USC Viterbi
School of Engineering

3D PHASE RETRIEVAL AT NANO-SCALE VIA ACCELERATED WIRTINGER FLOW

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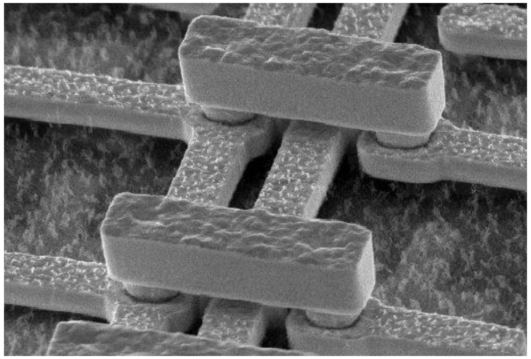
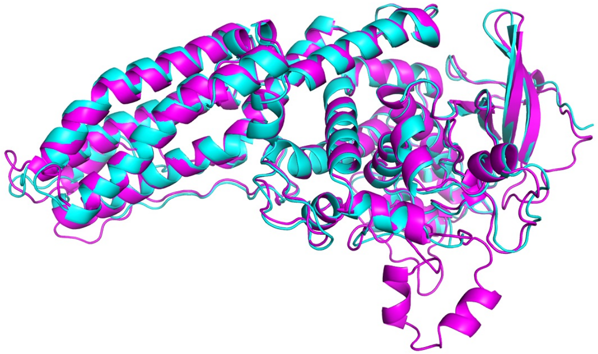
MOTIVATION

- 3D nano-scale imaging applications:
  - nano-technology
  - electronics industry (quality control)
  - protein structures
  - biological specimen





- Challenges:
  - high energy beams
  - special devices needed
  - information loss
  - extremely long acquisition time (up to 1000 days!)

