CTA200H Assignment 3

Zeinab Imani

Question 1: Mandelbrot Set

To explore the Mandelbrot set, we iterated the complex quadratic map: $z_{i+1} = z_i^2 + c, z_0 = 0$ over the domain $-2 < x < 2$, $-2 < y < 2$, where $c = x + iy$. A function was implemented in `function.py` to compute the number of iterations it took for the sequence to diverge, using an escape threshold of 10 and a maximum of 100 iterations.

Two plots were generated:

- A binary image showing whether each point is bounded or diverges.
- A color-coded image showing how many iterations each point took to escape.

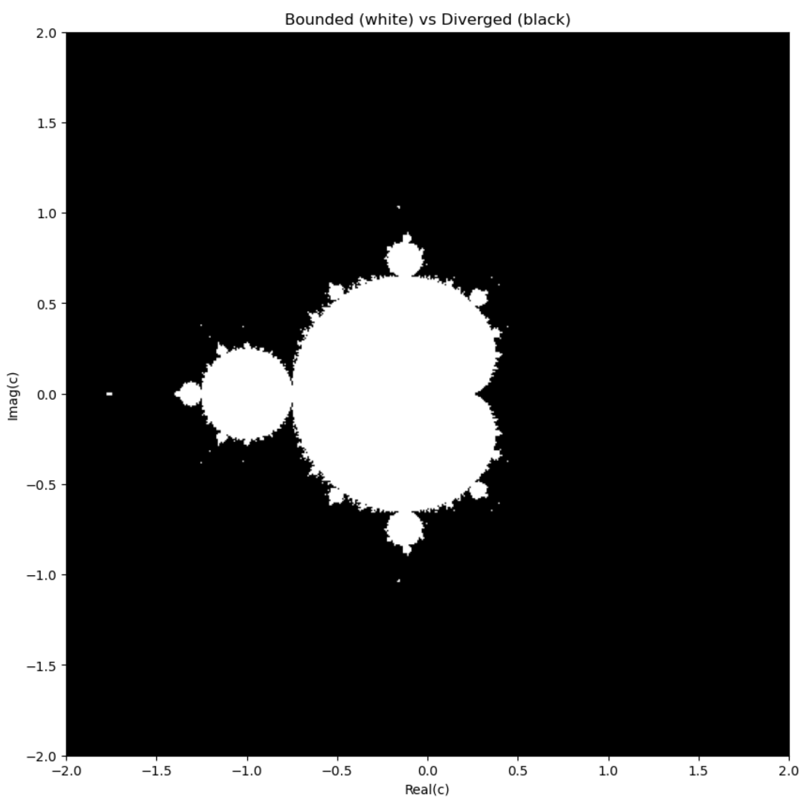

Figure 1: Binary visualization: white = bounded, black = diverged.

