A TSP-based MILP Model for Medium-Term Planning of Single-Stage Continuous Multiproduct Plants

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In this paper, we consider the problem of medium-term planning of single-stage continuous plants with a single processing unit that manufactures several products over a planning horizon of several weeks. Sequence-dependent changeover times and costs occur when switching from one type of product to another. To overcome the computational expensiveness of traditional slot-based models for large instances, a novel TSP-based (traveling salesman problem) mixed-integer linear programming (MILP) model is proposed that relies on a hybrid discrete/continuous time representation. The model is applied to an example of a real world polymer processing plant to illustrate its applicability. Finally, the proposed model is compared to recently published approaches through literature examples, and the results show that the computational performance of the proposed model is superior.

1. Introduction

The planning of process systems involves the procedures and processes of allocating the available resources and equipment over a period of time to perform a series of tasks required to manufacture one or more products. Recently, because of the increasing need for more flexible processing facilities to produce more than one product, several papers have addressed the planning problem of multiproduct plants, in which iterative decomposition approaches,[1−5] as well as rolling horizon approaches[6−8] have been proposed to avoid the exponential growth in the computational effort when planning horizons and model sizes increase. Particularly, Erdirik-Dogan and Grossmann[3] proposed a bilevel decomposition procedure that allows the optimization and integration of the planning and scheduling of single-stage single-unit continuous multiproduct plants producing several products with sequence-dependent transition times and costs.

There are also many literature works that have formulated the planning and scheduling problem of multiproduct plants with sequence-dependent changeovers as economic lot sizing problems (ELSP) or capacitated lot sizing problems (CLSP) (see for example, Oh and Karimi,[9] Sung and Maravelias[10]).

Many planning problems are based on continuous time representations (Erdirik-Dogan and Grossmann,[3] Zhu and Majozi,[11] Mendez and Cerda,[12] Castro et al.[13]). Recently published papers adopt a discrete/continuous time representation. Westerlund et al.[14] presented a mixed-time formulation for large scale industrial scheduling problems. Chen et al.[15] proposed a mixed integer linear programming (MILP) for medium-term planning of single-stage single-unit continuous mulitproduct plants based on a hybrid discrete/continuous time representation. In particular, the weeks of the planning horizon are modeled with a discrete time representation while within each week a continuous time representation is employed. This paper also adopts a similar hybrid time approach for the planning horizon but a different formulation is proposed. As an example in Figure 1, the total planning horizon is divided into W discrete weeks, and each week is formulated on a continuous time representation.

Usually in the literature, time slots are postulated in each time period (Erdirik-Dogan and Grossmann,[3,5] Chen et al.[15]). However, the introduction of binary variables to assign of products to time slots during each week increases significantly the size of the resulting optimization models, thus affecting their computational performance. These slot-based models always become intractable when a long planning horizon is considered. Thus, in some recent papers (Alle and Pinto,[16] Alle et al.[17]), TSP-based (traveling salesman problem) formulations are proposed, where binary variables to represent changeovers are used in a way similar to the classic formulation used to model TSP.

The objective of this paper is to develop a novel TSP-based model for medium-term planning of a single-stage plant with a single continuous processing unit producing several products with sequence-dependent changeovers based on a hybrid discrete/continuous time formulation.

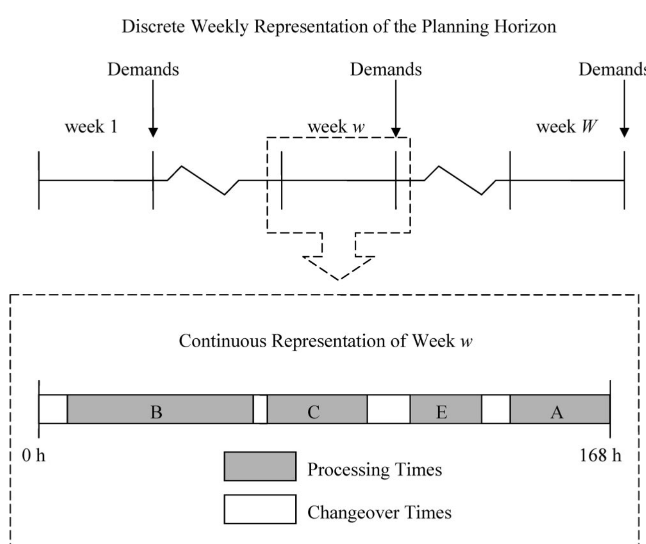

Figure 1. Hybrid discrete/continuous time representation.

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The paper is organized as follows. A detailed description of the problem is given in Section 2. Section 3 presents the mathematical formulation of the proposed model. In Section 4, an illustrative example is described and computational results are presented. The proposed model is compared with three recently published approaches (Erdirik-Dogan and Grossmann,[3,4] Chen et al.[15]) in Section 5. Finally, some concluding remarks are drawn in Section 6.

2. Problem Description

The work considers the optimal medium-term planning of a single-stage plant. The plant manufactures several types of products in one processing machine over a planning horizon. The total available processing time is divided into multiple weeks.

The customers place orders for one or more products. These demands are allowed to be delivered only at the end of each week, which is a key difference from ELSP, in which continuous demand rates are considered. The weekly demands allow the use of hybrid discrete/continuous time representation (see Figure 1). If the demand is not fulfilled at the desired time, late delivery is allowed. At the same time, backlog penalties are imposed on the plant operation.

The plant can also manufacture a larger amount of products than the demand in a time period. The limited inventory is allowed for product storage before sales.

Sequence-dependent changeover times and costs occur when switching production between different products.

Given are the demands, prices, processing rates, changeover unit costs and times, unit penalty costs, and inventory costs for each product. Here, the main optimization variables include decisions on the products to be produced during each week, processing schedule, production times, production amounts, and inventory and backlog levels over the planning horizon. The objective is to maximize the total profit, involving sales revenue, product changeover cost, backlog penalty cost, and inventory cost.

3. Mathematical Formulation

Due to the nature of the problem, a hybrid discrete/continuous time representation (Figure 1), based on the models of Casas-Liza and Pinto[18] and Chen et al.,[15] is applied over a planning horizon, in which the weeks of the planning horizon are modeled with a discrete time formulation and each week is represented by a continuous time formulation.

One key characteristic of the problem is that the sequence-dependent changeovers occur when switching from one product to another. Because of the sequence-dependent changeover times and costs, different sequences of the processing products produce different total profits, even if the processing times are fixed. Here, the planning of multiproduct plants can be taken as a TSP problem. In the classic TSP problem, a salesman is required to visit a number of cities in a sequence that minimizes the overall costs or time, and in the classic TSP formulation binary variables are used to represent the transition from one city to another.[19] Similarly, in a multiproduct plant, a number of products must be produced in a sequence that maximizes profits. So, similar to the binary variables in classic TSP formulation, binary variables Z_{ijw} and ZF_{ijw} are introduced to model the changeover from the production of product i to that of product j in week w and between two consecutive weeks $w-1$ and w, respectively.

Also, to avoid the occurrence of subtours in the sequence of the products, we introduced product ordering variables together with additional mathematical constraints to eliminate product subtours generation at the optimal solution.

The planning problem is formulated as a mixed-integer linear programming (MILP) optimization model with the following notation:

Indices

c = customers

i,j = products

w = weeks

Sets

C = set of customers

I,J = set of products

W = set of weeks

Parameters

CB_{ic} = unit backlog penalty cost of product i to customer c

CC_{ij} = changeover cost from product i to product j

CI_i = unit inventory cost of product i

D_{iw} = demand of product i in week w

M = a large number

PS_{ic} = unit selling price of product i to customer c

r_i = processing rate of product i

$V_i^{\max}$ = maximum storage of product i

$V_i^{\min}$ = minimum storage of product i

θ^L = lower bound for processing time in a week

θ^U = upper bound for processing time in a week

τ_{ij} = changeover time from product i to product j

Binary Variables

E_{iw} = 1 if product i is processed during week w; 0 otherwise.

