assignment08

November 21, 2018

1 Binary Classification

Name: ZHU GUANGYU
Student ID: 20165953
Github Repo: assignment08

In a *classification problem*, the outcome takes on only a finit number of values. In the simplest case, outcome has only two values, for example TRUE or FALSE. This is called the *binary classification problem*.

As in real-valued data fitting, we assume that an approxomate relation ship of the form $y \approx f(x)$ holds, where $f : R^n \to -1, +1$. The model $\hat{f}$ is called a *classifier*.

For a given data point x, y with predicted outcome $\hat{y} = \hat{f}(x)$, there are four possibilities:

- *True positive*: $y = +1$ and $\hat{y} = +1$.
- *True negative*: $y = -1$ and $\hat{y} = -1$.
- *False positive*: $y = -1$ and $\hat{y} = +1$.
- *False negative*: $y = +1$ and $\hat{y} = -1$.

1.1 Least squares classfier

Least squares is a very sople method for classification.

First, carry out ordinary real-valued least squares fitting of the outcome, ignoring for the moment that the outcome y takes on only the values $-1 and + 1$. We choose *basis functions* $f_1, \cdots, f_p$, and the *perameters* $\theta_1, \cdots, \theta_p$ so as to minimize the sum squared error

$$(y^1 - \tilde{f}(x^1))^2 + \cdots + (y^N - \tilde{f}(x^N))^2,$$

where $\tilde{f} = \theta_1 f_1(x) + \cdots \theta_p f_p(x)$. The function $\tilde{f}$ is the least squares fit over our data set, it is a number.

Our final classifier is

$$\hat{f}(x) = sign(\tilde{f}(x)),$$

We call this classifier the *least squares classifier*.

1.2 Use least squares classfier to do handwritten digit classification

Here, we define our sign function as

$$sign(x) = \begin{cases} +1 & if x \geq 0 \\ -1 & if x < 0 \end{cases}$$

and we have basis function(feature function):

$$f_i(x) = x_i$$

The partitioning function is

$$\tilde{f}(x, \theta) = \theta_1 f_1(x) + \theta_2 f_2(x) + \cdots + \theta_p f_p(x)$$

Change it to matrix form, we get $f \cdot \theta = y$. By *pseudo inverse* $(A^T A)^{-1} A$ we can find θ.

1.2.1 Read data sets

First, let import the data set. We have two data sets, one for training, one for testing. Each element is a image that has height 28 and width 28 pixels.

In [1]:
```python
import matplotlib.pyplot as plt
import numpy as np

file_data_train = "mnist_train.csv"
file_data_test  = "mnist_test.csv"

h_data_train    = open(file_data_train, "r")
h_data_test     = open(file_data_test, "r")

data_train      = h_data_train.readlines()
data_test       = h_data_test.readlines()

h_data_train.close()
h_data_test.close()

size_row    = 28    # height of the image
size_col    = 28    # width of the image

num_train   = len(data_train)   # number of training images
num_test    = len(data_test)    # number of testing images

# number of training images: 60000
# number of testing images: 10000
```

To reduce the bias, we need to normalize the data.

```
In [2]: #
        # normalize the values of the input data to be [0, 1]
        #
        def normalize(data):

            data_normalized = (data - min(data)) / (max(data) - min(data))

            return(data_normalized)
```

Normalize each pixel, and put image data into a 764*num_image matrix.

```
In [3]: #
        # make a matrix each column of which represents an images in a vector form
        #
        list_image_train    = np.empty((size_row * size_col, num_train), dtype=float)  # 764 *
        list_label_train    = np.empty(num_train, dtype=int)

        list_image_test     = np.empty((size_row * size_col, num_test), dtype=float)
        list_label_test     = np.empty(num_test, dtype=int)

        count = 0

        for line in data_train:

            line_data   = line.split(',')
            label       = line_data[0]
            im_vector   = np.asfarray(line_data[1:])  # convert to float type
            im_vector   = normalize(im_vector)

            list_label_train[count]     = label
            list_image_train[:, count]  = im_vector  # each column is a image

            count += 1

        count = 0

        for line in data_test:

            line_data   = line.split(',')
            label       = line_data[0]
            im_vector   = np.asfarray(line_data[1:])
            im_vector   = normalize(im_vector)

            list_label_test[count]      = label
            list_image_test[:, count]   = im_vector

            count += 1
```

