assignment10

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1 Binary Classification with Different Features (multiple number)

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In a *classification problem*, the outcome takes on only a finit number of values. In the simplest case, outcome has only two values, for example TRUE or FALSE. This is called the *binary classification problem*.

As in real-valued data fitting, we assume that an approxomate relation ship of the form $y \approx f(x)$ holds, where $f : R^n \to -1, +1$. The model $\hat{f}$ is called a *classifier*.

For a given data point x, y with predicted outcome $\hat{y} = \hat{f}(x)$, there are four possibilities:

- *True positive*: $y = +1$ and $\hat{y} = +1$.
- *True negative*: $y = -1$ and $\hat{y} = -1$.
- *False positive*: $y = -1$ and $\hat{y} = +1$.
- *False negative*: $y = +1$ and $\hat{y} = -1$.

Continue ith assignmen09, this time we use *least squares classifer* to build classifiers for each number in MNIST data set.

We still use feature functions $f_i = r_i^T x, r_i \sim N(0, \sigma)$, and try with varing the number of parameter p with the standard deviation $\sigma = 1$ of the random feature vectore r.

Since this time we want to do the classification for ten numbers, our sign function is change to:

$$argmax_n \tilde{f}_n(x)$$

1.1 Create Classifier

1.1.1 Import data set

We have two data sets, one for training, one for testing. Each element is a image that has height 28 and width 28 pixels.

In [1]:
```python
import matplotlib.pyplot as plt
import numpy as np

file_data_train = "mnist_train.csv"
file_data_test  = "mnist_test.csv"

h_data_train    = open(file_data_train, "r")
h_data_test     = open(file_data_test, "r")

data_train      = h_data_train.readlines()
data_test       = h_data_test.readlines()

h_data_train.close()
h_data_test.close()

size_row    = 28    # height of the image
size_col    = 28    # width of the image

num_train   = len(data_train)    # number of training images
num_test    = len(data_test)     # number of testing images

# number of training images: 60000
# number of testing images: 10000
```

To reduce the bias, we need to normalize the data.

In [2]:
```python
#
# normalize the values of the input data to be [0, 1]
#
def normalize(data):

    data_normalized = (data - min(data)) / (max(data) - min(data))

    return(data_normalized)
```

Normalize each pixel, and put image data into a $784 * num\_image$ matrix.

In [3]:
```python
#
# make a matrix each column of which represents an images in a vector form
#
list_image_train    = np.empty((size_row * size_col, num_train), dtype=float)  # 784 *
list_label_train    = np.empty(num_train, dtype=int)

list_image_test     = np.empty((size_row * size_col, num_test), dtype=float)
list_label_test     = np.empty(num_test, dtype=int)

count = 0

for line in data_train:
```

```
        line_data   = line.split(',')
        label       = line_data[0]
        im_vector   = np.asfarray(line_data[1:])  # convert to float type
        im_vector   = normalize(im_vector)

        list_label_train[count]      = label
        list_image_train[:, count]   = im_vector  # each column is a image

        count += 1

    count = 0

    for line in data_test:

        line_data   = line.split(',')
        label       = line_data[0]
        im_vector   = np.asfarray(line_data[1:])
        im_vector   = normalize(im_vector)

        list_label_test[count]       = label
        list_image_test[:, count]    = im_vector

        count += 1
```

1.1.2 Define feature functions generator

Depends on the given number of parameters we should generate correponde feature functions r. Each element of r is a vector with length $28 * 28$. The element of r_i is random number from a normal distribution.

```
In [4]: def generate_features(n, size):
            """Generate feature functions

            Argumengs:
                n(int): number of parameters
                size: number of elements of each vector
            Return:
                functions(2d matrix): feature function matrix
            """

            functions = []

            # for n, generate vectore with #size elements
            mean, sigma = 0, 1

            for _ in range(n):
                ri = np.random.normal(mean, sigma, size)
```

```
            functions.append(ri)

        return np.array(functions)
```

