Matlab Practical 2

GEOL647

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Problem 1.1: What is the analytic expression for the frequency response function of a simple seismometer with mass **m**, spring constant **k**, and coefficient of friction **D**?

Solution:In the Fourier domain, the ODE for seismometer motion is:

$$\omega^2\tilde{u}(\omega) = \frac{k}{m}\tilde{x}(\omega) + i\omega\frac{D}{m}\tilde{x}(\omega) - \omega^2\tilde{x}(\omega)$$

Where D is the damping term, u is the relative position of the Earth (input) and x is the relative position of the seismometer (output). So the frequency response function (FRF) is:

$$\tilde{X}(\omega) = \frac{\tilde{x}(\omega)}{\tilde{u}(\omega)} = \frac{\omega^2}{\frac{k}{m} + \frac{D}{m}i\omega - \omega^2}$$

Problem 1.2: Derive the amplitude response function of the seismometer from the FRF:

Solution:

$$ARF = \sqrt{|\tilde{X}(\omega)|} = \sqrt{\frac{\omega^2}{\frac{k}{m} + \frac{D}{m}i\omega - \omega^2}\frac{\omega^2}{\frac{k}{m} - \frac{D}{m}i\omega - \omega^2}}$$

$$ARF = \frac{\omega^2}{\sqrt{\left(\frac{k}{m} - \omega^2\right)^2 + \left(\frac{D}{m}\omega\right)^2}}$$

Problem 1.3: Derive the phase delay function of the seismometer from the FRF:

Solution:

$$\Theta(\omega) = \arctan\left(\frac{Im(\tilde{X}(\omega))}{Re(\tilde{X}(\omega))}\right)$$

To find this, we must rewrite $\tilde{X}(\omega)$ as:

$$\tilde{X}(\omega) = \frac{\omega^2}{\frac{k}{m} + \frac{D}{m}i\omega - \omega^2}\frac{\frac{k}{m} - \frac{D}{m}i\omega - \omega^2}{\frac{k}{m} - \frac{D}{m}i\omega - \omega^2} = \frac{-\omega^4 + \omega^2\frac{k}{m} - i\omega^3\frac{D}{m}}{\left[\frac{k}{m} - \omega^2\right]^2 + \left(\frac{D}{m}\omega\right)^2}$$

$$Im(\tilde{X}(\omega)) = \frac{-\omega^3\frac{D}{m}}{\left[\frac{k}{m} - \omega^2\right]^2 + \left(\frac{D}{m}\omega\right)^2}, \quad Re(\tilde{X}(\omega)) = \frac{-\omega^4 + \omega^2\frac{k}{m}}{\left[\frac{k}{m} - \omega^2\right]^2 + \left(\frac{D}{m}\omega\right)^2}$$

Plugging this in, we have:

$$\Theta(\omega) = \arctan\left(\frac{-\omega^3\frac{D}{m}}{-\omega^4 + \omega^2\frac{k}{m}}\right) = \arctan\left(\frac{-\omega\frac{D}{m}}{\frac{k}{m} - \omega^2}\right) = \arctan\left(\frac{-\omega D}{k - m\omega^2}\right)$$

Problem 2.1: Write the frequency response function for a seismometer with this type of feedback system. To do this, think about inputs and outputs. Because voltage is output by the system and ground acceleration is input, compute $V/\ddot{u}$.

$$(1)\ \ddot{y} = \ddot{u} - \ddot{x} \qquad (2)\ \ddot{x} = \beta V \qquad (3)\ V = \frac{KA\omega^2}{\omega^2 - 2i\epsilon\omega - \omega_0^2}\ddot{y}$$

Solution: To begin, we will rewrite certain terms to make things clearer:

$$\begin{aligned} V &= KA\tilde{X}_0\ddot{y} \\ \ddot{u} &= \ddot{y} + \ddot{x} = \ddot{y} + \beta KA\tilde{X}_0\ddot{y} \end{aligned}$$

