513, Homework 7. Zijie Zhang.

1(c) By the definition of condition number.

$$C_2(A) = \|A\|_2 \cdot \|A^{-1}\|_2$$

$$= \frac{\max\{\sigma \mid \sigma \text{ is an singular value of } A\}}{\min\{\sigma \mid \sigma \text{ is an singular value of } A\}}$$

$$= \frac{\max\{\sqrt{\lambda} \mid \lambda \in \sigma(A'A)\}}{\min\{\sqrt{\lambda} \mid \lambda \in \sigma(A'A)\}}$$

$$= \sqrt{\frac{\max(\sigma(A'A))}{\min(\sigma(A'A))}}$$

(d) In case A is symmetric.

a $(a>0)$ is singular value of A

$\Leftrightarrow$ $a \in \sigma(A)$ or $-a \in \sigma(A)$,

Thus. $$c_2(A) = \frac{\max\{|\lambda| : \lambda \in \sigma(A)\}}{\min\{|\lambda| : \lambda \in \sigma(A)\}}$$

(e) $A = \begin{pmatrix} -4 & 72 \\ 72 & -46 \end{pmatrix}$ $\sigma(A) = \{-100, 50\}$

$\sigma(A'A) = \{2500, 10000\}$

$$c_2(A) = \sqrt{\frac{10000}{2500}} = 2.$$

$$c_2(A) = \frac{100}{50} = 2$$

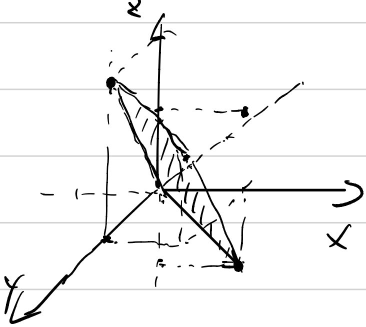
$A = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$ $\sigma(A) = \{1, 1\}$

$\sigma(A'A) = \{0.3820, 2.6180\}$

$c_2(A) = \sqrt{\frac{1}{1}} = 1 \neq c_2(A) = 6.8541$

2. $\operatorname{rank}(A) = 2 < 3$. So the range(A) is a 2-D plane in $\mathbb{R}^3$.

rang $(A) = \operatorname{span}\{(1,0,-1), (0,1,2)\}$



$Ax^* = \left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right)$

x^* can be $\left(-\frac{2}{3}, \frac{2}{3}, 0\right)$. $\left(0, -\frac{2}{3}, \frac{2}{3}\right)$ $\left(-\frac{1}{3}, 0, \frac{1}{3}\right)$

No, contradiction, Ax^* is unique.

A is not full rank, so the solution to $Ax^* = \tilde{w}$ can be multiple.