Matlab Practical 2

GEOL647

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Problem 1: What is the analytic expression for the frequency response function of a simple seismometer with mass **m**, spring constant **k**, and coefficient of friction **D**?

Solution:In the Fourier domain, the ODE for seismometer motion is:

$$\omega^2 \tilde{u}(\omega) = \frac{k}{m}\tilde{x}(\omega) + i\omega\frac{D}{m}\tilde{x}(\omega) - \omega^2\tilde{x}(\omega)$$

Where D is the damping term, u is the relative position of the Earth (input) and x is the relative position of the seismometer (output). So the frequency response function (FRF) is:

$$\tilde{X}(\omega) = \frac{\tilde{x}(\omega)}{\tilde{u}(\omega)} = \frac{\omega^2}{\frac{k}{m} + \frac{D}{m}i\omega - \omega^2}$$

Problem 2: Derive the amplitude response function of the seismometer from the FRF:

Solution:

$$ARF = \sqrt{|\tilde{X}(\omega)|} = \sqrt{\frac{\omega^2}{\frac{k}{m} + \frac{D}{m}i\omega - \omega^2}\frac{\omega^2}{\frac{k}{m} - \frac{D}{m}i\omega - \omega^2}}$$

$$ARF = \frac{\omega^2}{\sqrt{\left(\frac{k}{m} - \omega^2\right)^2 + \left(\frac{D}{m}\omega\right)^2}}$$

Problem 3: Derive the phase delay function of the seismometer from the FRF:

Solution:

$$\Theta(\omega) = \arctan\left(\frac{Im(\tilde{X}(\omega))}{Re(\tilde{X}(\omega))}\right)$$

To find this, we must rewrite $\tilde{X}(\omega)$ as:

$$\tilde{X}(\omega) = \frac{\omega^2}{\frac{k}{m} + \frac{D}{m}i\omega - \omega^2}\frac{\frac{k}{m} - \frac{D}{m}i\omega - \omega^2}{\frac{k}{m} - \frac{D}{m}i\omega - \omega^2} = \frac{-\omega^4 + \omega^2\frac{k}{m} - i\omega^3\frac{D}{m}}{\left[\frac{k}{m} - \omega^2\right]^2 + \left(\frac{D}{m}\omega\right)^2}$$

$$Im(\tilde{X}(\omega)) = \frac{-\omega^3\frac{D}{m}}{\left[\frac{k}{m} - \omega^2\right]^2 + \left(\frac{D}{m}\omega\right)^2}, \quad Re(\tilde{X}(\omega)) = \frac{-\omega^4 + \omega^2\frac{k}{m}}{\left[\frac{k}{m} - \omega^2\right]^2 + \left(\frac{D}{m}\omega\right)^2}$$

Plugging this in, we have:

$$\Theta(\omega) = \arctan\left(\frac{-\omega^3\frac{D}{m}}{-\omega^4 + \omega^2\frac{k}{m}}\right) = \arctan\left(\frac{-\omega\frac{D}{m}}{\frac{k}{m} - \omega^2}\right) = \arctan\left(\frac{-\omega D}{k - m\omega^2}\right)$$