CS513, Spring 21

Prof. Ron

HW #1, Factor=.5

Due February 9, 2021

Let Q be an $m \times m$ real matrix that satisfies, for every vector $v \in \mathbb{R}^m$,

$$\|Qv\|_2 = \|v\|_2. \qquad (X)$$

(a) Show that 1 is the only eigenvalue of $Q'Q$, i.e., show that $\sigma(Q'Q) = \{1\}$.

(b) Use (a) in order to show that the matrix Q in this question (i.e., the one that satisfies (X)) is *orthogonal*.

Note that the question establishes the inverse of the simpler statement we have seen in class: "Given an orthogonal Q and any vector v, $\|Qv\|_2 = \|v\|_2$."

Hints: You will surely need to use the fact that every matrix of the form $A'A$ is symmetric. Also, you are going to play with innerproducts, hence will surely need the already-known-to-be-exceptionally-useful identity

$$(Av, w) = (v, A'w)$$

that we mentioned in class.