have the same number of nodes (internal and leaf),
   - have the same structure (each corresponding node has the same number of child nodes, etc), and
   - each node contains the same data.