University of Wisconsin-Madison
ISyE/Math/CS/Stat 525: Linear Optimization
Spring 2021 Syllabus

1 Course Name and Number

Course name	Linear Optimization
Course number	ISyE/Math/CS/Stat 525

2 Credits and contact hours

Credits	3
Contact hours	2.5 hours/week

3 Canvas Course URL

https://canvas.wisc.edu/courses/230426

4 Course Designations and Attributes

Breadth	Natural Science
Level	Advanced
L&S Credit	Counts as Liberal Arts and Science credit in L&S

5 Meeting Time and Location

None

6 Required or elective

Elective

7 Instructional Mode

All online

8 UW-Madison Credit Hour Policy

Traditional Carnegie Definition

9 Instructors And Teaching Assistants

9.1 Instructor Title and Name	Prof. Alberto Del Pia
9.2 Office hours	Tuesday 9:00-10:00am and Thursday 4:00–5:00pm in Zoom
9.3 Instructor Email	delpia@wisc.edu
9.4 Teaching Assistant	Bainian Hao
9.5 TA Office Hours	Wednesday 10–11am and Friday 4:00–5:00pm in Zoom
9.6 TA Email	bhao8@wisc.edu

Prof. Del Pia's office hours are reserved for discussions on the material presented in the lectures and related theoretical questions. On the other hand, in the office hours held by the TA students can ask questions regarding exercises and assignments. All office hours take place in the Zoom meeting with ID 946 4024 8032 and Passcode

860910. Instructor and TA Office Hours take place from the day *instruction begins* (Jan 25, 2021) to the *last class day* (Apr 30, 2021), according to the UW-Madison Academic Calendar.

10 Official course description

Introduces optimization problems whose constraints are expressed by linear inequalities. Develops geometric and algebraic insights into the structure of the problem, with an emphasis on formal proofs. Presents the theory behind the simplex method, the main algorithm used to solve linear optimization problems. Explores duality theory and theorems of the alternatives.

11 Requisites

MATH 320, 340, 341, 375, or 443 or graduate/professional standing or member of the Pre-Masters Mathematics (Visiting International) Program

12 Learning Outcomes

12.1 Course Learning Outcomes

After taking this course, all students should achieve the following outcomes:

1. Use linear programming to formulate real world decision problems.
2. Understand the theory of linear programming, including geometric properties of polyhedra and duality.
3. Apply algorithms to solve linear programming problems, and demonstrate their correctness.

Moreover, graduate students should achieve the additional outcome:

4. Combine different proving techniques explored in class in an original way to show new results.

12.2 ABET Student Outcomes

The following ISyE ABET student outcomes are addressed in this course:

(a) Apply knowledge of mathematics, science, and engineering.
(b) Design and conduct experiments, as well as to analyze and interpret data.
(c) Design a system, component, or process to meet desired needs within realistic constraints such as economic, environmental, social, political, ethical, health and safety, manufacturability, and sustainability.
(d) Function on multidisciplinary teams.
(e) Identify, formulate, and solve engineering problems.
(h) The broad education necessary to understand the impact of engineering solutions in a global, economic, environmental, and societal context.
(k) An ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.

13 Brief List Of Topics To Be Covered

Following is a schedule of the topics of the course. This is subject to change.

Topic	Readings
Course Introduction	
Introduction	Ch. 1: Sections 1.1–1.4
The geometry of linear programming	Ch. 2: Sections 2.1–2.8
The simplex method – part 1	Ch. 3: Sections 3.1–3.3 and 1.6
The simplex method – part 2	Ch. 3: Sections 3.4–3.7
Duality theory	Ch. 4: Sections 4.1–4.6
Network flow problems	Ch. 7: Sections 7.1–7.3
Complexity of linear programming and the ellipsoid method	Ch. 8: Sections 8.1–8.5

14 Discussion Sessions

This course does not offer discussion sections.