1.1.3 Generate the matrix of feature funtions and data

Since we have got feature functions, now we can apply them on input data.

```
In [5]: def generate_tilde_matrix(feature_func, data, num_data):
            """Create matrix of feature function on data

            Arguments:
                feature_func(2d matrix): feature function matrix
                data: input image data
                num_data: number of input data
            Return:
                matrix of feature functions applied on image data
            """

            A = []

            for i in range(num_data):
                img = data[:, i]  # ith image(column)
                row = np.inner(feature_func, img)
                A.append(row)

            return np.array(A)
```

1.1.4 Compute each number's θ_n

For computing, we need to process label y which gives correspond number's image $+1$ and other number's image -1.

```
In [6]: # process label array
        def process_label(labels, number):
            result = []
            for label in labels:
                if label == number:
                    result.append(1)
                else:
                    result.append(-1)

            return result
```

Depends on $A\theta = y$, while A is the matrix of feature function apply on data set, θ is perameters, and y is the label.

Through pseudo inverse $(A^T A)^{-1} A$ we can compute the θ. Here we use `np.linalg.pinv` to get A^{-1}.

In [7]:
```python
def compute_theta(A, y):
    A_inv = np.linalg.pinv(A)
    theta = np.inner(A_inv, y)

    return theta
```

Because we need to which label is the best result for current number image, we need each number's parameter to do the comparision.

In [8]:
```python
def compute_all_thetas(A, image_labels):
    """Compute parameters for each number

    Generate correspond label list, then use it to compute theta.

    Argument:
        A(2d matrix): tilde matrix of feature funcion on input images
    Return:
        A list of parameters. Index correspond to number label.
    """

    parameters = []

    for i in range(10):
        label_processed = process_label(image_labels, i)
        theta = compute_theta(A, label_processed)
        parameters.append(theta)

    return parameters
```

1.1.5 Define classifier $\hat{f}(x)$

Now we have had feature functions, parameters, so we can create our classifier $\hat{f}(x) = argmax_n(\tilde{f}_n(x))$, where $\tilde{f}(x) = \theta_1 f_1(x) + \theta_2 f_2(x) + \cdots + \theta_p f_p(x)$

In [9]:
```python
def argmax(A, parameters, num_images):
    """Give back maximum argument's label

    Arguments:
        A(2d matrix): tilde matrix of feature function on input images
        parameters(list): list of parameters of each number image
    Return:
        prediction label of images
    """
    tilde_all = []

    # compute each images's tilde value correspond to diff parameter
    for parameter in parameters:
        f_tilde_n = np.inner(A, np.array(parameter))
        tilde_all.append(f_tilde_n)
```

```
        tilde_all = np.array(tilde_all)

        result = []
        # find maximum tilde value for each image
        # let its label be images's prediction
        for i in range(num_images):
            value_i = tilde_all[:, i]
            label = np.argmax(value_i)
            result.append(label)

        return np.array(result)
```

1.2 Create Confusion Matrix and F_1 Scores

1.2.1 Create confusion matrix

Let row of matrix be digits, column be the number of correspond predictions.

```
In [10]: def create_confusion_matrix(test_labels, predictions):
             """Build confusion matrix

             Matrix that indicates the number of classification for the digit

             Argument:
                 test_labels(1d array): correct label of input images
                 predictions(1d array): predicted label of input images
             Return:
                 2d matrix
             """

             matrix = np.zeros((10, 10), dtype=int)  # there are ten numbers

             length = len(test_labels)
             for i in range(length):
                 matrix[test_labels[i]][predictions[i]] += 1

             return matrix
```

1.2.2 Compute F_1 scores

$$F_1 score = 2 \cdot \frac{precision \cdot recall}{precision + recall},$$

where $precision = \frac{true\ positives}{true\ positives\ +\ false\ positives}$, $recall = \frac{true\ positives}{false\ negative\ +\ true\ positive}$

We compute each number's F_1 score then use their average value as finial score for current number of parameters.