Where $\tilde{X}_0$ is the FRF of an inertial seismometer. Now, solving for the FRF, we have:

$$\begin{aligned} \tilde{X}(\omega) &= \frac{V}{\ddot{u}} = \frac{KA\tilde{X}_0\ddot{y}}{\ddot{y} + \beta KA\tilde{X}_0\ddot{y}} \\ \tilde{X}(\omega) &= \frac{KA\tilde{X}_0}{1 + \beta KA\tilde{X}_0} \\ \tilde{X}(\omega) &= \frac{KA\omega^2}{\omega^2 - 2i\epsilon\omega - \omega_0^2 + \beta KA\omega^2} \\ \tilde{X}(\omega) &= \frac{KA\omega^2}{(1 + \beta KA)\omega^2 - 2i\epsilon\omega - \omega_0^2} \end{aligned}$$

Problem 2.2: Solve for the amplitude response function of the system (I'm going to call $ARF(\omega)$, $\xi(\omega)$ because I think it's cool):

Solution:

$$\begin{aligned} \xi(\omega) &= |\tilde{X}(\omega)| \\ \xi(\omega) &= \sqrt{\frac{KA\omega^2}{(1 + \beta KA)\omega^2 - 2i\epsilon\omega - \omega_0^2}\frac{KA\omega^2}{(1 + \beta KA)\omega^2 + 2i\epsilon\omega - \omega_0^2}} \\ \xi(\omega) &= \frac{KA\omega^2}{\sqrt{[(1 + \beta KA)\omega^2 - \omega_0^2]^2 + 4\epsilon^2\omega^2}} \end{aligned}$$

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%MP2 for GEOL647

clear;
clearvars;
%hello

%q1.4
freq = linspace(0.3,100,10000);
w = 2*pi*freq;


%q1.5
k_M = 400*pi^2;
D_M = 0.05;
ARF = w.^2 ./ (sqrt((k_M - w.^2) .^ 2 + (D_M .* w) .^ 2));
plot(w,ARF,'DisplayName',num2str(D_M))
xlabel('Frequency');ylabel('Amplitude Response')
%The seismometer is most sensitive at about 63 rad^-1, or 10 Hz

%q1.6
set(gca,'xscale','log','yscale','log')
%The sensitivity of the seismometer below the resonant is higher than the
sensitivity above the resonant frequency.

%q1.7
hold on;
xlabel('Frequency');ylabel('Amplitude Response');title('Amplitude Response
while varying damping coefficient')
for i = [0.3,1,3,10]
    ARF = w.^2 ./ (sqrt((k_M - w.^2) .^ 2 + (i .* w) .^ 2));
    plot(w,ARF,'DisplayName',num2str(i))
end

legend

%As D/M increases, the overshoot in the ARF at the resonant frequency is
decreased.

%q1.8
figure;
xlabel('Frequency');ylabel('Phase Delay');title('Phase Delay while varying
damping coefficient')
hold on;
for i = [0.05,0.3,1,3,10]
    PD = atan2(-w .* i,k_M - w .^ 2);
    plot(w,PD,'DisplayName',num2str(i))
end

legend
%Damping causes a "smoothing out" effect in the phase delay around the
resonant frequency. At low D/M, the phase delay occurs abruptly at the
resonant frequency, but an increase in D/M causes this change to occur more
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gradually.

%q1.9
%Based on the results of 7 and 8, a value of D/m that is somehwere in the
middle between 0.05 and 10, say, 3 would be the ideal balance between a
signal that is the most even, but also the least distorted.

%q1.10

figure;
xlabel('Frequency');ylabel('Amplitude Response');title('Amplitude Response,
varying fundamental frequency of spring')
set(gca,'xscale','log','yscale','log')
hold on;
D_M = 3;
for i = linspace(16*pi^2,16000*pi^2,10)
    ARF = w.^2 ./ (sqrt((i - w.^2) .^ 2 + (D_M .* w) .^ 2));
    plot(w,ARF,'DisplayName',num2str(i))
end
legend

figure;
xlabel('Frequency');ylabel('Phase Delay');title('Phase Delay, varying
fundamental frequency of spring')
hold on;
for i = linspace(16*pi^2,16000*pi^2,10)
    PD = atan2(-w .* D_M,i - w .^ 2);
    plot(w,PD,'DisplayName',num2str(i))
end
legend

%Changing k/M changes the resonant frequency of the seismometer, so it
shifts the peak and the dropoff for the ARF and the PD, respectively.

