

CS 760: Machine Learning - Fall 2020

Homework 1: Review

Due : 09/24/2020

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Problem 1

Proof. By the definition, \mathbb{R}^D is a subspace if for every $a, b \in \mathbb{R}$ and every $\mathbf{u}, \mathbf{v} \in \mathbb{R}^D$, $a\mathbf{u} + b\mathbf{v} \in \mathbb{R}^D$. It is trivial. □

Problem 2

Proof.

- (a) $\mathbf{x} = (-1, \dots, -1) \in \mathbb{R}^D$. The element-wise square roots is $(i, \dots, i) \notin \mathbb{R}^D$.
 \mathbb{R}^D is not closed under element-wise square roots.
- (b) \mathbb{C}^D . □

Problem 3

Proof. For every element $\mathbf{x}, \mathbf{y} \in \mathbb{U}$. They can be represented by a linear combination of $\mathbf{u}_1, \dots, \mathbf{u}_R$.

$$\mathbf{x} = \alpha_1 \mathbf{u}_1 + \dots + \alpha_R \mathbf{u}_R$$

$$\mathbf{y} = \beta_1 \mathbf{u}_1 + \dots + \beta_R \mathbf{u}_R$$

For every $a, b \in \mathbb{R}$,

$$a\mathbf{x} + b\mathbf{y} = (a\alpha_1 + b\beta_1)\mathbf{u}_1 + \dots + (a\alpha_R + b\beta_R)\mathbf{u}_R$$

$a\mathbf{x} + b\mathbf{y} \in \mathbb{U}$, so \mathbb{U} is a subspace. □

Problem 4

Proof.

- (a) By the *Bayes rule*,

$$\begin{aligned} & \mathbb{P}(\text{Have diabetes} | \text{These genes inactive}) \\ &= \frac{\mathbb{P}(\text{These genes inactive} | \text{Have diabetes}) \cdot \mathbb{P}(\text{Have diabetes})}{\mathbb{P}(\text{These genes inactive})} \end{aligned}$$

- (b) I need to know $\mathbb{P}(\text{These genes inactive})$.
- (c) If the probability that these three genes inactive is very small, I should be concerned. □

Problem 5

Proof.

$$\mathbb{P}(x|\theta) = \begin{cases} \theta e^{-\theta(x-t_0)} & , x \geq t_0 \\ 0 & , x \leq t_0 \end{cases}$$

- t_0 is the minimal time delay: $\mathbb{P}(x \leq t_0|\theta) = 0$
- Larger delays are rarer than shorter ones: For all $x, y \in [t_0, \infty]$, $x < y$,

$$\begin{aligned} \frac{\mathbb{P}(y|\theta)}{\mathbb{P}(x|\theta)} &= \frac{\theta e^{-\theta(x-t_0)}}{\theta e^{-\theta(y-t_0)}} \\ &= e^{-\theta(x-y)} \\ &< 1 \end{aligned}$$

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□

Problem 6

Proof.

□

Problem 7

Proof.

□