1.1 let $n=|E|$, $m=|V|$ then,
the binary integer problem is :

$$\min\{cx : x\in S\},\quad S=\{x\in\{0,1\}^n : Ax\le b,\ T=\{e: x_e=1\} \text{ is a spannig tree}\}$$

$$c=(c_1,\cdots,c_n),\quad c_i=w(i)$$
$$b=(k_1,\cdots,k_m)^T$$
$$A=(A_{ij})_{m\times n}\qquad A_{ij}=\begin{cases}1, & \text{if } x_j\in\delta(i)\\ 0, & \text{if } x_j\notin\delta(i)\end{cases}$$

1.2 pf: Suppose $T\subseteq G$ is a minimum weight spanning tree, x^* is the optimal solution of MST, x^0 is the optimal solution of DCMST
Since x^* is the solution of $\min\{c\cdot x : x\in P\}$,
$P=\{x\in\{0,1\}^n : T=\{e: x_e=1\} \text{ is a spanning tree}\}$
since $S\subseteq P$, we can know that $cx^*\le cx^0$
which means it's a lower boud on the optimal value

1.3 Algorithm for solving MDCST problem:

1. solve the MST problem and get a minimum weight spanning tree T
2. for all the node $i\in V$, if $|\delta(i)\cap T|>k_i$, delet $(k_i-|\delta(i)\cap T|)$ edges that adjacent to i from $G=(V,E)$. we got a subgraph $G'\subseteq G$
3. solve the MST problem in this subgraph

```
Instance: ftv47.atsp
optimal objective value: 1.776000000000e+03
the solution time: 1.114s
the number of nodes explored in the branch-and-bound tree: 3411
the root relaxation objective value: 1.655816e+03

Instance: si175.atsp
optimal objective value: 2.173000000000e+04
the solution time: 300.0s
the number of nodes explored in the branch-and-bound tree: 6541
the root relaxation objective value: 2.025148e+04
the ending optimality gap: 2.2365%



[Most fractional]
optimal objective value: 1.776000000000e+03
the solution time: 180.0s
the number of nodes explored in the branch-and-bound tree: 1013749
the root relaxation objective value: 1.655816e+03
the ending optimality gap: 0.6194%

[Strong branching]
optimal objective value: 1.776000000000e+03
the solution time: 2.164s
the number of nodes explored in the branch-and-bound tree: 2408
the root relaxation objective value: 1.655816e+03

[Turn off all cuts]
optimal objective value: 1.776000000000e+03
the solution time: 0.663s
the number of nodes explored in the branch-and-bound tree: 3197
the root relaxation objective value: 1.655816e+03




[Summary]
From the first test. We know that, the time it takes to solve the problem is related to
the scale of the data. Exponential rise.
From the second test. We can find that for the 'ftv47.atsp' instance. If I choose
BFS(Breath First Search) to explore the entire tree, it will take a lot of time, because
the search space for this problem is very huge. If I choose DFS(Deep First Search), with
the help of heuristic algorithm it will quickly solve it.
