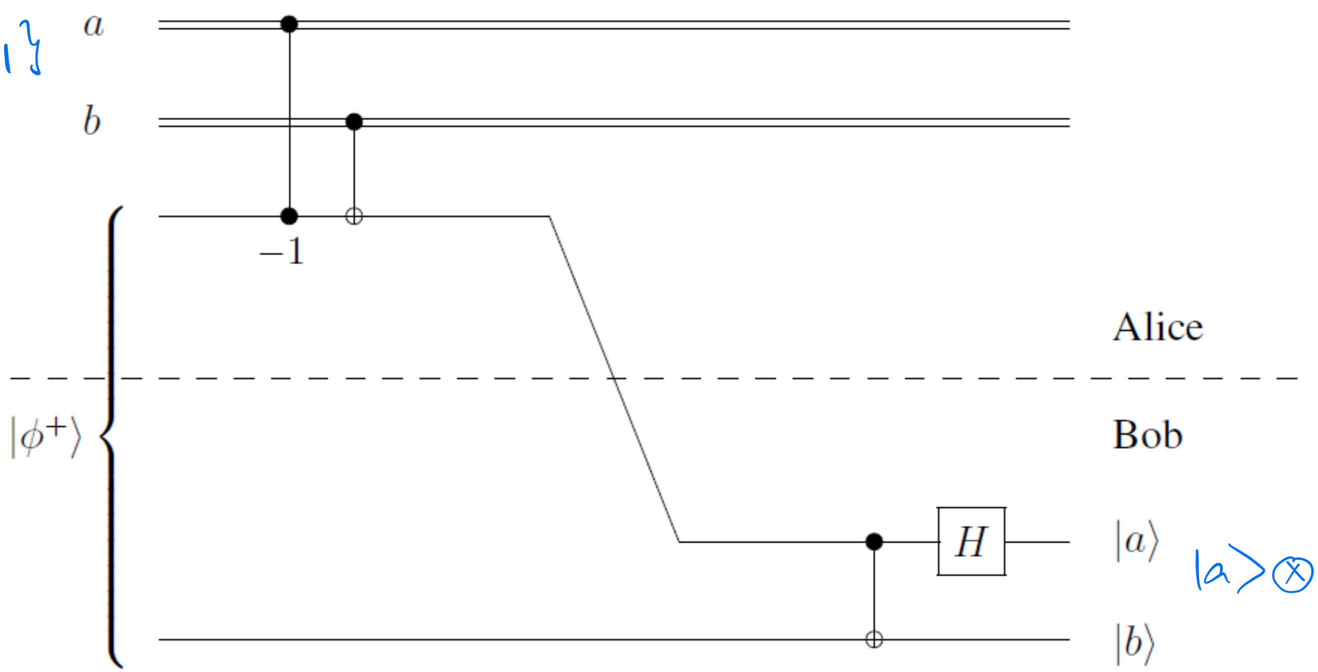
Superdense Coding

$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Holevo's Bound

$a, b \in \{0,1\}$

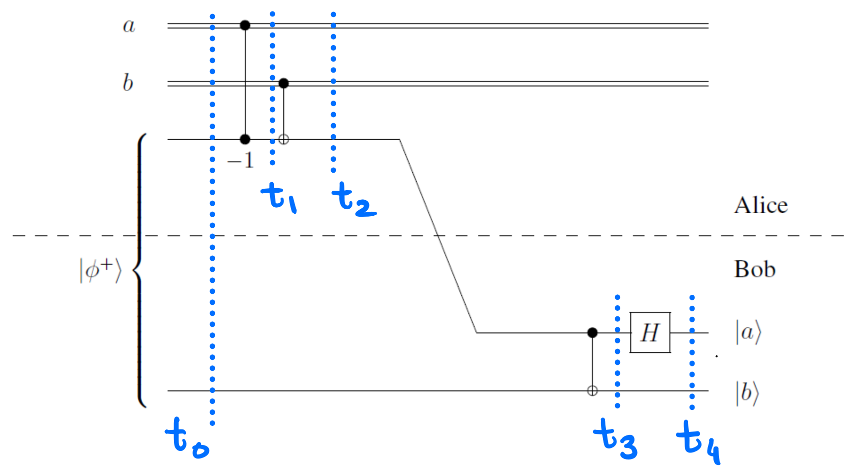


$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) = |\phi^+\rangle$

$|a\rangle \otimes |b\rangle$

Superdense Coding

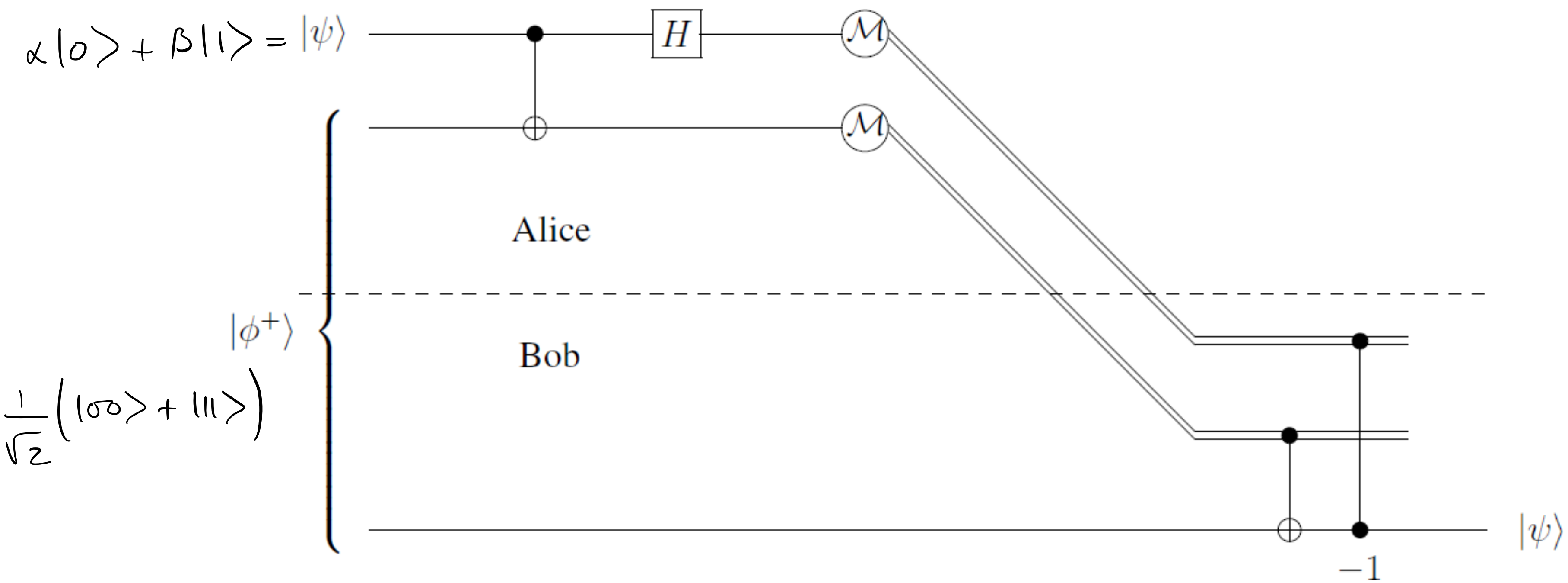
$$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$



$$Z|0\rangle = |0\rangle$$
$$Z|1\rangle = -|1\rangle$$
$$H|0\rangle = |+\rangle$$
$$H|1\rangle = |-\rangle$$

ab	State at t_1	State at t_2	State at t_3	State at t_4
00	$\|\varphi^+\rangle$	$\|\varphi^+\rangle$	$\frac{1}{\sqrt{2}}\left(\|00\rangle+\|10\rangle\right)=\|+\rangle\|0\rangle$	$\|00\rangle$
01	$\|\varphi^+\rangle$	$\frac{1}{\sqrt{2}}\left(\|10\rangle+\|01\rangle\right)$	$\frac{1}{\sqrt{2}}\left(\|11\rangle+\|01\rangle = \|+\rangle\|1\rangle\right.$	$\|01\rangle$
10	$\frac{1}{\sqrt{2}}\left(\|00\rangle-\|11\rangle\right)$	$\frac{1}{\sqrt{2}}\left(\|00\rangle-\|11\rangle\right)$	$\frac{1}{\sqrt{2}}\left(\|00\rangle-\|10\rangle\right)=\|-\rangle\|0\rangle$	$\|10\rangle$
11	$\frac{1}{\sqrt{2}}\left(\|00\rangle-\|11\rangle\right)$	$\frac{1}{\sqrt{2}}\left(\|10\rangle-\|01\rangle\right)$	$\frac{1}{\sqrt{2}}\left(\|1\rangle-\|0\rangle\right)\|1\rangle = -\|-\rangle\|1\rangle$	$-\|11\rangle$