1.2.2 Define feature function, then apply it on data

In [4]:
```
def f_i(x, i):
    return x[i]


def create_A(func, data, num_data, size):
    """Build f(x) matrix

    Apply feature function to each data image,
    get a matrix
    """

    A = []

    for i in range(num_data):
        img = data[:, i]  # ith image(column)
        row = [func(img, j) for j in range(size)]
        A.append(row)

    return np.array(A)

A = create_A(f_i, list_image_train, num_train, size_row*size_col)
```

1.2.3 Compute θ

We have $A\theta = y$, while A is the matrix of feature function apply on data set, θ is perameters, and y is the label.

Because we just want to separate 0 and other numbers, we need to process label y which gives 0's image $+1$ and other number's image -1.

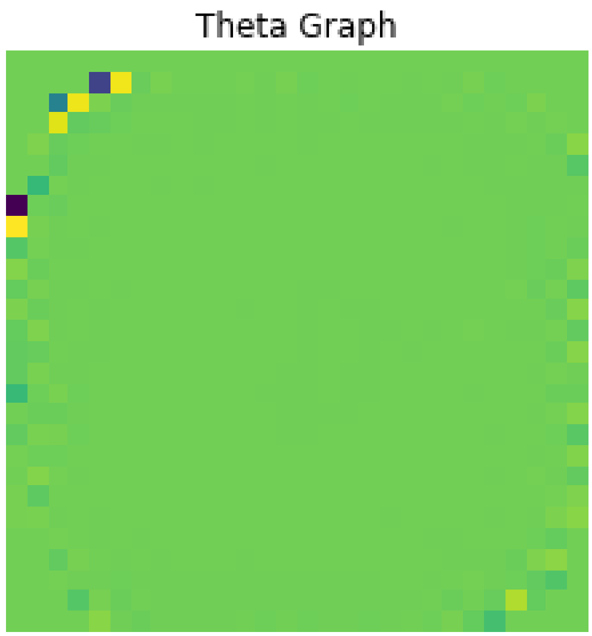
Through pseudo inverse $(A^T A)^{-1}A$ we can compute the θ. Here we use `np.linalg.pinv` to get A^{-1}.

In [5]:
```
# process label array
def process_label(labels):
    result = []
    for label in labels:
        if label == 0:
            result.append(1)
        else:
            result.append(-1)

    return result


def theta(A, y):
    A_inv = np.linalg.pinv(A)
    theta = np.inner(A_inv, y)
```

```
        return theta

label_train = process_label(list_label_train)
theta_arr = theta(A, label_train)
```

1.2.4 Plot θ graph

```
In [6]: theta_matrix = theta_arr.reshape((28, 28))

        plt.title("Theta Graph")
        plt.imshow(theta_matrix)
        plt.axis("off")
        plt.show()
```



1.2.5 Define classifier $\hat{f}(x)$

We can define˜since we have got θ.

Now we can create the classifier $\hat{f}(x) = sign(\tilde{f}(x))$, while

$$sign(x) = \begin{cases} +1 & if x \geq 0 \\ -1 & if x < 0 \end{cases}$$

```
In [7]: def sign_func(x):
            if x >= 0:
                return 1
```

```
        else:
            return -1


    def lsf(f_i, data, num_data, size, theta):
        # least squares fit
        A = create_A(f_i, data, num_data, size)
        return np.inner(data.T, theta)


    def classifier(f_i, data, num_data, size, theta, sign_func, lsf):
        f_tilde = lsf(f_i, data, num_data, size, theta)
        f_hat = list(map(sign_func, f_tilde))

        return np.array(f_hat)
```

1.2.6 Prediction of testing set

Put testing data set into our classifier we can get the prediction value.

```
In [8]: prediction = classifier(f_i, list_image_test, num_test, size_row*size_col, theta_arr, s
```

1.2.7 Count predicted outcome: TP, FP, TN, and FN

We have already got the prediction by our classifier $\hat{f}(x)$, now let's compare it with the label of testing data set to check how it works.