F_{iw} = 1 if product i is the first one in week w; 0 otherwise.

L_{iw} = 1 if product i is the last one in week w; 0 otherwise.

Z_{ijw} = 1 if product i immediately precedes product j during week w; 0 otherwise.

ZF_{ijw} = 1 if the changeover between weeks $w-1$ and w is from product i to j; 0 otherwise.

Variables

O_{iw} = order index of product i during week w

P_{iw} = amount of product i produced during week w

S_{ciw} = sales volume of product i to customer c during week w

T_{iw} = processing time of product i during week w

V_{iw} = inventory volume of product i at the end of week w

Δ_{ciw} = backlog of product i to customer c at the end of week w

Π = total profit

Objective Function. The profit of the plant is equal to the sales revenue minus operating costs, involving changeover cost, backlog cost, and inventory cost

$$\Pi = \sum_c \sum_i \sum_w PS_{ic} \cdot S_{ciw} - \sum_i \sum_{j \neq i} \sum_w CC_{ij} \cdot Z_{ijw} - \sum_i \sum_{j \neq i} \sum_{w \in W-\{1\}} CC_{ij} \cdot ZF_{ijw} - \sum_c \sum_i \sum_w CB_{ic} \cdot \Delta_{ciw} - \sum_i \sum_w CI_i \cdot V_{iw} \quad (1)$$

Assignment Constraints. Assuming that each week comprises the processing of at least one product, the first and last products to be processed during each week are assigned

$$\sum_i F_{iw} = 1, \quad \forall w \in W \quad (2)$$

$$\sum_i L_{iw} = 1, \quad \forall w \in W \quad (3)$$

A product cannot be assigned as the first or last one if the product is not processed in a week, i.e., if $E_{iw} = 0$, then F_{iw} and L_{iw} should be forced to be 0

$$F_{iw} \le E_{iw}, \quad \forall\, i \in I,\ w \in W \tag{4}$$

$$L_{iw} \le E_{iw}, \quad \forall\, i \in I,\ w \in W \tag{5}$$

Changeover Constraints. Changeovers refer to production switches between two different types of products. In the planning horizon, changeovers may occur within a week or between two consecutive weeks. Binary variables Z_{ijw} represent the changeovers within a week, while binary variables ZF_{ijw} represent the changeovers between two weeks.

For changeovers within a week, if a product is the first one processed, then no product is processed precedent to this product in the same week. Also, if a product is to be processed, but it is not the first one, in a week, then there is exactly one product precedent to this product in the same week

$$\sum_{i \ne j} Z_{ijw} = E_{jw} - F_{jw}, \quad \forall\, j \in J,\ w \in W \tag{6}$$

If a product is the last one processed in a week, then no product is processed following this product in the same week. Also, if a product is to be processed, but it is not the last one, in a week, then there is exactly one product following this product in the same week:

$$\sum_{j \ne i} Z_{ijw} = E_{iw} - L_{iw}, \quad \forall\, i \in I,\ w \in W \tag{7}$$

Note that from constraints 6 and 7, there is no changeover from or to a product that is not to be processed. Figure 2 shows an example of changeover with two products A and B within week w.

For changeovers between two consecutive weeks, if product j is the first one to be processed in week w, there is exactly one changeover from a product at week $w - 1$ to product j. Also, if product i is the last one to be processed in week $w - 1$, there is exactly one changeover to a product at the beginning of week w. If a product is not the first or the last one processed, then there is no changeover involving the product between two weeks.

$$\sum_{i} ZF_{ijw} = F_{jw}, \quad \forall\, j \in J,\ w \in W - \{1\} \tag{8}$$

$$\sum_{j} ZF_{ijw} = L_{i,w-1}, \quad \forall\, i \in I,\ w \in W - \{1\} \tag{9}$$

Here, we assume the changeover between week $w - 1$ and w occurs at the beginning of week w. Figure 3 is an example of changeover from product A to B between weeks $w - 1$ and w.

It should be noticed that the last product processed in week $w - 1$ may be the same product as the one processed first in week w. But such cases do not incur changeover times and costs.

It should be added that variables ZF_{ijw} are treated as continuous, $0 \le ZF_{ijw} \le 1$, as the relevant changeover terms are minimized in the objective function.

Subtour Elimination Constraints. The mentioned constraints have the potential drawback of generating solution with subcycles. When a subcycle is present, the solution of the model is an infeasible schedule (see Figure 4b). So, subtour elimination

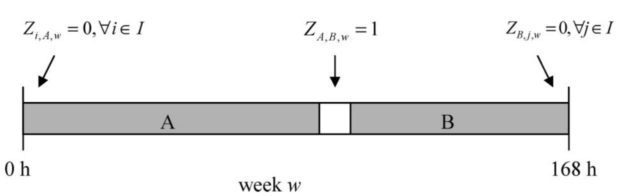

Figure 2. Changeovers within 1 week.

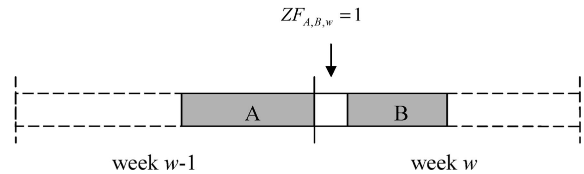

Figure 3. Changeover between 2 weeks.

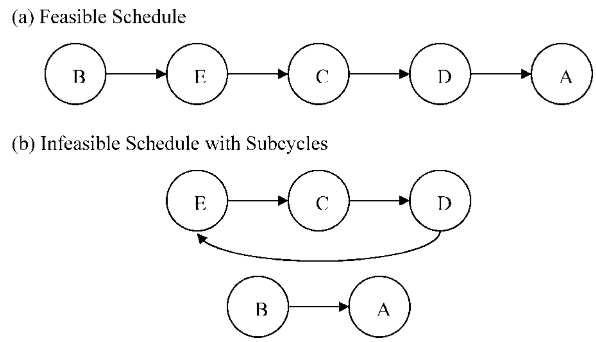

Figure 4. Feasible schedule and infeasible schedule with subcycle.

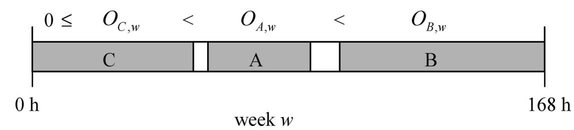

Figure 5. Order indices within 1 week.

constraints are needed to generate feasible schedules (see Figure 4a).

To avoid subtours, positive integer variables O_{iw} are introduced to define the order in which each product is processed in a week. The later a product is processed, the greater its order index is, as shown in Figure 5.

Here, we assume that if product i is processed precedent product j in week w, the order index of product j is at least one higher than that of product i:

$$O_{jw} - (O_{iw} + 1) \ge -M \cdot (1 - Z_{ijw}), \quad \forall\, i, j \in I,\ j \ne i,\ w \in W \tag{10}$$

Also, when a product is not processed, its order index is equal to zero

$$O_{iw} \le M \cdot E_{iw}, \quad \forall\, i \in I,\ w \in W \tag{11}$$

Constraints 10 and 11 guarantee that subtours do not exist in any feasible optimal solution, as similar subtour elimination constraints given in the classic TSP formulation.[19,20] A theorem on the effect of constraint 10 on is shown and proven in Appendix A.

