

Theory of Probability I

COURSE INFORMATION

Theory of Probability I

MATH 733 001 (3 Credits) 2020 Fall (1212) [1212]

Description

An introduction to measure theoretic probability and stochastic processes. Topics include foundations, independence, zero-one laws, laws of large numbers, convergence in distribution, characteristic functions, central limit theorems, random walks, conditional expectations. Enroll Info: Familiarity with basic measure theory (e.g. MATH 629 or 721) or concurrent registration in MATH 721 is strongly recommended.

Prerequisite(s)

Graduate/professional standing or member of the Pre-Masters Mathematics (Visiting International) Program

Instruction Mode

Online Only

Section Level Com B

False

Department: Mathematics **College:** Letters and Science

Canvas Course URL

https://canvas.wisc.edu/



2020 Fall (1212) [1212]

Term Start Date: Wednesday, 2-Sep-2020 Term End Date: Monday, 1-Feb-2021

Location and Schedule: ONLINE TR 1:00 PM-2:15 PM

CRN: 600011797

How the Credit Hours are Met

This class meets for two 75-minute class periods each week over the semester (remotely) and carries the expectation that students will work on course learning activities (reading, writing, problem sets, studying, etc) for about 3 hours out of classroom for every class period. The syllabus includes more information about meeting times and expectations for student work.

INSTRUCTORS AND TEACHING ASSISTANTS

Instructor



Instructor Availability

Office hours: Tu 4-5pm, F 3-4pm, or by appointment. Office hours will be held remotely via Google Meet, BBCU, or some other video-conferencing tool.

GRADING AND COURSE MATERIALS

Course Website, Learning Management System and Instructional Tools

- The course website provides a brief overview of the course, but all course material will be posted on Canvas.
- The live lectures will be broadcasted via BBCU. We might switch to Zoom when that tool becomes approved and implemented by the university.
- All live lectures will be recorded in BBCU, and the recorded lectures will be made available to students in Canvas.
- All homework assignments will have to be submitted via Canvas. You will either need to scan your work, or submit a version prepared with LaTeX.
- We will use Piazza for online discussion. The system is catered to getting you help fast and efficiently from classmates and myself. Discussing math via email is somewhat cumbersome, so I encourage you to post your questions on Piazza, instead of emailing them.



State, explain, and apply the axioms, principal results, definitions, and theorems of modern measure theoretic probability theory.

S32850



Analyze asymptotic problems in probability theory using the law of large numbers, the central limit theorem, and the Borel-Cantelli lemmas.

S32851



Master the mathematically rigorous definitions of conditional probabilities and conditional expectations, learn various ways for computing them, and apply them in he theory of martingales.

S32852

Grading

Course grades will be based on biweekly home work assignments (25%), class participation (15%), a midterm exam (30%) and the final exam (30%).

Required Textbook, Software, & Other Course Materials

We will use the textbook Richard Durrett: Probability: Theory and Examples, 5th edition, 2019

(Note that the author posted a digital copy of a version of the textbook on his webpage.)

There are several good textbooks on probability and it might help to have a look around. Here is a list of textbooks that could be used for extra reading:

- Olav Kallenberg: Foundations of Modern Probability. 2nd edition, Springer, 2002
- William Feller. An introduction to probability theory and its applications. Vol. I. Third edition. John Wiley and Sons Inc., New York, 1968.
- David Williams. Probability with martingales. Cambridge Mathematical Textbooks. Cambridge University Press, Cambridge, 1991.
- Patrick Billingsley. Probability and measure. Wiley Series in Probability and Mathematical Statistics. John Wiley & Sons
 Inc., New York, 1995.

Students should consult the technology guidelines and recommendation provided by UW-Madison.