15 Laboratory Sessions

This course does not offer laboratory sections.

16 Required Textbook, Software & Other Course Materials

There is no required textbook. The recommended textbook is *Introduction to Linear Optimization* by Dimitris Bertsimas and John N. Tsitsiklis, 1997. ISBN: 978-1-886529-19-9

Grading

The course grade will be based on a weighted average of these components.

- 40% Homework.
- 30% Midterm exam.
- 30% Final exam.

In order to schedule the Midterm exam please fill out the poll http://whenisgood.net/cr3rgm5 by January 30. The final exam will take place in the course slot in final week, which is May 6, 5:05pm–7:05pm. Both exams will be online. Students who have a conflict with either of the exam times should notify me in the first week of class.

The midterm exam covers Ch. 1, Ch. 1, and Ch. 3 – part 1. The final exam is cumulative, but mainly focused on the second part of the course (Ch. 3 – part 2, Ch. 4, and Ch. 7).

Undergraduate students will be graded on a curve consistent with standards in existing courses in ISyE, Math, CS, and Stat. Specifically:

- A (approximately 25%): Requires consistent high performance across homeworks and exams, demonstrating good grasp of modeling and algorithmic concepts.
- AB (approximately 15%): Requires solid performance on most class work, not quite at the standard of A.
- B (approximately 25%): Requires moderate performance on most classwork, demonstrating a grasp of most concepts in the curriculum.
- BC (approximately 10%): Requires acceptable performance on most classwork.
- C (approximately 15%): Requires acceptable performance on a majority of classwork, with some significant gaps in understanding of a minority of concepts and possible several missed homeworks.
- D (approximately 5%): Requires marginally satisfactory performance.
- F (typically less than 5%): Unacceptable performance.

Graduate students will be graded separately.

Homework

Homework will be given roughly every two weeks, and is a required part of the course. The homework assignments will be posted on Canvas and should be submitted electronically in pdf format in Canvas. Due to the large size of this class, it is not possible to grade the assignments in full. Therefore, a random subset of exercises will be selected to be graded. Examples of correct solutions will be provided, and especially for exercises that aren't graded, it is important that the students self-check their solutions and make sure they understand the correct solutions. Late submission policy: 20% of total points will be deducted per hour.

Students are encouraged to work in groups of two on the assignments. Groups must work independently of each other, may not share answers with each other, and solutions must not be copied from the internet or other sources. If improper collaboration on an assignment is detected, *all groups* involved will automatically receive a 0 on the assignment in question. In any assignment, students must properly give credit to any outside resources they use (such as books, papers, etc.). Please note that as the exams must be done independently, it is in each student's best interest to take an active role in all assignment's exercises.

Some questions in the assignments are specific to undergraduate and graduate students. The exercises for undergraduate students are designed to test basic understanding of the topic. The exercises for graduate students are more complex and require the students to combine different proving techniques explored in class in an original way.

The table below contains all the homework due dates.

Homework #	Topic	Available on	Due on
1	Ch. 1	week 2 (Feb. 1)	end of week 3 (Feb. 13)
2	Ch. 2	week 4 (Feb. 15)	end of week 5 (Feb. 27)
3	Ch. 3 part 1	week 6 (Mar. 1)	end of week 8 (Mar. 20)
4	Ch. 3 part 2	week 9 (Mar. 22)	end of week 10 (Apr. 3)
5	Ch. 4	week 11 (Apr. 5)	end of week 12 (Apr. 17)
6	Ch. 7	week 13 (Apr. 19)	end of week 14 (May 1)

Other Course Information

Required previous knowledge

This is a proof-based course, therefore a prerequisite is a good understanding of mathematical arguments. This includes the basic vocabulary for mathematical statements, and the basic principles for proving statements. All these concepts are presented in the document "Introduction to mathematical arguments" by Michael Hutchings, which is available in Canvas. The only way to further develop skills with writing proofs is by experience. We will do formal proofs in the course, and students will be expected to prove things in assignments and on exams.