%q1.11

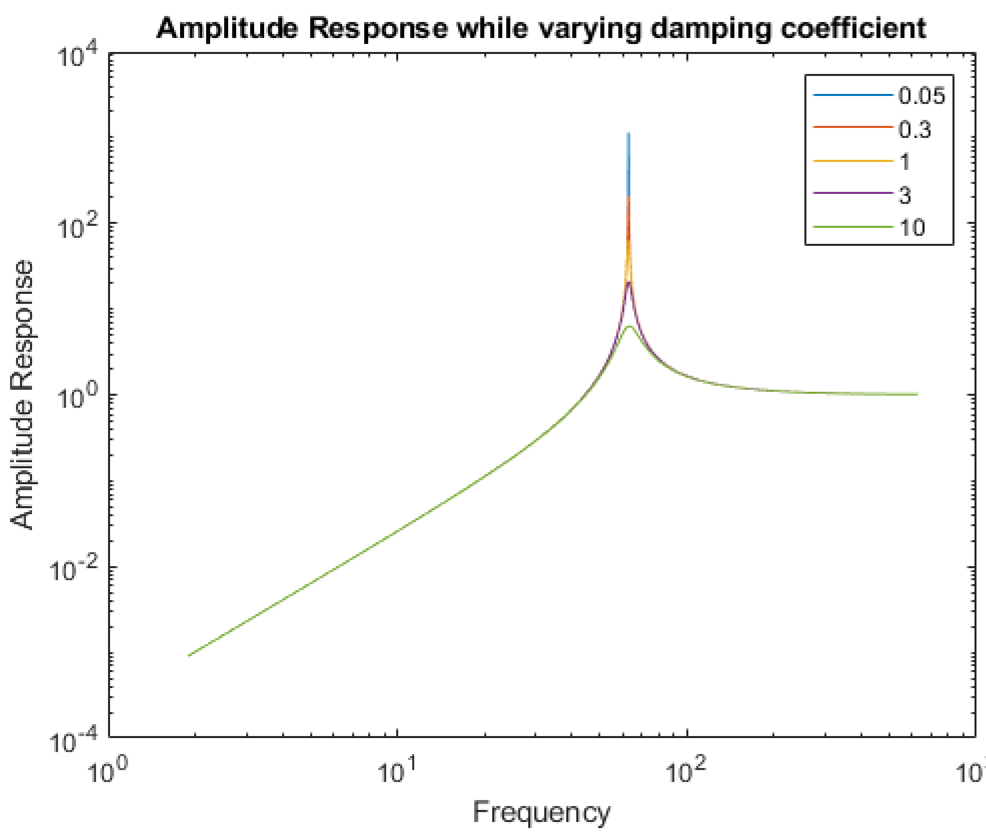
%To maximize sensitivity for long period motion (i.e. low frequency), I
would reduce k and increase m. D does not changes the sensitivity, but
rather the distortion of the signal.