EXAMS, QUIZZES, PAPERS & OTHER MAJOR GRADED WORK

Homework & Other Assignments

- Assignments will be posted and collected on the Canvas page.
- No late assignments will be accepted.
- **Observe rules of academic integrity.** Handing in plagiarized work, whether copied from a fellow student or off the web, is not acceptable. Plagiarism cases will lead to sanctions.
- Neatness and clarity are essential. Write one problem per page except in cases of very short problems. You are strongly
 encouraged to use LaTeX (or Overleaf) to typeset your solutions. Handwritten solutions will need to be scanned, it is your
 responsibility to check that the scanned file is readable, and your work is legible.
- It is not trivial to learn to write solutions. You have to write enough to show that you understand the flow of ideas and that you are not jumping to unjustified conclusions, but not too much to get lost in details. If you are unsure of the appropriate level of detail to include, you can separate some of the technical details as "Lemmas" and put them at the end of the solution. A good rule of thumb is if the grader needs to pick up a pencil to check your assertion, you should have proved it. The grader can deduct points in such cases.
- You can use basic facts from analysis and measure theory in your homework, and the theorems we cover in class without
 reproving them. If you find a helpful theorem or passage in another book, do not copy the passage but use the idea to
 write up your own solution. If you do use other literature for help, cite your sources properly. However, it is better to attack
 the problems with your own resources instead of searching the literature or the internet. The purpose of the homework is
 to strengthen your problem solving skills, not literature search skills.
- It is valuable to discuss ideas for homework problems with other students. But it is not acceptable to write solutions together or to copy another person's solution. In the end you have to hand in your own **personal** work. Similarly, finding solutions on the internet is tantamount to cheating. It is the same as copying someone else's solution.

Exams, Quizzes, Papers & Other Major Graded Work

Participation grade

One of the most challenging aspects of remote instruction is to have appropriate interaction between the students and between the students and the instructor. I included a participation component in the course grade to encourage you generate interactions.

You can obtain full credit for your participation grade component by engaging in sufficiently many of the following activities:

- Resolving another student's math question on Piazza.
- Posting the correct solution of a homework problem on Piazza after the submission deadline.
- Having a short one-on-one meeting with me to present the solution of (non-homework) problem.
- Answering or asking questions during the live lectures.
- Engaging in a meaningful discussion on Piazza on the class material.

Exams

We will have two proctored online exams: a midterm exam and a final exam. These will be open book exams,

but no outside collaboration (other students or sources from the internet) will be allowed. Most likely we will use Honorlock for the online proctoring. Please consult this page for more information on Honorlock, in particular check the minimum system requirements. Please reach out to me as soon as possible if you do not think that you can use Honorlock, we will figure out an alternate accommodation for you.

These are the scheduled times for the exams:

- Midterm 1: Wednesday, October 21, 8pm-10pm.
- Final Exam: Tuesday, December 15, 12:25pm-2:25pm.

If you are not expected to be available for one of these exams then please get in touch with me as soon as possible.

Honorlock

Honorlock is the campus-supported proctoring tool. For FAQ's about Honorlock

Additional resources about Honorlock include:

- See suggested syllabus language for use of Honorlock
- For Instructors: Honorlock Quick Guide and UW-Madison's Knowledge Base documentation
- For Students: Getting Started for Students

Privacy of Student Information and Digital Proctoring Statement

The privacy and security of faculty, staff and students' personal information is a top priority for UW-Madison. The university carefully reviews and vets all campus-supported teaching and learning tools, including proctoring tools and takes necessary steps to ensure that tool providers prioritize proper handling of sensitive data in alignment with FERPA, industry standards and best practices.

Under the Family Educational Rights and Privacy Act (FERPA – which protects the privacy of student education records), student consent is not required for the university to share with Honorlock those student education records necessary for carrying out the proctoring service. 34 CFR 99.31(a)(1)(i)(B). FERPA specifically allows universities to treat vendors as school officials and to share student education records with them where they perform services for the university and are subject to FERPA requirements governing the use and redisclosure of personally identifiable information from education records. Honorlock is FERPA compliant and is bound by the terms of its agreement with the university to comply with FERPA's restrictions on the use of student education records.

Privacy of Student Records and the Usage of Audio Recorded Lectures

See information about privacy of student records and the usage of audio-recorded lectures.

Usage of Audio Recorded Lectures Statement

Lecture materials and recordings for [insert class name] are protected intellectual property at UW- Madison. Students in this course may use the materials and recordings for their personal use related to participation in this class. Students may also take notes solely for their personal use. If a lecture is not already recorded, you are not authorized to record my lectures without my permission unless you are considered by the university to be a qualified student with a disability requiring accommodation. [Regent Policy Document 4-1] Students may not copy or have lecture materials and recordings outside of class, including posting on internet sites or selling to commercial entities. Students are also prohibited from providing or selling their personal notes to anyone else or being paid for taking notes by any person or commercial firm without the instructor's express written permission. Unauthorized use of these copyrighted lecture materials and recordings constitutes copyright infringement and may be addressed under the university's policies, UWS Chapters 14 and 17, governing student academic and non-academic misconduct

OTHER COURSE INFORMATION

Other Course Information

Course description:

This is the first semester of a two-semester graduate-level introduction to probability theory and it also serves as a stand-alone introduction to the subject. The course will focus on the basics of probability and cover at least the following topics: foundations (probability spaces and existence of processes), independence, zero-one laws, laws of large numbers, weak convergence and the central limit theorem, conditional expectations and their properties, and martingales (convergence theorem and basic properties).

