SP21 COMPSCI 513 - Homework 3

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(a)

$$
\begin{aligned}
|\lambda I - A| &= \begin{vmatrix} \lambda - 2 & 2 \\ -1 & \lambda - 1 \end{vmatrix} \\
&= (\lambda - 2)(\lambda - 1) + 2 \\
&= \lambda^2 - 3\lambda + 4
\end{aligned}
$$

Let $|\lambda I - A| = 0$, we have

$$\lambda_1 = 1.5 - \frac{\sqrt{7}}{2} i$$

$$\lambda_2 = 1.5 + \frac{\sqrt{7}}{2} i$$

The spectrum is $\{1.5 - \frac{\sqrt{7}}{2}i, 1.5 + \frac{\sqrt{7}}{2}i\}$. The spectral radius is 2.

(b)

$$\|A\|_1 = \max_{1 \leqslant i \leqslant n} \|a_i\|_1 = 3$$

$$\|A\|_2 = \sqrt{\rho(A'A)}$$

$$A'A = \begin{bmatrix} 5 & -3 \\ -3 & 5 \end{bmatrix} \rho(A'A) = \max\{2, 8\} = 8$$

$$\|A\|_2 = \sqrt{\rho(A'A)} = 2\sqrt{2}$$

$$\|A\|_\infty = \|A'\|_1 = 4$$

(c) The eigenvectors of AA' is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ The eigenvectors of AA' is $\begin{bmatrix} -0.7071 & -0.7071 \\ -0.7071 & 0.7071 \end{bmatrix}$

So, the left singular vectors is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ the right singular vectors is $\begin{bmatrix} -0.7071 & -0.7071 \\ -0.7071 & 0.7071 \end{bmatrix}$

the singular values of A is $\begin{bmatrix} \sqrt{2} & 0 \\ 0 & 2\sqrt{2} \end{bmatrix}$.

(d) Consider $\Sigma_1 = \begin{bmatrix} -\sqrt{2} & 0 \\ 0 & 2\sqrt{2} \end{bmatrix}$. It is an orthogonal matrix. Let

$$B = U\Sigma_1 V'$$

We have

$$B = \begin{bmatrix} -2 & 2 \\ 1 & 1 \end{bmatrix}$$

Check it: $B'B = \begin{bmatrix} 5 & -3 \\ -3 & 5 \end{bmatrix} = A'A$, $BB' = \begin{bmatrix} 8 & 0 \\ 0 & 2 \end{bmatrix} = AA'$

(e) Find the schur decompositions of $A'A$.

$$A'A = \begin{bmatrix} 5 & -3 \\ -3 & 5 \end{bmatrix}$$

We know the eigenvectors of $A'A$ is $\begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix}$ So, the 3 different decompositions are

$$U_1 = \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}, T_1 = \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix}$$

$$U_2 = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}, T_2 = \begin{bmatrix} 8 & 0 \\ 0 & 2 \end{bmatrix}$$

$$U_3 = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}, T_3 = \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix}$$