We should notice that the order indices obtained from constraints 10 and 11 do not guarantee values of successive integers. If the latter is required, the following constraints should be included

$$F_{iw} \le O_{iw} \le \sum_{j} E_{jw}, \quad \forall\, i \in I,\ w \in W \tag{12}$$

Note the constraints 11 and 12 force the product order indices to take values between 1 and the total number of products selected for week w.

Alternatively, the following term can be subtracted in the objective function $\varepsilon \cdot \sum_i \sum_w O_{iw}$, where ε is a small number.

Timing Constraints. For each product processed in a week, its duration must be restricted between the lower and upper availability bounds (θ^{L} and θ^{U}, respectively)

$$\theta^{\mathrm{L}} \cdot E_{iw} \le T_{iw} \le \theta^{\mathrm{U}} \cdot E_{iw}, \quad \forall\, i \in I,\ w \in W \tag{13}$$

Also, the total processing and changeover time in a week should not exceed the total available time

$$\sum_i T_{iw} + \sum_i ((Z_{ijw} + ZF_{ijw}) \cdot \tau_{ij}) \le \theta^{\mathrm{U}}, \quad \forall\, w \in W - \{1\} \tag{14}$$

$$\sum_i T_{iw} + \sum_i (Z_{ijw} \cdot \tau_{ij}) \le \theta^{\mathrm{U}}, \quad \forall\, w \in \{1\} \tag{15}$$

Production Constraints. The product amount produced per week is simply given by

$$P_{iw} = r_i \cdot T_{iw}, \quad \forall\, i \in I,\ w \in W \tag{16}$$

Backlog Constraints. The backlog of a product to a customer in a week is defined as the backlog at the previous week plus the demand in this week, minus the sales volume to the customer:

$$\Delta_{ciw} = \Delta_{c,i,w-1} + D_{ciw} - S_{ciw}, \quad \forall\, c \in C,\ i \in I,\ w \in W \tag{17}$$

Inventory Constraints. The inventory of a product in a week is defined as the inventory at the previous week plus the amount produced, minus the total sales volume of the product to all customers:

$$V_{iw} = V_{i,w-1} + P_{iw} - \sum_c S_{ciw}, \quad \forall\, i \in I,\ w \in W \tag{18}$$

The amounts of products to be stored are bounded by minimum and maximum capacities

$$V_i^{\min} \le V_{i,w} \le V_i^{\min}, \quad \forall\, i \in I,\ w \in W \tag{19}$$

The single-stage multiproduct plant is formulated as a MILP model that is described by constraints 2–11, 13–19 with eq 1 as the objective function.

4. An Illustrative Example

To illustrate the applicability of the proposed model, we consider one example of a real world polymer processing plant, which is an extension of the example discussed in Chen et al.[15] In the example, 10 types of products (A−J) are manufactured by a single-stage plant. Weekly demands for each product (see Table 1) are ordered from 10 customers (C1−C10) for a period of 8 weeks. The processing rate is 110 ton/week for each product.

Table 1. Weekly Demands by the Customers (ton)

		weekly demands							
customers	products	1	2	3	4	5	6	7	8
C1, C5	A	5				5			
	C	2	2	2	2	3	3	3	3
C2, C6	D	3		3		3		3	
	E	5		5		5		5	
	H		12				12		12
C3, C7, C9	B	4				4			
	G			5					
	J		6		6		6		6
C4, C8, C10	A	7				7			
	B		5		5			5	
	C	5			5			5	
	D	10					10		
	E	11		11		11		11	
	F	8			8			8	
	G	4		4		4		4	
	H	1	1	1	3	3	3	1	1
	I	5	5	5	5	5	5	5	5
	J		3		3	3		3	

Table 2. Changeover Times (min)

from/to	A	B	C	D	E	F	G	H	I	J
A		45	45	45	60	80	30	25	70	55
B	55		55	40	60	80	80	30	30	55
C	60	100		100	75	60	80	80	75	75
D	60	100	30		45	45	45	60	80	100
E	60	60	55	30		35	30	35	60	90
F	75	75	60	100	75		100	75	100	60
G	80	100	30	60	100	85		60	100	65
H	60	60	60	60	60	60	60		60	60
I	80	80	30	30	60	70	55	85		100
J	100	100	60	80	80	30	45	100	100	

Table 3. Product Selling Prices ($/ton)

	A	B	C	D	E	F	G	H	I	J
prices	10	12	13	12	15	10	8	14	7	15

Table 4. Solution Results of 4-, 6-, and 8-Week Cases

	4-Week Case		
time horizon (weeks)	4	sales revenue ($)	6050.2
no. of eqs	1193	changeover cost ($)	114.2
no. of continuous variables	1261	backlog cost ($)	493.7
no. of binary variables	480	inventory cost ($)	3.5
computational time (CPU s)	3.5	total profit ($)	5438.8
	6-Week Case		
time horizon (weeks)	6	sales revenue ($)	9111.6
no. of equations	1799	changeover cost ($)	185.8
no. of continuous variables	1941	backlog cost ($)	781.3
no. of binary variables	720	inventory cost ($)	9.7
computational time (CPUs)	28	total profit ($)	8134.8
	8-Week Case		
time horizon (weeks)	8	sales revenue ($)	12 035.3
no. of eqs	2405	changeover cost ($)	254.2
no. of continuous variables	2621	backlog cost ($)	1125.7
no. of binary variables	960	inventory cost ($)	0.6
computational time (CPU s)	160	total profit ($)	10 654.9

The total available processing time in each week is 168 h. The minimum processing time for a product in any week is 5 h. The changeover times (in minutes) are shown in Table 2. The changeover costs are proportional to the changeover times (in hours) by a factor of 10. For example, the changeover cost from product A to B is 45 × 10/60 = $7.5.

Table 3 shows the product prices for all customers, except for customer C10, who is 50% higher. The unit inventory and backlog costs are 10 and 20% of product prices, respectively.

Here, we consider three cases of the example, with planning horizons of 4, 6, and 8 weeks, respectively. The models are implemented in GAMS 22.6[21] using solver CPLEX 11.0[22] on a Pentium 4 3.40 GHz, 1.00 GB RAM machine. The optimality gap is set to be 0%, and the computational time is limited to 3600 s. The solution results are shown in Table 4, and the detailed schedules corresponding to the optimal solutions of three cases are shown in Figures 6−8, from which we can see that the proposed model is able to generate optimal schedules within 3 min, even for the case with a planning horizon of 8 weeks.

In the optimal solution of the 8-week case, only product J has an inventory of 0.42 ton at the end of week 4. In Table 5, the optimal weekly aggregate sales and backlogs of the 8-week case are shown.

Now, we focus on the optimal schedules over the first 4 weeks of all 3 cases. From Figures 6−8, we can see that the sequence of the products over the first 4 weeks of the 6-week case is

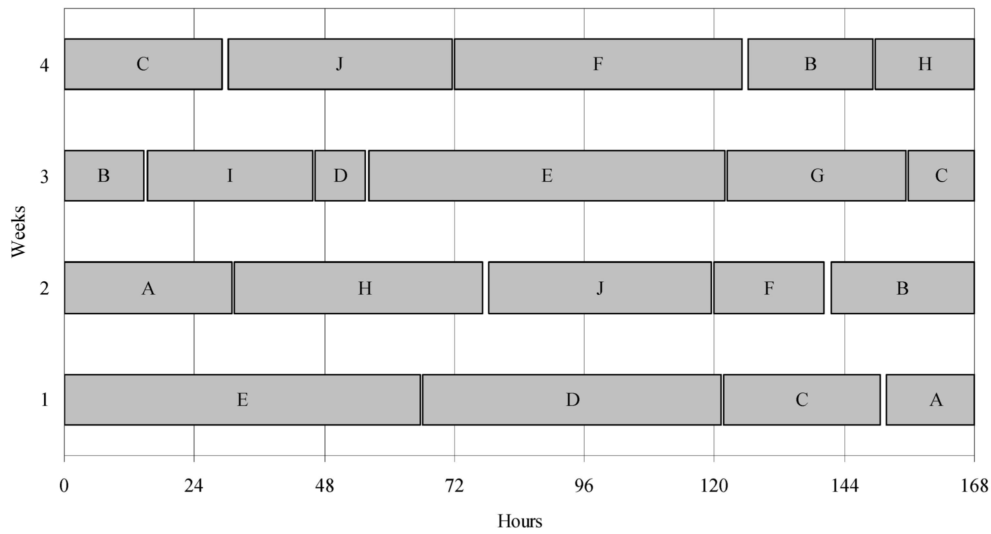

Figure 6. Optimal production schedule of 4-week case.

