

소비자이론 (4)

미시경제이론

조남운

Topics

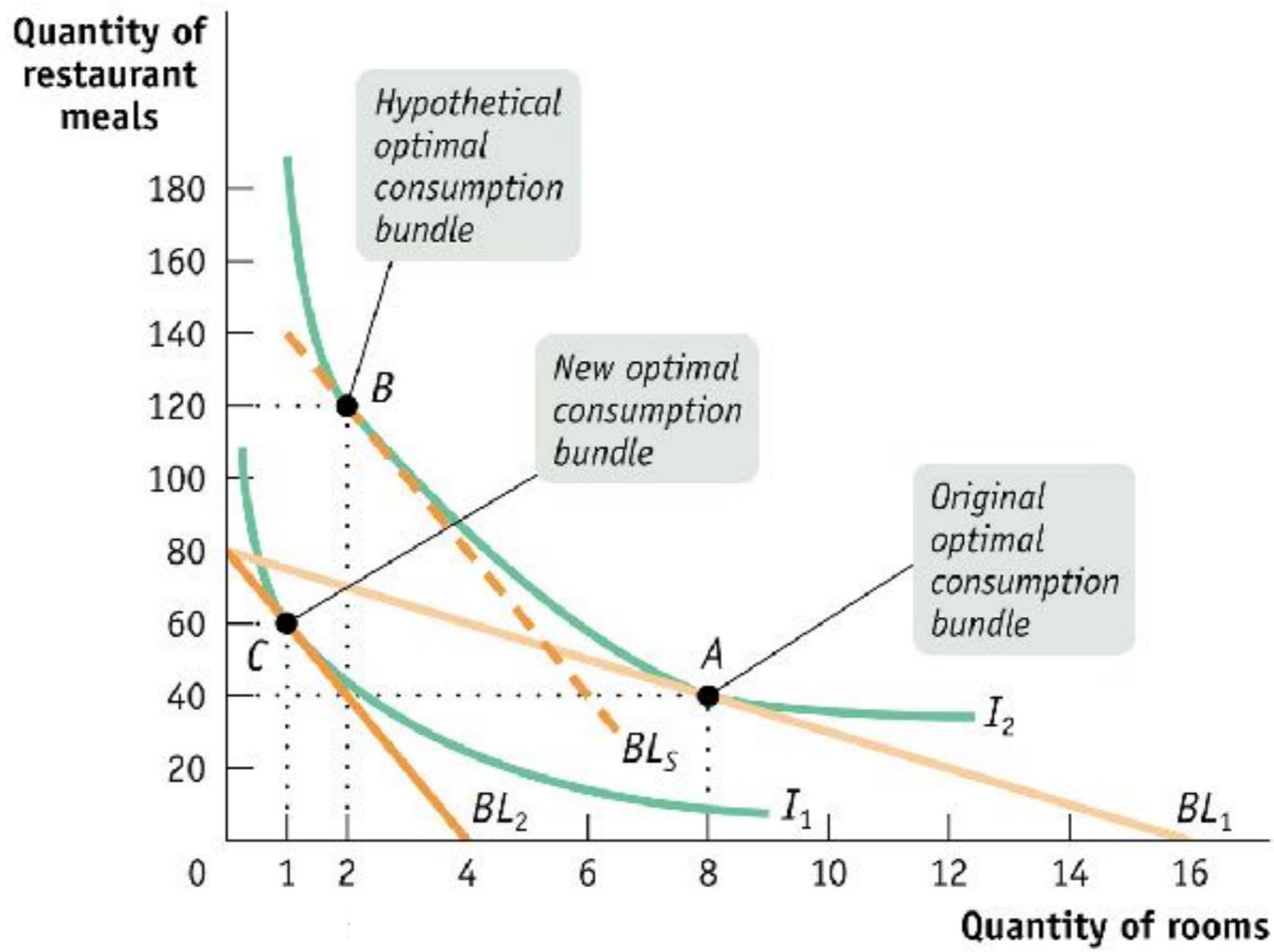
- 수요법칙
- 소비자후생

수요법칙

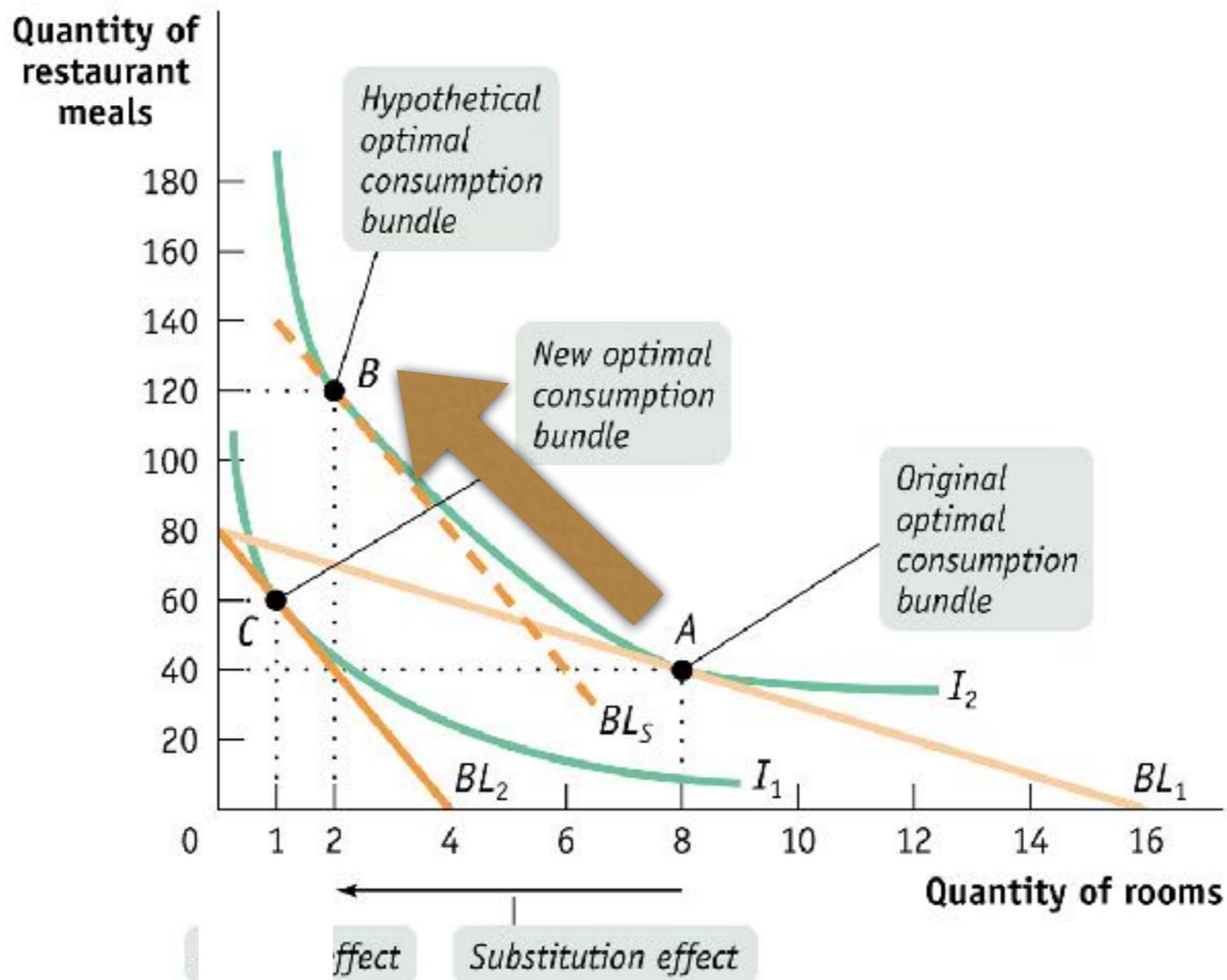
Law of Demand

Review: Hicks Decomposition (정상재)

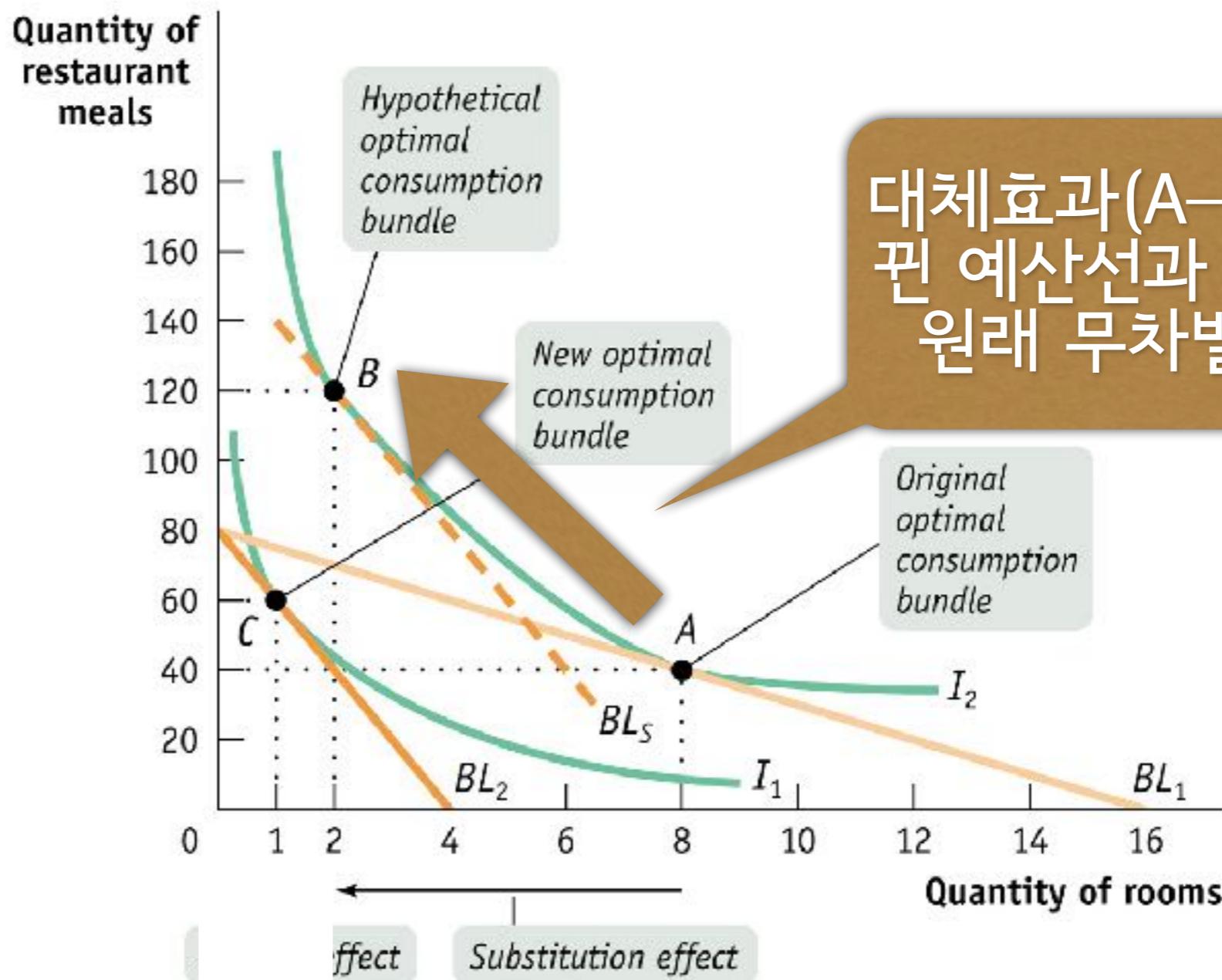
Review: Hicks Decomposition (정상재)



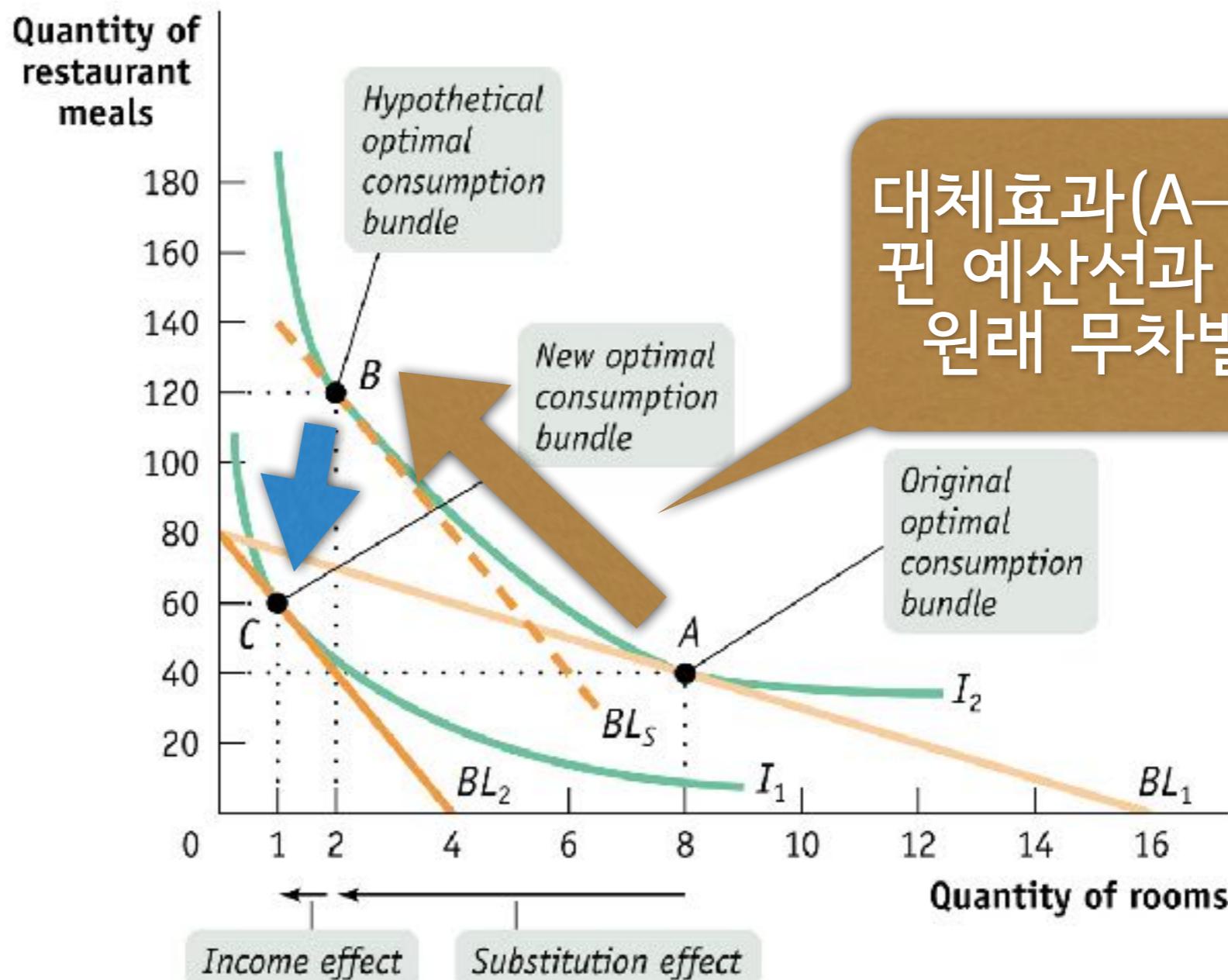
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Review: Hicks Decomposition (정상재)

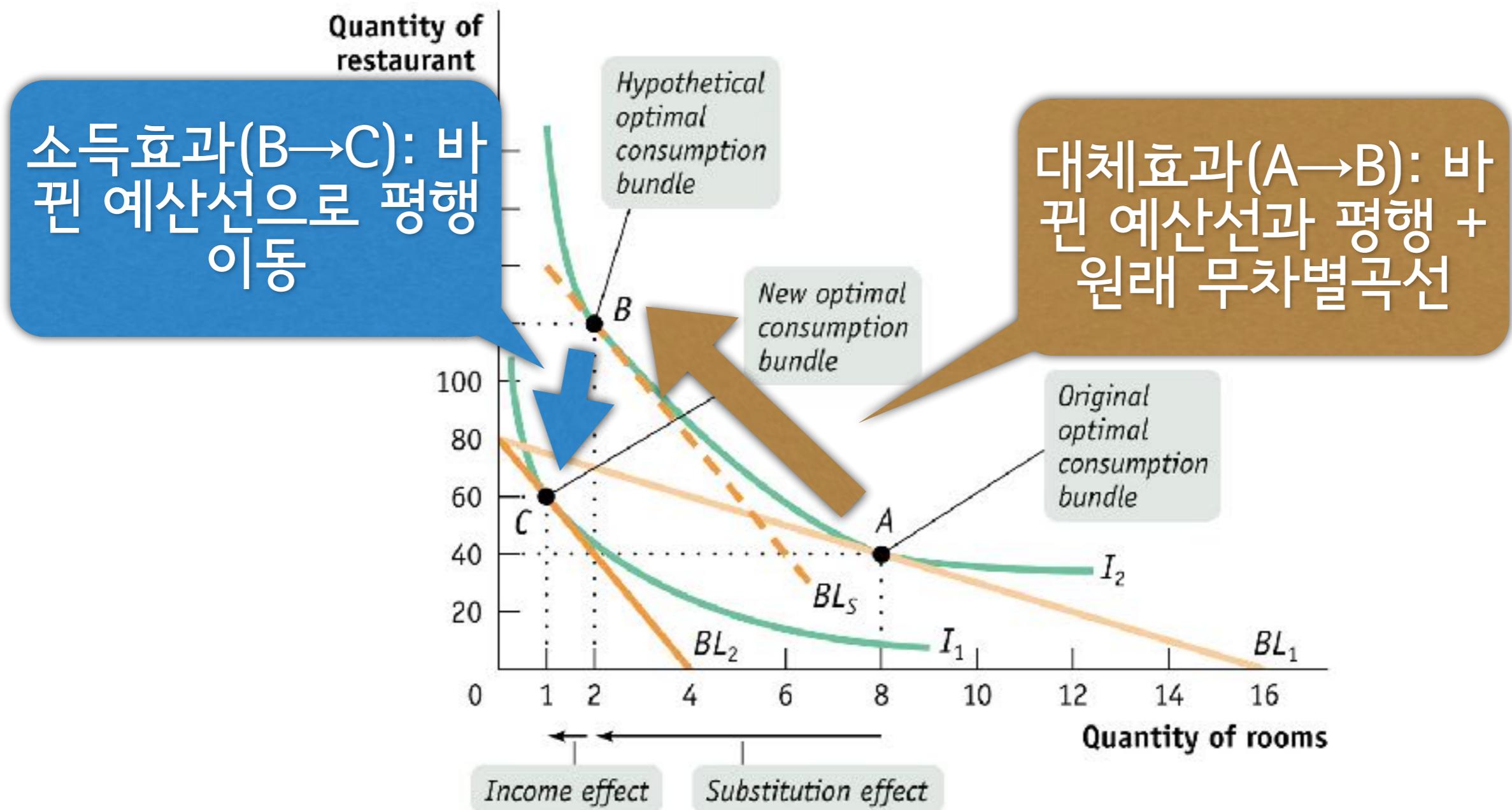


Review: Hicks Decomposition (정상재)

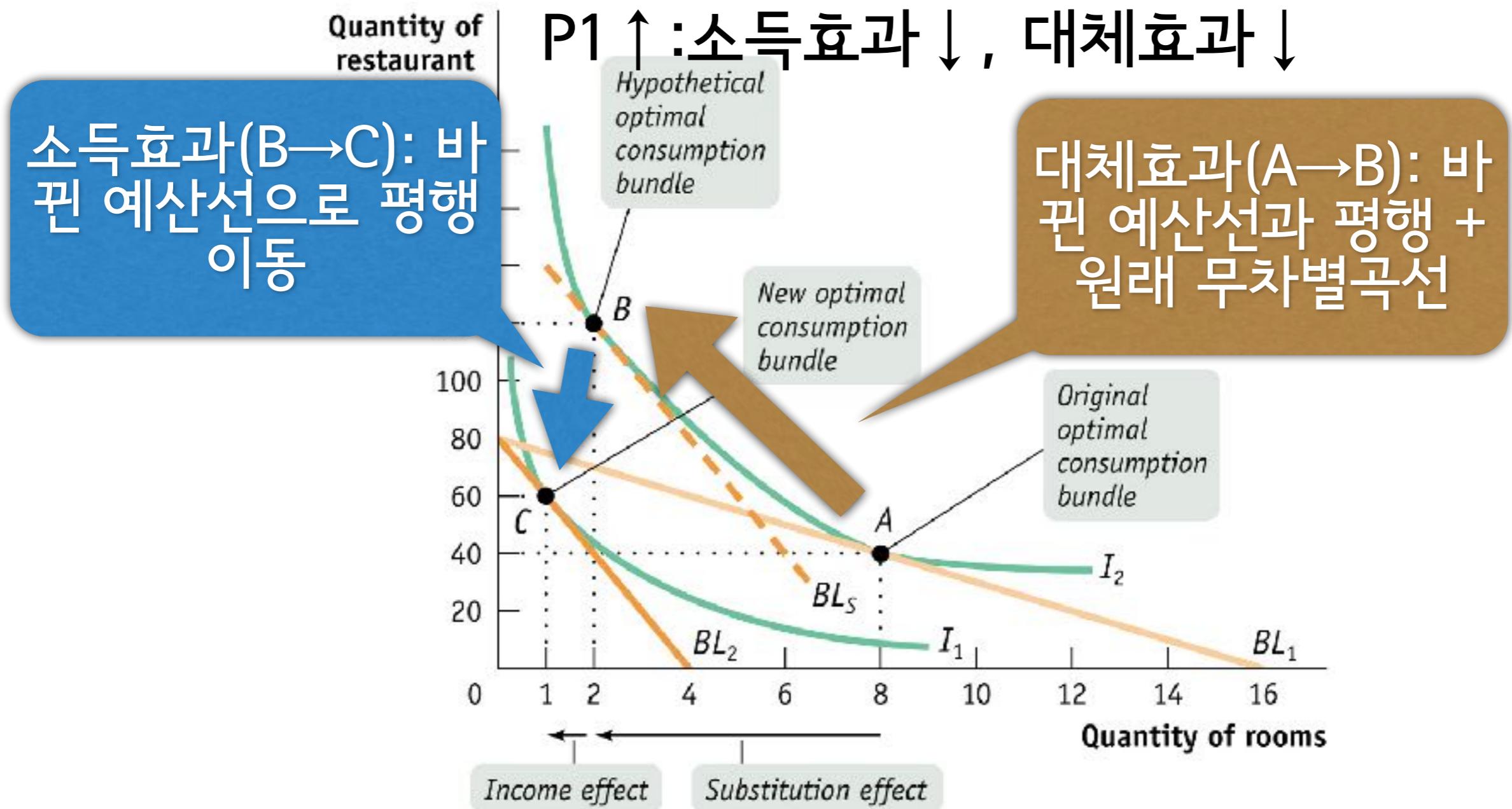


대체효과($A \rightarrow B$): 바뀐 예산선과 평행 + 원래 무차별곡선

Review: Hicks Decomposition (정상재)

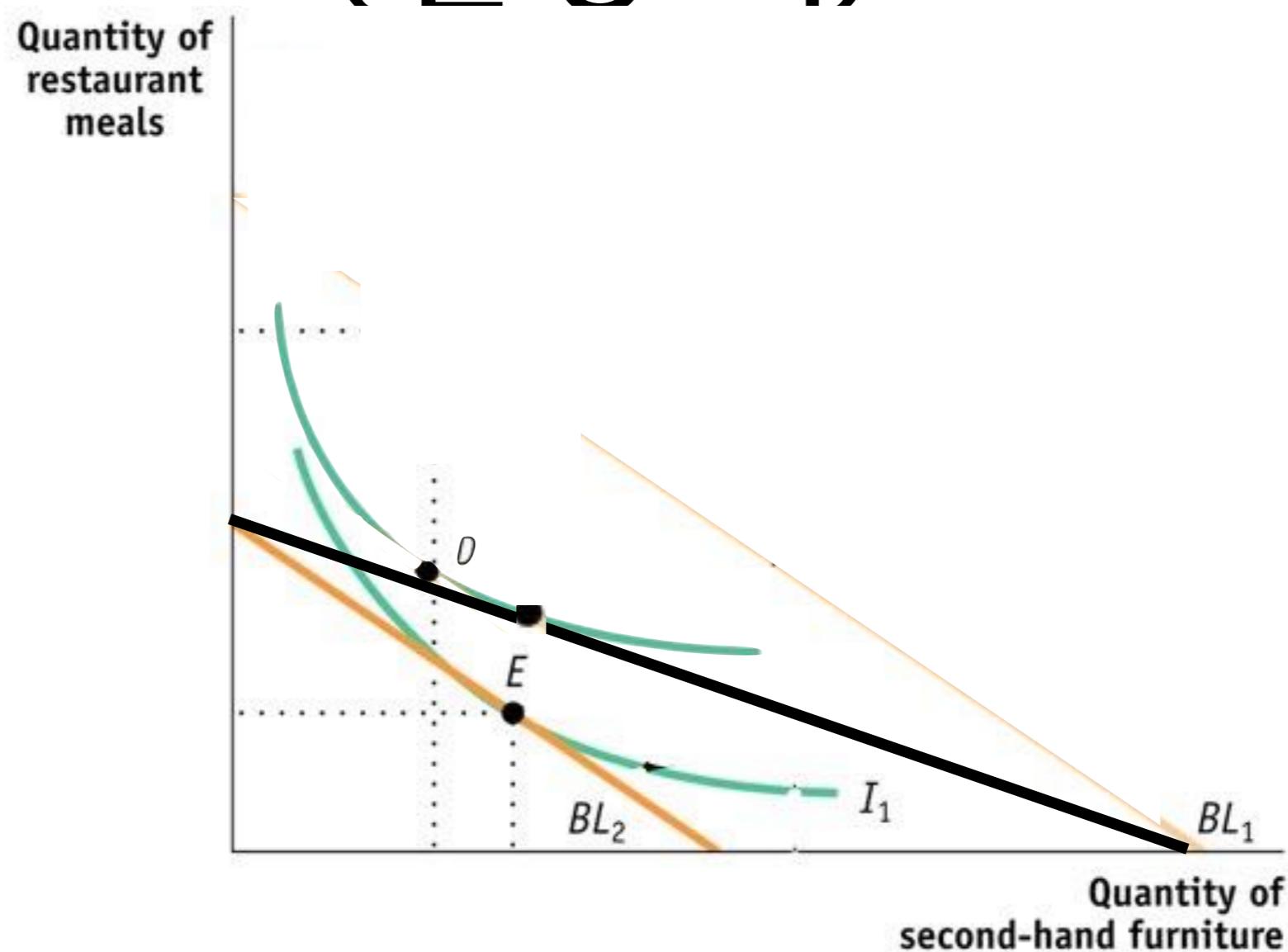


Review: Hicks Decomposition (정상재)

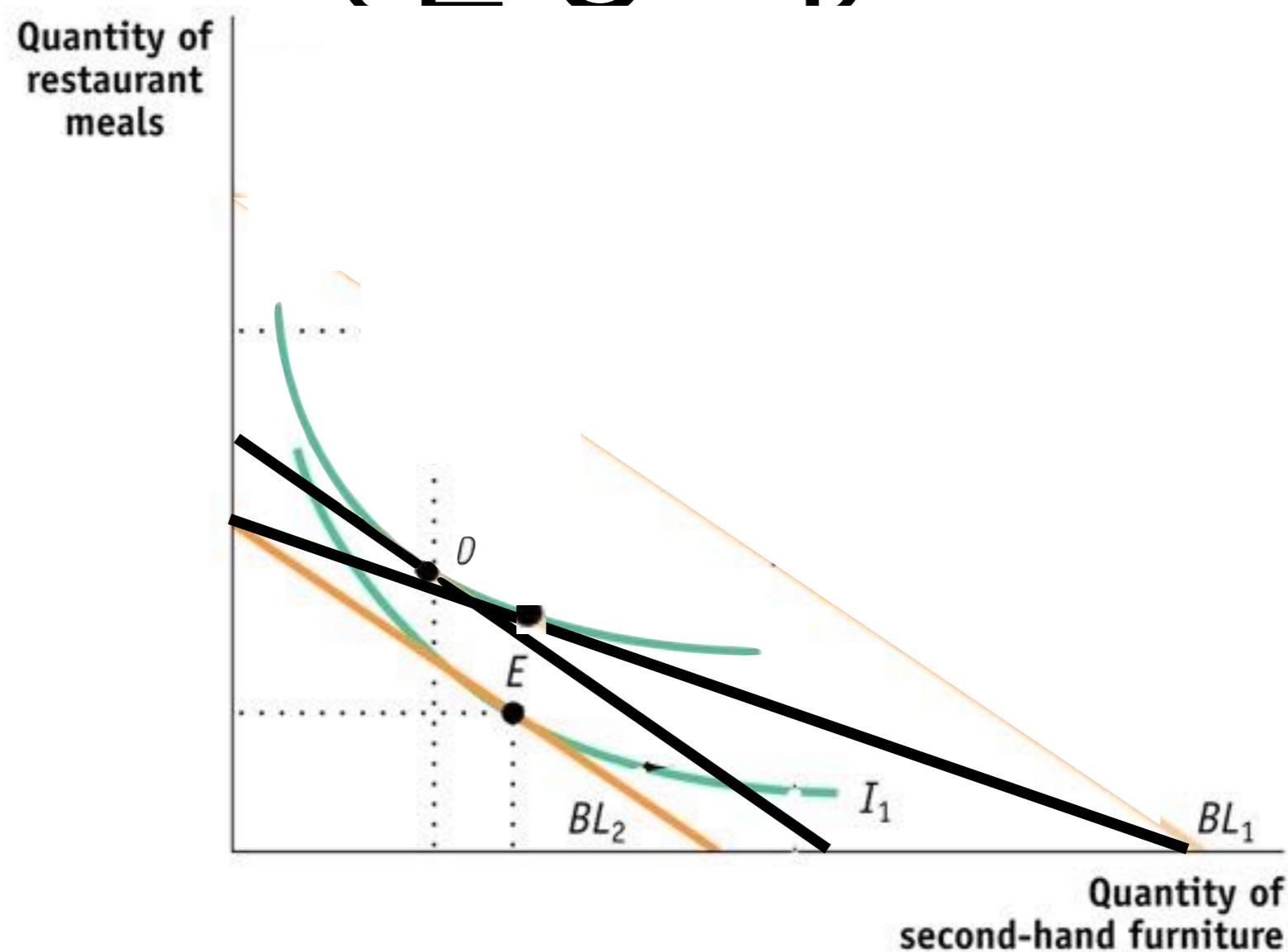


Hicks Decomposition (열등재)

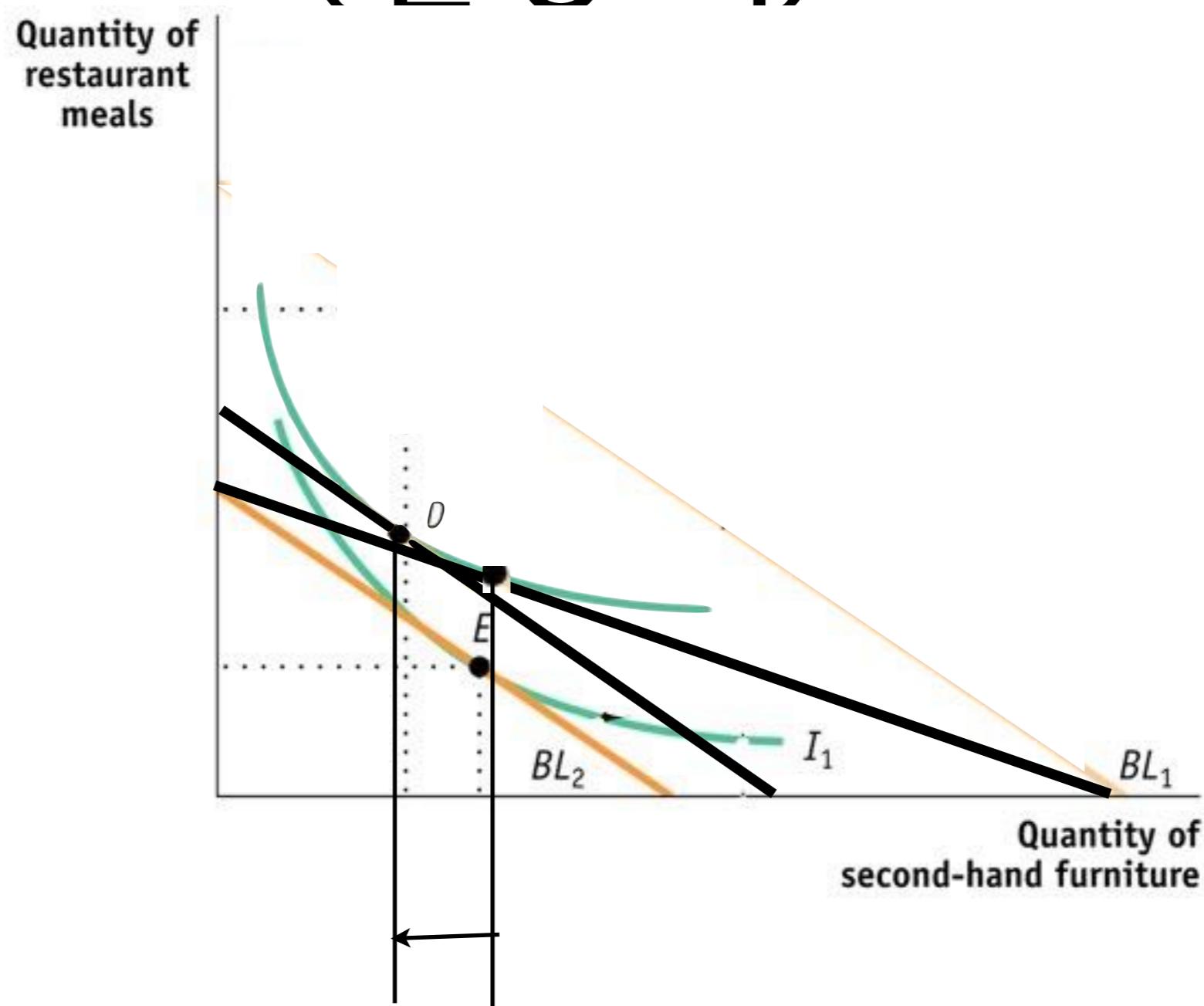
Hicks Decomposition (열등자)