```

```
import sys
from itertools import combinations
import gurobipy as gp
from gurobipy import GRB

# Parse argument

if len(sys.argv) < 2:
    print('Usage: atsp.py instance')
    sys.exit(1)

# Read the data

f = open(sys.argv[1])

nnodestr = f.readline().split()
n = int(nnodestr[0])

dist = {}
strcosts = f.read().split()
for i in range(n):
        for j in range(n):
                dist[i,j] = int(strcosts[i*n+j])

m = gp.Model()

# Create variables

x = m.addVars(n, n, vtype=GRB.BINARY, name='x')
u = m.addVars(n, lb=2, ub=n, vtype=GRB.INTEGER, name='u')

m.setObjective(gp.quicksum(dist[i,j]*x[i,j] for i in range(n) for j in range(n)),
GRB.MINIMIZE)

# Initial constraint
for i in range(n):
    m.addConstr(gp.quicksum(x[i,j] for j in range(n) if j!=i)==1)
for j in range(n):
    m.addConstr(gp.quicksum(x[i,j] for i in range(n) if i!=j)==1)

# Position constraint
for i in range(1, n):
    for j in range(1, n):
        m.addConstr(u[i]-u[j]+1<=(n-1)*(1-x[i,j]))
m.Params.timelimit=180.0
# m.Params.varbranch=2
# m.Params.varbranch=3
# m.Params.cuts=0

m.optimize()
```

```
Academic license - for non-commercial use only - expires 2021-04-26
Using license file /home/zijie/gurobi.lic
Changed value of parameter timelimit to 300.0
   Prev: inf  Min: 0.0  Max: inf  Default: inf
Gurobi Optimizer version 9.1.1 build v9.1.1rc0 (linux64)
Thread count: 8 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 2305 rows, 2352 columns and 11045 nonzeros
Model fingerprint: 0x72774eba
Variable types: 0 continuous, 2352 integer (2304 binary)
Coefficient statistics:
  Matrix range     [1e+00, 5e+01]
  Objective range  [7e+00, 1e+08]
  Bounds range     [1e+00, 5e+01]
  RHS range        [1e+00, 5e+01]
Presolve removed 47 rows and 49 columns
Presolve time: 0.01s
Presolved: 2258 rows, 2303 columns, 10998 nonzeros
Variable types: 0 continuous, 2303 integer (2256 binary)
Found heuristic solution: objective 7533.0000000

Root relaxation: objective 1.655816e+03, 199 iterations, 0.00 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0 1655.81560    0   61 7533.00000 1655.81560  78.0%     -    0s
     0     0 1725.46809    0   77 7533.00000 1725.46809  77.1%     -    0s
H    0     0                    1891.0000000 1725.46809  8.75%     -    0s
     0     0 1725.46809    0   73 1891.00000 1725.46809  8.75%     -    0s
     0     0 1737.50355    0   66 1891.00000 1737.50355  8.12%     -    0s
     0     0 1738.17021    0   60 1891.00000 1738.17021  8.08%     -    0s
     0     0 1738.17021    0   69 1891.00000 1738.17021  8.08%     -    0s
     0     0 1740.11348    0   46 1891.00000 1740.11348  7.98%     -    0s
     0     0 1740.11348    0   63 1891.00000 1740.11348  7.98%     -    0s
     0     0 1740.11348    0   71 1891.00000 1740.11348  7.98%     -    0s
     0     0 1740.11348    0   72 1891.00000 1740.11348  7.98%     -    0s
     0     0 1740.11348    0   73 1891.00000 1740.11348  7.98%     -    0s
     0     0 1740.11348    0   79 1891.00000 1740.11348  7.98%     -    0s
     0     0 1741.42222    0   76 1891.00000 1741.42222  7.91%     -    0s
     0     0 1741.42222    0   81 1891.00000 1741.42222  7.91%     -    0s
     0     0 1745.00000    0   67 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   70 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   85 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   85 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   37 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   34 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   36 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   36 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   33 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   33 1891.00000 1745.00000  7.72%     -    0s
H    0     0                    1808.0000000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   31 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   62 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   61 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   62 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   73 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   73 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   83 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   82 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   64 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   77 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   41 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   27 1808.00000 1745.00000  3.48%     -    0s