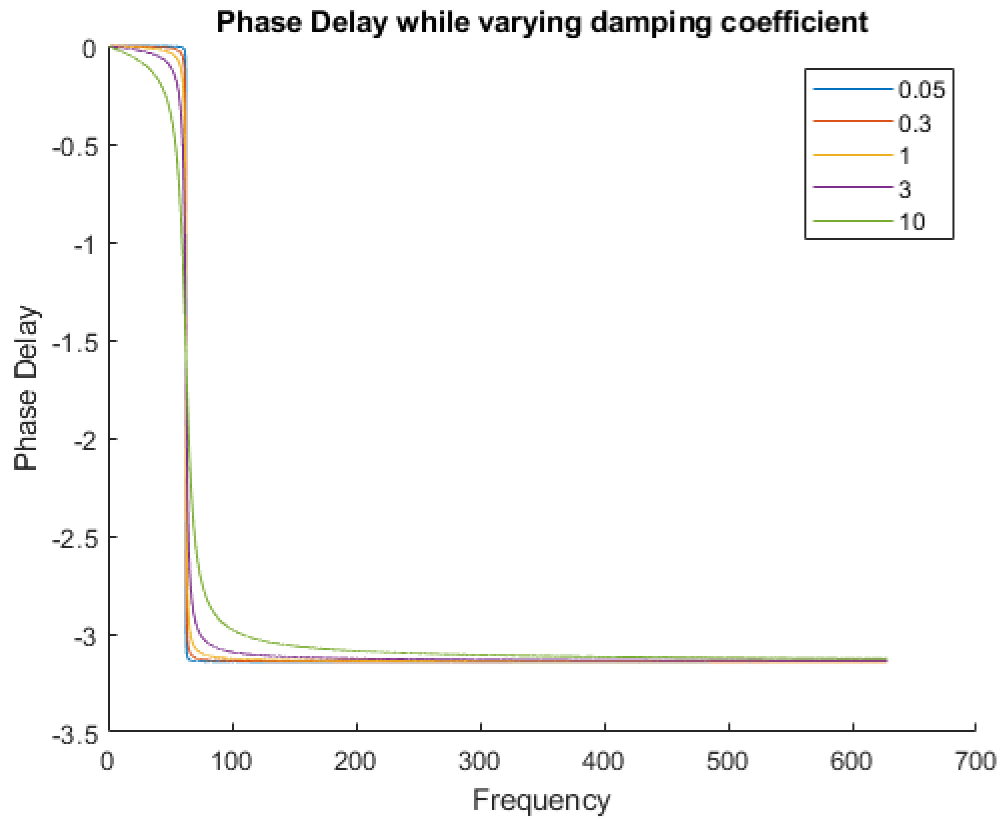
%q2.3
w_0 = 20*pi;
ep = 1;
bt = 1;
figure;
xlabel('Frequency');ylabel('Amplitude Respone');title('Force Feedback
Seismometer')
set(gca,'xscale','log','yscale','log');
hold on;
for AK = linspace(0.1,30,10)
    ARF = (AK .* w .^2) ./ (sqrt((((1+bt*AK) .* w .^2) - w_0^2) .^2 +
(4*ep^2 .* w .^2)));
    plot(w,ARF,'DisplayName',num2str(AK))
end
```

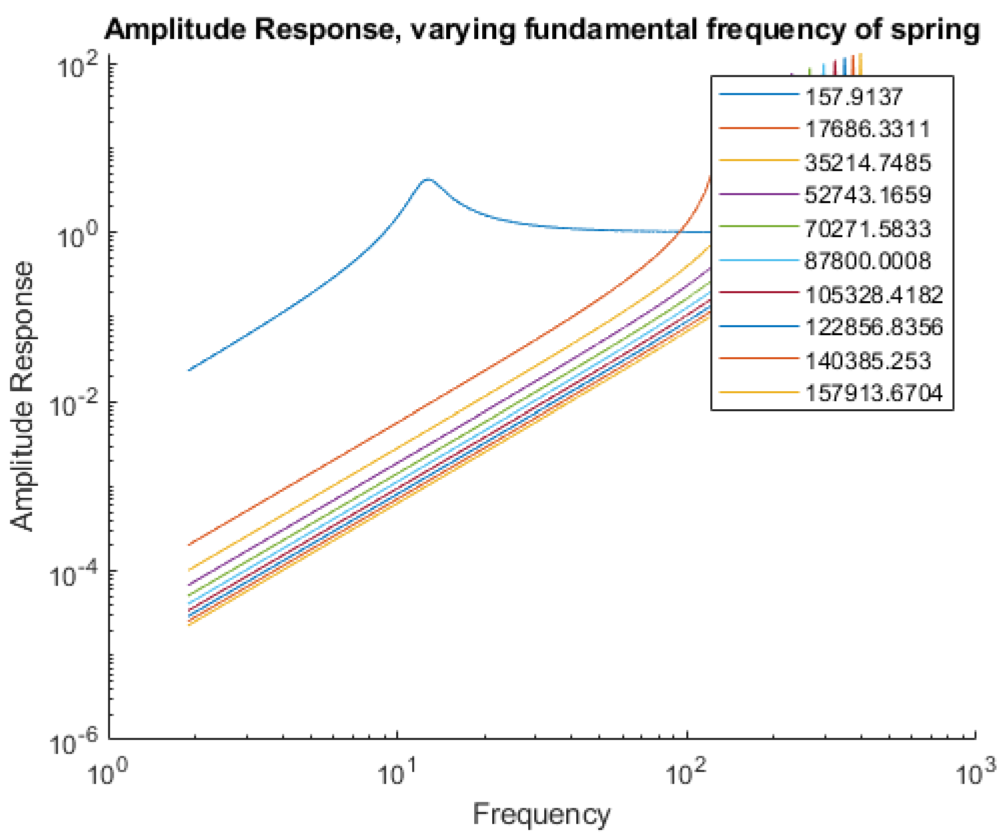
```
legend