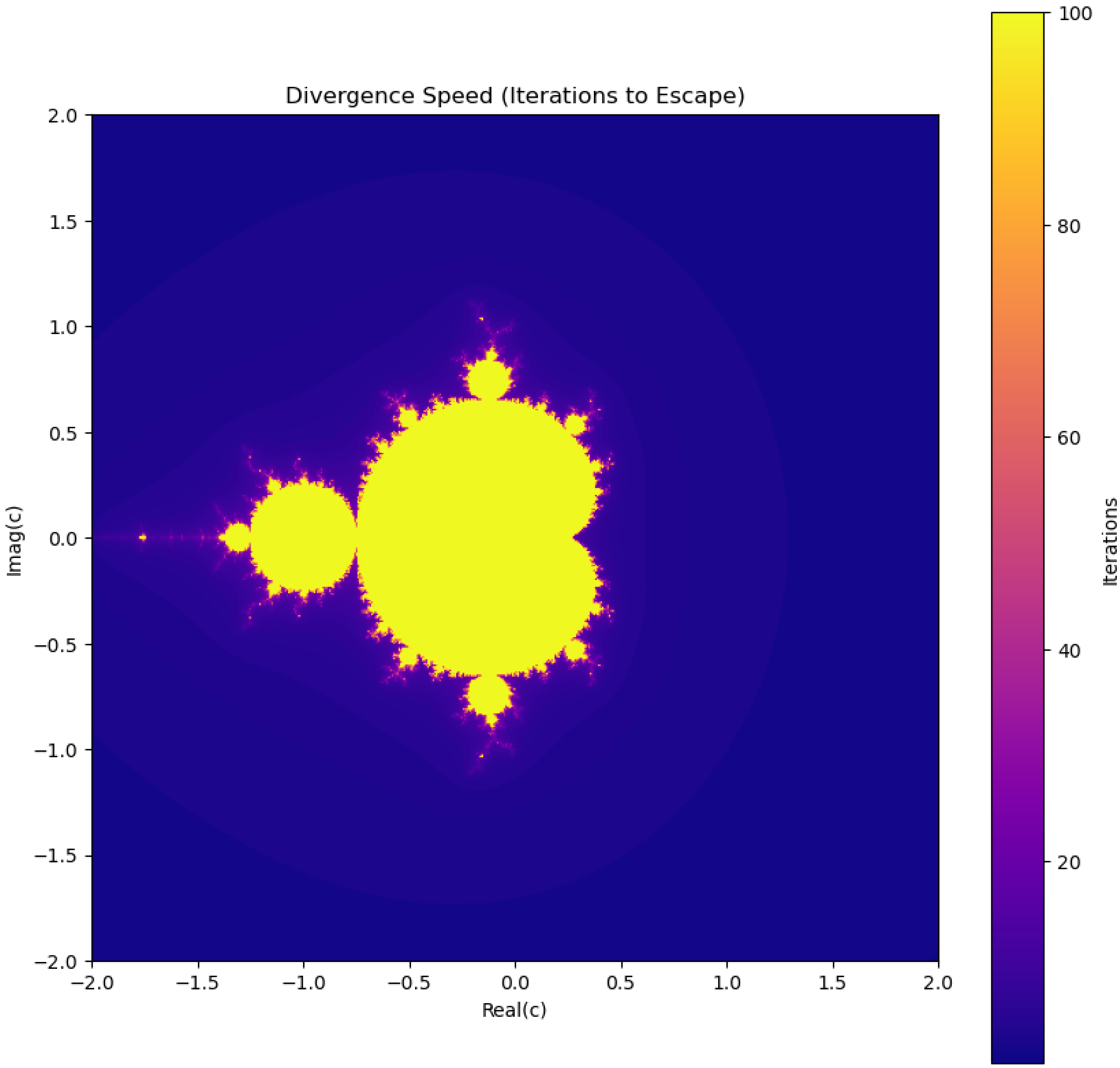

Figure 2: Color map: divergence speed (number of iterations to escape).

Question 2: The Lorenz Attractor

We numerically solved the Lorenz system:

$$\dot{X} = \sigma(Y - X)$$
$$\dot{Y} = rX - Y - XZ$$
$$\dot{Z} = -bZ + XY$$

with parameters $\sigma = 10$, $r = 28$, $b = \frac{8}{3}$, and initial condition $W_0 = [0, 1, 0]$, using `solve_ivp`.

Figure 1: Y vs Time

We plotted $Y(t)$ over three intervals: t = 0–10, 10–20, and 20–30.

