CS 760: Machine Learning - Fall 2020

Homework 1: Review

Due : 09/24/2020

Zijie Zhang

September 21, 2020

Problem 1

Proof. By the definition, $\mathbb{R}^D$ is a subspace if for every $a,\ b \in \mathbb{R}$ and every $\boldsymbol{u}, \boldsymbol{v} \in \mathbb{R}^D$, $a\boldsymbol{u} + b\boldsymbol{v} \in \mathbb{R}^D$.
It is trivial. $\square$

Problem 2

Proof.

(a) $\boldsymbol{x} = (-1, \cdots, -1) \in \mathbb{R}^D$. The element-wise square roots is $(i, \cdots, i) \notin \mathbb{R}^D$.
$\mathbb{R}^D$ is not closed under element-wise square roots.

(b) $\mathbb{C}^D$.

$\square$

Problem 3

Proof. For every element $\boldsymbol{x},\ \boldsymbol{y} \in \mathbb{U}$. They can be represented by a linear combination of $\boldsymbol{u}_1, \cdots, \boldsymbol{u}_R$.

$$\boldsymbol{x} = \alpha_1 \boldsymbol{u}_1 + \cdots + \alpha_R \boldsymbol{u}_R$$

$$\boldsymbol{y} = \beta_1 \boldsymbol{u}_1 + \cdots + \beta_R \boldsymbol{u}_R$$

For every $a,\ b \in \mathbb{R}$,

$$a\boldsymbol{x} + b\boldsymbol{y} = (a\alpha_1 + b\beta_1)\boldsymbol{u}_1 + \cdots + (a\alpha_R + b\beta_R)\boldsymbol{u}_R$$

$a\boldsymbol{x} + b\boldsymbol{y} \in \mathbb{U}$, so $\mathbb{U}$ is a subspace. $\square$

Problem 4

Proof. $\square$

Problem 5

Proof. $\square$

Problem 6

Proof. $\square$

Problem 7

Proof. □