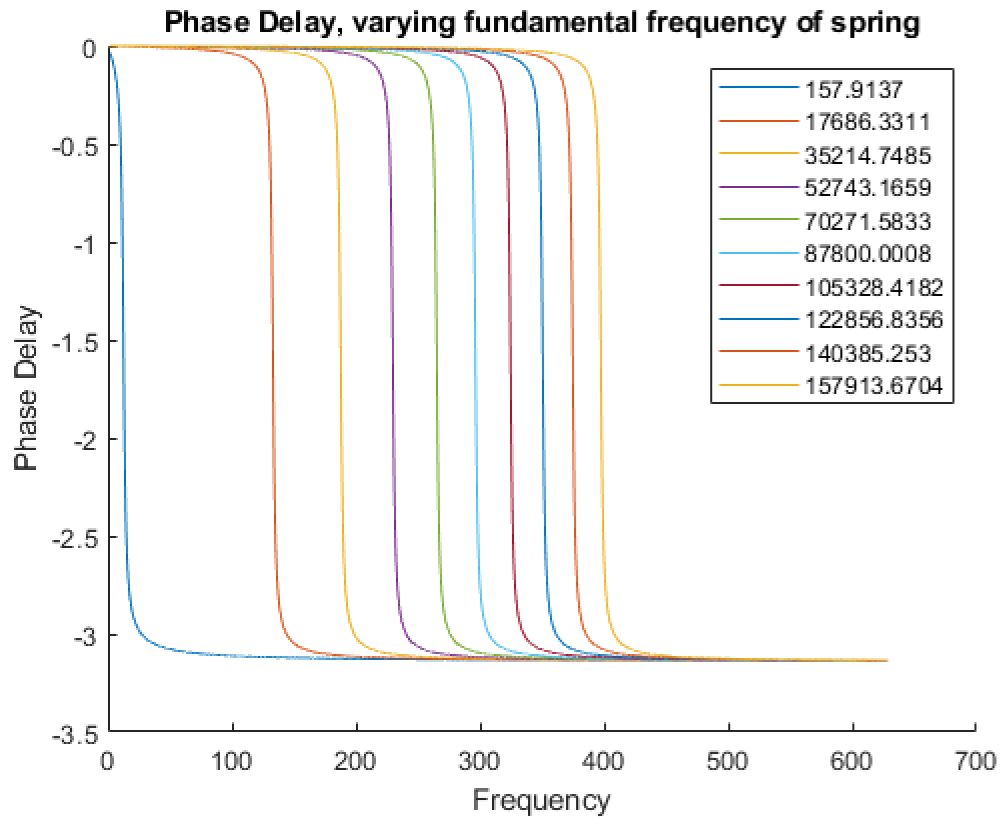
%q2.4
%As AK is increased, the resonant frequency decreases. This is different
from part 1, since the fundamental frequency w_0 does not need to change for
the resonant frequency to drop. The general shape of the amplitude response
function. however, is the same for both the inertial seismometer and the
force feedback seismometer.