The outcome are

- *True positive*: $y = +1$ and $\hat{y} = +1$.
- *True negative*: $y = -1$ and $\hat{y} = -1$.
- *False positive*: $y = -1$ and $\hat{y} = +1$.
- *False negative*: $y = +1$ and $\hat{y} = -1$.

Here, we still need to process the testing data label.

```
In [9]: def outcomes(label, prediction):
            """count outcomes of prediction

            Input:
                label(array): correct labels
                prediction(array): prediction of classifier
            Return:
                A dictionary contains the indices of each outcome type
                tp: true positive
                tn: true negative
                fp: false positive
                fn: flase negative
            """
```

```
    length = len(label)
    tp = []
    fp = []
    tn = []
    fn = []

    for i in range(length):
        if label[i] == 1 and prediction[i] == 1:
            tp.append(i)
        elif label[i] == 1 and prediction[i] == -1:
            fn.append(i)
        elif label[i] == -1 and prediction[i] == -1:
            tn.append(i)
        else:
            fp.append(i)

    outcome = {'TP': tp,
               'FP': fp,
               'TN': tn,
               'FN': fn}

    return outcome


label_test = process_label(list_label_test)
outcome_dic = outcomes(label_test, prediction)
```

Print out evaluation table.

```
In [10]: from prettytable import PrettyTable

         table = PrettyTable()
         for i, j in outcome_dic.items():
             table.add_column(i, [len(j)])

         print(' Evaluation Value Table')
         print(table)
```

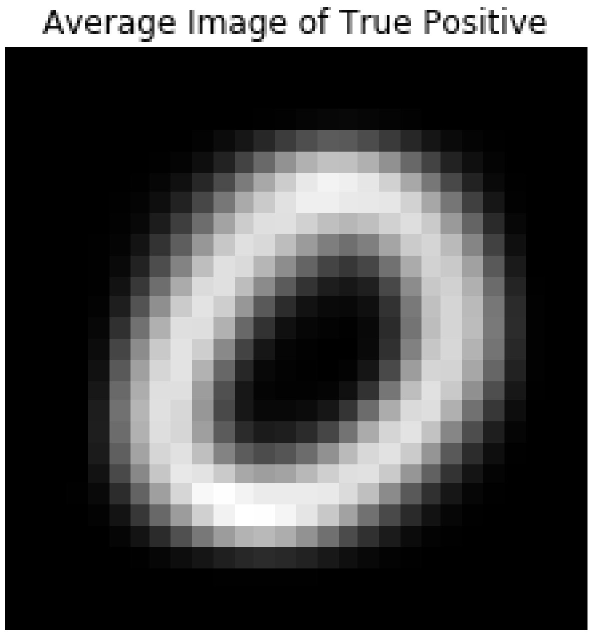
```
 Evaluation Value Table
+-----+----+------+----+
|  TP | FP |  TN  | FN |
+-----+----+------+----+
| 917 | 61 | 8959 | 63 |
+-----+----+------+----+
```

1.2.8 Plot average images

In [11]:
```python
def average_img(data, indices):
    # compute the average value of one outcome type

    size = 28 * 28
    sum_img = np.zeros(size)

    for index in indices:
        img = data[:, index]
        sum_img += img

    num_img = len(indices)

    return sum_img / num_img


def plot_graph(img, title):
    plt.title(title)
    plt.imshow(img, cmap='gray')
    plt.axis('off')
    plt.show()
```

