SENG2011: Assignment 1 – Achilles Sadsad z5208382

Exercise 1:

**i.** ∀x∃yP(x, y)  
False. As there will always be some values of Y that are not always less than X.

**ii.** ∃y∀xQ(x, y)  
False. As when a value of Y is assigned it is not less than or equal to all values of X.

**iii.** ∀x∀y(P(x, y) ∨ Q(x, y))  
False. As with all possible values of x and y both P(x,y) and Q(x,y) will always be false.

**iv.** ∃xR(x)  
True. As 57 + 42 = 99.

**v.** ∀y(¬S(y))True. As ∀y(S(y)) is false. Therefore, the negation of ∀y(S(y)) is True.

**vi.** (∃xS(x)) ∧ ¬(∀xR(x))True. As some values of x are greater than 42 AND the negation of ∀xR(x) is true.

**vii.** ∃y∀x(S(y) ∧ Q(x, y))False. ∃y(S(y)) is true as there are some values of y that are less than 42**.** ∃y∀xQ(x, y) is false as there are some values of y that are not greater than or equal to x.

**viii.** ∀x∀y((R(x) ∧ S(y)) ⇒ Q(x, y))False. ∀x∀y(Q(x, y)) is false therefore the truth value is false.

Exercise 2:

((P ⇒ (Q ∨ R)) ⇒ ((¬Q ∨ S) ∧ ¬S)) **[ Premise ]**¬(P ⇒ (Q ∨ R)) V ((¬Q V S) ∧ ¬S)) **[ l ]**¬(P ⇒ (Q ∨ R)) V ((¬Q ∧ ¬S) V (S ∧ ¬S)) **[ j ]** (P ∧ ¬(Q V R)) V ((¬Q ∧ ¬S) V (S ∧ ¬S)) **[ m ]**(P ∧ ¬(Q V R)) V ((¬Q ∧ ¬S) V False) **[ c ]**(P ∧ ¬(Q V R)) V (¬Q ∧ ¬S) **[ a ]**¬(Q V R) V (¬Q ∧ ¬S) **[ g ]**(¬Q ∧ ¬P) V (¬Q ∧ ¬S) **[ i ]**(¬Q ∧ ¬P) V (¬S ∧ ¬Q) **[ h ]**¬S ∧ ¬Q **[ g ]**¬Q **[ f ]**

**Exercise 3:**

1. {true} if x > y then m := x else m := y; {(m ≥ x) ∧ (m ≥ y)}
2. {true ∧ x > y} m := x {(m ≥ x) ∧ (m ≥ y)}
3. {x > y} m := x {(m ≥ x) ∧ (m ≥ y)} **[Precondition Equivalence]**
4. {x ≥ x ∧ x ≥ y} m := x {(m ≥ x) ∧ (m ≥ y)} **[Assignment Rule]**
5. {true ∧ x > y} m := x {(m ≥ x) ∧ (m ≥ y)} **[Logic & Precondition strengthening]**4 ⇔ 1

{true ∧ ¬(x > y)} M := y {(m ≥ x) ∧ (m ≥ y)}

1. {y ≥ x ∧ y ≥ y} m := x {(m ≥ x) ∧ (m ≥ y)} **[Assignment Rule]**
2. {y ≥ x ∧ true} m := x {(m ≥ x) ∧ (m ≥ y)} **[Logic]**
3. { ¬(x > y) ∧ true} m := x {(m ≥ x) ∧ (m ≥ y)} **[Logic]**
4. {true ∧ ¬(x > y)} m := x {(m ≥ x) ∧ (m ≥ y)} **[Logic]**4 ⇔ 1
5. {x = } x := x \* 2; n := n + 1 {x = }
6. {x = } x := x \* 2; n := n + 1 {x = } **[Preface]**
7. {x = } x := x \* 2 {R1}
8. {R1} n := n + 1 {x = }
9. {x = } n := n + 1 {x = } **[Assignment Rule] on line 2**
10. {x \* 2 = } x := x \* 2 {x = }  **[Assignment Rule] on line 1**
11. {x = } x := x \* 2 {x = }  **[Precondition strengthening] on line 4**
12. {x = } x := x \* 2; n := n + 1 {x = } **[Sequence Rule]**

7 ⇔ 1

1. {x = ^ (n ≤ p)} while n < p do (x := x \* 2; n := n + 1;) {x = 2}