CDI FOR 3D IMAGING

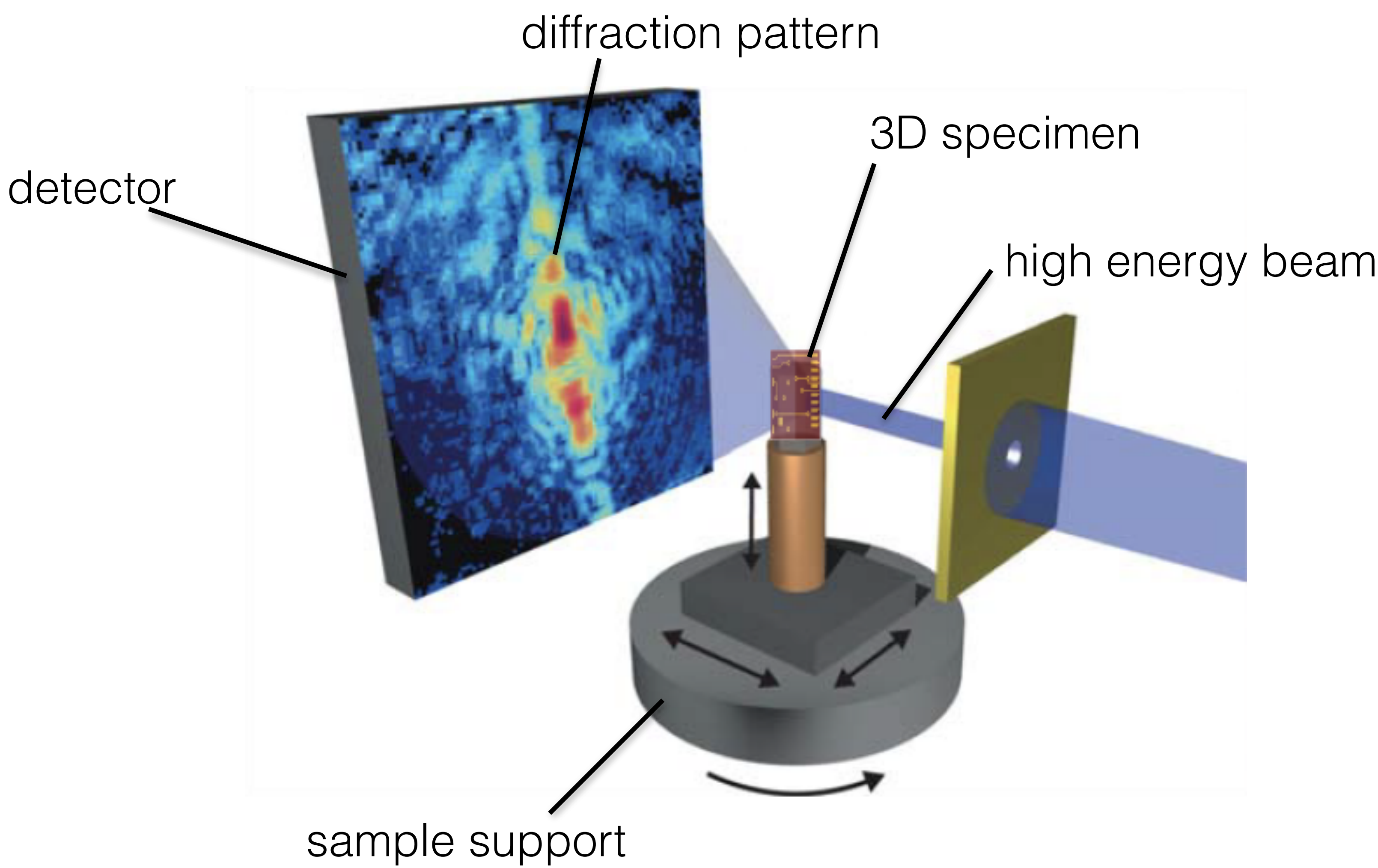


Illustration: Dierolf, M. et al. "Ptychographic X-ray computed tomography at the nanoscale." *Nature*, 2010

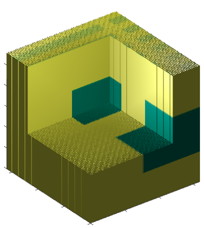
PHASELESS IMAGING IN 3D

- Object modeling:

$$x = d + ib$$

complex refractive index | phase shift | attenuation

- Forward model:

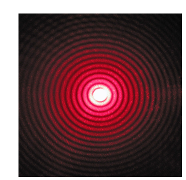


3D object (complex)

wave propagation through object

2D exit wave (complex)

