Math 733 - Fall 2020

Homework 3

Due: 10/11, 10pm

Zijie Zhang

October 9, 2020

1. (a) *Proof.*

$$X \sim B(n,p) \Rightarrow P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$Y \sim B(m,p) \Rightarrow P(X=k) = \binom{m}{k} p^k (1-p)^{m-k}$$

Then

$$\begin{aligned}
P(X+Y=k) &= \sum_{i=0}^{k} P(X=i, Y=k-i) \\
&= \sum_{i=0}^{k} P(X=i) \cdot P(Y=k-i) \\
&= \sum_{i=0}^{k} \binom{n}{i} p^i (1-p)^{n-i} \cdot \binom{m}{k-i} p^{k-i} (1-p)^{m-k+i} \\
&= p^k (1-p)^{m+n-k} \sum_{i=0}^{k} \binom{n}{i} \binom{m}{k-i} \\
&= \binom{n+m}{k} p^k (1-p)^{m+n-k}
\end{aligned}$$

Thus,

$$X+Y \sim B(n+m, p)$$

□

(b) *Proof.*

$$X \sim \text{Poisson}(\lambda) \Rightarrow P(X=k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

$$Y \sim \text{Poisson}(\mu) \Rightarrow P(Y=k) = \frac{\mu^k}{k!} e^{-\mu}$$

Then

$$
\begin{aligned}
P(X+Y=k) &= \sum_{i=0}^{k} P(X=i, Y=k-i) \\
&= \sum_{i=0}^{k} P(X=i) \cdot P(Y=k-i) \\
&= \sum_{i=0}^{k} \frac{\lambda^i}{i!} e^{-\lambda} \cdot \frac{\mu^{k-i}}{(k-i)!} e^{-\mu} \\
&= e^{-(\lambda+\mu)} \sum_{i=0}^{k} \frac{\lambda^i}{i!} \frac{\mu^{k-i}}{(k-i)!} \\
&= \frac{(\lambda+\mu)^k}{k!} e^{-(\lambda+\mu)}
\end{aligned}
$$

Thus,

$$X + Y \sim \text{Poisson}(\lambda + \mu)$$

□

2. *Proof.* □
3. *Proof.* □
4. *Proof.* □
5. *Proof.* □
6. *Proof.* □