H    0     0                    1782.0000000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   32 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   49 1782.00000 1745.00000  2.08%     -    0s
```

```
     0     0 1745.00000    0   18 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   73 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   45 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   59 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   63 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   67 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   74 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   57 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   57 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   31 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   19 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   24 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   32 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   19 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   31 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   31 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   31 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   31 1782.00000 1745.00000  2.08%     -    0s
     0     2 1750.00000    0   31 1782.00000 1750.00000  1.80%     -    0s
*  153    80              17    1777.0000000 1750.00000  1.52%  10.3    0s
*  658   241              19    1776.0000000 1752.03673  1.35%   8.6    0s

Cutting planes:
  Learned: 8
  Gomory: 13
  Cover: 18
  Implied bound: 46
  Clique: 2
  MIR: 37
  StrongCG: 1
  Inf proof: 16
  Zero half: 5
  RLT: 21
  Relax-and-lift: 27

Explored 3411 nodes (31121 simplex iterations) in 1.02 seconds
Thread count was 8 (of 8 available processors)

Solution count 6: 1776 1777 1782 ... 7533

Optimal solution found (tolerance 1.00e-04)
Best objective 1.776000000000e+03, best bound 1.776000000000e+03, gap 0.0000%
```

```
Academic license - for non-commercial use only - expires 2021-04-26
Using license file /home/zijie/gurobi.lic
Changed value of parameter timelimit to 300.0
   Prev: inf  Min: 0.0  Max: inf  Default: inf
Gurobi Optimizer version 9.1.1 build v9.1.1rc0 (linux64)
Thread count: 8 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 30626 rows, 30800 columns and 151380 nonzeros
Model fingerprint: 0x4ba55dc7
Variable types: 0 continuous, 30800 integer (30625 binary)
Coefficient statistics:
  Matrix range     [1e+00, 2e+02]
  Objective range  [7e+01, 1e+07]
  Bounds range     [1e+00, 2e+02]
  RHS range        [1e+00, 2e+02]
Presolve removed 174 rows and 176 columns
Presolve time: 0.14s
Presolved: 30452 rows, 30624 columns, 151206 nonzeros
Variable types: 0 continuous, 30624 integer (30450 binary)

Deterministic concurrent LP optimizer: primal and dual simplex
Showing first log only...

Concurrent spin time: 0.00s

Solved with dual simplex

Root relaxation: objective 2.025148e+04, 775 iterations, 0.08 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0 20251.4828    0  297          - 20251.4828      -     -    0s
     0     0 20924.7816    0  410          - 20924.7816      -     -    1s
     0     0 20924.7816    0  400          - 20924.7816      -     -    1s
     0     0 20964.2011    0  401          - 20964.2011      -     -    1s
     0     0 21011.8621    0  403          - 21011.8621      -     -    2s
     0     0 21088.0115    0  429          - 21088.0115      -     -    2s
     0     0 21089.0000    0  409          - 21089.0000      -     -    2s
     0     0 21089.0000    0  413          - 21089.0000      -     -    2s
     0     0 21089.0000    0  422          - 21089.0000      -     -    2s
     0     0 21089.0000    0  418          - 21089.0000      -     -    3s
     0     0 21089.0000    0  423          - 21089.0000      -     -    3s