propagation to far field

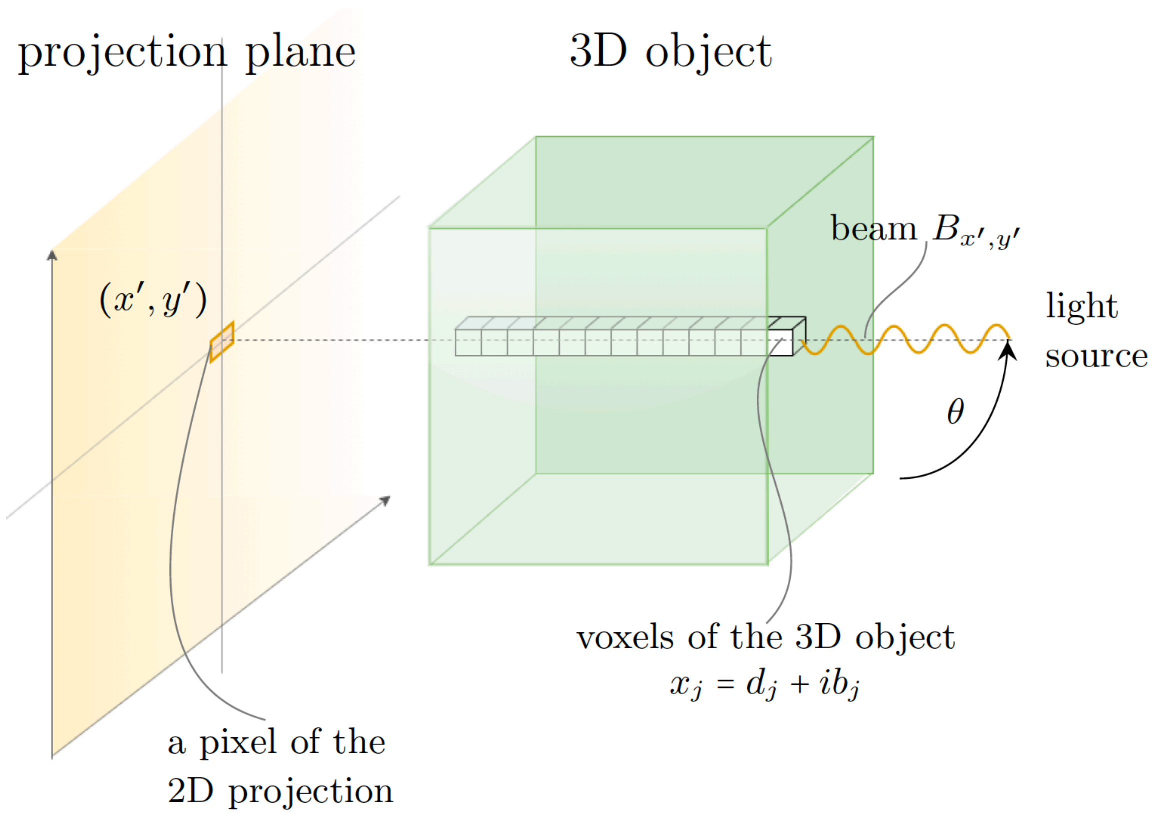


2D intensity measurements (real)

3D OBJECT TO 2D EXIT WAVE

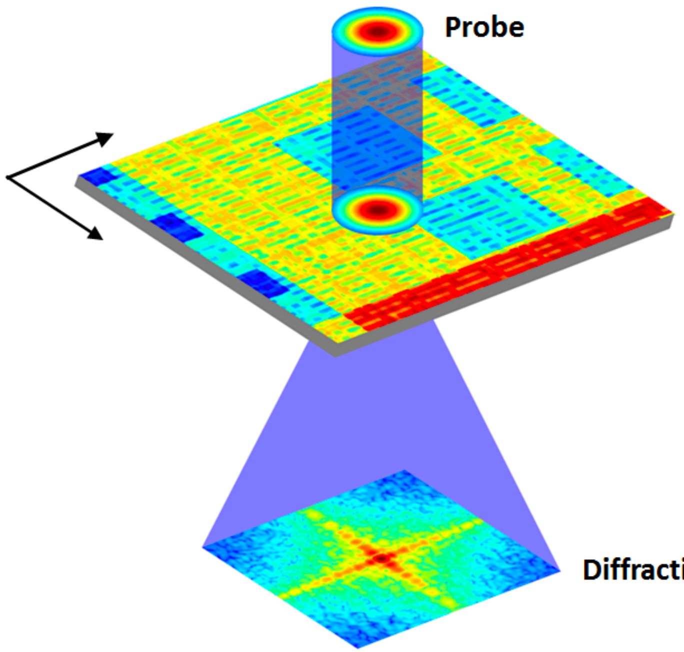
- Nonlinear propagation model:

$$g_\theta(x) = exp\left(\frac{2\pi i}{\lambda}\int_{-\infty}^{\infty}(d+ib)dz_\theta\right)$$



EXIT WAVE TO MEASUREMENT

- Propagation to far field:
  - linear model: probe + Fourier transform



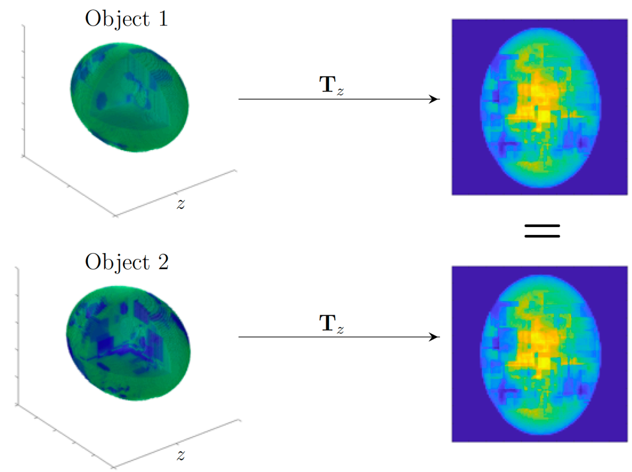
$$\mathbf{A} = \begin{bmatrix} \mathbf{F} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{F} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{F} & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{F} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{\mathrm{supp}(\mathbf{p}_1)} \mathrm{diag}\left(\mathbf{p}_1\right) \\ \mathbf{I}_{\mathrm{supp}(\mathbf{p}_2)} \mathrm{diag}\left(\mathbf{p}_2\right) \\ \vdots \\ \mathbf{I}_{\mathrm{supp}(\mathbf{p}_K)} \mathrm{diag}\left(\mathbf{p}_K\right) \end{bmatrix}$$