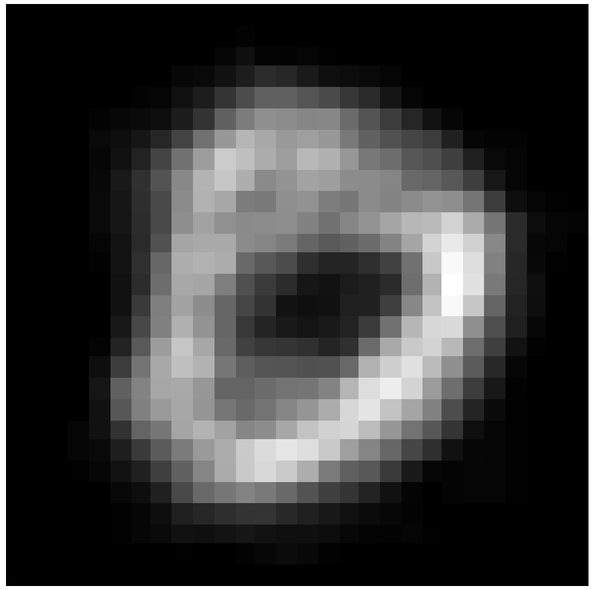
True positive

In [12]:
```python
# True positive
tp = average_img(list_image_test, outcome_dic['TP'])
tp_matrix = tp.reshape((28, 28))

plot_graph(tp_matrix, 'Average Image of True Positive')
```



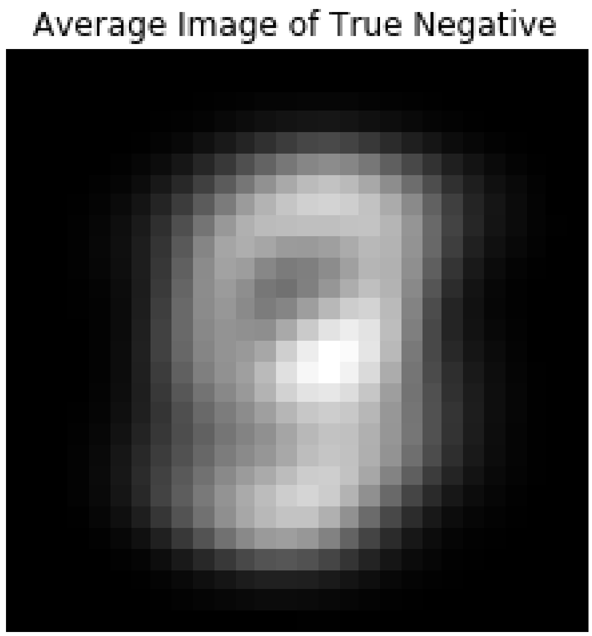
False Positive

```
In [13]: # False positive
         fp = average_img(list_image_test, outcome_dic['FP'])
         fp_matrix = fp.reshape((28, 28))

         plot_graph(fp_matrix, 'Average Image of False Positive')
```



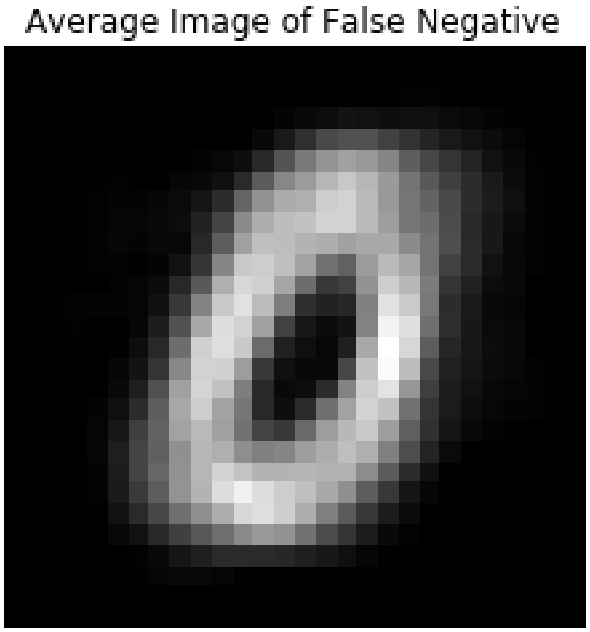
True Negative

```
In [14]: # True negative
         tn = average_img(list_image_test, outcome_dic['TN'])
         tn_matrix = tn.reshape((28, 28))

         plot_graph(tn_matrix, 'Average Image of True Negative')
```



False Negative

```
In [15]: # False negative
         fn = average_img(list_image_test, outcome_dic['FN'])
         fn_matrix = fn.reshape((28, 28))

         plot_graph(fn_matrix, 'Average Image of False Negative')
```



From above images we can tell, both *True Positive* and *False Negative*'s images are clear zero images.

True negative mixed number 1 to 9, so the image is blurred.