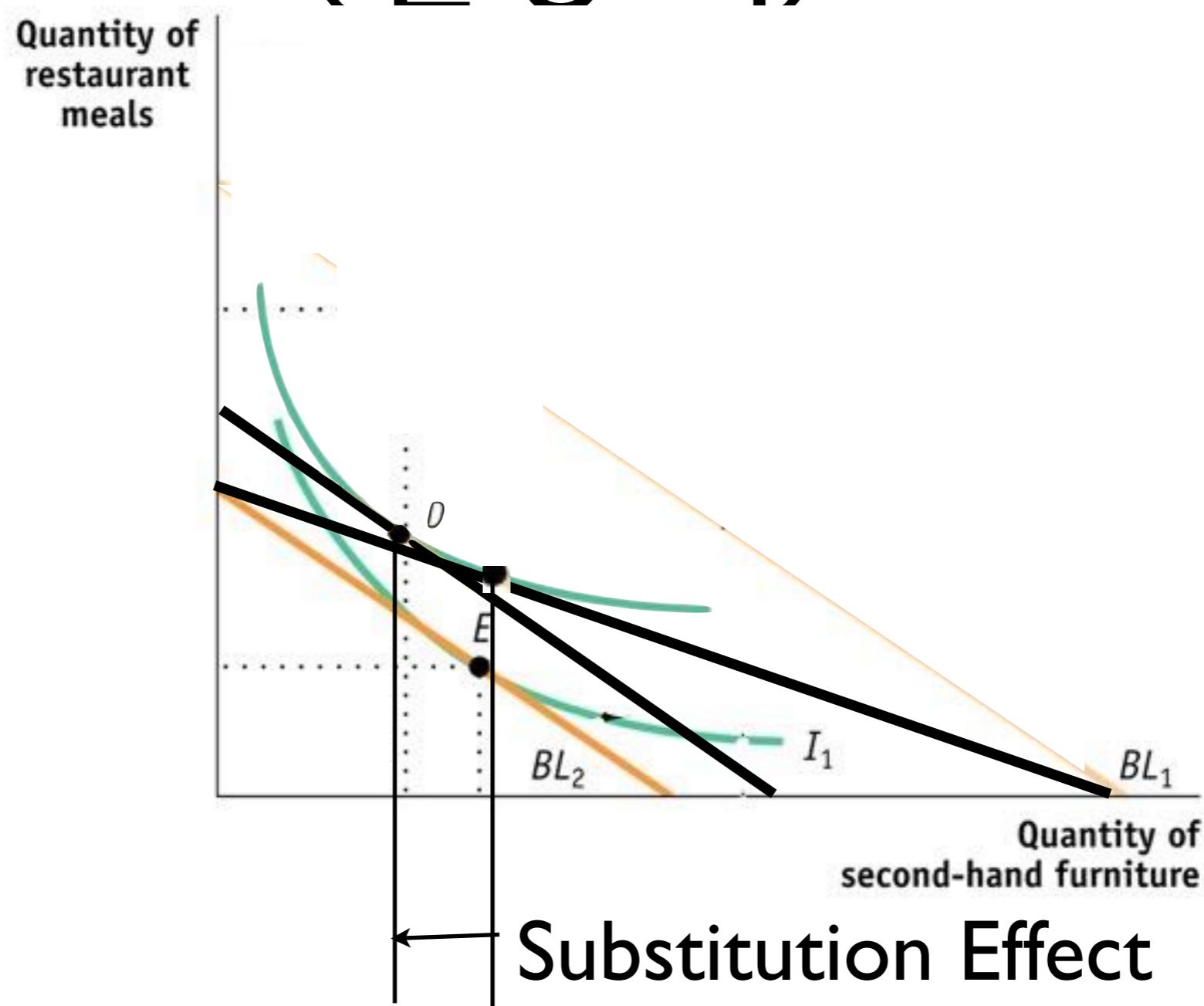
Hicks Decomposition (열등자)



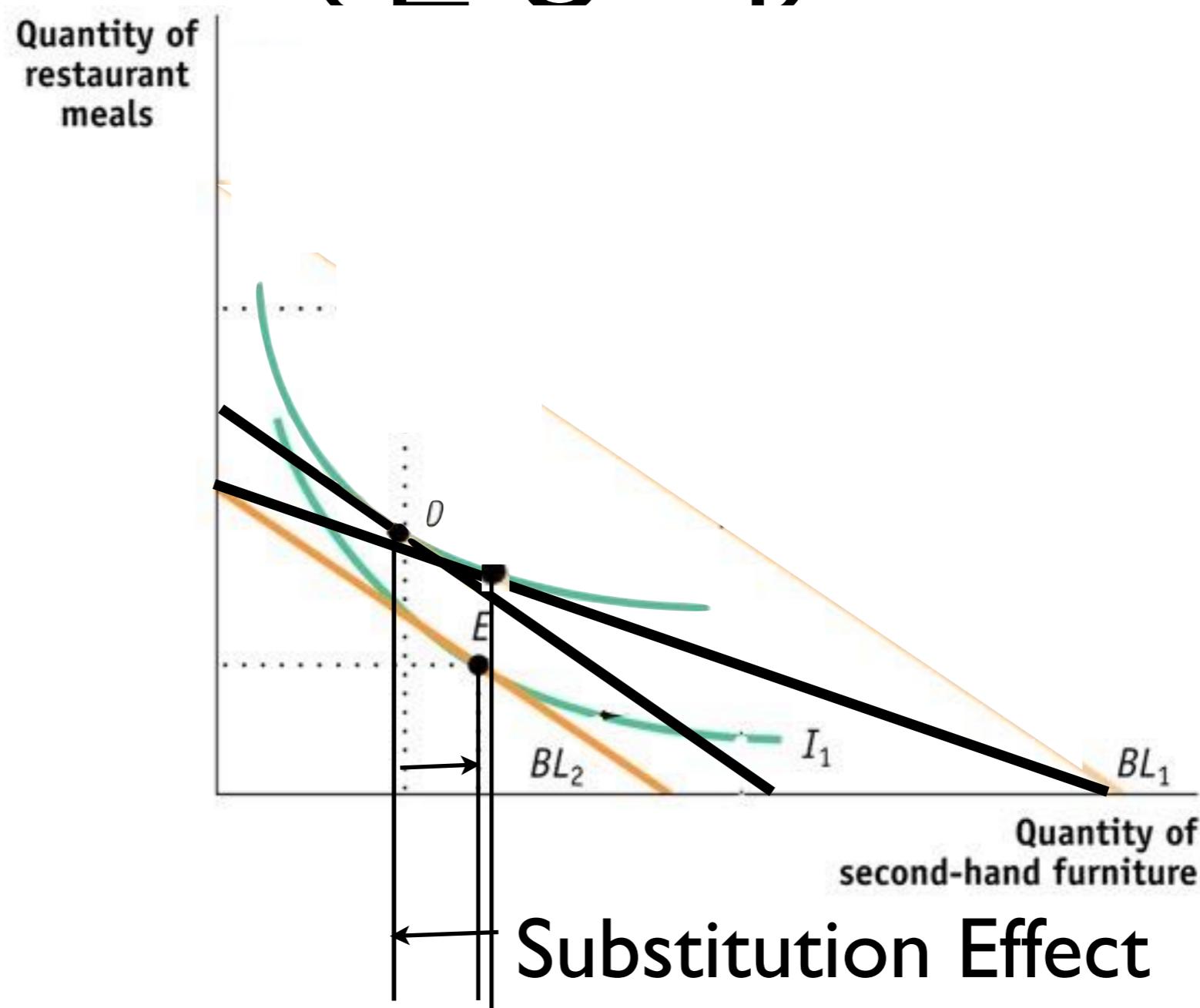
Hicks Decomposition (열등자)



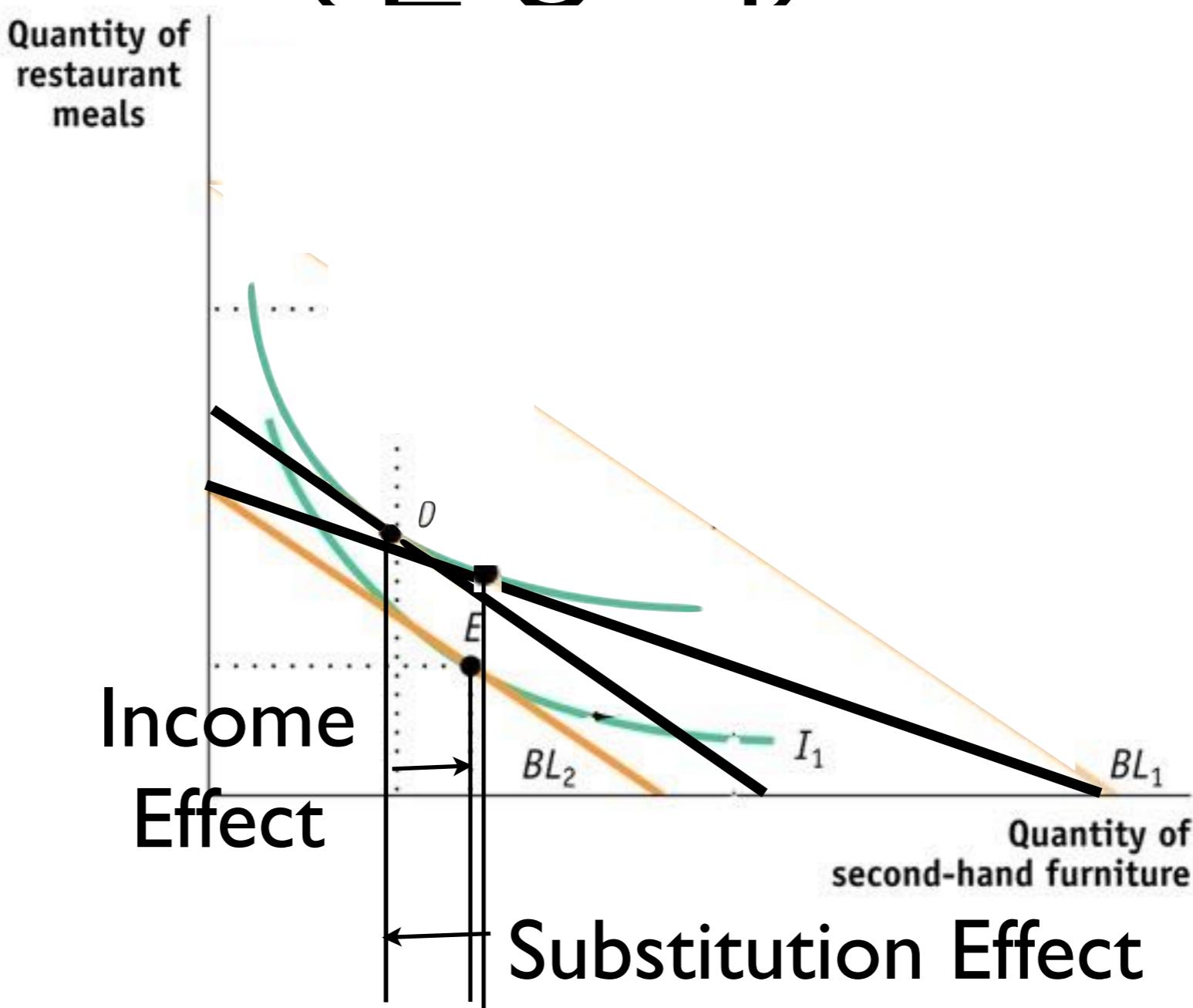
Hicks Decomposition (열등재)



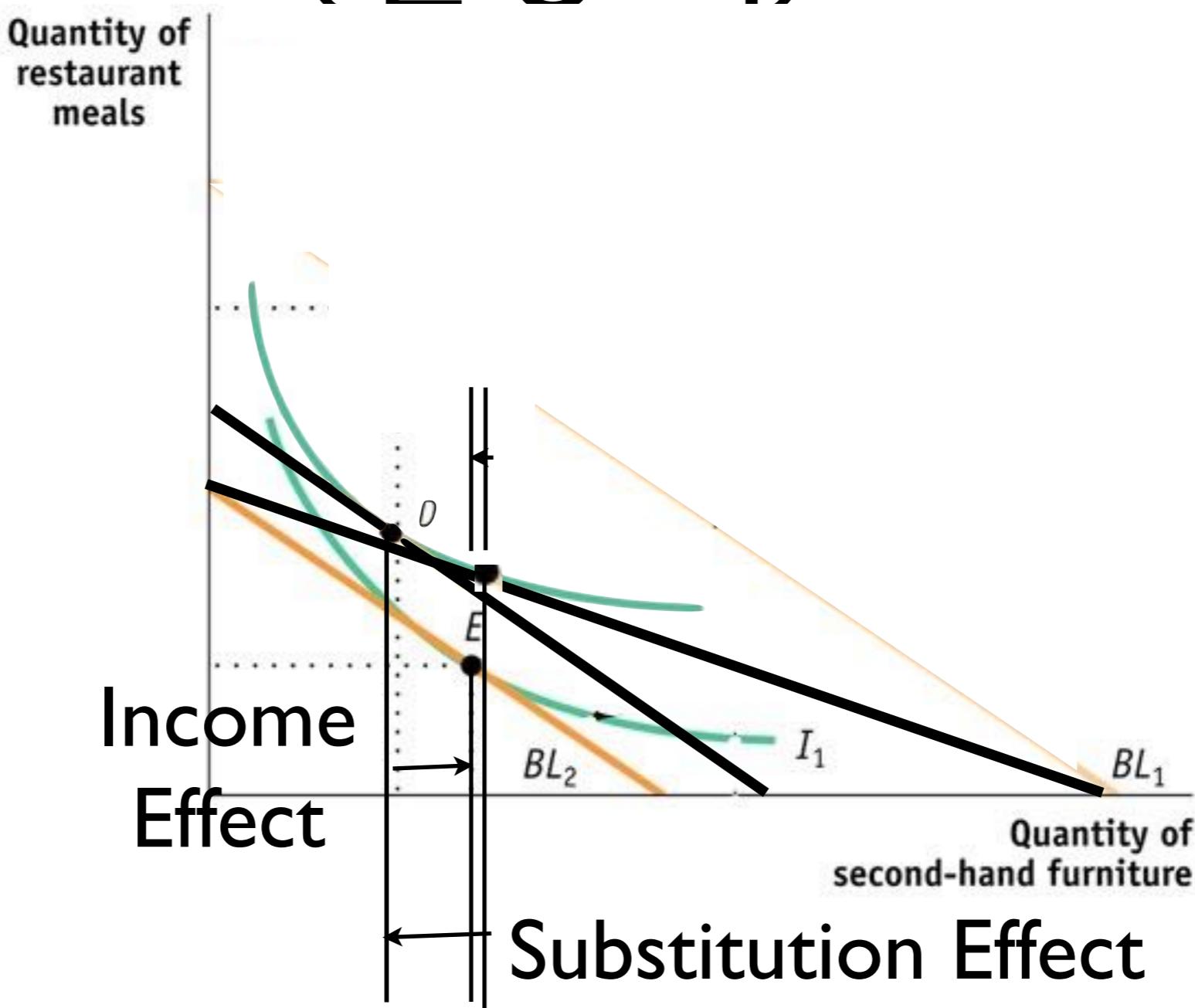
Hicks Decomposition (열등재)



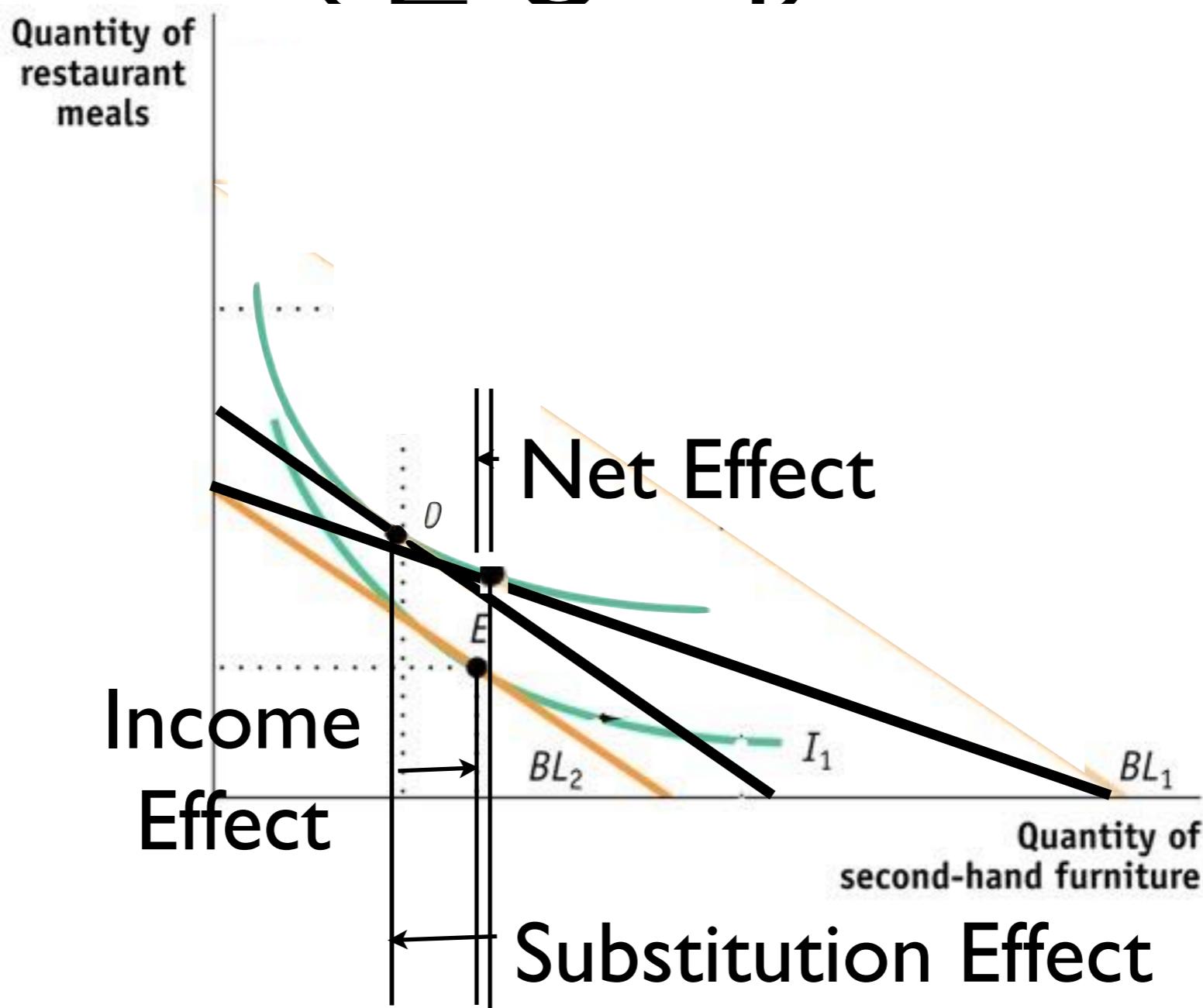
Hicks Decomposition (열등재)



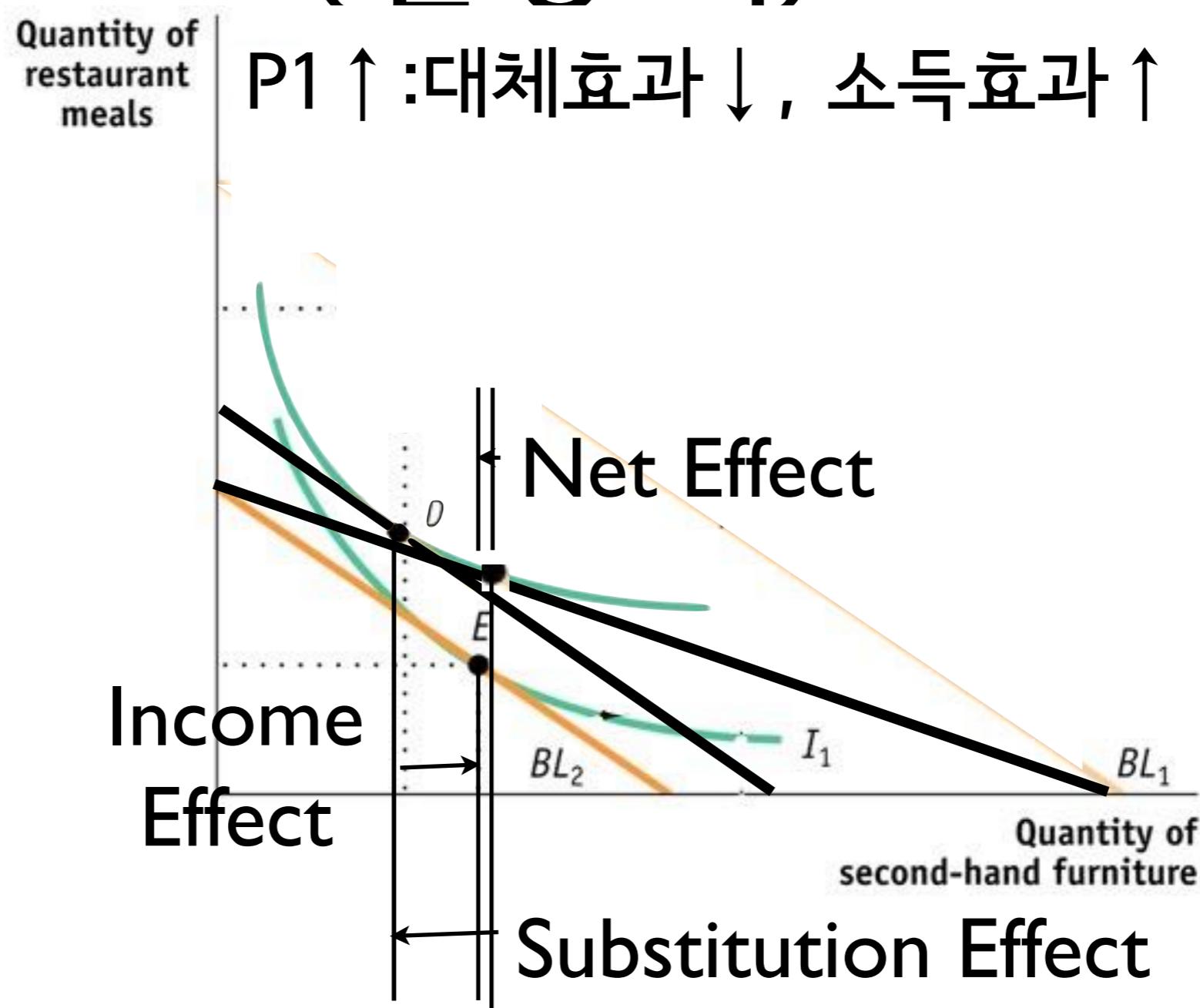
Hicks Decomposition (열등재)



Hicks Decomposition (열등재)

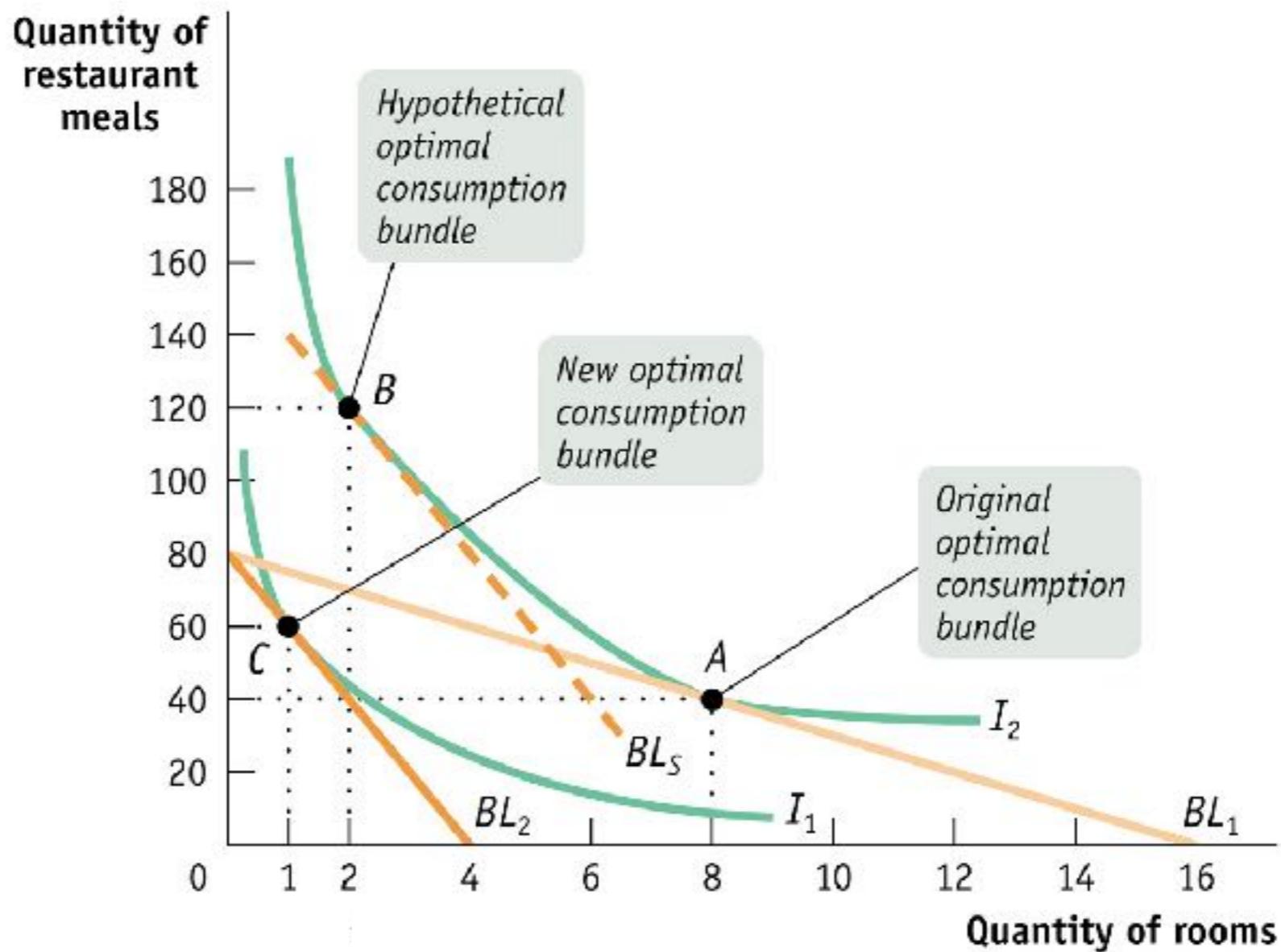


Hicks Decomposition (열등재)

