Problem Solving

Learning Goals

- What is “problem solving”?
- Big picture: understanding ➔ search
- Why problem representations matter
- Types of problems & problem-solving approaches

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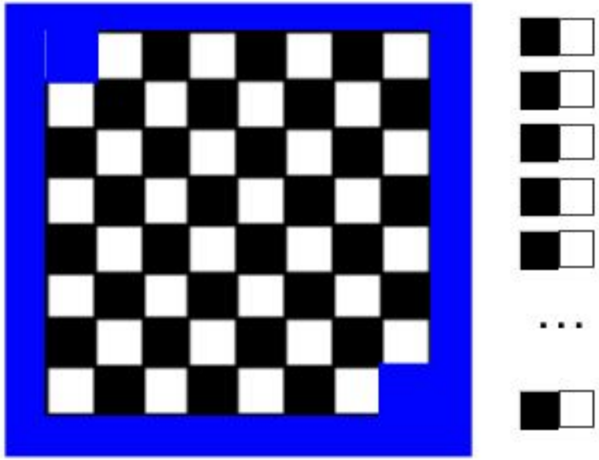
- We first form a mental picture of the problem. We then explore possible moves within that picture to get closer to the goal.
- A good **representation** makes the goal, the allowed moves, and the constraints easy to see. A poor representation hides these and makes progress slow.

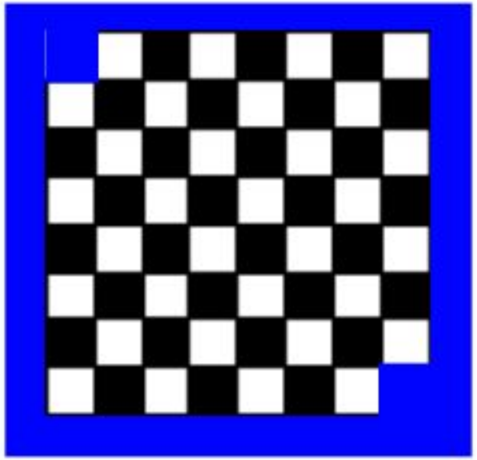
The Mutilated Checkerboard Problem

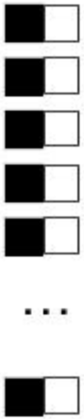
If we remove two diagonally opposite corners, can the board still be covered with dominoes (that is, rectangles made of one white and one black square)?



The Matchmaker Problem

There is a matchmaker in a village. There are 32 single women and 32 single men. There is a fire and 2 of the single men die. Will the matchmaker be able to pair all the single women with all the single men?





Types of Problems

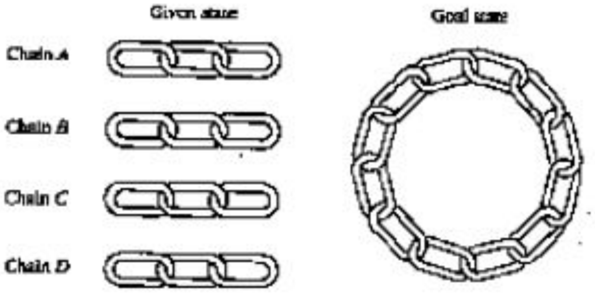
- Well-defined: clear initial state, goal, operators, constraints (e.g., Tower of Hanoi)
  - Solve for x: $7(x + 2) + 3 = 38$
- Ill-defined: goals/constraints ambiguous, success criteria unclear (e.g., designing a policy)
  - What things would you take from your house if it caught on fire?
- Insight problems: require re-representing the problem; solution can feel sudden (“Aha!”)
  - Describe how to cut a hole in a 3x5 card big enough to poke your head through.

Solving Well-Defined Problems

- IGOR helps us analyze a problem: Initial state, Goal state, Operators (moves), and Restrictions (rules).

Silveria (1971) The "cheap-necklace" problem.

You are given four separate pieces of chain that are each three links in length. It costs 2 cents to open a link and 3 cents to close a link. All links are closed at the beginning of the problem. Your goal is to join all 12 links of chain into a single circle at a cost of no more than 15 cents.



The Candle Problem

Using tacks and a box of matches, how can you mount the candle on the wall?

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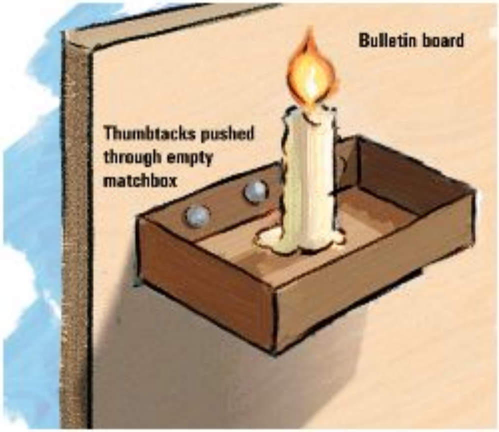

ACTIVITY 2.3

Identify the initial state, goal state, operators and restrictions for the following coin problem:

Starting with an arrangement of coins in Figure 2.5:

FIGURE 2.5A The eight coins problem

rearrange them so that they end up in the following arrangement:

FIGURE 2.5B The eight coins problem

A coin can only move to an empty space adjacent to it. A coin can jump over only ONE coin of either colour. Silver (white) coins can only move to the right and copper (black) coins to the left.

