

SP21 COMPSI 513 - Homework 4

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Q1

- (a) (λ, v) is an eigenpair of B means

$$Bv = \lambda v$$

We know that B is invertible, then B^{-1} exists. It gives

$$B^{-1}Bv = B^{-1}\lambda v = \lambda B^{-1}v = v$$

$$B^{-1}v = \lambda^{-1}v$$

This means $(1/\lambda, v)$ is an eigenpair of B^{-1} .
The entire proof is equivalent and reversible.

- (b) By definition,

$$\|A^{-1}\|_2 = \sqrt{\rho((AA')^{-1})}$$

i.e.

$$\|A^{-1}\|_2 = \sqrt{\max\{|\lambda| : \lambda \in \sigma((AA')^{-1})\}}$$

By the statement in (a)

$$\|A^{-1}\|_2 = \sqrt{\max\{|1/\lambda| : \lambda \in \sigma(AA')\}} = 1/\sqrt{\min\{|\lambda| : \lambda \in \sigma(AA')\}}$$

$\|A^{-1}\|_2$ is the absolute value of reciprocal of the singular value of A with the smallest absolute value.

Q2

- (a) (λ, v) is an eigenpair of $A'A$ if and only if $(\sqrt{\lambda}, v)$ or $(-\sqrt{\lambda}, v)$ is an eigenpair of A .

```

1 clc;
2 A = [ 3.1448  7.1228  5.4348  5.6298  4.9838  5.8348;
3       7.1228 23.6488 11.3778 12.6298 11.9528 18.4238;
4       5.4348 11.3778 18.3368 16.8318 10.2888 11.6888;
5       5.6298 12.6298 16.8318 20.4998 10.2888 12.7538;
6       4.9838 11.9528 10.2888 10.2888 11.8898  9.8458;
7       5.8348 18.4238 11.6888 12.7538  9.8458 16.1818];
8 [V,D] = eig(A);
9 A'*A
10 [V1,D1] = eig(A'*A)

format short
>> Q2
A =
3.144800000000000 7.122800000000000 5.434800000000000 5.629800000000000 4.983800000000000 5.834800000000000
7.122800000000000 23.648800000000000 11.377800000000000 12.629800000000000 11.952800000000000 18.423800000000000
5.434800000000000 11.377800000000000 18.336800000000000 16.831800000000000 10.288800000000000 11.688800000000000
5.629800000000000 12.629800000000000 16.831800000000000 20.499800000000000 10.288800000000000 12.753800000000000
4.983800000000000 11.952800000000000 10.288800000000000 10.288800000000000 11.889800000000000  9.845800000000000
5.834800000000000 18.423800000000000 11.688800000000000 12.753800000000000  9.845800000000000 16.181800000000000

V =
0.651835181889366 -0.789882886334948 -0.982148895460485 -0.177363738667598 0.4208857788731424 0.182279195359129
-0.49868498252083 -0.329368413187534 0.975286442821836 0.188883749887638 0.613432928848738 0.388748818283483
-0.475756668702571 -0.823486517811887 0.732752921616651 -0.168867778888884 -0.48664871141931 0.431877653383411
-0.14313181762616 -0.872849486177719 -0.55725438849317 -0.533187258413951 0.4586183882262031 0.466618882820791
-0.848515154749581 -0.292398161638822 -0.363776376515668 -0.434537799328884 -0.451712973318881 0.381168841643234
0.554214551382793 0.544188896937979 0.898197386913926 0.334852961293881 0.386163959483682 0.441240351574134

D =
0.818543519413941 0 0 0 0 0
0 1.432434261307756 0 0 0 0
0 0 3.14195578848488 0 0 0
0 0 0 4.594857543388457 0 0
0 0 0 0 12.46588951573242 0
0 0 0 0 0 17.764897788623178

[V1,D1] = eig(A'*A)

format short
>> Q2
A =
3.1448 7.1228
7.1228 23.6488

V =
-0.6000 -0.8000
0.8000 -0.6000

D =
-100.0000 0
0 50.0000

V1 =
-0.8000 -0.6000
-0.6000 0.8000

D1 =
2500 0
0 18000

```

(b) If (λ, v) is an eigenpair of $A'A$, then we have

$$A'A v = \lambda v$$

We know that A is symmetric, $A'A = A^2$, which means, we can find the square root of A^2 . Obviously, every eigenvalue is unique by the special property. Then, it gives

$$A v = \sqrt{\lambda} v \quad \text{or} \quad A v = -\sqrt{\lambda} v$$

If $(\sqrt{\lambda}, v)$ or $(-\sqrt{\lambda}, v)$ is an eigenpair of A . Consider $(\sqrt{\lambda}, v)$ is an eigenpair of A , it gives

$$A v = \sqrt{\lambda} v, A' v = \sqrt{\lambda} v$$

$$A'A v = \lambda v$$

(c) Check with MATLAB.

True.

```

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-0.8000 -0.6000
-0.6000 0.8000

D1 =
2500 0
0 18000

```

(d)

$$A = \begin{bmatrix} 1 & 3 \\ 3 & -1 \end{bmatrix}$$

$$V = \begin{bmatrix} 0.5847 & -0.8112 \\ -0.8112 & -0.5847 \end{bmatrix}$$

$$D = \begin{bmatrix} -3.1623 & 0 \\ 0 & 3.1623 \end{bmatrix}$$

$$A'A = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

Obviously, this theorem does not hold.