assignment05

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1 Convolution

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Convolution is an important operations in image processing, and usually is used for blurring, sharpening, embossing, edge detection.

An image actually is an array of pixel values. If we regart it as a continuous function of spatial coodinates, this function is the brightness of light impinging onto the camera sensor. Partial derivatives of this continuous function can be used to measure the edges of objects in the image.

1.1 Edges

Since an edge is an abrupt change of image intensity, we change find the abrupt by computing the derivatives of an image.

Let's use x to represent horizontal, y to represent vertical. The *partial derivative* of function $f(x, y)$ is

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\Delta y \to 0} \frac{f(x, y + \Delta x) - f(x, y)}{\Delta x}$$

$$\frac{\partial f(x,y)}{\partial y} = \lim_{\Delta y \to 0} \frac{f(x, y + \Delta y) - f(x, y)}{\Delta y}$$

As we said before, we regart the image matrix as a continuous function, then each element of the matrix is the value of $f(x, y)$ and Δx (Δy) is 1.

This means the partial derivatives of the function is the substracion between two neighbor pixels. In this situation, *convolution* can help us to get the partial derivative matrix.

1.2 Implementation

Let's say we get a matrix like this:

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} & \dots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \dots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{d1} & x_{d2} & x_{d3} & \dots & x_{dn} \end{bmatrix}$$

The partial deravitive of *ith* row is $x_{i(j+1)} - x_{ij}$. To get this result we need define our convolution kernel to

$$\begin{bmatrix} 1 & -1 \end{bmatrix}$$

The partial deravitive of *jth* column is $x_{(i+1)j} - x_{ij}$, thus we define our convolution kernel to

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

However, the problem is if we use the 1×2 or 2×1 filters to do the computation, they will shift the image by half a pixel. To avoid this, we need to use kernels that have odd number of elements since the symmetric property.

We change the kernel to:

$$\begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

and,

$$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

Let's start coding.
First import packages that we need.

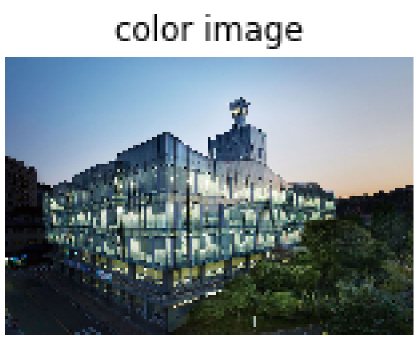
```
In [1]: import matplotlib.pyplot as plt
        import numpy as np
        from scipy import signal
        from skimage import io, color
        from skimage import exposure
```

Read the image file and get the gray image.

```
In [2]: file_image = 'cau.jpg'

        im_color = io.imread(file_image)
        im_gray = color.rgb2gray(im_color)
```

Plot out the input image.

```
In [3]: p1 = plt.subplot(1, 2, 1)
        p1.set_title('color image')
        plt.imshow(im_color)
        plt.axis('off')

        p2 = plt.subplot(1, 2, 2)
        p2.set_title('gray image')
        plt.imshow(im_gray, cmap='gray')
        plt.axis('off')

        plt.show()
```



Since the image data is represented by a 2 dimentional matrix, we need to extend our derivative which we defined before into 2 dimentinal.

Then, we get the convolution kernel for x-direction derivative:

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

and convolution kernel for y-direction derivative:

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

```
In [4]: # x-axis kernel
        ker_x = np.array([
                          [1, 0, -1],
                          [1, 0, -1],
                          [1, 0, -1]])

        # y-axis kernel
        ker_y = np.array([[1, 1, 1],
                          [0, 0, 0],
                          [-1, -1, -1]])
```

Use these two kernels to do convolution with the `gray image`, we can get the derivative.

Here we use `scipy.signal.convolve2d` function directly.

Because at the boundary there are some mapping part to kernel are empty, we need some paddings to handle the problem. `boundary` keyword indicates the way how we handle the problem.

