# SP21 COMPSCI 513 - Homework 5

Zijie Zhang

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## Q1

All the necessary calculation details are in the next four pages.

[CODE]

```matlab
clear
% Declare Matrix Z
Z = [1      2      3;
     4      5      6;
     7      8      7;
     4      2      3;
     4      2      2]
e = eye(3);
E = eye(5);

% First Step
A0 = Z;
x = A0 * e(:, 1)
y = norm(x,2) * E(:, 1)
w = (x-y) / norm(x-y, 2)
H1 = E - 2 * w * w'
A1 = H1 * A0

% Second Step
x = A1 * e(:, 2)
y = x(1) * E(:, 1) + norm(x(2:5),2) * E(:, 2)
w = (x-y) / norm(x-y, 2)
H2 = E - 2 * w * w'
A2 = H2 * A1

% Third Step
x = A2 * e(:, 3);
y = x(1) * E(:, 1) + x(2) * E(:, 2) + norm(x(3:5),2) * E(:, 3)
w = (x-y) / norm(x-y, 2)
H3 = E - 2 * w * w'
A3 = H3 * A2

% Calc q and r
q = H1 * H2 * H3
r = H3 * H2 * H1 * Z

% Use qr routine
[Q R] = qr(Z)
```

[OUTPUT]

```
>> Q1

Z =

     1      2      3
     4      5      6
     7      8      7
     4      2      3
     4      2      2


x =

     1
     4
     7
     4
     4


y =
```

```
    9.8995
         0
         0
         0
         0
```

w =

```
   -0.6704
    0.3013
    0.5273
    0.3013
    0.3013
```

H1 =

```
    0.1010    0.4041    0.7071    0.4041    0.4041
    0.4041    0.8184   -0.3178   -0.1816   -0.1816
    0.7071   -0.3178    0.4438   -0.3178   -0.3178
    0.4041   -0.1816   -0.3178    0.8184   -0.1816
    0.4041   -0.1816   -0.3178   -0.1816    0.8184
```

A1 =

```
    9.8995    9.4954    9.6975
         0    1.6311    2.9897
    0.0000    2.1044    1.7320
         0   -1.3689   -0.0103
         0   -1.3689   -1.0103
```

x =

```
    9.4954
    1.6311
    2.1044
   -1.3689
   -1.3689
```

y =

```
    9.4954
    3.2919
         0
         0
         0
```

w =

```
         0
   -0.5023
    0.6364
   -0.4140
   -0.4140
```

H2 =

```
    1.0000         0         0         0         0
         0    0.4955    0.6393   -0.4158   -0.4158
         0    0.6393    0.1900    0.5269    0.5269
```

```
         0   -0.4158    0.5269    0.6572   -0.3428
         0   -0.4158    0.5269   -0.3428    0.6572
```

A2 =

```
    9.8995    9.4954    9.6975
    0.0000    3.2919    3.0129
    0.0000    0.0000    1.7026
    0.0000    0.0000    0.0089
    0.0000   -0.0000   -0.9911
```

y =

```
    9.6975
    3.0129
    1.9701
         0
         0
```

w =

```
         0
         0
   -0.2606
    0.0086
   -0.9654
```

H3 =

```
    1.0000         0         0         0         0
         0    1.0000         0         0         0
         0         0    0.8642    0.0045   -0.5031
         0         0    0.0045    0.9999    0.0167
         0         0   -0.5031    0.0167   -0.8641
```

A3 =

```
    9.8995    9.4954    9.6975
    0.0000    3.2919    3.0129
   -0.0000    0.0000    1.9701
    0.0000    0.0000   -0.0000
   -0.0000    0.0000    0.0000
```

q =

```
    0.1010    0.3162    0.5420    0.3408   -0.6928
    0.4041    0.3534    0.5162   -0.5730    0.3422
    0.7071    0.3906   -0.5248    0.2684    0.0028
    0.4041   -0.5580    0.3871    0.5006    0.3534
    0.4041   -0.5580   -0.1204   -0.4825   -0.5273
```