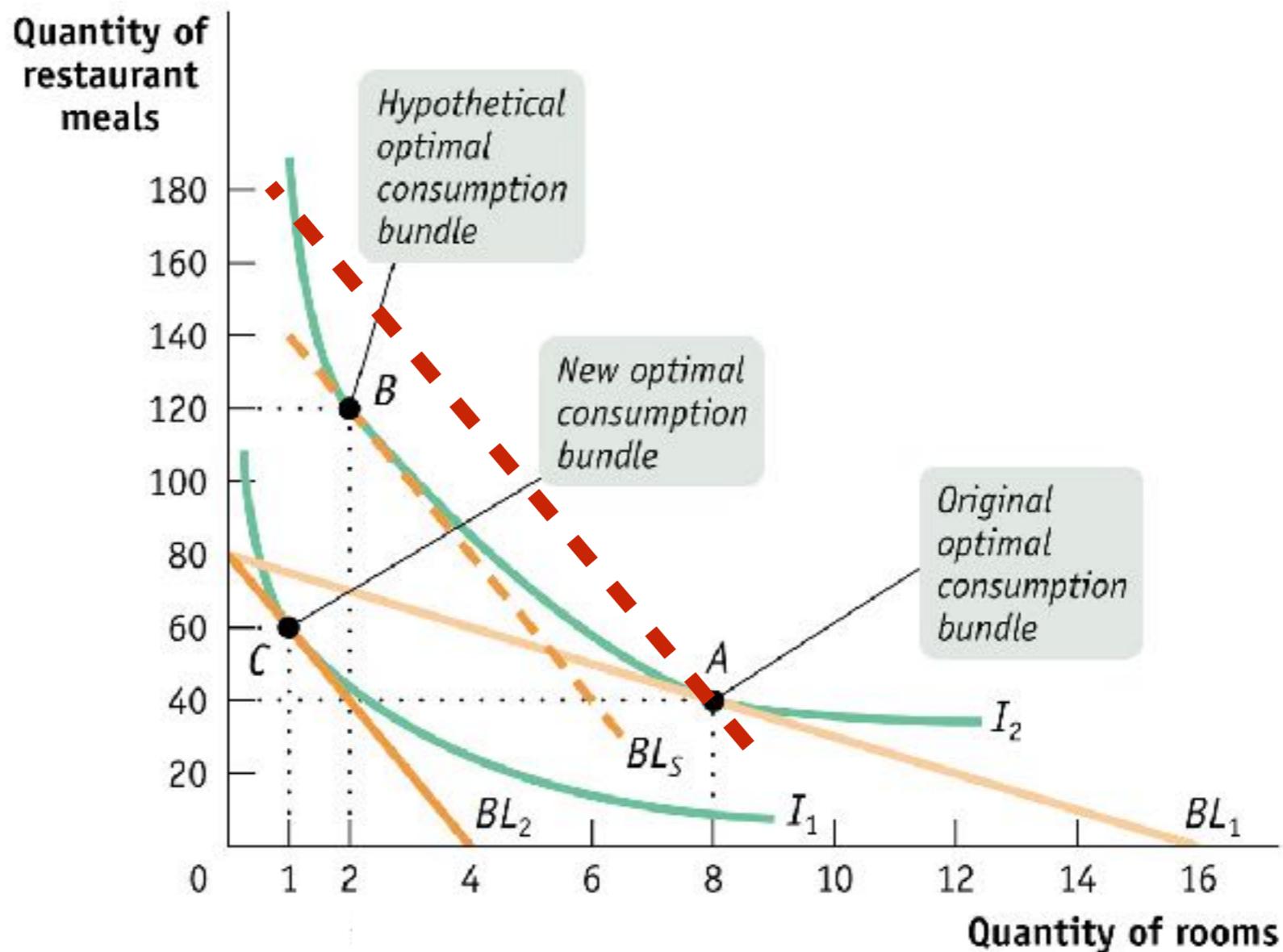


Hicks versus Slutsky Decomposition

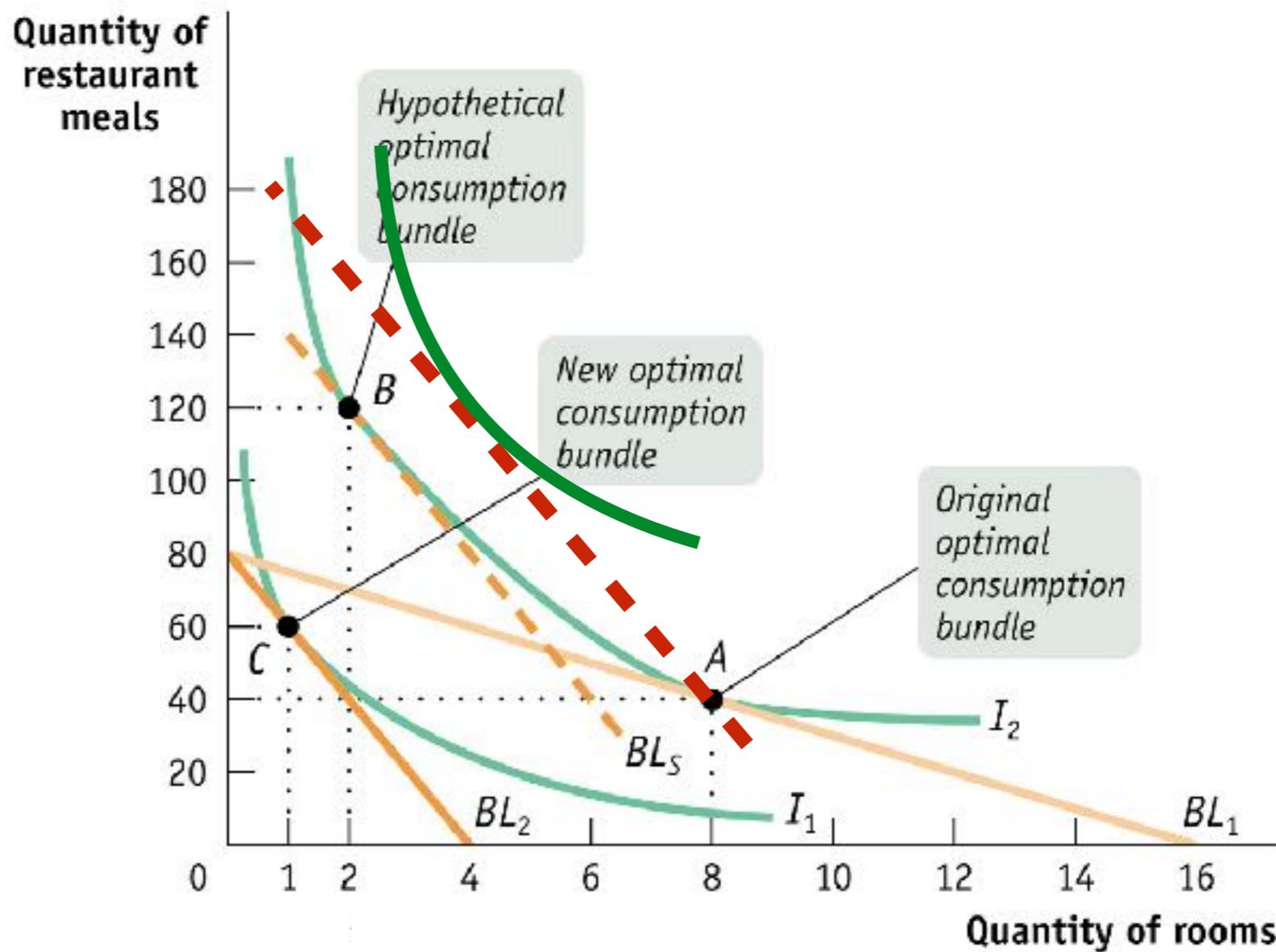
Hicks versus Slutsky Decomposition



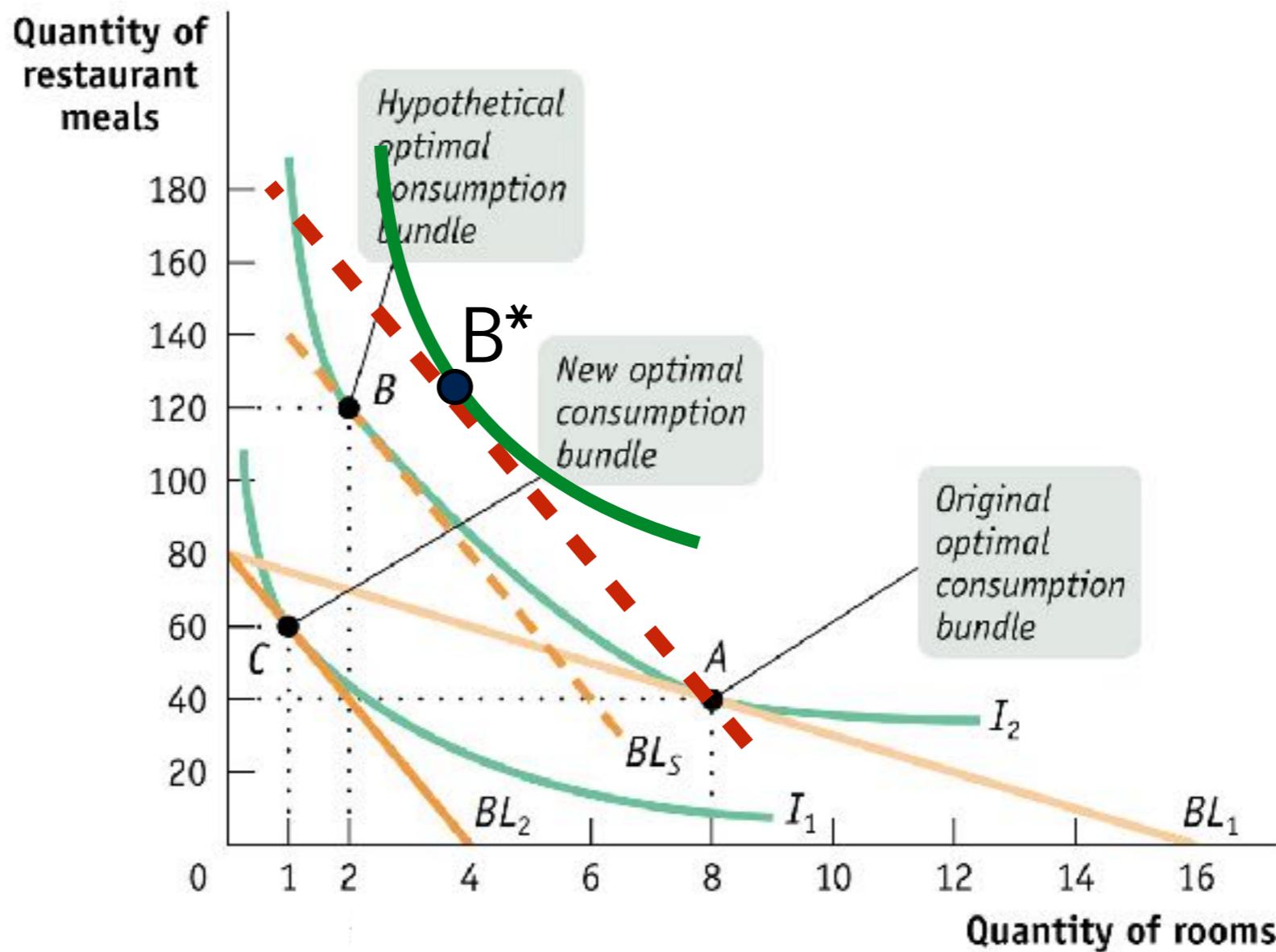
Hicks versus Slutsky Decomposition



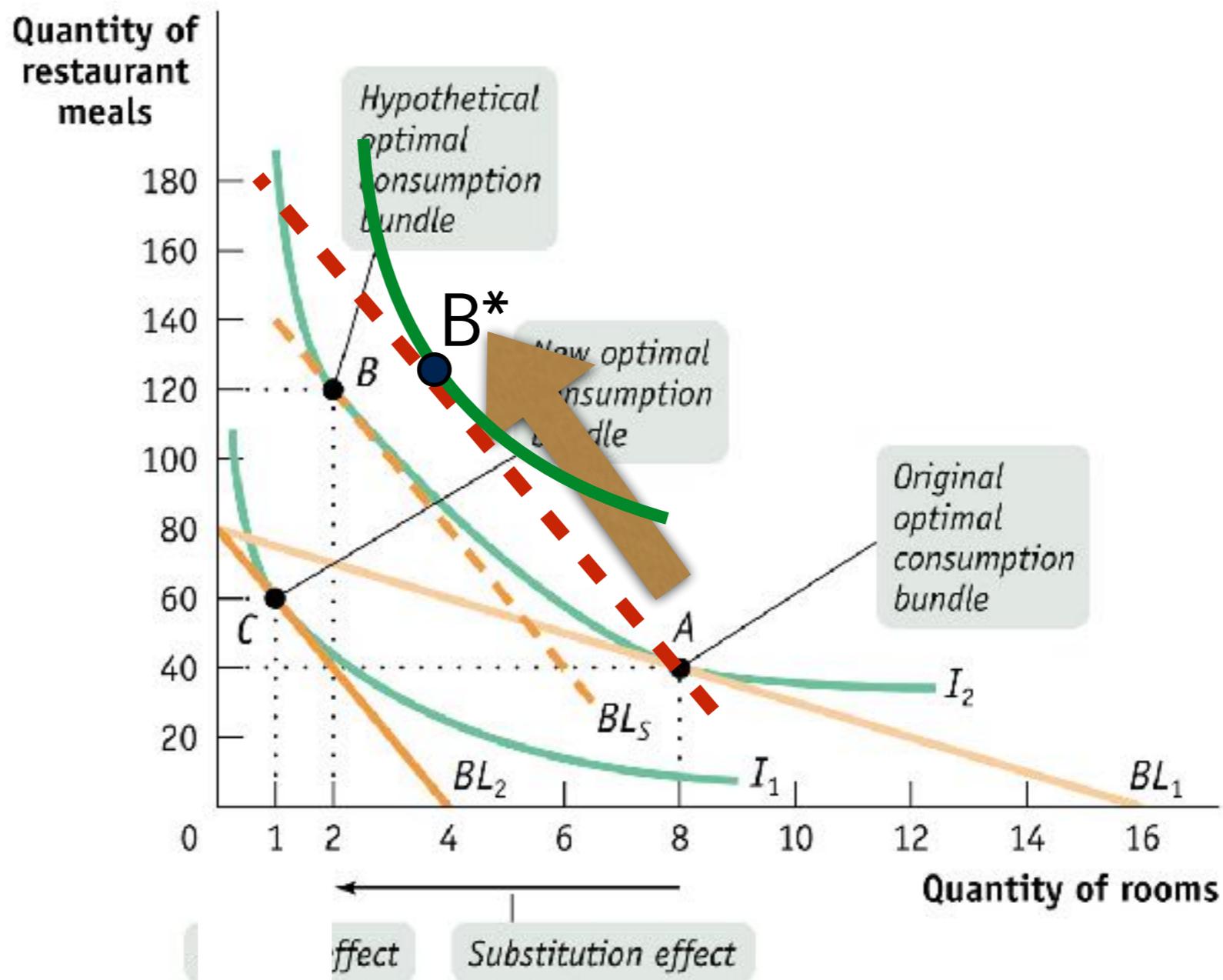
Hicks versus Slutsky Decomposition



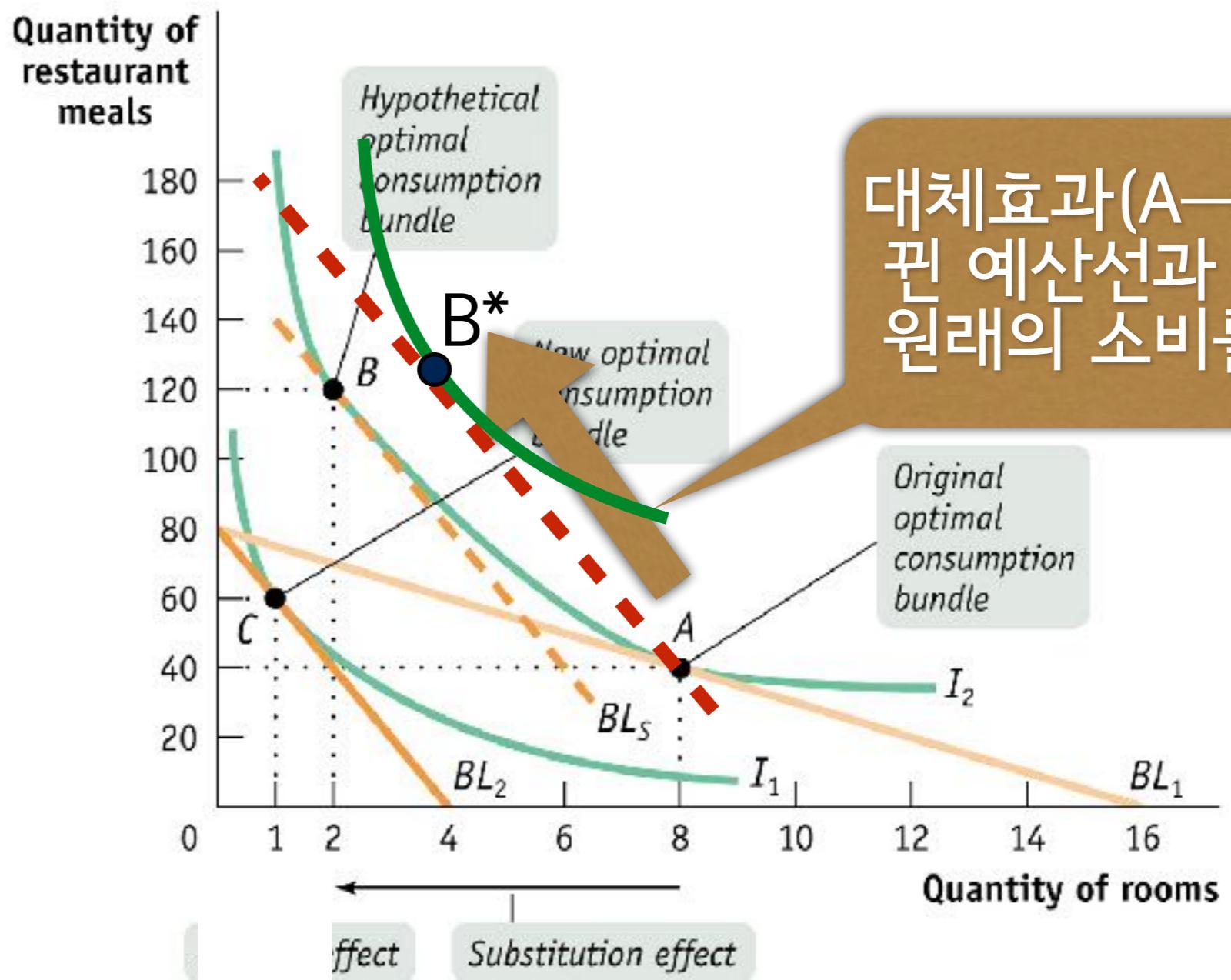
Hicks versus Slutsky Decomposition



Hicks versus Slutsky Decomposition

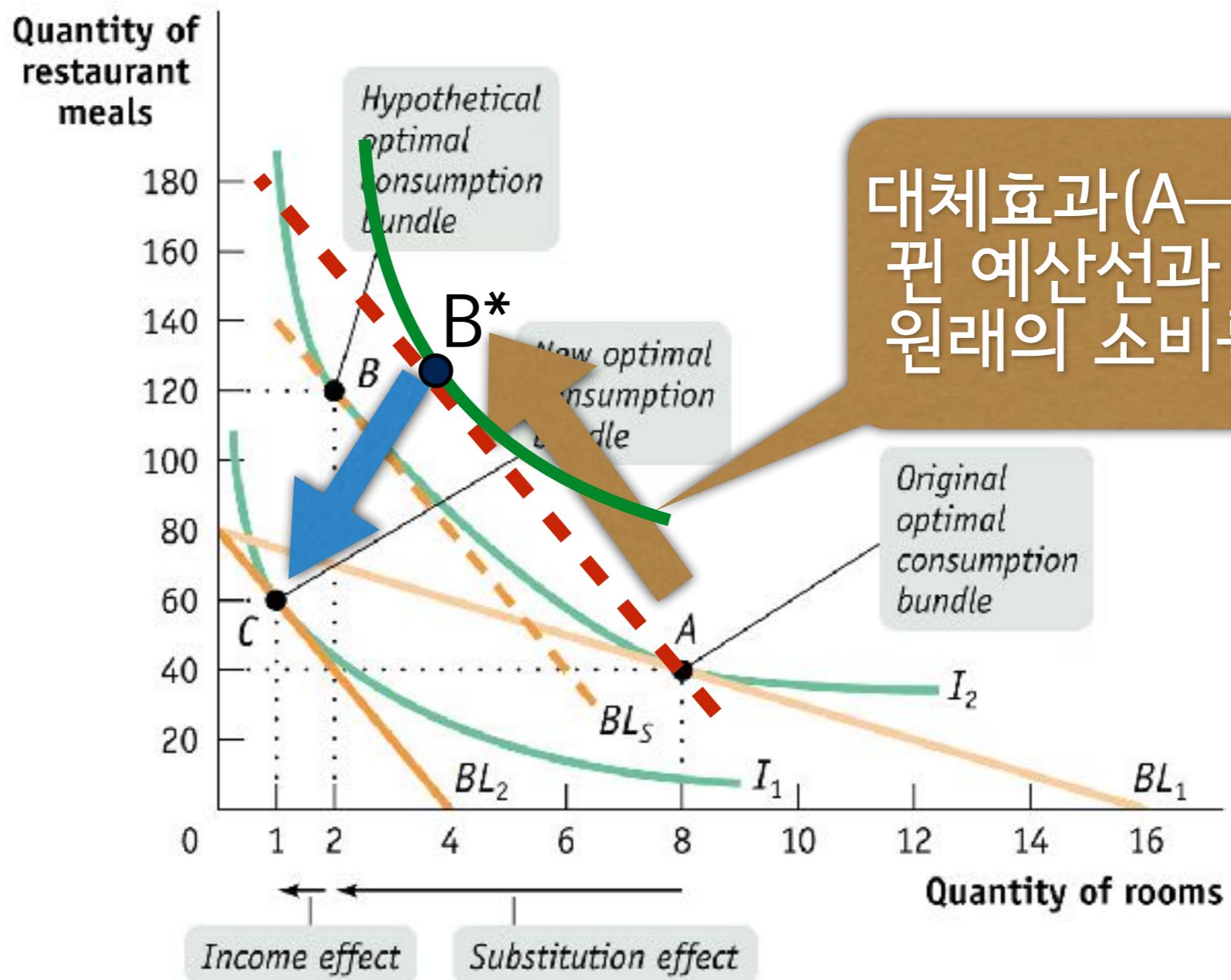


Hicks versus Slutsky Decomposition



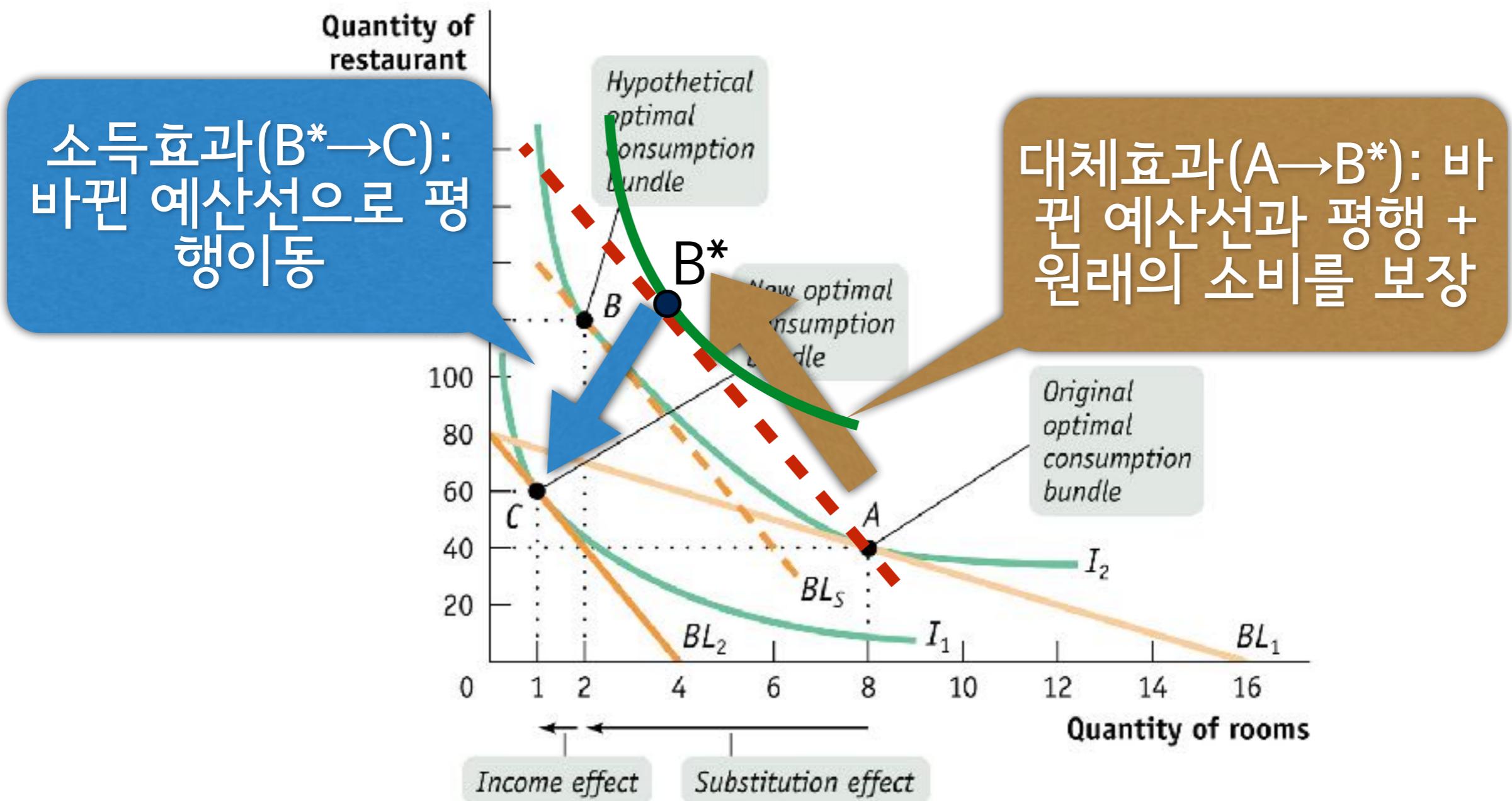
대체효과($A \rightarrow B^*$): 바뀐 예산선과 평행 + 원래의 소비를 보장

Hicks versus Slutsky Decomposition

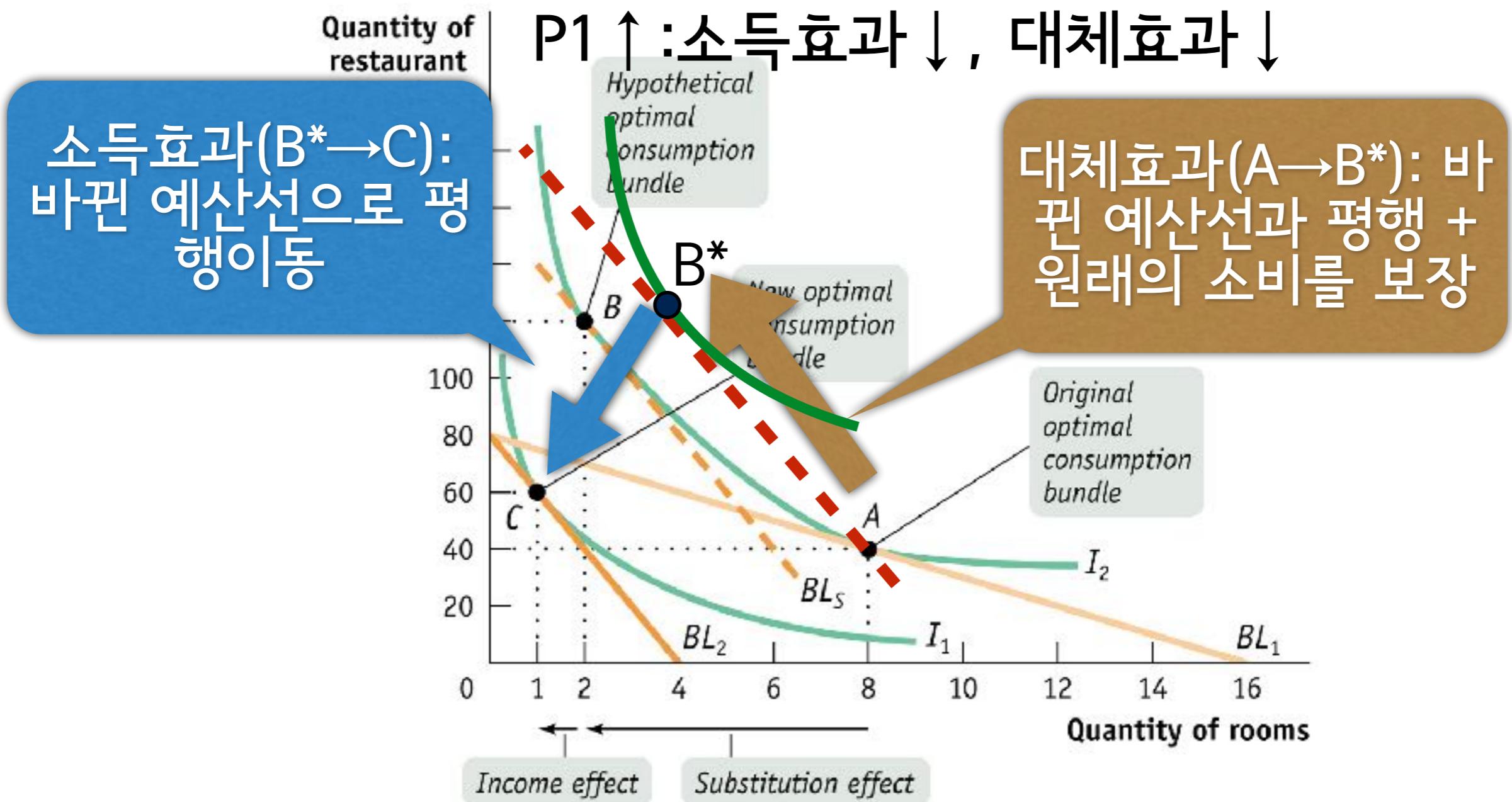


대체효과($A \rightarrow B^*$): 바뀐 예산선과 평행 + 원래의 소비를 보장

Hicks versus Slutsky Decomposition

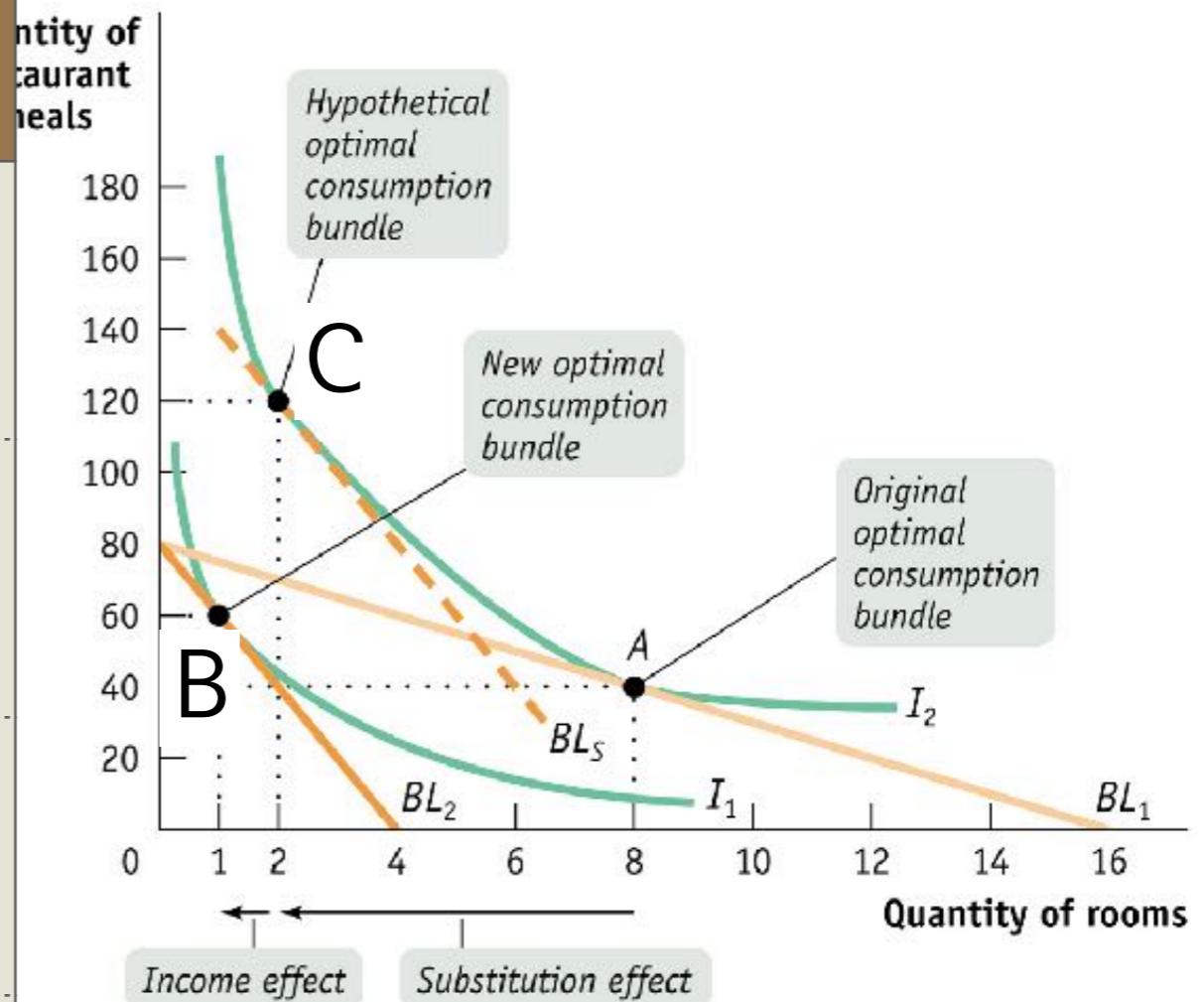


Hicks versus Slutsky Decomposition

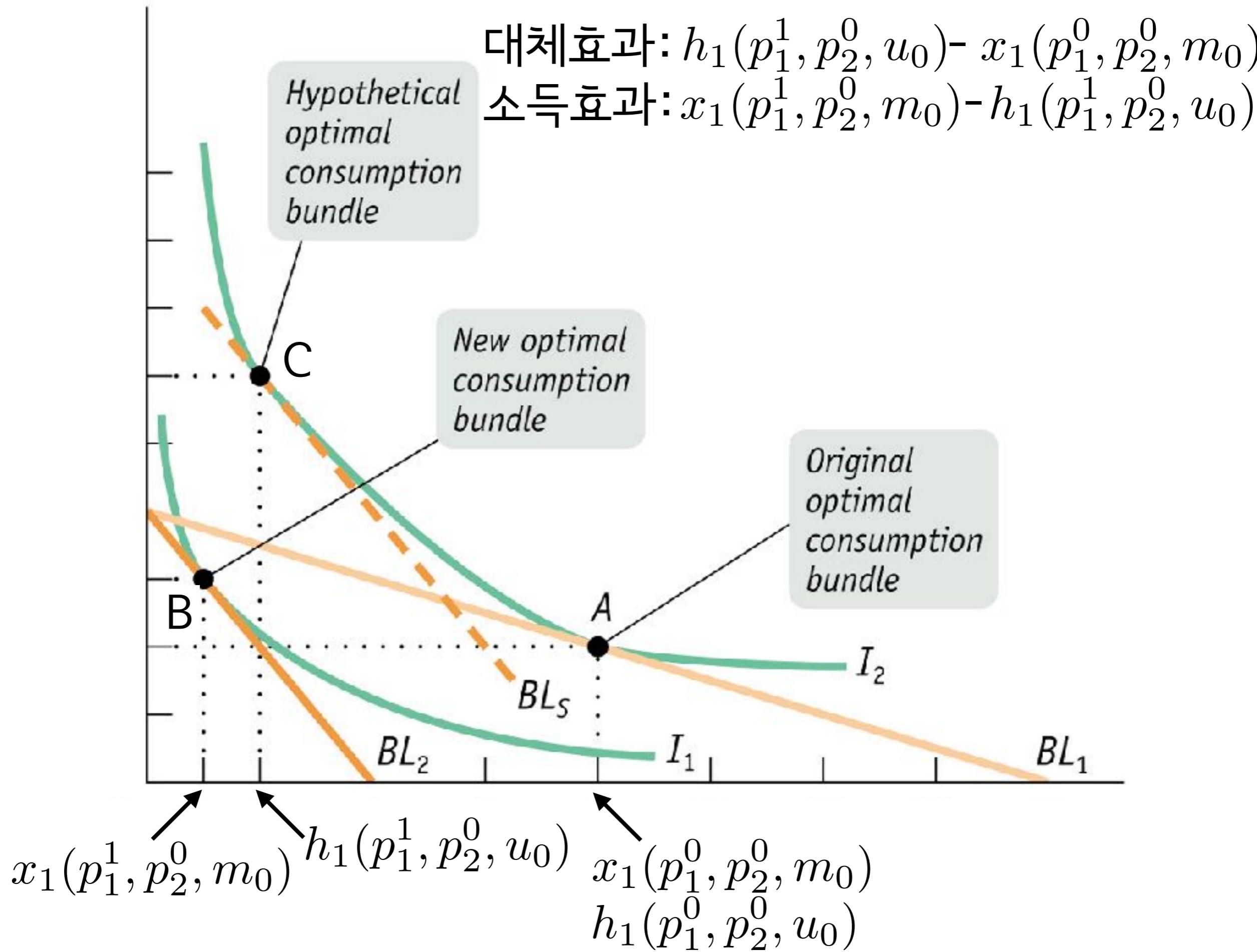


Hicks Decomposition: 수리적 분석 (p_1 가격변화)