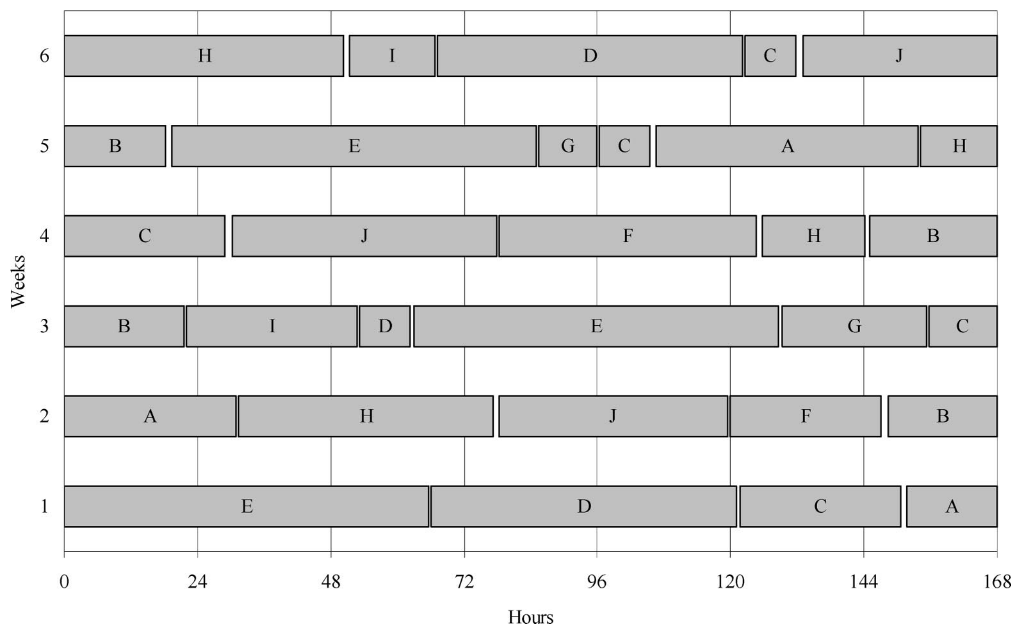

Figure 7. Optimal production schedule of 6-week case.

different from those of the 4-week and 8-week cases, and the differences result from the last two products processed in week 4. In the 4- and 8-week cases, product H is the last one produced in week 4, whereas product B is the last one produced in week 4 in the 6-week case.

We can also see that except for the products B and H in week 4, the optimal sequences over the first 4 weeks are the same in all three cases, while the processing times are different for the same product in different cases, such as products F and B in week 2, products B, I, and G in week 3, and products J and F in week 4. The reason for such differences in sequences and processing times is that the length of the overall planning horizon and associated product demands affect the scheduling decisions.

On the basis of the above observations, the advantages of the proposed single-level MILP approach are emphasized by applying the following hierarchical scheme: solve the 4-week case; fix the schedule (sequence and timings) over 4 weeks; and solve 6- and 8-week cases in reduced spaces. The comparative results between the proposed approach and the hierarchical scheme are shown in Table 6. It can clearly been seen that the profit decreases in both cases. Moreover, if the hierarchical scheme is applied over a rolling horizon fashion, profit may decrease significantly.

5. Comparison to Literature Models

In this section, the computational efficiency of the proposed MILP model is demonstrated by comparing it with those introduced by Erdirik-Dogan and Grossmann[3,4] and Chen et al.,[15] in which Erdirik-Dogan and Gorssmann[4] proposed a bilevel decomposition approach for the scheduling and planning of continuous multiproduct plant with parallel units. Here, we only consider the single-unit case of model and compare the proposed model with the first iteration of the approach. The details of the models proposed by Erdirik-Dogan and Grossmann[3] and Chen et al.[15] are described in Appendices B and C, respectively. Since the lower level problem proposed by Erdirik-Dogan and Grossmann[4] is an extension of the model in Appendix B, this paper only gives the details of its upper level problem and integer cuts, which are described in Appendix D.

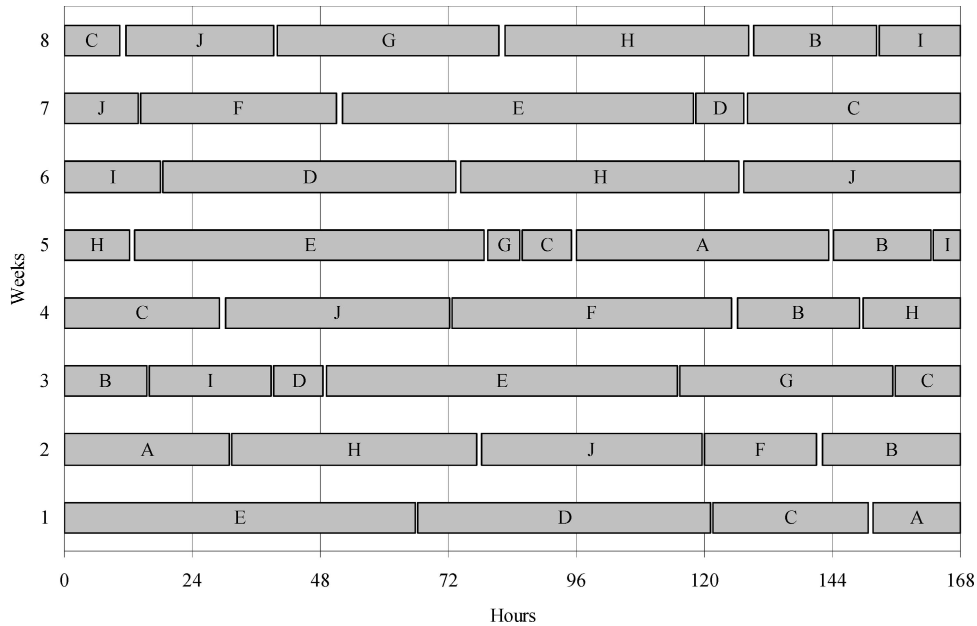

Figure 8. Optimal production schedule of 8-week case.

Table 5. Optimal Aggregate Sales and Backlogs of 8-Week Case (ton)

		weeks							
		1	2	3	4	5	6	7	8
optimal weekly aggregate sales	A	10.7	20.3			31.0			
	B		16.9	10.1	15.0	12.0			15.0
	C	19.0		8.0	19.0	6.0		26.2	6.8
	D	36.0		6.0			36.0	6.0	
	E	43.0		43.0		43.0		43.0	
	F		13.7		34.3			24.0	
	G			26.1		4.0			27.3
	H		30.0		12.0	7.9	34.1		30.0
	I			15.0		3.3	11.7		10.0
	J		27.0		27.0	0.4	26.6	9.0	18.0
optimal weekly aggregate backlogs	A	20.3							
	B	12.0	10.1					15.0	
	C		4.0				6.0	0.8	
	D					6.0			
	E								
	F	24.0	10.3	10.3					
	G	12.0	12.0	12.9	12.9	20.9	20.9	32.9	5.6
	H	3.0		3.0		1.1		3.0	
	I	15.0	30.0	30.0	45.0	56.7	60.0	75.0	80.0
	J					8.6			

Table 6. Objectives of the Proposed Approach and the Hierarchical Scheme

	proposed approach	hierarchical scheme
6-week case	8134.8	8131.5
8-week case	10 654.9	10 647.3

Here, we compare the above three models using two examples. Example 1 was introduced by Erdirik-Dogan and Grossmann.[3] Example 2 is the one shown in Section 4. Also, all models are run in GAMS 22.6[21] with solver CPLEX 11.0[22] on a Pentium 4 3.40 GHz, 1.00 GB RAM machine. All models are run with terminating tolerance of 0.00% and the time limit of 3600 s. For the same representation and a fair comparison of their solution performance among the four MILP models, few modifications are made to the three literature models.

