CS513, Spring 21 Prof. Ron

HW #8, Factor=.75

Due March 17, 2021

(1)

In this question, you try to find a low degree polynomial fit to some given data.

(i) Write a Matlab code that, given a row vector T of size m, and a number k, generates a matrix A as follows:

$$A(i,j) = T(i)^{j-1}, \quad i = 1:m, \quad j = 1:k+1.$$

(you lose 5 points (out of 75) if you use two loops for generating A, and you receive 5 extra points if you use no loops at all for that.)

(ii) Write now a code that: (a) given a function f defined on an interval [a,b], it evaluates the function at m equally spaced points T on that interval (use Matlab's linspace command), and then tries to find a polynomial of degree k

$$p(x) = \sum_{j=0}^{k} a(j)x^{j}$$

that approximates f best, in the sense that the least squares error

$$\sum_{t \in T} (p(t) - f(t))^2$$

is minimized. You lose 10 points if you use Matlab's least square solver $A \setminus b$ here.

(iii) Now run your code on the functions $\tan(x/4)$ and |x-3| and another function of your own choice (take [a,b] = [0,4]). Take small values of k (e.g., 4 or 6), and reasonable values of m (e.g., 20 or 40). In order to assess the quality of your fit, you may plot the graph of the original function and the graph of its polynomial fit in the same window.

Turn in your code, your output (or a sample of it, if you ran many examples), and any conclusions you could have made.

You will get an extra 5 points if you use here code from recent (another) homework in this assignment and compute error norms in your code.