

1.1 let  $n=|E|$ ,  $m=|V|$  then,  
the binary integer problem is :

$$\min \{cx : x \in S\}, S = \{x \in \{0,1\}^n : Ax \leq b, T = \{e : x_e = 1\} \text{ is a spanning tree}\}$$

$$C = (c_1, \dots, c_n), c_i = w(i)$$

$$b = (k_1, \dots, k_m)^T$$

$$A = (A_{ij})_{m \times n} \quad A_{ij} = \begin{cases} 1, & \text{if } x_j \in \delta(i) \\ 0, & \text{if } x_j \notin \delta(i) \end{cases}$$

1.2 pf: suppose  $T \subseteq G$  is a minimum weight spanning tree,  $x^*$  is  
the optimal solution of MST,  $x^0$  is the optimal solution of DCMST  
Since  $x^*$  is the solution of  $\min \{cx : x \in P\}$ ,  
 $P = \{x \in \{0,1\}^n : T = \{e : x_e = 1\} \text{ is a spanning tree}\}$   
Since  $S \subseteq P$ , we can know that  $Cx^* \leq Cx^0$   
which means it's a lower bound on the optimal value

1.3 Algorithm for solving MDCST problem:

1. solve the MST problem and get a minimum weight spanning tree  $T$
2. for all the node  $i \in V$ , if  $|\delta(i) \cap T| > k_i$ , delete  $(k_i - |\delta(i) \cap T|)$  edges that adjacent to  $i$  from  $G = (V, E)$ . we got a subgraph  $G' \subseteq G$
3. solve the MST problem in this subgraph