Types of Problem Solving

1. **Well-defined problem solving:** The problem has a clear starting point, a clear goal, allowed moves, and rules you must follow, so you can analyze it with IGOR (Initial, Goal, Operators, Restrictions) and work through it step by step.
2. **Heuristic search and difference-reduction:** When trying every option is impossible, you use smart shortcuts that move you closer to the goal without checking all paths.
   a. Hill-climbing: You choose the next step that seems to bring you closer to the goal right now.
   b. Means–ends analysis: You set subgoals that shrink the gap between where you are and where you want to be, and you pick actions that achieve each subgoal.
3. **Insight problem solving:** You make progress by seeing the problem in a new way, and once the new representation "clicks," the solution becomes clear and quick. For example, a tricky algebra step or a tiling puzzle may suddenly make sense when you redraw it or restate it.
4. **Transfer (analogical) problem solving:** You solve a new problem by noticing how it is structurally similar to one you have solved before and by mapping the earlier solution to the new case. For example, you might use the logic of a science experiment you know to plan a history investigation with the same structure.
5. **Creative problem solving:** You generate and refine new ideas or new representations when routine methods are not enough, and you combine pieces in original ways to reach a workable solution. For example, you may brainstorm multiple models, test the most promising one, and then adapt it to fit the constraints.

Class Activity

- In pairs, do three things:
- Choose one approach to problem solving
- Define it in your own words
- Name two school subjects + concrete topics where it applies and explain how
    - Example: Mathematics — applying Pythagorean theorem (algorithmic/strong method).
- Please make sure to hand in your work

Effects of Direct Instruction and Discovery Learning

Constructivism

Constructivism

- Constructivism: knowledge is “constructed” by learners rather than transmitted by teachers.
- "The premise of constructivism implies that the knowledge students construct on their own, for example, is more valuable than the knowledge modeled for them; told to them; or shown, demonstrated, or explained to them by a teacher"
- Knowledge discovered by students themselves is more valuable and lasting.
- Piaget: “Each time one prematurely teaches a child something he could have discovered for himself, that child is kept from inventing it and consequently from understanding it completely.”
- Discovery learning emphasizes ownership and authenticity of learning.

Direct Instruction vs. Discovery Learning

Direct Instruction vs. Discovery Learning

- Direct Instruction:
    - Teacher-centered.
    - Goals, materials, examples, explanations, and pace are teacher-controlled.
    - Emphasizes clarity, modeling, and guided practice.
- Discovery Learning:
    - Student-centered.
    - Minimal teacher guidance.
    - Learners explore, test, and infer principles on their own.
- Key Distinction: Both involve active learning — difference lies in level of guidance and explanation.

Direct Instruction vs. Discovery Learning

- Discovery Learning:
  - Promotes deep understanding and independent reasoning.
  - Fosters ownership and authentic learning experiences.
  - Encourages long-term transfer of knowledge.
- Direct Instruction:
  - Most scientific and mathematical knowledge is taught, not discovered.
  - More efficient for teaching multistep or abstract skills.
  - Reduces confusion, errors, and inconsistent feedback.

Aim of the Study

- Overall Aim:
  - To evaluate how different instructional approaches (direct instruction vs. discovery learning) affect children's acquisition and transfer of a fundamental science skill: the Control of Variables Strategy (CVS).
- Central Question:
  - If children achieve mastery of a new procedure, does the way they reached that mastery matter for their ability to transfer what they've learned?
- Research Questions:
  - Is direct instruction more effective than discovery learning in teaching children to design and interpret simple, unconfounded experiments (CVS)?
  - Do children who master CVS perform better on authentic reasoning tasks — such as evaluating science-fair posters — than those who do not?
  - Does the learning path (direct instruction or discovery learning) affect how well children transfer what they've learned to new contexts?

Subject Matter

- Skill taught: Control of Variables Strategy (CVS).
  - It’s a way of designing experiments where you change only one factor at a time while keeping all other factors the same.
  - This helps you find out which variable actually causes the change you observe.
  - For example, if you want to test whether ramp steepness affects how far a ball rolls, you must keep the ball type, ramp surface, and length constant.
- Why important:
  - Enables valid causal reasoning.
  - Foundational for scientific thinking and experimentation.

Participants

- 112 children from 4 elementary schools (one all-girls).
- Grades 3 and 4.
- Age range: ~8.4 to 10.6 years.
- Random assignment to either Direct Instruction or Discovery Learning conditions.