Students must have working knowledge of linear algebra, including set theory, vectors and matrices, matrix inversion, subspaces and bases, and affine subspaces. An overview of these concepts can be found in any textbook or lecture notes on linear algebra. It can also be found in Section 1.5 of the 525 recommended textbook: "Introduction to Linear Optimization" by D. Bertsimas, and J.N. Tsitsiklis.

In this course there will be no computer programming, therefore no knowledge of programming languages is necessary.

Course web site

The Canvas web site is where lecture slides, homework assignments, homework solutions, and grades will be posted. Students will use the Canvas web site also to submit the assignments electronically in pdf format and to view the TA's comments.

The Canvas web site contains also two documents that cover some of the prerequisites. For basic proof techniques, see the document "Introduction to mathematical arguments." For mathematical notation, see the document "Math symbols cheat sheet."

The Canvas web site also contains a "Discussions" section. When students have a question related to this course, they should first check this discussions section to see if it has already been answered, and if not, post their question there. This way, all students benefit from the question and the answer. Students are also allowed, and encouraged, to answer questions posted by other students. Students are allowed to post questions anonymously so that the class will not know who asked the question.

Video lectures

The course will be delivered in an asynchronous way through pre-recorded video lectures. The videos will be uploaded in YouTube in the following playlist. New videos will be available each Monday throughout the duration of the course. On average, each week, the videos amount to a total of 1 hour and 13 minutes. It is useful for students to subscribe to the channel and turn on notifications.

The videos are topic based, therefore their length varies from as little as five minutes to more than one hour. The first two weeks of class are dedicated to Chapter 1 (total length 2:09:25). In week 3 and week 4 we discuss Chapter 2 (total length 2:53:42). In weeks 5, 6, and 7 we study Chapter 3 part 1 (total length 3:24:26). In weeks 8 and 9 we discuss Chapter 3 part 2 (total length 2:05:22). In weeks 10 and 11 we discuss Chapter 4 (total length 2:22:05). In weeks 12 and 13 we study Chapter 7 (total length 2:09:26). Finally, in the last week we study Chapter 8 (total length 1:21:00). The total playlist length amounts to 17:00:32, including the introductory video.

The next table indicates the topic and length of each video, and when each video will be available in YouTube.

Video #	Topic	Readings	Available in YouTube	Length
0	Course introduction		week 1 (Jan. 25)	35:06
1	Variants of the linear programming problem	Section 1.1	week 1	57:13
2	Examples of LP problems	Section 1.2	week 2 (Feb. 1)	33:24
3	Piecewise linear convex functions	Section 1.3	week 2	15:09
4	Graphical representation and solution	Section 1.4	week 2	23:39
5	Polyhedra and convex sets	Section 2.1	week 3 (Feb. 8)	14:34
6	Extreme points, vertices, and basic feasible solutions	Section 2.2	week 3	48:04
7	Polyhedra in standard form	Section 2.3	week 3	30:16
8	Degeneracy	Section 2.4	week 4 (Feb. 15)	18:12
9	Existence of extreme points	Section 2.5	week 4	13:03
10	Optimality of extreme points	Section 2.6	week 4	18:49
11	Representation of polytopes	Section 2.7	week 4	5:42
12	Projections of polyhedra: Fourier-Motzkin elimination	Section 2.8	week 4	25:02
13	Optimality conditions	Section 3.1	week 5 (Feb. 22)	42:01
14	Development of the simplex method	Section 3.2	week 6 (Mar. 1)	49:30
15	Algorithms and operation counts	Section 1.6	week 6	20:43
16	Implementations of the simplex method	Section 3.3	week 7 (Mar. 8)	1:32:12
17	Anticycling: lexicography and Bland's rule	Section 3.4	week 8 (Mar. 15)	41:18
18	Finding an initial basic feasible solution	Section 3.5	week 8	37:19
19	Column geometry and the simplex method	Section 3.6	week 9 (Mar. 22)	19:13
20	Computational efficiency of the simplex method	Section 3.7	week 9	27:32
21	Motivation	Section 4.1	week 10 (Mar. 29)	18:25
22	The dual problem	Section 4.2	week 10	24:52
23	The duality theorem	Section 4.3	week 10	47:40
24	Optimal dual variables as marginal costs	Section 4.4	week 11 (Apr. 5)	7:11
25	Standard form problems and the dual simplex method	Section 4.5	week 11	32:19
26	Farkas' lemma and linear inequalities	Section 4.6	week 11	11:38
27	Graphs	Section 7.1	week 12 (Apr. 12)	27:39
28	Formulation of the network flow problem	Section 7.2	week 12	20:32
29	The network simplex algorithm	Section 7.3	week 13 (Apr. 19)	1:21:15
30	Efficient algorithms and computational complexity	Section 8.1	week 14 (Apr. 26)	14:31
31	The key geometric result behind the ellipsoid method	Section 8.2	week 14	8:11
32	The ellipsoid method for the feasibility problem	Section 8.3	week 14	28:41
33	The ellipsoid method for optimization	Section 8.4	week 14	10:22
34	Problems with exponentially many constraints	Section 8.5	week 14	19:15