From confusion matrix M, we can tell, for each index i:

- $M[i][i]$ is the True Positive of number i,

- sum of $row[i]$ is True Posive adds False Nagetive
- sum of $column[i]$ is True Posive adds False Positive

```
In [11]: def get_f1(M):

             f1_scores = []

             for i in range(10):  # there are ten numbers
                 tp = M[i][i]
                 fn = sum(M[i]) - tp
                 fp = sum(M[:, i]) - tp
                 f1_scores.append(compute_f1(tp, fn, fp))

             return sum(f1_scores) / 10


         def compute_f1(tp, fn, fp):
             precision = tp / (tp + fp)
             recall = tp / (fn + tp)

             return 2*precision*recall / (precision+recall)
```

1.2.3 Combine all parts together

```
In [17]: def classificate(p,
                          image_train, num_trian, labels_train,
                          image_test, num_test, labels_test):

             size_img = 28 * 28

             # generate features
             r = generate_features(p, size_img)

             # generate training img tilde matrix
             A_train = generate_tilde_matrix(r, image_train, num_train)

             # compute parameters
             parameters = compute_all_thetas(A_train, labels_train)

             # generate testing img tilde matrix
             A_test = generate_tilde_matrix(r, image_test, num_test)

             # get predictions
             predictions = argmax(A_test, parameters, num_test)

             # generate confusion matrix
             confusion = create_confusion_matrix(labels_test, predictions)
```

```
    # compute F1 score
    f1 = get_f1(confusion)

    return confusion, f1
```

1.3 Test With Different p

Now let's try different number of parameters to see how F_1 score changes.

1.3.1 Functions for present results

```
In [13]: from prettytable import PrettyTable

         def print_table(M):
             totals = []
             table = PrettyTable()

             table.add_column(" ", [0, 1, 2, 3, 4, 5, 6, 7, 8, 9])
             for i in range(10):
                 totals.append(sum(M[i]))
                 table.add_column(str(i), M[:,i].flatten())
             table.add_column("Total", totals)

             print(table)
```

1.3.2 Try different number of parameters

Now let's try different number parameters.

Here we use logarithm $2^n, n \in [1, 10]$ number of parameters.

```
In [14]: num_paras = [2**n for n in range(1, 11)]

         f1_history = []

         for p in num_paras:
             confusion, f1 = classificate(p,
                 list_image_train, num_train, list_label_train,
                 list_image_test, num_test, list_label_test)

             f1_history.append(round(f1, 4))

             print('Confusion Table with {} parameters'.format(p))
             print_table(confusion)
             print('\n\n')
```

```
/home/ziggy/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:15: RuntimeWarning: in
  from ipykernel import kernelapp as app
```

Confusion Table with 2 parameters

	0	1	2	3	4	5	6	7	8	9	Total
0	209	178	0	0	2	0	0	591	0	0	980
1	433	207	0	1	48	0	0	446	0	0	1135
2	200	131	0	0	26	0	0	675	0	0	1032
3	208	218	0	2	71	0	0	511	0	0	1010
4	103	24	0	0	61	0	0	794	0	0	982
5	208	166	0	2	18	0	0	498	0	0	892
6	106	25	0	0	10	0	0	817	0	0	958
7	152	30	0	0	2	0	0	844	0	0	1028
8	310	177	0	1	14	0	0	472	0	0	974
9	151	18	0	0	21	0	0	819	0	0	1009

Confusion Table with 4 parameters

	0	1	2	3	4	5	6	7	8	9	Total
0	304	66	97	48	62	23	164	205	4	7	980
1	3	902	88	1	39	60	13	18	10	1	1135
2	67	66	421	13	197	6	157	94	8	3	1032
3	195	109	214	42	115	65	171	92	4	3	1010
4	21	74	168	8	469	50	141	36	8	7	982
5	120	106	62	30	168	134	180	86	3	3	892
6	77	48	168	21	102	28	449	51	9	5	958
7	249	68	165	4	47	59	149	279	7	1	1028
8	31	304	161	17	251	17	93	61	31	8	974
9	36	246	158	3	257	89	98	81	32	9	1009

Confusion Table with 8 parameters

	0	1	2	3	4	5	6	7	8	9	Total
0	135	200	74	171	30	2	194	20	146	8	980
1	22	961	53	37	0	2	25	1	34	0	1135
2	29	125	339	171	124	1	97	20	110	16	1032
3	23	88	66	630	15	3	139	9	33	4	1010
4	15	15	72	12	620	1	68	93	18	68	982
5	72	137	16	148	39	47	159	101	165	8	892
6	15	41	58	20	39	0	740	32	11	2	958
7	19	94	112	8	166	1	101	445	42	40	1028

