SENG2011: Assignment 1 – Achilles Sadsad z5208382

Exercise 1:

i. $\forall x \exists y P(x, y)$

False. As there will always be some values of Y that are not always less than X.

ii. $\exists y \forall x Q(x, y)$

False. As when a value of Y is assigned it is not less than or equal to all values of X.

iii. $\forall x \forall y (P(x, y) \vee Q(x, y))$

False. As with all possible values of x and y both P(x,y) and Q(x,y) will always be false.

iv. $\exists x R(x)$

True. As 57 + 42 = 99.

v. $\forall y (\neg S(y))$

True. As $\forall y(S(y))$ is false. Therefore, the negation of $\forall y(S(y))$ is True.

vi. $(\exists x S(x)) \wedge \neg(\forall x R(x))$

True. As some values of x are greater than 42 AND the negation of $\forall x R(x)$ is true.

vii. $\exists y \forall x (S(y) \wedge Q(x, y))$

False. $\exists y(S(y))$ is true as there are some values of y that are less than 42**.** $\exists y \forall x Q(x, y)$ is false as there are some values of y that are not greater than or equal to x.

viii. $\forall x \forall y ((R(x) \wedge S(y)) \Rightarrow Q(x, y))$

False. $\forall x \forall y(Q(x, y))$ is false therefore the truth value is false.

Exercise 2:

$((P \Rightarrow (Q \vee R)) \Rightarrow ((\neg Q \vee S) \wedge \neg S))$ **[Premise]**
$\neg(P \Rightarrow (Q \vee R)) \vee ((\neg Q \vee S) \wedge \neg S))$ **[l]**
$\neg(P \Rightarrow (Q \vee R)) \vee ((\neg Q \wedge \neg S) \vee (S \wedge \neg S))$ **[j]**
$(P \wedge \neg(Q \vee R)) \vee ((\neg Q \wedge \neg S) \vee (S \wedge \neg S))$ **[m]**
$(P \wedge \neg(Q \vee R)) \vee ((\neg Q \wedge \neg S) \vee$ False) **[c]**
$(P \wedge \neg(Q \vee R)) \vee (\neg Q \wedge \neg S)$ **[a]**
$\neg(Q \vee R) \vee (\neg Q \wedge \neg S)$ **[g]**
$(\neg Q \wedge \neg P) \vee (\neg Q \wedge \neg S)$ **[i]**
$(\neg Q \wedge \neg P) \vee (\neg S \wedge \neg Q)$ **[h]**
$\neg S \wedge \neg Q$ **[g]**
$\neg Q$ **[f]**

Exercise 3:

i. {true} if x > y then m := x else m := y; {(m ≥ x) ∧ (m ≥ y)}

1. {true ∧ x > y} m := x {(m ≥ x) ∧ (m ≥ y)}
2. {x > y} m := x {(m ≥ x) ∧ (m ≥ y)} **[Precondition Equivalence]**
3. {x ≥ x ∧ x ≥ y} m := x {(m ≥ x) ∧ (m ≥ y)} **[Assignment Rule]**
4. {true ∧ x > y} m := x {(m ≥ x) ∧ (m ≥ y)} **[Logic & Precondition strengthening]**

   4 ⇔ 1

{true ∧ ¬(x > y)} M := y {(m ≥ x) ∧ (m ≥ y)}

1. {y ≥ x ∧ y ≥ y} m := x {(m ≥ x) ∧ (m ≥ y)} **[Assignment Rule]**
2. {y ≥ x ∧ true} m := x {(m ≥ x) ∧ (m ≥ y)} **[Logic]**
3. { ¬(x > y) ∧ true} m := x {(m ≥ x) ∧ (m ≥ y)} **[Logic]**
4. {true ∧ ¬(x > y)} m := x {(m ≥ x) ∧ (m ≥ y)} **[Logic]**

   4 ⇔ 1

ii. {x = 2^n} x := x * 2; n := n + 1 {x = 2^n}

1. {x = 2^n} x := x * 2; n := n + 1 {x = 2^n} **[Preface]**
2. {x = 2^n} x := x * 2 {R1}
3. {R1} n := n + 1 {x = 2^n}
4. {x = 2^n} n := n + 1 {x = 2^n} **[Assignment Rule] on line 2**
5. {x * 2 = 2^n} x := x * 2 {x = 2^n} **[Assignment Rule] on line 1**
6. {x = 2^n} x := x * 2 {x = 2^n} **[Precondition strengthening] on line 4**
7. {x = 2^n} x := x * 2; n := n + 1 {x = 2^n} **[Sequence Rule]**

   7 ⇔ 1

iii. {x = 2^n ^ (n ≤ p)} while n < p do (x := x * 2; n := n + 1;) {x = 22^p}