  - magnitude-only measurements

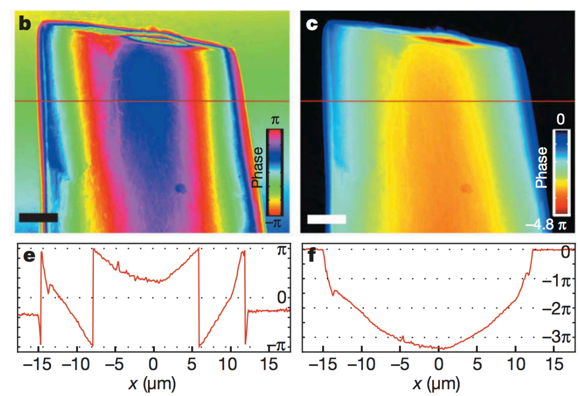
$$\mathbf{y}_\ell = |\mathbf{A}\mathbf{g}_\ell|$$

SOURCES OF AMBIGUITY

- Ambiguity of tomography:
  - Radon-transform has non-trivial null space
  - Given projection may correspond to infinitely many objects
  - Resolution: sufficient number of angles



- Phase ambiguity:
  - Unknown global phase
  - Phase wrapping: $2\pi k$ phase shift
  - Voxel-level ambiguity: invariance to λk shifts in the real part of voxels



Dierolf, M. et al. "Ptychographic X-ray computed tomography at the nanoscale." *Nature*, 2010

OPTIMIZATION PROBLEM

- Non-convex minimization problem:

$$\hat{\mathbf{x}} = arg \min_{\mathbf{x} \in \mathbb{C}^N} \quad \underbrace{\mathscr{L}(\mathbf{x})}_{\text{data consistency}} + \lambda_{TV} \underbrace{\mathsf{TV}_{3D}(\mathbf{x}; \mathbf{w})}_{\text{regularizer}}$$

- Data consistency:
  - solution is consistent with measurements

$$\mathscr{L}(\mathbf{x}) = \sum_{l=1}^{L} \| \; \mathbf{y}_\ell - |\mathbf{A}\mathbf{g}_\ell(\mathbf{x})| \; \|_2^2$$

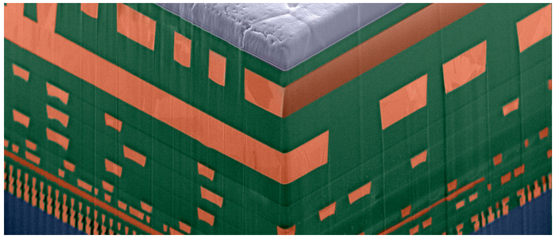
- Regularizer:
  - incorporates prior knowledge
  - enforces piecewise constant structure

BENEFITS OF TV REGULARIZATION

- Benefits of total variation regularization:

1. Accelerates data acquisition

2. Incorporates the structure of ICs



3. Alleviates the effect of phase wrapping
   - Ambiguity of λk shifts in the real part of each voxel
   - TV penalizes abrupt jumps between voxels
   - Approximate ambiguity as constant
   - Correction:

$$\hat{d}_F = \hat{d} - c \cdot \mathbf{1}$$

3D-AWF

- Steps:

  1. Acceleration $\mathbf{y}_{\tau+1} = \mathbf{x}_\tau + \beta_\tau(\mathbf{x}_\tau - \mathbf{x}_{\tau-1}) - \mu_\tau \nabla \mathscr{L}(\mathbf{x}_\tau + \beta_\tau(\mathbf{x}_\tau - \mathbf{x}_{\tau-1}))$