의미	표현
효용극대화 소비량	$x_i(p_1, p_2, m)$
지출극소화 소비량	$h_i(p_1, p_2, u)$
시초상태 (A)	p_1^0, p_2^0, m_0, u_0
검토 상황($A \rightarrow B$)	$p_1^0 \rightarrow p_1^1$



*편의상 주텍스트와 notation 일치시킴



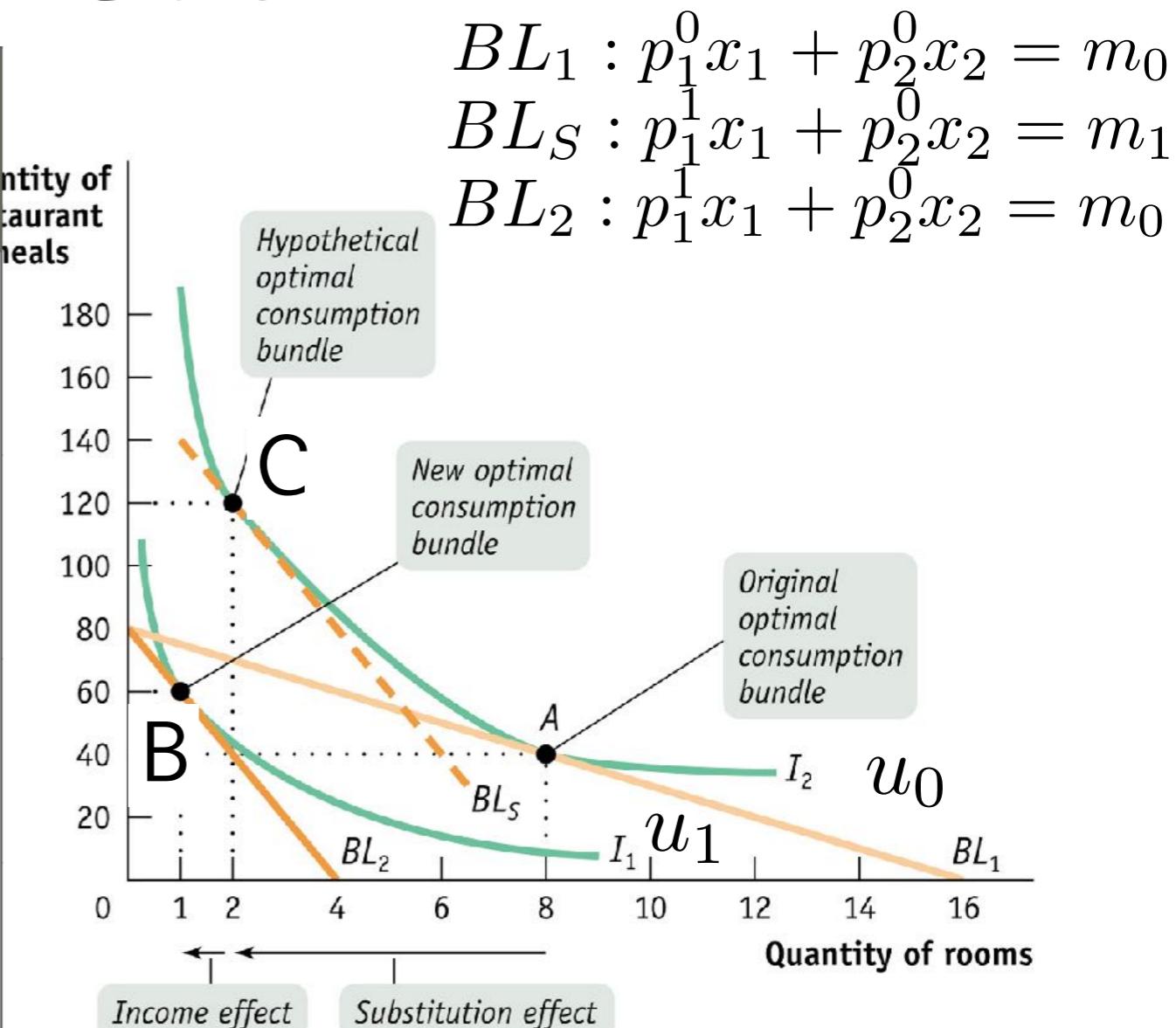
소비자후생

가격 변화와 소비자 후생

- 상품1의 가격이 변할 때 그로 인해 효용이 변화
 - 가격상승 \Rightarrow 효용감소, 가격하락 \Rightarrow 효용증가
 - 효용의 서수적 성격으로 인해 효용변화를 양적으로 표현하는데 어려움 존재
- 지출함수의 아이디어를 통해 가격변화로 인한 만족감 변화의 정도를 화폐단위로 표시하고자 함
 - 모든 u 에 대해 보상수요가 존재함을 이용
 - 각 u 에 대한 보상수요량을 위한 지출 e 계산

Hicks Decomposition Revisited:

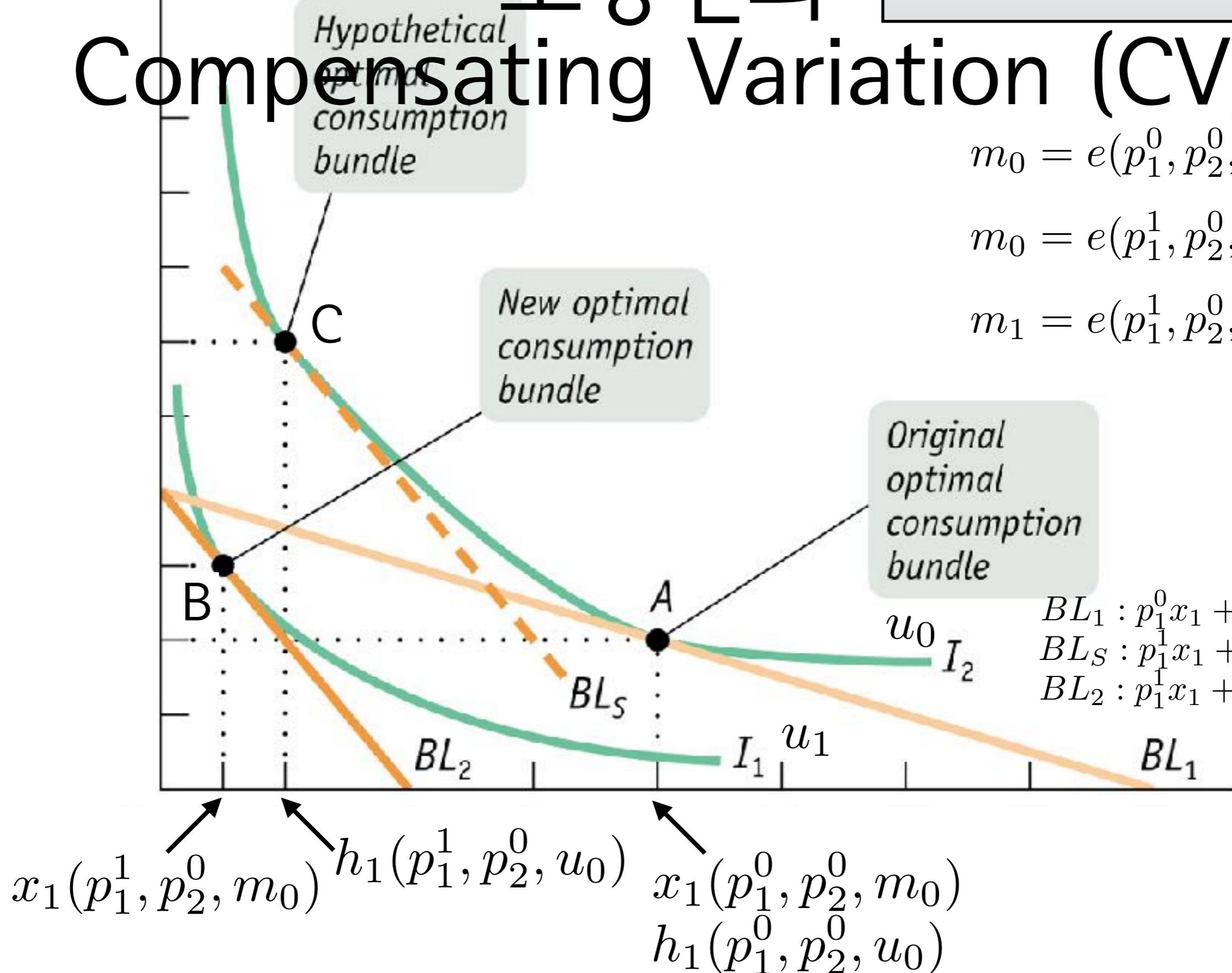
의미	표현
효용극대화 소비량	$x_i(p_1, p_2, m)$
지출극소화 소비량	$h_i(p_1, p_2, u)$
시초상태 (A)	p_1^0, p_2^0, m_0, u_0
검토 상황(A→B)	$p_1^0 \rightarrow p_1^1$
효용변화(A→B)	$u_0 \rightarrow u_1$



*편의상 주텍스트와 notation 일치시킴

$$CV := m_1 - m_0$$

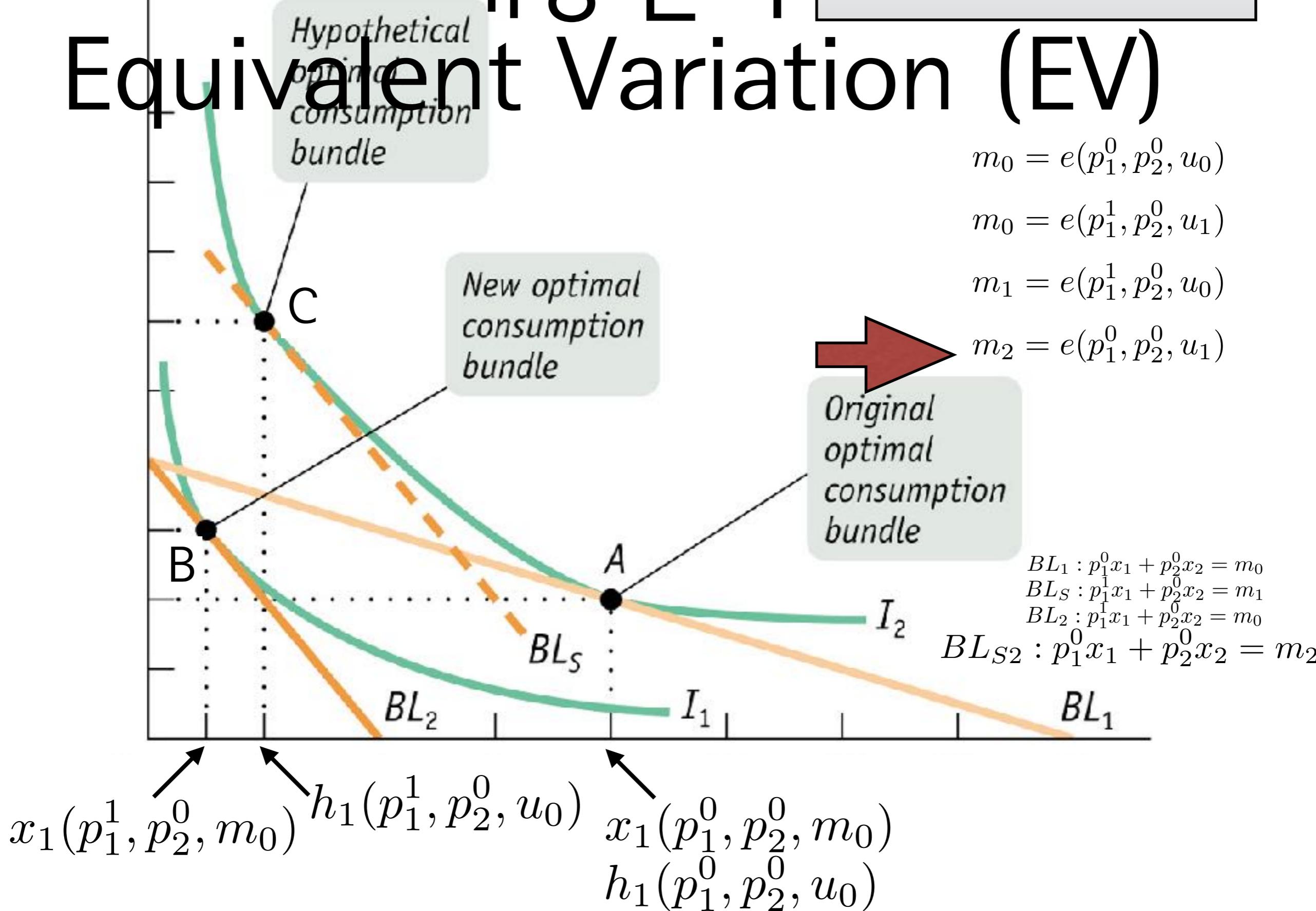
보상변화 Compensating Variation (CV)



대등변화

$$EV := m_2 - m_0$$

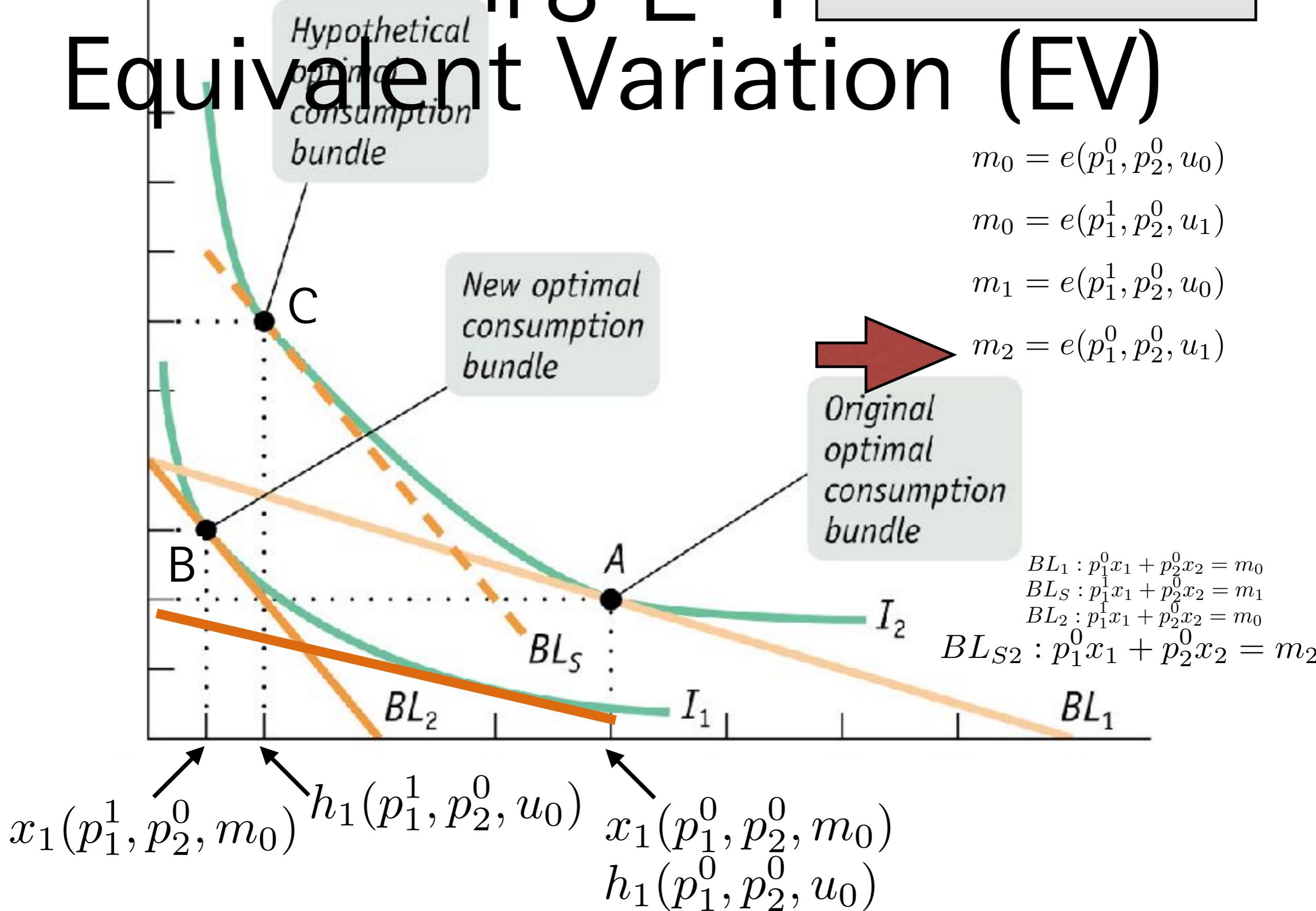
Equivalent Variation (EV)



대등변화

$$EV := m_2 - m_0$$

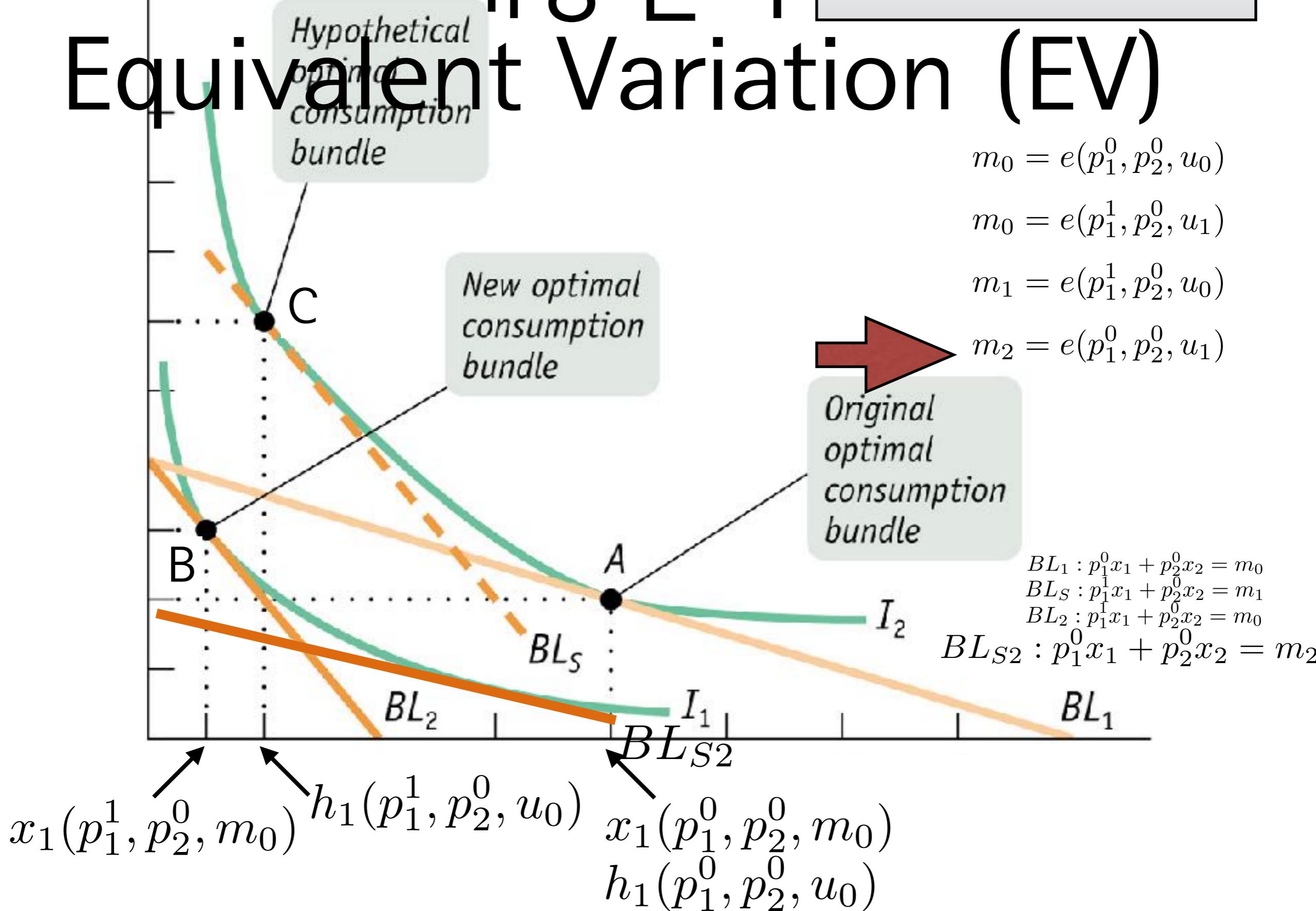
Equivalent Variation (EV)



대등변화

$$EV := m_2 - m_0$$

Equivalent Variation (EV)



보상변화와 대등변화

- 아이디어는 비슷하지만 두 값은 부호만 같고 크기는 다름
 - 가격변화로 효용 증가시 +, 감소시 -
- 도출에 필요한 지출함수 e 는 효용의 측정을 전제로 함 \Rightarrow 실제 계산에 한계 존재
- 대안: 소비자잉여

소비자잉여 Consumer Surplus

지불용의와 수요곡선

Willingness to Pay and Demand cv.

- 지불용의(WTP):
 - 어떤 재화를 구매할 때 지불하고자 하는 최대금액
 - 유보가격(reserve price)라고도 함
- 모든 수요자의 지불용의수준 조사
⇒ 수요계획 도출

수요자의 지불용의목록

수요자의 지불용의목록



수요자의 지불용의목록

잠재수요자	지불용의(\$)
Aleisha	59
Brad	45
Claudia	35
Darren	25
Edwina	10



수요계획구하기

Demand Schedule

수요계획구하기

Demand Schedule

잠재수요자	지불용의(\$)
Aleisha	59
Brad	45
Claudia	35
Darren	25
Edwina	10

수요계획구하기

Demand Schedule

잠재수요자	지불용의(\$)
Aleisha	59
Brad	45
Claudia	35
Darren	25
Edwina	10

수요계획구하기

Demand Schedule

잠재수요자	지불용의(\$)
Aleisha	59
Brad	45
Claudia	35
Darren	25
Edwina	10

가격(\$)	

수요계획구하기

Demand Schedule

잠재수요자	지불용의(\$)
Aleisha	59
Brad	45
Claudia	35
Darren	25
Edwina	10

가격(\$)	수요량(EA)

수요계획구하기

Demand Schedule

잠재수요자	지불용의(\$)
Aleisha	59
Brad	45
Claudia	35
Darren	25
Edwina	10

가격(\$)	수요량(EA)
over 59	

수요계획구하기

Demand Schedule

잠재수요자	지불용의(\$)
Aleisha	59
Brad	45
Claudia	35
Darren	25
Edwina	10

가격(\$)	수요량(EA)
over 59	0

수요계획구하기

Demand Schedule

잠재수요자	지불용의(\$)
Aleisha	59
Brad	45
Claudia	35
Darren	25
Edwina	10

가격(\$)	수요량(EA)
over 59	0
46~59	

수요계획구하기

Demand Schedule

잠재수요자	지불용의(\$)
Aleisha	59
Brad	45
Claudia	35
Darren	25
Edwina	10

가격(\$)	수요량(EA)
over 59	0
46~59	1

수요계획구하기

Demand Schedule

잠재수요자	지불용의(\$)
Aleisha	59
Brad	45
Claudia	35
Darren	25
Edwina	10

가격(\$)	수요량(EA)
over 59	0
46~59	1
36~45	

수요계획구하기

Demand Schedule

잠재수요자	지불용의(\$)
Aleisha	59
Brad	45
Claudia	35
Darren	25
Edwina	10

가격(\$)	수요량(EA)
over 59	0
46~59	1
36~45	2

수요계획구하기

Demand Schedule

잠재수요자	지불용의(\$)
Aleisha	59
Brad	45
Claudia	35
Darren	25
Edwina	10

가격(\$)	수요량(EA)
over 59	0
46~59	1
36~45	2
26~35	

수요계획구하기

Demand Schedule

잠재수요자	지불용의(\$)
Aleisha	59
Brad	45
Claudia	35
Darren	25
Edwina	10

가격(\$)	수요량(EA)
over 59	0
46~59	1
36~45	2
26~35	3

수요계획구하기

Demand Schedule

잠재수요자	지불용의(\$)
Aleisha	59
Brad	45
Claudia	35
Darren	25
Edwina	10

가격(\$)	수요량(EA)
over 59	0
46~59	1
36~45	2
26~35	3
11~25	

수요계획구하기

Demand Schedule

잠재수요자	지불용의(\$)
Aleisha	59
Brad	45
Claudia	35
Darren	25
Edwina	10

가격(\$)	수요량(EA)
over 59	0
46~59	1
36~45	2
26~35	3
11~25	4

수요계획구하기

Demand Schedule

잠재수요자	지불용의(\$)
Aleisha	59
Brad	45
Claudia	35
Darren	25
Edwina	10

가격(\$)	수요량(EA)
over 59	0
46~59	1
36~45	2
26~35	3
11~25	4
below 10	

수요계획구하기

Demand Schedule

잠재수요자	지불용의(\$)
Aleisha	59
Brad	45
Claudia	35
Darren	25
Edwina	10

가격(\$)	수요량(EA)
over 59	0
46~59	1
36~45	2
26~35	3
11~25	4
below 10	5

수요곡선도출

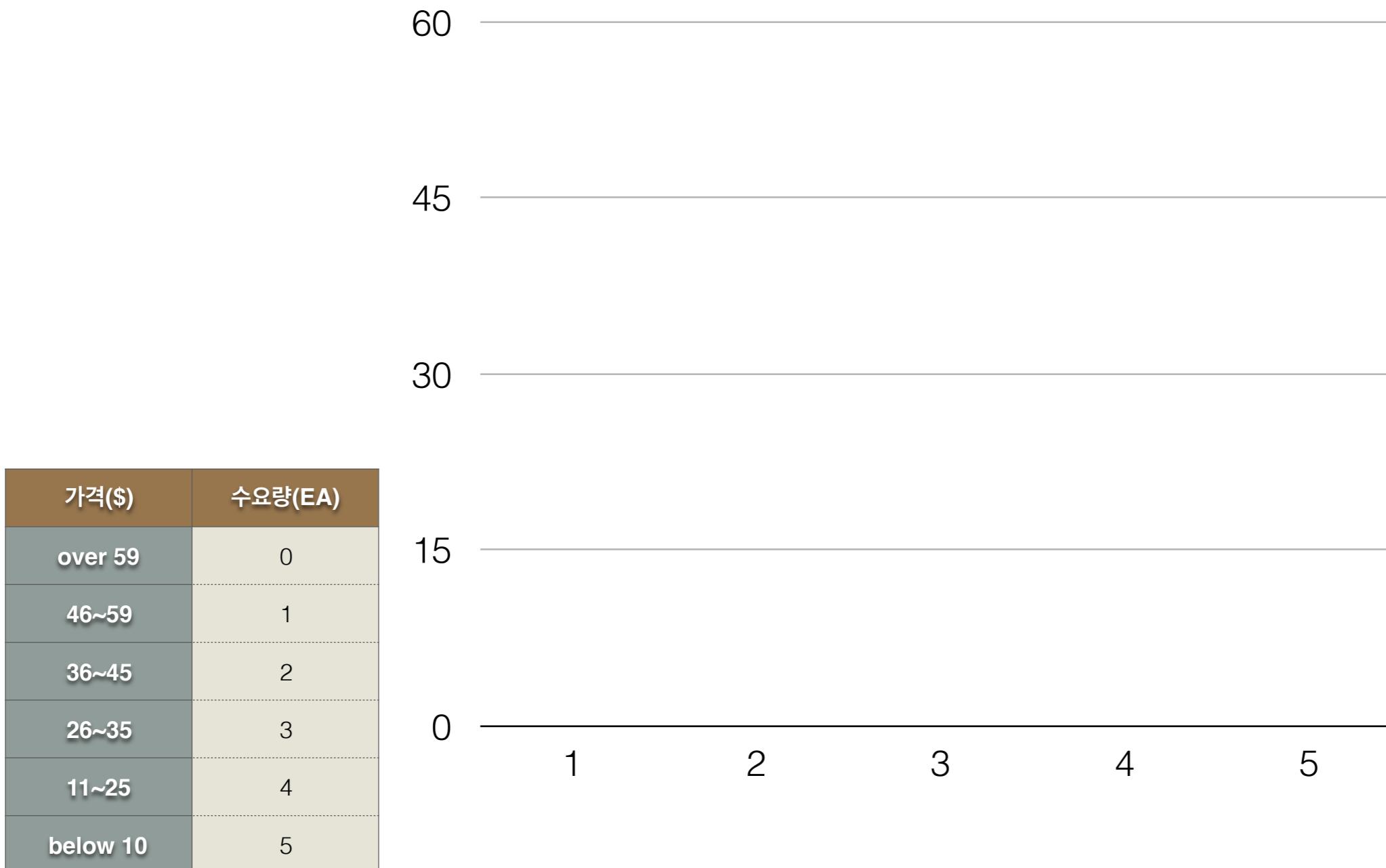
Demand Curve

가격(\$)	수요량(EA)
over 59	0
46~59	1
36~45	2
26~35	3
11~25	4
below 10	5

수요곡선도출 Demand Curve

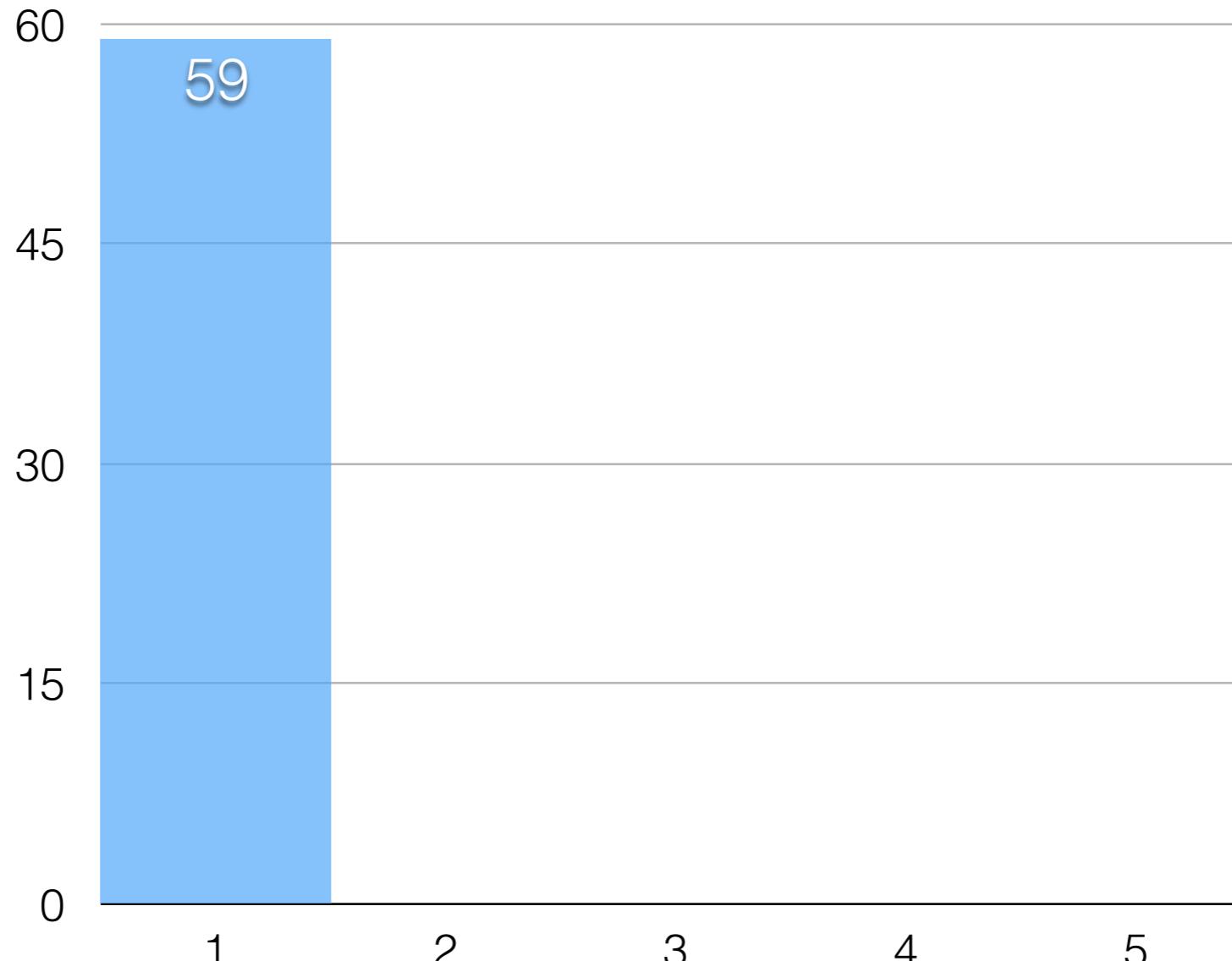
가격(\$)	수요량(EA)
over 59	0
46~59	1
36~45	2
26~35	3
11~25	4
below 10	5