8	50	97	90	80	63	3	163	59	347	22	974
9	23	31	82	14	457	0	74	77	48	203	1009

Confusion Table with 16 parameters

	0	1	2	3	4	5	6	7	8	9	Total
0	862	2	39	22	1	2	16	4	32	0	980
1	0	1091	2	6	7	1	4	9	15	0	1135
2	77	160	540	48	27	2	56	33	69	20	1032
3	55	82	19	696	24	7	35	20	56	16	1010
4	41	85	52	15	587	1	49	56	32	64	982
5	142	96	39	277	84	48	43	21	122	20	892
6	176	39	63	48	78	3	504	13	25	9	958
7	117	157	26	29	63	1	33	472	59	71	1028
8	52	80	49	145	62	4	25	21	517	19	974
9	65	95	50	38	170	0	37	111	43	400	1009

Confusion Table with 32 parameters

	0	1	2	3	4	5	6	7	8	9	Total
0	894	5	5	14	6	9	25	10	12	0	980
1	0	1081	27	4	1	0	9	4	6	3	1135
2	18	109	709	53	22	0	51	22	40	8	1032
3	8	42	38	786	9	25	34	26	23	19	1010
4	12	38	25	36	648	1	44	55	12	111	982
5	169	60	21	109	29	219	81	77	94	33	892
6	62	31	29	8	40	11	772	0	3	2	958
7	17	43	33	38	19	8	3	826	10	31	1028
8	25	140	21	127	18	23	36	34	527	23	974
9	30	30	23	47	74	3	8	122	15	657	1009

Confusion Table with 64 parameters

	0	1	2	3	4	5	6	7	8	9	Total
0	923	0	9	3	0	3	27	2	13	0	980
1	1	1078	8	11	1	2	9	3	22	0	1135

2	30	95	740	38	21	2	30	26	43	7	1032
3	10	33	38	806	3	28	15	41	20	16	1010
4	3	39	14	3	767	3	27	8	23	95	982
5	34	16	13	120	22	508	46	41	67	25	892
6	25	14	17	1	30	8	857	1	3	2	958
7	6	48	23	5	21	0	2	871	13	39	1028
8	25	78	29	56	19	28	35	43	626	35	974
9	20	19	10	14	93	8	2	91	24	728	1009

Confusion Table with 128 parameters

	0	1	2	3	4	5	6	7	8	9	Total
0	934	1	3	3	2	8	18	3	7	1	980
1	0	1093	5	5	1	0	6	2	23	0	1135
2	20	62	820	20	14	1	27	23	37	8	1032
3	7	19	25	861	4	18	12	27	22	15	1010
4	1	25	10	1	860	2	12	3	14	54	982
5	28	13	11	99	24	564	33	22	73	25	892
6	20	11	12	0	16	13	880	0	6	0	958
7	4	42	19	11	16	0	3	877	6	50	1028
8	15	55	12	39	20	37	20	22	734	20	974
9	19	14	9	14	59	4	4	70	14	802	1009

Confusion Table with 256 parameters

	0	1	2	3	4	5	6	7	8	9	Total
0	937	0	4	3	1	7	18	1	8	1	980
1	0	1102	2	2	1	1	5	2	20	0	1135
2	15	64	807	23	16	0	43	24	39	1	1032
3	4	18	30	876	2	14	8	24	20	14	1010
4	0	23	8	1	866	3	12	1	12	56	982
5	16	17	10	94	18	602	24	21	66	24	892
6	17	9	13	0	21	16	873	0	9	0	958
7	5	38	16	11	17	0	2	885	4	50	1028
8	16	55	8	35	27	43	19	14	733	24	974
9	18	12	4	16	62	0	2	62	8	825	1009