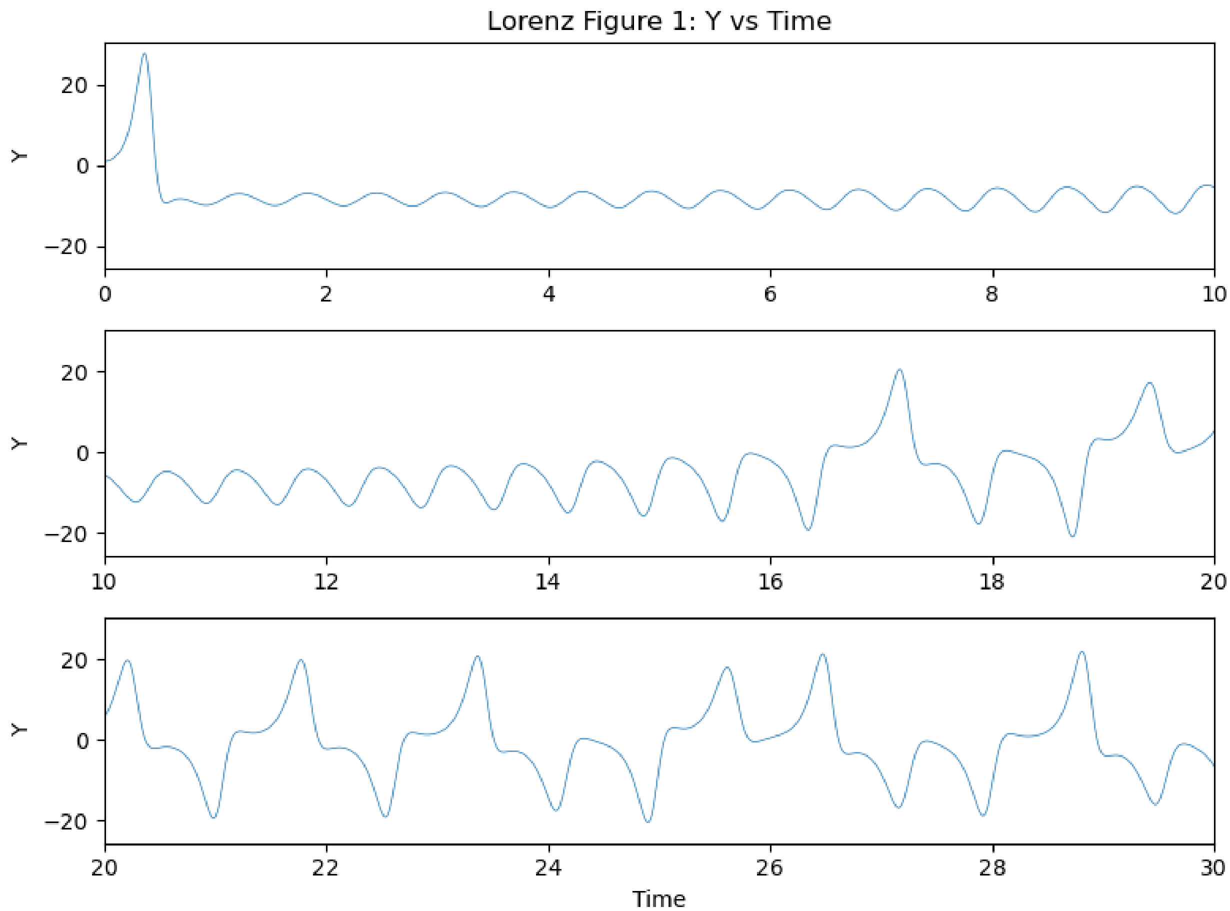

Figure 3: Lorenz Figure 1: Y vs Time over three 10-second segments.

Figure 2: Phase Space Projections

We plotted:

- Top: Z vs Y
- Bottom: Y vs X

for t in [14, 19], including markers labeled 14 to 19 at integer time steps.