수요곡선도출 Demand Curve



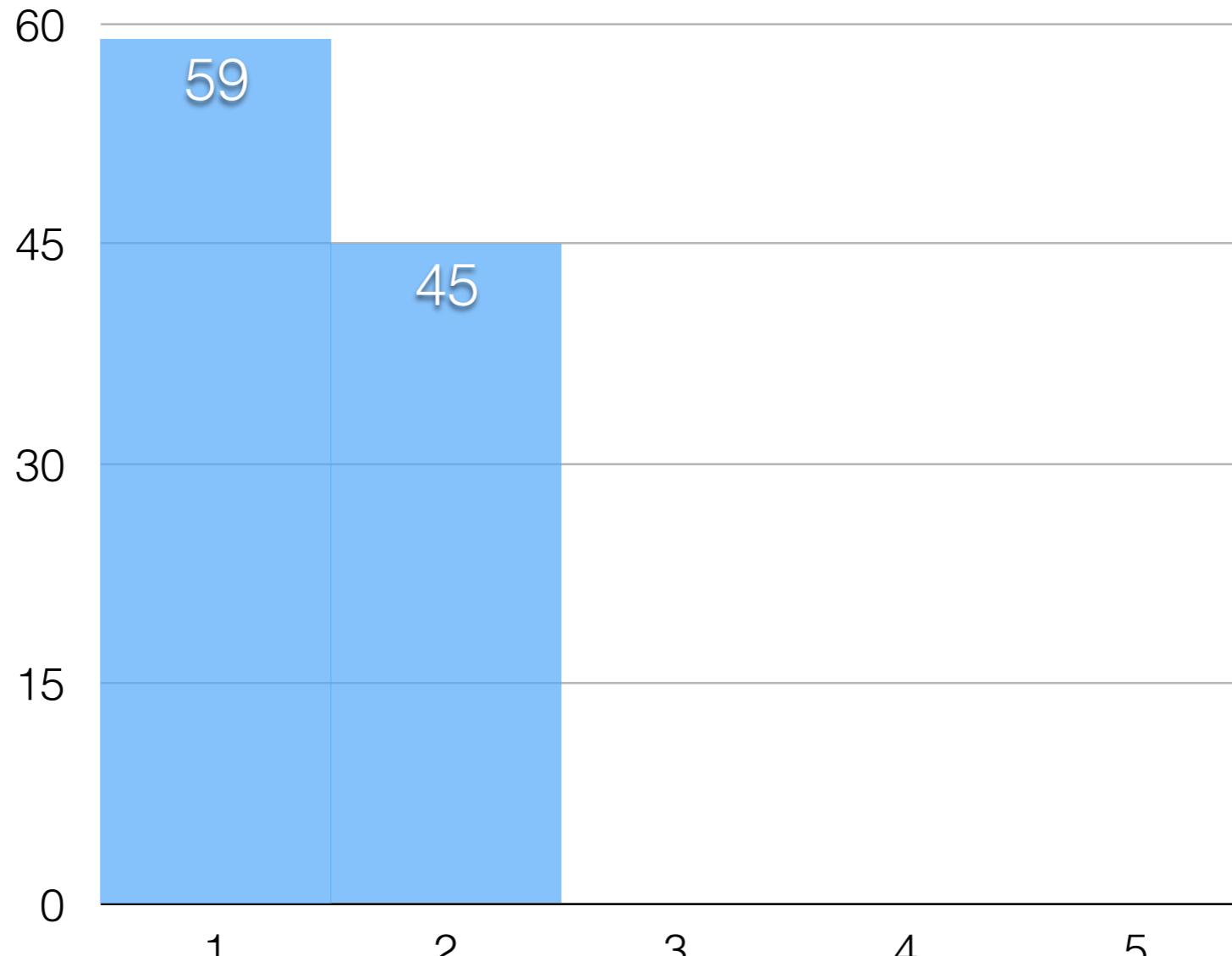
수요곡선도출 Demand Curve

가격(\$)	수요량(EA)
over 59	0
46~59	1
36~45	2
26~35	3
11~25	4
below 10	5



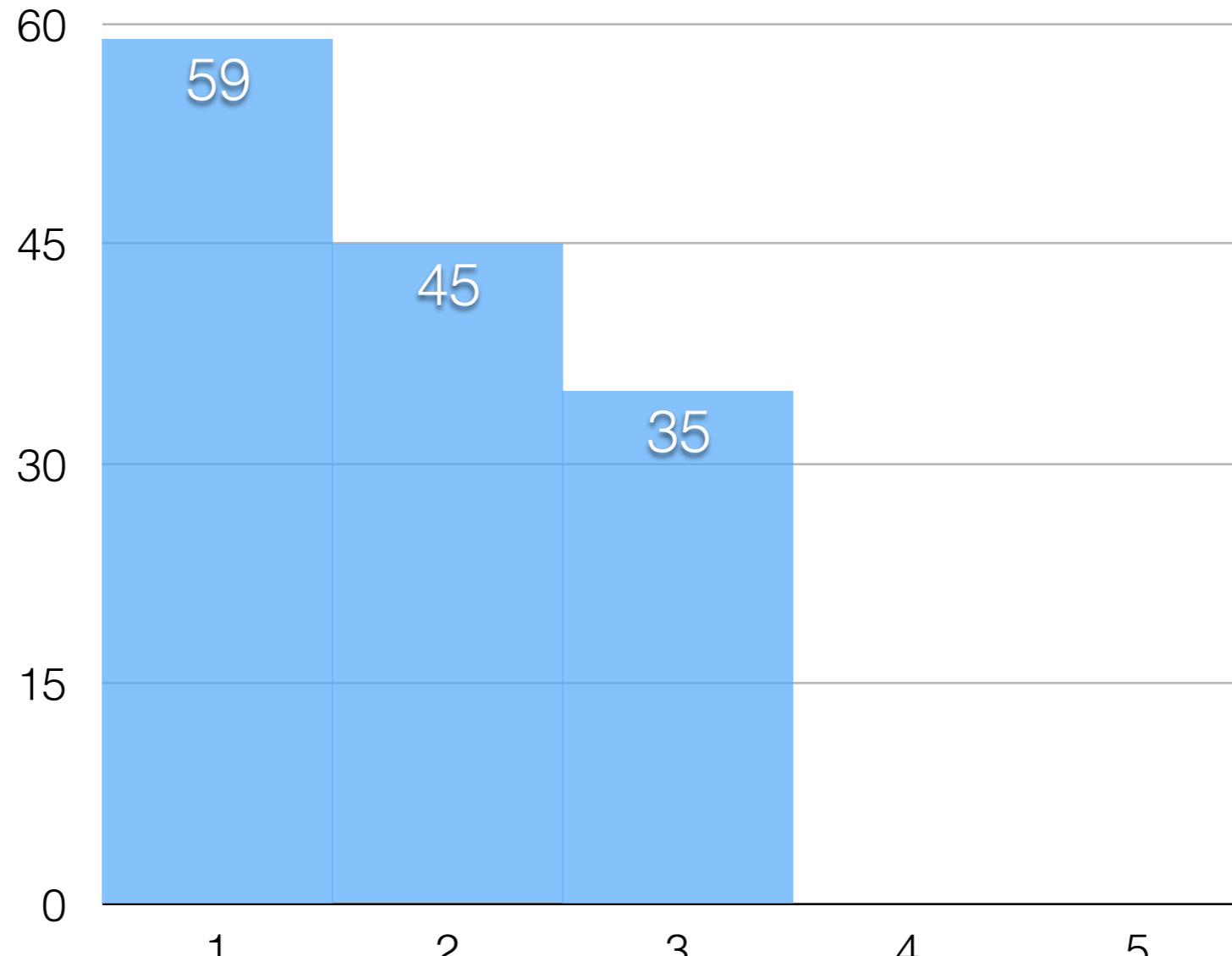
수요곡선도출 Demand Curve

가격(\$)	수요량(EA)
over 59	0
46~59	1
36~45	2
26~35	3
11~25	4
below 10	5

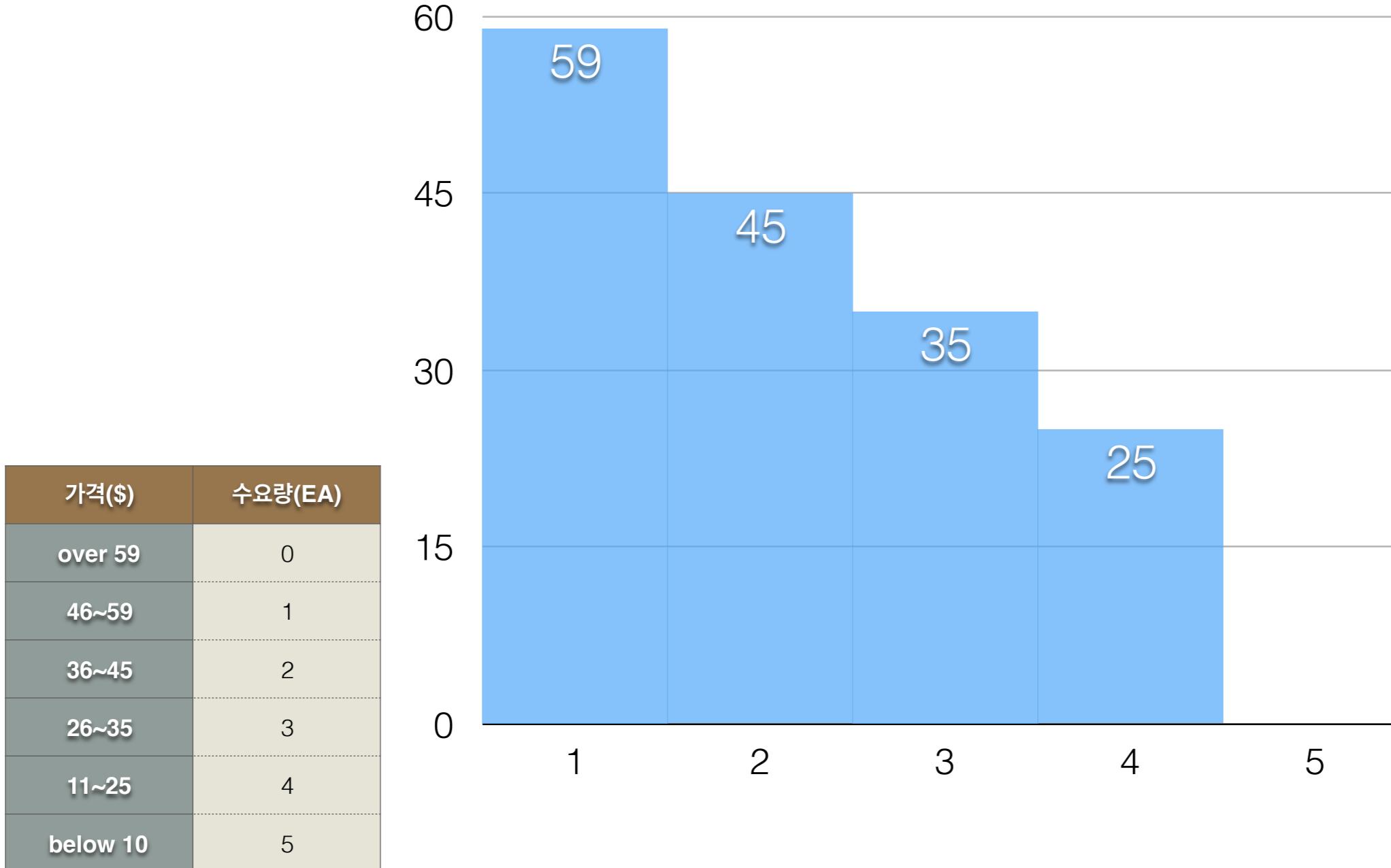


수요곡선도출 Demand Curve

가격(\$)	수요량(EA)
over 59	0
46~59	1
36~45	2
26~35	3
11~25	4
below 10	5

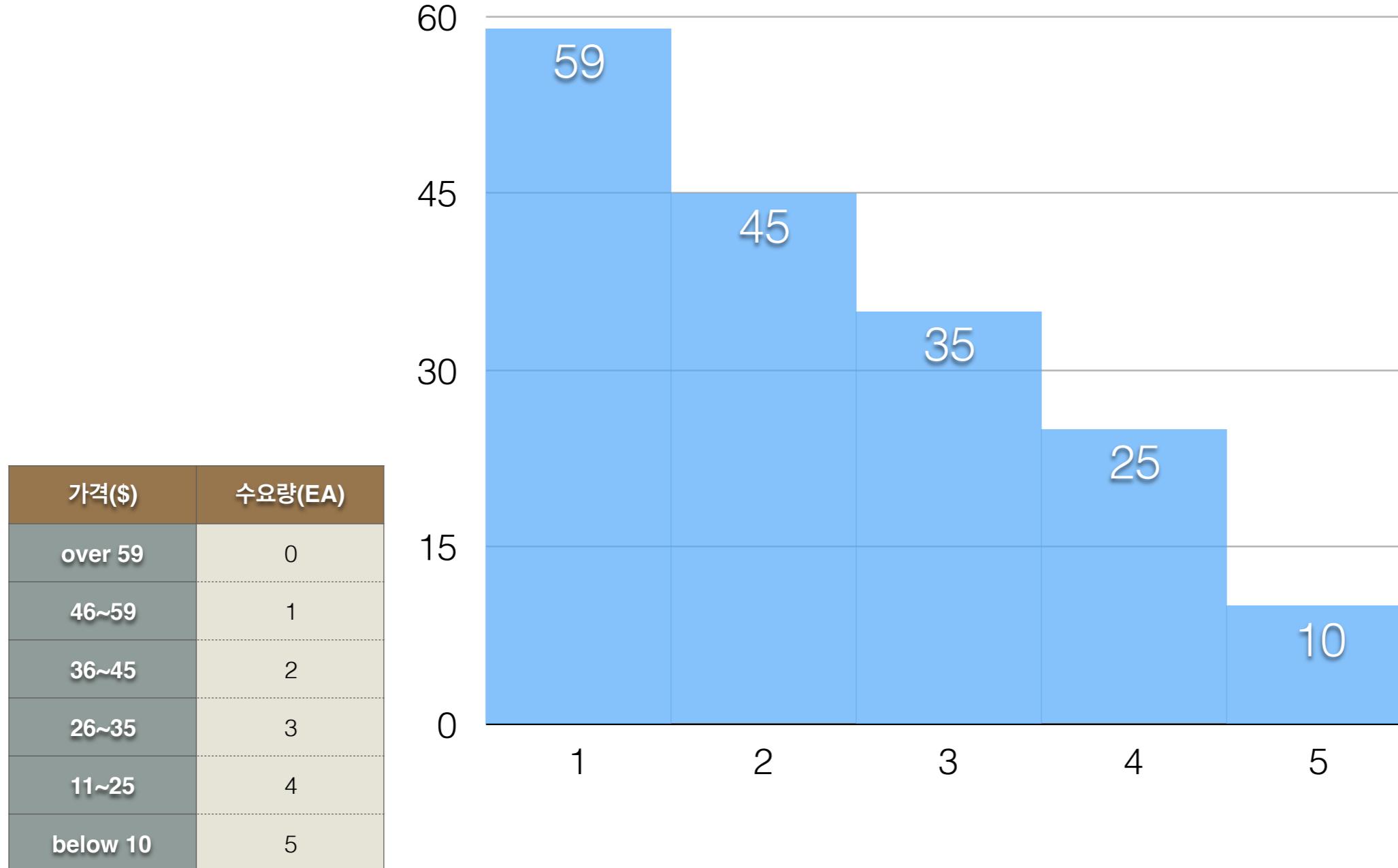


수요곡선도출 Demand Curve



수요곡선도출

Demand Curve



소비자잉여 Consumer Surplus

- 어떤 상품에 대해서 소비자마다 지불용의 수준은 다르지만, 시장 판매가격은 같다는 사정으로 인해 발생
- 정의식: (개별)소비자잉여 \equiv 지불용의 - 구매가격
- 총소비자잉여: 개별소비자잉여의 총합
- 소비자잉여는 개별소비자잉여일수도, 총소비자잉여 일수도 있으므로 문맥에 따라 구별

(개별)소비자잉여 $_i \equiv$ 지불용의 $_i$ – 구매가격

$$\text{총소비자잉여} = \sum_{i \in D} \text{소비자잉여}_i$$

시장가격이 \$30인 경우

시장가격이 \$30인 경우

잠재수요자	지불용의(\$)	구매가격(\$)	개별소비자잉여(\$)
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시장가격이 \$30인 경우

잠재수요자	지불용의(\$)	구매가격(\$)	개별소비자잉여(\$)
Aleisha	59	30	29

시장가격이 \$30인 경우

잠재수요자	지불용의(\$)	구매가격(\$)	개별소비자잉여(\$)
Aleisha	59	30	29
Brad	45	30	15

시장가격이 \$30인 경우

잠재수요자	지불용의(\$)	구매가격(\$)	개별소비자잉여(\$)
Aleisha	59	30	29
Brad	45	30	15
Claudia	35	30	5

시장가격이 \$30인 경우

잠재수요자	지불용의(\$)	구매가격(\$)	개별소비자잉여(\$)
Aleisha	59	30	29
Brad	45	30	15
Claudia	35	30	5
Darren	25	30	-

시장가격이 \$30인 경우

잠재수요자	지불용의(\$)	구매가격(\$)	개별소비자잉여(\$)
Aleisha	59	30	29
Brad	45	30	15
Claudia	35	30	5
Darren	25	30	-
Edwina	10	30	-

시장가격이 \$30인 경우

잠재수요자	지불용의(\$)	구매가격(\$)	개별소비자잉여(\$)
Aleisha	59	30	29
Brad	45	30	15
Claudia	35	30	5
Darren	25	30	-
Edwina	10	30	-
total	-	-	49

시장가격이 \$30인 경우

잠재수요자	지불용의(\$)	구매가격(\$)	개별소비자잉여(\$)
Aleisha	59	30	29
Brad	45	30	15
Claudia	35	30	5
Darren	25	30	-
Edwina	10	30	-
total	-	-	49