     0     0 21089.0000    0  425          - 21089.0000      -     -    3s
     0     0 21089.0000    0  313          - 21089.0000      -     -    3s
     0     0 21089.0000    0  374          - 21089.0000      -     -    4s
     0     0 21089.0000    0  308          - 21089.0000      -     -    4s
     0     0 21089.0000    0  357          - 21089.0000      -     -    4s
     0     0 21089.0000    0  317          - 21089.0000      -     -    5s
     0     0 21089.0000    0  366          - 21089.0000      -     -    5s
     0     0 21089.0000    0  311          - 21089.0000      -     -    5s
     0     0 21089.0000    0  350          - 21089.0000      -     -    5s
H    0     0                    25218.000000 21089.0000  16.4%     -    6s
     0     0 21089.0000    0  299 25218.0000 21089.0000  16.4%     -    6s
     0     0 21089.0000    0  293 25218.0000 21089.0000  16.4%     -    6s
H    0     0                    24332.000000 21140.0000  13.1%     -    7s
     0     2 21140.0000    0  293 24332.0000 21140.0000  13.1%     -    7s
H   31    40                    24083.000000 21140.0000  12.2%  81.9   11s
H   32    40                    22610.000000 21140.0000  6.50%  80.4   11s
H   34    40                    22376.000000 21140.0000  5.52%  77.8   11s
   252   256 21218.6839   34  321 22376.0000 21140.0000  5.52%  31.2   15s
H  256   256                    22361.000000 21140.0000  5.46%  30.8   15s
   968  1017 21540.2069  137  269 22361.0000 21140.0000  5.46%  17.5   20s
  2032  2038 22217.5549  288  228 22361.0000 21140.0000  5.46%  14.9   25s
  3111  3095 21233.7706   50  392 22361.0000 21140.0000  5.46%  13.9   30s
  4428  4328 21865.7407  242  339 22361.0000 21140.0000  5.46%  13.2   35s
  5587  5476 22275.5833  408  231 22361.0000 21140.0000  5.46%  12.8   40s
```

```
  6341  6018 21389.0418   42  301 22361.0000 21140.0000  5.46%  13.2   45s
H 6501  6000                    22346.000000 21140.0000  5.40%  13.2   45s
H 6502  5946                    22325.000000 21140.0000  5.31%  13.2   45s
  6503  5941 21259.7761   24  293 22325.0000 21140.0000  5.31%  13.2   65s
  6505  5942 21442.9495  105  293 22325.0000 21140.0000  5.31%  13.2   71s
H 6505  5644                    21744.000000 21140.0000  2.78%  13.2   91s
  6509  5647 21275.5805  121  336 21744.0000 21161.0780  2.68%  13.2   95s
  6510  5648 21320.2790   88  333 21744.0000 21175.7437  2.61%  13.2  100s
  6513  5650 21744.0000  222  367 21744.0000 21203.8660  2.48%  13.2  107s
  6516  5652 21230.0529   22  403 21744.0000 21214.9286  2.43%  13.2  112s
  6517  5652 21708.4387  224  416 21744.0000 21225.7429  2.38%  13.2  117s
  6519  5654 21744.0000  662  481 21744.0000 21227.2500  2.38%  13.2  121s
  6520  5654 21744.0000  328  375 21744.0000 21233.9163  2.35%  13.2  126s
  6521  5655 21740.1312   82  386 21744.0000 21233.9997  2.35%  13.2  132s
  6522  5656 21447.1397   83  329 21744.0000 21235.4996  2.34%  13.2  142s
  6523  5656 21744.0000  243  407 21744.0000 21235.5070  2.34%  13.2  148s
  6524  5657 21744.0000  303  371 21744.0000 21235.5070  2.34%  13.2  158s
  6525  5658 21235.5070   12  422 21744.0000 21235.5070  2.34%  13.2  165s
  6526  5658 21447.3756  179  428 21744.0000 21236.6188  2.33%  13.2  174s
  6528  5660 21744.0000  387  438 21744.0000 21240.1196  2.32%  13.2  175s
  6529  5660 21434.9536   96  441 21744.0000 21240.1711  2.32%  13.2  184s
  6530  5661 21744.0000  191  304 21744.0000 21240.7996  2.31%  13.2  193s
  6531  5662 21744.0000  260  391 21744.0000 21240.7996  2.31%  13.2  201s
  6532  5662 21599.2371  108  401 21744.0000 21242.5249  2.31%  13.2  210s
  6533  5663 21669.4712  208  432 21744.0000 21242.7042  2.31%  13.2  219s
  6534  5664 21744.0000  330  380 21744.0000 21243.1665  2.30%  13.2  226s
  6536  5665 21571.7471  123  486 21744.0000 21243.6270  2.30%  13.2  234s
  6537  5666 21744.0000  376  505 21744.0000 21243.7217  2.30%  13.2  242s
  6538  5666 21744.0000  267  498 21744.0000 21243.8174  2.30%  13.2  251s
H 6538  5382                    21730.000000 21243.8797  2.24%  13.2  271s
  6540  5384 21261.8996   35  484 21730.0000 21243.8797  2.24%  13.2  284s
