Lecture 9: Application of QR-factorization

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Outline

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Linear systems overview

Linear system

A is a given square invertible $m \times m$, matrix, $b \in R^m$.
Problem: Find (the unique) x for which

$$Ax = b.$$

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There are two classes of methods for solving invertible systems:

- Direct Methods.
- Indirect Methods.

Linear systems overview

Factorization

All the direct methods for solving a linear system are based on factorization.

The basic idea of factorization

Step I: Factor A into $A = BC$.
Step II: Solve the system $By = b$.
Step III: Solve the system $Cx = y$.
Then

$$Ax = (BC)x = B(Cx) = By = b.$$

The basic idea of factorization

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Works whenever we have algorithms for solving systems based on B and based on C.

Solving by QR-factorization

Step I: Factor $A = QR$.
Step II: Solve the system $Qy = b$.
Step III: Solve the system $Rx = y$.

Solving by QR-factorization

Solving $Qy = b$:

$$y = Q'b.$$

Solving by QR-factorization

Solving $Rx = y$: By induction on m:

- If $m = 1$, the system is a trivial 1×1.
- Suppose that we know how to solve upper triangular invertible systems $m \times m$. Assume that your system A is $(m+1) \times (m+1)$. Let A_1 be the system obtained from A by removing the first row and the first column of A. A_1 is $m \times m$ and invertible. By solving it you find $x(2:m)$. Substitute that solution to the first equation of A and find $x(1)$.
- This is *back substitution*.