  2. Proximal map $\mathbf{x}_{\tau+1} = prox_{TV}(\mathbf{y}_{\tau+1})$

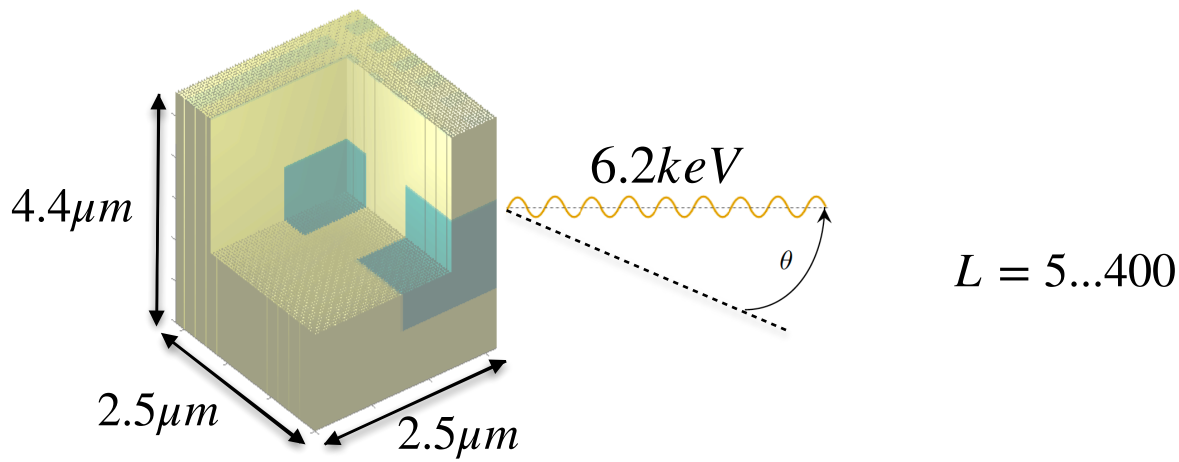
- Convergence theory

  Let $\mathbf{x}^*$ be a global minimum of $\mathscr{L}(\mathbf{x})$. If we run 3D-AWF updates with step size sufficiently small and $\beta_\tau = 0$, then we have

$$\min_{\tau \in \{1,2,\ldots,T\}} \|prox_{TV}(\mathbf{y}_\tau) - \mathbf{y}_\tau\| \leq \mu \frac{\mathscr{L}(\mathbf{x}_0) - \mathscr{L}(\mathbf{x}^*)}{T+1}$$

