ISyE/Math/CS/Stat 525 – Linear Optimization
Spring 2021
Homework 1

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Exercise 1

Proof. For every $x, y \in \mathbb{R}^n$ and every $\lambda \in [0, 1]$

$$\begin{aligned} f(\lambda x + (1-\lambda)y) &= \sum_{i=1}^{m} f_i(\lambda x + (1-\lambda)y) \\ &\leqslant \sum_{i=1}^{m} \lambda f_i(x) + (1-\lambda) f_i(y) \\ &= \lambda \sum_{i=1}^{m} f_i(x) + (1-\lambda) \sum_{i=1}^{m} f_i(y) \\ &= \lambda f(x) + (1-\lambda) f(y) \end{aligned}$$

f is convex. $\square$

Exercise 8

Proof. (a)

$$\begin{aligned} \min \quad & c'(x^+ - x^-) + d'y \\ \text{s.t.} \quad & A(x^+ - x^-) + By + z - b = 0 \\ & x^+ + x^- - y = 0 \\ & x^+, x^-, z \geqslant 0 \end{aligned}$$

(b) For every feasible solution to the original problem,

$$(x, y) = (x^+ - x^-, y)$$

There exists a feasible solution to reformulation problem.

$\square$

Exercise 9

Proof.

$$
\begin{aligned}
\min \quad & \|I - I^*\|_2 \\
\text{s.t.} \quad & I = A * P \\
\text{where} \quad & A_{ij} = a_{ij}, P_j = p_j
\end{aligned}
$$

□

Exercise 11

Proof. □

Exercise 12

Proof. □