Midterm Exam 1

GEOL647

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Problem 1: You are given a magnetometer with inductance $L = 3$, capacitance, $C = \frac{1}{2}$, and resistance $R = 4$ and told that the frequency response function of the magnetometer is therefore given by $\tilde{X}(\omega) = \omega^2/(-3\omega^2 + 2 + i3\omega)$.

1. Calculate the amplitude response function for this magnetometer. (show all work)

   Solution:

$$
\begin{aligned}
ARF &= |X\tilde{(}\omega)| \\
ARF &= \sqrt{\frac{\omega^2}{-3\omega^2 + 2 + i3\omega} \frac{\omega^2}{-3\omega^2 + 2 - i3\omega}} \\
ARF &= \sqrt{\frac{\omega^4}{(2 - 3\omega^2)^2 + 9\omega^2}} \\
ARF &= \omega^2 \sqrt{\frac{1}{(2 - 3\omega^2)^2 + 9\omega^2}}
\end{aligned}
$$

2. Explain what the amplitude response function tells you about how the output of the magnetometer relates to the variations in the magnetic field. Be as specific as possible for full credit. Refer to the amplitude and frequency of inputs and outputs.

   Solution: The basic formula for the frequency response (and thus the amplitude response) of an instrument is $Output = \tilde{X}(\omega) * Input$. From this, we know that the amplitude response function tells us how much the amplitude of the input signal (e.g. the magnetic field) is amplified at any given frequency to produce the output signal (e.g. squiggles on a computer screen). Two key pieces of information we can get from this function are 1) the range of frequencies to which the instrument is most effectively sensitive, and 2) the resonant frequency at which the instrument is *most* sensitive (often catastrophically so!).

3. Calculate the approximate resonant frequency of this magnetometer (use symbols and numbers from the FRF, and, if you'd like, you can consider R to be negligible to make your computation simpler).

   Solution: In order to calculate the resonant frequency, we must determine where $\tilde{X}(\omega)$ is at a maximum. This occurs when the denominator $((2 - 3\omega^2)^2 + 9\omega^2)$ is close to zero. So, we set this equal to zero (disregarding the $9\omega^2$ term that relates to resistance) and solve for ω to get an approximation.

$$
\begin{aligned}
(2 - 3\omega^2)^2 &= 0 \\
\omega^2 &= \frac{2}{3} \\
\omega_0 &= \sqrt{\frac{2}{3}}
\end{aligned}
$$

4. How does the resonant frequency relate to the amplitude response function? How does changing R affect the amplitude response of the magnetometer? Be specific.

   Solution: The resonant frequency is the frequency at which the amplitude response function is a maximum. Changing R on a magnetometer has a similar effect to changing the damping coefficient on a seismometer; it will flatten the peak of the amplitude response function around the resonant frequency, making the amplification of the signal at this frequency less catastrophic.