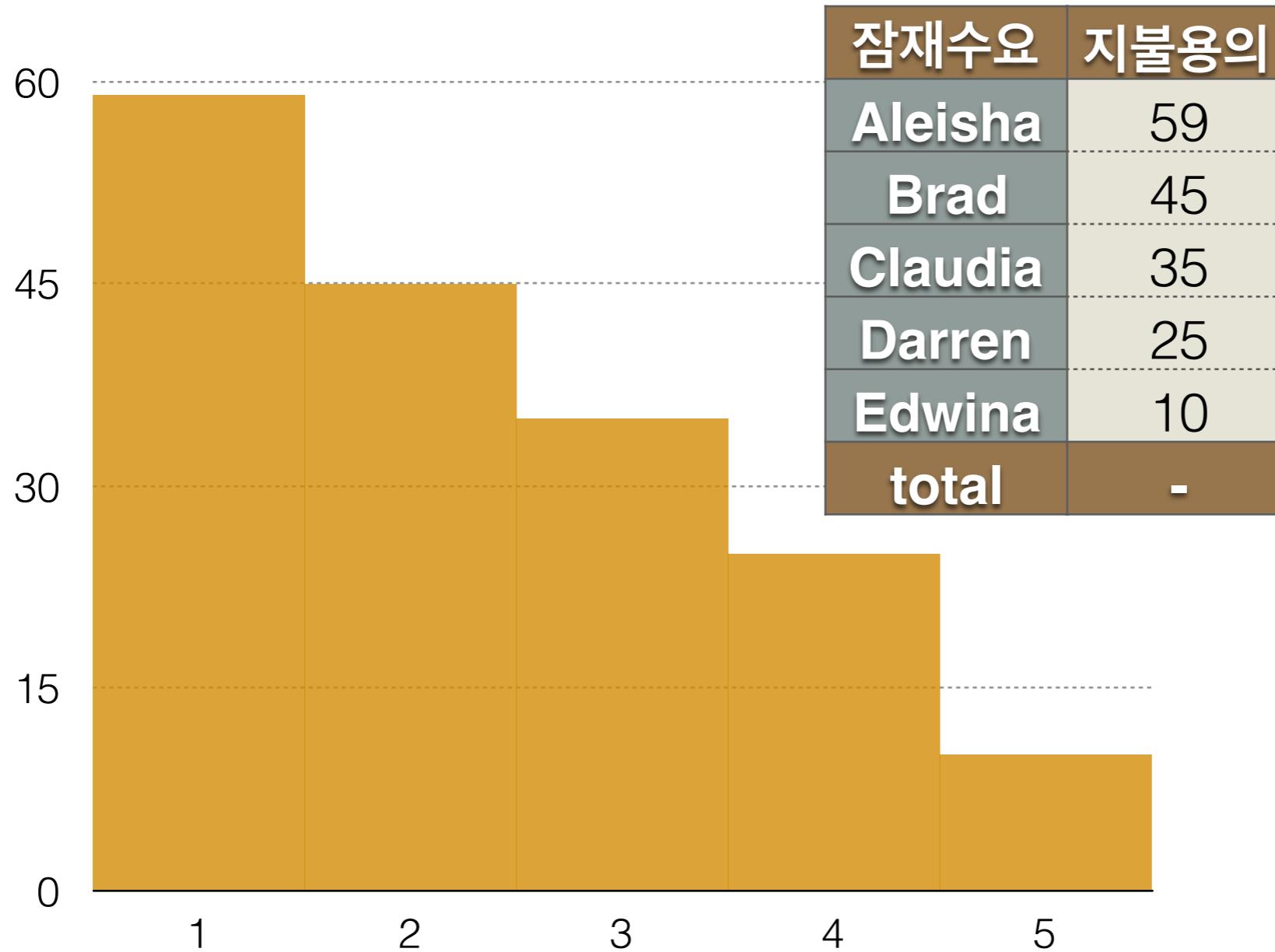
총 소비
자잉여

소비자잉여 Consumer Surplus

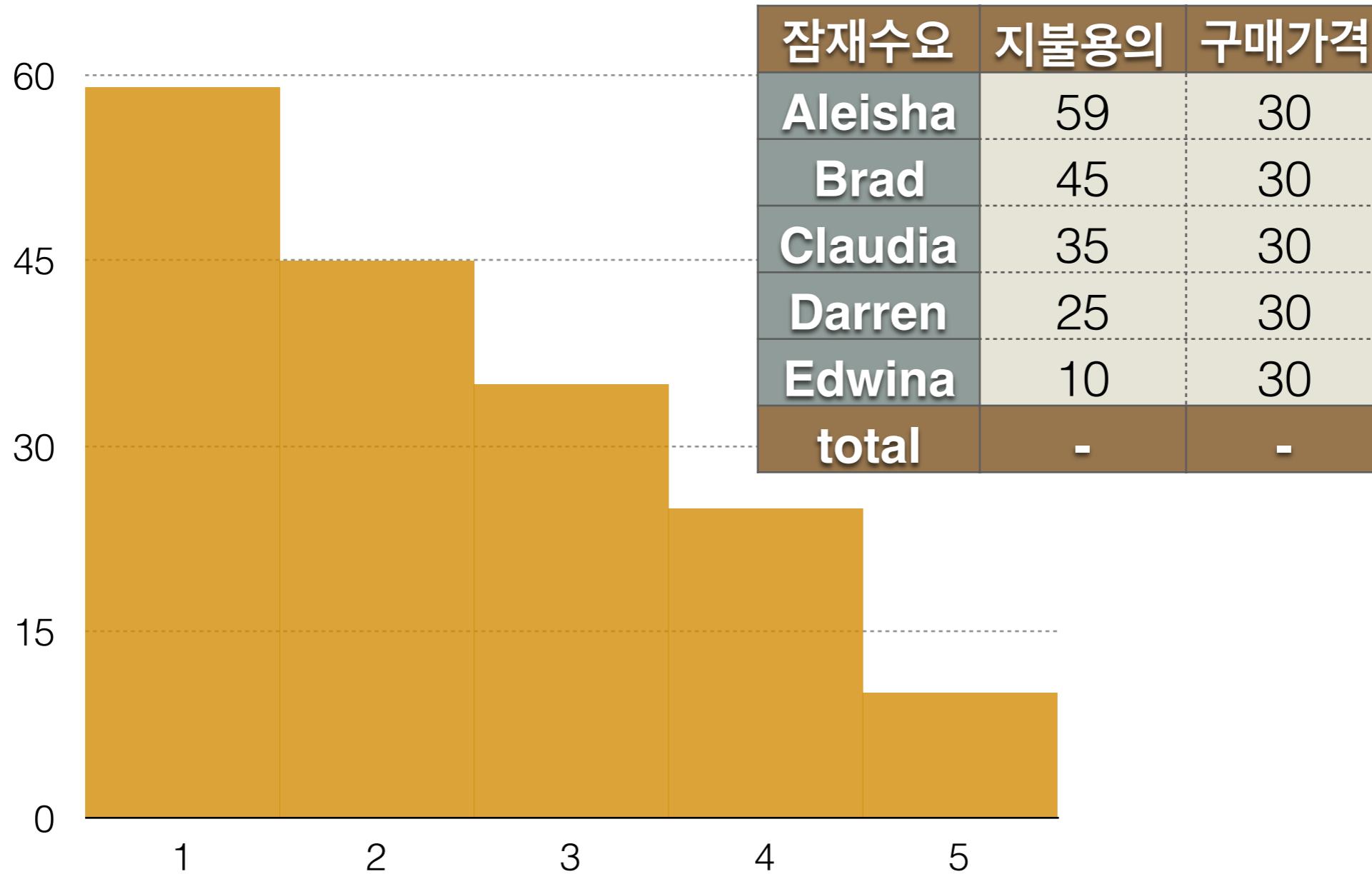
소비자잉여 Consumer Surplus

잠재수요	지불용의
Aleisha	59
Brad	45
Claudia	35
Darren	25
Edwina	10
total	-

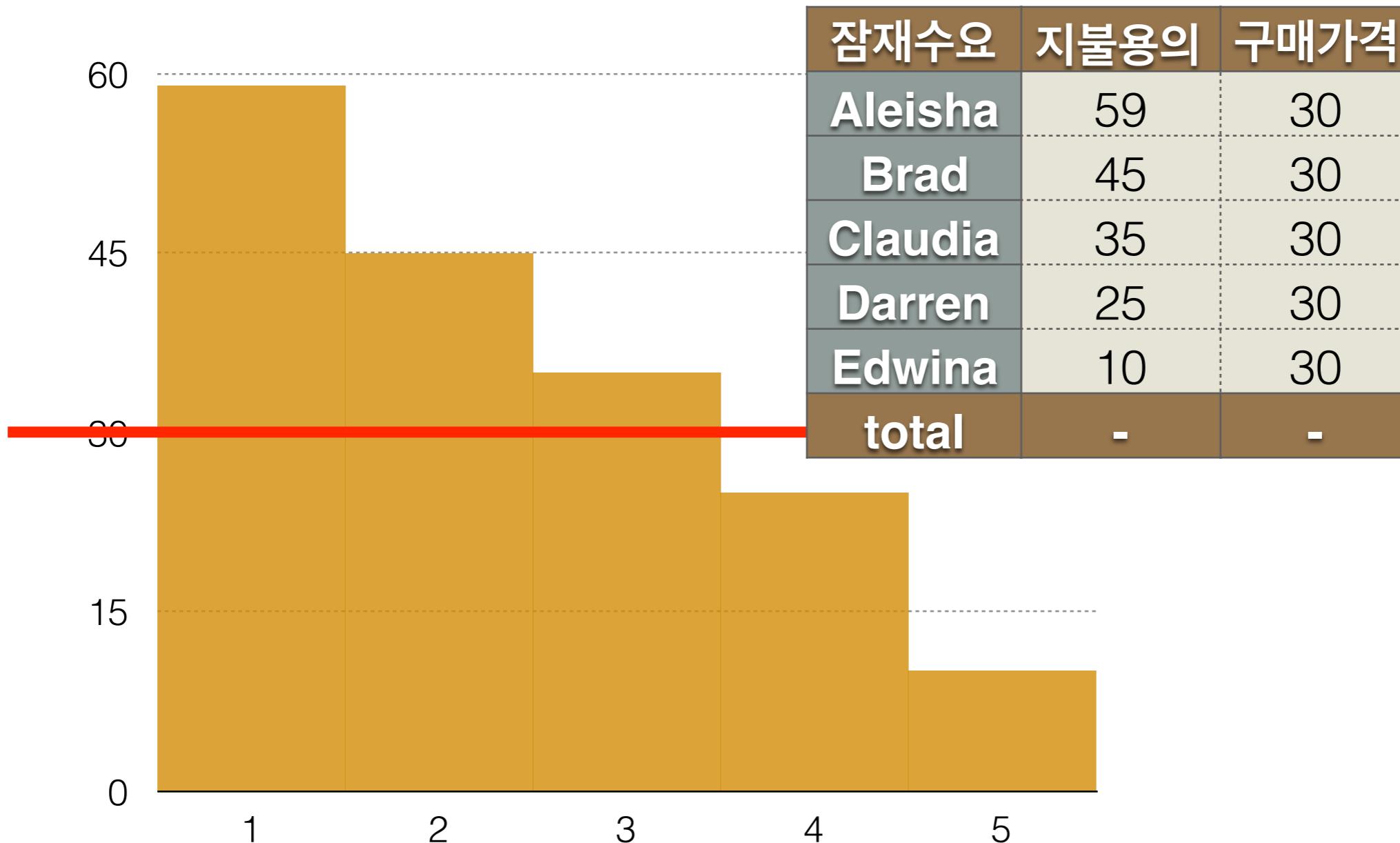
소비자잉여 Consumer Surplus



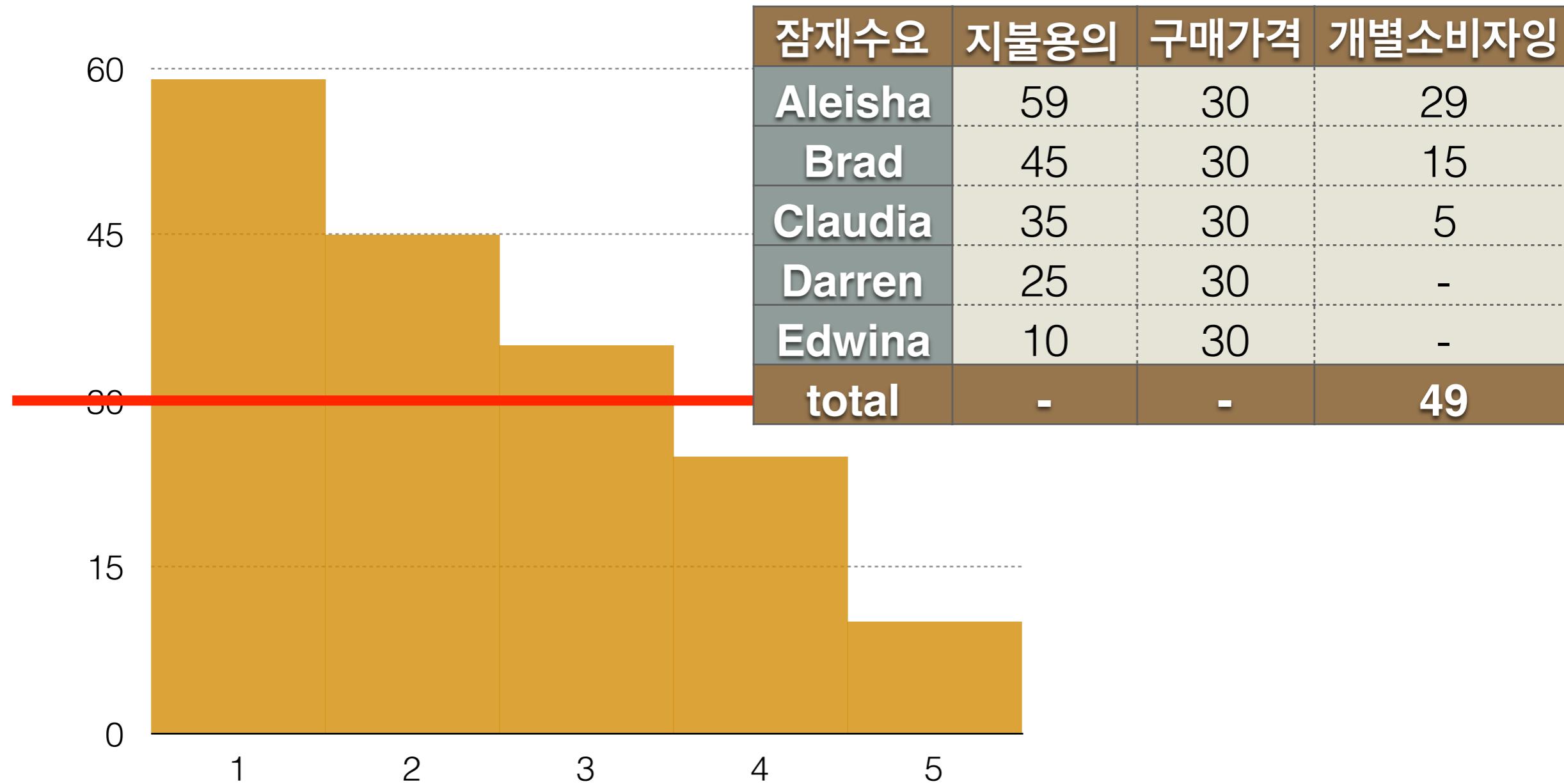
소비자잉여 Consumer Surplus



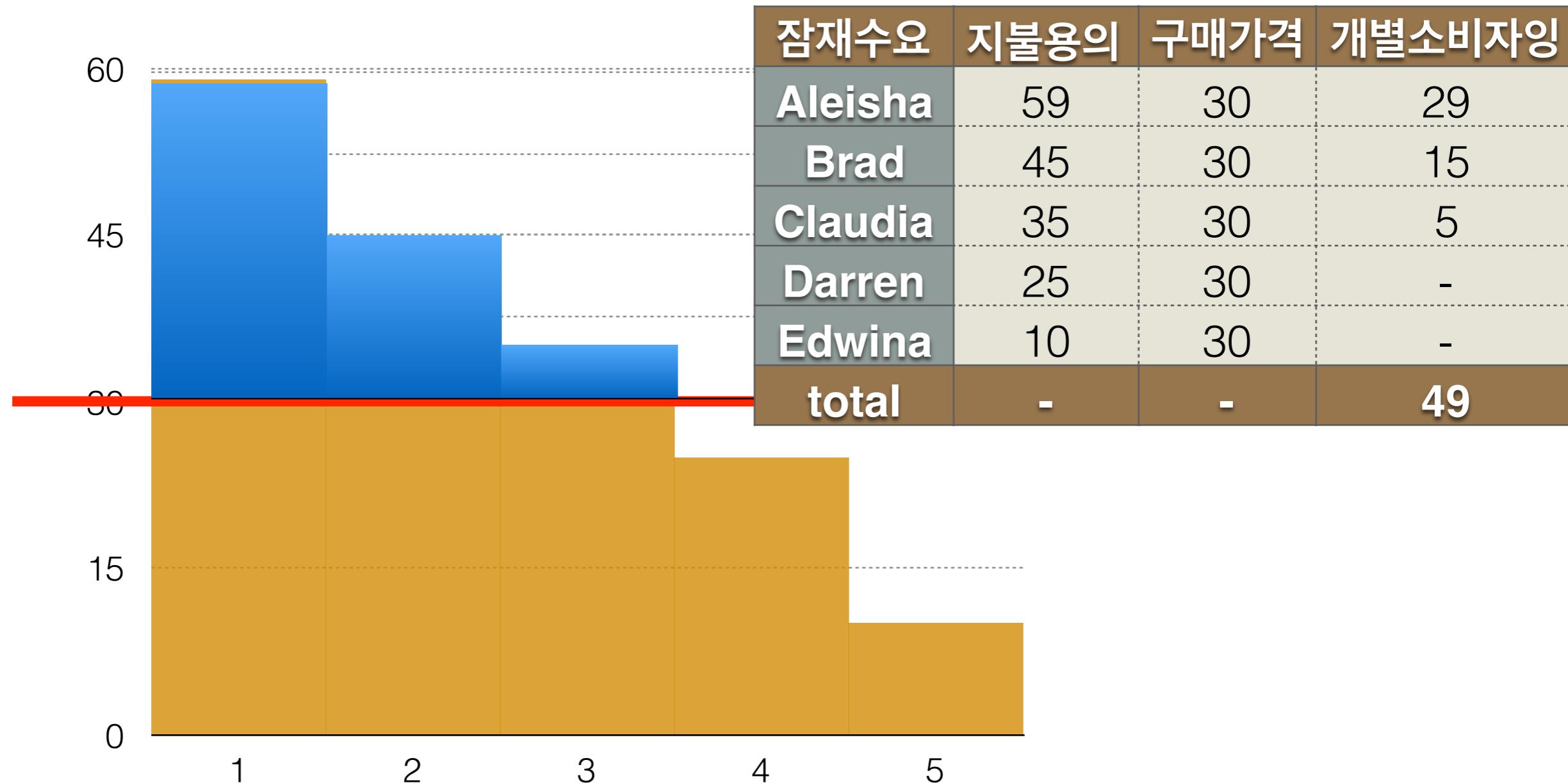
소비자잉여 Consumer Surplus



소비자잉여 Consumer Surplus

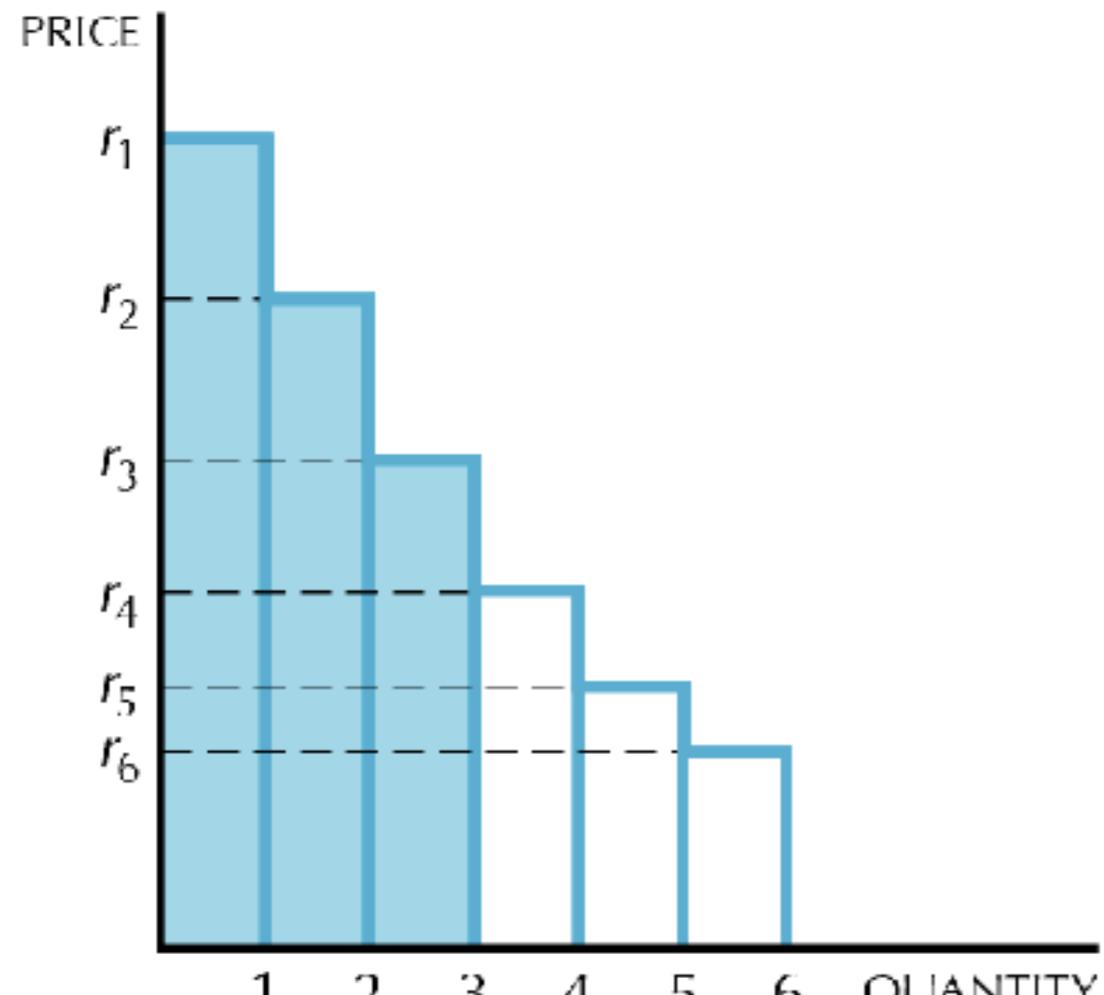


소비자잉여 Consumer Surplus

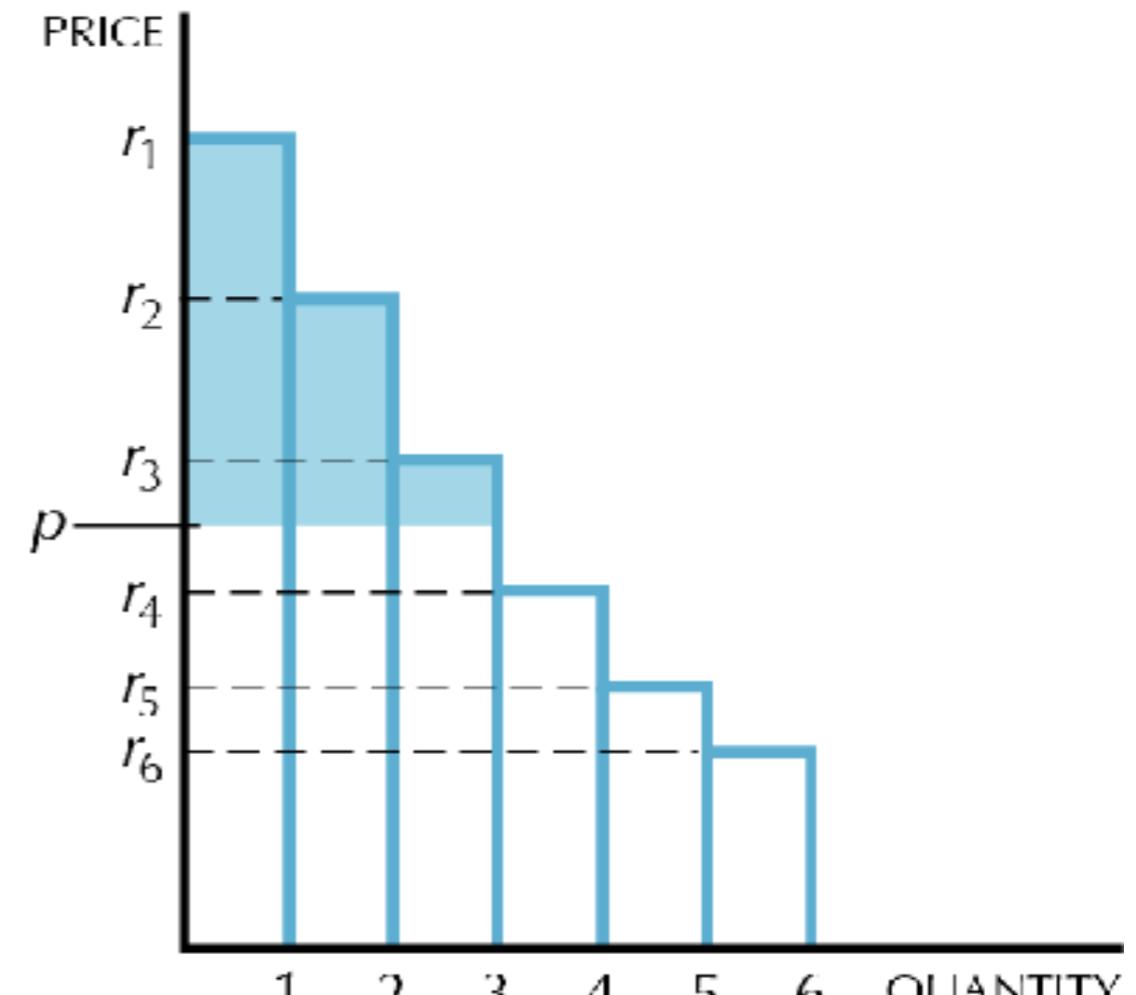


Huge Consumers

Huge Consumers



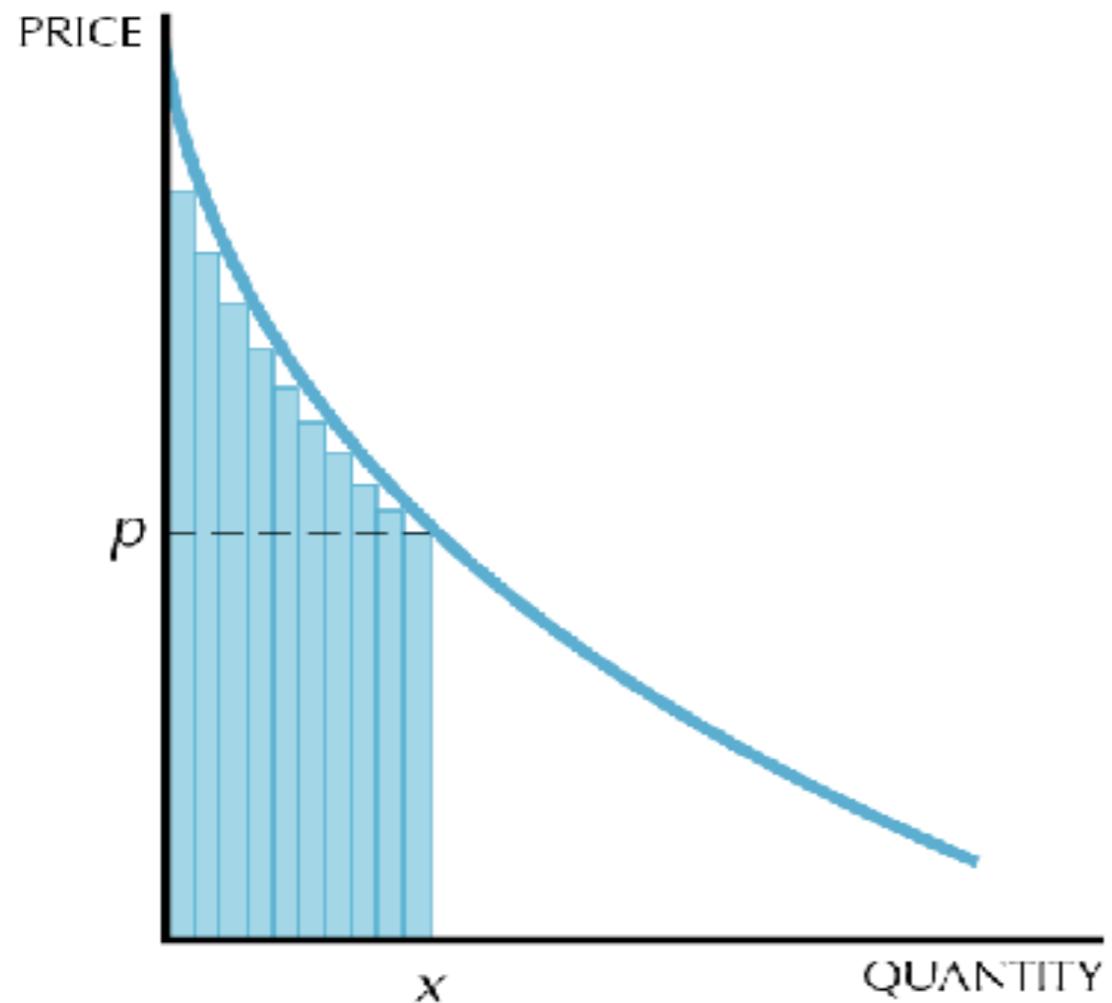
A Gross surplus



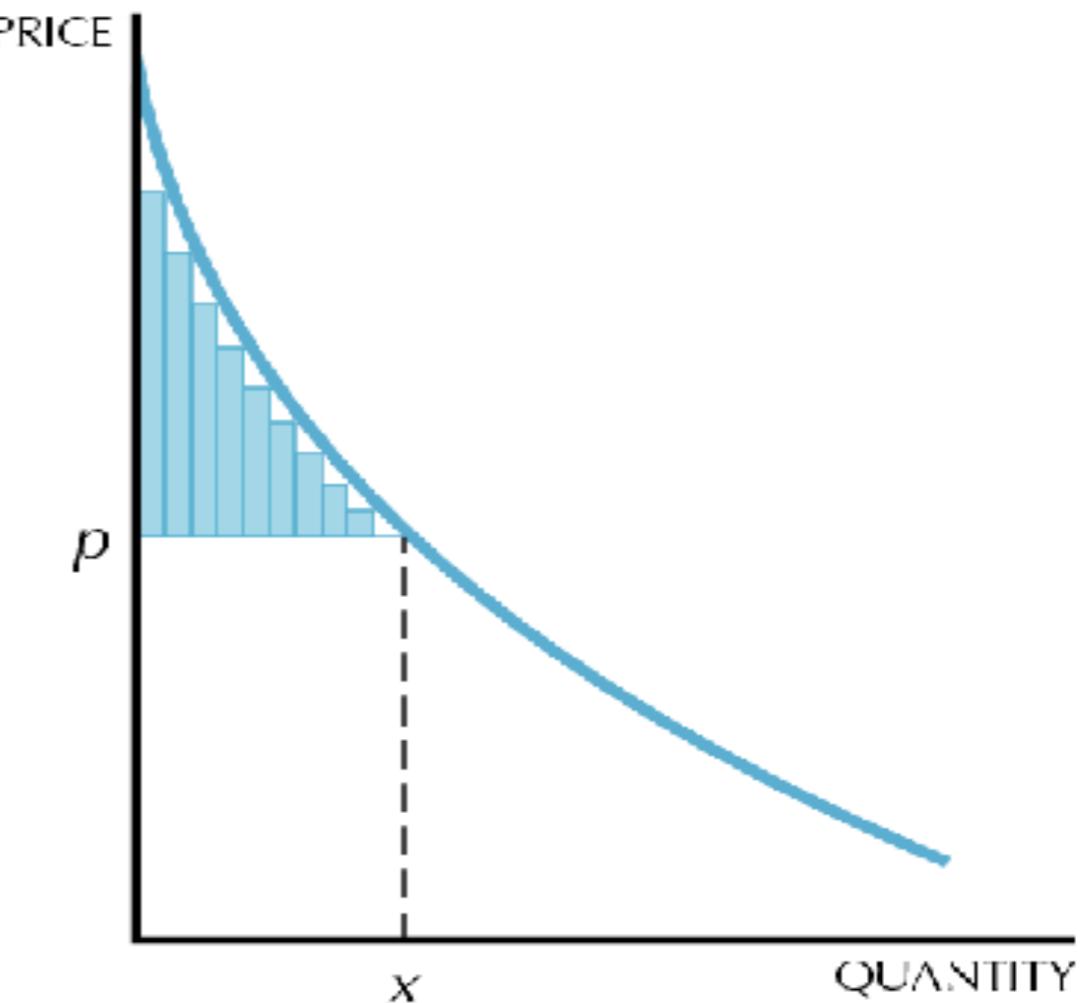
B Net surplus

Figure 14.1 Reservation prices and consumer's surplus

Huge Consumers



A Approximation to gross surplus

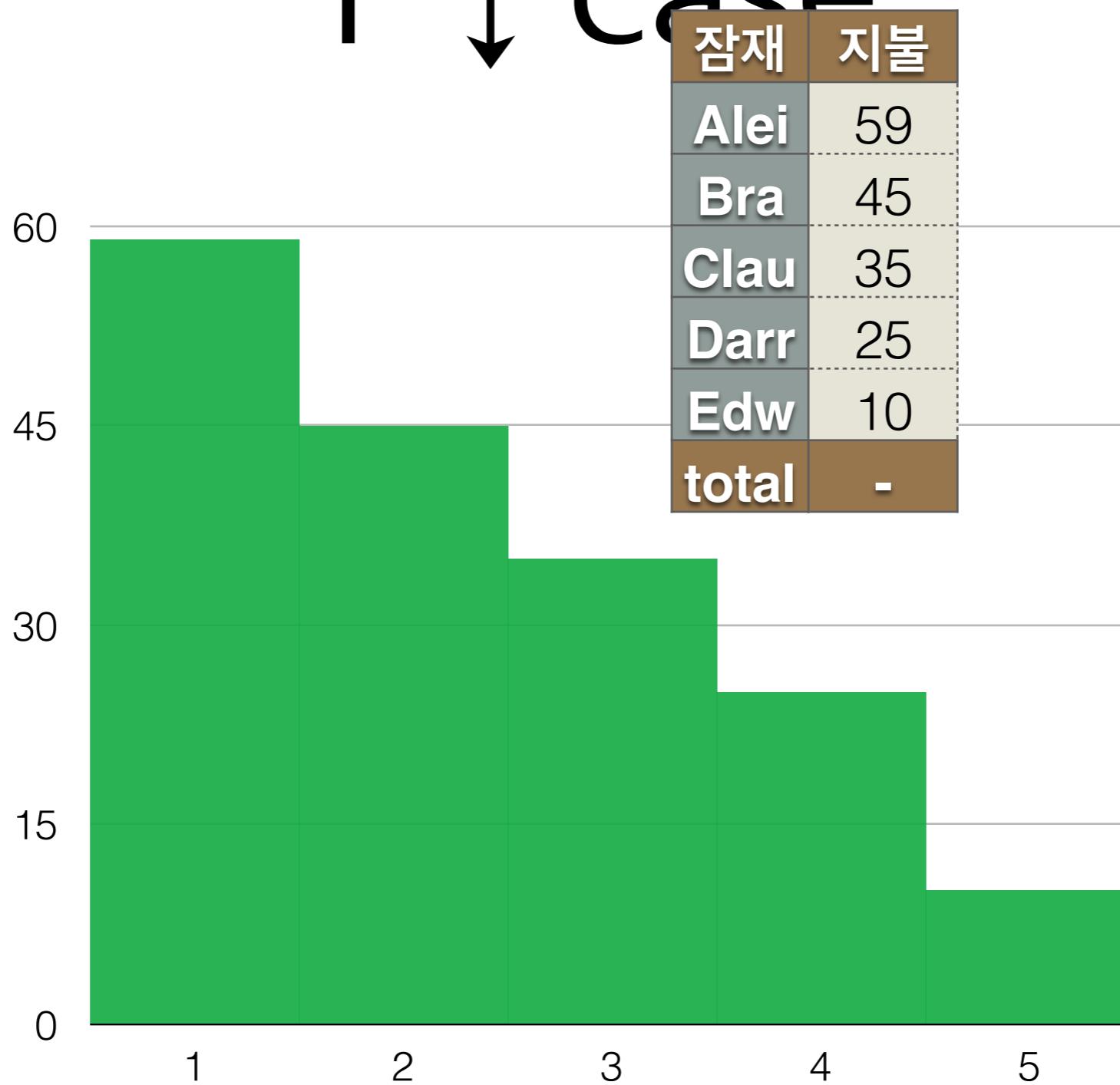


B Approximation to net surplus

Figure 14.2 Approximating a continuous demand

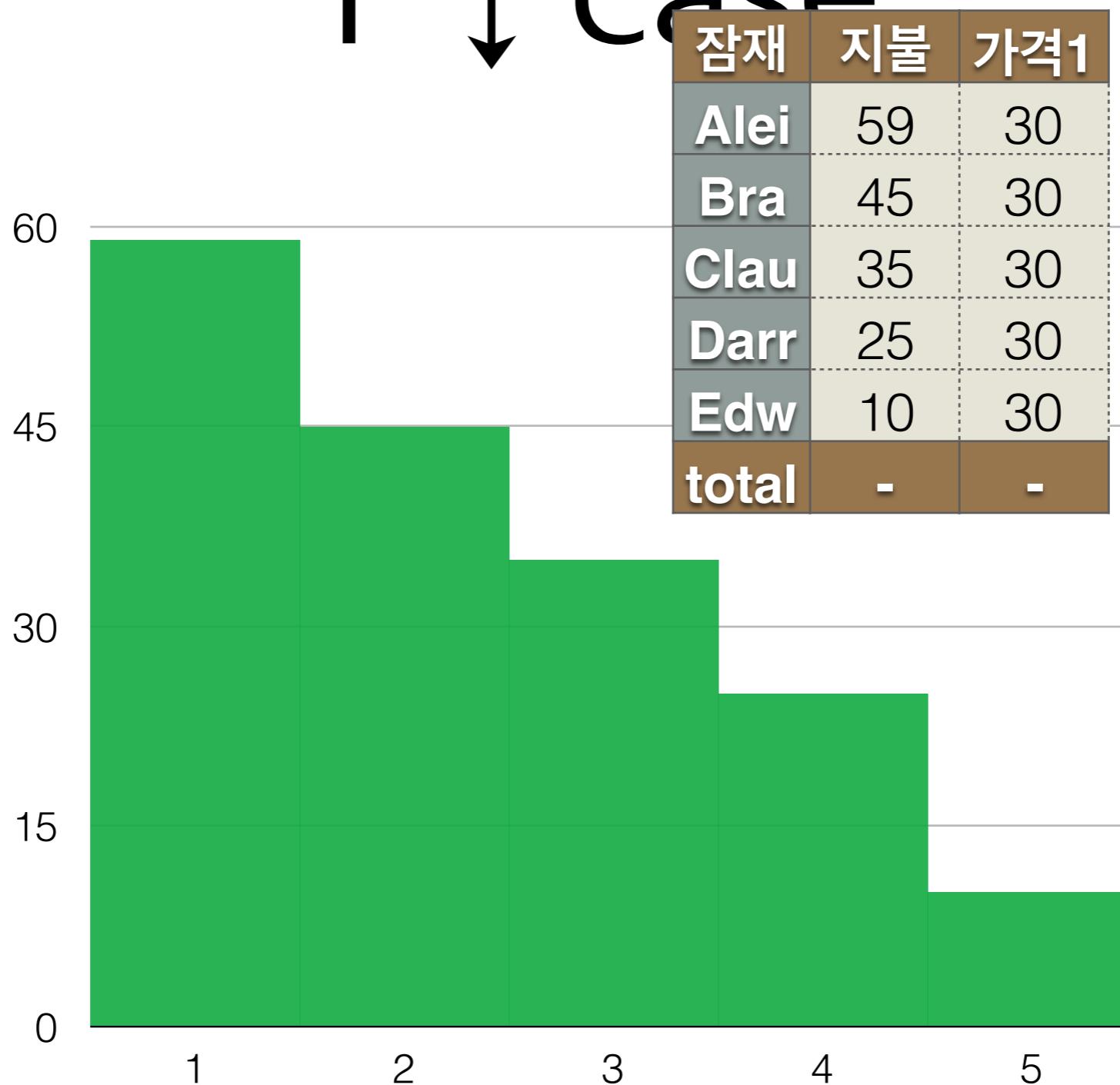
Consumer Surplus: P ↓ case

Consumer Surplus: P ↓ case



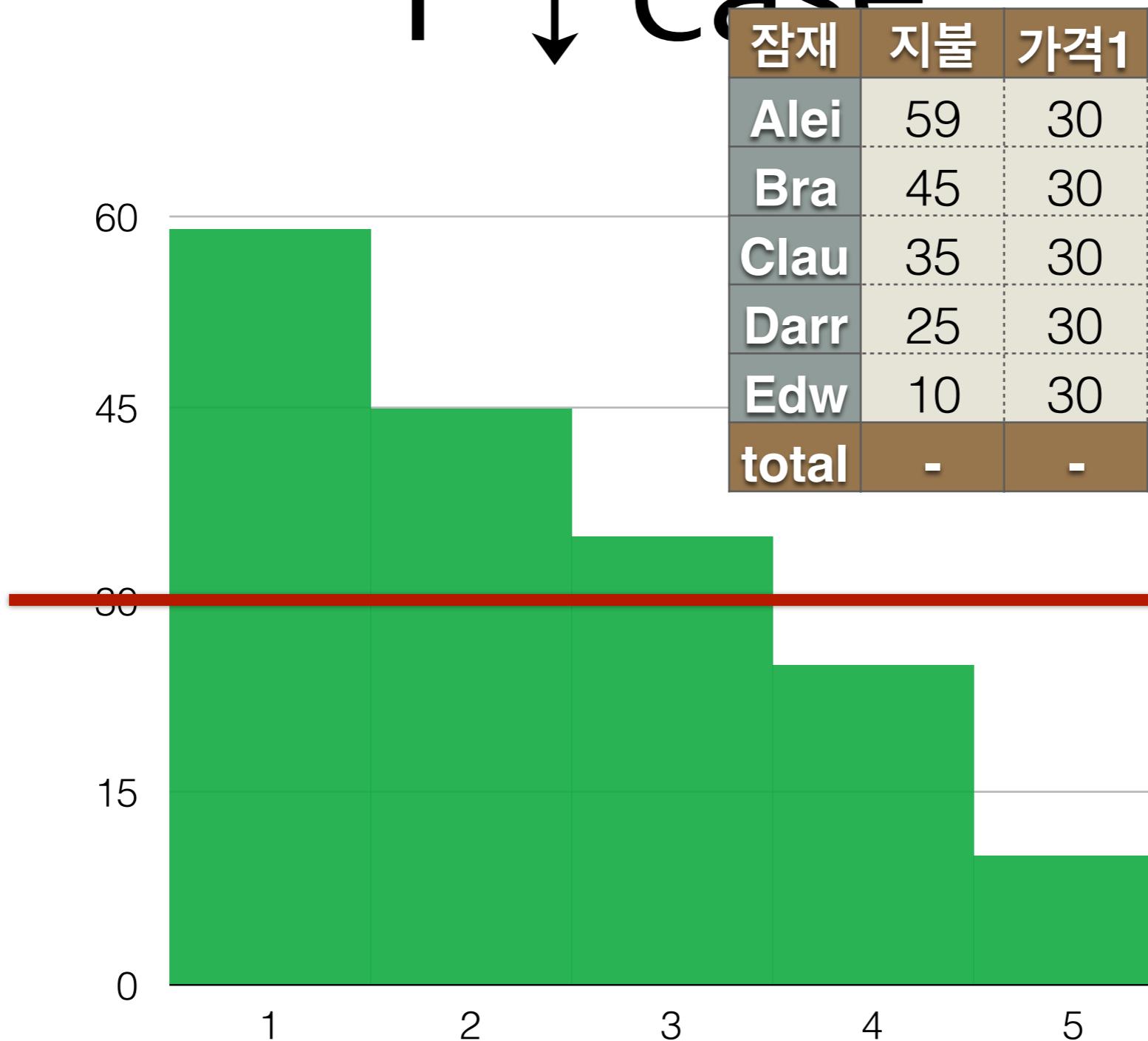
Consumer Surplus:

P ↓ case

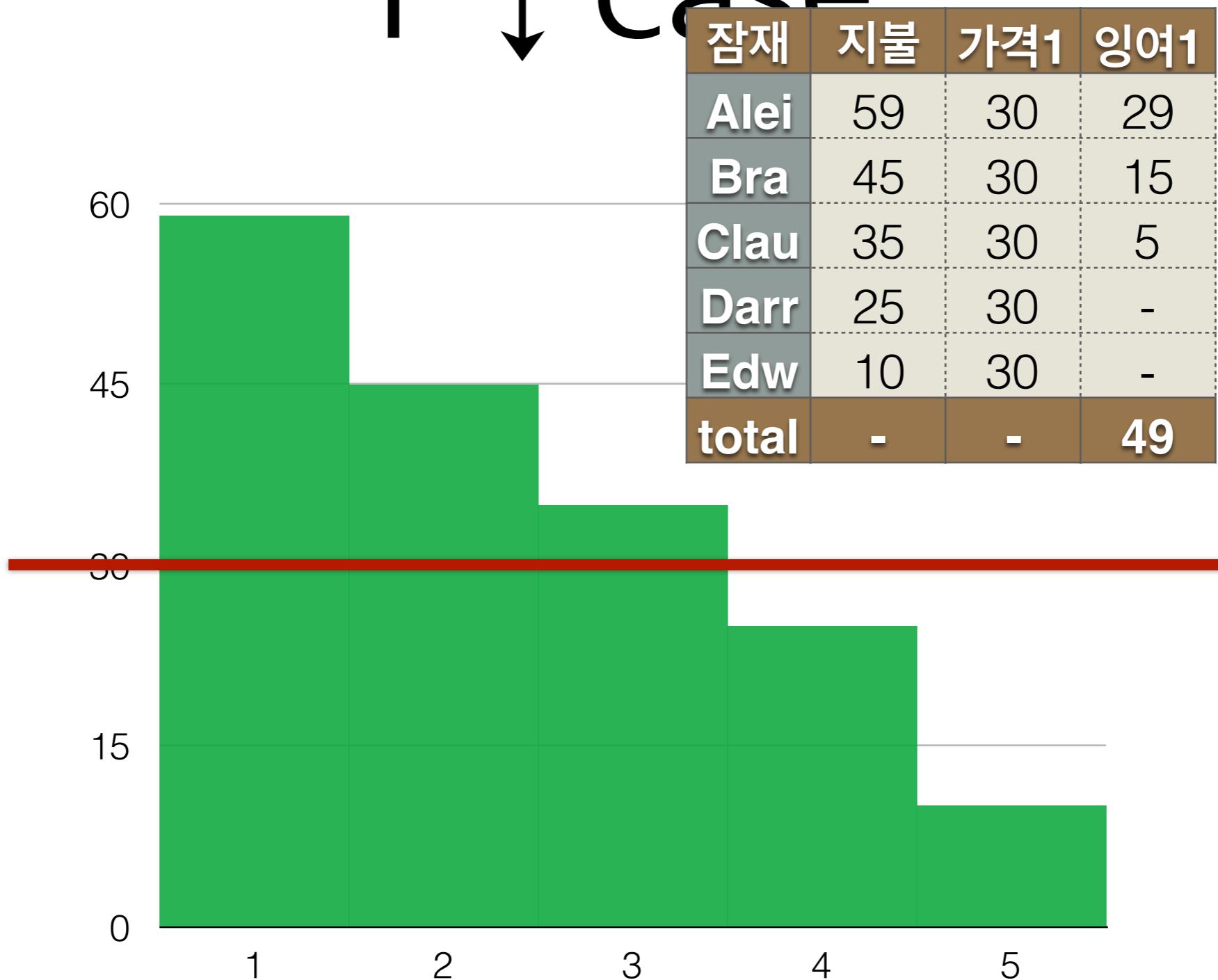


Consumer Surplus:

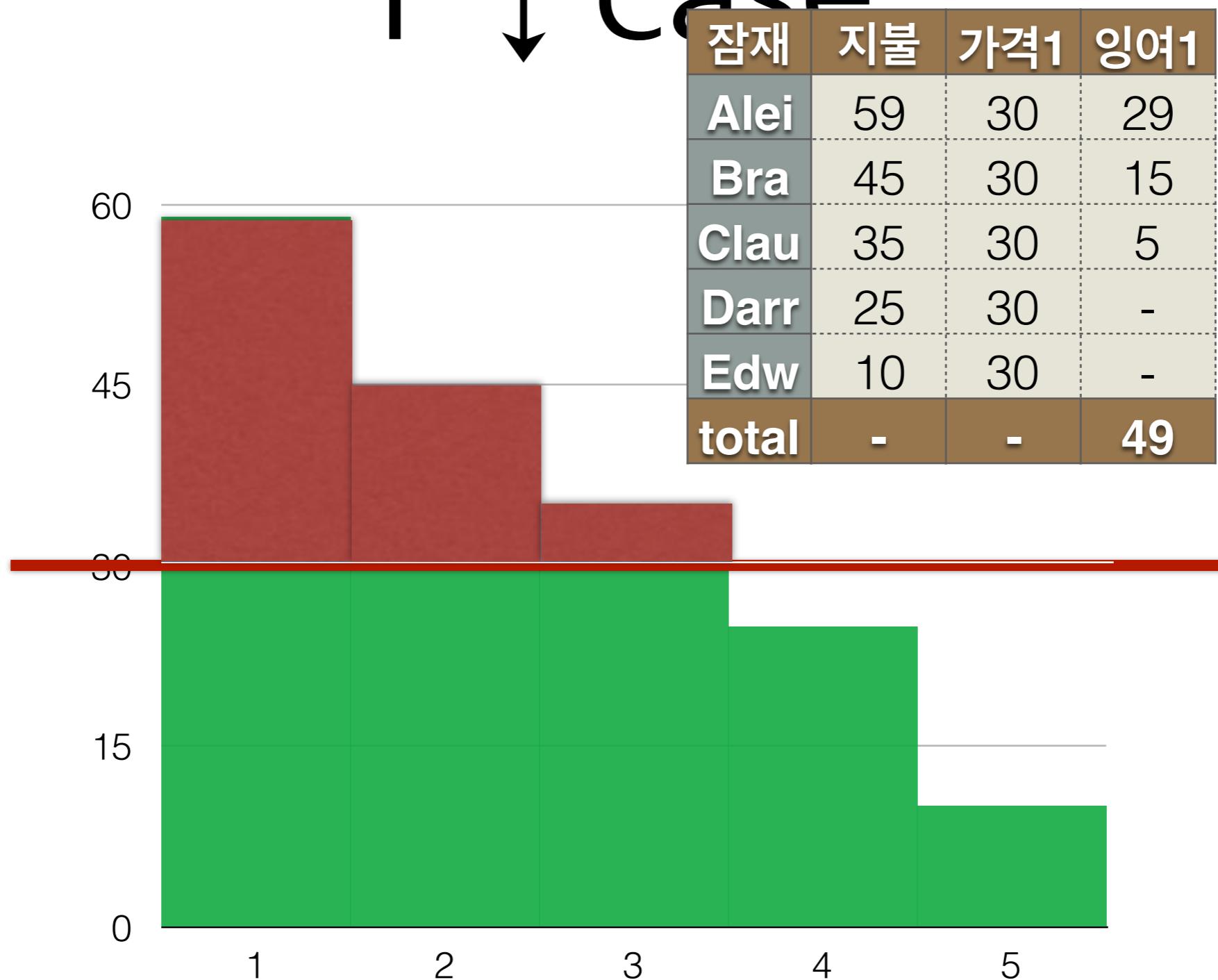
P ↓ case



Consumer Surplus: P ↓ case

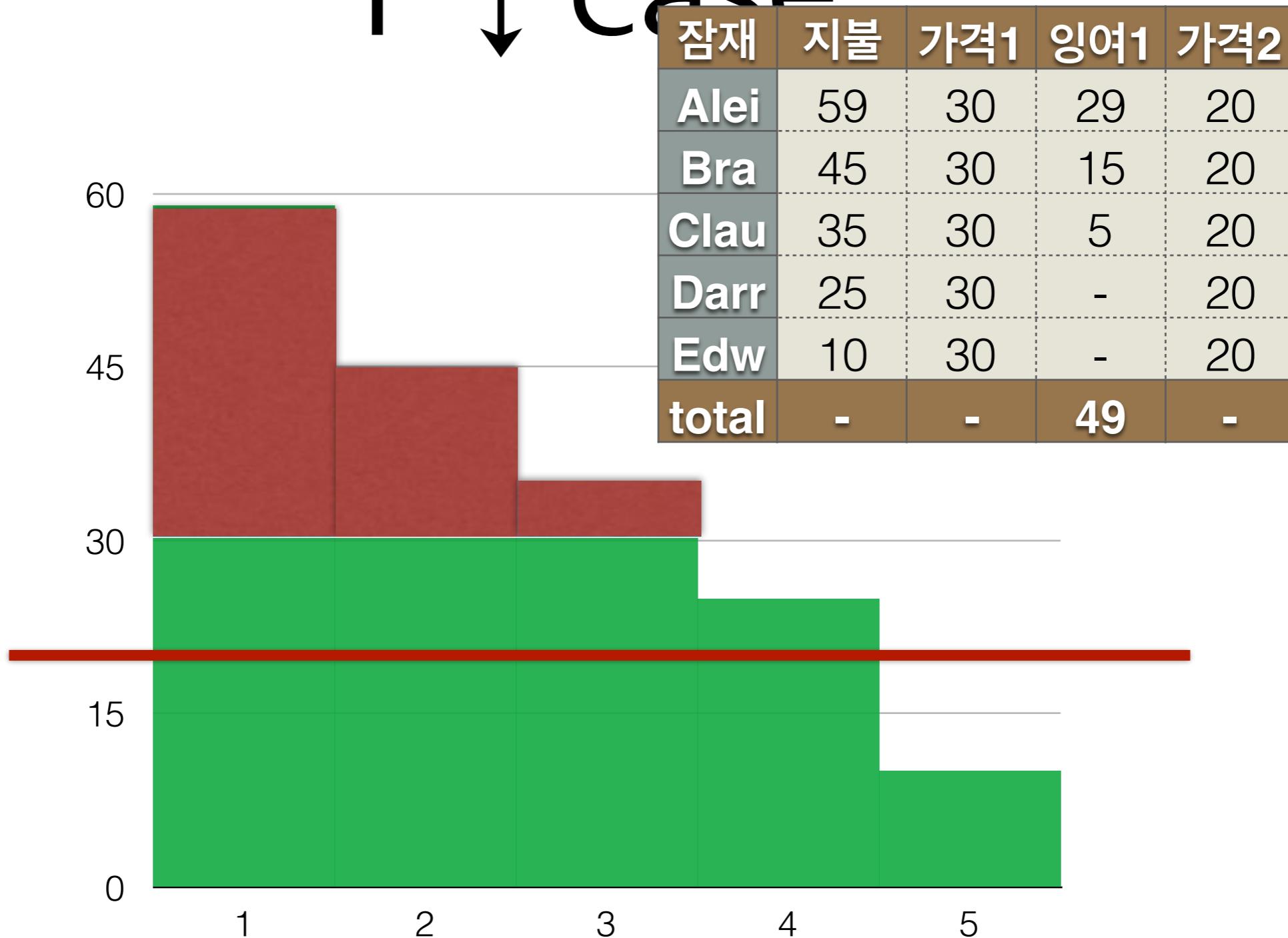


Consumer Surplus: P ↓ case



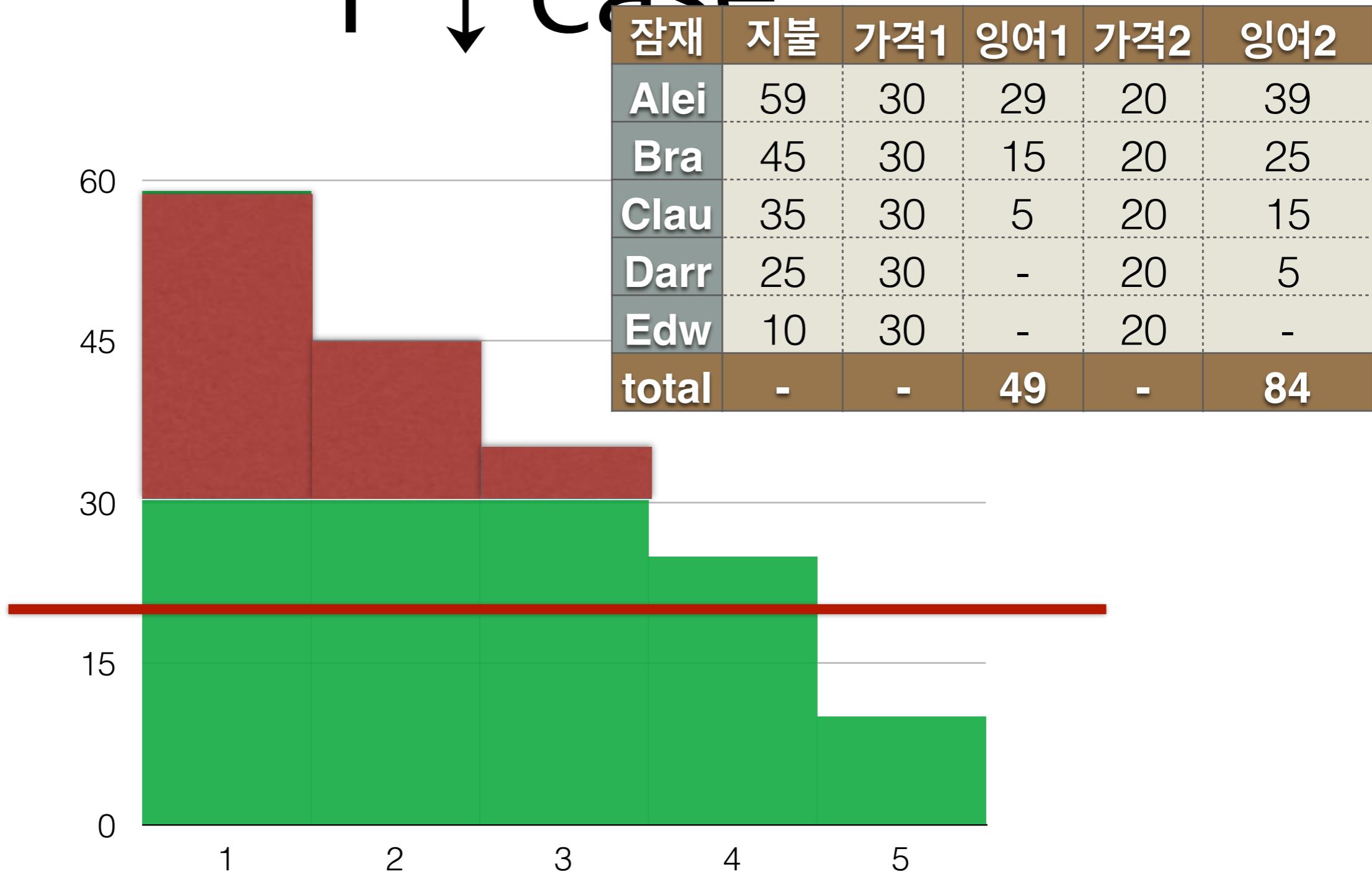
Consumer Surplus:

P ↓ case



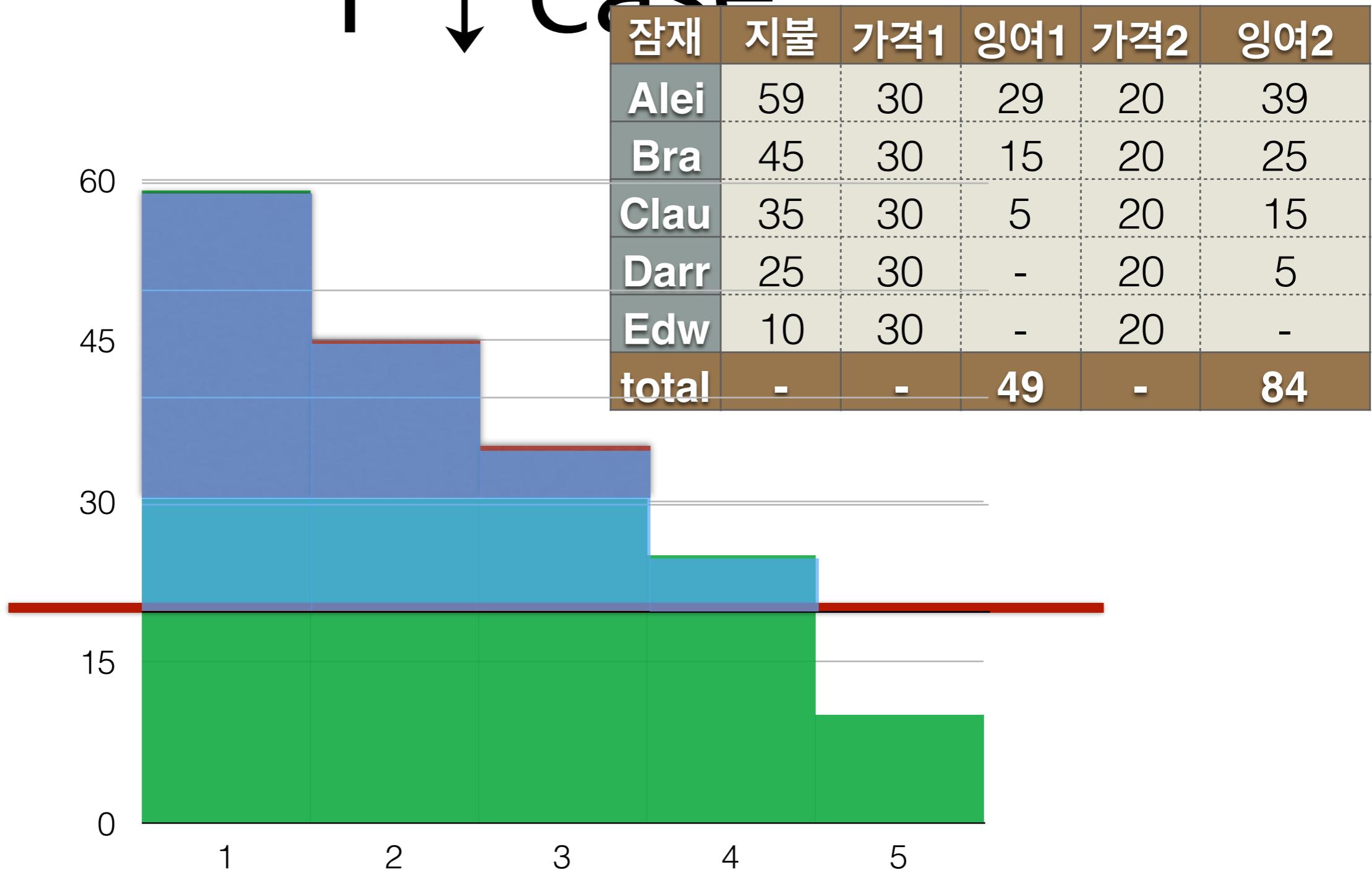
Consumer Surplus:

P ↓ case



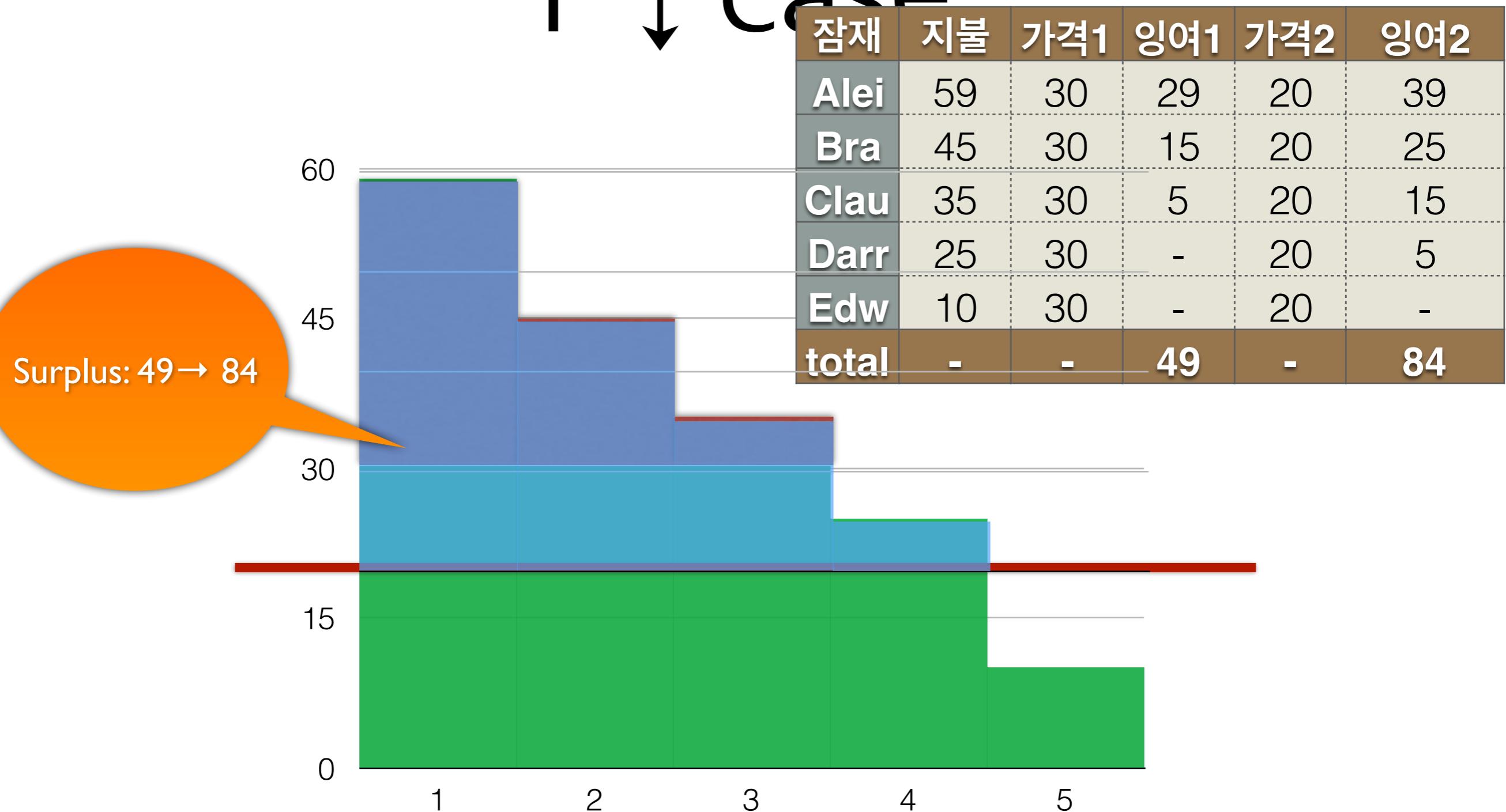
Consumer Surplus:

P ↓ case



Consumer Surplus:

P ↓ case

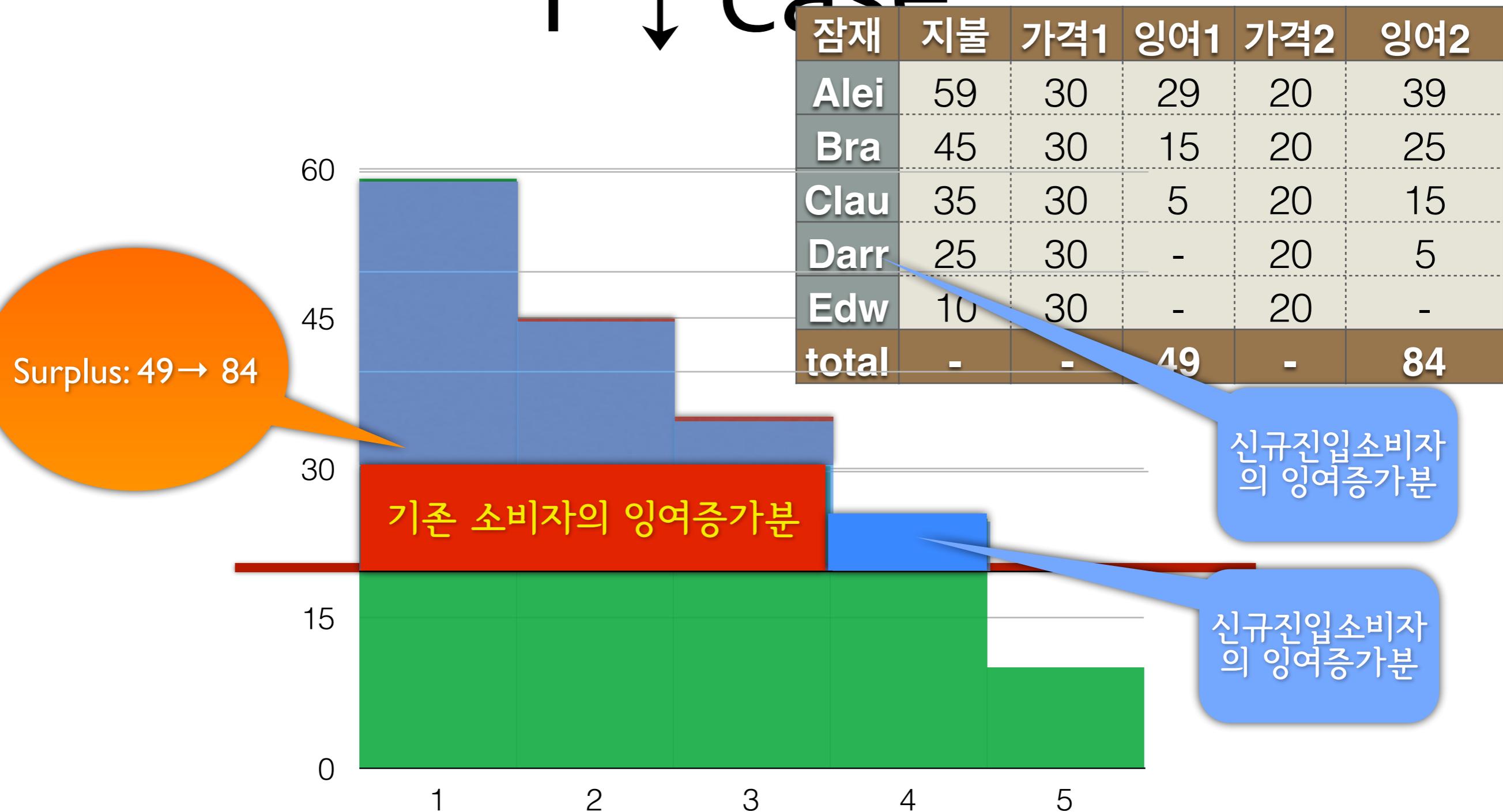


Consumer Surplus: P ↓ case



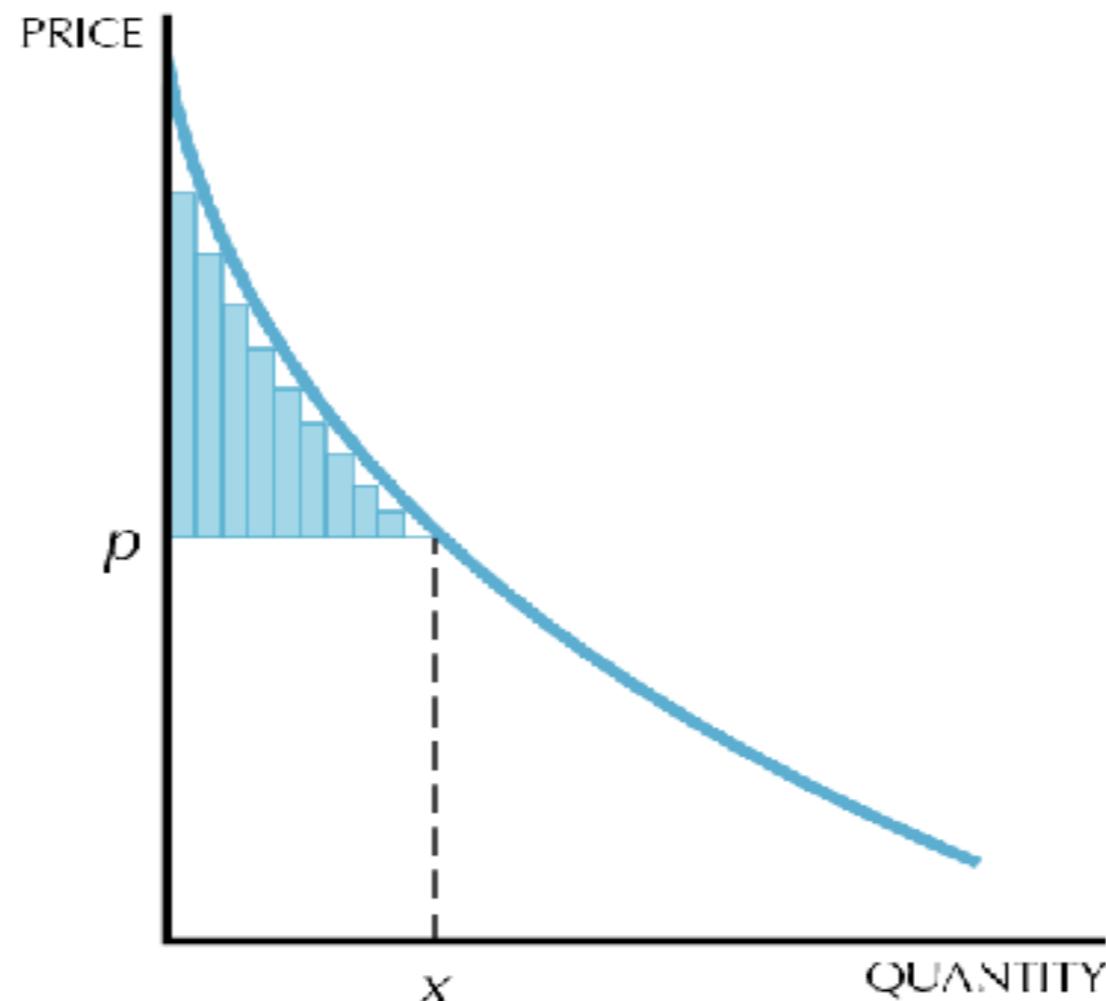
Consumer Surplus:

P ↓ case



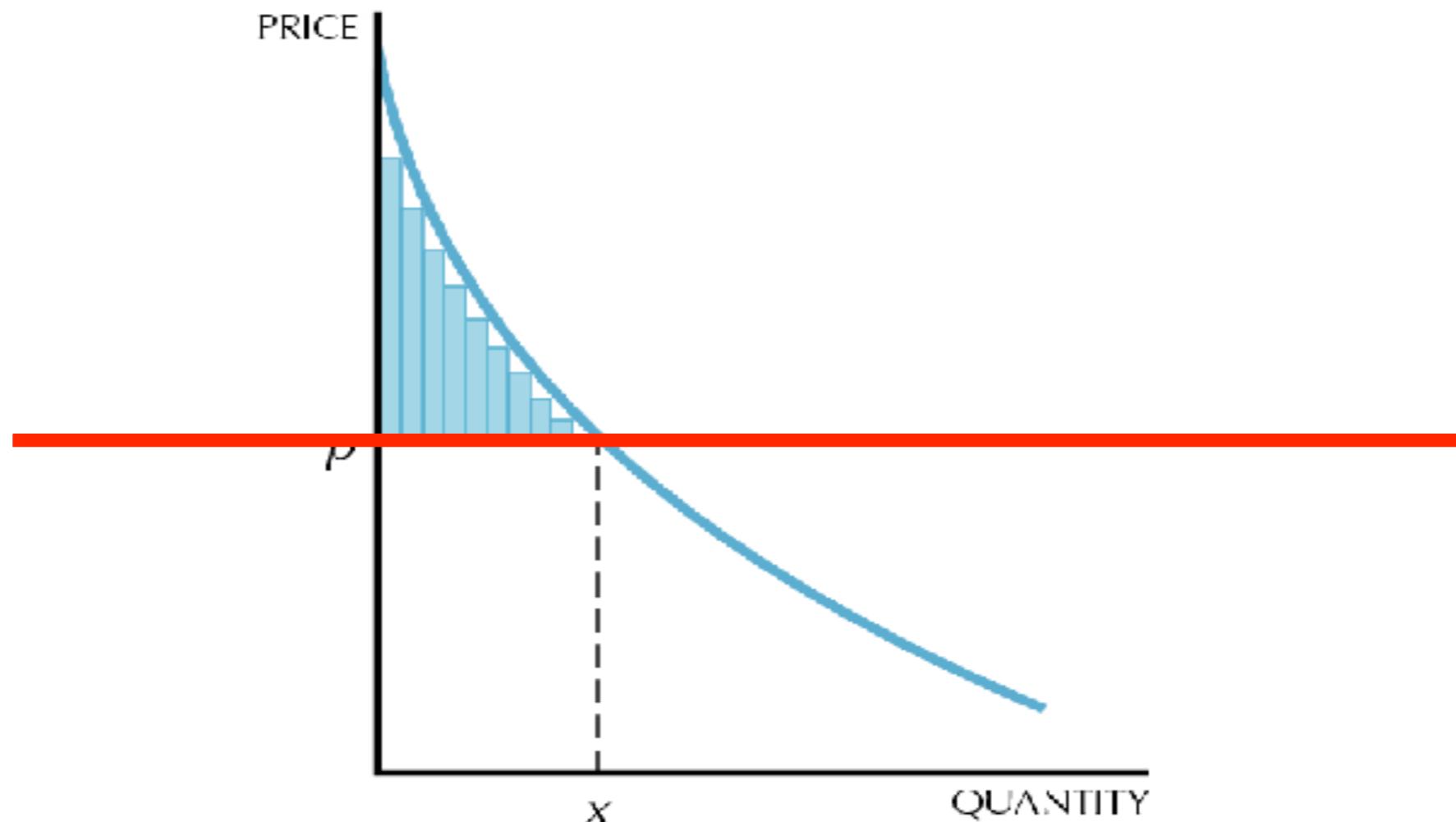
소비자 수가 많을 경우

소비자 수가 많을 경우



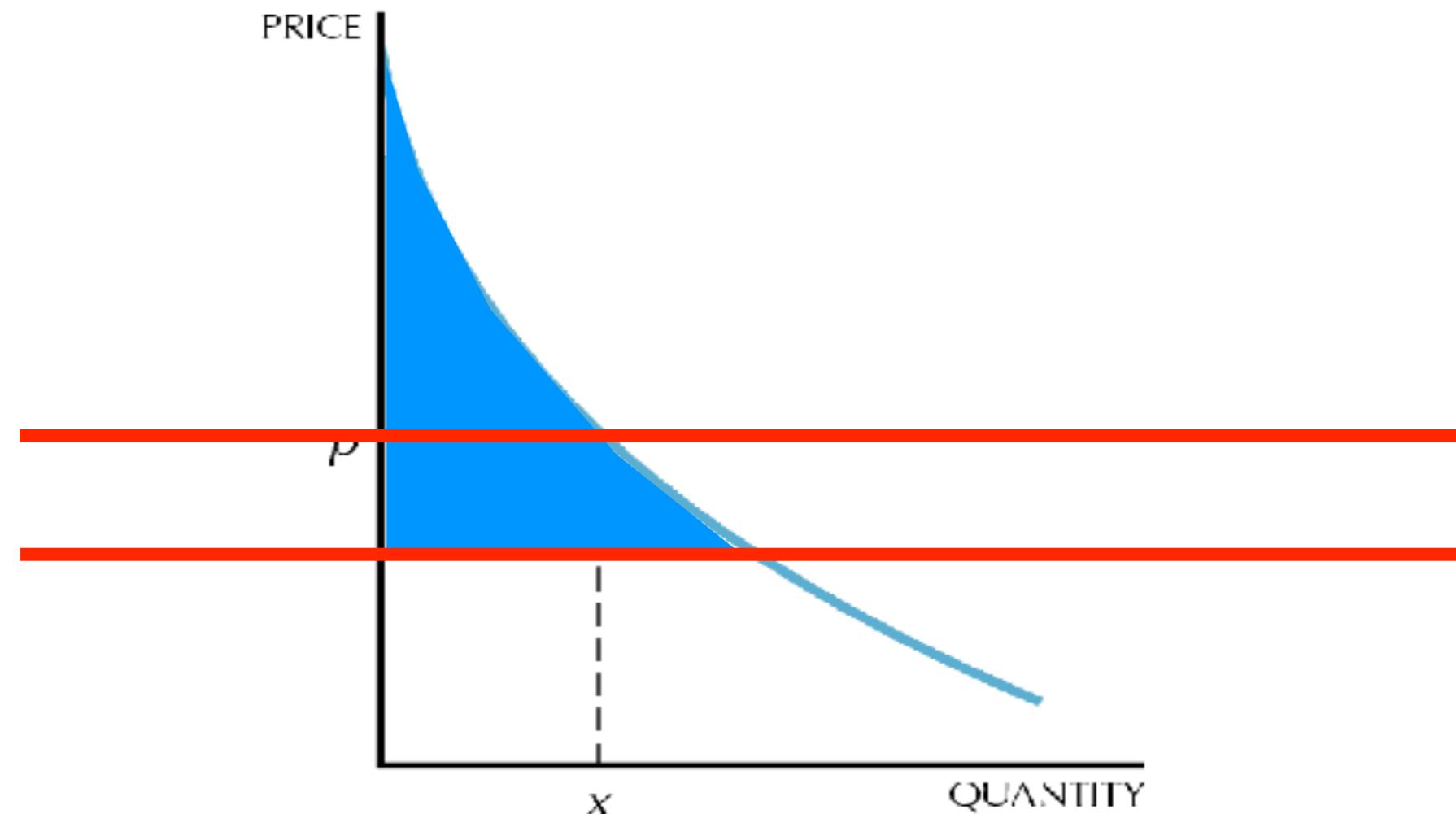
B Approximation to net surplus

소비자 수가 많을 경우



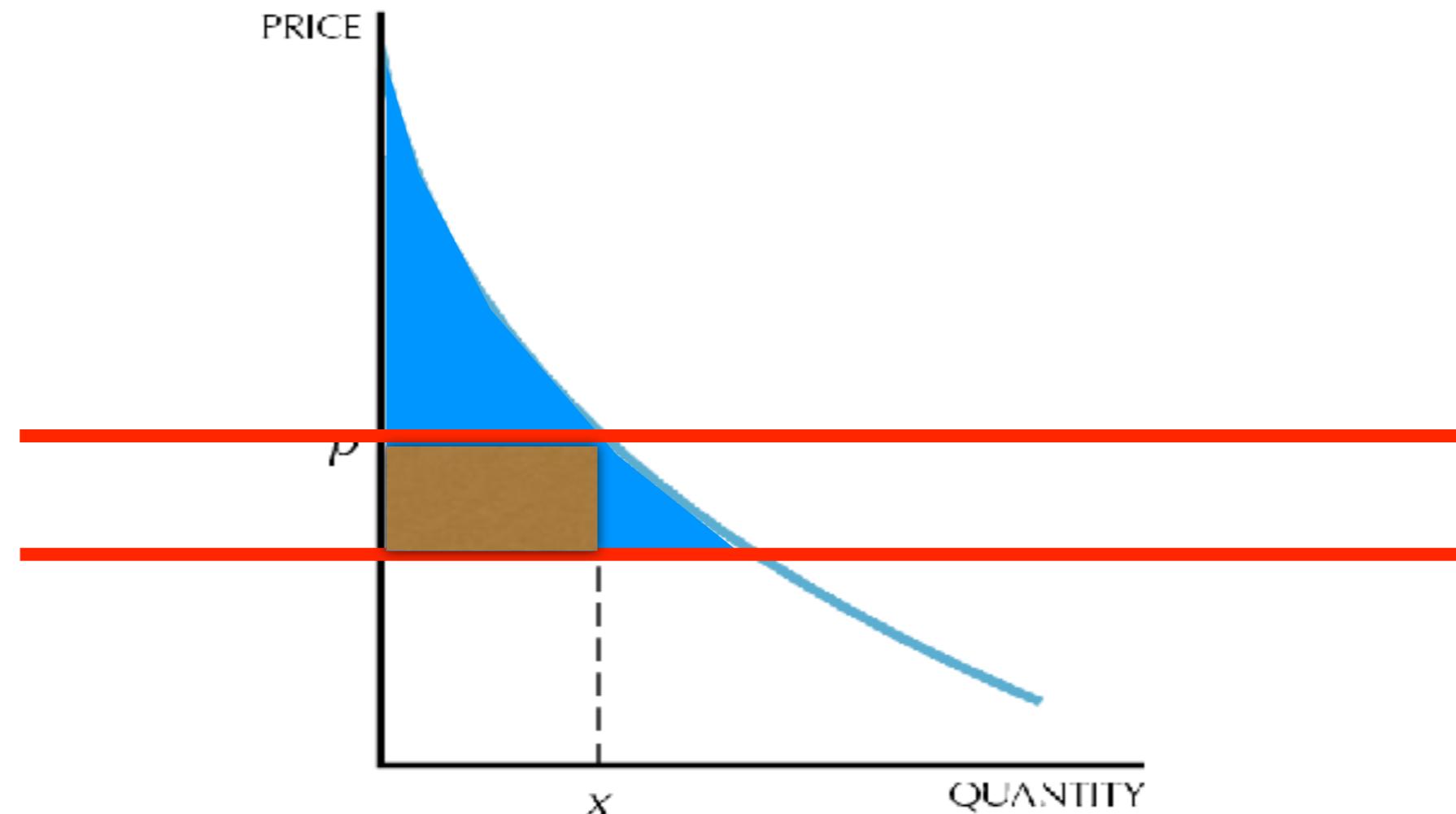
B Approximation to net surplus

소비자 수가 많을 경우



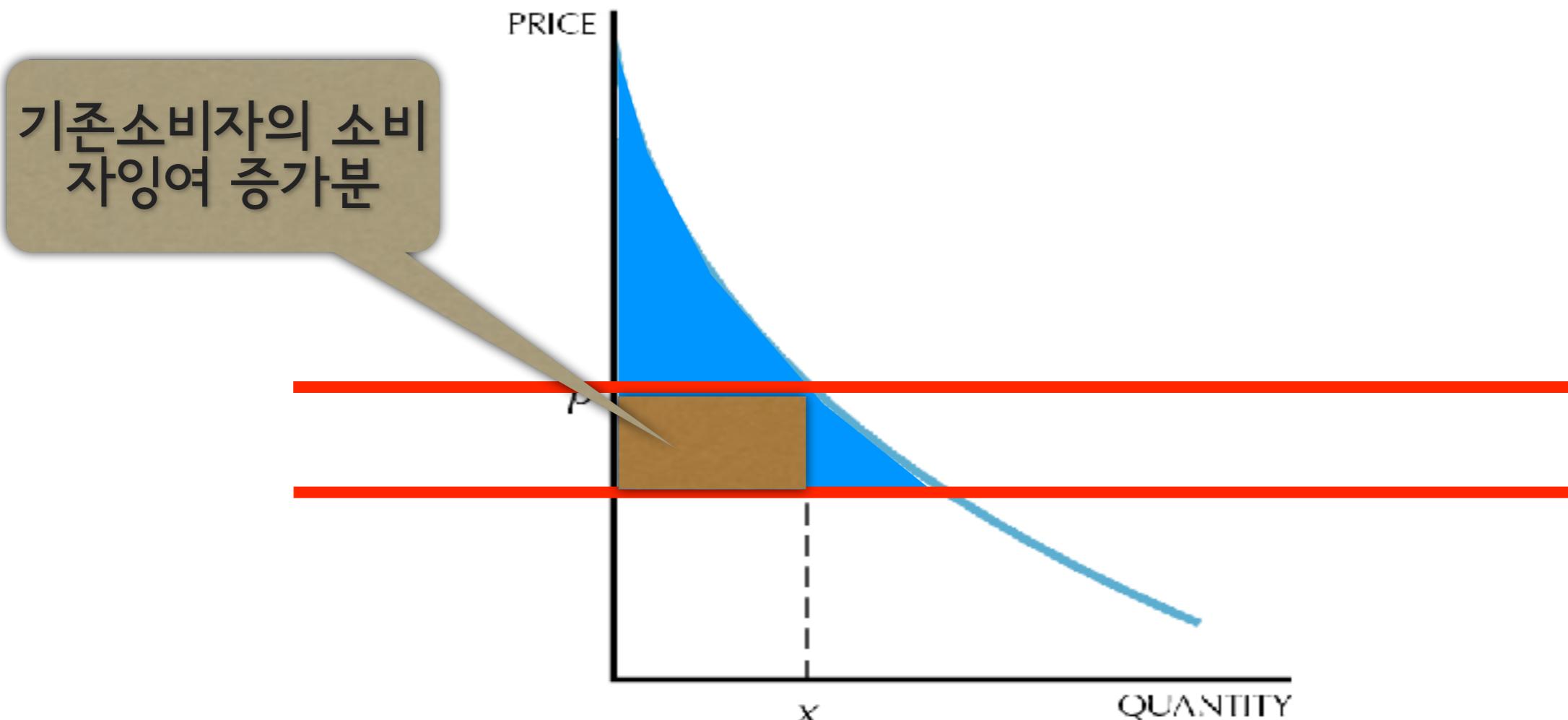
B Approximation to net surplus

소비자 수가 많을 경우



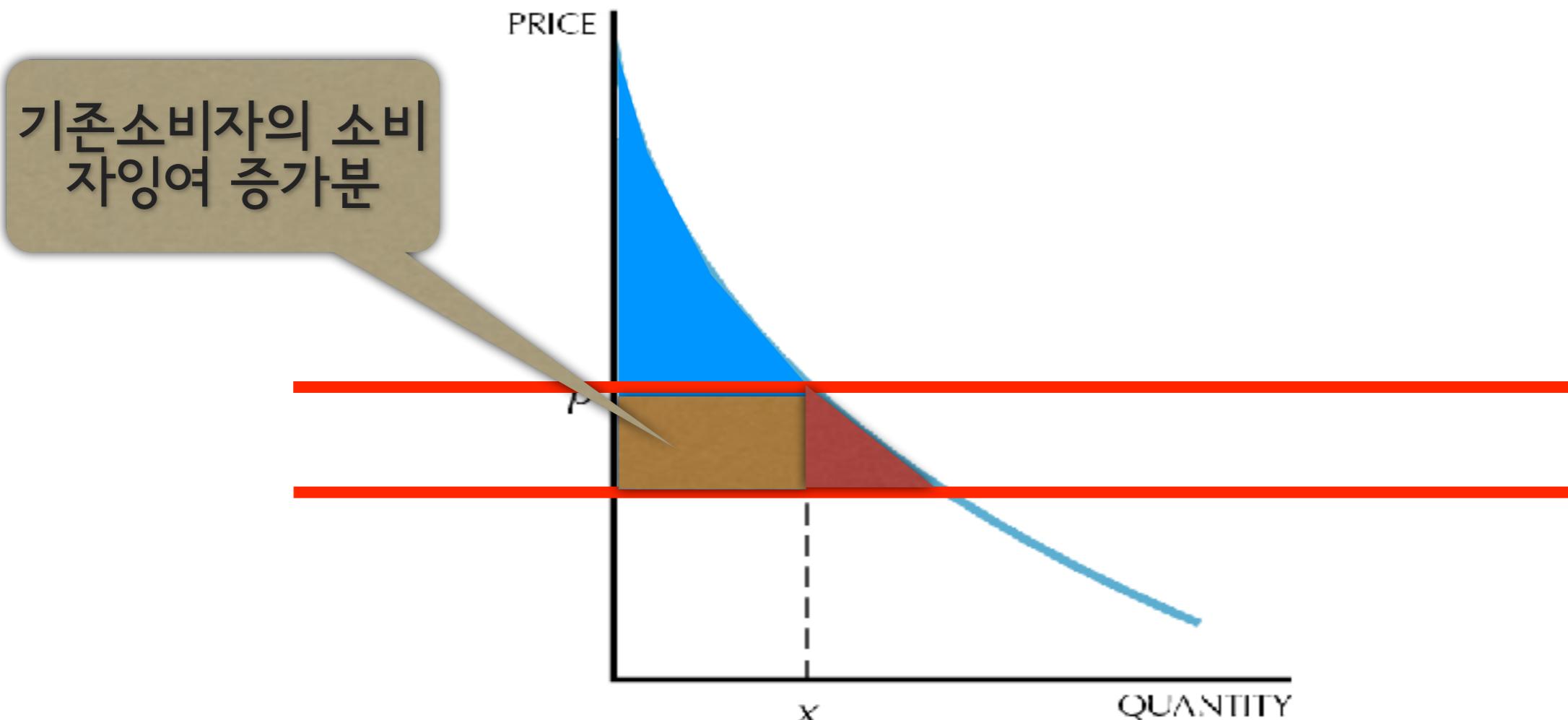
B Approximation to net surplus

소비자 수가 많을 경우



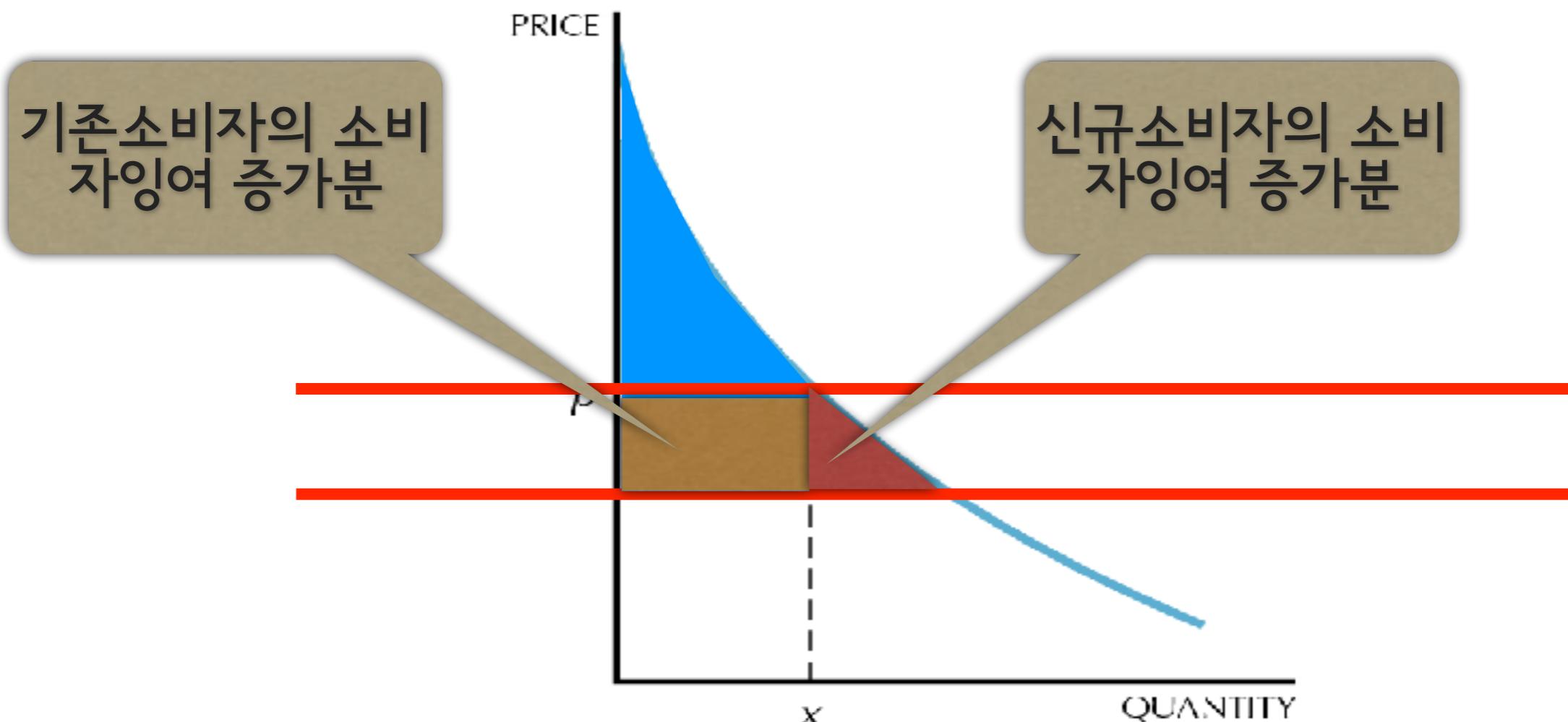
B Approximation to net surplus

소비자 수가 많을 경우



B Approximation to net surplus

소비자 수가 많을 경우



B Approximation to net surplus

연습: 제약학에서의 극대화

Consumers' Problem의 일반화

$$\arg \max_{x_1, x_2} u(x_1, x_2) \quad s.t. \quad \bar{p}_1 x_1 + \bar{p}_2 x_2 \leq \bar{m}$$

- N=2, 1 Inequality Constraint case
- 제약부등식을 만족하면서 u 함수를 가장 크게 만드는 x_1, x_2 를 찾는 문제
 - 함의: 예산 범위 내에서 가장 큰 만족을 주는 소비집합을 찾는 문제

$$\arg \max_{x_1, x_2} f(x_1, x_2) \quad s.t. \quad g(x_1, x_2) \leq \bar{b}$$

일반적 절차

$$\arg \max_{x_1, x_2} u(x_1, x_2)$$

** 부등제약은 직접
풀 수 없다

- STEP1: 제약없는 극대화 문제를 푼다 (non binding solution)
 - 이 문제의 해가 부등제약을 충족하는지 검토하여 충족한다면 해의 후보
- STEP2: 부등제약(\geq)을 등제약($=$)으로 바꾸고 등제약 하에서의 극대화 문제를 푼다. (binding solution)
 - 이 문제의 해는 해의 후보
$$\arg \max_{x_1, x_2} u(x_1, x_2) \quad s.t. \quad \bar{p}_1 x_1 + \bar{p}_2 x_2 = \bar{m}$$
- 위에서 구한 해의 후보들 중 가장 높은 값을 찾기

라그랑지안 함수

$$\arg \max_{x_1, x_2} f(x_1, x_2) \quad s.t. \quad g(x_1, x_2) \leq \bar{b}$$

$$L(\mathbf{x}, \lambda) := f(\mathbf{x}) + \lambda(\bar{b} - g(\mathbf{x}))$$

- 부등제약 하에서의 극대화 문제를 풀기 위해 고안된 함수
- 앞에서 언급한 일반적 절차는 이 라그랑지안 함수의 1계 조건 (FOC), 2계 조건 (SOC)을 검토함으로써 해결할 수 있음
 - 사실상 라그랑지안 함수의 1계 조건, 2계 조건이 이 일반 절차를 유도하는 것임

First Order Conditions

Inequality Constraints: Main Concept

Inequality constrained solution = [Equality Constrained solution] (corner solution, binding) or [Unconstrained solution] (internal solution, not binding)

Theorem (18.3)

f, g are C^1 function on \mathbb{R}^2 and \mathbf{x}^* max(min)imizes f on the inequality constraint set $g(\mathbf{x}) \leq b$. If $g(\mathbf{x}^*) = b$, and $Dg_{\mathbf{x}}(\mathbf{x}^*) \neq \mathbf{0}$, There is a multiplier λ^* satisfying:

- ① $L(\mathbf{x}, \lambda) := f(\mathbf{x}) + \lambda(\bar{b} - g(\mathbf{x}))$
- ② $DL_{\mathbf{x}, \lambda}(\mathbf{x}^*, \lambda) = \mathbf{0}$
- ③ $\lambda^*(\bar{b} - g(\mathbf{x}^*)) = 0$
- ④ $\lambda^* \geq 0$
- ⑤ $\bar{b} - g(\mathbf{x}^*) \geq 0$

Second Order Condition

Sufficient SOC: Calculation Procedure

Suppose $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{H} \in \mathbb{R}^m$, and NCDQ holds.

- ① Form a Lagrangian function L

$$L := f(\mathbf{x}) + \mu(\mathbf{c} - \mathbf{H})$$

- ② Get points \mathbf{x}^* , μ^* satisfy FOCs
- ③ Make a bordered Hessian

$$H := D^2 L_{\mu, \mathbf{x}}(\mathbf{x}^*, \mu^*) = \begin{pmatrix} \mathbf{0} & D\mathbf{H}_{\mathbf{x}}(\mathbf{x}^*) \\ D\mathbf{H}_{\mathbf{x}}(\mathbf{x}^*)^T & D^2 L_{\mathbf{x}}(\mathbf{x}^*, \mu^*) \end{pmatrix}$$

- ④ If H is PD, then \mathbf{x}^* is strict local min. If ND, \mathbf{x}^* is strict local max.
 - (a) If $\text{sign}(\det H) = \text{sign}((-1)^m)$ and all $n - m$ LPMs have same sign, H is PD on the constraint set
 - (b) If $\text{sign}(\det H) = \text{sign}((-1)^n)$ and following $n - m$ LPMs alternates in sign, H is ND on the constraint set

Second Order Condition

Sufficient SOC: Calculation Procedure

Suppose $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{H} \in \mathbb{R}^m$, and NCDQ holds.

- ① Form a Lagrangian function L

$$L := f(\mathbf{x}) + \mu(\mathbf{c} - \mathbf{H})$$

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Calculation Procedure

- STEP 1: Build Lagrangian function
- STEP 2: Make all points satisfying
 - [STEP 2-1-1] FOCs, [STEP 2-1-2] SOC of Lagrangian
 - [STEP 2-2] Irregular
- STEP 3: Find max among above candidates

Example 1

$$\arg \max_{\mathbf{x}} f(\mathbf{x}) \quad s.t. \quad x_1 + x_2 \leq 1$$

$$f(\mathbf{x}) = x_1^2 - x_1x_2 + x_2^2$$

Step 1: Building Lagrangian

$$L(\mathbf{x}, \lambda) := f(\mathbf{x}) + \lambda(\bar{b} - g(\mathbf{x}))$$

$$\arg \max_{\mathbf{x}} f(\mathbf{x}) \quad s.t. \quad x_1 + x_2 \leq 1$$

$$f(\mathbf{x}) = x_1^2 - x_1x_2 + x_2^2$$

$$L = x_1^2 - x_1x_2 + x_2^2 + \lambda(1 - x_1 - x_2)$$

L은 모든 점에서 미분 가능하므로 irregular point 존재하지 않음

STEP2-1-1: Solve FOCs

- ② $DL_{\mathbf{x}, \lambda}(\mathbf{x}^*, \lambda) = 0$
- ③ $\lambda^*(\bar{b} - g(\mathbf{x}^*)) = 0$
- ④ $\lambda^* \geq 0$
- ⑤ $\bar{b} - g(\mathbf{x}^*) \geq 0$

$$D \lambda (2x_1 - x_2 - \lambda) \quad -x_1 + 2x_2 - \lambda = (0 \quad 0)$$

$$\lambda(1 - x_1 - x_2) = 0, \quad \lambda \geq 0, \quad x_1 + x_2 \leq 1.$$

Case 1: lambda = 0 인 경우 (Non binding case)

(c) (10 points) 위 FOC를 만족하는 모든 점(점)을 찾으라.

$$\textcircled{1} L(x_1, x_2, \lambda) = x_1^2 - x_1 x_2 + x_2^2 + \lambda(1 - x_1 - x_2)$$

10

$$D_L(\lambda) = \begin{pmatrix} 2x_1 - x_2 - \lambda & -x_1 + 2x_2 - \lambda \end{pmatrix} = \textcircled{0} \text{ 이므로}$$

$$\begin{cases} 2x_1 - x_2 = \lambda \\ 2x_2 - x_1 = \lambda \end{cases} \text{이나 } \begin{cases} 2x_1 - x_2 = \lambda \\ x_1 - 2x_2 = \lambda \end{cases}$$

$$\lambda(1 - x_1 - x_2) = 0 \text{에서}, \lambda = 0 \text{일 때 } 2x_1 = x_2, 2x_2 = x_1 \\ \therefore x_1 = x_2 = 0$$

$$\lambda = 0 \text{ 일 때 } x_1^* = (0, 0)$$

OK 수용

Case 2: lambda > 0 인 경우 (Binding case)

시사국제금융세미나 A반

시사국제금융세미나 A반 기말시험

2016년 가을학기

(e) (10 points) 위 FOC를 만족하는 모든 점(들)을 찾으라.

$$\begin{cases} 1 - \lambda x_1 - \lambda x_2 = 0 \\ 2x_1 - x_2 - \lambda = 0 \\ -x_1 + 2x_2 - \lambda = 0 \end{cases}$$

$$\begin{aligned} 2x_1 - x_2 &= \lambda \rightarrow 2x_1 - x_2 = -x_1 + 2x_2 \\ -x_1 + 2x_2 &= \lambda \rightarrow 3x_1 = 3x_2 \rightarrow x_1 = x_2 \end{aligned}$$

$$\rightarrow 1 - x_1 - x_1 = 0 \rightarrow x_1 = \frac{1}{2}, x_2 = \frac{1}{2} // \text{case ①}$$

* NDC & 확인
 $(-1 \ -1)$ 이므로 Q를 Pass 할.

$$\begin{array}{r} 1 - (-2) \\ -2 - (-1) \\ \hline = 3 \end{array}$$

송영수

$$1 - (-2)$$

Case 2: lambda > 0 인 경우 (Binding case)

시사국제금융세미나 A반

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$$\begin{aligned} 2x_1 - x_2 &= \lambda \rightarrow 2x_1 - x_2 = -x_1 + 2x_2 \\ -x_1 + 2x_2 &= \lambda \quad \rightarrow 3x_1 = 3x_2 \rightarrow x_1 = x_2 \end{aligned}$$

$$\rightarrow 1 - x_1 - x_1 = 0 \rightarrow x_1 = \frac{1}{2}, x_2 = \frac{1}{2} // \text{case ①}$$



Q를 Pass 할.

송영수

$$\begin{array}{r} 1 - (-2) \\ -2 - (-1) \\ \hline = 3 \end{array}$$

$$1 - (-2)$$

STEP2-1-2: SOC

If H is PD, then \mathbf{x}^* is strict local min. If ND, \mathbf{x}^* is strict local max.

10

송영식

$1 - (-2)$

(d) (10 points) 위에서 찾은 점(들)에 대하여 2계조건 (SOC)를 검토하라.

① $\begin{pmatrix} 0 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$ $\rightarrow m=1 \rightarrow 1, 1\text{개의 LPM}_1$
 $n=2$ $\rightarrow LPM_{(3)} = 0 \cdot (-1)^{(1+1)(1+2)} \cdot \begin{vmatrix} 0 & -1 \\ -1 & 2 \end{vmatrix} + (-1) \cdot (-1)^{(2+1)} \cdot \begin{vmatrix} -1 & -1 \\ 1 & 2 \end{vmatrix}$

$\lambda \neq 0$ 일 때: case ①

$$= 0 + (-1) + (-3) = -4 < 0$$

② $\lambda = 0$ 일 때: case ②
 $\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \rightarrow LPM_{(2)} = (2 \times 2) + ((-1) \times (-1)) = 4 - 1 = 3 > 0$
 $LPM_{(1)} = 2 > 0$

$\therefore \lambda \neq 0$ 일 때와 $\lambda = 0$ 일 때 모두 PD이다.

No Solution

3e

(e) (10 points) 위에서 찾은 점(들)이 주어진 극대화문제의 해가 될 수 있는지 논하라.

10

(e). (10 points) 위에서 찾은 점(들)이 주어진 극대화문제의 해가 될 수 있는지 논하라.

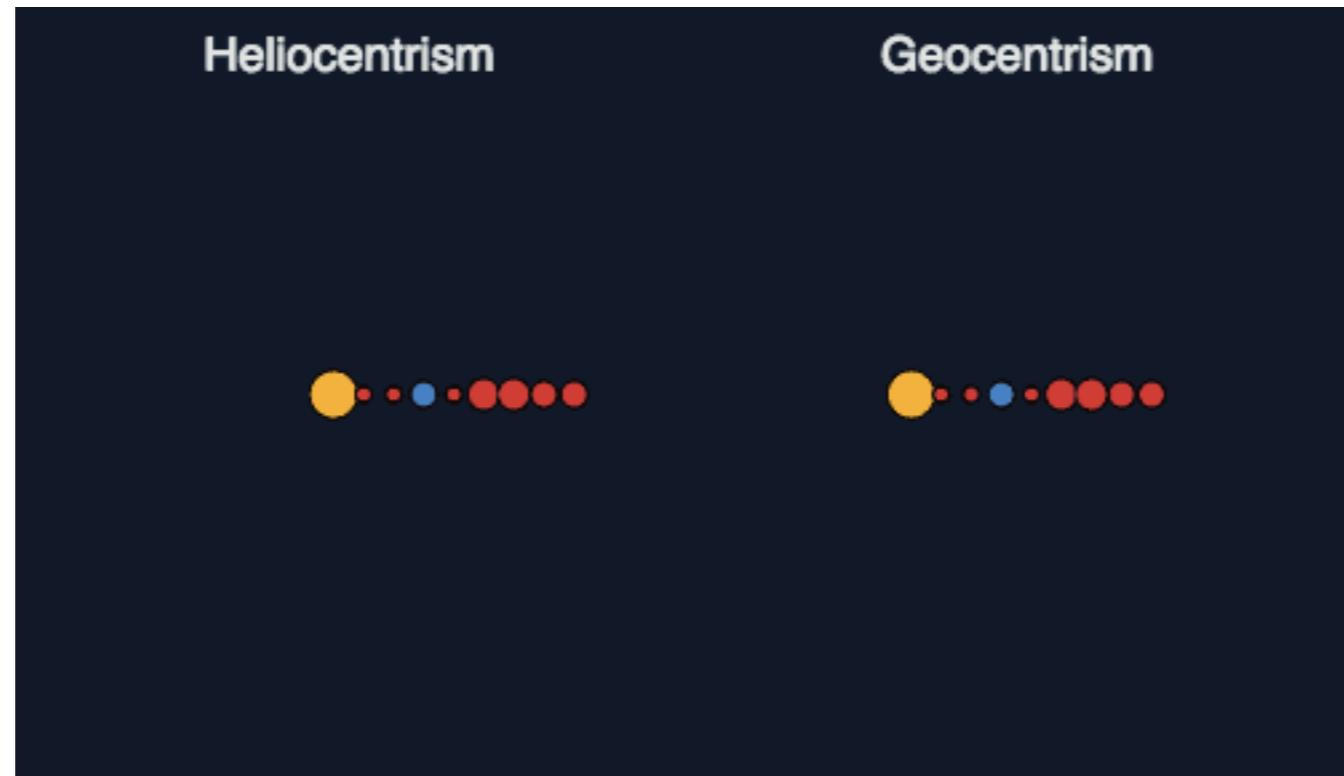
$\lambda \neq 0$ 일 때의 $x_1 = \frac{1}{2}, x_2 = \frac{1}{2}$ }, 이 해들(점들)은 주어진
 $\lambda = 0$ ", $x_1 = 0, x_2 = 0$

극대화 문제의 해가 될 수 없다.

STEP3: 결론

- 후보가 없으므로 답 없음

수고하셨습니다!



수고하셨습니다!

