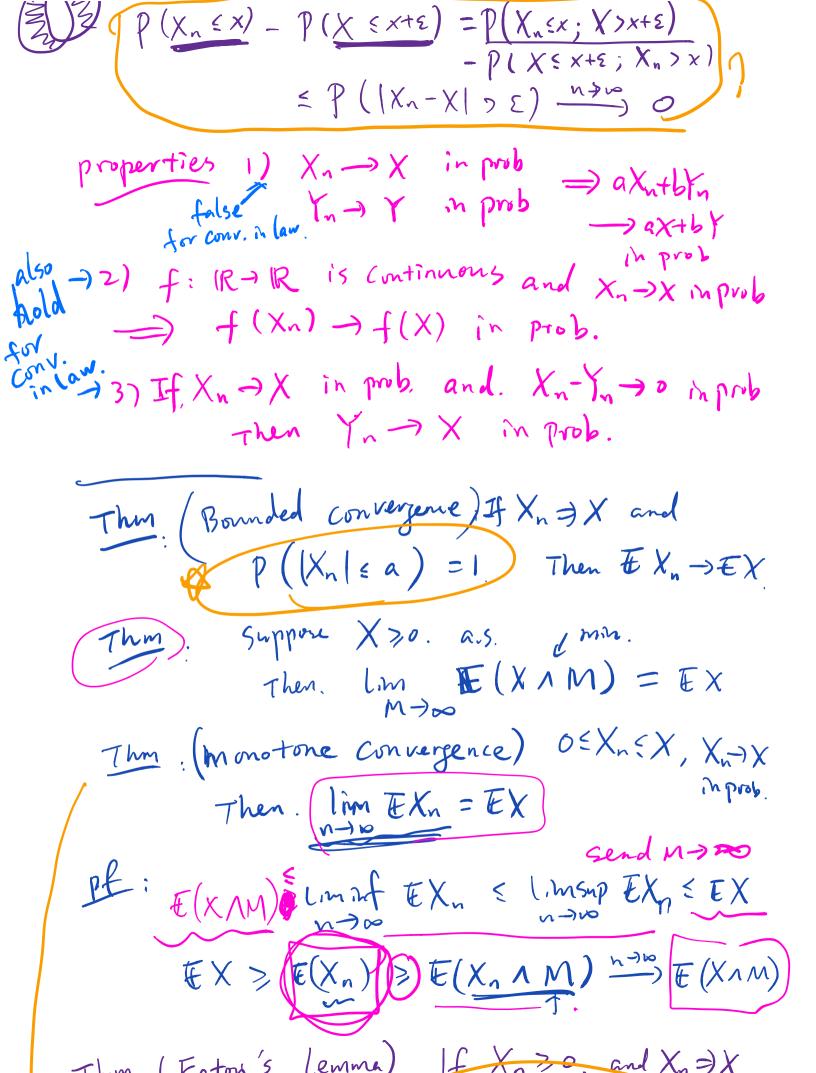


for non-discrete r.v. X. When $X \in \left(\frac{k}{n}, \frac{k+1}{n}\right)$, then $\frac{k}{n} = \frac{k}{n}$ Sit, $Y_n < X \le Z_n$ $|Z_n - Y_n| \le \frac{1}{n}$ $\mathbb{E} \times \stackrel{\text{def}}{=} \lim_{n \to \infty} \mathbb{E}(Y_n) = \lim_{n \to \infty} \mathbb{E}(2_n)$ When EX exists, we say X is integrable properties of E Linearity: E(aX+bY) = aE(X)+bE(Y) Monotonicity: X> (a.s) =) EX > EY. Convergence of random variables. (a) $X_n \rightarrow X$ a.s (b) (a) (a) (b) (b) (a) (b) (b(b) $X_n \rightarrow X$ in prob. (c) $\lim_{n \to \infty} P(|X_n - X| > \varepsilon) = 0$ (ε) $(x_n \rightarrow X)$ in distribution (in law) $\begin{cases} \lim_{n\to\infty} P(X = x) = P(X = x) \\ = F_{X}(x) \end{cases}$ for all x at which Fx is continuous. \overline{hm} , (a) = (b) = (c)proof. (b) =) (c) need to show $\begin{cases} \limsup_{x \to \infty} P(X_n \in X) \leq P(X_n \in X + \epsilon) \leq \\ \liminf_{x \to \infty} P(X_n \in X) > P(X_n \in X + \epsilon) \end{cases}$



ther liminf EXn > EX Thm (Dominated convergence) Assume Xn => X |Xn | < Yn. and EYn > EY < 00 E(Xn) -) E(X) liming EXn & langup EXn & Ex. EXZ E(Xn) Z E[Xn /M] => E[X/M] > [[X M] & land EX E[XXM] = EX