r =

```
    9.8995    9.4954    9.6975
         0    3.2919    3.0129
   -0.0000   -0.0000    1.9701
    0.0000         0    0.0000
   -0.0000   -0.0000   -0.0000
```

```
Q =

  -0.1010   -0.3162    0.5420   -0.6842   -0.3577
  -0.4041   -0.3534    0.5162    0.3280    0.5812
  -0.7071   -0.3906   -0.5248    0.0094   -0.2683
  -0.4041    0.5580    0.3871    0.3656   -0.4918
  -0.4041    0.5580   -0.1204   -0.5390    0.4695


R =

  -9.8995   -9.4954   -9.6975
        0   -3.2919   -3.0129
        0         0    1.9701
        0         0         0
        0         0         0
```

## Q2

(a) Take a look at the structure of the $H$ matrix.

$$H = I - 2ww'$$

When compute $Hv$, we have

$$Hv = (I - 2ww')v = v - 2ww'v$$

Here, the computational complexity of $w'v$ is $O(m)$, the computational complexity of $v - 2w(w'v)$ is $O(m)$. So the final computational complexity is still $O(m)$.

In conclusion, this algorithm is more efficient.

(b) [CODE] fast_qr.m

```
1 function [x] = fast_qr(A,b)
2     [n m] = size(A);
3     e = eye(m);
4     E = eye(n);
5     for i=1:m
6         x = A * e(:, i);
7         y = zeros(n,1);
8         for j=1:i-1
9             y = y + x(j) * E(:, j);
10        end
11        y = y + norm(x(i:n),2) * E(:, i);
12        w = (x-y) / norm(x-y, 2);
13        A = A - 2*w*(w'*A);
14        b = b - 2*w*(w'*b);
15     end
16     x = zeros(m,1);
17     for i=m:-1:1
18         t = b(i);
19         for j=i+1:m
20             t = t - x(j)*A(i,j);
21         end
22         x(i) = t / A(i,i);
23     end
24 end
```

(c) Run examples:

```
Editor - /home/zijie/Documents/2021Spring/513/Assignments/hw5/r_example.m
  Q1.m  ×  fast_qr.m  ×  r_example.m  ×  +
1 -    A = rand(5)
2 -    b = rand(5, 1)
3 -    x = A \ b
4 -    x = fast_qr(A, b)
```

Command Window
```
>> r_example

A =

    0.2858    0.0759    0.1299    0.1622    0.6020
    0.7572    0.0540    0.5688    0.7943    0.2630
    0.7537    0.5308    0.4694    0.3112    0.6541
    0.3804    0.7792    0.0119    0.5285    0.6892
    0.5678    0.9340    0.3371    0.1656    0.7482


b =

    0.4505
    0.0838
    0.2290
    0.9133
    0.1524


x =

    1.7489
   -0.3097
   -2.8814
    0.3636
    0.4809


x =

    1.7489
   -0.3097
   -2.8814
    0.3636
    0.4809
```

```
Editor - /home/zijie/Documents/2021Spring/513/Assignments/hw5/r_example.m
  Q1.m  ×  fast_qr.m  ×  r_example.m  ×  +
1 -    A = rand(5)
2 -    b = rand(5, 1)
3 -    x = A \ b
4 -    x = fast_qr(A, b)
```

Command Window
```
>> r_example

A =

    0.8258    0.1067    0.8687    0.4314    0.1361
    0.5383    0.9619    0.0844    0.9106    0.8693
    0.9961    0.0046    0.3998    0.1818    0.5797
    0.0782    0.7749    0.2599    0.2638    0.5499
    0.4427    0.8173    0.8001    0.1455    0.1450


b =

    0.8530
    0.6221
    0.3510
    0.5132
    0.4018


x =

   -0.8635
   -0.6374
    1.3066
    0.8661
    0.9214


x =

   -0.8635
   -0.6374
    1.3066
    0.8661
    0.9214
```

(d) Take a look at the code above.
Suppose $A$ is $n \times m$.
Calculate $x, y, w$ cost $O(n)$
Calculate new $A$ cost $O(mn)$
Calculate new $b$ cost $O(m)$
With $m$ times iteration, the total calculation complexity is $O(m^2 n)$.