5.1. Modifications. First, because of the similar nature of the models proposed by Erdirik-Dogan and Grossmann[3,4] (E-D&G1 and E-D&G2 for short, respectively), we compare the proposed model with the E-D&G1 and the upper level problem of E-D&G2 simultaneously. There are seven differences between the proposed model and the other two models. Three involve the revenue and cost terms in the objective function. The others involve the sales, inventory, and time constraints. First, the proposed model includes backlog cost that is not present in the E-D&G1 and E-D&G2 models. Second, the E-D&G1 and E-D&G2 models both contain processing cost, which is not involved in the proposed model. Third, the E-D&G1 and E-D&G2 models do not consider multiple customers, while the proposed model considers the revenue and backlog cost from multiple customers. Fourth, the proposed model represents the inventory constraints on a weekly basis (constraint 18), whereas the E-D&G1 and E-D&G2 models both utilize a linear overestimate of the inventory curve (B.13-B.16 and D.4−D.7). Fifth, the proposed model permits the occurrence of backlog (constraint 17), whereas all demands in the E-D&G1 and E-D&G2 models must be satisfied (B.17 and D.8). Sixth, the proposed model forces the processing time for a product in a week to exceed the minimum processing time (constraint 13), whereas there is no such constraints in the E-D&G1 and

Table 7. Model and Solution Statistics of Four Models for Example 1

	E-D&G1	CPP	E-D&G2 (upper/ lower level)	proposed
		4-Week Case		
no. of equations	1139	479	456/779	323
no. of continuous variables	936	696	205/961	196
no. of binary variables	120	120	260/120	140
total profit ($)	235 550	235 550	235 550/235 550	235 550
optimality gap (%)	0.0	0.0	0.0/0.0	0.0
computational time (CPUs)	1321	43	1.9 (1.5/0.4)	0.7
		8-Week Case		
no. of eqs	2303	967	916/1563	655
no. of continuous variables	2136	1396	409/1921	416
no. of binary variables	240	240	520/240	280
total profit ($)	470 520	471 330	471 350/471 350	471 350
optimality gap (%)	1.1	0.4	0.0/0.0	0.0
computational time (CPUs)	3600	3600	135.7 (135/0.7)	83

E-D&G2 models. Last, the E-D&G1 model does not allow the production idle time except changeover (B.10), whereas the proposed model has no restriction on it.

To make a precise comparison, five modifications are made to both the E-D&G1 model and the upper level problem of the E-D&G2 model. First, the operating cost terms are removed from the objective function of each model. Second, a backlog cost term is added to the objective function of each model, and constraints B.17 and D.8 are replaced by constraint 17. Third, multiple customers are considered in the revenue term in the objective function for each model. Fourth, the inventory constraints B.13–B.16 and D.4–D.7 are both replaced by constraint 18, and the inventory cost term in the objective function is modified. Thus, the objective function (B.1) of the E-D&G1 model becomes

$$z^p = \sum_c \sum_i \sum_t p_{cit} \cdot S_{cit} - \sum_i \sum_t c_i^{\mathrm{inv}} \cdot V_{it} - \sum_c \sum_i \sum_t c_{ic}^{\mathrm{bac}} \cdot \Delta_{cit} - \sum_i \sum_k \sum_l \sum_t c_{ik}^{\mathrm{trans}} Z_{iklt} - \sum_t \sum_i \sum_k c_{ik}^{\mathrm{trans}} TRT_{ikt} \quad (20)$$

And the objective function of the upper level problem (D.1) of the E-D&G2 model becomes:

$$z^p = \sum_c \sum_i \sum_t CP_{cit} \cdot S_{cit} - \sum_i \sum_t \mathrm{CINV}_{it} \cdot V_{it} - \sum_c \sum_i \sum_t CB_{ic} \cdot \Delta_{cit} - \sum_t \sum_m \sum_{i \in I(m)} \sum_{k \in I(m)} \mathrm{CTRANS}_{ik} \cdot ZP_{ikmt} - \sum_t \sum_m \sum_{i \in I_m} \sum_{k \in I_m} \mathrm{CTRANS}_{ik} \cdot (ZZZ_{ikmt} - ZZP_{ikmt}) \quad (21)$$

Fifth, the following constraints are added to the E-D&G1 model and the upper level problem of the E-D&G2 model:

$$\theta_{it} \geq \theta^{\mathrm{L}} \cdot YOP_{it}, \quad \forall\, i \in N,\ t \in HTot \quad (22)$$

To allow idle time in the schedule, another modification added to the E-D&G1 model is that the constraint B.10 is modified as

$$Te_{lt} + \sum_i \sum_k \tau_{ik} TRT_{ikt} \leq Ts_{ll,t+1}, \quad \forall\, t \in H_{\mathrm{Tot}},\ l = N, ll = 1 \quad (23)$$

The modifications made to the lower level problem of the E-D&G2 model are the same as those made to the E-D&G1 model.

Now, we compare the proposed model with the model in Chen et al.[5] (CCP for short). There is no difference between the presentations of the two models, so no modification is made to the CPP model.

5.2. Example 1. Example 1, which was discussed in the work of Erdirik-Dogan and Grossmann,[3] consists of five types of products (A−E). The problem has a set of high demands and a set of low demands for a period of eight weeks. Only the set of high demands is used in the comparison. The original example does not include backlog penalty cost, which is assumed to be 20% of product prices in the comparison. Two cases, with a planning horizon of 4 and 8 weeks, respectively, are considered. Table 7 shows the solution results of the four models.

It is observed that for the 4-week case, all models are able to achieve global optimality. The same optimal objective value obtained by the four models. At the same time, the E-D&G1 model uses over 1000 s to find the optimal solution, the CPP model takes over 40 s to reach optimality, and the bilevel E-D&G2 approach requires around two seconds, while the proposed model requires only less than 1 s to find the globally optimal schedule.

From Table 7, we can also see that both the E-D&G1 model and the CPP model cannot find the global optimal solution of 8-week case in the specified time limit, although the CCP model generates a very good approximation of the optimal schedule. However, the E-D&G2 model and the proposed model reach global optimality, in which the former takes over 130 s, and the latter uses about 80 s. The results show that the proposed model has superior computational performance than the other three models.

5.3. Example 2. In Example 2, we also consider three cases with a planning horizon of 4, 6, and 8 weeks. The solution results of the four models are shown in Table 8.

From the comparison, we can see that the proposed model is capable of finding the global optimal solution to all three cases within 200 s, even for the 8-week case. However, the E-D&G1 model and the CPP model cannot reach global optimality within the specified time limit, even for the smallest-size case with a 4-week planning horizon. Between the two models, CCP model has shown a better computational performance than E-D&G1 model for all cases. The E-D&G2 model generates the global optimal schedule only for the 4-week case, whereas for the other two cases, although the upper level problems can be solved in less than 120 s, the lower level problems cannot automatically terminate within the specified time limit. The E-D&G2 model can find better solution than the E-D&G1 model and the CPP model.

