SP21 COMPSCI 513 - Homework 4

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Q1

(a) (λ, v) is an eigenpair of B means

$$Bv = \lambda v$$

We know that B is invertible, then B^{-1} exists. It gives

$$B^{-1}Bv = B^{-1}\lambda v = \lambda B^{-1}v = v$$

$$B^{-1}v = \lambda^{-1}v$$

This means $(1/\lambda, v)$ is an eigenpair of B^{-1}.
The entire proof is equivalent and reversible.

(b) By definition,

$$\left\|A^{-1}\right\|_2 = \sqrt{\rho((AA')^{-1})}$$

i.e.

$$\left\|A^{-1}\right\|_2 = \sqrt{\max\{|\lambda| : \lambda \in \sigma((AA')^{-1})\}}$$

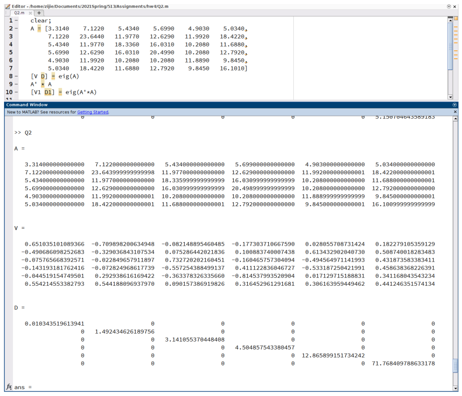
By the statement in (a)

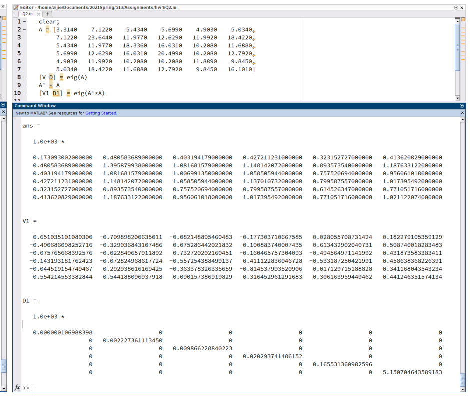
$$\left\|A^{-1}\right\|_2 = \sqrt{\max\{|1/\lambda| : \lambda \in \sigma(AA')\}} = 1/\sqrt{\min\{|\lambda| : \lambda \in \sigma(AA')\}}$$

$\left\|A^{-1}\right\|_2$ is the absolute value of reciprocal of the singular value of A with the smallest absolute value.

Q2

(a) (λ, v) is an eigenpair of $A'A$ if and only if $(\sqrt{\lambda}, v)$ or $(-\sqrt{\lambda}, v)$ is an eigenpair of A.





(b) If (λ, v) is an eigenpair of $A'A$, then we have

$$A'Av = \lambda v$$

We know that A is symmetric, $A'A = A^2$, which means, we can find the square root of A^2. Obviously, every eigenvalue is unique by the special property. Then, it gives

$$Av = \sqrt{\lambda}v \quad \text{or} \quad Av = -\sqrt{\lambda}v$$

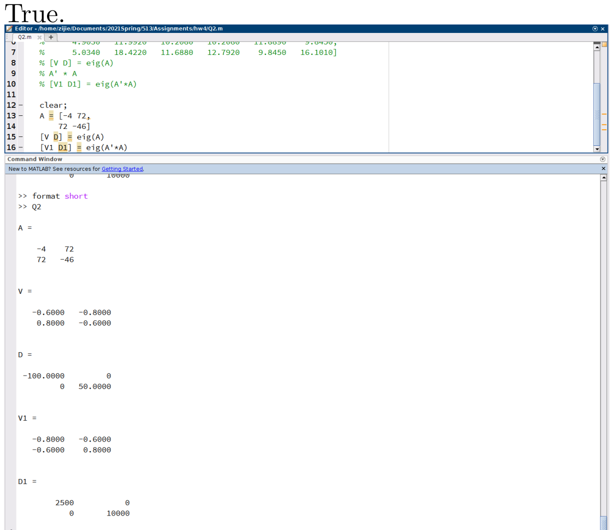
If $(\sqrt{\lambda}, v)$ or $(-\sqrt{\lambda}, v)$ is an eigenpair of A. Condsider $(\sqrt{\lambda}, v)$ is an eigenpair of A, it gives

$$Av = \sqrt{\lambda}v, A'v = \sqrt{\lambda}v$$

$$A'Av = \lambda v$$

(c) Check with MATLAB.

True.



(d)

$$A = \begin{bmatrix} 1 & 3 \\ 3 & -1 \end{bmatrix}$$

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V = [0.5847      -0.8112
     -0.8112     -0.5847]
D = [-3.1623           0
     0            3.1623]
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$$A'A = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

Obviously, this theorem does not hold.