Design

- Day 1: Learning and assessment using ramp apparatus.
  - Children manipulated ramp steepness, surface, length, and ball type.
  - Task: test how variables affect distance balls roll.
  - Conditions:
    - Direct Instruction: teacher demonstrated confounded/unconfounded experiments, explained reasoning.
    - Discovery: children explored freely with no feedback or explanation.
- Day 2: Transfer task; evaluate two real science-fair posters for quality of research design and conclusions.

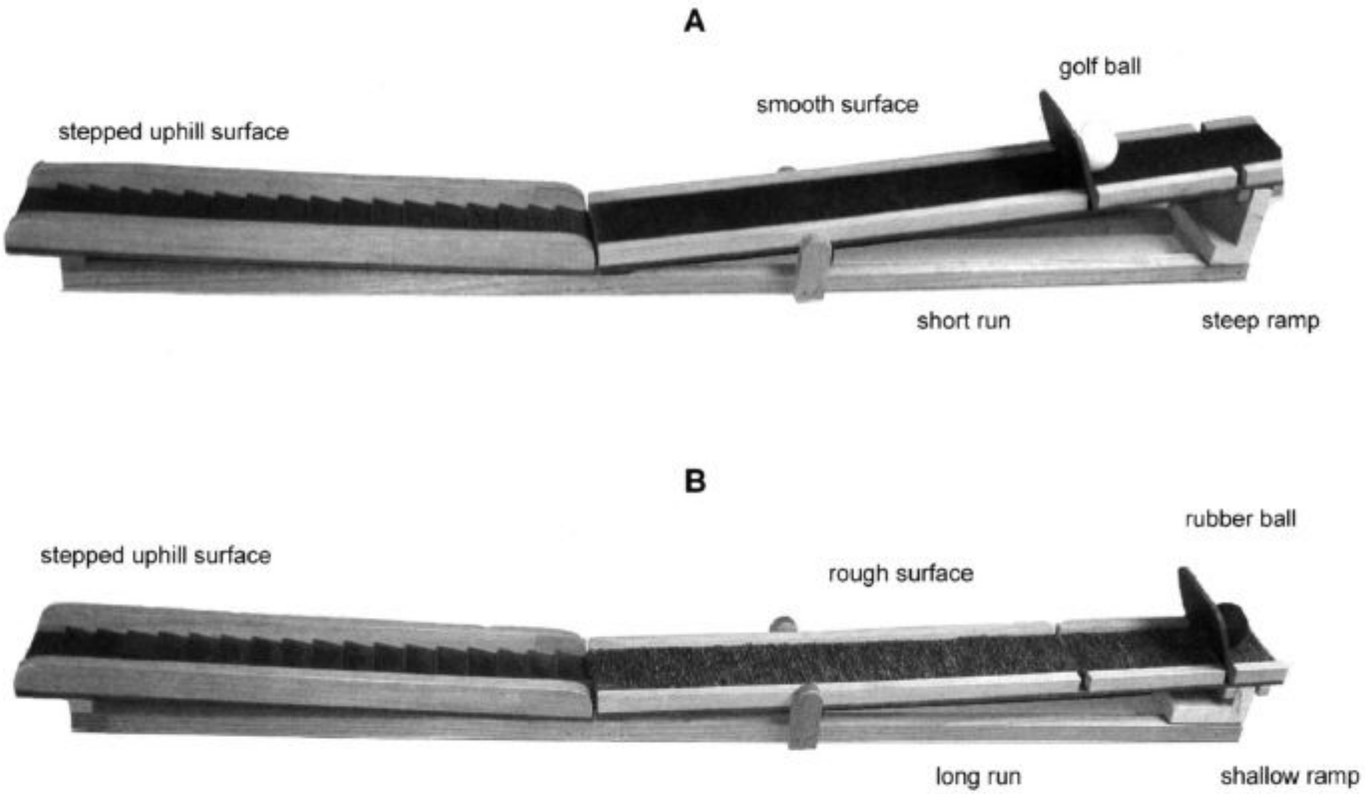

Fig. 1. The ramps used during the exploration and assessment phases. On each of the two ramps, children could vary the steepness, surface, and length of the ramp, as well as the type of ball. The confounded experiment depicted here contrasts (a) a golf ball on a steep, smooth, short ramp with (b) a rubber ball on a shallow, rough, long ramp.

Results

- Direct Instruction group:
  - 40% correct prior to instruction to 80% correct following instruction
  - 77% reached mastery
- Discovery Learning group:
  - No significant improvement
  - Only 23% reached mastery
- Clear advantage for direct instruction

Results

- Children who mastered CVS — regardless of how they learned it — performed equally well on poster evaluations.
- Those who failed to master CVS performed poorly, regardless of learning path.
- “What is learned is more important than how it is taught.”

Key Takeaways

- Direct instruction was more efficient in teaching CVS.
- Discovery learning produced smaller gains but worked for some learners.
- Mastery, not method, predicted transfer.
- Challenges assumption that discovery learning leads to better generalization.
- “Extreme” versions of instruction
- Connecting this study to broader debates about “guided discovery” and “inquiry-based learning.”