CS 760: Machine Learning - Fall 2020

Homework 1: Review

Due : 09/24/2020

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Problem 1

Proof. By the definition, $\mathbb{R}^D$ is a subspace if for every $a,\ b \in \mathbb{R}$ and every $\boldsymbol{u}, \boldsymbol{v} \in \mathbb{R}^D$, $a\boldsymbol{u} + b\boldsymbol{v} \in \mathbb{R}^D$.
It is trivial. □

Problem 2

Proof.

(a) $\boldsymbol{x} = (-1, \cdots, -1) \in \mathbb{R}^D$. The element-wise square roots is $(i, \cdots, i) \notin \mathbb{R}^D$.
$\mathbb{R}^D$ is not closed under element-wise square roots.

(b) $\mathbb{C}^D$.

□

Problem 3

Proof. For every element $\boldsymbol{x},\ \boldsymbol{y} \in \mathbb{U}$. They can be represented by a linear combination of $\boldsymbol{u}_1, \cdots, \boldsymbol{u}_R$.

$$\boldsymbol{x} = \alpha_1 \boldsymbol{u}_1 + \cdots + \alpha_R \boldsymbol{u}_R$$

$$\boldsymbol{y} = \beta_1 \boldsymbol{u}_1 + \cdots + \beta_R \boldsymbol{u}_R$$

For every $a,\ b \in \mathbb{R}$,

$$a\boldsymbol{x} + b\boldsymbol{y} = (a\alpha_1 + b\beta_1)\boldsymbol{u}_1 + \cdots + (a\alpha_R + b\beta_R)\boldsymbol{u}_R$$

$a\boldsymbol{x} + b\boldsymbol{y} \in \mathbb{U}$, so $\mathbb{U}$ is a subspace. □

Problem 4

Proof.

(a) By the *Bayes rule*,

$$\begin{aligned}&\mathbb{P}(\text{Have diabetes}|\text{These genes inactive})\\ &\quad= \frac{\mathbb{P}(\text{These genes inactive}|\text{Have diabetes}) \cdot \mathbb{P}(\text{Have diabetes})}{\mathbb{P}(\text{These genes inactive})}\end{aligned}$$

(b) I need to know $\mathbb{P}(\text{These genes inactive})$.

(c) If the probability that these three genes inactive is very small, I should be concerned.

□

Problem 5

Proof.

$$\mathbb{P}(x|\theta) = \begin{cases} \theta e^{-\theta(x-t_0)} & , x \geqslant t_0 \\ 0 & , x \leqslant t_0 \end{cases}$$

- *t_0 is the minimal time delay*: $\mathbb{P}(x \leqslant t_0|\theta) = 0$
- *Larger delays are rarer than shorter ones*: For all $x,\ y \in [t_0, \infty],\ x < y$,

$$\begin{aligned} \frac{\mathbb{P}(y|\theta)}{\mathbb{P}(x|\theta)} &= \frac{\theta e^{-\theta(x-t_0)}}{\theta e^{-\theta(y-t_0)}} \\ &= e^{-\theta(x-y)} \\ &< 1 \end{aligned}$$

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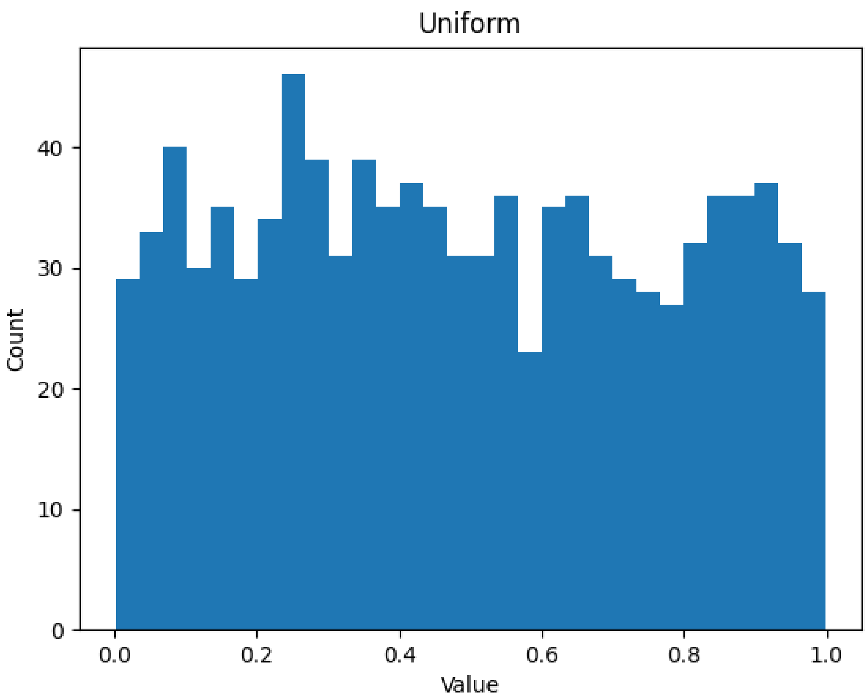
□

Problem 6

Proof.



(a) It doesn't look fairly uniform.

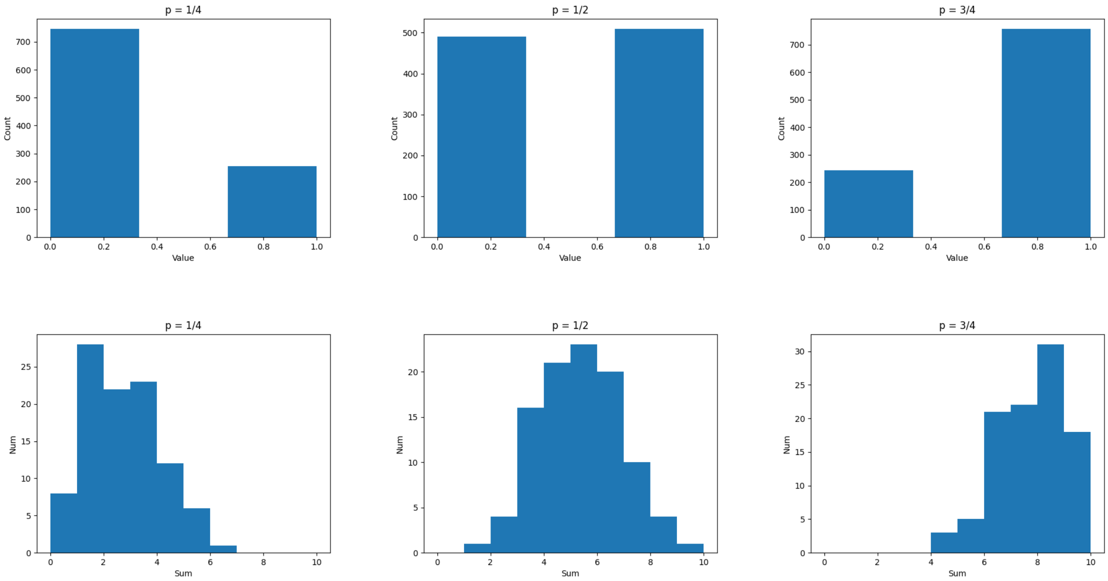
(b) $y_i \sim$ Bernoulli(p)

(c) Yes.

(d) $z_k \sim$ B(n,p)

(e) Yes.

□



Problem 7

Proof. □