  6541  5384 21730.0000  453  335 21730.0000 21243.8797  2.24%  13.1  299s

Cutting planes:
  Learned: 9
  Gomory: 65
  Implied bound: 6
  Projected implied bound: 1
  Clique: 1
  MIR: 35
  Flow cover: 41
  Zero half: 27
  RLT: 2
  Relax-and-lift: 1

Explored 6541 nodes (111973 simplex iterations) in 300.05 seconds
Thread count was 8 (of 8 available processors)

Solution count 10: 21730 21744 22325 ... 25218

Time limit reached
Best objective 2.173000000000e+04, best bound 2.124400000000e+04, gap 2.2365%
```

Problem 2.1.2

```
Academic license - for non-commercial use only - expires 2021-04-26
Using license file /home/zijie/gurobi.lic
Changed value of parameter timelimit to 180.0
   Prev: inf  Min: 0.0  Max: inf  Default: inf
Changed value of parameter varbranch to 2
   Prev: -1  Min: -1  Max: 3  Default: -1
Gurobi Optimizer version 9.1.1 build v9.1.1rc0 (linux64)
Thread count: 8 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 2305 rows, 2352 columns and 11045 nonzeros
Model fingerprint: 0x72774eba
Variable types: 0 continuous, 2352 integer (2304 binary)
Coefficient statistics:
  Matrix range     [1e+00, 5e+01]
  Objective range  [7e+00, 1e+08]
  Bounds range     [1e+00, 5e+01]
  RHS range        [1e+00, 5e+01]
Presolve removed 47 rows and 49 columns
Presolve time: 0.01s
Presolved: 2258 rows, 2303 columns, 10998 nonzeros
Variable types: 0 continuous, 2303 integer (2256 binary)
Found heuristic solution: objective 7533.0000000

Root relaxation: objective 1.655816e+03, 199 iterations, 0.00 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0 1655.81560    0   61 7533.00000 1655.81560  78.0%     -    0s
     0     0 1725.46809    0   77 7533.00000 1725.46809  77.1%     -    0s
H    0     0                    1891.0000000 1725.46809  8.75%     -    0s
     0     0 1725.46809    0   73 1891.00000 1725.46809  8.75%     -    0s
     0     0 1737.50355    0   66 1891.00000 1737.50355  8.12%     -    0s
     0     0 1738.17021    0   60 1891.00000 1738.17021  8.08%     -    0s
     0     0 1738.17021    0   69 1891.00000 1738.17021  8.08%     -    0s
     0     0 1740.11348    0   46 1891.00000 1740.11348  7.98%     -    0s
     0     0 1740.11348    0   63 1891.00000 1740.11348  7.98%     -    0s
     0     0 1740.11348    0   71 1891.00000 1740.11348  7.98%     -    0s
     0     0 1740.11348    0   72 1891.00000 1740.11348  7.98%     -    0s
     0     0 1740.11348    0   73 1891.00000 1740.11348  7.98%     -    0s
     0     0 1740.11348    0   79 1891.00000 1740.11348  7.98%     -    0s
     0     0 1741.42222    0   76 1891.00000 1741.42222  7.91%     -    0s
     0     0 1741.42222    0   81 1891.00000 1741.42222  7.91%     -    0s
     0     0 1745.00000    0   67 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   70 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   85 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   85 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   37 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   34 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   36 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   36 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   33 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   33 1891.00000 1745.00000  7.72%     -    0s
H    0     0                    1808.0000000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   31 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   62 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   61 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   62 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   73 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   73 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   83 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   82 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   64 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   77 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   41 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   27 1808.00000 1745.00000  3.48%     -    0s
H    0     0                    1782.0000000 1745.00000  2.08%     -    0s
```

```