Confusion Table with 512 parameters

	0	1	2	3	4	5	6	7	8	9	Total
0	941	0	2	2	1	8	15	2	7	2	980
1	0	1105	2	2	1	1	5	2	17	0	1135
2	18	61	812	26	16	0	37	18	39	5	1032
3	4	15	22	890	2	16	10	21	19	11	1010
4	0	22	6	2	871	4	9	1	15	52	982
5	21	16	6	87	17	623	20	13	67	22	892
6	19	10	11	0	20	17	872	0	9	0	958
7	4	37	17	8	21	1	1	879	4	56	1028
8	17	53	10	31	26	40	16	13	746	22	974
9	18	11	4	15	68	0	1	78	12	802	1009

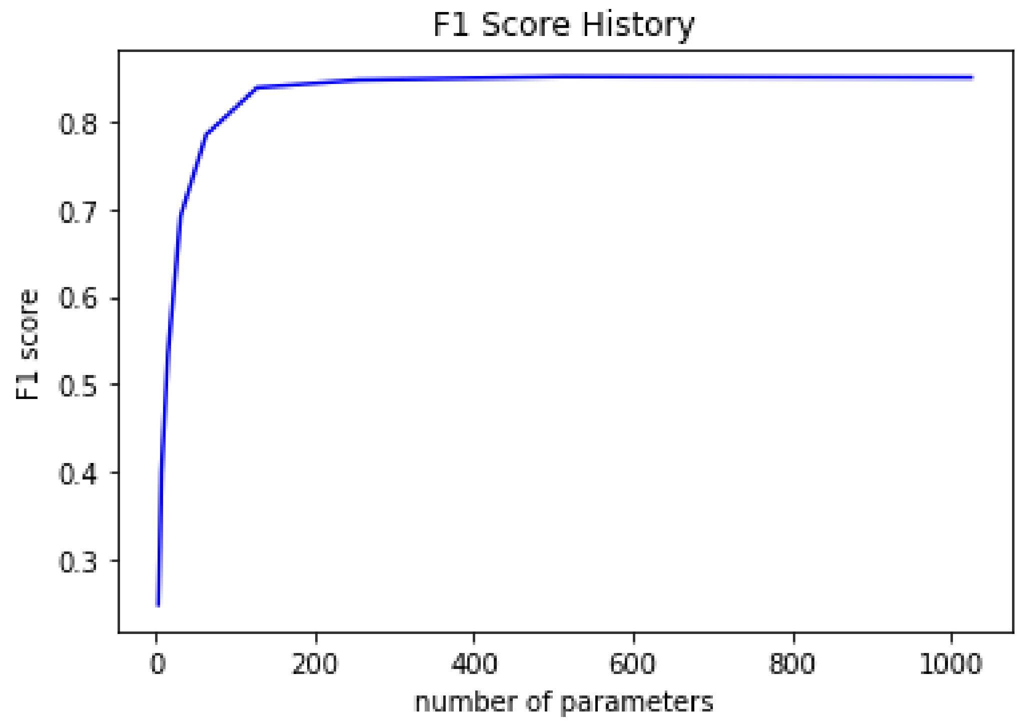
Confusion Table with 1024 parameters

	0	1	2	3	4	5	6	7	8	9	Total
0	942	0	2	2	1	7	15	2	7	2	980
1	0	1107	2	2	1	1	5	2	15	0	1135
2	17	56	809	28	16	0	42	21	39	4	1032
3	4	15	26	887	2	14	9	21	21	11	1010
4	0	23	6	3	872	5	10	2	13	48	982
5	20	17	2	84	19	624	22	13	69	22	892
6	17	9	10	0	21	20	872	0	9	0	958
7	5	38	18	8	20	0	1	877	3	58	1028
8	17	54	9	32	27	42	15	12	743	23	974
9	18	10	2	15	72	1	1	77	13	800	1009

In [16]:

```
plt.title("F1 Score History")
plt.plot(num_paras, f1_history, 'b-')
plt.xlabel('number of parameters')
plt.ylabel('F1 score')
plt.show()

t = PrettyTable(num_paras)
t.add_row(f1_history)
print(t)
```



2	4	8	16	32	64	128	256	512	1024
nan	0.2482	0.3999	0.5342	0.692	0.7847	0.8386	0.8474	0.8512	0.8503

When number of parameters is small, sometimes we cannot even get valid F_1 score, because True Positive and False Posive can both be zero.

When have 128 parameters, we almost get one of the best score, after that point F_1 score still increases but not changes much.