Related courses

- ISyE 323: Operations Research – Deterministic Modeling
- ISyE/Math/CS 425: Introduction to Combinatorial Optimization
- CS/ECE/ISyE 524: Introduction to Optimization
- CS/ISyE 526: Advanced Linear Programming
- ISyE/Math/CS/Stat 726: Nonlinear Optimization I
- ISyE/Math/CS 728: Integer Optimization

Academic Integrity

By enrolling in this course, each student assumes the responsibilities of an active participant in UW-Madison's community of scholars in which everyone's academic work and behavior are held to the highest academic integrity standards. Academic misconduct compromises the integrity of the university. Cheating, fabrication, plagiarism, unauthorized collaboration, and helping others commit these acts are examples of academic misconduct, which can result in disciplinary action. This includes but is not limited to failure on the assignment/course, disciplinary probation, or suspension. Substantial or repeated cases of misconduct will be forwarded to the Office of Student Conduct & Community Standards for additional review. For more information, refer to https://conduct.students.wisc.edu/academic-integrity/

Accommodations For Students With Disabilities

McBurney Disability Resource Center syllabus statement: "The University of Wisconsin-Madison supports the right of all enrolled students to a full and equal educational opportunity. The Americans with Disabilities Act (ADA), Wisconsin State Statute (36.12), and UW-Madison policy (Faculty Document 1071) require that students with disabilities be reasonably accommodated in instruction and campus life. Reasonable accommodations for students with disabilities is a shared faculty and student responsibility. Students are expected to inform faculty of their need for instructional accommodations by the end of the third week of the semester, or as soon as possible after a disability has been incurred or recognized. Faculty, will work either directly with the student or in coordination with the McBurney Center to identify and provide reasonable instructional accommodations. Disability information, including instructional accommodations as part of a student's educational record, is confidential and protected under FERPA."
http://mcburney.wisc.edu/facstaffother/faculty/syllabus.php

Diversity & Inclusion

Institutional statement on diversity: "Diversity is a source of strength, creativity, and innovation for UW-Madison. We value the contributions of each person and respect the profound ways their identity, culture, background, experience, status, abilities, and opinion enrich the university community. We commit ourselves to the pursuit of excellence in teaching, research, outreach, and diversity as inextricably linked goals.

The University of Wisconsin-Madison fulfills its public mission by creating a welcoming and inclusive community for people from every background – people who as students, faculty, and staff serve Wisconsin and the world."
https://diversity.wisc.edu/

Disrespectful behavior or comments directed toward any group or individual will be addressed by the instructor.