     0     0 1745.00000    0   32 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   49 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   18 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   73 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   45 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   59 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   63 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   67 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   74 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   57 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   57 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   31 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   19 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   24 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   32 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   19 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   31 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   31 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   31 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   31 1782.00000 1745.00000  2.08%     -    0s
     0     2 1745.00000    0   31 1782.00000 1745.00000  2.08%     -    0s
H  533   433                     1777.0000000 1746.80000  1.70%   6.7    1s
 24613  9986 1773.84217   54   72 1777.00000 1758.00335  1.07%  17.0    5s
 44857 15553 1763.66667   45   70 1777.00000 1758.00335  1.07%  17.4   10s
H44860 14776                     1776.0000000 1758.00335  1.01%  17.4   10s
 66620 17155     cutoff  111      1776.00000 1758.00335  1.01%  16.5   15s
 85797 16738     cutoff   93      1776.00000 1765.00000  0.62%  16.4   20s
 111770 15590 1765.00000   98   51 1776.00000 1765.00000  0.62%  16.2   25s
 135977 15399 1765.00000  125   94 1776.00000 1765.00000  0.62%  16.9   30s
 162381 17874     cutoff  102      1776.00000 1765.00000  0.62%  17.5   35s
 182871 19862 infeasible  112      1776.00000 1765.00000  0.62%  17.6   40s
 198141 22338     cutoff  101      1776.00000 1765.00000  0.62%  18.0   45s
 215314 26375 1765.00000  104   37 1776.00000 1765.00000  0.62%  18.2   50s
 243995 31852 1765.00000  112   81 1776.00000 1765.00000  0.62%  18.4   55s
 271196 34364 1771.27306  127   60 1776.00000 1765.00000  0.62%  18.8   60s
 301019 38415 1765.00000  119   87 1776.00000 1765.00000  0.62%  19.1   65s
 329750 41077 infeasible  116      1776.00000 1765.00000  0.62%  19.3   70s
 358067 43174 infeasible  127      1776.00000 1765.00000  0.62%  19.6   75s
 387699 45768 1769.18729   93   60 1776.00000 1765.00000  0.62%  19.7   80s
 418493 47259 1765.00000   90   52 1776.00000 1765.00000  0.62%  19.9   85s
 447473 50679 1771.21497   94   47 1776.00000 1765.00000  0.62%  20.1   90s
 479919 52855 infeasible  110      1776.00000 1765.00000  0.62%  20.2   95s
 513167 54100     cutoff   94      1776.00000 1765.00000  0.62%  20.4  100s
 544845 54324     cutoff  126      1776.00000 1765.00000  0.62%  20.4  105s
 580687 55511 1765.00000   93   49 1776.00000 1765.00000  0.62%  20.4  110s
 611231 57369 1765.00000   71   97 1776.00000 1765.00000  0.62%  20.5  115s
 643158 59404 infeasible   91      1776.00000 1765.00000  0.62%  20.5  120s
 673849 61837 infeasible   77      1776.00000 1765.00000  0.62%  20.5  125s
 705223 65206 1765.00000   90   62 1776.00000 1765.00000  0.62%  20.4  130s
 736986 67068     cutoff   76      1776.00000 1765.00000  0.62%  20.3  135s
 766062 68646 1765.60640   68   65 1776.00000 1765.00000  0.62%  20.3  140s
 796686 69335 infeasible  118      1776.00000 1765.00000  0.62%  20.3  145s
 828284 70065 infeasible  127      1776.00000 1765.00000  0.62%  20.3  150s
 858657 70675 1770.36076  115   88 1776.00000 1765.00000  0.62%  20.2  155s
 890141 71871 1765.00000   66   40 1776.00000 1765.00000  0.62%  20.2  160s
 921286 71751 infeasible  116      1776.00000 1765.00000  0.62%  20.2  165s
 952324 71116 infeasible  119      1776.00000 1765.00000  0.62%  20.2  170s
 982742 70246     cutoff   90      1776.00000 1765.00000  0.62%  20.1  175s
 1013397 69034 1769.35592   84   79 1776.00000 1765.00000  0.62%  20.1  180s

Cutting planes:
  Learned: 5
  Gomory: 22
  Cover: 58