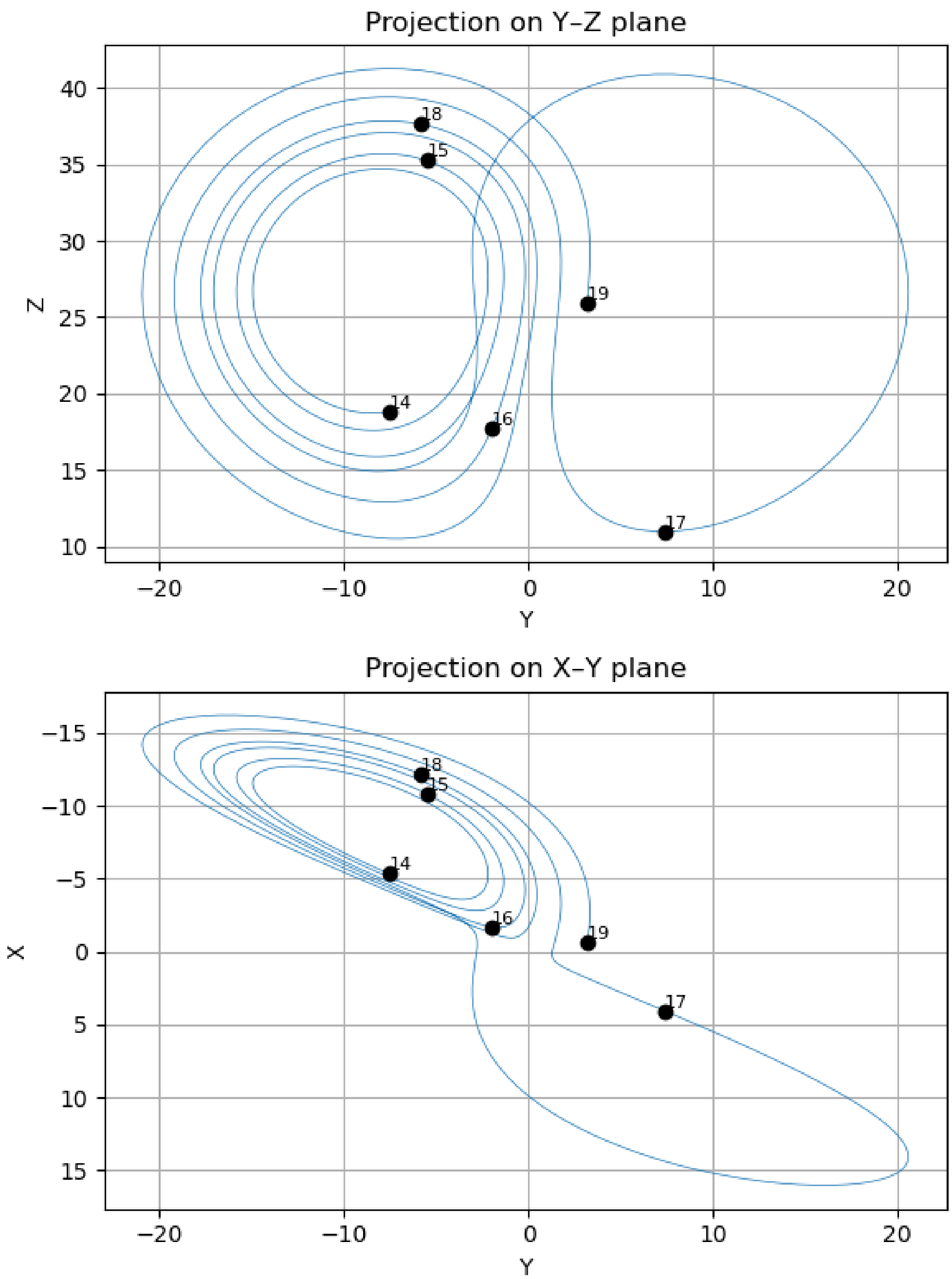

Figure 4: Lorenz Figure 2: Phase projections with time labels.

Divergence from Nearby Initial Conditions

We perturbed the initial condition by adding a small value to the second component:

$$W_0' = W_0 + [0, 1 \times 10^{-8}, 0] = [0, 1.00000001, 0]$$

We then solved the Lorenz equations again and computed the Euclidean distance between the two trajectories over time:

$$d(t) = \|\mathbf{W}(t) - \mathbf{W}'(t)\|$$

We plotted this distance on a semilog scale (logarithmic y-axis, linear x-axis). The nearly straight line confirms that the divergence grows exponentially—an important signature of chaotic behavior and sensitivity to initial conditions.

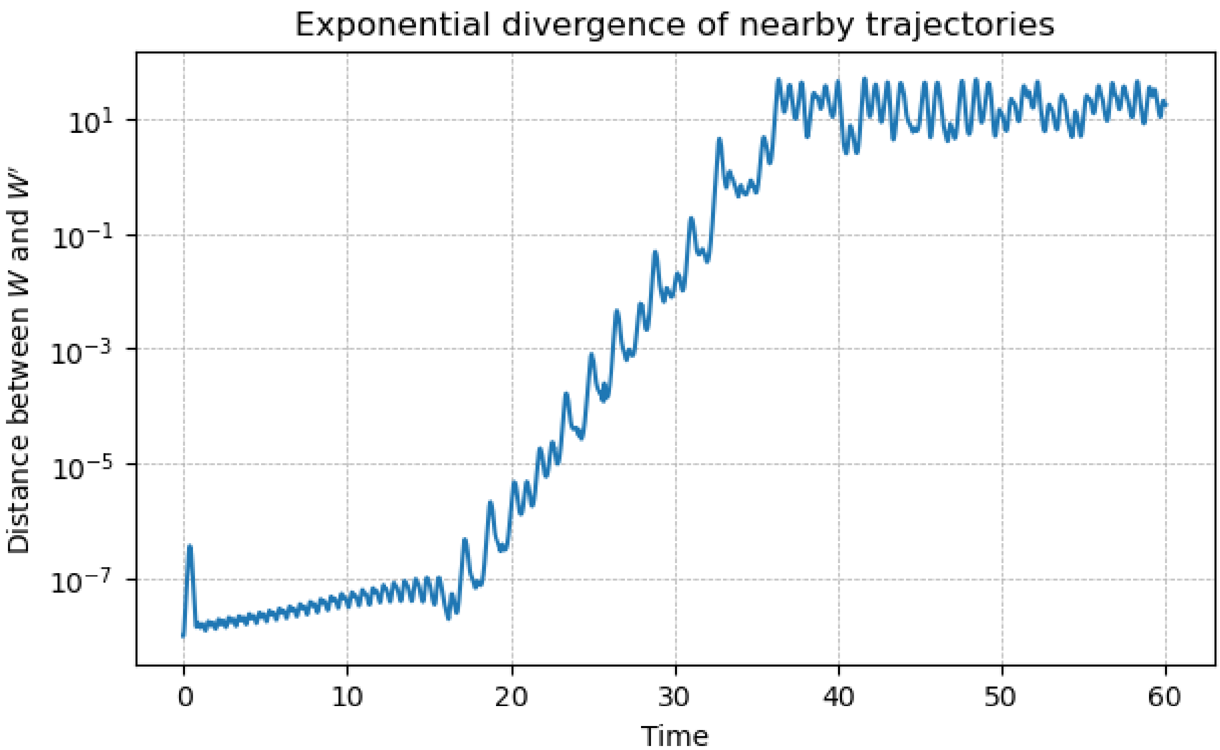

Figure 5: Exponential divergence of nearby trajectories.