Here, when implementing the models in GAMS, variables ZF_{ijw} in the proposed model, variables Z_{iklt} and TRT_{ikt} in the

Table 8. Model and Solution Statistics of Four Models for Example 2

model	E-D&G1	CPP	E-D&G2 (upper/lower level)	proposed
		4-Week Case		
no. of eqs	6384	1909	1651/2,904	1193
no. of continuous variables	6141	5311	1325/6141	1261
no. of binary variables	440	440	920/440	480
total profit ($)	5354	5422	5448/5439	5439
optimality gap (%)	6. 0	1.4	0.0/0.0	0.0
computational time (CPUs)	3600	3600	390 (10/380)	3.5
		6-Week Case		
no. of eqs	9626	2873	2481/4366	1799
no. of continuous variables	9261	7971	1987/9261	1941
no. of binary variables	660	660	1380/660	720
total profit ($)	7889	8045	8148/8102	8135
optimality gap (%)	8.3	3.2	0.0/1.6	0.0
computational time (CPUs)	3600	3600	3639 (39/3600)	28
		8-Week Case		
no. of eqs	12 868	3837	3311/5828	2405
no. of continuous variables	12 381	10 631	2649/12381	2621
no. of binary variables	880	880	1840/880	960
total profit ($)	10110	10531	10667/10642	10655
optimality gap (%)	11.0	4.1	0.0/1.7	0.0
computational time (CPUs)	3600	3600	3713 (113/3600)	160

E-D&G1 model, variables Z_{iklt} in the CPP model, variables ZZZ_{ikmt} in the E-D&G2 model are treated as continuous variables between 0 and 1. Model statistics in Tables 7 and 8 show that the proposed model has much fewer equations and continuous variables than the other three models, especially the E-D&G1 model. These models have a similar number of binary variables, except for the upper level problem of the E-D&G2 model.

6. Concluding Remarks

A novel mixed-integer linear programming model for medium-term planning of single-stage continuous multiproduct plants with one processing unit has been presented in this paper. The model is based on a hybrid discrete/continuous time representation, in which the planning horizon comprises a discrete number of weeks and each week is modeled by a continuous time representation. Because of the similar nature of the problem with the TSP, a formulation similar to the one used to model changeovers in the classic TSP is introduced. Also, to eliminate subtours in the schedule, we also propose integer variables representing the ordering of the products and the subtour elimination constraints.

An example of real world polymer processing plant is used to illustrate the applicability of the proposed model. Finally, the proposed TSP-based model has been compared favorably with recent literature,[3,4,15] exhibiting a much improved computational performance than the slot-based formulation for both examples investigated.

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Appendix A: Theorem on Subtour Elimination

Theorem: Constraint 10 eliminates subtours in the feasible solutions.

Proof: Assume in a feasible solution that there is a cyclic sequence consisting of k products $i_1, i_2, ..., i_k$, in week w, where $k \geq 2$.

So, we have $Z_{i_1 i_2 w} = Z_{i_2 i_3 w} = = Z_{i_{k-1} i_k w} = Z_{i_k i_1 w} = 1$.

From constraint 10, we obtain

$O_{i_2 w} - O_{i_1 w} \geq 1,$

$O_{i_3 w} - O_{i_2 w} \geq 1,$

...

$O_{i_k w} - O_{i_{k-1} w} \geq 1,$

$O_{i_1 w} - O_{i_k w} \geq 1.$

By adding the above k constraints together, we get $O_{i_1 w} - O_{i_1 w} = 0 \geq k$, which is a contradiction. So, there is no subtour in the feasible solutions.

Appendix B: E-D&G1 Model

The model proposed by Erdirik-Dogan and Grossmann,[3] for the simultaneous planning and scheduling of single-stage single-unit continuous muliproduct plants is a multiperiod MILP model based on a continuous time representation.

Nomenclature

Indices

i, k = product indices; $i, k = 1,..., N$

l, ll = time slot indices; $l, ll = 1,..., N$

t = time period indices; $t = 1,..., H_{\text{Tot}}$

Parameters

r_i = production rates of product i

d_{it} = demand of product i in period t

τ_{ik} = transition time from product i to product k

$INVI_{i0}$ = initial inventory level of product i

c_{inv} = inventory cost

c_{it}^{oper} = operating cost for product i in period t

c_{ik}^{trans} = transition cost from product i to k

p_{it} = selling price of product i in period t

H_t = duration of the tth time period

HTot = time at the end of the planning horizon

Variables

NY_{it} = number of slots that product i is assigned in period t

$\tilde{X}_{ilt}$ = amount produced of product i in slot l of period t

X_{it} = amount produced of product i in period t

$\tilde{\theta}_{ilt}$ = production time of product i in slot l of period t

θ_{it} = production time of product i in period t

Ts_{lt} = start time of slot l in period t

Te_{lt} = end time of slot l in period t

INV_{it} = inventory level of product i at the end of time period t

$INVO_{it}$ = final inventory of product i at time t after the demands are satisfied

Area_{it} = area below the inventory time graph for product i at period t

S_{it} = sales of product i in period t

Z^{P} = total profit over a given time horizon

Binary Variables

W_{ilt} = 1 if product i is assigned to slot l of period t; 0 otherwise.

YOP_{it} = 1 if product i is assigned to period t; 0 otherwise.

Z_{iklt} = 1 if product i is followed by product k in slot l of period t; 0 otherwise.

TRT_{ikt} = 1 if product i is followed by product k at the end of period t; 0 otherwise.

Mathematical Formulation

Objective Function:

$$z^p = \sum_i\sum_t p_{it}S_{it} - c_{inv}\sum_i\sum_t \text{Area}_{it} - \sum_i\sum_t c_{it}^{\text{oper}}X_{it} - \sum_i\sum_k\sum_l\sum_t c_{ik}^{trans}Z_{iklt} - \sum_t\sum_i\sum_k c_{ik}^{\text{trans}}\text{TRT}_{ikt} \quad \text{(B.1)}$$

Assignment and Processing Times:

$$\sum_i W_{ilt} = 1 \quad l \in N,\ t \in H\text{Tot} \quad \text{(B.2)}$$

$$0 \le \tilde{\theta}_{ilt} \le H_t W_{ilt} \quad i \in N,\ l \in N,\ t \in H\text{Tot} \quad \text{(B.3)}$$

$$\theta_{it} = \sum_l \tilde{\theta}_{ilt} \quad i \in N,\ t \in H\text{Tot} \quad \text{(B.4)}$$

$$\tilde{X}_{ilt} = r_i\tilde{\theta}_{ilt} \quad i \in N,\ l \in N,\ t \in H\text{Tot} \quad \text{(B.5)}$$

$$X_{it} = \sum_l \tilde{X}_{ilt} \quad i \in N,\ t \in H\text{Tot} \quad \text{(B.6)}$$

Transitions:

$$Z_{iklt} \ge W_{ilt} + W_{k,l+1,t} - 1 \quad i \in N,\ k \in N,\ l \in N,\ t \in H\text{Tot} \quad \text{(B.7)}$$