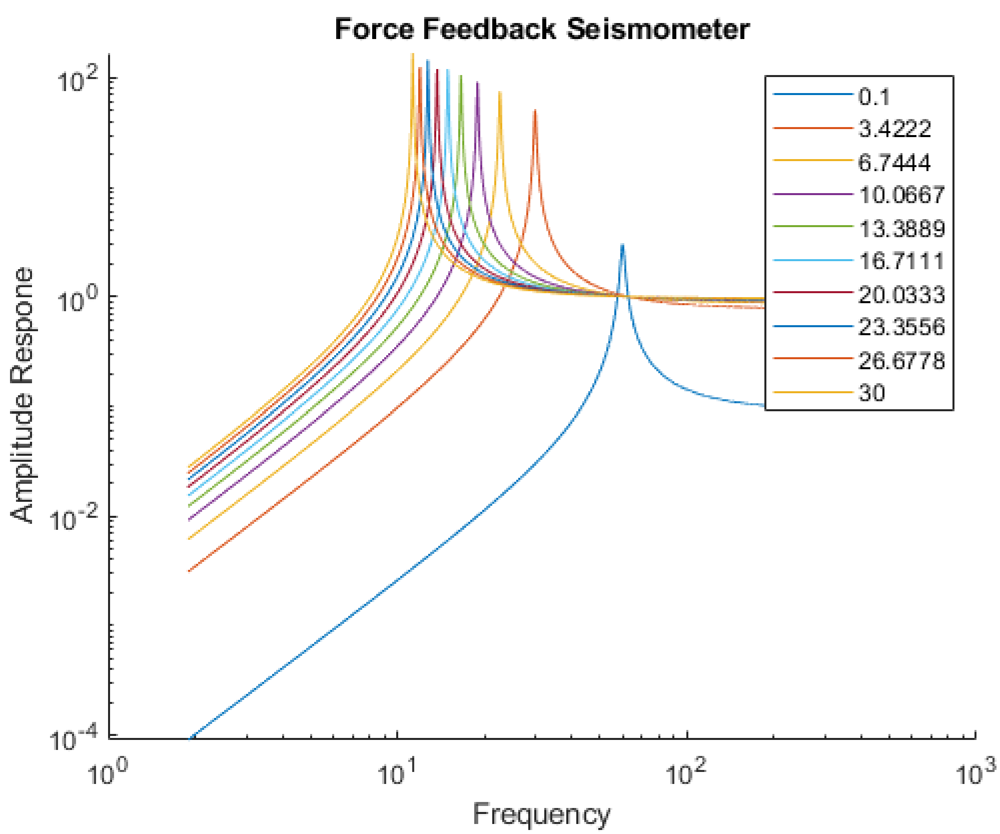
%q2.5
% A force feedback system gives you a way of easily tuning the fundamental
frequency of your seismometer without adjusting any of the mechanical
components of the system (e.g. spring constant and damping coefficient)
```




Phase Delay while varying damping coefficient
0.05
0.3
1
3
10
Phase Delay
Frequency
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
0
100
200
300
400
500
600
700


Amplitude Response, varying fundamental frequency of spring
157.9137
17686.3311
35214.7485
52743.1659
70271.5833
87800.0008
105328.4182
122856.8356
140385.253
157913.6704
Amplitude Response
Frequency
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{0}
10^{1}
10^{2}
10^{3}


Phase Delay, varying fundamental frequency of spring
Phase Delay
Frequency
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
0
100
200
300
400
500
600
700
157.9137
17686.3311
35214.7485
52743.1659
70271.5833
87800.0008
105328.4182
122856.8356
140385.253
157913.6704


Force Feedback Seismometer
Amplitude Respone
Frequency
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{0}
10^{1}
10^{2}
10^{3}
0.1
3.4222
6.7444
10.0667
13.3889
16.7111
20.0333
23.3556
26.6778
30

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