Prerequisites:

Measure theory is a basic tool for this course. A suitable background can be obtained from Math 629 or Math 721. Chapter 1 in Durrett covers the measure theory needed. We will very briefly review some measure theory at the beginning of the semester, but it will be expected that all students in the course are familiar with the basics. Prior exposure to elementary probability theory could be useful, but it is not required.

Course content:

We cover selected portions of Chapters 1-4 of Durrett. This is a rough course outline:

Weeks 1-2: Foundations, properties of probability spaces

Weeks 3-5: Independence, 0-1 laws, strong law of large numbers

Weeks 6-10: Characteristic functions, weak convergence and the central limit theorem

Weeks 11-15: Conditional expectation, Martingales

The course continues in the spring semester as Math 734 covering topics such as Markov chains, stationary processes, ergodic theory, and Brownian motion.

If you would like to learn more probability...

Check out the Probability Seminar, the Graduate Probability Seminar and the Statistics Seminar for talks that might interest you. Have a look at the wiki page of our probability group to learn more about the probabilists at UW-Madison. Consider attending the 2020 Midwest Probability Colloquium (this will be held online this year).

ADDITIONAL COURSE INFORMATION AND ACADEMIC POLICIES



OTHER IN-PERSON INSTRUCTIONAL REQUIREMENTS

Practice physical distancing (staying at least six feet apart from others), in both indoor and outdoor spaces, including when entering and exiting building and instructional spaces.

- Monitor symptoms using the COVID-19 Symptom Tracker daily and, if symptoms exist, stay home and
 immediately get tested. Free testing is available to the entire campus community including students. Find more
 information on testing including on-campus testing locations, getting test results and what to do if you test
 positive for COVID-19.
- Follow specific guidance on classroom seating and furniture use. "Sit Here Signs" will be placed in classrooms
 to indicated where students should sit, as well as floor decals to indicate where furniture should be placed and
 remain.
- Limit the sharing of materials (papers, books, writing utensils, calculators, etc.) with others in class. Any materials brought to class must be taken with you when leaving the classroom.
- Food and beverages are not allowed in instructional spaces. For students who may need access to food or beverages during class (e.g., medical condition, other accommodation or circumstance) may do so while wearing face coverings. If this will be a reoccurring siutation, students should discuss their on-going need with their instructor.
- Carefully observe and follow health and safety signs posted inside and outside the classroom.
- Course start and stop times are not staggered. At their discretion, instructors may start or end class a few
 minutes off schedule to avoid congestion in the halls.

Additionally, students should:

 Clean desks and seats before and after class with the provided classroom supplies and wipe off hands with disinfectant wipes. The provided clearning supplies will be safe for skin contact; gloves are not needed. See guidance for cleaning classrooms.

- Exit the classroom as quickly as possible to allow the next section to transition in safely
- Be aware of and sensitive to others around you, particularly those who may be struggling or having difficulties.



HOW TO SUCCEED IN THIS COURSE

Provide information on how students can best succeed in the course. For instance, instructors might include suggestions about completing assignments or studying for exams. Instructors might also include resource links to other campus services such as:

- University Health Services
- Undergraduate Academic Advising and Career Services
- Office of the Registrar
- Office of Student Financial Aid
- Dean of Students Office



STUDENTS' RULES, RIGHTS & RESPONSIBILITIES

During the global COVID-10 pandemic, we must prioritize our collective health and safety to keep ourselves, our campus, and our community safe. As a university community, we must work together to prevent the spread of the virus and to promote the collective health and welfare of our campus and surrounding community. Rights & Responsibilities



UW-MADISON BADGER PLEDGE

Badger Pledge



UW-MADISON FACE COVERING GUIDELINES

UW-Madison face covering guidelines. While on campus all employees and students are required to wear appropriate and properly fitting face coverings while present in any campus building unless working alone in a laboratory or office space.