Timing Relations:

$$Te_{li} = Ts_{lt} + \sum_i \tilde{\theta}_{ilt} + \sum_i\sum_k \tau_{ik}Z_{iklt} \quad l \in N,\ t \in H\text{Tot} \quad \text{(B.8)}$$

$$TRT_{ikt} \ge W_{ilt} + W_{kll,t+1} - 1 \quad i \in N,\ k \in N,\ l \in N,\ \text{ll} = 1 \quad \text{(B.9)}$$

$$Te_{lt} + \sum_i\sum_k \tau_{ik}TRT_{ikt} = Ts_{ll,t+1} \quad t \in H\text{Tot}l = N,\ \text{ll} = 1 \quad \text{(B.10)}$$

$$Te_{lt} = Ts_{l+1,t} \quad l \ne N,\ t \in H\text{Tot} \quad \text{(B.11)}$$

$$Te_{Nt} \le HT_t \quad t \in H\text{Tot} \quad \text{(B.12)}$$

Inventory:

$$INV_{it} = INVI_{i0} + \sum_l r_i\tilde{\theta}_{ilt} \quad i \in N,\ t = 1 \quad \text{(B.13)}$$

$$INV_{it} = INVO_{it-1} + \sum_l r_i\tilde{\theta}_{ilt} \quad i \in N,\ t \ne 1 \quad \text{(B.14)}$$

$$INVO_{it} = INV_{it} - S_{it} \quad i \in N,\ t \in H\text{Tot} \quad \text{(B.15)}$$

$$\text{Area}_{it} \ge \text{INVO}_{i,t-1}H_t + r_i\theta_{it}H_t \quad i \in N,\ t \in H\text{Tot} \quad \text{(B.16)}$$

Demand:

$$S_{it} \ge d_{it} \quad i \in N,\ t \in H\text{Tot} \quad \text{(B.17)}$$

Degeneracy Prevention:

$$NY_{it} = \sum_l W_{ilt} \quad i \in N,\ t \in H\text{Tot} \quad \text{(B.18)}$$

$$YOP_{it} \ge W_{ilt} \quad i \in N,\ l \in N,\ t \in H\text{Tot} \quad \text{(B.19)}$$

$$YOP_{it} \le NY_{it} \le NYOP_{it} \quad i \in N,\ t \in H\text{Tot} \quad \text{(B.20)}$$

$$NY_{it} \ge N - [(\sum_i YOP_{it}) - 1] - (1 - W_{i,l,t}) \quad i \in N,\ t \in H\text{Tot} \quad \text{(B.21)}$$

$$NY_{it} \le N - [(\sum_i YOP_{it}) - 1] + M(1 - W_{i,l,t}) \quad i \in N,\ t \in H\text{Tot} \quad \text{(B.22)}$$

Appendix C: CPP Model

The model proposed by Chen et al.[15] for the medium-term planning of single-stage single-unit continuous muliproduct plants is a MILP model based on a hybrid discrete/continuous time representation.

Nomenclature

Indices

c = customers

i,j = products

k = time slots

w = weeks

Sets

C = customers

I,J = products

K_w = time slots in week w

W = weeks

Parameters

$CB_{c,i}$ = backlog cost of product i to customer c

$CI_{i,w}$ = inventory cost of product i in week w

$CT_{i,j}$= transition cost from product i to product j

$D_{c,i,w}$ = demand of product i from customer c in week w

$PS_{c,i}$ = price of product i to customer c

r_i = processing rate of product i

$V_i^{\max}$ = maximum storage of product i

$V_i^{\min}$ = minimum storage of product i

θ^{L} = lower bound for the processing time

θ^{U} = upper bound for the processing time

$\tau_{i,j}$ = changeover time from product i to product j

Variables

Pro = operating profit

$P_{i,w}$ = production of product i in week w

$S_{c,i,w}$ = sales of product i to customer c in week w

$T_{k,w}$ = end time of slot k in week w

$V_{i,w}$ = volume of product i in week w

$\Delta_{c,i,w}$ = backlog of product i for customer c in week w

$\theta_{i,k,w}$ = processing time of product i in slot k during week w

Binary Variables

$E_{i,w}$ = 1 if product i is produced in week w; 0 otherwise.

$y_{i,k,w}$ = 1 if product i is processed in time slot k during week w; 0 otherwise.

$Z_{i,j,k,w}$ = 1 if product i (slot k − 1) precedes product j (slot k) in week w; 0 otherwise.

Mathematical Formulation

Objective Function:

$$\text{Pro} = \sum_i\sum_w[\sum_c(PS_{i,c}S_{c,i,w} - CB_{i,c}\Delta_{c,i,w}) - (\sum_j\sum_{k\in K_w} CT_{i,j}Z_{i,j,k,w} + CI_{i,w}V_{i,w})] \quad \text{(C.1)}$$

Assignment Constraints:

$$\sum_i y_{i,k,w} = 1 \quad k \in K_w,\ w \in W \quad \text{(C.2)}$$

Timing Constraints:

$$T_{0,w} = 0 \quad T_{|K_w|,w} = 168 \quad w \in W \quad \text{(C.3)}$$

$$0 \le \theta_{i,k,w} \le \theta^{\text{U}}y_{i,k,w} \quad i \in I,\ k \in K_w,\ w \in W \quad \text{(C.4)}$$

$$\sum_{k\in K_w} \theta_{i,k,w} \ge \theta^{\text{L}}E_{i,w} \quad i \in I,\ w \in W \quad \text{(C.5)}$$

$$T_{k,w} - T_{k-1,w} = \sum_i(\theta_{i,k,w} + \sum_i \tau_{j,i}\cdot Z_{j,i,k,w}) \quad \forall\, k \in K_w,\ w \in W \quad \text{(C.6)}$$

Transition Constraints:

$$\sum_j Z_{i,j,k,w} = y_{i,k-1,w} \quad i \in I,\ k \in K_w - \{1\},\ w \in W \quad \text{(C.7)}$$

$$\sum_i Z_{i,j,k,w} = y_{j,k,w} \quad j \in J,\ k \in K_w - \{1\},\ w \in W \tag{C.8}$$

$$\sum_j Z_{i,j,1,w+1} = y_{i,K_w,w} \quad i \in I,\ w \in W \tag{C.9}$$

$$\sum_i Z_{i,j,1,w+1} = y_{j,1,w+1} \quad j \in J,\ w \in W \tag{C.10}$$

Process and Storage Capacity Constraints:

$$P_{i,w} = r_i \sum_{k \in K_w} \theta_{i,k,w} \quad i \in I,\ w \in W \tag{C.11}$$

$$V_i^{\min} \le V_{i,w} \le V_i^{\max} \quad i \in I,\ w \in W \tag{C.12}$$

Inventory and Demand Constraints:

$$V_{i,w} = V_{i,w-1} + P_{i,w} - \sum_c S_{c,i,w} \quad i \in I,\ w \in W \tag{C.13}$$

$$\Delta_{c,i,w} = \Delta_{c,i,w-1} + D_{c,i,w} - S_{c,i,w} \quad c \in C, i \in I,\ w \in W \tag{C.14}$$

Degeneracy Prevention Constraints:

$$\sum_k y_{i,k,w} \le E_{i,w} + (K_w - 1) y_{i,K_w,w} \quad i \in I,\ w \in W \tag{C.15}$$

$$E_{i,w} \ge y_{i,K_w,w} \quad i \in I,\ w \in W \tag{C.16}$$

$$\sum_{j \ne i} \sum_k (Z_{i,j,k,w} + Z_{j,i,k,w}) \le 2 - y_{i,K_w,w} \quad i \in I,\ w \in W \tag{C.17}$$

Appendix D: Upper Level Problem of E-D&G2 Model

In the bilevel decomposition algorithm proposed by Erdirik-Dogan and Grossmann,[4] the original MILP model of simultaneous planning and scheduling of single-stage multiproduct continuous plants with parallel units is decomposed into an upper level planning and a lower level scheduling problem, in which the lower level problem is an extension of the single unit model proposed by Erdirik-Dogan and Grossmann[3] (See Appendix B).

