Homework 2: Problems (Due April 11)

Remark: for [programing] problems, you are supposed to write a brief report about the solution with code attached. You are also asked to submit a source code. Exercises and Computer Problems are taken from the textbook.

(1) Exercises 3.1

(2) Exercises 3.18 (a), (b), and (c)

(3) [Programming] Implement Algorithm 3.2 (Classical Gram-Schmidt Orthogonalization). With this algorithm, you then use QR factorization to solve for the linear least square problem below

$$\boldsymbol{A}\boldsymbol{x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \cong \begin{bmatrix} 1237 \\ 1941 \\ 2417 \\ 711 \\ 1177 \\ 475 \end{bmatrix} = \boldsymbol{b},$$

(4) A is a square and invertible matrix, show that $||A^{-1}||_2 = 1/\sigma_{\min}$, where $\sigma_{\min}$ is the minimum singular value of matrix A.

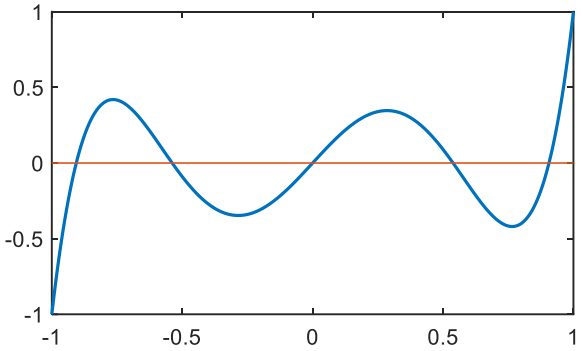
(5) Consider $f(x) = x^2 - 1$, take $x_0 = 2$ and apply Newton's method with 4 iterations and show the iteration history. Take $x_0 = -2$ and repeat the computation. What do you find?

(6) Consider $f(x) = x^2 - 1$, take $x_0 = 3, x_1 = 2$ and apply secant method with 6 iterations and show the iteration history.

(7) [Programming]: Consider Legendre polynomial $L_5(x) = (63x^5 - 70x^3 + 15x)/8$.

1) Use bisection methods to find all five zeros of the polynomial
2) Combine bisection and Newton's method to speed up your iterations.

Hint: the zeros of $L_5(x)$ is located within the interval [-1,1]. In addition, the zeros are between subsequent local maximum and minimum.



(8) Exercises 6.4 (a), (d)

(9) [Programming] Implement Algorithm 6.1 (Golden section search) and use the algorithm to solve for the minimum of the function $f(x) = 1 - 0.3xe^{-x^2}$ with initial bracket [0,2]. (Hint: you can use Example 6.8 to test your implementation of the algorithm)

(10) [Programming] Computer Problems 6.9 (a) and (b).