  Implied bound: 8
  Projected implied bound: 6
```

```
  Clique: 3
  MIR: 58
  StrongCG: 1
  Flow cover: 193
  Inf proof: 86
  Zero half: 32
  RLT: 10
  Relax-and-lift: 30

Explored 1013749 nodes (20363177 simplex iterations) in 180.01 seconds
Thread count was 8 (of 8 available processors)

Solution count 6: 1776 1777 1782 ... 7533

Time limit reached
Best objective 1.776000000000e+03, best bound 1.765000000000e+03, gap 0.6194%
```

```
Academic license - for non-commercial use only - expires 2021-04-26
Using license file /home/zijie/gurobi.lic
Changed value of parameter timelimit to 180.0
   Prev: inf  Min: 0.0  Max: inf  Default: inf
Changed value of parameter varbranch to 3
   Prev: -1  Min: -1  Max: 3  Default: -1
Gurobi Optimizer version 9.1.1 build v9.1.1rc0 (linux64)
Thread count: 8 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 2305 rows, 2352 columns and 11045 nonzeros
Model fingerprint: 0x72774eba
Variable types: 0 continuous, 2352 integer (2304 binary)
Coefficient statistics:
  Matrix range     [1e+00, 5e+01]
  Objective range  [7e+00, 1e+08]
  Bounds range     [1e+00, 5e+01]
  RHS range        [1e+00, 5e+01]
Presolve removed 47 rows and 49 columns
Presolve time: 0.01s
Presolved: 2258 rows, 2303 columns, 10998 nonzeros
Variable types: 0 continuous, 2303 integer (2256 binary)
Found heuristic solution: objective 7533.0000000

Root relaxation: objective 1.655816e+03, 199 iterations, 0.00 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0 1655.81560    0   61 7533.00000 1655.81560  78.0%     -    0s
     0     0 1725.46809    0   77 7533.00000 1725.46809  77.1%     -    0s
H    0     0                    1891.0000000 1725.46809  8.75%     -    0s
     0     0 1725.46809    0   73 1891.00000 1725.46809  8.75%     -    0s
     0     0 1737.50355    0   66 1891.00000 1737.50355  8.12%     -    0s
     0     0 1738.17021    0   60 1891.00000 1738.17021  8.08%     -    0s
     0     0 1738.17021    0   69 1891.00000 1738.17021  8.08%     -    0s
     0     0 1740.11348    0   46 1891.00000 1740.11348  7.98%     -    0s
     0     0 1740.11348    0   63 1891.00000 1740.11348  7.98%     -    0s
     0     0 1740.11348    0   71 1891.00000 1740.11348  7.98%     -    0s
     0     0 1740.11348    0   72 1891.00000 1740.11348  7.98%     -    0s
     0     0 1740.11348    0   73 1891.00000 1740.11348  7.98%     -    0s
     0     0 1740.11348    0   79 1891.00000 1740.11348  7.98%     -    0s
     0     0 1741.42222    0   76 1891.00000 1741.42222  7.91%     -    0s
     0     0 1741.42222    0   81 1891.00000 1741.42222  7.91%     -    0s
     0     0 1745.00000    0   67 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   70 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   85 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   85 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   37 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   34 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   36 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   36 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   33 1891.00000 1745.00000  7.72%     -    0s
     0     0 1745.00000    0   33 1891.00000 1745.00000  7.72%     -    0s
H    0     0                    1808.0000000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   31 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   62 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   61 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   62 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   73 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   73 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   83 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   82 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   64 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   77 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   41 1808.00000 1745.00000  3.48%     -    0s
     0     0 1745.00000    0   27 1808.00000 1745.00000  3.48%     -    0s
H    0     0                    1782.0000000 1745.00000  2.08%     -    0s
```

```