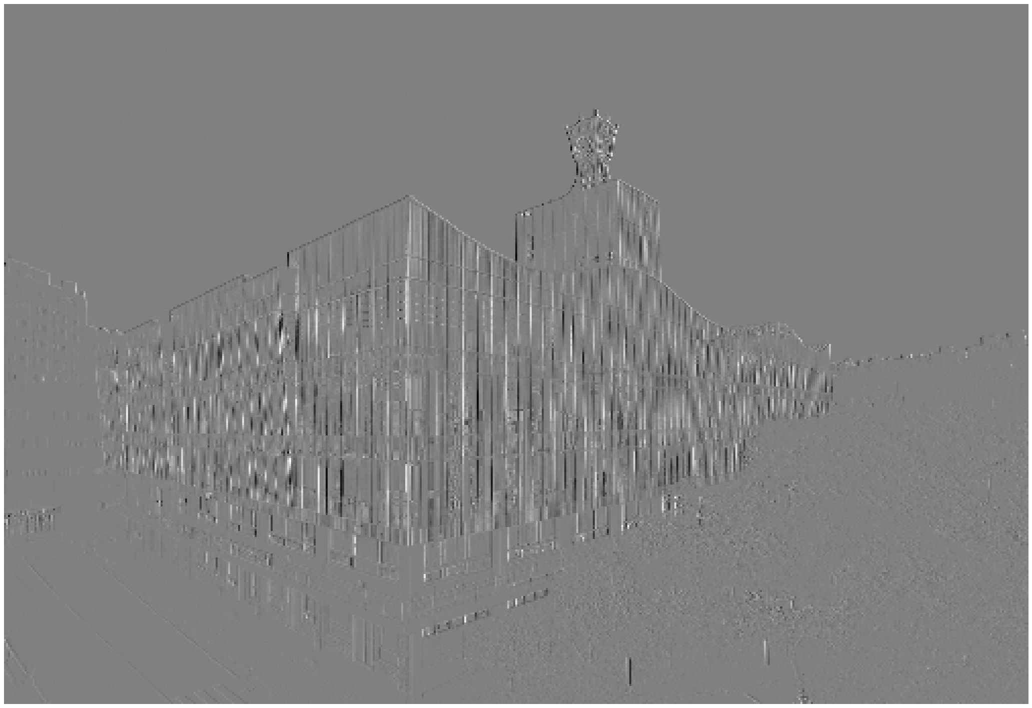
The `mode` keyword indicates the size of the output. Here we choose get the same size as the inputs.

```
In [7]: # derivative x-axis
        de_x = signal.convolve2d(
            im_gray, ker_x, boundary='symm', mode='same')

        # derivative y-axis
        de_y = signal.convolve2d(
            im_gray, ker_y, boundary='symm', mode='same')
```

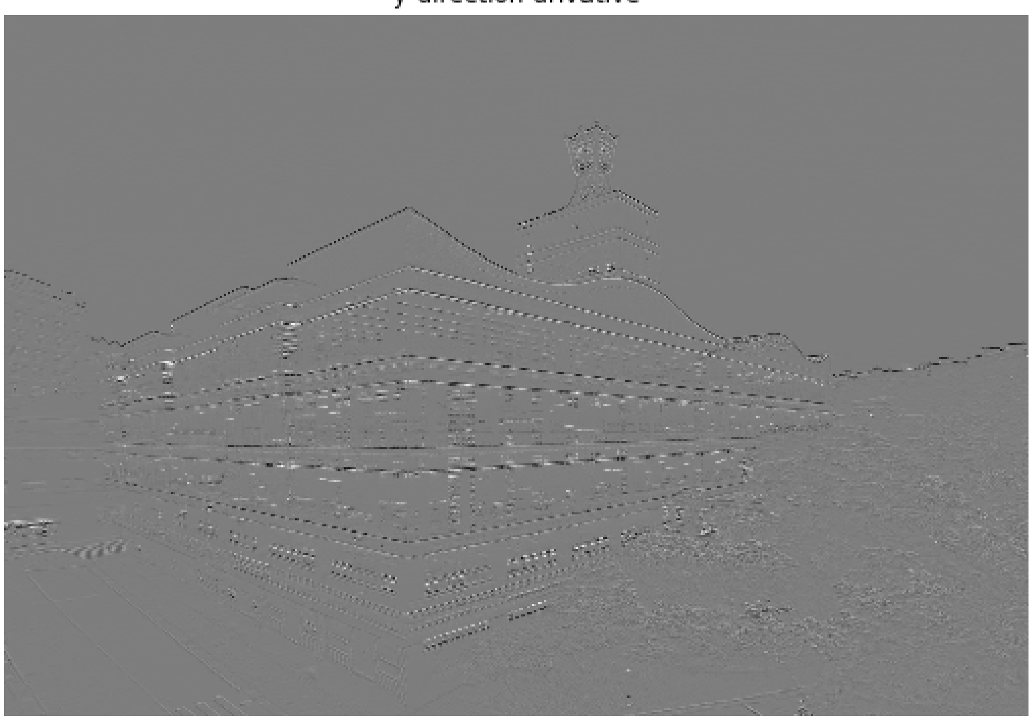
Plot the result:

```
In [8]: plt.figure(2, figsize=(9,12))
        plt.title('x-direction derivative')
        plt.imshow(de_x, cmap='gray')
        plt.axis('off')

        plt.figure(3, figsize=(9,12))
        plt.title('y-direction drivative')
        plt.imshow(de_y, cmap='gray')
        plt.axis('off')

        plt.show()
```

