CS513, Spring 21
Prof. Ron

HW #3, Factor=1

Due Feb. 16, 2021

You are given the matrix

$$A = \begin{pmatrix} 2 & -2 \\ 1 & 1 \end{pmatrix}. \tag{א}$$

All your computations below should be without the aid of any computational software (like `Matlab`), unless you are told explicitly otherwise - so you will need to show the details of your work!

(a) Find the spectrum and the spectral radius of this matrix (show your work). Use then `Matlab`'s `eig` and `abs` routines to check your answers.

Note, with $\sigma(A)$ the spectrum of the square matrix A, the *spectral radius* $\rho(A)$ is defined by

$$\rho(A) := \max\{|\lambda| : \lambda \in \sigma(A)\}.$$

(b) Compute the 1-, 2-, and ∞- norms of A above (show your work). Use then `Matlab`'s `norm` routine to check your answers.

(c) Find the left singular vectors, right singular vectors, and the singular values of A. Check that A as well as A' map right/left singular vectors to left/right singular vectors. Based on your findings above, write the SVD of A, and compare it with `Matlab`'s command `svd`.

(d) Find a matrix B, such that $B \neq A$, $B \neq -A$, but, nonetheless,

$$B'B = A'A, \quad BB' = AA'.$$

For this part, you may use any computational platform as you see fit - but you should be able to do it using just manual computations.

(e) Select from the matrices mentioned above any matrix which is *symmetric* (obviously not A itself...). Find 3 different Schur decompositions for that matrix.

Final remark: The computations above are overlapping, i.e., an output for one part may provide almost the complete solution to another. You surely may not need to show twice your work if you compute the same vectors/numbers that you already computed.