     0     0 1745.00000    0   32 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   49 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   18 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   73 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   45 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   59 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   63 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   67 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   74 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   57 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   57 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   31 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   19 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   24 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   32 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   19 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   31 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   31 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   31 1782.00000 1745.00000  2.08%     -    0s
     0     0 1745.00000    0   31 1782.00000 1745.00000  2.08%     -    0s
     0     2 1745.00000    0   31 1782.00000 1745.00000  2.08%     -    0s
H  215    94                    1776.0000000 1750.01716  1.46%  10.8    1s

Cutting planes:
  Learned: 8
  Gomory: 8
  Cover: 3
  Implied bound: 41
  Clique: 2
  MIR: 39
  StrongCG: 1
  Inf proof: 2
  Zero half: 3
  RLT: 19
  Relax-and-lift: 30

Explored 2408 nodes (23940 simplex iterations) in 2.09 seconds
Thread count was 8 (of 8 available processors)

Solution count 5: 1776 1782 1808 ... 7533

Optimal solution found (tolerance 1.00e-04)
Best objective 1.776000000000e+03, best bound 1.776000000000e+03, gap 0.0000%
```

```
Academic license - for non-commercial use only - expires 2021-04-26
Using license file /home/zijie/gurobi.lic
Changed value of parameter timelimit to 180.0
   Prev: inf  Min: 0.0  Max: inf  Default: inf
Changed value of parameter cuts to 0
   Prev: -1  Min: -1  Max: 3  Default: -1
Gurobi Optimizer version 9.1.1 build v9.1.1rc0 (linux64)
Thread count: 8 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 2305 rows, 2352 columns and 11045 nonzeros
Model fingerprint: 0x72774eba
Variable types: 0 continuous, 2352 integer (2304 binary)
Coefficient statistics:
  Matrix range     [1e+00, 5e+01]
  Objective range  [7e+00, 1e+08]
  Bounds range     [1e+00, 5e+01]
  RHS range        [1e+00, 5e+01]
Presolve removed 47 rows and 49 columns
Presolve time: 0.01s
Presolved: 2258 rows, 2303 columns, 10998 nonzeros
Variable types: 0 continuous, 2303 integer (2256 binary)
Found heuristic solution: objective 7533.0000000

Root relaxation: objective 1.655816e+03, 199 iterations, 0.00 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0 1655.81560    0   61 7533.00000 1655.81560  78.0%     -    0s
     0     0 1674.57447    0   77 7533.00000 1674.57447  77.8%     -    0s
H    0     0                     1891.0000000 1674.57447  11.4%     -    0s
     0     0 1725.44681    0   78 1891.00000 1725.44681  8.75%     -    0s
     0     0 1725.44681    0   77 1891.00000 1725.44681  8.75%     -    0s
     0     0 1725.44681    0   62 1891.00000 1725.44681  8.75%     -    0s
     0     0 1736.44681    0   60 1891.00000 1736.44681  8.17%     -    0s
     0     0 1736.44681    0   68 1891.00000 1736.44681  8.17%     -    0s
     0     0 1736.44681    0   74 1891.00000 1736.44681  8.17%     -    0s
     0     0 1736.44681    0   59 1891.00000 1736.44681  8.17%     -    0s
H    0     0                     1776.0000000 1736.44681  2.23%     -    0s
     0     0 1736.44681    0   62 1776.00000 1736.44681  2.23%     -    0s
     0     0 1736.44681    0   66 1776.00000 1736.44681  2.23%     -    0s
     0     0 1736.44681    0   65 1776.00000 1736.44681  2.23%     -    0s
     0     2 1736.44681    0   63 1776.00000 1736.44681  2.23%     -    0s

Cutting planes:
  Learned: 9

Explored 3197 nodes (20908 simplex iterations) in 0.57 seconds
Thread count was 8 (of 8 available processors)

Solution count 3: 1776 1891 7533

Optimal solution found (tolerance 1.00e-04)
Best objective 1.776000000000e+03, best bound 1.776000000000e+03, gap 0.0000%
```