Face Coverings During In-person Instruction Statement (COVID-19)

Individuals are expected to wear a face covering while inside any university building. Face coverings must be worn correctly (i.e., covering both your mouth and nose) in the building if you are attending class in person. If any student is unable to wear a face-covering, an accommodation may be provided due to disability, medical condition, or other legitimate reason.

Students with disabilities or medical conditions who are unable to wear a face covering should contact the McBurney Disability Resource Center or their Access Consultant if they are already affiliated. Students requesting an accommodation unrelated to disability or medical condition, should contact the Dean of Students Office.

Students who choose not to wear a face covering may not attend in-person classes, unless they are approved for an accommodation or exemption. All other students not wearing a face covering will be asked to put one on or leave the classroom. Students who refuse to wear face coverings appropriately or adhere to other stated requirements will be reported to the Office of Student Conduct and Community Standards and will not be allowed to return to the classroom until they agree to comply with the face covering policy. An instructor may cancel or suspend a course inperson meeting if a person is in the classroom without an approved face covering in position over their nose and mouth and refuses to immediately comply.



QUARANTINE OR ISOLATION DUE TO COVID-19

Student should continually monitor themselves for COVID-19 symptoms and get tested for the virus if they have symptoms or have been in close contact with someone with COVID-19. Student should reach out to instructors as soon as possible if they become ill or need to isolate or quarantine, in order to make alternate plans for how to proceed with the course. Students are strongly encouraged to communicate with their instructor concerning their illness and the anticipated extent of their absence from the course (either in-person or remote). The instructor will work with the student to provide alternative ways to complete the course work.



COURSE EVALUATIONS

Indicate how students can evaluate the course. For example:

Students will be provided with an opportunity to evaluate this course and your learning experience. Student participation is an integral component of this course, and your feedback is important to me. I strongly encourage you to participate in the course evaluation.

Digital Course Evaluation (AEFIS)

For instructors using the campus digital course evaluation survey tool, AEFIS.

UW-Madison now uses an online course evaluation survey tool, AEFIS. In most instances, you will receive an official email two weeks prior to the end of the semester when your course evaluation is available. You will receive a link to log into the course evaluation with your NetID where you can

complete the evaluation and submit it, anonymously. Your participation is an integral component of this course, and your feedback is important to me. I strongly encourage you to participate in the course evaluation.



ACADEMIC CALENDAR & RELIGIOUS OBSERVANCES

See: https://secfac.wisc.edu/academic-calendar/#religious-observances



ACADEMIC INTEGRITY STATEMENT

Instructors should discuss academic integrity with students early and often. For suggested ways to engage students in these discussions, see the College of Letters and Science Remote Teaching Toolkit.

By virtue of enrollment, each student agrees to uphold the high academic standards of the University of Wisconsin-Madison; academic misconduct is behavior that negatively impacts the integrity of the institution. Cheating, fabrication, plagiarism, unauthorized collaboration, and helping others commit these previously listed acts are examples of misconduct which may result in disciplinary action. Examples of disciplinary action include, but is not limited to, failure on the assignment/course, written reprimand, disciplinary probation, suspension, or expulsion.



ACCOMMODATIONS FOR STUDENTS WITH DISABILITIES

STATEMENT

The University of Wisconsin-Madison supports the right of all enrolled students to a full and equal educational opportunity. The Americans with Disabilities Act (ADA), Wisconsin State Statute (36.12), and UW-Madison policy (Faculty Document 1071) require that students with disabilities be reasonably accommodated in instruction and campus life. Reasonable accommodations for students with disabilities is a shared faculty and student responsibility. Students are expected to inform faculty [me] of their need for instructional accommodations by the end of the third week of the semester, or as soon as possible after a disability has been incurred or recognized. Faculty [I], will work either directly with the student [you] or in coordination with the McBurney Center to identify and provide reasonable instructional accommodations. Disability information, including instructional accommodations as part of a student's educational record, is confidential and protected under FERPA. (See: McBurney Disability Resource Center)



DIVERSITY & INCLUSION STATEMENT

https://diversity.wisc.edu/is a source of strength, creativity, and innovation for UW-Madison. We value the contributions of each person and respect the profound ways their identity, culture, background, experience, status, abilities, and opinion enrich the university community. We commit ourselves to the pursuit of excellence in teaching, research, outreach, and diversity as inextricably linked goals.

The University of Wisconsin-Madison fulfills its public mission by creating a welcoming and inclusive community for people from every background – people who as students, faculty, and staff serve Wisconsin and the world.