EXPERIMENTAL SETUP

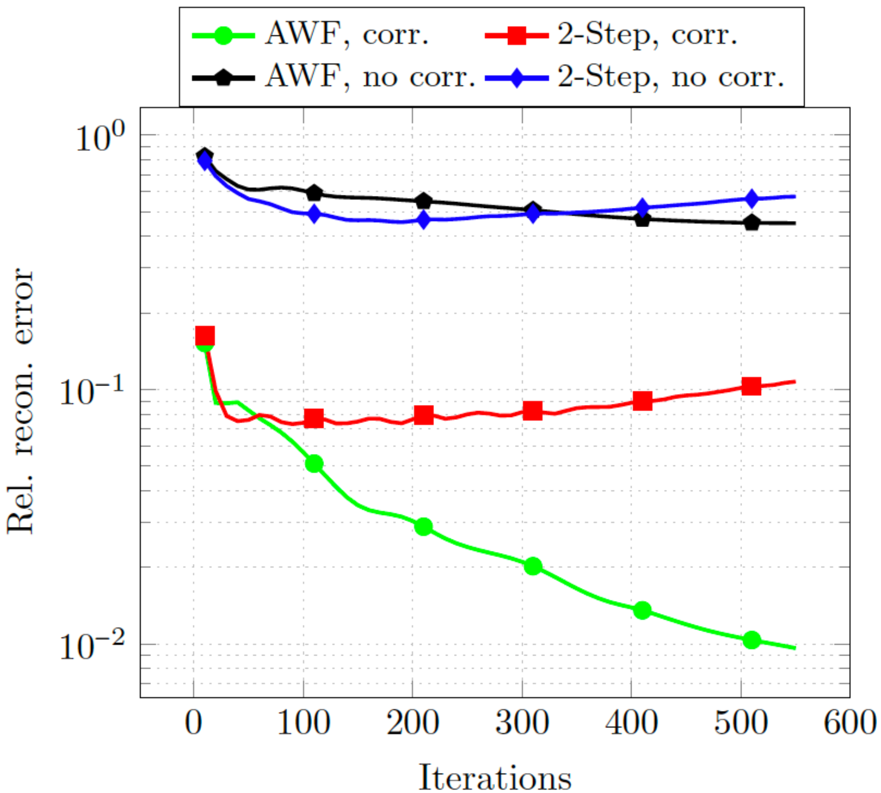
- Test object



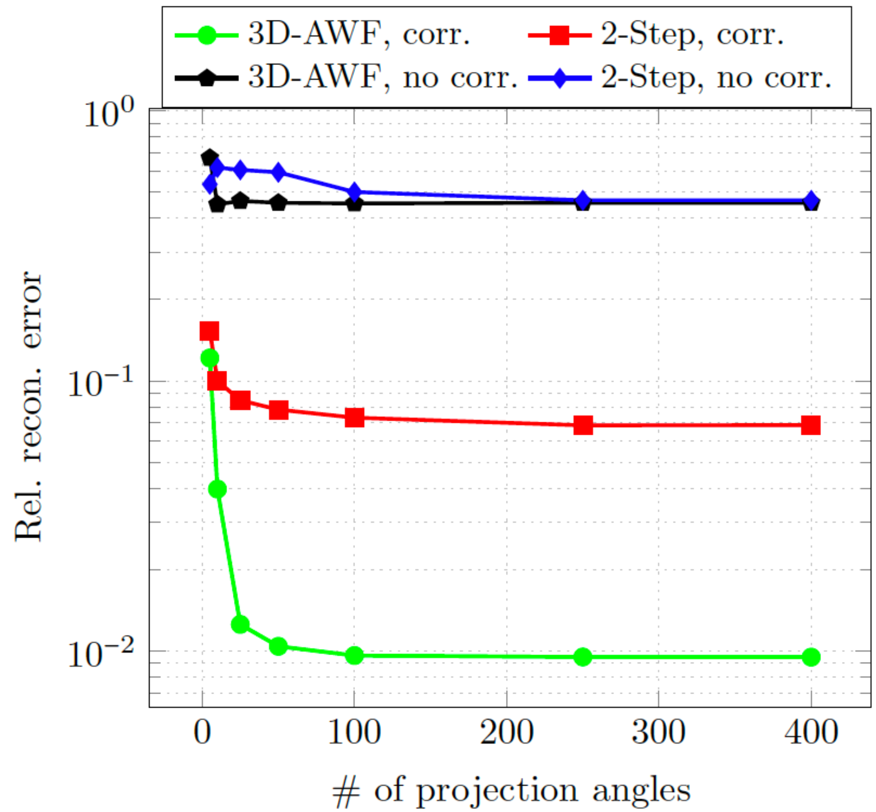
- Compare 3D-AWF with 2-step approach
  - 3D-AWF: reconstruct object directly from measurements via non-linear propagation model
  - 2-Step:
    1. Phase retrieval to reconstruct exit waves
    2. Tomography, using linear approximation

$$exp(\frac{2\pi i}{\lambda}\boldsymbol{T}_\ell \boldsymbol{x}) \approx 1 + \frac{2\pi i}{\lambda}\boldsymbol{T}_\ell \boldsymbol{x}$$

EXPERIMENTAL RESULTS (1)