Quantum Teleportation

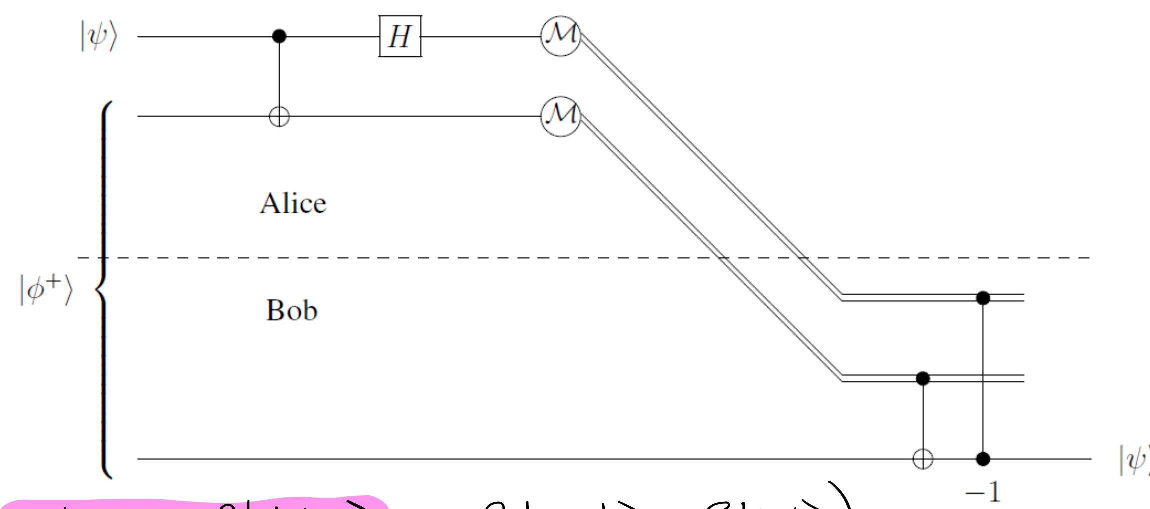


Quantum Teleportation

$$|\psi\rangle \otimes |\varphi^+\rangle = (\alpha|0\rangle + \beta|1\rangle)\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$= \frac{1}{\sqrt{2}}(\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle)$$

$$\xrightarrow{CNOT \otimes \mathbb{1}} \frac{1}{\sqrt{2}}(\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle)$$



$$\xrightarrow{H \otimes \mathbb{1} \otimes \mathbb{1}}$$

$$= \frac{1}{2}(\alpha|000\rangle + \alpha|100\rangle + \alpha|011\rangle + \alpha|111\rangle + \beta|010\rangle - \beta|110\rangle + \beta|001\rangle - \beta|101\rangle)$$

$$= \frac{1}{2}\Big(|00\rangle\underbrace{(\alpha|0\rangle + \beta|1\rangle)}_{\substack{\mathbb{1} \\ |\psi\rangle}} + |01\rangle\underbrace{(\alpha|1\rangle + \beta|0\rangle)}_{\substack{X(\alpha|1\rangle + \beta|0\rangle) \\ |\psi\rangle = \alpha|0\rangle + \beta|1\rangle}} + |10\rangle\underbrace{(\alpha|0\rangle - \beta|1\rangle)}_{\substack{Z(\alpha|0\rangle - \beta|1\rangle) \\ = \alpha|0\rangle + \beta|1\rangle}} + |11\rangle\underbrace{(\alpha|1\rangle - \beta|0\rangle)}_{\substack{ZX(\alpha|1\rangle - \beta|0\rangle) \\ Z(\alpha|0\rangle - \beta|1\rangle) \\ \alpha|0\rangle + \beta|1\rangle}}\Big)$$