# Lecture 11: Least squares, OD I

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# Outline

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1. Overdetermined system
   - Overdetermined system and the least square notion
   - Abstraction of least squares

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# Problem: Overdetermined systems

**Definition: Overdetermined (OD) systems**

We are given a system

$$Ax = b.$$

It is overdetermined if it does not have an exact solution

# Problem: Overdetermined systems

**Definition: OD Least Squares**

We are given an OD system

$$Ax = b.$$

The vector $x^* \in R^m$ is the Least Square Solution to the system if

$$||Ax^* - b||_2 \leq ||Ax - b||_2, \quad \forall x \in R^m.$$

# The problem restated as an approximation problem

- We work in some vector space $V$.
  In our case $V = R^m$.
- We have $v \in V$ that we need to approximate.
  In our case, $v = b$, the right hand side.
- $W$ is some subspace of $V$.
  In our case, $W$ is the range of the matrix $A$. Since we know that our equation has no solutions, it must be that $W$ is a proper subspace of $V$.
- We seek $w^* \in W$ such that

$$||w^* - v||_2 \leq ||w - v||_2, \quad \forall w \in W.$$

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## The main characterization theorem

Let $w_0 \in W$ be such

$$v - w_0 \perp W.$$