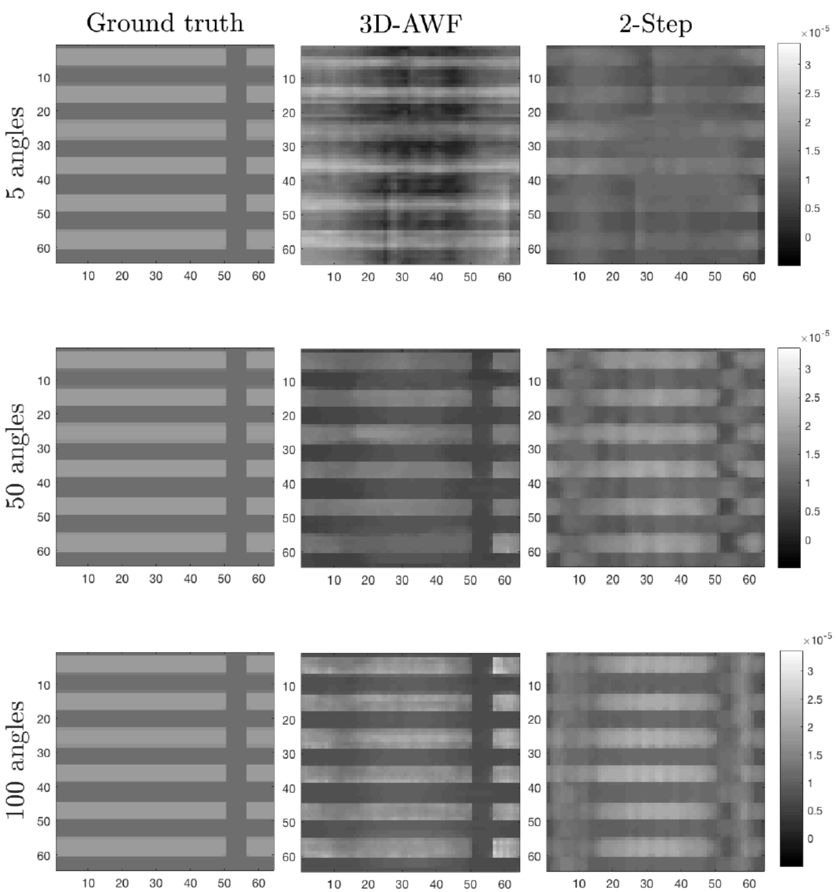


Evolution of reconstruction error throughout reconstruction with L=100.

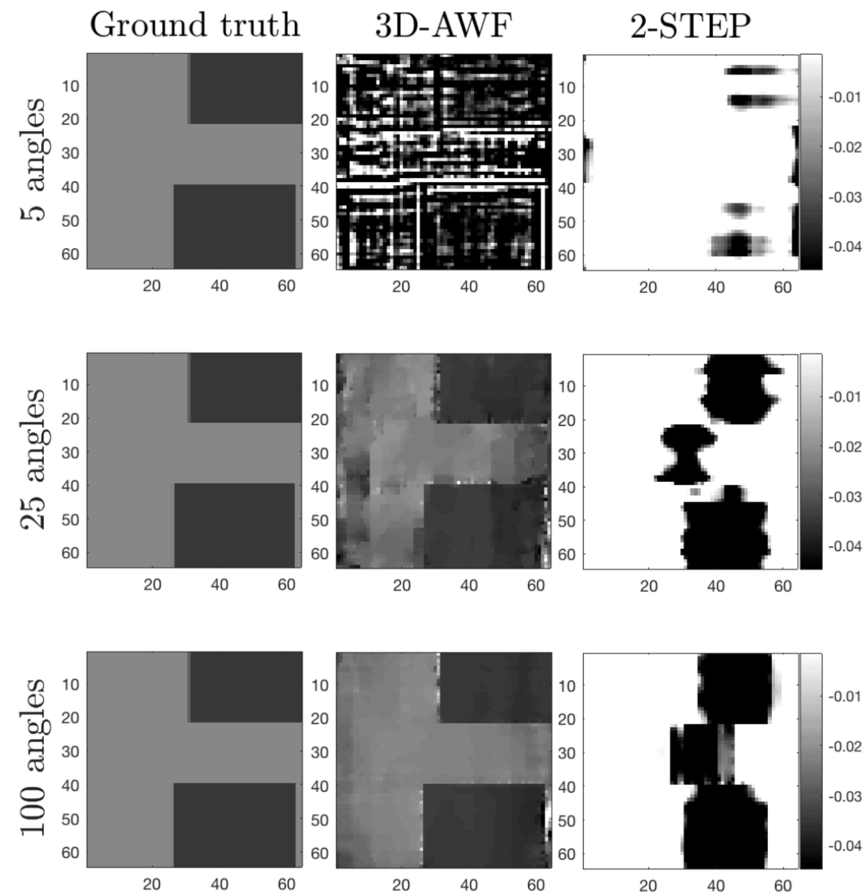


Reconstruction error v. number of illumination angles

EXPERIMENTAL RESULTS (2)



Magnitude reconstruction of a slice.



Phase reconstruction of a slice.

EXPERIMENTAL RESULTS (3)