x-direction derivative



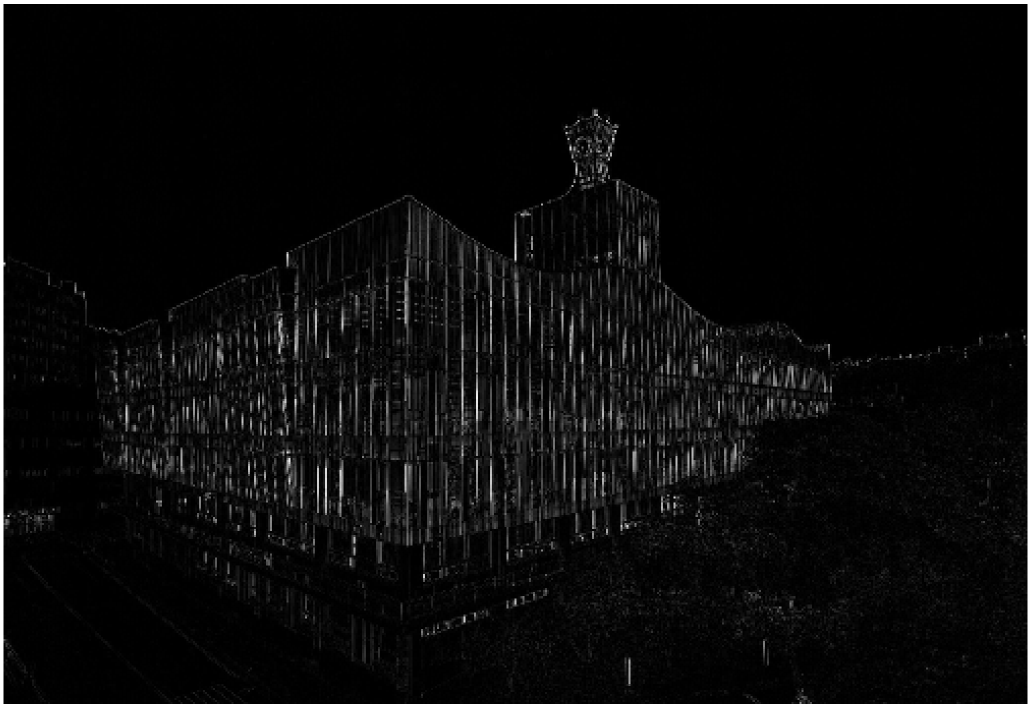
y-direction drivative



The result of derivative is mixed by positive and negative values. Convert them to absolute value, then plot again to see the result.

```
In [9]: # absolute value of gradient
        def deriv_abs(deriv):
            n_row = len(deriv)
            n_col = len(deriv[0])
            result = np.empty([n_row, n_col])

            for i in range(n_row):
                for j in range(n_col):
                    result[i][j] = abs(deriv[i][j])

            return result

        abs_x = deriv_abs(de_x)
        abs_y = deriv_abs(de_y)

        plt.figure(4, figsize=(9,12))
        plt.title('Absolute value of x-direction derivative')
        plt.imshow(abs_x, cmap='gray')
        plt.axis('off')

        plt.figure(5, figsize=(9,12))
        plt.title('Absolute value of y-direction drivative')
        plt.imshow(abs_y, cmap='gray')
        plt.axis('off')

        plt.show()
```

Absolute value of x-direction derivative



Absolute value of y-direction drivative