In the decomposition approach, the upper level problem yields a valid upper bound on the profit, while, by excluding the products that were not selected by the upper level problem for each unit at each period, the lower level problem is solved to yield a lower bound on the profit. The two subproblems are solved iteratively. Integer cuts are used to exclude the current assignment and generate new solutions. Finally, the solution of lower level problem becomes the final solution after convergence is achieved.

It should be noticed that for the single-unit case, the number of units considered is 1, i.e., Card(m) = 1, and all products can be processed on the unit, i.e., $I_m = I$.

Nomenclature

Indices
i,k = products
m = units
t = time periods
$\bar{t}$ = last time period
Sets
I = set of products
I_m = set of products that can be processed on unit m
M = set of units
M_i = set of units that can process product i
Parameters
CINV_{it} = inventory cost of product i in period t
COP_{it} = operating cost of product i in period t
CP_{it} = selling price of product i in period t
CTRANS_{ikm} = transition cost of changing the production from product i to k in unit m
d_{it} = demand of product i at the end of period t
H_t = duration of the t th time period
INVI_i = initial inventory of product i
r_{im} = production rate of product i in unit m
τ_{ikm} = transition time from product i to product k in unit m
Variables
Area_{it} = overestimate of the area below the inventory time graph for product i at the end of period t
INV_{it} = inventory level of product i at the end of time period t
INVO_{it} = inventory level of product i at the end of period t after demands are satisfied
S_{it} = Sales of product i at the end of period t
TRNP_{mt} = total transition time for unit m within each time period
$\tilde{X}_{imt}$ = amount of product i produced in unit m during period t
$\tilde{\theta}_{imt}$ = production time of product i produced in unit m during period t
Binary Variables
XF_{imt} = 1 if product i the first product in unit m during period t
XL_{imt} = 1 if product i the last product in unit m during period t
YP_{imt} = assignment of product i to unit m during period t
ZP_{ikmt} = 1 if product i precedes product k in unit m during time period t
ZZP_{ikmt} = 1 if the link between products i and k are broken
ZZZ_{ikmt} = transition variable denoting the changeovers across adjacent periods

Mathematical Formulation

Objective Function:

$$\mathrm{Profit} = \sum_i \sum_t \mathrm{CP}_{it} S_{it} - \sum_i \sum_t \mathrm{CINV}_{it} \mathrm{Area}_{it} - \sum_i \sum_t \mathrm{COP}_{it} \tilde{X}_{it} - \sum_t \sum_m \sum_{i \in I_m} \sum_{k \in I_m} \mathrm{CTRANS}_{ik} (\mathrm{ZP}_{ikmt} - \mathrm{ZZP}_{ikmt}) - \sum_t \sum_m \sum_{i \in I_m} \sum_{k \in I_m} \mathrm{CTRANS}_{ik} \cdot \mathrm{ZZZ}_{ikmt} \tag{D.1}$$

Assignment and Production Constraints:

$$\tilde{\theta}_{imt} \le H_t \cdot \mathrm{YP}_{imt} \quad \forall i \in I_m, m, t \tag{D.2}$$

$$\tilde{X}_{imt} = r_{im} \cdot \tilde{\theta}_{imt} \quad \forall i \in I_m, m, t \tag{D.3}$$

Inventory Balance and Costs:

$$\mathrm{INV}_{it} = \mathrm{INVI}_i + \sum_{m \in M_i} r_{im} \tilde{\theta}_{imt} \quad \forall i,\ t = 1 \tag{D.4}$$

$$\mathrm{INV}_{it} = \mathrm{INVO}_{i,t-1} + \sum_{m \in M_i} r_{im} \tilde{\theta}_{imt} \quad \forall i,\ t \ne 1 \tag{D.5}$$

$$\mathrm{INVO}_{it} = \mathrm{INV}_{it} - S_{it} \quad \forall i,\ t \tag{D.6}$$

$$\mathrm{Area}_{it} \ge \mathrm{INVO}_{i,t-1} H_t + \Big(\sum_{m \in M_i} r_{im} \tilde{\theta}_{imt}\Big) H_t \quad \forall i,\ t \tag{D.7}$$

Demand:

$$S_{it} \ge d_{it} \quad \forall i,\ t \tag{D.8}$$

Sequencing Constraints:

$$\mathrm{YP}_{imt} = \sum_{k \in I_m} \mathrm{ZP}_{ikmt} \quad \forall i \in I_m,\ m,\ t \tag{D.9}$$

$$\mathrm{YP}_{kmt} = \sum_{i \in I_m} \mathrm{ZP}_{ikmt} \quad \forall k \in I_m,\ m,\ t \tag{D.10}$$

$$\sum_{i \in I_m} \sum_{k \in I_m} \mathrm{ZZP}_{ikmt} = 1 \quad \forall m,\ t \tag{D.11}$$

$$\mathrm{ZZP}_{ikmt} \le \mathrm{ZP}_{ikmt} \quad \forall i \in I_m,\ k \in I_m,\ m,\ t \tag{D.12}$$

$$\mathrm{YP}_{imt} \geq \mathrm{ZP}_{iimt} \quad \forall\, i \in I_m,\ m,\ t \tag{D.13}$$

$$\mathrm{ZP}_{iimt} + \mathrm{YP}_{kmt} \leq 1 \quad \forall\, i \in I_m,\ k \in I_m,\ i \neq k,\ m,\ t \tag{D.14}$$

$$\mathrm{ZP}_{iimt} \geq \mathrm{YP}_{imt} - \sum_{k \neq i, k \in I_m} \mathrm{YP}_{kmt} \quad \forall\, i \in I_m,\ m,\ t \tag{D.15}$$

$$\mathrm{TRNP}_{mt} = \sum_{i \in I_m}\sum_{k \in I_m} \tau_{ik}\mathrm{ZP}_{ikmt} - \sum_{i \in I_m}\sum_{k \in I_m} \tau_{ik}\mathrm{ZZP}_{ikmt} \quad \forall\, m,\ t \tag{D.16}$$

$$\mathrm{XF}_{kmt} \geq \sum_{i \in I_m} \mathrm{ZZP}_{ikmt} \quad \forall\, k \in I_m,\ m,\ t \tag{D.17}$$

$$\mathrm{XL}_{kmt} \geq \sum_{k \in I_m} \mathrm{ZZP}_{ikmt} \quad \forall\, i \in I_m,\ m,\ t \tag{D.18}$$

$$\sum_{i \in I_m} \mathrm{XF}_{imt} = 1 \quad \forall\, m,\ t \tag{D.19}$$

$$\sum_{i \in I_m} \mathrm{XL}_{imt} = 1 \quad \forall\, m,\ t \tag{D.20}$$

$$\sum_{k \in I_m} \mathrm{ZZZ}_{ikmt} = \mathrm{XL}_{imt} \quad \forall\, i \in I_m,\ m,\ t \tag{D.21}$$

$$\sum_{i \in I_m} \mathrm{ZZZ}_{ikmt} = \mathrm{XF}_{k,m,t+1} \quad \forall\, k \in I_m,\ m,\ t \in T - \{\bar{t}\} \tag{D.22}$$

Time Balance:

$$\sum_{i \in I_m} \tilde{\theta}_{imt} + \mathrm{TRNP}_{mt} - \sum_{i \in I_m}\sum_{k \in I_m} (\tau_{ik}\mathrm{ZZZ}_{ikmt}) \leq H_t \quad \forall\, m,\ t \tag{D.23}$$

Integer Cuts:

$$\sum_{(i,t) \in Z_1^r} \mathrm{YP}_{imt} - \sum_{(i,t) \in Z_0^r} \mathrm{YP}_{imt} \leq |Z_0^r|\text{-}1 \tag{D.24}$$

where $Z_0^r = \{i,t|YP_{imt}^r = 0\}$ and $Z_1^r = \{i,t|YP_{imt}^r = 1\}$.

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